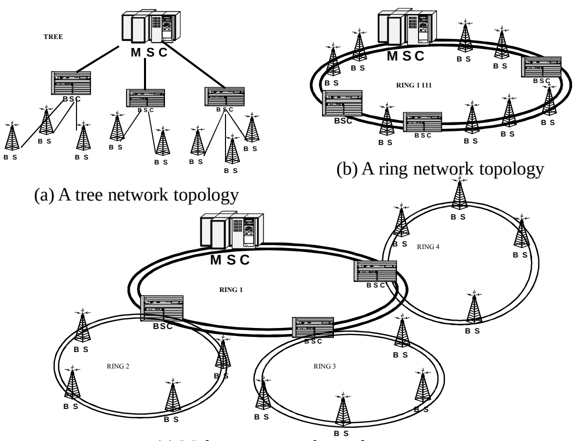
Network Reliability

Network Topology



(a) A tree network topology

(b) A ring network topology

(c) Multi-ring network topology

Network Reliability

- A network can be modeled by a probabilistic graph

$$G = \{V, E, p\}$$

- The probability of a link failure is (1-*p*). All links are assumed having independent failure.
- The probability of network functioning is

$$R(G) = Pr\left\{\bigcup_{i=1}^{n_t} S_i\right\}$$

where S_i is the event in which spanning tree T_i is operational and n_t is the number of spanning trees in G.

$$R\left(G^*_{n,e}\right) = \max\{R(G_{n,e}) \mid G = (V, E, p), |V| = n, |E| = e\}$$

- For spanning tree, $|E| = |V|-1$

$$R\left(G^*_{n,n-1}\right) = p^{n-1}$$

- For ring network: $R\left(G^*_{n,n}\right) = p^n + np^{n-1}(1-p)$
- For multi-ring network:

$$R\left(mG^*_{(n_1,n_1)(n_2,n_2)\ldots,(n_m,n_m)}\right) = \prod_{i=1}^{m}\left\{ p^{n_i} + \left(n_i p^{n_i-1}\right)(1-p) \right\}$$

where m = number of sub-rings + 1 (for the main ring)

- The number of nodes in the network is equal to sum of number of nodes in each sub-ring plus number of nodes in the main ring which do not have sub-rings.

$$n = \left(\sum_{i=2}^{m} n_i\right) + \left(n_1 - (m-1)\right)$$

where n_1 = number of nodes in the main ring

- It is easy to show that

$$R\left(G^*_{n,n-1}\right) < R\left(G^*_{n,n}\right) < R\left(mG^*_{(n_1,n_1)(n_2,n_2)\ldots,(n_m,n_m)}\right)$$

Reliability Analysis

Network Topology	Reliability
Tree	$R\left(G^*_{16,16}\right) = 0.514728$
Ring	$R\left(G^*_{16,15}\right) = 0.685302$
Multi-ring: $(n_2, n_3, n_4) = (5, 5, 6)$	$R\left(4G^*_{(4,4)(5,5),(5,5),(6,6)}\right) = 0.708224$
Multi-ring: $(n_2, n_3, n_4) = (4, 6, 6)$	$R\left(4G^*_{(4,4)(4,4),(6,6),(6,6)}\right) = 0.704611$
Multi-ring: $(n_2, n_3, n_4) = (4, 5, 7)$	$R\left(4G^*_{(4,4)(4,4),(5,5),(7,7)}\right) = 0.701479$