Karin Metzgar
HW 10
ASTE 546

Analytical Solution

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{-\rho}{\varepsilon_0} \qquad \rho = \varepsilon_0 \sin\left(\frac{x}{L} 2\pi m\right)$$

$$c = \frac{2\pi m}{L}$$

$$\rho = \varepsilon_0 \sin(cx)$$

$$\frac{\partial^2 \phi}{\partial x^2} = -\frac{\varepsilon_0 \sin(cx)}{\varepsilon_0}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -\sin(cx)$$

$$\int \frac{\partial \phi}{\partial x^2} = -\int \sin(cx)$$

$$\int \frac{\partial \phi}{\partial x} = \int -\frac{1}{c}\cos(cx) + C_1$$

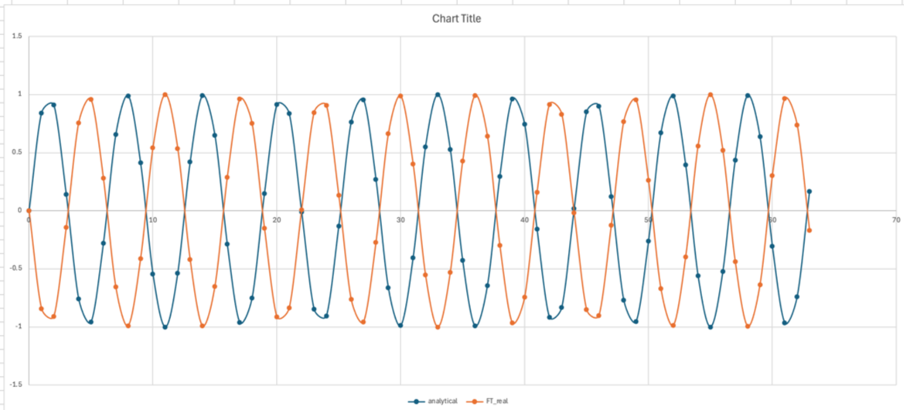
$$\phi(x) = \frac{1}{c^2}\sin(cx) + C_1 x + C_2$$

$$\phi(0) = 0$$

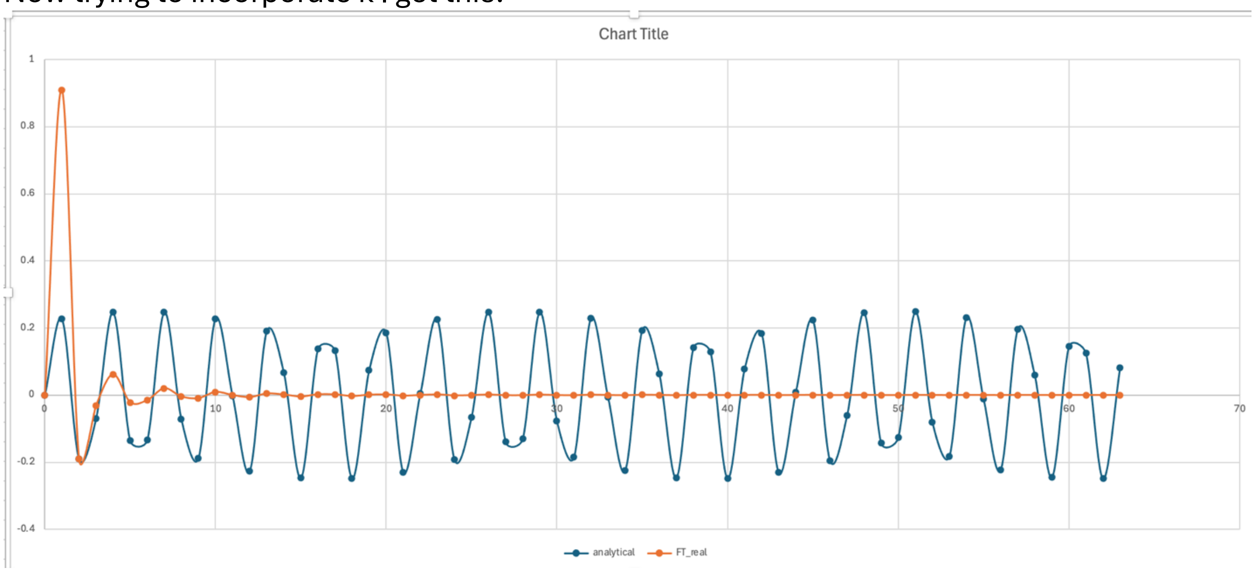
$$\boxed{\phi(x) = \frac{1}{c^2}\sin(cx) + C_1 x}$$

For my first attempt, without incorporating k with a m=1 gives the inverse of the analytical solution.

x	analytical	FT_real	FT_imaginary
0	0	-1.33E-15	-7.28E-16
1	0.841471	-0.841471	-3.51E-15
2	0.909297	-0.909297	-4.08E-16
3	0.14112	-0.14112	4.07E-15
4	-0.756802	0.756802	5.33E-15
5	-0.958924	0.958924	-3.98E-16
6	-0.279415	0.279415	2.92E-15
7	0.656987	-0.656987	-2.65E-15
8	0.989358	-0.989358	-1.08E-15
9	0.412118	-0.412118	2.76E-15
10	-0.544021	0.544021	1.11E-15
11	-0.99999	0.99999	5.45E-15
12	-0.536573	0.536573	-8.82E-16
13	0.420167	-0.420167	-6.00E-15
14	0.990607	-0.990607	-6.72E-16
15	0.650288	-0.650288	2.76E-15
16	-0.287903	0.287903	4.38E-15
17	-0.961397	0.961397	2.94E-15
18	-0.750987	0.750987	2.65E-15
19	0.149877	-0.149877	-1.97E-16
20	0.912945	-0.912945	-3.24E-15
21	0.836656	-0.836656	1.70E-16
22	-0.0088513	0.00885131	3.69E-15
23	-0.84622	0.84622	-3.23E-15
24	-0.905578	0.905578	2.36E-15
25	-0.132352	0.132352	2.75E-15
26	0.762558	-0.762558	-2.21E-15
27	0.956376	-0.956376	-2.59E-15
28	0.270906	-0.270906	8.30E-16
29	-0.663634	0.663634	4.26E-15
30	-0.988032	0.988032	1.17E-15
31	-0.404038	0.404038	1.14E-15
32	0.551427	-0.551427	-1.71E-15
33	0.999912	-0.999912	1.54E-15
34	0.529083	-0.529083	-9.57E-16
35	-0.428183	0.428183	-4.63E-15

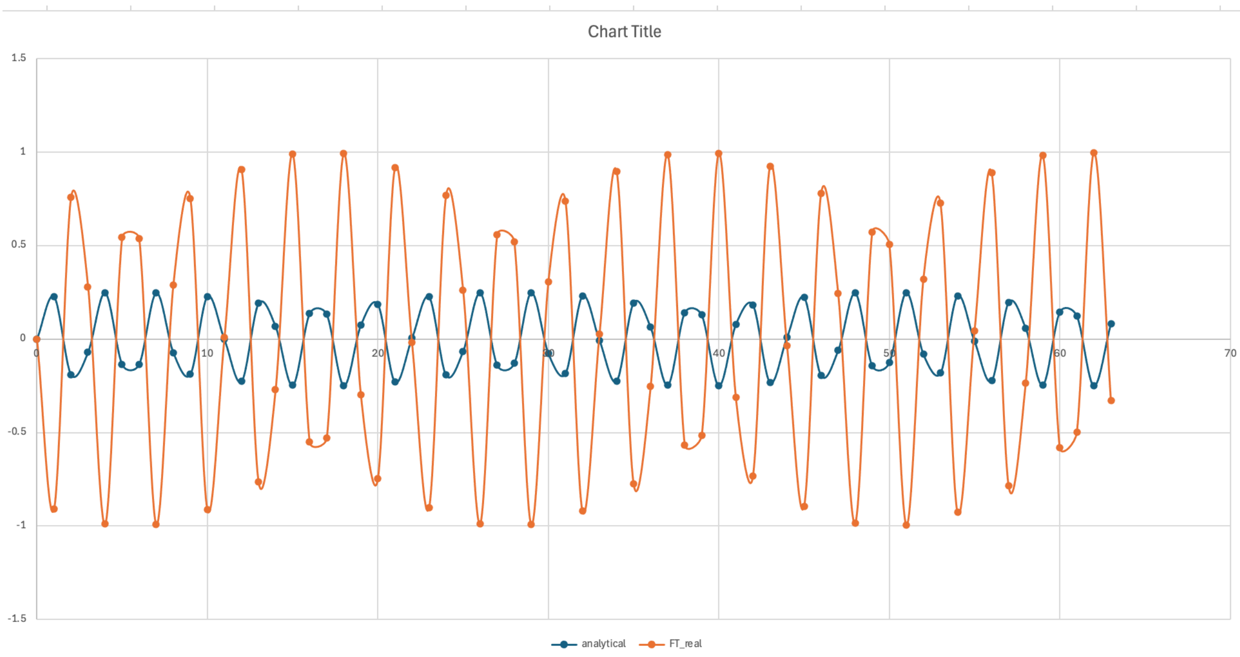


Now trying to incorporate k I get this:

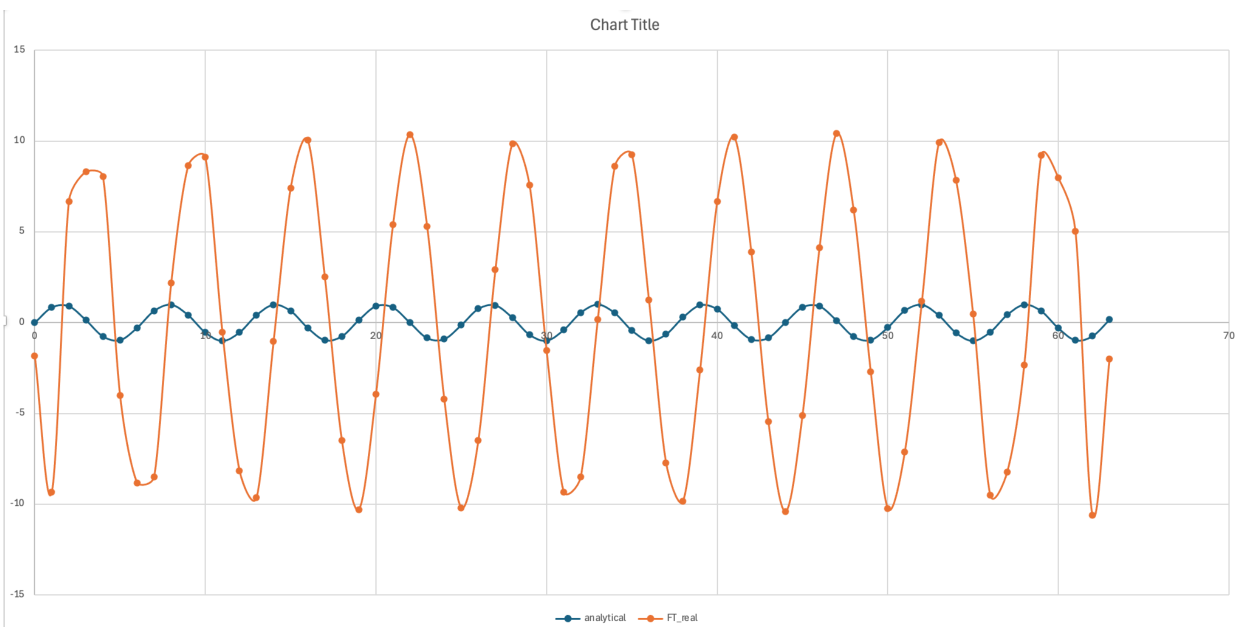


Maybe this needs to be incorporated to the Phase Space equation before it gets inverse FT'ed back to time space ..

My attempt to add the k term to the DFT function didn't work, but this is the result with the k term code removed. The problem is definitely the implementation of the k term because this looks correct .. except a -k term would scale it properly.

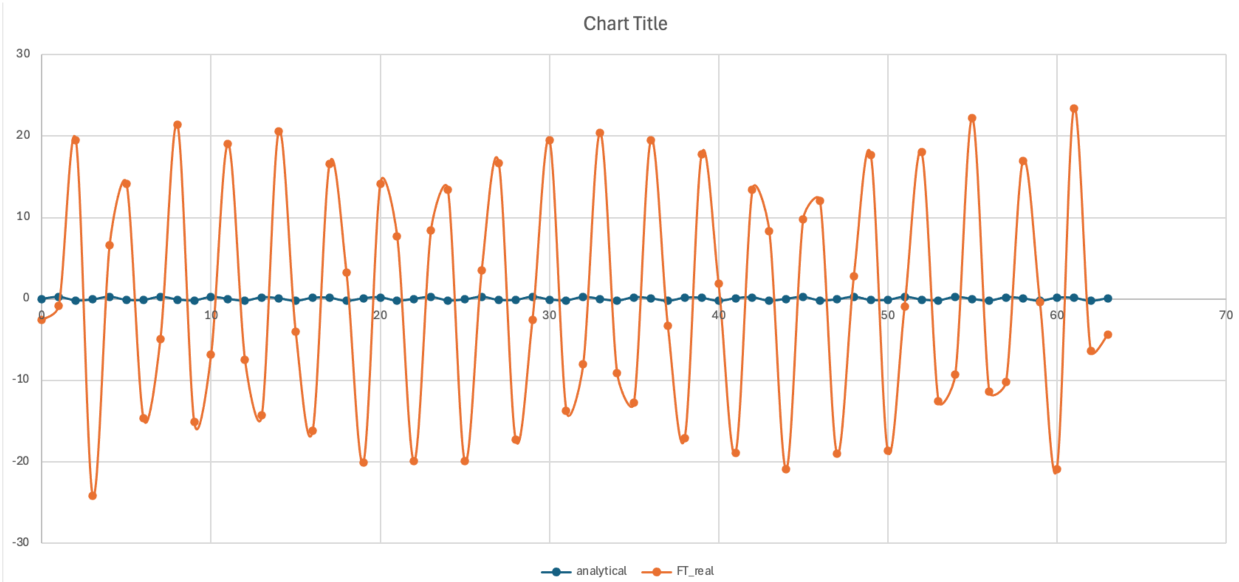


Well I thought I incorporated k correctly, like it's done in the lecture and link but for m=1 I get this:

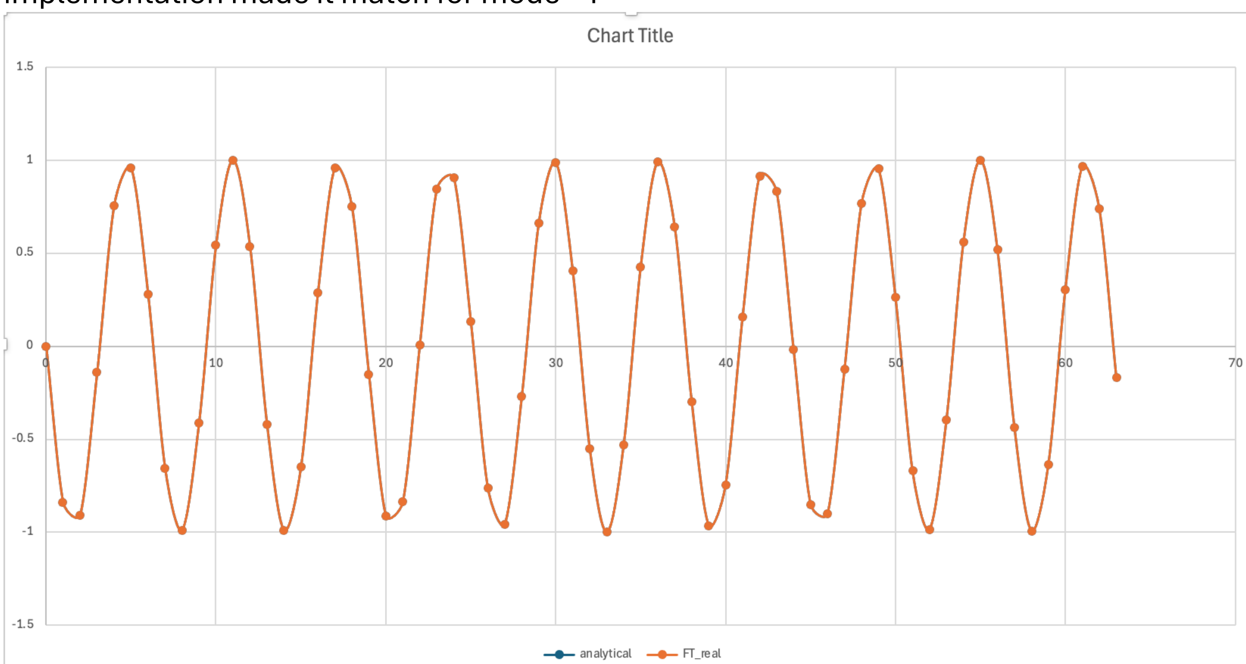


So FT is still being scaled incorrectly

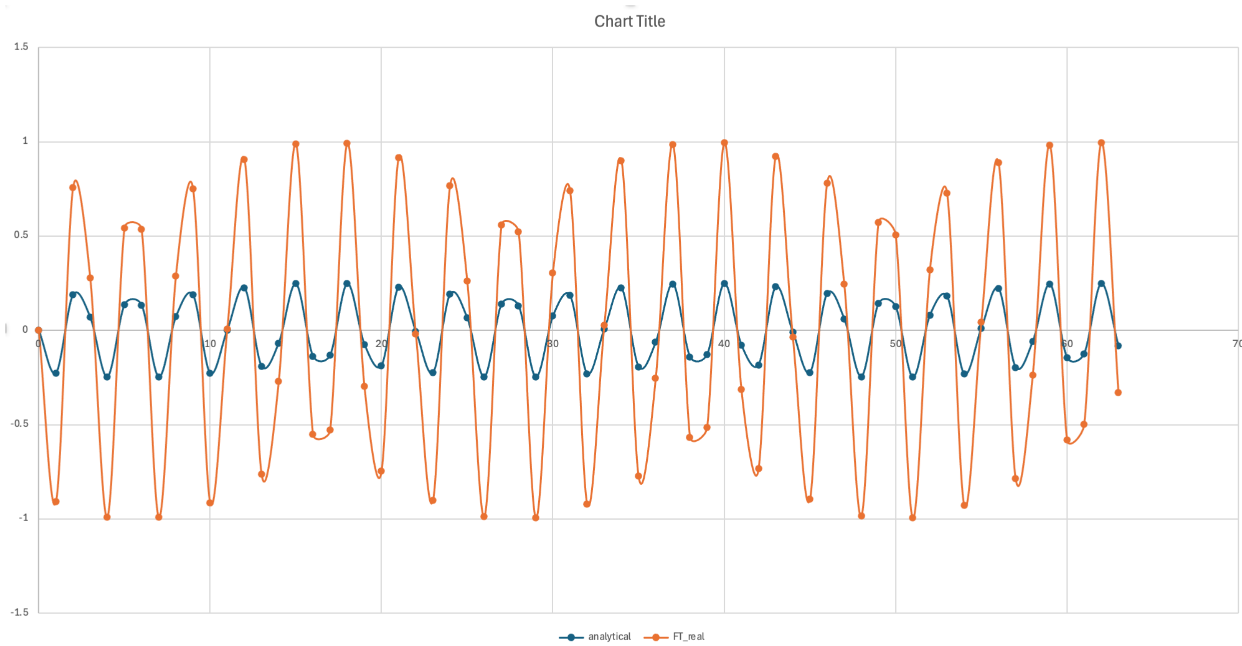
For m = 2 I get:
So now the amplitude and the period seems to be a little off…



Fixing the negative that was missing in my analytical solution and removing the i*k implementation made it match for mode =1



However we can see that with higher modes the amplitude doesn’t scale correctly:



GOT IT TO WORK

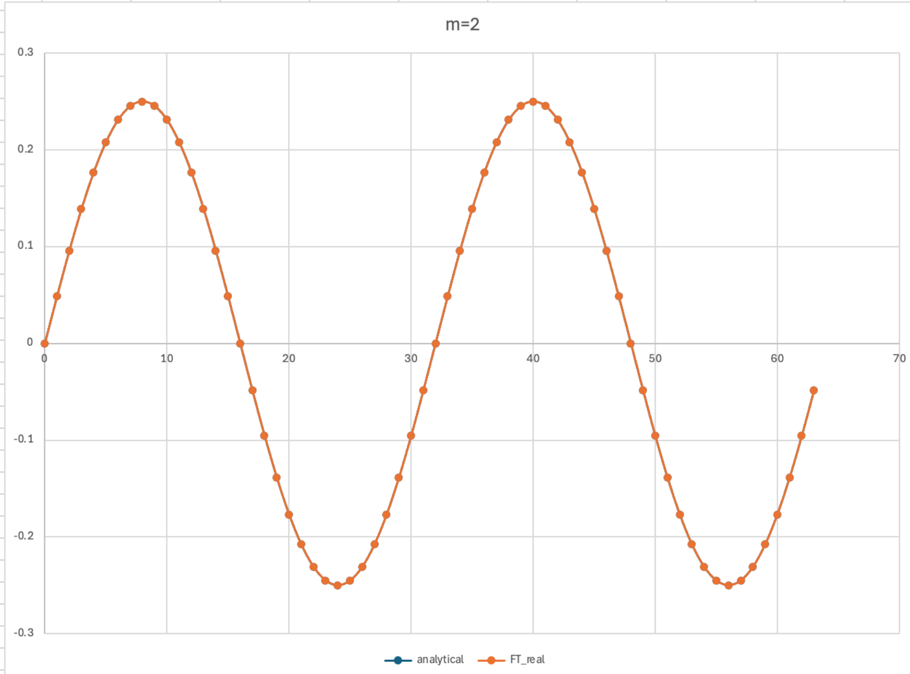
I did not correctly implement x = I *dx in function initially

```
    double dx;
    dx = L / N;
    double x;

    double c = 2 * M_PI * mode / L;
    for (int i = 0; i < N; i++) {
        x = i * dx;
        rho[i] = eps0 * sin(c * x);
        function[i] = -rho[i] / eps0;
    }
```

M = 2

x	analytical	FT_real
0	0	-1.71E-15
1	0.0487726	0.0487726
2	0.0956709	0.0956709
3	0.138893	0.138893
4	0.176777	0.176777
5	0.207867	0.207867
6	0.23097	0.23097
7	0.245196	0.245196
8	0.25	0.25
9	0.245196	0.245196
10	0.23097	0.23097
11	0.207867	0.207867
12	0.176777	0.176777
13	0.138893	0.138893
14	0.0956709	0.0956709
15	0.0487726	0.0487726
16	3.06162e-17	1.8003e-15
17	-0.0487726	-0.0487726
18	-0.0956709	-0.0956709
19	-0.138893	-0.138893
20	-0.176777	-0.176777
21	-0.207867	-0.207867
22	-0.23097	-0.23097
23	-0.245196	-0.245196
24	-0.25	-0.25
25	-0.245196	-0.245196
26	-0.23097	-0.23097
27	-0.207867	-0.207867
28	-0.176777	-0.176777
29	-0.138893	-0.138893
30	-0.0956709	-0.0956709
31	-0.0487726	-0.0487726



M = 4

x	analytical	FT_real
0	0	-6.31E-16
1	0.0239177	0.0239177
2	0.0441942	0.0441942
3	0.0577425	0.0577425
4	0.0625	0.0625
5	0.0577425	0.0577425
6	0.0441942	0.0441942
7	0.0239177	0.0239177
8	7.65404e-18	6.7609e-17
9	-0.0239177	-0.0239177
10	-0.0441942	-0.0441942
11	-0.0577425	-0.0577425
12	-0.0625	-0.0625
13	-0.0577425	-0.0577425
14	-0.0441942	-0.0441942
15	-0.0239177	-0.0239177
16	-1.53E-17	-5.09E-16
17	0.0239177	0.0239177
18	0.0441942	0.0441942
19	0.0577425	0.0577425
20	0.0625	0.0625
21	0.0577425	0.0577425
22	0.0441942	0.0441942
23	0.0239177	0.0239177
24	2.29621e-17	-3.62E-16
25	-0.0239177	-0.0239177
26	-0.0441942	-0.0441942
27	-0.0577425	-0.0577425
28	-0.0625	-0.0625
29	-0.0577425	-0.0577425
30	-0.0441942	-0.0441942