

1 Panel data models

Problem 1

Suppose that the random effects model $y_{it} = x'_{it}\beta + \eta_i + v_{it}$ is to be estimated with a panel in which the groups have different numbers of observations. Let T_i be the number of observations in group i . Show that the pooled least squares estimator is unbiased and consistent despite this complication.

Problem 2

Consider $y_{it} = x'_{it}\beta + \eta_i + v_{it}$, $i = 1, \dots, N$, $t = 1, \dots, T$, where $v_{it} \sim \mathcal{N}(0, \sigma^2)$ and $\beta = 0$. Write out the likelihood for estimating η_i and σ^2 , and show that the MLE estimator $\hat{\sigma}^2$ is biased when $T < \infty$.

Problem 3

Consider $y_{it} = \mathbb{1}\{x_{it}\beta + \eta_i + v_{it} \geq 0\}$, where the errors v_{it} have the logistic cdf. Consider $T = 2$, $x_{i1} = 0$ and $x_{i2} = 1$, and show that the sufficient statistic for η_i is $y_{i1} + y_{i2} = 1$, i.e. conditioning on $y_{i1} + y_{i2} = 1$ implies that the MLE does not depend on η_i .

Problem 4

Derive the bias of the OLS estimator for α in a dynamic panel of the form $y_{it} = \alpha y_{it-1} + \eta_i + v_{it}$. Are there any conditions on α that should hold for the estimator to be well-defined?