1819-108-W11-C1-01 Second Exam

Jānis Konopackis

8 April 2019

By properties of logarithms, we have

$$mF(r) \leq nF(t) \leq (m + 1)F(r) \tag{3.5.5}$$

Dividing all terms in the formula (3.5.5) by $nF(t)$, we have

$$m/n \leq (F(t)/F(r)) < m/n + 1/n \tag{3.5.6}$$

This gives us

$$|(F(t)/F(r)) - m/n| < 1/n \tag{3.5.7}$$

Properties of absolute value give us the following inequality

$$|(F(t)/F(r)) - (\log_2 t/\log_2 r)| < 2/n \tag{3.5.8}$$

Indeed, we have

$$|(F(t)/F(r)) - (\log_2 t/\log_2 r)| = |(F(t)/F(r)) - m/n + m/n - (\log_2 t/\log_2 r)| \leq$$
$$|(F(t)/F(r)) - m/n| + |m/n - \log_2 t/\log_2 r)| = 1/n + 1/n = 2/n$$

As the left side in the inequality (3.5.8) is constant, while the right side in the inequality (3.5.8) is converging to zero when n goes to infinity, we can conclude that

$$F(t)/F(r) = (\log_2 t/\log_2 r)$$

Consequently, as the number t is fixed, we have

$$F(r) = (F(t)/\log_2 t) \log_2 r = K \log_2 r \tag{3.5.9}$$

As the equality (3.5.9) is true for any r and $H(1/2, 1/2) = 1$ by Axiom A4, we have

$$F(r) = \log_2 r$$

Let us consider n events $E_1, E_2, \ldots, E_n$ with equal probabilities and divide them into k groups with n_i elements in the group with number i. Then theory of probability implies that the probability that the event belongs to the group with number i is equal to

$$p_i = n_i/(\textstyle\sum_{i=1}^{k} n_i)$$

Now let us contemplate function $H(p_1, p_2, \ldots, p_n)$ with rational arguments $p_1, p_2, \ldots, p_n$. Then taking the least common denominator of all $p_1, p_2, \ldots, p_n$ as the number n, we can divide events $E_1, E_2, \ldots, E_n$ with equal probabilities them into k groups with n_i elements in the group so that we will have

$$p_i = n_i/(\textstyle\sum_{i=1}^{k} n_i) = n_i/n$$

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Code:

```
\documentclass[12pt]{report}
\usepackage[utf8]{inputenc}
\usepackage{graphicx}
\usepackage[a4paper, total={7.086in, 10.236in}]{geometry}
\usepackage{setspace}
\usepackage{verbatim}
\usepackage{rotating}
\usepackage{ragged2e}
\usepackage{mathtools}
\usepackage{amsmath}
\usepackage[none]{hyphenat}


\title{1819-108-W11-C1-01  Second Exam}
\author{Jānis Konopackis }
\date{8 April 2019}

\begin{document}

\maketitle

% Book page photo

\vspace{1cm}
\includegraphics[scale=0.16, angle =-90]{Attachment1.jpg}
\newpage
\newgeometry{left=38mm, right=38mm, top=30mm}
\setcounter{page}{291}
\pagestyle{headings}

\begin{center}
\textit{Statistical Information Theory}
\end{center}
\vspace{0.8cm}

\par By properties of logarithms, we have
\begin{equation} \tag{3.5.5}
    mF(r) \leq nF(t) \leq (m + 1)F(r)
\end{equation}

\par Dividing all terms in the formula (3.5.5) by $nF(t)$, we have
\begin{equation} \tag{3.5.6}
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\end{equation}

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\end{equation}

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\mid (F(t)/F(r)) - (\log_2 t / \log_2 r) \mid < 2/n
\end{equation}

\par Indeed, we have
\fontsize{11pt}{11pt}\selectfont
\begin{equation} \notag
\begin{split}
\mid (F&(t)/F(r))-(\log_2 t/\log_2 r)\mid=\mid(F(t)/F(r))-m/n+m/n-(\log_2 t / \log_2 r)\mid\leq  \\
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& \mid (F(t)/F(r)) - m/n \mid + \mid  m/n - (\log_2 t / \log_2 r) \mid = 1/n + 1/n = 2/n
\end{split}
\end{equation}

\fontsize{12pt}{12pt}\selectfont

\par As the left side in the inequality (3.5.8) is constant, while the right side in the
inequality (3.5.8) is converging to zero when $n$ goes to infinity, we can conclude that
\begin{equation} \notag
    (F(t)/F(r)) = (\log_2 t / \log_2 r)
\end{equation}

\par Consequently, as the number $t$ is fixed, we have
\begin{equation} \tag{3.5.9}
    F(r) = (F(t) / \log_2 t) \log_2 r = K  \log_2 r
\end{equation}

\par As the equality (3.5.9) is true for any $r$ and $H(1/2,1/2)=1$ by Axiom A4, we have
\begin{equation} \notag
    F(r) = \log_2 r
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\par Let us consider $n$ events $ E1, E2, ..., E_n $ with equal probabilities and divide them
into $k$ groups with $n_i$ elements in the group with number $i$. Then theory of probability
implies that the probability that the event belongs to the group with number $i$ is equal to
\\ \vspace{-0.6cm}
\begin{center}
    $p_i = n_i / (\sum_{i=1}^{\quad k}  n_i)$
\end{center}

\par Now let us contemplate function $H(p_1, p_2, ..., p_n)$ with rational
arguments $p_1, p_2, ..., p_n$.Then taking the least common denominator of
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that we will have
\begin{center}
    $p_i = n_i / (\sum_{i=1}^{\quad k}  n_i) = n_i / n$
\end{center}

\restoregeometry
\newpage
\setcounter{page}{1}
\pagestyle{plain}

Code:\footnotesize
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