Discrete Mathematics

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Chapter 1

Introduction

Propositional logic

Propositional logic (and mathematics, in general) studies propositions: **declarative sentences** (a sentence that declares a fact) that is either true or false, but not both.

Example 1.1. Propositions:

1. Toronto is the capital of Canada (false but a declarative sentence nonetheless)
2. 1+1=2
3. 2+2=3
4. 3 is a prime number

The following are not propositions:

Not declarative:

1. What time is it?
2. Read this carefully.

Neither true or false:

1. $x + 1 - 2$
2. $x + y - z$

We use letters to denote propositions: **p, q, r,** .... Now propositions (called compound propositions) are constructed by combining one or more propositions using logical operators.

Negation (NOT)

If p is a proposition, its negation is denoted by $\neg p$.

"It's not the case that p". "The negation of p".

Example 1.2. p: "My PC runs Linux"

$\neg p$: "It's not the case that my PC runs Linux" $\Rightarrow$ "My PC doesn't run Linux"

Example 1.3. p: $1 + 1 = 2$ $\neg$ p: $1 + 1 \neq 2$

$\Rightarrow$ $\neg p$ is true iff (if and only if) p is false.

Conjunction (AND)

Let p, q be two propositions $\Rightarrow p \wedge q$ "p and q".

$p \wedge q$ is true iff p and q are true.

Remark. Sometimes the word "but" is used instead of "and". For example: 2 is even but 3 is odd.

Disjunction (OR)

Let p, q be two propositions $\Rightarrow p \vee q$ "p or q".

$p \vee q$ is true iff p is true, q is true or both are.

This correspondos to the **inclusive or** in English.

Remark. The **exclusive or**, it is not possible to have both propositions. For example: soup or salad comes as an entrée, it most certainly means that the customer cannot have both soup or salad.

Conditional statement / Implication

Let p, q be two propositions $\Rightarrow p \rightarrow q$ "if p, then q".

Because of its essential role in mathematical reasoning, a variety of terminology is used to express $p \rightarrow q$:

- if p and then q
- if p, q $\rightarrow$ p implies q
- q if p $\rightarrow$ p only if q
- q when p
- if p, q

$p \rightarrow q$ is false when p (the hypothesis / antecedent) is true and q (the consequence / conclusion) is false; otherwise, it is true.

Useful way to understand its truth value: A pledge many politicians make when running for office: "If I'm elected, I will lower taxes". It is only when the politician is elected that (..) not lower taxes that it can be said he has broken his pledge.

Remark. Note that this definition is more general tthan the meaning attached to such statements in English: there needs to be no relationship between p and q: "If the Moon is made of cheese, then 2+3=4".

Biconditional statement / Bi-implication

$q \leftrightarrow q$. It's equivalent to p