Untitled

November 3, 2023

1 MNIST Handwriting Recognition from Scratch with NumPy

[1]:
```
from tensorflow import keras
from sklearn.metrics import accuracy_score
import numpy as np
```

[2]:
```
# loading dataset

(X_train, y_train), (X_test, y_test) = keras.datasets.mnist.load_data()
```

[3]:
```
print(X_train.shape)
print(X_test.shape)
```

```
(60000, 28, 28)
(10000, 28, 28)
```

[4]:
```
# reshape to flatten the data
X_train = X_train.reshape(X_train.shape[0], X_train.shape[1]*X_train.shape[2])
X_test = X_test.reshape(X_test.shape[0], X_test.shape[1]*X_test.shape[2])
```

[5]:
```
# maximum value in dataset
print(max(X_train[0]))
```

```
255
```

[6]:
```
# scaling
X_train = X_train/255
X_test = X_test/255
```

[7]:
```
def sigmoid(x):
    return 1 / (1 + np.exp(-x))
```

[8]:
```
def softmax(x):
    exp_x = np.exp(x - np.max(x, axis=1, keepdims=True))
    return exp_x / exp_x.sum(axis=1, keepdims=True)
```

[9]:
```
# Initialize weights and biases
input_size = 784
hidden_size = 128
```

```python
output_size = 10

W1 = np.random.randn(input_size, hidden_size)
b1 = np.zeros((1, hidden_size))
W2 = np.random.randn(hidden_size, output_size)
b2 = np.zeros((1, output_size))
```

[10]:
```python
# Hyperparameters
learning_rate = 1
epochs = 1000
loss_threshold = 0.5
```

[11]:
```python
# Training loop
for epoch in range(epochs):
    # Forward pass
    z1 = X_train.dot(W1) + b1
    a1 = sigmoid(z1)
    z2 = a1.dot(W2) + b2
    a2 = softmax(z2)

    # Loss
    loss = -np.sum(np.log(a2[np.arange(len(X_train)), y_train])) / len(X_train)

    # Backpropagation
    d2 = a2
    d2[np.arange(len(X_train)), y_train] -= 1
    d1 = (d2.dot(W2.T)) * (a1 * (1 - a1))

    W2 -= learning_rate * a1.T.dot(d2) / len(X_train)
    b2 -= learning_rate * np.sum(d2, axis=0, keepdims=True) / len(X_train)
    W1 -= learning_rate * X_train.T.dot(d1) / len(X_train)
    b1 -= learning_rate * np.sum(d1, axis=0, keepdims=True) / len(X_train)

    print(f'Epoch {epoch + 1}/{epochs}, Loss: {loss}')

    if loss <= loss_threshold:
        break
```

```
Epoch 1/1000, Loss: 11.450051587077485
Epoch 2/1000, Loss: 8.578540171694577
Epoch 3/1000, Loss: 6.273806161975045
Epoch 4/1000, Loss: 5.2555476502361795
Epoch 5/1000, Loss: 4.183392011380041
Epoch 6/1000, Loss: 3.6810628245993784
Epoch 7/1000, Loss: 3.3607578872242825
Epoch 8/1000, Loss: 3.108751060503716
Epoch 9/1000, Loss: 2.887312422153707
Epoch 10/1000, Loss: 2.713919044607191
```

```
Epoch 11/1000, Loss: 2.5508879454522115
Epoch 12/1000, Loss: 2.422895833467487
Epoch 13/1000, Loss: 2.299152444204859
Epoch 14/1000, Loss: 2.1995027041230473
Epoch 15/1000, Loss: 2.103561662130783
Epoch 16/1000, Loss: 2.0235212553717394
Epoch 17/1000, Loss: 1.9475638856981672
Epoch 18/1000, Loss: 1.8819391009881454
Epoch 19/1000, Loss: 1.8204432600105247
Epoch 20/1000, Loss: 1.7657228887315022
Epoch 21/1000, Loss: 1.7147868516938307
Epoch 22/1000, Loss: 1.6684670894649503
Epoch 23/1000, Loss: 1.62540293068508
Epoch 24/1000, Loss: 1.5856694654658519
Epoch 25/1000, Loss: 1.5486392986445494
Epoch 26/1000, Loss: 1.5141489542680986
Epoch 27/1000, Loss: 1.4818701677377237
Epoch 28/1000, Loss: 1.4516080842630228
Epoch 29/1000, Loss: 1.4231555378153102
Epoch 30/1000, Loss: 1.396346485196879
Epoch 31/1000, Loss: 1.371031138925402
Epoch 32/1000, Loss: 1.3470786112709543
Epoch 33/1000, Loss: 1.3243737161941362
Epoch 34/1000, Loss: 1.3028138923986583
Epoch 35/1000, Loss: 1.2823082246355275
Epoch 36/1000, Loss: 1.2627752615150338
Epoch 37/1000, Loss: 1.2441421053357036
Epoch 38/1000, Loss: 1.2263431073531992
Epoch 39/1000, Loss: 1.2093190844943484
Epoch 40/1000, Loss: 1.1930164758870179
Epoch 41/1000, Loss: 1.177386724491567
Epoch 42/1000, Loss: 1.1623856842238856
Epoch 43/1000, Loss: 1.1479731382493876
Epoch 44/1000, Loss: 1.1341123590279054
Epoch 45/1000, Loss: 1.120769731835171
Epoch 46/1000, Loss: 1.1079144188504753
Epoch 47/1000, Loss: 1.0955180662961383
Epoch 48/1000, Loss: 1.0835545467284073
Epoch 49/1000, Loss: 1.0719997339841159
Epoch 50/1000, Loss: 1.060831306377085
Epoch 51/1000, Loss: 1.0500285746180837
Epoch 52/1000, Loss: 1.039572330828598
Epoch 53/1000, Loss: 1.0294447155045
Epoch 54/1000, Loss: 1.0196290996412267
Epoch 55/1000, Loss: 1.010109979771795
Epoch 56/1000, Loss: 1.0008728840408603
Epoch 57/1000, Loss: 0.9919042878322496
Epoch 58/1000, Loss: 0.9831915377072035
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```
Epoch 59/1000, Loss: 0.9747227826353921
Epoch 60/1000, Loss: 0.9664869116446796
Epoch 61/1000, Loss: 0.9584734971512596
Epoch 62/1000, Loss: 0.950672743334427
Epoch 63/1000, Loss: 0.9430754390157171
Epoch 64/1000, Loss: 0.9356729145796003
Epoch 65/1000, Loss: 0.9284570025398359
Epoch 66/1000, Loss: 0.9214200014088183
Epoch 67/1000, Loss: 0.9145546425703001
Epoch 68/1000, Loss: 0.9078540598895227
Epoch 69/1000, Loss: 0.9013117618217031
Epoch 70/1000, Loss: 0.8949216058016186
Epoch 71/1000, Loss: 0.8886777747154506
Epoch 72/1000, Loss: 0.8825747552721699
Epoch 73/1000, Loss: 0.8766073181064218
Epoch 74/1000, Loss: 0.8707704994585282
Epoch 75/1000, Loss: 0.8650595842901375
Epoch 76/1000, Loss: 0.8594700907062843
Epoch 77/1000, Loss: 0.8539977555661976
Epoch 78/1000, Loss: 0.8486385211760552
Epoch 79/1000, Loss: 0.8433885229669904
Epoch 80/1000, Loss: 0.838244078071005
Epoch 81/1000, Loss: 0.8332016747159668
Epoch 82/1000, Loss: 0.8282579623686593
Epoch 83/1000, Loss: 0.8234097425618556
Epoch 84/1000, Loss: 0.8186539603477271
Epoch 85/1000, Loss: 0.8139876963255702
Epoch 86/1000, Loss: 0.8094081591969342
Epoch 87/1000, Loss: 0.8049126788057812
Epoch 88/1000, Loss: 0.8004986996254067
Epoch 89/1000, Loss: 0.7961637746574829
Epoch 90/1000, Loss: 0.7919055597118541
Epoch 91/1000, Loss: 0.7877218080386078
Epoch 92/1000, Loss: 0.7836103652865449
Epoch 93/1000, Loss: 0.7795691647644677
Epoch 94/1000, Loss: 0.7755962229837491
Epoch 95/1000, Loss: 0.7716896354624792
Epoch 96/1000, Loss: 0.7678475727731088
Epoch 97/1000, Loss: 0.7640682768169672
Epoch 98/1000, Loss: 0.7603500573103554
Epoch 99/1000, Loss: 0.7566912884681091
Epoch 100/1000, Loss: 0.7530904058716251
Epoch 101/1000, Loss: 0.7495459035093556
Epoch 102/1000, Loss: 0.7460563309787195
Epoch 103/1000, Loss: 0.7426202908392571
Epoch 104/1000, Loss: 0.7392364361076701
Epoch 105/1000, Loss: 0.7359034678861682
Epoch 106/1000, Loss: 0.7326201331162419
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Epoch 107/1000, Loss: 0.7293852224506491
Epoch 108/1000, Loss: 0.7261975682370021
Epoch 109/1000, Loss: 0.7230560426068917
Epoch 110/1000, Loss: 0.7199595556649653
Epoch 111/1000, Loss: 0.716907053772817
Epoch 112/1000, Loss: 0.7138975179229112
Epoch 113/1000, Loss: 0.7109299621980879
Epoch 114/1000, Loss: 0.708003432312469
Epoch 115/1000, Loss: 0.705117004229801
Epoch 116/1000, Loss: 0.7022697828554656
Epoch 117/1000, Loss: 0.69946090079854
Epoch 118/1000, Loss: 0.6966895172004085
Epoch 119/1000, Loss: 0.6939548166265537
Epoch 120/1000, Loss: 0.6912560080182285
Epoch 121/1000, Loss: 0.6885923237008211
Epoch 122/1000, Loss: 0.6859630184457981
Epoch 123/1000, Loss: 0.6833673685832017
Epoch 124/1000, Loss: 0.6808046711617787
Epoch 125/1000, Loss: 0.6782742431539023
Epoch 126/1000, Loss: 0.6757754207025649
Epoch 127/1000, Loss: 0.6733075584078312
Epoch 128/1000, Loss: 0.670870028650249
Epoch 129/1000, Loss: 0.668462220948853
Epoch 130/1000, Loss: 0.6660835413515083
Epoch 131/1000, Loss: 0.663733411855481
Epoch 132/1000, Loss: 0.6614112698562459
Epoch 133/1000, Loss: 0.659116567622667
Epoch 134/1000, Loss: 0.656848771796821
Epoch 135/1000, Loss: 0.6546073629168454
Epoch 136/1000, Loss: 0.652391834961313
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Epoch 138/1000, Loss: 0.6480364623460374
Epoch 139/1000, Loss: 0.645895669019438
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Epoch 142/1000, Loss: 0.639615417019295
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Epoch 147/1000, Loss: 0.6295966798447422
Epoch 148/1000, Loss: 0.6276566024932054
Epoch 149/1000, Loss: 0.625736875350013
Epoch 150/1000, Loss: 0.6238371420524015
Epoch 151/1000, Loss: 0.6219570546970073
Epoch 152/1000, Loss: 0.6200962735971463
Epoch 153/1000, Loss: 0.618254467048277
Epoch 154/1000, Loss: 0.6164313111012769
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Epoch 155/1000, Loss: 0.6146264893431844
Epoch 156/1000, Loss: 0.6128396926850773
Epoch 157/1000, Loss: 0.6110706191567856
Epoch 158/1000, Loss: 0.6093189737081454
Epoch 159/1000, Loss: 0.6075844680165234
Epoch 160/1000, Loss: 0.6058668203003551
Epoch 161/1000, Loss: 0.6041657551384554
Epoch 162/1000, Loss: 0.6024810032948751
Epoch 163/1000, Loss: 0.6008123015490853
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Epoch 165/1000, Loss: 0.5975220245626874
Epoch 166/1000, Loss: 0.5958999515004633
Epoch 167/1000, Loss: 0.5942929325874141
Epoch 168/1000, Loss: 0.5927007323059763
Epoch 169/1000, Loss: 0.5911231202365548
Epoch 170/1000, Loss: 0.5895598709199967
Epoch 171/1000, Loss: 0.5880107637240864
Epoch 172/1000, Loss: 0.5864755827139349
Epoch 173/1000, Loss: 0.5849541165261527
Epoch 174/1000, Loss: 0.5834461582466984
Epoch 175/1000, Loss: 0.5819515052922983
Epoch 176/1000, Loss: 0.5804699592953443
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Epoch 178/1000, Loss: 0.5775454151146672
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Epoch 186/1000, Loss: 0.5663345412818165
Epoch 187/1000, Loss: 0.5649851797678925
Epoch 188/1000, Loss: 0.5636468360989988
Epoch 189/1000, Loss: 0.5623193576665866
Epoch 190/1000, Loss: 0.5610025947855986
Epoch 191/1000, Loss: 0.5596964006219974
Epoch 192/1000, Loss: 0.5584006311225547
Epoch 193/1000, Loss: 0.5571151449468303
Epoch 194/1000, Loss: 0.5558398034012736
Epoch 195/1000, Loss: 0.5545744703753733
Epoch 196/1000, Loss: 0.5533190122797902
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Epoch 198/1000, Loss: 0.550837198770185
Epoch 199/1000, Loss: 0.5496105882528982
Epoch 200/1000, Loss: 0.5483933423484469
Epoch 201/1000, Loss: 0.547185339209922
Epoch 202/1000, Loss: 0.5459864591782135
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Epoch 203/1000, Loss: 0.5447965847321555
Epoch 204/1000, Loss: 0.5436156004401363
Epoch 205/1000, Loss: 0.5424433929131194
Epoch 206/1000, Loss: 0.541279850759021
Epoch 207/1000, Loss: 0.5401248645383887
Epoch 208/1000, Loss: 0.5389783267213338
Epoch 209/1000, Loss: 0.5378401316456648
Epoch 210/1000, Loss: 0.5367101754761768
Epoch 211/1000, Loss: 0.5355883561650543
Epoch 212/1000, Loss: 0.534474573413341
Epoch 213/1000, Loss: 0.5333687286334376
Epoch 214/1000, Loss: 0.5322707249125911
Epoch 215/1000, Loss: 0.5311804669773343
Epoch 216/1000, Loss: 0.5300978611588453
Epoch 217/1000, Loss: 0.529022815359191
Epoch 218/1000, Loss: 0.5279552390184244
Epoch 219/1000, Loss: 0.5268950430825059
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Epoch 221/1000, Loss: 0.524796443551661
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Epoch 223/1000, Loss: 0.5227263332827584
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Epoch 226/1000, Loss: 0.5196731444625161
Epoch 227/1000, Loss: 0.5186689565495782
Epoch 228/1000, Loss: 0.5176714104020431
Epoch 229/1000, Loss: 0.5166804304235612
Epoch 230/1000, Loss: 0.5156959422181309
Epoch 231/1000, Loss: 0.5147178725660867
Epoch 232/1000, Loss: 0.5137461494006405
Epoch 233/1000, Loss: 0.5127807017849608
Epoch 234/1000, Loss: 0.5118214598897805
Epoch 235/1000, Loss: 0.5108683549715117
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Epoch 237/1000, Loss: 0.5089802863919476
Epoch 238/1000, Loss: 0.5080451904818521
Epoch 239/1000, Loss: 0.5071159670106898
Epoch 240/1000, Loss: 0.5061925523520909
Epoch 241/1000, Loss: 0.5052748838441421
Epoch 242/1000, Loss: 0.5043628997707569
Epoch 243/1000, Loss: 0.5034565393434672
Epoch 244/1000, Loss: 0.5025557426836288
Epoch 245/1000, Loss: 0.5016604508050282
Epoch 246/1000, Loss: 0.5007706055968837
Epoch 247/1000, Loss: 0.4998861498072317
```

[12]:
```
z1 = X_test.dot(W1) + b1
a1 = sigmoid(z1)
z2 = a1.dot(W2) + b2
a2 = softmax(z2)
```

[13]:
```
predicted_labels = np.argmax(a2, axis=1)
```

[14]:
```
predicted_labels
```

[14]:
```
array([7, 2, 1, …, 4, 5, 6], dtype=int64)
```

[15]:
```
accuracy_score(y_test, predicted_labels)
```

[15]:
```
0.8649
```