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Pruning Algorithm for Multi-Objective Optimization with Decision Maker's Preferences of System Redundancy Allocation Problem

Naruemon Wattanapongsakorn
Tipwimol Sooktip
King Mongkut's University of Technology Thonburi
Bangkok, Thailand

KMUTT

Outline

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- Related Work
- Problem Statement:
  Pruning Algorithm According to The Decision Maker's Preferences
- Redundancy Allocation Problem
- Preference-Based Ranking Method
- Experimental Result
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Introduction to Multi-Objective Optimization

The mathematical formulation

Maximize/Minimize $f_i(\mathbf{x})$ for $i = 1, 2, \ldots, n$

Subject to $g_j(\mathbf{x}) \geq 0 \quad j = 1, 2, \ldots, J$

$h_k(\mathbf{x}) = 0 \quad k = 1, 2, \ldots, K$

$x_i^{(L)} \leq x_i \leq x_i^{(U)}$

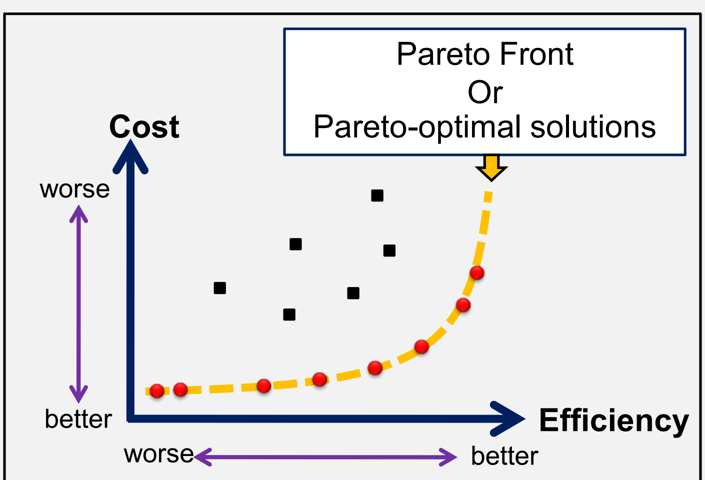
where $\mathbf{x}$ is a vector of n decision variable(s):

$\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$.

$\mathbf{f}(\mathbf{x}) = (f_1\,(x), f_2\,(x), \ldots, f_M\,(x))^T$.

Multi-Objective Optimization Characteristics

Figure 1 The Pareto front or Pareto-optimal solutions



Related Work

- There exists many searching algorithms such as genetic algorithm, non-dominated sorting genetic algorithm-II (NSGA-II), particle swarm optimization algorithm, ant colony algorithm, etc.
- A challenging task of multi-objective optimization is to select the appropriate solutions among a large number of non-dominated solutions.
- Preference-based methods according to the Decision maker (DM)'s preferences should be considered to identify the preferred solution(s).

Related Work

- A non-numerical ranking preference (NNRP) method [1]
  - The DM gives all relations of ordering objective functions.
    - For example, "the objective 1 is more important than objective 2; objective 2 is more important than objective 3".
    - The ranked objectives functions is: $f_1 \succ f_2 \succ f_3$
  - The weights are generated randomly
    - A relative order is $w_1 > w_2 > w_3$
    - Likelihood of different weight combination

$$\boxed{f = w_1 f_1(x) + \ldots + w_n f_n(x)}$$

  - Suitable for 3-4 objective functions
- An expanding of NNRP method [2] is presented for post-Pareto optimality method to solve a 5-objective problem.
  - Require large amount of mathematical calculation to derive the weights.

Problem Statement

- Which solution is the most appropriate one?
- A large possible solution set

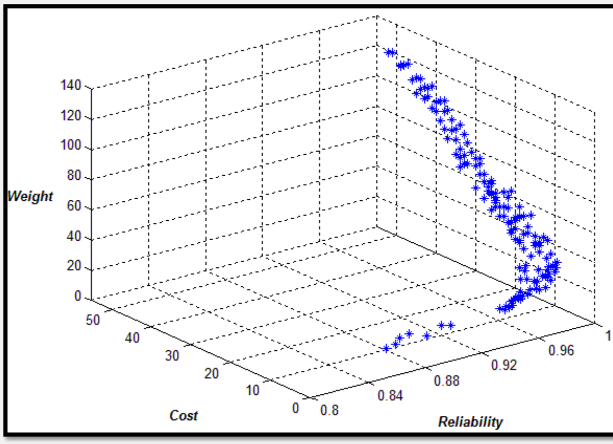
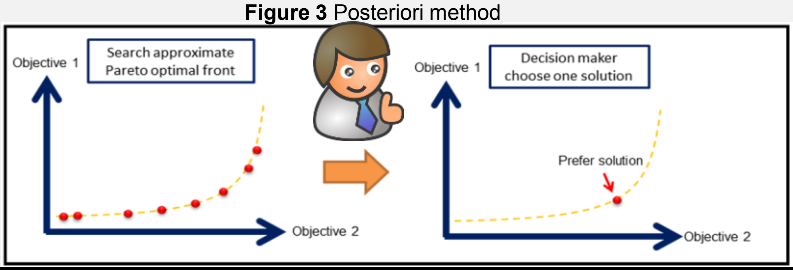


Figure 2 The approximated Pareto-optimal solutions in 3-objective plane. Each point represent the possible alternative in objective space.

Problem Statement

- Propose a preference-based ranking method as a pruning algorithm for multi-objective optimization.
- Consider a posteriori pruning approach.
- Help the DM identify the prefer solutions from the approximation of the Pareto front according to the DM's preferences.
- The parameter requirements
  - A ranking priority of the objective functions.

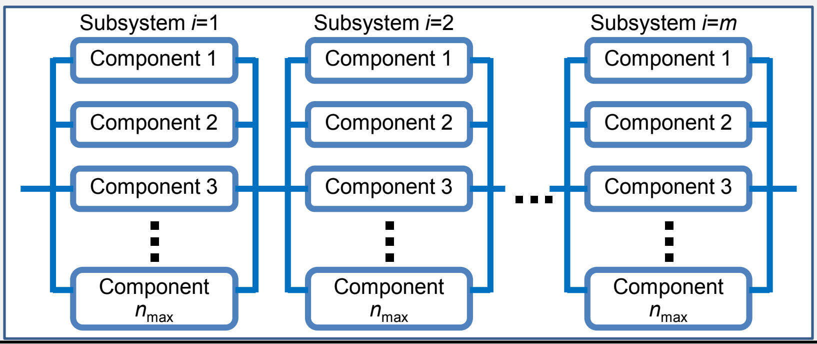
Figure 3 Posteriori method



Redundancy Allocation Problem

- Determining an optimal system design choice by allocating redundant components from the design alternatives.
- The problem can be very complex when mixing of non-identical components is allowed in each subsystem

Figure 4 A redundancy allocation problem with series-parallel structure



Notation

R_{sys}	system reliability
C_{sys}	system cost
W_{sys}	system weight
R_i	reliability in subsystem i
r_{ij}	reliability of the j^{th} component in subsystem i
c_{ij}	cost of the j^{th} component in subsystem i
w_{ij}	weight of the j^{th} component in subsystem i
$\mathbf{x}$, $\mathbf{x}_i$	a vector which defines the number of components type j in subsystem i
x_{ij}	number of components of type j in subsystem i
n_{max}	number of maximum components in subsystem i
t_i	number of component type choices in subsystem i
m	number of subsystems connected in series

Multi-objective Optimization of Series-Parallel System Model

$$\max\ R_{sys}(\mathbf{x}) = \prod_{i=1}^{m} R_i(\mathbf{x}_i)$$

$$\min\ C_{sys}(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{t_i} c_{ij}x_{ij}$$

$$\min\ W_{sys}(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{t_i} w_{ij}x_{ij}$$

s.t.

$$1 \le \sum_{j=1}^{t_i} x_{ij} \le n_{\max}$$

$$R_i(\mathbf{x}) = 1 - \prod_{j=1}^{t_i}(1 - R_{ij}(\mathbf{x}))^{x_{ij}}$$

where $x_{ij} \in \{0,\ 1,\ 2,\ \ldots, n_{\max}\},\ i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, t_i$

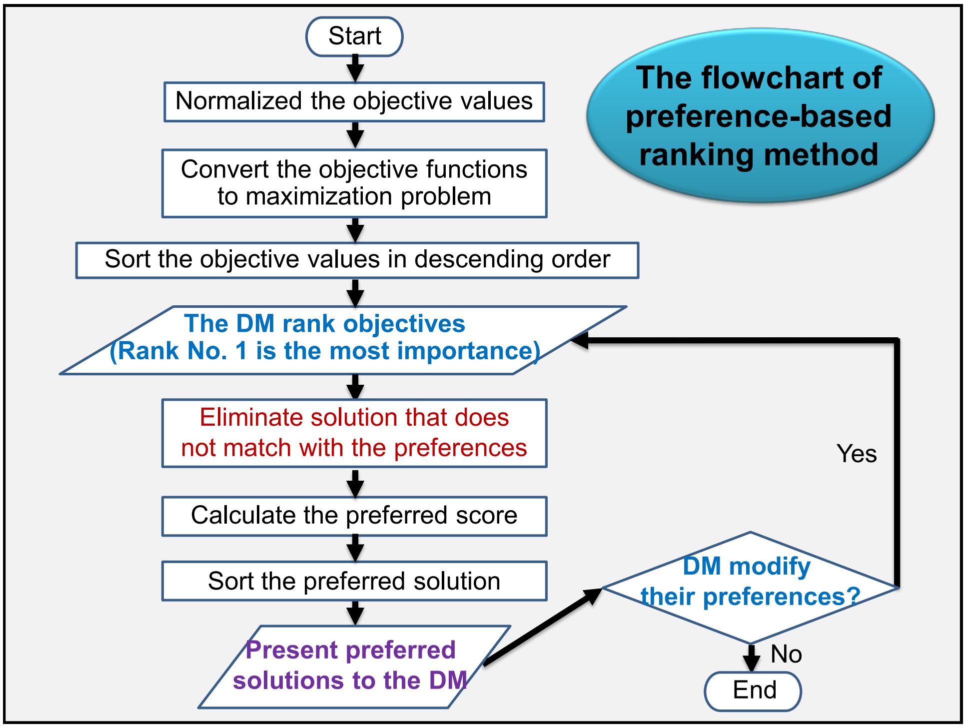
Problem Assumption

- Mixing of non-identical component types is allowed.
- NSGA-II is used as the searching algorithm.
- The alternative component choices have s-independent component.
- The components and the system have only two possible states: work or else fail.

Table I. Available Component Types for Each Subsystem

Subsystem i	Design alternative j														
	Component Choice 1			Component Choice 2			Component Choice 3			Component Choice 4			Component Choice 5		
	r_{ij}	c_{ij}	w_{ij}	r_{ij}	c_{ij}	w_{ij}	r_{ij}	c_{ij}	w_{ij}	r_{ij}	c_{ij}	w_{ij}	r_{ij}	c_{ij}	w_{ij}
1	0.95	2	5	0.93	1	4	0.91	2	2	0.90	1	3	0.95	2	8
2	0.99	4	4	0.98	3	6	0.97	1	5	0.96	2	7	-	-	-
3	0.90	6	5	0.85	5	4	0.82	3	3	0.79	3	5	0.99	2	4

Note: r = reliability, c = cost, w = weight,
"-" means that a design alternative is not available



Preference-Based Ranking Method

- The DM gives **ranking priority** of the objective functions.
  - Rank #1 means the highest priority for the objective function.
- The experiments vary the ranking priority preferences of the objective functions into four cases as following:
  - $\text{Reliability} \succ \text{Cost} \succ \text{Weight}$
  - $\text{Cost} \succ \text{Reliability} \succ \text{Weight}$
  - $\text{Weight} \succ \text{Reliability} \succ \text{Cost}$
  - $\text{Reliability} \succ \text{Weight} \succ \text{Cost}$

Parameter Setting for NSGA-II

- Encoding: The chromosomes are represented as decimal strings. Each gene in the chromosomes represent the number of selected component type *j* for subsystem *i*, (x_{ij}) where i ={1, 2, 3, …, *m*} and j ={1, 2, 3, …, t_i }.

	Subsystem 1					Subsystem 2				Subsystem 3				
Component type	1	2	3	4	5	1	2	3	4	1	2	3	4	5
x_{ij}	0	0	2	0	0	1	0	0	0	0	0	0	0	1

- Selection: Binary tournament selection
- Crossover: SBX crossover
- Mutation: Polynomial mutation operators

Table II. The parameter setting of NSGA-II

Parameter	Value
Population size	200
Mutation probability	0.07
Crossover probability	0.9
Max generation	1000

Pareto-optimal solutions

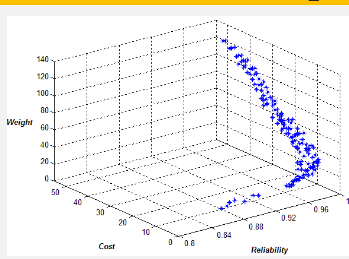


Figure 5 The approximated Pareto-optimal solutions in 3-objective plane.

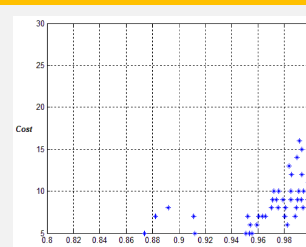


Figure 7 The approximated Pareto-optimal solutions with **reliability and cost.**

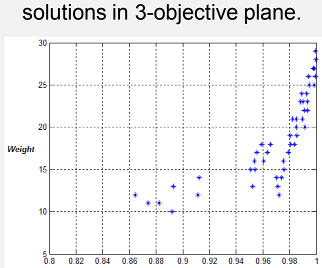


Figure 6 The approximated Pareto-optimal solutions with **reliability and weight.**

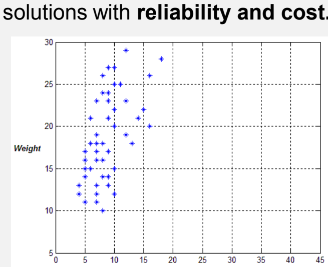
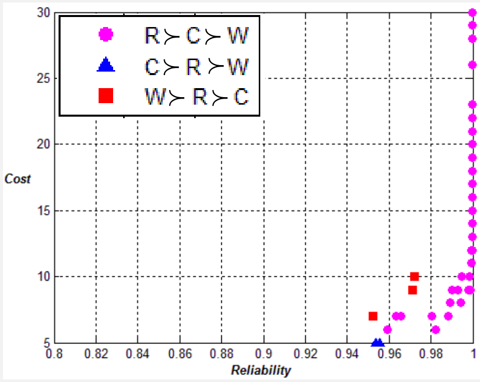


Figure 8 The approximated Pareto-optimal solutions with **cost and weight.**

Experimental Results

The preferred solutions for 3 preferences



The preferred solutions for R≻W≻C

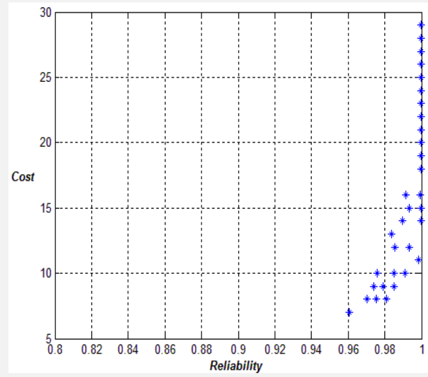


Table III. The preferred solutions for W≻R≻C

			Subsystem 1 Choice					Subsystem 2 Choice				Subsystem 3 Choice					
Reliability	Cost	Weight	1	2	3	4	5	1	2	3	4	1	2	3	4	5	Prefer score
0.972161	10	12	0	0	2	0	0	1	0	0	0	0	0	0	0	1	0.213636
0.971279	9	13	0	0	1	1	0	1	0	0	0	0	0	0	0	1	0.153788
0.952522	7	13	0	0	2	0	0	0	0	1	0	0	0	0	0	1	0.070455

Conclusion

- A pruning algorithm with preference information for multi-objective optimization problems is proposed.
  - The non-numerical ranking preference method
  - Applying after the Pareto Front is obtained by an efficient multi-objective optimization algorithm, NSGA-II.
- Solving redundancy allocation problem
- Identifying the preferred solution set without using quantitative numerical preference parameters.

Conclusion (2)

Research Contribution

- The DM can see the whole picture of the Pareto Front before making a decision for final choice.
- Helping the DM identify appropriate solution(s) among a large set of Pareto-optimal solutions and also satisfy the DM's preferences which are specified as raking/relative priority of the objective functions.
- The DM does not have to be an expert in the problem solving.

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