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# **DETERMINATION OF THE THERMAL NEUTRON FLUX IN THE CORE OF THE REACTOR**

## **Measurement guide**

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## 1. INTRODUCTION

The nuclei of certain naturally occurring isotopes can be transformed into radioactive ones by exposing the material to neutron radiation, and the activity of the radioactive products produced can be measured by means of appropriate counter system. In addition to the factors determined by the conditions of measurement, this activity is affected only by the neutron flux in the point of irradiation and by the activation cross section of the target material which in the present case, is the neutron detector. Provided the activation cross section is known, the neutron flux can be determined by measurement of the activity of the sample irradiated. As the activation cross section of a number of materials depends on the neutron energy in different way this method will yield information also on the energy distribution of neutrons.

In the present measurement the thermal neutron flux will be determined using activation detectors.

## 2. THEORY

### 2.1. Neutron spectrum and neutron flux distribution in thermal reactors

Fast neutrons, the energy distribution of which can be seen in Fig. 1., are produced in fission reactions. In thermal nuclear reactors, the fast neutrons gradually slow down due primarily to the collisions with the atomic nuclei of the moderator, and a neutron spectrum schematically depicted in Fig. 2. is formed. Let the density of the neutrons with energies between  $E$  and  $E+dE$  near a point  $\underline{r}$  of the detector be  $n(\underline{r}, E)dE$ . Thus, the definition of *neutron flux* is as follows:

$$\Phi(\underline{r}, E) = v n(\underline{r}, E), \quad (1)$$

where  $v$  is the velocity of a neutron with energy  $E$ . As far as this measurement is concerned, the most important part of the neutron spectrum is the thermal spectrum, which can approximately be described by the Maxwell distribution:

$$\Phi(\underline{r}, E) = \frac{E}{(kT)^2} \exp \left\{ -\frac{E}{kT} \right\}, \quad E \leq E_{th} \quad (2)$$

where  $E_{th}$  is the more or less arbitrarily chosen upper limit of the thermal region (usually 0.625 eV),  $k$  is the Boltzmann constant, and  $T$  is a quantity with dimension of temperature characteristic of the thermal neutron spectrum.<sup>1</sup>

By *thermal neutron flux* we mean the following integral:

$$\Phi_{th}(\underline{r}) = \int_0^{E_{th}} \Phi(\underline{r}, E) dE, \quad (3)$$

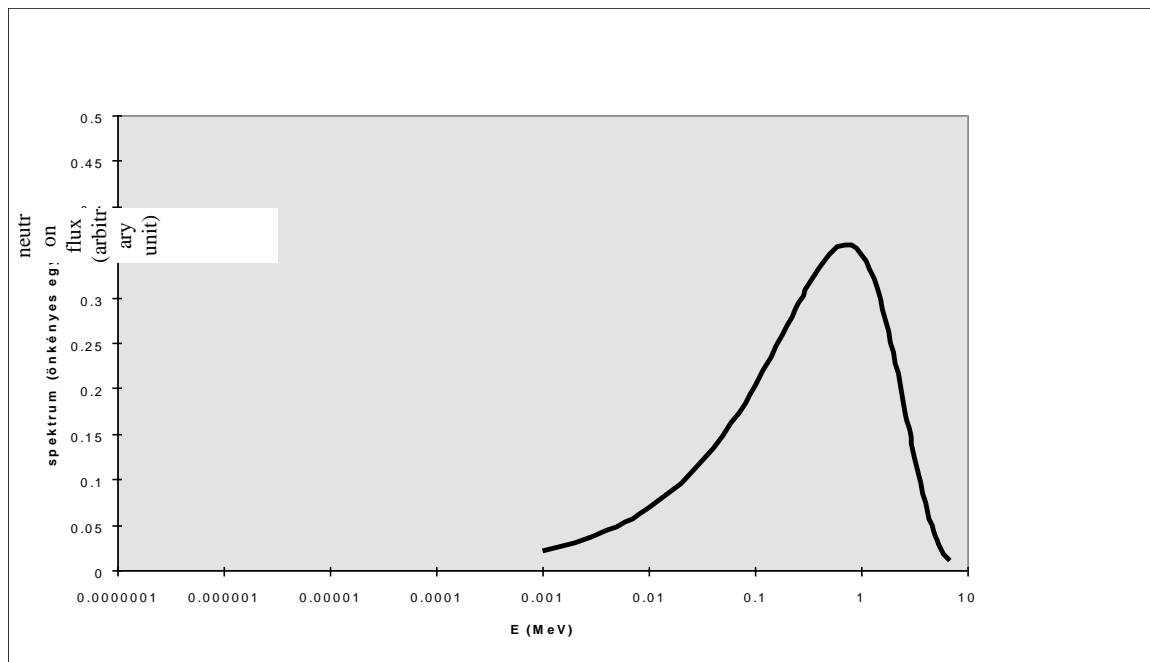
which is often written in the following form:

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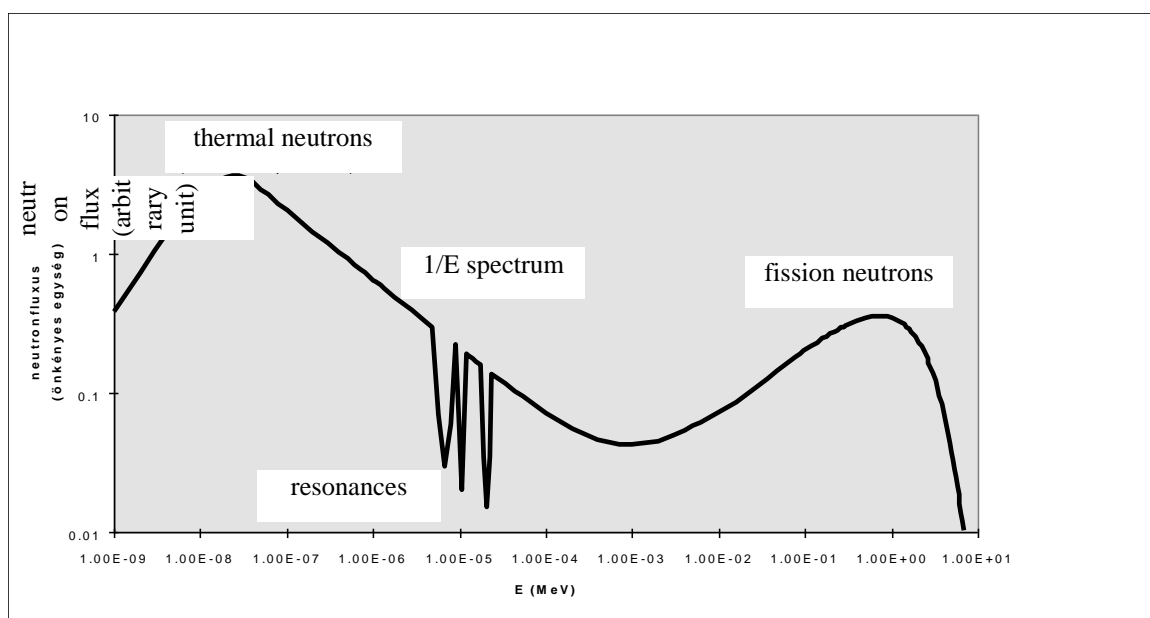
<sup>1</sup> It is usually called *neutron temperature*, which is slightly higher than the absolute temperature of the reactor.

$$\Phi_{th}(\underline{r}) = v \cdot n_{th}(\underline{r}), \quad (3a)$$

where  $n_{th}$  is the density of the thermal neutrons around point  $\underline{r}$ , and  $v$  is the average velocity of the thermal neutrons (see below).



**Figure 1.** The energy spectrum of the fission neutrons



**Figure 2.** The schematic energy spectrum of a thermal reactor

In thermal reactors, the spatial change of the neutron flux is significant. In case of a uniform bare (without reflector) slab reactor core, the spatial distribution of the thermal neutron flux, according to the reactor theory, is given by the following relationship [1]:

$$\Phi_{th}(x, y, z) = \Phi_{th}(0, 0, 0) \cdot \cos\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) \cos\left(\frac{\pi z}{c}\right), \quad (4)$$

where  $a = a' + 2\delta$ ,  $b = b' + 2\delta$ ,  $c = c' + 2\delta$ ,

$a'$ ,  $b'$ ,  $c'$  – actual physical dimensions of the core,

$\delta$  – extrapolation length.

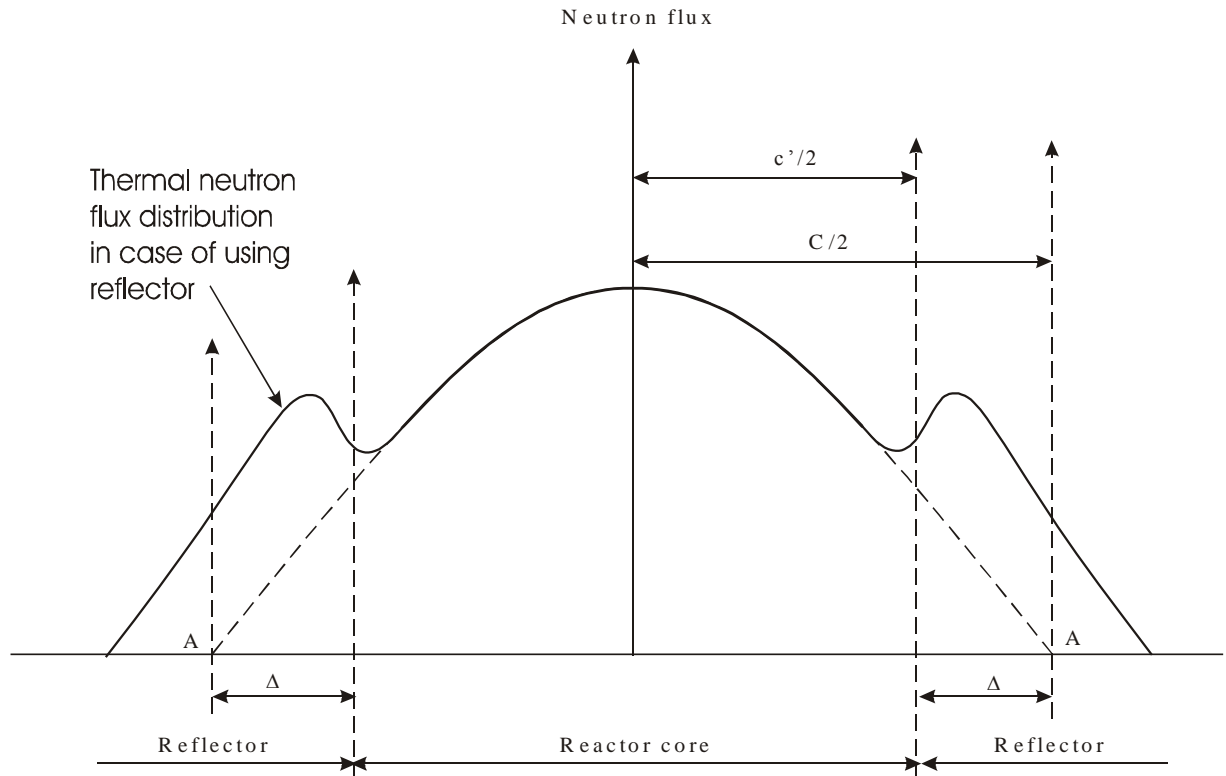
Accordingly, the distribution of neutron flux along axis  $z$  at a given place  $x_0, y_0$  can be written:

$$\Phi_{th}(x_0, y_0, z) = \Phi_0 \cdot \cos\left(\frac{\pi z}{c}\right) = \Phi_0 \varphi(z), \quad (5)$$

where

$$\Phi_0 = \Phi_{th}(0, 0, 0) \cdot \cos\left(\frac{\pi x_0}{a}\right) \cos\left(\frac{\pi y_0}{b}\right).$$

The non-uniform neutron flux distribution is unfavorable, among others as it results in a non-uniform burn-up of the fuel rods, the central rods burning up soon due to the high flux density while rods arranged in the edge of the core are hardly burned up. This problem may be eased by using a reflector surrounding the reactor by a material scattering a considerable part of neutrons leaking through the reactor boundary back into the core. As a result, the thermal neutron flux increases with explicit reflector peaks in the thermal neutron flux distribution on the edge of the reactor (Fig. 3.)



**Figure 3.** Thermal neutron flux distribution in a reflected reactor

Use of a reflector improves economy of the reactor construction, as less fuel is required to achieve the critical mass. The so-called reflector saving means the decrease of the linear dimensions of the critical reactor due to the reflector. Neglecting the extrapolation length it can be determined

at a good approximation by fitting a function (in the present case a cosine function) to the neutron flux distribution developing in the core of the reactor and extrapolating the curve to the reflector area. The extrapolated boundary of the bare critical reactor equivalent to the reflected reactor i.e. which would produce the same neutron flux distribution within the core as does the reflected reactor would be at the zero value of this curve (i.e. at zero flux (Fig.3.)).

Hence, reflector saving is understood as the difference between the linear dimensions of the equivalent bare critical reactor and the reflected critical reactor:

$$\Delta = \frac{C - c'}{2}, \quad (6)$$

where  $C$  – distance between the zero value points of the cosine curve fitted to the neutron flux distribution inside the core,

$c'$  – actual physical dimension of the core.

## 2.2. Determination of the thermal neutron flux by activation method

### 2.2.1. Principle of the method

Irradiating an activation detector in thermal neutron flux the reaction rate during activation is expressed by

$$R = \Phi_{th} \sigma_{act} N_T, \quad (7)$$

where  $\Phi_{th}$  – thermal neutron flux [ $m^{-2}s^{-1}$ ],

$\sigma_{act}$  – microscopic activation cross section [ $m^2$ ],

$N_T$  – number of target atoms in the sample.

The rate of change of radioactive atoms during irradiation is the difference between the rates of production and decay:

$$\frac{dN(t)}{dt} = \Phi_{th} \sigma_{act} N_T - \lambda N(t), \quad (8)$$

where  $\lambda$  – decay constant of the radioactive nuclei produced [ $s^{-1}$ ],

$t$  – time [s].

Be  $N(0) = 0$  at the start of irradiation. Then the solution of the differential equation (8) using (7):

$$N(t) = \frac{R}{\lambda} (1 - e^{-\lambda t}). \quad (9)$$

The activity of the detector at the end of irradiation enduring time  $T$ :

$$A(T) = \lambda N(T) = R (1 - e^{-\lambda T}). \quad (10)$$

The activity at time  $\tau$  after finishing the irradiation:

$$A(T, \tau) = R (1 - e^{-\lambda T}) e^{-\lambda \tau}. \quad (11)$$

These relationships will, however, be accurate only if each atom of the sample is irradiated by the same neutron flux. In case of a foil of non-negligible thickness, the average neutron flux will be lower inside it ( $\overline{\Phi_{th}}$ ) than on the surface ( $\Phi_{th}^0$ ), due to neutron absorption. Their ratio is the self-shielding factor:

$$G_{th} = \frac{\overline{\Phi}_{th}}{\Phi_{th,0}} \quad (12)$$

Finally, the time function of the detector activity is obtained by using the above formulae:

$$A(T, \tau) = \Phi_{th} G_{th} \sigma_{act} N_T (1 - e^{-\lambda T}) e^{-\lambda \tau} \quad (13)$$

### 2.2.2. Determination of the thermal neutron flux from the activity of the irradiated detector

In the thermal neutron energy range, the velocity distribution of neutrons follows the Maxwell-Boltzmann distribution according to which the most probable velocity of the neutrons is:

$$v_0 = \sqrt{\frac{2kT}{m}} \quad (14)$$

where  $k$  – Boltzmann constant,

$T$  – neutron temperature [K],

$m$  – mass of neutron [g].

At room temperature ( $20^\circ\text{C} = 293\text{ K}$ )  $v_0 = 2200\text{ m/s}$  which corresponds to an energy of  $0.025\text{ eV}$ . Again, according to the Maxwell-Boltzmann distribution, the average velocity of neutrons:

$$\overline{v} = \frac{2}{\sqrt{\pi}} v_0 \quad (15)$$

The reaction rate defined by (7) describes the activation process for irradiation in a monoenergetic neutron flux. As the thermal neutron velocities are different and the cross sections are dependent on the neutron velocity  $v$ , the activation process would be correctly described by:

$$R = \int_0^\infty \Phi(v) \sigma_{act}(v) N_T dv \quad (16)$$

Generally, this would be seriously complicate procedure and, therefore, the integral above is substituted by the product in (7) where the values  $\Phi_{th}$  and  $\sigma_{act}$  will be determined in concordance with each other in such a way that their product yields the actual reaction rate according to (16). For instance, it is customary to define neutron flux  $\Phi_{th,0} = n \cdot v_0$ , where  $v_0$  is the most probable velocity of neutrons according to (14),  $n$  is the total density of thermal neutrons. In this case, the reaction rate for a  $1/v$  detector ( $\sigma \sim 1/v$ ) and non-distorted Maxwell-Boltzmann spectrum will be obtained from:

$$R = n v_0 \sigma_{0,act} N_T \quad (17)$$

where  $\sigma_{0,act}$  – activation cross section belonging to the most probable velocity of Maxwell-Boltzmann distribution.

Defining the thermal neutron flux as  $\overline{\Phi}_{th} = n \cdot \overline{v}$  (where  $\overline{v}$  is the average velocity of thermal neutrons according to (15)), involves a cross section  $\sigma_{\overline{v},act}$  which, in case of a cross section ' $1/v$ ' and a neutron temperature  $T_n$  is given by:

$$\sigma_{\overline{v},act}(T_n) = \sigma_{0,act} \frac{\sqrt{\pi}}{2} \sqrt{\frac{T_0}{T_n}} \quad (18)$$

and  $T_0 = 293\text{ K}$ .

The number of target nuclei  $N_T$ :

$$N_T = \frac{m \cdot \alpha \cdot L}{A} \quad (19)$$

where  $m$  – mass of sample [g],  
 $\alpha$  – abundance of target isotope,  
 $A$  – mass number of target isotope [g],  
 $L$  –  $6.02 \cdot 10^{23}$ .

From the detector activity in (13), only the activity induced by thermal neutrons shall be taken into account.

Irradiating the detector at a place where also a significant epithermal flux is present and/or the epithermal activation cross section of the detector is significant compared to the thermal activation cross section, then a part of the activity obtained is due to the epithermal neutrons. Therefore a correction has to be applied to determine the contribution of the epithermal neutron flux to the activity produced. This is possible because the neutron absorption cross section of cadmium is very high at thermal energies (Fig. 4.), while negligible in the epithermal energy range. Hence, if the detector covered with cadmium of appropriate thickness, then thermal neutrons will be practically fully absorbed by the cadmium so that the activity of detector will be induced only by epithermal neutrons of energies above about 0.4 to 0.7 eV, depending on thickness of the cadmium cover. The activity induced by the thermal neutrons is simple to determine by irradiating and measuring the activity of two uniform detectors, one bare and the other covered with cadmium, in the same circumstances:

$$A_{th} = A_b - A_{Cd}, \quad (20)$$

where  $A_b$  – activity of the bare detector,  
 $A_{Cd}$  – activity of the cadmium-covered detector.

It is also customary to define the so-called cadmium ratio, the ratio of the activities of bare and cadmium-covered detectors:

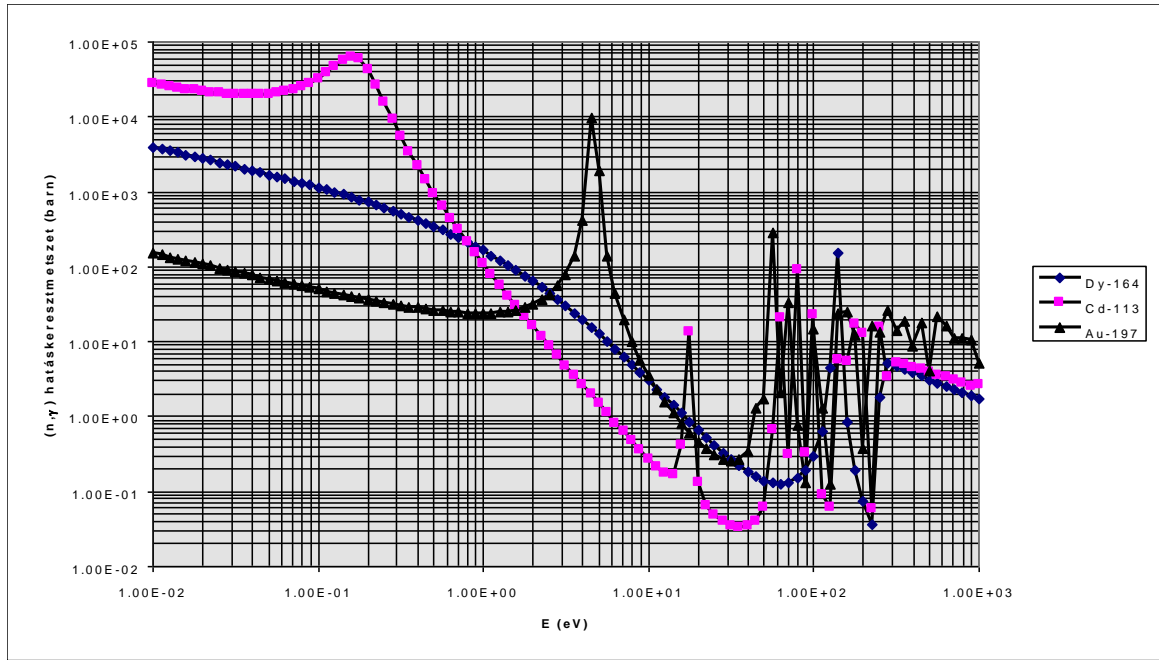
$$R_{Cd} = \frac{A_b}{A_{Cd}}. \quad (21)$$

On the basis of (20) and (21), the ratio of activity induced by thermal neutrons to the total activity of the detector:

$$\frac{A_{th}}{A_b} = 1 - \frac{1}{R_{Cd}}. \quad (22)$$

In case of detector materials with activation cross section following the  $1/v$  law through the whole neutron energy range of interest, the value of  $R_{Cd}$  is very high and thus, according to (22), the share of epithermal neutrons in the activity induced in the detector may be neglected.





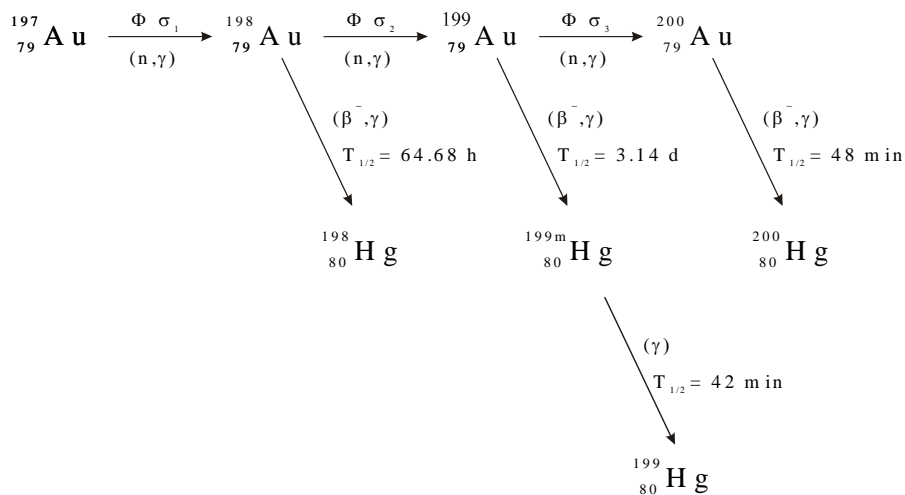
**Figure 4.** Total microscopic cross section of Dy, Au and Cd vs. neutron energy

Using a bare detector and a cadmium covered detector of identical masses, the thermal neutron flux is calculated from relationships (13), (18), (19) and (20) as follows:

$$\Phi_{th} = \frac{1}{N_T \sigma_{v,act}(T_n) G_{th}} \frac{e^{\lambda \tau}}{1 - e^{-\lambda \tau}} [A_b(\tau) - A_{Cd}(\tau)] \quad (23)$$

### 2.2.3. Determination of the absolute value of thermal neutron flux

The following series of reactions will take place in gold irradiated by neutrons:

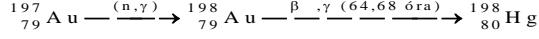


Of this multi-step reaction series, actually it is sufficient to take the first one into consideration because the activation of  ${}^{198}\text{Au}$  by neutron capture can be neglected because of the experimental conditions ( $\Phi_{th}$ ,  $\sigma_{act}$ , irradiation time etc., see later).

In the range of thermal neutron energies, the activation cross section of  ${}^{197}\text{Au}$  follows the law

$1/\nu$  but it has a number of resonance peaks in the resonance region (see Fig. 4.) stressing the role of cadmium correction according to (19).

The absolute value of the thermal neutron flux can be determined by gold activation. The absorption cross section of the  $^{197}\text{Au}$  isotope is also depicted in Fig. 4. The following reaction series takes place in gold irradiated by neutrons:



As depicted in Fig. 5.,  $\beta$  and  $\gamma$  particles are formed simultaneously in the 64.68-hour half-life decay of  $^{198}\text{Au}$ . (More precisely, 95.62% of the  $\beta$  decays is followed by the emission of a 411.8 keV  $\gamma$ -photon.) Thus, the absolute activity of gold can be determined with a coincidence counter.

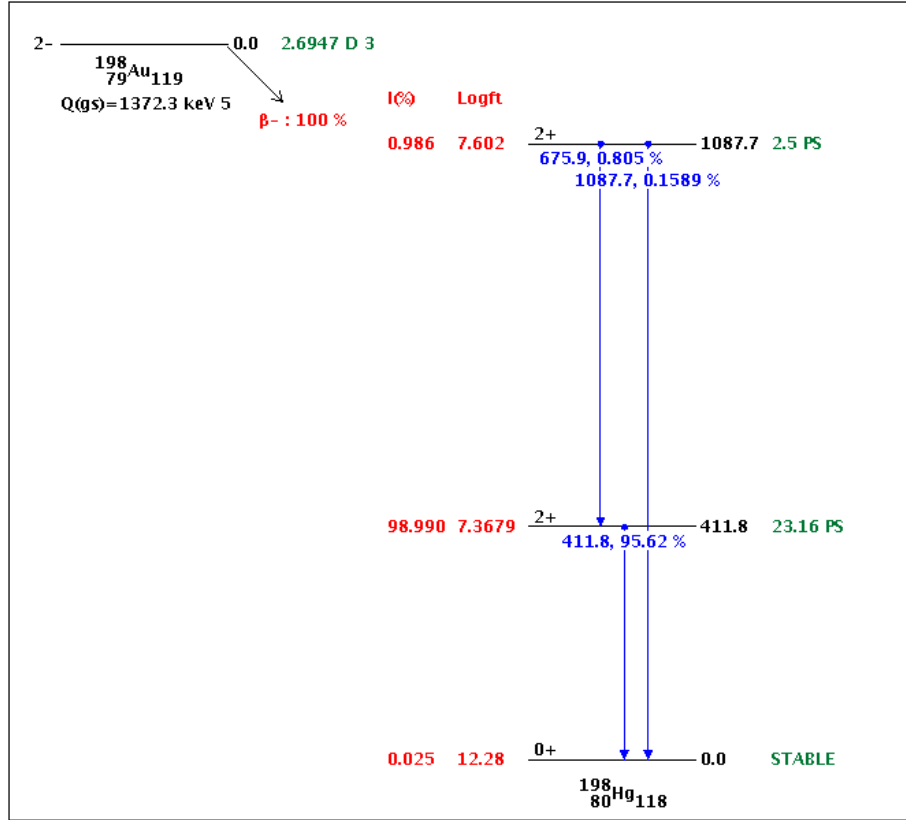


Figure 5. The decay scheme of  $^{198}\text{Au}$  [5]

In order to achieve this, an instrument is used that is equipped both with a scintillation beta detector and a scintillation gamma detector. The outgoing signals of the beta and the gamma channels are connected to a counter and to the two inputs of a coincidence circuit. Thus, the number of the beta particles and the gamma photons is recorded, as well as the number of the coincidence events occurring during the time of measurement. Let  $n_\beta$ ,  $n_\gamma$  and  $n_{co}$  denote the number of beta particles, gamma photons and coincidence events per second, respectively. Using these symbols, the following relations can be written:

$$n_\beta = \eta_\beta A ; \quad (24a)$$

$$n_\gamma = \eta_\gamma k_\gamma A ; \quad (24b)$$

$$n_{co} = \eta_\beta \eta_\gamma k_\gamma A \quad (24c)$$

where  $\eta_\beta$  and  $\eta_\gamma$  are the efficiency of the beta and the gamma detectors,  $k_\gamma$  is the frequency of the  $\gamma$ -line, and  $A$  is the number of decays per second in the foil, i.e. the activity of the sample. The following expression can be derived:

$$A \text{ (Bq)} = \frac{n_\beta \cdot n_\gamma}{n_{co}} \quad (25)$$

It can be seen that the activity of the sample can be obtained without knowing the efficiency values.

The measuring instrument is composed of a beta and a gamma scintillation detector, two single channel analyzers (differential discriminators), three counters and a coincidence circuit. The signals from the detector arrive in the single channel analyzers, with the aid of which the required range can be selected both from the beta and the gamma spectra. (This is important for reducing the background.) The coincidence circuit, also called the AND gate, has (at least) two inputs and one output. A pulse only appears on the output if signals arrive at the inputs simultaneously. Of course, the time difference of the temporal coincidence of the signals has to be specified: signals arriving at the coincidence counter with a time difference smaller than a given time interval  $\tau$  are detected as simultaneous;  $\tau$  is called the *resolving time* of the instrument. Two of the three counters are connected directly to the output of the single channel analyzer and count  $n_\gamma$  and  $n_\beta$ , while the third one counts  $n_{co}$ . The counters are synchronized so that they can be turned on simultaneously and count for the same amount of time.

During the measurement, the fact that the beta detector is to some extent also sensitive to gamma radiation has to be taken into consideration. This is the so-called *gamma sensitivity* of the beta detector, which can be determined by placing a beta absorber (a plastic foil) on the side of the sample that faces the beta detector. The counting rate  $n_{\beta\gamma}$  in the  $\beta$  channel obtained in this way is the gamma sensitivity of the beta detector.

Another correction has to be applied because of random coincidences. The number  $n_{rd}$  of the *random coincidences* per unit time is:

$$n_{rd} = 2\tau n_\beta n_\gamma \quad (26)$$

where  $\tau$  is the unlock time of the coincidence instrument, while  $n_\beta$  and  $n_\gamma$  are the counting rates in the beta and the gamma channels. Accordingly, the activity of the foil is:

$$A = \frac{(n_\beta - n_{\beta\gamma})(n_\gamma - h_\gamma)}{n_{ko} - n_{rd} - n_{\beta,\gamma,co}} = \frac{n'_\beta n'_\gamma}{n'_{co}} \quad (27)$$

where  $h_\gamma$  is the counting rate of the gamma background (cps),  $n_{\beta\gamma}$  is the counting rate measured in the beta branch when applying the beta absorber (this includes the value of the beta background  $h_\beta$  as well),  $n_{\beta,\gamma,co}$  is the coincidence counting rate when applying the absorber, while the primed quantities are the corrected beta, gamma and coincidence counting rates, respectively.

In the case of the usually applied 'AND' type coincidence circuits, resolving time  $\tau$  is the sum of the duration of the signals on the input of the coincidence circuit. Since the signals, which are 0.5  $\mu$ s long standard logic pulses (TTL), arrive at the inputs from the single channel analyzers, resolving time can be considered 0.5  $\mu$ s. Based on expressions (24c) and (25), it can be easily seen that if the condition

$$A = \frac{1}{2\tau}$$

were met, the number of random coincidences would be equal to that of true coincidences. In our case, we would need a 0.51 MBq activity source for this to be true. Since the activity of the source

we use is lower than that by several orders of magnitude,  $n_{\text{nd}}$  can in most practical cases be neglected.

In the range of thermal neutron energies, the activation cross section of  $^{197}\text{Au}$  follows the law  $1/v$ , but it has a number of resonance peaks in the resonance region (see Fig. 6.). The highest resonance occurs at 4.47 eV, where  $\sigma_c = 9890$  barn. Therefore, cadmium correction according to (20) plays an important role here. Thus, the activity of an irradiated foil has to be determined both with and without a Cd cover.

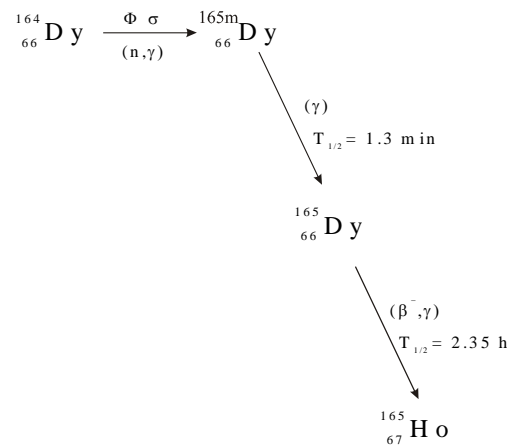
Table 1. summarizes the measurements to be performed in order to determine the absolute value of the thermal neutron flux.

Table 1. The measurements to be performed in order to determine the absolute value of the thermal neutron flux using the coincidence method

	without $\beta$ -shield			with $\beta$ -shield		
	$\beta$	$\gamma$	coincidence	$\beta$	$\gamma$	coincidence
bare foil	$n_\beta$	$n_\gamma$	$n_{co}$	$n_{\beta\gamma}$		$n_{\beta\gamma,co}$
Cd-claded foil	$n_\beta$	$n_\gamma$	$n_{co}$	$n_{\beta\gamma}$		$n_{\beta\gamma,co}$
no foil		$h_\gamma$				

#### 2.2.4. Determination of the relative distribution of thermal neutron flux

The following reaction is applied to determine the relative spatial distribution of the thermal neutron flux:



The activation cross section of dysprosium is very large and approximately  $1/v$ -dependent throughout the whole neutron energy range. Accordingly, the contribution of epithermal neutrons to its activity is negligible. Hence, the activity of such detectors is proportional to the thermal neutron flux and, therefore, the counts during the measurement can be directly used to calculate the relative thermal neutron flux distribution. The simplest way to provide identical experimental conditions (identical volume of samples, identical irradiation time etc.) is to use a wire sample to be activated and measuring the activity of wire sections. However, the activity of the sample decreases continuously in the course of measurement, which has to be compensated during the measurement or correction has to be applied for decay in evaluating the results. In this exercise, we prefer compensation.

The activity of the detector at the start of the measurement can be written on the basis of

formula (13):

$$A(z, \tau) = \Phi_{th}(z) \cdot G_{th} \sigma_{act} N_T (1 - e^{-\lambda T}) e^{-\lambda \tau} \quad (28)$$

Provided the activity of the detector is not reasonably large compared with the dead time of the counting apparatus, the measured intensity  $I(\tau)$  will be proportional to the activity of the sample:

$$I(\tau) = \eta A(\tau) \quad (29)$$

where  $\eta$  – efficiency of counting apparatus.

The measurement delivers the total number of counts during a finite measuring period  $t_m$ :

$$B = \int_{\tau}^{\tau + t_m} I(t) dt \quad (30)$$

or

$$B = \int_0^{t_m} \eta A(\tau) e^{-\lambda t} dt \quad (31)$$

Hence:

$$B = \frac{\eta A(\tau)(1 - e^{-\lambda t_m})}{\lambda} \quad (32)$$

The activity of the sample:

$$A(\tau) = \frac{B \lambda}{\eta (1 - e^{-\lambda t_m})} \quad (33)$$

If the measuring period is short ( $\lambda t_m \approx 0$ ), then  $B \approx \eta A(\tau) t_m$ , and  $A(\tau) \approx B / (\eta t_m)$ . The number of counts during the measurement can be determined using formulas (28) and (33):

$$B(z, \tau) = \Phi_{th}(z) \left\{ \frac{G_{th} \sigma_{act} N_T (1 - e^{-\lambda T})}{\lambda} \right\} \cdot (1 - e^{-\lambda t_m}) e^{-\lambda \tau} \quad (34)$$

The subproduct in figure brackets is the same for each point of the wire. In order to obtain values that are proportional to the thermal neutron flux from the number of counts, the values of the function

$$F(t_m, t) = (1 - e^{-\lambda t_m}) \cdot e^{-\lambda t} \quad (35)$$

have to be determined for every single data point. For this purpose,  $t_m$  is fixed at an appropriate value at the beginning of the measurement, and the time of the end of each measurement is recorded when scanning the parts of the wire. Thus, knowing the half-life of  $^{165}\text{Dy}$  (based on [6] it is  $2.334 \pm 0.001$  hours), the correction according to (35) can be calculated.

### 3. OBJECTIVE OF THE MEASUREMENT

In this exercise, the relative distribution of the thermal neutron flux will be determined along a vertical axis of the core, using the method described in chapter 2.2.4. Then, the vertical reflector savings are determined. A wire of Dy-Al alloy containing 10% of Dy is used for the measurement of the neutron flux distribution. The absolute value of thermal neutron flux is determined by gold activation detector.

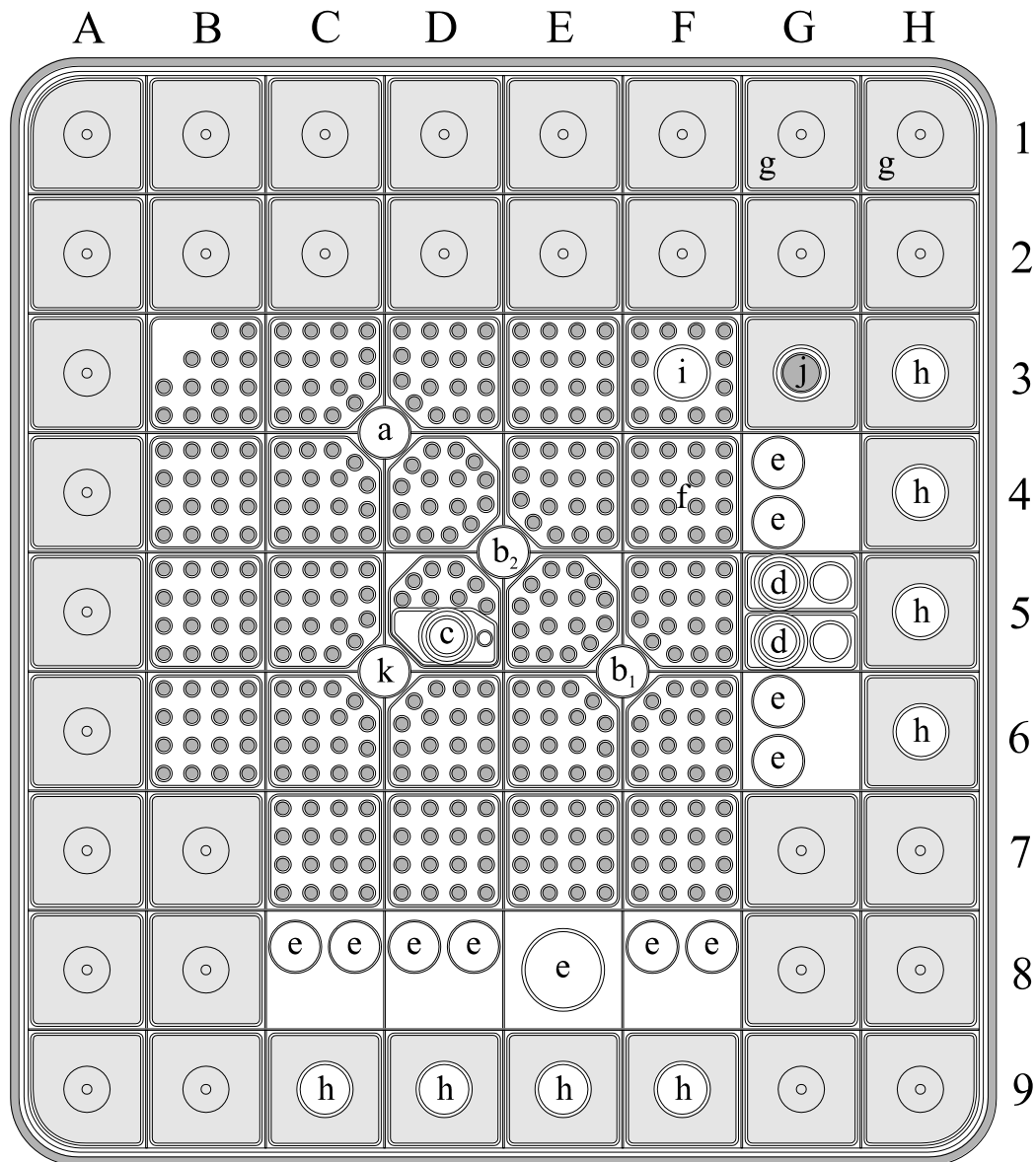
#### 4. EQUIPMENT AND MATERIALS NEEDED

- Reactor + pneumatic rabbit system for sample delivery
- Plexiglass rod to hold the wire
- Coincidence instrument
- Wire activity measuring apparatus (controlled by a PC)
- Wire of Dy-Al alloy
- Foil of Dy-Al alloy
- Gold foils
- Al and Cd capsules for covering the foils
- Radiation protection equipment (rubber gloves, pincers etc.)

##### 4.1. Description of the apparatus to measure the wire activity

A wire of Dy-Al alloy is used to determine the spatial distribution of the thermal neutron flux. The activity as a function of position along the Dy-Al wire is determined with a so-called wire activity measuring apparatus. Since the wire is positioned along the axial coordinate  $z$ , instead of writing  $B(\underline{r}, t)$  in expression (34), we now write  $B(z, t)$ . The wire activity measuring apparatus consists of the following parts:

- tooth-racked mounting rail in which the wire is fastened
- lead tower through which the wire holding rail can be pushed
- scintillation beta measuring head fitted into the lead tower, and a copper collimator below it so that it ‘sees’ only a few mm part of the wire
- stepper motor which makes forwarding the rail below the detector possible
- stepper motor control unit connected to a computer via a USB port
- digital spectrum analyser (Canberra DSA 1000) for processing the signals of the beta measuring head, also connected to the computer
- computer with data acquisition software (GenieMot)



**Figure 6.** The cross section of the reactor core

a – automatic control rod;  $b_1$ ,  $b_2$  – safety rods; c – fast pneumatic rabbit system; d – thermal rabbit system; e – vertical irradiating channels in water; f – fuel assemblies; g – graphite reflector elements; h – vertical irradiating channels in graphite; i – vertical irradiating channel in fuel assembly; j – neutron source; k – manual control rod.

## 5. PROCEDURE

### 5.1. Measurement of the relative distribution of the thermal neutron flux

In preparing the measurement, the wire is placed in a groove of the plexiglass rod. The moderation properties of plexiglass are similar to those of water, and thus the disturbance of the flux distribution of the active zone will be negligible.

The plexiglass rod is then placed by the operator to core position E6 (neighbouring D5) of the non-operating reactor, and the irradiation is performed. Simultaneously, the Dy foil is also activated in the rabbit system in core position D5. Since the wire used is made of Dy-Al alloy, the

samples have to rest for at least 20 minutes after irradiation to permit the interfering activity of Al to decay. (The half-life of the  $^{28}\text{Al}$  is 2.24 minutes.) Then the wire is removed from the plexiglass rod and placed into the wire activity measuring apparatus. In the measurement,  $\beta^-$  particles from the decay of  $^{165}\text{Dy}$  are detected.

First the Dy foils irradiated with and without a cadmium cover are measured with the aid of the beta measuring head. By comparing the number of counts, it can be verified that the assumption that only a negligibly small part of the activity originates from the reactions induced by the epithermal neutrons is true. Thus, the activity distribution of the wire irradiated without the cover will indeed yield the distribution of the thermal flux.

Subsequently, the holding rail has to be pushed into the starting position and the measurement points and measurement time have to be set on the computer. The spectrum range in which the numbers of counts are summarized has to be set as well. At the end of the measurement an ASCII file is obtained, the rows of which contain the position, the number of counts acquired during the time of measurement in the pre-selected spectrum range, and the time of completion of the measurement in the given position.

## 5.2. Measurement to determine the absolute value of thermal neutron flux

In the irradiation channel in core position D5 two gold foils of identical size and mass – one bare while the other in cadmium cover – are irradiated. The foils in a polyethylene sample holder are forwarded to the operating reactor by means of the pneumatic rabbit system joining the irradiation channel. Record the time of the end of irradiation and measure time  $t$  elapsed until the measurement of the activity of the individual foils. Take the activity of the sample corresponding to the  $A_{\text{bare}}$  and  $A_{\text{Cd}}$  values in (20) with the coincidence counter.

## 6. EVALUATION OF MEASUREMENT DATA

### 6.1. Evaluation of the relative distribution of the thermal neutron flux

According to expression (35), the decay time correction has to be performed on the numbers of counts measured in the individual points. The position dependence of the corrected data points obtained in this way is proportional to that of the thermal neutron flux distribution (cf. (34)). Plot the data points, then determine the axial reflector saving using formula (6) by fitting a cosine function to the central section of the measured curve according to formula (5).<sup>2</sup>

#### 6.1.1. Error analysis of the relative flux measurement

It is known from probability theory that the distribution of the number of counts follows the Poisson distribution. Consequently, the variance of the individual numbers of counts is equivalent to their expected value. The latter is approximated by the numbers of counts, so the following expression is usually true:

$$D^2(B_i) = M(B_i) \approx B_i \quad (36)$$

This approximative formula is valid with the same condition as the measurement method itself: dead time and the effects of the background can both be neglected. The simplest way of correcting dead time is providing the time of measurement  $t_m$  as live time. However, it is important to make sure that dead time does not exceed a few percent even in the case of the highest numbers of counts.

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<sup>2</sup> Usually a suitable curve fitting software is needed to perform this task.



## 6.2. Error analysis of the absolute value of the thermal neutron flux

Error analysis of the absolute value of the thermal neutron flux is considerably more difficult than that of the relative distribution. First of all, two types of errors need to be differentiated between: the *statistical error* and the *systematic error*.<sup>3</sup>

### 6.2.1. Estimation of statistical error

On the assumption that the distribution of the number of counts follows the Poisson distribution, the variance of the activities calculated based on expression (24) can be written using the well-known formula for the propagation for uncertainty (e.g. [6]), after neglecting  $n_{rnd}$ :

$$D^2(A) = (n_\beta + n_{\beta\gamma}) \left( \frac{n'_\gamma}{n'_{ko}} \right)^2 + (n_\gamma + h_\gamma) \left( \frac{n'_\beta}{n'_{ko}} \right)^2 + (n_{ko} + n_{\beta\gamma,ko}) \left( \frac{n'_\beta n'_\gamma}{n'_{ko}{}^2} \right)^2 \quad (37)$$

From this relation, relative standard deviation can be written in the following form:

$$r(A) = \frac{D(A)}{A} = \sqrt{\frac{(n_\beta + n_{\beta\gamma})}{n'_{\beta}{}^2} + \frac{(n_\gamma + h_\gamma)}{n'_{\gamma}{}^2} + \frac{(n_{ko} + n_{\beta\gamma,ko})}{n'_{ko}{}^2}} = \sqrt{r^2(n'_\beta) + r^2(n'_\gamma) + r^2(n'_{ko})} \quad (38)$$

It can thus be seen that the relative standard deviations of the individual numbers of counts are added up. Since of the three measured values  $n'_{ko}$  will expectedly be an order of magnitude smaller than the other two, it will affect the statistical error of the activity the most. Therefore, the time of measurement has to be chosen so that  $n'_{ko}$  will also be sufficiently high.

With the exception of  $N_d$  the quantities in formula (23) are calculated quantities, the uncertainties of which are systematic errors (see below). The accuracy with which  $N_d$  is known is practically equivalent to that of the mass  $M$  of the foils (cf. formula (19)).<sup>4</sup> Thus, the relative standard deviation of the thermal neutron flux is:

$$\frac{D(\Phi_{th})}{\Phi_{th}} \approx \sqrt{\frac{D^2(M)}{M^2} + \frac{D^2(A_{bare}) + D^2(A_{Cd})}{(A_{bare} - A_{Cd})^2}} \quad (39)$$

### 6.2.2. Systematic error

By performing the measurement carefully, the statistical error expressed in relation (30) can be reduced to 1÷2%. It is more difficult to reduce systematic errors, which can be as high as 10%. The most important errors are the following:

- Self-shielding factor  $G$  is hard to calculate. This is on the one hand due to the fact that formula (12) is just approximative (especially in the case of the cadmium-covered foil), on the other hand cross section  $\sigma_t$  in the formula is difficult to calculate.
- A similarly great number of assumptions are needed for the calculation of the product  $\bar{v} \cdot \bar{\sigma}_d$  in formula (17). A few examples to illustrate the problems which may arise: the neutron spectrum

<sup>3</sup> In mathematical statistics these terms are known as *standard deviation* and *bias*. The terms above are colloquial and not accepted in mathematics.

<sup>4</sup> The masses of the bare and the cadmium-covered foil may be different. In this case the formula for calculating the error is much more complicated. For simplicity, we assume that their masses are equivalent.

to be used in formula (16) is only known from calculations, the cross section of the ratio is only known with a finite accuracy, etc.

Decay constant  $\lambda$  in formula (13) is known with a finite accuracy only. However, the effect of this is only in the order of magnitude of one thousandth (e.g. the half life of  $^{198}\text{Au}$  based on [6] is  $2.6948 \pm 0.0012$  days). Further inaccuracies stem from the fact that the activation factor containing  $T$  'assumes' that at the beginning of activation the sample gets in the irradiation position, and back, instantaneously. This can be corrected if instead of equation (8), a differential equation considering the real time dependence of the neutron flux during the movement of the sample is solved. However, if the time of activation lasts 10 s or longer, this correction is below 1%.

In the end, it can be seen that because of the numerous systematic errors we cannot actually speak about measuring the neutron flux. What we can achieve is the approximative determination of the neutron flux, the value of which is strongly dependent on the assumptions made during the evaluation.

## 7. KNOWLEDGE CHECK

1. How do we define the scalar neutron flux? What is its dimension?
2. Characterize the spatial distribution of thermal neutron flux in a bare and a reflected reactor!
3. What is the principle of determination of the thermal neutron flux by activation method?
4. Why do we use cadmium cover for one of the gold samples and how do we define the cadmium ratio?
5. Why is it not necessary to use cadmium cover for the dysprosium (Dy-Al alloy) wire?
6. How is the absolute value of the thermal neutron flux determined?
7. What statistical and systematic errors occur when determining the value of the thermal neutron flux?

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