1819-108-W6-C1-01
First Exam

Jānis Konopackis

4 March 2019

We also note that often the same general method, used in the above example for proving the uniqueness theorem for Poisson's equation, can be employed to prove the uniqueness (or otherwise) of solutions to other equations and boundary conditions.

20.8 Exercises

20.1 Determine whether the following can be written as functions of $p = x^2 + 2y$ only, and hence whether they are solutions of (20.8):

(a) $x^2(x^2 - 4) + 4y(x^2 - 2) + 4(y^2 - 1)$;
(b) $x^4 + 2x^2y + y^2$;
(c) $[x^4 + 4x^2y + 4y^2 + 4]/[2x^4 + x^2(8y + 1) + 8y^2 + 2y]$.

20.2 Find partial differential equations satisfied by the following functions $u(x, y)$ for all arbitrary functions f and all arbitrary constants a and b:

(a) $u(x, y) = f(x^2 - y^2)$;
(b) $u(x, y) = (x - a)^2 + (y - b)^2$;
(c) $u(x, y) = y^n f(y/x)$;
(d) $u(x, y) = f(x + ay)$.

20.3 Solve the following partial differential equations for $u(x, y)$ with the boundary conditions given:

(a) $x\dfrac{\partial u}{\partial x} + xy = u, \quad u = 2y$ on the line $x = 1$;

(b) $1 + x\dfrac{\partial u}{\partial y} = xu, \quad u(x, 0) = x$.

20.4 Find the most general solutions $u(x, y)$ of the following equations, consistent with the boundary conditions stated:

(a) $y\dfrac{\partial u}{\partial x} - x\dfrac{\partial u}{\partial y} = 0, \quad u(x, 0) = 1 + \sin x$;

(b) $i\dfrac{\partial u}{\partial x} = 3\dfrac{\partial u}{\partial y}, \quad u = (4 + 3i)x^2$ on the line $x = y$;

(c) $\sin x \sin y\dfrac{\partial u}{\partial x} + \cos x \cos y\dfrac{\partial u}{\partial y} = 0, \quad u = \cos 2y$ on $x + y = \pi/2$;

(d) $\dfrac{\partial u}{\partial x} + 2x\dfrac{\partial u}{\partial y} = 0, \quad u = 2$ on the parabola $y = x^2$.

20.5 Find solutions of

$$\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} = 0$$

for which (a) $u(0, y) = y$ and (b) $u(1, 1) = 1$.

20.6 Find the most general solutions $u(x, y)$ of the following equations consistent with the boundary conditions stated:

(a) $y\dfrac{\partial u}{\partial x} - x\dfrac{\partial u}{\partial y} = 3x, \quad u = x^2$ on the line $y = 0$;

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20.4 Find the most general solutions $u(x, y)$ of the following equations, consistent with the boundary conditions started:

(a) $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0, \quad u(x, 0) = 1 + sinx$;
(b) $i\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial y}, \quad u = (4 + 3i)x^2$ on the line $x = y$;
(c) $\sin x \sin y\frac{\partial u}{\partial x} + \cos x \cos y\frac{\partial u}{\partial y} = 0, \quad u = cos2y$ on $x + y = \pi/2$;
(d) $\frac{\partial u}{\partial x} + 2x\frac{\partial u}{\partial y} = 0, \quad u = 2$ on the parabola $y = x^2$.

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(a) $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y}, \quad u = x^2$ on the line $y = 0$;

Code:

```
\documentclass[12pt]{report}
\usepackage[utf8]{inputenc}
\usepackage{graphicx}
\usepackage[a4paper, total={7.086in, 10.236in}]{geometry}
\usepackage{setspace}
\usepackage{enumitem}
\usepackage{verbatim}

\title{1819-108-W5-C1-01 \\ First Exam }
\author{Jānis Konopackis }
\date{4 March 2019}

\begin{document}

\maketitle

Example Nr. 691 % Book page photo

\vspace{1cm}
\includegraphics[scale=0.16]{page.jpg}
\newpage
\newgeometry{left=25mm, right=30mm, top=15mm}
\begin{center}
\setcounter{page}{707}

20.8 Exercises
\end{center}
\vspace{0.25cm}
\hline

\vspace{0.7cm}

\par We also note that often the same general method, used in the above example for proving the
uniqueness theorem for Poisson‘s equation, can be employ to prove the uniqueness (or otherwise) of
solutions to other equations and boundary conditions.
\vspace{1cm}
\begin{center}
\textbf{20.8 Exercises}
\end{center}
\vspace{0.3cm}

\begin{enumerate}[labelsep=1cm]
\item[20.1]  Determine whether the following can be written as functions of $p=x^2 + 2y$ only, and
 hence whether they are solutions of (20.8):
\end{enumerate}

\begin{enumerate}
    \begin{enumerate}
    \item $x^2(x^2-4)+4y(x^2-2)+4(y^2-1)$;
    \item $x^4+2x^2y+y^2$;
    \item $[x^4+4x^2y+4y^2+4]/[2x^4+x^2(8y+1)+8y^2+2y]$.
    \end{enumerate}
\end{enumerate}

\begin{enumerate}[labelsep=1cm]
\item[20.2] Find partial differential equations satisfied by the following functions $u(x,y)$ for
all arbitrary functions $f$ and all constants $a$ and $b$ :
\end{enumerate}
\begin{enumerate}
    \begin{enumerate}
```

```
    \item $u(x,y) = f(x^2-y^2)$;
    \item $u(x,y) = (x-a)^2 + (y-b)^2$;
    \item $u(x,y) = y^nf(y/x)$;
    \item $u(x,y) = f(x+ay)$.
    \end{enumerate}
\end{enumerate}

\begin{enumerate}[labelsep=1cm]
\item[20.3] Solve the following partial differential equations for $u(x,y)$ with
the boundary conditions given :
\end{enumerate}
\begin{enumerate}
    \begin{enumerate}
    \item $x\frac{\partial u}{\partial x}+xy = u,\quad\quad\quad u = 2y
    \mbox{ on the line } x = 1$;

\setlength{\parskip}{0.6em}

    \item $1+x\frac{\partial u}{\partial y} = xu,\quad\quad\quad u(x,0) = x$.
    \end{enumerate}
\end{enumerate}

\begin{enumerate}[labelsep=1cm]
\item[20.4] Find the most general solutions $u(x,y)$ of the following equations, consistent
with the boundary conditions started:
\end{enumerate}

\begin{enumerate}
    \begin{enumerate}
    \item $y\frac{\partial u}{\partial x}-x\frac{\partial u}{\partial y} = 0, \quad u(x,0) = 1+sinx$;
\setlength{\parskip}{0.6em}
    \item $i\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial y} , \quad u = (4+3i)x^2
    \mbox{ on the line } x = y$;
\setlength{\parskip}{0.6em}
    \item $\mbox{sin }x\mbox{ sin }y\frac{\partial u}{\partial x}+\mbox{cos }x
    \mbox{ cos }y\frac{\partial u}{\partial y} = 0,\quad u=cos2y \mbox{ on } x+y = \pi/2$;
\setlength{\parskip}{0.6em}
    \item  $\frac{\partial u}{\partial x}+2x\frac{\partial u}{\partial y} = 0, \quad u = 2
    \mbox{ on the parabola } y = x^2$.
    \end{enumerate}
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\begin{enumerate}[labelsep=1cm]
\item[20.5] Find solutions of
$$\frac{1}{x}\frac{\partial u}{\partial x}+\frac{1}{y}\frac{\partial u}{\partial y} = 0$$
for which (a) $u(0,y) = y$ and (b) $u(1, 1)=1$.
\end{enumerate}

\begin{enumerate}[labelsep=1cm]
\item[20.6] Find the most general solutions $u(x, y)$ of the following equations consistent with
the  boundary conditions stated :
\end{enumerate}

\begin{enumerate}
    \begin{enumerate}
    \item $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} , \quad u = x^2
    \mbox{ on the line } y = 0$;
    \end{enumerate}
\end{enumerate}
\restoregeometry
\newpage
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