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Predicting the punching strength of conventional slab-column specimens

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A method for predicting the punching strength of conventional slab-column specimens is derived from rational concepts of the various modes of failure. These modes can be classified broadly as either flexural or shear, depending on whether failure is initiated by the yielding of the reinforcement (flexure), crushing of the concrete (flexure) or by internal diagonal cracking (shear). The ultimate flexural capacity is related to the yield moment by an analytically based linear interpolative moment factor. This factor depends on the slab ductility which controls the degree of yielding in the slab at failure. The ultimate shear capacity is based on a semi-empirical relationship for vertical shear stress on a critical section close to the column perimeter. The proposed method of prediction is relatively simple to use and is shown to give significantly better correlation with test results than methods advocated by the present codes of practice and other procedures.

Notation

L	slab span ($= 2{\cdot}5a$)
M	moment per unit width
M_b	bending moment of resistance
$M_{(bal)}$	balanced moment of resistance
P	applied load
P_{flex}	ultimate load predicted by yield-line theory
P_p	predicted punching strength
P_T	measured ultimate load of test specimen
P_{vf}	predicted flexural punching strength by proposed method
P_{vs}	predicted shear punching strength by proposed method
U	cube compressive strength of concrete
a	distance between supports of conventional slab specimen ($= 0{\cdot}4L$)
c	length of column side
d	average effective depth to tensile reinforcement
f'_c	cylinder compressive strength of concrete (taken as 80% of the cube compressive strength)
f_y	yield stress of bonded reinforcement
h	overall depth of section
k_b	ratio of applied load to internal bending moment at column periphery
k_t	ratio of applied load to ultimate moment of resistance at failure
k_{yl}	moment factor for overall tangential yielding
r_f	reduction coefficient to allow for column shape

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r_s radius of conventional slab specimen
r_y radius of yield zone
s side length of conventional slab specimen
x depth of concrete compression zone
ρ reinforcement ratio
$\rho_{(bal)}$ reinforcement ratio at balanced moment of resistance

Introduction

The present understanding of punching failure in reinforced concrete flat slab structures is based largely on experimental studies of the behaviour and strength of the column and portion of slab within the elastic line of contraflexure (Fig. 1). This isolated unit is what is referred to as the conventional slab-column specimen. Consequently, the design provisions incorporated in the various building codes[1, 2, 3] are a direct result of the empirical procedures derived from tests on such specimens.

2. Unfortunately in the UK and in the USA the development of design approaches has followed different routes, the British codes[1, 2] being based primarily on the work of Regan[4] and the American code[3] being based primarily on the work of Moe[5]. Thus, various inconsistencies exist between the two codes. For example, the maximum permissible shear stress is taken on different critical perimeters around the column: in the British code the shear stress depends on the level of reinforcement and the concrete strength whereas in the American code the shear stress depends only on the concrete strength. This usually results in very different predictions of the punching strength for the same specimen. Thus because of the inconsistencies between these empirically based design approaches it is desirable to achieve a better understanding of the mechanisms of punching failure and to develop a more reliable method of predicting the punching strength.

3. In this Paper a method of prediction is developed from rational concepts of the modes of failure exhibited by conventional slab-column specimens. The basis for the development of the proposed method of predicting the punching strength is now reviewed briefly.

Basis of method

4. Experiments have shown that the ultimate load carrying capacity of conventional slab-column specimens is reached when a truncated cone or pyramid of concrete is punched through the slab by the loaded column.[6] Although punching failure occurs finally by the concrete shearing in the highly stressed compression zone adjacent to the column, the penultimate deformations depend primarily on the flexural characteristics of the slab and thus many researchers[5–9] have considered it necessary to examine both the flexural and shearing actions in the slab at failure.

5. Long and Bond[10] proposed the use of an approximate method of analysis to determine the moments in the slab at failure. This involved the interpolation between two idealized models of the slab-column connection at failure. Masterson and Long[11] later developed this approach and proposed a simplified finite element model for local slab conditions at ultimate failure. Their idealized representation of the slab-column connection was equivalent to the theory of development of local plasticity at the column periphery. By relating the applied load to the internal moment at failure and by making an appropriate allowance for dowel and

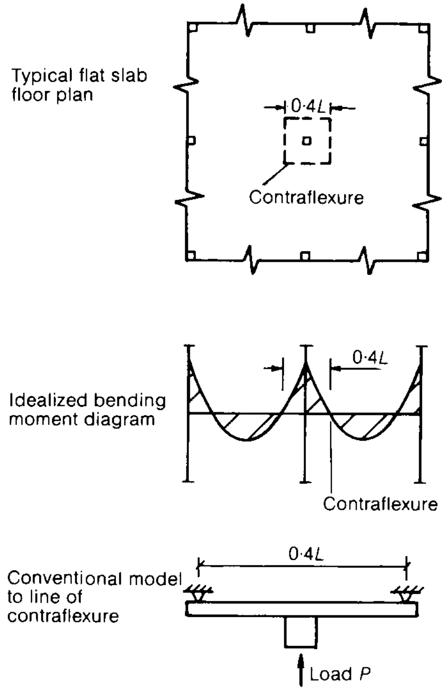

Fig. 1. Conventional slab-column test specimen

tensile membrane effects, the punching strength was found to be well predicted for the majority of realistic slab-column specimens tested by various researchers.

6. Long[12] later formulated a two-phase design procedure in which the punching strength was predicted as the lesser of either a flexural or shear criterion of failure. Although this approach was found to give good correlation with a wide range of test results there was some divergence between the predictions and the test results for slab-column specimens with low levels of reinforcement. Thus, it was considered that this two-phase approach formulated by Long could be improved by a more rational treatment of the flexural mode of punching failure.

Development of method—flexural punching strength

7. The behaviour of lightly reinforced slab-column specimens is characterized by considerable yielding of the reinforcement prior to punching. In fact, the lower the level of reinforcement the more the spread of yielding approaches the full yield-line pattern. Conversely, in heavily reinforced slabs, yielding becomes more localized and the failure mode approaches that of localized compression failure of the concrete around the column. Thus, depending on the slab ductility, the flexural punching strength of a reinforced concrete slab-column specimen must be somewhere between the yield-line capacity and the load to cause localized compression failure. This relationship between the flexural punching strength and the spread of yielding into the slab is illustrated in Fig. 2. The flexural punching strength for the

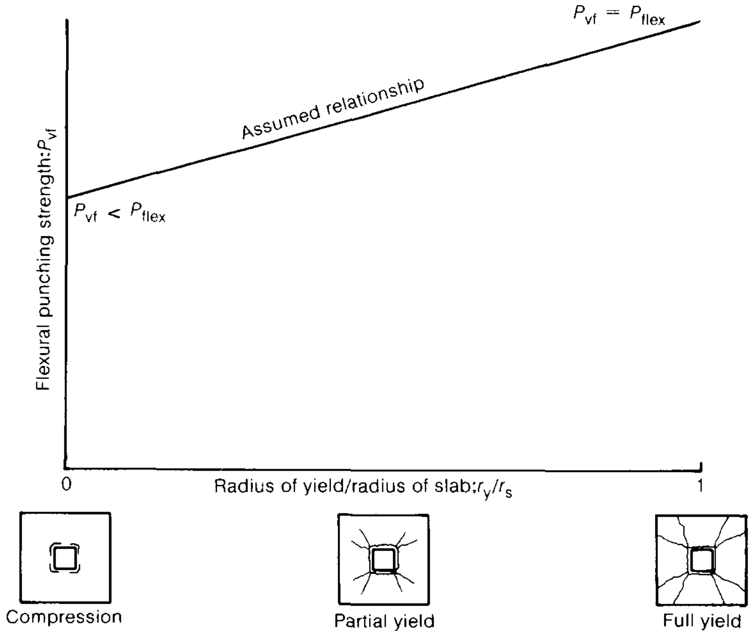

Fig. 2. Relationship between flexural punching strength and spread of yielding in slab

three cases of full yielding, localized compression and partial yielding can be considered as follows:

Full-yielding

8. The yield-line pattern giving the minimum load for conventional slab-column specimens is shown in Fig. 3. The flexural punching load required for this mode of failure is given by

$$P_{flex} = k_{yl} M_b \tag{1}$$

From consideration of the virtual work done by the actions on the yield lines the solution for this mechanism is

$$k_{yl} = 8\left(\frac{s}{a - c} - 0{\cdot}172\right) \tag{2}$$

Localized compression failure

9. In over-reinforced slabs punching will occur due to localized compression failure of the concrete when the internal moment at the column periphery attains its ultimate value. Thus, it is necessary to know the ratio of the applied column load to the internal moment at this critical section. This ratio can be termed the elastic moment factor as there is no plasticity beyond the column periphery. Therefore, the relationship between the applied load and the internal moment at the column periphery can be based on elastic plate analysis and is given by

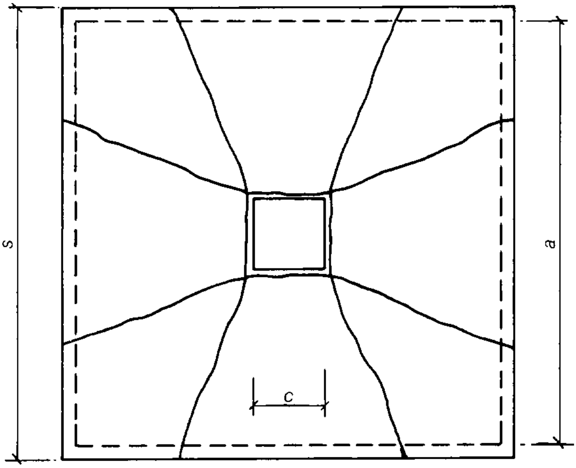

Fig. 3. Yield-line pattern for conventional slab-column specimen

$$P = k_b M_{(bal)} \qquad (3)$$

Long[12] derived a simplified relationship for k_b from finite element analyses and this is shown in Fig. 4. It can be seen that a slightly improved relationship for the elastic moment is given by the expression

$$k_b = \frac{25}{\left(\log_e \frac{2{\cdot}5a}{c}\right)^{1.5}} \qquad (4)$$

Partial yielding

10. The load at which punching occurs after partial yielding of the slab lies between the load to cause full yielding and the load to cause localized plasticity around the column. This load can be related to the yield moment by

$$P = k_t M_b \qquad (5)$$

Brotchie[13] has developed the general elastic–plastic relationship between the applied load and the radius of the yield zone for a simply supported circular plate under concentric ring loading. This relationship is illustrated in Fig. 5 and it can be seen that the moment factor for partial yielding k_t can be closely approximated by linearly interpolating between the moment factor for full yielding k_{y1} and the elastic moment factor k_b. Hence

$$k_t = k_b + (k_{yl} - k_b) r_y / r_s \qquad (6)$$

Slab ductility

11. In § 10 it was shown that the column load is directly related to the radius of the yield zone in the slab. Thus, if the radius of the yield zone at failure can be deduced then the flexural punching strength can be predicted. The radius of the

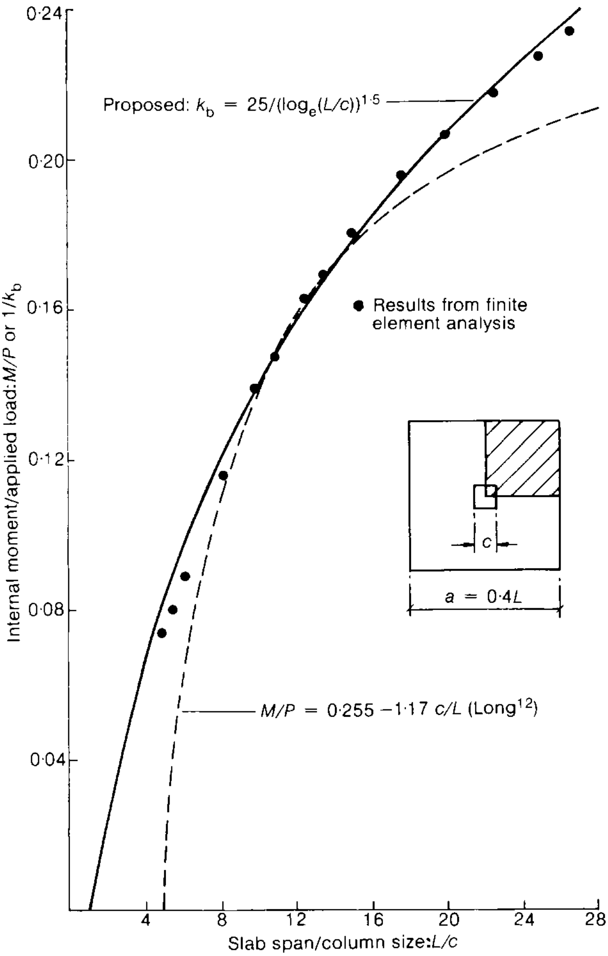

Fig. 4. Relationship between applied load and internal moment at column periphery

yield zone at failure is dependent on the ductility of the slab and it is widely accepted[14] that a measure of this is given by the ratio $\rho/\rho_{(\mathrm{bal})}$. However, as the calculation of $\rho_{(\mathrm{bal})}$ is quite lengthy it is proposed that a more convenient ductility parameter, i.e. $M_b/M_{(\mathrm{bal})}$, is used; M_b can be calculated from a widely accepted equation proposed by the American Concrete Institute.[3]

$$M_b = \rho f_y\, d^2(1 - 0{\cdot}59(\rho f_y/f'_c)) \tag{7}$$

$M_{(\mathrm{bal})}$ can be calculated simply from the empirical expression first proposed by Whitney[15] and is given by

$$M_{(\mathrm{bal})} = 0{\cdot}333 f'_c d^2 \tag{8}$$

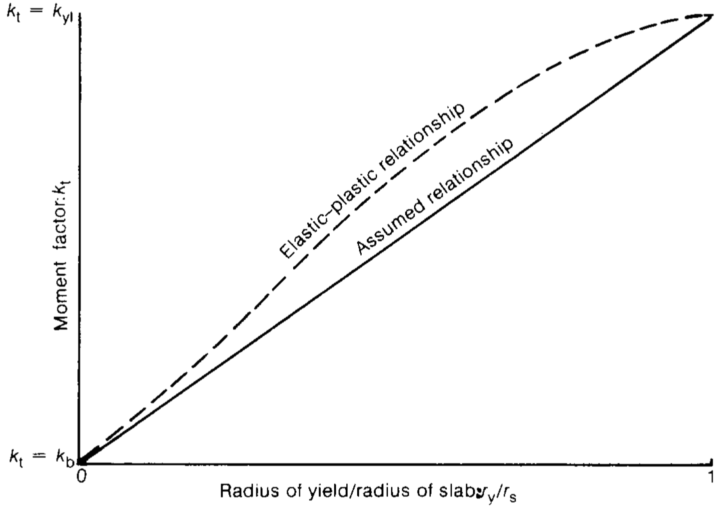

Fig. 5. Relationship between moment factor and spread of yielding in slab

12. As the development of the full yield-line pattern at failure requires a very low level of reinforcement in the slab (i.e. M_b approaches zero) it is proposed that the ratio $M_b/M_{(bal)}$ is taken as zero for full yielding. Conversely, for punching failure initiated by localized plasticity of concrete around the column a very high level of reinforcement is required (i.e. calculated value of $M_b \geqslant M_{(bal)}$) and therefore it is proposed that the ratio $M_b/M_{(bal)}$ is taken as unity for localized compression failure of the concrete. Thus, for punching failure to occur after partial yielding, the ductility parameter $M_b/M_{(bal)}$ must be between zero and unity. For simplicity it is proposed that the radius of the yield zone at failure can be taken as being linearly dependent on the slab ductility parameter, and thus

$$r_y/r_s = 1 - (M_b/M_{(bal)}) \tag{9}$$

This assumption will be tested later in relation to the results of tests reported in the literature. The moment factor for partial yielding now becomes

$$k_t = k_{yl} - (k_{yl} - k_b)(M_b/M_{(bal)}) \tag{10}$$

Column shape factor

13. It has been found that the punching strength of slab-column specimens with square columns is significantly reduced by the presence of stress concentrations at the column corners (e.g. Vanderbilt[16]). Regan[17] has suggested that connections which have circular columns are approximately 15% stronger than square columns of equivalent perimeters. It is, however, logical to assume that the effect of stress concentrations is least for punching at full yield and greatest for punching due to localized compression of the concrete. Thus, it is proposed to incorporate the column shape factor r_f in the interpolated moment factor such that when $M_b/M_{(bal)}$ approaches zero, k_t approaches k_{yl} and when $M_b/M_{(bal)}$ approaches unity, k_t approaches k_b/r_f. For conventional slab-column specimens the interpolated moment factor is therefore

$$k_t = k_{yl} - \left(k_{yl} - \frac{k_b}{r_f}\right)\frac{M_b}{M_{(bal)}} \tag{11}$$

The appropriate column shape factors are $r_f = 1{\cdot}0$ for circular columns and $r_f = 1{\cdot}15$ for square columns. It should also be possible to determine appropriate shape factors for rectangular columns of various aspect ratios.

13. The flexural punching strength of conventional slab-column specimens can therefore be predicted from the expression

$$P_{vf} = \left(k_{yl} - \left(k_{yl} - \frac{k_b}{r_f}\right)\frac{M_b}{M_{(bal)}}\right)M_b \not> \frac{k_b}{r_f} M_{(bal)} \tag{12}$$

Long[12] has shown that punching may also be precipitated by internal diagonal cracking before the flexural punching strength is attained. This shear mode of punching failure is examined in the following section.

Development of method—shear punching strength

14. The shear mode of punching failure is precipitated by internal diagonal tension cracking prior to the development of yielding of the reinforcement or crushing of the concrete[12]. Thus, as the shear punching strength must be dependent on the tensile strength of the concrete, the method of predicting the shear punching strength is based on an empirical relationship.

15. Long and Bond[10] have shown that the critically stressed concrete is on the neutral plane as there is no beneficial influence of direct compression at this level. The vertical shear stress is also dispersed before reaching the neutral plane and the critical section is therefore at some distance from the column periphery. This dispersion is dependent on the ratio of the size of column to the slab effective depth and it has been suggested[6] that this ratio may be taken into account by assuming a pseudo-critical section at one half of the slab effective depth from the column periphery (Fig. 6). This would seem reasonable in view of the fact that the shear mode of punching failure is more likely in heavily reinforced slabs in which the depth of the compression zone may well approach one half of the slab effective depth.

16. The depth of the concrete compression zone must also have an influence on the shear punching strength. From Fig. 7 it can be seen that for realistic concrete strengths and levels of reinforcement the neutral axis factor x/d is approximately dependent on $(100\rho)^{0{\cdot}25}$.

17. As it is widely accepted[18] that the tensile strength of concrete is directly related to the square root of the compressive strength it is proposed that the nominal ultimate shear stress is dependent on $\sqrt{f'_c}$. Taking into account the shape factor for square columns ($r_f = 1{\cdot}15$) the shear punching strength of conventional slab-column specimens has been determined on an empirical basis as

$$P_{vs} = 1{\cdot}66\sqrt{f'_c}(c + d)\, d(100\rho)^{0{\cdot}25} \tag{13}$$

The punching strength for the two principal modes of punching failure—flexure and shear, can now be calculated. Thus, the predicted punching strength is given by the lesser of these two values. A summary of the proposed procedure for predicting the punching strength of conventional slab-column connections is given in Fig. 8.

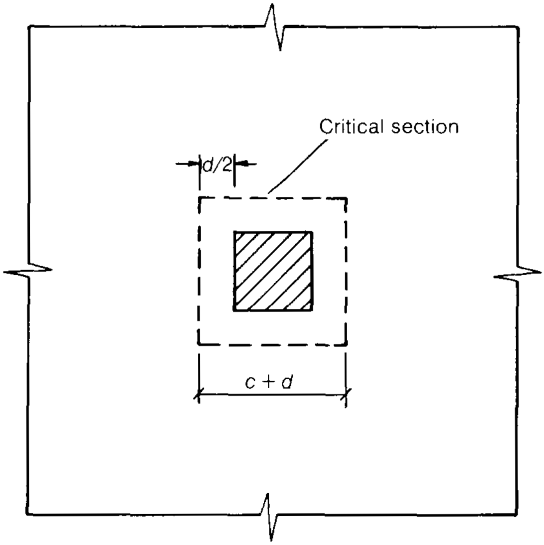

Fig. 6. Assumed critical section for shear mode of punching

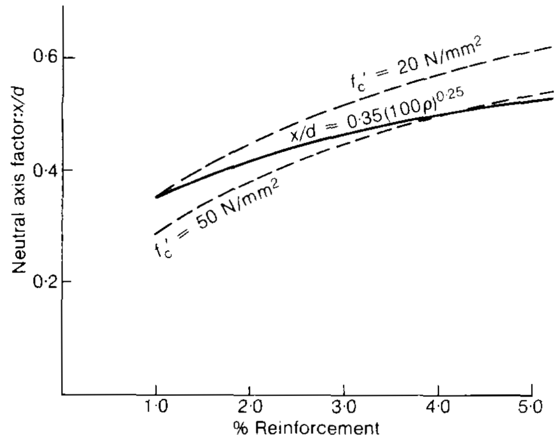

Fig. 7. Approximate relationship for neutral axis factor

Experimental programme

Test specimens

18. A realistic configuration of flat slab was selected and 27 conventional slab-column specimens subjected to concentric vertical loading were constructed and tested to failure. The series of tests comprised 1/4 scale square slabs extending to the nominal line of contraflexure, taken as being at $0{\cdot}2L$ from the column centre. A model concrete mix which had a realistic ratio of tensile to compressive

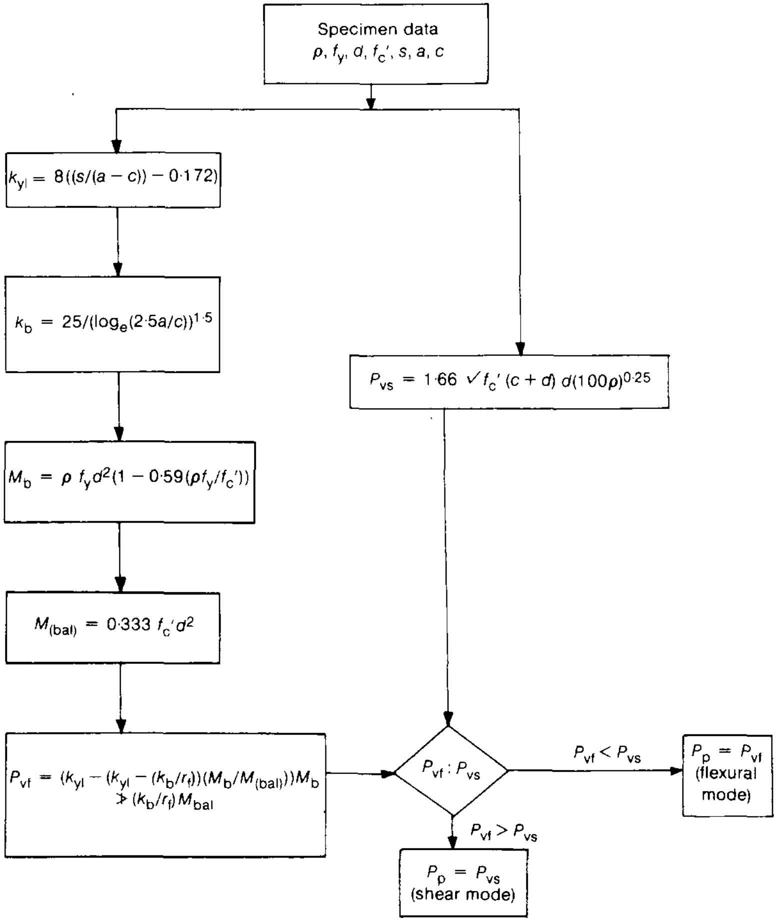

Fig. 8. Proposed procedure for predicting the punching strength of conventional slab-column connections

strength (approximately 9%) was utilized and there was little variation about the mean cube compressive strength of 40 N/mm^2. Tension reinforcement only was included in the models and this was varied over the range 0·4–2·0% for span/depth (L/h) ratios from 25–35. The reinforcement had a well defined yield point with no strain hardening (Fig. 9) and these properties were consistent throughout the test programme. Model dimensions are shown in Fig. 10 and the principal details of the slabs are presented in Table 1.

Test procedure

19. Each specimen was loaded incrementally until punching failure occurred. Measurements of deflexion, reinforcement strain and concrete strain at the column periphery were recorded at each stage of loading. The slabs were tested with the

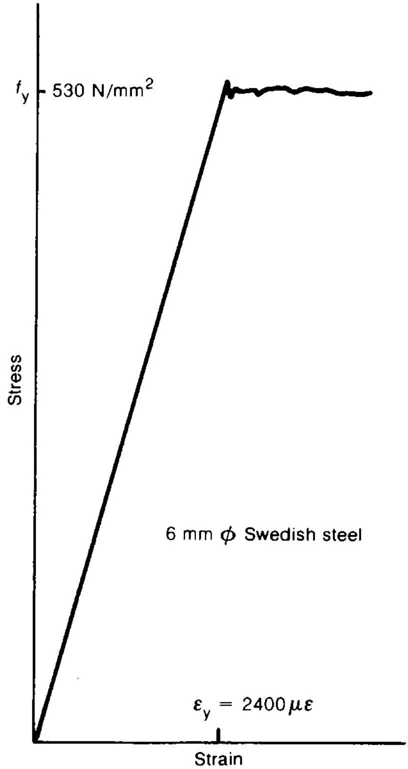

Fig. 9. Properties of model reinforcement

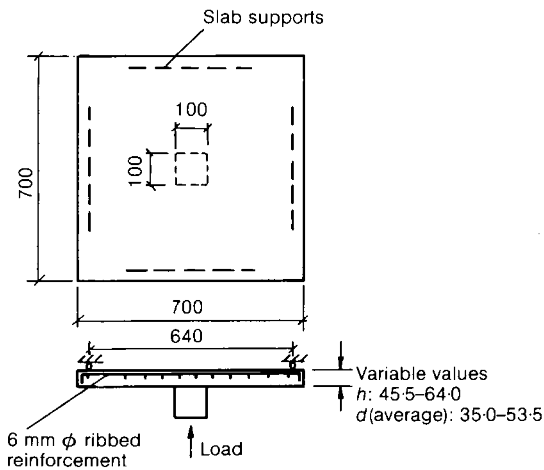

Fig. 10. Rankin's slab-column models[19]

Table 1. Results of conventional slab-column specimens tested by Rankin[19]

Model	h mm	d mm	ρ, %	U, N/mm^2	P_{TEST}, kN
1	51·0	40·5	0·423	38·4	36·42
2	51·0	40·5	0·558	38·4	49·08
3	51·0	40·5	0·691	38·4	56·55
4	51·0	40·5	0·821	43·5	56·18
5	51·0	40·5	0·883	43·5	57·27
6	51·0	40·5	1·026	43·5	65·58
7	51·0	40·5	1·163	37·1	70·94
8	51·0	40·5	1·292	37·1	71·09
9	51·0	40·5	1·454	37·1	78·60
10	51·0	40·5	0·517	37·4	43·59
11	51·0	40·5	0·802	37·4	55·00
12	51·0	40·5	1·107	37·4	67·06
13	51·0	40·5	0·601	42·5	49·39
14	51·0	40·5	0·691	42·5	52·45
15	51·0	40·5	1·994	42·5	84·84
1A	57·0	46·5	0·442	36·0	45·19
2A	57·0	46·5	0·691	36·0	66·24
3A	57·0	46·5	1·293	36·0	89·72
4A	57·0	46·5	1·992	38·6	97·43
1B	45·5	35·0	0·423	47·1	28·85
2B	45·5	35·0	0·690	47·1	37·63
3B	45·5	35·0	1·292	47·1	56·67
4B	45·5	35·0	1·994	38·6	72·52
1C	64·0	53·5	0·423	34·8	62·74
2C	64·0	53·5	0·690	40·5	87·86
3C	64·0	53·5	1·288	40·5	124·14
4C	64·0	53·5	1·993	34·8	125·94

tension face up (Fig. 11) to enable observations of crack development to be made. Fig. 12 shows the small number of large flexural cracks which develop before failure in lightly reinforced slabs and are typical of flexural punching failure after significant yielding has occurred. Conversely, Fig. 13 shows the large number of fine flexural cracks which develop before failure in heavily reinforced slabs and are typical of shear punching failure which occurs before yielding of the reinforcement.

Test results

20. The punching strengths of three sub-series of tests are plotted against the main variable of the percentage of reinforcement in Fig. 14. As the level of reinforcement is increased the punching strength is also significantly increased. For example, doubling the level of reinforcement from 0·6% to 1·2% increased the punching strength by 53% for the thin slabs ($L/h = 35$) and 44% for the thick slabs ($L/h = 25$). The punching strength of the thick slabs also levels off at a lower reinforcement percentage than for the thin slabs. This trend is consistent with the proposed concepts and is caused by the transition from the flexural to the shear mode of punching failure.

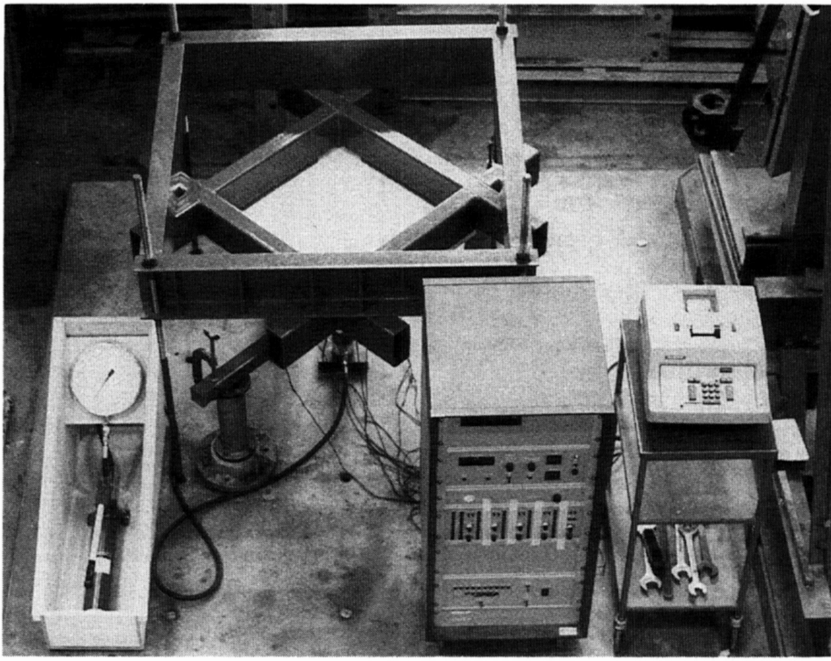

Fig. 11. Test arrangement for Rankin's tests[19]

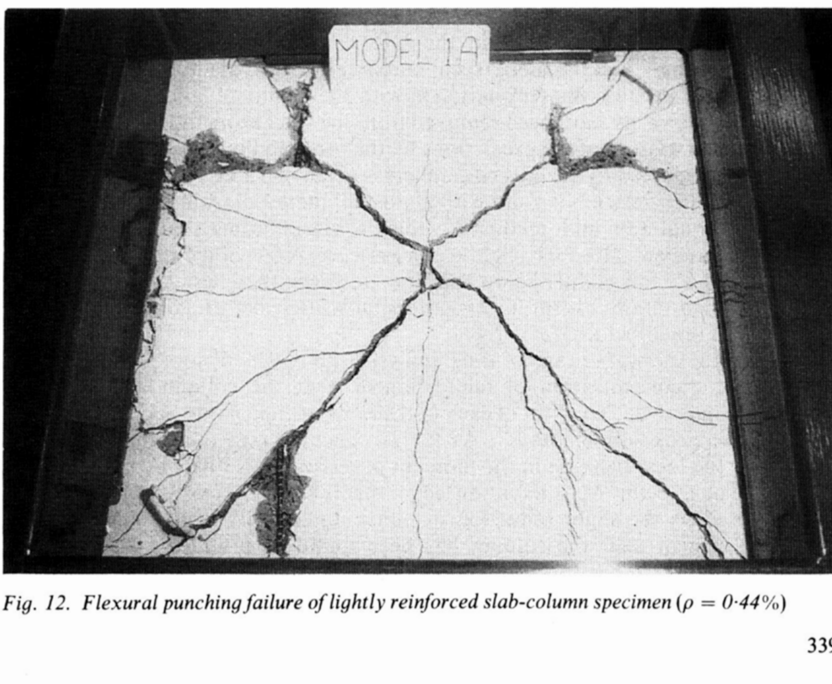

Fig. 12. Flexural punching failure of lightly reinforced slab-column specimen ($\rho = 0{\cdot}44\%$)

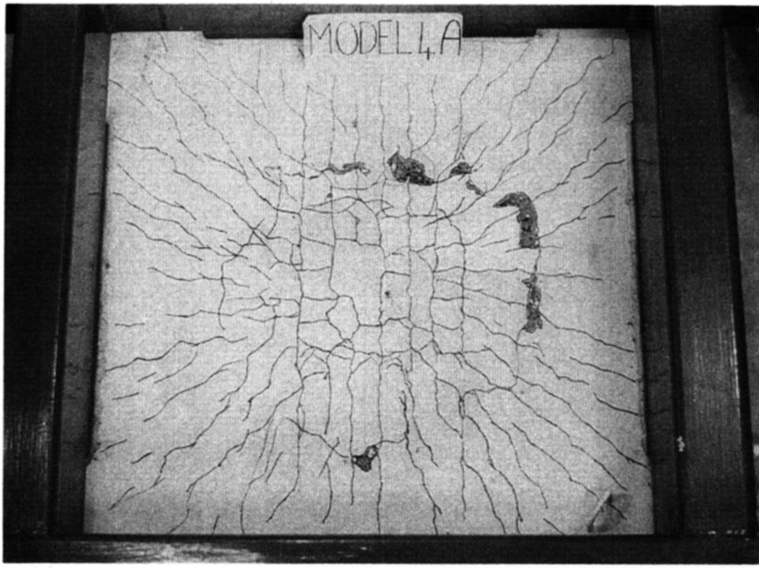

Fig. 13. Shear punching failure of heavily reinforced slab-column specimen (ρ = 2·0%)

Comparison of predictions with test results

21. The proposed method of predicting the punching strength of conventional slab-column specimens is compared with the test results of Rankin[19] and many of those reported in the literature[5, 8, 20–25]. The predicted punching strengths given by the present code procedures in the UK[1,2] and the USA,[2] the two-phase approach of Long[12] and the more recent empirically based method of Regan[17] are also compared. Because the correlation is with the results of laboratory tests the partial factors of safety have been removed from the various methods.

22. The correlation of the predictions by the proposed method with the results of 120 tests conducted by various researchers is given in Table 2 and shown in Fig. 15. The overall correlation is good with almost all the results falling slightly above the line of equal test and predicted values. This indicates that the proposed method is consistent, although slightly conservative. A few of the results of Elstner and Hognestad[20] fall slightly below this line; however, these test results appear to vary for no real reason except for experimental scatter and are for unrealistically high levels of reinforcement ($\rho > 2{\cdot}4\%$).

23. Within the wide range of tests analysed the results of some tests on slabs which have a concentration of reinforcement near the column and eccentric loading are included. The effect of each of these can be taken into account satisfactorily in the proposed method[19]. As far as concentrating the reinforcement is concerned, the local increase in the moment of resistance is offset by reduced slab ductility. Thus, to the Authors' knowledge, this is the only existing method that actually predicts the slight reduction in punching capacity which concentrating the reinforcement near the column has been found to produce[5]. To allow for eccentric loading, it has been found that this can be satisfactorily achieved by utilizing the capacity reduction factor proposed by Long[12].

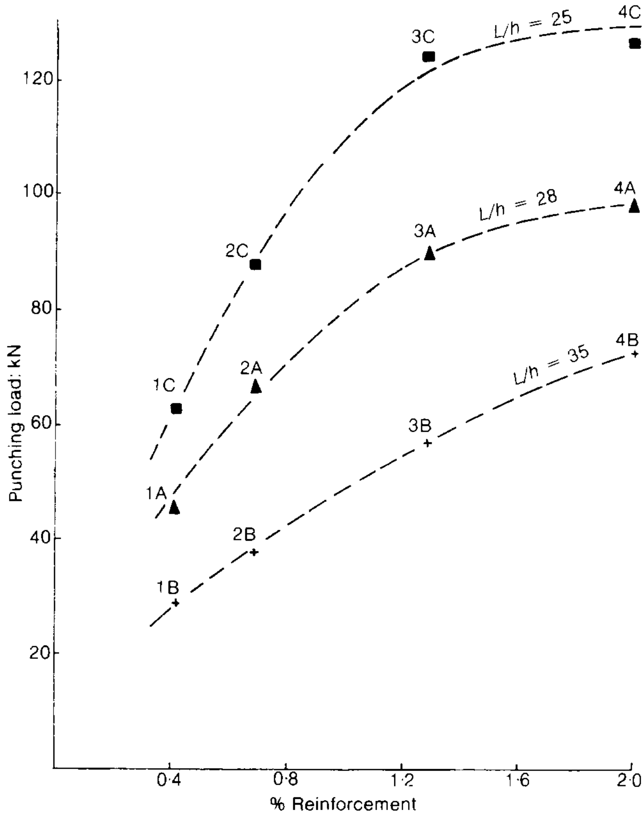

Fig. 14. Variation of punching load with level of reinforcement and span/depth ratio—Rankin's tests[19]

24. On examination of the predicted modes of failure it was found that of all the tests analysed only 10% punched due to localized compression failure of the concrete at the column periphery. Of the remaining tests approximately one half were predicted as flexural yield punching failures and one half as shear punching failures. Indeed, all slabs with a reinforcement index $\rho f_y/f'_c$ of less than 0·1 were predicted to punch in the yield mode. This highlights the importance of recognizing both the flexural and shear modes of punching failure for the normal range of reinforcement levels in flat slab structures.

25. For simplicity, the predictions by the proposed method were compared with the predictions by the other methods for concentrically loaded, uniformly reinforced slabs with square columns. This amounted to the correlation of each method with 101 test results from various sources. This information is presented in Table 3 which shows that the correlation given by the proposed method is significantly better than that given by the other procedures. The mean ratio of the test to predicted failure load by the proposed method is 1·189 with a coefficient of variation equal to 10·6%. This coefficient of variation corresponds to an improvement

Table 2. Correlation of prediction by proposed method with test results from various sources

Source of test results	Number of tests	Range of P_T/P_p	Average P_T/P_p	Coefficient of variation
Moe[5]	7	1·078–1·291	1·184	0·061
Moe*	6	0·989–1·336	1·190	0·124
Moe†	9	1·036–1·254	1·152	0·067
Kinnunen and Nylander[8]	4	1·029–1·217	1·101	0·080
Regan[25]	4	1·060–1·264	1·178	0·075
Rankin[19]	27	1·095–1·502	1·278	0·082
Elstner and Hognestad[20]	23	0·843–1·446	1·101	0·140
Base[21]	9	1·092–1·285	1·216	0·055
Taylor and and Hayes[22]	8	1·075–1·299	1·193	0·055
Dragosavic and Van den Beukel[23]	15	1·096–1·413	1·223	0·067
Criswell[24]	8	1·014–1·149	1·057	0·042

* Concentrated reinforcement.
† Eccentric loading.

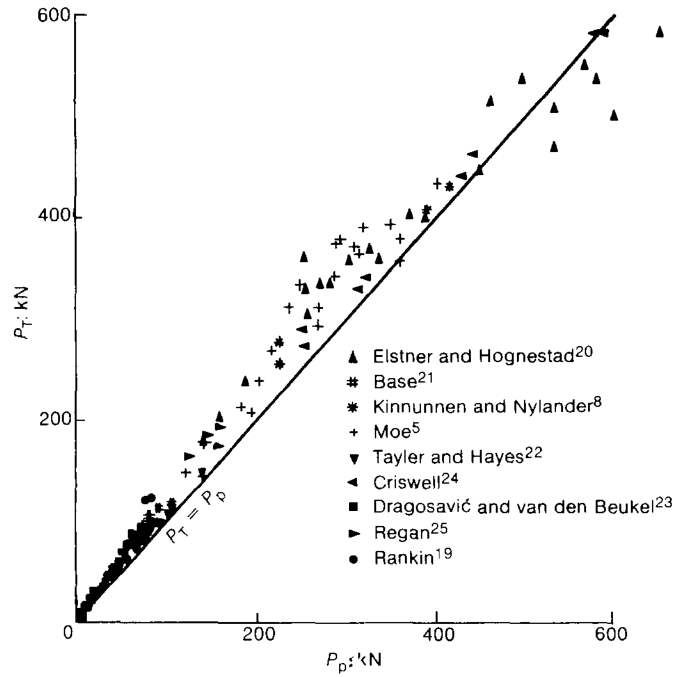

Fig. 15. Correlation of test results with punching strength predicted by proposed method

Table 3. Correlation of various methods of prediction with the results of 101 tests on uniformly reinforced slabs with square columns

Method of prediction	Mean ratio P_T/P_p	Coefficient of variation
Authors	1·189	0·106
Long[12]	1·212	0·148
Regan[17]	1·305	0·189
BS 8110[2]	1·262	0·183
CP 110[1]	1·339	0·195
American Concrete Institute[3]	1·435	0·198

of approximately 28% over the next coefficient of variation (14·8%) given by Long's method[12] and an improvement of approximately 44% over the average coefficient of variation given by the methods of CP 110[1], BS 8110[2] and Regan[17]. The mean ratio of the test to predicted punching strength by the proposed method is also slightly closer to unity than by the methods advocated by the British and American codes[1–3]. In addition, the predictions by the British code methods[1,2] would normally be divided by a partial factor of safety ($\gamma_m = 1{\cdot}25$) which would make the average ratio of test to predicted punching strengths even greater.

26. The American code method[3] gives a coefficient of variation of approximately the same magnitude as the British code methods[1,2]. However, the mean ratio of the test to predicted punching strengths is slightly higher by the American code method because the latter does not take into account the effect of the level of reinforcement on the punching strength.

27. The predictions by the various methods are shown in comparison with the main series of test results obtained by Rankin[19] in Fig. 16. The methods advocated by the British codes—CP 110 and BS 8110[2], the American code—American Concrete Institute[3], and Regan[17] do not allow adequately for the level of reinforcement, especially for realistic levels of up to about 1·5%. These methods also require the use of yield-line theory to give safe predictions for levels of reinforcement of less than about 0·4%. It is also evident that there is little difference in the predictions by the new British code BS 8110[2] and the older CP 110[1], although the new method does have the benefit of utilizing a slightly more logical shear perimeter at 1·5 d from the column face instead of 1·5 h as before.

28. The predictions by the proposed method, however, follow the trend of results quite well, particularly for realistic levels of reinforcement. In addition, the flexural punching predictions are conservative as the method interpolates between the upper bound plastic solution (yield-line theory) and the lower bound elastic solution (Long's approach[12]). However, unlike Long's method[12], it is unnecessary to include an empirical factor to allow for dowel and tensile membrane effects. This is because the proposed method is based on a more rational treatment of the flexural mode of punching failure.

Discussion

29. It has been shown that the proposed method for predicting the punching strength of conventional slab-column specimens gives good correlation with a wide range of test results. Thus, the classification of punching failure into the two

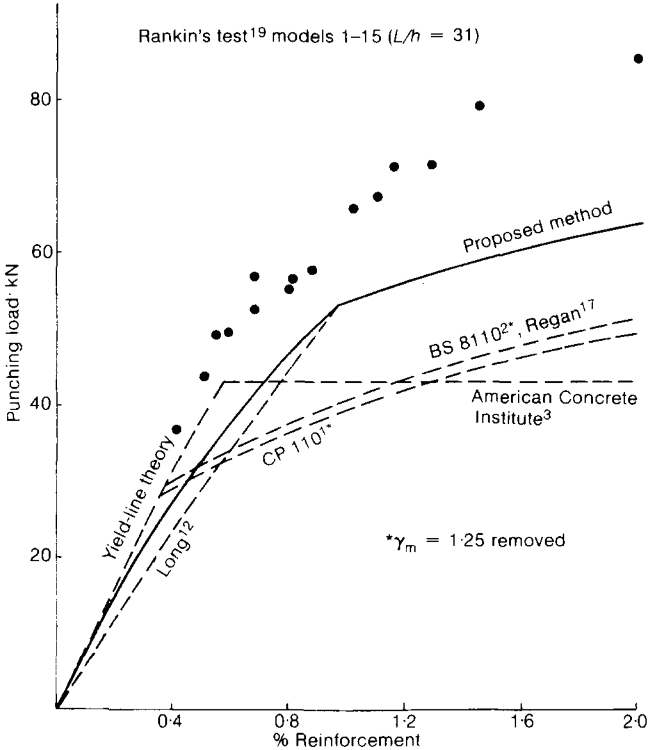

Fig. 16. Comparison of various methods of prediction with Rankin's test results[19]

principal modes of flexure and shear has proven to be a valid and worthwhile approach. Furthermore, the procedure is simple enough for use by designers and the use of the $M_b/M_{(bal)}$ concept allows some quantitative control over the ductility of the slab at failure. The method could be utilized to deal with slabs and columns of rectangular geometry and various boundary conditions. The effect of concentrating the reinforcement near the column and eccentric loading can also be taken into account.

30. The importance of the two principal modes of punching failure is that for a slab of a given span/depth ratio, the flexural punching strength can be increased effectively by increasing the level of reinforcement until the mode of punching failure becomes shear at which point the strength of the concrete becomes more important.

31. The adoption of this two-phase approach for the prediction of the punching strength of slab-column connections would mean that more efficient use could be made of the strength of the reinforcement and the concrete. Although simple to use, present code methods[1–3] do not give nearly as consistent predictions as this approach. Furthermore, because of their different empirical bases the British[1, 2] and American[3] code predictions differ widely for the same slab-column

connections. Thus, in order to produce more economical and safer designs of flat slab structures it would evidently be well worth recognizing both the flexural and shear modes of punching failure in future code provisions.

Conclusions

32. In view of the good correlation of the proposed method with a wide range of test results, the following conclusions concerning the punching strength of slab-column connections can be drawn.

(*a*) The two principal modes of punching failure can be classified as flexure and shear depending on whether failure is initiated by yielding of the reinforcement (flexure), crushing of the concrete (flexure) or internal diagonal cracking (shear).

(*b*) The proposed method for predicting the punching strength of conventional slab-column specimens is a rational extension of the work of Long[12]. It has been shown to give significantly better correlation with a wide range of test results than not only the British[1, 2] and American[3] code procedures but also the methods of Long[12] and Regan[17].

(*c*) The method of prediction in the new British code BS 8110[2] gives a slight improvement in correlation with test results compared with the older method in CP 110[1]. The American method of prediction advocated by the American code[3] gives a slightly higher average ratio of test to predicted punching strength and a coefficient of variation approximately the same as CP 110[1].

(*d*) Future code provisions to predict the punching strength of slab-column connections should recognize the importance of the flexural and shear modes of punching failure to produce more consistent and economical designs.

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