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Two-Way Shear Strength of Slab-Column Connections: Reexamination of ACI 318 Provisions

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In typical lightly-reinforced slab-column connections, extensive flexural yielding is likely to occur before the computed punching shear capacity is reached. The basic ACI 318 two-way shear strength provision has not changed since 1963 and was developed based on a statistical analysis of test results on scaled specimens that were believed to have failed in shear. Several researchers showed that the use of the basic ACI two-way shear strength provision yields results that were unconservative compared with the two-way shear strength of slabs in experimental tests. This paper shows that the applicability of the ACI 318 provisions for typical lightly-reinforced slabs is questionable.

Keywords: building code; flat plate; slab-column connection; two-way shear strength.

INTRODUCTION

Punching shear failures due to insufficient two-way shear strength of slab-column connections may result in a progressive collapse of a building.[1] Failures of flat-plate structures initiated by punching shear failure, including that of the Sampoong Department Store[2] that occurred in 1995, indicate that two-way shear strength of slab-column connections and the mechanics of punching shear failure have not been well understood. Park and Gamble[3] indicated that the actual behavior of the failure region of a cracked slab is extremely complex and design provisions used are empirical simplifications of the real behavior. Bari[4] reports that there are significant variations among different empirical treatments.

ACI 318-08[5] defines the basic nominal two-way shear strength V_c of an interior slab-column connection with a square column and normalweight concrete as

$$V_c = 4 \times \sqrt{f_c'} \times b_o \times d \text{ (U.S. units)} \quad (1)$$

$$V_c = 0.33 \times \sqrt{f_c'} \times b_o \times d \text{ (SI units)}$$

where d is the average depth of slab reinforcement, b_o is the critical shear perimeter located at a distance $d/2$ away from the edge of the column or from the outermost shear reinforcement, and f_c' is the concrete compressive strength. The ACI 318 provision for basic two-way shear strength of a slab (Eq. (1)) has not changed since 1963. In addition, the ACI provision is simpler than provisions in several other building codes, as discussed later in this paper. The simple provision was derived from relatively complex expressions.

RESEARCH SIGNIFICANCE

ACI 318 provisions for evaluating the strength of slab-column connections are evaluated in light of test data. To achieve this goal, the historical development of basic ACI two-way shear strength provisions was studied as the first step. Second, the strengths of slab-column connections were estimated using the ACI 318 provisions and those estimates were compared with results of tests conducted on slab-column connections. These tests included two 2/3-scale slab-column connection specimens tested at the Ferguson Structural Engineering Laboratory of the University of Texas during the course of this study. In this study: 1) a summary of the historical development of ACI 318 provisions for two-way shear strength is provided; 2) current code provisions are evaluated; and 3) the results of tests conducted on slab-column connections are summarized. It is recommended that the two-way shear strength of lightly-reinforced connections is reduced. A value of ($V_c = 2\sqrt{f_c'}\, b_o d$) represents a lower bound on the data.

SLAB-COLUMN CONNECTIONS

Characteristics of typical flat-plate structures

Flat-plate structural systems consist of slabs that are supported directly on columns without any beams, drop panels, or column capitals. Durrani et al.[6] indicated that in the central and eastern regions of the U.S., there are many older flat-slab buildings designed and detailed to resist gravity loads only. The floor slabs in these buildings can be categorized as lightly reinforced. In this paper, the lightly-reinforced slabs refer to the slabs with a less than 1% flexural reinforcement ratio in the column strip ($\rho_{column\ strip} < 1\%$). Sherif and Dilger[7] reported that most slabs in flat plate structures have a flexural reinforcement ratio of less than 1%. Structural drawings of several flat-plate structures located in the Western U.S. were examined in this study. Those drawings show that the use of 0.5% flexural reinforcement ratio in the column strips and no shear reinforcement was typical.[1]

Failure mode of slab-column connections

A reinforced concrete slab-column connection can reach its capacity and fail in two modes: punching shear prior to or after the widespread flexural yielding of longitudinal reinforcement.[8-10] Independent of whether the connections fail in punching shear prior to or after the complete formation of a yield-line mechanism, failure always occurs when the loaded area punches through the slab. The failure surface has the form of a truncated cone or pyramid with a minimum cross section at least as large as the patched loaded area.[11]

Even though some researchers explicitly classified the failure mode of slabs as punching shear failure and flexural failure,[12-14] many researchers did not explicitly differentiate between punching shear and flexural failure. Gesund and

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Kaushik[13] conducted a systematic investigation into the relationship between the calculated flexural strength P_{flex} and the measured failure load P_{test} of the slabs under concentric loading. They found that the arithmetic mean of P_{flex}/P_{test} for 106 tests reported as punching shear failures was 1.02, with a standard deviation of 0.25. Regan and Braestrup[15] indicated that a substantial proportion of the test results reported in the literature as punching shear failures exhibited ultimate loads that did not differ significantly from the flexural capacities.

Regardless of whether punching failure occurs before or after the slab yields in flexure, failures in slab-column connections look the same: the column together with a portion of the slab pushes through the slab (footing) or the slab pushes down around a column. Therefore, the failures were labeled punching failure. For connections of normal proportions and with usual amounts of flexural reinforcement, a yield-line mechanism will precede punching shear failure.[16]

HISTORIC DEVELOPMENT OF ACI 318 PROVISIONS

Joint Committee of 1924[17]

In 1924, the ACI code committee[17] recommended that the calculated shear stress v and the allowable shear stress are given in Eq. (2) and (3), respectively

$$v = \frac{V}{bjd} \tag{2}$$

$$v = 0.02f'_c(1 + n) \le 0.03f'_c \tag{3}$$

where V is the shear force, b is the critical shear perimeter located at a distance of (t – 1.5 in. [38.1 mm]) from the periphery of the loaded area, jd is the distance between the centroid of compression and tension force, t is the slab thickness, f'_c is the concrete compressive strength (psi), and n is the ratio of the flexural reinforcement area crossing directly through the loaded area (column or column capital) to the total flexural reinforcement area in the slab. The report by the Joint Committee of 1924[17] was also adopted by ACI as standard specifications and only minor changes have been made with respect to shear and diagonal tension in slabs and footings since then.

ACI 318-41,[18] ACI 318-47,[19] and ACI 318-51[20]

The three editions of the ACI 318 Code from 1941 to 1951 have the same provisions for two-way shear strength and acknowledge that the shear strength is sensitive to the amount of flexural reinforcement. The shear stress v as a measure of diagonal tension is computed using Eq. (2), and the allowable v for two-way slabs is:

- $0.03f'_c$ if at least 50% of the total negative flexural reinforcement in the column strip passes through the periphery.
- $0.025f'_c$ if 25% or less of the total negative flexural reinforcement in the column strip passes through the periphery.

The allowable v for footings is $0.03f'_c \le 75$ psi (0.52 MPa). As recommended by the Joint Committee of 1924, the critical shear perimeter is located at a distance of (t – 1.5 in. [38.1 mm]) from the periphery of the loaded area.

ACI 318-56[21]

ACI 318-56[21] introduced the maximum limit of 100 and 85 psi (0.69 and 0.59 MPa) for the allowable v (that is, $0.03f'_c \le 100$ psi [0.69 MPa] and $0.025f'_c \le 85$ psi [0.59 MPa]). The critical shear perimeter is located at a distance d away from the loaded area. The allowable v for footings is still $0.03f'_c \le 75$ psi (0.52 MPa).

ACI 318-63[22]

The provisions of ACI 318-63[22] were developed on the basis of the recommendations by Joint ACI-ASCE Committee 426, Shear and Diagonal Tension. Significant changes to shear provisions introduced in ACI 318-63 were[22]:

1. ACI 318-63[22] was the first edition of ACI 318 that contained an ultimate strength design criteria for shear. ACI 318-63[22] prescribed the use of both load factors and capacity reduction factors ϕ;

2. Diagonal tension for concrete was stated as a function of $\sqrt{f'_c}$. Joint ACI-ASCE Committee 326 recommended that v was a function of $\sqrt{f'_c}$ and the ratio of the column size to the effective slab depth c/d. The committee pointed out, however, that the variable of c/d could also be taken into account by using a critical perimeter $d/2$ away from the loaded area. For simplicity, especially for irregular column shapes and slabs with openings near the column, ACI 318-63[22] adopted the following approach: v was independent of c/d and equal to $4\sqrt{f'_c}$;

3. The critical shear perimeter was located at $d/2$ away from the loaded area. Commentary of ACI 318-63[22] indicated that while the true pyramidal failure surface was at 45 degrees to the neutral axis, the stresses on this surface were complex (containing both shear and bending forces). For simplicity, a vertical section on which the tangential component was caused only by shear was selected. Such a section was located at a distance of $d/2$ from the loaded area;

4. The factor j was eliminated; and

5. Long and narrow slabs or footings, acting as a one-way beam and a two-way member, respectively, were differentiated.

ACI 318-63[22] stated that the nominal ultimate shear strength v_u in slabs and footings is

$$v_u = \frac{V_u}{b_o d} \tag{4}$$

where V_u is the total factored shear force and b_o is the critical shear perimeter located at $d/2$ away from the loaded area. Without shear reinforcement

$$v_u \le 4\phi\sqrt{f'_c} \tag{5}$$

where ϕ is the capacity reduction factor (0.85 for shear).

ACI 318 since 1971[5,23-29]

The ACI 318 provisions for basic two-way shear strength of slab (Eq. (4) and (5)) have not changed since 1963, except in 2002,[29] when the ϕ-factor was reduced to 0.75.

PREVIOUS RESEARCH ON TWO-WAY SHEAR RESISTANCE OF SLABS

Richart[30,31]

Richart[30,31] reported tests of 24 wall footings and 132 column footings supported on a bed of steel springs simulating soil pressure. Most of the specimens tested by Richart[30,31] were 7 ft (2.1 m) square footings. Richart[30,31] found that the reinforcing steel in the footings with 0.2% and 0.4% flexural reinforcement ratio yielded before punching failure occurred. These footings developed extensive cracking and finally failed in diagonal tension at relatively low shear stresses v ($2\sqrt{f_c'}$ to $3.2\sqrt{f_c'}$ psi [$0.17\sqrt{f_c'}$ to $0.27\sqrt{f_c'}$ MPa]), evaluated at the critical section d away from the column face). He referred to punching shear failure as a secondary failure (after the yielding of flexural reinforcement), and explained that the secondary failure occurred because yielding of the steel produced large cracks, which then reduced the concrete section resisting shear. He also found that footings with ρ = 0.56% and 0.75% clearly failed in diagonal tension, at shear stress (v) levels that varied between $2.9\sqrt{f_c'}$ and $3.5\sqrt{f_c'}$ psi ($0.24\sqrt{f_c'}$ and $0.29\sqrt{f_c'}$ MPa) (evaluated at the critical section d away from the column face).

Hognestad[32]

Hognestad[32] concluded that the majority of the footings failed after local yielding of the flexural reinforcement, but before reaching the ultimate flexural load from a yield-line analysis. He recognized that the flexural and shear strength were interrelated and introduced the parameter $\phi_o = V_{shear}/V_{flex}$, where V_{shear} is the ultimate shear capacity of the slab, and V_{flex} is the ultimate flexural capacity. Based on Richart's footing test results,[30,31] Hognestad[32] proposed the following empirical equation

$$v = \frac{V}{bjd} = \left(0.035 + \frac{0.07}{\phi_o}\right)f_c' + 130 \text{ psi} \tag{6}$$

(however, $0.38 \le d/c \le 1.14$; 2000 psi [13.79 MPa] $\le f_c' \le$ 5000 psi [34.47 MPa]) where j = 7/8, b is the circumference of the loaded area, d is the effective depth of slab, f_c' is the concrete cylinder strength (psi), and ϕ_o is

$$\phi_o = \frac{A + \sqrt{A^2 + 0.28BC}}{2BC} \tag{7}$$

$$A = 0.035 + \frac{130}{f_c'} \tag{8}$$

$$B = \frac{V_{flex}}{\frac{7}{8}d4cf_c'} \tag{9}$$

$$V_{flex} = \frac{8a}{(a-c)^2}A_s df_y\left(1 - \frac{1}{2}\frac{\rho f_y}{f_c'}\right) \tag{10}$$

$$C = \frac{a^2 - (c+2d)^2}{a^2} \tag{11}$$

where a is the width of the slab or footing, c is the column dimension, and f_y is the yield strength of steel reinforcing bars.

Elstner and Hognestad[33]

Elstner and Hognestad[33] found that the final failure of slabs with flexural reinforcement ratios that varied from 1.15% to 3.7% was by the column punching through the slab. When shear stresses v were evaluated at a distance $d/2$ away from the column, v varied from $4\sqrt{f_c'}$ to $7.4\sqrt{f_c'}$ psi ($0.33\sqrt{f_c'}$ to $0.62\sqrt{f_c'}$ MPa). In most cases, such punching occurred after initial yielding of the reinforcement in the vicinity of the column. A flexural failure, however, was observed for the slabs with 0.5% and 1% flexural reinforcement (when v was evaluated at a distance $d/2$ away from the column, v varied from $2.1\sqrt{f_c'}$ to $3.5\sqrt{f_c'}$ psi [$0.18\sqrt{f_c'}$ to $0.29\sqrt{f_c'}$ MPa]).

After reanalyzing Richart's test results,[30,31] Elstner and Hognestad[33] indicated that v computed at the column face was a better measure of shear strength than that computed at a distance d away from the column faces. They also revised the earlier Hognestad empirical formula (Eq. (6)) as follows

$$v = \frac{V}{bjd} = \frac{333 \text{ psi}}{f_c'} + \frac{0.046}{\phi_o} \tag{12}$$

where j = 7/8. They also found that a concentration of 50% of the flexural reinforcement directly over a column did not increase the shear strength and compression reinforcement had no effect on the ultimate shear strength.

Whitney[34]

Whitney[34] reviewed Richart's[30,31] and Elstner and Hognestad's[33] test results and suggested that the conventional shear formula ($v = V/b_jd = k(f_c')$) was not acceptable because the shear strength was not a function of concrete strength alone, but depended largely on the amount of flexural reinforcement and its efficiency. He indicated that the conventional shear formula was too conservative for cases with a large ρ value and relatively unsafe with a light ρ value. He also found that using the critical section at a distance $d/2$ away (instead of d away) from the column face gave the most consistent results for all slab depths.

Whitney[34] proposed the following expression

$$v = \frac{V}{bd} = 100 \text{ psi} + 0.75\frac{m_u}{d^2}\sqrt{\frac{d}{l_s}} \tag{13}$$

where $b = 4(c + d)$ (critical shear perimeter is at a distance $d/2$ away from the loaded area), l_s is the shear span, and m_u is the ultimate moment capacity per unit width of slab near the column defined as follows.

For under-reinforced slabs

$$m_u = \rho d^2 f_y\left(1 - \frac{\rho f_y}{1.7f_c'}\right) \tag{14}$$

For over-reinforced slabs

$$m_u = \frac{d^2 f_c'}{3} \quad (15)$$

The customary term j was omitted from Eq. (13) because the value of v was calculated empirically and the average value for the full depth was considered to be as good as any other.

Whitney[34] explained that there were two different types of failure: gradual and sudden. The gradual type of failure occurred after flexural reinforcement yielded and caused excessive cracking that eventually reduced the shear strength until the column punched through the slab. The sudden type of failure occurred before any of the flexural reinforcement yielded. This sudden failure could be caused by over-reinforcement in flexure (resulting in destruction of the compression zone around the column) or bond/anchorage failure (because of insufficient embedment length or very close spacing of the reinforcing bars). In explaining a mechanism of failure, Whitney[34] indicated that the horizontal component of the shear force on the pyramid of rupture must be resisted by the flexural reinforcement passing through the pyramid. Whitney[34] stated that this horizontal component was limited by the yield strength of flexural reinforcement. As the reinforcement yielded, three failure mechanisms could happen: 1) flexural cracks could extend up from the steel into the pyramid until they finally precipitated a shear failure; 2) if the slab was over-reinforced, the compression zone around the column crushed and resulted in sudden punching; or 3) if the steel was not properly anchored, it slipped and permitted sudden punching.

Joint ACI-ASCE Committee 326[11] commented that because the test results of specimens with relatively high flexural strengths were omitted in the study leading to Eq. (13), this equation could only apply in cases of nearly balanced design (that is, when ϕ_o is close to unity). It can be seen from Eq. (13) that v can be increased by increasing ρ inside the pyramid of rupture. Shifting the flexural reinforcement from the outside of the pyramid to the inside also increases ρ inside the pyramid of rupture and, hence, increases v.

Moe[35]

Moe[35] suggested that the shear strength was proportional to $\sqrt{f_c'}$ instead of f_c' to reflect the fact that shear failures are controlled primarily by tensile splitting. Based on the test results of the slabs with varying degrees of concentration of the flexural reinforcement inside the pyramid of rupture, Moe[35] found that V_{flex} is a better indicator of the shear strength than m_u, which was used by Whitney[34] (Eq. (13)). Moe,[35] however, indicated that the magnitude of V_{flex} had in itself no direct physical relation to the mechanism of failure. Rather, it reflected several other important influences, such as distribution of cracking, amount of the elongation of the tensile reinforcement, magnitude of the compressive stresses in the critical section, and the depth of neutral axis at failure.

Moe believed that the interaction between shear and flexural strength could be approximated by a straight line as follows

$$\frac{V}{V_o} + C\frac{V}{V_{flex}} = 1 \quad (16)$$

He assumed that $V_o = Abd\sqrt{f_c'}$, where b is the critical shear perimeter at a distance of $d/2$ away from the loaded area. Moe[35] also believed that the shear strength is sensitive to c/d and assumed a linear variation.

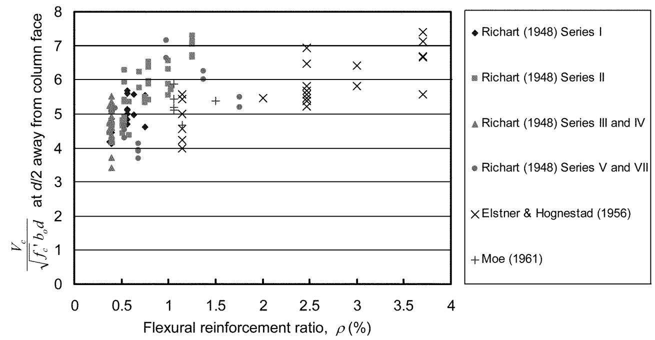


Fig. 1—Test results used in Moe's[35] statistical analysis.

Based on a statistical analysis of 37 slab and 106 footing test results (shown in Fig. 1) of Richart,[30,31] Elstner and Hognestad,[33] and his own tests, Moe[35] proposed Eq. (17). All slab and footing specimens were 7 x 7 ft (2.1 x 2.1 m) or smaller, and had flexural reinforcement ratios that varied between 0.39% and 3.7%

$$v = \frac{V}{bd} = \frac{15\sqrt{f_c'}\left(1 - 0.075\frac{c}{d}\right)}{1 + 5.25\frac{bd\sqrt{f_c'}}{V_{flex}}} \quad (17)$$

where V_{flex} is the shear force at the calculated ultimate flexural capacity of the slab using the yield line theory. Using a definition of $\phi_o = V/V_{flex}$, Eq. (17) can be reorganized as follows

$$v = \frac{V}{bd} = \left[15\left(1 - 0.075\frac{c}{d}\right) - 5.25\phi_o\right]\sqrt{f_c'} \quad (18)$$

Only the specimens that were believed to have failed in shear were included in Moe's[35] statistical analysis ($\phi_o < 1.0$) and are shown in Fig. 1. One-hundred and six of Richart's[30,31] footing test results are included in Fig. 1. It should be noted that 22 tests (with $0.2\% \leq \rho \leq 0.4\%$) out of 156 footing tests conducted by Richart,[30,31] which were believed to have failed in flexure, were excluded from Moe's[35] statistical analysis. Of the 38 slab tests conducted by Elstner and Hognestad,[33] only 34 results were included in Moe's[35] statistical analysis. Four slabs (with $0.5\% \leq \rho \leq 1\%$ and $2.1\sqrt{f_c'}$ psi $\leq v \leq 3.5\sqrt{f_c'}$ psi [$0.18\sqrt{f_c'}$ MPa $\leq v \leq 0.29\sqrt{f_c'}$ MPa] where v was evaluated at a distance $d/2$ away from the column face) were believed to have failed in flexure and were excluded from Moe's[35] statistical analysis. Perhaps the most important contribution of Moe's[35] study stems from his effort to explicitly include the effect of flexural reinforcement through the term V_{flex}.

Based on the ultimate strengths of slabs and footings obtained from relatively short-duration tests and considering the average strength, rather than the minimum, Moe[35] also developed design equations. Because slabs failing in flexure resisted loads considerably greater than the flexural capacity, as computed using the yield line theory, Moe[35] assumed $V = 1.1V_{flex}$ as the point of balanced design (that is, the value at which the flexural and shear strengths are equal).

To ensure that the flexural failure always governs over the shear failure, Moe[35] proposed that v must be limited to the following values

$$v = \left(9.23 - 1.12\frac{c}{d}\right)\sqrt{f_c'} \text{ for } c/d \le 3 \quad (19)$$

$$v = \left(2.5 + 10\frac{c}{d}\right)\sqrt{f_c'} \text{ for } c/d > 3 \quad (20)$$

Equations (19) and (20) were developed on the basis of tests on slabs and footings with c/d between 0.9 and 3.1.

Joint ACI-ASCE Committee 326[11]

Joint ACI-ASCE Committee 326[11] reviewed Moe's[35] equation (Eq. (18)) and believed that ϕ_o could be eliminated from Eq. (18) by substituting $\phi_o = 1.0$ because, in a practical design, V_{shear} should exceed V_{flex} (that is, $\phi_o \ge 1.0$). It should be noted from Eq. (18) that the shear strength v decreases as ϕ_o increases. Because Eq. (18) was derived based on the test data with $\phi_o \le 1.0$, substituting $\phi_o = 1.0$ was conservative. This simplification resulted in the following equation

$$v = \frac{V}{bd} = \left(9.75 - 1.125\frac{c}{d}\right)\sqrt{f_c'} \quad (21)$$

The committee, however, believed that Eq. (21) could not be applied for all cases encountered in practical design because of the following reasons:

1. When the load was applied to a slab over a very small area (that is, b and c/d were very small), v would approach $9.75\sqrt{f_c'}$ but V would approach zero; and
2. When c/d was large (that is, columns with drop panels), v would approach zero.

Based on a conservative fit to 198 available test results with $\phi_o \le 1.0$, Joint ACI-ASCE Committee 326[11] then proposed the following equation

$$v = \frac{V}{bd} = 4\left(\frac{d}{c} + 1\right)\sqrt{f_c'} \quad (22)$$

where b is the periphery of the loaded area. It should be noted that, because Eq. (22) was derived based on a conservative fit of the test data with $\phi_o \le 1.0$, the applicability of Eq. (22) to the cases where $\phi_o > 1.0$ is questionable. Because Eq. (18) shows that v decreases as ϕ_o increases, it is expected that Eq. (22) is unconservative when it is applied to the cases where $\phi_o > 1.0$, which is typical in lightly-reinforced slabs.

To avoid an open interpretation on the value of c for irregular columns or columns with openings, and to propose a design recommendation that was consistent with the ACI 318-56[21] concept, the committee simplified Eq. (22) into the following equation

$$v = \frac{V}{b_o d} = 4\sqrt{f_c'} \quad (23)$$

where b_o is the critical section located at a distance $d/2$ from the loaded area.

In the discussion of the paper by Joint ACI-ASCE Committee 326,[11] Diaz de Cossio[36] considered that the lower limit of $4\sqrt{f_c'}$ psi ($0.33\sqrt{f_c'}$ MPa) at $d/2$ from the loaded area was reasonable and on the safe side for most common cases. Diaz de Cossio's[36] test results of 22 one-way slabs (reinforced in tension only) with ρ values varying between 1.85% and 2.81% had an average v (measured at $d/2$ away from the loaded area) of $3.65\sqrt{f_c'}$ psi ($0.3\sqrt{f_c'}$ MPa) with a coefficient of variation of 7.4%. He stated, however, that it was likely that two-way slabs with significantly larger width-to-depth ratio than that of his specimens would have higher strengths than those measured in his tests. Hence, it can be seen that $4\sqrt{f_c'}$ psi ($0.3\sqrt{f_c'}$ MPa) was not considered to be a lower limit of the shear stress, but more like an average stress.

Joint ACI-ASCE Committee 326[11] also indicated that concentration of reinforcement over the column had advantages in flexure (that is, increasing the slab stiffness and reducing the stresses in the flexural reinforcement in the vicinity of the column) and therefore should be encouraged. The committee did not, however, tie a requirement for flexural reinforcement to the design requirements for shear.

Guralnick and LaFraugh[37]

Guralnick and LaFraugh[37] tested a 3/4-scale flat-plate test specimen having overall dimensions of 45 x 45 ft (13.7 x 13.7 m) consisting of nine 15 x 15 ft (4.6 x 4.6 m) panels arranged three-by-three. The amounts of top flexural reinforcement in all interior connections were 0.73% in the column strip and 1.5% within the ($c + 2h$) region.

Failure occurred when one of the interior columns punched through the slab at a load of 85% of the two-way shear capacity computed using ACI 318-63[22] (that is, $0.85 \times 4\sqrt{f_c'}\, b_o d$). The measured failure load of the test structure was 1.05 times the predicted yield-line failure load. Immediately before failure, the average steel strain at the four faces of the column was approximately 0.01, which was seven times greater than the yield strain.

Magura and Corley[38]

Magura and Corley[38] reported the results of the test conducted on the waffle slab roof of the Rathskeller Building constructed for the 1964-1965 New York World's Fair. The roof of the structure was a 2 ft (0.61 m) thick waffle slab supported on columns, approximately 30 ft (9.1 m) on centers. The building was designed to meet the provisions of ACI 318-56[21] and the roof was designed for a live load of 300 lb/ft^2 (14.4 kPa) and an average computed dead load of 220 lb/ft^2 (10.5 kPa).

In one of the tests, Connection C4 (one of the interior connections that had a 26 x 26 in. [660 x 660 mm] column and flexural reinforcement ratios within the column strip of 0.5% in the North-South direction and 1.9% in the East-West direction) was loaded concentrically up to failure. Connection C4 failed in shear before reaching its flexural capacity. The structure behaved elastically until failure occurred. The connection failed at a load that was 16% greater than that estimated using ACI 318-63.[22] The measured failure load, however, was 20% lower than that estimated using Moe's[35] equation (Eq. (17)).

Criswell[14,39]

Criswell[14,39] tested several connections with low flexural reinforcement ratios and some of his test results are summarized in Table 1. Criswell[14,39] found that a punching failure could occur at loads considerably below the ACI Code values. The total factored shear force V_u of the connections with $\rho = 0.75\%$ were approximately the same as V_{flex}, whereas values of V_u with $\rho = 1.5\%$ were lower than V_{flex}.

Criswell[14,39] indicated that because ACI 318-63[22] and Moe's[35] equations were derived using only test results with $\phi_o < 1.0$ and failing primarily in shear, the applicability of those equations to the connections with $\rho = 0.75\%$, which failed in flexure, was questionable. Criswell[14,39] stated, "…the strengths of the connections with smaller ρ values were primarily controlled by the flexural capacity even though a punching failure did develop before the connections displayed large ductility. Such failures could be considered as flexural-shear or secondary shear failure..."

Joint ACI-ASCE Committee 426[40]

Joint ACI-ASCE Committee 426[40] indicated that v at failure for lightly-reinforced slabs with a square column could be less than $4\sqrt{f_c'}$ psi ($0.33\sqrt{f_c'}$ MPa) if the slabs developed large deflections prior to the punching failure.

Hawkins and Mitchell[41]

Hawkins and Mitchell[41] reported that if a slab was properly designed according to ACI 318-77[24] concepts, the flexural strength could be slightly less than the shear strength and, therefore, the ACI 318-77[24] provisions attempted to define the punching shear strength for the onset of large rotations. The design based on ACI 318-77[24] was conservatively presumed to correspond to $\phi_o = 1.0$. Hawkins and Mitchell[41] indicated that if a connection is forced to develop rotations larger than those at which the flexural capacity is first reached, a punching failure occurs unless the shear stress is limited to $2\sqrt{f_c'}$ psi ($0.167\sqrt{f_c'}$ MPa) or shear reinforcement is provided.

Moehle et al.[42]

Moehle et al.[42] recommended that the shear strength of a connection be reduced to 3/4 of the value given by ACI 318 (for both basic formulas and with large critical shear area) if extensive yielding is anticipated.

Joint ACI-ASCE Committee 352[43]

Joint ACI-ASCE Committee 352[43] reported that connections subjected to widespread flexural yielding exhibited shear strengths lower than those failing in shear prior to flexural yielding because in-plane restraint significantly decreases when the flexural reinforcement yields. The committee recommended a reduction factor C_v of 0.75 in cases where flexural yielding is anticipated.

Yamada et al.[44]

Yamada et al.[44] reported that their control specimen (6.6 x 6.6 x 7.9 in. [168 x 168 x 200 mm]) failed in punching shear at the ultimate load that was only 92% of that estimated by the ACI 318 Code. The properties of the control specimen were as follows: 1) no shear reinforcement; 2) 1.23% slab top reinforcement (No. 4 bars, f_y = 116 ksi [799.8 MPa]); and 3) 0.62% slab bottom reinforcement (No. 4 bars, f_y = 116 ksi [799.8 MPa]).

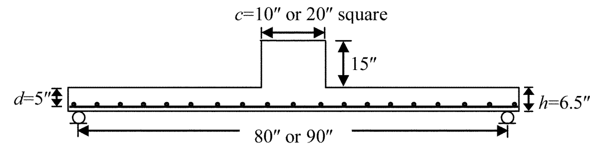
BUILDING CODE PROVISIONS FOR TWO-WAY SHEAR STRENGTH OF INTERIOR SLAB-COLUMN CONNECTIONS

The basic two-way shear provisions of several major building codes for interior slab-column connections without shear reinforcement under concentric load (that is, without moment transfer) are summarized in this section. Only square columns and normal-density concrete are considered.

Without shear reinforcement, the nominal two-way shear strength of reinforced concrete members, V_n, is equal to the concrete contribution V_c. All code recommendations on punching use nominal shear stresses calculated by dividing the shear force by an area equal to the product of the length of a critical perimeter and the effective depth of the slab. The codes differ in regard to the distance between the column face and the perimeter, and in the expressions used to define the limiting value of the stress, the effect of flexural reinforcement, and the size effect. Reviews of codes are given in Hallgren,[45] Bari,[4] *fib*,[46] Albrecht,[47] Salna et al.,[48] and several papers in ACI SP-232.[49]

Table 1—Criswell's test results

Specimen	Measured c/d	ρ, %	V_u/V_{ACI}	V_u/V_{flex}
S2075-1	2.1	0.79	0.85	1.02
S2075-2	2.1	0.78	0.83	0.95
S4075-1	4.0	0.75	0.62	1.01
S4075-2	4.1	0.77	0.56	0.99
S4150-1	4.1	1.50	0.92	0.89
S4150-2	4.1	1.50	0.92	0.87



Note: V_u is observed failure load, V_{ACI} is calculated failure load using ACI Code, and V_{flex} is calculated load using yield line theory.

Different building code provisions for V_c are summarized in Table 2. A special provision to account for a reduction in nominal shear strength due to increasing ratios of critical shear perimeter to effective depth is not included. To make the comparison easier, a consistent set of symbols is used for all provisions.

In general, V_c can be expressed as follows

$$V_c = v_c \times \xi \times \kappa_\rho \times b_o \times d \tag{24}$$

where v_c is the nominal shear strength, ξ is the size effect factor, κ_ρ is the longitudinal flexural reinforcement factor, b_o is the critical shear perimeter, and d is the effective depth.

The European codes use a characteristic strength f_{ck} instead of a specified concrete strength f_c'. Gardner[50] reported that f_c' can be related to f_{ck} as follows

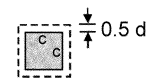
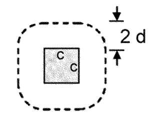
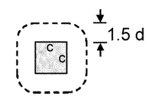
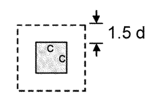
$$f_{ck} = f_c' - 1.60 \text{ MPa} \tag{25}$$

Building code provisions: comparison

As shown in Table 2, not all code provisions account for ρ as a factor affecting two-way shear strength. To compare the sensitivity of two-way shear strength with the change in ρ according to different code provisions, a prototype slab-column connection without shear reinforcement and with a 24 in. (610 mm) square column, 9 in. (230 mm) slab thickness, 7 in. (180 mm) effective depth, and 4000 psi (28 MPa) specified concrete cylinder strength was analyzed. Figure 2 shows the estimated two-way shear strength of an interior connection in the prototype structure as a function of ρ.

The shear strength varies from approximately 480 kN (107.9 kips) using German Code DIN 1045-1[51] to over 1100 kN (247.3 kips) using Canadian Standards CSA A23.3-04[52] for slabs with a 0.5% flexural reinforcement ratio. Some of these differences may be reduced if load or understrength factors are included. The variations, however, indicate the diverging approaches used for the code equations.

Table 2—Code provisions for basic two-way shear strength

	General equation: $V_c = v_c \times \xi \times \kappa_\rho \times b_o \times d$					
	v_c		ξ		κ_ρ	b_o
	Shear strength		Size effect			
Building codes	SI units	U.S. units	SI units	U.S. units	Reinforcement ratio	Critical shear perimeter
ACI 318-08	$0.33\sqrt{f_c'}$	$4\sqrt{f_c'}$	—	—	—	 $b_o = 4(c + d)$
CSA A23.3-04	$0.38\sqrt{f_c'}$	$4.6\sqrt{f_c'}$	For $d > 300$ mm: $1300/(1000 + d)$	For $d > 11.8$ in.: $51.2/(39.4 + d)$	—	
AS 3600-1994	$0.34\sqrt{f_c'}$	$4.1\sqrt{f_c'}$	—	—	—	
IS:456	$0.25\sqrt{f_{ck,\,cube}}$	$3\sqrt{f_{ck,\,cube}}$	—	—	—	
Eurocode 2-2003 and CEB-FIP MC 90	$0.18(f_{ck})^{1/3}$	$5(f_{ck})^{1/3}$	$\left(1+\sqrt{\dfrac{200}{d}}\right) \le 2.0$	$\left(1+\sqrt{\dfrac{7.9}{d}}\right) \le 2.0$	$(100\rho)^{1/3}$ $\rho \le 0.02$	 $b_o = 4(c + \pi d)$
DIN 1045-1	$0.14(f_{ck})^{1/3}$	$3.9(f_{ck})^{1/3}$				 $b_o = 4c + 3\pi d$
BS 8110-97	For $f_{ck,cube} > 25$ MPa: $0.79\left(\dfrac{f_{ck,\,cube}}{25}\right)^{1/3}$ $f_{ck,cube} \le 40$ MPa	For $f_{ck,cube} > 3600$ psi: $115\left(\dfrac{f_{ck,\,cube}}{3600}\right)^{1/3}$ $f_{ck,cube} \le 5800$ psi	$\left(\dfrac{400}{d}\right)^{1/4} \le 1.0$	$\left(\dfrac{15.7}{d}\right)^{1/4} \le 1.0$	$(100\rho)^{1/3}$ $\rho \le 0.03$	 $b_o = 4(c + 3d)$

Note: f_c' is specified concrete cylinder compressive strength, f_{ck} is characteristic concrete cylinder compressive strength, $f_{cube} \approx 1.25f_c'$, d is average depth of slab reinforcement, and c is column dimension.

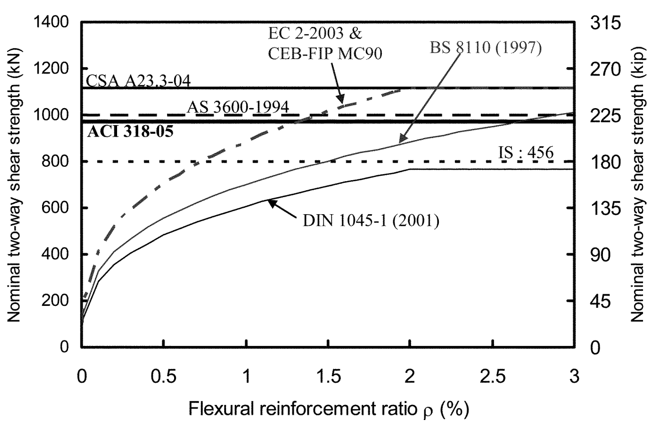


Fig. 2—Two-way shear strength according to different code provisions.

EFFECT OF FLEXURAL REINFORCEMENT ON PUNCHING SHEAR STRENGTH

Flexural reinforcement ratio

There have been conflicting opinions on whether the flexural reinforcement ratio ρ has an effect on the two-way shear strength of slab-column connections, V_c. Marzouk and Hussein,[53] Gardner and Shao,[54] and Sherif and Dilger[7] concluded from their test results that V_c is a function of ρ. Vanderbilt[55] showed that doubling ρ from 1 to 2% resulted in only a modest increase in V_c. Elstner and Hognestad[33] and Moe,[35] however, indicated that increasing ρ near the column did not increase V_c. The concentration of reinforcement resulted in ρ = 7 and 6.3%[33] and ρ = 1.5, 2.3, and 3.5%.[35] Whitney[34] and Alexander and Simmonds[56] pointed out that these earlier investigations did not show the benefits of increasing ρ by concentrating the flexural steel because the specimens failed due to bond failure of closely spaced bars.

Regan[57] indicated that ρ may affect punching resistance in several ways:

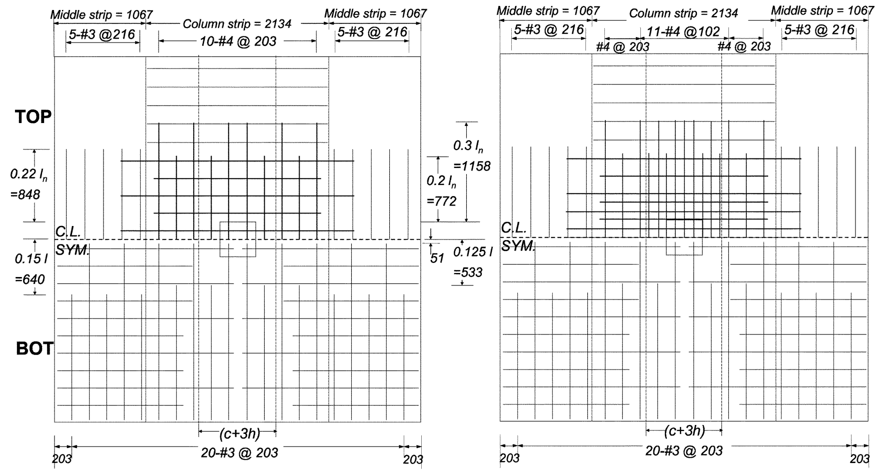
1. An increase of ρ should increase the depth of the compression zone and thus the area of uncracked concrete available to support shear forces. It should also reduce the crack width, thus improving the transfer of forces by aggregate interlock, and increase dowel action;

2. An increase of ρ should enhance the restraint available in the plane of the slab, and therefore increase the two-way shear strength. Hawkins and Mitchell,[41] however, indicated that the available restraint (due to membrane action) around the connection can diminish if flexural reinforcement yields. Therefore, the nominal ultimate shear strength of connections transferring shear decreases as the extent of yielding in the slab flexural reinforcement increases.

Yitzhaki[12] indicated that the relative amount of ρ with respect to the balanced reinforcement ratio ρ_{bal} (ρ_{bal} was defined as the ρ to make the punching shear strength equal to the flexural strength) can affect mode of failure. When $\rho < \rho_{bal}$, slabs would fail in flexure and increasing ρ is very effective to increase punching resistance. When $\rho > \rho_{bal}$, slabs would fail in punching. In this case, punching resistance was insensitive to ρ and increasing ρ to increase punching resistance would be uneconomical. Gardner[58] also indicated that while increasing ρ increases the punching resistance, the behavior of the connection becomes more brittle.

Concentration of reinforcement toward column or loaded area

Joint ACI-ASCE Committee 426[40] indicated that a concentration of reinforcement toward the column or loaded area does not improve the shear strength. The committee, however, encouraged the concentration of reinforcement in



Specimen G0.5
0.5% slab top steel within (c+3h)

Specimen G1.0
1.0% slab top steel within (c+3h)

Fig. 3—Details of slab reinforcement.

the column region because it enhances the flexural behavior of the slab under service loads.

Regan[57] reported that for practical arrangements of bars, Moe's[35] tests and the Concrete Industry Research and Information Association (CIRIA) results showed decreases of strength by roughly 6% with increasing concentration, compared with those for slabs with uniform steel. Regan and Braestrup[15] concluded that concentrating the reinforcement is not beneficial. In extreme cases, the results showed that it can even be harmful because excessive concentration leaves large radial sectors almost unreinforced.

Rankin and Long[9] indicated that the local increase of moment capacity due to concentration of reinforcement is offset by the reduction of slab ductility. McHarg et al.[59] concluded that the concentration of the top mat of flexural reinforcement results in a higher punching shear resistance, higher post-cracking stiffness, a more uniform distribution of strains in the top bars, and smaller cracks at all levels of loading compared with companion specimens with a uniform distribution of top reinforcement.

EXPERIMENTAL PROGRAM

To evaluate the two-way shear strength of a slab-column connection, two 2/3-scale interior slab-column connections (Specimens G0.5 and G1.0, shown in Fig. 3) were tested. The test specimens were 14 ft (4.3 m) square and had 6 in. (150 mm) thick slabs supported on 16 in. (400 mm) square columns.

Test specimens represent an interior flat-plate slab-column connection of a prototype structure that was designed using ACI 318-71.[23] The prototype structure for Specimen G0.5 was assumed to have office occupancy, a live load of 50 lb/ft^2 (2.4 kPa), partition and additional dead load of 20 lb/ft^2 (0.96 kPa), 21 ft (6.4 m) span length, 24 in. (610 mm) square column, and 9 in. (230 mm) slab thickness. The slab had 0.5% top reinforcement in the column strip and 0.25% reinforcement elsewhere, which were common in flat plate structures built in 1970s. Specimen G1.0 had 1% top reinforcement between lines that are 1.5 × (slab thickness) outside opposite faces of the column (a width of (c + 3h)), which is typical in modern flat-plate structures. All slabs had the same bottom reinforcement and no shear reinforcement was used. Grade 60 deformed reinforcement satisfying ASTM A706-06 requirements and 4000 psi (28 MPa) concrete were used in

Table 3—Test specimens and results

Specimen	ρ_{top} within $(c + 3h)^*$	f'_c, psi (MPa)	V,† kip (kN)	ACI b_o,‡ in. (mm)	$v = V/(b_o d)$, psi (MPa)
G0.5	0.5	4550 (31.4)	69.9 (310.9)	84 (2134)	$2.47\sqrt{f'_c}$ $(0.206\sqrt{f'_c})$
G1.0	1.0	4070 (28.1)	90.2 (401.2)	84 (2134)	$3.37\sqrt{f'_c}$ $(0.281\sqrt{f'_c})$

*c = 16 in. (406.4 mm) (column dimension); h = 6 in. (152.4 mm) (slab thickness).
†V is punching load.
‡b_o = 84 in. (2133.6 mm) was calculated for the critical perimeter d/2 away from column face (d = 5 in. [127 mm]).
Note: ρ_{top} is percent slab top steel and v is failure shear stress.

the experimental program. The actual concrete compressive strengths for both specimens are shown in Table 3. The details of slab reinforcement are shown in Fig. 3. Longitudinal reinforcement was placed in perpendicular directions and satisfied the minimum length requirements of Section 13.5.6 of ACI 318-71.[23] The top and bottom reinforcement in the lateral loading direction had a clear cover of 0.5 in. (13 mm). The average depth of slab reinforcement, d, was 5 in. (130 mm).

Figure 4 shows the setup used to test the specimens under monotonically increasing concentric vertical loads applied upward through the column. The positions of the vertical struts were selected using results of nonlinear finite element analyses conducted on the prototype structure to produce conditions similar to those under uniform loading on the slab.

Figure 5 shows the gravity load versus vertical displacement curves around the column for punching shear tests. The gravity load capacity and the shear stress at failure calculated at the critical shear perimeter v_c are summarized in Table 3. At failure, v_c of Specimens G0.5 and G1.0 reached $2.47\sqrt{f'_c}$ and $3.37\sqrt{f'_c}$ psi ($0.206\sqrt{f'_c}$ and $0.281\sqrt{f'_c}$ MPa), respectively. The measured strengths were only 63 and 85% of the strength estimated using ACI 318-08[5] expression (Eq. (1)). This observation is consistent with the test results of a 45 ft (13.7 m) square flat-plate structure.[37] The results of the tests conducted in this experimental study also indicate that two-way shear capacity of the connections were sensitive to the amount of flexural reinforcement within (c + 3h) region.

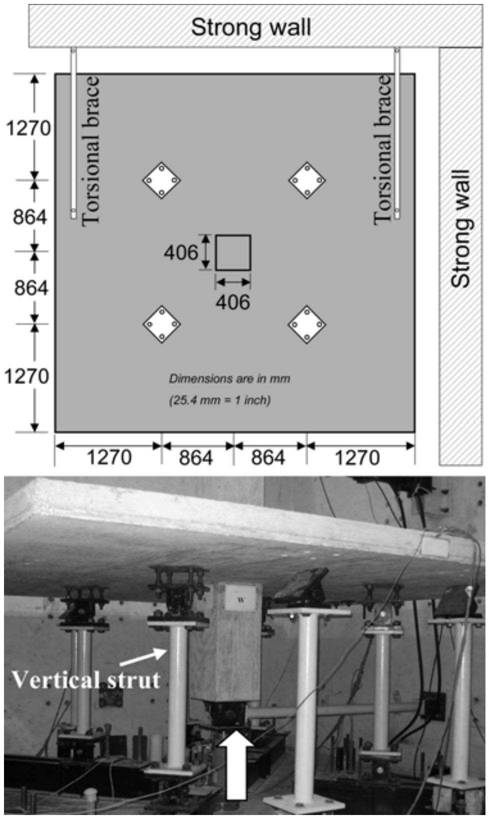


Fig. 4—Test setup.

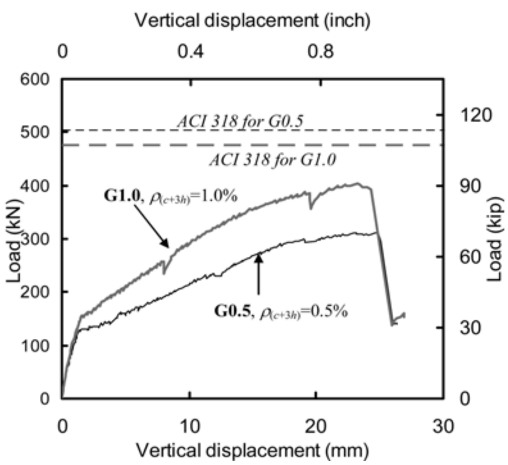


Fig. 5—Load-versus-deformation curves.

Measured versus estimated two-way shear strengths

The estimated two-way shear strengths using different codes for control Specimens G0.5 and G1.0 are compared with the measured strengths in Fig. 6. As can be seen in Fig. 6, only DIN 1045-1[51] gave conservative estimates of the two-way shear strength. All building codes that did not consider flexural reinforcement influence on the two-way shear strength[5,25,60,61] estimated that Specimen G1.0 had a lower two-way shear strength than Specimen G0.5 because Specimen G1.0 had somewhat lower concrete strength (Table 3). As expected, the other building codes that considered the influence of flexural reinforcement[51,62-64] estimated that Specimen G1.0 had a higher two-way shear strength than Specimen G0.5 because Specimen G1.0 had a higher percentage of flexural reinforcement.

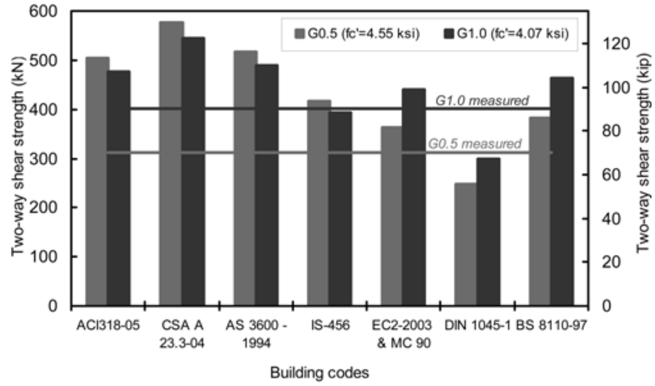


Fig. 6—Estimated strength from building codes versus measured strength.

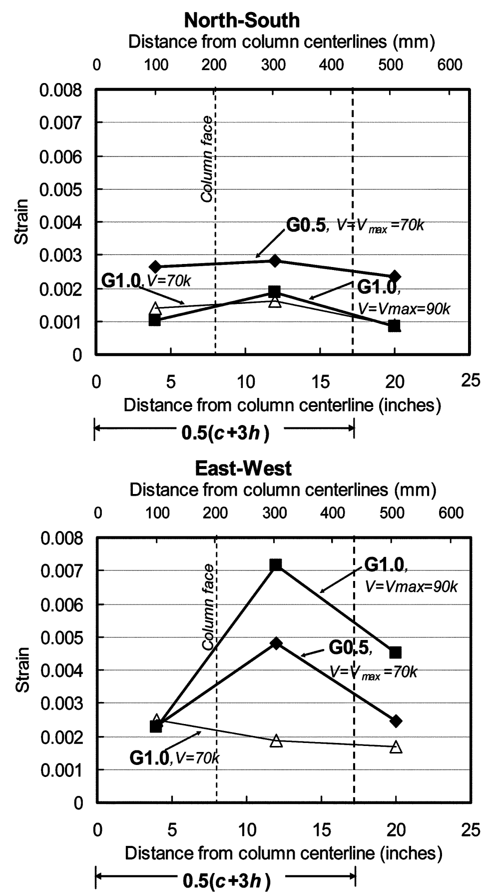


Fig. 7—Reinforcing bar strains.

Effect of flexural reinforcement ratio

In Fig. 7, the strains in reinforcing bars running in both the North-South and East-West directions at the maximum load V_{max} of both Specimens G0.5 and G1.0 are compared. At $V = V_{max}$, reinforcing bars within (c + 3h) region in both Specimens G0.5 and G1.0 yielded. The fact that flexural yielding preceded punching shear failure of Specimens G0.5 and G1.0 was consistent with the observations of failure made during testing.[1] Figure 7 also shows that the reinforcing bars outside the (c + 3h) region did not reach strains as high as those within the (c + 3h) region.

The strains in reinforcing bars of Specimen G1.0 at V = 70 kips (the maximum load of Specimen G0.5) are also shown in Fig. 7. At that load, the strains in Specimen G1.0

were generally half those in Specimen G0.5. This indicates that for a given load level, the reinforcing bar strain decreased as the percentage of flexural reinforcement increased. Smaller strains mean smaller crack widths and a more important contribution from aggregate interlock to the shear strength. Therefore, in lightly-reinforced slab-column connections (that is, with 1% flexural reinforcement or less), increasing the amount of flexural reinforcement within (c + 3h) region will result in a reduction of reinforcement strains and improvements in the shear strength.

CONCLUSIONS

Based on an extensive literature review, the following observations can be made:

1. The simple expression that gives the basic two-way shear strength in the current ACI provision ($V_c = 4\sqrt{f_c'}\, b_o d$) has not changed since 1963 and was developed from a relatively complex empirical equation proposed by Moe.[35] It should be noted that Moe's[35] empirical equation was based on a statistical analysis of 106 footing test results reported by Richart,[30,31] 34 slab test results from Elstner and Hognestad,[33] and by Moe[35] that were considered to have failed in shear ($\phi_o \leq 1.0$, where $\phi_o = V/V_{flex}$). Test results that were considered to have failed in flexure were excluded from Moe's[35] statistical analysis. Because Moe's[35] equation was derived based on test data with $\phi_o \leq 1.0$, it does not apply to the cases where $\phi_o > 1.0$;

2. It is clear that results from tests with $\phi_o \leq 1.0$ were used to arrive at the basic two-way shear strength expression ($V_c = 4\sqrt{f_c'}\, b_o d$). The ACI 318 provisions were conservatively presumed to correspond to $\phi_o = 1.0$. Because the shear strength decreases as ϕ_o increases, the applicability of the ACI provisions for typical lightly-reinforced slabs ($\rho_{column\ strip} < 1\%$) is questionable because such slabs have ϕ_o values larger than 1.0; and

3. There are significant variations among code provisions. Even for the codes that account for the influence of flexural reinforcement on the two-way shear strength, the influence of flexural reinforcement is accounted for in different ways. Code provisions are almost exclusively empirical and were derived by examining experimental results, which were very sensitive to test setups and specimen details. Because test setup, specimens, and reinforcement details varied among research projects, there are considerable divergences among the code provisions.

Based on the results of experimental research conducted at the University of Texas at Austin, the following observations can be made:

1. The capacity of Specimen G0.5 that represents a slab-column connection typical of flat-plate structures built in the 1970s was significantly overestimated (between 17 and 86%) by ACI 318-08,[5] CSA A.23.3-04,[52] AS 3600-1994,[60] IS-456,[61] EC2-2003,[62] MC-90,[63] and BS 8110-97.[64] Because the two-way shear capacity was sensitive to ρ, the differences were larger for code provisions that did not explicitly consider ρ as a parameter affecting the two-way shear strength.[5,52,60,61] Unlike other building codes, DIN 1045-1[51] provided a 20% conservative estimate of the capacity for the connection tested; and

2. The capacity of Specimen G1.0 (which represents a slab-column connection in typical flat-plate structures built to meet current standards) was also overestimated (between 9 and 36%) by all but DIN 1045-1.[51]

RECOMMENDATIONS

The ACI 318 provision for two-way shear strength ($V_c = 4\sqrt{f_c'}\, b_o d$) has not changed since 1963. The overwhelming evidence gathered from the literature[14,37,39-44] and obtained in the experimental program[1] illustrates that the use of ACI 318 provisions for lightly-reinforced slab-column connections is questionable. The current ACI provision for two-way shear strength does not reflect the shear strength after significant yielding of flexural reinforcement, which is the case in typical lightly-reinforced slab-column connections. Based on the experimental evidence reviewed, the two-way shear strength of lightly reinforced connections should be reduced. A value of ($V_c = 2\sqrt{f_c'}\, b_o d$) represents a lower bound on the data.

ACKNOWLEDGMENTS

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