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Course: **Statistics**

1 Basic probability theory

Exercise 1.1. For each of the following experiments, describe the sample space.

(a) Toss a coin four times.

(b) Count the number of insect-damaged leaves on a plant.

(c) Measure the lifetime (in hours) of a particular brand of light bulb.

(d) Record the weights of 10-day-old rats.

(e) Observe the proportion of defectives in a shipment of electronic components.

Exercise 1.2. Verify the following identities:

(a) $A \setminus B = A \setminus (A \cap B) = A \cap B^c$,

(b) $B = (B \cap A) \cup (B \cap A^c)$,

(c) $B \setminus A = B \cap A^c$,

(d) $A \cup B = A \cup (B \cap A^c)$.

Exercise 1.3. For events A and B, find formulas for the probabilities of the following events in terms of the quantities $\mathbb{P}(A)$, $\mathbb{P}(B)$, and $\mathbb{P}(A \cap B)$:

(a) either A or B or both,

(b) either A or B but not both,

(c) at least one of A or B,

(d) at most one of A or B.

Exercise 1.4. Consider two different setups:

(a) A fair dice is cast until a 6 appears. What is the probability that it must be cast more than five times?

(b) Prove that if $\mathbb{P}(A) > 0$ and $\mathbb{P}(B) > 0$, then:

- if A and B are mutually exclusive, they cannot be independent,
- if A and B are independent, they cannot be mutually exclusive.

Exercise 1.5. Two coins, one with $\mathbb{P}(\text{head}) = u$ and one with $\mathbb{P}(\text{head}) = w$, are to be tossed together independently. Let

$$p_0 = \mathbb{P}(0 \text{ heads occur}), \quad p_1 = \mathbb{P}(1 \text{ heads occur}), \quad p_2 = \mathbb{P}(2 \text{ heads occur}).$$

Can u and w be chosen such that $p_0 = p_1 = p_2$? Prove your answer.

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Exercise 1.6. Consider telegraph signals ”dot” and ”dash” sent in the proportion 3:4, where erratic transmissions cause a dot to become a dash with probability $1/4$ and a dash to become a dot with probability $1/3$.

(a) If a dash is received, what is the probability that a dash has been sent? If a dot is received, what is the probability that a dot has been sent?

(b) Assuming independence between signals, if the message dot-dot was received, what is the probability distribution of the four possible messages that could have been sent?