Laborator 4

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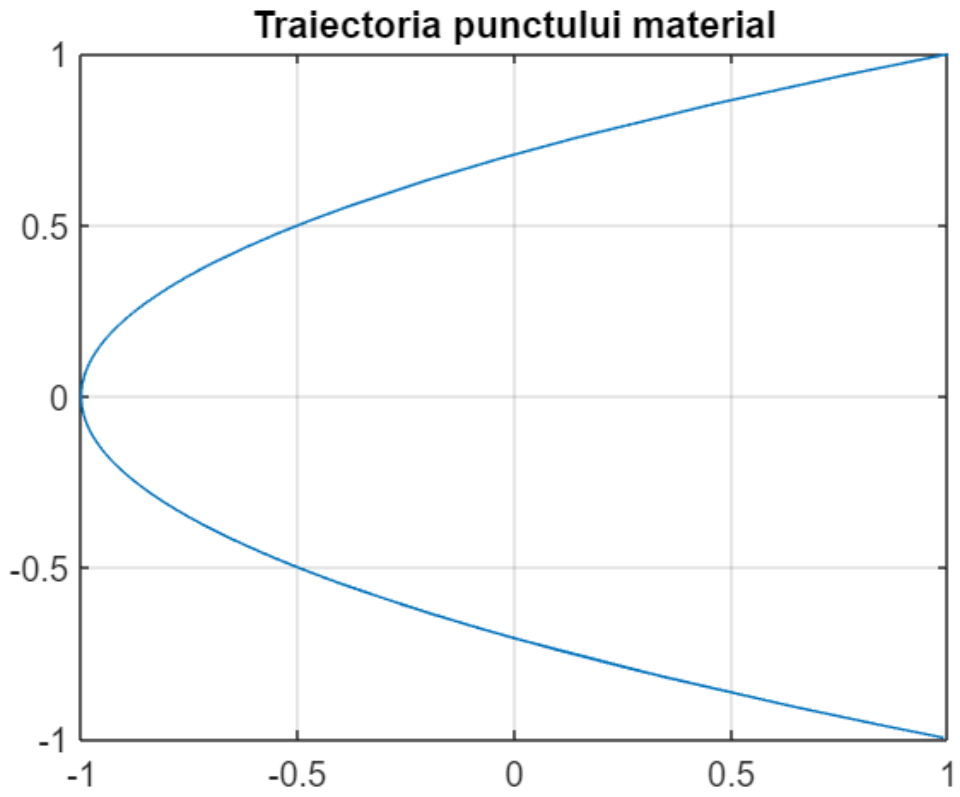
Problema 1

```
syms x(t) y(t);

x_0=1;
dx_0=0;
f=-4.*x;
a=0;
b=5;

Dx=diff(x);
x_eq=diff(x,t,2)==f;
x_cond=x(a)==x_0;
Dx_cond=Dx(a)==dx_0;
x_conds=[x_cond Dx_cond];
x_sol(t)=dsolve(x_eq,x_conds);

x=matlabFunction(x_sol);

y_0=1;
dy_0=0;
g=-y;

Dy=diff(y);
y_eq=diff(y,t,2)==g;
y_cond=y(a)==y_0;
Dy_cond=Dy(a)==dy_0;
y_conds=[y_cond Dy_cond];
y_sol(t)=dsolve(y_eq,y_conds);

y=matlabFunction(y_sol);

fplot(x,y,[a,b])
grid on
title("Traiectoria punctului material")
```



Problema 2

a) $M_0(0,0), \alpha = \frac{\pi}{3}, v_0 = 8$

```
syms x(t) y(t);

v_0=8;
alfa=pi/3;
x_0=0;
dx_0=v_0.*cos(alfa);
f=0;
t_0=0;

Dx=diff(x);
x_eq=diff(x,t,2)==f;
x_cond=x(t_0)==x_0;
Dx_cond=Dx(t_0)==dx_0;
x_conds=[x_cond Dx_cond];
x_sol(t)=dsolve(x_eq,x_conds);

x=matlabFunction(x_sol);

g=9.8;
y_0=0;
dy_0=v_0.*sin(alfa);
```

```
f1=-g;

Dy=diff(y);
y_eq=diff(y,t,2)==f1;
y_cond=y(t_0)==y_0;
Dy_cond=Dy(t_0)==dy_0;
y_conds=[y_cond Dy_cond];
y_sol(t)=dsolve(y_eq,y_conds);

y=matlabFunction(y_sol);

v_x=diff(x_sol);
v_y=diff(y_sol);
v=sqrt(v_x.^2+v_y.^2);
eq=y(t)==0;
assume(t>0)
t_f=solve(eq);
d=x(t_f)-x_0
```

```
d =
```

$$\frac{160\sqrt{3}}{49}$$

```
h_max=t_f
```

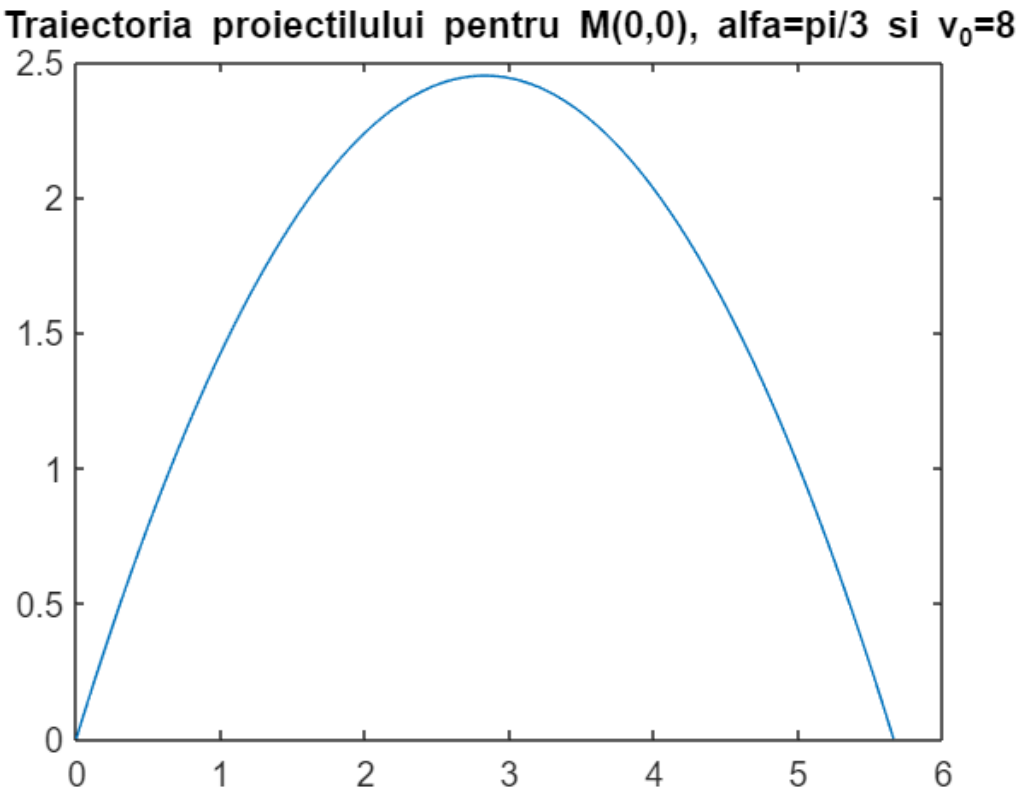
```
h_max =
```

$$\frac{40\sqrt{3}}{49}$$

```
v_f=v(t_f)
```

```
v_f = 8
```

```
t=linspace(t_0,t_f);

plot(x(t),y(t))
title("Traiectoria proiectilului pentru M(0,0), alfa=pi/3 si v_0=8")
```



b) $M_0(0, 10), \alpha = \frac{\pi}{4}, v_0 = 15$

```
syms x(t) y(t)

v_0=15;
alfa=pi/4;
x_0=0;
dx_0=v_0.*cos(alfa);
f=0;
t_0=0;

Dx=diff(x);
x_eq=diff(x,t,2)==f;
x_cond=x(t_0)==x_0;
Dx_cond=Dx(t_0)==dx_0;
x_conds=[x_cond Dx_cond];
x_sol(t)=dsolve(x_eq,x_conds);

x=matlabFunction(x_sol);

g=9.8;
y_0=10;
dy_0=v_0.*sin(alfa);
f1=-g;
```

```
Dy=diff(y);
y_eq=diff(y,t,2)==f1;
y_cond=y(a)==y_0;
Dy_cond=Dy(a)==dy_0;
y_conds=[y_cond Dy_cond];
y_sol(t)=dsolve(y_eq,y_conds);

y=matlabFunction(y_sol);

v_x=diff(x_sol);
v_y=diff(y_sol);
v=sqrt(v_x.^2+v_y.^2);
eq=y(t)==0;
assume(t>0)
t_f=solve(eq);
d=x(t_f)-x_0
```

```
d =
```

$$\frac{15\sqrt{2}\left(\frac{5\sqrt{2}\sqrt{617}}{98}+\frac{75\sqrt{2}}{98}\right)}{2}$$

```
h_max=t_f
```

```
h_max =
```

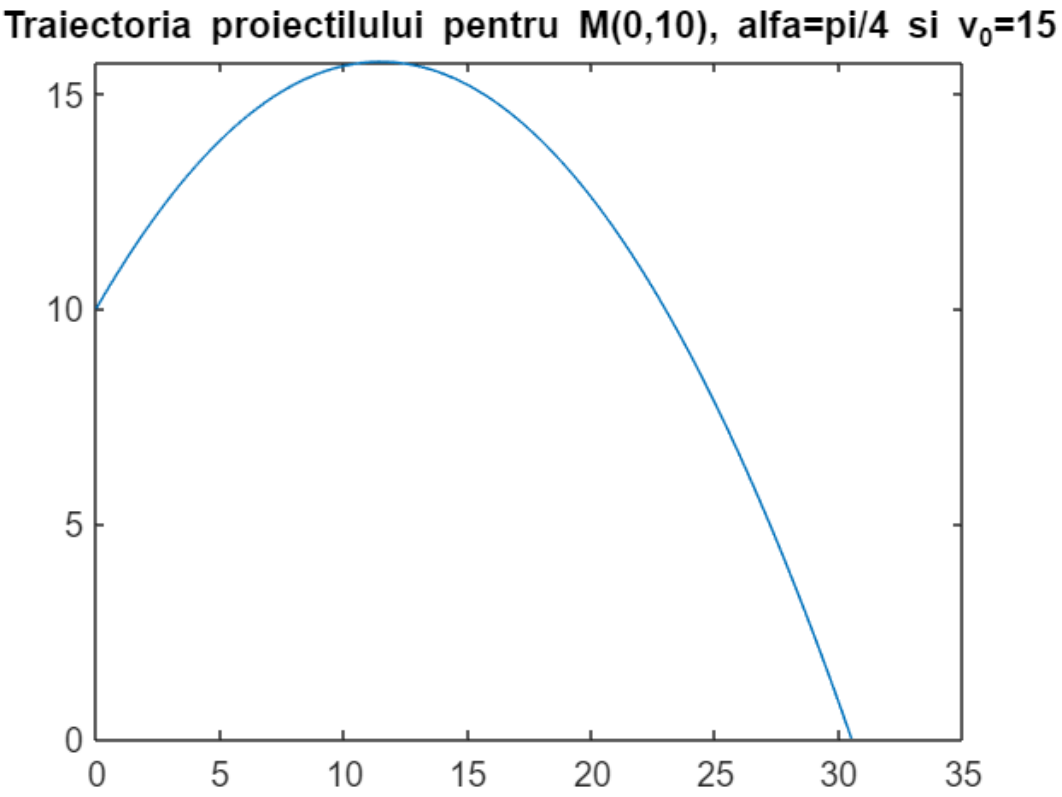
$$\frac{5\sqrt{2}\sqrt{617}}{98}+\frac{75\sqrt{2}}{98}$$

```
v_f=v(t_f)
```

```
v_f =
```

$$\sqrt{421}$$

```
t=linspace(t_0,t_f);

plot(x(t),y(t))
title("Traiectoria proiectilului pentru M(0,10), alfa=pi/4 si v_0=15")
```



c) $M_0(0,40), \alpha = 0, v_0 = 5$

```
syms x(t) y(t)

v_0=5;
alfa=0;
x_0=0;
dx_0=v_0.*cos(alfa);
f=0;
t_0=0;

Dx=diff(x);
x_eq=diff(x,t,2)==f;
x_cond=x(t_0)==x_0;
Dx_cond=Dx(t_0)==dx_0;
x_conds=[x_cond Dx_cond];
x_sol(t)=dsolve(x_eq,x_conds);

x=matlabFunction(x_sol);

g=9.8;
y_0=40;
dy_0=v_0.*sin(alfa);
f1=-g;

Dy=diff(y);
y_eq=diff(y,t,2)==f1;
```

```
y_cond=y(a)==y_0;
Dy_cond=Dy(a)==dy_0;
y_conds=[y_cond Dy_cond];
y_sol(t)=dsolve(y_eq,y_conds);

y=matlabFunction(y_sol);

v_x=diff(x_sol);
v_y=diff(y_sol);
v=sqrt(v_x.^2+v_y.^2);
eq=y(t)==0;
assume(t>0)
t_f=solve(eq);
d=x(t_f)-x_0
```

```
d =
```

$$\frac{100}{7}$$

```
h_max=t_f
```

```
h_max =
```

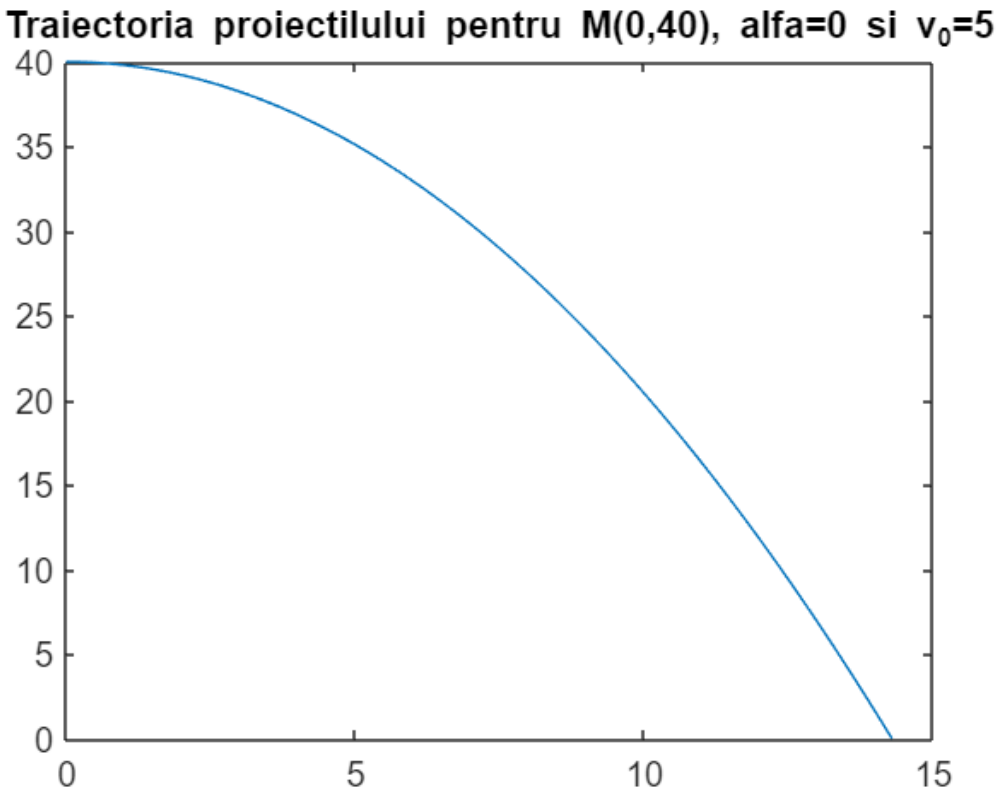
$$\frac{20}{7}$$

```
v_f=v(t_f)
```

```
v_f =
```

$$\sqrt{809}$$

```
t=linspace(t_0,t_f);

plot(x(t),y(t))
title("Traiectoria proiectilului pentru M(0,40), alfa=0 si v_0=5")
```



d) $M_0(10, 30), \alpha = \frac{\pi}{2}, v_0 = 3$

```
syms x(t) y(t)

v_0=3;
alfa=pi/2;
x_0=10;
dx_0=v_0.*cos(alfa);
f=0;
t_0=0;

Dx=diff(x);
x_eq=diff(x,t,2)==f;
x_cond=x(t_0)==x_0;
Dx_cond=Dx(t_0)==dx_0;
x_conds=[x_cond Dx_cond];
x_sol(t)=dsolve(x_eq,x_conds);

x=matlabFunction(x_sol);

g=9.8;
y_0=30;
dy_0=v_0.*sin(alfa);
f1=-g;
```

```
Dy=diff(y);
y_eq=diff(y,t,2)==f1;
y_cond=y(a)==y_0;
Dy_cond=Dy(a)==dy_0;
y_conds=[y_cond Dy_cond];
y_sol(t)=dsolve(y_eq,y_conds);

y=matlabFunction(y_sol);

v_x=diff(x_sol);
v_y=diff(y_sol);
v=sqrt(v_x.^2+v_y.^2);
eq=y(t)==0;
assume(t>0)
t_f=solve(eq);
d=x(t_f)-x_0
```

```
d =
```

$$\frac{37258182000161335\ \sqrt{597}}{19876761411578637015468306260295 68}+\frac{111774546000484005}{19876761411578637015468306260295 68}$$

```
h_max=t_f
```

```
h_max =
```

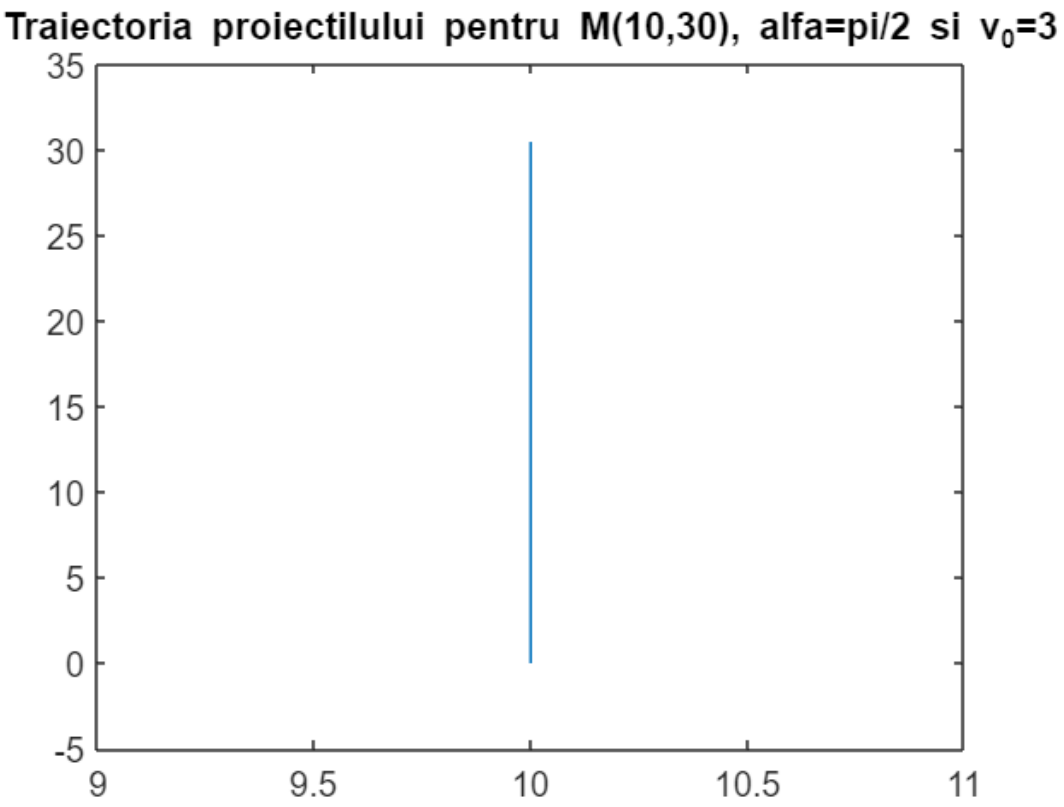
$$\frac{5\ \sqrt{597}}{49}+\frac{15}{49}$$

```
v_f=v(t_f)
```

```
v_f =
```

$$\frac{\sqrt{24559155518019000179162915049668457675039458751726904464624017052 1}}{20282409603651670423947251286016}$$

```
t=linspace(t_0,t_f);

plot(x(t),y(t))
title("Traiectoria proiectilului pentru M(10,30), alfa=pi/2 si v_0=3")
```



Problema 3

```
h=40;
v_0=5;
k=0.1;
g=9.8;
tf=3;

syms x(t) y(t);

x(t)=(v_0./k).*(1-(exp(1)).^(-k.*t));
y(t)=(-g./(k.^2)).*(exp(1)).^(-k.*t)-(g./k).*t+(g./(k.^2))+h;

v_x(t)=v_0.*exp(1).^(-k.*t);
v_y(t)=(g./k).*exp(1).^(-k.*t)-(g./k);
v=sqrt(v_x.^2+v_y.^2);
eq=y(t)==0;
assume(t>0)
t_f=solve(eq);
d=x(t_f)-x_0
```

```
d =
```

$$40-50\left(\frac{3060513257434037}{1125899906842624}\right)^{\frac{W_0\left(10\,e^{\frac{2550\log(2)}{49}-\frac{51\log(3060513257434037)}{49}}\left(5\log(2)-\frac{\log(3060513257434037)}{10}\right)\right)}{50\log(2)-\log(3060513257434037)}-\frac{51}{49}}$$

```
h_max=t_f
```

h_max =

$$\frac{510}{49} - \frac{10\,W_0\left(10\,e^{\frac{2550\log(2)}{49}-\frac{51\log(3060513257434037)}{49}}\left(5\log(2)-\frac{\log(3060513257434037)}{10}\right)\right)}{50\log(2)-\log(3060513257434037)}$$

```
v_f=v(t_f)
```

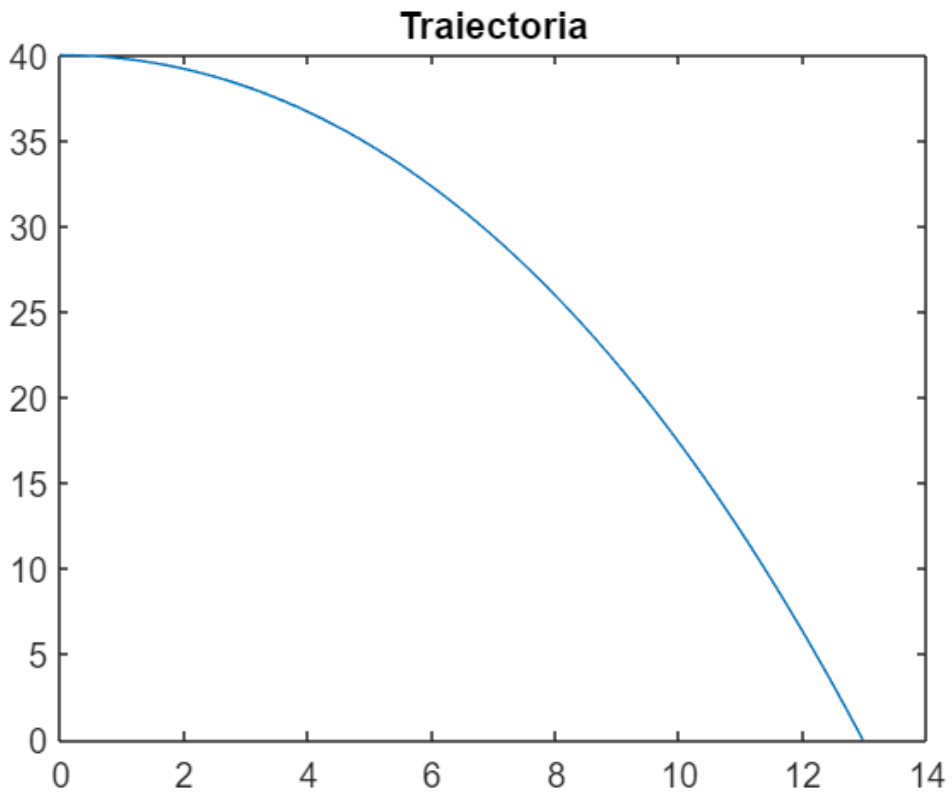
v_f =

$$\sqrt{25\left(\frac{3060513257434037}{1125899906842624}\right)^{\frac{2\sigma_1}{50\log(2)-\log(3060513257434037)}-\frac{102}{49}}+\left(98\left(\frac{3060513257434037}{1125899906842624}\right)^{\frac{\sigma_1}{50\log(2)-\log(306051325}}\right.}$$

where

$$\sigma_1 = W_0\left(10\,e^{\frac{2550\log(2)}{49}-\frac{51\log(3060513257434037)}{49}}\left(5\log(2)-\frac{\log(3060513257434037)}{10}\right)\right)$$

```
t=linspace(t_0,t_f);

plot(x(t),y(t))
title("Traiectoria")
```



Observ că rezultatele obținute la Problema 3 sunt aproape identice cu cele obținute la Problema 2 c). Diferența dintre cele două locuri de aterizare (și probabil a vitezei care scade mai repede în mediul natural, decât în vid) se datorează frecării cu aerul.