Performance of a Base-Isolated Building with System Parameter Uncertainty Subjected to a Stochastic Earthquake

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Base isolation has long been established as an effective tool for improving the seismic performance of structures. The effect of parameter uncertainty on the performance of base isolated structure is investigated in the present study. With the aid of the matrix perturbation theory and first-order Taylor series expansion, the total probability concept is used to evaluate the unconditional response of the system under parameter uncertainty. To do so, the conditional second-order information of responses are obtained by time domain nonlinear random vibration analysis through stochastic linearization. The implications of parametric uncertainty are illustrated in terms of the responses of interest in design applications. The lead rubber bearing isolator, isolating a multistoried building frame, is considered for numerical elucidation. It is observed that, although the randomness in a seismic event dominates, the uncertainty in the system parameters also affects the stochastic responses of the system. Particularly, the variance of the stochastic responses due to parameter uncertainty is notable.

1. INTRODUCTION

Vibration control technologies are widely acclaimed amongst researchers and practicing engineers as a viable alternative to traditional seismic design, which relies on energy dissipation through inelastic deformations of structural elements under earthquake-induced vibrations. In contrast to the traditional design, passive/active vibration control strategies substantially reduce the structural responses to ensure minimal damage to structures. A reduction of response is achieved through control systems, using base isolation (BI), tuned mass dampers, liquid column vibration absorbers, etc. Among these, BI systems have been used and globally accepted as an effective technology to reduce the seismic effects on strategically important structures as well as in retrofitting. In a BI system, the building rests on a system of isolators uncoupling the building from the horizontal component of the ground motion to effectively reduce the seismic load transmission to the structure. Various BI devices, such as rubber bearings (RB), lead rubber bearings (LRB), high-damping rubber bearings (HDRB), friction pendulums (FP), and resilient friction bearing isolators (R-FBI), are conventionally adopted for seismic protection of buildings, bridges, and other infrastructural facilities. These devices use different materials and design strategies to disconnect the superstructure motion from the ground. The effectiveness of BI systems and their performances has been extensively studied.[1–4] Several studies on stochastic response of base-isolated structures under random earthquakes are also notable.[5–7] These studies provide important insight into the behaviour of structures with BI systems. It is well established that the response of BI systems largely depends on the characteristics of the isolator, such as the yield strength for the LRB and RB types of isolator, optimal damping for R-FBI, and so on. Earlier studies have also provided parameters to ensure optimal performances.[7–9] In fact, studies on the optimum design of such systems are well known.[8–10] However, most of these works are based on deterministic descriptions of the parameters, characterizing the mechanical model of the superstructure-BI system as well as the stochastic load model for the earthquake. A major limitation of the deterministic approach is that the uncertainties in the performance-related decision variables cannot be included in the parameters for the process of optimization. Yet, the efficiency of such a system may be drastically reduced if the parameters are off tuned to the vibrating mode for which it is designed to suppress because of the unavoidable presence of uncertainty in the system parameters. Therefore, the passive vibration control of structures using BI system with uncertain parameters has attracted the interest of the vibration control community.

The developments in the field of passive vibration control by using various passive devices and considering system parameter uncertainty have been improved by many researchers.[11–18] However, this is not the case for BI systems. Studies on the performance of BI systems in connection with passive vibration control strategy are very limited. Benfratello et al. indicated that the effect of uncertainty on the response of structure with regard to base isolators and the ground motion filter parameters cannot be ignored.[19] Kawano et al. studied the effect of uncertain parameters on the nonlinear dynamic response of the BI structure in the framework of Monte Carlo simulation methods.[20] It has been demonstrated that uncertain parameters play a significant role on the maximum responses of the BI system. Scruggs et al. proposed a probability-based active control synthesis for seismic isolation of an eight storey base-isolated benchmark structure using uncertain model parameters.[21] Zhou, Wen, and Cai[22] and Zhou and Wen[23] presented two adaptive back-stepping control algorithms for the active seismic protection of building structures using an uncertain hysteretic system. Though studies on the performance of BI systems supplemented by the active vibration control strategies are extensive, the effect of uncertain parameters on the responses and performance of BI systems with passive vibration control is limited.

Thus, in the present study, the effect of system parameter uncertainty on the performance of BI systems is evaluated under a stochastic earthquake load. The response evaluation involves consideration of uncertainty in the properties of the isolated superstructure, isolator, and ground motion characteristics. With the aid of the matrix perturbation theory using first-order Taylor series expansion, the total probability concept is used to evaluate the unconditional response of structures under parameter uncertainty.[24] For this, the conditional second-order information of responses are obtained using time domain analysis of nonlinear random vibration by stochastic linearization. Subsequently, the root mean square (RMS) of the top-floor displacement and acceleration (considered to be the performance index) are obtained to study the effect of system parameter uncertainty. Numerical analysis elucidates the effects of parameter uncertainty on the stochastic responses of interest. The implication of the parametric uncertainty is demonstrated in terms of the disparity between the conditional and unconditional stochastic responses and their associated variances.

2. FORMULATIONS

2.1. Response of a Base-Isolated Structure under a Random Earthquake

The structure considered in the present study is idealized as a shear frame isolated by an LRB type of isolator. The idealization of the structure and the LRB is shown in Figs. 1(a) and (b), respectively. Since the BI system substantially reduces the structural response, the isolated structure can reasonably be assumed to behave linearly. The damping of the superstructure is assumed to be the viscous type. The energy is dissipated through the hysteresis of LRB by large shear deformation and by yielding of the lead core. Therefore, the force-deformation behaviour of the LRB is highly nonlinear and is idealized as bilinear (Fig. 1(c))[25–27] with associated parameters such as yield displacement (q), yield strength (F_Y), pre-yield (k_b), and post-yield stiffness (αk_b). The structure is considered to be excited by the horizontal component of the ground motion only.

The equation of motion for an n-storied superstructure can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{r}(\ddot{x}_g + \ddot{x}_b); \qquad (1)$$

in which $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{C}$ are the mass, stiffness*, and damping matrices of the structure, respectively, of order $n \times n$; $\mathbf{x} = [x_1 \quad x_2 \quad \dots \quad x_n]^T$ is the displacement vector of the superstructure containing the lateral displacement of any floor relative to the isolator, shown in Fig. 1(a). The symbol $\mathbf{r}$ is the influence coefficient vector implying the pseudo-elastic deformation of the respective floor due to a unit deformation of the ground. The symbol $\ddot{x}_b$ is the acceleration of the isolator with respect to the ground and $\ddot{x}_g$ is the earthquake ground acceleration. The details of these matrices are presented in the appendix.

The governing equation of motion for the isolator mass (Fig. 1(b)) can be written as

$$m_b\ddot{x}_b + c_b\dot{x}_b + F_b - c_1\dot{x}_1 - k_1x_1 = -m_b\ddot{x}_g; \qquad (2)$$

where m_b is the mass of the base, F_b is the restoring force of the isolator, c_b is the viscous damping in the rubber of the LRB, and k_1 and c_1 are the stiffness and damping of the first storey of the superstructure, respectively.

The bilinear force-deformation behaviour is adopted in this study to model the LRB in which the force-deformation behaviour is expressed by the differential Bouc-Wen model.[25,26] Following this, the isolator restoring force can be expressed as

$$F_b(x_b, \dot{x}_b, Z) = \alpha k_b x_b + (1-\alpha)F_y Z; \qquad (3)$$

where k_b is the initial elastic stiffness, x_b and $\dot{x}_b$ are the relative displacement and velocity, α is the ratio of post- to pre-yield stiffness (referred as rigidity ratio), and F_y is the yield strength of the LRB. The variable Z is a variable quantifying the hysteretic behaviour of the isolator. Substituting Eq. (3) in Eq. (2) and dividing by m_b, the equation reduces to

$$\ddot{x}_b + \frac{c_b}{m_b}\dot{x}_b + \alpha\frac{k_b}{m_b}x_b + \frac{(1-\alpha)F_y}{m_b}Z - \frac{c_1}{m_b}\dot{x}_1 - \frac{k_1}{m_b}x_1 = -\ddot{x}_g. \qquad (4)$$

The variable Z is governed by the differential equation

$$q\dot{Z} = -\gamma|\dot{x}_b|Z|Z|^{\eta-1} - \beta\dot{x}_b|Z|^{\eta} + \delta\dot{x}_b; \qquad (5)$$

where q is the yield displacement of the isolator. The five parameters β, γ, η, α, and δ in Eq. (5) control the shape of the hysteresis loop. The variable η controls the transition from the elastic to plastic phase; when $\eta \to \infty$ (infinity), the model becomes elasto-plastic. The nature of the hysteretic behaviour is controlled by β; $\beta > 0$ implies hardening and $\beta < 0$ results in softening. Presently, the parameters are adopted as $\alpha = 0.05$, $\beta = \gamma = 0.5$, $\delta = 1$, and $\eta = 1$ to correspond the bilinear behaviour. However, such choices lead to smooth transition from the elastic to plastic state, which is adequately taken to be close enough to sharp bilinear behaviour.

The post-yield stiffness of the LRB (αk_b) is selected to provide the specific time period of isolation (T_b), given by

$$T_b = 2\pi\sqrt{\frac{M}{\alpha k_b}}; \qquad (6)$$

where $M = \sum_{i=1}^{n} m_i + m_b$ is the total mass of the isolation-superstructure system, which is the sum of all the floor mass

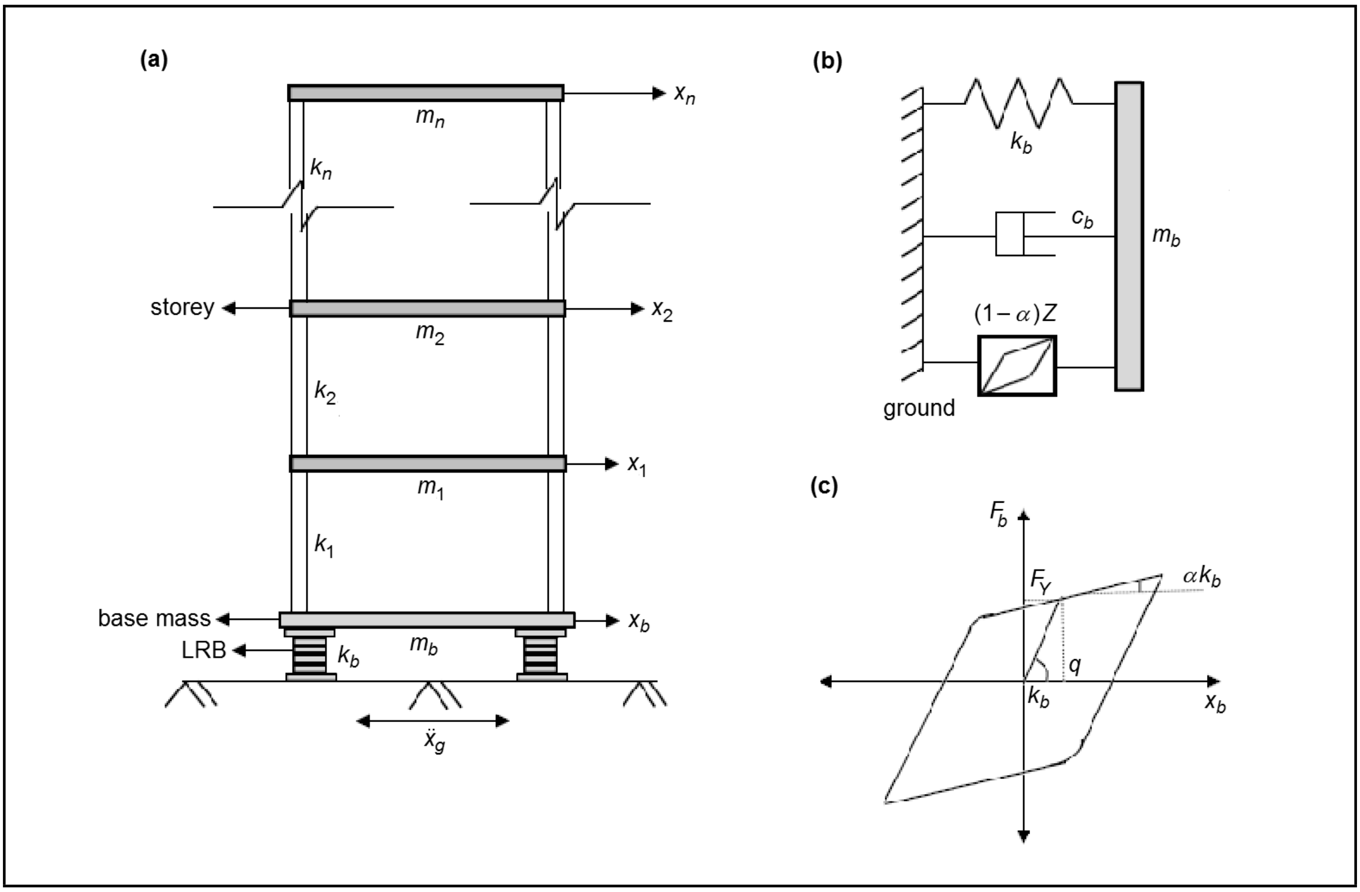

Figure 1. (a) Schematic of the base-isolated structure; (b) Mechanical model of the LRB; and (c) Idealized bilinear hysteresis of the LRB under cyclic loading.

(m_i) and the isolator mass (m_b). The viscous damping (c_b) in the rubber of the LRB is written as

$$c_b = 2\xi_b M\omega_b; \tag{7}$$

where ξ_b is the damping ratio and ω_b is the frequency ($\omega_b = 2\pi/T_b$) of the isolator. The variable q is the displacement corresponding to the yield strength, conveniently normalized (F_0) with respect to the weight ($W = Mg$) of the isolation-structure system:

$$F_0 = \frac{F_Y}{W}; \tag{8}$$

g is the gravitational acceleration. The characteristic parameters of the isolator are, therefore, the time period of isolation (T_b), viscous damping (ξ_b), and the normalized yield strength (F_0). These parameters dictate the performance of isolation in seismic vibration mitigation. The response quantities of the BI system varies monotonically with respect to the isolator time period (i.e., increasing time period reduces the response and vice versa).[7,8] The response also reduces monotonically with increasing damping for the LRB.[7,8] However, depending on the type of BI system (e.g., R-FBI), this trend might not be the same, and instead, the optimal value of damping minimizes the response.[7,8] In the present study, the isolator time period is fixed at 2 s, and the viscous damping in the isolator is taken as 10%.

The nonlinear force-deformation characteristic in Eq. (5) of the LRB is too complicated to be readily incorporated in the state-space formulation for the evaluation of the stochastic responses and the associated sensitivity statistics. This has been facilitated through stochastic linearization by writing a stochastically equivalent linear form of the nonlinear Eq. (5) as[27,28]

$$q\dot{Z} + C_e\dot{x}_b + K_e Z = 0; \tag{9}$$

where C_e and K_e are the equivalent damping and stiffness values obtained from the least-square error minimization among the nonlinear equation (Eq. (5)) and the linear equation (Eq. (9)). Presently, for $\eta = 1$, the closed form expressions of the equivalent linear damping and stiffness values are adopted as[27]

$$C_e = \sqrt{\frac{2}{\pi}}\left\{\gamma\frac{\mathrm{E}[\dot{x}_b Z]}{\sqrt{\mathrm{E}[\dot{x}_b^2]}} + \beta\sqrt{\mathrm{E}[Z^2]}\right\} - \delta; \tag{10a}$$

$$K_e = \sqrt{\frac{2}{\pi}}\left\{\gamma\sqrt{\mathrm{E}[\dot{x}_b^2]} + \beta\frac{\mathrm{E}[\dot{x}_b Z]}{\sqrt{\mathrm{E}[Z^2]}}\right\}; \tag{10b}$$

in which E[ ] is the expectation operator. In stochastic linearization, the responses $(x_b, \dot{x}_b)$ of the system are assumed to be jointly Gaussian. This assumption might not be correct, given that the system is nonlinear in the presence of bilinear hysteresis. However, it is demonstrated that this assumption does not result in serious error so far as the stochastic response evaluation is concerned.[27,28]

In evaluation of the stochastic response of a structure under a random earthquake, unlike the deterministic time histories, stochastic models are employed to describe the underlying stochastic process. Presently, the well-known Kanai-Tajimi model is considered;[29,30] the power spectral density of the ground motion, $S_{\ddot{x}_g}(\omega)$, in this model is expressed as

$$S_{\ddot{x}_g}(\omega) = S_0 \left(\frac{1 + 4\xi_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2 + 4\xi_g^2(\omega/\omega_g)^2} \right); \quad (11)$$

where S_0 is the white-noise intensity of the rock bed excitation responsible for the seismic event; ω_g and ξ_g are the characteristic frequency and damping of the soil media over the rock bed and underlying the building. The variable ω is the frequency component of the ground motion. The parameter S_0 is related to the RMS ground acceleration ($\ddot{u}_{g\,\max}$) of the earthquake as[31]

$$S_0 = \frac{2\xi_g \ddot{u}_{g\,\max}}{\pi(1+4\xi_g^2)\omega_g}. \quad (12)$$

The state-space formulation can include the white-noise type of ground excitation directly in the formulation, whereas the coloured-noise type of excitation (as in the Kanai-Tajimi model) can be included in the formulation by incorporating the equations for the Kanai-Tajimi filter within the dynamic equations of motion for the superstructure and the isolator. These equations convert the rock bed white noise to colour while passing through the filter. The equations for the Kanai-Tajimi filter can be expressed as

$$\ddot{x}_g = \ddot{x}_f + \ddot{w}; \quad (13a)$$

$$\ddot{x}_f + 2\xi_g\omega_g\dot{x}_f + \omega_g^2 x_f = -\ddot{w}. \quad (13b)$$

Substituting Eq. (13b) in Eq. (13a),

$$\ddot{x}_g = -2\xi_g\omega_g\dot{x}_f - \omega_g^2 x_f; \quad (14)$$

in which $\ddot{w}$ is the white-noise intensity at the rock bed with power spectral density of S_0. The variables $\ddot{x}_f$, $\dot{x}_f$, and x_f are the response of the Kanai-Tajimi filter. The seismic motion, thus, is introduced in the formulation through incorporating Eq. (13b) and substituting the expression $\ddot{x}_g$ from Eq. (14) in the rest of the equations.

The above equations are now rearranged to represent the state-space form. Multiplying both sides of Eq. (1) with $\mathbf{M}^{-1}$ and substituting the expression with $(\ddot{x}_g + \ddot{x}_b)$ from Eq. (4), Eq. (1) can be rewritten as

$$\ddot{\mathbf{x}} = -\mathbf{M}^{-1}\mathbf{C}\dot{\mathbf{x}} - \mathbf{M}^{-1}\mathbf{K}\mathbf{x} + \mathbf{r}\left(\frac{c_b}{m_b}\dot{x}_b + \alpha\frac{k_b}{m_b}x_b + \frac{(1-\alpha)F_Y}{m_b}Z - \frac{c_1}{m_b}\dot{x}_1 - \frac{k_1}{m_b}x_1 \right). \quad (15)$$

Similarly, Eq. (2) for the base mass/isolator can be rewritten by dividing both sides by base mass m_b and substituting the expression $\ddot{x}_g$ from the filter Eq. (14) as

$$\ddot{x}_b = -\frac{c_b}{m_b}\dot{x}_b - \alpha\frac{k_b}{m_b}x_b - \frac{(1-\alpha)F_Y}{m_b}Z + \frac{c_1}{m_b}\dot{x}_1 + \frac{k_1}{m_b}x_1 + 2\xi_g\omega_g\dot{x}_f + \omega_g^2 x_f. \quad (16)$$

Equation (9) for Z can be rewritten as

$$\dot{Z} = -\frac{C_e}{q}\dot{x}_b - \frac{K_e}{q}Z. \quad (17)$$

Also, the filter equations from Eq. (13b) can be expressed as

$$\ddot{x}_f = -2\xi_g\omega_g\dot{x}_f - \omega_g^2 x_f - \ddot{w}. \quad (18)$$

The state variables are introduced in a state vector as

$$\mathbf{Y} = \begin{bmatrix} \mathbf{x}^T & x_b & Z & x_f & \dot{\mathbf{x}}^T & \dot{x}_b & \dot{x}_f \end{bmatrix}^T. \quad (19)$$

Equations (15)–(18) can be expressed in state-space form as

$$\frac{d}{dt}\mathbf{Y} = \mathbf{A}\mathbf{Y} + \mathbf{w}; \quad (20)$$

where $\mathbf{A}$ is the augmented system matrix and

$$\mathbf{w} = \begin{bmatrix} \mathbf{0} & 0 & 0 & 0 & \mathbf{0} & 0 & -\ddot{w} \end{bmatrix}^T. \quad (21)$$

In the equations above, the vector $\mathbf{x}$ has a dimension equal to the number of structural degrees of freedom (n), and $\mathbf{Y}$ has a size of $(2n+5)$. The structure of the augmented system matrix $\mathbf{A}$ is provided in the Appendix.

The response of the system can be evaluated by solving Eq. (20). In stochastic dynamic analysis, the statistics, such as covariance of responses, are of interest. It can be shown that the covariance matrix $\mathbf{C_{YY}}$ of the response vector $\mathbf{Y}$ (assumed as Markovian) evolves following an equation of the form[32]

$$\frac{d}{dt}\mathbf{C_{YY}} = \mathbf{A}\mathbf{C_{YY}}^T + \mathbf{C_{YY}}\mathbf{A}^T + \mathbf{S_{ww}}. \quad (22)$$

$\mathbf{C_{YY}}$ has dimensions of $(2n+5, 2n+5)$ with its terms as

$$C_{Y_iY_j} = \mathrm{E}[Y_iY_j]. \quad (23)$$

The matrix $\mathbf{S_{ww}}$ contains a term that quantifies the intensity of the white-noise excitation at the rock bed, denoted as S_0. Following the structure of $\mathbf{w}$, matrix $\mathbf{S_{ww}}$ has all terms that equal zero, except the last diagonal, which is $2\pi S_0$.

The covariance of responses can be obtained by solving Eq. (22). It should be noted that even though the stochastic linearization is adopted for the nonlinear isolator behaviour, the system still shows the nonlinear characteristics because the equivalent linear stiffness and damping are still functions of the responses

$$C_e = f(\dot{x}_b, Z); \qquad K_e = g(\dot{x}_b, Z); \quad (24)$$

where f and g refer to the nonlinear functions of the response quantities $\dot{x}_b$ and Z, the isolator velocity, and hysteretic displacement, respectively (as in Eq. (10a) and (10b)). In solving Eq. (22), these terms are modified in each iteration, following the response statistics of the previous steps. Iterations stops (converges) when the response from two successive steps are practically identical.

The equations for response statistics of their derivative process (such as acceleration $\ddot{\mathbf{x}}$, $\ddot{x}_b$) are obtained as

$$\mathbf{C_{\dot{Y}\dot{Y}}} = \mathbf{A}\mathbf{C_{YY}}\mathbf{A}^T + \mathbf{S_{ww}}. \quad (25)$$

The RMS responses are obtained from their covariance:

$$\sigma_{Y_i} = \sqrt{C_{Y_iY_i}}. \quad (26)$$

The absolute top-floor acceleration ($\ddot{u}_n$) and the relative top-floor displacement (x_n) are two important design parameters for a BI system. The RMS of the top-floor displacement response is given by

$$\sigma_{x_n} = \sqrt{C_{YY}(n,n)}; \tag{27}$$

and the top-floor acceleration ($\ddot{u}_n$) is obtained by summing up the relative top-floor ($\ddot{x}_n$), isolator-base ($\ddot{x}_b$), and the ground acceleration ($\ddot{x}_g$) as

$$\ddot{u}_n = \ddot{x}_g + \ddot{x}_b + \ddot{x}_n. \tag{28}$$

Then, the RMS top-floor acceleration can be written as

$$\sigma_{\ddot{u}_n\ddot{u}_n} = \sqrt{C_{\dot{Y}\dot{Y}}(2n{+}3,2n{+}3) + C_{\dot{Y}\dot{Y}}(2n{+}4,2n{+}4) + C_{\dot{Y}\dot{Y}}(2n{+}5,2n{+}5)}\ ; \tag{29}$$

where $\sigma^2_{\ddot{x}_g} = C_{\dot{Y}\dot{Y}}(2n{+}3,2n{+}3)$, $\sigma^2_{\ddot{x}_b} = C_{\dot{Y}\dot{Y}}(2n{+}4,2n{+}4)$, and $\sigma^2_{\ddot{x}_n} = C_{\dot{Y}\dot{Y}}(2n+5,2n+5)$ are the variance of ground acceleration, acceleration of the isolator, and top-floor acceleration, respectively.

Subsequently, whenever they are discussed, the displacement and accelerations represent the motion of the top floor in the superstructure. The stochastic dynamic analysis, presented herein, is based on an earthquake load modelled as a stationary stochastic process. Extending this analysis to a non-stationary earthquake model will be straight forward. However, doing so will involve evaluating the time-dependent response statistics.

2.2. Sensitivity of Stochastic Response under Parametric Uncertainty

The stochastic response evaluation presented above assumes that the system parameters are deterministic.[33] However, uncertainties in the system parameters may lead to large and unexpected excursion of responses, causing drastic reduction in accuracy and precision of safety evaluation.[34] In design of such a system, apart from the stochastic nature of earthquake loading, the uncertainties with regard to these parameters are expected to be influential. The sensitivities of the stochastic responses with respect to the uncertain parameters are essential in considering the effects of parametric uncertainty. The formulation to obtain the sensitivities is presented.

The random variability is reasonably assigned to the parameters of the isolator, the structure, and in the earthquake load, denoted by

$$\boldsymbol{\theta} = \begin{bmatrix} k & c & k_b & c_b & F_Y & \xi_g & \omega_g & S_0 \end{bmatrix}^T; \tag{30}$$

where $\boldsymbol{\theta}$ is the vector of random design parameters, k is the uniform storey stiffness, c is the uniform damping of each storey, and the other parameters are defined earlier. For simplicity of presentation, uniform storey stiffness and damping are considered herein. However, the proposed formulation is not restricted to such an assumption and can easily be applied for varying values of k and c along different stories. First-order sensitivity of the base of Eq. (22) with respect to the i-th parameter θ_i is written as

$$\frac{d}{dt}\frac{\partial \mathbf{C_{YY}}}{\partial\theta_i} = \mathbf{A}\frac{\partial \mathbf{C_{YY}}}{\partial\theta_i}^T + \frac{\partial \mathbf{C_{YY}}}{\partial\theta_i}\mathbf{A}^T + \mathbf{B}; \tag{31}$$

in which $\frac{\partial \mathbf{C_{YY}}}{\partial\theta_i}$ is the sensitivity of response covariance ($\mathbf{C_{YY}}$) with respect to the parameter θ_i, and $\mathbf{B}$ is defined by

$$\mathbf{B} = \frac{\partial \mathbf{A}}{\partial\theta_i}\mathbf{C_{YY}}^T + \mathbf{C_{YY}}\frac{\partial \mathbf{A}}{\partial\theta_i}^T + \frac{\partial \mathbf{S_{ww}}}{\partial\theta_i}. \tag{32}$$

It should be noted that Eq. (31) has the same form as Eq. (22) and can be solved similarly. The sensitivity of the time derivative processes (i.e., acceleration) can be obtained with

$$\frac{\partial \mathbf{C_{\dot{Y}\dot{Y}}}}{\partial\theta_i} = \mathbf{A}\frac{\partial \mathbf{C_{YY}}}{\partial\theta_i}\mathbf{A}^T + \mathbf{B}_1; \tag{33}$$

where

$$\mathbf{B}_1 = \mathbf{A}\mathbf{C_{YY}}\frac{\partial \mathbf{A}}{\partial\theta_i}^T + \frac{\partial \mathbf{A}}{\partial\theta_i}\mathbf{C_{YY}}\mathbf{A}^T + \frac{\partial \mathbf{S_{ww}}}{\partial\theta_i}. \tag{34}$$

The second-order sensitivity is obtained by further differentiating Eq. (33) with respect to the parameter θ_j. After rearranging the terms, the equation for the second-order sensitivity becomes

$$\frac{d}{dt}\frac{\partial^2 \mathbf{C_{YY}}}{\partial\theta_i\partial\theta_j} = \mathbf{A}\frac{\partial^2 \mathbf{C_{YY}}}{\partial\theta_i\partial\theta_j}^T + \frac{\partial^2 \mathbf{C_{YY}}}{\partial\theta_i\partial\theta_j}\mathbf{A}^T + \mathbf{C}; \tag{35}$$

where $\mathbf{C}$ is given as

$$\mathbf{C} = \frac{\partial \mathbf{A}}{\partial\theta_j}\frac{\partial \mathbf{C_{YY}}}{\partial\theta_i}^T + \frac{\partial \mathbf{C_{YY}}}{\partial\theta_i}\frac{\partial \mathbf{A}}{\partial\theta_j}^T + \frac{\partial^2 \mathbf{A}}{\partial\theta_i\partial\theta_j}\mathbf{C_{YY}}^T + \frac{\partial \mathbf{A}}{\partial\theta_i}\frac{\partial \mathbf{C_{YY}}}{\partial\theta_j}^T + \mathbf{C_{YY}}\frac{\partial^2 \mathbf{A}}{\partial\theta_i\partial\theta_j}^T + \frac{\partial \mathbf{C_{YY}}}{\partial\theta_j}\frac{\partial \mathbf{A}}{\partial\theta_i}^T + \frac{\partial^2 \mathbf{S_{ww}}}{\partial\theta_i\partial\theta_j}. \tag{36}$$

The statistics for the respective time derivative processes are also expressed as

$$\frac{\partial^2 \mathbf{C_{\dot{Y}\dot{Y}}}}{\partial\theta_i\partial\theta_j} = \mathbf{A}\frac{\partial^2 \mathbf{C_{\dot{Y}\dot{Y}}}}{\partial\theta_i\partial\theta_j}\mathbf{A}^T + \mathbf{C}_1; \tag{37}$$

where $\mathbf{C}_1$ is given by

$$\mathbf{C}_1 = \mathbf{A}\frac{\partial \mathbf{C_{YY}}}{\partial\theta_i}\frac{\partial \mathbf{A}}{\partial\theta_j}^T + \frac{\partial \mathbf{A}}{\partial\theta_j}\frac{\partial \mathbf{C_{YY}}}{\partial\theta_i}\mathbf{A}^T + \frac{\partial \mathbf{A}}{\partial\theta_j}\mathbf{C_{YY}}\frac{\partial \mathbf{A}}{\partial\theta_i}^T + \mathbf{A}\frac{\partial \mathbf{C_{YY}}}{\partial\theta_j}\frac{\partial \mathbf{A}}{\partial\theta_i}^T + \mathbf{A}\mathbf{C_{YY}}\frac{\partial^2 \mathbf{A}}{\partial\theta_i\partial\theta_j}^T + \frac{\partial^2 \mathbf{A}}{\partial\theta_i\partial\theta_j}\mathbf{C_{YY}}\mathbf{A}^T + \frac{\partial \mathbf{A}}{\partial\theta_i}\mathbf{C_{YY}}\frac{\partial \mathbf{A}}{\partial\theta_j}^T + \frac{\partial \mathbf{A}}{\partial\theta_i}\frac{\partial \mathbf{C_{YY}}}{\partial\theta_j}\mathbf{A}^T + \frac{\partial^2 \mathbf{S_{ww}}}{\partial\theta_i\partial\theta_j}. \tag{38}$$

Equations (31) and (35) can be solved similarly for first- and second-order sensitivity, respectively. It should be mentioned that some of the matrices involved in Eq. (35) for evaluating $\mathbf{C}$ are null. This is because $\mathbf{A}$ and $\mathbf{S_{ww}}$ have zero-th/first-order terms, which are a function of random variables θ_i, which vanishes after first-/second-order differentiation with respect to θ_i.

The sensitivities of the RMS responses are obtained by differentiating Eq. (26) with respect to the i-th random parameter as

$$\frac{\partial\sigma_{Y_m}}{\partial\theta_i} = \frac{1}{2}\frac{1}{\sqrt{C_{Y_mY_m}}}\frac{\partial C_{Y_mY_m}}{\partial\theta_i};$$
$$\frac{\partial^2\sigma_{Y_m}}{\partial\theta_i\partial\theta_j} = \frac{1}{2}\frac{1}{\sqrt{C_{Y_mY_m}}}\left[\frac{\partial^2 C_{Y_mY_m}}{\partial\theta_i\partial\theta_j} - \frac{1}{2}\frac{1}{C_{Y_mY_m}}\left(\frac{\partial C_{Y_mY_m}}{\partial\theta_i}\right)^2\right]. \tag{39}$$

In the above equation, Y_m is the response; σ_{Y_m} is the RMS response of Y_m. The terms $\frac{\partial \sigma_{Y_m}}{\partial \theta_i}$ and $\frac{\partial^2 \sigma_{Y_m}}{\partial \theta_i \partial \theta_j}$ represent the first- and second-order sensitivity with respect to the parameters θ_i and θ_j, respectively.

It should be mentioned here that the system parameter matrix $\mathbf{A}$, defined in the Appendix, is an explicit function of uncertain model parameters, $\boldsymbol{\theta}$. Thus, the derivatives can be directly obtained by differentiating with respect to each parameter. However, the formulation does not impose any limit to the number of elements or degrees of freedom used in the analysis. But with an increasing number of elements, the matrices will be bigger to accommodate the computational demand. For more complex super-structural systems, involving finite element modelling for response evaluation, the matrix $\mathbf{A}$ cannot be obtained explicitly. For implicitly generated element mass, stiffness, and the damping matrix of the system, the differentiation can be carried out through a sequence of calculations or, alternatively, by finite difference approximation and can subsequently be used to obtain the partial derivative of $\mathbf{A}$. There are important practical considerations for computing derivatives that are required in structural sensitivity analysis.[34,35]

2.3. Parameter Uncertainty and Unconditional Stochastic Response

The stochastic response of a structure under earthquake loading depends on the system parameters and can be expanded around the mean value of the uncertain parameters (with the assumption that the random variability is small) using the Taylor series. The random system parameter (θ_i) can be viewed as the superposition of the deterministic mean component ($\overline{\theta_i}$) with a zero mean deviatoric component ($\Delta\theta_i$). Thus, the Taylor series expansion of the RMS response at the mean value of the random parameters can be written as

$$\sigma_{Y_m} = \sigma_{Y_m}(\overline{\theta_i}) + \sum_{i=1}^{nv} \frac{\partial \sigma_{Y_m}}{\partial \theta_i}\Delta\theta_i + \frac{1}{2}\sum_{i=1}^{nv}\sum_{i=1}^{nv} \frac{\partial^2 \sigma_{Y_m}}{\partial \theta_i \partial \theta_j}\Delta\theta_i\Delta\theta_j; \tag{40}$$

in which nv is the total number of random variables involved, and the derivative of the respective response quantities are the sensitivity terms addressed in the previous section. Assuming that the uncertain random variables are uncorrelated, the quadratic approximation provides the expected value of the unconditional RMS response as

$$\sigma_{Y_m} = \sigma_{Y_m}(\overline{\theta_i}) + \frac{1}{2}\sum_{i=1}^{nv} \frac{\partial^2 C_{Y_m Y_m}}{\partial \theta_i^2}\sigma_{\theta_i}^2; \tag{41}$$

where $\sigma_{\theta_i}^2$ is the standard deviation of the i-th random parameter. The linear approximation of the Taylor series expansion furnishes the variance of the RMS response:

$$\text{var}\left[\sigma_{Y_m}\right] = \sigma_{\sigma_{Y_m}}^2 = \sum_{i=1}^{nv}\left(\frac{\partial \sigma_{Y_m}}{\partial \theta_i}\right)^2 \sigma_{\theta_i}^2. \tag{42}$$

Such responses are referred to as unconditional because the condition that the structural parameters to be deterministic has been relaxed while estimating such responses.[33]

Table 1. Statistical properties of the random system parameters.

Parameters	Mean	Coefficient of variation	Distribution
storey stiffness (k)	5830 kN/m	5%–15%	normal
storey damping (c)	264499.5 kNs/m		normal
isolator stiffness (k_b)	56791296 kN/m		normal
isolator damping (c_b)	10%		normal
isolator yield-strength (F_Y)	5% of total weight (W)		normal
ground damping (ξ_g)	60%		normal
ground frequency (ω_g)	5 rad/s		normal
seismic intensity (S_0)	0.05 m/s^2		normal

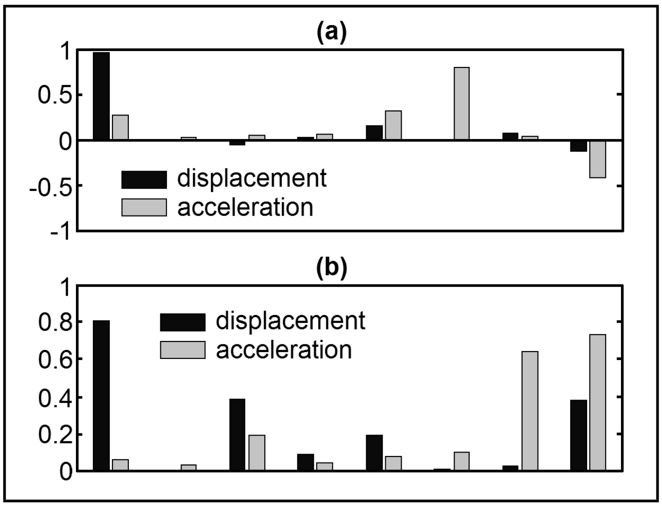

Figure 2. The normalized (a) second-order and (b) first-order terms dictating the contribution of individual stochastic parameters in affecting the unconditional response and their variances.

3. NUMERICAL ILLUSTRATIONS

A five-storied shear building ($n = 5$) is studied to investigate the effects of parametric uncertainty on the performance of a BI system subject to stochastic earthquake. The mass, stiffness, and damping (m_i, k_i, ξ_i) of each floor are assumed to be identical. The values of stiffness and mass for each storey are assigned to provide the desired value of time period (T) to the superstructure (varying from 0.1 s to 1 s) employed for parametric study. The uniform viscous damping ratio (ξ) is taken as 2%, unless specifically mentioned. The LRB is characterized with a mass ratio (m_b/m) of 1, the viscous damping ratio (ξ_b) of 10%, and the normalized yield strength (F_0) of 0.05, unless otherwise specified. The parameters characterizing the random earthquakes are ω_g, ξ_g, and S_0. All random parameters in the study are assumed to be statistically independent, normally distributed, and the properties are listed in Table 1.

In prior to present the response of the BI system under parameter uncertainty, the relative importance of the random system parameters are studied through a bar chart sensitivity analysis (see Figs. 2(a) and 2(b)) in order to assess the contribution of the individual random variable in affecting the responses (shown in Eqs. (41) and (42)). In these figures, the first- and second-order sensitivity terms are multiplied by the variance of the respective random variables. While plotting, the terms are normalized with respect to the square root of sum of all squared terms. For first-order terms (governing the variance of

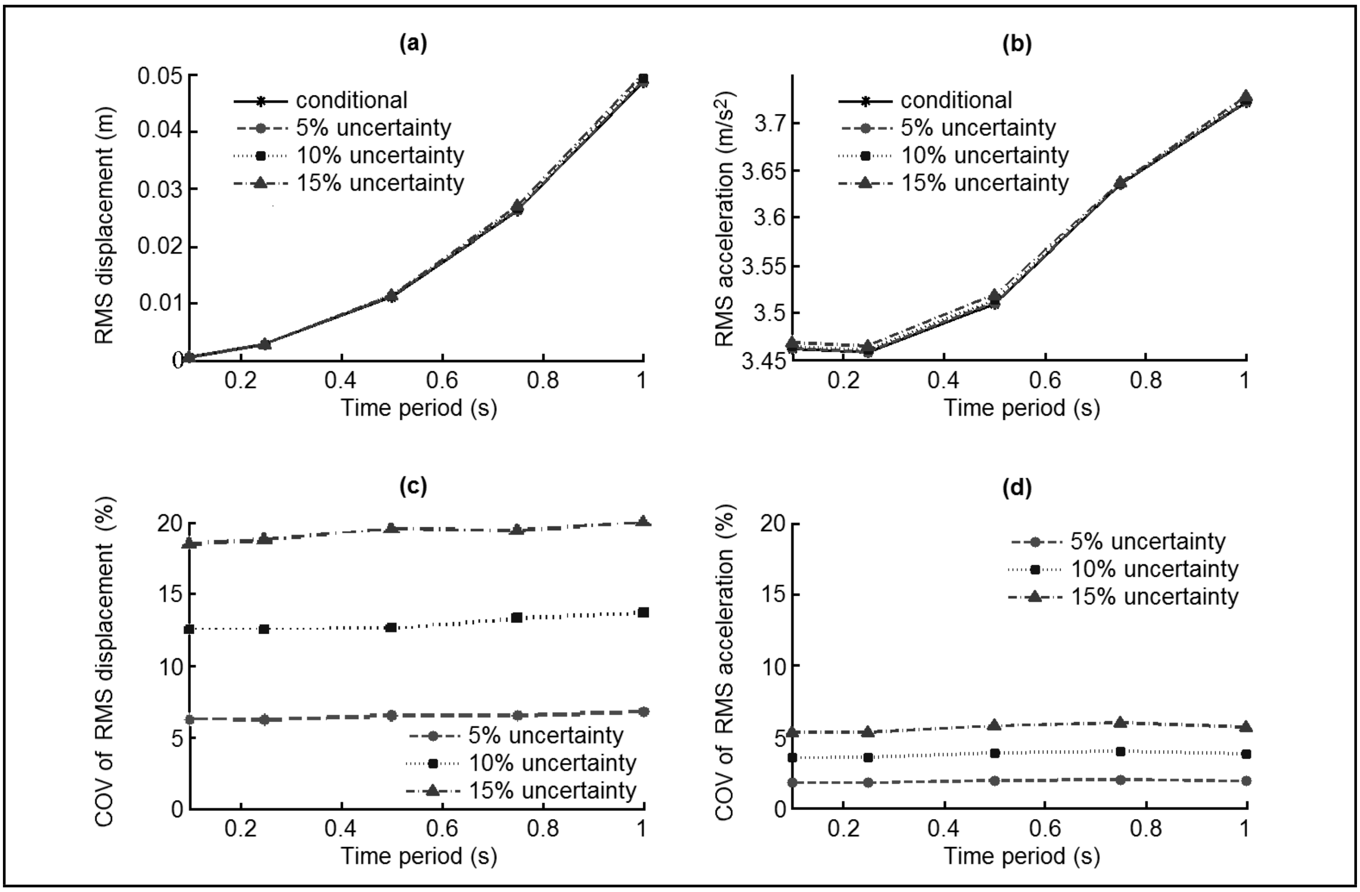

Figure 3. Conditional and unconditional RMS top-floor (a) displacement and (b) acceleration, and coefficient of variation of top-floor (c) displacement and (d) acceleration with varied superstructure flexibilities and different degrees of uncertainty.

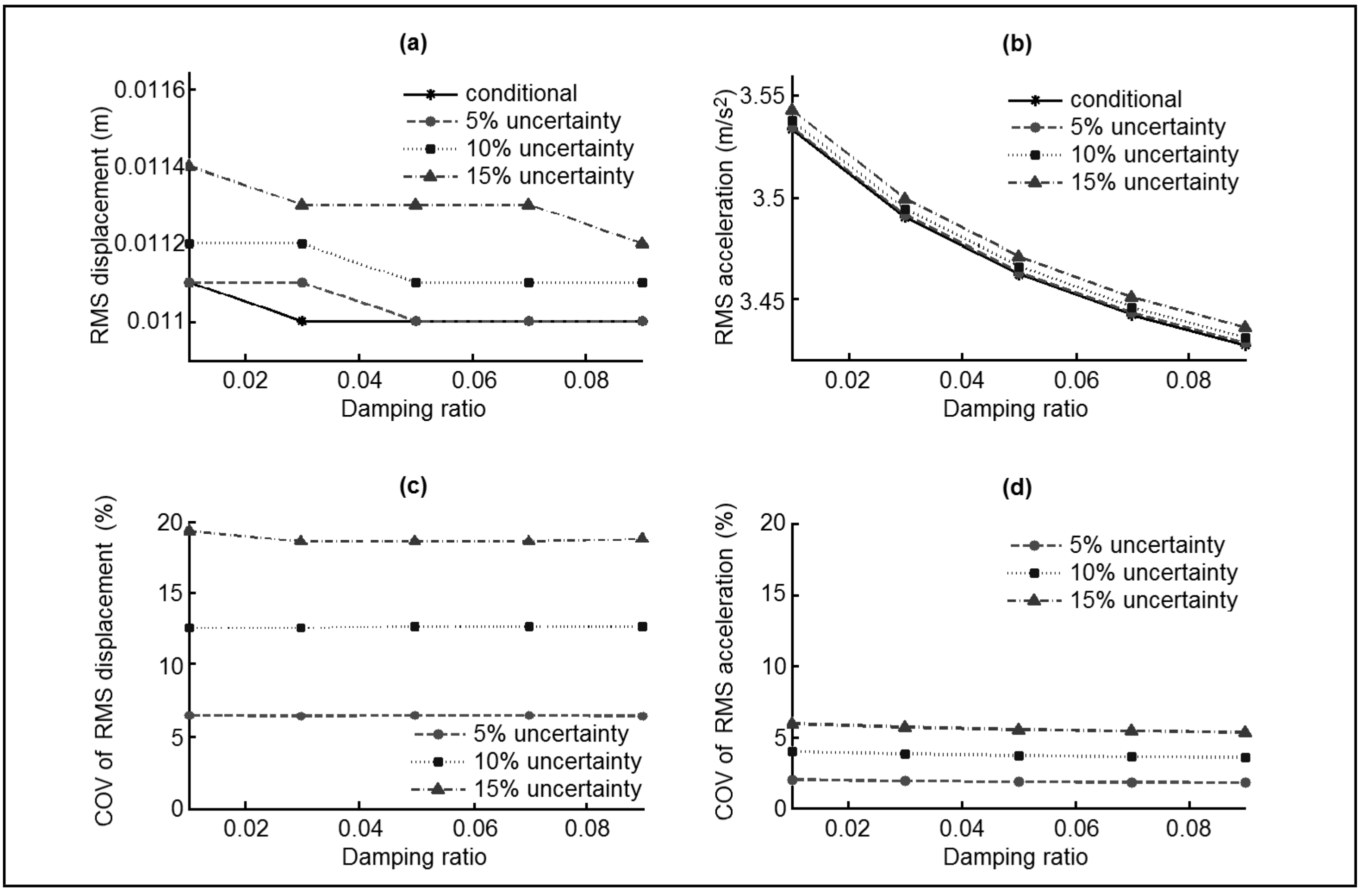

Figure 4. Conditional and unconditional RMS top-floor (a) displacement and (b) acceleration, and coefficient of variation of top-floor (c) displacement and (d) acceleration with varied superstructure damping for different degrees of uncertainty.

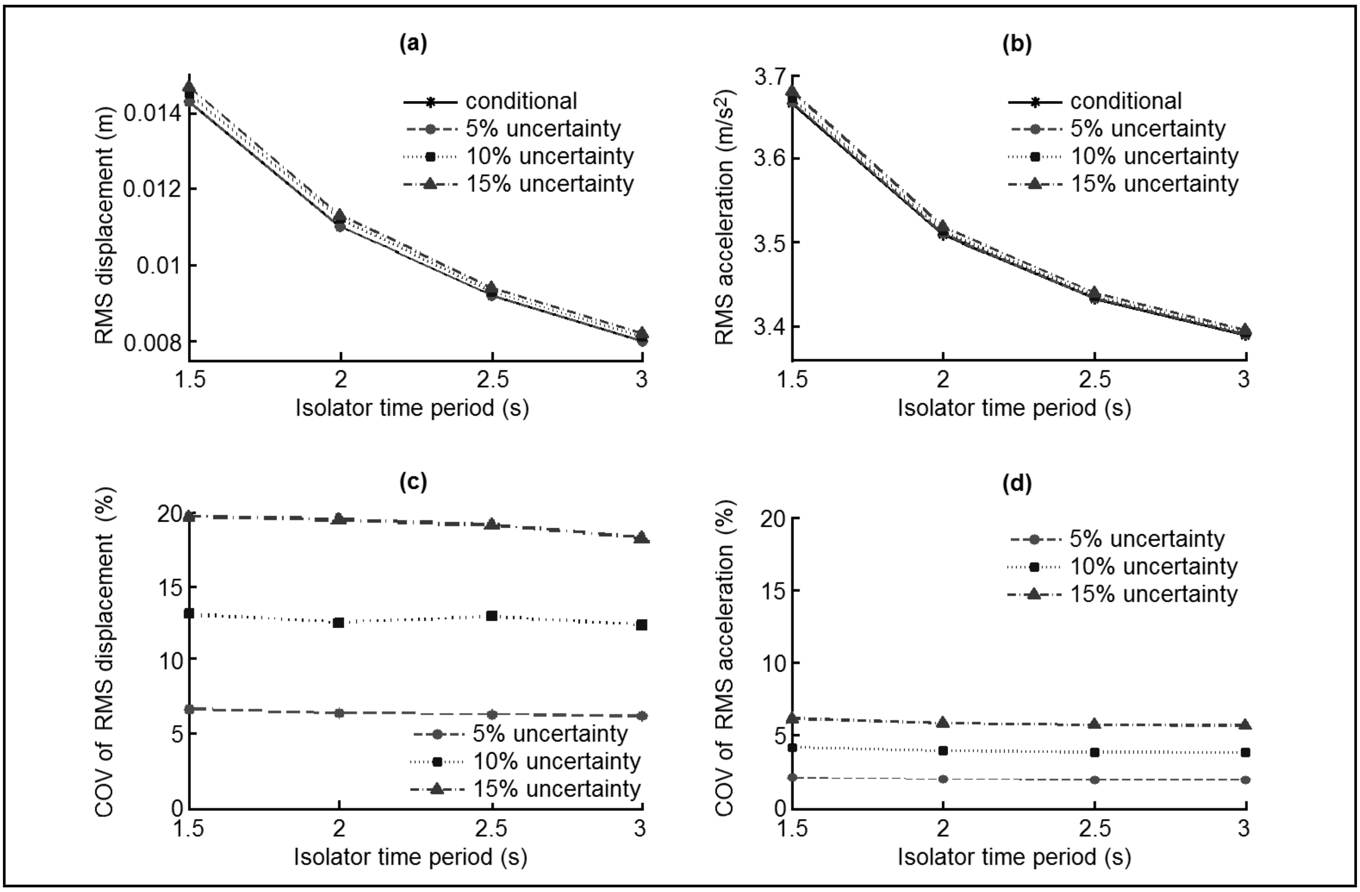

Figure 5. Conditional and unconditional RMS top-floor (a) displacement and (b) acceleration, and coefficient of variation of top-floor (c) displacement and (d) acceleration with varied isolation time period and different degrees of uncertainty.

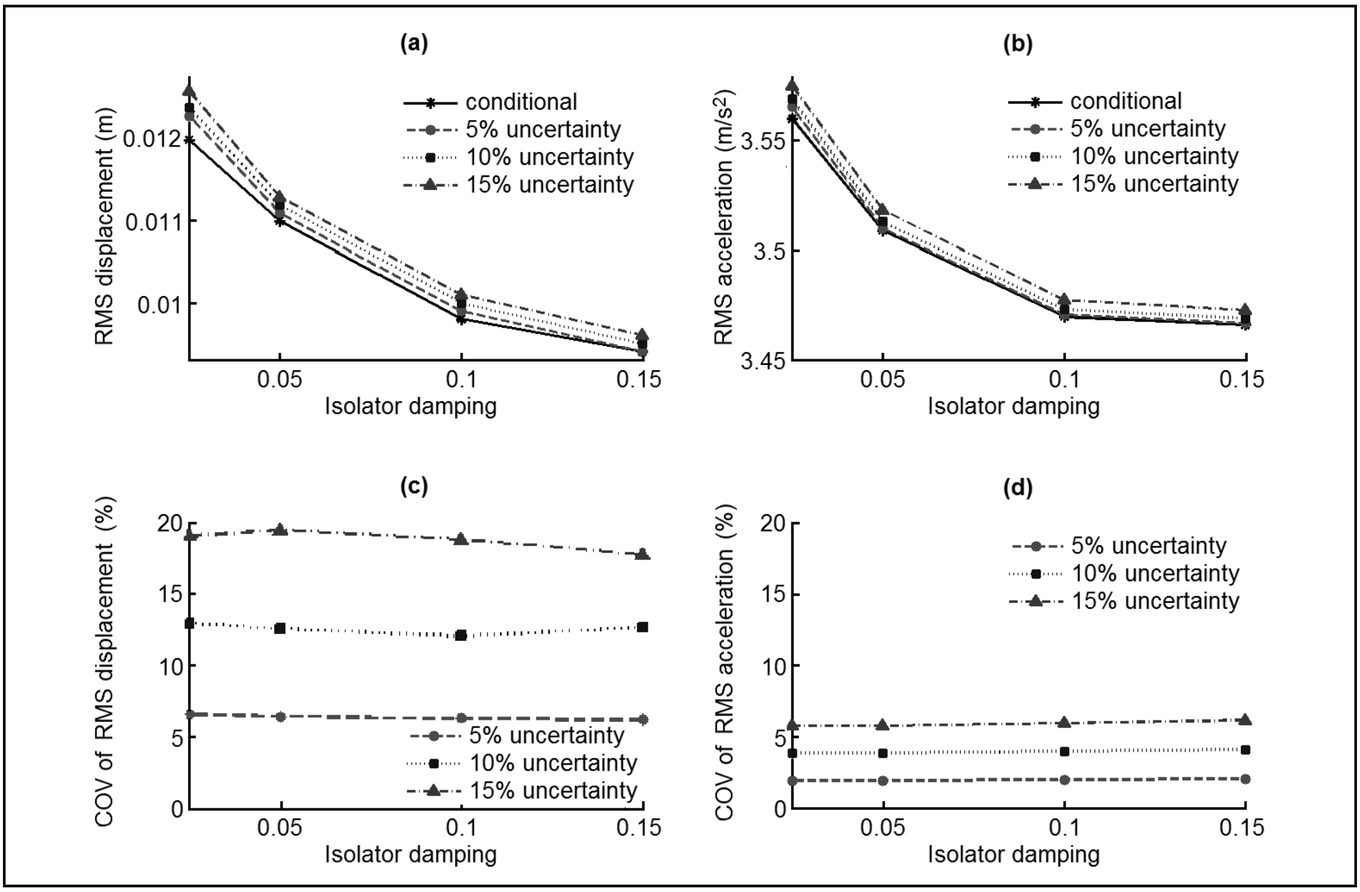

Figure 6. Conditional and unconditional RMS top-floor (a) displacement and (b) acceleration, and coefficient of variation of top-floor (c) displacement and (d) acceleration with varied isolation damping and different degrees of uncertainty.

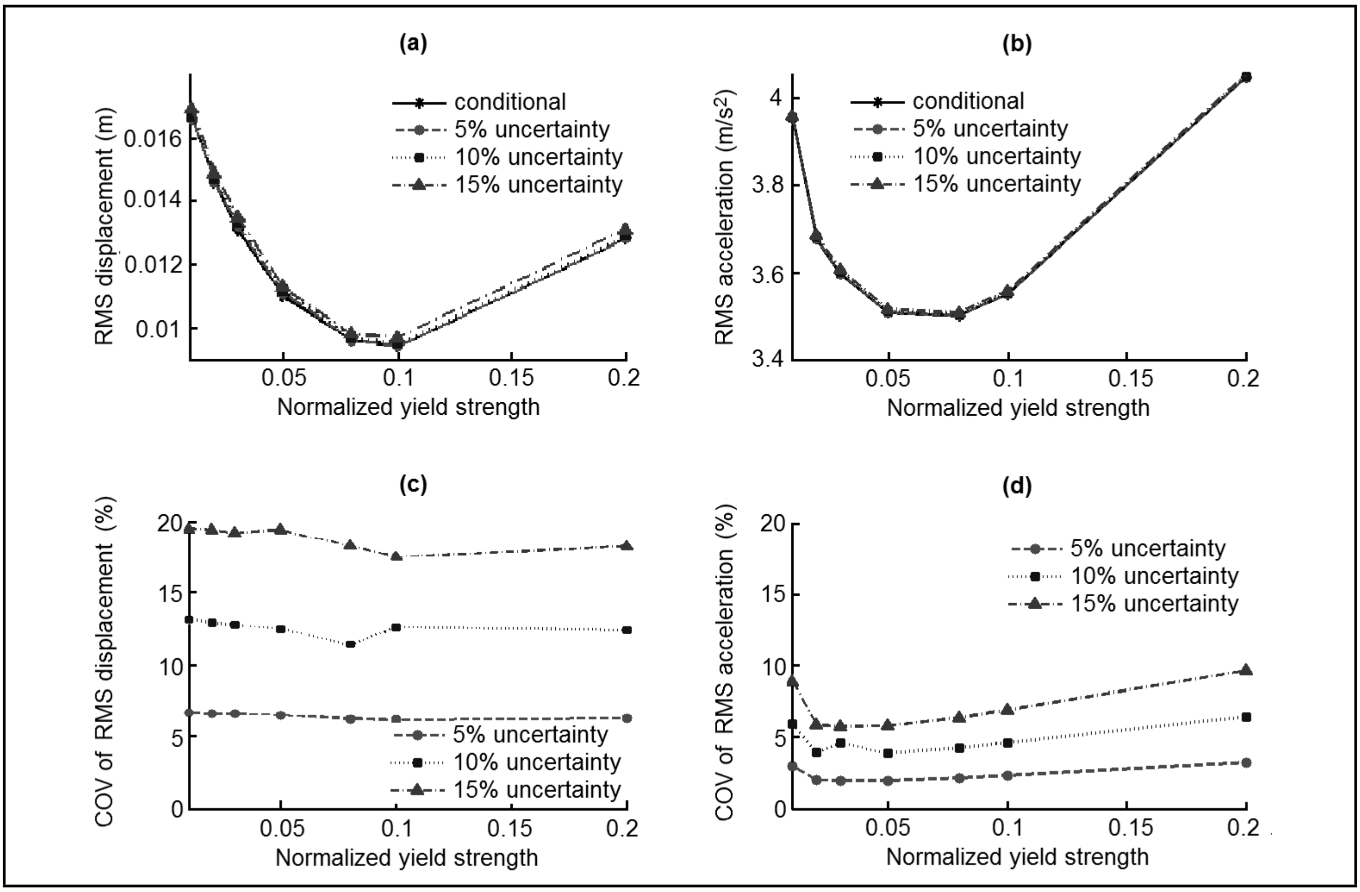

Figure 7. Conditional and unconditional RMS top-floor (a) displacement and (b) acceleration, and coefficient of variation of top-floor (c) displacement and (d) acceleration with varied yield strength of isolator and different degrees of uncertainty.

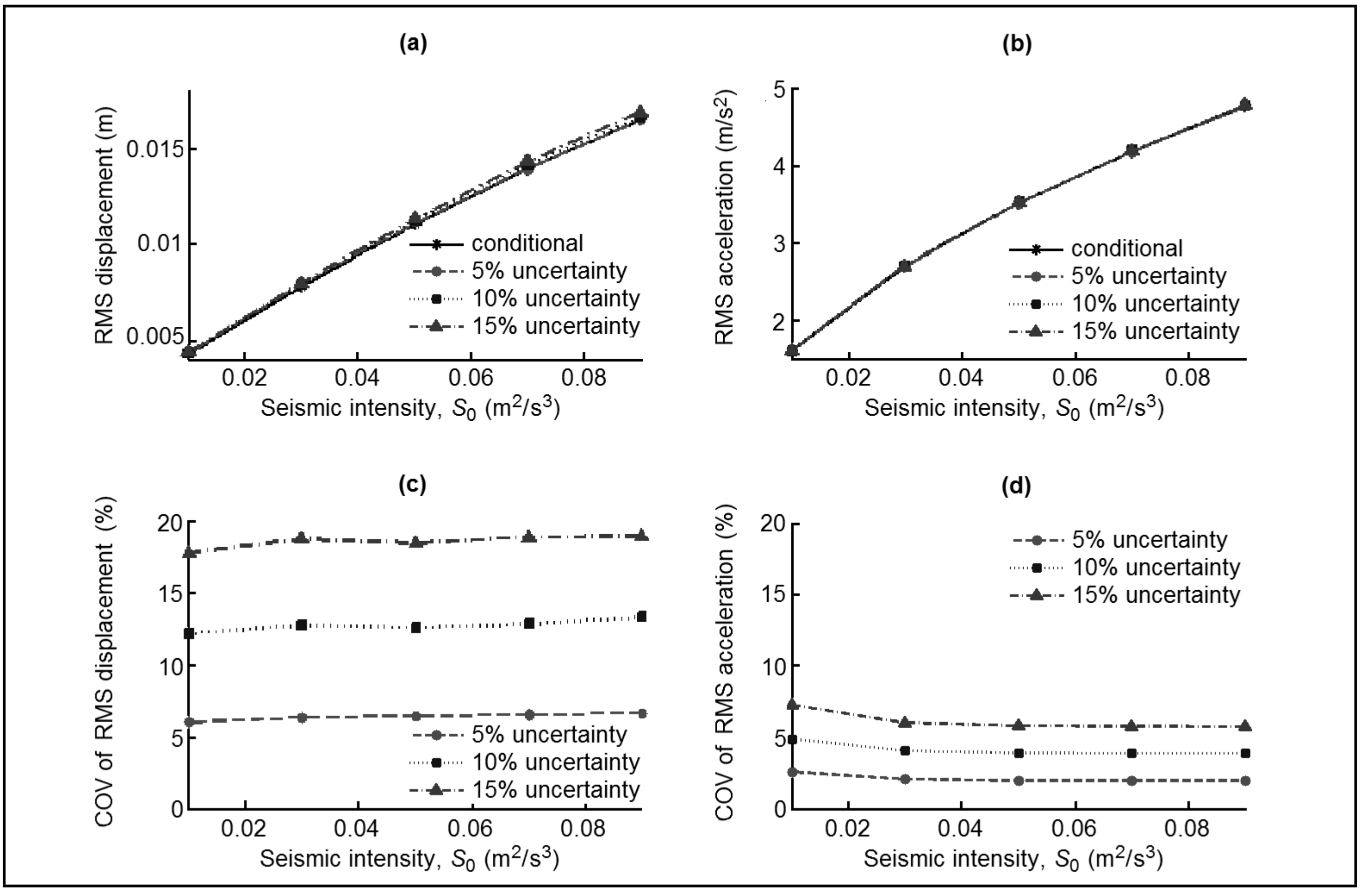

Figure 8. Conditional and unconditional RMS top-floor (a) displacement and (b) acceleration, and coefficient of variation of top-floor (c) displacement and (d) acceleration with varied seismic intensity and different degrees of uncertainty.

unconditional response), this is expressed as

$$\frac{(\partial f/\partial x_i)\sigma_{x_i}}{\sqrt{\sum_{i=1}^{nv}(\partial f/\partial x_i)^2\sigma_{x_i}^2}}; \tag{43}$$

and a similar expression is used for contribution of the second-order terms (governing the unconditional response). Figure 2 clearly indicates that the superstructure stiffness is the most important parameter as far as the top-floor displacement responses are concerned, whereas the top-floor acceleration response is primarily affected by the uncertainty in the ground frequency (ω_g) and intensity of the earthquake (S_0). The second-order sensitivity of both the responses with respect to the seismic intensity is negative. It is noteworthy that the first-order sensitivity of top-floor acceleration with respect to the ground frequency is much higher than the ground damping, whereas for the second-order sensitivity, the trend is the opposite. These observations conform to the fact that the displacement response is governed by the system stiffness, whereas the floor acceleration, being a measure of the amount of seismic force transmitted to the structure, is dictated by the seismic ground motion.

Though the uncertainty in the excitation process dominates the performance of the BI system, the uncertainties in the system parameters, in particular, the superstructure stiffness, damping, and yield strength of the LRB have noticeable influence and cannot be neglected. With this nature of the parametric randomness and the relative importance, the conditional and unconditional response of the system and the associated coefficient of variation are studied subsequently.

The conditional and the unconditional responses of the BI system are presented for varied superstructure flexibilities in Figs. 3(a) and 3(b). The superstructure flexibilities are expressed in terms of the time period of the superstructure. The parametric uncertainty increases the RMS displacement and acceleration, shown in Figs. 3(a) and 3(b). The increase in the RMS acceleration is, however, lesser than the RMS displacement. This difference might be the result of the absolute acceleration including ground acceleration, which is irrespective of the parametric uncertainty. The coefficient of variation for the respective unconditional responses are shown in Figs. 3(c) and 3(d). It should be noted that the coefficient of variation increases with increasing randomness for the range of considered superstructure flexibilities. The coefficient of variation for the RMS displacement is almost three orders of magnitude higher compared to that of RMS acceleration.

The variations of the conditional and unconditional responses of the BI system are shown with respect to the varied damping of the superstructure in Figs. 4(a) and 4(b). It is important to note that the deterministic system underestimates the superstructure RMS displacement and acceleration. The difference increases with an increasing level of uncertainty. Observing the parametric variations of responses with the superstructure damping, it can be inferred that the RMS acceleration response is more sensitive to the variation of damping. However, the change of coefficient of variation of the acceleration with an increasing level of randomness in system parameters is less than that of the displacement, implying less sensitivity of the acceleration to uncertainty.

The conditional and unconditional responses are presented with varied time period of isolation in Figs. 5(a)–(d). These figures also show that the unconditional responses are greater than the respective conditional responses. The coefficient of variation of the acceleration is seen to be a few orders of magnitude less than that of displacement.

The parametric variation of the responses and their coefficients of variation are shown with respect to isolator damping in Figs. 6(a)–(d). As described earlier, for different level of randomness in the system parameters, the unconditional responses and associated coefficients of variation are affected differently. The displacement is more affected by the uncertainty compared to the acceleration, as shown in terms of the unconditional responses and associated coefficients of variation. The effect of uncertainty uniformly affects the disparities among the conditional and unconditional responses. The coefficient of variation of the unconditional responses does not change significantly with the variations of isolator damping, confirmed by Figs. 6(c) and (d).

The responses of the BI system are further studied with regard to varied yield strength of the isolator in Figs. 7(a)–(d). As described earlier, the conditional responses are lower compared to the respective unconditional responses. It is noteworthy that the isolator yield strength presents the optimal value for which acceleration/displacement is the minimum. Obviously, the optimal yield strength values for minimum displacement and acceleration are not necessarily identical. It should be noted that the associated coefficient of variation of the displacement is almost irrespective of the parametric variations of the yield strength (Fig. 7(c)), whereas the coefficient of variation for the acceleration shows a definite trend with respect to varying isolator yield strength (Fig. 7(d)). The coefficient of variation of the unconditional acceleration response decreases first and then starts increasing with increasing yield strength of the isolator, which is unlikely for the variations of the responses presented so far. Comparing Figs. 7(b) and 7(d), it is observed that the optimal values of the yield strength corresponding to the minimum RMS acceleration might not ensure that the respective coefficient of variation is also at the minimum. Thus, designers must deal with two conflicting objective functions, i.e., the minimization of the unconditional RMS response and its variance. This problem can formally be addressed by formulating it as bi-objective optimization in order to minimize the unconditional RMS response as well as its dispersion (coefficient of variation) simultaneously—referred to as robust design optimization (RDO). The optimum LRB parameters in RDO are achieved from a set of Pareto optimal solutions by ensuring the desired level of robustness in the design. The issue is emphasized, though not studied, in this paper.

It is noteworthy that the trend in Figs. 7(a) and (b) might not necessarily be the same as that observed in an isolated structure subject to a particular ground motion time history, even though for large numbers of ground motion time histories, the average trend should match that of the stochastic response in an ensem-

Table 2. Disparities among the conditional and unconditional responses for ranges of parameter variations.

System parameters	Lower and upper value	Uncertainty (cov, %)	Discrepancy among cond. and uncond. top-floor displacement (%)		Discrepancy among cond. and uncond. top-floor acceleration (%)	
			Lower	Upper	Lower	Upper
Structural period	0.1 1.0	5	0.27	0.41	0.020	0.019
		10	1.10	1.43	0.08	0.07
		15	2.48	3.08	0.19	0.17
Structural damping	0.01 0.09	5	0.00	0.00	0.03	0.03
		10	0.90	0.90	0.90	0.11
		15	2.70	1.81	0.27	0.25
Isolator period	1.5 2.5	5	0.00	0.00	0.044	0.02
		10	1.40	1.25	0.17	0.08
		15	2.80	2.50	0.39	0.18
Isolator yield strength	0.01 0.20	5	0.0	0.0	0.015	0.017
		10	0.60	0.78	0.055	0.066
		15	1.80	2.34	0.12	0.15
Isolator damping	0.025 0.10	5	2.5	0.0	0.15	0.02
		10	3.33	1.06	0.25	0.08
		15	5.00	2.13	0.42	0.20
Seismic intensity	0.01 0.09	5	0.00	0.00	0.018	0.030
		10	0.00	0.60	0.025	0.12
		15	2.32	2.42	0.20	0.27

ble sense. This is due to the wide disparities which exist in the power spectrum of a particular ground motion time history and the idealized Kanai-Tajimi spectra, employed in this stochastic analysis.

The behaviour is further studied for earthquake scenario of different intensities, shown in Figs. 8(a)–(d). With increasing seismic intensity, the RMS response increases for obvious reasons. The coefficient of variation of the unconditional displacement response is almost three times higher than the corresponding acceleration response. However, the coefficient of variation for both is insensitive to the variations in the seismic intensities (Figs. 8(c) and (d)).

The discrepancies among the conditional and unconditional responses and the associated coefficients of variation have been presented, along with the parametric variations of responses with the structural and isolator parameters. Though the variations of the response coefficients of variation are quite obvious from the plots, the response values and associated disparities (in %) are presented in Table 2 to provide more precise estimates of the disparities. The particular cases presented here correspond to the minimum and maximum value of the respective parameters considered in the parametric study. Table 2 shows that the differences among the conditional and unconditional RMS displacement responses could be as high as 5%, whereas for acceleration response, the difference is around 0.5%. The associated coefficients of variation are around 20% for displacement and 8% for acceleration (Figs. 7(c), 7(d), 8(c), and 8(d)). These differences (particularly for displacement) are substantial, considering the fact that the peak value of the responses could be as high as three to four times (peak factor) of the respective RMS values.[31,32] Thus, system parameter uncertainty is a critical issue in response evaluation and design of BI systems in order to ensure the desired level of seismic safety.[35]

4. CONCLUSIONS

The effects of system parameter uncertainty on the performance of BI systems subject to a stochastic earthquake load has been examined. This study incorporates the effects of system parameter uncertainty in the response evaluation through perturbation-based analysis of the dynamic equations of motion. The degree of parameter uncertainty is assumed to be small so that the linear first-order perturbation analysis is valid. The responses are found to be in parity with the results obtained from the usual random vibration analysis assuming deterministic system parameters. However, a definite change in the responses occurs when the effects of system parameter uncertainty are included. In general, the efficiency of a BI system tends to decrease as the level of uncertainty increases, though efficiency is not completely eliminated. Therefore, even though the randomness in the seismic events dominates, the random variations of the system parameters play a significant role. The randomness in the structural stiffness governs the unconditional RMS displacements and associated coefficient of variation, whereas the randomness in the characteristic ground frequency and intensity governs the unconditional RMS acceleration response and coefficient of variation. The unconditional RMS responses are always greater than (floor displacement, acceleration) the conditional responses. The discrepancies among the conditional and unconditional RMS responses are observed to be 5% for the superstructure displacement, with the maximum possible value of isolator damping. The respective coefficient of variation of the unconditional responses for the top-floor superstructure displacement and acceleration are around 19% and 6%, respectively. With the observed discrepancies amongst the conditional and unconditional responses and coefficient of variation, the peak value of the respective responses could be significantly different from the conditional one. Thus, disregarding the system uncertainty might lead to an unsafe design. From the parametric variations of the responses with respect to the isolator yield strength, it is observed that the optimal yield strength exists to ensure the best performance, which might be affected by the uncertainty. The optimum value of the yield strength might be based on two mutually conflicting objective function containing the minimization of both the unconditional responses as well as the associated standard deviations, thus making the problem ideal for reliability-based robust optimization. However, this aspect requires further study.

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$$
\begin{bmatrix}
0 & \cdots & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\
\vdots & n \times n & \vdots & \vdots & n \times 3 & \vdots & \vdots & n \times n & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\
0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\
0 & 3 \times n & 0 & 0 & -\frac{k_e}{q} & 0 & 0 & 3 \times n & 0 & -\frac{c_e}{q} & 0 \\
0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\
M^{-1}K_{11} - \frac{k_1}{m_b} & \cdots & M^{-1}K_{1n} & \alpha \frac{k_b}{m_b} & \frac{(1-\alpha)F_Y}{m_b} & 0 & M^{-1}C_{11} - \frac{c_1}{m_b} & \cdots & M^{-1}C_{1n} & \frac{c_b}{m_b} & 0 \\
\vdots & M^{-1}K_{ij} - \delta_{i1} \frac{k_1}{m_b} & \vdots & \vdots & \vdots & \vdots & \vdots & M^{-1}C_{ij} - \delta_{i1} \frac{c_1}{m_b} & \vdots & \vdots & \vdots \\
M^{-1}K_{n1} - \frac{k_1}{m_b} & \cdots & M^{-1}K_{nn} & \alpha \frac{k_b}{m_b} & \frac{(1-\alpha)F_Y}{m_b} & 0 & M^{-1}C_{n1} - \frac{c_1}{m_b} & \cdots & M^{-1}C_{nn} & \frac{c_b}{m_b} & 0 \\
\frac{k_1}{m_b} & \cdots & 0 & -\alpha \frac{k_b}{m_b} & -\frac{(1-\alpha)F_Y}{m_b} & \omega_g^2 & \frac{c_1}{m_b} & \cdots & 0 & -\frac{c_b}{m_b} & 2\xi_g \omega_g \\
0 & \cdots & 0 & 0 & 0 & -\omega_g^2 & 0 & \cdots & 0 & 0 & -2\xi_g \omega_g
\end{bmatrix}
\quad \text{(A.1)}
$$

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APPENDIX

The structure of the augmented system matrix (Eqs. (20) and (22)) $\mathbf{A}$ for a n-storied shear building is given in Eq. A.1 All the parameters in the matrices have already been defined in the main text. The variable δ_{ij} is the Kronecker's delta defined as $\delta_{ij} = 1$ when $i = j$ and $\delta_{ij} = 0$ when $i \neq j$. The augmented stiffness ($\mathbf{M}^{-1}\mathbf{K}$) and damping ($\mathbf{M}^{-1}\mathbf{C}$) matrices are indicated in the respective block of dimension $n \times n$.

The mass matrix is diagonal containing the storey mass in each diagonal term. The stiffness and damping matrices take the following form:

$$
\mathbf{K} = \begin{bmatrix}
k_1 + k_2 & -k_2 & \cdots & \cdots & \cdots \\
-k_2 & k_2 + k_3 & \cdots & \cdots & \cdots \\
\cdots & -k_i & k_i + k_{i+1} & -k_{i+1} & \cdots \\
\cdots & \cdots & \cdots & k_n + k_{n-1} & -k_n \\
\cdots & \cdots & \cdots & -k_n & k_n
\end{bmatrix}
\quad \text{(A.2)}
$$

$$
\mathbf{C} = \begin{bmatrix}
c_1 + c_2 & -c_2 & \cdots & \cdots & \cdots \\
-c_2 & c_2 + c_3 & \cdots & \cdots & \cdots \\
\cdots & -c_i & c_i + c_{i+1} & -c_{i+1} & \cdots \\
\cdots & \cdots & \cdots & c_n + c_{n-1} & -c_n \\
\cdots & \cdots & \cdots & -c_n & c_n
\end{bmatrix}
\quad \text{(A.3)}
$$

in which k_i and c_i are the stiffness and damping of the i-th storey of the building. The damping for the i-th storey is expressed as

$$c_i = 2\xi \sqrt{k_i m_i}; \quad \text{(A.4)}$$

where ξ is the viscous damping ratio of the superstructure.

The matrix for the rock bed seismic motion, characterized by the white noise of intensity S_0 is expressed as

$$
\mathbf{S_{ww}} = \begin{bmatrix}
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 2\pi S_0
\end{bmatrix}
\quad \text{(A.5)}
$$

which is a square matrix of dimension $(2n + 5)$.