LEE BOUNDS WITH MULTILAYERED SAMPLE SELECTION

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Abstract. This paper investigates the causal effect of job training on wage rates in the presence of firm heterogeneity. When training affects worker sorting to firms, sample selection is no longer binary but is "multilayered". This paper extends the canonical Heckman (1979) sample selection model - which assumes selection is binary - to a setting where it is multilayered, and shows that in this setting Lee bounds set identifies a total effect that combines a weighted-average of the causal effect of job training on wage rates across firms with a weighted-average of the contrast in wages between different firms for a fixed level of training. Thus, Lee bounds set identifies a policy-relevant estimand only when firms pay homogeneous wages and/or when job training does not affect worker sorting across firms. We derive sharp closed-form bounds for the causal effect of job training on wage rates at each firm which leverage information on firm-specific wages. We illustrate our partial identification approach with an empirical application to the Job Corps Study. Results show that while conventional Lee bounds are strictly positive, our within-firm bounds include 0 showing that canonical Lee bounds may be capturing a pure sorting effect of job training.

Keywords: job training, sample selection, unordered treatments

JEL subject classification: C12, C14, C21 and C26.

Date: This present version of the paper is September 6, 2024. We thank seminar audiences at Carnegie Mellon University, National University of Singapore, University of Toronto, as well as the Rochester Labor Economics conference. We thank Stephen Claassen for providing outstanding research assistance.

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1. Introduction

Governments allocate substantial funds to job training programs that are designed to improve worker skills. The United States (U.S.) federal government spends roughly 19 billion U.S. dollars annually on employment and training programs.[1] Federal agencies administer roughly 40 employment and training programs to assist job seekers in gaining employment. Against this backdrop, there is an ongoing debate about the appropriate level of spending on such programs. Advocates argue that they help close the "skills gap" and address worker shortages while critics argue that they are ineffective and socially wasteful.

At the center of the debate is the longstanding question of whether job training has a causal effect on the labor market outcomes of participants. This question has garnered significant interest from both academics and policymakers. Knowing the answer to this question is essential for determining whether to continue spending on training programs. Substantial progress on answering this question has been made as a result of randomized evaluations of job training programs. These evaluations have been the subject of several comprehensive meta analyses (see Heckman et al. 1999 and Card et al. 2010, 2018).

To date, most evaluations of training programs have focused on total earnings as the primary outcome of analysis. While this focus is surely important for answering some questions, for others it is important to narrow the focus. Earnings naturally reflect both labor supply decisions (employment and hours margin) and wage rates. To better understand whether job training raises worker skills and welfare, standard economic models show that it is important to focus on the latter.[23] However, identifying the causal effect of job training on wage rates is empirically challenging due to the

[1]See the Council of Economic Advisers 2019 report: "Government Employment and Training Programs: Asessing the Evidence on their Performance" (The Council of Economic Advisers 2019).

[2]Labor supply decisions could also be impacted by job training via an increase in human capital. In particular, workers with higher skills are more likely to be offered – and accept – better paying jobs.

[3]Hendren and Sprung-Keyser (2020) measure the willingness to pay for job training using the treatment effect on total earnings. This assumes that all increases in earnings stem from returns to human capital (higher wage rate), not from higher levels of labor supply.

well known sample selection problem (Heckman 1979). This arises since a researcher only observes wages of the employed and the likelihood of employment can itself be impacted by job training.

In a seminal contribution, Lee (2009) showed that one can partially identify the causal effect of job training on wages for the always-employed under the Imbens and Angrist (1994) ("IA") monotonicity assumption. His key insight was to reduce the partial identification problem in this framework to the one considered by Horowitz and Manski (1995) – the problem of finding sharp bounds for the mean of an unobserved potential outcome that is a component of an observed mixing distribution with set identified mixing probabilities. IA's monotonicity condition delivers point identification of both the mixing weight (i.e., the share of always-employed) and the mean of the "untrained" potential outcome for the always-employed. Therefore, to obtain sharp bounds on the causal effect of job training on wages, one only needs to find the sharp bounds on the mean of the "trained" potential outcome for the always-employed. Lee's bounding approach has become influential in empirical research.[4]

In developing his approach, Lee focused on the potential for job training to affect labor supply along the extensive margin (work vs no work). While it is important to know whether training raises employment, a key question that is of interest to policymakers is whether job training improves labor market outcomes by raising job quality. In this case, training may raise earnings by matching workers to "better jobs". For example, this is a key feature of President Biden's workforce training initiative – the "American Rescue Plan's Good Jobs Challenge" – which prioritizes job quality and is designed to ensure individuals can access good jobs.

Despite the emphasis on job quality by policymakers, the academic literature on job training programs has mostly ignored firms. In the standard competitive model considered by Lee and most of the training literature, the firm an individual works for does not matter for wages. Yet, there is growing empirical evidence that demonstrates

[4]In a survey of the literature which we detail later in this section, we counted 56 papers published in 'top 5' general interest economic journals – the American Economic Review, Econometrica, the Journal of Political Economy, the Quarterly Journal of Economics and the Review of Economic Studies – that cited Lee (2009) with 42 of them having empirically implemented Lee bounds to address sample selection.

the importance of firms for wage determination.[5] This research highlights the importance of having a "good job" which can be interpreted as working at a "good firm" that offers a higher wage for all its employees. Thus, an open question is whether job training raises earnings by moving participants into higher-paying firms.

The presence of firm heterogeneity and the potential for worker sorting raises several new questions of interest. First, what estimand do "Lee bounds" partially identify when there is firm heterogeneity in wages? This question cannot be addressed with the canonical Heckman (1979) sample selection model since this assumes that sample selection is binary, i.e., job training can increase employment but has no effect on sorting to firms. The first contribution of this paper is to extend the standard sample selection model to a setting where sample selection is multilayered, and show that the conventional Lee bounds set identifies (for the population of always-employed) a total effect that combines a weighted-average of the causal effect of job training on wage rates across firms (we label this the "within-firm effect") with a weighted-average of the contrast in wages between different firms for a fixed level of training (we label this the "sorting effect").

Second, is it possible to separate the within-firm wage effect of job training from the sorting effect in the presence of heterogeneous firms? There are several reasons why one would want to separately identify these effects. First, some features of job training programs affect sorting (job search assistance) whereas others affect skill acquisition (classroom and vocational training). Thus, the decomposition could potentially highlight which investments – job search assistance or classroom training – are effective for raising wages and thus improve targeting. Second, the within-firm wage effect is arguably better able to shed light on the causal effect of *employer-sponsored* job training. Third, for a welfare analysis of job training programs, it is important to focus on the direct wage effects of job training since labor supply effects have second-order effects on utility (via the envelope theorem) (see Hendren

[5]See, for example, Abowd et al. (1999), Card et al. (2013), Song et al. (2019), Bonhomme et al. (2019) and Bonhomme et al. (2023).

and Sprung-Keyser 2020).[6] Thus, the within-firm wage effect is potentially the more welfare-relevant causal effect of interest.

The second contribution of this paper is to derive sharp bounds on the within-firm wage effect. Our bounding approach proceeds in two steps. In the first step, we derive sharp, closed-form bounds on the *response type* probabilities.[7] In deriving these bounds, we exploit a unique feature of our setting which is that (unlike in the traditional instrumental variables framework), the exclusion restriction does not hold since job training can have a direct causal effect on the outcome (wages). We show that this feature implies that the distribution of response types does not depend on the outcome (wage) distribution and allows us to derive closed-form bounds on the distribution of response types.[8] The second step provides closed-form bounds on the treatment effects as a function of the sharp bounds on the response types derived in the first step. This step involves extending the Horowitz and Manski (1995) approach (which involves a single-equation mixture model with two components) to our setting which involves two mixture model equations with unknown weights that are interdependent across the equations. Importantly, we show that while this two-step approach provides an easy and tractable way to construct closed-form bounds, it does not entail any loss of information, and provides sharp bounds. Finally, we consider a set of restrictions on response types and show that they naturally lead to tighter bounds on the treatment effects of interest.

We next consider an empirical application using the randomized evaluation of Job Corps following Lee (2009). We classify firms into observable firm types taking advantage of the fact that in the publicly available survey data, there are direct measures of firm amenities, such as the availability of health insurance, paid vacation and retirement or pension benefits. We demonstrate in these data that, on average, firms that offer these amenities pay higher wages than firms that do not. We also show that the wage distribution for firms that offer amenities stochastically dominates the

[6]This logic requires that the government is increasing spending on job training by a sufficiently small amount.

[7]The *response type* represents the pair of firms that an individual would choose to work at if she were externally assigned to the control group or the treatment group, respectively.

[8]While we derive closed-form bounds on the distribution of response types, we show that one can obtain them equivalently using a linear programming approach.

wage distribution for firms that do not, in both the treatment and control groups. We then go on to show that being randomly assigned to Job Corps leads individuals to work at firms with better job amenities compared to the control group. This combined evidence suggests that sample selection is multilayered and motivates our implementation of sharp bounds to these data. We replicate the findings from Lee (2009). Our estimates reveal that while the conventional Lee bounds are strictly positive ($[0.042, 0.043]$), our multilayered bounds for the within-firm wage effect (which hold the sorting effect constant) include 0. This suggests that Lee bounds may be capturing a pure sorting effect of job training rather than a direct human capital effect.

Our partial identification approach can be applied to any setting where there is multilayered sample selection. In Appendix D, we discuss a literature review we conducted based on all papers published in 'top 5' general interest economic journals from 2008 to 2023 that cited Lee (2009). The purpose of our review was to classify the nature of the sample selection in these papers; specifically, whether it was binary or multilayered. In total, 42 papers empirically implemented Lee bounds and 7 of them featured multilayered selection. To apply Lee bounds in these settings, researchers collapsed the sample selection problem to a single dimension. As we show in this paper, simplifying sample selection in this manner does not leverage all features of the data and may affect the interpretation of the causal estimand of interest.

Our paper builds on and contributes to the following literatures. First, there is a large literature on active labor market programs which is reviewed in Heckman et al. (1999) and Card et al. (2010, 2018). Our contribution to this literature is to examine whether and to what extent worker sorting to firms affects the wage impacts of job training. To the best of our knowledge, there are only a few empirical analyses that have examined the impact of training on worker sorting to firms. Andersson et al. (2022) find suggestive evidence of a positive impact of training on firm characteristics, as well as effects on industry of employment. Another related study is Katz et al. (2022), who evaluate sectoral-based training programs. Examining evidence from randomized evaluations of programs that combine upfront screening, occupational and soft skills training, wraparound services, and target low-wage workers, Katz et al.

(2022) find substantial and persistent earnings gains after training. Regarding mechanisms, the paper interprets the earnings gain as driven in part by the sorting of workers to higher-paying industries and occupations. However, it does not provide a framework to isolate the sorting effect as a causal mechanism. Lastly, Schochet et al. (2008) evaluate the impact of Job Corps on the sorting of workers to jobs with different amenities, such as availability of health insurance and retirement or pension benefits and report positive impacts. However, this paper does not disentangle the effects on these characteristics for those who would be employed in any case from a selection effect coming from the impact of the treatment on employment.

Second, our paper relates to the literature which documents firm heterogeneity in wages cited above. Firms have been shown to matter for wage inequality (Abowd et al. 1999), the cyclicality of wages and early career progression (Card et al. 2013), earnings losses of displaced workers (Lachowska et al. 2020; Schmieder et al. 2023), and gender (Card et al. 2016) and racial wage gaps (Gerard et al. 2021). Our contribution to this literature is to examine the role of firms for understanding the wage effect of job training.

Third, our paper relates to econometric approaches that address the sample selection problem. The Heckman (1979) sample selection model has been extended in various dimensions. First, a series of papers, including Gallant and Nychka (1987), Newey et al. (1990), and Ahn and Powell (1993), propose estimation and inference methods that relax the normality assumption imposed by Heckman (1979); see Li and Racine (2007, Chapter 10) for a review of such extensions. Second, Lee (2009) extends Heckman (1979) by relaxing the instrumental variable exclusion restriction and derives bounds on the parameters of interest. Honoré and Hu (2020) study a semi-parametric version of Lee's model. Additionally, Semenova (2020) and Olma (2021) propose various approaches for inference on Lee's bounds conditional on (potentially continuous) covariates. To our knowledge, this paper represents the first attempt to extend the seminal Heckman (1979) sample selection model to multilayered settings. While the focus of our paper is primarily on firms, which we consider as the main layer of interest, our analysis can be extended in various directions. For instance, one could consider occupation as a layer and examine the returns to occupation while controlling for sorting, similar to the approach taken by Gottschalk et al. (2014).

Finally, one can view the firm as a "mediator" in the context of the literature on mediation analysis (see, for example, Robins and Greenland (1992) and Pearl (2001)). Traditionally, most of this literature abstracts from sample selection where the outcome is not observed at some values of the mediator. A recent exception is Zuo et al. (2022), who consider the identification of direct and indirect effects within a mediation analysis framework, when both the outcome and mediator are missing. Their analysis focuses on point-identification under various assumptions including the abstract and non-falsifiable assumption of completeness;[9] some of those assumptions do not apply to the setting considered here. For instance, in our setting, whether the wage is observed can depend on wage rate offered by firms (the mediator), and this situation is ruled out by their assumptions. Our paper complements Zuo et al. (2022) by providing partial identification of direct and indirect effect without imposing a completeness assumption. Our maintained assumptions are transparent and directly apply to the primitives of our model. Our approach accommodates an endogenous mediator and allows the outcome to be non-randomly missing, even conditional on covariates.

The rest of the paper is organized as follows. Section 2 considers the multilayered sample selection problem both in a parametric model along the lines of Heckman (1979) and a more general treatment effects framework. It defines the key causal estimands of interest. Section 3 considers the causal interpretation of Lee bounds in the presence of multilayered sample selection and presents the general decomposition. Section 4 derives the sharp bounds of a large class of parameters of interest in the multilayered sample selection model. Section 5 considers simulations of the model assuming there are two types of firms. Section 6 is the empirical application that implements the sharp bounds for Job Corps. Section 7 concludes. Appendix B presents all proofs for the paper.

2. Analytical Framework

2.1. **Multilayered Sample Selection: A parametric model.** Since Heckman's seminal work in 1979, the sample selection model has been conceptualized as follows:

[9]For an in-depth review of completeness, see D'Haultfoeuille (2011), and also Canay et al. (2013).

$$
\begin{aligned}
Y &= \begin{cases} \alpha Z + \beta X' + U, & \text{if } D = 1, \\ \text{unobserved}, & \text{if } D = 0, \end{cases} && (2.1) \\
D &= 1\{\eta Z + \theta X' + V > 0\}, && (2.2)
\end{aligned}
$$

where D captures the binary sample selection model (i.e., D is equal to 1 if employed and 0 if not) and Y represents the outcome which is observed only when D is equal to 1 (i.e., Y is the observed wage when employed). The latent variables in the model are denoted as (U, V), X is a vector of observed exogenous covariates, and $Z \in \{0, 1\}$ is a binary variable that respects the conditional independence assumption: $(U, V) \perp Z|X$ (i.e., job training, captured by the binary variable Z, is randomly assigned). In the Heckman sample selection model, identification of the parameters in the outcome equation requires at least one variable that is independent of the latent variables but is excluded from the outcome equation. In model (2.1, 2.2) this is equivalent to assuming α to be equal to 0, implying that Z is excluded from the outcome equation. In this context, Z becomes a valid instrumental variable to consistently estimate β, satisfying both the independence and exclusion restrictions.

However, as highlighted by Lee (2009) and recognized more generally, in certain cases, the exclusion restriction may be violated implying that $\alpha \neq 0$. In this case, α is potentially a parameter of primary interest. For instance, in the job training example, participating in training could boost an individual's human capital, directly affecting their wage rate. Consequently, α can be interpreted as the causal effect of job training on the wage rate which is the key parameter studied by Lee (2009). The primary methodological contribution of Lee is to provide a method that allows researchers to partially identify the causal effect of job training on the wage rate (i.e., α) in the presence of sample selection (i.e., when training can affect labor supply via η).

A key assumption in the parametric model above is the sample selection problem is binary: individuals are either employed or unemployed. If job training affects not only whether an individual works but also which firm they work at, the sample selection problem becomes multilayered. We now generalize the seminal Heckman sample selection model (2.1, 2.2) to allow for a richer model of labor supply where individuals choose layers, i.e. firms. We refer to this extended model as the "*parametric multilayered selection model*":

$$
\begin{aligned}
Y &= \begin{cases} \alpha_K Z + \beta_K X' + U_K, & \text{if } D = K, \\ \quad\vdots & \quad\vdots \\ \alpha_1 Z + \beta_1 X' + U_1, & \text{if } D = 1, \\ \text{unobserved}, & \text{if } D = 0, \end{cases} && (2.3) \\
D &= \arg \max_{d \in \{0,1,\ldots,K\}} \{\eta_d Z + \theta_d X' + V_d\} && (2.4)
\end{aligned}
$$

where $\eta_0 Z + \theta_0 X' + V_0 = 0$. In this model, each layer (D) represents a distinct firm, with corresponding parameters α_d, β_d, and latent variable U_d. Expected utility for a given firm d is given by $\eta_d Z + \theta_d X' + V_d$. The utility of the outside option (i.e., unemployment) is $\eta_0 Z + \theta_0 X' + V_0 = 0$. The worker selects the firm with the highest expected utility.

In the parametric multilayered sample selection model, α_d is the causal effect of job training on the wage rate within firm d. We refer to this causal effect as the "within-firm effect" for layer d. The vector $(\eta_1, \ldots, \eta_K)$ encompasses parameters reflecting the causal effect of job training on different firm labor supplies, which determines worker sorting across firms. In the next section, we demonstrate that Lee bounds do not separately identify the within-firm effects $(\alpha_1, \ldots, \alpha_K)$ from the sorting effects summarized in $(\eta_1, \ldots, \eta_K)$. As discussed in the introduction, this has potential policy implications, since recovering these diverse causal channels allows policymakers to efficiently allocate resources across different types of training programs.

2.2. **Multilayered Sample Selection: Generalized version using the potential outcome model.** Let $(\Omega, \mathcal{F}, P)$ be a probability space, where we interpret Ω as the population of interest, and $\omega \in \Omega$ as a generic individual in the population. Let $Y_{z,d}(\omega)$ be the potential outcome (i.e., potential wage) if agent ω is externally assigned to the treatment group $z \in \{0, 1\}$ (i.e., job training) and to a specific layer $d \in \{0, \ldots, K\}$, where $d = 0$ denotes the layer for which the outcome is not observed (i.e., $Y_{z,0}(\omega)$ is not observed).[10] D_z denotes the potential layer the individual selects if externally assigned to the treatment group $z \in \{0,1\}$. Denote the realized outcome by $Y \in \mathcal{Y} \subseteq \mathbb{R}$ and the realized layer by $D \in \{0,1,...,K\}$. Let Z be the assigned treatment group, and let X be a vector of covariates. We assume that (D,Z,X) is observed for every individual, but the realized outcome Y is only observed if $D \neq 0$ (i.e., if the individual is employed). This implies the following model:[11]

$$\begin{aligned} Y &= \sum_{d=1}^{K} \left[Y_{1,d}Z + Y_{0,d}(1-Z)\right] 1\{D=d\}, && (2.5)\\ D &= D_1 Z + D_0(1-Z), && (2.6) \end{aligned}$$

along with the following conditional independence assumption.

Assumption 1 (Conditional Random Assignment)**.** *Individuals are randomly assigned to a treatment group.* $\{(Y_{z,d}, D_z) : d \in \{0,1,...,K\}, z \in \{0,1\}\} \perp Z|X$.

The outcome equation (2.5) collapses to Lee (2009)'s outcome model when there is no heterogeneity across the layered potential outcomes, i.e. $Y_{z,d} = Y_z$ for $d \in \{1,...,K\}$. In this case, we have:

$$\begin{aligned} Y = \sum_{d=1}^{K} \left[Y_{1,d}Z + Y_{0,d}(1-Z)\right] 1\{D=d\} &= \left[Y_1 Z + Y_0(1-Z)\right] \sum_{d=1}^{K} 1\{D=d\} \\ &= \left[Y_1 Z + Y_0(1-Z)\right] 1\{D \neq 0\}. \end{aligned}$$

What causal interpretation should be given to Lee bounds in the presence of multilayer sample selection, where $Y_{z,d} \neq Y_{z,d'}$, for $d,d' \in \{1,...,K\}$? Depending on the researcher's interest, various causal estimands of interests could be defined. Before defining our parameters of interest, we show that there is a link between the causal effects in our multilayered framework and the ones typically considered in the mediation analysis literature. We then use this link to characterize our key estimands below.

[10] In our empirical application, we will assume that the layer corresponds to a firm's type, where the type is constructed based on a firm's observable characteristics.

[11] Strictly speaking, equation (2.5) implies that the outcome $Y_{z,d}(\omega) = 0$ when $D = 0$, but it should really be interpreted as $Y_{z,d}(\omega)$ is unobserved.

2.3. **Direct and indirect effects in presence of sample selection.** In our model, particularly in equation (2.5), a notable connection exists to the literature on mediation analysis, as discussed by Pearl (2001) and others. The graphical representation of the outcome equation in our model takes the form:[12]

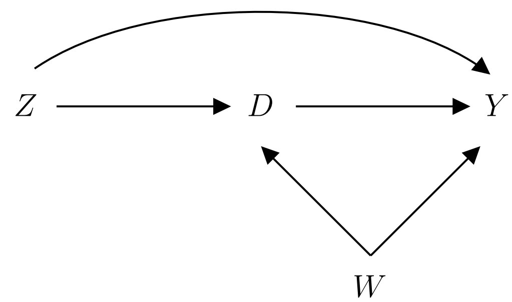


FIGURE 1. DAG of causal relationships between variables in our model.

In the context of mediation analysis, where Z represents the randomized treatment, D is conceptualized as the "mediator", and Y denotes the outcome, our model allows the treatment (job training) to influence the outcome through two channels: a direct channel and an indirect channel that traverses through the mediator. W is a vector of latent unobserved variables often called confounding variables which affect simultaneously D and Y making D an endogenous variable. In the parametric model, we have $W \equiv (U_1, ..., U_K, V_1, ..., V_K)$. In our framework, the mediator corresponds to the firm where the individual would be employed if they were externally assigned to job training.

In the mediation analysis literature, two categories of causal estimands have garnered attention: "direct effects" and "indirect effect". Focusing on the former, two types of "direct effects" have been conceptualized. First, the *control direct effect* (CDE) is defined as:

$$\text{CDE}(d) \equiv \mathbb{E}[Y_{1,d} - Y_{0,d}]. \tag{2.7}$$

This captures the causal effect of job training on earnings within a specific firm d when the firm is held fixed. It is equivalent to the within-firm wage effect for layer d. In Lee's (2009) terminology, the CDE corresponds to the causal impact of job

[12]For the sake of clarity, this graph simplifies the discussion by omitting sample selection.

training on the wage rate illustrated by the curved arrow in Figure 1. This parameter is the primary focus of Lee (2009).

The CDE can vary significantly across firms (d), reflecting the potential heterogeneous impact of job training on earnings for different firms. The CDEs are useful when the policymaker is interested primarily in the impact of job training on wages at a specific firm. More generally, policymakers may also be interested in understanding the overall impact of training on wages at firms that workers naturally choose when trained. The second type of direct effect – the "natural direct effect" (NDE) – is introduced to capture this notion:

$$\text{NDE} \equiv \mathbb{E}[Y_{1,D_1} - Y_{0,D_1}] = \sum_{d=0}^{K} \mathbb{E}[Y_{1,d} - Y_{0,d}|D_1 = d] \times \mathbb{P}[D_1 = d] \tag{2.8}$$

where $Y_{z,D_{z'}} \equiv \sum_{d=0}^{K} Y_{z,d} 1\{D_{z'} = d\}$ for $z, z' \in \{0,1\}$, and $d \in \{0, ..., K\}$. The expression $Y_{1,D_1}(\omega) - Y_{0,D_1}(\omega)$ represents the causal impact of job training on earnings for the specific firm that worker ω would have selected if she had been externally assigned to receive job training. The NDE is essentially the average of these individual effects.

Turning to indirect effects, the "natural indirect effect" (NIE) is defined as:

$$\begin{aligned} \text{NIE} &\equiv \mathbb{E}[Y_{0,D_1} - Y_{0,D_0}] \\ &= \sum_{d=0,d'=0:d\neq d'}^{K,K} \mathbb{E}[Y_{0,d} - Y_{0,d'}|D_0 = d', D_1 = d] \times \mathbb{P}[D_0 = d', D_1 = d]. \end{aligned} \tag{2.9}$$

The term $Y_{0,d} - Y_{0,d'}$ represents the wage contrast between firms d and d' in the absence of job training. However, rather than specifying the pair of firms (d, d'), we can examine this wage difference at the "natural representative" firms D_1 and D_0, resulting in $Y_{0,D_1} - Y_{0,D_0}$. The indirect effect aims to capture the causal impact of job training on the outcome purely due to the shift in firms; it can be seen as the influence of job training transitioning through a change in the firm. As highlighted by Pearl (2009), the empirical relevance of the indirect effect estimand is controversial and questionable. Implementing an intervention that would suppress the direct effect of Z on Y while allowing the indirect channel through D is not realistic. Nevertheless, it remains a key parameter in the mediation analysis literature.

In the context of sample selection, the outcome is observable only when $D \neq 0$. We can categorize the population into four major groups: $\Omega = \{\omega : D_0(\omega) = 0, D_1(\omega) = 0\} \cup \{\omega : D_0(\omega) > 0, D_1(\omega) = 0\} \cup \{\omega : D_0(\omega) = 0, D_1(\omega) > 0\} \cup \{\omega : D_0(\omega) > 0, D_1(\omega) > 0\}$. For the first group, we never observe outcomes, regardless of training status. For the second group, only outcomes under job training are never observed, whereas for the third group, only outcomes when not assigned to job training are never observed. In these three groups, if we are unwilling to assume that the outcome is missing at random (or selection on observable only) or impose parametric assumptions, the observed data cannot provide information on the causal effect for individuals belonging to those groups. We refrain from imposing such stringent restrictions and focus solely on the causal effects for the final group, the subpopulation $\{\omega : D_0(\omega) > 0, D_1(\omega) > 0\}$.

It is useful to further partition our population of interest $\{\omega : D_0(\omega) > 0, D_1(\omega) > 0\}$ into finer groups, which we label *response types*, i.e. $\{\omega : D_0(\omega) > 0, D_1(\omega) > 0\} = \cup_{\{d,d' \in \{1,...,K\}\}} \{\omega \in \Omega : D_1(\omega) = d, D_0(\omega) = d'\}$.[13] Response types are defined by the pair of firms that individual ω would choose to work for if externally assigned to the control group or the treatment group. Formally, the response type is defined as the random variable $T = (D_0, D_1)$ and $\mathcal{T}$ represents its support.

We now introduce two pivotal parameters, the Local Controlled Direct Effect (LCDE) and the Local Controlled Indirect Effect (LCIE):

$$\text{LCDE}(d|t) = \mathbb{E}[Y_{1,d} - Y_{0,d}|T = t], d \in \{1, ..., K\}, \text{ and } t \in \mathcal{T} \tag{2.10}$$

and

$$\text{LCIE}(z, d, d'|t) = \mathbb{E}[Y_{z,d} - Y_{z,d'}|T = t], d \in \{1, ..., K\}, \text{ and } t \in \mathcal{T} \tag{2.11}$$

The individual CDEs may vary across individuals, i.e. $Y_{0,d}(\omega) - Y_{0,d'}(\omega) \neq Y_{0,d}(\omega') - Y_{0,d'}(\omega')$ for $\omega \neq \omega'$. By considering the LCDE, we allow for heterogeneity of the CDE across response types. In certain instances, a specific LCDE would be more policy relevant than the CDE itself. Both the LCDE and CDE exhibit their own policy relevance, akin to the extensive debate in the instrumental variable (IV) literature

[13]See Heckman and Pinto (2018) for a more detailed discussion on the advantages of such a partition.

regarding the empirical relevance between the average treatment effect (ATE) versus Local ATE (LATE). This analogy extends to the LCIE.

We now show that the "sample selection" versions of the CDE, NDE, and NIE can be perceived as a weighted average of $\mathrm{LCDE}(d|t)$ or $\mathrm{LCIE}(z, d, d'|t)$. By "sample selection" version, we mean that the effects are defined to be conditional on being always employed, i.e. $\{\omega : D_0(\omega) > 0, D_1(\omega) > 0\}$.

$$\mathbb{E}[Y_{1,d} - Y_{0,d}|D_0 > 0, D_1 > 0] = \sum_{l=1,l'=1}^{K,K} \mathrm{LCDE}(d|l, l') \times \mathbb{P}[T = (l, l')|D_0 > 0, D_1 > 0],$$

$$\mathbb{E}[Y_{1,D_1} - Y_{0,D_1}|D_0 > 0, D_1 > 0] = \sum_{d=1,d'=1}^{K,K} \mathrm{LCDE}(d|d', d) \times \mathbb{P}[T = (d', d)|D_0 > 0, D_1 > 0],$$

$$\mathbb{E}[Y_{0,D_1} - Y_{0,D_0}|D_0 > 0, D_1 > 0] = \sum_{d=1,d'=1:d\neq d'}^{K,K} \mathrm{LCIE}(0, d, d'|d', d) \times \mathbb{P}[T = (d', d)|D_0 > 0, D_1 > 0].$$

As illustrated, $\mathrm{LCDE}(d|t)$ or $\mathrm{LCIE}(z, d, d'|t)$ represent more primitive parameters compared to CDE, NDE, and NIE. This paper will focus particularly on identifying $\mathrm{LCDE}(d|t)$. It is worth noting that, in the absence of individual heterogeneity, whenever $Y_{z,d}(\omega) = Y_{z,d}(\omega')$ for $\omega \neq \omega'$, we have $\mathrm{LCDE}(d|t) = \mathrm{LCDE}(d) = \alpha_d$, as in the parametric version of the model. Moreover, since the outcome is never observed when $D = 0$, hereafter we use the following notation $Y_{z,D_{z'}} \equiv \sum_{d=1}^{K} Y_{z,d} 1\{D_{z'} = d\}$ for $z, z' \in \{0, 1\}$.

Remark 1. *In Lee (2009), training assignment Z is a randomly assigned treatment that fails to satisfy the exclusion restriction required to address sample selection using standard methods. In our more general setting, while Z remains a treatment of interest, it also plays the role of being an instrument for D. In some settings, the impact of D on the outcome may be of independent interest, and our results also apply to such settings. Moreover, since our results apply even when there is no sample selection (i.e. Y is always observed, $P(D = 0) = 0$), our approach generalizes the IV model to settings where the instrument does not satisfy the exclusion restriction.*

3. The Causal Interpretation of Lee's bounds in the presence of Multilayered Sample Selection

First, notice that the generalized multilayered selection model i.e., equations (2.5, 2.6) implies the following:

$$\begin{aligned} Y &= \left[Y_{1,D_1} Z + Y_{0,D_0}(1-Z)\right], && (3.1)\\ 1\{D>0\} &= 1\{D_1>0\}Z + 1\{D_0>0\}(1-Z). && (3.2) \end{aligned}$$

In addition, Lee imposes the following monotonicity assumption:

Assumption 2 (Conditional Lee's Monotonicity Assumption)**.** *We impose the following restriction:* $\mathbb{P}\left[1\{D_1>0\} \geq 1\{D_0>0\} | \, X\right] = 1$ *a.s.*

This assumption means that being assigned to the treatment group can never lower employment and this applies uniformly for all agents in the population. In our general framework with multilayered sample selection, this assumption requires that all agents are more likely to join an employment layer when assigned to treatment. Since Lee's monotonicity assumption is only required to hold conditional on X, it can be modified to allow the direction of monotonicity to vary across different values of X. Such a modification does not present a challenge for our identification analysis, which holds X fixed throughout, but inference methods need to be adapted to accommodate such an assumption, especially when X is continuous. For further details on such adaptations, see Słoczyński (2020) and Semenova (2020), which provides inference methods that are valid under such assumptions.

All the remaining analysis, results, and assumptions should be understood as implicitly conditioning on $X = x$ for some value x of the vector of observed covariates, X, which will generally be suppressed in the notation.

Lemma 1 (Lee Bounds)**.** *Under Assumptions 1 and 2 Lee bounds set identifies the following estimand* $\mathbb{E}[Y_{1,D_1} - Y_{0,D_0} | D_0 > 0, D_1 > 0]$*:*

$$\underline{\theta}^{\ell} \leq \mathbb{E}[Y_{1,D_1} - Y_{0,D_0} | D_0 > 0, D_1 > 0] \leq \overline{\theta}^{\ell} \quad (3.3)$$

where

(i) *For continuous outcome:*

$$\underline{\theta}^{\ell} \equiv \mathbb{E}[Y|D>0, Z=1, Y \le F^{-1}_{Y|D>0,Z=1}(p)] - \mathbb{E}[Y|D>0, Z=0], \tag{3.4}$$

$$\overline{\theta}^{\ell} \equiv \mathbb{E}[Y|D>0, Z=1, Y \ge F^{-1}_{Y|D>0,Z=1}(1-p)] - \mathbb{E}[Y|D>0, Z=0], \tag{3.5}$$

(ii) *For binary outcome:*

$$\underline{\theta}^{\ell} \equiv \max\left\{0, 1 - \frac{1}{p}P[Y=0|D>0, Z=1]\right\} - \mathbb{E}[Y|D>0, Z=0], \tag{3.6}$$

$$\overline{\theta}^{\ell} \equiv \min\left\{1, \frac{1}{p}\mathbb{P}[Y=1|D>0, Z=1]\right\} - \mathbb{E}[Y|D>0, Z=0], \tag{3.7}$$

with $F^{-1}_W(u) \equiv \inf\{w \in \mathbb{R} : \mathbb{P}(W \le w) \ge u\}$ *for* $u \in [0,1]$ *and* $p \equiv \frac{\mathbb{P}(D>0|Z=0)}{\mathbb{P}(D>0|Z=1)}$.

Lemma 1 shows that in the presence of heterogeneous firms, Lee's identification approach bounds the following estimand $\mathbb{E}[Y_{1,D_1} - Y_{0,D_0}|D_0 > 0, D_1 > 0]$. What is the causal interpretation of this estimand? The following lemma sheds light on this.

Lemma 2 (Decomposition). *Assuming the generalized multilayered sample selection model, we have the following decomposition:*

(i) *General decomposition:*

$$\begin{aligned}
&\mathbb{E}[Y_{1,D_1} - Y_{0,D_0}|D_0 > 0, D_1 > 0] \\
&\qquad = \underbrace{\sum_{d=1,d'=1}^{K,K} LCDE(d|d',d) \times \mathbb{P}[T=(d',d)|D_0>0, D_1>0]}_{\mathbb{E}[Y_{1,D_1} - Y_{0,D_1}|D_0>0, D_1>0]} \\
&\qquad\quad + \underbrace{\sum_{d=1,d'=1:d\neq d'}^{K,K} LCIE(0,d,d'|d',d) \times \mathbb{P}[T=(d',d)|D_0>0, D_1>0]}_{\mathbb{E}[Y_{0,D_1} - Y_{0,D_0}|D_0>0, D_1>0]}
\end{aligned} \tag{3.8}$$

(ii) *No mediation effect (No firm-specific wage rate, i.e.,* $Y_{z,d} = Y_{z\bullet}$*) or no sorting across firms, i.e.,* $\mathbb{P}[T=(d',d)|D_0>0, D_1>0] = 0$ *for* $d \neq d'$.

$$\begin{aligned}
&\mathbb{E}[Y_{1,D_1} - Y_{0,D_0}|D_0 > 0, D_1 > 0] \\
&\quad = \sum_{d=1,d'=1}^{K,K} \mathbb{E}[Y_{1\bullet} - Y_{0\bullet}|D_1 = d, D_0 = d']\mathbb{P}(D_1 = d, D_0 = d'|D_0 > 0, D_1 > 0) \\
&\quad = \mathbb{E}[Y_{1\bullet} - Y_{0\bullet}|D_0 > 0, D_1 > 0] \quad (3.9)
\end{aligned}$$

(iii) *No direct effect (i.e., $Y_{z,d} = Y_{\bullet d}$).*

$$\begin{aligned}
&\mathbb{E}[Y_{1,D_1} - Y_{0,D_0}|D_0 > 0, D_1 > 0] \\
&= \sum_{d=1,d'=1:d\neq d'}^{K,K} \mathbb{E}[Y_{\bullet d} - Y_{\bullet d'}|D_0 = d', D_1 = d] \times \mathbb{P}[D_0 = d', D_1 = d|D_0 > 0, D_1 > 0] \\
&= \mathbb{E}[Y_{\bullet D_1} - Y_{\bullet D_0}|D_0 > 0, D_1 > 0] \quad (3.10)
\end{aligned}$$

Lemma 2 (i) shows that in the presence of firm heterogeneity, Lee's partial identification approach establishes bounds for a total effect. This total effect combines the sample selection version of the NDE and the NIE (e.g., conditional on $D_0 > 0$ and $D_1 > 0$), with each possessing distinct interpretations. Importantly, as discussed above, the NDE and NIE do not hold the mediator (firm) D fixed. The NDE is an average of causal effects of job training at each firm weighted by fraction of workers choosing that firm under job training. The NIE is an average of causal effects of firm on wages (in the no job training counterfactual scenario) weighted by the share of response types choosing those firms. Thus, without additional assumptions, this approach does not allow one to separately identify the CDEs (the within-firm wage effects) from the labor supply effects or sorting effects that transit through D. Lemma 2 (ii) shows that when there are no mediation effects (or no heterogeneity in wages across firms), i.e., $Y_{z,d} = Y_{z\bullet}$ as assumed in Lee (2009) and illustrated in Figure 2, the NIE vanishes while the NDE reduces to the CDE which is the target parameter in Lee's framework.[14] Finally, Lemma 2 (iii) reveals that in the absence of a direct effect of job training on earnings (i.e., $Y_{z,d} = Y_{\bullet d}$) as depicted in Figure 3, Lee bounds

[14]Notice that Figures 2 and 3 are drawn for the subpopulation of always observed, i.e. $\{\omega : D_0(\omega) > 0, D_1(\omega) > 0\}$.

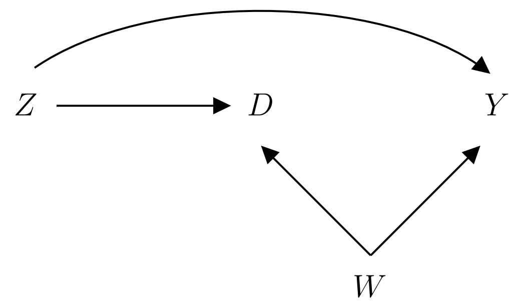


Figure 2. DAG when there are no mediation effects.

capture the effect of job training on earnings coming exclusively from the sorting of individuals into different firms.

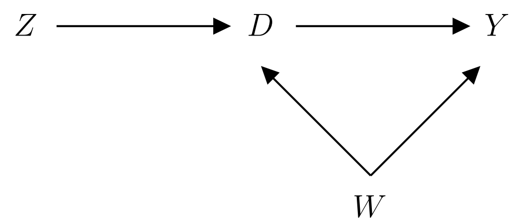


Figure 3. DAG when there is no direct effect.

This shows that in general, interpreting Lee bounds as being informative about the human capital effect of job training is problematic unless there is clear empirical evidence of the absence of mediation effects. Unfortunately, Lee's approach does not provide a means to assess this. Given these challenges, the next section introduces an alternative partial identification approach that is designed to overcome these limitations and aims to partially identify the true causal impact of job training on wage rates.

4. Sharp bounds in the multilayered sample selection model

In this section, we develop a partial identification strategy to recover the parameters $\mathrm{LCDE}(d|t)$ and $\mathrm{LCIE}(z, d, d'|t)$ which will allow us to isolate the within-firm effect of job training from the sorting effect. Under Assumption 1, the response type T is independent of Z. Assumption 2 restricts the response type support. For instance, under Assumption 2, $\mathbb{P}[T = (d, 0)] = 0$ for $d \in \{1, ..., K\}$. We denote by

$f_{Y_{z,d}|D,Z}(y|d',z')$ the conditional density of $Y_{z,d}$ given $\{D=d', Z=z'\}$ and assume that it is absolutely continuous respect to a dominating measure μ on $Y_{z,d}$. We note that $f_{Y_{z,d},D|Z}(y,d|z) \equiv f_{Y_{z,d}|D,Z}(y|d,z)\mathbb{P}(D=d|Z=z)$. For $d,d' \in \{1,...,K\}$ and $z \in \{0,1\}$, and any $y \in \mathcal{Y}$ we have the following:

$$f_{Y|D=d,Z=z}(y) = f_{Y_{z,d}|D_z}(y|d) = \sum_{d'=1}^{K} \frac{\mathbb{P}(D_z=d, D_{1-z}=d')}{\mathbb{P}(D=d|Z=z)} \times f_{Y_{z,d}|D_z,D_{1-z}}(y|d,d') \quad (4.1)$$

where the first equality holds under Assumption 1.

More precisely, under Assumption 1, the following system of equations characterizes the empirical content of the multilayered sample selection model:

$$f_{Y,D=d|Z=1}(y) = \sum_{d'=0}^{K} \mathbb{P}[T=(d',d)] \times f_{Y_{1,d}|T}(y|d',d) \quad (4.2)$$

$$f_{Y,D=d|Z=0}(y) = \sum_{d'=0}^{K} \mathbb{P}[T=(d,d')] \times f_{Y_{0,d}|T}(y|d,d') \quad (4.3)$$

and this holds for any $d,d' \in \{1,...,K\}$ and $y \in \mathcal{Y}$. The left-hand side of equations (4.2) and (4.3) are observed while the individual types, i.e. $\mathbb{P}[T=(d,d')]$ and the conditional potential outcome distributions, i.e. $f_{Y_{z,d}|T}(y|d',d)$ on the right-hand side of the equations are unknown. For a given d, the number of unknown quantities $2K+1+2(K+1)|\mathcal{Y}|$ is bigger than the number of equations $2|\mathcal{Y}|$. We therefore have an under-determined system of linear equations with unknown coefficients. As such, it is only possible to set identify these parameters. The identified set of unknown parameters could naturally shrink if the researcher is willing to impose additional assumptions such as Assumption 2. For instance, under Assumption 2, $\mathbb{P}[T=(d,0)]=0$ for $d \in \{1,...,K\}$ which implies that $\mathbb{P}[T=(d,0)]f_{Y_{z,d}|T}(y|d,0)=0$ for $d \in \{1,...,K\}$. Consequently, for a fixed d, this leads to a reduction of $|\mathcal{Y}|+1$ in the total number of unknown parameters while keeping fixed the same number of equations. As a result, the system of equations becomes more tightly constrained. When the support of Y, i.e. $\mathcal{Y}$, is finite, the system of equations (4.2)-(4.3) could be solved using a linear programming method, with the drawback that the linear programming approach does not provide intuition about the source of identification power.[15] More importantly, the linear programming approach can no longer be used when Y is continuous, as is the case in our empirical application. To address this issue, we develop a two-step identification approach. The first step provides sharp bounds on the response types. This step involves only the distribution on (D, Z) which has finite support in our framework and then can be solved using a linear programming approach since it does not involve Y which could have continuous support. The second step provides closed-form bounds on the treatment effects of interest as functions of the sharp bounds on the response types computed in the first step. We show that these two steps provide sharp bounds on our parameters of interest.

4.1. **Step 1: Sharp bounds on the response types.** In this step, we focus on the partial identification of the distribution of the response types. Integrating equations (4.2) and (4.3) over the whole support of $\mathcal{Y}$ we obtain the following system of equations:

$$\mathbb{P}(D = d|Z = 1) = \sum_{d'=0}^{K} \mathbb{P}[T = (d', d)] \tag{4.4}$$

$$\mathbb{P}(D = d|Z = 0) = \sum_{d'=0}^{K} \mathbb{P}[T = (d, d')] \tag{4.5}$$

In general, in the standard IV model, the distribution of response types depends on the full joint distribution of the observed data (Y, D, Z), not just on the distribution of (D, Z).[16] This complexity happens because in the IV framework, the exclusion restriction is imposed, i.e., $Y_{z,d} = Y_{\bullet d}$. Indeed, when this restriction is imposed, the response-type conditional density of $Y_{\bullet d}$ appears in both equations (4.2) and (4.3), and integrating each equation separately can lead to a loss of information on the response-type probabilities (leading to non-sharp bounds). In the absence of the

[15]If the researcher is interested in analyzing a discrete outcome and wishes to explore this avenue further, she could employ the inferential method developed by Fang et al. (2023).

[16]This has been pointed out by Huber et al. (2017) and is also implicit in the results of Kitagawa (2021). See Theorem 3 in Vayalinkal (2024) for a result characterizing the relationship between outcome distributions and the identified set of response-type probabilities.

exclusion restriction however, each response-type conditional density $f_{Y_{z,d}|T}$ in the system of equations (4.2) and (4.3) only appears in one equation and so the integration step can be performed without losing any information on response-type probabilities. Therefore, we show that in our model, sharp bounds on the response types are entirely characterized by the distribution of (D, Z) which justifies proceeding in two steps.

Lemma 3. *Consider the model (2.5, 2.6). Under Assumption 1, the sharp characterization of the response types is given by equations (4.4, 4.5).*

The researcher may also seek to apply additional restrictions on the distribution of response types, including but not limited to Assumption 2. For example, one could assume that there are more upward switchers than downward switchers or more stayers than downward switchers. We consider such restrictions as a possible auxillary assumption. We designate the set of linear constraints that can be applied to the response types as $\mathcal{R}_T$.

Assumption 3. *[Restriction on response types] Consider that the layers (i.e., firms) are ordered.*

(i) *[Strong Monotonicity]* $1\{D_1(\omega) = d\} \geq 1\{D_0(\omega) = d'\}$ *for* $d \geq d'$*, or equivalently* $\mathbb{P}[T = (d', d)] = 0$ *for* $d \geq d'$.
(ii) *[More upward switchers than downward switchers]*
$$\mathbb{P}[T = (d, d')] \geq \mathbb{P}[T = (d', d)] \text{ for } d \geq d'.$$
(iii) *[More stayers than downward switchers]*
$$\mathbb{P}[T = (d, d)] \geq \mathbb{P}[T = (d', d)] \text{ for } d \geq d'.$$

A noteworthy aspect of Assumptions 2 and 3 is that these restrictions can seamlessly integrate into equations (4.4)-(4.5) as supplementary linear constraints. Consequently, the process of recovering response types that conform to all these behavioral restrictions simplifies to a feasible linear programming problem. For instance, researchers may choose $\mathcal{R}_T = \{\text{Assumption 2}\}$, $\mathcal{R}_T = \{\text{Assumption 2, Assumption 3}(i)\}$, or $\mathcal{R}_T = \{\text{Assumption 2, Assumption 3}\}$.

As mentioned in Lemma 3, equations (4.4)-(4.5) sharply characterize the restrictions on the distribution of T imposed by the model (2.5, 2.6). Therefore, the identified set for response-type probabilities, under model (2.5, 2.6), Assumptions 1, and responses type restrictions $\mathcal{R}_T$, can be written as

$$\Theta_I(\mathcal{R}_T) \equiv \{\mathbb{P}[T = (d, d')] : d, d' \in \{0, ..., K\} \text{ such that equations } (4.4) - (4.5) \text{ and } \mathcal{R}_T \text{ hold}\}.$$

We have also the following result:

Lemma 4. *Consider the model (2.5, 2.6). Assumption 1 and $\mathcal{R}_T$ are jointly rejected by the data if and only if $\Theta_I(\mathcal{R}_T) = \emptyset$.*

Lemma 4 holds significant practical implications. It shows that assessing the validity of model assumptions does not depend on knowledge of the outcome distribution. This property greatly simplifies the implementation of a falsification test for our model. In other words, once we can find a distribution of type that aligns with the model assumptions and the observed data on (D, Z), it is always possible to find a corresponding distribution of potential outcomes $f_{Y_{z,d}|T}(y|d', d)$ that would rationalize the observed joint distribution of (Y, D, Z).

For simplicity, we introduce the shorthand notation, $p_{d,d'} \equiv \mathbb{P}[T = (d, d')]$, and $\gamma^z_{d,d'} \equiv \frac{p_{d,d'}}{\mathbb{P}(D=d|Z=z)}$. When $\Theta_I(\mathcal{R}_T) \neq \emptyset$, let $\underline{p}^r_{d,d'}$ denote the infimum over all probability values for $p_{d,d'}$ that belongs to $\Theta_I(\mathcal{R}_T)$. Subsequently, we can define: $\underline{\gamma}^{z,r}_{d,d'} = \frac{\underline{p}^r_{d,d'}}{\mathbb{P}(D=d|Z=z)}$. The "$r$" superscript is used to emphasize the focus on the probability of the response type under the restrictions $\mathcal{R}_T$.

For all the potential choices of $\mathcal{R}_T$ considered above, $\Theta_I(\mathcal{R}_T)$ is the set of non-negative solutions to a linear system. Therefore, $\underline{\gamma}^{z,r}_{d,d'}$ for $d, d' \in \{0, ..., K\}$ and $z \in \{0, 1\}$ can be obtained as the solution to a linear program. Since the linear system of interest here is generally small, it is also possible to obtain an analytic solution for $\underline{p}^r_{d,d'}$ (and therefore for $\underline{\gamma}^{z,r}_{d,d'}$) via Fourier-Motzkin elimination. The details for both the computational and analytic approaches are presented in Appendix A.

4.2. **Step 2: Sharp bounds on the treatment effects.** As evident from equations (4.2) and (4.3), the conditional observed distribution of earnings, $F_{Y|D,Z}(y|d, z)$, can be expressed as a finite mixture of the conditional potential outcome distributions given the response types, $F_{Y_{z,d}|T}(y|l, l')$. More, precisely we have:

$$f_{Y|D=d,Z=1}(y) = \sum_{d'=0}^{K} \gamma^1_{d',d} \times f_{Y_{1,d}|T}(y|d',d) \tag{4.6}$$

$$f_{Y|D=d,Z=0}(y) = \sum_{d'=0}^{K} \gamma^0_{d,d'} \times f_{Y_{0,d}|T}(y|d,d') \tag{4.7}$$

The unknowns in this mixture are the weights, $\gamma^z_{d,d'}$ for $z \in \{0,1\}$, and $d, d' \in \{0, ..., T\}$. In Lee (2009), Assumption 2 implies that the weights are point-identified, and establishing identification reduces to recovering the mean average of the mixture components. However, in our scenario, the mixture weights are not point identified under Assumption 2. Even when we broaden Assumption 2 with Assumption 3, point identification is still not achieved. This under-identification issue primarily arises due to the presence of numerous unobserved types stemming from the multiple layers, i.e., firms. Nonetheless, we can still derive informative bounds on these weights, as elaborated in the preceding subsection.

Horowitz and Manski (1995) proposed sharp bounds on the distribution of mixture components in a single-equation mixture model with two components, where the weights are unknown but researchers possess non-trivial bounds for these weights, and Cross and Manski (2002) extended these results to single-equation models with many components. The empirical content of our model, however, is characterized by a set of systems of mixture equations, one system for each $d \in \{1, \ldots, K\}$, each with up to $(K+1)^2$ components. Importantly, in our setting, the weights are unknown and shared across these systems, introducing a cross-equation dependence not present in Horowitz and Manski (1995) or Cross and Manski (2002). We derive the identified set for the weights and then extend the approaches of Horowitz and Manski (1995) and Cross and Manski (2002) to this more general case, deriving closed form bounds on our key parameters of interest.

Moreover, Lee demonstrated that for continuous outcomes, the bounds proposed by Horowitz and Manski (1995) can be equivalently expressed as a mean of a truncated distribution. We extend Lee's results by introducing a generalized truncated mean representation that applies regardless of the outcome's distribution, whether it be continuous, discrete, or mixed.

Hereafter, to simplify our notation and enhance readability, we introduce the following notation: For any d and z we have: $\underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\gamma; d, z) \equiv \mathbb{E}[F^{-1}_{Y|D=d,Z=z}(U)|U \leq \gamma]$, and $\overline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\gamma; d, z) \equiv \mathbb{E}[F^{-1}_{Y|D=d,Z=z}(U)|U \geq 1-\gamma]$. We define y_L as the lower bound of the support of Y and y_U as the upper bound.[17]

Before stating the main result, we note the following. When the outcome is continuously distributed $\underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\gamma; d, z)$ is exactly equal to the truncated mean used in Lee (2009) i.e.,

$$\mathbb{E}[F^{-1}_{Y|D=d,Z=z}(U)|U \leq \gamma] = \mathbb{E}[Y|D = d, Z = z, Y \leq F^{-1}_{Y|D=d,Z=z}(\gamma)].$$

When the outcome is binary we have

$$\mathbb{E}[F^{-1}_{Y|D=d,Z=z}(U)|U \leq \gamma] = \max\left\{0, 1 - \frac{1}{\gamma}P[Y = 0|D = d, Z = z]\right\}.$$

This novel formulation provides a general truncation formula that applies to any type of outcomes, continuous, discrete, or mixed.

Theorem 1. *Suppose that Assumptions 1, and restrictions $\mathcal{R}_T$ hold. Whenever $\Theta_I(\mathcal{R}_T) \neq \emptyset$, then the following bounds are pointwise sharp:*

(i) *Local Controlled Direct Effect (LCDE):*

$$\underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{1,r}_{d,d}; d, 1) - \overline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{0,r}_{d,d}; d, 0) \leq LCDE(d|d, d) \leq \overline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{1,r}_{d,d}; d, 1) - \underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{0,r}_{d,d}; d, 0),$$

$$\underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{1,r}_{d',d}; d, 1) - y_U \leq LCDE(d|d', d) \leq \overline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{1,r}_{d',d}; d, 1) - y_L, \text{ for } d \neq d',$$

$$y_L - \overline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{0,r}_{d,d'}; d, 0) \leq LCDE(d|d, d') \leq y_U - \underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{0,r}_{d,d'}; d, 0) \text{ for } d \neq d'.$$

(ii) *Local Controlled Indirect Effect (LCIE).*

$$\underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{1,r}_{l,d}; d, 1) - y_U \leq LCIE(1, d, d'|l, d) \leq \overline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{1,r}_{l,d}; d, 1) - y_L, \text{ for } d \neq d' \text{ and any } l,$$

$$\underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{0,r}_{d,l}; d, 0) - y_U \leq LCIE(0, d, d'|d, l) \leq \overline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}(\underline{\gamma}^{0,r}_{d,l}; d, 0) - y_L, \text{ for } d \neq d', \text{ and any } l.$$

[17]Note that these bounds need not be finite.

(iii) *Aggregate LCDE:*

$$\inf_{\substack{\{(p_{d,d}:d\in\{l,\ldots,l'\})\\ \in\Theta_I(\mathcal{R}_T)\}}} \sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d'=l}^{l'} p_{d',d'}} \left[\underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}\left(\frac{p_{d,d}}{\mathbb{P}(D=d|Z=1)};d,1\right) - \overline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}\left(\frac{p_{d,d}}{\mathbb{P}(D=d|Z=0)};d,0\right)\right]$$

$$\leq \sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d'=l}^{l'} p_{d',d'}} LCDE(d|d,d) \leq$$

$$\sup_{\substack{\{(p_{d,d}:d\in\{l,\ldots,l'\})\\ \in\Theta_I(\mathcal{R}_T)\}}} \sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d'=l}^{l'} p_{d',d'}} \left[\overline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}\left(\frac{p_{d,d}}{\mathbb{P}(D=d|Z=1)};d,1\right) - \underline{\mathbb{E}}_{F^{-1}_{Y|D,Z}}\left(\frac{p_{d,d}}{\mathbb{P}(D=d|Z=0)};d,0\right)\right].$$

The derivation of the bounds in Theorem 1 comes from extending Horowitz and Manski (1995) bounding approach summarized in Lemma 5 in Appendix A. However, demonstrating their sharpness presents a considerably more intricate challenge. This involves showing that solving equations (4.2) to (4.3) for all $y \in \mathcal{Y}$ and $d \in \{1, ..., K\}$ while imposing the restrictions defined in $\mathcal{R}_T$ consistently yields the same information as the closed-form bounds presented in Theorem 1. As explained above, the absence of the IV exclusion restrictions facilitates this result.

Theorem 1 (i) indicates that, without additional assumptions on the potential outcome distributions, the derived bounds can at best determine the direction (sign) of the within-firm effect at layer d solely for individuals remaining with firm d under any treatment assignment Z, i.e., $\mathbb{E}[Y_{1,d} - Y_{0,d}|T = (d, d)]$. This finding is somewhat intuitive given that these "stayers" are equivalent to the so-called "always-employed" in Lee's model where firm heterogeneity is not taken into account. The bounds for those who switch firms due to treatment ("switchers"), such as $\mathbb{E}[Y_{1,d}-Y_{0,d}|T = (d, d')]$ and $\mathbb{E}[Y_{1,d} - Y_{0,d}|T = (d', d)]$ for $d \neq d'$ always include 0. This is the case because the observed data (Y, D, Z) does not reveal any information on the following unobserved counterfactuals $\mathbb{E}[Y_{0,d}|T = (d', d)]$ and $\mathbb{E}[Y_{0,d}|T = (d, d')]$.

Similarly, Theorem 1 (ii) reveals that in the absence of extra restrictions on the potential outcome distributions, it is impossible to identify the sign of the LCIE$(z, d, d'|t) = \mathbb{E}[Y_{z,d} - Y_{z,d'}|T = t]$. This underscores the inherent challenges in identifying some specific treatment effects without imposing further assumptions on the outcome distributions.

Finally, Theorem 1 (iii) presents the closed-form bounds that correspond to the weighted-average of the LCDE$(d|d,d)$, $\sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d=l}^{l'} p_{d,d}}$LCDE$(d|d,d)$. These bounds are sharp and take into account the interdependence between eqs (4.6) and (4.7). A naive approach to deriving bounds would be to construct a weighted average of the bounds of the LCDE$(d|d,d)$ proposed in Theorem 1 (i). However, these bounds would not be sharp since they would not account for the interdependence across equations.

4.2.1. *Inference.* Inference on the causal parameters considered in Theorem 1 can often be performed using existing methods for inference on parameters bounded by truncated conditional expectations. All three sets of bounds in Theorem 1 are known functions of the (D,Z)-conditional expectations of Y truncated above or below at particular quantiles. Inference in such settings is complicated by the need to estimate two nuisance parameters: the conditional quantile functions themselves, and the truncation quantile level.

In parts (i) and (ii) of Theorem 1, the truncation quantile levels are of the form $\underline{\gamma}_{d,d'}^{z,r} = \frac{\underline{p}_{d,d'}^{r}}{\mathbb{P}(D=d|Z=z)}$ and these can generally be estimated at faster rate than the second step (truncated conditional expectation).[18] Moreover, each bound involves either only one truncated conditional expectation, or truncated conditional expectations that can be independently estimated. Therefore, inference on the parameters considered in Theorem 1(i)-(ii) can be performed by plugging in the estimators of Lee (2009), Semenova (2020), or Olma (2021) for the truncated conditional expectation(s), and adapting the inference approaches discussed there (subject to the regularity conditions outlined there). These approaches allow for conditioning on, and aggregating over, the covariates X, which can lead to much tighter bounds than the unconditional case, as noted by Lee (2009). The approach proposed in Lee (2009) applies when X is finitely-supported, whereas the approaches developed by Semenova (2020) and Olma (2021) allows X to be continuous.

[18]This can be shown when a closed form expression for $\underline{p}_{d,d'}^{r}$ has been derived, as we do in our examples below, but also holds true if $\underline{p}_{d,d'}^{r}$ is estimated via linear programming over the estimate of the set $\Theta_I(\mathcal{R}_T)$ obtained by plugging in sample analogue estimators for the propensity scores.

Inference on the "aggregate" parameters considered in part (iii) of Theorem 1 is more complicated since the truncation quantile level to be estimated depends on the solution to a higher dimensional optimization problem involving the outcome distributions. We defer the development of estimation and inference methods for this case to future work.

4.3. **2 Firm Types Case: A numerical illustration.** To provide intuition for our bounding approach, in this section we consider a scenario involving two types of firms. We assume that firms are identical within each type. Firms are categorized as either high type (H) or low type (L). In our empirical application, the instrument Z corresponds to assignment to job training, firms that offer health insurance are classified as type H, and firms that do not are classified as type L. Our objective is to investigate whether the instrument has a causal effect on wages within each firm type. Under Assumption 2, the support of possible response types is:

$$\mathcal{T} := \{(0,0),(0,L),(0,H),(L,L),(H,H),(L,H),(H,L)\}.$$

Thus, the always-employed (AE) encompasses four distinct response types:

$$\{D_0 > 0, D_1 > 0\} = \{(L,L),(H,H),(L,H),(H,L)\} \equiv AE.$$

Lee bounds applied to this setting correspond to:

$$\begin{aligned}
&\underline{\theta}^{\ell} \leq \frac{p_{L,L}+p_{H,L}}{\mathbb{P}(AE)}\mathbb{E}[Y_{1,L}-Y_{0,L}|T\in\{(L,L),(H,L)\}]\\
&+\frac{p_{H,H}+p_{L,H}}{\mathbb{P}(AE)}\mathbb{E}[Y_{1,H}-Y_{0,H}|T\in\{(H,H),(L,H)\}]+\frac{p_{L,H}}{\mathbb{P}(AE)}\mathbb{E}[Y_{0,H}-Y_{0,L}|T=(L,H)\}]+\\
&\qquad\qquad\frac{p_{H,L}}{\mathbb{P}(AE)}\mathbb{E}[Y_{0,L}-Y_{0,H}|T=(H,L)\}]\leq\overline{\theta}^{\ell}.
\end{aligned}$$

To establish bounds on our causal effects of interest, we first characterize identification of the response-type probabilities: $\{p_t : t \in \mathcal{T}\}$. Using information on (D, Z) only, $\{p_t : t \in \mathcal{T}\}$ has to satisfy eqs (4.4, 4.5) from step 1 above. In addition, if we impose Assumption 2, i.e., $\mathcal{R}_T = \{\text{Assumption 2}\}$ we can show that the identified set for the response types in this simple case is characterized by the following set of equations:

$$p_{0,0} = 1 - \mathbb{P}\left(D = H \mid Z = 1\right) - \mathbb{P}\left(D = L \mid Z = 1\right), \tag{4.8}$$

$$p_{0,L} = \mathbb{P}\left(D = L \mid Z = 1\right) - \mathbb{P}\left(D = H \mid Z = 0\right) + p_{H,H} - p_{L,L}, \tag{4.9}$$

$$p_{0,H} = \mathbb{P}\left(D = H \mid Z = 1\right) - \mathbb{P}\left(D = L \mid Z = 0\right) + p_{L,L} - p_{H,H}, \tag{4.10}$$

$$p_{L,H} = \mathbb{P}\left(D = L \mid Z = 0\right) - p_{L,L}, \tag{4.11}$$

$$p_{H,L} = P\left(D = H \mid Z = 0\right) - p_{H,H}, \tag{4.12}$$

$$\max\{0, \mathbb{P}(D = H|Z = 0) - \mathbb{P}(D = L|Z = 1)\} \leq$$
$$p_{H,H} \leq \min\{\mathbb{P}(D = H|Z = 0), \mathbb{P}(D = H, Z = 1)\} \tag{4.13}$$

$$\max\{0, \mathbb{P}(D = L|Z = 0) - \mathbb{P}(D = H|Z = 1)\} \leq$$
$$p_{L,L} \leq \min\{\mathbb{P}(D = L|Z = 0), \mathbb{P}(D = L, Z = 1)\}, \tag{4.14}$$

$$\mathbb{P}(D = 0|Z = 1) \leq \mathbb{P}(D = 0|Z = 0), \tag{4.15}$$

$$\mathbb{P}(D \neq 0|Z = 0) \leq \mathbb{P}(D \neq 0|Z = 1). \tag{4.16}$$

More precisely, we can show that

$$\Theta_I(\mathcal{R}_T) = \{\{p_t : t \in \mathcal{T} \text{ such that eqs (4.8) to (4.16) are satisfied}\}\}.$$

We can easily see that $\Theta_I(\mathcal{R}_T) \neq \emptyset$ if and only if the eqs (4.15) and (4.16) hold. Having defined the identified set for response types, we proceed to construct bounds on our treatment effects of interest. In this particular case, we demonstrate that:

$$\underline{\gamma}^z_{H,H} = \frac{\max\{0, \mathbb{P}(D = H|Z = 0) - \mathbb{P}(D = L|Z = 1)\}}{\mathbb{P}(D = H|Z = z)}, \text{ for } z \in \{0, 1\},$$
$$\underline{\gamma}^z_{L,L} = \frac{\max\{0, \mathbb{P}(D = L|Z = 0) - \mathbb{P}(D = H|Z = 1)\}}{\mathbb{P}(D = L|Z = z)}, \text{ for } z \in \{0, 1\},$$

One can then apply the closed-form formula from Theorem 1 to establish bounds on the treatment effects of interest. A unique aspect of our methodology is that imposing additional restrictions on response types does not alter the bounds in Theorem 1. These bounds remain consistent, with only $\underline{\gamma}^z_{d,d}$ adjusting according to the new restrictions.

In the numerical illustration below, we describe $\Theta_I(\mathcal{R}_T)$ and demonstrate how imposing further assumptions on response types can significantly refine $\Theta_I(\mathcal{R}_T)$. More, precisely we consider the following additional assumptions:

(i) $p_{H,H} \geq p_{H,L}$, which implies that remaining within a high-type firm is more probable than transitioning from a high-type to a low-type firm as a consequence of the treatment.
(ii) $\min_t p_t = p_{H,L}$, indicating that the smallest proportion of response type consists of individuals moving from a high-type to a low-type firm due to the treatment.
(iii) $p_{H,L} = 0$, implying an absence of transitions from high-type to low-type firms as a result of treatment.

It is important to note that $\{p_t : t \in (0,0), (0,L), (0,H), (L,H), (H,L)\}$ are determined once $p_{H,H}$ and $p_{L,L}$ are fixed, which are themselves identified within a set. Hence, our forthcoming discussion will primarily focus on illustrating the projections of $\Theta_I(\mathcal{R}_T)$ with respect to the dimensions of $p_{H,H}$ and $p_{L,L}$.

4.3.1. *Data Generating Process.* We first generate propensity scores to be consistent with the data in Lee (2009). This is reported in Table 1.

Job Training	$P(D = L \mid Z = z)$	$P(D = H \mid Z = z)$
$Z = 1$	0.302886	0.408114
$Z = 0$	0.313959	0.373041

TABLE 1. Probabilities of employment at firms of type L and H, contingent on the treatment status Z.

The data are generated such that the true values for $p_{H,H}$, and $p_{L,L}$ are:

$$\begin{aligned} p_{H,H} &= \mathbb{P}(D = H \mid Z = 0) = 0.373041, \\ p_{L,L} &= 0.278886. \end{aligned}$$

Next, we randomly generate the outcomes.[19] For each type $t \in \mathcal{T}$, denote by D_z their employment status when externally assigned $Z = z$. The conditional distributions of $\exp(Y_{z,D_z}) \mid T = t$ for each $t \in \mathcal{T}$ are assumed to follow Lognormal$(\mu_{z|t}, \sigma_{z|t})$ distributions. Here, $\sigma_{z|t} = 1$ for all combinations of z and t indicating that variability only arises through $\mu_{z|t}$ across types. We present two distinct potential earnings distribution models as described in Table 2 and Table 3.

t	Dist. of $\exp(Y_{1,D_1})\|T = t$	Dist. of $\exp(Y_{0,D_0})\|T = t)$
$(0, L)$	Lognormal$(9.5, 1)$	Lognormal$(9.5, 1)$
$(0, H)$	Lognormal$(11.5, 1)$	Lognormal$(9.5, 1)$
(L, H)	Lognormal$(16.5, 1)$	Lognormal$(9.5, 1)$
(H, L)	Lognormal$(9.75, 1)$	Lognormal$(9.6, 1)$
(L, L)	Lognormal$(9.5, 1)$	Lognormal$(9.5, 1)$
(H, H)	Lognormal$(14.5, 1)$	Lognormal$(14.5, 1)$

Table 2. Design 1

t	Dist. of $\exp(Y_{1,D_1})\|T = t$	Dist. of $\exp(Y_{0,D_0})\|T = t)$
$(0, L)$	Lognormal$(10.5, 1)$	Lognormal$(9.5, 1)$
$(0, H)$	Lognormal$(12.5, 1)$	Lognormal$(9.5, 1)$
(L, H)	Lognormal$(14.5, 1)$	Lognormal$(9.5, 1)$
(H, L)	Lognormal$(10.5, 1)$	Lognormal$(10.5, 1)$
(L, L)	Lognormal$(10.5, 1)$	Lognormal$(9.5, 1)$
(H, H)	Lognormal$(14, 1)$	Lognormal$(12, 1)$

Table 3. Design 2

[19]By construction, the outcomes (wages) simulated here are independent of the dataset used in Lee (2009).

4.3.2. *Simulations Results.* We begin by exploring the geometry of $\Theta_I(\mathcal{R}_T)$, and demonstrate how incorporating further assumptions regarding response types can significantly shrink its shape.

Parameter	True value	Assumptions	Bounds Lower	Bounds Upper
$E(Y_{1,D_1} - Y_{0,D_0} \mid AE)$	0.3574	Assumptions 1-2	0.0949	0.4368
$E(Y_{1,H} - Y_{0,H} \mid T = (H,H))$	0.000	Assumptions 1-2	-2.8937	3.5221
		Assumptions 1-2 and $P(T = (H,H)) \geq P(T = (H,L))$	-0.8070	1.1686
		Assumptions 1-2 and $\min_{\tau} P(T = \tau) = P(T = (H,L))$	-0.0909	0.4261
		Assumptions 1-2 and $P(T = (H,L)) = 0$	-0.0340	0.3725
$E(Y_{1,L} - Y_{0,L} \mid T = (L,L))$	0.000	Assumptions 1-2	*Trivial Bounds*	
		Assumptions 1-2 and $P(T = (H,H)) \geq P(T = (H,L))$	-2.3615	2.3378
		Assumptions 1-2 and $\min_{\tau} P(T = \tau) = P(T = (H,L))$	-0.3678	0.3883
		Assumptions 1-2 and $P(T = (H,L)) = 0$	-0.3678	0.3883

TABLE 4. Results for Design 1.

Figures 5 and 6, along with Tables 4 and 5, present the outcomes for designs 1 and 2, respectively. Initially, we compute the Lee bounds, which set identifies the total effect $\mathbb{E}(Y_{1,D_1} - Y_{0,D_0} \mid D_0 > 0, D_1 > 0)$, as in Lemma 2.

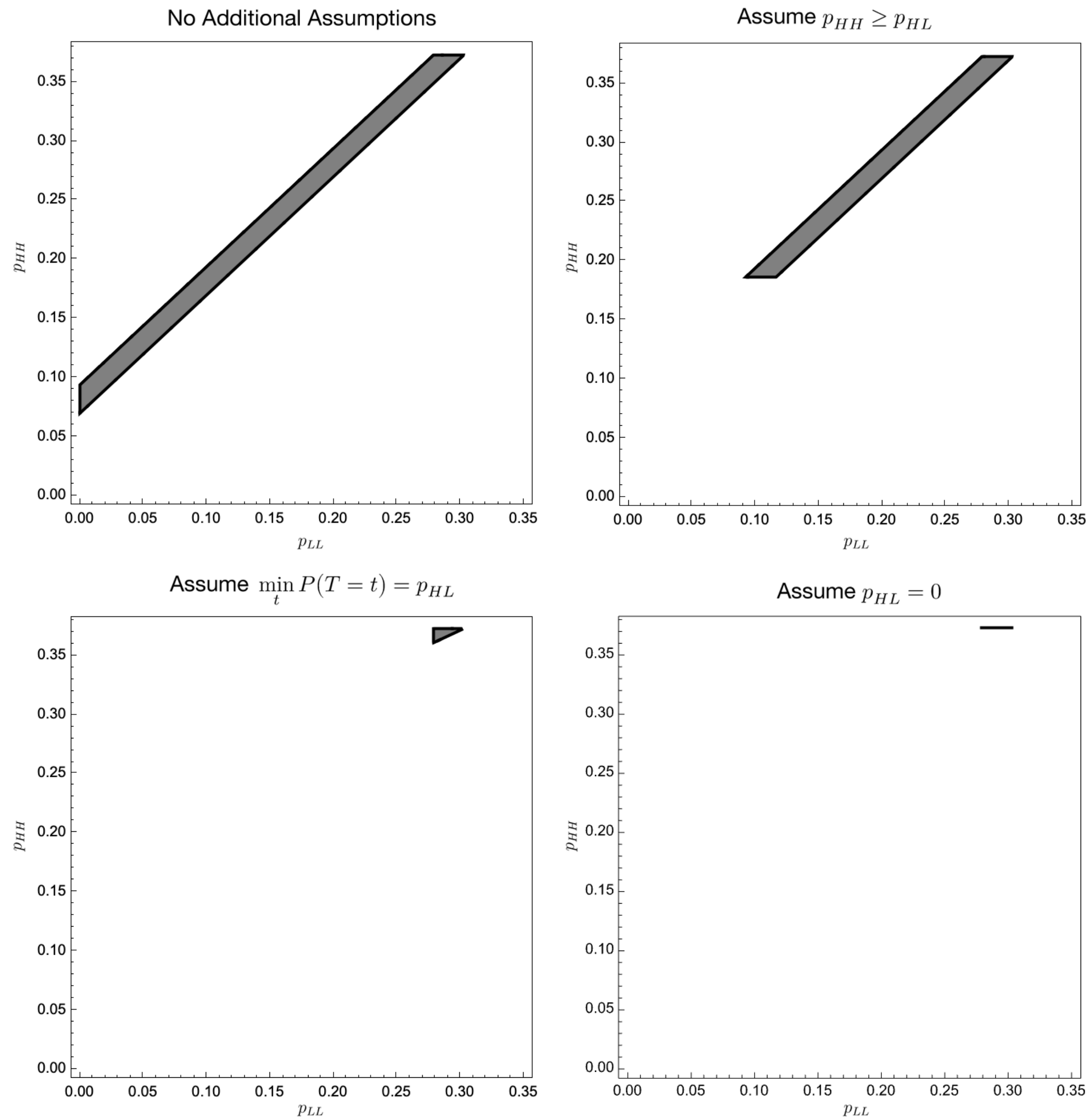


FIGURE 4. Identified set for $p_{L,L}, p_{H,H}$ in both simulated DGPs. The first panel delineates the identified set under the base assumption $\mathcal{R}_T =$ Assumption 2. Subsequent panels illustrate the refinement achieved by imposing additional assumptions.

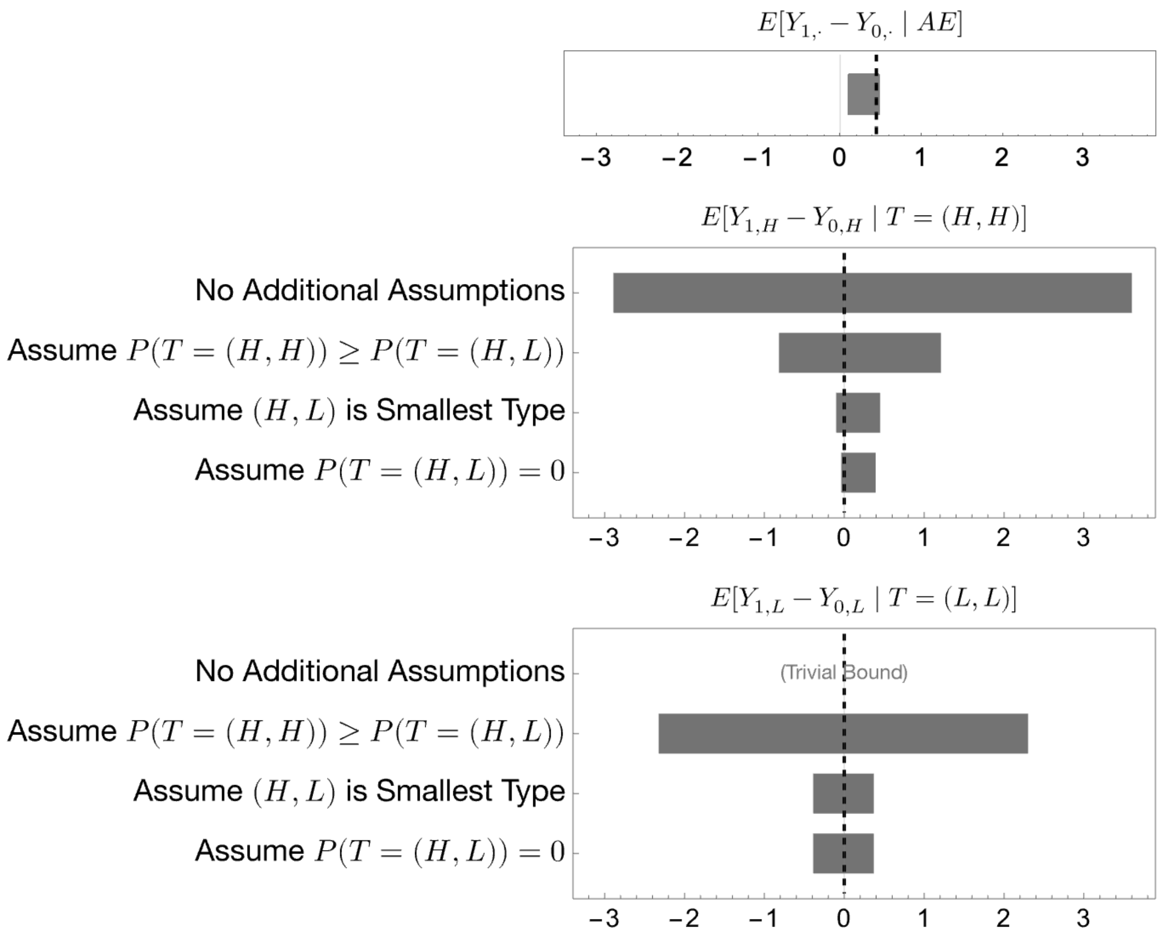


FIGURE 5. Results for design 1. The dashed black line represents the true value of the parameter.

Within the framework of design 1, these bounds lie entirely within the positive quadrant, excluding 0. When ignoring firm heterogeneity in wages, this suggests that job training increases wage rates for the always-employed, i.e., $AE = \{(L,L),(H,H),(L,H),(H,L)\}$. However, this DGP is consistent with the fact that the true within-firm causal effect for $t \in \{(L,L),(H,H),(L,H),(H,L)\}$ is 0, i.e. $\mathbb{E}(Y_{1,H} - Y_{0,H}|T = t) = \mathbb{E}(Y_{1,L} - Y_{0,L}|T = t) = 0 \ \forall \ t \in \{(L,L),(H,H),(L,H),(H,L)\}$. The positive effect recovered by Lee bounds purely captures the effect of job training on sorting (labor supply) rather than a change in wages. This distinction is further clarified through the equation below, which illustrates that whenever $\mathbb{E}(Y_{1,H} - Y_{0,H}|T = (H,H)) =$

Parameter	True value	Assumptions	Bounds	
			Lower	Upper
$E(Y_{1,D_1} - Y_{0,D_0} \mid AE)$	1.7472	Assumptions 1-2	1.5743	1.8288
$E(Y_{1,H} - Y_{0,H} \mid T = (H,H))$	2.000	Assumptions 1-2	−0.8437	4.9352
		Assumptions 1-2 and $P(T = (H,H)) \geq P(T = (H,L))$	1.1719	2.8840
		Assumptions 1-2 and $\min_\tau P(T = \tau) = P(T = (H,L))$	1.8246	2.2560
		Assumptions 1-2 and $P(T = (H,L)) = 0$	1.8727	2.2080
$E(Y_{1,L} - Y_{0,L} \mid T = (L,L))$	1.000	Assumptions 1-2	*Trivial Bounds*	
		Assumptions 1-2 and $P(T = (H,H)) \geq P(T = (H,L))$	−1.2638	3.3657
		Assumptions 1-2 and $\min_\tau P(T = \tau) = P(T = (H,L))$	0.6728	1.4179
		Assumptions 1-2 and $P(T = (H,L)) = 0$	0.6728	1.4179

Table 5. Results of Design 2.

$\mathbb{E}(Y_{1,L} - Y_{0,L} \mid T = (L,L)) = 0$, the observed changes in wages captured by Lee's bounds are primarily a consequence of sorting induced by job training:

$$\mathbb{E}(Y_{1,D_1} - Y_{0,D_0} \mid D_0 > 0, D_1 > 0) = \frac{p_{L,H}}{\mathbb{P}(AE)} \mathbb{E}[Y_{1,H} - Y_{0,L} \mid T = (L,H)\}] + \frac{p_{H,L}}{\mathbb{P}(AE)} \mathbb{E}[Y_{1,L} - Y_{0,H} \mid T = (H,L)\}].$$

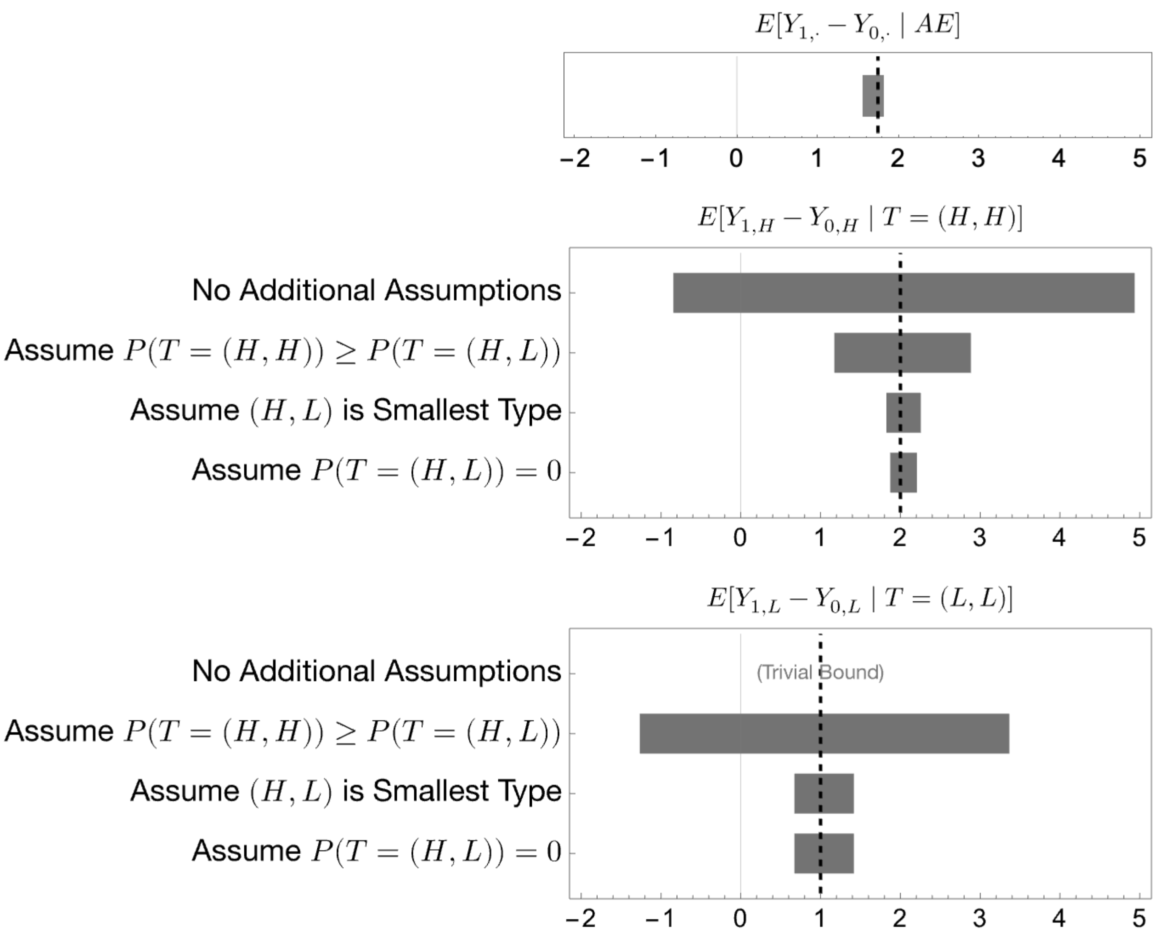


FIGURE 6. Results of Design 2. The dashed black line represents the true value of the parameter.

Applying our bounds on the within-firm effects, we can provide a more accurate assessment of the impact of job training on wages. The bounds for $\mathbb{E}(Y_{1,H} - Y_{0,H}|T = (H,H))$ and $\mathbb{E}(Y_{1,L} - Y_{0,L}|T = (L,L))$ always include 0 across the different scenarios corresponding to different assumptions on response types, suggesting there is not enough evidence in the data to support the hypothesis that job training has a causal effect on wages.

In design 2, the Lee bounds also lie entirely within the positive quadrant, excluding 0. However, in this case, our DGP is consistent with the fact that the true within-firm causal effects for $t \in \{(L,L), (H,H), (L,H), (H,L)\}$ are strictly positive. Interestingly, our bounds for $\mathbb{E}(Y_{1,H} - Y_{0,H}|T = (H,H))$ and $\mathbb{E}(Y_{1,L} - Y_{0,L}|T = (L,L))$ lie entirely within the positive quadrant, excluding 0 when we restrict the response types. This shows that in this case, our bounds are informative enough to reveal that job training directly influences wages.

5. Empirical Application: Job Corps Study

Job Corps is the largest residential career training program in the U.S. and has trained more than two million individuals since its inception under the Economic Opportunity Act of 1964. Today, Job Corps trains over 60,000 enrollees per year, at roughly 131 Job Corps centers nationwide, with an estimated cost of 34,301 USD per enrollee and 57,312 USD per graduate (Liu et al. 2020).[20] During the mid to late 1990s, the U.S. Department of Labor funded a randomized evaluation of Job Corps which was completed by Mathematica Policy Research, Inc. Existing evaluations of the Job Corps Study include Schochet et al. (2001), Schochet et al. (2008), Lee (2009) and Blanco et al. (2013). This section describes the Job Corps program, the randomized evaluation of the program and reports our multilayered bounds.

5.1. **Job Corps Program.** Job Corps is free for participants and targets disadvantaged individuals ages 16 to 24 with the aim to assist these individuals to become more responsible, employable and productive citizens (Johnson et al. 1999).[21] Job Corps is an intensive and comprehensive program. The typical participant will complete Job Corps over a span of 30 weeks. The vast majority will live at the local Job Corps centre during this time and complete 440 hours of academic instruction and 700 hours of vocational training. Job Corps centres also provide job search assistance

[20]At the time of the National Job Corps Study, the average cost of Job Corps was 14,000 USD per enrollee (Burghardt et al. 2001).

[21]At the time of the randomized evaluation, applicants were required to meet nine criteria to be eligible for Job Corps in addition to the age criterion: (i) be a US resident; (ii) be economically disadvantaged; (iii) need additional education or training; (iv) live in an environment characterized by a disruptive home life, limited job opportunities, high crime rates; (v) be free of health concerns; (vi) be free of behavioral problems; (vii) have suitable arrangements for any dependents (if applicable); (viii) have received parental consent (if a minor); (ix) have capabilities and aspirations to complete the program (Johnson et al. 1999).

upon participant completion of the program, although these placement services are more limited in scope.[22]

5.2. **Job Corps Randomized Evaluation.** The Job Corps Study randomized 80,883 eligible individuals who applied to Job Corps for the first time between November 1994 and December 1995 into two groups: (i) 5,977 individuals in the control group who were embargoed from participating in Job Corps for three years and (ii) 74,906 individuals in the treatment group.[23] The control group could still complete non-Job Corps training and many did. Schochet et al. (2001) report that nearly 72% of the control group completed education or training in the 48 months following randomization. Out of the 74,906 individuals assigned to treatment, 9,409 individuals were randomly selected for data collection and all control individuals were selected for data collection. Therefore, the final observed sample of participants is 15,386 individuals who were interviewed at the time of random assignment and then 12 months, 30 months and 48 months after random assignment.

5.3. **Data and Variable Definitions.**

5.3.1. *Data and Sample Construction.* We use the public-release data of the National Job Corps Study (Schochet et al. 2003). The original sample size in the National Job Corps Study is 15,386 individuals which is comprised of 5,977 control individuals and 9,409 treatment individuals. We impose two sample restrictions to address missing values due to interview non-response and sample attrition over time. The first sample restriction, which follows Lee (2009), is to only keep individuals who have non-missing values for weekly earnings and hours for every week following random assignment. Restricting the sample in this manner decreases sample size to 9,145 individuals (= 3,599 control units + 5,546 treated units).

[22]In the Job Corps randomized evaluation, only 40% of treated individuals stated receiving placement services and only 41% of these individuals stated finding a job as a result of these placement services. This finding is not unique to the study period and was a noted weakness of the Job Corps program at the time of the evaluation (Johnson et al. 1999).

[23]The study randomized some subpopulations into the treatment group with known, different probabilities. Therefore analyzing the data requires the use of design weights as provided in the public use dataset (Schochet et al. 2003).

As discussed below, we do not observe the firm identity in our data and thus we rely on a classification of firms into types based on the provision of amenities. Therefore, we further restrict our sample by only keeping individuals who have non-missing amenity values for the weeks of interest (90, 135, 180 and 208). To preserve sample size, we impose this sample restriction on a per-amenity basis.[24] When classifying firms based on the provision of health insurance, this results in a final sample size of 6,403 individuals (= 2,540 control units + 3,863 treated units).

5.3.2. *Key Variable Definitions.* The three key variables of interest are employment, hourly wage and job amenities for employed individuals. We follow Lee (2009) by defining employment in a week based on whether an individual has positive earnings in that week and defining the hourly wage in a week by dividing weekly earnings by weekly hours worked. We observe the following job amenities in our data: health insurance, paid sick leave, paid vacation, child care assistance, flexible hours, employer-provided transportation, retirement or pension benefits, dental plan, tuition reimbursement or training course. Amenity status at a job is defined based on individuals' survey responses.[25]

5.3.3. *Summary Statistics.* Table 6 presents summary statistics for the sample of individuals who have non-missing values for weekly earnings and hours for every week following random assignment. Means and standard deviations for a number of baseline and post-randomization variables are reported separately by treatment status. Consistent with successful randomization in the National Job Corps Study, the table shows that there are no statistical differences in the means of demographic,

[24]In some cases, individual's hold multiple jobs. For the purpose of assigning amenities, Schochet et al. (2008) select a unique job by choosing the one with the maximum job tenure. For some individuals, we are unable to identify an individual's unique job in the weeks of interest since the job tenure is identical across the jobs. We drop these individuals which decreases the sample size from 9,145 individuals to 8,379 individuals. Among the remaining 8,379 individuals, 95% of the employed hold only one job in the weeks of interest.

[25]In certain cases, we follow Schochet et al. (2008) convention of "imputing" certain amenities based on an individual's job type. For example, if an individual is in the military, they are coded as having health insurance, even if they report that they did not have health insurance or did not answer the interview question.

education, background and baseline employment/income variables across treatment and control groups. This finding aligns with the previous evaluations of the National Job Corps Study.

Table 6 also shows economically and statistically significant differences in employment and earnings outcomes by treatment status, post randomization. We see that 52 weeks after randomization, treatment group hours and earnings are lower than the control group but 104 weeks after randomization, treatment group hours and earnings exceed the control group. After 208 weeks, treatment group hours and earnings are approximately 8% and 14% higher, respectively, than the control group. These differences are statistically significant and are consistent with the previous evaluations of the National Job Corps Study.

5.4. **Classification of Firm Type and Differential Worker Sorting.**

5.4.1. *Classification of Firm Type Based on Job Amenities.* Motivated by the importance of firm amenity provision in the workforce (e.g, Jones 2005; Maestas et al. 2017, 2023) we classify firm type based on the provision of health insurance, paid vacation and pension/retirement benefits.[26] At the time of the National Job Corps study, there were no legal requirements for firms to provide any of these amenities and, conditional on firm provision, federal law generally prohibited discriminatory provision across workers (United States Equal Employment Opportunity Commission 2009).[27]

[26]Maestas et al. (2017) find that benefits are highly unequally distributed throughout the workforce, specifically for younger workers, with non-college graduates being less likely to have benefits compared to college graduates. Eligibility criteria for Job Corps explicitly requires individuals to be young (ages 16-24) and in need of additional education/training (not college educated). Therefore, benefit provision is a particularly useful classification of firm type for this sample. Maestas et al. (2017) further show that the vast majority of workers rate health insurance, paid vacation and retirement benefits as being essential fringe benefits.

[27]The relevant federal laws at the time of the National Job Corps Study included: Title VII of the Civil Rights Act of 1964; the Age Discrimination in Employment Act of 1967; Title I and Title V of the Americans with Disabilities Act of 1990 (United States Equal Employment Opportunity Commission 2009).

	Control		Treated		Difference	
	Mean	S.D.	Mean	S.D.	Difference	S.E.
Female	0.46	0.50	0.45	0.50	-0.01	0.01
Age at baseline	18.35	2.10	18.44	2.16	0.09	0.05
White, non-Hispanic	0.26	0.44	0.27	0.44	0.00	0.01
Black, non-Hispanic	0.49	0.50	0.49	0.50	0.00	0.01
Hispanic	0.17	0.38	0.17	0.37	-0.00	0.01
Other race/ethnicity	0.07	0.26	0.07	0.26	-0.00	0.01
Never married	0.92	0.28	0.92	0.28	0.00	0.01
Married	0.02	0.15	0.02	0.14	-0.00	0.00
Living together	0.04	0.20	0.04	0.19	-0.00	0.00
Separated	0.02	0.14	0.02	0.15	0.00	0.00
Has child	0.19	0.39	0.19	0.39	-0.00	0.01
Number of children	0.27	0.64	0.27	0.65	0.00	0.01
Education	10.11	1.54	10.11	1.56	0.01	0.03
Mother's education	11.46	2.59	11.48	2.56	0.02	0.06
Father's education	11.54	2.79	11.39	2.85	-0.15	0.08
Ever arrested	0.25	0.43	0.25	0.43	-0.00	0.01
Household income						
<3,000	0.25	0.43	0.25	0.44	0.00	0.01
3,000-6,000	0.21	0.41	0.21	0.40	-0.00	0.01
6,000-9,000	0.11	0.32	0.12	0.32	0.00	0.01
9,000-18,000	0.24	0.43	0.24	0.43	-0.00	0.01
>18,000	0.18	0.39	0.18	0.38	-0.00	0.01
Personal income						
<3,000	0.79	0.41	0.79	0.41	-0.00	0.01
3,000-6,000	0.13	0.34	0.13	0.33	-0.00	0.01
6,000-9,000	0.05	0.21	0.05	0.22	0.01	0.00
>9,000	0.03	0.18	0.03	0.17	-0.00	0.00
At baseline						
Have job	0.19	0.39	0.20	0.40	0.01	0.01
Mths. empl. prev. yr.	3.53	4.24	3.60	4.25	0.07	0.09
Had job, prev. yr.	0.63	0.48	0.63	0.48	0.01	0.01
Earnings, prev. yr.	2810.48	4435.62	2906.45	6401.33	95.97	117.10
Usual hours/week	20.91	20.70	21.82	21.05	0.91	0.45
Usual weekly earn.	102.89	116.46	110.99	350.61	8.10	5.09
Post randomization						
Week 52 hours	17.78	23.39	15.30	22.68	-2.49	0.49
Week 104 hours	21.98	26.08	22.64	26.25	0.67	0.56
Week 156 hours	23.88	26.15	25.88	26.57	2.00	0.56
Week 208 hours	25.83	26.25	27.79	25.74	1.95	0.56
Week 52 earn.	103.80	159.89	91.55	149.28	-12.25	3.33
Week 104 earn.	150.41	210.24	157.42	200.27	7.02	4.42
Week 156 earn.	180.88	224.43	203.71	239.80	22.84	4.94
Week 208 earn.	200.50	230.66	227.91	250.22	27.41	5.11
Total 4 yr. earn.	30006.69	26893.60	30800.41	26437.39	793.72	571.83
Sample size	3599		5546		9145	

Notes: Weekly earnings calculated as the sum of total earnings in a given week and are not conditional on employment (i.e., includes 0s for the unemployed).

Table 6. Summary statistics by treatment status

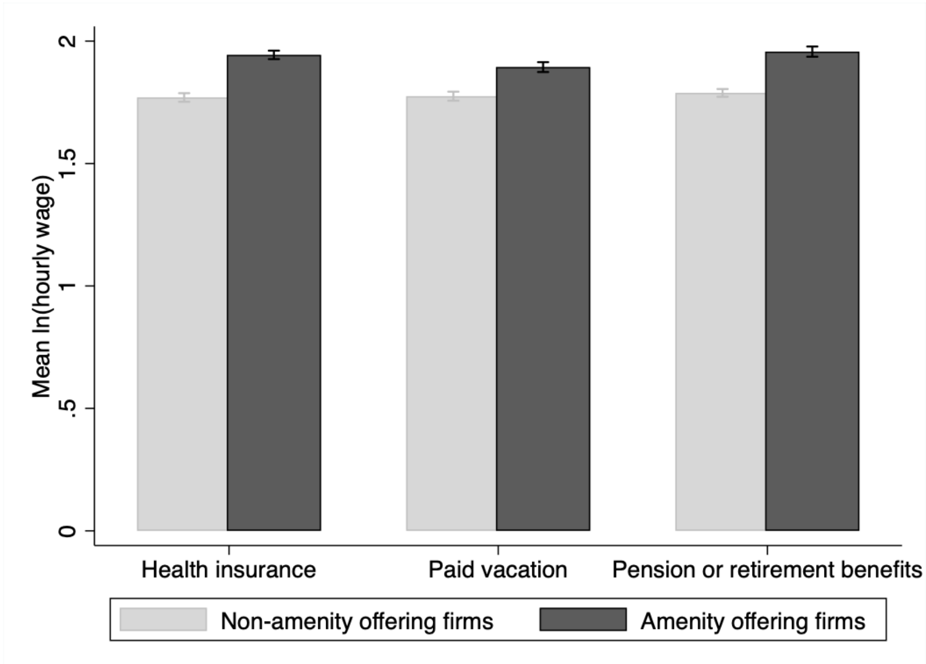
Figure 7 presents the mean log wage at week 90 according to whether firms provide amenities.[28] At week 90 (208), mean wages at firms that offer amenities are approximately 15% (18%) higher compared to firms that do not. Figure 8 presents the empirical cumulative distribution functions of log wage by firm type, classifying firms based on the provision of health insurance, for treatment and control groups at week 90. For both treated and control units, the distribution of log wages for firms that provide health insurance stochastically dominates the distribution of log wages for firms that do not. This evidence suggests that firms that provide amenities pay higher wages than firms that do not.[29] A natural follow-up question is whether Job Corps affects the sorting of workers into amenity-providing firms.

5.4.2. *Differential Sorting of Treatment and Control Workers.* Lee (2009) focuses on the potential for job training to affect labor supply along the extensive margin but ignores an additional margin of labor supply: firm choice. If there is scope for job training to affect worker sorting to firms, sample selection is multilayered. Table 7 presents the probability of working at a firm (conditional on employment) that provides observable amenities, at week 90, by treatment status. The evidence shows that treated individuals are more likely to work at firms with job amenities in all but one case (and we also find that this trend persists across all weeks). This is consistent with the evidence presented in Schochet et al. (2008).[30]

[28]In this section, descriptive tables and figures presented are for week 90 which follows the preferred specification in Lee (2009). Tables and figures for other weeks of interest (135, 180 and 208) are presented in Appendix C.1.

[29]Of course, it is possible that firms pay compensating differentials which causally reduce wages. The evidence presented here shows that the across-firm variation dominates the within-firm variation. This is consistent with evidence in Lamadon et al. (2022) who show that the high-amenities firms are also the more productive firms.

[30]In Appendix C.2 we focus on the amenities we use to classify firm types – health benefits H, retirement/pension benefits R and paid vacation V – and categorize jobs into eight mutually exclusive categories based on the amenities available. Appendix Table 13 presents the distribution of workers by the amenity category their job falls into at week 90. Treated workers are approximately 15% more likely to work in jobs that offer all amenities and 10% less likely to work in jobs that offer no amenities. Reinforcing the finding from Figure 7, hourly wages are approximately 22% higher in jobs with all amenities compared to jobs with no amenities. These trends persist qualitatively across all weeks.



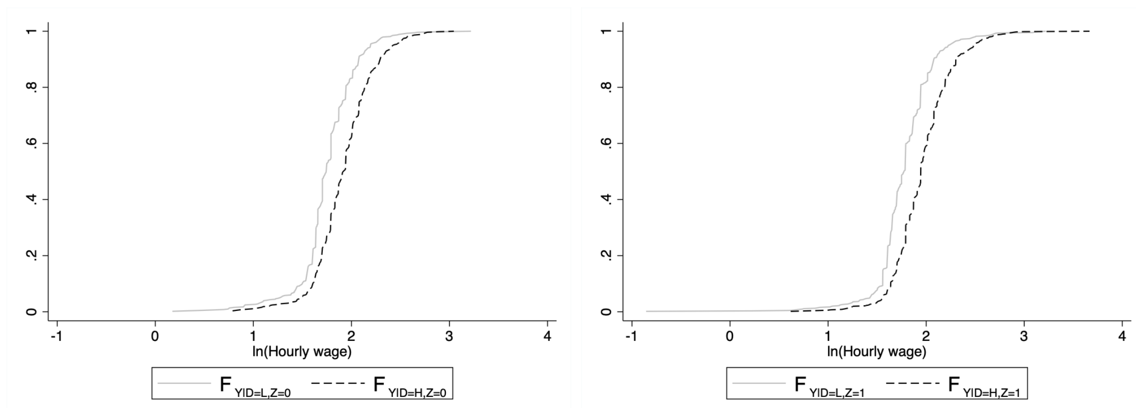
Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

FIGURE 7. Mean ln(hourly Wage) by firm amenity provision at week 90

Taken together, Figures 7 and 8 and Table 7 show: (i) firms that provide amenities pay higher wages than firms that do not and (ii) Job Corps affects the sorting of workers into amenity-providing firms. We therefore conclude that Job Corps training not only affects labor supply along the extensive margin but also along the additional margin of firm choice. As a result, sample selection is multilayered motivating the use of our bounds.

5.5. **Multilayered Bounds.**

5.5.1. *Lee Bounds.* As a first step, we replicate the bounds reported in Lee (2009) which, as discussed above, target the parameter $\mathbb{E}[Y_{1,D_1} - Y_{0,D_0}|D_0 > 0, D_1 > 0]$. In Appendix C.3 we report Lee's bounds for weeks 90, 135, 180 and 208 along with the trimming proportion $p \equiv \mathbb{P}(AE)$, e.g., the share of the always-employed among



(a) control units (b) treated units

Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

FIGURE 8. Cumulative distribution function by firm type at week 90, amenity=health

	Control		Treated		Difference	
	Mean	S.D.	Mean	S.D.	Difference	S.E.
Health insurance	0.4860	0.5000	0.5072	0.5001	0.0211	0.0186
Paid sick leave	0.3922	0.4884	0.4251	0.4945	0.0329	0.0183
Paid vacation	0.5407	0.4985	0.5800	0.4937	0.0393	0.0180
Childcare assistance	0.1311	0.3376	0.1422	0.3494	0.0111	0.0127
Flexible hours	0.5330	0.4991	0.5568	0.4969	0.0237	0.0183
Employer-provided transportation	0.1973	0.3981	0.1876	0.3905	-0.0097	0.0147
Pension or retirement benefits	0.3670	0.4822	0.3938	0.4887	0.0268	0.0180
Dental plan	0.3863	0.4871	0.4288	0.4950	0.0425	0.0182
Tuition reimbursement	0.2212	0.4152	0.2602	0.4389	0.0390	0.0158
Employment	0.4368	0.4961	0.4391	0.4963	0.0022	0.0111
Sample size	3288		5091		8379	

Notes: Control and treatment probabilities have interpretation as $P[D = H|D > 0, Z = z]$ for $z \in \{0, 1\}$, respectively, when classifying firms as type H if they provide a given amenity.

TABLE 7. Probability of working at amenity-providing firm at week 90 (conditional on employment)

individuals receiving job training. Lee focuses on week 90 which produces the tightest bounds $[0.0468, 0.0484]$.[31]

[31]As we detail in Appendix C.3, these are Lee's bounds when treating ln(hourly wage) as a continuous variable as we do throughout this paper. Lee (2009) uses vingtiles of ln(hourly wage) which produces bounds $[0.0423, 0.0428]$.

5.5.2. *Identified Sets for Response Types.* We now consider the scenario with two firm types, denoted as L and H. Firms are classified based on the provision of health insurance with H denoting firms that offer health insurance and L denoting firms that do not.[32] As in Section 5, we always impose Assumptions 1-2 and then sequentially impose the restrictions in Assumption 3: (i) $p_{H,H} \geq p_{H,L}$ (more stayers than downward switchers) (ii) $\min_t \mathbb{P}[T = t] = p_{H,L}$ (more upward switchers than downward switchers) and (iii) $p_{H,L} = 0$ (strong monotonicity).

Table 8 presents the estimated propensity scores for each week of interest from the National Job Corps Study.[33] As expected, in all weeks $\mathbb{P}[D > 0|Z = 1] > \mathbb{P}[D > 0|Z = 0]$ showing that treated individuals are more likely to be employed. The table also shows that $\mathbb{P}[D = H|D > 0, Z = 1] > \mathbb{P}[D = H|D > 0, Z = 0]$ so that individuals who receive Job Corps training have a higher propensity to be employed at firms which offer health insurance than individuals who do not receive Job Corps training, conditional on employment, consistent with the evidence presented in Table 7.

	$\mathbb{P}[D = H\|Z = 0]$	$\mathbb{P}[D = H\|Z = 1]$	$\mathbb{P}[D = L\|Z = 0]$	$\mathbb{P}[D = L\|Z = 1]$
Week 90	0.2239	0.2372	0.2361	0.2229
Week 135	0.2758	0.3037	0.2415	0.2414
Week 180	0.2941	0.3313	0.2462	0.2512
Week 208	0.3142	0.3559	0.2513	0.2509

TABLE 8. Propensity scores by week. Amenity=health insurance

Using the week 90 propensity scores from Table 8, Figure 9 presents the identified set for $(p_{L,L}, p_{H,H})$. Naturally, incorporating additional restrictions on the response

[32]Results for classifying firms based on the provision of pension/retirement benefits and paid vacation are presented in Appendix C.4 - C.5.

[33]Our sample restriction to keep observations with non-missing amenity values for the weeks of interest drops only employed individuals. This restriction mechanically reduces the propensity scores. To ensure comparability with Lee (2009), we rescale our estimated propensity scores such that the probabilities of employment by treatment status are the same as the ones reported in Lee (2009).

types leads to a sharpening of the identified sets.[34] Having characterized the identified set of response-type probabilities, we now present our multilayered bounds.

5.5.3. *Multilayered Bounds.* Recall that under Assumption 2, the support of possible response types is:

$$\mathcal{T} := \{(0,0), (0,L), (0,H), (L,L), (H,H), (L,H), (H,L)\}.$$

The always-employed (AE) definition used in Lee (2009) therefore combines four different response types: $\{D_0 > 0, D_1 > 0\} = \{(L,L), (H,H), (L,H), (H,L)\} \equiv AE$. We focus on the bounds for stayers, defined as the response types (H,H) and (L,L).

Figure 10 presents our multilayered bounds for $\mathbb{E}[Y_{1,H} - Y_{0,H}|T = (H,H)]$ and $\mathbb{E}[Y_{1,L} - Y_{0,L}|T = (L,L)]$ for weeks 90, 135, 180 and 208. We illustrate the bounds in the baseline case (only Assumptions 1 and 2 are imposed) and also when we sequentially impose the following restrictions: (i) more stayers than downward switchers, (ii) more upward switchers than downward switchers and (iii) strong monotonicity. As already discussed, the Lee bounds for week 90 are $[0.0468, 0.0484]$. Focusing on the type H firms (firms that offer health insurance), our estimates indicate that $\mathbb{E}[Y_{1,H} - Y_{0,H}|T = (H,H)] \in [-2.1415, 2.3907]$. Assuming more stayers than downward switchers tightens these bounds to $[-0.4214, 0.5020]$. Further assuming that (H,L) is the smallest response type tightens them further, to $[-0.0023, 0.0754]$. Finally, assuming strong monotonicity narrows these bounds to $[-0.0018, 0.0750]$.

We find a similar pattern of results for the L-type bounds. These patterns persist across all weeks of interest and also when classifying firms based on the provision of alternative amenities (paid vacation and retirement/pension benefits) as shown in Appendix Figures 28 and 29. Table 9 report our estimated bounds across weeks.[35]

[34]In the bottom two panels of the this figure, we shrink the scale of the axis to see more clearly the identified set.

[35]Appendix Tables 21 and 22 provide our estimated bounds when classifying firms based on the provision of paid vacation and retirement/pension benefits, respectively.

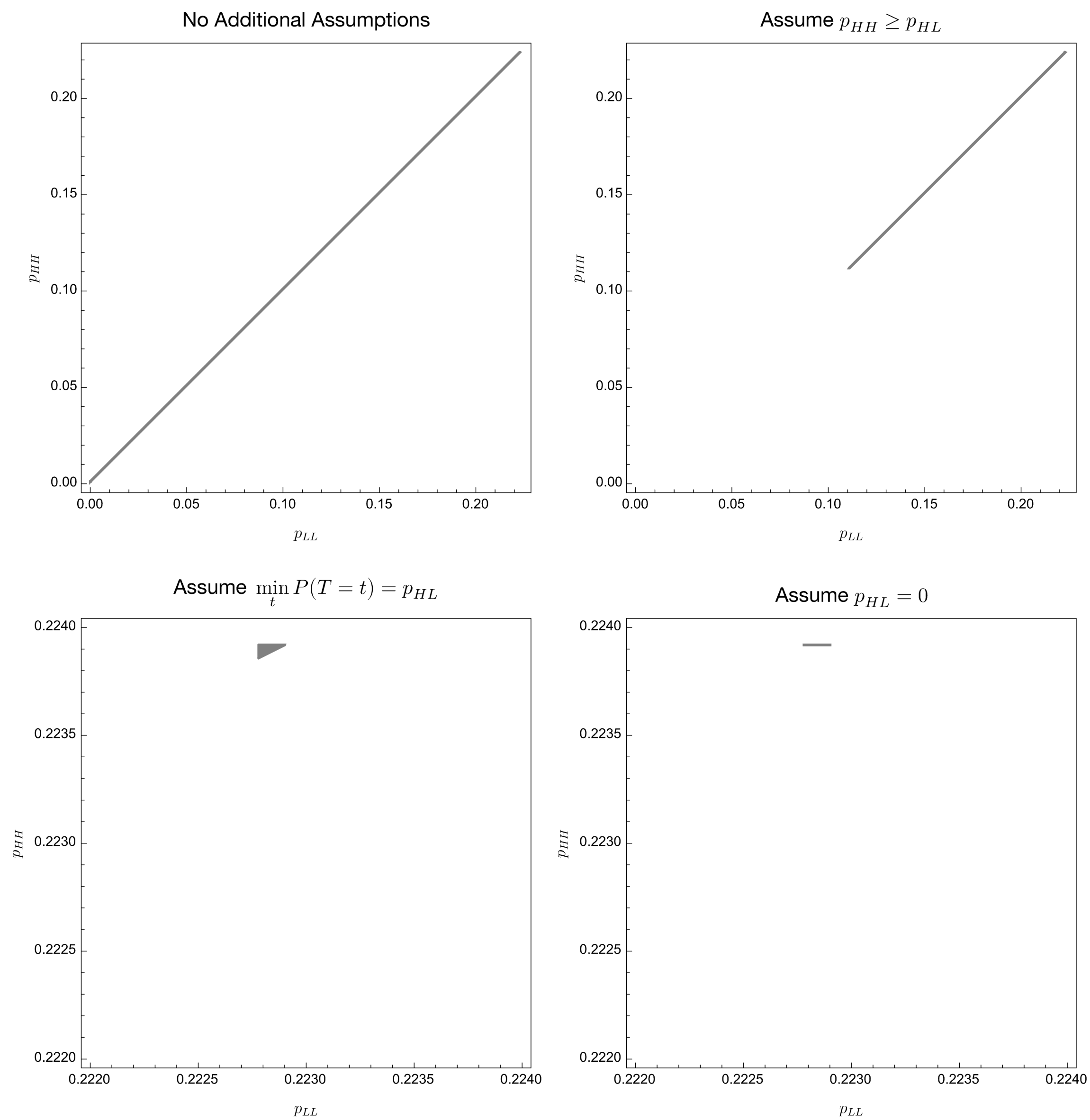


FIGURE 9. Identified set for $p_{L,L}, p_{H,H}$ at week 90. Amenity=health insurance. The first panel illustrates the identified set when $\mathcal{R}_T$ = {Assumption 2}. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2. The scale of the axis in the bottom two panels is shrunk to see more clearly the identified set.

Thus, while the conventional Lee bounds are strictly positive, our bounds for the within-firm effects include 0 even under strong assumptions on the response types. This suggests that Lee bounds may be capturing a pure sorting response to job training rather than a direct wage effect.

			$\mathbb{E}(Y_{1,H} - Y_{0,H} \vert T = (H,H))$		$\mathbb{E}(Y_{1,L} - Y_{0,L} \vert T = (L,L))$	
Week 90	$p^*_{H,H}$	$p^*_{L,L}$	lower	upper	lower	upper
Baseline	0.0010	0.0000	-2.1415	2.3907		
$p_{H,H} \geq p_{H,L}$	0.1120	0.1108	-0.4214	0.5020	-0.4002	0.4542
(H,L) is smallest type	0.2239	0.2228	-0.0023	0.0754	-0.0191	0.0673
$p_{H,L} = 0$	0.2239	0.2228	-0.0018	0.0750	-0.0191	0.0673
Week 135						
Baseline	0.0344	0.0000	-1.1228	1.1454		
$p_{H,H} \geq p_{H,L}$	0.1379	0.0758	-0.5075	0.5433	-0.6064	0.7090
(H,L) is smallest type	0.2619	0.2137	-0.1135	0.1369	-0.1180	0.1672
$p_{H,L} = 0$	0.2758	0.2137	-0.0529	0.0732	-0.1180	0.1672
Week 180						
Baseline	0.0429	0.0000	-1.0454	1.1410		
$p_{H,H} \geq p_{H,L}$	0.1471	0.0620	-0.5063	0.5525	-0.7710	0.8503
(H,L) is smallest type	0.2730	0.2090	-0.1421	0.1704	-0.1806	0.2110
$p_{H,L} = 0$	0.2941	0.2090	-0.0552	0.0854	-0.1806	0.2110
Week 208						
Baseline	0.0633	0.0000	-0.8821	0.9895		
$p_{H,H} \geq p_{H,L}$	0.1571	0.0525	-0.4888	0.5670	-0.9059	0.8730
(H,L) is smallest type	0.2935	0.2096	-0.1217	0.1797	-0.1914	0.2082
$p_{H,L} = 0$	0.3142	0.2096	-0.0430	0.1016	-0.1914	0.2082

Notes: Treatment bounds are for ln(hourly wage); hourly wage calculated as weekly earnings divided by weekly hours for the employed. p^*_t is the minimum value of p_t over the identified set for response-types under the given assumption.

TABLE 9. Multilayered bounds by week. Amenity=health insurance.

6. CONCLUSION

This paper develops a new methodology to partially identify the causal effect of job training on wages in the presence of multilayered sample selection. We define new treatment effects that operate within firms and between firms and provide a new

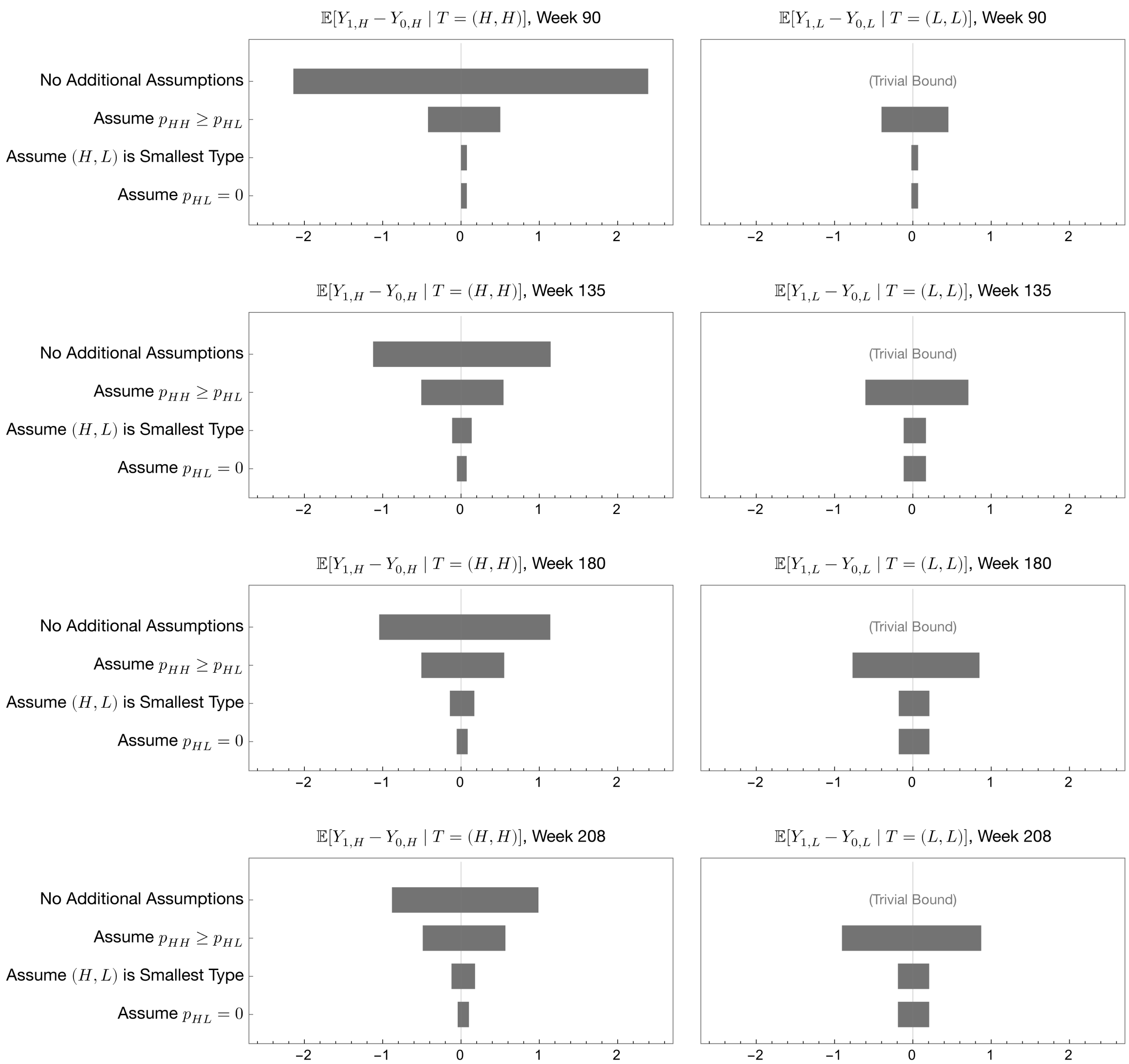


FIGURE 10. Multilayered bounds by week. Amenity=health insurance. Notes: Treatment bounds are for ln(hourly wage); hourly wage calculated as weekly earnings divided by weekly hours for the employed.

identification approach which extends Horowitz and Manski (1995) bounds. As a proof of concept, we show how to empirically implement these bounds by considering an application to the Job Corps Study.

While we consider our approach in the context of job training where a layer corresponds to a firm, we view it as naturally extending to other settings. In particular, it applies to any setting where sample selection is multilayered. As an example, consider a setting where a researcher is interested in estimating the causal effect of a tuition subsidy on labor market outcomes.[36] The subsidy may have an effect on the type of institution that an individual enrolls in and graduates from. If earnings depend on institutional quality, part of the earnings effects of the subsidy could reflect the value-added of institutions that are affected by the subsidy.

While our framework has primarily focused on nonparametric (partial) identification, we are currently re-examining the classic parametric sample selection approach from Heckman (1979) in the context of multilayered sample selection, as well as its semi-parametric version discussed in Honoré and Hu (2020). By imposing additional structure on the unobservables, this approach has the potential to significantly tighten the bounds and may achieve point identification of causal effects.

[36]Bettinger et al. (2019) evaluate the impact of California's state-based financial aid on long-run earnings.

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Appendix A. Additional Results

Lemma 5. *Consider a vector of random variables $(W, W_1, ..., W_K)$ satisfying the following condition for all w:*

$$F_W(w) = \sum_{k=0}^{K} \gamma_k F_{W_k}(w) \tag{A.1}$$

where $\gamma_k \geq 0$, $F_{W_k}(\cdot)$ represents the cumulative distribution function (CDF) of W_k, and $F_W(\cdot)$ represents the CDF of W.

The results are as follows:

(i) *The bounds for a fixed component of the mixture are point-wise sharp and given by:*

$$\mathbb{E}[F_W^{-1}(U)|U \leq \gamma_k] \leq \mathbb{E}[W_k] \leq \mathbb{E}[F_W^{-1}(U)|U \geq 1-\gamma_k], \tag{A.2}$$

and this for $k \in \{1, ..., K\}$ where $U \sim$ Uniform$[0,1]$.

(ii) *The bounds for a weighted average of fixed components of the mixture are point-wise sharp and given by:*

$$\mathbb{E}\left[F_W^{-1}(U)|U \leq \sum_{k=l}^{l'} \gamma_k\right] \leq \sum_{k=l}^{l'} \frac{\gamma_k}{\sum_{k=l}^{l'} \gamma_k} \mathbb{E}[W_k] \leq \mathbb{E}\left[F_W^{-1}(U)|U \geq 1-\sum_{k=l}^{l'} \gamma_k\right], \tag{A.3}$$

Lemma 5(i) extends the Horowitz and Manski (1995) bounds to scenarios involving mixtures with more than two components. Lemma 5(ii) introduces novel bounds on the weighted average of certain mixture components spanning l to l'. It is noteworthy that

$$\mathbb{E}\left[F_W^{-1}(U)|U \leq \sum_{k=l}^{l'} \gamma_k\right] \geq \sum_{k=l}^{l'} \frac{\gamma_k}{\sum_{k=l}^{l'} \gamma_k} \mathbb{E}[F_W^{-1}(U)|U \leq \gamma_k],$$

indicating that utilizing the point-wise bounds from Lemma 5(i) (Horowitz and Manski bounds) to establish bounds on the weighted average incurs some information loss.

Appendix B. Proofs

B.1. Proof of Lemma 2.

Proof. Note that (ii) and (iii) follow immediately from (i), so it suffices to show (i). First, notice that

$$\mathbb{E}\left[Y_{1,D_1} - Y_{0,D_0} | D_0 > 0, D_1 > 0\right] = \mathbb{E}\left[Y_{1,D_1} - Y_{0,D_1} | D_0 > 0, D_1 > 0\right] \tag{B.1}$$

$$+ \mathbb{E}\left[Y_{0,D_1} - Y_{0,D_0} | D_0 > 0, D_1 > 0\right] \ . \tag{B.2}$$

Next, we have that

$$\begin{aligned}
&\mathbb{E}\left[Y_{1,D_1} - Y_{0,D_1} | D_0 > 0, D_1 > 0\right] \\
&= \sum_{d=1,d'=1}^{K,K} \mathbb{E}\left[Y_{1,D_1} - Y_{0,D_1} | D_0 = d', D_1 = d\right] \mathbb{P}\left(D_0 = d', D_1 = d | D_0 > 0, D_1 > 0\right) \\
&= \sum_{d=1,d'=1}^{K,K} \mathbb{E}\left[Y_{1,d} - Y_{0,d} | D_0 = d', D_1 = d\right] \mathbb{P}\left(D_0 = d', D_1 = d | D_0 > 0, D_1 > 0\right) \\
&= \sum_{d=1,d'=1}^{K,K} LCDE\left(d | d', d\right) \mathbb{P}\left(T = \left(d', d\right) | D_0 > 0, D_1 > 0\right) \ ,
\end{aligned}$$

and similarly,

$$\begin{aligned}
&\mathbb{E}\left[Y_{0,D_1} - Y_{0,D_0} | D_0 > 0, D_1 > 0\right] \\
&= \sum_{d=1,d'=1}^{K,K} \mathbb{E}\left[Y_{0,D_1} - Y_{0,D_0} | D_0 = d', D_1 = d\right] \mathbb{P}\left(D_0 = d', D_1 = d | D_0 > 0, D_1 > 0\right) \\
&= \sum_{d=1,d'=1}^{K,K} \mathbb{E}\left[Y_{0,d} - Y_{0,d'} | D_0 = d', D_1 = d\right] \mathbb{P}\left(D_0 = d', D_1 = d | D_0 > 0, D_1 > 0\right) \\
&= \sum_{d=1,d'=1,d\neq d'}^{K,K} \mathbb{E}\left[Y_{0,d} - Y_{0,d'} | D_0 = d', D_1 = d\right] \mathbb{P}\left(D_0 = d', D_1 = d | D_0 > 0, D_1 > 0\right) \\
&= \sum_{d=1,d'=1,d\neq d'}^{K,K} LCIE\left(0, d, d' | d', d\right) \mathbb{P}\left(T = \left(d', d\right) | D_0 > 0, D_1 > 0\right) \ ,
\end{aligned}$$

and plugging these values back into (B.1) immediately implies the result. □

B.2. **Proof of Lemma 4.**

Proof. By the definition of T, it is clear that the response-type probabilities satisfy the eqs (4.4, 4.5). It suffices to show that given any solution $\left(\boldsymbol{p}_{(d,d')} : (d',d) \in \{0,\ldots,K\}^2\right) \geq 0$ such that $\sum_{d=0,d'=0}^{K,K} \boldsymbol{p}_{(d,d')} = 1$ and satisfying, for every $d \in \{0,\ldots,K\}$,

$$\mathbb{P}(D = d|Z = 1) = \sum_{d'=0}^{K} \boldsymbol{p}_{(d,d')} \tag{B.3}$$

$$\text{and } \mathbb{P}(D = d|Z = 0) = \sum_{d'=0}^{K} \boldsymbol{p}_{(d,d')} , \tag{B.4}$$

there exists a joint distribution Q of $((Y_{0,d} : d \in \{0,\ldots,K\}), (Y_{1,d} : d \in \{0,\ldots,K\}), D_0, D_1, Z)$ such that

$$\left(\mathbb{P}_Q\left[T = (d,d')\right] : (d',d) \in \{0,\ldots,K\}^2\right) = \left(\boldsymbol{p}_{(d,d')} : (d',d) \in \{0,\ldots,K\}^2\right) ,$$

and Q induces a distribution of (Y,D,Z) under (2.5, 2.6) and Assumption 1, that is consistent with the observed data. Since Y is not observed when $D = 0$, set $Y|D = 0, Z = 1$ and $Y|D = 0, Z = 0$ to arbitrary distributions, so that we can treat (Y,D,Z) as observed. We will now construct a Q that induces this distribution.

Define $\mathcal{Y}_{z,d} := \operatorname{supp}(Y|D = d, Z = z)$ and, for each $d \in \{0,\ldots,K\}$, $z \in \{0,1\}$, and $(d',d'') \in \{0,\ldots,K\}^2$ define the CDF $F_{(d',d'')}^{(z,d)}$ as

$$F_{(d',d'')}^{(z,d)}(y) = P\left(Y \leq y|D = d, Z = z\right) ,$$

and note that it does not depend on (d',d''). Next, for every $(d',d'') \in \{0,\ldots,K\}^2$, let $C_{(d',d'')}$ be an arbitrary copula of dimension $|\{0,1\} \times \{0,\ldots,K\}|$. Finally, define Q as

$$Q\left(y,t,z\right) := C_t\left(\left(F_t^{(z,d)}\left(y_{(z,d)}\right) : (z,d) \in \{0,1\} \times \{0,\ldots,K\}\right)\right) \times \boldsymbol{p}_{(t)} \times P\left(Z = z\right) ,$$

for any $y \in \prod_{(z,d)\in\{0,1\}\times\{0,\ldots,K\}} \mathcal{Y}_{z,d}$, $t \in \{0,\ldots,K\}^2$, and $z \in \{0,1\}$, where $Q(y,t,z)$ is shorthand for

$$Q\left((Y_{z,d} : (z,d) \in \{0,1\} \times \{0,\ldots,K\}) \leq y, (D_0,D_1) = t, Z = z\right)$$

Next, for all $(d',d'') \in \{0,\ldots,K\}^2$, let the conditional joint distribution

$$Q\left(Y_{z,d} \leq y_{z,d}|T = (d',d'')\right) .$$

By construction, Q satisfies Assumption 1 and $\left(\mathbb{P}_Q\left[T=(d,d')\right]:(d',d)\in\{0,\ldots,K\}^2\right)=\left(\boldsymbol{p}_{(d,d')}:(d',d)\in\{0,\ldots,K\}^2\right)$. The result now follows immediately by noting that Q induces the observed data distribution under (2.5, 2.6) since, for any $y\in\mathcal{Y}_{z,d}, d\in\{0,\ldots,K\}$ and $z\in\{0,1\}$, we have that

$$\begin{aligned}
Q\left(Y_{z,d}\leq y, D_z=d, Z=z\right) &= Q\left(Y_{z,d}\leq y|D_z=d,Z=z\right)Q\left(D_z=d|Z=z\right)Q\left(Z=z\right)\\
&= Q\left(Y_{z,d}\leq y|D_z=d\right)Q\left(D_z=d\right)P\left(Z=z\right)\\
&= P\left(Z=z\right)\sum_{d'=1}^{K,K}Q\left(Y_{z,d}\leq y|D_z=d,D_{1-z}=d'\right)\left((1-z)\boldsymbol{p}_{(d,d')}+z\boldsymbol{p}_{(d',d)}\right)\\
&= P\left(Z=z\right)\sum_{d'=1}^{K,K}\left((1-z)F_{(d,d')}^{(z,d)}\left(y\right)\boldsymbol{p}_{(d,d')}+zF_{(d',d)}^{(z,d)}\left(y\right)\boldsymbol{p}_{(d',d)}\right)\\
&= P\left(Z=z\right)P\left(Y\leq y|D=d,Z=z\right)\sum_{d'=1}^{K,K}\left((1-z)\boldsymbol{p}_{(d,d')}+z\boldsymbol{p}_{(d',d)}\right)\\
&= P\left(Z=z\right)P\left(Y\leq y|D=d,Z=z\right)P\left(D=d|Z=z\right)\\
&= P\left(Y\leq y,D=d,Z=z\right)
\end{aligned}$$

where the second equality follows from Q satisfying Assumption 1, and the penultimate equality follows from $\boldsymbol{p}$ satisfying (B.3) and (B.4). □

B.3. **Proof of Lemma 4.**

Proof. Given Lemma 3 above, this result follows immediately from Theorem 3 in Vayalinkal (2024). Since our proof of Lemma 3 is constructive, however, we can also argue directly, as follows. Note that both parts below proceed by showing the contrapositive.

($\Longleftarrow$) If Assumption 1 and $\mathcal{R}_T$ are consistent with the data, then there exists a joint distribution Q of $\left(\left(Y_{0,d}:d\in\{0,\ldots,K\}\right),\left(Y_{1,d}:d\in\{0,\ldots,K\}\right),D_0,D_1,Z\right)$ that is consistent with the observed data distribution such that the response-type probabilities induced by Q is in $\Theta_I(\mathcal{R}_T)$ and so $\Theta_I(\mathcal{R}_T)\neq\emptyset$.

($\Longrightarrow$) Suppose that $\Theta_I(\mathcal{R}_T)\neq\emptyset$, then there exists $\boldsymbol{p}=\left(\boldsymbol{p}_{(d,d')}:(d',d)\in\{0,\ldots,K\}^2\right)\in\Theta_I(\mathcal{R}_T)$. Now, our proof of Lemma 3 shows that we can construct a joint distribution Q of $\left(\left(Y_{0,d}:d\in\{0,\ldots,K\}\right),\left(Y_{1,d}:d\in\{0,\ldots,K\}\right),D_0,D_1,Z\right)$ such that Q satisfies

Assumption 1 and induces the observed data distribution under (2.5, 2.6), and

$$\left(\mathbb{P}_Q\left[T=(d,d')\right] : (d',d) \in \{0,\dots,K\}^2\right) = \left(\boldsymbol{p}_{(d,d')} : (d',d) \in \{0,\dots,K\}^2\right) ,$$

which implies that Q also satisfies $\mathcal{R}_T$, as required, since $\boldsymbol{p} \in \Theta_I(\mathcal{R}_T)$. □

B.4. **Proof of Lemma 5.**

Proof. Since (i) is just a special case of (ii), it suffices to show (ii). Let W and $\{W_k\}_{k=0}^K$ be defined on the common probability space $(\Omega, \Sigma, \mathrm{P})$, and take values in the set $\mathcal{W} \subseteq \mathbb{R}$, equipped with the Borel sigma algebra and a probability measure μ. Moreover, let μ be such that W and $\{W_k\}_{k=0}^K$ are μ-integrable and have densities with respect to μ. Denote the μ-density of W by f_W, and denote the μ-density of W_k by f_{W_k}, for $k \in \{0,\dots,K\}$.

First, we show that the bounds are valid. Define $\bar{\gamma} := \sum_{k=l}^{l'} \gamma_k$ and let $U \sim$ Uniform$[0,1]$. Now, suppose that

$$\mathbb{E}\left[F_W^{-1}(U) | U \leq \bar{\gamma}\right] > \sum_{k=l}^{l'} \frac{\gamma_k}{\bar{\gamma}} \mathbb{E}[W_k] ,$$

then there must exist w such that

$$\mathbb{P}\left(F_W^{-1}(U) \leq w | U \leq \bar{\gamma}\right) < \sum_{k=l}^{l'} \frac{\gamma_k}{\bar{\gamma}} \int_{(-\infty,w]} f_{W_k}(x)\ d\mu(x) = \frac{1}{\bar{\gamma}} \int_{(-\infty,w]} \sum_{k=l}^{l'} \gamma_k f_{W_k}(x)\ d\mu(x) .$$

Now, note that

$$\begin{aligned}\mathbb{P}\left(F_W^{-1}(U) \leq w | U \leq \bar{\gamma}\right) &= \mathbb{P}\left(F_W^{-1}(\bar{\gamma}U) \leq w\right) = \mathbb{P}\left(\bar{\gamma}U \leq F_W(w)\right) \\ &= \frac{F_W(w)}{\bar{\gamma}} = \frac{\int_{(-\infty,w]} f_W(x)\ d\mu(x)}{\bar{\gamma}} ,\end{aligned}$$

but this implies that

$$\int_{(-\infty,w]} \sum_{k=0}^{K} \gamma_k f_{W_k}(x)\ d\mu(x) = \int_{(-\infty,w]} f_W(x)\ d\mu(x) < \int_{(-\infty,w]} \sum_{k=l}^{l'} \gamma_k f_{W_k}(x)\ d\mu(x) ,$$

which, in turn, implies that

$$\int_{(-\infty,w]} \sum_{k=0}^{l-1} \gamma_k f_{W_k}(x) + \sum_{k=l'+1}^{K} \gamma_k f_{W_k}(x)\ d\mu(x) < 0 ,$$

a contradiction. Therefore, the lower bound is valid. The validity of the upper bound follows from the analogous argument.

Define $\underline{\gamma} := \bar{\gamma} + \sum_{k'=0}^{l-1} \gamma_{k'}$. Now, sharpness of the lower bound follows immediately by defining each F_{W_k} as follows

$$F_{W_k}(w) := \begin{cases} \mathbb{P}\left(F_W^{-1}(U) \leq w | U \in \left(\sum_{k'=l}^{k-1} \gamma_{k'}, \sum_{k'=l}^{k} \gamma_{k'}\right)\right) & \text{if } k \in \{l, \ldots, l'\} \\ \mathbb{P}\left(F_W^{-1}(U) \leq w | U \in \left(\bar{\gamma} + \sum_{k'=0}^{k-1} \gamma_{k'}, \bar{\gamma} + \sum_{k=0}^{k} \gamma_{k'}\right)\right) & \text{if } k \in \{0, \ldots, l-1\} \\ \mathbb{P}\left(F_W^{-1}(U) \leq w | U \in \left(\underline{\gamma} + \sum_{k'=l'+1}^{k-1} \gamma_{k'}, \underline{\gamma} + \sum_{k'=l'+1}^{k} \gamma_{k'}\right)\right) & \text{all other } k \end{cases}.$$

Sharpness of the upper bound follows from the analogous construction. □

B.5. **Proof of Theorem 1.**

Proof. Define

$$\mathbf{f}_{(z,d)|d,d'}(y) \equiv \mathbb{P}[T = (d', d)] f_{Y_{z,d}|T}(y|d', d) \ ,$$

and also define the shorthand notation

$$\mathbf{f}_{z|d,d'}(y) \equiv \mathbb{P}[T = (d', d)] f_{Y_{z,T(z)}|T}(y|d', d) \ .$$

Consider the vector of weighted response-type conditional densities

$$\mathbf{f}(y) \equiv \left(\mathbf{f}_{(z,d'')|d,d'}(y) : d, d', d'' \in \{0, \ldots, K\}, z \in \{0, 1\}\right) \ .$$

First, note that $\mathbf{f}$ must satisfy (4.2) and (4.3), i.e. we must have that

$$\begin{aligned} f_{Y,D=d|Z=1}(y) &= \sum_{d'=0}^{K} \mathbb{P}[T = (d', d)] f_{Y_{1,d}|T}(y|d', d) = \sum_{d'=0}^{K} \mathbf{f}_{1|d',d}(y) \\ f_{Y,D=d|Z=0}(y) &= \sum_{d'=0}^{K} \mathbb{P}[T = (d, d')] f_{Y_{0,d}|T}(y|d, d') = \sum_{d'=0}^{K} \mathbf{f}_{0|d,d'}(y) \end{aligned}$$

for all $d \in \{1, \ldots, K\}$. Second, we must also have that there exists a $p := (p_t : t \in \text{supp}(T)) \in \Theta_I(\mathcal{R}_T)$ such that $\int_{y_L}^{y_U} \mathbf{f}_{(z,d'')|d',d}(y) \; d\mu(y) = p_{d',d}$ for all $z \in \{0, 1\}$ and $d, d', d'' \in \{0, \ldots, K\}$. Finally, we must have that each component of $\mathbf{f}$ is a non-negative function supported on (a subset of) $[y_L, y_U]$

The remainder of the proof proceeds in two parts. We first show that these conditions are sharp (i.e. they define the identified set of $\mathbf{f}$). We then show that this allows us to complete the proof using the sharp bounds on mixture components given in Lemma 5.

A Preliminary: Identified set of $\mathbf{f}$. Note that for any type t and $z \in \{0,1\}$, $f_{Y_{z,d''}|T}(y|t)$ is independent of the data whenever $d'' \neq t(z)$, and so, is only constrained to be a density that is supported over (a subset of) $[y_L, y_U]$, the support of $Y_{z,d''}$; this immediately implies that the sharp identification region for the expectation of any such component is simply $[y_L, y_U]$. Given Lemma 3 above, it now follows from Theorem 3.2 in Vayalinkal (2024) that these conditions are sharp, i.e. any $\mathbf{f}$ satisfying these conditions is consistent with the data. We summarize the argument here, as follows.

First, note that the observed data depends only on the (i) the distribution of Z (F_Z), (ii) the marginal distribution of D_z for each $z \in \{0,1\}$, and (iii) the conditional marginal distribution of $Y_{z,d}$ given $D_z = d, Z = z$ for all $d \in \{1,\ldots,K\}$ and $z \in \{0,1\}$. For any joint distribution $((Y_{z,d} : d \in \{0,\ldots,K\}, z \in \{0,1\}), T, Z) \sim Q$, let $\mathbf{f}_Q$ be the vector of weighted response-type conditional densities implied by Q. Given $\mathbf{f}$ satisfying the conditions above, we construct a Q with $\mathbf{f}_Q = \mathbf{f}$ as follows: define $Q_Z = F_Z$, $Q(T = t) = \int_{\mathcal{Y}} \mathbf{f}_{1|t}(y) d\mu(y)$, and define

$$\begin{aligned} &Q\left(Y_{0,0} \le y_{0,0}, \ldots, Y_{0,K} \le y_{0,K}, Y_{1,0} \le y_{1,0}, \ldots, Y_{1,K} \le y_{1,K}, T = t, Z = z\right) \\ &= \left(\prod_{k=0}^{K} \int_{y_L}^{y_{0,k}} \mathbf{f}_{(0,k)|t}(y) d\mu(y)\right) \left(\prod_{k=0}^{K} \int_{y_L}^{y_{1,k}} \mathbf{f}_{(0,k)|t}(y) d\mu(y)\right) Q\left(T = t\right) Q\left(Z = z\right) \ . \end{aligned}$$

The above construction assumes that the potential outcome distributions are independent given T, but any dependence structure (copula) can be used, after conditioning on a value of T. Suppose we are given a Q such that $\mathbf{f}_Q$ satisfies the conditions above. By construction, $Q_Z = F_Z$, $Q(D_z = d) = \sum_{t:t(z)=d} Q(T = t) = \sum_{t:t(z)=d} \int_{y_L}^{y_U} \mathbf{f}_{z|t}(y) d\mu(y) = P(D = d|Z = z)$ for all $d \in \{1,\ldots,K\}$ and $z \in \{0,1\}$. This also implies $Q(D_z = 0) = P(D = 0|Z = z)$ by the definition of Θ_I and, finally, we have that for any $z \in \{0,1\}, d \in \{1,\ldots,K\}$

$$\begin{aligned} Q\left(Y_{z,d} \le y | D_z = d, Z = z\right) &= \sum_{t:t(z)=d} \int_{y_L}^{y} \mathbf{f}_{z|t}(y) d\mu(y) = \int_{y_L}^{y} \sum_{t:t(z)=d} \mathbf{f}_{z|t}(y) d\mu(y) \\ &= \int_{y_L}^{y} f_{Y,D=d|Z=z}(y) d\mu(y) = P(Y \le y | D = d, Z = z) \ , \end{aligned}$$

as required, showing that the above conditions define the identified set for $\mathbf{f}$.

Remainder of proof. Since the two mixtures given by (4.2) and (4.3) do not share any components, the above result reduces the problem of finding bounds on the conditional expectation of $Y_{d,z}$ given T to the problem of finding sharp bounds on expectations of mixture components. Therefore, we now complete the proof using the results given in Lemma 5, as follows.

Proof of (i) and (ii): For any type (d',d) and any $z \in \{0,1\}$, the weighted conditional density $\mathbb{P}[T=(d',d)] \times f_{Y_{z,T(z)}|T}(y|d',d)$ only appears in at most one of (4.2) and (4.3). Therefore, sharp bounds on the expectation of any such component can be obtained as the bounds of Horowitz and Manski (1995) (HM), which are the bounds provided in Lemma 5(i) evaluated at the smallest feasible value of γ_k. This immediately implies the validity and sharpness of (ii) and the last two parts of (i) above. This also implies that the bounds in the first part of (i) are valid, as follows: (I) LCDE$(d|d,d)$ consists of the difference in expectation of two such components, and (II) the lower (upper) bound is given by the HM lower (upper) bound of the first component minus the HM upper (lower) bound of the second component. For sharpness, first note that LCDE$(d|d,d)$ consists of the difference in expectation of two components, each of which belongs to a *different* mixture of the two defined by (4.2) and (4.3). Since these two mixtures do not share any components (only weights), the two HM bounds can be attained jointly whenever the weights $\underline{\gamma}_{d,d}^{1,r}$ and $\underline{\gamma}_{d,d}^{0,r}$ are jointly feasible. The result now follows by noting that $\underline{\gamma}_{d,d}^{1,r}$ and $\underline{\gamma}_{d,d}^{0,r}$ are jointly feasible if and only if $\mathbb{P}(D=d|Z=1)\underline{\gamma}_{d,d}^{1,r} = \mathbb{P}(D=d|Z=0)\underline{\gamma}_{d,d}^{0,r} = \underline{p}_{d,d'}^{r}$ belongs to the identified set for $p_{d,d'}$ which is true by definition of $\underline{p}_{d,d'}^{r}$.

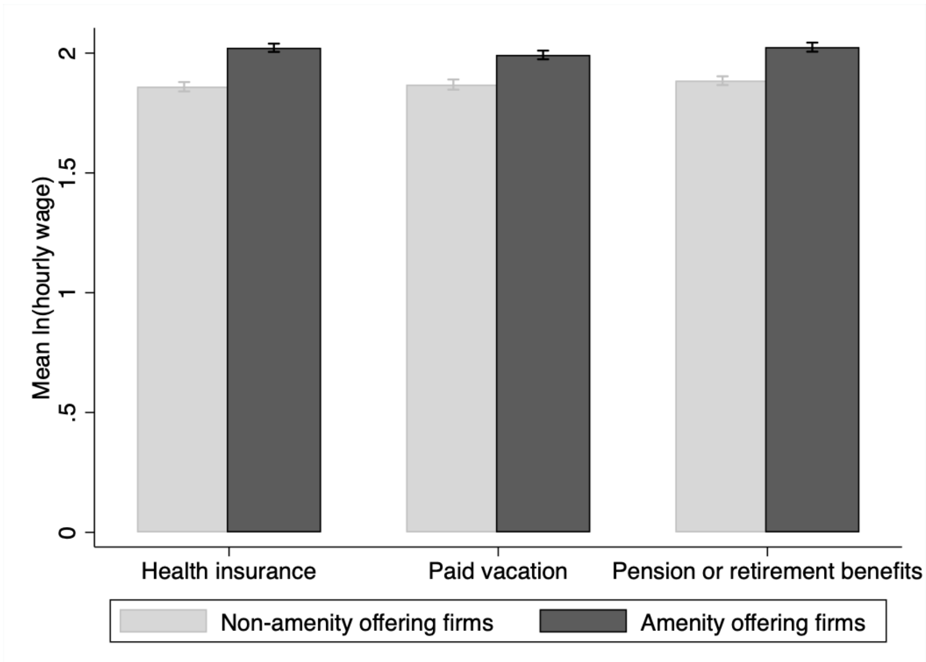
Proof of (iii): Finally, for (iii), note that

$$\begin{aligned}
\sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d=l}^{l'} p_{d,d}} \text{LCDE}(d|d,d) &= \sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d=l}^{l'} p_{d,d}} \mathbb{E}[Y_{1,d} - Y_{0,d}|T=(d,d)] \\
= \sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d=l}^{l'} p_{d,d}} \mathbb{E}[Y_{1,d}|T=(d,d)] &- \sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d=l}^{l'} p_{d,d}} \mathbb{E}[Y_{0,d}|T=(d,d)] \\
= \sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d=l}^{l'} p_{d,d}} \int_{\mathcal{Y}} y \mathbf{f}_{1|d,d}(y)\ d\mu(y) &- \sum_{d=l}^{l'} \frac{p_{d,d}}{\sum_{d=l}^{l'} p_{d,d}} \int_{\mathcal{Y}} y \mathbf{f}_{0|d,d}(y)\ d\mu(y)\ ,
\end{aligned}$$

where the first term is the expectation of the aggregation of components of the mixture (4.2) and the second term is the expectation of the aggregation of components of the mixture (4.3); we can now use the same argument as above, but now based on Lemma 5(ii), as follows. First, suppose that $\{(p_{d,d} : l \leq d \leq l') \mid p \in \Theta_I(\mathcal{R}_T)\}$ is a singleton, so that the weights are known and there is no optimization required: sharpness now follows immediately from Lemma 5(ii) since the two mixtures do not share any components. Finally, when this is not the case, the sharp bounds are obtained by maximizing (resp. minimizing) the pointwise (in $(p_{d,d} : l \leq d \leq l')$) upper (resp. lower) bound over the identified set for $(p_{d,d} : l \leq d \leq l')$, which is exactly the bounds given in (iii). $\square$

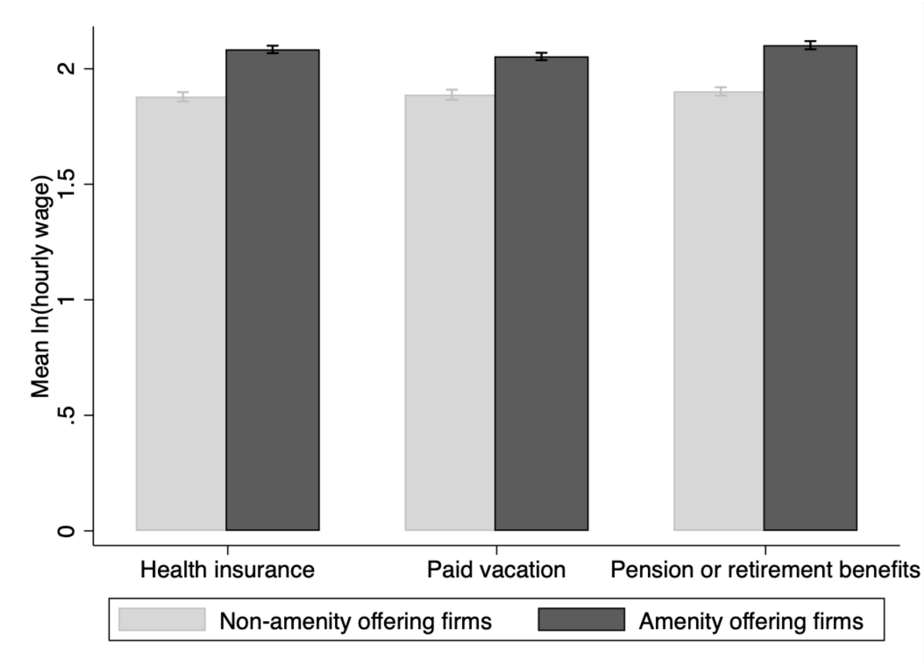
Appendix C. Additional Empirical Tables and Figures

C.1. **Descriptive figures and tables.** Appendix C.1 provides descriptive figures and tables from Section 5 for weeks 135, 180 and 208.



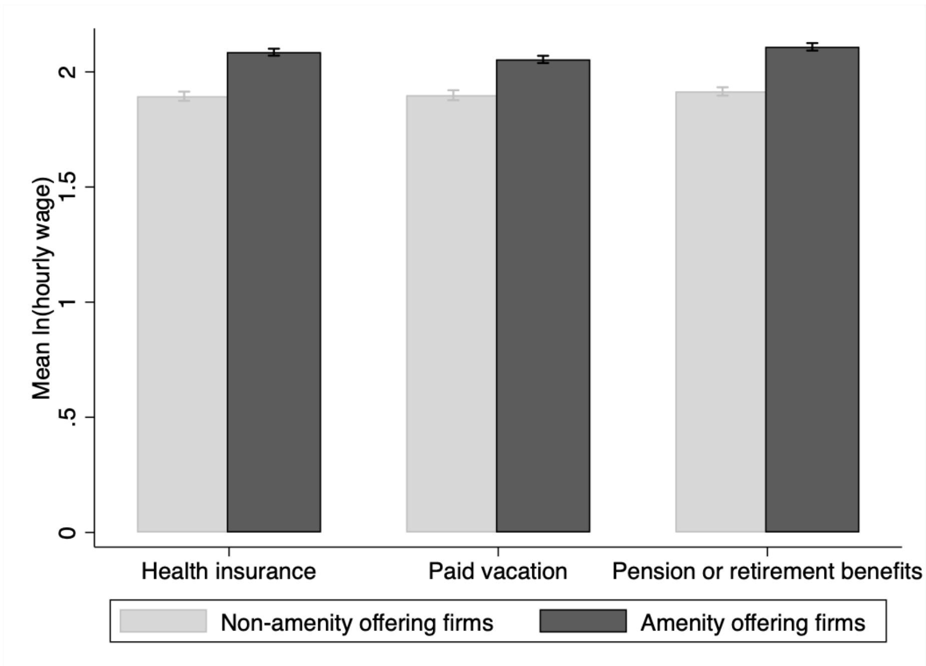
Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

Figure 11. Mean ln(hourly Wage) by firm amenity provision at week 135



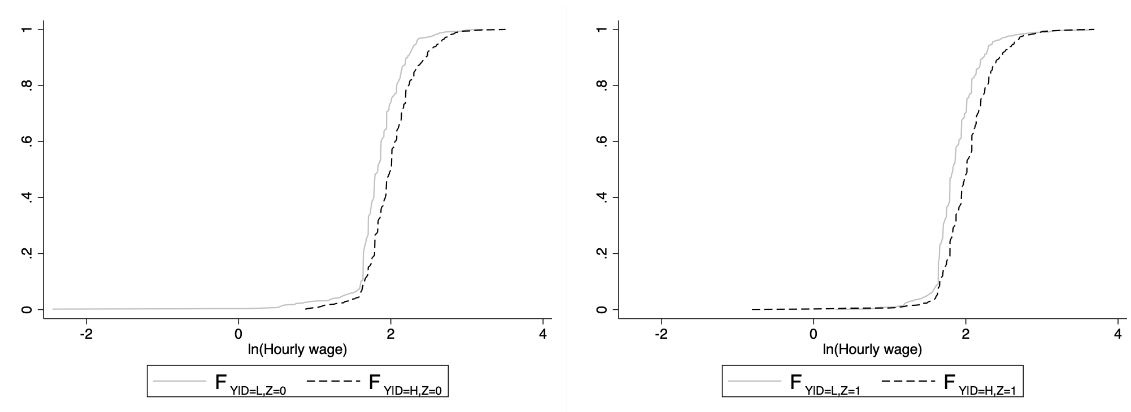
Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

FIGURE 12. Mean ln(hourly Wage) by firm amenity provision at week 180



Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

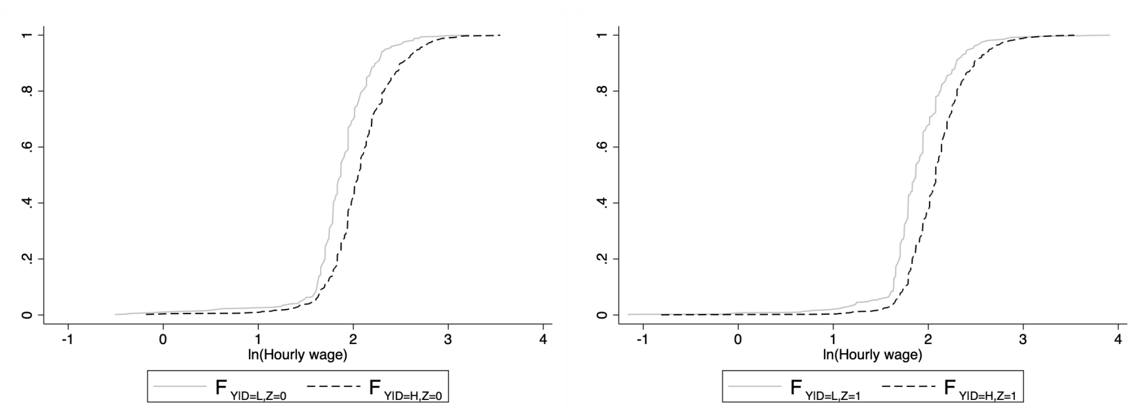
Figure 13. Mean ln(hourly Wage) by firm amenity provision at week 208



(a) control units (b) treated units

Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

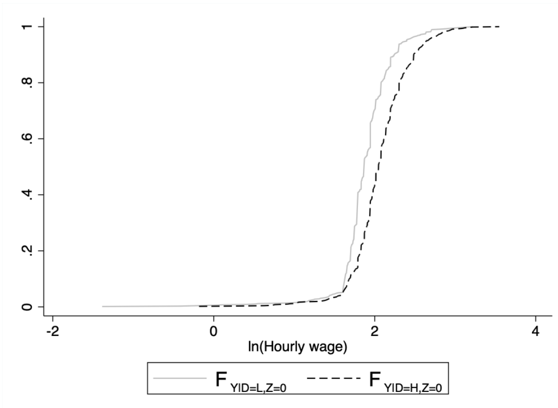
FIGURE 14. Cumulative distribution function by firm type at week 135, amenity=health



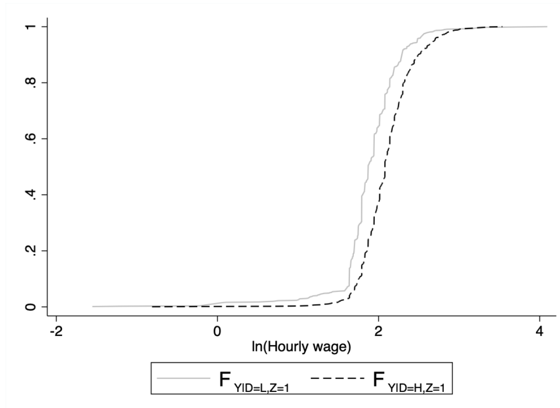
(a) control units (b) treated units

Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

FIGURE 15. Cumulative distribution function by firm type at week 180, amenity=health



(a) control units



(b) treated units

Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

FIGURE 16. Cumulative distribution function by firm type at week 208, amenity=health

	Control		Treated		Difference	
	Mean	S.D.	Mean	S.D.	Difference	S.E.
Health insurance	0.5375	0.4988	0.5514	0.4975	0.0139	0.0177
Paid sick leave	0.4614	0.4987	0.4552	0.4981	-0.0061	0.0177
Paid vacation	0.6067	0.4887	0.5954	0.4909	-0.0114	0.0171
Childcare assistance	0.1254	0.3313	0.1411	0.3482	0.0157	0.0120
Flexible hours	0.5600	0.4966	0.5794	0.4938	0.0194	0.0174
Employer-provided transportation	0.1899	0.3924	0.1867	0.3898	-0.0032	0.0139
Pension or retirement benefits	0.4355	0.4960	0.4428	0.4968	0.0073	0.0176
Dental plan	0.4745	0.4995	0.4733	0.4994	-0.0012	0.0178
Tuition reimbursement	0.2574	0.4374	0.2883	0.4531	0.0309	0.0158
Employment	0.4923	0.5000	0.5242	0.4995	0.0319	0.0112
Sample size	3288		5091		8379	

TABLE 10. Probability of working at amenity-providing firm at week 135 (conditional on employment)

	Control		Treated		Difference	
	Mean	S.D.	Mean	S.D.	Difference	S.E.
Health insurance	0.5503	0.4976	0.5754	0.4944	0.0251	0.0155
Paid sick leave	0.4639	0.4988	0.4952	0.5001	0.0313	0.0156
Paid vacation	0.6286	0.4833	0.6341	0.4818	0.0055	0.0147
Childcare assistance	0.1423	0.3495	0.1596	0.3663	0.0173	0.0111
Flexible hours	0.5622	0.4963	0.5839	0.4930	0.0217	0.0153
Employer-provided transportation	0.1841	0.3877	0.1974	0.3981	0.0133	0.0123
Pension or retirement benefits	0.4393	0.4965	0.4754	0.4995	0.0361	0.0156
Dental plan	0.4726	0.4994	0.5001	0.5001	0.0274	0.0156
Tuition reimbursement	0.2676	0.4428	0.3047	0.4604	0.0371	0.0141
Employment	0.5216	0.4996	0.5651	0.4958	0.0435	0.0111
Sample size	3288		5091		8379	

TABLE 11. Probability of working at amenity-providing firm at week 180 (conditional on employment)

	Control		Treated		Difference	
	Mean	S.D.	Mean	S.D.	Difference	S.E.
Health insurance	0.5619	0.4963	0.5945	0.4911	0.0326	0.0151
Paid sick leave	0.4752	0.4995	0.5187	0.4997	0.0435	0.0153
Paid vacation	0.6363	0.4812	0.6488	0.4774	0.0126	0.0143
Childcare assistance	0.1441	0.3513	0.1680	0.3739	0.0239	0.0110
Flexible hours	0.6038	0.4892	0.6088	0.4881	0.0049	0.0147
Employer-provided transportation	0.1877	0.3906	0.2038	0.4029	0.0160	0.0121
Pension or retirement benefits	0.4500	0.4976	0.4883	0.5000	0.0382	0.0152
Dental plan	0.4882	0.5000	0.5175	0.4998	0.0293	0.0153
Tuition reimbursement	0.2951	0.4562	0.3150	0.4646	0.0199	0.0140
Employment	0.5510	0.4975	0.5902	0.4918	0.0393	0.0111
Sample size	3288		5091		8379	

Table 12. Probability of working at amenity-providing firm at week 208 (conditional on employment)

C.2. **Distribution of Job Types.** Appendix C.2 presents the distribution of workers by the amenity category their job falls into, as discussed in Footnote 30, for all weeks of interest (90, 135, 180 and 208). The sample size in these distributions decreases to 6,232 individuals (=2,454 control units + 3,778 treated units). Constructing these distributions requires restricting the sample to workers who have non-missing amenity status for all three amenities simultaneously across weeks of interest. This is a stronger restriction than only requiring non-missing amenity status across weeks on a per amenity basis, as is the case for our primary sample of analysis.

	Control		Treated		Difference		Pooled	
	Mean	S.D.	Mean	S.D.	Difference	S.E.	ln(Hourly wage)	S.D.
H=1, R=1, V=1	0.2914	0.4547	0.3352	0.4722	0.0438	0.0198	1.9757	0.3192
H=1, R=1, V=0	0.0303	0.1715	0.0192	0.1375	-0.0110	0.0068	1.9308	0.3114
H=1, R=0, V=1	0.1123	0.3160	0.1073	0.3096	-0.0050	0.0135	1.8983	0.2501
H=1, R=0, V=0	0.0500	0.2180	0.0485	0.2149	-0.0015	0.0093	1.8232	0.2830
H=0, R=1, V=1	0.0288	0.1672	0.0341	0.1817	0.0054	0.0074	1.8834	0.2968
H=0, R=1, V=0	0.0195	0.1383	0.0129	0.1130	-0.0066	0.0055	1.8118	0.2250
H=0, R=0, V=1	0.0758	0.2648	0.0913	0.2881	0.0155	0.0118	1.7606	0.3116
H=0, R=0, V=0	0.3920	0.4885	0.3514	0.4776	-0.0406	0.0208	1.7558	0.3098
Sample size	2454		3778				6232	

Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

TABLE 13. Distribution of job types at week 90 (conditional on employment)

	Control		Treated		Difference		Pooled	
	Mean	S.D.	Mean	S.D.	Difference	S.E.	ln(Hourly wage)	S.D.
H=1, R=1, V=1	0.3598	0.4802	0.3806	0.4857	0.0208	0.0192	2.0418	0.3401
H=1, R=1, V=0	0.0262	0.1597	0.0270	0.1620	0.0008	0.0064	2.0489	0.3276
H=1, R=0, V=1	0.1118	0.3152	0.1007	0.3011	-0.0110	0.0123	1.9947	0.3217
H=1, R=0, V=0	0.0345	0.1825	0.0445	0.2063	0.0101	0.0076	1.8928	0.3839
H=0, R=1, V=1	0.0302	0.1713	0.0284	0.1661	-0.0019	0.0067	1.9083	0.2917
H=0, R=1, V=0	0.0222	0.1475	0.0188	0.1357	-0.0035	0.0057	1.8744	0.3267
H=0, R=0, V=1	0.0843	0.2779	0.0625	0.2421	-0.0218	0.0106	1.8479	0.3182
H=0, R=0, V=0	0.3311	0.4709	0.3376	0.4730	0.0064	0.0188	1.8545	0.3716
Sample size	2454		3778				6232	

Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

TABLE 14. Distribution of job types at week 135 (conditional on employment)

	Control		Treated		Difference		Pooled	
	Mean	S.D.	Mean	S.D.	Difference	S.E.	ln(Hourly wage)	S.D.
H=1, R=1, V=1	0.3648	0.4816	0.3899	0.4878	0.0251	0.0177	2.1175	0.3441
H=1, R=1, V=0	0.0202	0.1409	0.0324	0.1772	0.0122	0.0057	2.1131	0.3140
H=1, R=0, V=1	0.1208	0.3260	0.1045	0.3059	-0.0163	0.0117	2.0218	0.3153
H=1, R=0, V=0	0.0386	0.1928	0.0381	0.1915	-0.0005	0.0070	1.8936	0.4835
H=0, R=1, V=1	0.0289	0.1677	0.0306	0.1721	0.0016	0.0062	1.9896	0.3512
H=0, R=1, V=0	0.0114	0.1060	0.0183	0.1340	0.0069	0.0043	1.9150	0.3633
H=0, R=0, V=1	0.0851	0.2792	0.0711	0.2571	-0.0140	0.0099	1.8927	0.3296
H=0, R=0, V=0	0.3301	0.4704	0.3151	0.4647	-0.0149	0.0171	1.8606	0.4033
Sample size	2454		3778				6232	

Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

TABLE 15. Distribution of job types at week 180 (conditional on employment)

	Control		Treated		Difference		Pooled	
	Mean	S.D.	Mean	S.D.	Difference	S.E.	ln(Hourly wage)	S.D.
H=1, R=1, V=1	0.3663	0.4820	0.4083	0.4916	0.0420	0.0172	2.1271	0.3271
H=1, R=1, V=0	0.0256	0.1579	0.0298	0.1702	0.0043	0.0057	2.0602	0.3268
H=1, R=0, V=1	0.1198	0.3248	0.1073	0.3096	-0.0125	0.0113	2.0046	0.3436
H=1, R=0, V=0	0.0418	0.2003	0.0387	0.1929	-0.0031	0.0070	1.9273	0.4623
H=0, R=1, V=1	0.0291	0.1681	0.0276	0.1639	-0.0015	0.0059	1.9958	0.3719
H=0, R=1, V=0	0.0133	0.1148	0.0170	0.1293	0.0037	0.0043	1.9189	0.3324
H=0, R=0, V=1	0.0875	0.2827	0.0717	0.2580	-0.0158	0.0097	1.9253	0.2689
H=0, R=0, V=0	0.3166	0.4653	0.2996	0.4582	-0.0170	0.0163	1.8768	0.4223
Sample size	2454		3778				6232	

Notes: Hourly wage calculated as weekly earnings divided by weekly hours for the employed.

TABLE 16. Distribution of job types at week 208 (conditional on employment)

C.3. **Replication of Lee (2009) Bounds.** As discussed in Section 5.5.1, Appendix Table 17 reports our replication of the bounds reported in Lee (2009) for weeks 90, 135, 180 and 208 along with the trimming proportion $p \equiv \mathbb{P}(AE)$, e.g., the share of the always-employed among individuals receiving job training. We do not report bounds for week 45 since we discovered that the monotonicity assumption is violated.

Table 17 reports Lee's bounds when treating ln(hourly wage) as a continuous variable (as we do throughout the paper). All quantities are very close to the estimates in Lee (2009). There is a small difference that arises in the bounds due to Lee's use of vingtiles of ln(hourly wage). Table 18 shows that when we use vingtiles of ln(hourly wage), the bounds are identical to the ones reported in Lee (2009).

				$\mathbb{E}[Y_{1,D_1} - Y_{0,D_0} \| D_0 > 0, D_1 > 0]$	
	$\mathbb{P}[D > 0 \| Z = 0]$	$\mathbb{P}[D > 0 \| Z = 1]$	p	lower	upper
Week 90	0.4600	0.4601	0.0003	0.0468	0.0484
Week 135	0.5173	0.5451	0.0509	-0.0072	0.0842
Week 180	0.5403	0.5825	0.0724	-0.0325	0.0901
Week 208	0.5655	0.6068	0.0680	-0.0217	0.0989

Notes: Treatment bounds are for ln(hourly wage); hourly wage calculated as weekly earnings divided by weekly hours for the employed. Propensity scores and trimming proportion are numerically equivalent to Lee; slight numerical difference in bounds occurs as Lee uses vingtiles of ln(hourly wage) and we do not. See Table 18 for identical treatment bounds to Lee.

Table 17. Lee's bounds: continuous ln(hourly wage)

				$\mathbb{E}[Y_{1,D_1} - Y_{0,D_0} \| D_0 > 0, D_1 > 0]$	
	$\mathbb{P}[D > 0 \| Z = 0]$	$\mathbb{P}[D > 0 \| Z = 1]$	p	lower	upper
Week 90	0.4600	0.4601	0.0003	0.0423	0.0428
Week 135	0.5173	0.5451	0.0509	-0.0159	0.0757
Week 180	0.5403	0.5825	0.0724	-0.0325	0.0868
Week 208	0.5655	0.6068	0.0680	-0.0194	0.0933

Notes: Treatment bounds are for vingtiles of ln(hourly wage); hourly wage calculated as weekly earnings divided by weekly hours for the employed.

Table 18. Lee's bounds: vingtiles of ln(hourly wage)

C.4. **Identified Sets for Response Types.** Appendix C.4 provides propensity scores for when classifying firm type based on the provision of paid vacation and retirement/pension benefits. Appendix C.4 also provides the identified sets for $p_{L,L}, p_{H,H}$ for all weeks and amenities, with the exception of week 90 for health insurance which is provided in the main text.

	$\mathbb{P}[D = H \mid Z = 0]$	$\mathbb{P}[D = H \mid Z = 1]$	$\mathbb{P}[D = L \mid Z = 0]$	$\mathbb{P}[D = L \mid Z = 1]$
Week 90	0.2470	0.2673	0.2129	0.1928
Week 135	0.3102	0.3209	0.2071	0.2241
Week 180	0.3334	0.3589	0.2069	0.2236
Week 208	0.3516	0.3839	0.2139	0.2229

Table 19. Propensity scores by week, amenity: paid vacation

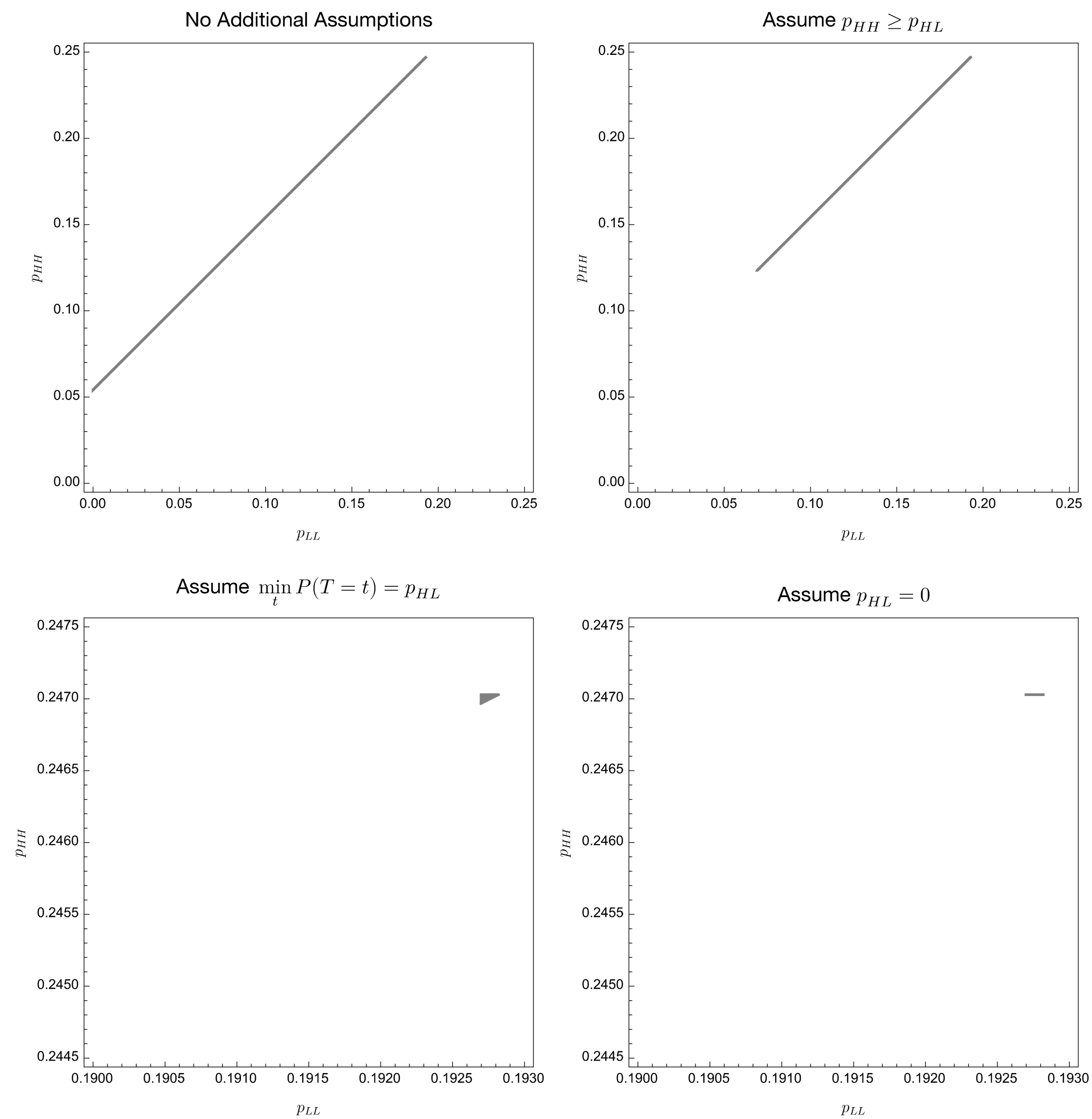


FIGURE 17. Identified set for $p_{L,L}, p_{H,H}$ at week 90. Amenity=paid vacation. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2. The scale of the bottom two panels is increased to focus on the upper right portion of the identified set.

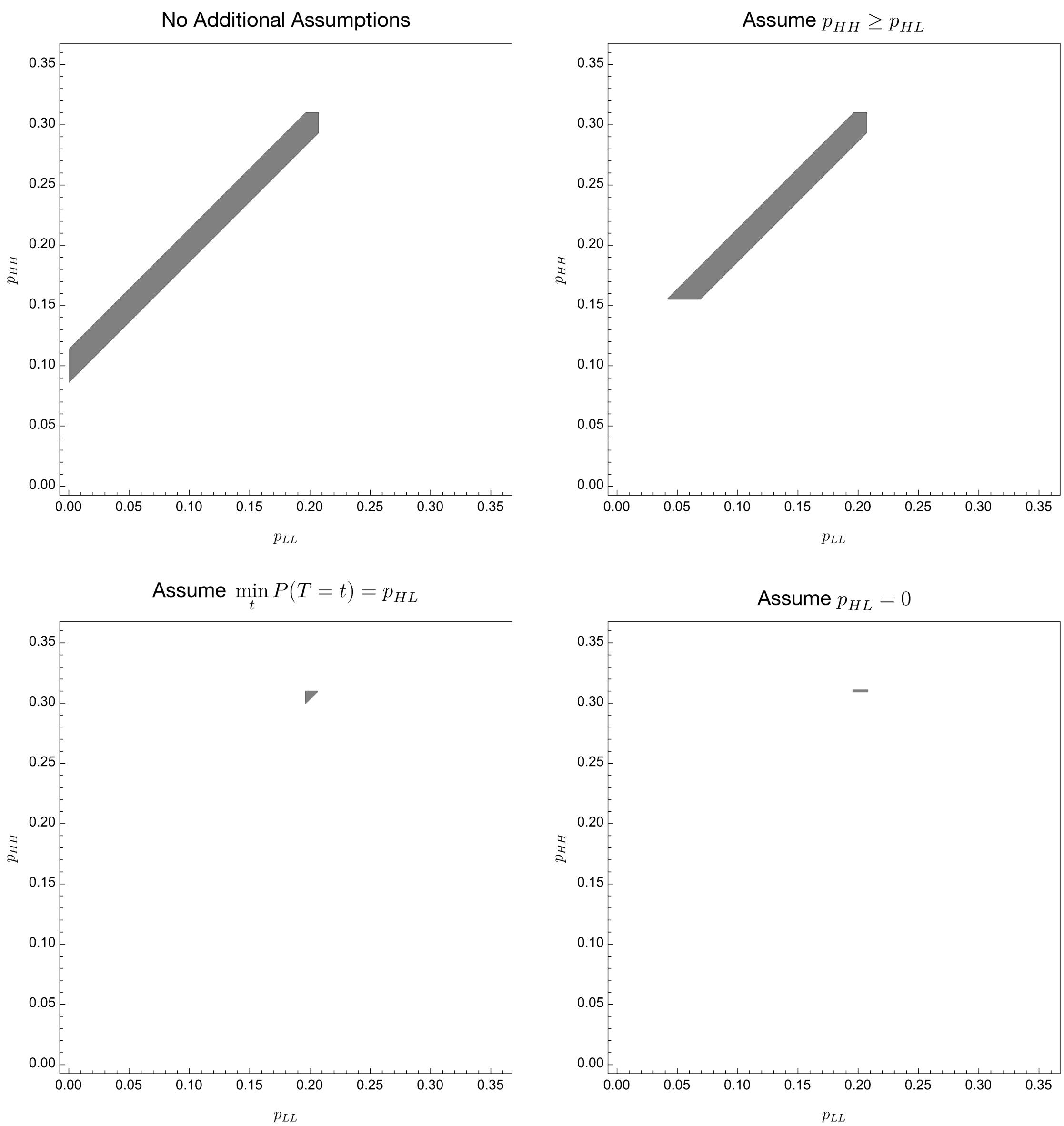


FIGURE 18. Identified set for $p_{L,L}, p_{H,H}$ at week 135. Amenity=paid vacation. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2.

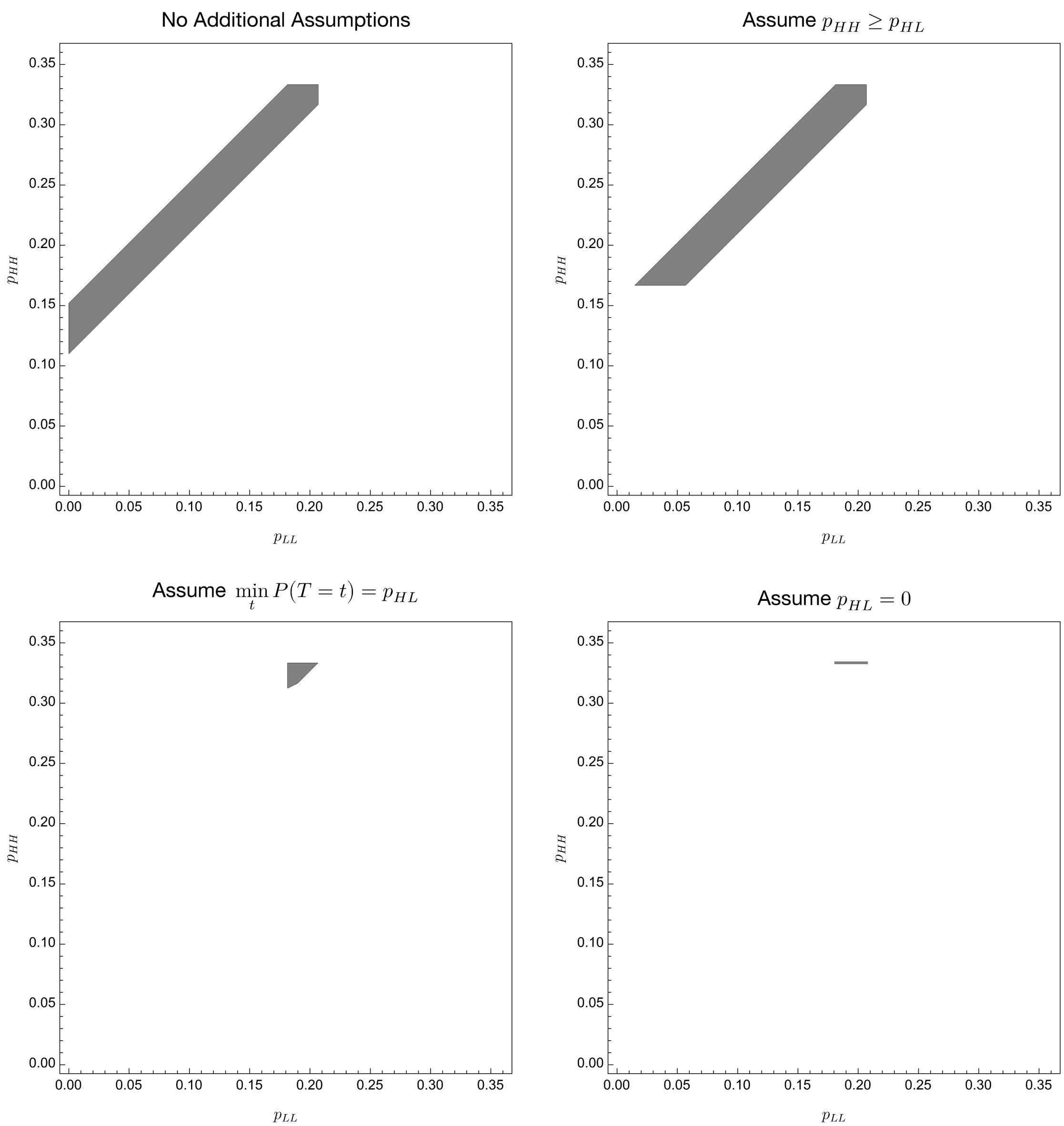


FIGURE 19. Identified set for $p_{L,L}, p_{H,H}$ at week 180. Amenity=paid vacation. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2.

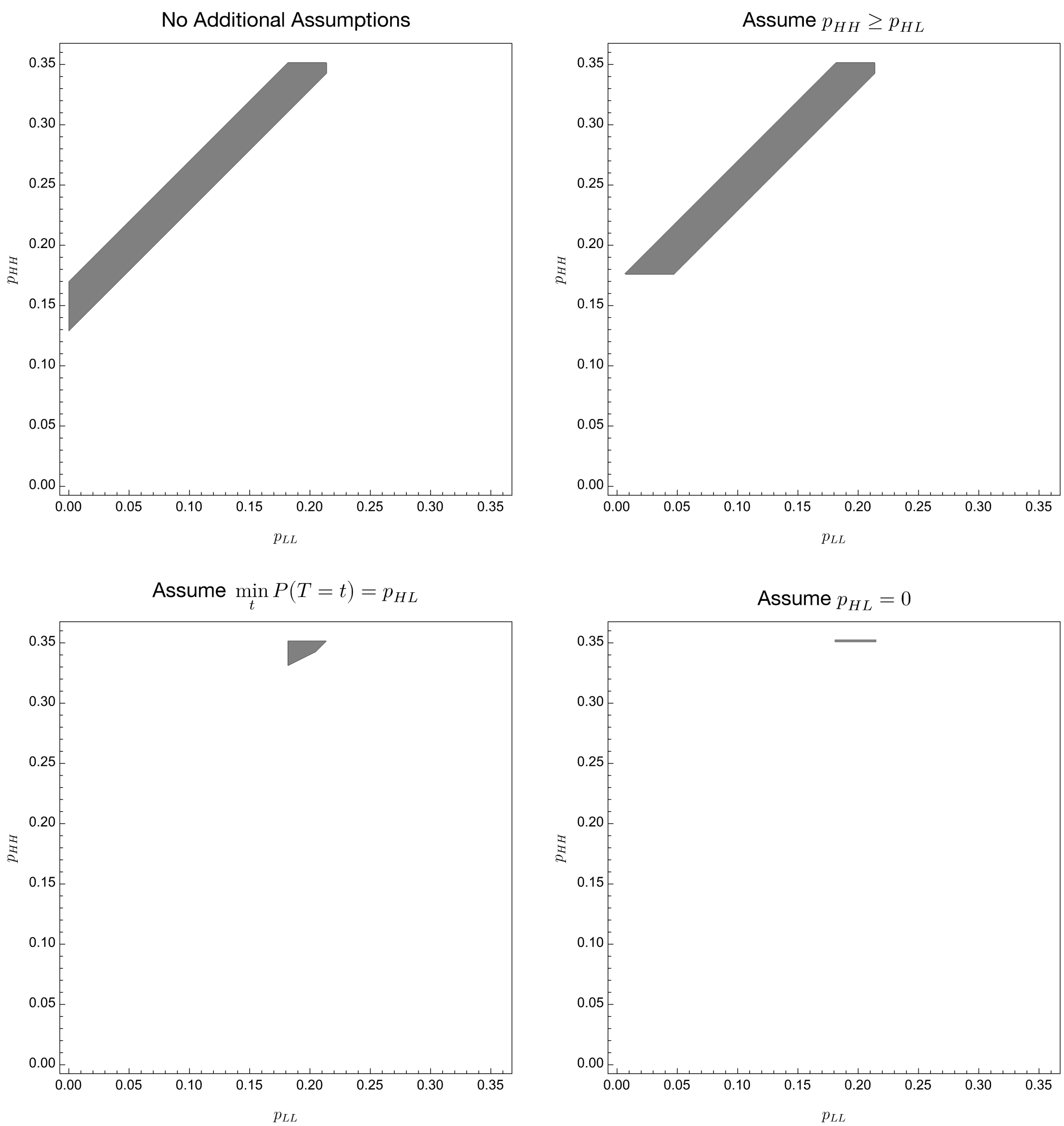


FIGURE 20. Identified set for $p_{L,L}, p_{H,H}$ at week 208. Amenity=paid vacation. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2.

	$\mathbb{P}[D=H\|Z=0]$	$\mathbb{P}[D=H\|Z=1]$	$\mathbb{P}[D=L\|Z=0]$	$\mathbb{P}[D=L\|Z=1]$
Week 90	0.1706	0.1852	0.2894	0.2749
Week 135	0.2275	0.2475	0.2898	0.2976
Week 180	0.2309	0.2748	0.3094	0.3076
Week 208	0.2460	0.2930	0.3195	0.3138

TABLE 20. Propensity scores by week, amenity: retirement/pension benefits

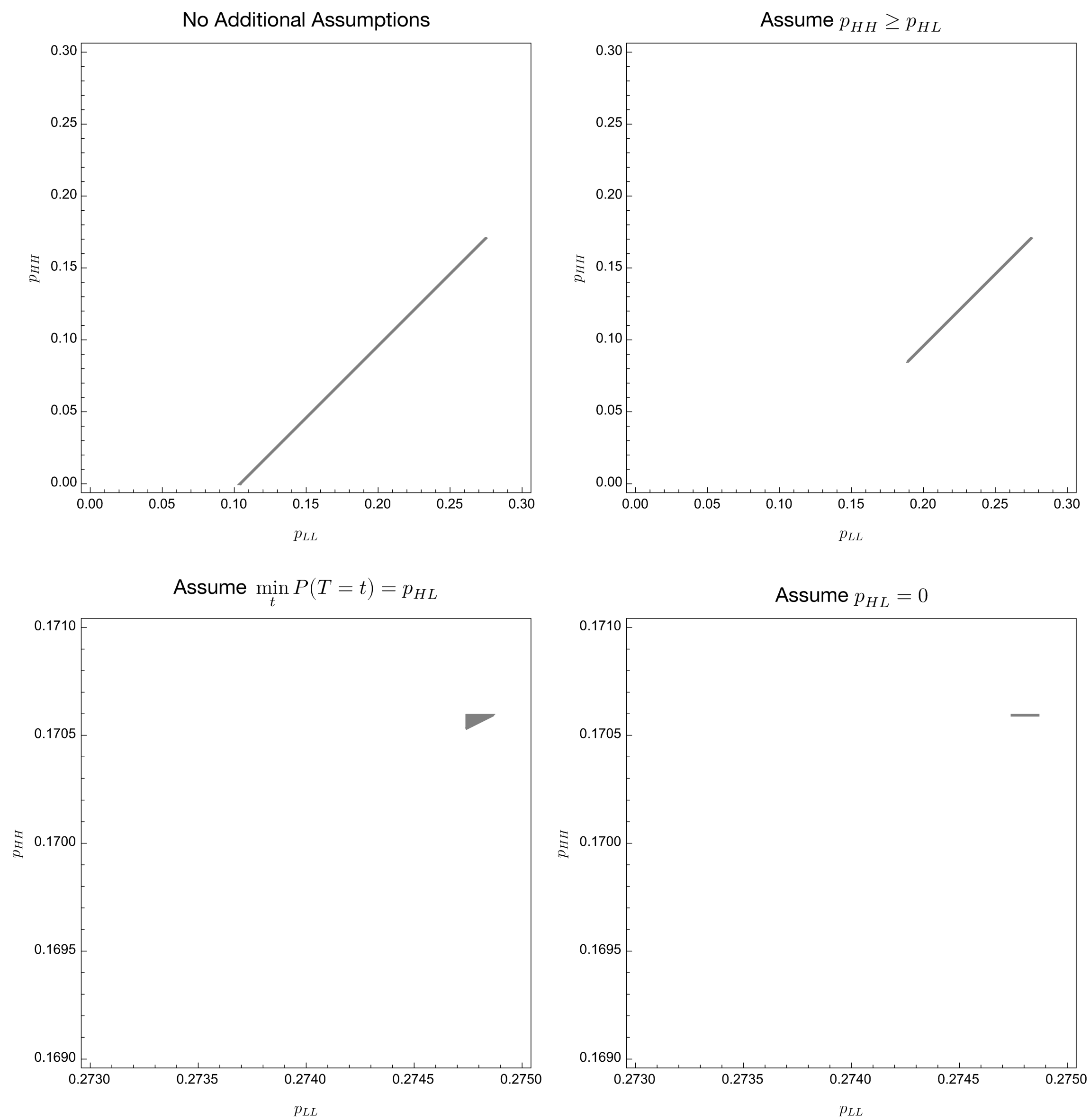


FIGURE 21. Identified set for $p_{L,L}, p_{H,H}$ at week 90. Amenity=retirement/pension benefits. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2. The scale of the bottom two panels is increased to focus on the upper right portion of the identified set.

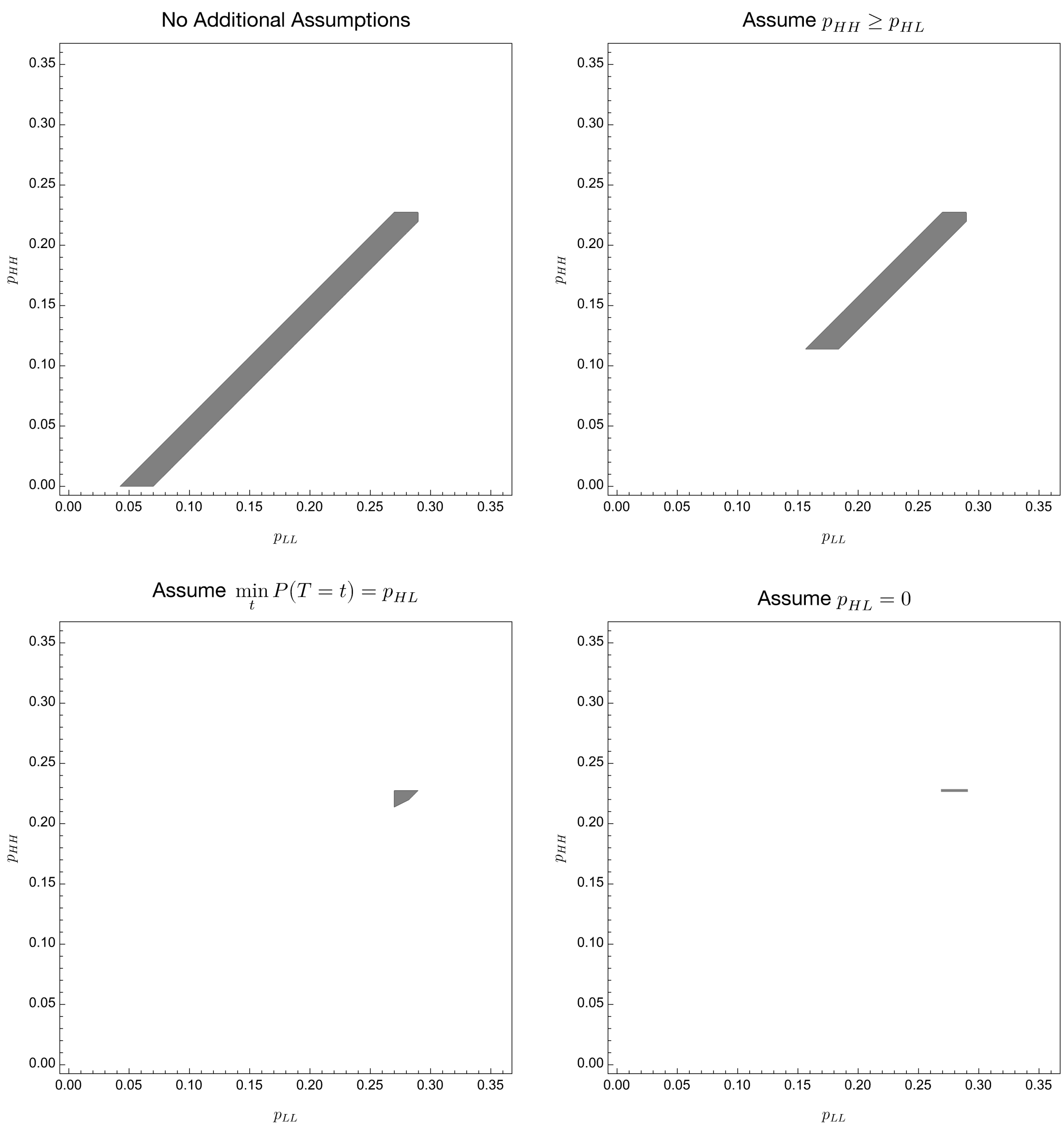


FIGURE 22. Identified set for $p_{L,L}, p_{H,H}$ at week 135. Amenity=retirement/pension benefits. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2.

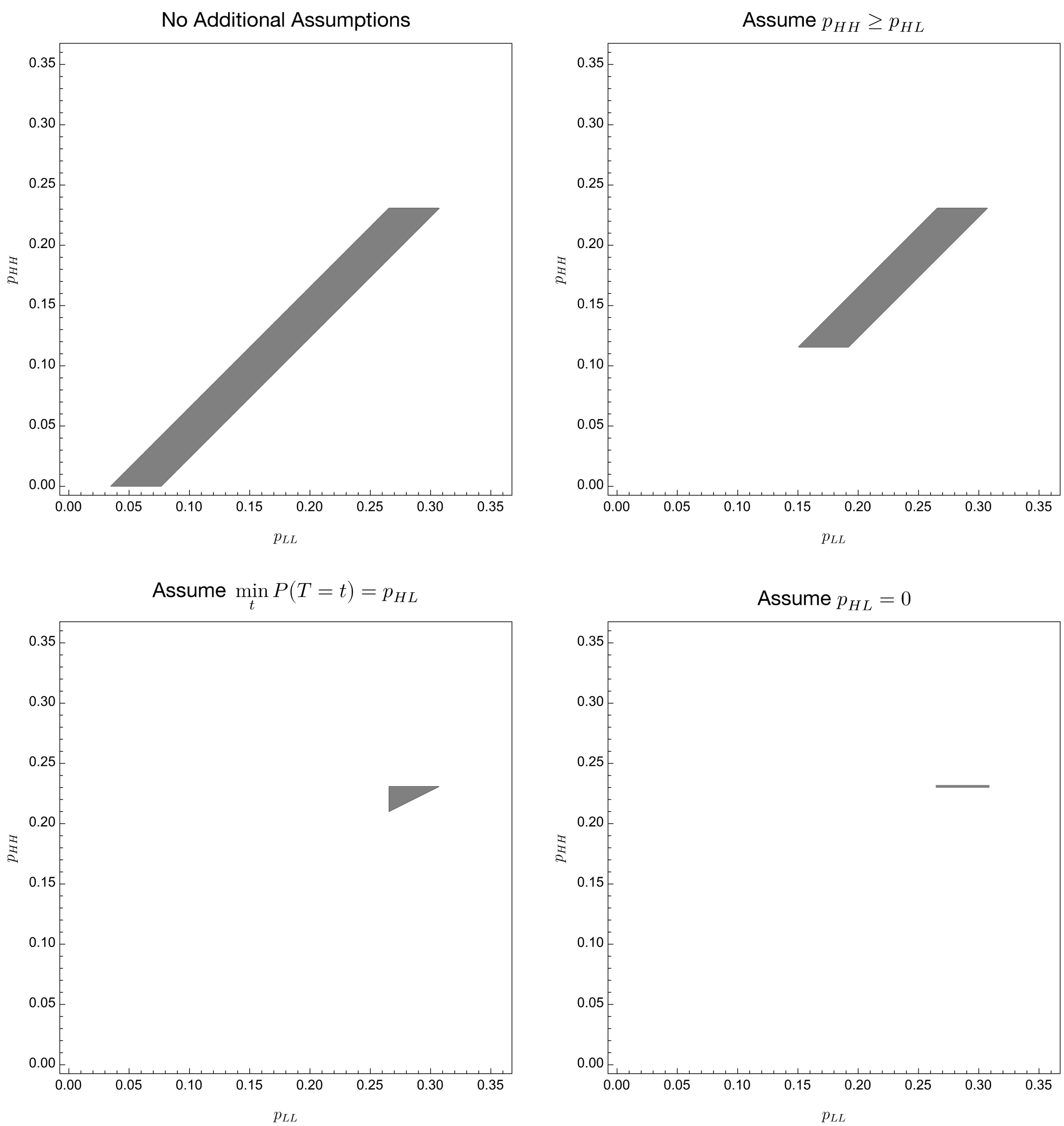


FIGURE 23. Identified set for $p_{L,L}, p_{H,H}$ at week 180. Amenity=retirement/pension benefits. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2.

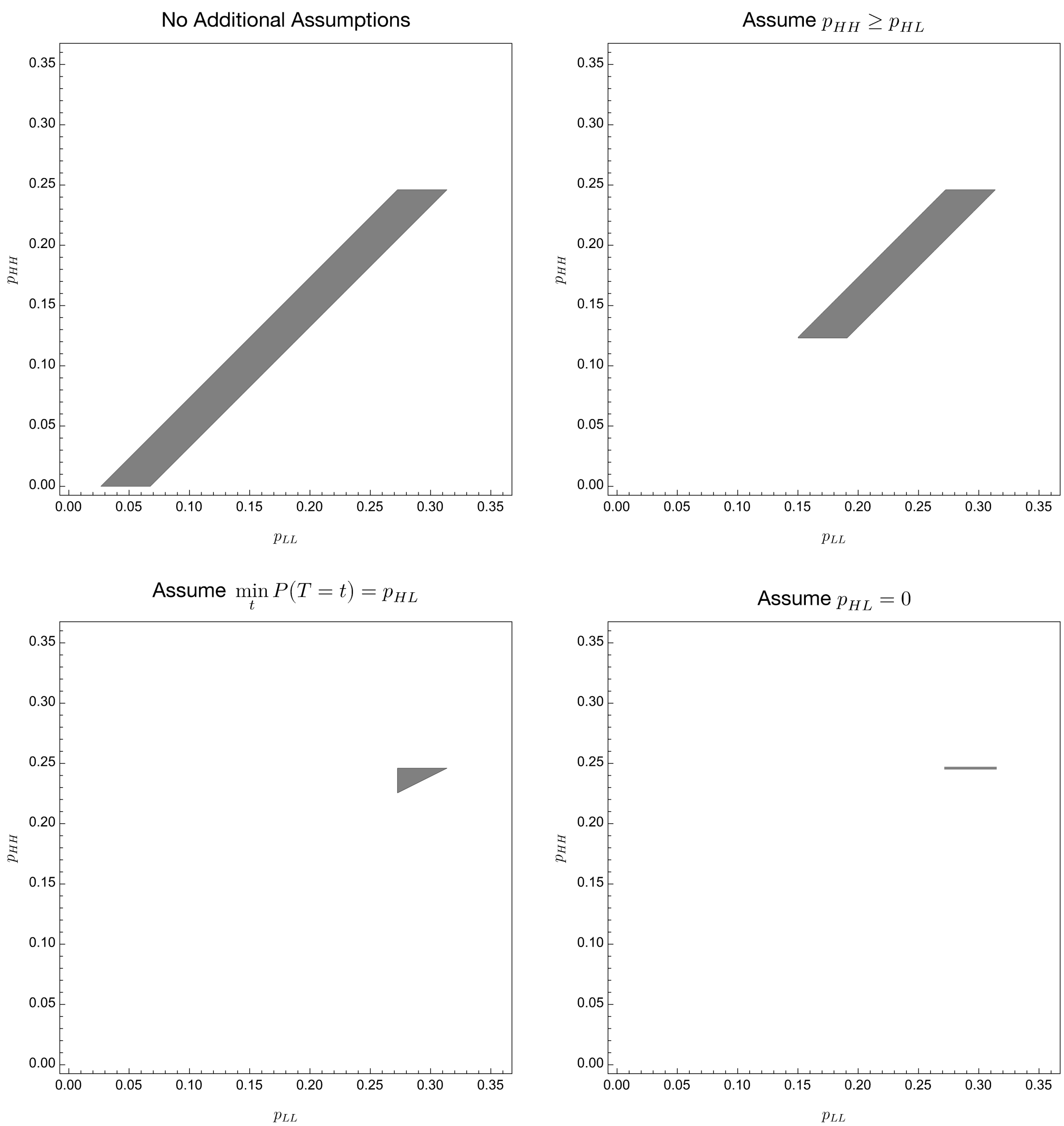


FIGURE 24. Identified set for $p_{L,L}, p_{H,H}$ at week 208. Amenity=retirement/pension benefits. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2.

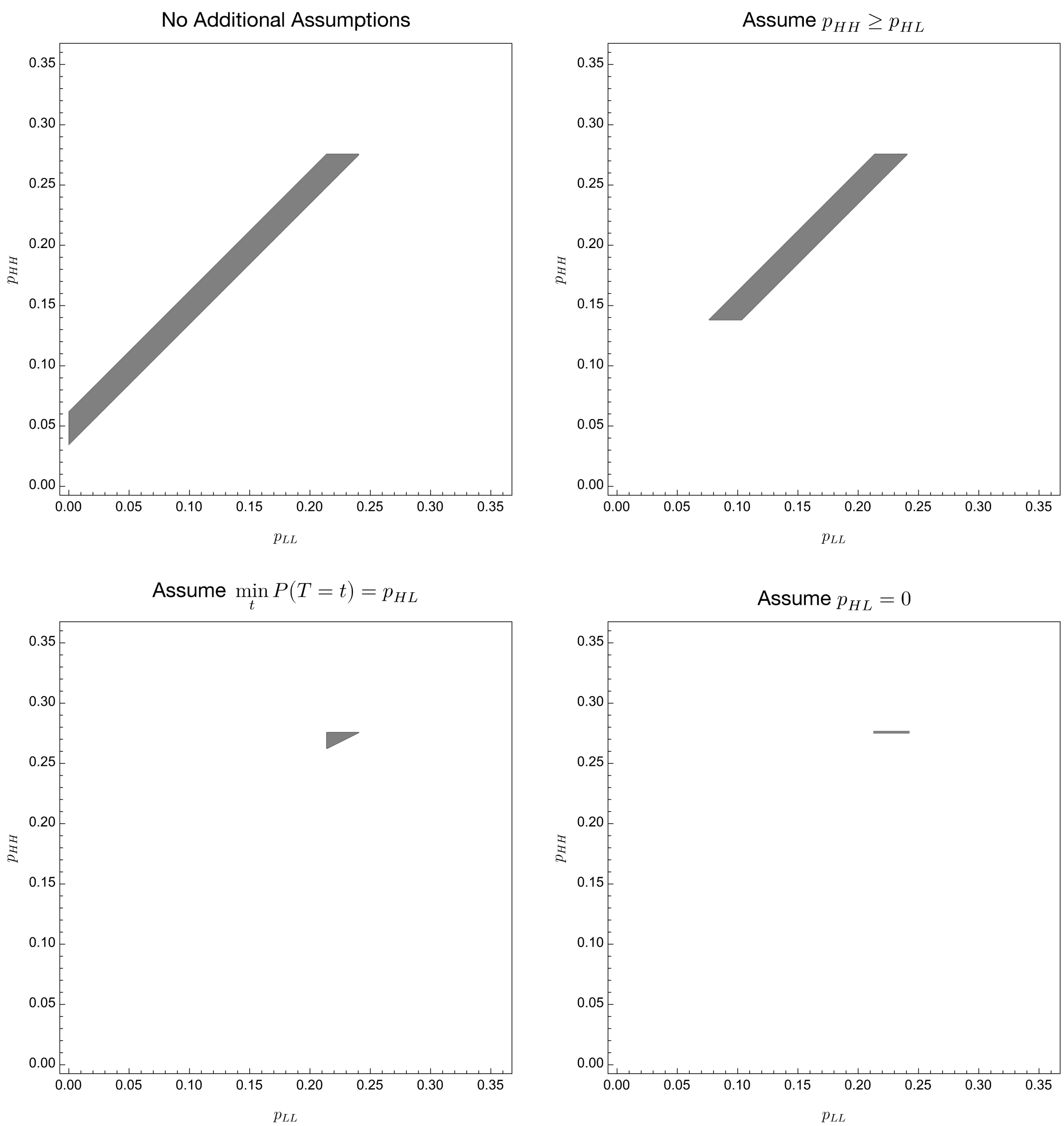


FIGURE 25. Identified set for $p_{L,L}, p_{H,H}$ at week 135. Amenity=health insurance. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2.

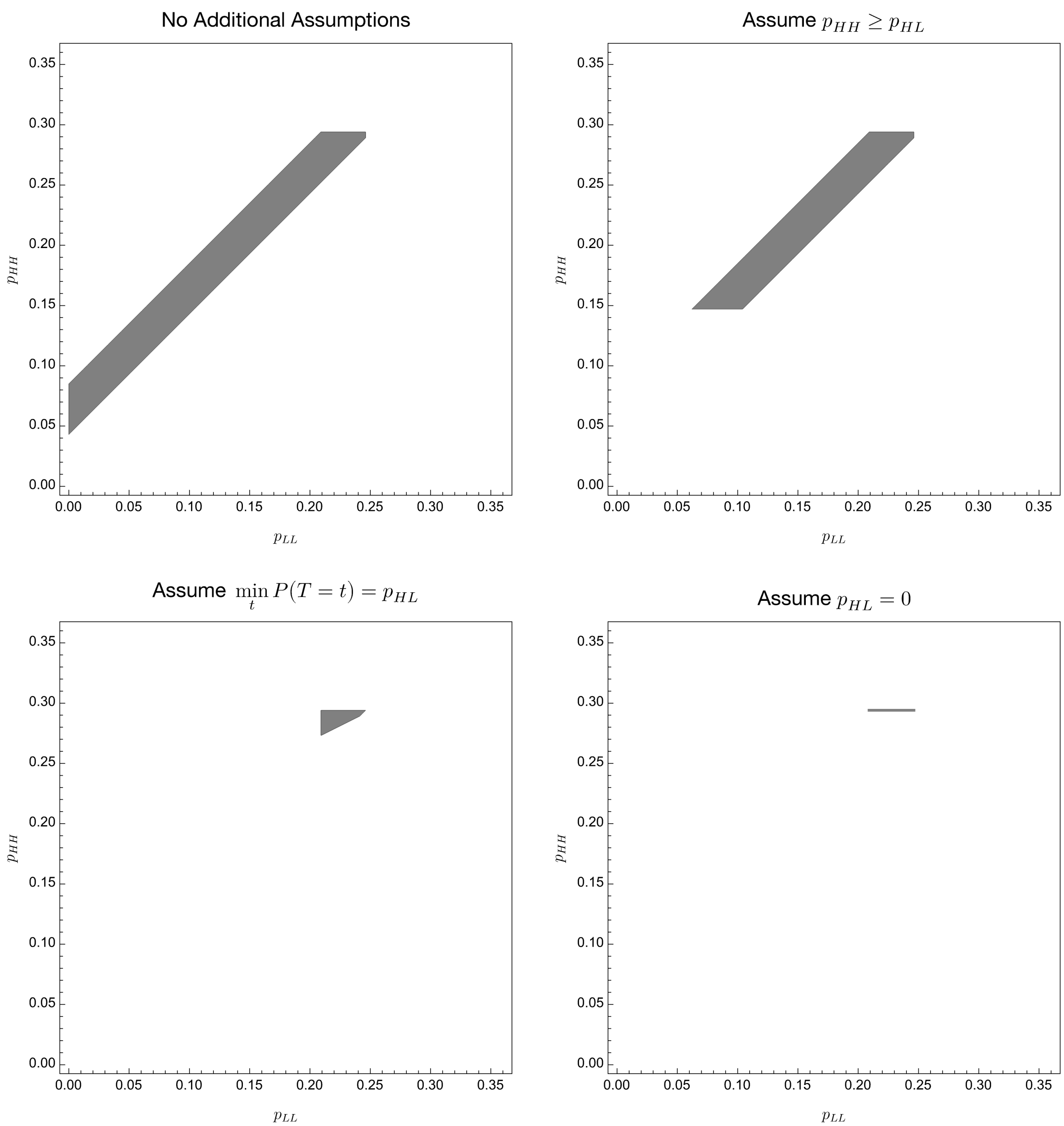


FIGURE 26. Identified set for $p_{L,L}, p_{H,H}$ at week 180. Amenity=health insurance. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption } 2\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2.

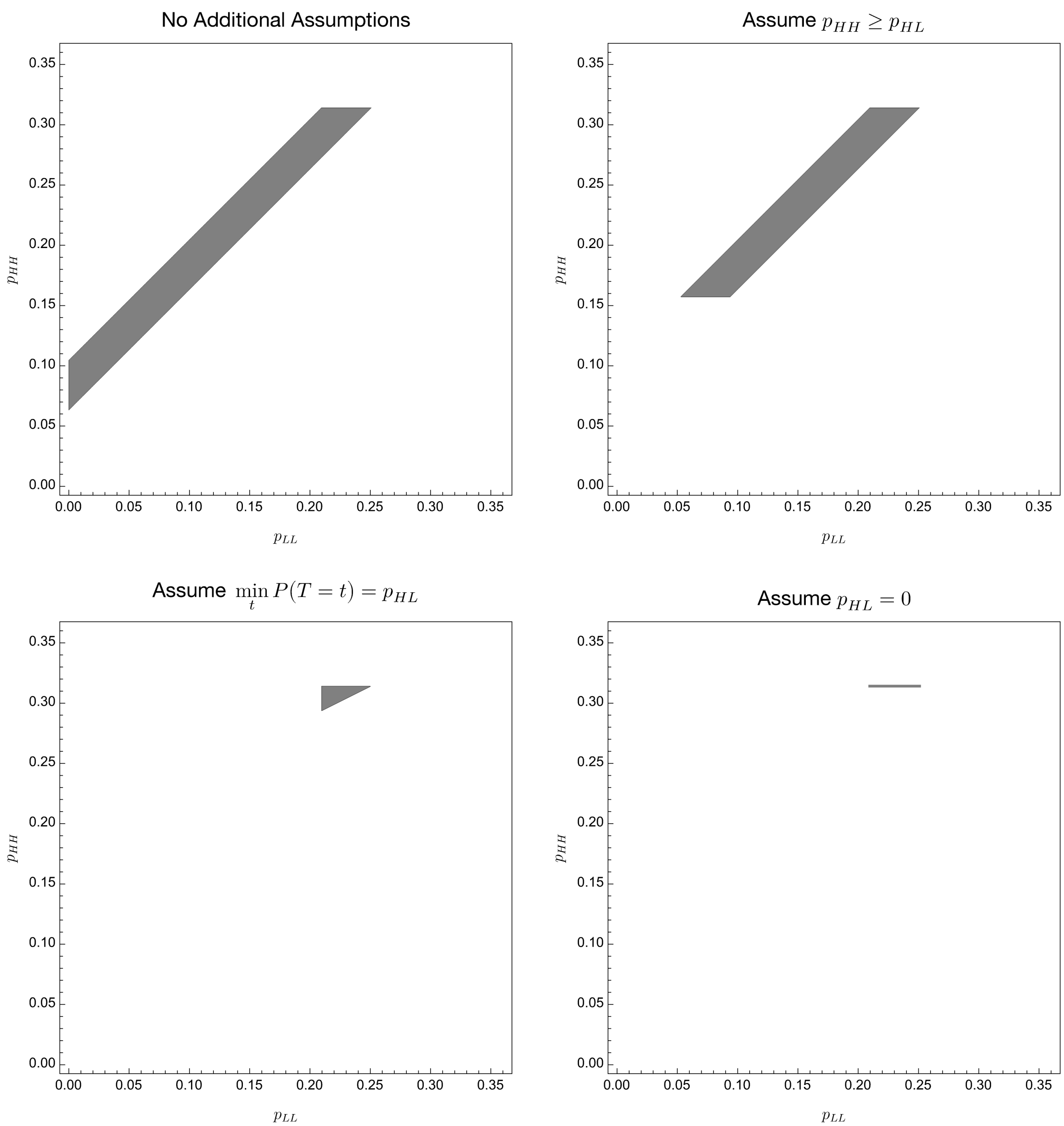


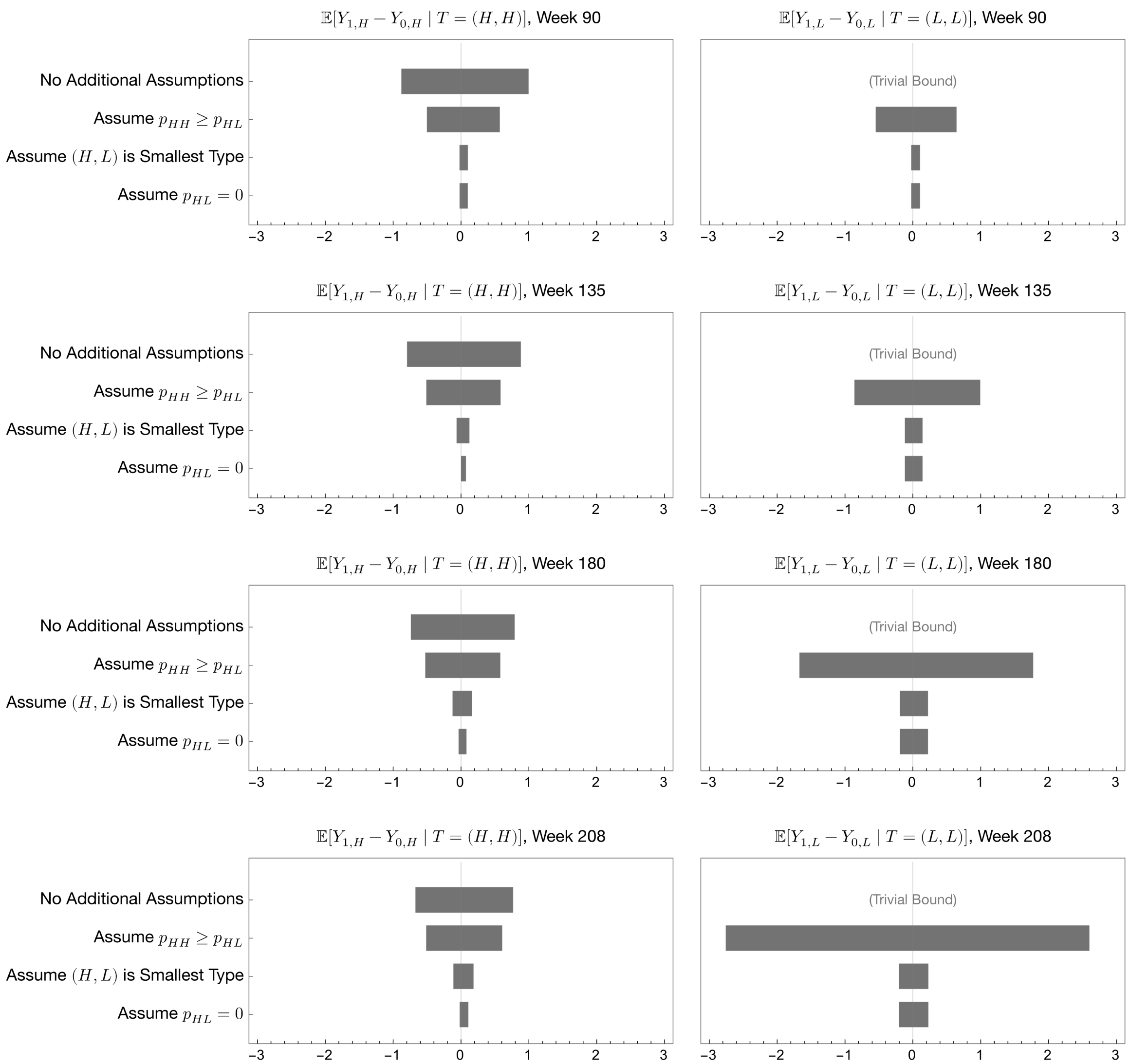
FIGURE 27. Identified set for $p_{L,L}, p_{H,H}$ at week 208. Amenity=health insurance. The first panel illustrates the identified set when $\mathcal{R}_T = \{\text{Assumption 2}\}$. The remaining panels illustrate the identified set of additional assumptions imposed over Assumption 2.

C.5. **Multilayered Bounds.** Appendix C.5 provides multilayered bounds for when classifying firm type based on the provision of paid vacation and retirement/pension benefits.

			$\mathbb{E}(Y_{1,H} - Y_{0,H} \mid T = (H,H))$		$\mathbb{E}(Y_{1,L} - Y_{0,L} \mid T = (L,L))$	
Week 90	p^*_{HH}	p^*_{LL}	lower	upper	lower	upper
Baseline	0.0542	0.0000	-0.8794	0.9952		
$p_{H,H} \geq p_{H,L}$	0.1235	0.0692	-0.5033	0.5722	-0.5478	0.6430
(H,L) is smallest type	0.2470	0.1927	-0.0228	0.1002	-0.0242	0.1061
$p_{H,L} = 0$	0.2470	0.1927	-0.0218	0.0995	-0.0242	0.1061
Week 135						
Baseline	0.0860	0.0000	-0.7956	0.8818		
$p_{H,H} \geq p_{H,L}$	0.1551	0.0413	-0.5108	0.5835	-0.8611	0.9932
(H,L) is smallest type	0.2994	0.1964	-0.0655	0.1239	-0.1186	0.1420
$p_{H,L} = 0$	0.3102	0.1964	0.0012	0.0725	-0.1186	0.1420
Week 180						
Baseline	0.1099	0.0000	-0.7394	0.7904		
$p_{H,H} \geq p_{H,L}$	0.1667	0.0147	-0.5265	0.5794	-1.6717	1.7737
(H,L) is smallest type	0.3123	0.1814	-0.1244	0.1629	-0.1909	0.2229
$p_{H,L} = 0$	0.3334	0.1814	-0.0361	0.0806	-0.1909	0.2229
Week 208						
Baseline	0.1287	0.0000	-0.6719	0.7683		
$p_{H,H} \geq p_{H,L}$	0.1758	0.0058	-0.5136	0.6070	-2.7579	2.6008
(H,L) is smallest type	0.3310	0.1816	-0.1124	0.1847	-0.2046	0.2294
$p_{H,L} = 0$	0.3516	0.1816	-0.0206	0.1083	-0.2046	0.2294

Notes: Treatment bounds are for ln(hourly wage); hourly wage calculated as weekly earnings divided by weekly hours for the employed. p^*_t is the minimum value of p_t over the identified set for response-types under the given assumption.

TABLE 21. Multilayered bounds by week, amenity: paid vacation.



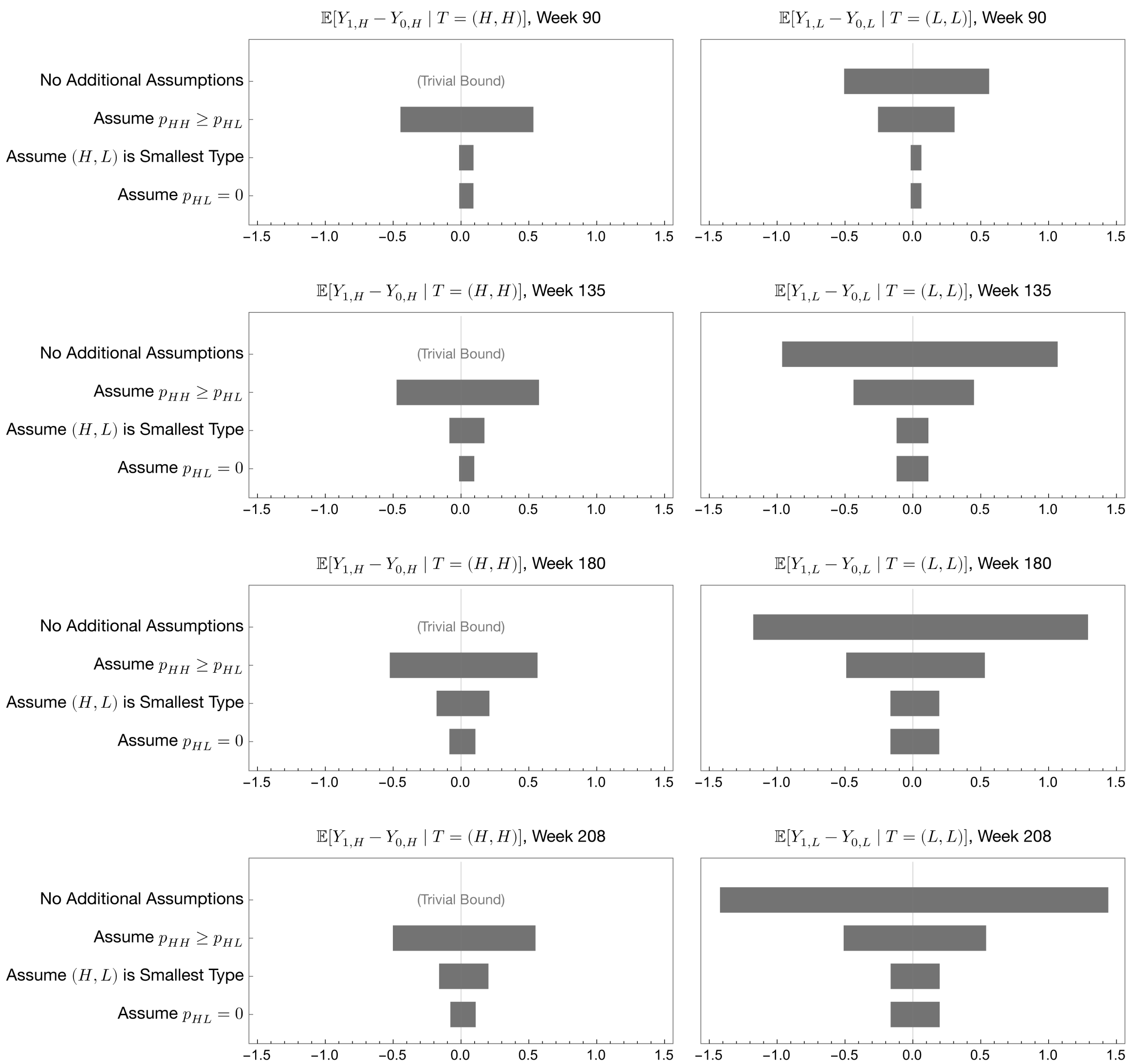
Notes: Treatment bounds are for ln(hourly wage); hourly wage calculated as weekly earnings divided by weekly hours for the employed.

FIGURE 28. Multilayered bounds by week. Amenity=paid vacation.

			$\mathbb{E}(Y_{1,H} - Y_{0,H} \mid T = (H,H))$		$\mathbb{E}(Y_{1,L} - Y_{0,L} \mid T = (L,L))$	
Week 90	p^*_{HH}	p^*_{LL}	lower	upper	lower	upper
Baseline	0.0000	0.1042			-0.5064	0.5623
$p_{H,H} \geq p_{H,L}$	0.0853	0.1894	-0.4465	0.5337	-0.2568	0.3074
(H,L) is smallest type	0.1705	0.2747	-0.0141	0.0921	-0.0161	0.0628
$p_{H,L} = 0$	0.1706	0.2747	-0.0135	0.0916	-0.0161	0.0628
Week 135						
Baseline	0.0000	0.0423			-0.9636	1.0670
$p_{H,H} \geq p_{H,L}$	0.1138	0.1561	-0.4749	0.5746	-0.4374	0.4505
(H,L) is smallest type	0.2136	0.2699	-0.0854	0.1729	-0.1203	0.1144
$p_{H,L} = 0$	0.2275	0.2699	-0.0143	0.0977	-0.1203	0.1144
Week 180						
Baseline	0.0000	0.0345			-1.1762	1.2907
$p_{H,H} \geq p_{H,L}$	0.1155	0.1500	-0.5247	0.5641	-0.4911	0.5305
(H,L) is smallest type	0.2098	0.2655	-0.1802	0.2097	-0.1655	0.1947
$p_{H,L} = 0$	0.2309	0.2655	-0.0856	0.1062	-0.1655	0.1947
Week 208						
Baseline	0.0000	0.0265			-1.4213	1.4398
$p_{H,H} \geq p_{H,L}$	0.1230	0.1495	-0.5019	0.5491	-0.5103	0.5400
(H,L) is smallest type	0.2254	0.2725	-0.1615	0.2023	-0.1640	0.1975
$p_{H,L} = 0$	0.2460	0.2725	-0.0784	0.1084	-0.1640	0.1975

Notes: Treatment bounds are for ln(hourly wage); hourly wage calculated as weekly earnings divided by weekly hours for the employed. p^*_t is the minimum value of p_t over the identified set for response-types under the given assumption.

TABLE 22. Multilayered bounds by week, amenity: retirement/pension benefits.



Notes: Treatment bounds are for ln(hourly wage); hourly wage calculated as weekly earnings divided by weekly hours for the employed.

FIGURE 29. Multilayered bounds by week. Amenity=pension benefits.

Appendix D. 'Top 5' papers (potentially) collapsing multilayered selection to single layered selection

In a literature survey which we detail in Section 1, we counted 56 papers published in 'top 5' general interest economic journals that cited Lee (2009) and 42 that empirically implemented Lee bounds. Table 23 details 7 of these papers that feature multilayered selection, where researchers simplified the sample selection problem by collapsing it to a single dimension.[37]

Paper			**Sample Selection**	
Application	Treatment	Outcomes	Layer addressed	Additional layer
Daruich et al. (2023)				
Study 2001 Italian reform lifting constraints on the employment of temporary contract workers but maintaining employment protection laws for permanent contract employees; exploit staggered implementation across collective bargaining agreements.	See "Application".	Individuals' earnings.	Only observe employment outcomes after labor market entry; utilize Lee bounds to address that the reform affects the entry margin into the labor market.	Conditional on labor market entry, altering temporary contract worker regulations likely also affects worker sorting across industries and/or firms; indeed this paper finds meaningful changes in the shares of temporary contract workers in certain industries.

[37]Only outcomes for which Lee's bounds are estimated are noted in column 3.

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Application	Treatment	Outcomes	Layer addressed	Additional layer
Cullen and Pakzad-Hurson (2023)				
Study state-level laws in the U.S. protecting the right of private sector workers to discuss salary information with co-workers.	See “Application”.	Worker wages.	Treatment effects could be driven by compositional changes of private sector workers if high-paid (low-paid) workers disproportionately leave (join) the private sector; estimate Lee bounds to address this challenge.	Conditional on private sector entry, treatment may also affect the sorting of workers across industries or firms within the private sector; for example, knowledge of co-workers’ salaries could cause workers to sort to firms with flatter pay hierarchies (i.e., fairness concerns).

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Application	Treatment	Outcomes	Layer addressed	Additional layer
Bianchi and Giorcelli (2022)				
Study long-term and spillover effects of management interventions on firm performance; estimate effect of the Training Within Industry program, a U.S. government training program intended to be provided to all firms involved in war production between 1940 and 1945.	Exploit that constraints resulted in only 7% of applicant firms receiving full training, 48% receiving no training and the remainder receiving partial training; this paper compares applicant firms who received training to applicant firms who did not.	Firm total factor productivity.	Estimate Lee bounds to address the higher attrition rate of untrained firms; treated firms had 90% survival rate at least 10 years following treatment whereas control firms only had 64% survival rate.	Conditional on firm survival, training may also affect the sorting of firms across industries or other important dimensions. This is particularly plausible in this paper's setting where: (i) trained firms undertook structural changes transforming them into larger and more complex organizations; (ii) this paper estimates treatment bounds for outcomes observed post-Second World War, when many firms would have plausibly switched industries (i.e., left war production).

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Application	Treatment	Outcomes	Layer addressed	Additional layer
Fink et al. (2020)				
Complete experiment offering subsidized loans to randomly selected villages in rural Zambia where farmers suffer from liquidity constraints in the months prior to harvest (lean season).	Offered cash and maize loans at the start of the lean season (January) with repayment due at harvest (July) in either cash, maize or both.	Individual- and village-level earnings.	Estimate Lee bounds to address that the likelihood of entering the labor market decreases with the loan treatment.	Conditional on entry to the labor market, treatment has potential to affect the types of jobs individuals accept; this is particularly plausible in this paper's setting as labor sales occur within villages between better- and worse-off farmers at individually negotiated rates.

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Application	Treatment	Outcomes	Layer addressed	Additional layer
Giorcelli (2019)				
Study long-run effects of management on firm performance; estimate effects of US Technical Assistance and Productivity Program (USTAPP) which provided management training and technologically advanced machines to Italian firms from 1952 to 1958.	Exploit unexpected USTAPP budget cut which occurred after the firm application submission and review period which resulted in many firms not receiving training; compare applicant firms who received training to those who did not.	Firm-level: sales, number of employees, total factor productivity revenue.	Estimate Lee bounds to address the treatment-control difference in firm survival probability.	As in Bianchi and Giorcelli (2022), conditional on firm survival, management training likely affects the sorting of firms across industries and/or other important dimensions.

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Application	Treatment	Outcomes	Layer addressed	Additional layer
Fisman et al. (2017)				
Estimate effect of cultural proximity (e.g., shared codes, language, religion) on loan outcomes for lenders and borrowers using dyadic data on religion and caste for lending officers and borrowers from a state-owned Indian bank.	Exploit lending officer rotation policy providing variation in officer-borrower matching.	Cultural group-level: amount of debt received, total number of borrowers and average loan size.	Estimate Lee bounds as column 3 outcomes are only observed conditional on a group receiving credit.	Conditional on a group receiving credit, "same group matches" also plausibly affect the type of loans a group receive. For example, "same group match" borrowers may receive favorable loan terms; this paper indeed notes the potential for these effects but is constrained by data limitations.
Blanco et al. (2013)				
Estimate bounds on average and quantile treatment effects of Job Corps.	Job Corps program; see description in Section 5.1.	Worker wages.	Estimate Lee bounds to address that Job Corps training may affect labor supply along the extensive margin.	See Section 5.4.2.

Table 23: 'Top 5' papers potentially collapsing multilayered sample selection to a single dimension