RTR108 Final Exam

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May 2019

TABLE 3
Transshipment Representation of Shortest-Path Problem and Optimal Solution (1)

	Node					
Node	**2**	**3**	**4**	**5**	**6**	**Supply**
1	4 (1)	3	M	M	M	1
2	0	M	3	2 (1)	M	1
3	M	0 (1)	M	3	M	1
4	M	M	0 (1)	M	2	1
5	M	M	M	0	2 (1)	1
Demand	1	1	1	1	1	

To illustrate the preceding ideas, we formulate the balanced transportation problem associated with finding the shortest path from node 1 to node 6 in Figure 2. We want to send one unit from node 1 to node 6. Node 1 is a supply point, node 6 is a demand point, and nodes 2, 3, 4, and 5 will be transshipment points. Using $s = 1$, we obtain the balanced transportation problem shown in Table 3. This transportation problem has two optimal solutions:

1 $z = 4 + 2 + 2 = 8, x_{12} = x_{25} = x_{56} = x_{33} = x_{44} = 1$ (all other variables equal 0). This solution corresponds to the path 1–2–5–6.

2 $z = 3 + 3 + 2 = 8, x_{13} = x_{35} = x_{56} = x_{22} = x_{44} = 1$ (all other variables equal 0). This solution corresponds to the path 1–3–5–6.

REMARK After formulating a shortest-path problem as a transshipment problem, the problem may be solved easily by using LINGO or a spreadsheet optimizer. See Section 7.1 for details.

PROBLEMS

Group A

1 Find the shortest path from node 1 to node 6 in Figure 3.

2 Find the shortest path from node 1 to node 5 in Figure 4.

3 Formulate Problem 2 as a transshipment problem.

4 Use Dijkstra's algorithm to find the shortest path from node 1 to node 4 in Figure 5. Why does Dijkstra's algorithm fail to obtain the correct answer?

FIGURE 4
Network for Problem 2

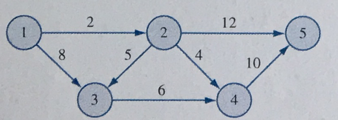
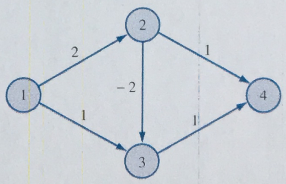


FIGURE 5
Network for Problem 4



5 Suppose it costs \$10,000 to purchase a new car. The annual operating cost and resale value of a used car are shown in Table 4. Assuming that one now has a new car, determine a replacement policy that minimizes the net costs of owning and operating a car for the next six years.

Table 3
Transshipment Representation
of Shortest-Path Problem and
Optimal Solution (1)

Node	2	3	Node 4	5	6	Supply
1	1					1
2			1		1	
3	1				1	
4		1			1	
5					1	1
Demand	1	1	1	1	1	

To illustrate the preceding ideas, we formulate the balanced transportation problem associated with finding the shortest path from node 1 to node 6 in Figure 2. We want to send one unit from node 1 to node 6. Node 1 is a supply point, node 6 is a demand point, and nodes 2,3,4 and 5 will be transshipment points. Using $s = 1$, we obtain the balanced transportation problem shown in Table 3. This transportation problem has two optimal solutions:

1. $z = 4 + 2 + 2 = 8, x_{12} = x_{25} = x_{56} = x_{33} = x_{44} = 1$ (all other variables equal 0). This solution corresponds to the path 1-2-5-6.

2. $z = 3 + 3 + 3 + 2 = 8, x_{13} = x_{35} = x_{56} = x_{22} = x_{44} = 1$ (all other variables equal 0). This solution corresponds to the path 1-3-5-6.

After formulating a shortest-path problem as a transshipment problem, the problem may be solved easily by using LINGO ar a spreadsheet optimizer. See Section 7.1 for details.

PROBLEMS

Group A

1. Find the shortest path from node 1 to node 6 in Figure 3.

2. Find the shortest path from node 1 to node 5 in Figure 4.

3. Formulate Problem 2 as a transshipment problem.

4. Use Dijkstra‘s algorithm to find the shortest path from node 1 to node 4 in Figure 5. Why does Dijkstra‘s algorithm fail to obtain the correct answer?

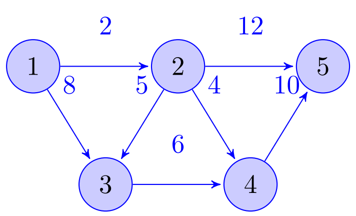
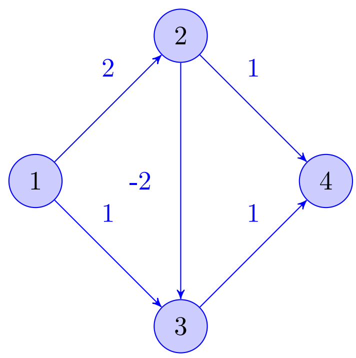


FIGURE 5

Network for problem 4



5. Suppose it costs $10,000 to purchase a new car. The annual operating cost and resale value of a used car are shown in Table 4. Assuming that one now has a new car, determine a replacement policy that minimizes the net costs of owning and operating a cer for the next six years.

```
 \documentclass{report}
\usepackage[utf8]{inputenc}
\usepackage{geometry}
 \geometry{a4paper,total={170mm,282mm},left=20mm,top=5mm,}
\usepackage{colortbl}
\usepackage{latexsym}
\usepackage{array}
\newcolumntype{M}[1]{>{\centering\arraybackslash}m{#1}}
\newcolumntype{N}{@{}m{0pt}@{}}
\usepackage[table,xcdraw]{xcolor}
\usepackage{multicol}
%\setlength{\columnsep}{1cm}
\usepackage{enumitem,color}
\usepackage{tikz,pgfplots}
\usetikzlibrary{automata,arrows,calc,positioning}


\setlist[enumerate]{before=\setupmodenumerate}

\newif\ifcitem
\newcommand{\setupmodenumerate}{
  \global\citemfalse
  \let\origmakelabel\makelabel
  \def\citem##1{\global\citemtrue\def\cecolor{##1}\item}
  \def\makelabel##1{%
    \origmakelabel{\ifcitem\color{\cecolor}\fi##1}
    \global\citemfalse}%
}


\title{RTR108 Final Exam}
\author{Jānis Konopackis }
\date{May 2019}

\begin{document}

\maketitle

\newpage
\pagestyle{empty}



\reveresemarginpar{Table 3 \\
Transshipment Representation \\
of Shortest-Path Problem and \\
Optimal Solution (1)}
\vspace{1cm}
\begin{flushright}

\begin{table}[h]\centering

 \begin{tabular}{M{1.3cm}M{1.2cm}M{1.2cm}M{1.2cm}M{1.2cm}M{1.2cm}M{2cm}}
 \multicolumn{1}{c}{{\color[HTML]{2B36BB} \textbf{Node}}} &
 \multicolumn{1}{c}{{\color[HTML]{2B36BB} \textbf{2}}} &
 \multicolumn{1}{c}{{\color[HTML]{2B36BB} \textbf{3}}} &
 \multicolumn{1}{c}{{\color[HTML]{2B36BB} \textbf{Node 4}}} &
 \multicolumn{1}{c}{{\color[HTML]{2B36BB} \textbf{5}}} &
 \multicolumn{1}{c}{{\color[HTML]{2B36BB} \textbf{6}}} &
 \multicolumn{1}{c}{{\color[HTML]{2B36BB} \textbf{Supply}}} \\\cline{1-6}

 \multicolumn{1}{c|}{{\color[HTML]{2B36BB} \textbf{1}}} &
 \multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{} &
 \multicolumn{1}{c|}{}&\multicolumn{1}{c|}{} &
 \multicolumn{1}{c|}{} & \multicolumn{1}{c}{1} \\ [18pt]\cline{2-6}

 \multicolumn{1}{c|}{{\color[HTML]{2B36BB} \textbf{2}}}
```

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 & \multicolumn{1}{c|}{} & \multicolumn{1}{c|}{}&\multicolumn{1}{c|}{1}
 &\multicolumn{1}{c|}{} & \multicolumn{1}{c|}{1} \\ [18pt]\cline{2-6}

\multicolumn{1}{c|}{{\color[HTML]{2B36BB} \textbf{3}}} &
\multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{}&
\multicolumn{1}{c|}{} &\multicolumn{}{c|}{} &
\multicolumn{1}{c|}{1} \\ [18pt]\cline{2-6}

\multicolumn{1}{c|}{{\color[HTML]{2B36BB} \textbf{4}}} &
\multicolumn{1}{c|}{} & \multicolumn{1}{c|}{1}&
\multicolumn{1}{c|}{} &\multicolumn{}{c|}{} &
\multicolumn{1}{c|}{1} \\ [18pt]\cline{2-6}

\multicolumn{1}{c|}{{\color[HTML]{2B36BB} \textbf{5}}} & \multicolumn{1}{c|}{}
& \multicolumn{1}{c|}{} & \multicolumn{1}{c|}{}&\multicolumn{1}{c|}{}
&\multicolumn{}{c|}{1} & \multicolumn{1}{c}{1} \\ [18pt]\cline{1-7}

\multicolumn{1}{c}{{\color[HTML]{2B36BB} \textbf{Demand}}} & 1 &  1& 1& 1& 1&  \\
\end{tabular}

\end{table}
\end{flushright}


\par To illustrate the preceding ideas, we formulate the balanced
transportation problem associated with finding the shortest path
from node 1 to node 6 in Figure 2. We want to send one unit from
node 1 to node 6. Node 1 is a supply point, node 6 is a demand point,
and nodes 2,3,4 and 5 will be transshipment points. Using $s = 1$, we
obtain the balanced transportation problem shown in Table 3.
This transportation problem has two optimal solutions:


\begin{enumerate}[labelsep=0.4cm]

\citem{blue}[1] $z = 4 + 2 + 2 = 8,x_{12} =x_{25}=x_{56}=x_{33}=x_{44}=1 $
(all other variables equal 0).\\
This solution corresponds to the path 1-2-5-6.
\citem{blue}[2]$z = 3 + 3 + 3 + 2 =8,x_{13} =x_{35}=x_{56}=x_{22}=x_{44}=1$
(all other variables equal 0).\\
This solution corresponds to the path 1-3-5-6.
\end{enumerate}

\color{blue}

After formulating a shortest-path problem as a transshipment problem,
the problem may be solved easily by using LINGO ar a spreadsheet optimizer.
See Section 7.1 for details.

\color{black}

\vspace{5 mm}

\arrayrulecolor{blue}\hline
\begin{multicols}{2}
\begin{flusleft}


\section*{PROBLEMS}{\color[HTML]{2B36BB} \textbf{Group A}}
\end{flusleft}

\begin{enumerate}[labelsep=0.3cm]
\citem{blue}[1] Find the shortest path from node 1 to node 6 in Figure 3.
\citem{blue}[2] Find the shortest path from node 1 to node 5 in Figure 4.
\citem{blue}[3] Formulate Problem 2 as a transshipment problem.
\citem{blue}[4] Use Dijkstra‘s algorithm to find the shortest path from
node 1 to node 4 in Figure 5. Why does Dijkstra‘s algorithm fail to obtain the correct answer?
```

```
\end{enumerate}

\tikzset{node/.style={black, draw=blue, fill=blue!20, minimum size=2em,}}

    \begin{tikzpicture}[>=stealth',shorten >=1pt,auto,node distance=1.2cm,scale=1,
    transform shape,align=center,minimum size=3em]

    \node[state,node]     (1)                                {1};
    \node[state,node]     (2)    [right=of 1]               {2};
    \node[state,node]     (3)    [below=of $(1)!0.5!(2)$]  {3};
    \node[state,node]     (4)    [right=of 3]          {4};
    \node[state,node]     (5)    [right=of 2]         {5};


    \path[->,blue] (1) edge node[above] {2} (2)
              (1) edge node[above] {8} (3)
              (2) edge node[above] {12} (5)
              (4) edge node[above] {10} (5)
              (2) edge node[above] {5} (3)
              (2) edge node[above] {4} (4)
              (3) edge node [above]{6} (4);

    \end{tikzpicture}

\columnbreak
\color[HTML]{2B36BB} \textbf{FIGURE 5}\\

\color[rgb]{0,0,0}\textbf{Network for problem 4}
\tikzset{node/.style={black, draw=blue, fill=blue!20, minimum size=2em,}}

    \begin{tikzpicture}[>=stealth',auto,node distance=2cm,scale=1,
    transform shape,align=center,minimum size=3em]

    \node[state,node]     (1)                                {1};
    \node[state,node]     (2)    [above right=of 1]               {2};
    \node[state,node]     (3)    [below right=of 1]  {3};
    \node[state,node]     (4)    [above right=of 3]          {4};



    \path[->,blue] (1) edge node[above] {2} (2)
              (1) edge node[above] {1} (3)
              (2) edge node[left] {-2} (3)
              (2) edge node[above] {1} (4)
              (3) edge node [above]{1} (4);

    \end{tikzpicture}
\begin{enumerate}[labelsep=0.3cm]
\citem{blue}[5] Suppose it costs \$10,000 to purchase a new car.
The annual operating cost and resale value of a used car are shown in Table 4.
Assuming that one now has a new car, determine a replacement policy
that minimizes the net costs of owning and operating a cer for the next six years.

\end{enumerate}

\end{multicols}
\setcounter{page}{\color[HTML]{2B36BB} \textbf{418}}
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code...
```