Chapter 1 Solutions

***1.1** With V = (base area) · (height)

$$V = \pi r^2 \cdot h$$

and $\rho = \dfrac{m}{V}$, we have

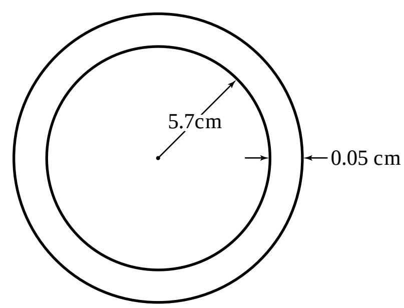
$$\rho = \frac{m}{\pi r^2 h} = \frac{1\ \text{kg}}{\pi (19.5\ \text{mm})^2\ 39.0\ \text{mm}} \left(\frac{10^9\ \text{mm}^3}{1\ \text{m}^3}\right)$$

$$\rho = \boxed{2.15 \times 10^4\ \ \text{kg/m}^3}$$

1.2 $\rho = \dfrac{M}{V} = \dfrac{M}{\frac{4}{3}\pi R^3}$

$$\rho = \frac{3(5.64 \times 10^{26}\ \text{kg})}{4\pi (6.00 \times 10^7\ \text{m})^3} = \boxed{623\ \text{kg/m}^3}$$

1.3 $V_{Cu} = V_0 - V_i = \dfrac{4}{3}\ \pi (r_o^3 - r_i^3)$



$$V_{Cu} = \frac{4}{3}\ \pi\left[(5.75\ \text{cm})^3 - (5.70\ \text{cm})^3\right] = 20.6\ \text{cm}^3$$

$$\rho = \frac{m}{V}$$

$$m = \rho V = (8.92\ \text{g/cm}^3)(20.6\ \text{cm}^3) = \boxed{184\ \text{g}}$$

1.4 $V = V_o - V_i = \dfrac{4}{3}\ \pi\ (r_2^3 - r_1^3)$

$$\rho = \frac{m}{V},\ \ \text{so}\ \ m = \rho V = \rho\left(\frac{4}{3}\pi\right)(r_2^3 - r_1^3) = \boxed{\frac{4\pi\rho\ (r_2^3 - r_1^3)}{3}}$$

***1.5** (a) The number of moles is $n = m/M$, and the density is $\rho = m/V$. Noting that we have 1 mole,

$$V_{1\ \text{mol}} = \frac{m_{\text{Fe}}}{\rho_{\text{Fe}}} = \frac{n_{\text{Fe}} M_{\text{Fe}}}{\rho_{\text{Fe}}} = \frac{(1\ \text{mol})(55.8\ \text{g/mol})}{7.86\ \text{g/cm}^3} = \boxed{7.10\ \text{cm}^3}$$

(b) In 1 mole of iron are N_A atoms:

$$V_{1\text{ atom}} = \frac{V_{1\text{ mol}}}{N_A} = \frac{7.10\text{ cm}^3}{6.02\times10^{23}\text{ atoms/mol}} = 1.18\times10^{-23}\text{ cm}^3$$

$$= \boxed{1.18\times10^{-29}\text{ m}^3}$$

(c) $d_{\text{atom}} = \sqrt[3]{1.18\times10^{-29}\text{ m}^3} = 2.28\times10^{-10}\text{ m} = \boxed{0.228\text{ nm}}$

(d) $V_{1\text{ mol U}} = \dfrac{(1\text{ mol})(238\text{ g/mol})}{18.7\text{ g/cm}^3} = \boxed{12.7\text{ cm}^3}$

$$V_{1\text{ atom U}} = \frac{V_{1\text{ mol U}}}{N_A} = \frac{12.7\text{ cm}^3}{6.02\times10^{23}\text{ atoms/mol}} = 2.11\times10^{-23}\text{ cm}^3$$

$$= \boxed{2.11\times10^{-29}\text{ m}^3}$$

$$d_{\text{atom U}} = \sqrt[3]{V_{1\text{ atom U}}} = \sqrt[3]{2.11\times10^{-29}\text{ m}^3} = 2.77\times10^{-10}\text{ m} = \boxed{0.277\text{ nm}}$$

***1.6** $r_2 = r_1\sqrt[3]{5} = (4.50\text{ cm})(1.71) = \boxed{7.69\text{ cm}}$

1.7 Use m = molar mass / N_A and 1 u = 1.66×10^{-24} g

(a) For He, $m = \dfrac{4.00\text{ g/mol}}{6.02\times10^{23}\text{ mol}^{-1}} = \boxed{6.64\times10^{-24}\text{ g} = 4.00\text{ u}}$

(b) For Fe, $m = \dfrac{55.9\text{ g/mol}}{6.02\times10^{23}\text{ mol}^{-1}} = \boxed{9.29\times10^{-23}\text{ g} = 55.9\text{ u}}$

(c) For Pb, $m = \dfrac{207\text{ g/mol}}{6.02\times10^{23}\text{ mol}^{-1}} = \boxed{3.44\times10^{-22}\text{ g} = 207\text{ u}}$

Goal Solution

Calculate the mass of an atom of (a) helium, (b) iron, and (c) lead. Give your answers in atomic mass units and in grams. The molar masses are 4.00, 55.9, and 207 g / mol, respectively, for the atoms given.

Gather information: The mass of an atom of any element is essentially the mass of the protons and neutrons that make up its nucleus since the mass of the electrons is negligible (less than a 0.05% contribution). Since most atoms have about the same number of neutrons as protons, the atomic mass is approximately double the atomic number (the number of protons). We should also expect that the mass of a single atom is a very small fraction of a gram ($\sim 10^{-23}$ g) since one mole (6.02×10^{23}) of atoms has a mass on the order of several grams.

Organize: An atomic mass unit is defined as 1 / 12 of the mass of a carbon-12 atom (which has a molar mass of 12.0 g / mol), so the mass of any atom in atomic mass units is simply the numerical value of the molar mass. The mass in grams can be found by multiplying the molar mass by the mass of one atomic mass unit (u):

$$1 \text{ u} = 1.66 \times 10^{-24} \text{ g.}$$

Analyze: For He, $m = 4.00 \text{ u} = (4.00 \text{ u})(1.66 \times 10^{-24} \text{ g / u}) = 6.64 \times 10^{-24} \text{ g}$

For Fe, $m = 55.9 \text{ u} = (55.9 \text{ u})(1.66 \times 10^{-24} \text{g / u}) = 9.28 \times 10^{-23} \text{ g}$

For Pb, $m = 207 \text{ u} = (207 \text{ u})(1.66 \times 10^{-24} \text{ g / u}) = 3.44 \times 10^{-22} \text{ g}$

Learn: As expected, the mass of the atoms is larger for bigger atomic numbers. If we did not know the conversion factor for atomic mass units, we could use the mass of a proton as a close approximation: $1\text{u} \approx m_p = 1.67 \times 10^{-24}$ g.

***1.8** $\Delta n = \dfrac{\Delta m}{M} = \dfrac{3.80 \text{ g} - 3.35 \text{ g}}{197 \text{ g / mol}} = 0.00228 \text{ mol}$

$$\Delta N = (\Delta n)N_{\text{A}} = (0.00228 \text{ mol})(6.02 \times 10^{23} \text{ atoms / mol}) = 1.38 \times 10^{21} \text{ atoms}$$

$$\Delta t = (50.0 \text{ yr})(365 \text{ d / yr})(24.0 \text{ hr / d})(3600 \text{ s / hr}) = 1.58 \times 10^{9} \text{ s}$$

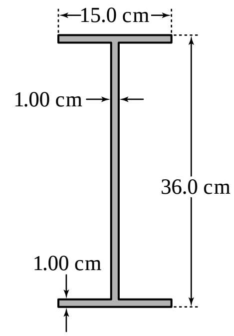
$$\frac{\Delta N}{\Delta t} = \frac{1.38 \times 10^{21} \text{ atoms}}{1.58 \times 10^{9} \text{ s}} = \boxed{8.72 \times 10^{11} \text{ atoms / s}}$$

1.9 (a) $m = \rho L^3 = (7.86 \text{ g / cm}^3)(5.00 \times 10^{-6} \text{ cm})^3 = \boxed{9.83 \times 10^{-16} \text{ g}}$

(b) $N = m\left(\dfrac{N_{\text{A}}}{\text{Molar mass}}\right) = \dfrac{(9.83 \times 10^{-16} \text{ g})(6.02 \times 10^{23} \text{ atoms / mol})}{55.9 \text{ g / mol}}$

$$= \boxed{1.06 \times 10^{7} \text{ atoms}}$$

1.10 (a) The cross-sectional area is



$$A = 2(0.150 \text{ m})(0.010 \text{ m}) + (0.340 \text{ m})(0.010 \text{ m})$$

$$= 6.40 \times 10^{-3} \text{ m}^2$$

The volume of the beam is

$$V = AL = (6.40 \times 10^{-3} \text{ m}^2)(1.50 \text{ m}) = 9.60 \times 10^{-3} \text{ m}^3$$

Thus, its mass is $m = \rho V = (7.56 \times 10^3 \text{ kg/m}^3)(9.60 \times 10^{-3} \text{ m}^3)$ $= \boxed{72.6 \text{ kg}}$

(b) Presuming that most of the atoms are of iron, we estimate the molar mass as $M = 55.9 \text{ g/mol} = 55.9 \times 10^{-3} \text{ kg/mol}$. The number of moles is then

$$n = \frac{m}{M} = \frac{72.6 \text{ kg}}{55.9 \times 10^{-3} \text{ kg/mol}} = 1.30 \times 10^3 \text{ mol}$$

The number of atoms is

$$N = nN_A = (1.30 \times 10^3 \text{ mol})(6.02 \times 10^{23} \text{ atoms/mol}) = \boxed{7.82 \times 10^{26} \text{ atoms}}$$

***1.11** (a) $n = \dfrac{m}{M} = \dfrac{1.20 \times 10^3 \text{ g}}{18.0 \text{ g/mol}} = 66.7 \text{ mol}$, and

$$N_{\text{pail}} = nN_A = (66.7 \text{ mol})(6.02 \times 10^{23} \text{ molecules/mol})$$

$$= \boxed{4.01 \times 10^{25} \text{ molecules}}$$

(b) Suppose that enough time has elapsed for thorough mixing of the hydrosphere.

$$N_{\text{both}} = N_{\text{pail}}\left(\frac{m_{\text{pail}}}{M_{\text{total}}}\right) = (4.01 \times 10^{25} \text{ molecules})\left(\frac{1.20 \text{ kg}}{1.32 \times 10^{21} \text{ kg}}\right), \text{ or}$$

$$N_{\text{both}} = \boxed{3.65 \times 10^4 \text{ molecules}}$$

1.12 r, a, b, c and s all have units of L.

$$\left[\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}\right] = \sqrt{\frac{L \times L \times L}{L}} = \sqrt{L^2} = \boxed{L}$$

Thus, the equation is dimensionally consistent.

1.13 The term s has dimensions of L, a has dimensions of LT^{-2}, and t has dimensions of T. Therefore, the equation, $s = ka^m t^n$ has dimensions of

$$L = (LT^{-2})^m (T)^n \quad \text{or} \quad L^1 T^0 = L^m T^{n-2m}$$

The powers of L and T must be the same on each side of the equation. Therefore,

$$L^1 = L^m \text{ and } \boxed{m = 1}$$

Likewise, equating terms in T, we see that $n - 2m$ must equal 0. Thus,

$$\boxed{n = 2m = 2}$$

The value of k, a dimensionless constant, $\boxed{\text{cannot be obtained by dimensional analysis}}$.

1.14 $$\left[2\pi\sqrt{\frac{l}{g}}\right] = \sqrt{\frac{L}{L/T^2}} = \sqrt{T^2} = \boxed{T}$$

1.15 (a) $\boxed{\text{This is incorrect}}$ since the units of $[ax]$ are m^2/s^2, while the units of $[v]$ are m/s.

(b) $\boxed{\text{This is correct}}$ since the units of $[y]$ are m, and $\cos(kx)$ is dimensionless if $[k]$ is in m^{-1}.

1.16 Inserting the proper units for everything except G,

$$\left[\frac{\text{kg m}}{\text{s}^2}\right] = \frac{G[\text{kg}]^2}{[\text{m}]^2}$$

Multiply both sides by $[\text{m}]^2$ and divide by $[\text{kg}]^2$; the units of G are

$$\boxed{\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}}$$

1.17 One month is $1 \text{ mo} = (30 \text{ day})(24 \text{ hr/day})(3600 \text{ s/hr}) = 2.592 \times 10^6 \text{ s}$

Applying units to the equation,

$$V = (1.50 \text{ Mft}^3/\text{mo})t + (0.00800 \text{ Mft}^3/\text{mo}^2)t^2$$

Since $1 \text{ Mft}^3 = 10^6 \text{ ft}^3$,

$$V = (1.50 \times 10^6 \text{ ft}^3/\text{mo})t + (0.00800 \times 10^6 \text{ ft}^3/\text{mo}^2)t^2$$

Converting months to seconds,

$$V = \frac{1.50 \times 10^6 \text{ ft}^3/\text{mo}}{2.592 \times 10^6 \text{ s/mo}}\, t + \frac{0.00800 \times 10^6 \text{ ft}^3/\text{mo}^2}{(2.592 \times 10^6 \text{ s/mo})^2}\, t^2$$

Thus, $\boxed{V[\text{ft}^3] = (0.579 \text{ ft}^3/\text{s})t + (1.19 \times 10^{-9} \text{ ft}^3/\text{s}^2)t^2}$

***1.18** Apply the following conversion factors:

1 in = 2.54 cm, 1 d = 86400 s, 100 cm = 1 m, and 10^9 nm = 1 m

$$\left(\frac{1}{32} \text{ in/day}\right)\frac{(2.54 \text{ cm/in})(10^{-2} \text{ m/cm})(10^9 \text{ nm/m})}{86400 \text{ s/day}} = \boxed{9.19 \text{ nm/s}}$$

This means the proteins are assembled at a rate of many layers of atoms each second!

1.19 Area A = (100 ft)(150 ft) = 1.50×10^4 ft^2, so

$$A = (1.50 \times 10^4 \text{ ft}^2)(9.29 \times 10^{-2} \text{ m}^2/\text{ft}^2) = \boxed{1.39 \times 10^3 \text{ m}^2}$$

<u>Goal Solution</u>

A rectangular building lot is 100 ft by 150 ft. Determine the area of this lot in m^2.

G: We must calculate the area and convert units. Since a meter is about 3 feet, we should expect the area to be about A ≈ (30 m)(50 m) = 1 500 m^2.

O: Area = Length × Width. Use the conversion: 1 m = 3.281 ft.

A: $A = L \times W = (100 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)(150 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 1\,390 \text{ m}^2$

L: Our calculated result agrees reasonably well with our initial estimate and has the proper units of m^2. Unit conversion is a common technique that is applied to many problems.

1.20 (a) $V = (40.0 \text{ m})(20.0 \text{ m})(12.0 \text{ m}) = 9.60 \times 10^3 \text{ m}^3$

$$V = 9.60 \times 10^3 \text{ m}^3 (3.28 \text{ ft}/1 \text{ m})^3 = \boxed{3.39 \times 10^5 \text{ ft}^3}$$

(b) The mass of the air is

$$m = \rho_{\text{air}} V = (1.20\ \text{kg/m}^3)(9.60 \times 10^3\ \text{m}^3) = 1.15 \times 10^4\ \text{kg}$$

The student must look up weight in the index to find

$$F_g = mg = (1.15 \times 10^4\ \text{kg})(9.80\ \text{m/s}^2) = 1.13 \times 10^5\ \text{N}$$

Converting to pounds,

$$F_g = (1.13 \times 10^5\ \text{N})(1\ \text{lb}/4.45\ \text{N}) = \boxed{2.54 \times 10^4\ \text{lb}}$$

***1.21** (a) Seven minutes is 420 seconds, so the rate is

$$r = \frac{30.0\ \text{gal}}{420\ \text{s}} = \boxed{7.14 \times 10^{-2}\ \text{gal/s}}$$

(b) Converting gallons first to liters, then to m^3,

$$r = \left(7.14 \times 10^{-2}\ \frac{\text{gal}}{\text{s}}\right)\left(\frac{3.786\ \text{L}}{1\ \text{gal}}\right)\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)$$

$$r = \boxed{2.70 \times 10^{-4}\ \text{m}^3/\text{s}}$$

(c) At that rate, to fill a 1-m^3 tank would take

$$t = \left(\frac{1\ \text{m}^3}{2.70 \times 10^{-4}\ \text{m}^3/\text{s}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right) = \boxed{1.03\ \text{hr}}$$

1.22 $v = \left(5.00\ \dfrac{\text{furlongs}}{\text{fortnight}}\right)\left(\dfrac{220\ \text{yd}}{1\ \text{furlong}}\right)\left(\dfrac{0.9144\ \text{m}}{1\ \text{yd}}\right)\left(\dfrac{1\ \text{fortnight}}{14\ \text{days}}\right)\left(\dfrac{1\ \text{day}}{24\ \text{hrs}}\right)\left(\dfrac{1\ \text{hr}}{3600\ \text{s}}\right)$

$= \boxed{8.32 \times 10^{-4}\ \text{m/s}}$

This speed is almost 1 mm/s; so we might guess the creature was a snail, or perhaps a sloth.

1.23 It is often useful to remember that the 1600-m race at track and field events is approximately 1 mile in length. To be precise, there are 1609 meters in a mile. Thus, 1 acre is equal in area to

$$(1\ \text{acre})\left(\frac{1\ \text{mi}^2}{640\ \text{acres}}\right)\left(\frac{1609\ \text{m}}{\text{mi}}\right)^2 = \boxed{4.05 \times 10^3\ \text{m}^2}$$

1.24 Volume of cube = L^3 = 1 quart (Where L = length of one side of the cube.) Thus,

$$L^3 = (1\text{ quart})\left(\frac{1\text{ gallon}}{4\text{ quarts}}\right)\left(\frac{3.786\text{ liters}}{1\text{ gallon}}\right)\left(\frac{1000\text{ cm}^3}{1\text{ liter}}\right) = 946\text{ cm}^3, \text{ and}$$

$$L = \boxed{9.82\text{ cm}}$$

1.25 The mass and volume, in SI units, are

$$m = (23.94\text{ g})\left(\frac{1\text{ kg}}{1000\text{ g}}\right) = 0.02394\text{ kg}$$

$$V = (2.10\text{ cm}^3)(10^{-2}\text{ m/cm})^3 = 2.10 \times 10^{-6}\text{ m}^3$$

Thus, the density is

$$\rho = \frac{m}{V} = \frac{0.02394\text{ kg}}{2.10 \times 10^{-6}\text{ m}^3} = \boxed{1.14 \times 10^4\text{ kg/m}^3}$$

<u>Goal Solution</u>

A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm^3. From these data, calculate the density of lead in SI units (kg/m^3).

G: From Table 1.5, the density of lead is $1.13 \times 10^4\text{ kg/m}^3$, so we should expect our calculated value to be close to this number. This density value tells us that lead is about 11 times denser than water, which agrees with our experience that lead sinks.

O: Density is defined as mass per volume, in $\rho = \frac{m}{V}$. We must convert to SI units in the calculation.

A: $\rho = \dfrac{23.94\text{ g}}{2.10\text{ cm}^3}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)\left(\dfrac{100\text{ cm}}{1\text{ m}}\right)^3 = 1.14 \times 10^4\text{ kg/m}^3$

L: At one step in the calculation, we note that *one million* cubic centimeters make one cubic meter. Our result is indeed close to the expected value. Since the last reported significant digit is not certain, the difference in the two values is probably due to measurement uncertainty and should not be a concern. One important common-sense check on density values is that objects which sink in water must have a density greater than 1 g/cm^3, and objects that float must be less dense than water.

1.26 (a) We take information from Table 1.1:

$$1 \text{ LY} = (9.46 \times 10^{15} \text{ m})\left(\frac{1 \text{ AU}}{1.50 \times 10^{11} \text{ m}}\right) = \boxed{6.31 \times 10^4 \text{ AU}}$$

(b) The distance to the Andromeda galaxy is

$$2 \times 10^{22} \text{ m} = (2 \times 10^{22} \text{ m})\left(\frac{1 \text{ AU}}{1.50 \times 10^{11} \text{ m}}\right) = \boxed{1.33 \times 10^{11} \text{ AU}}$$

1.27 $N_{\text{atoms}} = \dfrac{m_{\text{Sun}}}{m_{\text{atom}}} = \dfrac{1.99 \times 10^{30} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = \boxed{1.19 \times 10^{57} \text{ atoms}}$

1.28 1 mi = 1609 m = 1.609 km; thus, to go from mph to km/h, multiply by 1.609.

(a) 1 mi/h = $\boxed{1.609 \text{ km/h}}$

(b) 55 mi/h = $\boxed{88.5 \text{ km/h}}$

(c) 65 mi/h = 104.6 km/h. Thus, Δv = $\boxed{16.1 \text{ km/h}}$

1.29 (a) $\left(\dfrac{6 \times 10^{12} \text{ \$}}{1000 \text{ \$/s}}\right)\left(\dfrac{1 \text{ hr}}{3600 \text{ s}}\right)\left(\dfrac{1 \text{ day}}{24 \text{ hr}}\right)\left(\dfrac{1 \text{ yr}}{365 \text{ days}}\right) = \boxed{190 \text{ years}}$

(b) The circumference of the Earth at the equator is $2\pi(6378 \times 10^3 \text{ m}) = 4.01 \times 10^7$ m. The length of one dollar bill is 0.155 m so that the length of 6 trillion bills is 9.30×10^{11} m. Thus, the 6 trillion dollars would encircle the Earth

$$\frac{9.30 \times 10^{11} \text{ m}}{4.01 \times 10^7 \text{ m}} = \boxed{2.32 \times 10^4 \text{ times}}$$

Goal Solution

At the time of this book's printing, the U.S. national debt is about \$6 trillion. (a) If payments were made at the rate of \$1 000 per second, how many years would it take to pay off a \$6-trillion debt, assuming no interest were charged? (b) A dollar bill is about 15.5 cm long. If six trillion dollar bills were laid end to end around the Earth's equator, how many times would they encircle the Earth? Take the radius of the Earth at the equator to be 6 378 km. (*Note:* Before doing any of these calculations, try to guess at the answers. You may be very surprised.)

(a)

G: \$6 trillion is certainly a large amount of money, so even at a rate of \$1000/second, we might guess that it will take a lifetime (~ 100 years) to pay off the debt.

O: Time to repay the debt will be calculated by dividing the total debt by the rate at which it is repaid.

A: $T = \frac{\$6\text{ trillion}}{\$1000/\text{s}} = \frac{\$6 \times 10^{12}}{(\$1000/\text{s})(3.16 \times 10^7\text{ s/yr})} = 190\text{ yr}$

L: OK, so our estimate was a bit low. $6 trillion really is a lot of money!

(b)
G: We might guess that 6 trillion bills would encircle the Earth at least a few hundred times, maybe more since our first estimate was low.

O: The number of bills can be found from the total length of the bills placed end to end divided by the circumference of the Earth.

A: $N = \frac{L}{C} = \frac{(6 \times 10^{12})(15.5\text{ cm})(1\text{ m}/100\text{ cm})}{2\pi\, 6.37 \times 10^6\text{ m}} = 2.32 \times 10^4$ times

L: OK, so again our estimate was low. Knowing that the bills could encircle the earth more than 20 000 times, it might be reasonable to think that 6 trillion bills could cover the entire surface of the earth, but the calculated result is a surprisingly small fraction of the earth's surface area!

1.30 (a) $(3600\text{ s/hr})(24\text{ hr/day})(365.25\text{ days/yr}) = \boxed{3.16 \times 10^7\text{ s/yr}}$

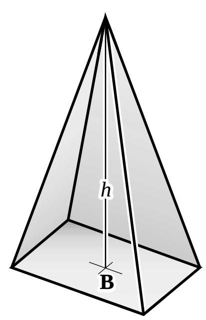
(b) $V_{mm} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5.00 \times 10^{-7}\text{ m})^3 = 5.24 \times 10^{-19}\text{ m}^3$

$$\frac{V_{\text{cube}}}{V_{mm}} = \frac{1\text{ m}^3}{5.24 \times 10^{-19}\text{ m}^3} = 1.91 \times 10^{18}\text{ micrometeorites}$$

This would take $\frac{1.91 \times 10^{18}\text{ micrometeorites}}{3.16 \times 10^7\text{ micrometeorites/yr}} = \boxed{6.05 \times 10^{10}\text{ yr}}$

1.31 $V = At$, so $t = \frac{V}{A} = \frac{3.78 \times 10^{-3}\text{ m}^3}{25.0\text{ m}^2} = \boxed{1.51 \times 10^{-4}\text{ m (or }151\ \mu\text{m)}}$

1.32 $V = \frac{1}{3}Bh = \frac{[(13.0\text{ acres})(43560\text{ ft}^2/\text{acre})]}{3}(481\text{ ft})$



$= 9.08 \times 10^7\text{ ft}^3$, or

$V = (9.08 \times 10^7\text{ ft}^3)\left(\frac{2.83 \times 10^{-2}\text{ m}^3}{1\text{ ft}^3}\right)$

$= \boxed{2.57 \times 10^6\text{ m}^3}$

1.33 $F_g = (2.50 \text{ tons/block})(2.00 \times 10^6 \text{ blocks})(2000 \text{ lb/ton}) = \boxed{1.00 \times 10^{10} \text{ lbs}}$

1.34 The area covered by water is

$$A_w = 0.700\, A_{\text{Earth}} = (0.700)(4\pi R_{\text{Earth}}^2) = (0.700)(4\pi)(6.37 \times 10^6 \text{ m})^2 = 3.57 \times 10^{14} \text{ m}^2$$

The average depth of the water is

$$d = (2.30 \text{ miles})(1609 \text{ m/l mile}) = 3.70 \times 10^3 \text{ m}$$

The volume of the water is

$$V = A_w d = (3.57 \times 10^{14} \text{ m}^2)(3.70 \times 10^3 \text{ m}) = 1.32 \times 10^{18} \text{ m}^3$$

and the mass is $m = \rho V = (1000 \text{ kg/m}^3)(1.32 \times 10^{18} \text{ m}^3) = \boxed{1.32 \times 10^{21} \text{ kg}}$

***1.35** SI units of volume are in m^3:

$$V = (25.0 \text{ acre-ft})\left(\frac{43560 \text{ ft}^2}{1 \text{ acre}}\right)\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right)^3 = \boxed{3.08 \times 10^4 \text{ m}^3}$$

***1.36** (a) $d_{\text{nucleus, scale}} = d_{\text{nucleus, real}}\left(\dfrac{d_{atom,\,scale}}{d_{\text{atom, real}}}\right)$

$$= (2.40 \times 10^{-15} \text{ m})\left(\frac{300 \text{ ft}}{1.06 \times 10^{-10} \text{ m}}\right)$$

$$= 6.79 \times 10^{-3} \text{ ft, or}$$

$$d_{\text{nucleus, scale}} = (6.79 \times 10^{-3} \text{ ft})(304.8 \text{ mm/1 ft}) = \boxed{2.07 \text{ mm}}$$

(b) $\dfrac{V_{\text{atom}}}{V_{\text{nucleus}}} = \dfrac{4\pi r_{\text{atom}}^3/3}{4\pi r_{\text{nucleus}}^3/3} = \left(\dfrac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3 = \left(\dfrac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3$

$$= \left(\frac{1.06 \times 10^{-10} \text{ m}}{2.40 \times 10^{-15} \text{ m}}\right)^3 = \boxed{8.62 \times 10^{13} \text{ times as large}}$$

1.37 The scale factor used in the "dinner plate" model is

$$S = \frac{0.25 \text{ m}}{1.0 \times 10^5 \text{ lightyears}} = 2.5 \times 10^{-6} \text{ m/lightyears}$$

The distance to Andromeda in the scale model will be

$$D_{\text{scale}} = D_{\text{actual}} S = (2.0 \times 10^6 \text{ lightyears})(2.5 \times 10^{-6} \text{ m/lightyears}) = \boxed{5.0 \text{ m}}$$

1.38 (a) $\dfrac{A_{\text{Earth}}}{A_{\text{Moon}}} = \dfrac{4\pi r_{\text{Earth}}{}^2}{4\pi r_{\text{Moon}}{}^2} = \left(\dfrac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^2 = \left(\dfrac{(6.37\times10^6\text{ m})(100\text{ cm/m})}{1.74\times10^8\text{ cm}}\right)^2 = \boxed{13.4}$

(b) $\dfrac{V_{\text{Earth}}}{V_{\text{Moon}}} = \dfrac{4\pi r_{\text{Earth}}{}^3/3}{4\pi r_{\text{Moon}}{}^3/3} = \left(\dfrac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^3 = \left(\dfrac{(6.37\times10^6\text{ m})(100\text{ cm/m})}{1.74\times10^8\text{ cm}}\right)^3 = \boxed{49.1}$

1.39 To balance, $m_{\text{Fe}} = m_{\text{Al}}$ or $\rho_{\text{Fe}}V_{\text{Fe}} = \rho_{\text{Al}}V_{\text{Al}}$

$$\rho_{\text{Fe}}\left(\frac{4}{3}\right)\pi r_{\text{Fe}}^3 = \rho_{\text{Al}}\left(\frac{4}{3}\right)\pi r_{\text{Al}}^3$$

$$r_{\text{Al}} = r_{\text{Fe}}\left(\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}\right)^{1/3}$$

$$r_{\text{Al}} = (2.00\text{ cm})\left(\frac{7.86}{2.70}\right)^{1/3} = \boxed{2.86\text{ cm}}$$

1.40 The mass of each sphere is

$$m_{\text{A1}} = \rho_{\text{Al}}V_{\text{Al}} = \frac{4\pi\rho_{\text{Al}}r_{\text{Al}}{}^3}{3} \quad\text{and}\quad m_{\text{Fe}} = \rho_{\text{Fe}}V_{\text{Fe}} = \frac{4\pi\rho_{\text{Fe}}r_{\text{Fe}}{}^3}{3}$$

Setting these masses equal,

$$\frac{4\pi\rho_{\text{Fe}}r_{\text{Fe}}^3}{3} = \frac{4\pi\rho_{\text{Fe}}r_{\text{Fe}}^3}{3} \quad\text{and}\quad \boxed{r_{\text{Al}} = r_{\text{Fe}}\sqrt[3]{\rho_{\text{Fe}}/\rho_{\text{Al}}}}$$

1.41 The volume of the room is $4\times4\times3 = 48\text{ m}^3$, while

the volume of one ball is $\dfrac{4\pi}{3}\left(\dfrac{0.038\text{ m}}{2}\right)^3 = 2.87\times10^{-5}\text{ m}^3$.

Therefore, one can fit about $\dfrac{48}{2.87\times10^{-5}} \sim \boxed{10^6}$ ping-pong balls in the room.

As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\dfrac{\pi\sqrt{2}}{6} = 0.74$ so that at least 26% of the space will be empty. Therefore, the above estimate reduces to $1.67\times10^6\times0.740 \sim 10^6$.

Goal Solution

Estimate the number of Ping-Pong balls that would fit into an average-size room (without being crushed). In your solution state the quantities you measure or estimate and the values you take for them.

G: Since the volume of a typical room is much larger than a Ping-Pong ball, we should expect that a very large number of balls (maybe a million) could fit in a room.

O: Since we are only asked to find an estimate, we do not need to be too concerned about how the balls are arranged. Therefore, to find the number of balls we can simply divide the volume of an average-size room by the volume of an individual Ping-Pong ball.

A: A typical room (like a living room) might have dimensions 15 ft × 20 ft × 8 ft. Using the approximate conversion 1 ft = 30 cm, we find

$$V_{\text{room}} \approx 15\text{ ft} \times 20\text{ ft} \times 8\text{ ft} = 2400\text{ ft}^3\left(\frac{30\text{ cm}}{1\text{ ft}}\right)^3 = 7 \times 10^7\text{ cm}^3$$

A Ping-Pong ball has a diameter of about 3 cm, so we can estimate its volume as a cube:

$$V_{\text{ball}} \approx (3 \times 3 \times 3)\text{ cm}^3 = 30\text{ cm}^3$$

The number of Ping-Pong balls that can fill the room is

$$N \approx \frac{V_{\text{room}}}{V_{\text{ball}}} = \frac{7 \times 10^7\text{ cm}^3}{30\text{ cm}^3} = 2 \times 10^6\text{ balls} \sim 10^6\text{ balls}$$

L: So a typical room can hold about a million Ping-Pong balls. This problem gives us a sense of how big a million really is.

***1.42** It might be reasonable to guess that, on average, McDonalds sells a $3\text{ cm} \times 8\text{ cm} \times 10\text{ cm} = 240\text{ cm}^3$ medium-sized box of fries, and that it is packed 3/4 full with fries that have a cross section of $1/2\text{ cm} \times 1/2\text{ cm}$. Thus, the typical box of fries would contain fries that stretched a total of

$$L = \left(\frac{3}{4}\right)\left(\frac{V}{A}\right) = \left(\frac{3}{4}\right)\left(\frac{240\text{ cm}^3}{(0.5\text{ cm})^2}\right) = 720\text{ cm} = 7.2\text{ m}$$

250 million boxes would stretch a total distance of $(250 \times 10^6\text{ box})(7.2\text{ m/box}) = 1.8 \times 10^9\text{ m}$. But we require an order of magnitude, so our answer is $\boxed{10^9\text{ m} = 1\text{ million kilometers}}$.

***1.43** A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make $(50\,000\text{ mi})(5280\text{ ft/mi})(1\text{ rev}/8\text{ ft}) = 3 \times 10^7\text{ rev} \sim \boxed{10^7\text{ rev}}$

1.44 A typical raindrop is spherical and might have a radius of about 0.1 inch. Its volume is then approximately 4×10^{-3} in^3. Since 1 acre = 43,560 ft^2, the volume of water required to cover it to a depth of 1 inch is

$$(1 \text{ acre})(1 \text{ inch}) = (1 \text{ acre} \cdot \text{in})\left(\frac{43{,}560 \text{ ft}^2}{1 \text{ acre}}\right)\left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right) \approx 6.3 \times 10^6 \text{ in}^3.$$

The number of raindrops required is

$$n = \frac{\text{volume of water required}}{\text{volume of a single drop}} \approx \frac{6.3 \times 10^6 \text{ in}^3}{4 \times 10^{-3} \text{ in}^3} = 1.6 \times 10^9 \sim \boxed{10^9}$$

***1.45** In order to reasonably carry on photosynthesis, we might expect a blade of grass to require at least $1/16$ in^2 = 43×10^{-5} ft^2. Since 1 acre = 43,560 ft^2, the number of blades of grass to be expected on a quarter-acre plot of land is about

$$n = \frac{\text{total area}}{\text{area per blade}} = \frac{(0.25 \text{ acre})(43{,}560 \text{ ft}^2/\text{acre})}{43 \times 10^{-5} \text{ ft}^2/\text{blade}}$$

$$= 2.5 \times 10^7 \text{ blades} \sim \boxed{10^7 \text{ blades}}$$

1.46 Since you have only 16 hours (57,600 s) available per day, you can count only \$57,600 per day. Thus, the time required to count \$1 billion dollars is

$$t = \frac{10^9 \text{ dollars}}{5.76 \times 10^4 \text{ dollars/day}}\left(\frac{1 \text{ year}}{365 \text{ days}}\right) = 47.6 \text{ years}$$

Since you are at least 18 years old, you would be beyond age 65 before you finished counting the money. It would provide a nice retirement, but a very boring life until then.

$\boxed{\text{We would not advise it.}}$

1.47 Assume the tub measure 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V = (0.5)(1.3 \text{ m})(0.5 \text{ m})(0.3 \text{ m}) = 0.10 \text{ m}^3$$

The mass of this volume of water is

$$m_{\text{water}} = \rho_{\text{water}} V = (1000 \text{ kg/m}^3)(0.10 \text{ m}^3) = 100 \text{ kg} \quad \boxed{\sim 10^2 \text{ kg}}$$

Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}} = \rho_{\text{copper}} V = (8930 \text{ kg/m}^3)(0.10 \text{ m}^3) = 893 \text{ kg} \quad \boxed{\sim 10^3 \text{ kg}}$$

***1.48** The typical person probably drinks 2 to 3 soft drinks daily. Perhaps half of these were in aluminum cans. Thus, we will estimate 1 aluminum can disposal per person per day. In the U.S. there are ~250 million people, and 365 days in a year, so $(250 \times 10^6 \text{ cans/day})(365 \text{ days/year}) \approx 10^{10}$ cans are thrown away or recycled each year. Guessing that each can weighs around 1/10 of an ounce, we estimate this represents

$$(10^{10} \text{ cans})(0.1 \text{ oz/can})(1 \text{ lb/16 oz})(1 \text{ ton/2000 lb}) \approx 3.1 \times 10^5 \text{ tons/year.}$$

$\boxed{\sim 10^5 \text{ tons}}$

1.49 Assume: Total population = 10^7; one out of every 100 people has a piano; one tuner can serve about 1,000 pianos (about 4 per day for 250 weekdays, assuming each piano is tuned once per year). Therefore,

$$\#\text{ tuners} \sim \left(\frac{1 \text{ tuner}}{1000 \text{ pianos}}\right)\left(\frac{1 \text{ piano}}{100 \text{ people}}\right)(10^7 \text{ people}) = \boxed{100}$$

1.50 (a) $\boxed{2}$ (b) $\boxed{4}$ (c) $\boxed{3}$ (d) $\boxed{2}$

1.51 (a) $\pi r^2 = \pi(10.5 \text{ m} \pm 0.2 \text{ m})^2$

$$= \pi\left[(10.5 \text{ m})^2 \pm 2(10.5 \text{ m})(0.2 \text{ m}) + (0.2 \text{ m})^2\right]$$

$$= \boxed{346 \text{ m}^2 \pm 13 \text{ m}^2}$$

(b) $2\pi r = 2\pi(10.5 \text{ m} \pm 0.2 \text{ m}) = \boxed{66.0 \text{ m} \pm 1.3 \text{ m}}$

1.52 (a)

$$\begin{array}{r} 756.?? \\ 37.2? \\ 0.83 \\ \underline{+\ 2.5?} \\ 796.53 \end{array} = \boxed{797}$$

(b) $0.0032 \text{ (2 s.f.)} \times 356.3 \text{ (4 s.f.)} = 1.14016 = \text{(2 s.f.)} \ \boxed{1.1}$

(c) $5.620 \text{ (4 s.f.)} \times \pi \text{ (> 4 s.f.)} = 17.656 = \text{(4 s.f.)} \ \boxed{17.66}$

1.53 $r = (6.50 \pm 0.20)\text{ cm} = (6.50 \pm 0.20) \times 10^{-2}\text{ m}$

$m = (1.85 \pm 0.02)\text{ kg}$

$$\rho = \frac{m}{\left(\frac{4}{3}\right)\pi r^3}$$ also,

$$\frac{\delta\rho}{\rho} = \frac{\delta m}{m} + \frac{3\delta r}{r}$$

In other words, the percentages of uncertainty are cumulative.

Therefore, $\frac{\delta\rho}{\rho} = \frac{0.02}{1.85} + \frac{3(0.20)}{6.50} = 0.103$

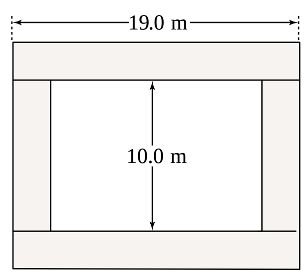
$$\rho = \frac{1.85}{\left(\frac{4}{3}\right)\pi(6.5 \times 10^{-2}\text{ m})^3} = \boxed{1.61 \times 10^3\text{ kg/m}^3}$$

and $\rho \pm \delta\rho = \boxed{(1.61 \pm 0.17) \times 10^3\text{ kg/m}^3}$

1.54 (a) $\boxed{3}$ (b) $\boxed{4}$ (c) $\boxed{3}$ (d) $\boxed{2}$

1.55 The distance around is 38.44 m + 19.5 m + 38.44 m + 19.5 m = 115.88 m, but this answer must be rounded to 115.9 m because the distance 19.5 m carries information to only one place past the decimal. $\boxed{115.9\text{ m}}$

1.56 $V = 2V_1 + 2V_2 = 2(V_1 + V_2)$



$V_1 = (17.0\text{ m} + 1.0\text{ m} + 1.0\text{ m})(1.0\text{ m})(0.09\text{ m}) = 1.70\text{ m}^3$

$V_2 = (10.0\text{ m})(1.0\text{ m})(0.090\text{ m}) = 0.900\text{ m}^3$

$V = 2(1.70\text{ m}^3) + 2(0.900\text{ m}^3) = \boxed{5.2\text{ m}^3}$

$$\left.\begin{aligned} \frac{\delta l_1}{l_1} &= \frac{0.12\text{ m}}{19.0\text{ m}} = 0.0063 \\ \frac{\delta w_1}{w_1} &= \frac{0.01\text{ m}}{1.0\text{ m}} = 0.010 \\ \frac{\delta t_1}{t_1} &= \frac{0.1\text{ cm}}{9.0\text{ cm}} = 0.011 \end{aligned}\right\} \frac{\delta V}{V} = 0.006 + 0.010 + 0.011 = 0.027 = \boxed{2.7\%}$$

***1.57** It is desired to find the distance x such that $\dfrac{x}{100\text{ m}} = \dfrac{1000\text{ m}}{x}$ (i.e., such that x is the same multiple of 100 m as the multiple that 1000 m is of x) .

Thus, it is seen that $x^2 = (100\text{ m})(1000\text{ m}) = 1.00 \times 10^5\text{ m}^2$, and therefore $x = \sqrt{1.00 \times 10^5\text{ m}^2} = \boxed{316\text{ m}}$.

1.58 The volume of oil equals $V = \dfrac{9.00 \times 10^{-7}\text{ kg}}{918\text{ kg/m}^3} = 9.80 \times 10^{-10}\text{ m}^3$. If the diameter of a molecule is d, then that same volume must equal $d(\pi r^2)$ = (thickness of slick)(area of oil slick) where $r = 0.418$ m. Thus,

$$d = \frac{9.80 \times 10^{-10}\text{ m}^3}{\pi\,(0.418\text{ m})^2} = \boxed{1.79 \times 10^{-9}\text{ m}}$$

1.59 $A_{\text{total}} = (N)(A_{\text{drop}}) = \left(\dfrac{V_{\text{total}}}{V_{\text{drop}}}\right)(A_{\text{drop}}) = \left(\dfrac{V_{\text{total}}}{4\pi r^3/3}\right)(4\pi r^2)$

$$= \left(\frac{3V_{\text{total}}}{r}\right) = 3\left(\frac{30.0 \times 10^{-6}\text{ m}^3}{2.00 \times 10^{-5}\text{ m}}\right) = \boxed{4.50\text{ m}^2}$$

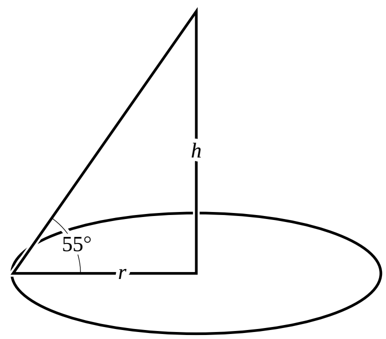
1.60

α' (deg)	α (rad)	$\tan(\alpha)$	$\sin(\alpha)$	difference	
15.0	0.262	0.268	0.259	3.47%	
20.0	0.349	0.364	0.342	6.43%	
25.0	0.436	0.466	0.423	10.2%	
24.0	0.419	0.445	0.407	9.34%	
24.4	0.426	0.454	0.413	9.81%	
24.5	0.428	0.456	0.415	9.87%	
24.6	0.429	0.458	0.416	9.98%	$\boxed{24.6°}$
24.7	0.431	0.460	0.418	10.1%	

1.61 $2\pi r = 15.0\text{ m}$ $r = 2.39\text{ m}$

$$\frac{h}{r} = \tan 55.0°$$

$$h = (2.39\text{ m})\tan(55.0°) = \boxed{3.41\text{ m}}$$


h
55°
r

***1.62** (a) $[V] = L^3$, $[A] = L^2$, $[h] = L$

$[V] = [A][h]$

$L^3 = L^3 L = L^3$. Thus, the equation is dimensionally correct.

(b) $V_{\text{cylinder}} = \pi R^2 h = (\pi R^2)h = Ah$, where $\boxed{A = \pi R^2}$

$V_{\text{rectangular object}} = \ell w h = (\ell w)h = Ah$, where $\boxed{A = \ell w}$

1.63 The actual number of seconds in a year is

$$(86{,}400 \text{ s / day})(365.25 \text{ day / yr}) = 31{,}557{,}600 \text{ s / yr}$$

The percentage error in the approximation is thus

$$\frac{\left|(\pi \times 10^7 \text{ s / yr}) - (31{,}557{,}600 \text{ s / yr})\right|}{31{,}557{,}600 \text{ s / yr}} \times 100\% = \boxed{0.449\%}$$

***1.64** From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$. Thus, since the atoms are separated by a distance

$L = 0.200$ nm, the diagonal planes are separated $\frac{1}{2}\sqrt{L^2 + L^2} = \boxed{0.141 \text{ nm}}$

***1.65** (a) The speed of flow may be found from

$$v = \frac{(\text{Vol rate of flow})}{(\text{Area: } \pi D^2/4)} = \frac{16.5 \text{ cm}^3/\text{s}}{\pi (6.30 \text{ cm})^2/4} = \boxed{0.529 \text{ cm/s}}$$

(b) Likewise, at a 1.35 cm diameter,

$$v = \frac{16.5 \text{ cm}^3/\text{s}}{\pi (1.35 \text{ cm})^2/4} = \boxed{11.5 \text{ cm/s}}$$

***1.66** $$t = \frac{V}{A} = \frac{V}{\pi D^2/4} = \frac{4(12.0 \text{ cm}^3)}{\pi (23.0 \text{ cm})^2} = 0.0289 \text{ cm}\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)\left(\frac{10^6 \ \mu\text{m}}{1 \text{ m}}\right) = \boxed{289 \ \mu\text{m}}$$

1.67 $$V_{20 \text{ mpg}} = \frac{(10^8 \text{ cars})(10^4 \text{ mi/yr})}{20 \text{ mi/gal}} = 5.0 \times 10^{10} \text{ gal/yr}$$

$$V_{25 \text{ mpg}} = \frac{(10^8 \text{ cars})(10^4 \text{ mi/yr})}{25 \text{ mi/gal}} = 4.0 \times 10^{10} \text{ gal/yr}$$

Fuel saved $= V_{25 \text{ mpg}} - V_{20 \text{ mpg}} = \boxed{1.0 \times 10^{10} \text{ gal/yr}}$

1.68 (a) 1 cubic meter of water has a mass

$$m = \rho V = (1.00 \times 10^{-3}\ \text{kg/cm}^3)(1.00\ \text{m}^3)(10^2\ \text{cm/m})^3 = \boxed{1000\ \text{kg}}$$

(b) As a rough calculation, we treat each item as if it were 100% water.

cell: $m = \rho V = \rho$ **Error!** $\pi R^3) = \rho$ **Error!** $\pi D^3)$

$$= (1000\ \text{kg/m}^3)\left(\frac{1}{6}\pi\right)(1.0 \times 10^{-6}\ \text{m})^3 = \boxed{5.2 \times 10^{-16}\ \text{kg}}$$

kidney:
$m = \rho V = \rho$ **Error!** $\pi R^3) = (1.00 \times 10^{-3}\ \text{kg/cm}^3)$**Error!** ^{3} = **Error!**

fly: $m = \rho\left(\frac{\pi}{4}D^2h\right)$

$$= (1 \times 10^{-3}\ \text{kg/cm}^3)\left(\frac{\pi}{4}\right)(2.0\ \text{mm})^2(4.0\ \text{mm})(10^{-1}\ \text{cm/mm})^3$$

$$= \boxed{1.3 \times 10^{-5}\ \text{kg}}$$

1.69 The volume of the galaxy is

$$\pi r^2 t = \pi(10^{21}\ \text{m})^2\ 10^{19}\ \text{m} \sim 10^{61}\ \text{m}^3$$

If the distance between stars is 4×10^{16} m, then there is one star in a volume on the order of $(4 \times 10^{16}\ \text{m})^3 \sim 10^{50}\ \text{m}^3$.

The number of stars is about $\dfrac{10^{61}\ \text{m}^3}{10^{50}\ \text{m}^3/\text{star}} \sim \boxed{10^{11}\ \text{stars}}$

1.70 The density of each material is $\rho = \dfrac{m}{V} = \dfrac{m}{\pi r^2 h} = \dfrac{4m}{\pi D^2 h}$

Al: $\rho = \dfrac{4(51.5\text{ g})}{\pi(2.52\text{ cm})^2(3.75\text{ cm})} = \boxed{2.75\ \dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(2.70\ \dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{2\%}$ smaller.

Cu: $\rho = \dfrac{4(56.3\text{ g})}{\pi(1.23\text{ cm})^2(5.06\text{ cm})} = \boxed{9.36\ \dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(8.92\ \dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{5\%}$ smaller.

Brass: $\rho = \dfrac{4(94.4\text{ g})}{\pi(1.54\text{ cm})^2(5.69\text{ cm})} = \boxed{8.91\ \dfrac{\text{g}}{\text{cm}^3}}$

Sn: $\rho = \dfrac{4(69.1\text{ g})}{\pi(1.75\text{ cm})^2(3.74\text{ cm})} = \boxed{7.68\ \dfrac{\text{g}}{\text{cm}^3}}$

Fe: $\rho = \dfrac{4(216.1\text{ g})}{\pi(1.89\text{ cm})^2(9.77\text{ cm})} = \boxed{7.88\ \dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(7.86\ \dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{0.3\%}$ smaller.

Chapter 2 Solutions

***2.1** (a) $\bar{v} = \boxed{2.30\ \text{m/s}}$

(b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\ \text{m} - 9.20\ \text{m}}{3.00\ \text{s}} = \boxed{16.1\ \text{m/s}}$

(c) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\ \text{m} - 0\ \text{m}}{5.00\ \text{s}} = \boxed{11.5\ \text{m/s}}$

2.2 (a) Displacement $= (8.50 \times 10^4\ \text{m/h})\left(\dfrac{35.0}{60.0}\ \text{h}\right) + 130 \times 10^3\ \text{m}$

$x = (49.6 + 130) \times 10^3\ \text{m} = \boxed{180\ \text{km}}$

(b) Average velocity $= \dfrac{\textit{displacement}}{\textit{time}} = \dfrac{180\ \text{km}}{\left[\dfrac{(35.0 + 15.0)}{60.0} + 2.00\right]\ \text{h}} = \boxed{63.4\ \text{km/h}}$

2.3 (a) $v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{10\ \text{m}}{2\ \text{s}} = \boxed{5\ \text{m/s}}$

(b) $v_{av} = \dfrac{5\ \text{m}}{4\ \text{s}} = \boxed{1.2\ \text{m/s}}$

(c) $v_{av} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5\ \text{m} - 10\ \text{m}}{4\ \text{s} - 2\ \text{s}} = \boxed{-2.5\ \text{m/s}}$

(d) $v_{av} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{-5\ \text{m} - 5\ \text{m}}{7\ \text{s} - 4\ \text{s}} = \boxed{-3.3\ \text{m/s}}$

(e) $v_{av} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{0 - 0}{8 - 0} = \boxed{0\ \text{m/s}}$

2.4 $x = 10t^2$

For $t(\text{s}) = 2.0 \quad 2.1 \quad 3.0$

$x(\text{m}) = 40 \quad 44.1 \quad 90$

(a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{50\ \text{m}}{1.0\ \text{s}} = \boxed{50.0\ \text{m/s}}$

(b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{4.1\ \text{m}}{0.1\ \text{s}} = \boxed{41.0\ \text{m/s}}$

2.5 (a) Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in $5.00\text{ m/s} = \dfrac{d}{t_1}$. Let t_2 represent the longer time for the return trip in $-3.00\text{ m/s} = -\dfrac{d}{t_2}$. Then the times are $t_1 = \dfrac{d}{(5.00\text{ m/s})}$ and $t_2 = \dfrac{d}{(3.00\text{ m/s})}$. The average speed is:

$$\bar{v} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d+d}{\dfrac{d}{(5.00\text{ m/s})}+\dfrac{d}{(3.00\text{ m/s})}} = \frac{2d}{\dfrac{(8.00\text{ m/s})d}{(15.0\text{ m}^2/\text{s}^2)}}$$

$$\bar{v} = \frac{2(15.0\text{ m}^2/\text{s}^2)}{8.00\text{ m/s}} = \boxed{3.75\text{ m/s}}$$

(b) She starts and finishes at the same point A.

With total displacement = 0, average velocity = $\boxed{0}$

2.6 (a) $\bar{v} = \dfrac{\text{Total distance}}{\text{Total time}}$

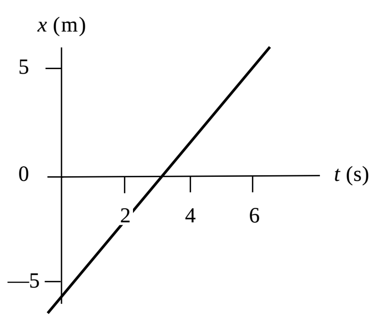
Let d be the distance from A to B.

Then the time required is $\dfrac{d}{v_1} + \dfrac{d}{v_2}$.

And the average speed is $\bar{v} = \dfrac{2d}{\dfrac{d}{v_1}+\dfrac{d}{v_2}} = \boxed{\dfrac{2v_1v_2}{v_1+v_2}}$

(b) With total displacement zero, her average velocity is $\boxed{0}$.

2.7 (a)



(b) $v = \text{slope} = \dfrac{5.00\text{ m} - (-3.00\text{ m})}{(6.00\text{ s} - 1.00\text{ s})} = \dfrac{8.00\text{ m}}{5.00\text{ s}} = \boxed{1.60\text{ m/s}}$

2.8 (a) At any time, t, the displacement is given by $x = (3.00\ \text{m/s}^2)t^2$.

Thus, at $t_i = 3.00$ s: $x_i = (3.00\ \text{m/s}^2)(3.00\ \text{s})^2 = \boxed{27.0\ \text{m}}$

(b) At $t_f = 3.00\ \text{s} + \Delta t : x_f = (3.00\ \text{m/s}^2)(3.00\ \text{s} + \Delta t)^2$, or

$$x_f = \boxed{27.0\ \text{m} + (18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2}$$

(c) The instantaneous velocity at $t = 3.00$ s is:

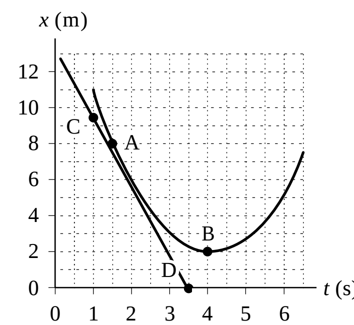
$$v = \lim_{\Delta t \to 0}\left(\frac{x_f - x_i}{\Delta t}\right) = \lim_{\Delta t \to 0}\ [(18.0\ \text{m/s}) + (3.00\ \text{m/s}^2)\Delta t], \text{ or}$$

$$v = \boxed{18.0\ \text{m/s}}$$

2.9 (a) at $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

at $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\ \text{m}}{(4 - 1.5)\ \text{s}} = -\frac{6.0\ \text{m}}{2.5\ \text{s}} = \boxed{-2.4\ \text{m/s}}$$



(b) The slope of the tangent line is found from points C and D.

$(t_C = 1.0\ \text{s}, x_C = 9.5\ \text{m})$ and $(t_D = 3.5\ \text{s}, x_D = 0)$,

$$v \cong \boxed{-3.8\ \text{m/s}}$$

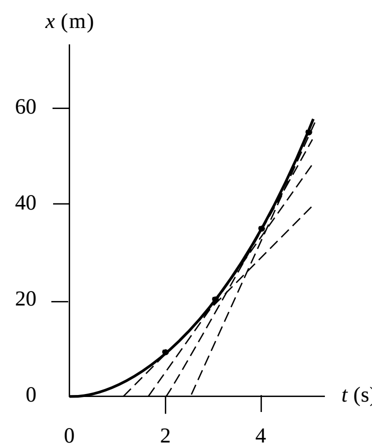
(c) The velocity is zero when x is a minimum. This is at $t \approx \boxed{4\ \text{s}}$.

2.10 (b) At $t = 5.0$ s, the slope is $v \cong \dfrac{58\ \text{m}}{2.5\ \text{s}} \cong \boxed{23\ \text{m/s}}$

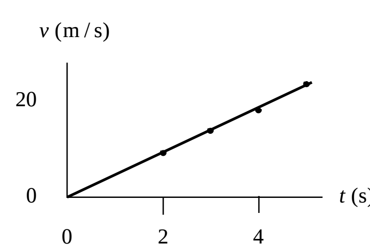
At $t = 4.0$ s, the slope is $v \cong \dfrac{54\ \text{m}}{3\ \text{s}} \cong \boxed{18\ \text{m/s}}$

At $t = 3.0$ s, the slope is $v \cong \dfrac{49\ \text{m}}{3.4\ \text{s}} \cong \boxed{14\ \text{m/s}}$

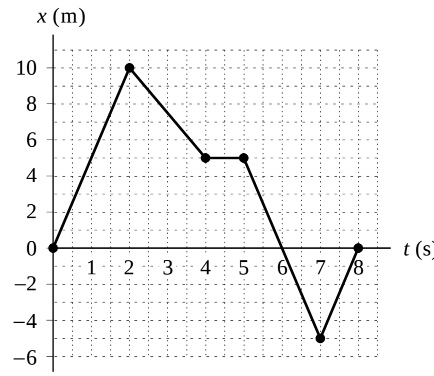
At $t = 2.0$ s, the slope is $v \cong \dfrac{36\ \text{m}}{4.0\ \text{s}} \cong \boxed{9.0\ \text{m/s}}$



(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \cong \dfrac{23\ \text{m/s}}{5.0\ \text{s}} \cong \boxed{4.6\ \text{m/s}^2}$



(d) Initial velocity of the car was $\boxed{\text{zero}}$.



2.11 (a) $v = \dfrac{(5-0)\text{ m}}{(1-0)\text{ s}} = \boxed{5\text{ m/s}}$

(b) $v = \dfrac{(5-10)\text{ m}}{(4-2)\text{ s}} = \boxed{-2.5\text{ m/s}}$

(c) $v = \dfrac{(5\text{ m}-5\text{ m})}{(5\text{ s}-4\text{ s})} = \boxed{0}$

(d) $v = \dfrac{0-(-5\text{ m})}{(8\text{ s}-7\text{ s})} = \boxed{+5\text{ m/s}}$

2.12 $\bar{a} = \dfrac{v_f - v_i}{t_f - t_i} = \dfrac{0-60.0\text{ m/s}}{15.0\text{ s}-0} = \boxed{-4.00\text{ m/s}^2}$

The negative sign in the result shows that the acceleration is in the negative x direction.

***2.13** Choose the positive direction to be the outward perpendicular to the wall.

$$v = v_i + at$$

$$a = \frac{\Delta v}{\Delta t} = \frac{22.0\text{ m/s}-(-25.0\text{ m/s})}{3.50\times10^{-3}\text{ s}} = \boxed{1.34\times10^{4}\text{ m/s}^2}$$

2.14 (a) Acceleration is constant over the first ten seconds, so at the end

$$v = v_i + at = 0 + (2.00\text{ m/s}^2)(10.0\text{ s}) = \boxed{20.0\text{ m/s}}$$

Then $a = 0$ so v is constant from $t = 10.0$ s to $t = 15.0$ s. And over the last five seconds the velocity changes to

$$v = v_i + at = 20.0\text{ m/s} - (3.00\text{ m/s}^2)(5.00\text{ s}) = \boxed{5.00\text{ m/s}}$$

(b) In the first ten seconds

$$x = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(2.00\text{ m/s}^2)(10.0\text{ s})^2 = 100\text{ m}$$

Over the next five seconds the position changes to

$$x = x_i + v_i t + \frac{1}{2}at^2 = 100\text{ m} + 20.0\text{ m/s}\,(5.00\text{ s}) + 0 = 200\text{ m}$$

And at $t = 20.0$ s

$$x = x_i + v_i t + \frac{1}{2}at^2 = 200\text{ m} + 20.0\text{ m/s}\,(5.00\text{ s}) + \frac{1}{2}(-3.00\text{ m/s}^2)(5.00\text{ s})^2 = \boxed{262\text{ m}}$$

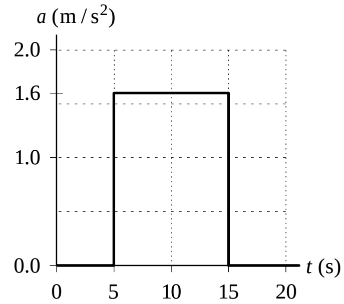
***2.15** (a) Acceleration is the slope of the graph of v vs t.

For $0 < t < 5.00$ s, $a = 0$

For 15.0 s $< t < 20.0$ s, $a = 0$

For 5.0 s $< t < 15.0$ s, $a = \dfrac{v_f - v_i}{t_f - t_i}$

$$a = \frac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60 \text{ m/s}^2$$



We can plot $a(t)$ as shown.

(b) $a = \dfrac{v_f - v_i}{t_f - t_i}$

(i) For 5.00 s $< t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s

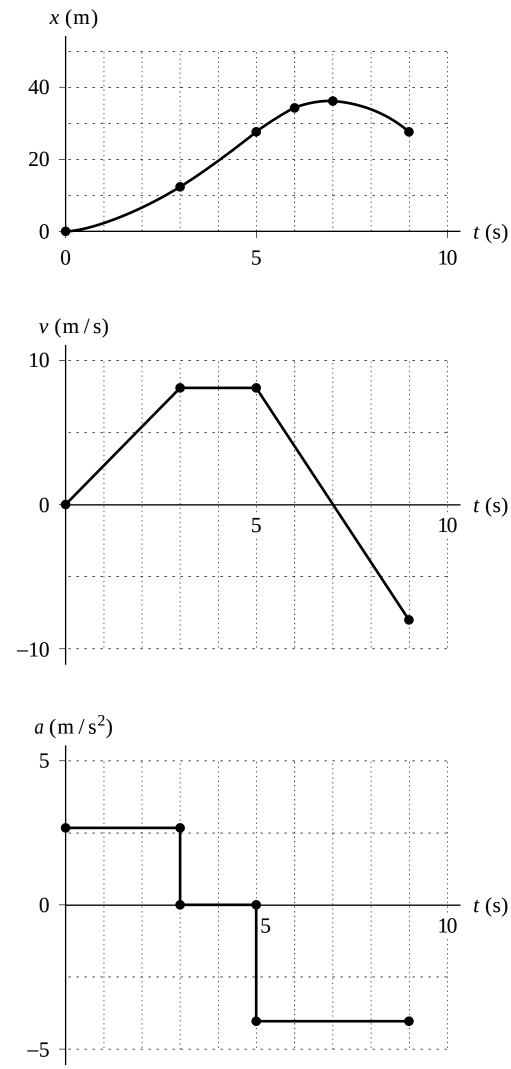
$t_f = 15.0$ s, $v_f = 8.00$ m/s;

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60 \text{ m/s}^2}$$

(ii) $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, $v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800 \text{ m/s}^2}$$

2.16 (a) See the Graphs at the right.



Choose $x = 0$ at $t = 0$

At $t = 3$ s, $x = \frac{1}{2}(8\text{ m/s})(3\text{ s}) = 12$ m

At $t = 5$ s, $x = 12\text{ m} + (8\text{ m/s})(2\text{ s}) = 28$ m

At $t = 7$ s, $x = 28\text{ m} + \frac{1}{2}(8\text{ m/s})(2\text{ s}) = 36$ m

(b) For $0 < t < 3$ s, $a = (8\text{ m/s})/3\text{ s} = 2.67\text{ m/s}^2$

For $3 < t < 5$ s, $a = 0$

(c) For $5\text{ s} < t < 9$ s, $a = -(16\text{ m/s})/4\text{ s} = \boxed{-4\text{ m/s}^2}$

(d) At $t = 6$ s, $x = 28\text{ m} + (6\text{ m/s})(1\text{ s}) = \boxed{34\text{ m}}$

(e) At $t = 9$ s, $x = 36\text{ m} + \frac{1}{2}(-8\text{ m/s})\,2\text{ s} = \boxed{28\text{ m}}$

2.17 $x = 2.00 + 3.00t - t^2$, $v = \frac{dx}{dt} = 3.00 - 2.00t$, $a = \frac{dv}{dt} = -2.00$

At $t = 3.00$ s:

(a) $x = (2.00 + 9.00 - 9.00)\text{ m} = \boxed{2.00\text{ m}}$

(b) $v = (3.00 - 6.00)\text{ m/s} = \boxed{-3.00\text{ m/s}}$

(c) $a = \boxed{-2.00\text{ m/s}^2}$

2.18 (a) At $t = 2.00$ s, $x = [3.00(2.00)^2 - 2.00(2.00) + 3.00]\text{ m} = 11.0$ m

At $t = 3.00$ s, $x = [3.00(9.00)^2 - 2.00(3.00) + 3.00]\text{ m} = 24.0$ m

so $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{24.0\text{ m} - 11.0\text{ m}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{13.0\text{ m/s}}$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}(3.00t^2 - 2.00t + 3.00) = (6.00t - 2.00) \text{ m/s}$$

At $t = 2.00$ s, $v = [6.00(2.00) - 2.00]$ m/s = $\boxed{10.0 \text{ m/s}}$

At $t = 3.00$ s, $v = [6.00(3.00) - 2.00]$ m/s = $\boxed{16.0 \text{ m/s}}$

(c) $$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{16.0 \text{ m/s} - 10.0 \text{ m/s}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{6.00 \text{ m/s}^2}$$

(d) At all times $a = \frac{d}{dt}(6.00 - 2.00) = \boxed{6.00 \text{ m/s}^2}$

(This includes both t= 2.00 s and t= 3.00 s).

2.19 (a) $$a = \frac{\Delta v}{\Delta t} = \frac{8.00 \text{ m/s}}{6.00 \text{ s}} = \boxed{\frac{4}{3} \text{ m/s}^2}$$

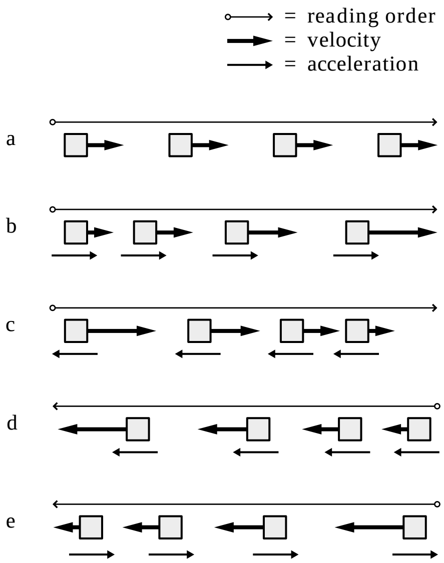
(b) Maximum positive acceleration is at t = 3 s, and is approximately $\boxed{2 \text{ m/s}^2}$

(c) a = 0, at $\boxed{t = 6 \text{ s}}$, and also for $\boxed{t > 10 \text{ s}}$

(d) Maximum negative acceleration is at t = 8 s, and is approximately

$\boxed{-1.5 \text{ m/s}^2}$

***2.20**



f One way of phrasing the answer: The spacing of the successive positions would change with less regularity. Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the acceleration vectors would vary in magnitude and direction.

***2.21** From $v_f^2 = v_i^2 + 2ax$, we have $(10.97 \times 10^3 \text{ m/s})^2 = 0 + 2a(220 \text{ m})$, so that

$\boxed{a = 2.74 \times 10^5 \text{ m/s}^2}$ which is $\boxed{2.79 \times 10^4 \text{ times } g}$

2.22 (a) Assuming a constant acceleration:

$$a = \frac{v_f - v_i}{t} = \frac{42.0 \text{ m/s}}{8.00 \text{ s}} = \boxed{5.25 \text{ m/s}^2}$$

(b) Taking the origin at the original position of the car,

$$x = \frac{1}{2}(v_i + v_f)\, t = \frac{1}{2}(42.0 \text{ m/s})(8.00 \text{ s}) = \boxed{168 \text{ m}}$$

(c) From $v_f = v_i + at$, the velocity 10.0 s after the car starts from rest is:

$$v_f = 0 + (5.25 \text{ m/s}^2)(10.0 \text{ s}) = \boxed{52.5 \text{ m/s}}$$

***2.23** (a) $x - x_i = \frac{1}{2}(v_i + v)\,t$ becomes $40\text{ m} = \frac{1}{2}(v_i + 2.80\text{ m/s})(8.50\text{ s})$

which yields $v_i = \boxed{6.61\text{ m/s}}$

(b) $a = \dfrac{v - v_i}{t} = \dfrac{2.80\text{ m/s} - 6.61\text{ m/s}}{8.50\text{ s}} = \boxed{-0.448\text{ m/s}^2}$

2.24 Suppose the unknown acceleration is constant as a car moving at $v_i = 35.0$ mi/h comes to a $v = 0$ stop in $x - x_i = 40.0$ ft. We find its acceleration from

$$v^2 = v_i^2 + 2a(x - x_i)$$

$$a = \frac{(v^2 - v_i^2)}{2(x - x_i)}$$

$$= \frac{0 - (35.0\text{ mi/h})^2}{2(40.0\text{ ft})}\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)^2\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2 = -32.9\text{ ft/s}^2$$

Now consider a car moving at $v_i = 70.0$ mi/h and stopping to $v = 0$ with $a = -32.9$ ft/s^2. From the same equation its stopping distance is

$$x - x_i = \frac{v^2 - v_i^2}{2a}$$

$$= \frac{0 - (70.0\text{ mi/h})^2}{2(-32.9\text{ ft/s}^2)}\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)^2\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2$$

$$= \boxed{160\text{ ft}}$$

2.25 Given $v_i = 12.0$ cm/s when $x_i = 3.00$ cm ($t = 0$), and at $t = 2.00$ s, $x = -5.00$ cm

$$\Delta x = v_i t + \left(\frac{1}{2}\right) at^2;$$

$$\Rightarrow \quad x - x_i = v_i t + \left(\frac{1}{2}\right) at^2;$$

$$-5.00 - 3.00 = 12.0(2.00) + \left(\frac{1}{2}\right) a\,(2.00)^2;$$

$$\Rightarrow \quad -8.00 = 24.0 + 2a$$

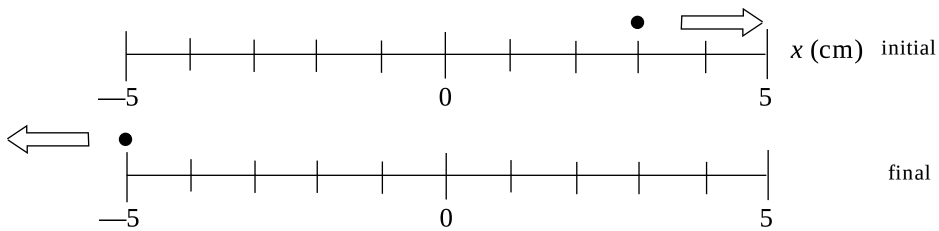
$$a = -\frac{32.0}{2} = \boxed{-16.0\text{ cm/s}^2}$$

Goal Solution

A body moving with uniform acceleration has a velocity of 12.0 cm / s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00s later is –5.00 cm, what is the magnitude of its acceleration?

G: Since the object must slow down as it moves to the right and then speeds up to the left, the acceleration must be negative and should have units of cm / s^2.

O: First we should sketch the problem to see what is happening:



Here we can see that the object travels along the x-axis, first to the right, slowing down, and then speeding up as it travels to the left in the negative x direction. We can show the position as a function of time with the notation: $x(t)$

$x(0) = 3.00$ cm, $x(2.00) = –5.00$ cm, and $v(0) = 12.0$ cm / s

A: Use the kinematic equation $x - x_i = v_i t + \frac{1}{2} at^2$, and solve for a.

$$a = \frac{2(x - x_i - v_i t)}{t^2}$$

$$a = \frac{2[-5.00 \text{ cm} - 3.00 \text{ cm} - (12.0 \text{ cm/s})(2.00 \text{ s})]}{(2.00 \text{ s})^2}$$

$$a = -16.0 \text{ cm/s}^2$$

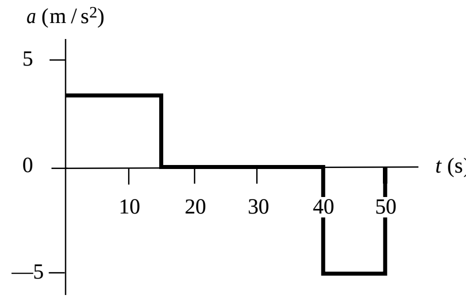
L: The acceleration is negative as expected and it has the correct units of cm / s^2. It also makes sense that the magnitude of the acceleration must be greater than 12 cm / s^2 since this is the acceleration that would cause the object to stop after 1 second and just return the object to its starting point after 2 seconds.

2.26 (a) Total displacement = area under the (v, t) curve from $t = 0$ to 50 s.

$$\Delta x = \frac{1}{2}(50\text{ m/s})(15\text{ s}) + (50\text{ m/s})(40 - 15)\text{s} + \frac{1}{2}(50\text{ m/s})(10\text{ s}) = \boxed{1875\text{ m}}$$

(b) From $t = 10$ s to $t = 40$ s, displacement (area under the curve) is

$$\Delta x = \frac{1}{2}(50\text{ m/s} + 33\text{ m/s})(5\text{ s}) + (50\text{ m/s})(25\text{ s}) = \boxed{1457\text{ m}}$$



(c) $0 \le t \le 15$ s: $a_1 = \dfrac{\Delta v}{\Delta t} = \dfrac{(50 - 0)\text{ m/s}}{15\text{ s} - 0} = \boxed{3.3\text{ m/s}^2}$

$15\text{ s} < t < 40\text{ s}$: $\boxed{a_2 = 0}$

$40\text{ s} \le t \le 50\text{ s}$: $a_3 = \dfrac{\Delta v}{\Delta t} = \dfrac{(0 - 50)\text{ m/s}}{50\text{ s} - 40\text{ s}} = \boxed{-5.0\text{ m/s}^2}$

(d) (i) $x_1 = 0 + \dfrac{1}{2}a_1t^2 = \dfrac{1}{2}(3.3\text{ m/s}^2)\,t^2$, or $\boxed{x_1 = (1.67\text{ m/s}^2)t^2}$

(ii) $x_2 = \dfrac{1}{2}(15\text{ s})\,[50\text{ m/s} - 0] + (50\text{ m/s})(t - 15\text{ s})$, or $\boxed{x_2 = (50\text{ m/s})t - 375\text{ m}}$

(iii) For $40\text{ s} \le t \le 50\text{ s}$, $x_3 = \begin{pmatrix}\text{area under } v \text{ vs } t \\ \text{from } t = 0 \text{ to } 40\text{ s}\end{pmatrix} + \dfrac{1}{2}a_3(t - 40\text{ s})^2 + (50\text{ m/s})(t - 40\text{ s})$

or $x_3 = 375\text{ m} + 1250\text{ m} + \dfrac{1}{2}(-5.0\text{ m/s}^2)(t - 40\text{ s})^2 + (50\text{ m/s})(t - 40\text{ s})$ which reduces to

$$\boxed{x_3 = (250\text{ m/s})t - (2.5\text{ m/s}^2)t^2 - 4375\text{ m}}$$

(e) $\bar{v} = \dfrac{\text{total displacement}}{\text{total elapsed time}} = \dfrac{1875\text{ m}}{50\text{ s}} = \boxed{37.5\text{ m/s}}$

***2.27** (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form $x = x_i + v_it + \dfrac{1}{2}at^2$ to recognize that:

$$x_i = 2.00\text{ m}, \quad v_i = 3.00\text{ m/s}, \quad \text{and} \quad a = -8.00\text{ m/s}^2$$

The velocity equation, $v = v_i + at$, is then $v = 3.00\text{ m/s} - (8.00\text{ m/s}^2)t$.

The particle changes direction when $v = 0$, which occurs at $t = \dfrac{3}{8}$ s.

The position at this time is:

$x = 2.00\text{ m} + (3.00\text{ m/s})$**Error!** s) $- (4.00\text{ m/s}^2)$**Error!** s)^2 = **Error!**

(b) From $x = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00 \text{ m/s})}{-8.00 \text{ m/s}^2} = \frac{3}{4} \text{ s and the velocity is}$$

$$v = 3.00 \text{ m/s} - (8.00 \text{ m/s}^2)\left(\frac{3}{4} \text{ s}\right) = \boxed{-3.00 \text{ m/s}}$$

2.28 $v_i = 5.20 \text{ m/s}$

(a) $v(t = 2.50 \text{ s}) = v_i + at = 5.20 \text{ m/s} + (3.00 \text{ m/s}^2)(2.50 \text{ s}) = \boxed{12.7 \text{ m/s}}$

(b) $v(t = 2.50 \text{ s}) = v_i + at = 5.20 \text{ m/s} + (-3.00 \text{ m/s}^2)(2.50 \text{ s}) = \boxed{-2.30 \text{ m/s}}$

2.29 (a) $x = \frac{1}{2}at^2$ (Eq 2.11)

$$400 \text{ m} = \frac{1}{2}(10.0 \text{ m/s}^2)\, t^2$$

$$t = \boxed{8.94 \text{ s}}$$

(b) $v = at$ (Eq 2.8)

$$v = (10.0 \text{ m/s}^2)(8.94 \text{ s}) = \boxed{89.4 \text{ m/s}}$$

2.30 (a) Take $t_i = 0$ at the bottom of the hill where $x_i = 0$, $v_i = 30.0 \text{ m/s}$, and $a = -2.00 \text{ m/s}^2$. Use these values in the general equation

$$x = x_i + v_i t + \frac{1}{2}at^2$$

to find $x = 0 + 30.0t \text{ m/s} + \frac{1}{2}(-2.00 \text{ m/s}^2)\, t^2$

when t is in seconds $x = \boxed{(30.0t - t^2)\text{ m}}$

To find an equation for the velocity, use

$$v = v_i + at = 30.0 \text{ m/s} + (-2.00 \text{ m/s}^2)t$$

$$v = \boxed{(30.0 - 2.00t) \text{ m/s}}$$

(b) The distance of travel x becomes a maximum, x_{max}, when $v = 0$ (turning point in the motion). Use the expressions found in part (a) for v to find the value of t when x has its maximum value:

From $v = (30.0 - 2.00t)$ m/s ,

$$v = 0 \quad \text{when} \quad t = 15.0 \text{ s}$$

Then $x_{max} = (30.0t - t^2)\text{ m} = (30.0)(15.0) - (15.0)^2 = \boxed{225 \text{ m}}$

2.31 (a) $v_i = 100$ m/s, $a = -5.00$ m/s^2

$$v^2 = v_i^2 + 2ax \qquad 0 = (100)^2 - 2(5.00)x$$

$$x = \boxed{1000 \text{ m}} \quad \text{and} \quad t = \boxed{20.0 \text{ s}}$$

(b) No, at this acceleration the plane would overshoot the runway.

***2.32** In the simultaneous equations

$$\begin{cases} v_x = v_{xi} + a_x t \\ x - x_i = \frac{1}{2}(v_{xi} + v_x)t \end{cases}$$

we have

$$\begin{cases} v_x = v_{xi} - (5.60 \text{ m/s}^2)(4.20 \text{ s}) \\ 62.4 \text{ m} = \frac{1}{2}(v_{xi} + v_x)4.20 \text{ s} \end{cases}$$

So substituting for v_{xi} gives

$$62.4 \text{ m} = \frac{1}{2}[v_x + (5.60 \text{ m/s}^2)(4.20 \text{ s}) + v_x]4.20 \text{ s}$$

$$14.9 \text{ m/s} = v_x + \frac{1}{2}(5.60 \text{ m/s}^2)(4.20 \text{ s})$$

$$v_x = \boxed{3.10 \text{ m/s}}$$

***2.33** Take any two of the standard four equations, such as

$$\begin{cases} v_x = v_{xi} + a_x t \\ x - x_i = \frac{1}{2}(v_{xi} + v_x)t \end{cases}$$

solve one for v_{xi}, and substitute into the other:

$$v_{xi} = v_x - a_x t$$

$$x - x_i = \frac{1}{2}(v_x - a_x t + v_x)\ t$$

Thus $\boxed{x - x_i = v_x t - \frac{1}{2}a_x t^2}$

Back in problem 32,

$$62.4\text{ m} = v_x(4.20\text{ s}) - \frac{1}{2}(-5.60\text{ m/s}^2)(4.20\text{ s})^2$$

$$v_x = \frac{62.4\text{ m} - 49.4\text{ m}}{4.20\text{ s}} = \boxed{3.10\text{ m/s}}$$

2.34 We assume the bullet is a cylinder. It slows down just as its front end pushes apart wood fibers.

(a) $a = \dfrac{v^2 - v_i^2}{2x} = \dfrac{(280\text{ m/s})^2 - (420\text{ m/s})^2}{2(0.100\text{ m})} = \boxed{-4.90 \times 10^5\text{ m/s}^2}$

(b) $t = \dfrac{0.100}{350} + \dfrac{0.020}{280} = \boxed{3.57 \times 10^{-4}\text{ s}}$

(c) $v_i = 420\text{ m/s}$, $v = 0$; $a = -4.90 \times 10^5\text{ m/s}^2$; $v^2 = v_i^2 + 2ax$

$$x = \frac{v^2 - v_i^2}{2a} = \frac{v_i^2}{2a} = -\frac{(420\text{ m/s})^2}{(-2 \times 4.90 \times 10^5\text{ m/s}^2)}$$

$$x = \boxed{0.180\text{ m}}$$

***2.35** (a) The time it takes the truck to reach 20.0 m/s is found from $v = v_i + at$,

solving for t yields $t = \dfrac{v - v_i}{a} = \dfrac{20.0\text{ m/s} - 0\text{ m/s}}{2.00\text{ m/s}^2} = 10.0\text{ s}$

The total time is thus 10.0 s + 20.0 + 5.00 s = $\boxed{35.0\text{ s}}$

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}\,t = \left(\frac{0 + 20.0}{2}\right)(10.0) = 100 \text{ m}$$

The distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) + 0 = 400 \text{ m}, \; a \text{ being 0 for this interval.}$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}\,t = \left(\frac{20.0 + 0}{2}\right)(5.00) = 50.0 \text{ m}$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50.0 = 550$ m, and the average velocity is given by

$$\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7 \text{ m/s}}$$

***2.36** Using the equation $x = v_i t + \frac{1}{2}at^2$ yields $x = 20.0(40.0) - 1.00(40.0)^2/2 = 0$, which is obviously wrong. The error occurs because the equation used is for uniformly accelerated motion, which this is not. The acceleration is -1.00 m/s^2 for the first 20.0 s and 0 for the last 20.0 s. The distance traveled in the first 20.0 s is:

$$x = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) - 1.00(20.0^2)/2 = 200 \text{ m}$$

During the last 20.0 s, the train is at rest. Thus, the total distance traveled in the 40.0 s interval is $\boxed{200 \text{ m}}$.

2.37 (a) $a = \dfrac{v - v_i}{t} = \dfrac{632(5280/3600)}{1.40} = \boxed{-662 \text{ ft/s}^2} = -202 \text{ m/s}^2$

(b) $x = v_i t + \frac{1}{2}at^2 = (632)(5280/3600)(1.40) - \frac{1}{2}662(1.40)^2 = \boxed{649 \text{ ft}} = \boxed{198 \text{ m}}$

2.38 We have $v_i = 2.00 \times 10^4$ m/s, $v = 6.00 \times 10^6$ m/s,

$$x - x_i = 1.50 \times 10^{-2} \text{ m}$$

(a) $x - x_i = \frac{1}{2}(v_i + v)\,t$

$$t = \frac{2(x - x_i)}{v_i + v} = \frac{2(1.50 \times 10^{-2} \text{ m})}{2.00 \times 10^4 \text{ m/s} + 6.00 \times 10^6 \text{ m/s}} = \boxed{4.98 \times 10^{-9} \text{ s}}$$

(b) $v^2 = v_i^2 + 2a(x - x_i)$

$$a = \frac{v^2 - v_i^2}{2(x - x_i)} = \frac{(6.00 \times 10^6 \text{ m/s})^2 - (2.00 \times 10^4 \text{ m/s})^2}{2(1.50 \times 10^{-2} \text{ m})} = \boxed{1.20 \times 10^{15} \text{ m/s}^2}$$

2.39 (a) Take initial and final points at top and bottom of the incline.

If the ball starts from rest, $v_i = 0$, $a = 0.500 \text{ m/s}^2$, $x - x_i = 9.00 \text{ m}$

Then $v^2 = v_i^2 + 2a(x - x_i) = 0^2 + 2(0.5\,00 \text{ m/s}^2)\,9.00 \text{ m}$

$$v = \boxed{3.00 \text{ m/s}}$$

(b) $x - x_i = v_i t + \frac{1}{2} a t^2$

$$9.00 \text{ m} = 0 + \frac{1}{2}(0.500 \text{ m/s}^2)\, t^2$$

$$t = \boxed{6.00 \text{ s}}$$

(c) Take initial and final points at the bottom of the planes and the top of the second plane, respectively. $v_i = 3.00 \text{ m/s}$ $v = 0$ $x - x_i = 15.00 \text{ m}$

$$v^2 = v_i^2 + 2a(x - x_i)$$

gives

$$a = \frac{(v^2 - v_i^2)}{2(x - x_i)} = \frac{[0 - (3.00 \text{ m/s})^2]}{2(15.0 \text{ m})}$$

$$= \boxed{-0.300 \text{ m/s}^2}$$

(d) Take initial point at the bottom of the planes and final point 8.00 m along the second: $v_i = 3.00 \text{ m/s}$ $x - x_i = 8.00 \text{ m}$ $a = -0.300 \text{ m/s}^2$

$$v^2 = v_i^2 + 2a(x - x_i)$$

$$= (3.00 \text{ m/s})^2 + 2(-0.300 \text{ m/s}^2)(8.00 \text{ m}) = 4.20 \text{ m}^2/\text{s}^2$$

$$v = \boxed{2.05 \text{ m/s}}$$

2.40 Take the original point to be when Sue notices the van. Choose the origin of the *x*-axis at Sue's car. For her we have

$$x_{is} = 0 \qquad v_{is} = 30.0 \text{ m/s} \qquad a_S = -2.00 \text{ m/s}^2$$

so her position is given by

$$x_s(t) = x_{is} + v_{is}\,t + \frac{1}{2}\,a_s t^2$$

$$= (30.0 \text{ m/s})t + \frac{1}{2}(-2.00 \text{ m/s}^2)\,t^2$$

For the van, $x_{iv} = 155$ m $\quad v_{iv} = 5.00$ m/s $\quad a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}\,a_v t^2 = 155 \text{ m} + (5.00 \text{ m/s})t + 0$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$

$$0 = t_c^2 - 25.0t_c + 155$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6 \text{ s or } \boxed{11.4 \text{ s}}$$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position 155 m + $(5.00 \text{ m/s})(11.4 \text{ s}) = \boxed{212 \text{ m}}$.

2.41 Choose the origin ($y = 0$, $t = 0$) at the starting point of the ball and take upward as positive. Then, $y_i = 0$, $v_i = 0$, and $a = -g = -9.80$ m/s². The position and the velocity at time t become:

$$y - y_i = v_i t + \frac{1}{2}\,at^2 \Rightarrow y = -\frac{1}{2}\,gt^2 = -\frac{1}{2}(9.80 \text{ m/s}^2)\,t^2$$

and $\quad v = v_i + at \Rightarrow v = -gt = -(9.80 \text{ m/s}^2)t$

(a) at $t = 1.00$ s: $y = -\frac{1}{2}(9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = \boxed{-4.90 \text{ m}}$

at $t = 2.00$ s: $y = -\frac{1}{2}(9.80 \text{ m/s}^2)(2.00 \text{ s})^2 = \boxed{-19.6 \text{ m}}$

at $t = 3.00$ s: $y = -\frac{1}{2}(9.80 \text{ m/s}^2)(3.00 \text{ s})^2 = \boxed{-44.1 \text{ m}}$

(b) at $t = 1.00$ s: $v = -(9.80 \text{ m/s}^2)(1.00 \text{ s}) = \boxed{-9.80 \text{ m/s}}$

at $t = 2.00$ s: $v = -(9.80 \text{ m/s}^2)(2.00 \text{ s}) = \boxed{-19.6 \text{ m/s}}$

at $t = 3.00$ s: $v = -(9.80 \text{ m/s}^2)(3.00 \text{ s}) = \boxed{-29.4 \text{ m/s}}$

***2.42** Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a = -g = -9.80 \text{ m/s}^2$. During the flight, Goff went 1 mile (1609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$$v^2 = v_i^2 + 2a(y - y_i) \Rightarrow 0 = v_i^2 - 2(9.80 \text{ m/s}^2)(1609 \text{ m}) \Rightarrow v_i = 178 \text{ m/s}$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$$y - y_i = v_i t + \frac{1}{2} a t^2 \Rightarrow 0 = (178 \text{ m/s})t - \frac{1}{2}(-9.80 \text{ m/s}^2)\, t^2$$

The root $t = 0$ describes launch; the other root, $t = 36.2$ s, describes his flight time. His rate of pay may then be found from

$$pay\ \ rate = \frac{\$1.00}{36.2 \text{ s}} = \left(0.0276 \frac{\$}{\text{s}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = \boxed{\$99.4/\text{h}}$$

2.43 (a) $y = v_i t + \frac{1}{2} a t^2$

$4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $\boxed{v_i = 10.0 \text{ m/s upward}}$

(b) $v = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s}$

$v = \boxed{4.68 \text{ m/s downward}}$

2.44 We have

$$y = -\frac{1}{2} g t^2 + v_i t + y_i$$

$$0 = -(4.90 \text{ m/s}^2)t^2 - (8.00 \text{ m/s})t + 30.0 \text{ m}$$

Solving for t, $\quad t = \dfrac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}$

Using only the positive value for t, we find $t = \boxed{1.79 \text{ s}}$

*__2.45__ The bill starts from rest $v_i = 0$ and falls with a downward acceleration of $9.80\ \text{m/s}^2$ (due to gravity). Thus, in 0.20 s it will fall a distance of

$$\Delta y = v_i t - \frac{1}{2} gt^2 = 0 - (4.90\ \text{m/s}^2)(0.20\ \text{s})^2 = -0.20\ \text{m}$$

This distance is about twice the distance between the center of the bill and its top edge ($\cong$ 8 cm). $\boxed{\text{Thus, David will be unsuccessful}}$.

<u>Goal Solution</u>
Emily challenges her friend David to catch a dollar bill as follows. She holds the bill vertically, as in Figure P2.45, with the center of the bill between David's index finger and thumb. David must catch the bill after Emily releases it without moving his hand downward. If his reaction time is 0.2 s, will he succeed? Explain your reasoning.

G: David will be successful if his reaction time is short enough that he can catch the bill before it falls half of its length (about 8 cm). Anyone who has tried this challenge knows that this is a difficult task unless the catcher "cheats" by anticipating the moment the bill is released. Since David's reaction time of 0.2 s is typical of most people, we should suspect that he will not succeed in meeting Emily's challenge.

O: Since the bill is released from rest and experiences free fall, we can use the equation $y = \frac{1}{2} gt^2$ to find the distance y the bill falls in $t = 0.2$ s

A: $y = \frac{1}{2}(9.80\ \text{m/s}^2)(0.2\ \text{s})^2 = 0.196\ \text{m} > 0.08\ \text{m}$
Since the bill falls below David's fingers before he reacts, he will not catch it.

L: It appears that even if David held his fingers at the bottom of the bill (about 16 cm below the top edge), he still would not catch the bill unless he reduced his reaction time by tensing his arm muscles or anticipating the drop.

***2.46** At any time t, the position of the ball released from rest is given by $y_1 = h - \frac{1}{2} gt^2$. At time t, the position of the ball thrown vertically upward is described by $y_2 = v_i t - \frac{1}{2} gt^2$.

The time at which the first ball has a position of $y_1 = h/2$ is found from the first equation as $h/2 = h - \frac{1}{2} gt^2$, which yields $t = \sqrt{h/g}$. To require that the second ball have a position of $y_2 = h/2$ at this time, use the second equation to obtain $h/2 = v_i\sqrt{h/g} - \frac{1}{2} g(h/g)$. This gives the required initial upward velocity of the second ball as $\boxed{v_i = \sqrt{gh}}$.

2.47 (a) $v = v_i - gt$ (Eq. 2.8)

$v = 0$ when $t = 3.00$ s, $g = 9.80$ m/s^2,

$\therefore\ v_i = gt = (9.80\ \text{m/s}^2)(3.00\ \text{s}) = \boxed{29.4\ \text{m/s}}$

(b) $y = \frac{1}{2}(v + v_i)\, t = \frac{1}{2}(29.4\ \text{m/s})(3.00\ \text{s}) = \boxed{44.1\ \text{m}}$

Goal Solution

A baseball is hit such that it travels straight upward after being struck by the bat. A fan observes that it requires 3.00 s for the ball to reach its maximum height. Find (a) its initial velocity and (b) the maximum height it reaches.

G: We can expect the initial speed of the ball to be somewhat greater than the speed of the pitch, which might be about 60 mph (~30 m/s), so an initial upward velocity off the bat of somewhere between 20 and 100 m/s would be reasonable. We also know that the length of a ball field is about 300 ft. (~100m), and a pop-fly usually does not go higher than this distance, so a maximum height of 10 to 100 m would be reasonable for the situation described in this problem.

O: Since the ball's motion is entirely vertical, we can use the equation for free fall to find the initial velocity and maximum height from the elapsed time.

A: Choose the "initial" point when the ball has just left contact with the bat. Choose the "final" point at the top of its flight. In between, the ball is in free fall for $t = 3.00$ s and has constant acceleration, $a = -g = -9.80$ m/s^2. Solve the equation $v_{yf} = v_{yi} - gt$ for v_{yi} when $v_{yf} = 0$ (when the ball reaches its maximum height).

(a) $v_{yi} = v_{yf} + gt = 0 + (9.80\ \text{m/s}^2)(3.00\ \text{s}) = 29.4\ \text{m/s}$ (upward)

(b) The maximum height in the vertical direction is

$$y_f = v_{yi}\, t + \frac{1}{2} at^2 = (29.4\ \text{m/s})(3.00\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(3.00\ \text{s})^2 = 44.1\ \text{m}$$

L: The calculated answers seem reasonable since they lie within our expected ranges, and they have the correct units and direction. We must remember that it is possible to solve a problem like this correctly, yet the answers may not seem reasonable simply because the conditions stated in the problem may not be physically possible (e.g. a time of 10 seconds for a pop fly would not be realistic).

2.48 Take downward as the positive y direction.

(a) While the woman was in free fall,

$$\Delta y = 144 \text{ ft},\ v_i = 0, \text{ and } a = g = 32.0 \text{ ft/s}^2$$

Thus,

$$\Delta y = v_i t + \frac{1}{2} at^2 \rightarrow 144 \text{ ft} = 0 + (16.0 \text{ ft/s}^2)t^2$$

giving $t_{\text{fall}} = 3.00$ s.

Her velocity just before impact is:

$$v = v_i + gt = 0 + (32.0 \text{ ft/s}^2)(3.00 \text{ s}) = \boxed{96.0 \text{ ft/s}}\ .$$

(b) While crushing the box, $v_i = 96.0$ ft/s, $v = 0$, and $\Delta y = 18.0$ in $= 1.50$ ft.

Therefore, $a = \dfrac{v^2 - v_i^2}{2(\Delta y)} = \dfrac{0 - (96.0 \text{ ft/s})^2}{2(1.50 \text{ ft})} = -3.07 \times 10^3 \text{ ft/s}^2$,

or $\boxed{a = 3.07 \times 10^3 \text{ ft/s}^2 \text{ upward}}$

(c) Time to crush box:

$$\Delta t = \frac{\Delta y}{\bar{v}} = \frac{\Delta y}{(v + v_i)/2} = \frac{2(1.50 \text{ ft})}{0 + 96.0 \text{ ft/s}}$$

or $\boxed{\Delta t = 3.13 \times 10^{-2} \text{ s}}$

2.49 Time to fall 3.00 m is found from Eq. 2.11 with $v_i = 0$,

$$3.00 \text{ m} = \frac{1}{2}(9.80 \text{ m/s}^2)\ t^2; \qquad t = 0.782 \text{ s}$$

(a) With the horse galloping at 10.0 m/s, the horizontal distance is

$$vt = \boxed{7.82 \text{ m}}$$

(b) $t = \boxed{0.782 \text{ s}}$

2.50 Time to top = 10.0 s. $v = v_i - gt$

(a) At the top, $v = 0$. Then, $t = \frac{v_i}{g} = 10.0\text{ s}$ $v_i = \boxed{98.0\text{ m/s}}$

(b) $h = v_i t - \frac{1}{2}gt^2$

At $t = 10.0$ s, $h = (98.0)(10.0) - \frac{1}{2}(9.80)(10.0)^2 = \boxed{490\text{ m}}$

2.51 $v_i = 15.0\text{ m/s}$

(a) $v = v_i - gt = 0$

$$t = \frac{v_i}{g} = \frac{15.0\text{ m/s}}{9.80\text{ m/s}^2} = \boxed{1.53\text{ s}}$$

(b) $h = v_i t - \frac{1}{2}gt^2 = \frac{v_i^2}{2g} = \frac{225}{19.6}\text{ m} = \boxed{11.5\text{ m}}$

(c) At $t = 2.00$ s

$$v = v_i - gt = 15.0 - 19.6 = \boxed{-4.60\text{ m/s}}$$

$$a = -g = \boxed{-9.80\text{ m/s}^2}$$

2.52 $y = 3.00t^3$

At $t = 2.00$ s, $y = 3.00(2.00)^3 = 24.0$ m, and $v_y = \frac{dy}{dt} = 9.00t^2 = 36.0\text{ m/s}\uparrow$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)\ t^2$$

Setting $y_b = 0$, $0 = 24.0 + 36.0t - 4.90t^2$

Solving for t, (only positive values of t count), $\boxed{t = 7.96\text{ s}}$

2.53 (a) $J = \frac{da}{dt}$ = constant

$da = Jdt \qquad a = J \int dt = Jt + c_2$

but $a = a_i$ when $t = 0$ so $c_1 = a_i$,

Therefore, $\boxed{a = Jt + a_i}$

$a = \frac{dv}{dt}$; $dv = adt$

$v = \int adt = \int (Jt + a_i)dt = \frac{1}{2}Jt^2 + a_it + c_2$

but $v = v_i$ when $t = 0$, so $c_2 = v_i$ and $\boxed{v = \frac{1}{2}Jt^2 + a_it + v_i}$

$v = \frac{dx}{dt}$; $dx = vdt$

$x = \int vdt = \int \left(\frac{1}{2}Jt^2 + a_it + v_i\right) dt$

$x = \frac{1}{6}Jt^3 + \frac{1}{2}a_it^2 + v_it + c_3$

$x = x_i$ when $t = 0$, so $c_3 = x_i$

Therefore, $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_it^2 + v_it + x_i}$

(b) $a^2 = (Jt + a_i)^2 = J^2t^2 + a_i^2 + 2Ja_it$

$a^2 = a_i^2 + (J^2t^2 + 2Ja_it)$

$a^2 = a_i^2 + 2J\left(\frac{1}{2}Jt^2 + a_it\right)$

Recall the expression for v: $v = \frac{1}{2}Jt^2 + a_it + v_i$

So $(v - v_i) = \frac{1}{2}Jt^2 + a_it$

Therefore, $\boxed{a^2 = a_i^2 + 2J(v - v_i)}$

2.54 (a) $a = \frac{dv}{dt} = \frac{d}{dt}[-5.00\times10^7\, t^2 + 3.00\times10^5\, t]$

Error! $t + 3.00\times10^5$ m / s^2)

Take $x_i = 0$ at $t = 0$. Then $v = \frac{dx}{dt}$

$$x - 0 = \int_0^t v\,dt = \int_0^t (-5.00\times10^7\, t^2 + 3.00\times10^5 t)\,dt$$

$$x = -5.00\times10^7\,\frac{t^3}{3} + 3.00\times10^5\,\frac{t^2}{2}$$

$$\boxed{x = -(1.67\times10^7\ \text{m/s}^3)t^3 + (1.50\times10^5\ \text{m/s}^2)t^2}$$

(b) The bullet escapes when $a = 0$, at $-(10.0\times10^7\ \text{m/s}^3)t + 3.00\times10^5\ \text{m/s}^2 = 0$

$$t = \frac{3.00\times10^5\ \text{s}}{10.0\times10^7} = \boxed{3.00\times10^{-3}\ \text{s}}$$

(c) New $v = (-5.00\times10^7)(3.00\times10^{-3})^2 + (3.00\times10^5)(3.00\times10^{-3})$

$$v = -450\ \text{m/s} + 900\ \text{m/s} = \boxed{450\ \text{m/s}}$$

(d) $x = -(1.67\times10^7)(3.00\times10^{-3})^3 + (1.50\times10^5)(3.00\times10^{-3})^2$

$$x = -0.450\ \text{m} + 1.35\ \text{m} = \boxed{0.900\ \text{m}}$$

2.55 $a = \frac{dv}{dt} = -3.00v^2$, $v_i = 1.50$ m / s

Solving for v, $\frac{dv}{dt} = -3.00v^2$

$$\int_{v=v_i}^{v} v^{-2}dv = -3.00\int_{t=0}^{0} dt$$

$$-\frac{1}{v} + \frac{1}{v_i} = -3.00t \qquad \text{or} \qquad 3.00t = \frac{1}{v} - \frac{1}{v_i}$$

When $v = \frac{v_i}{2}$, $t = \frac{1}{3.00\,v_i} = \boxed{0.222\ \text{s}}$

2.56 (a) The minimum distance required for the motorist to stop, from an initial speed of 18.0 m/s, is

$$\Delta x = \frac{v^2 - v_i^2}{2a} = \frac{0 - (18.0\text{ m/s})^2}{2(-4.50\text{ m/s}^2)} = 36.0\text{ m}$$

Thus, the motorist can travel at most (38.0 m – 36.0 m) = 2.00 m before putting on the brakes if he is to avoid hitting the deer. The maximum acceptable reaction time is then

$$t_{\text{max}} = \frac{2.00\text{ m}}{v_i} = \frac{2.00\text{ m}}{18.0\text{ m/s}} = \boxed{0.111\text{ s}}$$

(b) In 0.300 s, the distance traveled at 18.0 m/s is

$$x = v_i t_1 = (18.0\text{ m/s})(0.300) = 5.40\text{ m}$$

∴ The displacement for an acceleration -4.50 m/s^2 is $38.0 - 5.40 = 32.6$ m.

$$v^2 = v_i^2 + 2ax = (18.0\text{ m/s})^2 - 2(4.50\text{ m/s}^2)(32.6\text{ m}) = 30.6\text{ m}^2/\text{s}^2$$

$$v = \sqrt{30.6} = \boxed{5.53\text{ m/s}}$$

2.57 The total time to reach the ground is given by

$$y - y_i = v_i t + \frac{1}{2}at^2$$

$$0 - 25.0\text{ m} = 0 + \frac{1}{2}(-9.80\text{ m/s}^2)\ t^2$$

$$t = \sqrt{\frac{2(25.0\text{ m})}{9.80\text{ m/s}^2}} = 2.26\text{ s}$$

The time to fall the first fifteen meters is found similarly:

$$-15.0\text{ m} = 0 - \frac{1}{2}(9.80\text{ m/s}^2)\ t_1^2$$

$$t_1 = 1.75\text{ s}$$

So $t - t_1 = 2.26\text{ s} - 1.75\text{ s} = \boxed{0.509\text{ s}}$ suffices for the last ten meters.

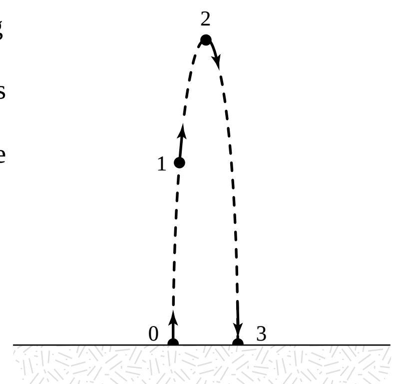
***2.58** The rate of hair growth is a velocity and the rate of its increase is an acceleration. Then $v_i = 1.04 \text{ mm/d}$ and $a = 0.132\left(\dfrac{\text{mm/d}}{\text{w}}\right)$. The increase in the length of the hair (i.e., displacement) during a time of $t = 5.00 \text{ w} = 35.0 \text{ d}$ is

$$\Delta x = v_i t + \frac{1}{2}at^2$$

$$\Delta x = (1.04 \text{ mm/d})(35.0 \text{ d}) + \frac{1}{2}(0.132 \text{ mm/d}\cdot\text{w})(35.0 \text{ d})(5.00 \text{ w})$$

or $\boxed{\Delta x = 48.0 \text{ mm}}$

2.59 Let path (#1) correspond to the motion of the rocket accelerating under its own power. Path (#2) is the motion of the rocket under the influence of gravity with the rocket still rising. Path (#3) is the motion of the rocket under the influence of gravity, but with the rocket falling. The data in the table is found for each phase of the rocket's motion.



(#1): $v^2 - (80.0)^2 = 2(4.00)(1000)$; therefore $v = 120 \text{ m/s}$

$120 = 80.0 + (4.00)t$ giving $t = 10.0 \text{ s}$

(#2): $0 - (120)^2 = 2(-9.80)\Delta x$ giving $\Delta x = 735 \text{ m}$

$0 - 120 = -9.80t$ giving $t = 12.2 \text{ s}$

This is the time of maximum height of the rocket.

(#3): $v^2 - 0 = 2(-9.80)(-1735)$

$v = -184 = (-9.80)t$ giving $t = 18.8 \text{ s}$

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0 \text{ s}}$

(b) $\Delta x_{\text{total}} = \boxed{1.73 \text{ km}}$

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	x	v	a
0	Launch	0	0	80	+4.00
#1	End Thrust	10.0	1000	120	+4.00
#2	Rise Upwards	22.2	1735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

2.60 Distance traveled by motorist = $(15.0 \text{ m/s})t$

Distance traveled by policeman = $\frac{1}{2}(2.00 \text{ m/s}^2)\, t^2$

(a) intercept occurs when $15.0t = t^2$ $\quad t = \boxed{15.0 \text{ s}}$

(b) v (officer) = $(2.00 \text{ m/s}^2)t = \boxed{30.0 \text{ m/s}}$

(c) x (officer) = $\frac{1}{2}(2.00 \text{ m/s}^2)\, t^2 = \boxed{225 \text{ m}}$

***2.61** Area A_1 is a rectangle. Thus, $A_1 = hw = v_i t$.

Area A_2 is triangular.

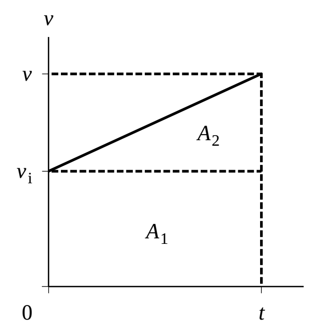
Therefore $A_2 = \frac{1}{2}\, bh = \frac{1}{2}\, t(v - v_i)$.

The total area under the curve is

$$A = A_1 + A_2 = v_i t + (v - v_i)t/2$$

and since $v - v_i = at$

$$\boxed{A = v_i t + \frac{1}{2} a t^2}$$



The displacement given by the equation is:

$x = v_i t + \frac{1}{2}\, at^2$, the same result as above for the total area.

2.62 $a_1 = 0.100 \text{ m/s}^2$, $a_2 = -0.500 \text{ m/s}^2$

$$x = 1000 \text{ m} = \frac{1}{2}\, a_1 t_1^2 + v_1 t_2 + \frac{1}{2}\, a_2 t_2^2$$

$t = t_1 + t_2$ and $v_1 = a_1 t_1 = -a_2 t_2$

$$1000 = \frac{1}{2}\, a_1 t_1^2 + a_1 t_1 \left(-\frac{a_1 t_1}{a_2}\right) + \frac{1}{2}\, a_2 \left(\frac{a_1 t_1}{a_2}\right)^2$$

$$1000 = \frac{1}{2}\, a_1 \left(1 - \frac{a_1}{a_2}\right) t_1^2$$

$$t_1 = \sqrt{\frac{20{,}000}{1.20}} = \boxed{129\text{ s}}$$

$$t_2 = \frac{a_1 t_1}{-a_2} = \frac{12.9}{0.500} \approx 26\text{ s}$$

Total time $= t = \boxed{155\text{ s}}$

2.63 (a) Let x be the distance traveled at acceleration a until maximum speed v is reached. If this achieved in time t_1 we can use the following three equations:

$$x = \frac{(v + v_i)}{2}\, t_1, \qquad 100 - x = v(10.2 - t_1) \quad \text{and} \quad v = v_i + at_1$$

The first two give $100 = \left(10.2 - \frac{1}{2} t_1\right) v = \left(10.2 - \frac{1}{2} t_1\right) a t_1$

$$a = \frac{200}{(20.4 - t_1)t_1}\ .$$

For Maggie $a = \dfrac{200}{(18.4)(2.00)} = \boxed{5.43\text{ m/s}^2}$

For Judy $a = \dfrac{200}{(17.4)(3.00)} = \boxed{3.83\text{ m/s}^2}$

(b) $v = at_1$

Maggie: $v = (5.43)(2.00) = \boxed{10.9\text{ m/s}}$

Judy: $v = (3.83)(3.00) = \boxed{11.5\text{ m/s}}$

(c) At the six-second mark $x = \frac{1}{2}\, at_1^2 + v(6.00 - t_1)$

Maggie: $x = \frac{1}{2}(5.43)(2.00)^2 + (10.9)(4.00) = 54.3\text{ m}$

Judy: $x = \frac{1}{2}(3.83)(3.00)^2 + (11.5)(3.00) = 51.7\text{ m}$

Maggie is ahead by $\boxed{2.62\text{ m}}$.

***2.64** Let the ball fall 1.50 m. It strikes at speed given by:

$$v_x^2 = v_{xi}^2 + 2a(x - x_i)$$

$$v_x^2 = 0 + 2(-9.80\ \mathrm{m/s^2})(-1.50\ \mathrm{m})$$

$$v_x = -5.42\ \mathrm{m/s}$$

and its stopping is described by

$$v_x^2 = v_{xi}^2 + 2a_x(x - x_i)$$

$$0 = (-5.42\ \mathrm{m/s})^2 + 2a_x(-10^{-2}\ \mathrm{m})$$

$$a_x = \frac{-29.4\ \mathrm{m^2/s^2}}{-2.00\times 10^{-2}\ \mathrm{m}} = +1.47\times 10^3\ \mathrm{m/s^2}$$

Its maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3\ \mathrm{m/s^2}}$.

2.65 Acceleration $a = 3.00\ \mathrm{m/s^2}$ Deceleration $a' = -4.50\ \mathrm{m/s^2}$

(a) Keeping track of speed and time for each phase of motion,

$v_0 = 0$, $v_1 = 12.0\ \mathrm{m/s}$ $\Delta t_{01} = 4.00\ \mathrm{s}$

$v_1 = 12.0\ \mathrm{m/s}$ $t_1 = 5.00\ \mathrm{s}$

$v_1 = 12.0\ \mathrm{m/s}$, $v_2 = 0$ $\Delta t_{12} = 2.67\ \mathrm{s}$

$v_2 = 0\ \mathrm{m/s}$, $v_3 = 18.0\ \mathrm{m/s}$ $\Delta t_{23} = 6.00\ \mathrm{s}$

$v_3 = 18.0\ \mathrm{m/s}$ $t_3 = 20.0\ \mathrm{s}$

$v_3 = 18.0\ \mathrm{m/s}$, $v_4 = 6.00\ \mathrm{m/s}$ $\Delta t_{34} = 2.67\ \mathrm{s}$

$v_4 = 6.00\ \mathrm{m/s}$ $t_4 = 4.00\ \mathrm{s}$

$v_4 = 6.00\ \mathrm{m/s}$, $v_5 = 0$ $\Delta t_{45} = 1.33\ \mathrm{s}$

$$\boxed{\Sigma t = 45.7\ \mathrm{s}}$$

(b) $x = \Sigma\, \bar{v}_i\, t_i = 6.00(4.00) + 12.0(5.00) + 6.00(2.67) + 9.00(6.00) + 18.0(20.0) + 12.0(2.67)$

$+ 6.00(4.00) + 3.00(1.33) = \boxed{574\ \mathrm{m}}$

(c) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{574\ \mathrm{m}}{45.7\ \mathrm{s}} = \boxed{12.6\ \mathrm{m/s}}$

(d) $t_{\text{WALK}} = \dfrac{2\Delta x}{v_{\text{WALK}}} = \dfrac{2(574\text{ m})}{(1.50\text{ m/s})} = \boxed{765\text{ s}}$

(about 13 minutes, and better exercise!)

2.66 (a) $d = \frac{1}{2}(9.80)\, t_1^2 \qquad d = 336t_2$

$t_1 + t_2 = 2.40$

$336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$

$$t_2 = \frac{359.5 \pm \sqrt{(359.5)^2 - 4(4.90)(28.22)}}{9.80}$$

$$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.0765\text{ s}$$

$\therefore d = 336t_2 = \boxed{26.4\text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of $\boxed{6.82\%}$.

2.67 (a) $y = v_{i1}t + \frac{1}{2}at^2 = 50.0 = 2.00t + \frac{1}{2}(9.80)\, t^2$

$t = \boxed{2.99\text{ s}}$ after the first stone is thrown.

(b) $y = v_{i2}t + \frac{1}{2}at^2$ and $t = 2.99 - 1.00 = 1.99$ s

substitute $50.0 = v_{i2}(1.99) + \frac{1}{2}(9.80)(1.99)^2$

$v_{i2} = \boxed{15.4\text{ m/s}}$ downward

(c) $v_1 = v_{i1} + at = -2.00 + (-9.80)(2.99) = \boxed{-31.3\text{ m/s}}$

$v_2 = v_{i2} + at = -15.3 + (-9.80)(1.99) = \boxed{-34.9\text{ m/s}}$

2.68 The time required for the car to come to rest and the time required to regain its original speed of 25.0 m/s are both given by $\Delta t = \dfrac{|\Delta v|}{|a|} = \dfrac{25.0\text{ m/s}}{2.50\text{ m/s}^2}$. The total distance the car travels in these two intervals is

$$x_{\text{car}} = \Delta x_1 + \Delta x_2 = \frac{(25.0\text{ m/s} + 0)}{2}(10.0\text{ s}) + \frac{(0 + 25.0\text{ m/s})}{2}(10.0\text{ s}) = 250\text{ m}$$

The total elapsed time when the car regains its original speed is

$$\Delta t_{\text{total}} = 10.0 \text{ s} + 45.0 \text{ s} + 10.0 \text{ s} = 65.0 \text{ s}$$

The distance the train has traveled in this time is

$$x_{\text{train}} = (25.0 \text{ m/s})(65.0 \text{ s}) = 1.63 \times 10^3 \text{ m}$$

Thus, the train is $1.63 \times 10^3 \text{ m} - 250 \text{ m} = \boxed{1.38 \times 10^3 \text{ m}}$ ahead of the car.

2.69 (a) We require $x_s = x_k$ when $t_s = t_k + 1.00$

$$x_s = \frac{1}{2}(3.50 \text{ m/s}^2)(t_k + 1.00)^2 = \frac{1}{2}(4.90 \text{ m/s}^2)(t_k)^2 = x_k$$

$$t_k + 1.00 = 1.183 t_k$$

$$t_k = \boxed{5.46 \text{ s}}$$

(b) $x_k = \frac{1}{2}(4.90 \text{ m/s}^2)(5.46 \text{ s})^2 = \boxed{73.0 \text{ m}}$

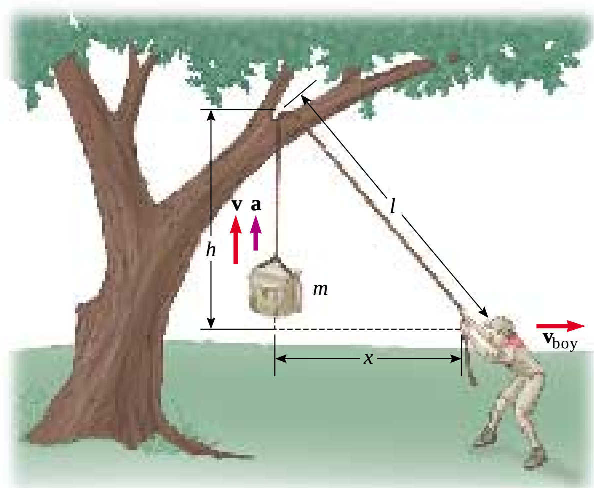
(c) $v_k = (4.90 \text{ m/s}^2)(5.46 \text{ s}) = \boxed{26.7 \text{ m/s}}$

$v_s = (3.50 \text{ m/s}^2)(6.46 \text{ s}) = \boxed{22.6 \text{ m/s}}$

2.70 (a) In walking a distance Δx, in a time Δt, the length of rope l is only increased by $\Delta x \sin\theta$.

$\therefore$ The pack lifts at a rate $\dfrac{\Delta x}{\Delta t}\sin\theta$.

$$v = \frac{\Delta x}{\Delta t}\sin\theta = v_{\text{boy}}\frac{x}{l} = \boxed{v_{\text{boy}}\frac{x}{\sqrt{x^2 + h^2}}}$$



(b) $$a = \frac{dv}{dt} = \frac{v_{\text{boy}}}{l}\frac{dx}{dt} + v_{\text{boy}}x\frac{d}{dt}\left(\frac{1}{l}\right)$$

$$a = v_{\text{boy}}\frac{v_{\text{boy}}}{l} - \frac{v_{\text{boy}}x}{l^2}\frac{dl}{dt}, \text{ but } \frac{dl}{dt} = v$$

$$\therefore a = \frac{v_{\text{boy}}^2}{l}\left(1 - \frac{x^2}{l^2}\right) = \frac{v_{\text{boy}}^2}{l}\frac{h^2}{l^2} = \boxed{\frac{h^2 v_{\text{boy}}^2}{(x^2 + h^2)^{3/2}}}$$

(c) $\dfrac{v_{\text{boy}}^2}{h}$, 0

(d) v_{boy}, 0

2.71 $h = 6.00\text{ m}$, $v_{\text{boy}} = 2.00\text{ m/s}$

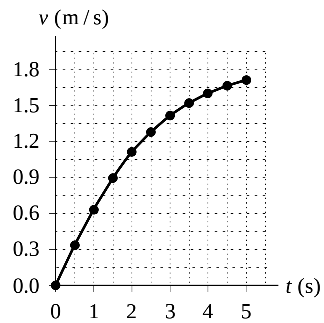
$$v = \frac{\Delta x}{\Delta t}\quad \sin\theta = v_{\text{boy}}\frac{x}{l} = \frac{v_{\text{boy}}x}{(x^2 + h^2)^{1/2}}$$

However $x = v_{\text{boy}}t$

$$\therefore\ v = \frac{v_{\text{boy}}^2 t}{(v_{\text{boy}}^2 t^2 + h^2)^{1/2}} = \frac{4t}{(4t^2 + 36)^{1/2}}$$

(a)

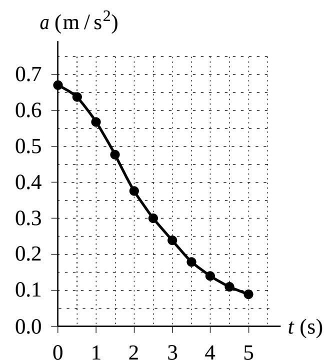
t(s)	v(m/s)
0	0
0.5	0.32
1	0.63
1.5	0.89
2	1.11
2.5	1.28
3	1.41
3.5	1.52
4	1.60
4.5	1.66
5	1.71



(b) From problem 2.70 above,

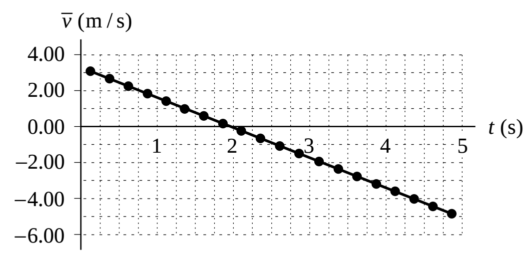
$$a = \frac{h^2 v_{\text{boy}}^2}{(x^2 + h^2)^{3/2}} = \frac{h^2 v_{\text{boy}}^2}{(v_{\text{boy}}^2 t^2 + h^2)^{3/2}} = \frac{144}{(4t^2 + 36)^{3/2}}$$

t(s)	a(m/s^2)
0	0.67
0.5	0.64
1	0.57
1.5	0.48
2	0.38
2.5	0.30
3	0.24
3.5	0.18
4	0.14
4.5	0.11
5	0.09



2.72

Time t(s)	Height h(m)	Δh (m)	Δt (s)	$\bar{v}$ (m/s)	midpt time t (s)
0.00	5.00				
		0.75	0.25	3.00	0.13
0.25	5.75				
		0.65	0.25	2.60	0.38
0.50	6.40				
		0.54	0.25	2.16	0.63
0.75	6.94				
		0.44	0.25	1.76	0.88
1.00	7.38				
		0.34	0.25	1.36	1.13
1.25	7.72				
		0.24	0.25	0.96	1.38
1.50	7.96				
		0.14	0.25	0.56	1.63
1.75	8.10				
		0.03	0.25	0.12	1.88
2.00	8.13				
		–0.06	0.25	–0.24	2.13
2.25	8.07				
		–0.17	0.25	–0.68	2.38
2.50	7.90				
		–0.28	0.25	–1.12	2.63
2.75	7.62				
		–0.37	0.25	–1.48	2.88
3.00	7.25				
		–0.48	0.25	–1.92	3.13
3.25	6.77				
		–0.57	0.25	–2.28	3.38
3.50	6.20				
		–0.68	0.25	–2.72	3.63
3.75	5.52				
		–0.79	0.25	–3.16	3.88
4.00	4.73				
		–0.88	0.25	–3.52	4.13
4.25	3.85				
		–0.99	0.25	–3.96	4.38
4.50	2.86				
		–1.09	0.25	–4.36	4.63
4.75	1.77				
		–1.19	0.25	–4.76	4.88
5.00	0.58				

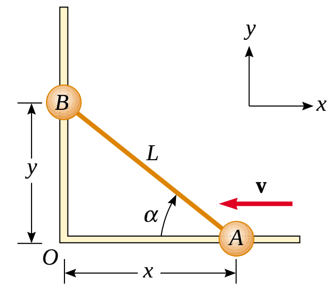


acceleration = slope of line is constant.

$\bar{a} = -1.63\ \text{m/s}^2 = \boxed{1.63\ \text{m/s}^2\ \text{downward}}$

2.73 The distance x and y are always related by $x^2 + y^2 = L^2$. Differentiating this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$



Now $\frac{dy}{dt}$ is v_B, the unknown velocity of B; and $\frac{dx}{dt} = -v$.

From the equation resulting from differentiation, we have

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v)$$

But $\frac{y}{x} = \tan\alpha$ so $v_B = \left(\frac{1}{\tan\alpha}\right)v$

When $\alpha = 60.0°$, $v_B = \frac{v}{\tan 60.0°} = \frac{v\sqrt{3}}{3} = \boxed{0.577v}$

Goal Solution

Two objects, A and B, are connected by a rigid rod that has a length L. The objects slide along perpendicular guide rails, as shown in Figure P2.73. If A slides to the left with a constant speed v, find the velocity of B when $\alpha = 60.0°$.

G: The solution to this problem may not seem obvious, but if we consider the range of motion of the two objects, we realize that B will have the same speed as A when $\alpha = 45$, and when $\alpha = 90$, then $v_B = 0$. Therefore when $\alpha = 60$, we should expect v_B to be between 0 and v.

O: Since we know a distance relationship and we are looking for a velocity, we might try differentiating with respect to time to go from what we know to what we want. We can express the fact that the distance between A and B is always L, with the relation: $x^2 + y^2 = L^2$. By differentiating this equation with respect to time, we can find $v_B = dy/dt$ in terms of $dx/dt = v_A = -v$.

A: Differentiating $x^2 + y^2 = L^2$ gives us $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

Substituting and solving for the speed of B: $v_B = \frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v)$

Now from the geometry of the figure, we notice that $\frac{y}{x} = \tan\alpha$, so $v_B = \frac{v}{\tan\alpha}$

When $\alpha = 60.0°$, $v_B = \frac{v}{\tan 60} = \frac{v}{\sqrt{3}} = 0.577v$ (B is moving up)

L: Our answer seems reasonable since we have specified both a magnitude and direction for the velocity of B, and the speed is between 0 and v in agreement with our earlier prediction. In this and many other physics problems, we can find it helpful to examine the limiting cases that define boundaries for the answer.

Chapter 3 Solutions

***3.1** $x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$

$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$

3.2 (a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2.00 - [-3.00]^2) + (-4.00 - 3.00)^2}$

$d = \sqrt{25.0 + 49.0} = \boxed{8.60\text{ m}}$

(b) $r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47\text{ m}}$

$\theta_1 = \tan^{-1}\left(-\dfrac{4.00}{2.00}\right) = \boxed{-63.4°}$

$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24\text{ m}}$

$\theta_2 = \boxed{135°}$ measured from + x axis.

3.3 We have $2.00 = r\cos 30.0°$

$r = \dfrac{2.00}{\cos 30.0°} = \boxed{2.31}$

and $y = r\sin 30.0° = 2.31\sin 30.0° = \boxed{1.15}$

3.4 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore

$x_1 = (2.50\text{ m})\cos 30.0°$, $y_1 = (2.50\text{ m})\sin 30.0°$, and

$(x_1, y_1) = \boxed{(2.17, 1.25)\text{ m}}$

$x_2 = (3.80\text{ m})\cos 120°$, $y_2 = (3.80\text{ m})\sin 120°$, and

$(x_2, y_2) = \boxed{(-1.90, 3.29)\text{ m}}$

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{16.6 + 4.16} = \boxed{4.55\text{ m}}$

3.5 The x distance out to the fly is 2.00 m and the y distance up to the fly is 1.00 m.

(a) We can use the Pythagorean theorem to find the distance from the origin to the fly,

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{(2.00\text{ m})^2 + (1.00\text{ m})^2} = \sqrt{5.00\text{ m}^2} = \boxed{2.24\text{ m}}$$

(b) $\theta = \text{Arctan}\left(\frac{1}{2}\right) = 26.6°$; $\mathbf{r} = \boxed{2.24\text{ m, }26.6°}$

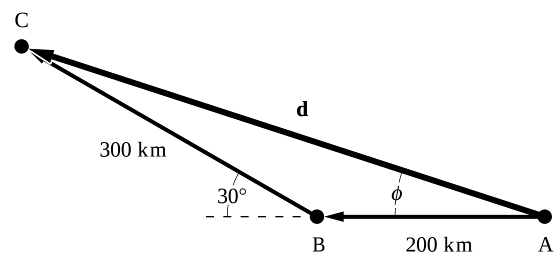
3.6 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \text{Arctan}\left(\frac{y}{x}\right)$

(a) The radius for this new point is $\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$ and its angle is

$$\text{Arctan}\left(\frac{y}{(-x)}\right) = \boxed{180° - \theta}$$

(b) $\sqrt{(-2x)^2 + (-2y)^2} = \boxed{2r}$ This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at angle $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2 + (-3y)^2} = \boxed{3r}$ This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at angle $\boxed{-\theta}$.



3.7 (a) The distance d from **A** to **C** is

$$d = \sqrt{x^2 + y^2}$$

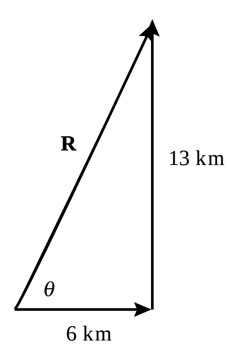
where $x = (200) + (300 \cos 30.0°) = 460$ km

and $y = 0 + (300 \sin 30.0°) = 150$ km

$$\therefore d = \sqrt{(460)^2 + (150)^2} = \boxed{484\text{ km}}$$

(b) $\tan \phi = \frac{y}{x} = \frac{150}{460} = 0.326$

$$\phi = \tan^{-1}(0.326) = \boxed{18.1°\text{ N of W}}$$

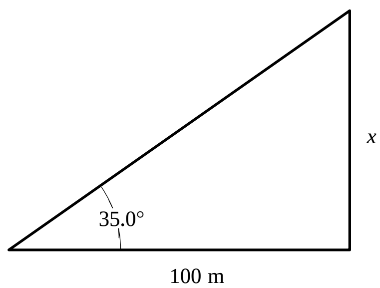


3.8 $R \cong \boxed{14\text{ km}}$

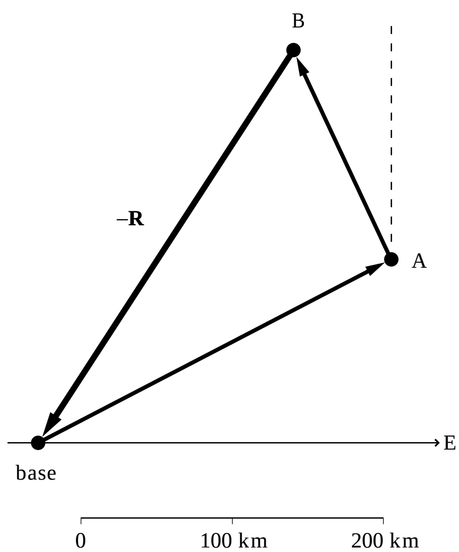
$\theta = \boxed{65°\text{ N of E}}$

3.9 $\tan 35.0° = \dfrac{x}{100 \text{ m}}$

$x = (100 \text{ m})(\tan 35.0°) = \boxed{70.0 \text{ m}}$

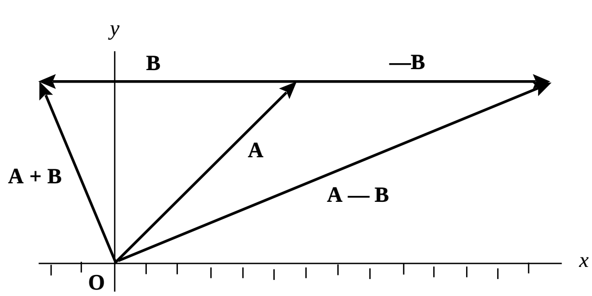


3.10 $-\mathbf{R} = \boxed{310 \text{ km at } 57° \text{ S of W}}$

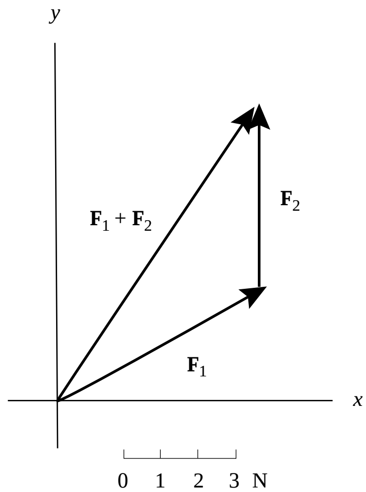


3.11 (a) Using graphical methods, place the tail of vector **B** at the head of vector **A**. The new vector **A** + **B** has a magnitude of 6.1 at 112° from the *x*-axis.

(b) The vector difference **A** – **B** is found by placing the negative of vector **B** at the head of vector **A**. The resultant vector **A** – **B** has magnitude 14.8 units at an angle of 22° from the + *x*-axis.



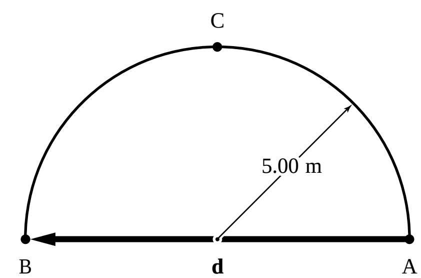
3.12 Find the resultant $\mathbf{F}_1 + \mathbf{F}_2$ graphically by placing the tail of $\mathbf{F}_2$ at the head of $\mathbf{F}_1$. The resultant force vector $\mathbf{F}_1 + \mathbf{F}_2$ is of magnitude $\boxed{9.5 \text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x\text{-axis}}$.



3.13 (a) $|\mathbf{d}| = |-10.0\mathbf{i}| = \boxed{10.0 \text{ m}}$ since the displacement is a straight line from point A to point B.

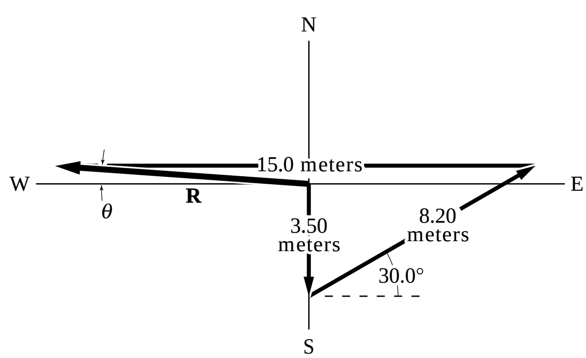
(b) The actual distance walked is not equal to the straight-line displacement. The distance follows the curved path of the semi-circle (ACB).

$$s = \left(\frac{1}{2}\right)(2\pi r) = 5\pi = \boxed{15.7 \text{ m}}$$



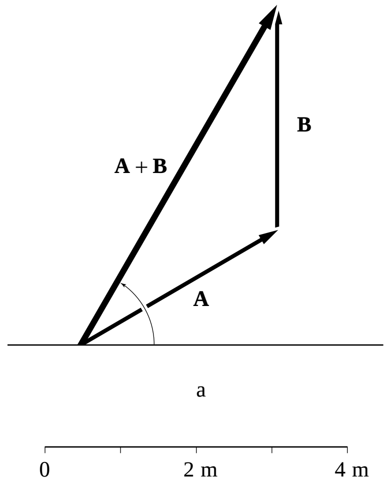
(c) If the circle is complete, **d** begins and ends at point A. Hence, $|\mathbf{d}| = \boxed{0}$.

3.14 Your sketch should be drawn to scale, and should look somewhat like that pictured below. The angle from the westward direction, θ, can be measured to be $\boxed{4° \text{ N of W}}$, and the distance R from the sketch can be converted according to the scale to be $\boxed{7.9 \text{ m}}$.

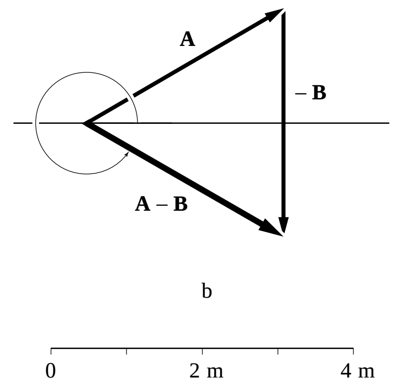


3.15 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor.

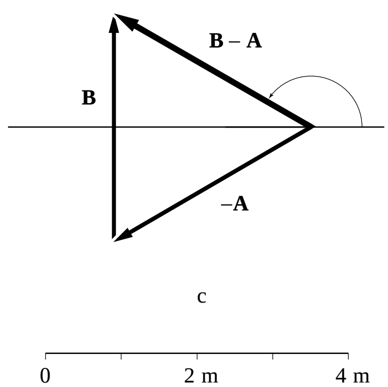
(a) **A** + **B** = 5.2 m at 60°



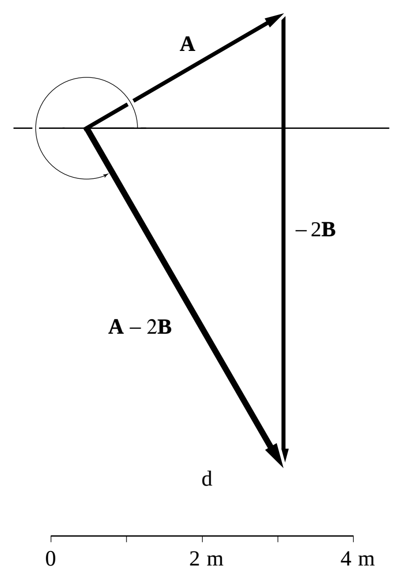
(b) **A** – **B** = 3.0 m at 330°



(c) **B** – **A** = 3.0 m at 150°



(d) **A** – 2**B** = 5.2 m at 300°

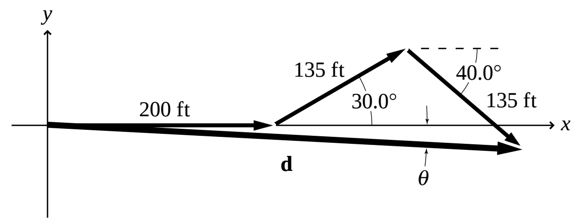


***3.16** (a) The large majority of people are standing or sitting at this hour. Their instantaneous foot-to-head vectors have upward vertical components on the order of 1 m and randomly oriented horizontal components. The citywide sum will be $\boxed{\sim 10^5 \text{ m upward}}$.

(b) Most people are lying in bed early Saturday morning. We suppose their beds are oriented north, south, east, west quite at random. Then the horizontal component of their total vector height is very nearly zero. If their compressed pillows give their height vectors vertical components averaging 3 cm, and if one-tenth of one percent of the population are on-duty nurses or police officers, we estimate the total vector height as

$\sim 10^5(0.03 \text{ m}) + 10^2(1 \text{ m})$ $\boxed{\sim 10^3 \text{ m upward}}$.

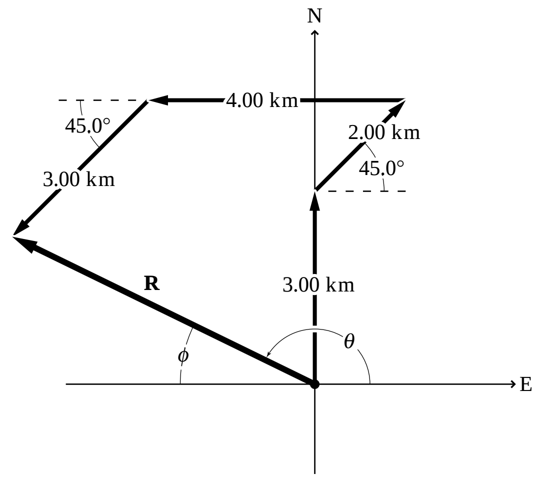
3.17 The scale drawing for the graphical solution should be similar to the figure at the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be



$\boxed{d \approx 420 \text{ ft and } \theta \approx -3°}$

3.18

x	y
0 km	3.00 km
1.41	1.41
–4.00	0
<u>–2.12</u>	<u>–2.12</u>
–4.71	2.29



$R = \sqrt{|x|^2 + |y|^2} = \boxed{5.24 \text{ km}}$

$\theta = \tan^{-1}\dfrac{y}{x} = 154°$ or $\boxed{\phi = 25.9° \text{ N of W}}$

3.19 Call his first direction the x direction.

$$R = 10.0 \text{ m } \mathbf{i} + 5.00 \text{ m}(-\mathbf{j}) + 7.00 \text{ m}(-\mathbf{i})$$

$$= 3.00 \text{ m } \mathbf{i} - 5.00 \text{ m } \mathbf{j}$$

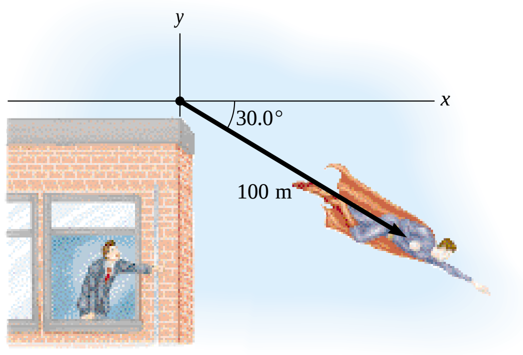
$$= \sqrt{(3.00)^2 + (5.00)^2} \text{ m at Arctan}\left(\frac{5}{3}\right) \text{ to the right}$$

$$R = \boxed{5.83 \text{ m at } 59.0° \text{ to the right from his original motion}}$$

3.20 Coordinates of super-hero are:

$$x = (100 \text{ m}) \cos(-30.0°) = \boxed{86.6 \text{ m}}$$

$$y = (100 \text{ m}) \sin(-30.0°) = \boxed{-50.0 \text{ m}}$$


y
x
30.0°
100 m

3.21 The person would have to walk $3.10 \sin(25.0°) = \boxed{1.31 \text{ km north}}$, and $3.10 \cos(25.0°) = \boxed{2.81 \text{ km east}}$.

3.22 $+x$ East, $+y$ North

$\Sigma x = 250 + 125 \cos 30° = 358$ m

$\Sigma y = 75 + 125 \sin 30° - 150 = -12.5$ m

$$d = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(358)^2 + (-12.5)^2} = 358 \text{ m}$$

$$\tan \theta = \frac{(\Sigma y)}{(\Sigma x)} = -\frac{12.5}{358} = -0.0349 \quad \theta = -2.00°$$

$$\boxed{\mathbf{d} = 358 \text{ m at } 2.00° \text{ S of E}}$$

***3.23** Let the positive x-direction be eastward, positive y-direction be vertically upward, and the positive z-direction be southward. The total displacement is then

$$\mathbf{d} = (4.80 \text{ cm } \mathbf{i} + 4.80 \text{ cm } \mathbf{j}) + (3.70 \text{ cm } \mathbf{j} - 3.70 \text{ cm } \mathbf{k})$$

or $$\mathbf{d} = 4.80 \text{ cm } \mathbf{i} + 8.50 \text{ cm } \mathbf{j} - 3.70 \text{ cm } \mathbf{k}$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2} \text{ cm} = \boxed{10.4 \text{ cm}}$

(b) Its angle with the y-axis follows from $\cos \theta = \frac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

3.24 $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_2\mathbf{k}$

$\mathbf{B} = 4.00\mathbf{i} + 6.00\mathbf{j} + 3.00\mathbf{k}$

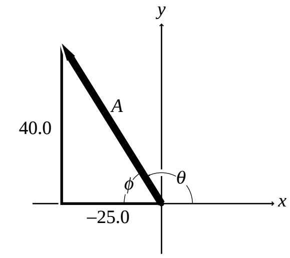
$$|\mathbf{B}| = \sqrt{(4.00)^2 + (6.00)^2 + (3.00)^2} = \boxed{7.81}$$

$$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°}$$

$$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°}$$

$$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°}$$

3.25 $A_x = -25.0 \quad A_y = 40.0$

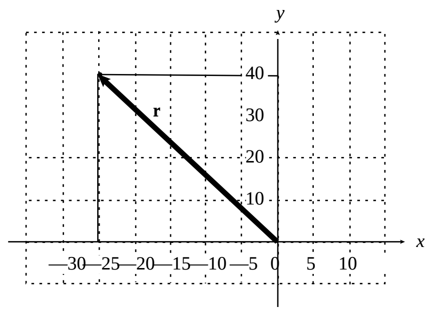


$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = 47.2$ units

From the triangle, we find that $|\phi| = 58.0°$, so that $\theta = 122°$

Goal Solution

A vector has an x component of –25.0 units and a y component of 40.0 units. Find the magnitude and direction of this vector.



G: First we should visualize the vector either in our mind or with a sketch. Since the hypotenuse of the right triangle must be greater than either the x or y components that form the legs, we can estimate the magnitude of the vector to be about 50 units. The direction of the vector appears to be about 120° from the $+x$ axis.

O: The graphical analysis and visual estimates above may be sufficient for some situations, but we can use trigonometry to obtain a more precise result.

A: The magnitude can be found by the Pythagorean theorem: $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{(-25.0 \text{ units})^2 + (40 \text{ units})^2} = 47.2 \text{ units}$$

We observe that $\tan\phi = \frac{y}{x}$ (if we consider x and y to both be positive) .

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°$$

The angle from the $+x$ axis can be found by subtracting from 180.

$$= 180 - 58 = 122°$$

L: Our calculated results agree with our graphical estimates. We should always remember to check that our answers are reasonable and make sense, especially for problems like this where it is easy to mistakenly calculate the wrong angle by confusing coordinates or overlooking a minus sign.
Quite often the direction angle of a vector can be specified in more than one way, and we must choose a notation that makes the most sense for the given problem. If compass directions were stated in this question, we could have reported the vector angle to be 32.0° west of north or a compass heading of 328°.

***3.26** The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$d_{\text{DCeast}} = d_{\text{DAeast}} + d_{\text{ACeast}} = 730 \cos 5.00° - 560 \sin 21.0° = 527$ miles.

$d_{\text{DCnorth}} = d_{\text{DAnorth}} + d_{\text{ACnorth}} = 730 \sin 5.00° + 560 \cos 21.0° = 586$ miles.

By the Pythagorean theorem, $d = \sqrt{(d_{\text{DCeast}})^2 + (d_{\text{DCnorth}})^2} = 788$ mi

Then $\tan\theta = \dfrac{d_{\text{DCnorth}}}{d_{\text{DCeast}}} = 1.11$ and $\theta = 48.0°$.

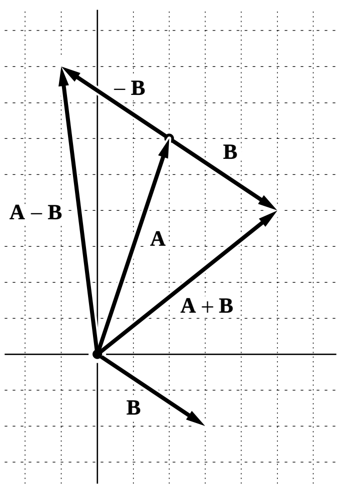
Thus, Chicago is $\boxed{\text{788 miles at 48.0° north east of Dallas}}$.

3.27 $x = d\cos\theta = (50.0\text{ m})\cos(120) = -25.0\text{ m}$

$y = d\sin\theta = (50.0\text{ m})\sin(120) = 43.3\text{ m}$

$\mathbf{d} = \boxed{(-25.0\text{ m})\mathbf{i} + (43.3\text{ m})\mathbf{j}}$

3.28 (a)



(b) $\mathbf{C} = \mathbf{A} + \mathbf{B} = 2.00\mathbf{i} + 6.00\mathbf{j} + 3.00\mathbf{i} - 2.00\mathbf{j} = \boxed{5.00\mathbf{i} + 4.00\mathbf{j}}$

$\mathbf{C} = \sqrt{25.0 + 16.0}$ at $\text{Arctan}\left(\dfrac{4}{5}\right)$

$\mathbf{C} = \boxed{6.40 \text{ at } 38.7°}$

$\mathbf{D} = \mathbf{A} - \mathbf{B} = 2.00\mathbf{i} + 6.00\mathbf{j} - 3.00\mathbf{i} + 2.00\mathbf{j} = \boxed{-1.00\mathbf{i} + 8.00\mathbf{j}}$

$\mathbf{D} = \sqrt{(-1.00)^2 + (8.00)^2}$ at $\text{Arctan}\left(\dfrac{8.00}{(-1.00)}\right)$

$\mathbf{D} = 8.06$ at $(180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$

3.29 $d = \sqrt{(x_1 + x_2 + x_3)^2 + (y_1 + y_2 + y_3)^2}$

$= \sqrt{(3.00 - 5.00 + 6.00)^2 + (2.00 + 3.00 + 1.00)^2} = \sqrt{52.0} = \boxed{7.21 \text{ m}}$

$\theta = \tan^{-1}\left(\frac{6.00}{4.00}\right) = \boxed{56.3°}$

3.30 $\mathbf{A} = -8.70\mathbf{i} + 15.0\mathbf{j}$ $\mathbf{B} = 13.2\mathbf{i} - 6.60\mathbf{j}$

$\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$

$3\mathbf{C} = \mathbf{B} - \mathbf{A} = 21.9\mathbf{i} - 21.6\mathbf{j}$

$\mathbf{C} = 7.30\mathbf{i} - 7.20\mathbf{j}$ or

$C_x = \boxed{7.30 \text{ cm}}$

$C_y = \boxed{-7.20 \text{ cm}}$

3.31 (a) $(\mathbf{A} + \mathbf{B}) = (3\mathbf{i} - 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = \boxed{2\mathbf{i} - 6\mathbf{j}}$

(b) $(\mathbf{A} - \mathbf{B}) = (3\mathbf{i} - 2\mathbf{j}) - (-\mathbf{i} - 4\mathbf{j}) = \boxed{4\mathbf{i} + 2\mathbf{j}}$

(c) $|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A} + \mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta_{|\mathbf{A} - \mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6°}$

3.32 Let **i** = east and **j** = north.

$\mathbf{R} = 3.00\text{b } \mathbf{j} + 4.00\text{b} \cos 45° \, \mathbf{i} + 4.00\text{b} \sin 45° \, \mathbf{j} - 5.00\text{b } \mathbf{i}$

$\mathbf{R} = -2.17\text{b } \mathbf{i} + 5.83\text{b } \mathbf{j}$

$\mathbf{R} = \sqrt{2.17^2 + 5.83^2}$ b at Arctan $\left(\frac{5.83}{2.17}\right)$ N of W

$= \boxed{\text{6.22 blocks at 110° counterclockwise from east}}$

3.33 $x = r\cos\theta$ and $y = r\sin\theta$, therefore:

(a) $x = 12.8\cos 150°$, $y = 12.8\sin 150°$, and $(x, y) = (-11.1\mathbf{i} + 6.40\mathbf{j})$ m

(b) $x = 3.30\cos 60.0°$, $y = 3.30\sin 60.0°$, and $(x, y) = (1.65\mathbf{i} + 2.86\mathbf{j})$ cm

(c) $x = 22.0\cos 215°$, $y = 22.0\sin 215°$, and $(x, y) = (-18.0\mathbf{i} - 12.6\mathbf{j})$ in

3.34 (a) $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C} = 2\mathbf{i} + 4\mathbf{j}$

$$|\mathbf{D}| = \sqrt{2^2 + 4^2} = \boxed{4.47\text{ m at } \theta = 63.4°}$$

(b) $\mathbf{E} = -\mathbf{A} - \mathbf{B} + \mathbf{C} = -6\mathbf{i} + 6\mathbf{j}$

$$|\mathbf{E}| = \sqrt{6^2 + 6^2} = \boxed{8.49\text{ m at } \theta = 135°}$$

3.35 $d_1 = (-3.50\mathbf{j})$ m

$d_2 = 8.20\cos 45.0°\mathbf{i} + 8.20\sin 45.0°\mathbf{j} = (5.80\mathbf{i} + 5.80\mathbf{j})$ m

$d_3 = (-15.0\mathbf{i})$ m

$\mathbf{R} = d_1 + d_2 + d_3 = (-15.0 + 5.80)\mathbf{i} + (5.80 - 3.50)\mathbf{j} = \boxed{(-9.20\mathbf{i} + 2.30\mathbf{j})\text{ m}}$

(or 9.20 m west and 2.30 m north)

The magnitude of the resultant displacement is

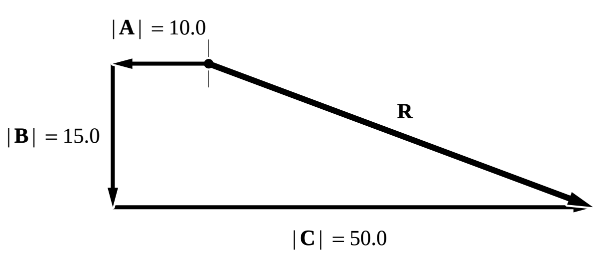
$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20)^2 + (2.30)^2} = \boxed{9.48\text{ m}}$$

The direction is $\theta = \text{Arctan}\left(\dfrac{2.30}{-9.20}\right) = \boxed{166°}$

3.36 Refer to the sketch

$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = -10.0\mathbf{i} - 15.0\mathbf{j} + 50.0\mathbf{i} = 40.0\mathbf{i} - 15.0\mathbf{j}$

$|\mathbf{R}| = [(40.0)^2 + (-15.0)^2]^{1/2} = \boxed{42.7\text{ yards}}$



3.37 (a) $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$$\mathbf{F} = 120\cos(60.0^\circ)\mathbf{i} + 120\sin(60.0^\circ)\mathbf{j} - 80.0\cos(75.0^\circ)\mathbf{i} + 80.0\sin(75.0^\circ)\mathbf{j}$$

$$\mathbf{F} = 60.0\mathbf{i} + 104\mathbf{j} - 20.7\mathbf{i} + 77.3\mathbf{j} = (39.3\mathbf{i} + 181\mathbf{j})\text{ N}$$

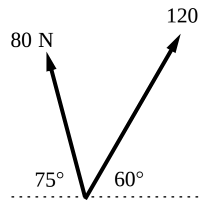
$$|\mathbf{F}| = \sqrt{(39.3)^2 + (181)^2} = \boxed{185\text{ N}}\ ;\ \theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8^\circ}$$

(b) $\mathbf{F}_3 = -\mathbf{F} = \boxed{(-39.3\mathbf{i} - 181\mathbf{j})\text{ N}}$

Goal Solution

The helicopter view in Figure P3.37 shows two people pulling on a stubborn mule. Find (a) the single force that is equivalent to the two forces shown and (b) the force that a third person would have to exert on the mule to make the resultant force equal to zero. The forces are measured in units of newtons.

G: The resultant force will be larger than either of the two individual forces, and since the two people are not pulling in exactly the same direction, the magnitude of the resultant should be less than the sum of the magnitudes of the two forces. Therefore, we should expect 120 N < R < 200 N. The angle of the resultant force appears to be straight ahead and perhaps slightly to the right. If the stubborn mule remains at rest, the ground must be exerting on the animal a force equal to the resultant **R** but in the opposite direction.



O: We can find **R** by adding the components of the two force vectors.

A: $\mathbf{F}_1 = (120\cos 60)\mathbf{i}\text{ N} + (120\sin 60)\mathbf{j}\text{ N} = 60.0\mathbf{i}\text{ N} + 103.9\mathbf{j}\text{ N}$

$\mathbf{F}_2 = -(80\cos 75)\mathbf{i}\text{ N} + (80\sin 75)\mathbf{j}\text{ N} = -20.7\mathbf{i}\text{ N} + 77.3\mathbf{j}\text{ N}$

$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = 39.3\mathbf{i}\text{ N} + 181.2\mathbf{j}\text{ N}$

$R = |\mathbf{R}| = \sqrt{(39.3)^2 + (181.2)^2} = 185\text{ N}$

The angle can be found from the arctan of the resultant components.

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{181.2}{39.3} = \tan^{-1}(4.61) = 77.8^\circ \text{ counterclockwise from the } +x \text{ axis}$$

The opposing force that the either the ground or a third person must exert on the mule, in order for the overall resultant to be zero, is 185 N at 258° counterclockwise from $+x$.

L: The resulting force is indeed between 120 N and 200 N as we expected, and the angle seems reasonable as well. The process applied to solve this problem can be used for other "statics" problems encountered in physics and engineering. If another force is added to act on a system that is already in equilibrium (sum of the forces is equal to zero), then the system may accelerate. Such a system is now a "dynamic" one and will be the topic of Chapter 5.

3.38

East x	North y
0 m	4.00 m
1.41	1.41
–0.500	–0.866
+0.914	4.55

$$|\mathbf{R}| = \sqrt{|x|^2 + |y|^2} = \boxed{4.64\text{ m at }78.6°\text{ N of E}}$$

3.39 $\mathbf{A} = 3.00$ m, $\theta_A = 30.0°$, $\mathbf{B} = 3.00$ m, $\theta_B = 90.0°$

$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60$ m, $A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50$ m

so, $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} = (2.60\mathbf{i} + 1.50\mathbf{j})$ m

$B_x = 0$, $B_y = 3.00$ m so $\mathbf{B} = 3.00\mathbf{j}$ m

$$\mathbf{A} + \mathbf{B} = (2.60\mathbf{i} + 1.50\mathbf{j}) + 3.00\mathbf{j} = \boxed{(2.60\mathbf{i} + 4.50\mathbf{j})\text{ m}}$$

***3.40** The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$ where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 268$ m/s. The position vector as a function of time is $\mathbf{P} = (268\text{ m/s})t\,\mathbf{i} + (7.60 \times 10^3\text{ m})\mathbf{j}$.

At $t = 45.0$ s, $\mathbf{P} = [1.21 \times 10^4\,\mathbf{i} + 7.60 \times 10^3\,\mathbf{j}]$ m. The magnitude is

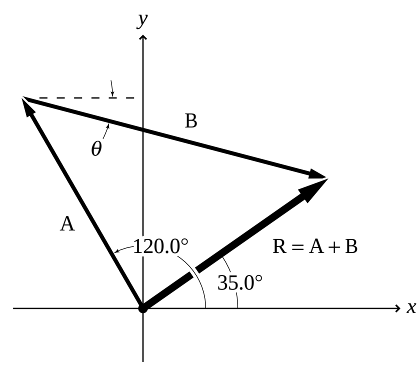
$$\mathbf{P} = \sqrt{(1.21 \times 10^4)^2 + (7.60 \times 10^3)^2}\text{ m} = \boxed{1.43 \times 10^4\text{ m}}$$

and the direction is

$$\theta = \text{Arctan}\left(\frac{7.60 \times 10^3}{1.21 \times 10^4}\right) =$$

$\boxed{32.2°\text{ above the horizontal}}$

3.41 We have $\mathbf{B} = \mathbf{R} - \mathbf{A}$



$$A_x = 150\cos 120° = -75.0\text{ cm}$$

$$A_y = 150\sin 120° = 130\text{ cm}$$

$$R_x = 140\cos 35.0° = 115\text{ cm}$$

$$R_y = 140\sin 35.0° = 80.3\text{ cm}$$

Therefore,

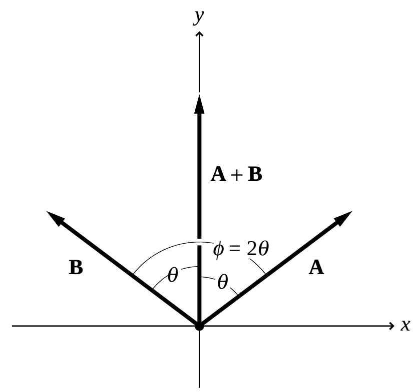
$$\mathbf{B} = [115 - (-75)]\mathbf{i} + [80.3 - 130]\mathbf{j} = (190\mathbf{i} - 49.7\mathbf{j})\text{ cm}$$

$$\left|\mathbf{B}\right| = [190^2 + (49.7)^2]^{1/2} = \boxed{196\text{ cm}}\ , \theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}$$

***3.42** Since **A** + **B** = 6.00**j**, we have $(A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} = 0\mathbf{i} + 6.00\,\mathbf{j}$ giving

$$A_x + B_x = 0, \text{ or } A_x = -B_x \qquad (1)$$

and $$A_y + B_y = 6.00 \qquad (2)$$



Since both vectors have a magnitude of 5.00, we also have:

$$A_x^2 + A_y^2 = B_x^2 + B_y^2 = (5.00)^2$$

From $A_x = -B_x$, it is seen that $A_x^2 = B_x^2$. Therefore $A_x^2 + A_y^2 = B_x^2 + B_y^2$ gives $A_y^2 = B_y^2$. Then $A_y = B_y$, and Equation (2) gives $A_y = B_y = 3.00$.

Defining θ as the angle between either **A** or **B** and the y axis, it is seen that

$$\cos\theta = \frac{A_y}{A} = \frac{B_y}{B} = \frac{3.00}{5.00} = 0.600$$

and $\theta = 53.1°$

The angle between **A** and **B** is then $\boxed{\phi = 2\theta = 106°}$.

3.43 (a) $\mathbf{A} = \boxed{8.00\mathbf{i} + 12.0\mathbf{j} - 4.00\,\mathbf{k}}$

(b) $\mathbf{B} = \mathbf{A}/4 = \boxed{2.00\mathbf{i} + 3.00\mathbf{j} - 1.00\mathbf{k}}$

(c) $\mathbf{C} = -3\mathbf{A} = \boxed{-24.0\mathbf{i} - 36.0\mathbf{j} + 12.0\mathbf{k}}$

3.44 $\mathbf{R} = 75.0\cos 240°\mathbf{i} + 75.0\sin 240°\mathbf{j} + 125\cos 135°\mathbf{i} + 125\sin 135°\mathbf{j} + 100\cos 160°\mathbf{i} + 100\sin 160°\mathbf{j}$

$\mathbf{R} = -37.5\mathbf{i} - 65.0\mathbf{j} - 88.4\mathbf{i} + 88.4\mathbf{j} - 94.0\mathbf{i} + 34.2\mathbf{j}$

$\mathbf{R} = \boxed{-220\mathbf{i} + 57.6\mathbf{j}}$

$\mathbf{R} = \sqrt{(-220)^2 + 57.6^2}$ at $\text{Arctan}\left(\frac{57.6}{220}\right)$ above the $-x$-axis

$\mathbf{R} = \boxed{227 \text{ paces at } 165°}$

3.45 (a) $\mathbf{C} = \mathbf{A} + \mathbf{B} = \boxed{(5.00\mathbf{i} - 1.00\mathbf{j} - 3.00\mathbf{k})\text{ m}}$

$$|\mathbf{C}| = \sqrt{(5.00)^2 + (1.00)^2 + (3.00)^2}\text{ m} = \boxed{5.92\text{ m}}$$

(b) $\mathbf{D} = 2\mathbf{A} - \mathbf{B} = \boxed{(4.00\mathbf{i} - 11.0\mathbf{j} + 15.0\mathbf{k})\text{ m}}$

$$|\mathbf{D}| = \sqrt{(4.00)^2 + (11.0)^2 + (15.0)^2}\ \text{m} = \boxed{19.0\ \text{m}}$$

***3.46** The displacement from radar station to ship is

$$\mathbf{S} = (17.3 \sin 136^\circ \mathbf{i} + 17.3 \cos 136^\circ \mathbf{j})\text{ km} = (12.0\mathbf{i} - 12.4\mathbf{j})\text{ km}$$

From station to plane, the displacement is

$$\mathbf{P} = (19.6 \sin 153^\circ \mathbf{i} + 19.6 \cos 153^\circ \mathbf{j} + 2.20\mathbf{k})\text{ km, or}$$

$$\mathbf{P} = (8.90\mathbf{i} - 17.5\mathbf{j} + 2.20\mathbf{k})\text{ km.}$$

(a) From plane to ship the displacement is

$$\mathbf{D} = \mathbf{S} - \mathbf{P} = \boxed{(3.12\mathbf{i} + 5.02\mathbf{j} - 2.20\mathbf{k})\text{ km}}$$

(b) The distance the plane must travel is

$$D = |\mathbf{D}| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2}\text{ km} = \boxed{6.31\text{ km}}$$

3.47 The hurricane's first displacement is $\left(\frac{41.0\text{ km}}{\text{h}}\right)(3.00\text{ h})$ at 60.0° N of W, and its second displacement is $\left(\frac{25.0\text{ km}}{\text{h}}\right)(1.50\text{ h})$ due North. With **i** representing east and **j** representing north, its total displacement is:

$$\left(41.0\,\frac{\text{km}}{\text{h}}\cos 60.0^\circ\right)(3.00\text{ h})(-\mathbf{i}) + \left(41.0\,\frac{\text{km}}{\text{h}}\sin 60.0^\circ\right)(3.00\text{ h})\,\mathbf{j}$$

$$+\left(25.0\,\frac{\text{km}}{\text{h}}\right)(1.50\text{ h})\,\mathbf{j} = 61.5\text{ km }(-\mathbf{i}) + 144\text{ km }\mathbf{j}$$

with magnitude $\sqrt{(61.5\text{ km})^2 + (144\text{ km})^2} = \boxed{157\text{ km}}$

***3.48** (a) $\mathbf{E} = (17.0\text{ cm})\cos 27.0^\circ \mathbf{i} + (17.0\text{ cm})\sin 27.0^\circ \mathbf{j}$

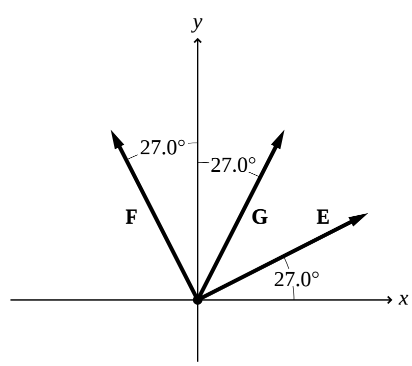
$\mathbf{E} = \boxed{(15.1\mathbf{i} + 7.72\mathbf{j})\text{ cm}}$

(b) $\mathbf{F} = -(17.0\text{ cm})\sin 27.0^\circ \mathbf{i} + (17.0\text{ cm})\cos 27.0^\circ \mathbf{j}$

$\mathbf{F} = \boxed{(-7.72\mathbf{i} + 15.1\mathbf{j})\text{ cm}}$

(c) $\mathbf{G} = +(17.0\text{ cm})\sin 27.0^\circ \mathbf{i} + (17.0\text{ cm})\cos 27.0^\circ \mathbf{j}$

$\mathbf{G} = \boxed{(+7.72\mathbf{i} + 15.1\mathbf{j})\text{ cm}}$



3.49 $A_x = -3.00$, $A_y = 2.00$

(a) $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} = \boxed{-3.00\mathbf{i} + 2.00\mathbf{j}}$

(b) $|\mathbf{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.00)^2 + (2.00)^2} = \boxed{3.61}$

$\tan\theta = \dfrac{A_y}{A_x} = \dfrac{2.00}{(-3.00)} = -0.667$, $\tan^{-1}(-0.667) = -33.7°$

θ is in the 2nd quadrant, so $\theta = 180° + (-33.7°) = \boxed{146°}$

(c) $\mathbf{R}_x = 0$, $\mathbf{R}_y = -4.00$, $\mathbf{R} = \mathbf{A} + \mathbf{B}$ thus $\mathbf{B} = \mathbf{R} - \mathbf{A}$ and

$B_x = R_x - A_x = 0 - (-3.00) = 3.00$, $B_y = R_y - A_y = -4.00 - 2.00 = -6.00$

Therefore, $\mathbf{B} = \boxed{3.00\mathbf{i} - 6.00\mathbf{j}}$

3.50 Let $+x$ = East, $+y$ = North,

x	y
300	0
–175	303
0	150
125	453

(a) $\theta = \tan^{-1}\dfrac{y}{x} = \boxed{74.6° \text{ N of E}}$

(b) $|\mathbf{R}| = \sqrt{x^2 + y^2} = \boxed{470 \text{ km}}$

3.51 Refer to Figure P3.51 in the textbook.

(a) $R_x = 40.0 \cos 45.0° + 30.0 \cos 45.0° = 49.5$

$R_y = 40.0 \sin 45.0° - 30.0 \sin 45.0° + 20.0 = 27.1$

$\mathbf{R} = \boxed{49.5\mathbf{i} + 27.1\mathbf{j}}$

(b) $|\mathbf{R}| = \sqrt{(49.4)^2 + (27.1)^2} = \boxed{56.4}$

$\theta = \tan^{-1}\left(\dfrac{27.1}{49.5}\right) = \boxed{28.7°}$

3.52 Taking components along **i** and **j**, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

$$-8.00a + 3.00b + 19.0 = 0$$

Solving simultaneously, $\boxed{a = 5.00,\ b = 7.00}$

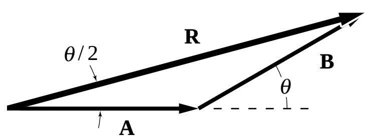
Therefore, $5.00\mathbf{A} + 7.00\mathbf{B} + \mathbf{C} = 0$

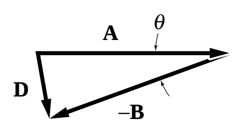
***3.53** Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\theta/2$, and $\theta/2$. The magnitude of **R** is then $R = 2A\cos(\theta/2)$. [*Hint:* apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity $(1 - \cos\theta) = 2\sin^2(\theta/2)$ gives the magnitude of **D** as $D = 2A\sin(\theta/2)$.

The problem requires that $R = 100D$.

Thus, $2A\cos(\theta/2) = 200A\sin(\theta/2)$. This gives $\tan(\theta/2) = 0.010$ and $\boxed{\theta = 1.15°}$.

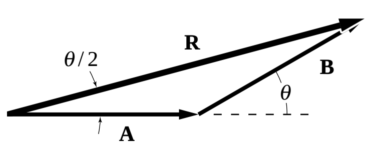


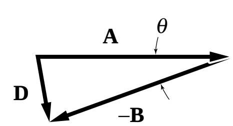


***3.54** Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\theta/2$, and $\theta/2$. The magnitude of **R** is then $R = 2A\cos(\theta/2)$. [*Hint:* apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity $(1 - \cos\theta) = 2\sin^2(\theta/2)$ gives the magnitude of **D** as $D = 2A\sin(\theta/2)$.

The problem requires that $R = nD$, or $\cos(\theta/2) = n\sin(\theta/2)$, giving $\boxed{\theta = 2\tan^{-1}(1/n)}$.





3.55 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

(b) $|\mathbf{R}| = \sqrt{R_x^2 + R_y^2 + R_Z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c) $\cos\theta_x = \dfrac{R_x}{|\mathbf{R}|} \Rightarrow \theta_x = \cos^{-1}\left(\dfrac{R_x}{|\mathbf{R}|}\right) = \boxed{57.7° \text{ from } + x}$

$\cos\theta_y = \dfrac{R_y}{|\mathbf{R}|} \Rightarrow \theta_y = \cos^{-1}\left(\dfrac{R_y}{|\mathbf{R}|}\right) = \boxed{74.5° \text{ from } + y}$

$\cos\theta_z = \dfrac{R_z}{|\mathbf{R}|} \Rightarrow \theta_z = \cos^{-1}\left(\dfrac{R_z}{|\mathbf{R}|}\right) = \boxed{36.7° \text{ from } + z}$

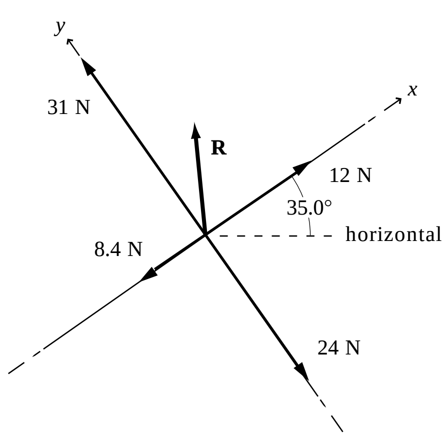
***3.56** Choose the $+x$-axis in the direction of the first force. The total force, in newtons, is then

$$12.0\mathbf{i} + 31.0\mathbf{j} - 8.40\mathbf{i} - 24.0\mathbf{j} = \boxed{(3.60\mathbf{i}) + (7.00\mathbf{j})\ \text{N}}$$

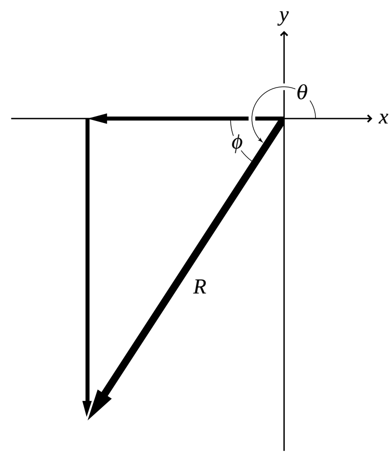
The magnitude of the total force is

$$\sqrt{(3.60)^2 + (7.00)^2}\ \text{N} = \boxed{7.87\ \text{N}}$$

and the angle it makes with our $+x$-axis is given by $\tan\theta = \dfrac{(7.00)}{(3.60)}$, $\theta = 62.8°$. Thus, its angle counterclockwise from the horizontal is $35.0° + 62.8° = \boxed{97.8°}$



3.57 $\mathbf{d}_1 = 100\mathbf{i}$ $\mathbf{d}_2 = -300\mathbf{j}$



$\mathbf{d}_3 = -150\cos(30.0°)\mathbf{i} - 150\sin(30.0°)\mathbf{j} = -130\mathbf{i} - 75.0\mathbf{j}$

$\mathbf{d}_4 = -200\cos(60.0°)\mathbf{i} + 200\sin(60.0°)\mathbf{j} = -100\mathbf{i} + 173\mathbf{j}$

$\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \mathbf{d}_4 = -130\mathbf{i} - 202\mathbf{j}$

$\left|\mathbf{R}\right| = [(-130)^2 + (-202)^2]^{1/2} = \boxed{240 \text{ m}}$

$\phi = \tan^{-1}\left(\frac{202}{130}\right) = 57.2°$

$\theta = 180 + \phi = \boxed{237°}$

***3.58** $d\mathbf{P}/dt = d(4\mathbf{i} + 3\mathbf{j} - 2t\,\mathbf{j})/dt = 0 + 0 - 2\mathbf{j} = \boxed{-(2.00 \text{ m/s})\mathbf{j}}$

The position vector at $t = 0$ is $4\mathbf{i} + 3\mathbf{j}$. At $t = 1$ s, the position is $4\mathbf{i} + 1\mathbf{j}$, and so on. The object is moving straight downward at 2 m/s, so

$d\mathbf{P}/dt$ represents $\boxed{\text{its velocity vector}}$.

3.59 $\mathbf{v} = v_x\mathbf{i} + v_y\,\mathbf{j} = (300 + 100\cos 30.0°)\mathbf{i} + (100\sin 30.0°)\mathbf{j}$

$\mathbf{v} = (387\mathbf{i} + 50.0\mathbf{j})$ mi/h

$\left|\mathbf{v}\right| = \boxed{390 \text{ mi/h at } 7.37° \text{ N of E}}$

3.60 (a) You start at $\mathbf{r}_1 = \mathbf{r}_A = 30.0 \text{ m } \mathbf{i} - 20.0 \text{ m } \mathbf{j}$.

The displacement to **B** is

$$\mathbf{r}_B - \mathbf{r}_A = 60.0\mathbf{i} + 80.0\mathbf{j} - 30.0\mathbf{i} + 20.0\mathbf{j} = 30.0\mathbf{i} + 100\mathbf{j}$$

You cover one–half of this, $15.0\mathbf{i} + 50.0\mathbf{j}$, to move to

$$\mathbf{r}_2 = 30.0\mathbf{i} - 20.0\mathbf{j} + 15.0\mathbf{i} + 50.0\mathbf{j} = 45.0\mathbf{i} + 30.0\mathbf{j}$$

Now the displacement from your current position to **C** is

$$\mathbf{r}_C - \mathbf{r}_2 = -10.0\mathbf{i} - 10.0\mathbf{j} - 45.0\mathbf{i} - 30.0\mathbf{j} = -55.0\mathbf{i} - 40.0\mathbf{j}$$

You cover one–third, moving to

$$\mathbf{r}_3 = \mathbf{r}_2 + \Delta\mathbf{r}_{23} = 45.0\mathbf{i} + 30.0\mathbf{j} + \frac{1}{3}(-55.0\mathbf{i} - 40.0\mathbf{j}) = 26.7\mathbf{i} + 16.7\mathbf{j}$$

The displacement from where you are to **D** is

$$\mathbf{r}_D - \mathbf{r}_3 = 40.0\mathbf{i} - 30.0\mathbf{j} - 26.7\mathbf{i} - 16.7\mathbf{j} = 13.3\mathbf{i} - 46.7\mathbf{j}$$

You traverse one–quarter of it, moving to

$$\mathbf{r}_4 = \mathbf{r}_3 + \frac{1}{4}(\mathbf{r}_D - \mathbf{r}_3) = 26.7\mathbf{i} + 16.7\mathbf{j} + \frac{1}{4}(13.3\mathbf{i} - 46.7\mathbf{j}) = 30.0\mathbf{i} + 5.00\mathbf{j}$$

The displacement from your new location to **E** is

$$\mathbf{r}_E - r_4 = -70.0\mathbf{i} + 60.0\mathbf{j} - 30.0\mathbf{i} - 5.00\mathbf{j} = -100\mathbf{i} + 55.0\mathbf{j}$$

of which you cover one–fifth, $-20.0\mathbf{i} + 11.0\mathbf{j}$, moving to

$$\mathbf{r}_4 + \Delta\mathbf{r}_{45} = 30.0\mathbf{i} + 5.00\mathbf{j} - 20.0\mathbf{i} + 11.0\mathbf{j} = 10.0\mathbf{i} + 16.0\mathbf{j}.$$

The treasure is at $\boxed{(10.0\text{ m}, 16.0\text{ m})}$

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\mathbf{r}_A + \frac{1}{2}(\mathbf{r}_B - \mathbf{r}_A) = \left(\frac{\mathbf{r}_A + \mathbf{r}_B}{2}\right)$$

then to

$$\frac{(\mathbf{r}_A + \mathbf{r}_B)}{2} + \frac{\mathbf{r}_C - (\mathbf{r}_A + \mathbf{r}_B)/2}{3} = \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3}$$

then to

$$\frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C}{3} + \frac{\mathbf{r}_D - (\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)/3}{4} = \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4}$$

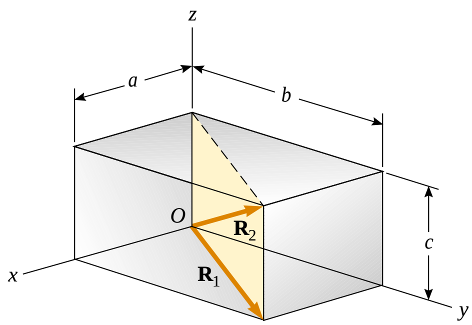
and at last to

$$\frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D}{4} + \frac{\mathbf{r}_E - (\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)/4}{5} = \frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E}{5}$$

This center of mass of the tree distribution is in the same location whatever order we take the trees in.

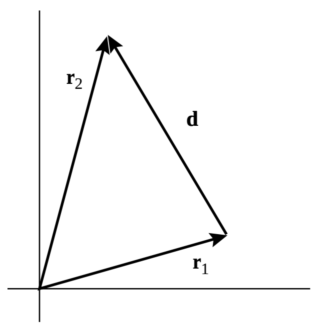
3.61 (a) From the picture $\mathbf{R}_1 = a\mathbf{i} + b\mathbf{j}$ and $\left|\mathbf{R}_1\right| = \sqrt{a^2 + b^2}$

(b) $\mathbf{R}_2 = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$. Its magnitude is $\sqrt{\left|\mathbf{R}_1\right|^2 + c^2} = \sqrt{a^2 + b^2 + c^2}$



3.62 (a) $\mathbf{r}_1 + \mathbf{d} = \mathbf{r}_2$ defines the displacement $\mathbf{d}$, so $\mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1$.

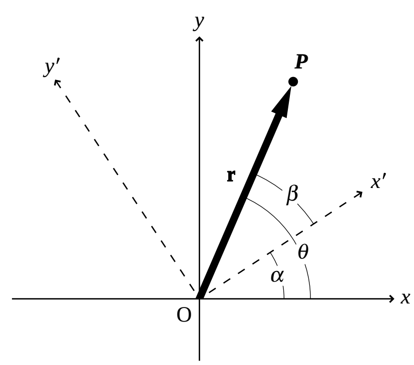
(b)



3.63 The displacement of point P is invariant under rotation of the coordinates.

Therefore, $r = r'$ and $r^2 = (r')^2$ or, $x^2 + y^2 = (x')^2 + (y')^2$

Also, from the figure, $\beta = \theta - \alpha$



$$\therefore \tan^{-1}\left(\frac{y'}{x'}\right) = \tan^{-1}\left(\frac{y}{x}\right) - \alpha$$

$$\frac{y'}{x'} = \frac{\left(\frac{y}{x}\right) - \tan\alpha}{1 + \left(\frac{y}{x}\right)\tan\alpha}$$

Which we simplify by multiplying top and bottom by $x\cos\alpha$. Then,

$x' = x\cos\alpha + y\sin\alpha$, $y' = -x\sin\alpha + y\cos\alpha$

Chapter 4 Solutions

***4.1**

x(m)	y(m)
0	– 3600
– 3000	0
– 1270	1270
– 4270 m	– 2330 m

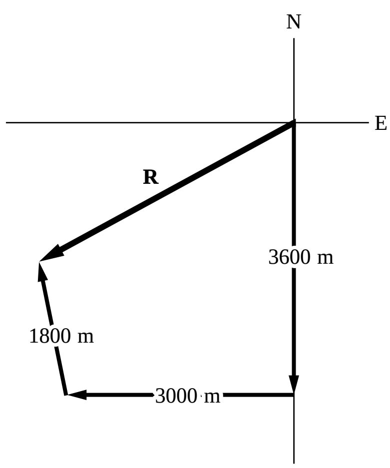
(a) Net displacement $= \sqrt{x^2 + y^2}$

$$= \boxed{4.87 \text{ km at } 28.6° \text{ S of W}}$$

(b) Average speed $= \dfrac{(20.0 \text{ m/s})(180 \text{ s}) + (25.0 \text{ m/s})(120 \text{ s}) + (30.0 \text{ m/s})(60.0 \text{ s})}{(180 \text{ s} + 120 \text{ s} + 60.0 \text{ s})}$

$$= \boxed{23.3 \text{ m/s}}$$

(c) Average velocity $= \dfrac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along } \mathbf{R}}$



4.2 (a) For the average velocity, we have

$$\bar{\mathbf{v}} = \left(\frac{x(4.00) - x(2.00)}{4.00 \text{ s} - 2.00 \text{ s}}\right)\mathbf{i} + \left(\frac{y(4.00) - y(2.00)}{4.00 \text{ s} - 2.00 \text{ s}}\right)\mathbf{j}$$

$$= \left(\frac{5.00 \text{ m} - 3.00 \text{ m}}{2.00 \text{ s}}\right)\mathbf{i} + \left(\frac{3.00 \text{ m} - 1.50 \text{ m}}{2.00 \text{ s}}\right)\mathbf{j}$$

$$\boxed{\bar{\mathbf{v}} = (1.00\mathbf{i} + 0.750\mathbf{j}) \text{ m/s}}$$

(b) For the velocity components, we have

$$v_x = \frac{dx}{dt} = a = 1.00 \text{ m/s}$$

$$v_y = \frac{dy}{dt} = 2ct = (0.250 \text{ m/s}^2)t$$

Therefore, $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = (1.00 \text{ m/s})\mathbf{i} + (0.250 \text{ m/s}^2)\, t\mathbf{j}$

$$\boxed{\mathbf{v}(2.00) = (1.00 \text{ m/s})\mathbf{i} + (0.500 \text{ m/s})\mathbf{j}}$$

and the speed is

$$\left|\mathbf{v}\right|_{t = 2.00 \text{ s}} = \sqrt{(1.00 \text{ m/s})^2 + (0.500 \text{ m/s})^2} = \boxed{1.12 \text{ m/s}}$$

4.3 (a) $\mathbf{r} = \boxed{18.0t\ \mathbf{i} + (4.00t - 4.90t^2)\mathbf{j}}$

(b) $\mathbf{v} = \boxed{(18.0 \text{ m/s})\mathbf{i} + [4.00 \text{ m/s} - (9.80 \text{ m/s}^2)t]\mathbf{j}}$

(c) $\mathbf{a} = \boxed{(-9.80 \text{ m/s}^2)\mathbf{j}}$

(d) $\mathbf{r}(3.00 \text{ s}) = \boxed{(54.0 \text{ m})\mathbf{i} - (32.1 \text{ m})\mathbf{j}}$

(e) $\mathbf{v}(3.00 \text{ s}) = \boxed{(18.0 \text{ m/s})\mathbf{i} - (25.4 \text{ m/s})\mathbf{j}}$

(f) $\mathbf{a}(3.00 \text{ s}) = \boxed{(-\ 9.80 \text{ m/s}^2)\mathbf{j}}$

4.4 (a) From $x = -5.00 \sin\omega t$, the x-component of velocity is

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt}\right)(-5.00\omega \sin\omega t) = -5.00\omega \cos\omega t$$

and $a_x = \dfrac{dv_x}{dt} = +5.00\omega^2 \sin\omega t$

similarly, $v_y = \left(\dfrac{d}{dt}\right)(4.00 - 5.00 \cos\omega t) = 0 + 5.00\omega \sin\omega t$

and $a_y = \left(\dfrac{d}{dt}\right)(5.00\omega \sin\omega t) = 5.00\omega^2 \cos\omega t$

At $t = 0$, $\mathbf{v} = -5.00\omega \cos 0\ \mathbf{i} + 5.00\omega \sin 0\ \mathbf{j} = \boxed{(-5.00\omega\ \mathbf{i} + 0\ \mathbf{j}) \text{ m/s}}$

and $\mathbf{a} = 5.00\omega^2 \sin 0\ \mathbf{i} + 5.00\omega^2 \cos 0\ \mathbf{j} = \boxed{(0\ \mathbf{i} + 5.00\omega^2\ \mathbf{j}) \text{ m/s}^2}$

(b) $\mathbf{r} = x\mathbf{i} + y\mathbf{j} = \boxed{(4.00\text{ m})\mathbf{j} + (5.00\text{ m})(-\sin\omega t\,\mathbf{i} - \cos\omega t\,\mathbf{j})}$

$\mathbf{v} = \boxed{(5.00\text{ m})\omega[-\cos\omega t\,\mathbf{i} + \sin\omega t\,\mathbf{j}]}$

$a = \boxed{(5.00\text{ m})\omega^2[\sin\omega t\,\mathbf{i} + \cos\omega t\,\mathbf{j}]}$

(c) The object moves in $\boxed{\text{a circle of radius 5.00 m centered at (0, 4.00 m)}}$.

4.5 (a) $\mathbf{v} = \mathbf{v}_\text{i} + \mathbf{a}t$

$$\mathbf{a} = \frac{(\mathbf{v} - \mathbf{v}_\text{i})}{t} = \frac{[(9.00\mathbf{i} + 7.00\mathbf{j}) - (3.00\mathbf{i} - 2.00\mathbf{j})]}{3.00} = \boxed{(2.00\mathbf{i} + 3.00\mathbf{j})\text{ m/s}^2}$$

(b) $\mathbf{r} = \mathbf{r}_\text{i} + \mathbf{v}_\text{i}t\frac{1}{2}\,\mathbf{a}t^2 = (3.00\mathbf{i} - 2.00\mathbf{j})t + \frac{1}{2}(2.00\mathbf{i} + 3.00\mathbf{j})\,t^2;$

$\boxed{x = (3.00t + t^2)\text{ m}}$ and $\boxed{y = (1.50t^2 - 2.00t)\text{ m}}$

4.6 (a) $v = \dfrac{dr}{dt} = \left(\dfrac{d}{dt}\right)(3.00\mathbf{i} - 6.00t^2\mathbf{j}) = \boxed{-12.0t\mathbf{j}\text{ m/s}}$

$a = \dfrac{dv}{dt} = \left(\dfrac{d}{dt}\right)(-12.0t\mathbf{j}) = \boxed{-12.0\mathbf{j}\text{ m/s}^2}$

(b) $\boxed{\mathbf{r} = (3.00\mathbf{i} - 6.00\mathbf{j})\text{ m};\ \mathbf{v} = -12.0\mathbf{j}\text{ m/s}}$

4.7 $\mathbf{v}_\text{i} = (4.00\mathbf{i} + 1.00\mathbf{j})$ m/s and $\mathbf{v}(20.0) = (20.0\mathbf{i} - 5.00\mathbf{j})$ m/s.

(a) $a_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{20.0 - 4.00}{20.0}\text{ m/s}^2 = \boxed{0.800\text{ m/s}^2}$

$a_y = \dfrac{\Delta v_y}{\Delta t} = \dfrac{-5.00 - 1.00}{20.0}\text{ m/s}^2 = \boxed{-0.300\text{ m/s}^2}$

(b) $\theta = \tan^{-1}\left[\dfrac{-0.300}{0.800}\right] = -20.6° = \boxed{339°\text{ from }+x\text{ axis}}$

(c) At $t = 25.0$ s,

$$x = x_i + v_{xi}t + \frac{1}{2}\,a_x t^2 = 10.0 + 4.00(25.0) + \frac{1}{2}(0.800)(25.0)^2 = \boxed{360\text{ m}}$$

$$y = y_i + v_{yi}t + \frac{1}{2}\,a_y t^2 = -4.00 + 1.00(25.0) + \frac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2°}$$

***4.8** $\mathbf{a} = 3.00\mathbf{j}\ \mathrm{m/s^2}$; $\mathbf{v}_i = 5.00\mathbf{i}\ \mathrm{m/s}$; $\mathbf{r}_i = 0\mathbf{i} + 0\mathbf{j}$

(a) $$\mathbf{r} = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 = \boxed{\left[5.00t\mathbf{i} + \frac{1}{2}3.00t^2\mathbf{j}\right]\mathrm{m}}$$

$$\mathbf{v} = \mathbf{v}_i + \mathbf{a}t = \boxed{(5.00\mathbf{i} + 3.00t\mathbf{j})\ \mathrm{m/s}}$$

(b) $$t = 2.00\ \mathrm{s},\ \mathbf{r} = (5.00)(2.00)\mathbf{i} + \frac{1}{2}(3.00)(2.00)^2\mathbf{j} = (10.0\mathbf{i} + 6.00\mathbf{j})\ \mathrm{m}$$

so $x = \boxed{10.0\ \mathrm{m}}$, $y = \boxed{6.00\ \mathrm{m}}$

$$\mathbf{v} = 5.00\mathbf{i} + (3.00)(2.00)\mathbf{j} = (5.00\mathbf{i} + 6.00\mathbf{j})\ \mathrm{m/s}$$

$$v = |\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81\ \mathrm{m/s}}$$

4.9 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x = v_{xi}t$. i.e., $t = \frac{x}{v_{xi}} = \frac{(1.40\ \mathrm{m})}{v_{xi}}$. In the same time it falls a distance 0.860 m with acceleration downward of $9.80\ \mathrm{m/s^2}$. Then using

$$y = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

we have

$$0 = 0.860\ \mathrm{m} - \frac{1}{2}(9.80\ \mathrm{m/s^2})\left(\frac{1.40\ \mathrm{m}}{v_{xi}}\right)^2$$

i.e., $$v_{xi} = \sqrt{\frac{(4.90\ \mathrm{m/s^2})(1.96\ \mathrm{m^2})}{0.860\ \mathrm{m}}} = \boxed{3.34\ \mathrm{m/s}}$$

(b) The vertical velocity component with which it hits the floor is

$$v_y = v_{yi} + a_y t = -(9.80\ \mathrm{m/s^2})\left(\frac{1.40\ \mathrm{m}}{3.34\ \mathrm{m/s}}\right) = -4.11\ \mathrm{m/s}$$

Hence, the angle θ at which the mug strikes the floor is given by

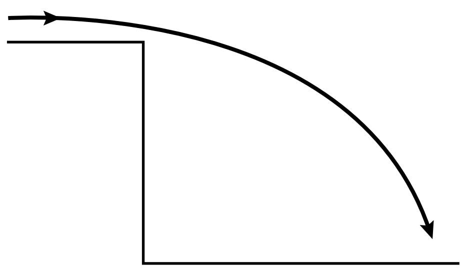
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9°}$$

Goal Solution

G: Based on our everyday experiences and the description of the problem, a reasonable speed of the mug would be a few m / s and it will hit the floor at some angle between 0 and 90°, probably about 45°.

O: We are looking for two different velocities, but we are only given two distances. Our approach will be to separate the vertical and horizontal motions. By using the height that the mug falls, we can find the time of the fall. Once we know the time, we can find the horizontal and vertical components of the velocity. For convenience, we will set the origin to be the point where the mug leaves the counter.

A: Vertical motion: $y = -0.860$ m, $v_{yi} = 0$, v_y = unknown, $a_y = -9.80$ m / s^2
Horizontal motion: $x = 1.40$ m, v_x = constant (unknown), $a_x = 0$



(a) To find the time of fall, we use the free fall equation: $y = v_{yi}t + \frac{1}{2}\,a_y t^2$

Solving: $-0.860\text{ m} = 0 + \frac{1}{2}(-9.80\text{ m/s}^2)\,t^2$ so that $t = 0.419$ s

Then $v_x = \frac{x}{t} = \frac{1.40\text{ m}}{0.419\text{ s}} = 3.34\text{ m/s}$

(b) As the mug hits the floor, $v_y = v_{yi} + a_y t = 0 - (9.8\text{ m/s}^2)(0.419\text{ s}) = -4.11\text{ m/s}$

The impact angle is $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4.11\text{ m/s}}{3.34\text{ m/}}\right) = 50.9°$ below the horizontal

L: This was a multi-step problem that required several physics equations to solve; our answers do agree with our initial expectations. Since the problem did not ask for the time, we could have eliminated this variable by substitution, but then we would have had to substitute the algebraic expression $t = 2y/g$ into two other equations. So in this case it was easier to find a numerical value for the time as an intermediate step. Sometimes the most efficient method is not realized until each alternative solution is attempted.

***4.10** The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x = v_{xi}t + \frac{1}{2}a_x t^2 = v_{xi}t + 0 \text{ and } y = v_{yi}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2$$

When the mug reaches the floor, $y = -h$, so $-h = -\frac{1}{2}gt^2$ which gives the time of impact as $t = \sqrt{\frac{2h}{g}}$.

(a) Since $x = d$ when the mug reaches the floor, $x = v_{xi}t$ becomes

$$d = v_{xi}\sqrt{\frac{2h}{g}}$$

giving the initial velocity as $\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$.

(b) Just before impact, the x-component of velocity is still $v_{xf} = v_{xi}$ while the y-component is $v_{yf} = v_{yi} + a_y t = 0 - g\sqrt{\frac{2h}{g}}$. Then the direction of motion just before impact is below the horizontal at an angle of $\theta = \tan^{-1}\left(\frac{|v_{yf}|}{v_{xf}}\right)$, or

$$\theta = \tan^{-1}\left(g\sqrt{\frac{2h}{g}} \bigg/ d\sqrt{\frac{g}{2h}}\right) = \boxed{\tan^{-1}\left(\frac{2h}{d}\right)}$$

4.11 (a) The time of flight of the first snowball is the nonzero root of

$$y = y_i + v_{yi}t_1 + \frac{1}{2}a_y t_1^2$$

$$0 = 0 + (25.0 \text{ m/s}) \sin 70.0° t_1 - \frac{1}{2}(9.80 \text{ m/s}^2)\, t_1^2$$

$$t_1 = \frac{2(25.0 \text{ m/s}) \sin 70.0°}{9.80 \text{ m/s}^2} = 4.79 \text{ s}$$

The distance to your target is

$$x - x_i = v_{xi}t_1 = (25.0 \text{ m/s}) \cos 70.0° \,(4.79 \text{ s}) = 41.0 \text{ m}$$

Now the second snowball we describe by

$$y = y_i + v_{yi}t_2 + \frac{1}{2}\ a_y t_2^2$$

$$0 = (25.0\text{ m/s})\sin\theta_2 t_2 - (4.90\text{ m/s}^2)t_2^2$$

$$t_2 = (5.10\text{ s})\sin\theta_2$$

$$x - x_i = v_{xi}t_2$$

$$41.0\text{ m} = (25.0\text{ m/s})\cos\theta_2\,(5.10\text{ s})\sin\theta_2 = (128\text{ m})\sin\theta_2\cos\theta_2$$

$$0.321 = \sin\theta_2\cos\theta_2$$

Using $\sin 2\theta = 2\sin\theta\cos\theta$ we can solve

$$0.321 = \frac{1}{2}\sin 2\theta_2 \qquad 2\theta_2 = \text{Arcsin } 0.643 \qquad \boxed{\theta_2 = 20.0°}$$

(b) The second snowball is in the air for time $t_2 = (5.10\text{ s})\sin\theta_2 = (5.10\text{ s})\sin 20.0° = 1.75\text{ s}$, so you throw it after the first by

$$t_1 - t_2 = 4.79\text{ s} - 1.75\text{ s} = \boxed{3.05\text{ s}}\,.$$

***4.12** $y = v_i(\sin 3.00°)t - \frac{1}{2}\ gt^2$, $v_y = v_i\sin 3.00° - gt$

When $y = 0.330$ m, $v_y = 0$ and $v_i\sin 3.00° = gt$

$$y = v_i(\sin 3.00°)\frac{v_i\sin 3.00°}{g} - \frac{1}{2}\ g\left(\frac{v_i\ \sin 3.00°}{g}\right)^2$$

$$y = \frac{v_i^2\sin^2 3.00°}{2g} = 0.330\text{ m}$$

$$\therefore\ v_i = \boxed{48.6\text{ m/s}}$$

The 12.6 m is unnecessary information.

***4.13** $x = v_{xi}t = v_i\cos\theta_i t$

$$x = (300\text{ m/s})(\cos 55.0°)(42.0\text{ s})$$

$$x = \boxed{7.23\times 10^3\text{ m}}$$

$$y = v_{yi}t - \frac{1}{2}\ gt^2 = v_i\ \sin\theta_i t - \frac{1}{2}\ gt^2$$

$$y = (300\text{ m/s})(\sin 55.0°)(42.0\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(42.0\text{ s})^2 = \boxed{1.68 \times 10^3\text{ m}}$$

***4.14** From Equation 4.14,

$$R = 15.0\text{ m},\ v_i = 3.00\text{ m/s},\ \theta_{\max} = 45.0°$$

$$\therefore g = \frac{v_i^2}{R} = \frac{9.00}{15.0} = \boxed{0.600\text{ m/s}^2}$$

4.15 $h = \dfrac{v_i^2 \sin^2 \theta_i}{2g}$; $R = \dfrac{v_i^2 (\sin 2\theta_i)}{g}$; $3h = R$,

so $\dfrac{3v_i^2 \sin^2 \theta_i}{2g} = \dfrac{v_i^2 (\sin 2\theta_i)}{g}$

or $\dfrac{2}{3} = \dfrac{\sin^2 \theta_i}{\sin 2\theta_i} = \dfrac{\tan \theta_i}{2}$

thus $\theta_i = \tan^{-1}\left(\dfrac{4}{3}\right) = \boxed{53.1°}$

4.16 (a) $x = v_{xi}t = (8.00 \cos 20.0°)(3.00) = \boxed{22.6\text{ m}}$

(b) Taking y positive downwards,

$$y = v_{yi}t + \frac{1}{2}gt^2$$

$$= 8.00(\cos 20.0°)3.00 + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\text{ m}}$$

(c) $10.0 = 8.00 \cos 20.0°\ t + \dfrac{1}{2}(9.80)\ t^2$

$4.90t^2 + 2.74t - 10.0 = 0$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\text{ s}}$$

4.17 $x = v_{xi}t$

$2000 \text{ m} = (1000 \text{ m/s}) \cos \theta_i t$

$$t = \frac{2.00 \text{ s}}{\cos \theta_i}$$

$$y = v_{yi}t + \frac{1}{2} a_y t^2$$

$$800 \text{ m} = (1000 \text{ m/s}) \sin \theta_i t - \frac{1}{2}(9.80 \text{ m/s}^2)\, t^2$$

$$800 \text{ m} = (1000 \text{ m/s}) \sin \theta_i \left(\frac{2.00 \text{ s}}{\cos \theta_i}\right) - \frac{1}{2}(9.80 \text{ m/s}^2)\left(\frac{2.00 \text{ s}}{\cos \theta_i}\right)^2$$

$$800 \text{ m} \cos^2 \theta_i = (2000 \text{ m}) \sin \theta_i \cos \theta_i - 19.6 \text{ m}$$

$$19.6 \text{ m} + 800 \text{ m} \cos^2 \theta_i = 2000 \text{ m} \sqrt{1 - \cos^2 \theta_i} \cos \theta_i$$

$$384 \text{ m}^2 + 31\,360 \text{ m}^2 \cos^2 \theta_i + 640\,000 \text{ m}^2 \cos^4 \theta_i$$

$$= 4\,000\,000 \text{ m}^2 \cos^2 \theta_i - 4\,000\,000 \text{ m}^2 \cos^4 \theta_i$$

$$4\,640\,000 \cos^4 \theta_i - 3\,968\,640 \cos^2 \theta_i + 384 = 0$$

$$\cos^2 \theta_i = \frac{3\,968\,640 \pm \sqrt{(3\,968\,640)^2 - 4(4\,640\,000)(384)}}{9\,280}$$

$\cos \theta_i = 0.925$ or 0.00984

$\theta_i = \boxed{22.4° \quad \text{or} \quad 89.4°}$ Both solutions are valid.

***4.18** The equation $y = (\tan \theta_i)x - \left(\dfrac{g}{2v_i^2 \cos^2 \theta_i}\right) x^2$ describes the trajectory of the projectile. When y is a maximum (at $x = x_h$), the slope is zero $\left(\text{ie., } \dfrac{dy}{dx} = 0 \text{ at } x = x_h\right)$. This gives $\left(\dfrac{dy}{dx}\right)_{x = x_h} = \tan \theta_i - \left(\dfrac{g}{2v_i^2 \cos^2 \theta_i}\right) 2x_h = 0$, so the x-coordinate at which the maximum height occurs is $\boxed{x_h = \dfrac{v_i^2 \sin \theta_i \cos \theta_i}{g}}$. The maximum-height point is halfway through the entire symmetrical trajectory. Thus, the horizontal range is $R = 2x_h = \dfrac{v_i^2\, 2 \sin \theta_i \cos \theta_i}{g} = \boxed{\dfrac{v_i^2 \sin 2\theta_i}{g}}$.

4.19 (a) We use Equation 4.12:

$$y = x \tan \theta_i - \frac{gx^2}{2v_i^2 \cos^2 \theta_i}$$

With $x = 36.0$ m, $v_i = 20.0$ m / s, and $\theta = 53.0°$, we find

$$y = (36.0 \text{ m})(\tan 53.0°) - \frac{(9.80 \text{ m/s}^2)(36.0 \text{ m})^2}{(2)(20.0 \text{ m/s})^2 \cos^2 53.0°} = 3.94 \text{ m}$$

The ball clears the bar by (3.94 – 3.05) m = $\boxed{0.889 \text{ m}}$.

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin \theta_i}{g} = \frac{(20.0 \text{ m/s})(\sin 53.0°)}{9.80 \text{ m/s}^2} = 1.63 \text{ s}$$

The time to travel 36.0 m horizontally is $t_2 = \dfrac{x}{v_{ix}}$

$$t_2 = \frac{36.0 \text{ m}}{(20.0 \text{ m/s})(\cos 53.0°)} = 2.99 \text{ s}$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

4.20 $(40.0 \text{ m/s})(\cos 30.0°)t = 50.0$ m. (Eq. 4.10)

The stream of water takes $t = 1.44$ s to reach the building, which it strikes at a height

$$y = v_{yi}t - \frac{1}{2}gt^2$$

$$= (40.0 \sin 30.0°)t - \frac{1}{2}(9.80)\, t^2 = (40.0)\left(\frac{1}{2}\right)(1.44) \quad - (4.90)(1.44)^2 = \boxed{18.7 \text{ m}}$$

4.21 From Equation 4.10, $x = v_{xi}t = (v_i \cos \theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \dfrac{d}{v_i \cos \theta_i}$. At this time, the altitude of the water is

$$y = v_{yi}t + \frac{1}{2}a_y t^2 = (v_i \sin \theta_i)\left(\frac{d}{v_i \cos \theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i \cos \theta_i}\right)^2$$

Therefore the water strikes the building at a height of $\boxed{y = d \tan \theta_i - \dfrac{gd^2}{2v_i^2 \cos^2 \theta_i}}$ above ground level.

4.22 The horizontal kick gives zero vertical velocity to the ball. Then its time of flight follows from

$$y = y_i + v_{yi}t + \frac{1}{2}\ a_y t^2$$

$$-40.0\text{ m} = 0 + 0 + \frac{1}{2}(-9.80\text{ m/s}^2)\ t^2$$

$$t = 2.86\text{ s}$$

The extra time 3.00 s – 2.86 s = 0.143 s is the time required for the sound she hears to travel straight back to the player. It covers distance

$$(343\text{ m/s})0.143\text{ s} = 49.0\text{ m} = \sqrt{x^2 + (40.0\text{ m})^2}$$

where x represents the horizontal distance the ball travels.

$$x = 28.3\text{ m} = v_{xi}t + 0t^2$$

$$\therefore\ v_{xi} = \frac{28.3\text{ m}}{2.86\text{ s}} = \boxed{9.91\text{ m/s}}$$

***4.23** From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80\text{ m/s}^2)(1.85 - 1.02)\text{ m}$. From this, $v_{yi} = 4.03\text{ m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation becomes

$$v_{yf}^2 = 0 + 2(-9.80\text{ m/s}^2)(0.900 - 1.85)\text{ m, giving } v_{yf} = -4.32\text{ m/s}$$

as the vertical velocity just before he lands.

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$ as follows:

$$-4.32\text{ m/s} = 4.03\text{ m/s} + (-9.80\text{ m/s}^2)t$$

or $\boxed{t = 0.852\text{ s}}$

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes $2.80\text{ m} = v_{xi}(0.852\text{ s})$, which yields $\boxed{v_{xi} = 3.29\text{ m/s}}$.

(c) $\boxed{v_{yi} = 4.03\text{ m/s}}$ See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\dfrac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\dfrac{4.03\text{ m/s}}{3.29\text{ m/s}}\right) = \boxed{50.8°}$.

(e) Similarly for the deer, the upward part of the flight gives

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i):\ 0 = v_{yi}^2 + 2(-9.80\text{ m/s}^2)(2.50 - 1.20)\text{ m}$$

so $v_{yi} = 5.04\text{ m/s}$

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields

$$v_{yf}^2 = 0 + 2(-9.80\text{ m/s}^2)(0.700 - 2.50)\text{ m}$$

and $v_{yf} = -5.94\text{ m/s}$

The hang time is then found as

$$v_{yf} = v_{yi} + a_y t:\ -5.94\text{ m/s} = 5.04\text{ m/s} + (-9.80\text{ m/s}^2)t$$

and $\boxed{t = 1.12\text{ s}}$

4.24 (a) $v = \dfrac{\Delta x}{\Delta t} = \dfrac{2\pi(3.84\times10^8\text{ m})}{[(27.3\text{ d})(24\text{ h/d})(3600\text{ s/h})]} = \boxed{1.02\times10^3\text{ m/s}}$

(b) Since v is constant and only direction changes,

$$a = \frac{v^2}{r} = \frac{(1.02\times10^3)^2}{(3.84\times10^8)} = \boxed{2.72\times10^{-3}\text{ m/s}^2}$$

4.25 $a_r = \dfrac{v^2}{r} = \dfrac{(20.0\text{ m/s})^2}{(1.06\text{ m})} = \boxed{377\text{ m/s}^2}$

The mass is unnecessary information.

4.26 $a = \dfrac{v^2}{R}$ $\qquad T = (24\text{ h})\left(\dfrac{3600\text{ s}}{\text{h}}\right) = 86400\text{ s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37\times10^6\text{ m})}{86400\text{ s}} = 463\text{ m/s}$$

$a = \dfrac{(463\text{ m/s})^2}{6.37\times10^6\text{ m}} = \boxed{0.0337\text{ m/s}^2}$ (directed toward the center of the Earth)

4.27 $r = 0.500\text{ m};\ v_t = \dfrac{2\pi r}{T} = \dfrac{2\pi(0.500\text{ m})}{(60.0\text{ s}/200\text{ rev})} = 10.47\text{ m/s}$ $\qquad \boxed{10.5\text{ m/s}}$

$$a = \frac{v^2}{r} = \frac{(10.47)^2}{0.5} = \boxed{219\text{ m/s}^2\text{ (inward)}}$$

***4.28** The centripetal acceleration is $a_r = \dfrac{v^2}{r}$, so the required speed is

$$v = \sqrt{a_r r} = \sqrt{1.40(9.80\text{ m/s}^2)(10.0\text{ m})} = 11.7\text{ m/s}$$

The period (time for one rotation) is given by $T = 2\pi r / v$ and the rotation rate is the frequency:

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{11.7\text{ m/s}}{2\pi(10.0\text{ m})} = \boxed{0.186\text{ s}^{-1}}$$

4.29. (a) $v = r\omega$

At 8.00 rev/s, $v = (0.600\text{ m})(8.00\text{ rev/s})(2\pi\text{ rad/rev}) = 30.2\text{ m/s} = 9.60\pi\text{ m/s}$

At 6.00 rev/s, $v = (0.900\text{ m})(6.00\text{ rev/s})(2\pi\text{ rad/rev}) = 33.9\text{ m/s} = 10.8\pi\text{ m/s}$

$\boxed{6.00\text{ rev/s}}$ gives the larger linear speed.

(b) Acceleration $= \dfrac{v^2}{r} = \dfrac{(9.60\pi\text{ m/s})^2}{0.600\text{ m}} = \boxed{1.52 \times 10^3\text{ m/s}^2}$

(c) At 6.00 rev/s, acceleration $= \dfrac{(10.8\pi\text{ m/s})^2}{0.900\text{ m}} = \boxed{1.28 \times 10^3\text{ m/s}^2}$

***4.30.** The satellite is in free fall. Its acceleration is due to the acceleration of gravity and is by effect a centripetal acceleration:

$$a_r = g$$

$$\frac{v^2}{r} = g$$

$$v = \sqrt{rg} = \sqrt{(6400 + 600)(10^3\text{ m})(8.21\text{ m/s}^2)} = \boxed{7.58 \times 10^3\text{ m/s}}$$

$$v = \frac{2\pi r}{T} \quad \text{and} \quad T = \frac{2\pi r}{v} = \frac{2\pi(7000 \times 10^3\text{ m})}{(7.58 \times 10^3\text{ m/s})} = \boxed{5.80 \times 10^3\text{ s}}$$

$$T = (5.80 \times 10^3\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 96.7\text{ min}$$

4.31 We assume the train is still slowing down at the instant in question.

$$a_r = \frac{v^2}{r} = 1.29\ \mathrm{m/s^2}$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0\ \mathrm{km/h})(10^3\ \mathrm{m/km})(1\ \mathrm{h}/3600\ \mathrm{s})}{15.0\ \mathrm{s}} = -0.741\ \mathrm{m/s^2}$$

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1.29\ \mathrm{m/s^2})^2 + (-0.741\ \mathrm{m/s^2})^2}$$

$$= \boxed{1.48\ \mathrm{m/s^2}\ \text{inward and}\ 29.9°\ \text{backward}}$$

Goal Solution

G: If the train is taking this turn at a safe speed, then its acceleration should be significantly less than g, perhaps a few $\mathrm{m/s^2}$ (otherwise it might jump the tracks!), and it should be directed toward the center of the curve and backward since the train is slowing.

O: Since the train is changing both its speed and direction, the acceleration vector will be the vector sum of the tangential and radial acceleration components. The tangential acceleration can be found from the changing speed and elapsed time, while the radial acceleration can be found from the radius of curvature and the train's speed.

A: First, let's convert the speeds to units from km/h to m/s:

$$v_i = 90.0\ \mathrm{km/h} = (90.0\ \mathrm{km/h})\left(\frac{10^3\ \mathrm{m}}{1\ \mathrm{km}}\right)\left(\frac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) = 25.0\ \mathrm{m/s}$$

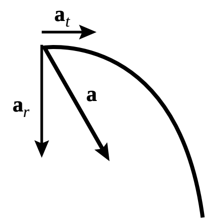
$$v_f = 50.0\ \mathrm{km/h} = (50.0\ \mathrm{km/h})\left(\frac{10^3\ \mathrm{m}}{1\ \mathrm{km}}\right)\left(\frac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) = 13.9\ \mathrm{m/s}$$

Tangential accel.: $a_t = \dfrac{\Delta v}{\Delta t} = \dfrac{13.9\ \mathrm{m/s} - 25.0\ \mathrm{m/s}}{15.0\ \mathrm{s}} = -0.741\ \mathrm{m/s^2}$ (backward)

Radial acceleration: $a_r = \dfrac{v^2}{r} = \dfrac{(13.9\ \mathrm{m/s})^2}{150\ \mathrm{m}} = 1.29\ \mathrm{m/s^2}$ (inward)

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(-0.741\ \mathrm{m/s^2})^2 + (1.29\ \mathrm{m/s^2})^2} = 1.48\ \mathrm{m/s^2}$$

$$\theta = \tan^{-1}\left(\frac{a_t}{a_r}\right) = \tan^{-1}\left(\frac{0.741\ \mathrm{m/s^2}}{1.29\ \mathrm{m/s^2}}\right) = 29.9°\ \text{(backwards from a radial line)}$$



L: The acceleration is clearly less than g, and it appears that most of the acceleration comes from the radial component, so it makes sense that the acceleration vector should point mostly toward the center of the curve and slightly backwards due to the negative tangential acceleration.

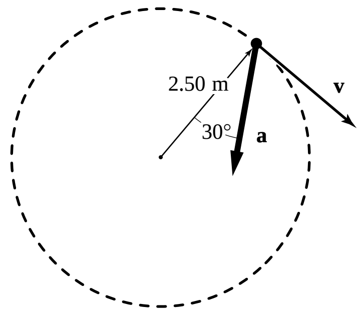
***4.32** (a) $a_t = \boxed{0.600\ \text{m/s}^2}$

(b) $a_r = \dfrac{v^2}{r} = \dfrac{(4.00\ \text{m/s})^2}{20.0\ \text{m}} = \boxed{0.800\ \text{m/s}^2}$

(c) $a = \sqrt{a_t^2 + a_r^2} = \boxed{1.00\ \text{m/s}^2}$

$\theta = \tan^{-1}\dfrac{a_r}{a_t} = \boxed{53.1°\ \text{inward from path}}$

4.33 $r = 2.50$ m, $a = 15.0\ \text{m/s}^2$



(a) $a_r = a\cos 30.0° = (15.0\ \text{m/s}^2)\cos 30.0° = \boxed{13.0\ \text{m/s}^2}$

(b) $a_r = \dfrac{v^2}{r}$

so $v^2 = ra_r = (2.50\ \text{m})(13.0\ \text{m/s}^2) = 32.5\ \text{m}^2/\text{s}^2$

$v = \sqrt{32.5}\ \text{m/s} = \boxed{5.70\ \text{m/s}}$

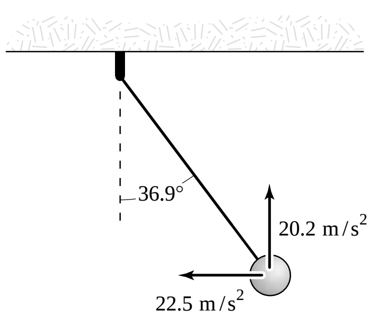
(c) $a^2 = a_t^2 + a_r^2$ so

$a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0\ \text{m/s}^2)^2 - (13.0\ \text{m/s}^2)^2} = \boxed{7.50\ \text{m/s}^2}$

4.34 (a) $a_{\text{top}} = \dfrac{v^2}{r} = \dfrac{(4.30\ \text{m/s})^2}{0.600\ \text{m}} = \boxed{30.8\ \text{m/s}^2\ \text{down}}$

(b) $a_{\text{bottom}} = \dfrac{v^2}{r} = \dfrac{(6.50\ \text{m/s})^2}{0.600\ \text{m}} = \boxed{70.4\ \text{m/s}^2\ \text{upward}}$

4.35 (a)



(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the radial acceleration:

$$a_r = (22.5\ \text{m/s}^2)\cos(90.0° - 36.9°) + (20.2\ \text{m/s}^2)\cos 36.9°$$

$a_r = \boxed{29.7 \text{ m/s}^2}$

(c) $a_r = \dfrac{v^2}{r}$

$v = \sqrt{a_r r} = \sqrt{29.7 \text{ m/s}^2 (1.50 \text{ m})} = 6.67 \text{ m/s}$ tangent to circle

$\mathbf{v} = \boxed{6.67 \text{ m/s at } 36.9° \text{ above the horizontal}}$

4.36 (a) $\mathbf{v}_\text{H} = 0 + \mathbf{a}_\text{H} t = (3.00\mathbf{i} - 2.00\mathbf{j}) \text{ m/s}^2 (5.00 \text{ s})$

$\mathbf{v}_\text{H} = (15.0\mathbf{i} - 10.0\mathbf{j}) \text{ m/s}$

$\mathbf{v}_\text{J} = 0 + \mathbf{a}_j t = (1.00\mathbf{i} + 3.00\mathbf{j}) \text{ m/s}^2 (5.00 \text{ s})$

$\mathbf{v}_\text{J} = (5.00\mathbf{i} + 15.0\mathbf{j}) \text{ m/s}$

$\mathbf{v}_\text{HJ} = \text{v}_\text{H} - \text{v}_\text{J} = (15.0\mathbf{i} - 10.0\mathbf{j} - 5.00\mathbf{i} - 15.0\mathbf{j}) \text{ m/s}$

$\mathbf{v}_\text{HJ} = (10.0\mathbf{i} - 25.0\mathbf{j}) \text{ m/s}$

$|\mathbf{v}_\text{HJ}| = \sqrt{(10.0)^2 + (25.0)^2} \text{ m/s} = \boxed{26.9 \text{ m/s}}$

(b) $\mathbf{r}_\text{H} = 0 + 0 + \frac{1}{2}\mathbf{a}_\text{H} t^2 = \frac{1}{2}(3.00\mathbf{i} - 2.00\mathbf{j}) \text{ m/s}^2 (5.00 \text{ s})^2$

$\mathbf{r}_\text{H} = (37.5\mathbf{i} - 25.0\mathbf{j}) \text{ m}$

$\mathbf{r}_\text{J} = \frac{1}{2}(1.00\mathbf{i} + 3.00\mathbf{j}) \text{ m/s}^2 (5.00 \text{ s})^2 = (12.5\mathbf{i} - 37.5\mathbf{j}) \text{ m}$

$\mathbf{r}_\text{HJ} = \mathbf{r}_\text{H} - \mathbf{r}_\text{J} = (37.5\mathbf{i} - 25.0\mathbf{j} - 12.5\mathbf{i} - 37.5\mathbf{j}) \text{ m}$

$\mathbf{r}_\text{HJ} = (25.0\mathbf{i} - 62.5\mathbf{j}) \text{ m}$

$|\mathbf{r}_\text{HJ}| = \sqrt{(25.0)^2 + (62.5)^2} \text{ m} = \boxed{67.3 \text{ m}}$

(c) $\mathbf{a}_\text{HJ} = \mathbf{a}_\text{H} - \mathbf{a}_\text{J} = (3.00\mathbf{i} - 2.00\mathbf{j} - 1.00\mathbf{i} - 3.00\mathbf{j}) \text{ m/s}^2$

$\mathbf{a}_\text{HJ} = \boxed{(2.00\mathbf{i} - 5.00\mathbf{j}) \text{ m/s}^2}$

4.37 Total time in still water $t = \frac{d}{v} = \frac{2000}{1.20} = \boxed{1.67 \times 10^3 \text{ s}}$

Total time = time upstream plus time downstream

$$t_{\text{up}} = \frac{1000}{(1.20 - 0.500)} = 1.43 \times 10^3 \text{ s}$$

$$t_{\text{down}} = \frac{1000}{(1.20 + 0.500)} = 588 \text{ s}$$

$$t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3 \text{ s}}$$

Goal Solution

G: If we think about the time for the trip as a function of the stream's speed, we realize that if the stream is flowing at the same rate or faster than the student can swim, he will never reach the 1.00 km mark even after an infinite amount of time. Since the student can swim 1.20 km in 1000 s, we should expect that the trip will definitely take longer than in still water, maybe about 2000 s (~30 minutes).

O: The total time in the river is the longer time upstream (against the current) plus the shorter time downstream (with the current). For each part, we will use the basic equation $t = d/v$, where v is the speed of the student relative to the shore.

A: $t_{\text{up}} = \frac{d}{v_{\text{student}} - v_{\text{stream}}} = \frac{1000 \text{ m}}{1.20 \text{ m/s} - 0.500 \text{ m/s}} = 1429 \text{ s}$

$t_{\text{dn}} = \frac{d}{v_{\text{student}} + v_{\text{stream}}} = \frac{1000 \text{ m}}{1.20 \text{ m/s} + 0.500 \text{ m/s}} = 588 \text{ s}$

Total time in river, $t_{\text{river}} = t_{\text{up}} + t_{\text{dn}} = 2.02 \times 10^3 \text{ s}$

In still water, $t_{\text{still}} = \frac{d}{v} = \frac{2000 \text{ m}}{1.20 \text{ m/s}} = 1.67 \times 10^3 \text{ s}$ therefore, $t_R = 1.21 t_{\text{still}}$

L: As we predicted, it does take the student longer to swim up and back in the moving stream than in still water (21% longer in this case), and the amount of time agrees with our estimation.

4.38 The bumpers are initially 100 m = 0.100 km apart. After time t the bumper of the leading car travels $40.0t$, while the bumper of the chasing car travels $60.0t$.

Since the cars are bumper-to-bumper at time t, we have

$$0.100 + 40.0t = 60.0t, \text{ yielding } t = 5.00 \times 10^{-3} \text{ h} = \boxed{18.0 \text{ s}}$$

4.39 $v = (150^2 + 30.0^2)^{1/2} = \boxed{153 \text{ km/h}}$

$\theta = \tan^{-1}\left(\frac{30.0}{150}\right) = \boxed{11.3°}$ north of west

4.40 For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$. Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{2L/c}{1 - v^2/c^2}}$$

For Beth, her cross-stream speed (both ways) is $\sqrt{c^2 - v^2}$

Thus, the total time for Beth is

$$t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \boxed{\frac{2L/c}{\sqrt{1 - v^2/c^2}}}$$

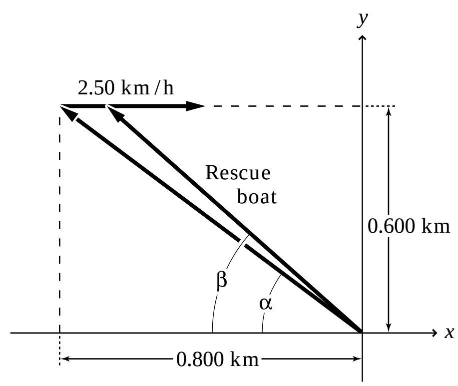
Since $1 - \frac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

4.41 α = Heading with respect to the shore

β = Angle of boat with respect to the shore

(a) The boat should always steer for the child at heading

$$\alpha = \tan^{-1}\frac{0.600}{0.800} = \boxed{36.9°}$$



(b) $v_x = 20.0 \cos\alpha - 2.50 = 13.5$ km/h

$v_y = 20.0 \sin\alpha = 12.0$ km/h

$$\beta = \tan^{-1}\left(\frac{12.0 \text{ km/h}}{13.5 \text{ km/h}}\right) = \boxed{41.6°}$$

(c) $t = \frac{d_y}{v_y} = \frac{0.600 \text{ km}}{12.0 \text{ km/h}} = 5.00 \times 10^{-2} \text{ h} = \boxed{3.00 \text{ min}}$

4.42 (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

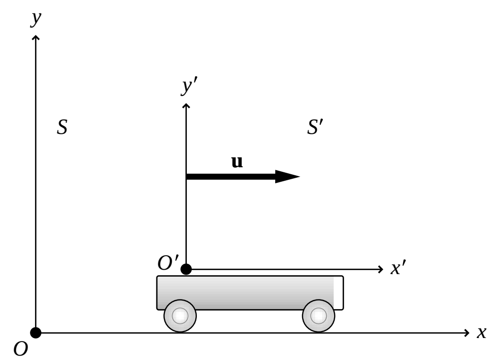
$$a = \sqrt{(2.50 \text{ m/s})^2 + (9.80 \text{ m/s})^2} = \boxed{10.1 \text{ m/s}^2}$$

$$\tan\theta = \frac{2.50 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 0.255$$

$$\theta = \boxed{14.3° \text{ to the south from the vertical}}$$

(b) $a = \boxed{9.80 \text{ m/s}^2 \text{ vertically downward}}$

4.43 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have



a

$$\frac{v_y'}{v_x'} = \tan 60.0° = \sqrt{3}$$

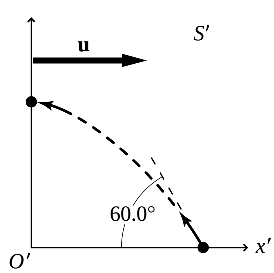
Then, because there is no x-motion in S, we can write $v_x = v_y' + u = 0$ so that $v_x' = -u = -10.0$ m/s. Hence the ball is thrown backwards in S'. Then,

$$v_y = v_y' = \sqrt{3}\left|v_x'\right| = 10.0\sqrt{3} \text{ m/s}$$

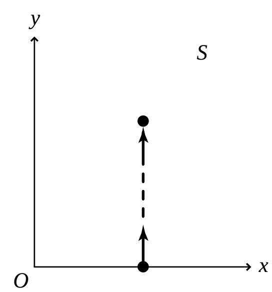
Using $v_y^2 = 2gh$ (from Eq. 4.13), we find

$$h = \frac{(10.0\sqrt{3} \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{15.3 \text{ m}}$$

The motion of the ball as seen by the student in S' is shown in diagram (b). The view of the professor in S is shown in diagram (c).



b



c

4.44 Equation 4.13: $h = \dfrac{v_i^2 \sin^2 \theta_i}{2g}$

Equation 4.14: $R = \dfrac{v_i^2 \sin 2\theta_i}{g} = \dfrac{2v_i^2 \sin \theta_i \cos \theta_i}{g}$

If $h = R/6$, Equation 4.13 yields $[v_i \sin \theta_i = \sqrt{gR/3}\,]$ (1)

Substituting the result given in Equation (1) above into Equation 4.14 gives

$$R = \frac{2(\sqrt{gR/3})v_i \cos \theta_i}{g}$$

which reduces to $[v_i \cos \theta_i = \frac{1}{2}\sqrt{3gR}\,]$ (2)

(a) From $v_{yf} = v_{yi} + a_y t$, the time to reach the peak of the path (where $v_{yf} = 0$) is found to be $t_{\text{peak}} = \dfrac{v_i \sin \theta_i}{g}$. Using Equation (1), this gives $t_{\text{peak}} = \sqrt{\dfrac{R}{3g}}$. The total time of the ball's flight is then

$$t_{\text{flight}} = 2t_{\text{peak}} = \boxed{2\sqrt{\frac{R}{3g}}}$$

(b) At the peak of the path, the ball moves horizontally with speed

$$v_{\text{peak}} = v_{xi} = v_i \cos \theta_i$$

Using Equation (1), this becomes $v_{\text{peak}} = \boxed{\frac{1}{2}\sqrt{3gR}}$.

(c) The initial vertical component of velocity is $v_{yi} = v_i \sin \theta_i$ and, from Equation (1), this is

$$v_{yi} = \boxed{\sqrt{gR/3}}$$

(d) Squaring Equations (1) and (2) and adding the results gives

$$v_i^2(\sin^2 \theta_i + \cos^2 \theta_i) = \frac{gR}{3} + \frac{3gR}{4} = \frac{13gR}{12}$$

Thus, the initial speed is $v_i = \boxed{\sqrt{\frac{13gR}{12}}}$.

(e) Dividing Equation (1) by (2) yields

$$\tan\theta_i = \frac{v_i \sin\theta_i}{v_i \cos\theta_i} = \left[\frac{(\sqrt{gR/3})}{\left(\frac{1}{2}\sqrt{3gR}\right)}\right] = \frac{2}{3}$$

Therefore, $\theta_i = \tan^{-1}\left(\frac{2}{3}\right) = \boxed{33.7°}$.

(f) For a given initial speed, the projection angle yielding maximum peak height is $\theta_i = 90.0°$. With the speed found in (d), Equation 4.13 then yields

$$h_{\text{max}} = \frac{(13\,gR/12)\sin^2 90.0°}{2g} = \boxed{\frac{13R}{24}}$$

(g) For a given initial speed, the projection angle yielding maximum range is $\theta_i = 45.0°$. With the speed found in (d), Equation 4.14 then gives

$$R_{\text{max}} = \frac{(13gR/12)\sin 90.0°}{g} = \boxed{\frac{13R}{12}}$$

4.45 At any time t, the two drops have identical y-coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2\,|\,x(t)\,| = 2(v_{xi}t) = 2(v_i \cos\theta_i)t = \boxed{2v_i t\cos\theta_i}$$

4.46 After the string breaks the ball is a projectile, for time t in

$$y = v_{yi}t + \frac{1}{2}\,a_y\,t^2$$

$$-1.20\text{ m} = 0 + \frac{1}{2}(-9.80\text{ m/s}^2)\,t^2$$

$$t = 0.495\text{ s}$$

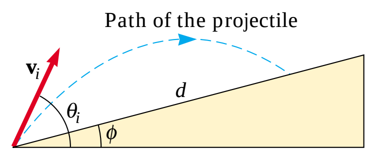
Its constant horizontal speed is

$$v_x = \frac{x}{t} = \frac{2.00\text{ m}}{0.495\text{ s}} = 4.04\text{ m/s}$$

so before the string breaks

$$a_c = \frac{v^2}{r} = \frac{(4.04\text{ m/s})^2}{0.300\text{ m}} = \boxed{54.4\text{ m/s}^2}$$

4.47 (a) $y = \tan(\theta_i)\, x - \dfrac{g}{2v_i^2 \cos^2(\theta_i)}\, x^2$



Setting $x = d\cos(\phi)$, and $y = d\sin(\phi)$, we have

$$d\sin(\phi) = \tan(\theta_i) d\cos(\phi) - \frac{g}{2v_i^2 \cos^2(\theta_i)} (d\cos(\phi))^2$$

Solving for d yields,

$$d = \frac{2v_i^2 \cos(\theta_i)[\sin(\theta_i)\cos(\phi) - \sin(\phi)\cos(\theta_i)]}{g\cos^2(\phi)}$$

or $\quad d = \boxed{\dfrac{2v_i^2 \cos(\theta_i)\sin(\theta_i - \phi)}{g\cos^2(\phi)}}$

(b) Setting $\dfrac{dd}{d\theta_i} = 0$ leads to $\boxed{\theta_i = 45° + \dfrac{\phi}{2}}$ and

$$\boxed{d_{\max} = \frac{v_i^2(1 - \sin\phi)}{g\cos^2\phi}}$$

4.48 (a)(b) Since the shot leaves the gun horizontally, the time it takes to reach the target is $t = \dfrac{x}{v_i}$.

The vertical distance traveled in this time is

$$y = -\frac{1}{2}gt^2 = -\frac{g}{2}\left(\frac{x}{v_i}\right)^2 = Ax^2$$

where $\boxed{A = -\dfrac{g}{2v_i^2}}$

(c) If $x = 3.00$ m, $y = -0.210$ m, then $A = \dfrac{-0.210}{9.00} = -2.33 \times 10^{-2}$

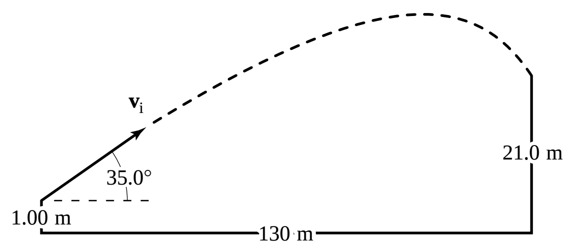
$$v_i = \sqrt{\frac{-g}{2A}} = \sqrt{\frac{-9.80}{-4.66 \times 10^{-2}}}\ \text{m/s} = \boxed{14.5\ \text{m/s}}$$

4.49 Refer to the sketch:

(a) & (b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i \cos 35.0°)t$

$\Delta y = v_{yi}t + \frac{1}{2}at^2$ substitution yields

$20.0 = (v_i \sin 35.0°)t + \frac{1}{2}(-9.80)\,t^2$



Solving the above gives $t = \boxed{3.81\text{ s}}$

$v_i = \boxed{41.7\text{ m/s}}$

(c) $v_y = v_i \sin\theta_i - gt$

$v_x = v_i \cos\theta_i$

At $t = 3.81$ s, $v_y = 41.7 \sin 35.0° - (9.80)(3.81) = \boxed{-13.4\text{ m/s}}$

$v_x = (41.7 \cos 35.0°) = \boxed{34.1\text{ m/s}}$

$v = \sqrt{v_x^2 + v_y^2} = \boxed{36.6\text{ m/s}}$

4.50 (a) The moon's gravitational acceleration is the bullet's centripetal acceleration:

(For the moon's radius, see endpapers of text.)

$a = \frac{v^2}{r}$

$\left(\frac{1}{6}\right) 9.80\text{ m/s}^2 = \frac{v^2}{1.74 \times 10^6\text{ m}}$

$v = \sqrt{2.84 \times 10^6\text{ m}^2/\text{s}^2} = \boxed{1.69\text{ km/s}}$

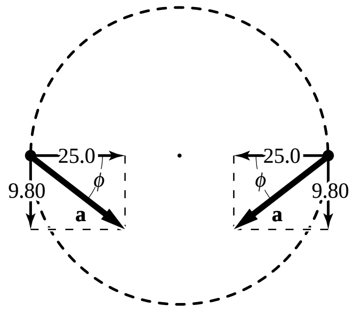
(b) $v = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v} = \frac{2\pi(1.74 \times 10^6\text{ m})}{1.69 \times 10^3\text{ m/s}} = 6.47 \times 10^3\text{ s} = \boxed{1.80\text{ h}}$

4.51 (a) $a_r = \frac{v^2}{r} = \frac{(5.00\text{ m/s})^2}{1.00\text{ m}} = \boxed{25.0\text{ m/s}^2}$

$a_T = g = \boxed{9.80\text{ m/s}^2}$

(b)



(c) $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(25.0\text{ m/s}^2)^2 + (9.80\text{ m/s})^2} = \boxed{26.8\text{ m/s}^2}$

$\phi = \tan^{-1}\left(\frac{a_t}{a_r}\right) = \tan^{-1}\frac{9.80\text{ m/s}^2}{25.0\text{ m/s}^2} = \boxed{21.4°}$

4.52 $x = v_{ix}t = v_i t \cos 40.0°$ Thus, when $x = 10.0$ m,

$t = \frac{10.0\text{ m}}{v_i \cos 40.0°}$

At this time, y should be 3.05 m – 2.00 m = 1.05 m.

Thus, $1.05\text{ m} = \frac{(v_i \sin 40.0°)\,10.0\text{ m}}{v_i \cos 40.0°} + \frac{1}{2}(-9.80\text{ m/s}^2)\left[\frac{10.0\text{ m}}{v_i \cos 40.0°}\right]^2$

From this, $v_i = \boxed{10.7\text{ m/s}}$

***4.53** At $t = 2.00$ s, $v_x = 4.00$ m/s

$v_y = -8.00\text{ m/s}$

$v = \sqrt{v_x^2 + v_y^2} = \boxed{8.94\text{ m/s}}$

$\theta = \tan^{-1}\frac{v_y}{v_x} = \boxed{-63.4°\text{, below horizontal}}$

4.54 The special conditions allowing use of Equation 4.14 apply.

For the ball thrown at 45.0°, $D = R_{45} = \dfrac{v_i^2\ 1}{g}$

For the bouncing ball, $D = R_1 + R_2 = \dfrac{v_i^2 \sin 2\theta}{g} + \dfrac{\left(\frac{v_i}{2}\right)^2 \sin 2\theta}{g}$ where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5} \qquad \boxed{\theta = 26.6°}$$

(b) The time for any symmetric parabolic flight is given by

$$y = v_{yi}t - \frac{1}{2}\ gt^2$$

$$0 = v_i \sin \theta_i t - \frac{1}{2}\ gt^2$$

If $t = 0$ is the time the ball is thrown, then $t = \dfrac{2v_i \sin \theta_i}{g}$ is the time at landing.

So, for the ball thrown at 45.0°

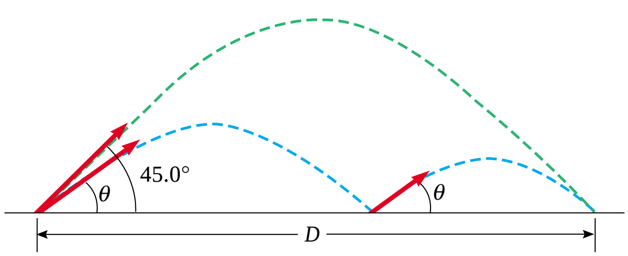
$$t_{45} = \frac{2v_i \sin 45.0°}{g}$$

For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2\left(\frac{v_i}{2}\right)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$$

The ratio of this time to that for no bounce is

$$\frac{3v_i \sin 26.6° / \text{g}}{2v_i \sin 45.0° / \text{g}} = \frac{1.34}{1.41} = \boxed{0.949}$$



4.55 From Equation 4.13, the maximum height a ball can reach is

$$h = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

For a throw straight up, $\theta_i = 90°$ and $h = \dfrac{v_i^2}{2g}$.

From Equation 4.14 the range a ball can be thrown is $R = \dfrac{v_i^2 \sin 2\theta}{g}$.

For maximum range, $\theta = 45°$ and $R = \dfrac{v_i^2}{g}$.

Therefore for the same v_i, $h = \dfrac{R}{2} = \dfrac{40.0 \text{ m}}{2} = \boxed{20.0 \text{ m}}$

4.56. Using the range equation (Equation 4.14)

$$R = \frac{v_i^2 \sin(2\theta_i)}{g}$$

the maximum range occurs when $\theta_i = 45°$, and has a value $R = \dfrac{v_i^2}{g}$.

Given R, this yields $v_i = \sqrt{gR}$

If the boy uses the same speed to throw the ball vertically upward, then

$$v_y = \sqrt{gR} - gt \qquad \text{and} \qquad y = \sqrt{gR}\, t - \frac{gt^2}{2}$$

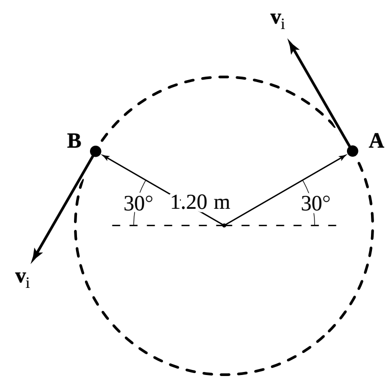
at any time, t.

At the maximum height, $v_y = 0$, giving $t = \sqrt{\dfrac{R}{g}}$, and so the maximum height reached is

$$y_{\max} = \sqrt{gR}\sqrt{\frac{R}{g}} - \frac{g}{2}\left(\sqrt{\frac{R}{g}}\right)^2 = R - \frac{R}{2} = \boxed{\frac{R}{2}}$$

4.57 Choose upward as the positive y-direction and leftward as the positive x-direction. The vertical height of the stone when released from A or B is

$$y_i = (1.50 + 1.20 \sin 30.0°)\text{ m} = 2.10\text{ m}$$



(a) The equations of motion after release at A are

$$v_y = v_i \sin 60.0° - gt = (1.30 - 9.80t)\text{ m/s}$$

$$v_x = v_i \cos 60.0° = 0.750\text{ m/s}$$

$$y = (2.10 + 1.30t - 4.90t^2)\text{ m}$$

$$\Delta x_A = (0.750t)\text{ m}$$

When $y = 0$, $t = \dfrac{-1.30 \pm \sqrt{(1.30)^2 + 41.2}}{-9.80} = 0.800\text{ s}$

Then, $\Delta x_A = (0.750)(0.800)\text{ m} = \boxed{0.600\text{ m}}$

(b) The equations of motion after release at point B are

$$v_y = v_i(-\sin 60.0°) - gt = (-1.30 - 9.80t)\text{ m/s}$$

$$v_x = v_i \cos 60.0° = 0.750\text{ m/s}$$

$$y_i = (2.10 - 1.30t - 4.90t^2)\text{ m}$$

When $y = 0$, $t = \dfrac{+1.30 \pm \sqrt{(-1.30)^2 + 41.2}}{-9.80} = 0.536\text{ s}$

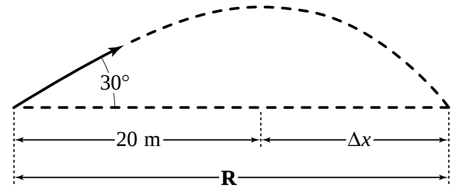
Then, $\Delta x_B = (0.750)(0.536)\text{ m} = \boxed{0.402\text{ m}}$

(c) $a_r = \dfrac{v^2}{r} = \dfrac{(1.50\text{ m/s})^2}{1.20\text{ m}} = \boxed{1.87\text{ m/s}^2\text{ toward the center}}$

(d) After release, $\mathbf{a} = -g\mathbf{j} = \boxed{9.80\text{ m/s}^2\text{ downward}}$

4.58 The football travels a horizontal distance

$$R = \frac{v_i^2 \sin(2\theta_i)}{g} = \frac{(20.0)^2 \sin(60.0°)}{9.80} = 35.3 \text{ m}$$



Time of flight of ball is

$$t = \frac{2v_i \sin\theta_i}{g} = \frac{2(20.0)\sin 30.0°}{9.80} = 2.04 \text{ s}$$

The receiver is Δx away from where the ball lands and $\Delta x = 35.3 - 20.0 = 15.3$ m.

To cover this distance in 2.04 s, he travels with a velocity

$$v = \frac{15.3}{2.04} = \boxed{7.50 \text{ m/s in the direction the ball was thrown}}$$

4.59 (a) $\Delta y = -\frac{1}{2} gt^2$; $\Delta x = v_i t$. Combine the equations eliminating t:

$$\Delta y = -\frac{1}{2} g\left(\frac{\Delta x}{v_i}\right)^2 \text{ from this } (\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right) v_i^2$$

$$\text{thus } \Delta x = v_i\sqrt{\frac{-2\,\Delta y}{g}} = 275\sqrt{\frac{-2(-300)}{9.80}} = 6.80 \times 10^3 \text{ m} = \boxed{6.80 \text{ km}}$$

(b) The plane has the same velocity as the bomb in the x direction.

Therefore, the plane will be $\boxed{\text{3000 m directly above the bomb}}$ when it hits the ground.

(c) When θ is measured from the vertical, $\tan\theta = \frac{\Delta x}{\Delta y}$; therefore,

$$\theta = \tan^{-1}\frac{\Delta x}{\Delta y} = \tan^{-1}\left(\frac{6800}{3000}\right) = \boxed{66.2°}$$

4.60 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - gx^2/2v_i^2$

(a) The elevation y_r of points on the rock is described by $y_r^2 + x^2 = R^2$. We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}$$

We get the strictest requirement for x approaching zero. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock:

$$1 > \frac{gR}{v_i^2} \qquad \boxed{v_i > \sqrt{gR}}$$

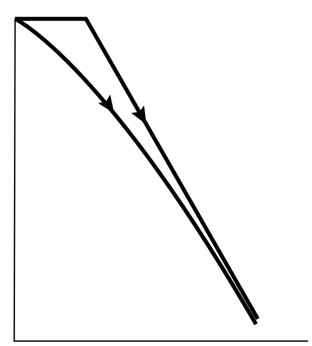
(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \frac{gx^2}{2gR}$ or $x = \sqrt{2}\,R$

The distance from the rock's base is $x - R = \boxed{(\sqrt{2} - 1)R}$

4.61 (a) From Part (C), the raptor dives for $6.34 - 2.00 = 4.34$ s

undergoing displacement 197 m downward and $(10.0)(4.34) = 43.4$ m forward.

$$v = \frac{\Delta d}{\Delta t} = \frac{\sqrt{(197)^2 + (43.4)^2}}{4.34} = \boxed{46.5 \text{ m/s}}$$



(b) $\alpha = \tan^{-1}\left(\frac{-197}{43.4}\right) = \boxed{-77.6°}$

(c) $197 = \frac{1}{2}gt^2 \quad \boxed{t = 6.34 \text{ s}}$

Goal Solution

G: We should first recognize that the hawk cannot instantaneously change from slow horizontal motion to rapid downward motion. The hawk cannot move with infinite acceleration. We assume that the time required for the hawk to accelerate is short compared to two seconds. Based on our everyday experiences, a reasonable diving speed for the hawk might be about 100 mph (~ 50 m/s) downwards and should last only a few seconds.

O: We know the distance that the mouse and hawk fall, but to find the diving speed of the hawk, we must know the time of descent. If the hawk and mouse both maintain their original horizontal velocity of 10 m/s (as they should without air resistance), then the hawk only needs to think about diving straight down, but to a ground-based observer, the path will appear to be a straight line angled less than 90° below horizontal.

A: The mouse falls a total vertical distance, $y = 200\text{ m} - 3.00\text{ m} = 197\text{ m}$

The time of fall is found from $y = v_{yi}t - \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2(197\text{ m})}{9.80\text{ m/s}^2}} = 6.34\text{ s}$$

To find the diving speed of the hawk, we must first calculate the total distance covered from the vertical and horizontal components. We already know the vertical distance, y, so we just need the horizontal distance during the same time (minus the 2.00 s late start).

$$x = v_{xi}(t - 2.00\text{ s}) = (10.0\text{ m/s})(6.34\text{ s} - 2.00\text{ s}) = 43.4\text{ m}$$

The total distance is $d = \sqrt{x^2 + y^2} = \sqrt{(43.4\text{ m})^2 + (197\text{ m})^2} = 202\text{ m}$

So the hawk's diving speed is $v_{\text{hawk}} = \dfrac{d}{t - 2.00\text{ s}} = \dfrac{202\text{ m}}{4.34\text{ s}} = 46.5\text{ m/s}$

At an angle of $\theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{197\text{ m}}{43.4\text{ m}}\right) = 77.6°$ below the horizontal

L: The answers appear to be consistent with our predictions, even thought it is not possible for the hawk to reach its diving speed in zero time. Sometimes we must make simplifying assumptions to solve complex physics problems, and sometimes these assumptions are not physically possible. Once the idealized problem is understood, we can attempt to analyze the more complex, real-world problem. For this problem, if we considered the realistic effects of air resistance and the maximum diving acceleration attainable by the hawk, we might find that the hawk could not catch the mouse before it hit the ground.

4.62 (1) Equation of bank (2) and (3) are the equations of motion

(1) $y^2 = 16x$ (2) $x = v_i t$ (3) $y = -\frac{1}{2} gt^2$

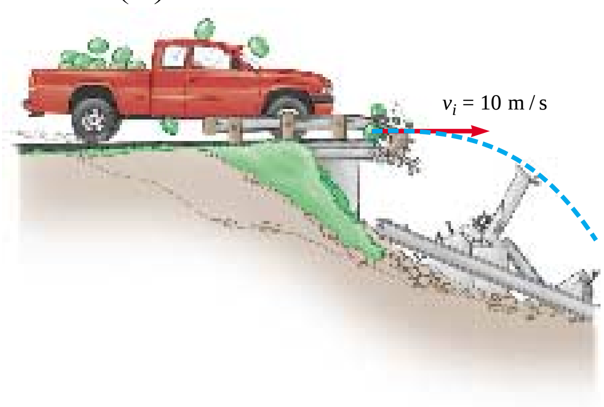
Substitute for t from (2) into (3) $y = -\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right)$

Equate y from the bank equation to y from the equations of motion:

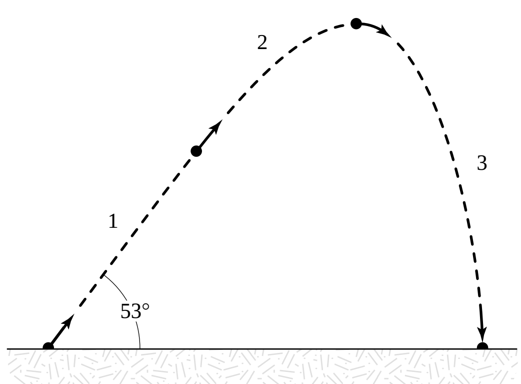
$$16x = \left[-\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2x^4}{4v_i^4} - 16x = x\left(\frac{g^2x^3}{4v_i^4} - 16\right) = 0$$

From this, $x = 0$ or $x^3 = \frac{64v_i^4}{g^2}$ and $x = 4\left(\frac{10^4}{9.80^2}\right)^{1/3} = \boxed{18.8 \text{ m}}$

Also, $y = -\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right) = \frac{1}{2}\frac{(9.80)(18.8)^2}{(10.0)^2} = \boxed{-17.3 \text{ m}}$



4.63 Consider the rocket's trajectory in 3 parts as shown in the sketch.



Our initial conditions give:

$a_y = 30.0 \sin 53.0° = 24.0 \text{ m/s}^2$; $a_x = 30.0 \cos 53.0° = 18.1 \text{ m/s}^2$

$v_{yi} = 100 \sin 53.0° = 79.9 \text{ m/s}$; $v_{xi} = 100 \cos 53.0° = 60.2 \text{ m/s}$

The distances traveled during each phase of the motion are given in the table.

Path #1: $v_{yf} - 79.9 = (24.0)(3.00)$ or $v_{yf} = 152$ m/s

$v_{xf} - 60.2 = (18.1)(3.00)$ or $v_{xf} = 114$ m/s

$$\Delta y = (79.9)(3.00) + \frac{1}{2}(24.0)(3.00)^2 = 347 \text{ m}$$

$$\Delta x = (60.2)(3.00) + \frac{1}{2}(18.1)(3.00)^2 = 262 \text{ m}$$

Path #2: $a_x = 0$, $v_{xf} = v_{xi} = 114$ m/s

$0 - 152 = -(9.80)t$ or $t = 15.5$ s

$\Delta x = (114)(15.5) = 1.77 \times 10^3$ m;

$$\Delta y = (152)(15.5) - \frac{1}{2}(9.80)(15.5)^2$$

$$= 1.17 \times 10^3 \text{ m}$$

Path #3: $(v_{yf})^2 - 0 = 2(-9.80)(-1.52 \times 10^3)$

or $v_{yf} = -173$ m/s

$v_{xf} = v_{xi} = 114$ m/s since $a_x = 0$

$-173 - 0 = -(9.80)t$ or $t = 17.6$ s

$\Delta x = (114)(17.7) = 2.02 \times 10^3$ m

	Path Part		
	#1	#2	#3
a_y	24.0	–9.80	–9.80
a_x	18.1	0.0	0.00
v_{yf}	152	0.0	–173
v_{xf}	114	114	114
v_{yi}	79.9	152	0.00
v_{xi}	60.2	114	114
Δy	347	1.17×10^3	-1.52×10^3
Δx	262	1.77×10^3	2.02×10^3
t	3.00	15.5	17.6

(a) $\Delta y(\text{max}) = \boxed{1.52 \times 10^3 \text{ m}}$

(b) $t(net) = 3.00 + 15.5 + 17.7 = \boxed{36.1 \text{ s}}$

(c) $\Delta x(net) = 262 + 1.77 \times 10^3 + 2.02 \times 10^3 = \boxed{4.05 \times 10^3 \text{ m}}$

4.64 Let V = boat's speed in still water and v = river's speed and let d = distance traveled upstream in $t_1 = 60.0$ min and t_2 = time of return. Then, for the log, $1000 \text{ m} = vt = v(t_1 + t_2)$, and for the boat, $d = (V - v)t_1$; $(d + 1000) = (V + v)t_2$; and $t = t_1 + t_2$

Combining the above gives

$$\frac{1000}{v} = \frac{d}{(V - v)} + \frac{d + 1000}{(V + v)}$$

Substituting for $d = (V - v)(3600)$ gives $v = \boxed{0.139 \text{ m/s}}$

4.65 (a) While on the incline:

$v^2 - v_i^2 = 2a\,\Delta x$ $\qquad$ $v - v_i = at$

$v^2 - 0 = 2(4.00)(50.0)$ $\qquad$ $20.0 - 0 = 4.00t$

$v = \boxed{20.0\text{ m/s}}$ $\qquad$ $t = \boxed{5.00\text{ s}}$

(b) Initial free-flight conditions give us

$v_{xi} = 20.0\cos 37.0° = 16.0\text{ m/s}$; $v_{yi} = -20.0\sin 37.0° = -12.0\text{ m/s}$

$v_x = v_{xi}$ since $a_x = 0$;

$$v_y = -(2a_y\,\Delta y + v_{yi}^2)^{1/2} = -[2(-9.80)(-30.0) + (-12.0)^2]^{1/2} = -27.1\text{ m/s}$$

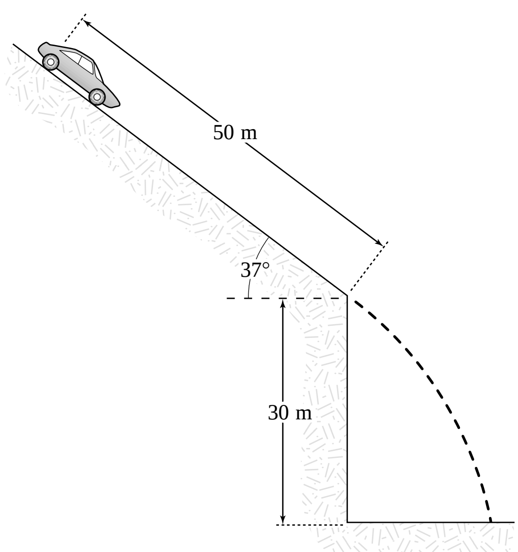
$$v = (v_x^2 + v_y^2)^{1/2} = [(16.0)^2 + (-27.1)^2]^{1/2}$$

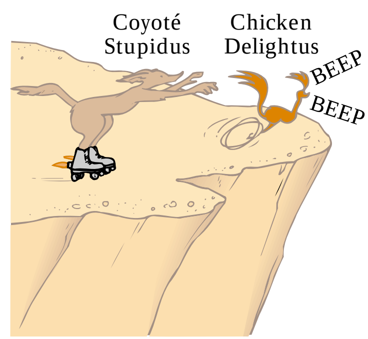
$$= \boxed{31.5\text{ m/s at }59.4°\text{ below the horizontal}}$$

(c) $t_1 = 5\text{ s}$; $t_2 = \dfrac{(v_y - v_{yi})}{a_y} = \dfrac{(-27.1 + 12.0)}{-9.80} = 1.54\text{ s}$

$t = t_1 + t_2 = \boxed{6.54\text{ s}}$

(d) $\Delta x = v_{xi}t_1 = (16.0)(1.54) = \boxed{24.6\text{ m}}$





4.66 (a) Coyote: $\Delta x = \frac{1}{2} at^2$; $70.0 = \frac{1}{2}(15.0)\ t^2$

Roadrunner: $\Delta x = v_i t$; $70.0 = v_i t$

Solving the above, we get $v_i = \boxed{22.9\ \text{m/s}}$ and $t = 3.06$ s

(b) At the edge of the cliff $v_{xi} = at = (15.0)(3.06) = 45.8$ m/s

$$\Delta y = \frac{1}{2} a_y t^2$$

Substituting we find $-100 = \frac{1}{2}(-9.80)\ t^2$

$$\Delta x = v_{xi} t + \frac{1}{2}\ a_x t^2 = (45.8)t + \frac{1}{2}(15.0)\ t^2$$

Solving the above gives $\Delta x = \boxed{360\ \text{m}}$ $\quad t = 4.52$ s

(c) For the Coyote's motion through the air

$$v_{xf} = v_{xi} + a_x t$$

$$v_{xf} = 45.8 + 15(4.52)$$

$$v_{xf} = \boxed{114\ \text{m/s}}$$

$$v_{yf} = v_{yi} + a_y t$$

$$= 0 - 9.80(4.52)$$

$$v_{yf} = \boxed{-44.3\ \text{m/s}}$$

4.67 (a) $\Delta x = v_{xi} t$, $\Delta y = v_{yi} t + \frac{1}{2}\ gt^2$,

$d \cos 50.0° = (10.0 \cos 15.0°)t$, and

$d \sin 50.0° = (10.0 \sin 15.0°)t + \frac{1}{2}(-9.80)\ t^2$

Solving the above gives

$$d = \boxed{43.2\ \text{m}} \quad t = 2.87\ \text{s}$$

(b) Since $a_x = 0$,

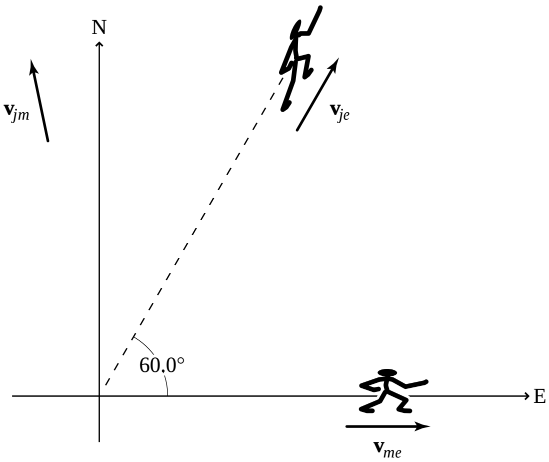
$$v_{xf} = v_{xi} = 10.0 \cos 15.0° = \boxed{9.66 \text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t = (10.0 \sin 15.0°) - (9.80)(2.87) = \boxed{-25.5 \text{ m/s}}$$

Air resistance would decrease the values of the range and maximum height.

As an air foil he can get some lift and increase his distance.

4.68 Define **i** to be directed East, and **j** to be directed North.



According to the figure, set

$\mathbf{v}_{\text{je}}$ = velocity of Jane, relative to the earth

$\mathbf{v}_{\text{me}}$ = velocity of Mary, relative to the earth

$\mathbf{v}_{\text{jm}}$ = velocity of Jane, relative to Mary,

Such that $\mathbf{v}_{\text{je}} = \mathbf{v}_{\text{jm}} + \mathbf{v}_{\text{me}}$

Solve for part (b) first. By the figure,

$$\mathbf{v}_{\text{je}} = [5.40(\cos 60.0°)\mathbf{i} + 5.40(\sin 60.0°)\mathbf{j}] \text{ m/s}$$

$$= (2.70\mathbf{i} + 4.68\mathbf{j}) \text{ m/s}$$

and $\mathbf{v}_{\text{me}} = 4.00\mathbf{i}$ m/s

So, (b) $\mathbf{v}_{\text{jm}} = \boxed{(-1.30\mathbf{i} + 4.68\mathbf{j}) \text{ m/s}}$

The distance between the two players increases at a rate of $|\mathbf{v}_{jm}|$:

$$|\mathbf{v}_{\text{jm}}| = \sqrt{(1.30)^2 + (4.68)^2} \text{ m/s} = 4.86 \text{ m/s}$$

(a) Therefore, $t = \dfrac{d}{\mathbf{v}_{\text{jm}}} = \dfrac{25.0 \text{ m}}{4.86 \text{ m/s}} = \boxed{5.14 \text{ s}}$

(c) After 4 s, $d = \mathbf{v}_{\text{jm}} t = (4.86 \text{ m/s})(4.00) = \boxed{19.4 \text{ m apart}}$

***4.69** Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius
60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6\text{ m})}{0.1\text{ s}} \cong 9\text{ m/s}$$

and its centripetal acceleration is

$$\frac{v^2}{r} \cong \frac{(9\text{ m/s})^2}{0.6\text{ m}} \boxed{\sim 10^2\text{ m/s}^2}$$

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

4.70 Find the highest elevation θ_H that will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest elevation θ_L that will clear the mountain peak; this will yield the maximum range under these conditions if both θ_H *and* θ_L are $> 45°$; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.

$$y = v_{yi}t - \frac{1}{2}\,gt^2 = v_i(\sin\,\theta)t - \frac{1}{2}\,gt^2$$

$$x = v_{xi}t = v_i\,(\cos\,\theta)t$$

Thus $t = \dfrac{x}{v_i\cos\,\theta}$

Substitute into the expression for y

$$y = v_i\,(\sin\,\theta)\frac{x}{v_i\cos\,\theta} - \frac{1}{2}\,g\,\textbf{Error!}$$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$ thus $y = x\tan\,\theta - \dfrac{gx^2}{2v_i^2}(\tan^2\theta + 1)$ and

$$0 = \frac{gx^2}{2v_i^2}\,\tan^2\theta - x\tan\,\theta + \frac{gx^2}{2v_i^2} + y$$

Substitute values, use the quadratic formula and find

$\tan\,\theta = 3.905$ or 1.197 which gives $\theta_H = 75.6°$ and $\theta_L = 50.1°$

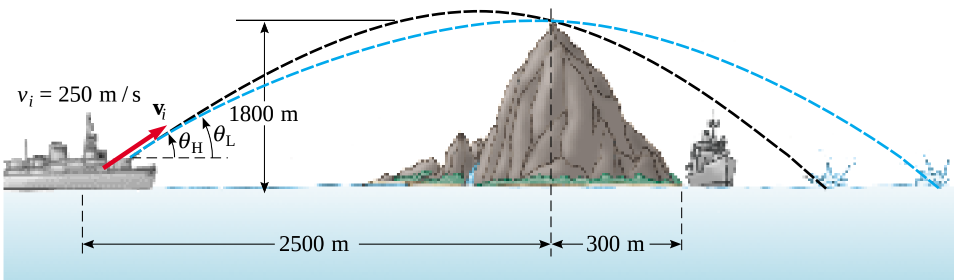
Range (at θ_H) $= \dfrac{v_i^2 \sin 2\theta_H}{g} = 3.07 \times 10^3$ m from enemy ship

$3.07 \times 10^3 - 2500 - 300 = 270$ m from shore

Range (at θ_L) $= \dfrac{v_i^2 \sin 2\theta_L}{g} = 6.28 \times 10^3$ m from enemy ship

$6.28 \times 10^3 - 2500 - 300 = 3.48 \times 10^3$ m from shore

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48 \times 10^3 \text{ m}}$ from the shore.



Chapter 5 Solutions

***5.1** For the same force F, acting on different masses

$$F = m_1 a_1 \quad \text{and} \quad F = m_2 a_2$$

(a) $\dfrac{m_1}{m_2} = \dfrac{a_2}{a_1} = \boxed{\dfrac{1}{3}}$

(b) $F = (m_1 + m_2)a = 4m_1 a = m_1(3.00 \text{ m/s}^2)$

$a = \boxed{0.750 \text{ m/s}^2}$

***5.2** $F = 10.0$ N, $m = 2.00$ kg

(a) $a = \dfrac{F}{m} = \dfrac{10.0 \text{ N}}{2.00 \text{ kg}} = \boxed{5.00 \text{ m/s}^2}$

(b) $F_g = mg = (2.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{19.6 \text{ N}}$

(c) $a = \dfrac{2F}{m} = \dfrac{2(10.0 \text{ N})}{2.00 \text{ kg}} = \boxed{10.0 \text{ m/s}^2}$

5.3 $m = 3.00$ kg, $\mathbf{a} = (2.00\mathbf{i} + 5.00\mathbf{j}) \text{ m/s}^2$

$\mathbf{F} = m\mathbf{a} = \boxed{(6.00\mathbf{i} + 15.0\mathbf{j}) \text{ N}}$

$\left|\mathbf{F}\right| = \sqrt{(6.00)^2 + (15.0)^2} \text{ N} = \boxed{16.2 \text{ N}}$

5.4 $m_{\text{train}} = 15{,}000{,}000$ kg

$F = 750{,}000$ N

$$a = \frac{F}{m} = \frac{75.0 \times 10^4 \text{ N}}{15.0 \times 10^6 \text{ kg}} = 5.00 \times 10^{-2} \text{ m/s}^2$$

$v_f = v_i + at \qquad v_i = 0$

$$t = \frac{v_f - v_i}{a} = \frac{(80.0 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s})}{5.00 \times 10^{-2} \text{ m/s}^2}$$

$\boxed{t = 444 \text{ s}}$

5.5 We suppose the barrel is horizontal.

$m = 5.00 \times 10^{-3}$ kg, $v_f = 320$ m/s, $v_i = 0$, $x = 0.820$ m

$$\bar{F}_x = m\bar{a} = m\frac{\Delta v}{\Delta t} \quad \text{(Eq. 5.2)}$$

Find Δt from Eq. 2.2 $\quad \Delta t = \dfrac{\Delta x}{\bar{v}} = \dfrac{0.820 \text{ m}}{160 \text{ m/s}} = 5.13 \times 10^{-3}$ s

$$\therefore \bar{F}_x = (5.00 \times 10^{-3} \text{ kg})\frac{(320 \text{ m/s})}{(5.13 \times 10^{-3})} = \boxed{312 \text{ N}}$$

Along with this force, which we assume is horizontal, exerted by the exploding gunpowder, the bullet feels a downward 49.0 mN force of gravity and an upward 49.0 mN force exerted by the barrel surface under it.

5.6 $F_g = mg = 1.40$ N, $m = 0.143$ kg

$v_f = 32.0$ m/s, $v_i = 0$, $\Delta t = 0.0900$ s

$\bar{v} = 16.0$ m/s $\qquad \bar{a} = \dfrac{\Delta v}{\Delta t} = 356$ m/s^2

(a) Distance $x = \bar{v}\,t = (16.0 \text{ m/s})(0.0900 \text{ s}) = \boxed{1.44 \text{ m}}$

(b) $\sum \mathbf{F} = m\mathbf{a}$

$\mathbf{F}_{\text{pitcher}} - 1.40 \text{ N}\mathbf{j} = (0.143 \text{ kg})(356\mathbf{i} \text{ m/s}^2)$

$\mathbf{F}_{\text{p}} = \boxed{(50.9\mathbf{i} + 1.40\mathbf{j}) \text{ N}}$

5.7 F_g = weight of ball = mg

$v_{\text{release}} = v$, time to accelerate = t

$$\boldsymbol{a} = \frac{\Delta v}{\Delta t} = \frac{v}{t} = \frac{v}{t}\,\mathbf{i}$$

(a) Distance $x = \bar{v}\,t = \left(\dfrac{v}{2}\right) t = \boxed{\dfrac{vt}{2}}$

(b) $\mathbf{F}_{\text{p}} - F_g\,\mathbf{j} = \dfrac{F_g}{g}\dfrac{v}{t}\,\mathbf{i}$

$$\mathbf{F}_{\text{p}} = \boxed{\frac{F_g v}{gt}\mathbf{i} + F_g\,\mathbf{j}}$$

***5.8** $F_g = mg$

$$1 \text{ pound} = (0.453\,592\,37 \text{ kg})(32.1740 \text{ ft/s}^2)\left(\frac{12.0 \text{ in}}{1 \text{ ft}}\right)\left(\frac{0.0254 \text{ m}}{1 \text{ in.}}\right) = \boxed{4.45 \text{ N}}$$

5.9 $m = 4.00 \text{ kg}$, $\mathbf{v}_i = 3.00\mathbf{i} \text{ m/s}$, $\mathbf{v}_8 = (8.00\mathbf{i} + 10.0\mathbf{j}) \text{ m/s}$, $t = 8.00 \text{ s}$

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{t} = \frac{(5.00\mathbf{i} + 10.0\mathbf{j})}{8.00} \text{ m/s}^2$$

$$\mathbf{F} = m\mathbf{a} = \boxed{(2.50\mathbf{i} + 5.00\mathbf{j}) \text{ N}}$$

$$F = \sqrt{(2.50)^2 + (5.00)^2} = \boxed{5.59 \text{ N}}$$

5.10 (a) Let the x-axis be in the original direction of the molecule's motion.

$$v_f = v_i + at$$

$$-670 \text{ m/s} = 670 \text{ m/s} + a(3.00 \times 10^{-13} \text{ s})$$

$$a = \boxed{-4.47 \times 10^{15} \text{ m/s}^2}$$

(b) For the molecule $\sum \mathbf{F} = m\mathbf{a}$. Its weight is negligible.

$$\mathbf{F}_{\text{wall on molecule}} = 4.68 \times 10^{-26} \text{ kg } (-4.47 \times 10^{15} \text{ m/s}^2)$$

$$= -2.09 \times 10^{-10} \text{ N}$$

$$\bar{\mathbf{F}}_{\text{molecule on wall}} = \boxed{+2.09 \times 10^{-10} \text{ N}}$$

5.11 (a) $F = ma$ and $v_f^2 = v_i^2 + 2ax$ or $a = \dfrac{v_f^2 - v_i^2}{2x}$

Therefore,

$$F = m\frac{(v_f^2 - v_i^2)}{2x}$$

$$F = (9.11 \times 10^{-31} \text{ kg})\frac{[(7.00 \times 10^5 \text{ m/s}^2)^2 - (3.00 \times 10^5 \text{ m/s})^2]}{(2)(0.0500 \text{ m})}$$

$$= \boxed{3.64 \times 10^{-18} \text{ N}}$$

(b) The weight of the electron is

$$F_g = mg = (9.11 \times 10^{-31}\text{ kg})(9.80\text{ m/s}^2) = \boxed{8.93 \times 10^{-30}\text{ N}}$$

The accelerating force is 4.08×10^{11} times the weight of the electron.

Goal Solution

G: We should expect that only a very small force is required to accelerate an electron, but this force is probably much greater than the weight of the electron if the gravitational force can be neglected.

O: Since this is simply a linear acceleration problem, we can use Newton's second law to find the force as long as the electron does not approach relativistic speeds (much less than 3×10^8 m/s), which is certainly the case for this problem. We know the initial and final velocities, and the distance involved, so from these we can find the acceleration needed to determine the force.

A: From $v_f^2 = v_i^2 + 2ax$ and $\sum F = ma$ we can solve for the acceleration and the force.

$$a = \frac{(v_f^2 - v_i^2)}{2x} \quad \text{and so} \quad \sum F = \frac{m(v_f^2 - v_i^2)}{2x}$$

(a) $$F = \frac{(9.11 \times 10^{-31}\text{ kg})\,[(7.00 \times 10^5\text{ m/s}^2) - (3.00 \times 10^5\text{ m/s})^2]}{(2)(0.0500\text{ m})} = 3.64 \times 10^{-18}\text{ N}$$

(b) The weight of the electron is

$$Fg = mg = (9.11 \times 10^{-31}\text{ kg})(9.80\text{ m/s}^2) = 8.93 \times 10^{-30}\text{ N}$$

The ratio of the accelerating force to the weight is $\dfrac{F}{F_g} = 4.08 \times 10^{11}$

L: The force that causes the electron to accelerate is indeed a small fraction of a newton, but it is much greater than the gravitational force. For this reason, it is quite reasonable to ignore the weight of the electron in electric charge problems.

5.12 (a) $F_g = mg = 120\text{ lb} = \left(4.448\ \dfrac{\text{N}}{\text{lb}}\right)(120\text{ lb}) = \boxed{534\text{ N}}$

(b) $m = \dfrac{F_g}{g} = \dfrac{534\text{ N}}{9.80\text{ m/s}^2} = \boxed{54.5\text{ kg}}$

5.13 $F_g = mg = 900\text{ N}$

$$m = \frac{900\text{ N}}{9.80\text{ m/s}^2} = 91.8\text{ kg}$$

$$(F_g)_{\text{on Jupiter}} = (91.8\text{ kg})(25.9\text{ m/s}^2) = \boxed{2.38\text{ kN}}$$

***5.14** Imagine a quick trip by jet, on which you do not visit the rest room and your perspiration is just canceled out by a glass of tomato juice. By subtraction, $(F_g)_p = mg_p$ and $(F_g)_C = mg_C$ give $\Delta F_g = m(g_P - g_C)$. For a person whose mass is 88.7 kg, the change in weight is

$$\Delta F_g = (88.7)(9.8095 - 9.7808) = \boxed{2.55\ \text{N}}$$

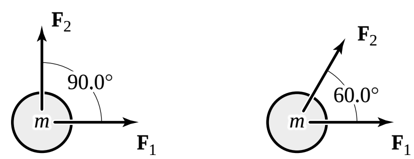
A precise balance scale, as in a doctor's office, reads the same in different locations because it compares you with the standard masses on its beams. A typical bathroom scale is not precise enough to reveal this difference.

5.15 (a) $\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = (20.0\mathbf{i} + 15.0\mathbf{j})$ N

$\sum \mathbf{F} = m\mathbf{a}$, $20.0\mathbf{i} + 15.0\mathbf{j} = 5.00\ \mathbf{a}$

$\mathbf{a} = (4.00\mathbf{i} + 3.00\ \mathbf{j})\ \text{m/s}^2$

or $\boxed{a = 5.00\ \text{m/s}^2}$ at $\boxed{\theta = 36.9°}$



a b

(b) $F_{2x} = 15.0 \cos 60.0° = 7.50$ N

$F_{2y} = 15.0 \sin 60.0° = 13.0$ N

$\mathbf{F}_2 = (7.50\mathbf{i} + 13.0\mathbf{j})$ N

$\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$= (27.5\mathbf{i} + 13.0\mathbf{j})\ \text{N} = m\mathbf{a} = 5.00\mathbf{a}$

$\mathbf{a} = (5.50\mathbf{i} + 2.60\mathbf{j})\ \text{m/s}^2$

5.16 We find acceleration: $\mathbf{r} - \mathbf{r}_i = \mathbf{v}_i t + \frac{1}{2}\ \mathbf{a}t^2$

$$4.20\ \text{m}\ \mathbf{i} - 3.30\ \text{m}\ \mathbf{j} = 0 + \frac{1}{2}\ \mathbf{a}(1.20\ \text{s})^2 = 0.720\ \text{s}^2\ \mathbf{a}$$

$\mathbf{a} = (5.83\ \mathbf{i} - 4.58\mathbf{j})\ \text{m/s}^2$

Now $\sum \mathbf{F} = m\mathbf{a}$ becomes

$$\mathbf{F}_2 = 2.80\ \text{kg}\ (5.83\mathbf{i} - 4.58\mathbf{j})\ \text{m/s}^2 + (2.80\ \text{kg})9.80\ \text{m/s}^2\ \mathbf{j}$$

$$\mathbf{F}_2 = \boxed{(16.3\mathbf{i} + 14.6\mathbf{j})\ \text{N}}$$

5.17 (a) You and earth exert equal forces on each other:

$$m_y g = M_e a_e$$

If your mass is 70.0 kg,

$$a_e = \frac{(70.0\text{ kg})(9.80\text{ m/s}^2)}{5.98 \times 10^{24}\text{ kg}} = \boxed{\sim 10^{-22}\text{ m/s}^2}$$

(b) You and planet move for equal times in $x = \frac{1}{2}at^2$. If the seat is 50.0 cm high,

$$\sqrt{\frac{2x_y}{a_y}} = \sqrt{\frac{2x_e}{a_e}}$$

$$x_e = \frac{a_e}{a_y}x_y = \frac{m_y}{m_e}x_y = \frac{70.0\text{ kg}}{5.98 \times 10^{24}\text{ kg}}(0.500\text{ m})\ \boxed{\sim 10^{-23}\text{ m}}$$

5.18 $F = \sqrt{(20.0)^2 + (10.0 - 15.0)^2} = 20.6\text{ N}$

$a = \dfrac{F}{m}$

$a = \boxed{5.15\text{ m/s}^2\text{ at }14.0°\text{ S of E}}$

***5.19** Choose the x-axis forward. Then

$$\sum F_x = ma_x$$

$$(2000\text{ N}\mathbf{i}) - (1800\text{ N}\mathbf{i}) = (1000\text{ kg})\mathbf{a}$$

$$\boxed{\mathbf{a} = 0.200\text{ m/s}^2\mathbf{i}}$$

(b) $x_f - x_i = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(0.200\text{ m/s}^2)(10.0\text{ s})^2 = \boxed{10.0\text{ m}}$

(c) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = 0 + (0.200\text{ m/s}^2\mathbf{i})(10.0\text{ s}) = \boxed{2.00\text{ m/s }\mathbf{i}}$

5.20 $\sum\mathbf{F} = m\mathbf{a}$ reads

$$(-2.00\mathbf{i} + 2.00\mathbf{j} + 5.00\mathbf{i} - 3.00\mathbf{j} - 45.0\mathbf{i})\text{ N} = m(3.75\text{ m/s}^2)\hat{\mathbf{a}}$$

where $\hat{\mathbf{a}}$ represents the direction of $\mathbf{a}$

$$(-42.0\mathbf{i} - 1.00\mathbf{j})\text{ N} = m(3.75\text{ m/s}^2)\hat{\mathbf{a}}$$

$$F = \sqrt{(42.0)^2 + (1.00)^2}\text{ N at Arctan}\left(\frac{1.00}{42.0}\right)\text{ below the }-x\text{-axis}$$

$$= m(3.75\text{ m/s}^2)\hat{\mathbf{a}}$$

$$F = 42.0\text{ N at }181° = m(3.75\text{ m/s}^2)\hat{\mathbf{a}}$$

For the vectors to be equal, their magnitudes and their directions must be equal:

(a) $\therefore$ $\boxed{\hat{\mathbf{a}} \text{ is at } 181°}$ counterclockwise from the x-axis

(b) $m = \dfrac{42.0 \text{ N}}{3.75 \text{ m/s}^2} = \boxed{11.2 \text{ kg}}$

(d) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = 0 + (3.75 \text{ m/s}^2 \text{ at } 181°)10.0 \text{ s}$

$= 37.5 \text{ m/s at } 181°$

$= 37.5 \text{ m/s} \cos 181°\mathbf{i} + 37.5 \text{ m/s} \sin 181°\mathbf{j}$

$\mathbf{v} = \boxed{(-37.5\mathbf{i} - 0.893\mathbf{j}) \text{ m/s}}$

(c) $|\mathbf{v}| = \sqrt{37.5^2 + 0.893^2} \text{ m/s} = \boxed{37.5 \text{ m/s}}$

***5.21** (a) $\boxed{15.0 \text{ lb up}}$ (b) $\boxed{5.00 \text{ lb up}}$ (c) $\boxed{0}$

***5.22** $v_x = \dfrac{dx}{dt} = 10t,\ v_y = \dfrac{dy}{dt} = 9t^2$

$a_x = \dfrac{dv_x}{dt} = 10,\ a_y = \dfrac{dv_y}{dt} = 18t$

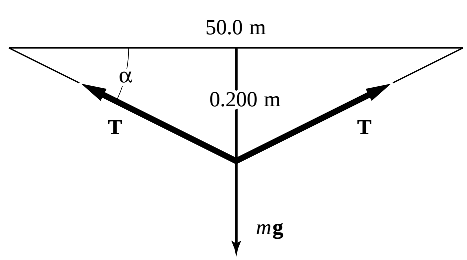
At $t = 2.00$ s, $a_x = 10.0 \text{ m/s}^2$, $a_y = 36.0 \text{ m/s}^2$

$F_x = ma_x = (3.00 \text{ kg})(10.0 \text{ m/s}^2) = 30.0 \text{ N}$

$F_y = ma_y = (3.00 \text{ kg})(36.0 \text{ m/s}^2) = 108 \text{ N}$

$F = \sqrt{F_x^2 + F_y^2} = \boxed{112 \text{ N}}$

5.23 $m = 1.00$ kg



$mg = 9.80$ N

$\tan \alpha = \dfrac{0.200 \text{ m}}{25.0 \text{ m}}$

$\alpha = 0.458°$

Balance forces,

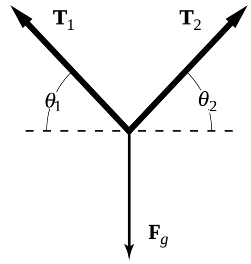
$2T \sin \alpha = mg$

$T = \dfrac{9.80 \text{ N}}{2 \sin \alpha} = \boxed{613 \text{ N}}$

5.24 $T_3 = F_g$ (1)

$T_1 \sin\theta_1 + T_2 \sin\theta_2 = F_g$ (2)

$T_1 \cos\theta_1 = T_2 \cos\theta_2$ (3)



Eliminate T_2 and solve for T_1,

$$T_1 = \frac{F_g \cos\theta_2}{(\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2)} = \frac{F_g \cos\theta_2}{\sin(\theta_1 + \theta_2)}$$

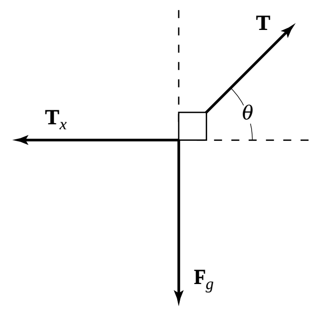
$$T_3 = F_g = \boxed{325\text{ N}}$$

$$T_1 = F_g \frac{\cos 25.0°}{\sin 85.0°} = \boxed{296\text{ N}}$$

$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} = (296\text{ N})\left(\frac{\cos 60.0°}{\cos 25.0°}\right) = \boxed{163\text{ N}}$$

5.25 See the solution for T_1 in Problem 5.24.

***5.26** (a) An explanation proceeding from fundamental physical principles will be best for the parents and for you. Consider forces on the bit of string touching the weight hanger as shown in the free-body diagram:



Horizontal Forces: $\sum F_x = ma_x \Rightarrow -T_x + T\cos\theta = 0$

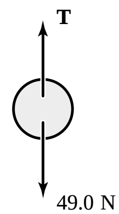
Vertical Forces: $\sum F_y = ma_y \Rightarrow -F_g + T\sin\theta = 0$

You need only the equation for the vertical forces to find that the tension in the string is given by $T = \boxed{\dfrac{F_g}{\sin\theta}}$. The force the child feels gets smaller, changing from T to $T\cos\theta$, while the counterweight hangs on the string. On the other hand, the kite does not notice what you are doing and the tension in the main part of the string stays constant. You do not need a level, since you learned in physics lab to sight to a horizontal line in a building. Share with the parents your estimate of the experimental uncertainty, which you make by thinking critically about the measurement, by repeating trials, practicing in advance and looking for variations and improvements in technique, including using other observers. You will then be glad to have the parents themselves repeat your measurements.

(b) $T = \dfrac{F_g}{\sin\theta} = \dfrac{(0.132\text{ kg})(9.80\text{ m/s}^2)}{\sin 46.3°} = \boxed{1.79\text{ N}}$

5.27 (a) Isolate either mass

$$T + mg = ma = 0$$



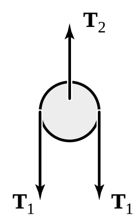
$$|T| = |mg|$$

The scale reads the tension T, so

$$T = mg = 5.00\text{ kg} \times 9.80\text{ m/s}^2$$

$$= \boxed{49.0\text{ N}}$$

(b) Isolate each mass

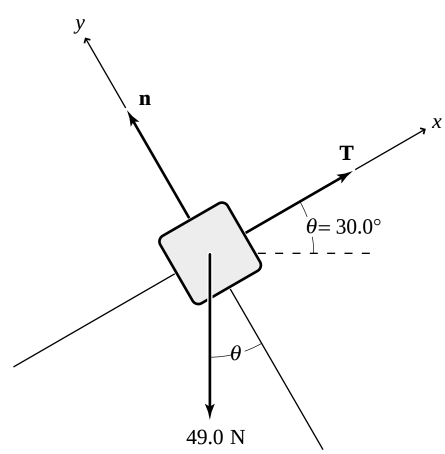


$$\mathbf{T}_2 + 2\mathbf{T}_1 = 0$$

$$T_2 = 2|T_1| = 2mg$$

$$= \boxed{98.0\text{ N}}$$

(c) $\sum\mathbf{F} = \mathbf{n} + \mathbf{T} + m\mathbf{g} = 0$



Take the component along the incline

$$\mathbf{n}_x + \mathbf{T}_x + m\mathbf{g}_x = 0$$

or $\quad 0 + T - mg\sin 30.0° = 0$

$$T = mg\sin 30.0° = \frac{mg}{2} = \frac{(5.00)(9.80)}{2}$$

$$= \boxed{24.5\text{ N}}$$

5.28 Let R represent the horizontal force of air resistance.

(a) $\sum\mathbf{F}_x = ma_x$ becomes $T\sin 40.0° - R = 0$

$\sum\mathbf{F}_y = ma_y$ reads $T\cos 40.0° - F_g = 0$

Then $T = \dfrac{mg}{\cos 40.0°} = \dfrac{6.08 \times 10^3\text{ N}}{\cos 40.0°} = 7.93 \times 10^3\text{ N}$

$$R = 7.93 \times 10^3\text{ N}\sin 40 = \boxed{5.10 \times 10^3\text{ N}}$$

(b) The value of R will be the same. Now

$$T \sin 7.00° - R = 0 \qquad T = \frac{5.10 \times 10^3 \text{ N}}{\sin 7.00°} = 4.18 \times 10^4 \text{ N}$$

$$T \cos 7.00° - mg = 0$$

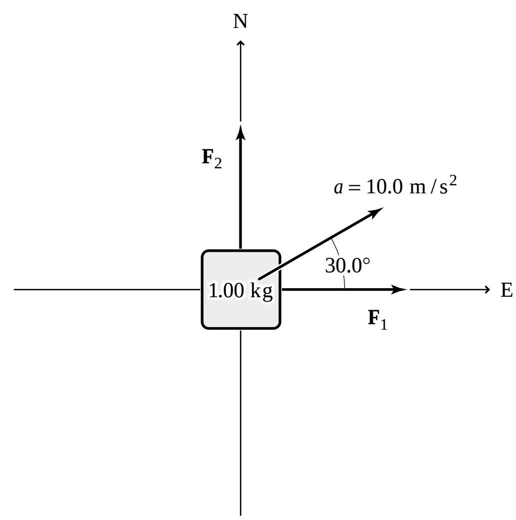
$$m = \frac{(4.18 \times 10^4 \text{ N}) \cos 7.00°}{9.80 \text{ m/s}^2} = 4.24 \times 10^3 \text{ kg}$$

$$m_{\text{water}} = 4.24 \times 10^3 \text{ kg} - 620 \text{ kg} = \boxed{3.62 \times 10^3 \text{ kg}}$$

5.29 Choosing a coordinate system with **i** East and **j** North.

$(5.00 \text{ N})\mathbf{j} + \mathbf{F}_1 = 10.0 \text{ N} \angle 30.0° = (5.00 \text{ N})\mathbf{j} + (8.66 \text{ N})\mathbf{i}$

$\therefore F_1 = \boxed{8.66 \text{ N (East)}}$



Goal Solution

G: The net force acting on the mass is $\sum F = ma = (1 \text{ kg})(10 \text{ m/s}^2) = 10$ N, so if we examine a diagram of the forces drawn to scale, we see that $F_1 \approx 9$ N directed to the east.

O: We can find a more precise result by examining the forces in terms of vector components. For convenience, we choose directions east and north along **i** and **j**, respectively.

A: $\mathbf{a} = [(10.0 \cos 30.0°)\mathbf{i} + (10.0 \sin 30.0°)\mathbf{j}] \text{ m/s}^2 = (8.66\mathbf{i} + 5.00\mathbf{j}) \text{ m/s}^2$

From Newton's second law, $\sum \mathbf{F} = m\mathbf{a} = (1.00 \text{ kg})[(8.66\mathbf{i} + 5.00\mathbf{j}) \text{ m/s}^2] = (8.66\mathbf{i} + 5.00\mathbf{j})$ N

And $\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$.

so $\mathbf{F}_1 = \sum \mathbf{F} - \mathbf{F}_2 = (8.66\mathbf{i} + 5.00\mathbf{j} - 5.00\mathbf{j})$ N $= 8.66\mathbf{i}$ N $= 8.66$ N to the east

L: Our calculated answer agrees with the prediction from the force diagram.

5.30 (a) The cart and mass accelerate horizontally.

$$\sum F_y = ma_y \quad +T\cos\theta - mg = 0$$

$$\sum F_x = ma_x \quad +T\sin\theta = ma$$

Substitute $T = \dfrac{mg}{\cos\theta}$

$$\frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = ma$$

$$\boxed{a = g\tan\theta}$$

(b) $a = (9.80\text{ m/s}^2)\tan 23.0° = \boxed{4.16\text{ m/s}^2}$

***5.31** Let us call the forces exerted by each person F_1 and F_2. Thus, for pulling in the same direction, Newton's second law becomes

$$F_1 + F_2 = (200\text{ kg})(1.52\text{ m/s}^2)$$

or $F_1 + F_2 = 304\text{ N}$ (1)

When pulling in opposite directions,

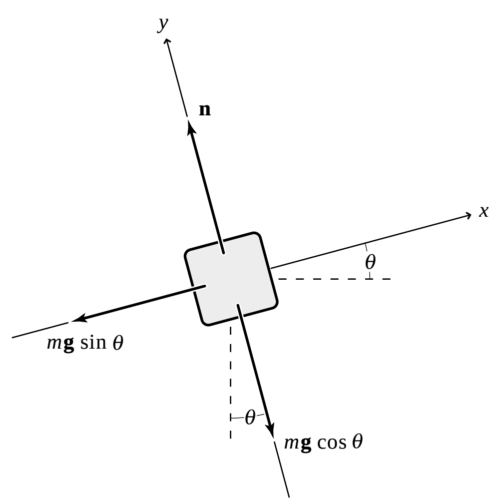
$$F_1 - F_2 = (200\text{ kg})(-0.518\text{ m/s}^2)$$

or $F_1 - F_2 = -104\text{ N}$ (2)

Solving simultaneously, we find

$$F_1 = \boxed{100\text{ N}}, \text{ and } F_2 = \boxed{204\text{ N}}$$

5.32 The two forces acting on the block are the normal force, n, and the weight, mg. If the block is considered to be a point mass and the x-axis is chosen to be parallel to the plane then the free body diagram will be as shown in the figure to the right. The angle θ is the angle of inclination of the plane. Applying Newton's second law for the accelerating system (and taking the direction of motion as the positive direction) we have



$$\sum F_y = n - mg\cos\theta = 0;\ n = mg\cos\theta$$

$$\sum F_x = -mg\sin\theta = ma;\ a = -g\sin\theta$$

(a) When $\theta = 15.0°$ $\quad a = \boxed{-2.54\text{ m/s}^2}$

(b) Starting from rest

$$v_f^2 = v_i^2 + 2ax$$

$$v_f = \sqrt{2ax} = \sqrt{(2)(-2.54\text{ m/s}^2)(-2.00\text{ m})} = \boxed{3.18\text{ m/s}}$$

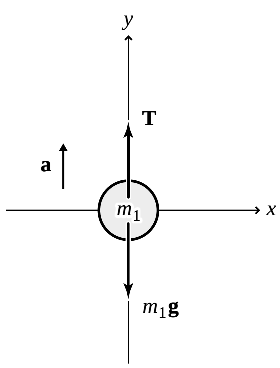
***5.33** $v_f^2 = v_i^2 + 2ax$

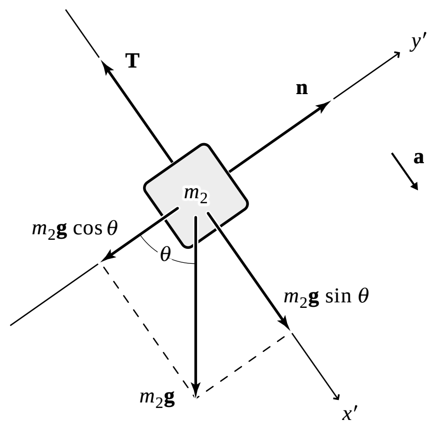
Taking $v = 0$, $v_i = 5.00$ m/s, and $a = -g\sin(20.0°)$ gives

$$0 = (5.00)^2 - 2(9.80)\sin(20.0°)x$$

or, $$x = \frac{25.0}{2(9.80)\sin(20.0°)} = \boxed{3.73\text{ m}}$$

5.34 With $m_1 = 2.00$ kg, $m_2 = 6.00$ kg and $\theta = 55.0°$,





(a) $\sum F_x = m_2 g\sin\theta - T = m_2 a$

and $T - m_1 g = m_1 a$

$$a = \frac{m_2 g\sin\theta - m_1 g}{m_1 + m_2} = \boxed{3.57\text{ m/s}^2}$$

(b) $T = m_1(a + g) = \boxed{26.7\text{ N}}$

(c) Since $v_i = 0$, $v_f = at = (3.57\text{ m/s}^2)(2.00\text{ s}) = \boxed{7.14\text{ m/s}}$

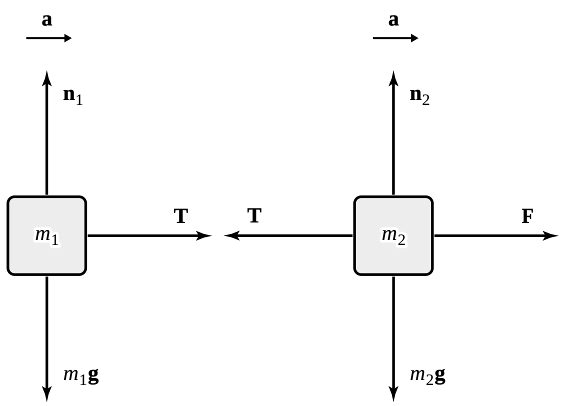
5.35 Applying Newton's second law to each block (motion along the x-axis).

For m_2: $\sum F_x = F - T = m_2 a$

For m_1: $\sum F_x = T = m_1 a$

Solving these equations for a and T, we find

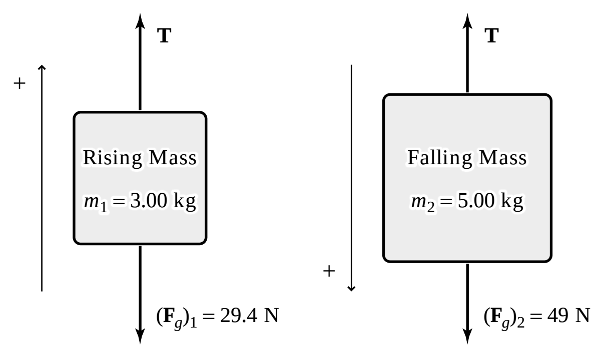
$$\boxed{a = \frac{F}{m_1 + m_2}} \quad \text{and} \quad \boxed{T = \frac{Fm_1}{m_1 + m_2}}$$



***5.36** First, consider the 3.00 kg rising mass. The forces on it are the tension, T, and its weight, 29.4 N. With the upward direction as positive, the second law becomes

$$\sum F_y = ma_y$$

$$T - 29.4\text{ N} = (3.00\text{ kg})a \qquad (1)$$



The forces on the falling 5.00 kg mass are its weight and T, and its acceleration is the same as that of the rising mass. Calling the positive direction down for this mass, we have

$$\sum F_y = ma_y$$

$$49\text{ N} - T = (5.00\text{ kg})a \qquad (2)$$

Equations (1) and (2) can be solved simultaneously to give

(a) the tension as $T = \boxed{36.8\text{ N}}$

(b) and the acceleration as $a = \boxed{2.45\text{ m/s}^2}$

(c) Consider the 3.00 kg mass. We have

$$y = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(2.45\ \text{m/s}^2)(1.00\ \text{s})^2 = \boxed{1.23\ \text{m}}$$

5.37 $T - m_1 g = m_1 a$ (1) Forces acting on 2.00 kg block

$F_x - T = m_2 a$ (2) Forces acting on 8.00 kg block

(a) Eliminate T and solve for a.

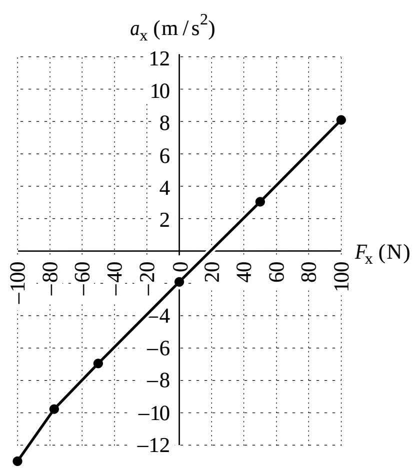
$$a = \frac{F_x - m_1 g}{m_1 + m_2} \qquad \boxed{a > 0 \text{ for } F_x > m_1 g = 19.6\ \text{N}}$$

(b) Eliminate a and solve for T.

$$T = \frac{m_1}{m_1 + m_2}(F_x + m_2 g) \qquad \boxed{T = 0 \text{ for } F_x \le -m_2 g = -78.4\ \text{N}}$$

(c)

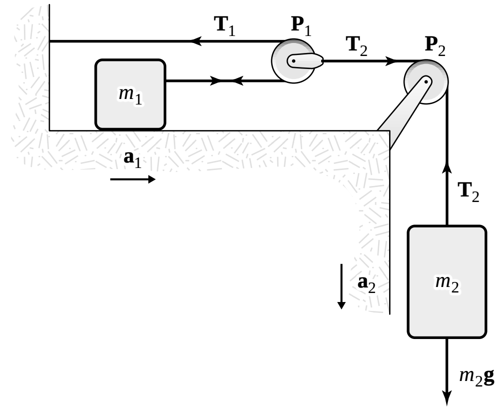
F_x, N	–100	–78.4	–50.0	0	50.0	100
a_x, m/s^2	–12.5	–9.80	–6.96	–1.96	3.04	8.04



5.38 (a) Pulley P_1 has acceleration a_2.

Since m_1 moves *twice* the distance P_1 moves in the same time, m_1 has twice the acceleration of P_1. i.e., $\boxed{a_1 = 2a_2}$

(b) From the figure, and using $F = ma$:



$$m_2g - T_2 = m_2a_2 \qquad (1)$$

$$T_1 = m_1a_1 = 2m_1a_2 \qquad (2)$$

$$T_2 - 2T_1 = 0 \qquad (3)$$

Equation (1) becomes $m_2g - 2T_1 = m_2a_2$

This equation combined with Equation (2) yields

$$\frac{2T_1}{m_1}\left(m_1 + \frac{m_2}{2}\right) = m_2g$$

$$\boxed{T_1 = \frac{m_1m_2}{2m_1 + \frac{1}{2}m_2}g} \quad \text{and} \quad \boxed{T_2 = \frac{m_1m_2}{m_1 + \frac{1}{4}m_2}g}$$

(c) From the values of T_1 and T_2 we find that

$$a_1 = \frac{T_1}{m_1} = \boxed{\frac{m_2g}{2m_1 + \frac{1}{2}m_2}}$$

$$a_2 = \frac{1}{2}a_1 = \boxed{\frac{m_2g}{4m_1 + m_2}}$$

5.39 First, we will compute the needed accelerations:

(1) Before it starts to move: $a_y = 0$.

(2) During the first 0.800 s: $a_y = \dfrac{v_y - v_{iy}}{t} = \dfrac{1.20\text{ m/s} - 0}{0.800\text{ s}} = 1.50\text{ m/s}^2$.

(3) While moving at constant velocity: $a_y = 0$.

(4) During the last 1.50 s: $a_y = \dfrac{v_y - v_{iy}}{t} = \dfrac{0 - 1.20\text{ m/s}}{1.50\text{ s}} = -0.800\text{ m/s}^2$.

Newton's second law is: $T = 706\text{ N} + (72.0\text{ kg})a_y$.

(a) When $a_y = 0$, $T = \boxed{706\text{ N}}$

(b) When $a_y = 1.50\text{ m/s}^2$, $T = \boxed{814\text{ N}}$

(c) When $a_y = 0$, $T = \boxed{706\text{ N}}$

(d) When $a_y = -0.800\text{ m/s}^2$, $T = \boxed{648\text{ N}}$

Goal Solution

G: Based on sensations experienced riding in an elevator, we expect that the man should feel slightly heavier when the elevator first starts to ascend, lighter when it comes to a stop, and his normal weight when the elevator is not accelerating. His apparent weight is registered by the spring scale beneath his feet, so the scale force should correspond to the force he feels through his legs (Newton's third law).

O: We should draw free body diagrams for each part of the elevator trip and apply Newton's second law to find the scale force. The acceleration can be found from the change in speed divided by the elapsed time.

A: Consider the free-body diagram of the man shown below. The force F is the upward force exerted on the man by the scale, and his weight is

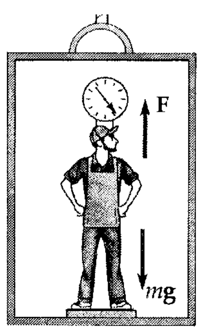
$$F_g = mg = (72.0 \text{ kg})(9.80 \text{ m/s}^2) = 706 \text{ N}$$

With $+y$ defined to be up, Newton's second law gives

$$\sum F_y = +F_s - F_g = ma$$

So the upward scale force is $F_s = 706 \text{ N} + (72.0 \text{ kg})$ **[Equation 1]**

Where a is the acceleration the man experiences as the elevator changes speed.



(a) Before the elevator starts moving, the acceleration of the elevator is zero ($a = 0$) so Equation 1 gives the force exerted by the scale on the man as 706 N (upward). Thus, the man exerts a downward force of 706 N on the scale.

(b) During the first 0.800 s of motion, the man's acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{(+1.20 \text{ m/s} - 0)}{0.800 \text{ s}} = 1.50 \text{ m/s}^2$$

Substituting a into Equation 1 then gives:

$$F_s = 706 \text{ N} + (72.0 \text{ kg})(+1.50 \text{ m/s}^2) = 814 \text{ N}$$

(c) While the elevator is traveling upward at constant speed, the acceleration is zero and Equation 1 again gives a scale force $F_s = 706$ N

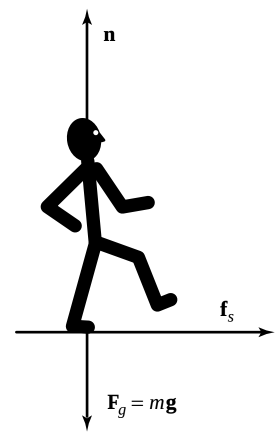
(d) During the last 1.50 s, the elevator starts with an upward velocity of 1.20 m / s, and comes to rest with an acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{0 - (+1.20 \text{ m/s})}{1.50 \text{ s}} = -0.800 \text{ m/s}^2$$

$$F_s = 706 \text{ N} + (72.0 \text{ kg})(-0.800 \text{ m/s}^2) = 648 \text{ N}$$

L: The calculated scale forces are consistent with our predictions. This problem could be extended to a couple of extreme cases. If the acceleration of the elevator were +9.8 m / s^2, then the man would feel twice as heavy, and if a = –9.8 m / s^2 (free fall), then he would feel "weightless", even though his true weight (Fg= mg) would remain the same.

***5.40** From Newton's third law, the forward force of the ground on the sprinter equals the magnitude of the friction force the sprinter exerts on the ground. If the sprinter's shoe is not to slip on the ground, this is a static friction force and its maximum magnitude is $f_{s,\,\max} = \mu_s mg$.

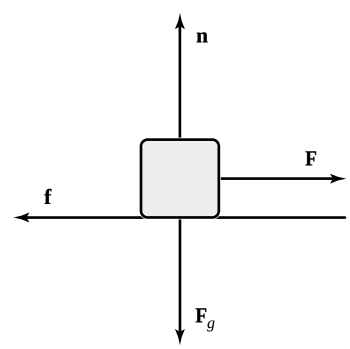


From Newton's second law applied to the sprinter, $f_{s,\,\max} = \mu_s mg = ma_{max}$ where $a_{\max}$ is the maximum forward acceleration the sprinter can achieve. From this, the acceleration is seen to be $a_{\max} = \mu_s g$. Note that the mass has canceled out.

If $\mu_s = 0.800$,

$$a_{\max} = 0.800(9.80 \text{ m/s}^2) = \boxed{7.84 \text{ m/s}^2 \text{ independent of the mass}}$$

5.41 For equilibrium: $f = F$ and $n = F_g$



Also, $f = \mu n$

i.e., $\mu = \dfrac{f}{n} = \dfrac{F}{F_g}$

$$\mu_s = \frac{75.0 \text{ N}}{(25.0)(9.80) \text{ N}} \quad \text{and} \quad \mu_k = \frac{60.0 \text{ N}}{(25.0)(9.80) \text{ N}}$$

$\mu_s = \boxed{0.306}$ $\mu_k = \boxed{0.245}$

***5.42** $F = \mu n = ma$ and in this case the normal force $n = mg$; therefore,

$$F = \mu mg = ma \quad \text{or} \quad \mu = \frac{a}{g}$$

The acceleration is found from

$$a = \frac{(v_f - v_i)}{t} = \frac{(80.0\ \text{mi/h})(0.447\ (\text{m/s})/(\text{mi/h}))}{8.00\ \text{s}} = 4.47\ \text{m/s}^2$$

Substituting this value into the expression for μ we find

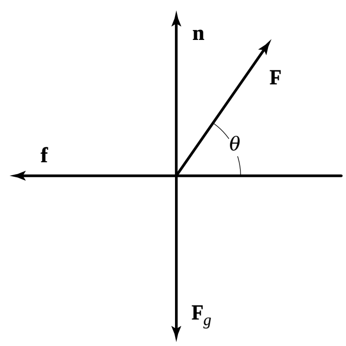
$$\mu = \frac{4.47\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \boxed{0.456}$$

5.43 $v_i = 50.0\ \text{mi/h} = 22.4\ \text{m/s}$

(a) $$x = \frac{v_i^2}{2\mu g} = \frac{(22.4\ \text{m/s})^2}{2(0.100)(9.80\ \text{m/s}^2)} = \boxed{256\ \text{m}}$$

(b) $$x = \frac{v_i^2}{2\mu g} = \frac{(22.4\ \text{m/s})^2}{2(0.600)(9.80\ \text{m/s}^2)} = \boxed{42.7\ \text{m}}$$

5.44 $m_{\text{suitcase}} = 20.0\ \text{kg}$, $F = 35.0\ \text{N}$



(a) $F\cos\theta = 20.0\ \text{N}$

$$\cos\theta = \frac{20.0}{35.0} = 0.571, \quad \boxed{\theta = 55.2°}$$

(b) $n = F_g - F\sin\theta = [196 - 35.0(0.821)]\ \text{N}$

$$\boxed{n = 167\ \text{N}}$$

5.45 $m = 3.00\ \text{kg}$, $\theta = 30.0°$, $x = 2.00\ \text{m}$, $t = 1.50\ \text{s}$

(a) $x = \frac{1}{2}at^2$

$$2.00\ \text{m} = \frac{1}{2}a(1.50\ \text{s})^2 \to a = \frac{4.00}{(1.50)^2} = \boxed{1.78\ \text{m/s}^2}$$

$\Sigma\mathbf{F} = \mathbf{n} + \mathbf{f} + m\mathbf{g} = m\mathbf{a}$

Along x: $0 - f + mg\sin 30.0° = ma \to f = m(g\sin 30.0° - a)$

Along y: $n + 0 - mg\sin 30.0° = 0 \to n = mg\cos 30.0°$

(b) $$\mu_k = \frac{f}{n} = \frac{m(g\sin 30.0° - a)}{mg\cos 30.0°} = \tan 30.0° - \frac{a}{g(\cos 30.0°)} = \boxed{0.368}$$

(c) $f = m(g \sin 30.0° - a) = (3.00)(9.80 \sin 30.0° - 1.78) = \boxed{9.37 \text{ N}}$

(d) $v_f^2 = v_i^2 + 2a(x_f - x_i)$ where $x_f - x_i = 2.00$ m

$v_f^2 = 0 + 2(1.78)(2.00) = 7.11 \text{ m}^2/\text{s}^2$

$v_f = \sqrt{7.11 \text{ m}^2/\text{s}^2} = \boxed{2.67 \text{ m/s}}$

***5.46** $-f + mg \sin \theta = 0$

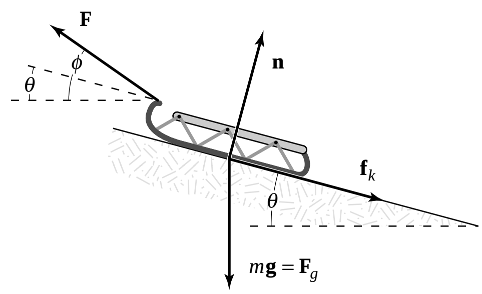
and $+n - mg \cos \theta = 0$

with $f = \mu n$ yield

$\mu_s = \tan \theta_c = \tan(36.0°) = \boxed{0.727}$

$\mu_k = \tan \theta_c = \tan(30.0°) = \boxed{0.577}$

5.47 $F_g = 60.0$ N



$\theta = 15.0°$

$\phi = 35.0°$

$F = 25.0$ N

(a) The sled is in equilibrium on the plane.

Resolving along the plane: $F \cos(\phi - \theta) = mg \sin \theta + f_k$.

Resolving $\perp$ plane: $n + F \sin(\phi - \theta) = mg \cos \theta$.

Also, $f_k = \mu_k n$

$$F \cos(\phi - \theta) - mg \sin \theta = \mu_k [mg \cos \theta - F \sin(\phi - \theta)]$$

$$25.0 \cos 20.0° - 60.0 \sin 15.0° = \mu_k (60.0 \cos 15.0° - 25.0 \sin 20.0°)$$

$$\mu_k = \boxed{0.161}$$

(b) Resolving $\perp$ to the plane: $n = mg \cos \theta$.

Along the plane we have $\Sigma F = ma$.

$$mg \sin \theta - f_k = ma$$

Also, $f_k = \mu_k n = \mu_k mg \cos \theta$.

So along the plane we have $mg \sin\theta - \mu_k mg \cos\theta = ma$

$$a = g(\sin\theta - \mu_k \cos\theta) = (9.80 \text{ m/s}^2)(\sin 15.0° - 0.161 \cos 15.0°)$$

$$= \boxed{1.01 \text{ m/s}^2}$$

***5.48** $mg \sin 5.00° - f = ma_x$ and $f = \mu mg \cos 5.00°$

$\therefore g \sin 5.00° - \mu g \cos 5.00° = a_x$

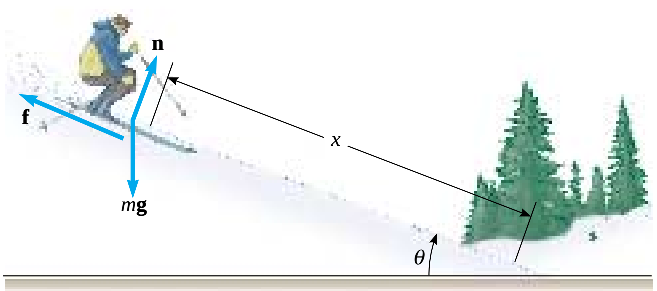
$a_x = g(\sin 5.00° - \mu \cos 5.00°) = -0.903 \text{ m/s}^2$

From Equation 2.12,

$$v_f^2 - v_i^2 = 2ax$$

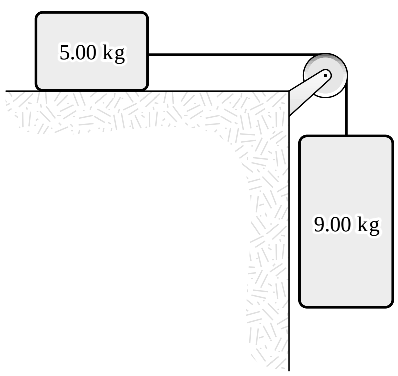
$$-(20.0)^2 = -2(0.903)x$$

$$\boxed{x = 221 \text{ m}}$$



5.49 $T - f_f = 5.00a$ (for 5.00 kg mass)

$9.00g - T = 9.00a$ (for 9.00 kg mass)



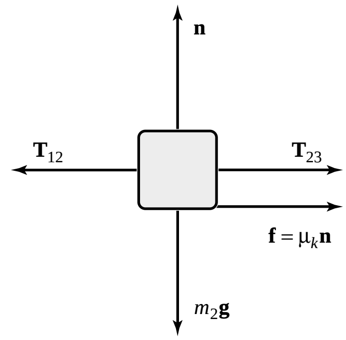
Adding these two equations gives:

$$9.00(9.80) - 0.200(5.00)(9.80) = 14.0a$$

$$a = 5.60 \text{ m/s}^2$$

$$\therefore T = 5.00(5.60) + 0.200(5.00)(9.80) = \boxed{37.8 \text{ N}}$$

5.50 Let a represent the positive magnitude of the acceleration $-a\mathbf{j}$ of m_1, of the acceleration $-a\mathbf{i}$ of m_2, and of the acceleration $+a\mathbf{j}$ of m_3. Call T_{12} the tension in the left rope and T_{23} the tension in the cord on the right.

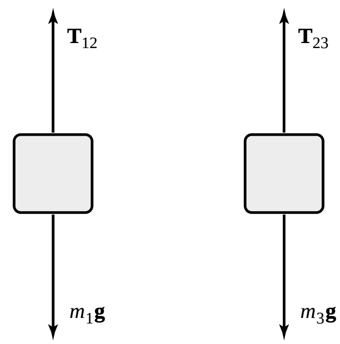


For m_1, $\Sigma F_y = ma_y \quad +T_{12} - m_1 g = -m_1 a$

For m_2, $\Sigma F_x = ma_x \quad -T_{12} + \mu_k n + T_{23} = -m_2 a$

and $\Sigma F_y = ma_y \quad n - m_2 g = 0$

for m_3, $\Sigma F_y = ma_y \quad T_{23} - m_3 g = +m_3 a$



we have three simultaneous equations

$$-T_{12} + 39.2\text{ N} = (4.00\text{ kg})a$$

$$+T_{12} - 0.350(9.80\text{ N}) - T_{23} = (1.00\text{ kg})a$$

$$+T_{23} - 19.6\text{ N} = (2.00\text{ kg})a$$

(a) Add them up:

$$+39.2\text{ N} - 3.43\text{ N} - 19.6\text{ N} = (7.00\text{ kg})a$$

$$a = \boxed{2.31\text{ m/s}^2\text{, down for } m_1\text{, left for } m_2\text{, and up for } m_3}$$

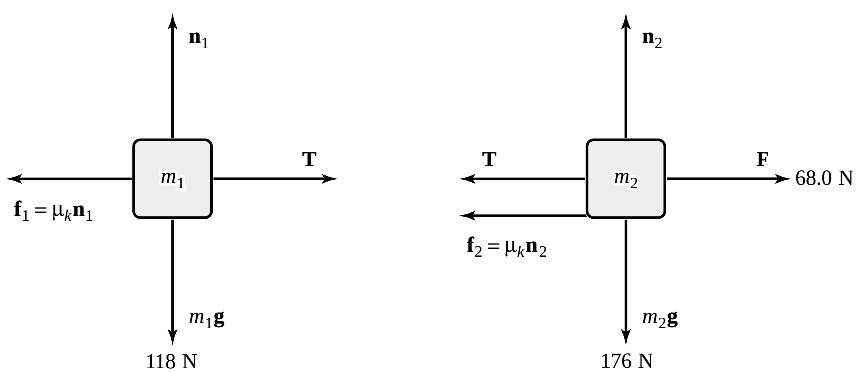
(b) Now $-T_{12} + 39.2\text{ N} = 4.00\text{ kg}(2.31\text{ m/s}^2)$

$$\boxed{T_{12} = 30.0\text{ N}}$$

and $T_{23} - 19.6\text{ N} = 2.00\text{ kg}(2.31\text{ m/s}^2)$

$$\boxed{T_{23} = 24.2\text{ N}}$$

5.51 (a)



(b) $68.0 - T - \mu m_2 g = m_2 a$ (Block #2)

$T - \mu m_1 g = m_1 a$ (Block #1)

Adding, $68.0 - \mu(m_1 + m_2)g = (m_1 + m_2)a$

$$a = \frac{68.0}{(m_1 + m_2)} - \mu g = \boxed{1.29\ \text{m/s}^2}$$

$$T = m_1 a + \mu m_1 g = \boxed{27.2\ \text{N}}$$

5.52 (a) The rope makes angle Arctan $\left(\frac{10.0\ \text{cm}}{40.0\ \text{cm}}\right) = 14.0°$

$\Sigma F_y = ma_y$ $+10.0\ \text{N}\sin 14.0° + n - 2.20\ \text{kg}(9.80\ \text{m/s}^2) = 0$

$n = 19.1\ \text{N}$

$f_k = \mu_k n = 0.400(19.1\ \text{N}) = 7.65\ \text{N}$

$\Sigma F_x = ma_x$ $+10.0\ \text{N}\cos 14.0° - 7.65\ \text{N} = (2.20\ \text{kg})a$

$a = \boxed{0.931\ \text{m/s}^2}$

(b) $\Sigma F_x = ma_x$

$10.0\ \text{N}\cos\theta - f_k = 0$

$10.0\ \text{N}\cos\theta = f_k = \mu_k n = 0.400[2.20\ \text{kg}(9.80\ \text{m/s}^2) - 10.0\ \text{N}\sin\theta]$

$10.0\ \text{N}\sqrt{1 - \sin^2\theta} = 8.62\ \text{N} - 4.00\ \text{N}\sin\theta$

$100 - 100\sin^2\theta = 74.4 - 69.0\sin\theta + 16.0\sin^2\theta$

$-116\sin^2\theta + 69.0\sin\theta + 25.6 = 0$

$$\sin\theta = \frac{-69.0 \pm \sqrt{(69.0)^2 - 4(25.6)(-116)}}{2(-116)}$$

$\sin\theta = -0.259$ or 0.854

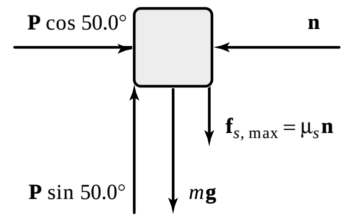
$\theta = -15.0°$ or $58.6°$

The negative root would refer to the pulley below the block.

We choose $\tan 58.6° = \frac{10.0\ \text{cm}}{x}$

$x = \boxed{6.10\ \text{cm}}$

***5.53** (Case 1, impending upward motion)



Setting $\sum F_x = 0$: $P \cos 50.0° - n = 0$

$$f_{s,\,\max} = \mu_s n = \mu_s P \cos 50.0°$$

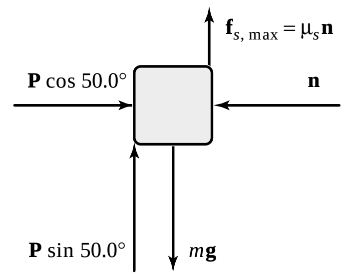
$$= 0.250(0.643)P = 0.161P$$

Setting $\sum F_y = 0$:

$$P \sin 50.0° - 0.161P - (3.00)(9.80) = 0$$

$$P_{\max} = \boxed{48.6 \text{ N}}$$

(Case 2, impending downward motion)



$f_{s,\,\max} = 0.161P$ as in Case 1.

Setting $\sum F_y = 0$:

$$P \sin 50.0° + 0.161P - (3.00)(9.80) = 0$$

$$P_{\min} = \boxed{31.7 \text{ N}}$$

5.54 $\sum \mathbf{F} = m\mathbf{a}$ gives the object's acceleration

$$\mathbf{a} = \frac{\sum F}{m} = \frac{(8.00\mathbf{i} - 4.00t\,\mathbf{j})\text{ N}}{2.00 \text{ kg}}$$

$$\mathbf{a} = (4.00 \text{ m/s}^2)\mathbf{i} - (2.00 \text{ m/s}^3)t\,\mathbf{j} = \frac{d\mathbf{v}}{dt}$$

Its velocity is

$$\int_{v_i}^{v} d\mathbf{v} = \mathbf{v} - \mathbf{v}_i = \mathbf{v} - 0 = \int_0^t \mathbf{a}\, dt$$

$$\mathbf{v} = \int_0^t [(4.00 \text{ m/s}^2)\mathbf{i} - (2.00 \text{ m/s}^3)t\,\mathbf{j}]\, dt$$

$$\mathbf{v} = (4.00t \text{ m/s}^2)\mathbf{i} - (1.00t^2 \text{ m/s}^3)\mathbf{j}$$

(a) We require $|\mathbf{v}| = 15.0$ m/s $|\mathbf{v}|^2 = 225 \text{ m}^2/\text{s}^2$

$$16.0t^2 \text{ m}^2/\text{s}^4 + 1.00t^4 \text{ m}^2/\text{s}^6 = 225 \text{ m}^2/\text{s}^2$$

$$1.00t^4 + 16.0 \text{ s}^2 t^2 - 225 \text{ s}^4 = 0$$

$$t^2 = \frac{-16.0 \pm \sqrt{(16.0)^2 - 4(-225)}}{2.00} = 9.00 \text{ s}^2$$

$$t = \boxed{3.00 \text{ s}}$$

Take $\mathbf{r}_i = 0$ at $t = 0$. The position is

$$\mathbf{r} = \int_0^t \mathbf{v}\,dt = \int_0^t [(4.00t \text{ m/s}^2)\mathbf{i} - (1.00t^2 \text{ m/s}^3)\mathbf{j}]dt$$

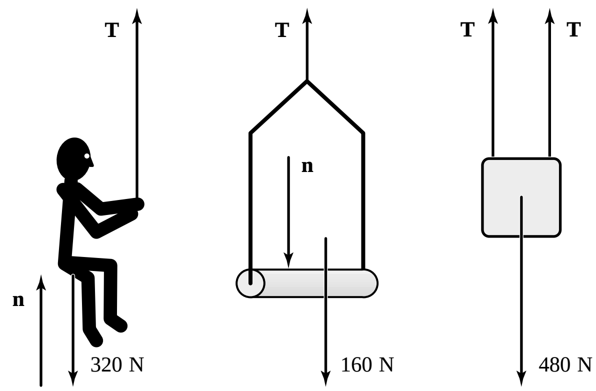
$$\mathbf{r} = (4.00 \text{ m/s}^2)\frac{t^2}{2}\,\mathbf{i} - (1.00 \text{ m/s}^3)\frac{t^3}{3}\,\mathbf{j}$$

at $t = 3$ s we evaluate

(c) $\mathbf{r} = \boxed{(18.0\mathbf{i} - 9.00\mathbf{j}) \text{ m}}$

(b) So $|\mathbf{r}| = \sqrt{(18.0)^2 + (9.00)^2} \text{ m} = \boxed{20.1 \text{ m}}$

5.55 (a)



(b) First consider Pat and the chair as the system. Note that *two* ropes support the system, and $T = 250$ N in each rope. Applying $\Sigma F = ma$

$$2T - 480 = ma \quad \text{where} \quad m = \frac{480}{9.80} = 49.0 \text{ kg}$$

Solving for a gives $a = \dfrac{(500 - 480)}{49.0} = \boxed{0.408 \text{ m/s}^2}$

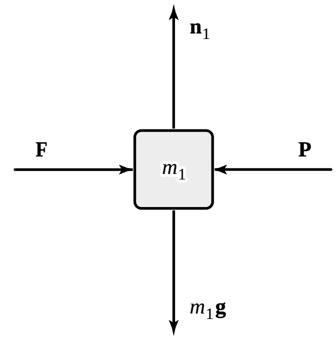
(c) $\Sigma F \text{ (on Pat)} = n + T - 320 = ma \quad \text{where} \quad m = \dfrac{320}{9.80} = 32.7 \text{ kg}$

$$n = ma + 320 - T = 32.7(0.408) + 320 - 250 = \boxed{83.3 \text{ N}}$$

***5.56** (a) $F = ma$

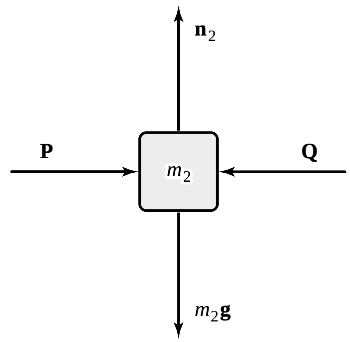
$18.0 = (2.00 + 3.00 + 4.00)a$

$a = \boxed{2.00\ \text{m/s}^2}$



(b) The force on each block can be found by knowing mass and acceleration:

$\Sigma F_1 = m_1 a = 2.00(2.00) = \boxed{4.00\ \text{N}}$



$\Sigma F_2 = m_2 a = 3.00(2.00) = \boxed{6.00\ \text{N}}$

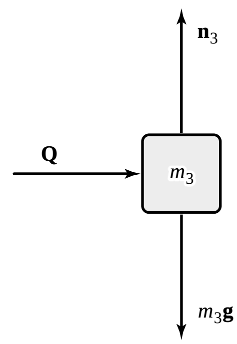
$\Sigma F_3 = m_3 a = 4.00(2.00) = \boxed{8.00\ \text{N}}$

(c) Therefore, $\Sigma F_1 = 4.00\ \text{N} = F - P$

$P = \boxed{14.0\ \text{N}}$

$\Sigma F_2 = 6.00\ \text{N} = P - Q$

$Q = \boxed{800\ \text{N}}$



***5.57** We find the diver's impact speed by anaylzing his free-fall motion:

$v_f^2 = v_i^2 + 2ax = 0^2 + 2(-9.80\ \text{m/s}^2)(-10.0\ \text{m})$

$v_f = -14.0\ \text{m/s}$

Now for the 2.00 s of stopping, we have

$v_f = v_i + at$

$0 = -14.0\ \text{m/s} + a(2.00\ \text{s}),\ a = +7.00\ \text{m/s}^2$

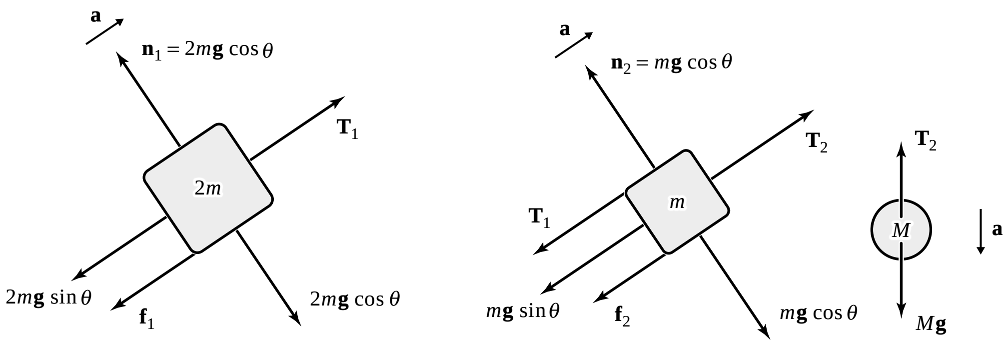
$\Sigma F_y = ma$

Call the force exerted by the water on the driver R.

$+R - (70.0\ \text{kg})(9.80\ \text{m/s}^2) = (70.0\ \text{kg})(7.00\ \text{m/s}^2)$

$R = \boxed{1.18\ \text{kN}}$

5.58



Applying Newton's second law to each object gives:

(1) $T_1 = f_1 + 2m(g\sin\theta + a)$ (2) $T_2 - T_1 = f_2 + m(g\sin\theta + a)$

and (3) $T_2 = M(g - a)$

Parts (a) and (b): Equilibrium ($\Rightarrow a = 0$) and frictionless incline ($\Rightarrow f_1 = f_2 = 0$) Under these conditions, the equations reduce to

(1) $\boxed{T_1 = 2mg\sin\theta}$ (2) $T_2 - T_1 = mg\sin\theta$ and (3') $T_2 = Mg$

Substituting (1') and (3') into equation (2') then gives $\boxed{M = 3m\sin\theta}$, so equation (3') becomes $\boxed{T_2 = 3mg\sin\theta}$.

Parts (c) and (d): $M = 6m\sin\theta$(double the value found above), and $f_1 = f_2 = 0$. With these conditions present, the equations become

$$T_1 = 2m(g\sin\theta + a) \quad T_2 - T_1 = m(g\sin\theta + a) \quad \text{and} \quad T_2 = 6m\sin\theta(g - a)$$

Solved simultaneously, these yield $\boxed{a = \dfrac{g\sin\theta}{1 + 2\sin\theta}}$,

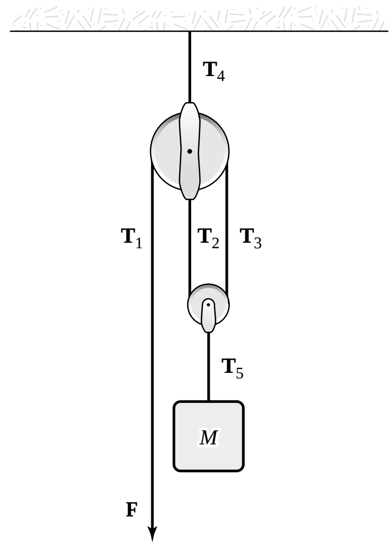
$\boxed{T_1 = 4mg\sin\theta\left(\dfrac{1+\sin\theta}{1+2\sin\theta}\right)}$ and $\boxed{T_2 = 6mg\sin\theta\left(\dfrac{1+\sin\theta}{1+2\sin\theta}\right)}$

Part (e): Equilibrium ($\Rightarrow a = 0$) and impending motion up the incline so $M = M_{\max}$ while $f_1 = 2\mu_s\, mg\cos\theta$ and $f_2 = \mu_s\, mg\cos\theta$, both directed down the incline. Under these conditions, the equations become $T_1 = 2mg(\sin\theta + \mu_s\cos\theta)$, $T_2 - T_1 = mg(\sin\theta + \mu_s\cos\theta)$, and $T_2 = M_{\max}\, g$ which yield $\boxed{M_{\max} = 3m(\sin\theta + \mu_s\cos\theta)}$.

Part (f): Equilibrium ($\Rightarrow a = 0$) and impending motion **down** the incline so $M = M_{\min}$ while $f_1 = 2\mu_s\, mg\cos\theta$ and $f_2 = \mu_s\, mg\cos\theta$, both directed **up** the incline. Under these conditions, the equations are $T_1 = 2mg(\sin\theta - \mu_s\cos\theta)$, $T_2 - T_1 = mg(\sin\theta - \mu_s\cos\theta)$, and $T_2 = M_{\min}\, g$ which yield $\boxed{M_{\min} = 3m(\sin\theta - \mu_s\cos\theta)}$.

Part (g): $\boxed{T_{2,\,\max} - T_{2,\,\min} = M_{\max}g - M_{\min}g = 6mg\mu_s \cos\theta}$

5.59 (a) First, we note that $F = T_1$. Next, we focus on the mass M and write $T_5 = Mg$. Next, we focus on the bottom pulley and write $T_5 = T_2 + T_3$. Finally, we focus on the top pulley and write $T_4 = T_1 + T_2 + T_3$.



Since the pulleys are massless and frictionless, $T_1 = T_3$, and $T_2 = T_3$. From this information, we have $T_5 = 2\,T_2$, so $T_2 = \dfrac{Mg}{2}$.

Then $\boxed{T_1 = T_2 = T_3 = \dfrac{Mg}{2}}$, and $\boxed{T_4 = \dfrac{3\,Mg}{2}}$, and $\boxed{T_5 = Mg}$

(b) Since $F = T_1$, we have $\boxed{F = \dfrac{Mg}{2}}$

5.60 (a) $\Sigma\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = (-9.00\mathbf{i} + 3.00\mathbf{j})\text{ N}$

Acceleration $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} = \dfrac{\Sigma\mathbf{F}}{m} = \dfrac{(-9.00\mathbf{i} + 3.00\mathbf{j})\text{ N}}{2.00\text{ kg}} = (-4.50\mathbf{i} + 1.50\mathbf{j})\text{ m/s}^2$

Velocity $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = \mathbf{v}_\text{i} + \mathbf{a}t = \mathbf{a}t$

$$\mathbf{v} = (-4.50\mathbf{i} + 1.50\mathbf{j})(\text{m/s}^2)(10\text{ s}) = \boxed{(-45.0\mathbf{i} + 15.0\mathbf{j})\text{ m/s}}$$

(b) The direction of motion makes angle θ with the x-direction.

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(-\frac{15.0\text{ m/s}}{45.0\text{ m/s}}\right)$$

$$\theta = -18.4° + 180° = \boxed{162°\text{ from }+x\text{-axis}}$$

(c) Displacement:

$$x\text{-displacement} = x - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = \left(\frac{1}{2}\right)(-4.50\text{ m/s}^2)(10.0\text{ s})^2 = -225\text{ m}$$

$$y\text{-displacement} = y - y_i = v_{yi}t + \frac{1}{2}a_y t^2 = \left(\frac{1}{2}\right)(+1.50\text{ m/s}^2)(10.0\text{ s})^2 = +75.0\text{ m}$$

$$\Delta\mathbf{r} = \boxed{(-225\mathbf{i} + 75.0\mathbf{j})\text{ m}}$$

(d) Position: $\equiv \mathbf{r} = \mathbf{r}_1 + \Delta\mathbf{r}$

$$\mathbf{r} = (-2.00\mathbf{i} + 4.00\mathbf{j}) + (-225\mathbf{i} + 75.0\mathbf{j}) = \boxed{(-227\mathbf{i} + 79.0\mathbf{j})\text{ m}}$$

5.61 (a) The crate is in equilibrium.
Let the normal force acting on it be n and the friction force, f_s.
Resolving vertically: $n = F_g + P\sin\theta$
Horizontally: $P\cos\theta = f_s$
But $f_s \le \mu_s n$

i.e., $P\cos\theta \le \mu_s(F_g + P\sin\theta)$

or $P(\cos\theta - \mu_s\sin\theta) \le \mu_s F_g$

Divide by $\cos\theta$: $P(1 - \mu_s\tan\theta) \le \mu_s F_g\sec\theta$

Then $\boxed{P_{\text{minimum}} = \dfrac{\mu_s F_g\sec\theta}{1 - \mu_s\tan\theta}}$

(b) $P = \dfrac{0.400(100\text{ N})\sec\theta}{1 - 0.400\tan\theta}$

θ (deg)	0.00	15.0	30.0	45.0	60.0
P (N)	40.0	46.4	60.1	94.3	260

5.62 (a) Following Example 5.6 $a = g\sin\theta = (9.80\text{ m/s}^2)\sin 30.0°$

$\boxed{a = 4.90\text{ m/s}^2}$

(b) The block slides distance x on the incline, with $\sin 30.0° = 0.500\text{ m}/x$ $x = 1.00\text{ m}$

$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0 + 2(4.90\text{ m/s}^2)(1.00\text{ m})$

$v_f = \boxed{3.13\text{ m/s}}$ after time $t_s = \dfrac{2x_f}{v_f} = \dfrac{2(1.00\text{ m})}{3.13\text{ m/s}} = 0.639\text{ s}$

Now in free fall $y_f - y_i = v_{yi}t + \dfrac{1}{2}a_y t^2$

$-2.00\text{ m} = (-3.13\text{ m/s})\sin 30.0° t - \dfrac{1}{2}(9.80\text{ m/s}^2)\,t^2$

$(4.90\text{ m/s}^2)t^2 + (1.56\text{ m/s})t - 2.00\text{ m} = 0$

$$t = \frac{-1.56\text{ m/s} \pm \sqrt{(1.56\text{ m/s})^2 - 4(4.90\text{ m/s}^2)(-2.00\text{ m})}}{9.80\text{ m/s}^2}$$

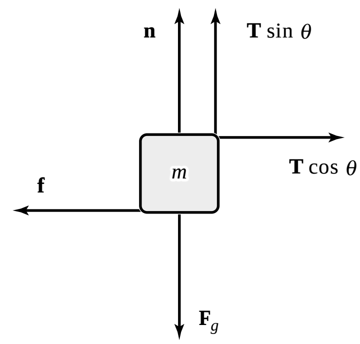
$t = 0.499$ s, the other root being unphysical.

(c) $x = v_x t = [(3.13\text{ m/s})\cos 30.0°](0.499\text{ s}) = \boxed{1.35\text{ m}}$

(d) total time $= t_s + t = 0.639\text{ s} + 0.499\text{ s} = \boxed{1.14\text{ s}}$

(e) The mass of the block makes no difference.

***5.63** With motion impending, $n + T\sin\theta - mg = 0$



$$f = \mu_s(mg - T\sin\theta)$$

and $T\cos\theta - \mu_s mg + \mu_s T\sin\theta = 0$

so $$T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$$

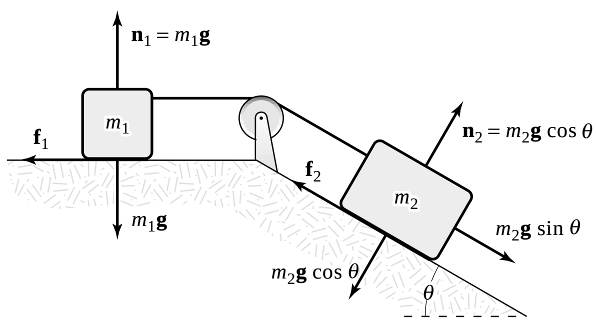
To minimize T, we maximize $\cos\theta + \mu_s\sin\theta$

$$\frac{d}{d\theta}(\cos\theta + \mu_s\sin\theta) = 0 = -\sin\theta + \mu_s\cos\theta$$

(a) $\theta = \text{Arctan}\,\mu_s = \text{Arctan}\,0.350 = \boxed{19.3°}$

(b) $$T = \frac{(0.350)(1.30\text{ kg})(9.80\text{ m/s}^2)}{\cos 19.3° + 0.350\sin 19.3°} = \boxed{4.21\text{ N}}$$

5.64



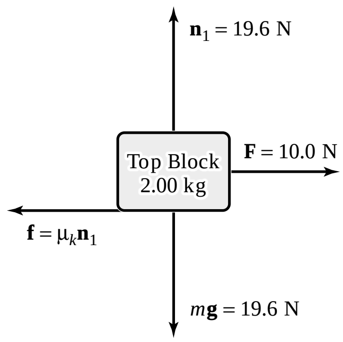
For the system to start to move when released, the force tending to move m_2 down the incline, $m_2 g\sin\theta$, must exceed the maximum friction force which can retard the motion:

$$f_{\max} = f_{1,\max} + f_{2,\max} = \mu_{s,1}n_1 + \mu_{s,2}n_2 = \mu_{s,1}m_1 g + \mu_{s,2}m_2 g\cos\theta$$

From Table 5.2, $\mu_{s,1} = 0.610$ (aluminum on steel) and $\mu_{s,2} = 0.530$ (copper on steel). With $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, $\theta = 30.0°$, the maximum friction force is found to be $f_{\max} = 38.9$ N. This exceeds the force tending to cause the system to move, $m_2 g\sin\theta = (6.00)(9.80)\sin 30.0° = 29.4$ N. Hence, $\boxed{\text{the system will not start to move when released}}$.

The friction forces increase in magnitudes until the total friction force retarding the motion, $f = f_1 + f_2$, equals the force tending to set the system in motion. That is until $\boxed{f = m_2 g\sin\theta = 29.4\text{ N}}$.

5.65 (a) First, draw a free-body diagram, (Fig. 1) of the top block.



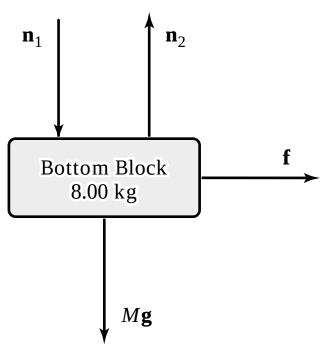
Since $a_y = 0$, $n_1 = 19.6$ N and $f_k = \mu_k n_1 = (0.300)(19.6) = 5.88$ N

$$\sum F_x = ma_T$$

gives 10.0 N – 5.88 N = (2.00 kg)a_T, or

$$a_T = 2.06 \text{ m/s}^2 \text{ (for top block)}$$

Now draw a free-body diagram (Fig. 2) of the bottom block and observe that



$$\sum F_x = Ma_B$$

gives $f = 5.88$ N = (8.00 kg)a_B, or

$$a_B = 0.753 \text{ m/s}^2 \text{ (for the bottom block)}$$

In time t, the distance each block moves (starting from rest) is

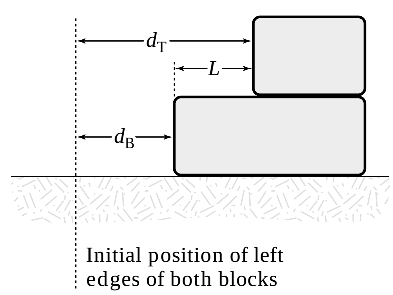
$$d_T = \frac{1}{2} a_T t^2 \quad \text{and} \quad d_B = \frac{1}{2} a_B t^2$$

For the top block to reach the right edge of the bottom block, it is necessary that

$$d_T = d_B + L \quad \text{or} \quad \text{(See Figure 3)}$$

$$\frac{1}{2}(2.06 \text{ m/s}^2)\, t^2 = \frac{1}{2}(0.735 \text{ m/s}^2)\, t^2 + 3.00 \text{ m}$$

which gives: $t = \boxed{2.13 \text{ s}}$

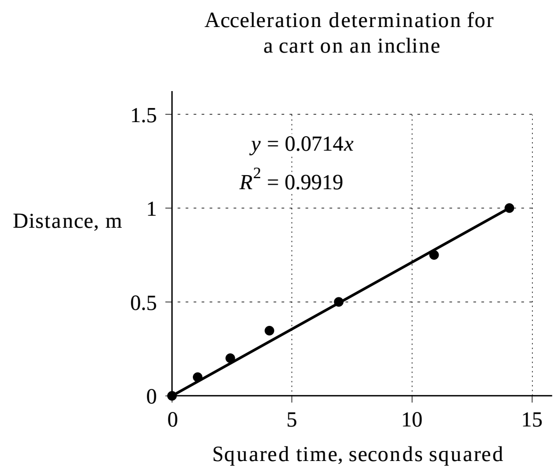


(b) From above,

$$d_B = \frac{1}{2}(0.735 \text{ m/s}^2)(2.13 \text{ s})^2 = \boxed{1.67 \text{ m}}$$

5.66

t(s)	t^2(s^2)	x(m)
0	0	0
1.02	1.040	0.100
1.53	2.341	0.200
2.01	4.040	0.350
2.64	6.970	0.500
3.30	10.89	0.750
3.75	14.06	1.00



From $x = \frac{1}{2}at^2$

the slope of a graph of x versus t^2 is $\frac{1}{2}a$,

and $a = 2 \times \text{slope} = 2(0.0714 \text{ m/s}^2) = \boxed{0.143 \text{ m/s}^2}$

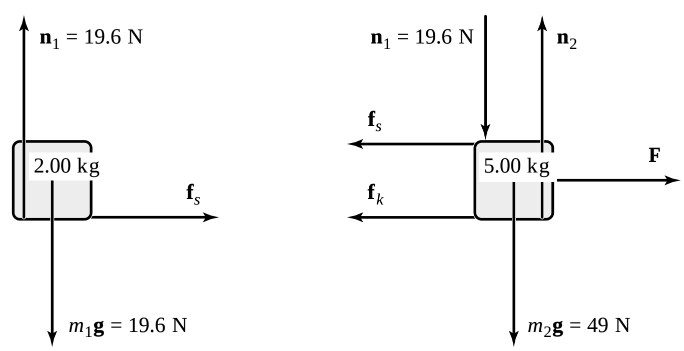
From $a' = g \sin\theta$, we obtain

$$a' = (9.80 \text{ m/s}^2)\left(\frac{1.774}{127.1}\right) = 0.137 \text{ m/s}^2, \text{ different by 4\%}$$

The difference is accounted for by the uncertainty in the data, which we may estimate from the third point as

$$\frac{0.350 - (0.0714)(4.04)}{0.350} = 18\%$$

5.67 (a)



The force of static friction between the blocks accelerates the 2.00 kg block.

(b) $\Sigma F = ma$, for both blocks together

$F - \mu n_2 = ma,$

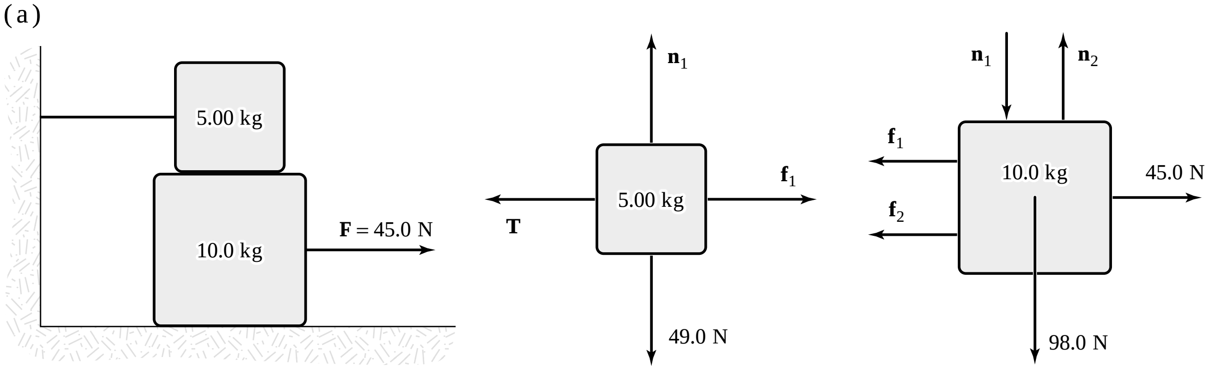
$F - (0.200)[(5.00 + 2.00)(9.80)] = (5.00 + 2.00)3.00$

Therefore $F = \boxed{34.7\text{ N}}$

(c) $f = \mu_1(2.00)(9.80) = m_1 a = 2.00(3.00)$

Therefore $\mu = \boxed{0.306}$

5.68 (a)



f_1 and n_1 appear in both diagrams as action-reaction pairs

(b) 5.00 kg: $\Sigma F_x = ma$ $n_1 = m_1 g = (5.00)(9.80) = 49.0\text{ N}$ $f_1 - T = 0$

$T = f_1 = \mu mg = (0.200)(5.00)(9.80) = \boxed{9.80\text{ N}}$

10.0 kg: $\Sigma F_x = ma$ $\Sigma F_y = 0$

$45.0 - f_1 - f_2 = 10.0a$ $n_2 - n_1 - 98.0 = 0$

$f_2 = \mu n_2 = \mu(n_1 + 98.0) = (0.20)(49.0 + 98.0) = 29.4\text{ N}$

$45 - 9.80 - 29.4 = 10.0a$

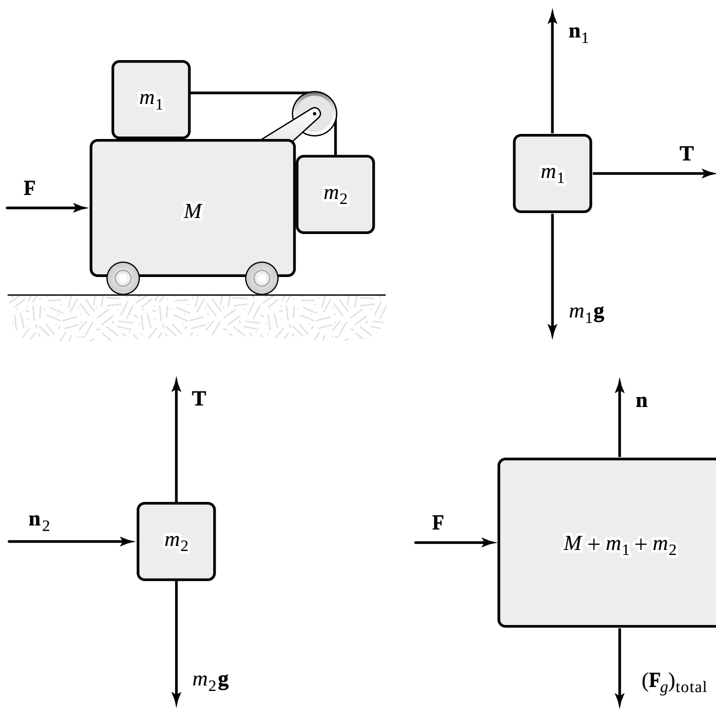
$a = \boxed{0.580\text{ m/s}^2}$

5.69 $\Sigma F = ma$

For m_1: $T = m_1 a \quad a = \dfrac{m_2 g}{m_1}$

For m_2: $T - m_2 g = 0$

For all 3 blocks: $F = (M + m_1 + m_2)a = \boxed{(M + m_1 + m_2)\left(\dfrac{m_2 g}{m_1}\right)}$



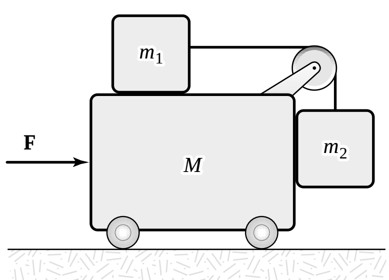
Goal Solution

Draw separate free-body diagrams for blocks m_1 and m_2.

Remembering that normal forces are always perpendicular to the contacting surface, and always **push** on a body, draw n_1 and n_2 as shown. Note that m_2 should be in **contact** with the cart, and therefore does have a normal force from the cart.

Remembering that ropes always **pull** on bodies in the direction of the rope, draw the tension force **T**.

Finally, draw the gravitational force on each block, which always points downwards.



G: What can keep m_2 from falling? Only tension in the cord connecting it with m_1. This tension pulls forward on m_1 to accelerate that mass. This acceleration should be proportional to m_2 and to g and inversely proportional to m_1, so perhaps $a = (m_2/m_1)g$ We should also expect the applied force to be proportional to the total mass of the system.

O: Use $\Sigma F = ma$ and the free-body diagrams above.

A: For m_2, $T - m_2 g = 0$ or $T = m_2 g$

For m_1, $T = m_1 a$ or $a = \dfrac{T}{m_1}$

Substituting for T, we have $a = \dfrac{m_2 g}{m_1}$

For all 3 blocks, $F = (M + m_1 + m_2)a$.

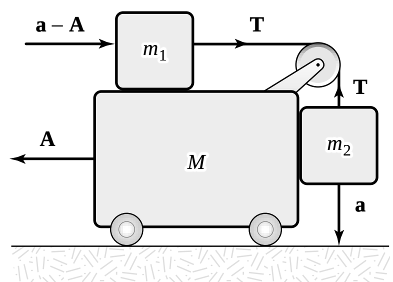
Therefore, $F = (M + m_1 + m_2)\left(\dfrac{m_2 g}{m_1}\right)$

L: Even though this problem did not have a numerical solution, we were still able to rationalize the algebraic form of the solution. This technique does not always work, especially for complex situations, but often we can think through a problem to see if an equation for the solution makes sense based on the physical principles we know.

5.70 (1) $m_1(a - A) = T \Rightarrow a = T/m_1 + A$

(2) $MA = R_x = T \Rightarrow A = T/M$

(3) $m_2 a = m_2 g - T \Rightarrow T = m_2(g - a)$



(a) Substitute the value for a from (1) into (3) and solve for T;

$$T = m_2[g - (T/m_1 + A)]$$

Substitute for A from (2);

$$T = m_2\left[g - \left(\frac{T}{m_1} + \frac{T}{M}\right)\right] = \boxed{m_2 g\left[\frac{m_1 M}{m_1 M + m_2(m_1 + M)}\right]}$$

(b) Solve (3) for a and substitute value of T

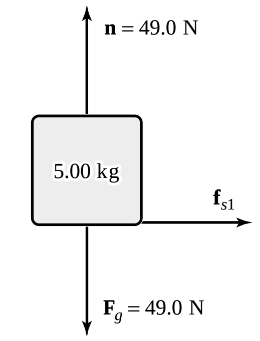
$$\boxed{a = \frac{m_2 g(M + m_1)}{m_1 M + m_2(M + m_1)}}$$

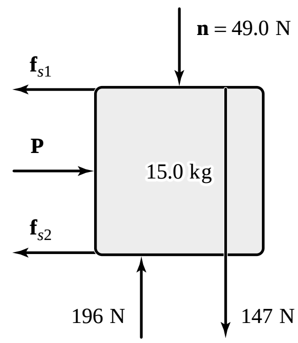
(c) From (2), $A = T/M$; Substitute the value of T

$$\boxed{A = \frac{m_1 m_2 g}{m_1 M + m_2(m_1 + M)}}$$

(d) $$\boxed{a - A = \frac{M m_2 g}{m_1 M + m_2(m_1 + M)}}$$

***5.71** (a) Motion impending





$f_{s1} = \mu n = 14.7\text{ N}$ $\qquad$ $f_{s2} = 0.500(196\text{ N}) = 98.0\text{ N}$

(b) $P = f_{s1} + f_{s2} = 14.7\text{ N} + 98.0\text{ N} = \boxed{113\text{ N}}$

(c) Once motion starts, kinetic friction acts.

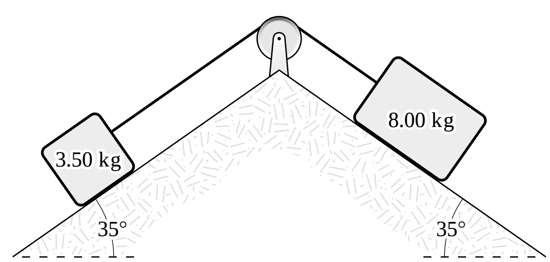
$$112.7\text{ N} - 0.100(49.0\text{ N}) - 0.400(196\text{ N}) = 15.0\text{ kg } a_2$$

$$a_2 = \boxed{1.96\text{ m/s}^2}$$

$$0.100(49.0\text{ N}) = 5.00\text{ kg } a_1$$

$$a_1 = \boxed{0.980\text{ m/s}^2}$$

5.72 Since it has a larger mass, we expect the 8.00 kg block to move down the plane. The acceleration for both blocks should have the same magnitude since they are joined together by a non-stretching string.



$\Sigma F_1 = m_1 a_1$: $\quad -m_1 g \sin 35.0° + T = m_1 a$

$\Sigma F_2 = m_2 a_2$: $\quad -m_2 g \sin 35.0° + T = -m_2 a$

and $\quad -(3.50)(9.80) \sin 35.0° + T = 3.50a$

$-(8.00)(9.80) \sin 35.0° + T = -8.00a$

$T = \boxed{27.4\text{ N}}$ $\qquad$ $a = \boxed{2.20\text{ m/s}^2}$

5.73 $\Sigma F_1 = m_1 a$: $-m_1 g \sin 35.0° - f_{k,1} + T = m_1 a$

(1) $-(3.50)(9.80) \sin 35.0° - \mu_k(3.50)(9.80) \cos 35.0° + T = (3.50)(1.50)$

$\Sigma F_2 = m_2 a$: $+m_2 g \sin 35.0° - f_{k,2} - T = m_2 a$

(2) $+(8.00)(9.80) \sin 35.0° - \mu_k(8.00)(9.80) \cos 35.0° - T = (8.00)(1.50)$

Solving equations (1) and (2) simultaneously gives

(a) $\boxed{\mu_k = 0.0871}$

(b) $\boxed{T = 27.4 \text{ N}}$

5.74 The forces acting on the sled are

(a) $T - F_f = ma$

$T - 500 \text{ N} = (100 \text{ kg})(1.00 \text{ m/s}^2)$

$T = \boxed{600 \text{ N}}$

(b) Frictional force pushes the horse forward.

$f - T = m_{\text{horse}} a$

$f - 600 \text{ N} = (500 \text{ kg})(1.00 \text{ m/s}^2)$

$f = \boxed{1100 \text{ N}}$

(c) $f - F_f = 600 \text{ N}$

$\Sigma m = 100 \text{ kg} + 500 \text{ kg}$

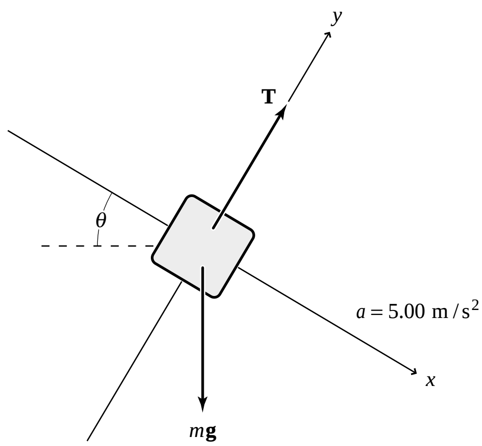
$$a = \frac{\Sigma F}{\Sigma m} = \frac{600 \text{ N}}{600 \text{ kg}} = \boxed{1.00 \text{ m/s}^2}$$

5.75 $mg \sin \theta = m(5.00 \text{ m/s}^2)$

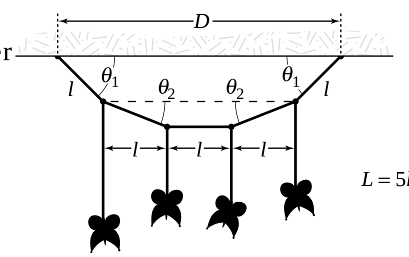
$\theta = \boxed{30.7°}$

$T = mg \cos \theta = (0.100)(9.80) \cos 30.7°$

$T = \boxed{0.843 \text{ N}}$



5.76 (a) Apply Newton's 2nd law to two points where butterflies are attached on either half of mobile (other half the same, by symmetry)



(1) $T_2 \cos \theta_2 - T_1 \cos \theta_1 = 0$

(2) $T_1 \sin \theta_1 - T_2 \sin \theta_2 - mg = 0$

(3) $T_2 \cos \theta_2 - T_3 = 0$

(4) $T_2 \sin \theta_2 - mg = 0$

Substituting (3) into (1) for $T_2 \sin \theta_2$ $T_1 \sin \theta_1 - mg - mg = 0$

Then $\boxed{T_1 = \dfrac{2mg}{\sin \theta_1}}$

Substitute (3) into (1) for $T_2 \cos \theta_2$ $T_3 - T_1 \cos \theta_1 = 0,$ $T_3 = T_1 \cos \theta_1$

Substitute value of T_1; $T_3 = 2mg \dfrac{\cos \theta_1}{\sin \theta_1} = \dfrac{2mg}{\tan \theta_1}$

From Eq. (4), $\boxed{T_2 = \dfrac{mg}{\sin \theta_2}}$

(b) We must find θ_2 and substitute for θ_2: $T_2 = \dfrac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\,\theta_1\right)\right]}$

divide (4) by (3);

$$\frac{T_2 \sin\,\theta_2}{T_2 \cos\,\theta_2} = \frac{mg}{T_3} \Rightarrow \tan\,\theta_2 = \frac{mg}{T_3}$$

Substitute value of $T_3 \Rightarrow \tan\,\theta_2 = \dfrac{mg\tan\,\theta_1}{2mg}$

$$\boxed{\theta_2 = \tan^{-1}\left(\frac{\tan\,\theta_1}{2}\right)}$$

(c) D is the total horizontal displacement of each string

$D = 2\,l\,\cos\theta_1 + 2\,l\,\cos\theta_2 + l$ and $L = 5\,l$

$$D = \frac{L}{5}\left\{2\cos\,\theta_1 + 2\cos\left[\tan^{-1}\left(\frac{1}{2}\tan\,\theta_1\right)\right] + 1\right\}$$

5.77 If all the weight is on the rear wheels,

(a) $F = ma \quad mg\mu_s = ma$

But $\Delta x = \dfrac{at^2}{2} = \dfrac{g\mu_s t^2}{2}$, so $\mu_s = \dfrac{2\Delta x}{gt^2}$

$$\mu_s = \frac{2(0.250\text{ mi})(1609\text{ m/mi})}{(9.80\text{ m/s}^2)(4.96\text{ s})^2} = \boxed{3.34}$$

(b) Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.

5.78 $\sum F_y = ma_y$: $n - mg\cos\theta = 0$, or

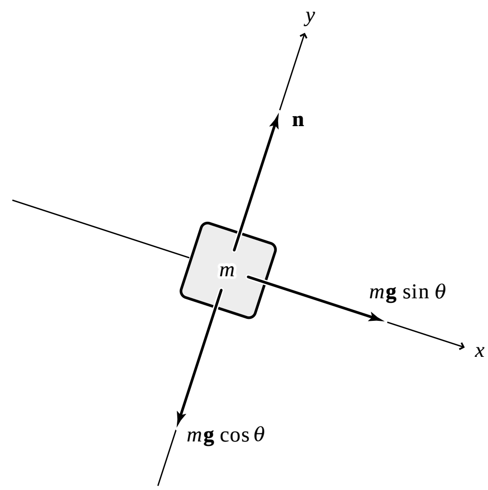
$n = (8.40)(9.80)\cos\theta$

$n = (82.3\text{ N})\cos\theta$

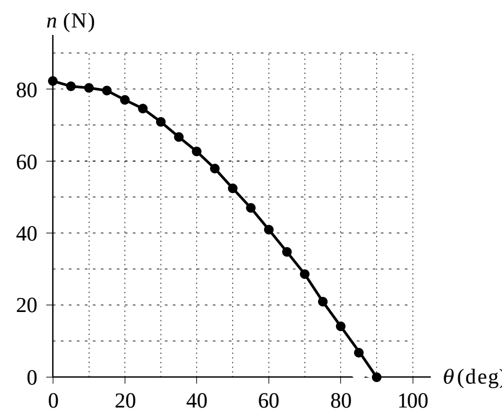
$\sum F_x = ma_x$: $mg\sin\theta = ma$, or

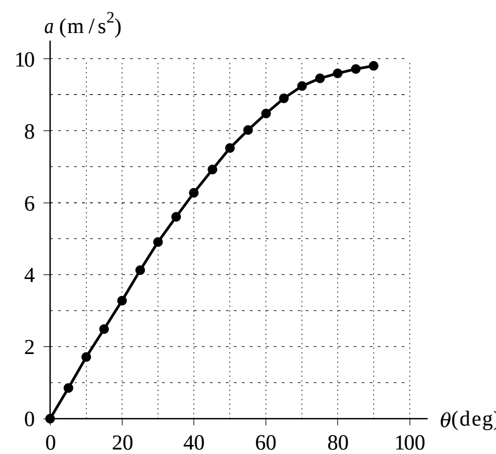
$a = g\sin\theta$

$a = (9.80\text{ m/s}^2)\sin\theta$



θ (deg)	n (N)	a (m/s^2)
0.00	82.3	0.00
5.00	82.0	0.854
10.0	81.1	1.70
15.0	79.5	2.54
20.0	77.4	3.35
25.0	74.6	4.14
30.0	71.3	4.90
35.0	67.4	5.62
40.0	63.1	6.30
45.0	58.2	6.93
50.0	52.9	7.51
55.0	47.2	8.03
60.0	41.2	8.49
65.0	34.8	8.88
70.0	28.2	9.21
75.0	21.3	9.47
80.0	14.3	9.65
85.0	7.17	9.76
90.0	0.00	9.80





At 0°, the normal force is the full weight and the acceleration is zero. At 90°, the mass is in free fall next to the vertical incline.

Chapter 6 Solutions

6.1 (a) Average speed $= \bar{v} = \dfrac{200\text{ m}}{25.0\text{ s}} = \boxed{8.00\text{ m/s}}$

(b) $F = \dfrac{mv^2}{r}$ where $r = \dfrac{200\text{ m}}{2\pi} = 31.8\text{ m}$

$$F = \frac{(1.50\text{ kg})(8.00\text{ m/s})^2}{31.8\text{ m}} = \boxed{3.02\text{ N}}$$

6.2 (a) $\Sigma F_x = ma_x$

$$T = \frac{mv^2}{r} = \frac{55.0\text{ kg }(4.00\text{ m/s})^2}{0.800\text{ m}} = \boxed{1100\text{ N}}$$

(b) The tension is $\boxed{\text{larger}}$ than her weight by

$$\frac{1100\text{ N}}{(55.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{2.04\text{ times}}$$

6.3 $m = 3.00$ kg: $r = 0.800$ m. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{\text{max}} = Mg = 25.0 \times 9.80 = 245\text{ N}$$

When the 3.00 kg mass rotates in a horizontal circle, the tension provides the centripetal force, so

$$T = \frac{mv^2}{r} = \frac{(3.00)v^2}{0.800}$$

Then $v^2 = \dfrac{rT}{m} = \dfrac{0.800T}{3.00} \le \dfrac{(0.800T_{\text{max}})}{3.00} = \dfrac{0.800 \times 245}{3.00} = 65.3\text{ m}^2/\text{s}^2$

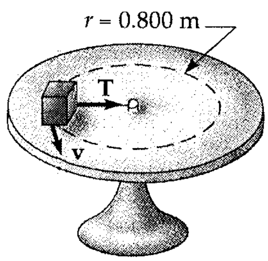
and $0 < v < \sqrt{65.3}$ or $\boxed{0 < v < 8.08\text{ m/s}}$

Goal Solution

The string will break if the tension T exceeds the test weight it can support,

$$T_{\max} = mg = (25.0\text{ kg})(9.80\text{ m/s}^2) = 245\text{ N}$$

As the 3.00-kg mass rotates in a horizontal circle, the tension provides the central force.



From $\sum F = ma$, $T = \dfrac{mv^2}{r}$

Then, $v \le \sqrt{\dfrac{rT_{\max}}{m}} = \sqrt{\dfrac{(0.800\text{ m})(245\text{ N})}{(3.00\text{ kg})}} = 8.08\text{ m/s}$

So the mass can have speeds between 0 and 8.08 m/s. ◊

6.4 (a) $F = \dfrac{mv^2}{r} = \dfrac{(9.11\times10^{-31}\text{ kg})(2.20\times10^{6}\text{ m/s})^2}{0.530\times10^{-10}\text{ m}} = \boxed{8.32\times10^{-8}\text{ N}}$ inward

(b) $a = \dfrac{v^2}{r} = \dfrac{(2.20\times10^{6}\text{ m/s})^2}{0.530\times10^{-10}\text{ m}} = \boxed{9.13\times10^{22}\text{ m/s}^2}$ inward

6.5 Neglecting relativistic effects. $F = ma_c = \dfrac{mv^2}{r}$

$$F = (2\times1.661\times10^{-27}\text{ kg})\,\frac{(2.998\times10^{7}\text{ m/s})^2}{(0.480\text{ m})} = \boxed{6.22\times10^{-12}\text{ N}}$$

6.6 (a) We require that $\dfrac{GmM_e}{r^2} = \dfrac{mv^2}{r}$ but $g = \dfrac{M_eG}{R_e^2}$

In this case $r = 2R_e$, therefore, $\dfrac{g}{4} = \dfrac{v^2}{2R_e}$ or $v = \sqrt{\dfrac{gR_e}{2}}$

$$v = \sqrt{\frac{(9.80\text{ m/s}^2)(6.37\times10^{6}\text{ m})}{2}} = \boxed{5.59\times10^{3}\text{ m/s}}$$

(b) $T = \dfrac{2\pi r}{v} = \dfrac{(2\pi)(2)(6.37\times10^{6}\text{ m})}{5.59\times10^{3}\text{ m/s}} = \boxed{239\text{ min}}$

(c) $F = \dfrac{GmM_e}{(2R_e)^2} = \dfrac{mg}{4} = \dfrac{(300\text{ kg})(9.80\text{ m/s}^2)}{4} = \boxed{735\text{ N}}$

6.7 The orbit radius is $r = 1.70\times10^{6}$ m + 100 km = 1.80×10^{6} m. Now using the information in Example 6.6,

$$\frac{GM_m m_s}{r^2} = \frac{m_s 2^2\pi^2 r^2}{rT^2} = m_s a$$

(a) $a = \dfrac{GM_m}{r^2} = \dfrac{(6.67\times10^{-11})(7.40\times10^{22})}{(1.80\times10^{6}\text{ m})^2} = \boxed{1.52\text{ m/s}^2}$

(b) $a = \dfrac{v^2}{r}$, $v = \sqrt{(1.52\text{ m/s}^2)(1.80\times10^{6}\text{ m})} = \boxed{1.66\text{ km/s}}$

(c) $v = \dfrac{2\pi r}{T}$, $T = \dfrac{2\pi(1.80\times10^{6})}{1.66\times10^{3}} = \boxed{6820\text{ s}}$

6.8 (a) Speed = distance/time. If the radius of the hand of the clock is r then

$$v = \frac{2\pi r}{T} \Rightarrow vT = 2\pi r$$

$$r_m = r_s \therefore\ T_m v_m = T_s v_s$$

where $v_m = 1.75 \times 10^{-3}$ m/s, $T_m = (60.0 \times 60.0)$s and $T_s = 60.0$ s.

$$v_s = \left(\frac{T_m}{T_s}\right) v_m = \left(\frac{3.60 \times 10^3 \text{ s}}{60.0 \text{ s}}\right)(1.75 \times 10^{-3} \text{ m/s}) = \boxed{0.105 \text{ m/s}}$$

(b) $v = \dfrac{2\pi r}{T}$ for the second hand, $r = \dfrac{vT}{2\pi} = \dfrac{(0.105 \text{ m/s})(60.0 \text{ s})}{2\pi} = 1.00$ m

Then $a_r = \dfrac{v^2}{r} = \dfrac{(0.105 \text{ m/s})^2}{1.00 \text{ m}} = \boxed{1.10 \times 10^{-2} \text{ m/s}^2}$

6.9 (a) $\boxed{\text{static friction}}$

(b) $ma\,\mathbf{i} = f\,\mathbf{i} + n\,\mathbf{j} + mg(-\mathbf{j})$

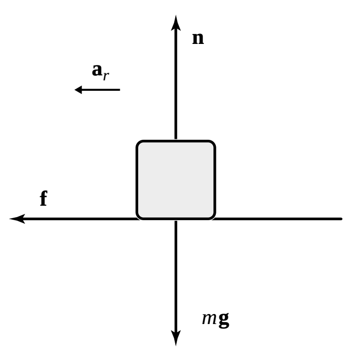
$\Sigma F_y = 0 = n - mg$

thus $n = mg$ and $\Sigma F_r = m\dfrac{v^2}{r} = f = \mu n = \mu mg$

Then $\mu = \dfrac{v^2}{rg} = \dfrac{(50.0 \text{ cm/s})^2}{(30.0 \text{ cm})(980 \text{ cm/s}^2)} = \boxed{0.0850}$

***6.10** $$a = \frac{v^2}{r} = \frac{\left[\left(86.5\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\right]^2}{61.0 \text{ m}}\left(\frac{1\,g}{9.80 \text{ m/s}^2}\right) = \boxed{0.966g}$$

6.11 $n = mg$ since $a_y = 0$



The centripetal force is the frictional force f.

From Newton's second law

$$f = ma_r = \frac{mv^2}{r}$$

But the friction condition is

$$f \le \mu_s n$$

i.e., $\dfrac{mv^2}{r} \le \mu_s mg$

$$v \le \sqrt{\mu_s rg} = \sqrt{(0.600)(35.0\ \text{m})(9.80\ \text{m/s}^2)}$$

$$v \le \boxed{14.3\ \text{m/s}}$$

6.12 (b) $v = \dfrac{235\ \text{m}}{36.0\ \text{s}} = \boxed{6.53\ \text{m/s}}$

The radius is given by $\dfrac{1}{4}\ 2\pi r = 235\ \text{m}$

$$r = 150\ \text{m}$$

(a) $\mathbf{a}_r = \left(\dfrac{v^2}{r}\right)$ toward center

$$= \frac{(6.53\ \text{m/s})^2}{150\ \text{m}}\ \text{at } 35.0° \text{ north of west}$$

$$= (0.285\ \text{m/s}^2)(\cos 35.0°(-\mathbf{i}) + \sin 35.0°\ \mathbf{j})$$

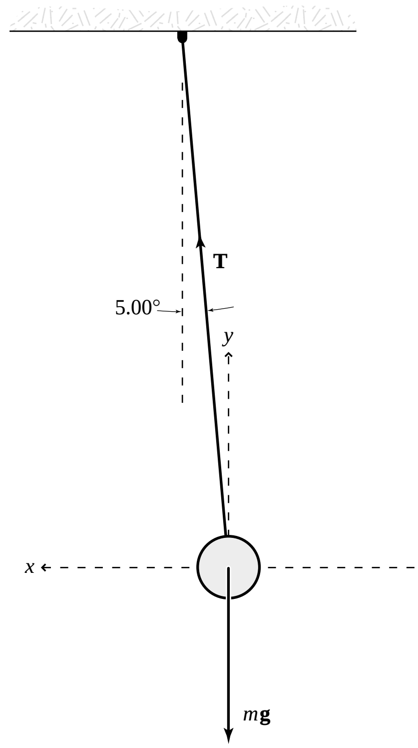
$$= \boxed{-0.233\ \text{m/s}^2\ \mathbf{i} + 0.163\ \text{m/s}^2\ \mathbf{j}}$$

(c) $\bar{\mathbf{a}} = \dfrac{(\mathbf{v}_\text{f} - \mathbf{v}_\text{i})}{t}$

$$= \frac{(6.53\ \text{m/s}\ \mathbf{j} - 6.53\ \text{m/s}\ \mathbf{i})}{36.0\ \text{s}}$$

$$= \boxed{-0.181\ \text{m/s}^2\ \mathbf{i} + 0.181\ \text{m/s}^2\ \mathbf{j}}$$

6.13 $T\cos 5.00° = mg = (80.0\text{ kg})(9.80\text{ m/s}^2)$



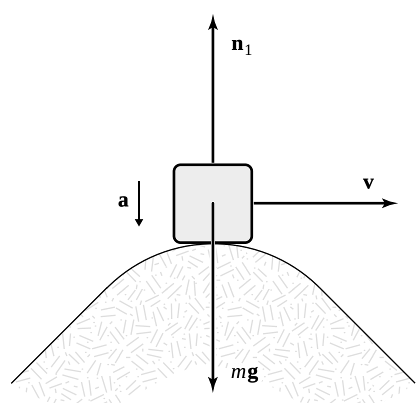
(a) $T = 787$ N

$T = \boxed{(68.6\text{ N})\mathbf{i} + (784\text{ N})\mathbf{j}}$

(b) $T\sin 5.00° = ma_r$

$\boxed{a_r = 0.857\text{ m/s}^2}$

6.14 (a) The reaction force n_1 represents the apparent weight of the woman



$F = ma$

i.e., $mg - n_1 = \dfrac{mv^2}{r}$, so $n_1 = mg - \dfrac{mv^2}{r}$

$$n_1 = 600 - \left(\frac{600}{9.80}\right)\frac{(9.00)^2}{11.0} = \boxed{149\text{ N}}$$

(b) If $n_1 = 0$, $mg = \dfrac{mv^2}{r}$

This gives $v = \sqrt{rg} = \sqrt{(11.0\text{ m})(9.80\text{ m/s}^2)} = \boxed{10.4\text{ m/s}}$

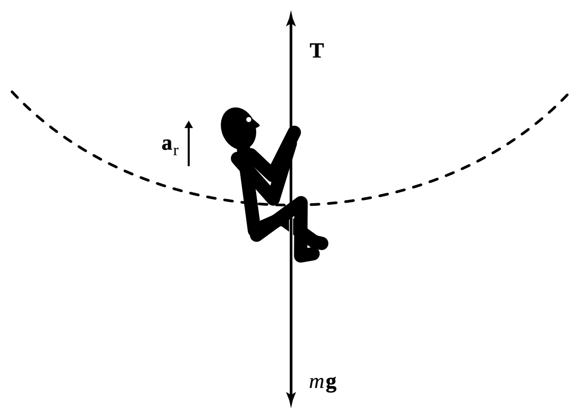
6.15 Let the tension at the lowest point be T.

$$F = ma$$

$$T - mg = ma_r = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right) = (85.0\text{ kg})\left[9.80\text{ m/s}^2 + \frac{(8.00\text{ m/s})^2}{10.0\text{ m}}\right] = 1.38\text{ kN} > 1000\text{ N}$$

$\boxed{\text{He doesn't make it across the river because the vine breaks.}}$

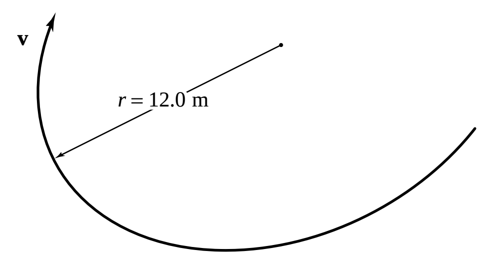


6.16 (a) $a_r = \frac{v^2}{r} = \frac{(4.00\text{ m/s})^2}{12.0\text{ m}} = \boxed{1.33\text{ m/s}^2}$

(b) $a = \sqrt{a_r^2 + a_T^2}$

$$a = \sqrt{(1.33)^2 + (1.20)^2} = \boxed{1.79\text{ m/s}^2}$$

at an angle $\theta = \tan^{-1}\left(\frac{a_r}{a_T}\right) = \boxed{47.9° \text{ inward}}$



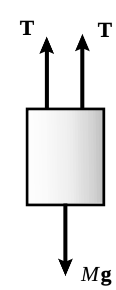
6.17 $M = 40.0$ kg, $R = 3.00$ m, $T = 350$ N

(a) $\Sigma F = 2T - Mg = \frac{Mv^2}{R}$

$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$

$$v^2 = [700 - (40.0)(9.80)]\left(\frac{3.00}{40.0}\right) = 23.1(\text{m}^2/\text{s}^2)$$

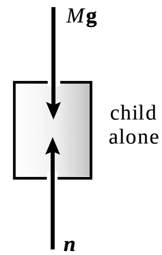
$\boxed{v = 4.81\text{ m/s}}$



child + seat

(b) $n - Mg = F = \dfrac{Mv^2}{R}$

$$n = Mg + \frac{Mv^2}{R} = 40.0\left(9.80 + \frac{23.1}{3.00}\right) = \boxed{700\ \text{N}}$$



Goal Solution

G: If the tension in each chain is 350 N at the lowest point, then the force of the seat on the child should just be twice this force or 700 N. The child's speed is not as easy to determine, but somewhere between 0 and 10 m/s would be reasonable for the situation described.

O: We should first draw a free body diagram that shows the forces acting on the seat and apply Newton's laws to solve the problem.

A: We can see from the diagram that the only forces acting on the system of child+seat are the tension in the two chains and the weight of the boy:

$\sum F = 2T - mg = ma$ where $a = \dfrac{v^2}{r}$ is the centripetal acceleration

$F = F_{\text{net}} = 2(350\ \text{N}) - (40.0\ \text{kg})(9.80\ \text{m/s}^2) = 308\ \text{N}$ upwards

$$v = \sqrt{\frac{F_{\max} r}{m}} = \sqrt{\frac{(308\ \text{N})(3.00\ \text{m})}{40.0\ \text{kg}}} = 4.81\ \text{m/s}\ \Diamond$$

The child feels a normal force exerted by the seat equal to the total tension in the chains.
$\mathbf{n} = 2(350\ \text{N}) = 700\ \text{N}$ (upwards) ◊

L: Our answers agree with our predictions. It may seem strange that there is a net upward force on the boy yet he does not move upwards. We must remember that a net force causes an acceleration, but not necessarily a motion in the direction of the force. In this case, the acceleration is due to a change in the direction of the motion. It is also interesting to note that the boy feels about twice as heavy as normal, so he is experiencing an acceleration of about 2g's.

6.18 (a) Consider the forces acting on the system consisting of the child and the seat:

$$\sum F_y = ma_y \Rightarrow 2T - mg = m\frac{v^2}{R}$$

$$v^2 = R\left(\frac{2T}{m} - g\right)$$

$$v = \boxed{\sqrt{R\left(\frac{2T}{m} - g\right)}}$$

(b) Consider the forces acting on the child alone:

$$\Sigma F_y = ma_y \Rightarrow n = m\left(g + \frac{v^2}{R}\right)$$

and from above, $v^2 = R\left(\frac{2T}{m} - g\right)$, so

$$n = m\left(g + \frac{2T}{m} - g\right) = \boxed{2T}$$

6.19 $\Sigma F_y = \frac{mv^2}{r} = mg + n$

But $n = 0$ at this minimum speed condition, so

$$\frac{mv^2}{r} = mg \Rightarrow v = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(1.00 \text{ m})} = \boxed{3.13 \text{ m/s}}$$

6.20 At the top of the vertical circle,

$$T = m\frac{v^2}{R} - mg$$

or $T = (0.400)\frac{(4.00)^2}{0.500} - (0.400)(9.80) = \boxed{8.88 \text{ N}}$

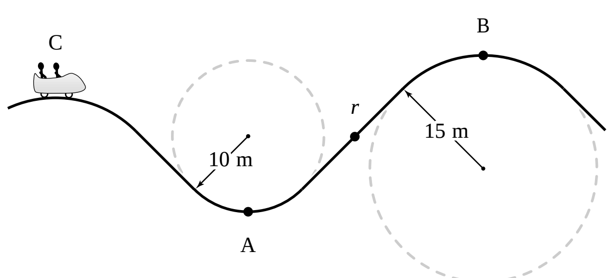
6.21 (a) $v = 20.0$ m/s, n = force of track on roller coaster, and $R = 10.0$ m.

$$\Sigma F = \frac{Mv^2}{R} = n - Mg$$

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500 \text{ kg})(9.80 \text{ m/s}^2) + \frac{(500 \text{ kg})(20.0 \text{ m/s}^2)}{10.0 \text{ m}}$$

$$n = 4900 \text{ N} + 20{,}000 \text{ N} = \boxed{2.49 \times 10^4 \text{ N}}$$



(b) At B, $n - Mg = -\dfrac{Mv^2}{R}$

The max speed at B corresponds to

$$n = 0$$

$$-Mg = -\frac{Mv_{\max}^2}{R} \Rightarrow v_{\max} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1 \text{ m/s}}$$

6.22 (a) $a_r = \dfrac{v^2}{r}$

$$r = \frac{v^2}{a_r} = \frac{(13.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{8.62 \text{ m}}$$

(b) Let n be the force exerted by rail.

Newton's law gives $Mg + n = \dfrac{Mv^2}{r}$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg\text{, downward}}$$

(c) $a_r = \dfrac{v^2}{r} = \dfrac{(13.0 \text{ m/s})^2}{20.0 \text{ m}} = \boxed{8.45 \text{ m/s}^2}$

If the force by the rail is n_1, then

$$n_1 + Mg = \frac{Mv^2}{r} = Ma_r$$

$n_1 = M(a_r - g)$ which is < 0,

since $a_r = 8.45 \text{ m/s}^2$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars. To be safe we must require n_1 to be positive. Then $a_r > g$. We need

$$\frac{v^2}{r} > g \quad \text{or} \quad v > \sqrt{rg} = \sqrt{(20.0 \text{ m})(9.80 \text{ m/s}^2)}$$

$$v > 14.0 \text{ m/s}$$

6.23 $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(3.00 \text{ m})}{(12.0 \text{ s})} = 1.57 \text{ m/s}$

(a) $a = \dfrac{v^2}{r} = \dfrac{(1.57 \text{ m/s})^2}{(3.00 \text{ m})} = \boxed{0.822 \text{ m/s}^2}$

(b) For no sliding motion,

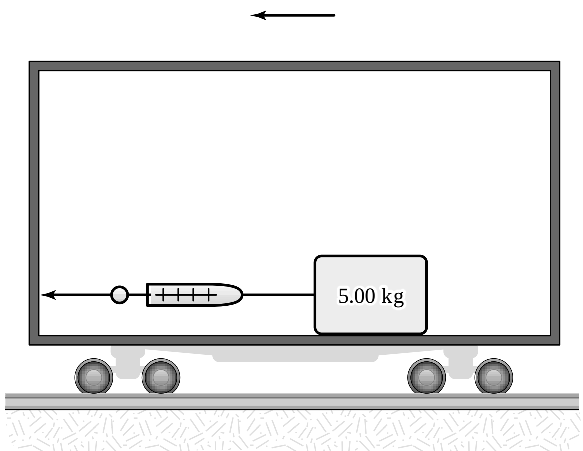
$$f_f = ma = (45.0\text{ kg})(0.822\text{ m/s}^2) = \boxed{37.0\text{ N}}$$

(c) $f_f = \mu mg,\ \mu = \dfrac{37.0\text{ N}}{(45.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.0839}$

6.24 (a) $\Sigma F_x = Ma,\ a = \dfrac{T}{M} = \dfrac{18.0\text{ N}}{5.00\text{ kg}} = \boxed{3.60\text{ m/s}^2}$ to the right.

(b) If v = const, $a = 0$, so $\boxed{T = 0}$ (This is also an equilibrium situation.)

(c) Someone in the car (noninertial observer) claims that the forces on the mass along x are T and a fictitious force ($-Ma$). Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x-direction.



6.25 $\sum F_x = T\sin\theta = ma_x$ (1)

$\sum F_x = T\cos\theta - mg = 0$

or $T\cos\theta = mg$ (2)

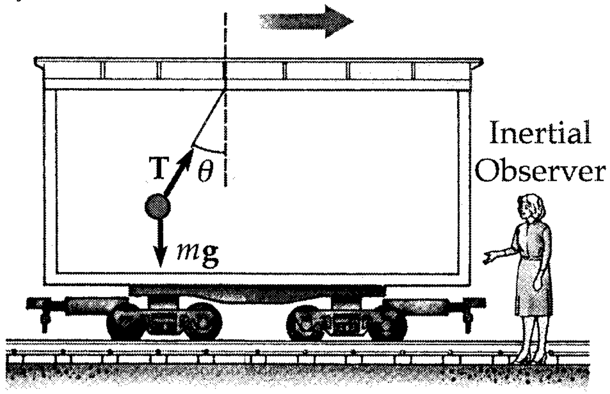
(a) Dividing (1) by (2) gives $\tan\theta = \dfrac{a_x}{g}$

$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right) = \tan^{-1}\left(\frac{3.00}{9.80}\right) = \boxed{17.0°}$$

(b) From (1), $T = \dfrac{ma_x}{\sin\theta} = \dfrac{(0.500\text{ kg})(3.00\text{ m/s}^2)}{\sin 17.0°} = \boxed{5.12\text{ N}}$

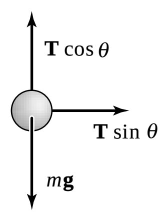
<u>Goal Solution</u>

G: If the horizontal acceleration were zero, then the angle would be 0, and if $a = g$, then the angle would be 45°, but since the acceleration is 3.00 m/s², a reasonable estimate of the angle is about 20°. Similarly, the tension in the string should be slightly more than the weight of the object, which is about 5 N.



O: We will apply Newton's second law to solve the problem.

A: The only forces acting on the suspended object are the force of gravity $m\mathbf{g}$ and the force of tension **T**, as shown in the free-body diagram. Applying Newton's second law in the *x* and *y* directions,



$\sum F_x = T \sin\theta = ma$ (1)

$\sum F_y = T\cos\theta - mg = 0$

or $T\cos\theta = mg$ (2)

(a) Dividing equation (1) by (2) gives

$$\tan\theta = \frac{a}{g} = \frac{3.00\text{ m/s}^2}{9.80\text{ m/s}^2} = 0.306$$

Solving for θ, $\theta = 17.0°$

(b) From Equation (1),

$$T = \frac{ma}{\sin\theta} = \frac{(0.500\text{ kg})(3.00\text{ m/s}^2)}{\sin(17.0°)} = 5.12\text{ N}$$

L: Our answers agree with our original estimates. This problem is very similar to Prob. 5.30, so the same concept seems to apply to various situations.

6.26 (a) $\sum F_r = ma_r$

$$mg = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$g = \frac{4\pi^2 R}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 R}{g}} = 2\pi\sqrt{\frac{6.37 \times 10^6 \text{ m}}{9.80 \text{ m/s}^2}} = 5.07 \times 10^3 = \boxed{1.41 \text{ h}}$$

(b) $$\text{speed increase factor} = \frac{v_{\text{new}}}{v_{\text{current}}} = \frac{\dfrac{2\pi R}{T_{\text{new}}}}{\dfrac{2\pi R}{T_{\text{current}}}} = \frac{T_{\text{current}}}{T_{\text{new}}} = \frac{24.0 \text{ h}}{1.41 \text{ h}} = \boxed{17.1}$$

6.27 $F_{\text{max}} = F_g + ma = 591 \text{ N}$

$F_{\text{min}} = F_g - ma = 391 \text{ N}$

(a) Adding, $2F_g = 982 \text{ N}$, $F_g = \boxed{491 \text{ N}}$

(b) Since $F_g = mg$, $m = \dfrac{491 \text{ N}}{9.80 \text{ m/s}^2} = \boxed{50.1 \text{ kg}}$

(c) Subtracting the above equations,

$2ma = 200 \text{ N}$ $\therefore$ $a = \boxed{2.00 \text{ m/s}^2}$

***6.28** In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r} = \frac{(5.70 \text{ m/s})^2}{2.40 \text{ m}} = 13.5 \text{ m/s}^2$$

In her own non-inertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2 + (v^2/r)^2} = \sqrt{(9.80)^2 + (13.5)^2} \text{ m/s}^2 = 16.7 \text{ m/s}^2$$

This is larger than g by a factor of $\dfrac{16.7}{9.80} = 1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

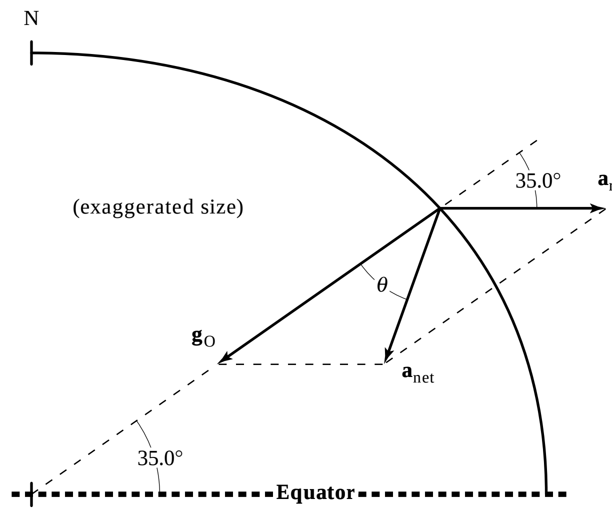
$$F = 1.71(55.0 \text{ N}) = \boxed{93.8 \text{ N}}$$

6.29 $a_r = \left(\frac{4\pi^2 R_e}{T^2}\right)\cos 35.0° = 0.0276\ \text{m/s}^2$

$(a_{\text{net}})_y = 9.80 - (a_r)_y = 9.78\ \text{m/s}^2$

$(a_{\text{net}})_x = 0.0158\ \text{m/s}^2$

$\theta = \arctan\frac{a_x}{a_y} = \boxed{0.0927°}$



***6.30** $m = 80.0$ kg, $v_T = 50.0$ m/s, $mg = \frac{D\rho A v_T^2}{2}$ $\therefore \frac{D\rho A}{2} = \frac{mg}{v_T^2} = 0.314$

(a) At $v = 30.0$ m/s

$$a = g - \frac{D\rho A v^2/2}{m} = 9.80 - \frac{(0.314)(30.0)^2}{80.0} = \boxed{6.27\ \text{m/s}^2\ \text{downward}}$$

(b) At $v = 50.0$ m/s, terminal velocity has been reached.

$$\Sigma F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{784\ \text{N directed up}}$$

(c) At $v = 30.0$ m/s

$$\frac{D\rho A v^2}{2} = (0.314)(30.0)^2 = \boxed{283\ \text{N}}\ \text{upward}$$

6.31 (a) $a = g - bv$

When $v = v_T$, $a = 0$ and $g = bv_T$.

$$b = \frac{g}{v_T}$$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus, $v_T = \frac{y}{t} = \frac{1.50\ \text{m}}{5.00\ \text{s}} = 0.300\ \text{m/s}$

Then $b = \frac{9.80\ \text{m/s}^2}{0.300\ \text{m/s}} = \boxed{32.7\ \text{s}^{-1}}$

(b) At $t = 0$, $v = 0$ and $a = g = \boxed{9.80 \text{ m/s}^2}$ down

(c) When $v = 0.150$ m/s,

$$a = g - bv = 9.80 \text{ m/s}^2 - (32.7 \text{ s}^{-1})(0.150 \text{ m/s}) = \boxed{4.90 \text{ m/s}^2} \text{ down}$$

***6.32** (a) $\rho = \dfrac{m}{V}$; $A = 0.0201 \text{ m}^2$; $R = \dfrac{1}{2}\rho A D v_t^2 = mg$

$$m = \rho V = (0.830 \text{ g/cm}^3)\left[\frac{4}{3}\pi (8.00 \text{ cm})^3\right] = 1.78 \text{ kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object,

$$v_t = \sqrt{\frac{2(1.78 \text{ kg})(9.80 \text{ m/s}^2)}{0.500(1.20 \text{ kg/m}^3)(0.0201 \text{ m}^2)}} = \boxed{53.8 \text{ m/s}}$$

(b) $v_f^2 = v_i^2 + 2gh = 0 + 2gh$

$$h = \frac{v_f^2}{2g} = \frac{(53.8 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{148 \text{ m}}$$

***6.33** Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward): $F = mg + bv$.

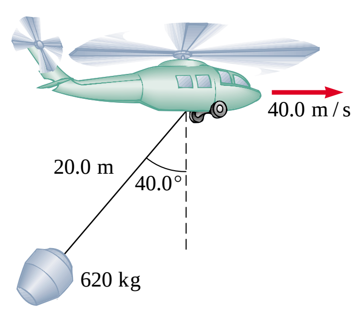
The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi\left(8.92 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right)(2.00 \times 10^{-2} \text{ m})^3 = 0.299 \text{ kg}$$

The applied force is then

$$F = mg + bv = (0.299)(9.80) + (0.950)(9.00 \times 10^{-2}) = \boxed{3.01 \text{ N}}$$

6.34 $\Sigma F_y = ma_y$



$+T \cos 40.0° - mg = 0$

$$T = \frac{(620 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 40.0°} = 7.93 \times 10^3 \text{ N}$$

$\Sigma F_x = ma_x$

$-R + T \sin 40.0° = 0$

$$R = (7.93 \times 10^3 \text{ N}) \sin 40.0° = 5.10 \times 10^3 \text{ N} = \frac{1}{2} D\rho A v^2$$

$$D = \frac{2R}{\rho A v^2} = \frac{2(5.10 \times 10^3 \text{ N})(\text{kg m/s}^2/\text{N})}{(1.20 \text{ kg/m}^2)3.80 \text{ m}^2 (40.0 \text{ m/s})^2} = \boxed{1.40}$$

6.35 (a) At terminal velocity,

$R = v_t b = mg$

$$\therefore b = \frac{mg}{v_t} = \frac{(3.00 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{(2.00 \times 10^{-2}\ \text{m/s})} = \boxed{1.47\ \text{N} \cdot \text{s/m}}$$

(b) From Equation 6.5, the velocity on the bead is

$v = v_t\,(1 - e^{-bt/m})$

$v = 0.630\,v_t$ when $e^{-bt/m} = 0.370$

or at time $t = -\left(\dfrac{m}{b}\right)\ln(0.370) = \boxed{2.04 \times 10^{-3}\ \text{s}}$

(c) At terminal velocity,

$R = v_t b = mg = \boxed{2.94 \times 10^{-2}\ \text{N}}$

***6.36** The resistive force is

$$R = \frac{1}{2}\,D\rho A v^2 = \frac{1}{2}(0.250)(1.20\ \text{kg/m}^3)(2.20\ \text{m}^2)(27.8\ \text{m/s})^2$$

$R = 255\ \text{N}$

$\boldsymbol{a} = -R/m = -(255\ \text{N})/(1200\ \text{kg}) = \boxed{-0.212\ \text{m/s}^2}$

6.37 (a) $v(t) = v_i e^{-ct}$

$v(20.0\ \text{s}) = 5.00 = v_i e^{-20.0c}$, $v_i = 10.0\ \text{m/s}$

So $5.00 = 10.0 e^{-20.0c}$

and $-20.0c = \ln\left(\dfrac{1}{2}\right)$

$$c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47 \times 10^{-2}\ \text{s}^{-1}}$$

(b) At $t = 40.0$ s

$$v = (10.0\text{ m/s})e^{-40.0c} = (10.0\text{ m/s})(0.250) = \boxed{2.50\text{ m/s}}$$

(c) $v = v_i e^{-ct}$

$$a = \frac{dv}{dt} = -cv_i e^{-ct} = \boxed{-cv}$$

6.38 $\Sigma F_x = ma_x$

$$-kmv^2 = ma_x = m\frac{dv}{dt}$$

$$-k\int_0^t dt = \int_{v_f}^{v} v^{-2}\,dv$$

$$-k(t-0) = \frac{v^{-1}}{-1}\Bigg|_{v_f}^{v} = -\frac{1}{v} + \frac{1}{v_f}$$

$$v = \boxed{\frac{v_f}{(1 + ktv_f)}}$$

***6.39** In $R = \frac{1}{2}D\rho Av^2$, we estimate that $D = 1.00$, $\rho = 1.20\text{ kg/m}^3$, $A = (0.100\text{ m})(0.160\text{ m}) = 1.60 \times 10^{-2}\text{ m}^2$ and $v = 27.0$ m/s. The resistance force is then

$$R = \frac{1}{2}(1.00)(1.20\text{ kg/m}^3)(1.60 \times 10^{-2}\text{ m}^2)(27.0\text{ m/s})^2 = 7.00\text{ N}$$

or $R \sim \boxed{10^1\text{ N}}$

***6.40** (a) At $v = v_t$, $a = 0$, $-mg - bv_t = 0$

$$v_t = \frac{-mg}{b} = -\frac{(3.00 \times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{3.00 \times 10^{-2}\text{ kg/s}} = \boxed{-0.980\text{ m/s}}$$

(b)

t(s)	x(m)	v(m / s)	F(mN)	a(m / s^2)
0	2	0	–29.4	–9.8
0.005	2	–0.049	–27.93	–9.31
0.01	1.999755	–0.09555	–26.534	–8.8445
0.015	1.9993	–0.13977	–25.2	–8.40
. . . we list the result after each tenth iteration				
0.5	1.990	–0.393	–17.6	–5.87
0.1	1.965	–0.629	–10.5	–3.51
0.15	1.930	–0.770	–6.31	–2.10
0.2	1.889	–0.854	–3.78	–1.26
0.25	1.845	–0.904	–2.26	–0.754
0.3	1.799	–0.935	–1.35	–0.451
0.35	1.752	–0.953	–0.811	–0.270
0.4	1.704	–0.964	–0.486	–0.162
0.45	1.65	–0.970	–0.291	–0.0969
0.5	1.61	–0.974	–0.174	–0.0580
0.55	1.56	–0.977	–0.110	–0.0347
0.6	1.51	–0.978	–0.0624	–0.0208
0.65	1.46	–0.979	–0.0374	–0.0125

Terminal velocity is never reached. The leaf is at 99.9% of v_t after 0.67 s. The fall to the ground takes about 2.14 s. Repeating with $\Delta t = 0.001$ s, we find the fall takes 2.14 s.

***6.41** (a) When $v = v_t$, $a = 0$, $\Sigma F = -mg + Cv_t^2 = 0$

$$v_t = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(4.80 \times 10^{-4} \text{ kg})(9.80 \text{ m/s}^2)}{2.50 \times 10^{-5} \text{ kg/m}}} = \boxed{-13.7 \text{ m/s}}$$

(b)

$t(\mathrm{s})$	$x(\mathrm{m})$	$v(\mathrm{m/s})$	$F(\mathrm{mN})$	$a(\mathrm{m/s^2})$
0	0	0	–4.704	–9.8
0.2	0	–1.96	–4.608	–9.5999
0.4	–0.392	–3.88	–4.3276	–9.0159
0.6	–1.168	–5.6832	–3.8965	–8.1178
0.8	–2.30	–7.3068	–3.3693	–7.0193
1.0	–3.77	–8.7107	–2.8071	–5.8481
1.2	–5.51	–9.8803	–2.2635	–4.7156
1.4	–7.48	–10.823	–1.7753	–3.6986
1.6	–9.65	–11.563	–1.3616	–2.8366
1.8	–11.96	–12.13	–1.03	–2.14
2	–14.4	–12.56	–0.762	–1.59
... listing results after each fifth step				
3	–27.4	–13.49	–0.154	–0.321
4	–41.0	–13.67	–0.0291	–0.0606
5	–54.7	–13.71	–0.00542	–0.0113

The hailstone reaches 99.95% of v_t after 5.0 s, 99.99% of v_t after 6.0 s, 99.999% of v_t after 7.4 s.

6.42 (a) At terminal velocity, $\sum F = 0 = -mg + Cv_t^2$.

$$C = \frac{mg}{v_t^2} = \frac{(0.142\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{(42.5\ \mathrm{m/s})^2} = \boxed{7.70 \times 10^{-4}\ \mathrm{kg/m}}$$

(b) $Cv^2 = (7.70 \times 10^{-4}\ \mathrm{kg/m})(36.0\ \mathrm{m/s})^2 = \boxed{0.998\ \mathrm{N}}$

(c)

Elapsed Time (s)	Altitude (m)	Speed (m/s)	Resistance Force (N)	Net Force (N)	Acceleration (m/s^2)
0.00000	0.00000	36.00000	-0.99849	–2.39009	–16.83158
0.05000	1.75792	35.15842	–0.95235	–2.34395	–16.50667
…					
2.95000	48.62327	0.82494	–0.00052	–1.39212	–9.80369
3.00000	48.64000	0.33476	–0.00009	–1.39169	–9.80061
3.05000	48.63224	–0.15527	0.00002	–1.39158	–9.79987
…					
6.25000	1.25085	–26.85297	0.55555	–0.83605	–5.88769
6.30000	–0.10652	–27.14736	0.56780	–0.82380	–5.80144

Maximum height is about $\boxed{49\text{ m}}$. It returns to the ground after about $\boxed{6.3\text{ s}}$ with a speed of approximately $\boxed{27\text{ m/s}}$.

6.43 (a) At constant velocity $\Sigma F = 0 = -mg + Cv_t^2$

$$v_t = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{0.200\text{ kg/m}}} = \boxed{-49.5\text{ m/s}}$$ with chute closed and

$$v_t = -\sqrt{\frac{(50.0\text{ kg})(9.80\text{ m/s})}{20.0\text{ kg/m}}} = \boxed{-4.95\text{ m/s}}$$ with chute open.

(b)

time(s)	height(m)	velocity(m/s)
0	1000	0
1	995	– 9.7
2	980	–18.6
4	929	– 32.7
7	812	– 43.7
10	674	– 47.7
10.1	671	– 16.7
10.3	669	– 8.02
11	665	– 5.09
12	659	– 4.95
50	471	– 4.95
100	224	– 4.95
145	0	– 4.95

6.44 (a)

time(s)	x(m)	y(m)
0	0	0
0.100	7.81	5.43
0.200	14.9	10.2
0.400	27.1	18.3
1.00	51.9	32.7
1.92	70.0	38.5
2.00	70.9	38.5
4.00	80.4	26.7
5.00	81.4	17.7
6.85	81.8	0

(b) range = $\boxed{81.8 \text{ m}}$

(c)

with θ	we find range
30.0°	86.410 m
35.0°	81.8 m
25.0°	90.181 m
20.0°	92.874 m
15.0°	93.812 m
10.0°	90.965 m
17.0°	93.732 m
16.0°	93.8398 m
15.5°	93.829 m
15.8°	93.839 m
16.1°	93.838 m
15.9°	93.8402 m

So we have maximum range at $\theta = \boxed{15.9°}$

***6.45** (a) At terminal speed, $\sum F = -mg + Cv^2 = 0$. Thus,

$$C = \frac{mg}{v^2} = \frac{(0.0460 \text{ kg})(9.80 \text{ m/s}^2)}{(44.0 \text{ m/s})^2} = \boxed{2.33 \times 10^{-4} \text{ kg/m}}$$

(b) We set up a spreadsheet to calculate the motion, try different initial speeds, and home in on $\boxed{53 \text{ m/s}}$ as that required for horizontal range of 155 m, thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s²)	y (m)	v_y (m/s)	a_y (m/s²)	$v=\sqrt{v_x^2+v_y^2}$ (m/s)	$\tan^{-1}(v_y/v_x)$ (deg)
0.0000	0.0000	45.6870	–10.5659	0.0000	27.4515	–13.6146	53.3000	31.0000
0.0027	0.1211	45.6590	–10.5529	0.0727	27.4155	–13.6046	53.2574	30.9822
…								
2.5016	90.1946	28.9375	–4.2388	32.5024	0.0235	–9.8000	28.9375	0.0466
2.5043	90.2713	28.9263	–4.2355	32.5024	–0.0024	–9.8000	28.9263	–0.0048
2.5069	90.3480	28.9150	–4.2322	32.5024	–0.0284	–9.8000	28.9151	–0.0563
…								
3.4238	115.2298	25.4926	–3.2896	28.3972	–8.8905	–9.3999	26.9984	–19.2262
3.4265	115.2974	25.4839	–3.2874	28.3736	–8.9154	–9.3977	26.9984	–19.2822
3.4291	115.3649	25.4751	–3.2851	28.3500	–8.9403	–9.3954	26.9984	–19.3382
…								
5.1516	154.9968	20.8438	–2.1992	0.0059	–23.3087	–7.0498	31.2692	–48.1954
5.1543	155.0520	20.8380	–2.1980	–0.0559	–23.3274	–7.0454	31.2792	–48.2262

(c) Similarly, the initial speed is $\boxed{42 \text{ m/s}}$. The motion proceeds thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s²)	y (m)	v_y (m/s)	a_y (m/s²)	$v=\sqrt{v_x^2+v_y^2}$ (m/s)	$\tan^{-1}(v_y/v_x)$ (deg)
0.0000	0.0000	28.7462	–4.1829	0.0000	30.8266	–14.6103	42.1500	47.0000
0.0035	0.1006	28.7316	–4.1787	0.1079	30.7754	–14.5943	42.1026	46.9671
…								
2.7405	66.3078	20.5484	–2.1374	39.4854	0.0260	–9.8000	20.5485	0.0725
2.7440	66.3797	20.5410	–2.1358	39.4855	–0.0083	–9.8000	20.5410	–0.0231
2.7475	66.4516	20.5335	–2.1343	39.4855	–0.0426	–9.8000	20.5335	–0.1188
…								
3.1465	74.4805	19.7156	–1.9676	38.6963	–3.9423	–9.7213	20.1058	–11.3077
3.1500	74.5495	19.7087	–1.9662	38.6825	–3.9764	–9.7200	20.1058	–11.4067
3.1535	74.6185	19.7018	–1.9649	38.6686	–4.0104	–9.7186	20.1058	–11.5056
…								
5.6770	118.9697	15.7394	–1.2540	0.0465	–25.2600	–6.5701	29.7623	–58.0731
5.6805	119.0248	15.7350	–1.2533	–0.0419	–25.2830	–6.5642	29.7795	–58.1037

The trajectory in (c) reaches maximum height 39 m, as opposed to 33 m in (b). In both, the ball reaches maximum height when it has covered about 57% of its range. Its speed is a minimum somewhat later. The impact speeds are both about 30 m/s.

6.46 (a) $\Sigma F_y = ma_y = \dfrac{mv^2}{R}$ down

$$+n - 1800\text{ kg }(9.80\text{ m/s}^2) = \frac{-(1800\text{ kg})(16.0\text{ m/s})^2}{42.0\text{ m}} = -1.10\times10^4\text{ N}$$

$$n = \boxed{6.67\times10^3\text{ N}}$$

(b) $0 - mg = \dfrac{-mv^2}{r}$

$$v = \sqrt{gr} = \sqrt{9.80\text{ m/s}^2\,(42.0\text{ m})} = \boxed{20.3\text{ m/s}}$$

6.47 (a) $\Sigma F_y = ma_y = \dfrac{mv^2}{R}$

$$mg - n = \frac{mv^2}{R}$$

$$n = \boxed{mg - \frac{mv^2}{R}}$$

(b) When $n = 0$, $mg = \dfrac{mv^2}{R}$

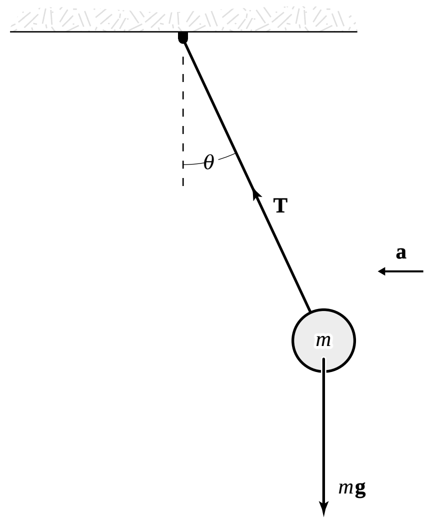
Then, $v = \boxed{\sqrt{gR}}$

6.48 $F = m\dfrac{v^2}{r}$

$$v = \sqrt{\frac{rF}{m}} = \sqrt{\frac{(5.30\times10^{-11}\text{ m})(8.20\times10^{-8}\text{ N})}{9.11\times10^{-31}\text{ kg}}} = 2.18\times10^6\text{ m/s}$$

$$\text{frequency} = (2.18\times10^6\text{ m/s})\left[\frac{1\text{ rev}}{2\pi(5.30\times10^{-11}\text{ m})}\right] = \boxed{6.56\times10^{15}\text{ rev/s}}$$

6.49 (a) While the car negotiates the curve, the accelerometer is at the angle θ.



Horizontally: $T \sin \theta = \dfrac{mv^2}{r}$

Vertically: $T \cos \theta = mg$

where r is the radius of the curve, and v is the speed of the car.

By division $\tan \theta = \dfrac{v^2}{rg}$. Then

$$a_r = \frac{v^2}{r} = g \tan \theta$$

$$a_r = (9.80 \text{ m/s}^2) \tan 15.0°$$

$$a_r = \boxed{2.63 \text{ m/s}^2}$$

(b) $r = \dfrac{v^2}{a_r} = \dfrac{(23.0 \text{ m/s})^2}{2.63 \text{ m/s}^2} = \boxed{201 \text{ m}}$

(c) $v^2 = rg \tan \theta = (201 \text{ m})(9.80 \text{ m/s}^2) \tan 9.00°$

$v = \boxed{17.7 \text{ m/s}}$

6.50 Take x-axis up the hill

$$\sum F_x = ma_x$$

$$+T \sin \theta - mg \sin \phi = ma$$

$$a = (T/m) \sin \theta - g \sin \phi$$

$$\sum F_y = ma_y$$

$$+T \cos \theta - mg \cos \phi = 0$$

$$T = mg \cos \phi / \cos \theta$$

$$a = g \cos \phi \sin \theta / \cos \theta - g \sin \phi$$

$$a = \boxed{g(\cos \phi \tan \theta - \sin \phi)}$$

6.51 (a) Since the 1.00 kg mass is in equilibrium, we have for the tension in the string,

$$T = mg = (1.00)(9.80) = \boxed{9.80\ \text{N}}$$

(b) The centripetal force is provided by the tension in the string. Hence,

$$F_r = T = \boxed{9.80\ \text{N}}$$

(c) Using $F_r = \dfrac{m_{\text{puck}} v^2}{r}$, we have $v = \sqrt{\dfrac{rF_r}{m_{\text{puck}}}} = \sqrt{\dfrac{(1.00)(9.80)}{0.250}} = \boxed{6.26\ \text{m/s}}$

6.52 (a) Since the mass m_2 is in equilibrium,

$$\sum F_y = T - m_2 g = 0$$

or $\quad T = \boxed{m_2 g}$

(b) The tension in the string provides the required centripetal force for the puck.

Thus, $F_r = T = \boxed{m_2 g}$

(c) From $F_r = \dfrac{m_1 v^2}{R}$, we have $v = \sqrt{\dfrac{RF_r}{m_1}} = \boxed{\sqrt{\left(\dfrac{m_2}{m_1}\right) gR}}$

6.53 (a) Since the centripetal acceleration of a person is downward (toward the axis of the earth), it is equivalent to the effect of a falling elevator.

Therefore, $F_g' = F_g - \dfrac{mv^2}{r}$ or $\boxed{F_g > F_g'}$

(b) At the poles $v = 0$, and $F_g' = F_g = mg = (75.0)(9.80) = \boxed{735\ \text{N}}$ down

At the equator, $F_g' = F_g - ma_r = 735\ \text{N} - (75.0)(0.0337)\ \text{N} = \boxed{732\ \text{N}}$ down

Goal Solution

G: Since the centripetal acceleration is a small fraction (~0.3%) of g, we should expect that a person would have an apparent weight that is just slightly less at the equator than at the poles due to the rotation of the Earth.

O: We will apply Newton's second law and the equation for centripetal acceleration.

A: (a) Let **n** represent the force exerted on the person by a scale, which is the "apparent weight." The true weight is $m\mathbf{g}$. Summing up forces on the object in the direction towards the Earth's center gives

$$mg - n = ma_c \qquad (1)$$

where $a_c = \dfrac{v^2}{R_x} = 0.0337 \text{ m/s}^2$

is the centripetal acceleration directed toward the center of the Earth.

Thus, we see that $n = m(g - a_c) < mg$

or $\quad mg = n + ma_c > n \quad \Diamond \qquad (2)$

(b) If $m = 75.0$ kg, $a_c = 0.0337 \text{ m/s}^2$, and $g = 9.800 \text{ m/s}^2$,

at the Equator: $\quad n = m(g - a_c) = (75.0 \text{ kg})(9.800 \text{ m/s}^2 - 0.0337 \text{ m/s}^2) = 732.5 \text{ N} \quad \Diamond$

at the Poles: $\quad n = mg = (75.0 \text{ kg})(9.800 \text{ m/s}^2) = 735.0 \text{ N} \quad \Diamond \qquad (a_c = 0)$

L: As we expected, the person does appear to weigh about 0.3% less at the equator than the poles. We might extend this problem to consider the effect of the earth's bulge on a person's weight. Since the earth is fatter at the equator than the poles, g is less than 9.80 m/s^2 at the equator and slightly more at the poles, but the difference is not as significant as from the centripetal acceleration. (Can you prove this?)

6.54 $\quad \sum F_x = ma_x \Rightarrow T_x = m\dfrac{v^2}{r} = m\dfrac{(20.4 \text{ m/s})^2}{(2.50 \text{ m})} = m(166 \text{ m/s}^2)$

$\sum F_y = ma_y \Rightarrow T_y - mg = 0$

or $\quad T_y = mg = m(9.80 \text{ m/s}^2)$

The total tension in the string is

$$T = \sqrt{T_x^2 + T_y^2} = m\sqrt{(166)^2 + (9.80)^2} = 50.0 \text{ N}$$

Thus, $m = \dfrac{50.0 \text{ N}}{\sqrt{(166)^2 + (9.80)^2} \text{ m/s}^2} = 0.300 \text{ kg}$

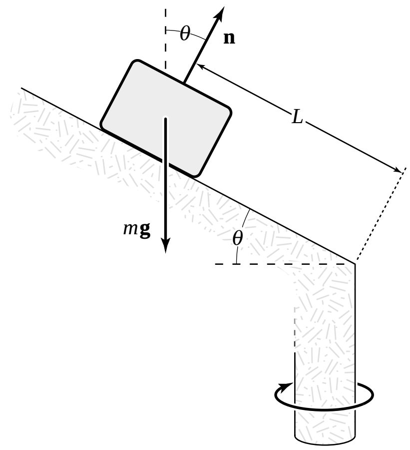
When the string is at the breaking point,

$$T_x = m\frac{v^2}{r} = (0.300 \text{ kg})\frac{(51.0 \text{ m/s}^2)}{(1.00 \text{ m})} = 780 \text{ N}$$

and $T_y = mg = (0.300 \text{ kg})(9.80 \text{ m/s}^2) = 2.94 \text{ N}$

Hence, $T = \sqrt{T_x^2 + T_y^2} = \sqrt{(780)^2 + (2.94)^2}\text{ N} = \boxed{780 \text{ N}}$

6.55 Let the angle that the wedge makes with the horizontal be θ. The equations for the mass m are



$$mg = n\cos\theta \quad \text{and} \quad n\sin\theta = \frac{mv^2}{r}$$

where $r = L\cos\theta$.

Eliminating n gives $\dfrac{n\cos\theta}{n\cos\theta} = \tan\theta = \dfrac{mv^2}{mg\,L\cos\theta}$

Therefore $v^2 = Lg\cos\theta\tan\theta = Lg\sin\theta$

$$\boxed{v = \sqrt{gL\sin\theta}}$$

6.56 (a) $v = 300 \text{ mi/h}\left(\dfrac{88.0 \text{ ft/s}}{60.0 \text{ mi/h}}\right) = 440 \text{ ft/s}$

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160 + \left(\frac{160}{32.0}\right)\frac{(440)^2}{1200} = \boxed{967 \text{ lb}}$$

(b) At the highest point, the force of the seat on the pilot is directed down and $F_g' = mg - m\dfrac{v^2}{r} = \boxed{-647 \text{ lb}}$

Since the plane is upside down, the seat exerts this downward force.

(c) When $F_g' = 0$, then $mg = \dfrac{mv^2}{R}$. If we vary the aircraft's R and v such that the above is true, then the pilot feels weightless.

6.57 Call the proportionality constant k:

$$a_r = k/r^2$$

$$v^2/r = k/r^2$$

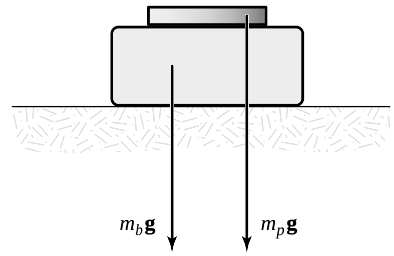
(a) $v = k^{1/2}r^{-1/2}$ so $\boxed{v \propto r^{-1/2}}$

(b) $v = 2\pi r / T = k^{1/2} r^{-1/2}$

$$T = \frac{2\pi r}{(k^{1/2} r^{-1/2})} = \left(\frac{2\pi}{k^{1/2}}\right) r^{3/2}$$

$$\boxed{T \propto r^{3/2}}$$

6.58 For the block to remain stationary, $\sum F_y = 0$ and $\sum F_x = ma_r$.

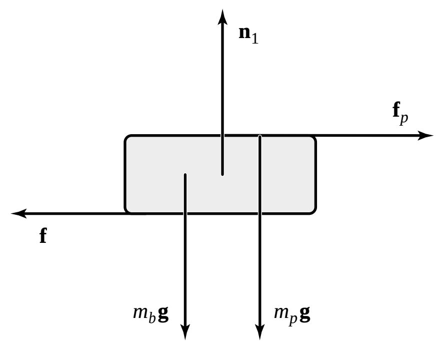


$$n_1 = (m_p + m_b)g \quad \text{so} \quad f \le \mu_{s1} n_1 = \mu_{s1}(m_p + m_b)g$$

At the point of slipping, the required centripetal force equals the maximum friction force:

$$\therefore \ (m_p + m_b)\frac{v_{\max}^2}{r} = \mu_{s1}(m_p + m_b)g$$

or $\quad v_{\max} = \sqrt{\mu_{s1} r g} = \sqrt{(0.750)(0.120)(9.80)} = 0.939 \text{ m/s}$

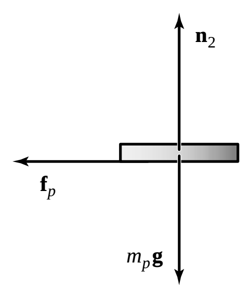


For the penny to remain stationary on the block:

$$\sum F_y = 0 \Rightarrow n_2 - m_p g = 0 \quad \text{or} \quad n_2 = m_p g$$

and $\quad \sum F_x = ma_r \Rightarrow f_p = m_p \dfrac{v^2}{r}$

When the penny is about to slip on the block, $f_p = f_{p,\,\max} = \mu_{s2} n_2$



or $\quad \mu_{s2} m_p g = m_p \dfrac{v_{\max}^2}{r}$

$$v_{\max} = \sqrt{\mu_{s2} r g} = \sqrt{(0.520)(0.120)(9.80)} = 0.782 \text{ m/s}$$

This is less than the maximum speed for the block, so the penny slips before the block starts to slip. The maximum rotation frequency is

$$\text{Max rpm} = \frac{v_{\max}}{2\pi r} = (0.782 \text{ m/s})\left[\frac{1 \text{ rev}}{2\pi(0.120 \text{ m})}\right]\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{62.2 \text{ rev/min}}$$

6.59 $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(9.00 \text{ m})}{(15.0 \text{ s})} = 3.77 \text{ m/s}$

(a) $a_r = \dfrac{v^2}{r} = \boxed{1.58 \text{ m/s}^2}$

(b) $F_{\text{low}} = m(g + a_r) = \boxed{455 \text{ N}}$

(c) $F_{\text{hi}} = m(g - a_r) = \boxed{329\ \text{N}}$

(d) $F_{\text{med}} = m\sqrt{g^2 + a_r^2} = \boxed{397\ \text{N}}$ at $\theta = \tan^{-1}\dfrac{a_r}{g} = \dfrac{1.58}{9.80} = \boxed{9.15°\ \text{inward}}$

***6.60** Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force supplies the centripetal force to cause the 3.00 m/s^2 centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

$$v = \sqrt{a_r r} = \sqrt{(3.00\ \text{m/s}^2)(60.0\ \text{m})} = 13.4\ \text{m/s}$$

The period of rotation comes from $v = \dfrac{2\pi r}{T}$

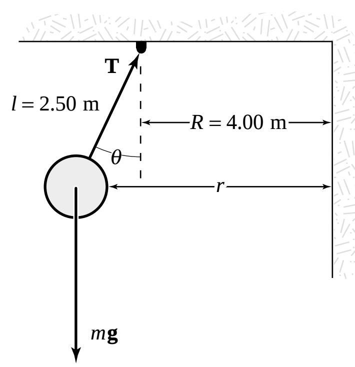
$$T = \frac{2\pi r}{v} = \frac{2\pi(60.0\ \text{m})}{13.4\ \text{m/s}} = 28.1\ \text{s}$$

so the frequency of rotation is

$$f = \frac{1}{T} = \frac{1}{28.1\ \text{s}} = \frac{1}{28.1\ \text{s}}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{2.14\ \text{rev/min}}$$

6.61 (a) The mass at the end of the chain is in vertical equilibrium.

Thus $T\cos\theta = mg$



Horizontally $T\sin\theta = ma_r = \dfrac{mv^2}{r}$

$$r = (2.50\sin\theta + 4.00)\ \text{m}$$

$$r = (2.50\sin 28.0° + 4.00)\ \text{m} = 5.17\ \text{m}$$

Then $a_r = \dfrac{v^2}{5.17\ \text{m}}$.

By division $\tan\theta = \dfrac{a_r}{g} = \dfrac{v^2}{5.17g}$

$$v^2 = 5.17g\tan\theta = (5.17)(9.80)(\tan 28.0°)\ \text{m}^2/\text{s}^2$$

$$v = \boxed{5.19\ \text{m/s}}$$

(b) $T\cos\theta = mg$

$$T = \frac{mg}{\cos\theta} = \frac{(50.0\ \text{kg})(9.80\ \text{m/s}^2)}{\cos 28.0°} = \boxed{555\ \text{N}}$$

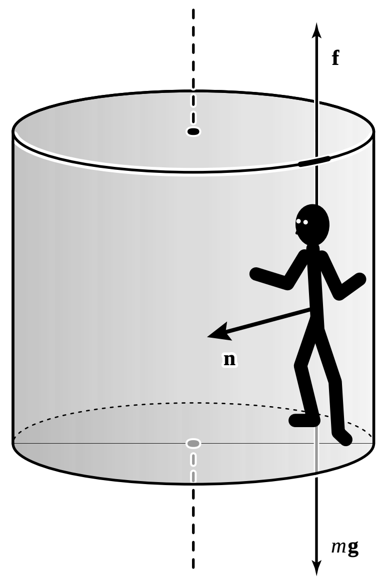
6.62 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \dfrac{2v}{g}$ where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \dfrac{2\pi R}{t}$, so $t = \dfrac{2v}{g} = \dfrac{2\pi R}{v}$

Thus, $v^2 = \pi R g$ and $\boxed{v = \sqrt{\pi R g}}$

(b) The putty is dislodged when F, the force holding it to the wheel is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}$$



6.63 (a) $n = \dfrac{mv^2}{R}$ $f - mg = 0$

$f = \mu_s n$ $v = \dfrac{2\pi R}{T}$

$$T = \boxed{\sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

(b) $T = \boxed{2.54\text{ s}}$

$$\#\frac{\text{rev}}{\text{min}} = \frac{1\text{ rev}}{2.54\text{ s}}\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{23.6\ \frac{\text{rev}}{\text{min}}}$$

***6.64** Let the x–axis point eastward, the y-axis upward, and the z-axis point southward.

(a) The range is $Z = \dfrac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{(9.80)(285)}{\sin 96.0°}} = 53.0\text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \dfrac{1}{2}a_y t^2$ as

$$0 = (53.0\text{ m/s})(\sin 48.0°)t - (4.90\text{ m/s}^2)t^2$$

giving $t = \boxed{8.04\text{ s}}$

(b) $v_{ix} = \dfrac{2\pi R_e \cos\phi_i}{86400\text{ s}} = \dfrac{2\pi(6.37\times 10^6\text{ m})\cos 35.0°}{86400\text{ s}} = \boxed{379\text{ m/s}}$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)(360°) = \left(\frac{285\text{ m}}{2\pi(6.37\times10^6\text{ m})}\right)(360°) = 2.56\times10^{-3}\text{ degrees}$$

The final latitude is then $\phi_f = \phi_i - \Delta\phi = 35.0° - 0.00256° = 34.9974°$.

The cup is moving eastward at a speed $v_{fx} = \dfrac{2\pi R_e \cos\phi_f}{86400\text{ s}}$, which is larger than the eastward velocity of the tee by

$$\Delta v_x = v_{fx} - v_{fi} = \frac{2\pi R_e}{86400\text{ s}}\left[\cos\phi_f - \cos\phi_i\right] = \frac{2\pi R_e}{86400\text{ s}}\left[\cos(\phi_i - \Delta\phi) - \cos\phi_i\right]$$

$$= \frac{2\pi R_e}{86400\text{ s}}\left[\cos\phi_i\cos\Delta\phi + \sin\phi_i\sin\Delta\phi - \cos\phi_i\right]$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and $\Delta v_x \approx \dfrac{2\pi R_e}{86400\text{ s}}\sin\phi_i\sin\Delta\phi$.

$$\Delta v_x \approx \frac{2\pi(6.37\times10^6\text{ m})}{86400\text{ s}}\sin 35.0°\sin 0.00256° = \boxed{1.19\times10^{-2}\text{ m/s}}$$

(d) $\Delta x = (\Delta v_x)t = (1.19\times10^{-2}\text{ m/s})(8.04\text{ s}) = 0.0955\text{ m} = \boxed{9.55\text{ cm}}$

***6.65** In $\sum F = m\dfrac{v^2}{r}$, both m and r are unknown but remain constant. Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\dfrac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2(130\text{ N}) = 215\text{ N}$$

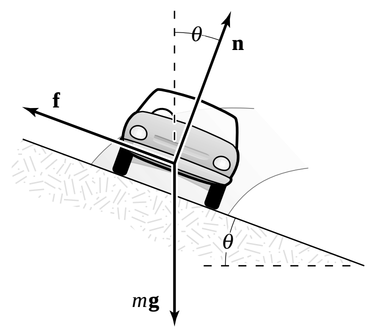
Symbolically, write $\sum F_{\text{slow}} = \left(\dfrac{m}{r}\right)(14.0\text{ m/s})^2$ and $\sum F_{\text{fast}} = \left(\dfrac{m}{r}\right)(18.0\text{ m/s})^2$.

Dividing gives $\dfrac{\sum F_{\text{fast}}}{\sum F_{\text{slow}}} = \left(\dfrac{18.0}{14.0}\right)^2$, or

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2\sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2(130\text{ N}) = \boxed{215\text{ N}}$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

6.66 (a) If the car is about to slip *down* the incline, f is directed up the incline.

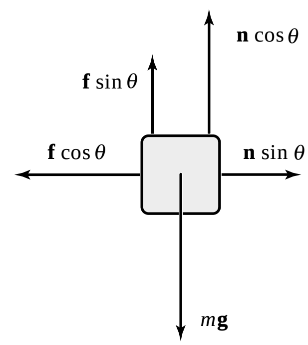


$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$ where $f = \mu_s n$ gives

$$n = \frac{mg}{\cos\theta\,(1+\mu_s\tan\theta)} \quad \text{and} \quad f = \frac{\mu_s mg}{\cos\theta\,(1+\mu_s\tan\theta)}$$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{\min}^2}{R}$ yields

$$\boxed{v_{\min} = \sqrt{\frac{Rg(\tan\theta - \mu_s)}{1+\mu_s\tan\theta}}}$$

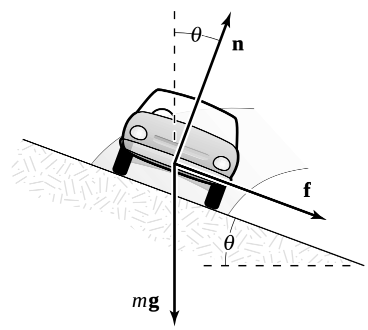


When the car is about to slip *up* the incline, f is directed down the incline. Then,
$\sum F_y = n\cos\theta - f\sin\theta - mg = 0$ with $f = \mu_s n$ yields

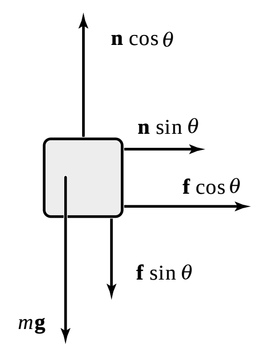
$$n = \frac{mg}{\cos\theta(1-\mu_s\tan\theta)} \quad \text{and} \quad f = \frac{\mu_s mg}{\cos\theta(1-\mu_s\tan\theta)}$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\dfrac{v_{\max}^2}{R}$, which gives

$$\boxed{v_{\max} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1-\mu_s\tan\theta}}}$$



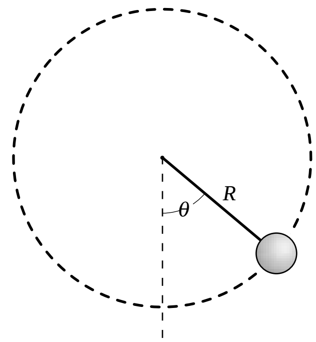
(b) If $v_{\min} = \sqrt{\dfrac{Rg(\tan\theta - \mu_s)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.



(c) $$v_{\min} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° - 0.100)}{1+(0.100)\tan 10.0°}} = \boxed{8.57\text{ m/s}}$$

$$v_{\max} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° + 0.100)}{1-(0.100)\tan 10.0°}} = \boxed{16.6\text{ m/s}}$$

***6.67** (a) The bead moves in a circle with radius $r = R \sin\theta$ at a speed of



$$v = \frac{2\pi r}{T} = \frac{2\pi R \sin\theta}{T}$$

The normal force has an inward radial component of $n \sin\theta$ and upward component of $n \cos\theta$.

$$\Sigma F_y = ma_y \Rightarrow n\cos\theta - mg = 0 \quad \text{or} \quad n = \frac{mg}{\cos\theta}$$

Then $\Sigma F_x = n\sin\theta = m\dfrac{v^2}{r}$ becomes $\left(\dfrac{mg}{\cos\theta}\right)\sin\theta = \dfrac{m}{R\sin\theta}\left(\dfrac{2\pi R \sin\theta}{T}\right)^2$, which reduces to

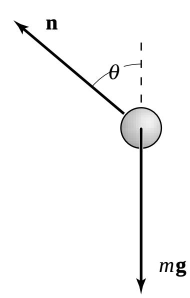
$\dfrac{g\sin\theta}{\cos\theta} = \dfrac{4\pi^2 R\sin\theta}{T^2}$. This has two solutions: (1) $\sin\theta = 0 \Rightarrow \theta = 0°$, and (2) $\cos\theta = \dfrac{gT^2}{4\pi^2 R}$.

If $R = 15.0$ cm and $T = 0.450$ s, the second solution yields

$$\cos\theta = \frac{(9.80 \text{ m/s}^2)(0.450 \text{ s})^2}{4\pi^2(0.150 \text{ m})} = 0.335 \quad \text{and} \quad \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution (2) above becomes



$$\cos\theta = \frac{(9.80 \text{ m/s}^2)(0.850 \text{ s})^2}{4\pi^2(0.150 \text{ m})} = 1.20, \text{ which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position.

6.68 At terminal velocity, the accelerating force of gravity is balanced by frictional drag:

$$mg = arv + br^2v^2$$

(a) $mg = 3.10 \times 10^{-9}\, v + 0.870 \times 10^{-10}\, v^2$

For water, $m = \rho V = (1000 \text{ kg/m}^8)\left(\frac{4}{3}\pi\right)(10^{-5})^3$

$$4.11 \times 10^{-11} = (3.10 \times 10^{-9})v + (0.870 \times 10^{-10})v^2$$

Assuming v is small, ignore the second term: $\boxed{v = 0.0132 \text{ m/s}}$

(b) $mg = 3.10 \times 10^{-8}\ v + 0.870 \times 10^{-8}\ v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$4.11 \times 10^{-8} = (3.10 \times 10^{-8})v + (0.870 \times 10^{-8})v^2$$

$$v = \frac{-3.10 \pm \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03 \text{ m/s}}$$

(c) $mg = 3.10 \times 10^{-7} v + 0.870 \times 10^{-6}\ v^2$

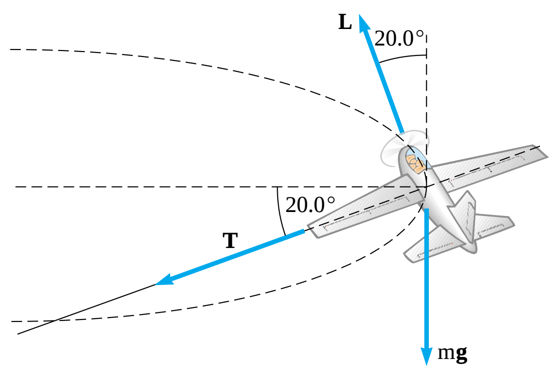
Assuming $v > 1$ m/s, and ignoring the first term:

$$4.11 \times 10^{-5} = 0.870 \times 10^{-6}\ v^2$$

$$v = \boxed{6.87 \text{ m/s}}$$

6.69 $\sum F_y = L_y - T_y - mg = L \cos 20.0° - T \sin 20.0° - 7.35 \text{ N} = ma_y = 0$

$$\sum F_x = L_x + T_x = L \sin 20.0° + T \cos 20.0° = m\frac{v^2}{r}$$



$$m\frac{v^2}{r} = (0.750 \text{ kg})\frac{(35.0 \text{ m/s})^2}{[(60.0 \text{ m}) \cos 20.0°]} = 16.3 \text{ N}$$

$$\therefore L \sin 20.0° + T \cos 20.0° = 16.3 \text{ N}$$

$$L \cos 20.0° - T \sin 20.0° = 7.35 \text{ N}$$

$$L + T\frac{\cos 20.0°}{\sin 20.0°} = \frac{16.3 \text{ N}}{\sin 20.0°}$$

$$L - T\frac{\sin 20.0°}{\cos 20.0°} = \frac{7.35 \text{ N}}{\cos 20.0°}$$

$$T(\cot 20.0° + \tan 20.0°) = \frac{16.3 \text{ N}}{\sin 20.0°} - \frac{7.35 \text{ N}}{\cos 20.0°}$$

$$T(3.11) = 39.8 \text{ N}$$

$$T = \boxed{12.8 \text{ N}}$$

6.70 $v = \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right]$

At $t \to \infty$, $v \to v_T = \frac{mg}{b}$

At $t = 5.54$ s,

$$0.500v_t = v_t\left[1 - \exp\left(\frac{-b(5.54\text{ s})}{9.00\text{ kg}}\right)\right]$$

$$\exp\left(\frac{-b(5.54\text{ s})}{9.00\text{ kg}}\right) = 0.500$$

$$\frac{-b(5.54\text{ s})}{9.00\text{ kg}} = \ln 0.500 = -0.693$$

$$b = \frac{9.00\text{ kg}(0.693)}{5.54\text{ s}} = 1.13\text{ m/s}$$

(a) $v_t = \frac{mg}{b} = \frac{9.00\text{ kg}(9.80\text{ m/s}^2)}{1.13\text{ kg/s}} = \boxed{78.3\text{ m/s}}$

(b) $0.750v_t = v_t\left[1 - \exp\left(\frac{-1.13t}{9.00\text{ s}}\right)\right]$

$$\exp\left(\frac{-1.13t}{9.00\text{ s}}\right) = 0.250$$

$$t = \frac{9.00\ (\ln 0.250)}{-1.13}\text{ s} = \boxed{11.1\text{ s}}$$

(c) $\frac{dx}{dt} = \left(\frac{mg}{b}\right)\left[1 - \exp\left(-\frac{bt}{m}\right)\right]$

$$\int_{x_0}^{x} dx = \int_0^t \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right] dt$$

$$x - x_0 = \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Big|_0^t$$

$$= \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right) - 1\right]$$

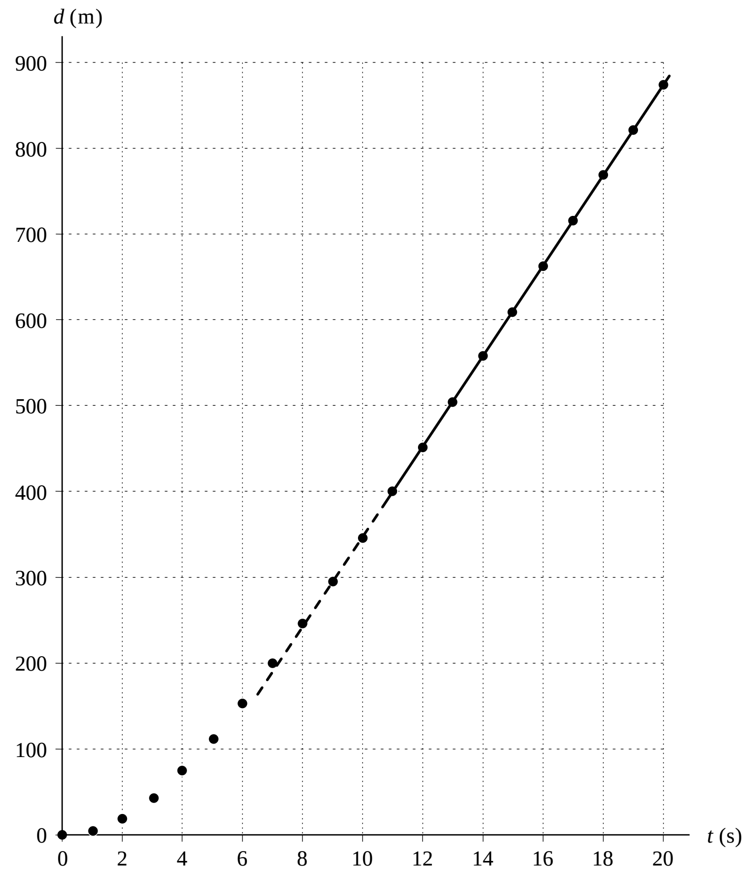
At t = 5.54 s,

$$x = (9.00\text{ kg})(9.80\text{ m/s}^2)\frac{5.54\text{ s}}{1.13\text{ kg/s}} + \left(\frac{(9.00\text{ kg})^2(9.80\text{ m/s}^2)}{(1.13\text{ kg/s})^2}\right)[\exp(-0.693) - 1]$$

$$x = 434\text{ m} + (626\text{ m})(-0.500) = \boxed{121\text{ m}}$$

6.71 (a)

t (s)	d (m)
1.00	4.88
2.00	18.9
3.00	42.1
4.00	73.8
5.00	112
6.00	154
7.00	199
8.00	246
9.00	296
10.0	347
11.0	399
12.0	452
13.0	505
14.0	558
15.0	611
16.0	664
17.0	717
18.0	770
19.0	823
20.0	876



(c) A straight line fits the points from t = 11.0 s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_t = slope = \frac{876\text{ m} - 399\text{ m}}{20.0\text{ s} - 11.0\text{ s}} = \boxed{53.0\text{ m/s}}$$

Chapter 7 Solutions

***7.1** $W = Fd = (5000\text{ N})(3.00\text{ km}) = \boxed{15.0\text{ MJ}}$

***7.2** The component of force along the direction of motion is

$$F\cos\theta = (35.0\text{ N})\cos 25.0° = 31.7\text{ N}$$

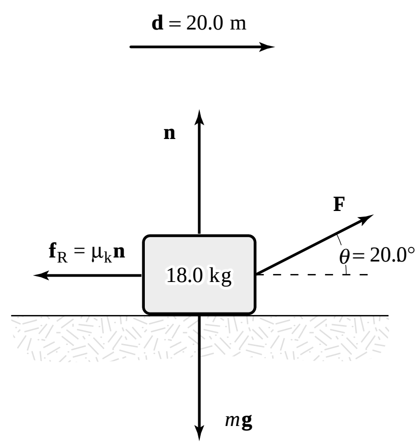
The work done by this force is

$$W = (F\cos\theta)d = (31.7\text{ N})(50.0\text{ m}) = \boxed{1.59\times 10^{3}\text{ J}}$$

7.3 (a) $W = mgh = (3.35\times 10^{-5})(9.80)(100)\text{ J} = \boxed{3.28\times 10^{-2}\text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28\times 10^{-2}\text{ J}}$

7.4 (a) $\Sigma F_y = F\sin\theta + n - mg = 0$



$n = mg - F\sin\theta$

$\Sigma F_x = F\cos\theta - \mu_k n = 0$

$$n = \frac{F\cos\theta}{\mu_k}$$

$$\therefore \quad mg - F\sin\theta = \frac{F\cos\theta}{\mu_k}$$

$$F = \frac{\mu_k mg}{\mu_k\sin\theta + \cos\theta}$$

$$F = \frac{(0.500)(18.0)(9.80)}{0.500\sin 20.0° + \cos 20.0°} = \boxed{79.4\text{ N}}$$

(b) $W_F = Fd\cos\theta = (79.4\text{ N})(20.0\text{ m})\cos 20.0° = \boxed{1.49\text{ kJ}}$

(c) $f_k = F\cos\theta = 74.6\text{ N}$

$$W_f = f_k d\cos\theta = (74.6\text{ N})(20.0\text{ m})\cos 180° = \boxed{-1.49\text{ kJ}}$$

7.5 (a) $W = Fd\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b) and (c) The normal force and the weight are both at 90° to the motion. Both do $\boxed{0}$ work.

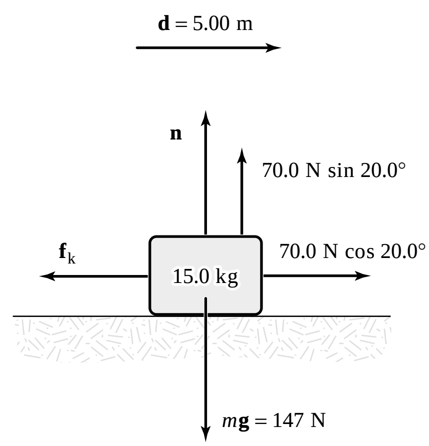
(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

7.6 $\sum F_y = ma_y$

$n + (70.0\text{ N})\sin 20.0° - 147\text{ N} = 0$

$n = 123\text{ N}$

$f_k = \mu_k n = 0.300\,(123\text{ N}) = 36.9\text{ N}$



(a) $W = Fd\cos\theta$

$= (70.0\text{ N})(5.00\text{ m})\cos 20.0° = \boxed{329\text{ J}}$

(b) $W = Fd\cos\theta = (123\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0\text{ J}}$

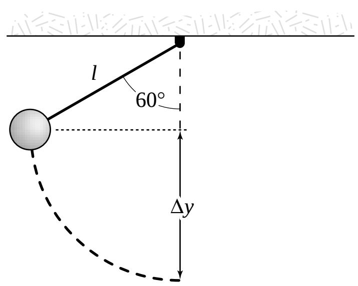
(c) $W = Fd\cos\theta = (147\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0}$

(d) $W = Fd\cos\theta = (36.9\text{ N})(5.00\text{ m})\cos 180° = \boxed{-185\text{ J}}$

(e) $\Delta K = K_f - K_i = \sum W = 329\text{ J} - 185\text{ J} = \boxed{+144\text{ J}}$

7.7 $W = mg(\Delta y) = mg(l - l\cos\theta)$

$= (80.0\text{ kg})(9.80\text{ m/s}^2)(12.0\text{ m})(1 - \cos 60.0°) = \boxed{4.70\text{ kJ}}$



7.8 $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$

7.9 $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = 7.00(4.00)\cos(130° - 70.0°) = \boxed{14.0}$

7.10 $\mathbf{A}\cdot\mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$

$\mathbf{A}\cdot\mathbf{B} = A_xB_x(\mathbf{i}\cdot\mathbf{i}) + A_xB_y(\mathbf{i}\cdot\mathbf{j}) + A_xB_z(\mathbf{i}\cdot\mathbf{k}) +$

$A_yB_x(\mathbf{j}\cdot\mathbf{i}) + A_yB_y(\mathbf{j}\cdot\mathbf{j}) + A_yB_z(\mathbf{j}\cdot\mathbf{k}) +$

$A_zB_x(\mathbf{k}\cdot\mathbf{i}) + A_zB_y(\mathbf{k}\cdot\mathbf{j}) + A_zB_z(\mathbf{k}\cdot\mathbf{k})$

$\mathbf{A} \cdot \mathbf{B} = \boxed{A_x B_x + A_y B_y + A_z B_z}$

7.11 (a) $W = \mathbf{F} \cdot \mathbf{d} = F_x x + F_y y = (6.00)(3.00)\text{ N} \cdot \text{m} + (-2.00)(1.00)\text{ N} \cdot \text{m} = \boxed{16.0\text{ J}}$

(b) $\theta = \cos^{-1}\dfrac{\mathbf{F} \cdot \mathbf{d}}{Fd} = \cos^{-1}\dfrac{16}{\sqrt{[(6.00)^2 + (-2.00)^2][(3.00)^2 + (1.00)^2]}} = \boxed{36.9°}$

7.12 $\mathbf{A} - \mathbf{B} = (3.00\mathbf{i} + \mathbf{j} - \mathbf{k}) - (-\mathbf{i} + 2.00\mathbf{j} + 5.00\mathbf{k})$

$\mathbf{A} - \mathbf{B} = 4.00\mathbf{i} - \mathbf{j} - 6.00\mathbf{k}$

$\mathbf{C} \cdot (\mathbf{A} - \mathbf{B}) = (2.00\mathbf{j} - 3.00\mathbf{k}) \cdot (4.00\mathbf{i} - \mathbf{j} - 6.00\mathbf{k})$

$= 0 + (-2.00) + (+18.0) = \boxed{16.0}$

7.13 (a) $\mathbf{A} = 3.00\mathbf{i} - 2.00\mathbf{j}$ $\mathbf{B} = 4.00\mathbf{i} - 4.00\mathbf{j}$

$\theta = \cos^{-1}\dfrac{\mathbf{A} \cdot \mathbf{B}}{AB} = \cos^{-1}\dfrac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\mathbf{B} = 3.00\mathbf{i} - 4.00\mathbf{j} + 2.00\mathbf{k}$ $\mathbf{A} = -2.00\mathbf{i} + 4.00\mathbf{j}$

$\cos\theta = \dfrac{\mathbf{A} \cdot \mathbf{B}}{AB} = \dfrac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}}$

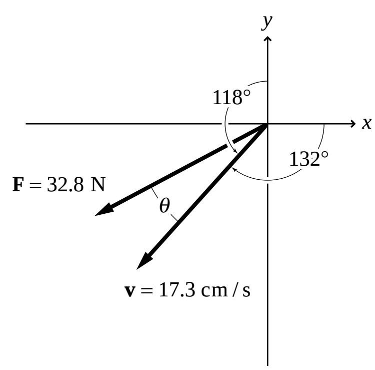
$\theta = \boxed{156°}$

(c) $\mathbf{A} = \mathbf{i} - 2.00\mathbf{j} + 2.00\mathbf{k}$ $\mathbf{B} = 3.00\mathbf{j} + 4.00\mathbf{k}$

$\theta = \cos^{-1}\left(\dfrac{A \cdot B}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00 + 8.00}{\sqrt{9.00} \cdot \sqrt{25.0}}\right) = \boxed{82.3°}$

***7.14** We must first find the angle between the two vectors. It is:

$\theta = 360° - 118° - 90.0° - 132° = 20.0°$



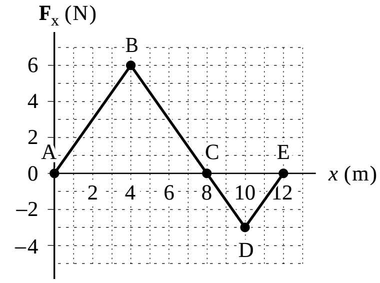
Then

$\mathbf{F} \cdot \mathbf{v} = Fv\cos\theta = (32.8\text{ N})(0.173\text{ m/s})\cos 20.0°$

or

$\mathbf{F} \cdot \mathbf{v} = 5.33\dfrac{\text{N} \cdot \text{m}}{\text{s}} = 5.33\dfrac{\text{J}}{\text{s}} = \boxed{5.33\text{ W}}$

7.15 $W = \int_i^f F dx$ = area under curve from x_i to x_f



(a) $x_i = 0$ $\quad$ $x_f = 8.00$ m

W = area of triangle $ABC = \left(\frac{1}{2}\right) AC \times$ altitude,

$$W_{0\to 8} = \left(\frac{1}{2}\right) \times 8.00 \text{ m} \times 6.00 \text{ N} = \boxed{24.0 \text{ J}}$$

(b) $x_i = 8.00$ m $\quad$ $x_f = 10.0$ m

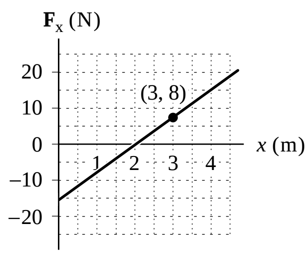
W = area of $\Delta CDE = \left(\frac{1}{2}\right) CE \times$ altitude,

$$W_{8\to 10} = \left(\frac{1}{2}\right) \times (2.00 \text{ m}) \times (-3.00 \text{ N}) = \boxed{-3.00 \text{ J}}$$

(c) $W_{0\to 10} = W_{0\to 8} + W_{8\to 10} = 24.0 + (-3.00) = \boxed{21.0 \text{ J}}$

***7.16** $F_x = (8x - 16)$ N

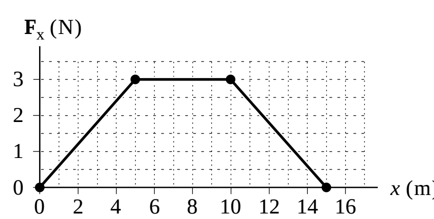
(a)



(b) $W_{\text{net}} = \dfrac{-(2.00 \text{ m})(16.0 \text{ N})}{2} + \dfrac{(1.00 \text{ m})(8.00 \text{ N})}{2} = \boxed{-12.0 \text{ J}}$

7.17 $W = \int F_x\, dx$ and

W equals the area under the Force-Displacement Curve



(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00 \text{ N})(5.00 \text{ m}) = \boxed{15.0 \text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$

7.18 $W = \int_i^f \mathbf{F} \cdot d\mathbf{s} = \int_0^{5\text{ m}} (4x\mathbf{i} + 3y\mathbf{j})\ \text{N} \cdot dx\mathbf{i}$

$$\int_0^{5\text{ m}} (4\text{ N/m})\, x\, dx + 0 = (4\text{ N/m})x^2/2 \Big|_0^{5\text{ m}} = \boxed{50.0\text{ J}}$$

***7.19** $k = \dfrac{F}{y} = \dfrac{Mg}{y} = \dfrac{(4.00)(9.80)\text{ N}}{2.50 \times 10^{-2}\text{ m}} = 1.57 \times 10^3\text{ N/m}$

(a) For 1.50 kg mass $y = \dfrac{mg}{k} = \dfrac{(1.50)(9.80)}{1.57 \times 10^3} = \boxed{0.938\text{ cm}}$

(b) Work $= \dfrac{1}{2}\ ky^2$

$$\text{Work} = \frac{1}{2}(1.57 \times 10^3\text{ N} \cdot \text{m})(4.00 \times 10^{-2}\text{ m})^2 = \boxed{1.25\text{ J}}$$

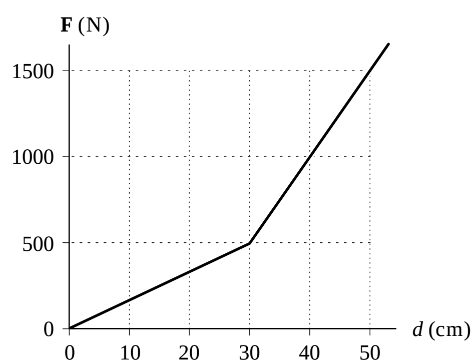
7.20 (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230\text{ N})}{(0.400\text{ m})} = \boxed{575\text{ N/m}}$$

(b) Work $= F_{\text{avg}}\, x = \dfrac{1}{2}(230\text{ N})(0.400\text{ m}) = \boxed{46.0\text{ J}}$

7.21 Compare an initial picture of the rolling car with a final picture with both springs compressed

$$K_i + \sum W = K_f$$



Use equation 7.11.

$$K_i + \frac{1}{2}\, k_1\,(x_{1i}^2 - x_{1f}^2) + \frac{1}{2}\, k_2\,(x_{2i}^2 - x_{2f}^2) = K_f$$

$$\frac{1}{2}\, m v_i^2 + 0 - \frac{1}{2}(1600\ \text{N/m})(0.500\ \text{m})^2 + 0 - \frac{1}{2}(3400\ \text{N/m})(0.200\ \text{m})^2 = 0$$

$$\frac{1}{2}(6000\ \text{kg})\, v_i^2 - 200\ \text{J} - 68.0\ \text{J} = 0$$

$$v_i = \sqrt{2 \times 268\ \text{J}/6000\ \text{kg}} = \boxed{0.299\ \text{m/s}}$$

7.22 (a) $W = \int_i^f \mathbf{F} \cdot d\mathbf{s}$

$$W = \int_0^{0.600\ \text{m}} (15000\ \text{N} + 10000\, x\ \text{N/m} - 25000\, x^2\ \text{N/m}^2)\, dx \cos 0°$$

$$W = 15{,}000x + \frac{10{,}000x^2}{2} - \frac{25{,}000x^3}{3}\Bigg|_0^{0.600}$$

$$W = 9.00\ \text{kJ} + 1.80\ \text{kJ} - 1.80\ \text{kJ} = \boxed{9.00\ \text{kJ}}$$

(b) Similarly,

$$W = (15.0\ \text{kN})(1.00\ \text{m}) + \frac{(10.0\ \text{kN/m})(1.00\ \text{m})^2}{2} - \frac{(25.0\ \text{kN/m}^2)(1.00\ \text{m})^3}{3}$$

$W = \boxed{11.7\ \text{kJ}}$, larger by 29.6%

7.23 $4.00\ \text{J} = \frac{1}{2}\, k(0.100\ \text{m})^2$

$\therefore k = 800\ \text{N/m}$

and to stretch the spring to 0.200 m requires

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\ \text{J} = \boxed{12.0\ \text{J}}$$

<u>Goal Solution</u>

G: We know that the force required to stretch a spring is proportional to the distance the spring is stretched, and since the work required is proportional to the force *and* to the distance, then $W \propto x^2$. This means if the extension of the spring is doubled, the work will increase by a factor of 4, so that for x = 20 cm, W = 16 J, requiring 12 J of additional work.

O: Let's confirm our answer using Hooke's law and the definition of work.

A: The linear spring force relation is given by Hooke's law: $\mathbf{F}_s = -k\mathbf{x}$

Integrating with respect to x, we find the work done by the spring is:

$$W_s = \int_{x_x}^{x_y} F_s\,dx = \int_{x_x}^{x_y} (-kx)dx = -\frac{1}{2}\,k\,(x_f^2 - x_i^2)$$

However, we want the work done *on* the spring, which is $W = -W_s = \frac{1}{2}\,k(x_f^2 - x_i^2)$

We know the work for the first 10 cm, so we can find the force constant:

$$k = \frac{2W_{0-10}}{x_{0-10}^2} = \frac{2(4.00\text{ J})}{(0.100\text{ m})^2} = 800\text{ N/m}$$

Substituting for k, x_i and x_f, the extra work for the next step of extension is

$$W = \left(\frac{1}{2}\right)(800\text{ N/m})\ \left[(0.200\text{ m})^2 - (0.100\text{ m})^2\right] = 12.0\text{ J}$$

L: Our calculated answer agrees with our prediction. It is helpful to remember that the force required to stretch a spring is proportional to the distance the spring is extended, but the work is proportional to the square of the extension.

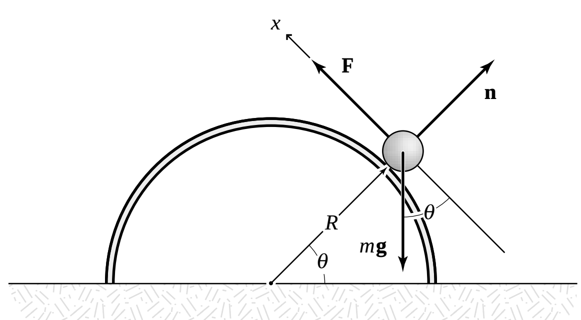
7.24 $W = \frac{1}{2}\,kd^2$

$$\therefore\ k = \frac{2W}{d^2}$$

$$\Delta W = \frac{1}{2}\,k(2d)^2 - \frac{1}{2}\,kd^2$$

$$\Delta W = \frac{3}{2}\,kd^2 = \boxed{3W}$$

7.25 (a) The radius to the mass makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x-axis, when we take the x-axis in the direction of motion tangent to the cylinder.



$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

(b) $W = \int_i^f \mathbf{F} \cdot d\mathbf{s}$

We use radian measure to express the next bit of displacement as $ds = r\,d\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg \cos\theta\, R d\theta = mgR \sin\theta \Big|_0^{\pi/2}$$

$$W = mgR\,(1 - 0) = \boxed{mgR}$$

***7.26** $[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg} \cdot \text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$

7.27 (a) $K_A = \frac{1}{2}(0.600\text{ kg})(2.00\text{ m/s})^2 = \boxed{1.20\text{ J}}$

(b) $\frac{1}{2}\, m v_B^2 = K_B$

$$v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = \boxed{5.00\text{ m/s}}$$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}\, m(v_B^2 - v_A^2)$

$$= 7.50\text{ J} - 1.20\text{ J} = \boxed{6.30\text{ J}}$$

***7.28** (a) $K = \frac{1}{2}\, mv^2 = \frac{1}{2}(0.300\text{ kg})(15.0\text{ m/s})^2 = \boxed{33.8\text{ J}}$

(b) $K = \frac{1}{2}(0.300)(30.0)^2 = \frac{1}{2}(0.300)(15.0)^2(4) = 4(33.8) = \boxed{135\text{ J}}$

7.29 $\mathbf{v}_i = (6.00\mathbf{i} - 2.00\mathbf{j})\text{ m/s}$

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0}\text{ m/s}$

$$K_i = \frac{1}{2}\, m v_i^2 = \frac{1}{2}(3.00\text{ kg})(40.0\text{ m}^2/\text{s}^2) = \boxed{60.0\text{ J}}$$

(b) $v = 8.00\mathbf{i} + 4.00\mathbf{j}$

$$v^2 = \mathbf{v} \cdot \mathbf{v} = 64.0 + 16.0 = 80.0\text{ m}^2/\text{s}^2$$

$$\Delta K = K - K_i = \frac{1}{2}\, m(v^2 - v_i^2) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0\text{ J}}$$

7.30 (a) $\Delta K = \sum W$

$\frac{1}{2}(2500 \text{ kg})\, v^2 = 5000 \text{ J}$

$v = \boxed{2.00 \text{ m/s}}$

(b) $W = \mathbf{F} \cdot \mathbf{d}$

$5000 \text{ J} = F(25.0 \text{ m})$

$F = \boxed{200 \text{ N}}$

7.31 (a) $\Delta K = \frac{1}{2}\, mv^2 - 0 = \sum W$, so

$v^2 = 2W/m$ and $v = \boxed{\sqrt{2W/m}}$

(b) $W = \mathbf{F} \cdot \mathbf{d} = F_x d \Rightarrow F_x = \boxed{W/d}$

7.32 (a) $\Delta K = K_f - K_i = \frac{1}{2}\, mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50 \text{ J})}{4.00 \text{ kg}}} = \boxed{1.94 \text{ m/s}}$

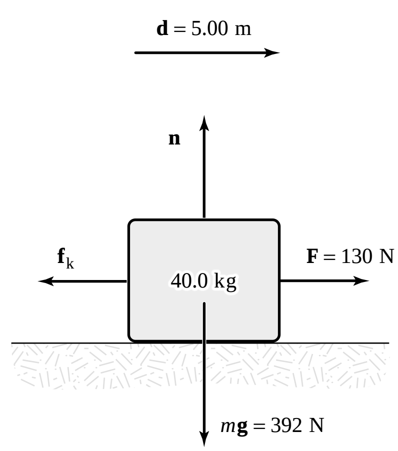
(b) $\Delta K = K_f - K_i = \frac{1}{2}\, mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.35 \text{ m/s}}$

(c) $\Delta K = K_f - K_i = \frac{1}{2}\, mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.87 \text{ m/s}}$

***7.33** $\sum F_y = ma_y$



$n - 392 \text{ N} = 0 \qquad n = 392 \text{ N}$

$f_k = \mu_k n = 0.300(392 \text{ N}) = 118 \text{ N}$

(a) $W_F = Fd\cos\theta = (130)(5.00)\cos 0° = \boxed{650 \text{ J}}$

(b) $W_{f_k} = f_k d\cos\theta = (118)(5.00)\cos 180° = \boxed{-588 \text{ J}}$

(c) $W_n = nd\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W$

$$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$$

(f) $v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

7.34 (a) $K_i + \sum W = K_f = \frac{1}{2}mv_f^2$

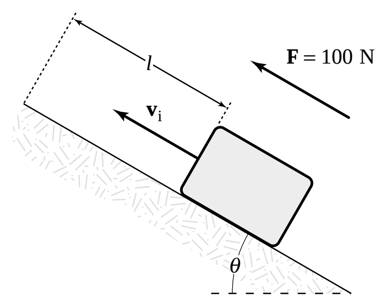
$$0 + \sum W = \frac{1}{2}(15.0\times 10^{-3}\text{ kg})(780\text{ m/s})^2 = \boxed{4.56\text{ kJ}}$$

(b) $F = \frac{W}{d\cos\theta} = \frac{4.56\times 10^3\text{ J}}{(0.720\text{ m})\cos 0°} = \boxed{6.34\text{ kN}}$

(c) $a = \frac{v_f^2 - v_i^2}{2x} = \frac{(780\text{ m/s})^2 - 0}{2(0.720\text{ m})} = \boxed{422\text{ km/s}^2}$

(d) $\sum F = ma = (15\times 10^{-3}\text{ kg})(422\times 10^3\text{ m/s}^2) = \boxed{6.34\text{ kN}}$

7.35 (a) $W_g = mgl\cos(90.0° + \theta) = (10.0\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m})\cos 110° = \boxed{-168\text{ J}}$



(b) $f_k = \mu_k n = \mu_k mg\cos\theta$

$W_f = -lf_k = l\mu_k mg\cos\theta\cos 180°$

$W_f = -(5.00\text{ m})(0.400)(10.0)(9.80)\cos 20.0° = \boxed{-184\text{ J}}$

(c) $W_F = Fl = (100)(5.00) = \boxed{500\text{ J}}$

(d) $\Delta K = \sum W = W_F + W_f + W_g = \boxed{148\text{ J}}$

(e) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65\text{ m/s}}$$

7.36 $\sum W = \Delta K = 0$

$$\int_0^L mg \sin 35.0°\, dl - \int_0^d kx\,dx = 0$$

$$mg \sin 35.0° (L) = \frac{1}{2} kd^2$$

$$d = \sqrt{\frac{2\, mg \sin 35.0°(L)}{k}}$$

$$= \sqrt{\frac{2(12.0 \text{ kg})(9.80 \text{ m/s}^2) \sin 35.0° (3.00 \text{ m})}{3.00 \times 10^4 \text{ N/m}}} = \boxed{0.116 \text{ m}}$$

7.37 $v_i = 2.00$ m/s $\quad \mu_k = 0.100$

$$\sum W = \Delta K$$

$$-f_k x = 0 - \frac{1}{2} mv_i^2$$

$$-\mu_k mgx = -\frac{1}{2} mv_i^2$$

$$x = \frac{v_i^2}{2\mu_k g} = \frac{(2.00 \text{ m/s})^2}{2(0.100)(9.80)} = \boxed{2.04 \text{ m}}$$

Goal Solution

G: Since the sled's initial speed of 2 m/s (~ 4 mph) is reasonable for a moderate kick, we might expect the sled to travel several meters before coming to rest.

O: We could solve this problem using Newton's second law, but we are asked to use the work-kinetic energy theorem: $W = K_f - K_i$, where the only work done on the sled after the kick results from the friction between the sled and ice. (The weight and normal force both act at 90° to the motion, and therefore do no work on the sled.)

A: The work due to friction is $W = -f_k d$ where $f_k = \mu_k mg$.

Since the final kinetic energy is zero, $W = \Delta K = 0 - K_i = -\frac{1}{2} mv_i^2$

Solving for the distance $d = \frac{mv_i^2}{2\mu_k mg} = \frac{v_i^2}{2\mu_k g} = \frac{(2.00 \text{ m})^2}{2(0.100)(9.80 \text{ m/s}^2)} = 2.04 \text{ m}$

L: The distance agrees with the prediction. It is interesting that the distance does not depend on the mass and is proportional to the square of the initial velocity. This means that a small car and a massive truck should be able to stop within the same distance if they both skid to a stop from the same initial speed. Also, doubling the speed requires 4 times as much stopping distance, which is consistent with advice given by transportation safety officers who suggest at least a 2 second gap between vehicles (as opposed to a fixed distance of 100 feet).

7.38 (a) $v_f = 0.01c = 10^{-2}(3.00 \times 10^8 \text{ m/s}) = 3.00 \times 10^6 \text{ m/s}$

$$K_f = \frac{1}{2} m v_f^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^6 \text{ m/s})^2 = \boxed{4.10 \times 10^{-18} \text{ J}}$$

(b) $K_i + Fd\cos\theta = K_f$

$$0 + F(0.360 \text{ m})\cos 0° = 4.10 \times 10^{-18} \text{ N} \cdot \text{m}$$

$$F = \boxed{1.14 \times 10^{-17} \text{ N}}$$

(c) $$a = \frac{\sum \mathbf{F}}{m} = \frac{1.14 \times 10^{-17} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} = \boxed{1.25 \times 10^{13} \text{ m/s}^2}$$

(d) $$x_f - x_i = \frac{1}{2}(v_i + v_f)\, t$$

$$t = \frac{2(x_f - x_i)}{(v_i + v_f)} = \frac{2(0.360 \text{ m})}{(3.00 \times 10^6 \text{ m/s})} = \boxed{2.40 \times 10^{-7} \text{ s}}$$

7.39 (a) $\sum W = \Delta K \Rightarrow fd\cos\theta = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$$f(4.00 \times 10^{-2} \text{ m})\cos 180° = 0 - \frac{1}{2}(5.00 \times 10^{-3} \text{ kg})(600 \text{ m/s})^2$$

$$f = \boxed{2.25 \times 10^4 \text{ N}}$$

(b) $$t = \frac{d}{\bar{v}} = \frac{4.00 \times 10^{-2} \text{ m}}{[0 + 600 \text{ m/s}]/2} = \boxed{1.33 \times 10^{-4} \text{ s}}$$

7.40 $\sum W = \Delta K$

$$m_1 gh - m_2 gh = \frac{1}{2}(m_1 + m_2)\, v_f^2 - 0$$

$$v_f^2 = \frac{2(m_1 - m_2)gh}{m_1 + m_2} = \frac{2(0.300 - 0.200)(9.80)(0.400)}{0.300 + 0.200} \frac{\text{m}^2}{\text{s}^2}$$

$$v_f = \sqrt{1.57} \text{ m/s} = \boxed{1.25 \text{ m/s}}$$

7.41 (a) $W_s = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2 = \frac{1}{2}(500)(5.00 \times 10^{-2})^2 - 0 = 0.625 \text{ J}$

$$\sum W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m v_f^2 - 0$$

so $$v_f = \sqrt{\frac{2(\sum W)}{m}} = \sqrt{\frac{2(0.625)}{2.00}} \text{ m/s} = \boxed{0.791 \text{ m/s}}$$

(b) $\sum W = W_s + W_f = 0.625\text{ J} + (-\mu_k mgd)$

$$= 0.625\text{ J} - (0.350)(2.00)(9.80)(5.00\times 10^{-2})\text{ J} = 0.282\text{ J}$$

$$v_f = \sqrt{\frac{2(\sum W)}{m}} = \sqrt{\frac{2(0.282)}{2.00}}\text{ m/s} = \boxed{0.531\text{ m/s}}$$

***7.42** A 1300-kg car speeds up from rest to 55.0 mi / h = 24.6 m / s in 15.0 s. The output work of the engine becomes its final kinetic energy,

$$\frac{1}{2}(1300\text{ kg})(24.6\text{ m/s})^2 = 390\text{ kJ}$$

with power $\dfrac{390000\text{ J}}{15.0\text{ s}}\ \boxed{\sim 10^4\text{ W}}$, around 30 horsepower.

7.43 Power $= \dfrac{W}{t} = \dfrac{mgh}{t} = \dfrac{(700\text{ N})(10.0\text{ m})}{8.00\text{ s}} = \boxed{875\text{ W}}$

7.44 Efficiency = e = useful energy output / total energy input. The force required to lift n bundles of shingles is their weight, nmg.

$$e = \frac{n\,mgh\cos 0°}{Pt}$$

$$n = \frac{ePt}{mgh} = \frac{(0.700)(746\text{ W})(7200\text{ s})}{(70.0\text{ kg})(9.80\text{ m/s}^2)(8.00\text{ m})} \times \frac{\text{kg}\cdot\text{m}^2}{\text{s}^3\cdot\text{W}} = \boxed{685\text{ bundles}}$$

7.45 $P_a = f_a v \Rightarrow f_a = \dfrac{P_a}{v} = \dfrac{2.24\times 10^4}{27.0} = \boxed{830\text{ N}}$

***7.46** (a) $\sum W = \Delta K$, but $\Delta K = 0$ because he moves at constant speed. The skier rises a vertical distance of (60.0 m) sin 30.0° = 30.0 m. Thus,

$$W_{\text{in}} = -W_g = (70.0\text{ kg})g(30.0\text{ m}) = \boxed{2.06\times 10^4\text{ J}} = \boxed{20.6\text{ kJ}}$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m / s is 30.0 s. Thus,

$$P_{\text{input}} = \frac{W}{\Delta t} = \frac{2.06\times 10^4\text{ J}}{30.0\text{ s}} = \boxed{686\text{ W}} = 0.919\text{ hp}$$

7.47 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}\, t = \left[\frac{0 + 1.75\ \text{m/s}}{2}\right](3.00\ \text{s}) = 2.63\ \text{m}$$

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + mgy_f - mgy_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + mg(\Delta y)$$

$$W = \frac{1}{2}(650\ \text{kg})(1.75\ \text{m/s})^2 - 0 + (650\ \text{kg})g(2.63\ \text{m}) = 1.77 \times 10^4\ \text{J}$$

Also, $W = \bar{P}\, t$

so $\bar{P} = \dfrac{W}{t} = \dfrac{1.77 \times 10^4\ \text{J}}{3.00\ \text{s}} = \boxed{5.91 \times 10^3\ \text{W}} = 7.92\ \text{hp}$

(b) When moving upward at constant speed ($v = 1.75$ m/s), the applied force equals the weight = $(650\ \text{kg})(9.80\ \text{m/s}^2) = 6.37 \times 10^3$ N.

Therefore, $P = Fv = (6.37 \times 10^3\ \text{N})(1.75\ \text{m/s}) = \boxed{1.11 \times 10^4\ \text{W}} = 14.9\ \text{hp}$

***7.48** *energy = power × time*

For the 28.0 W bulb:

Energy used = $(28.0\ \text{W})(1.00 \times 10^4\ \text{h}) = 280$ kilowatt · hrs

total cost = \$17.00 + (280 kWh)(\$0.080/kWh) = \$39.40

For the 100 W bulb:

Energy used = $(100\ \text{W})(1.00 \times 10^4\ \text{h}) = 1.00 \times 10^3$ kilowatt · hrs

bulb used $= \dfrac{1.00 \times 10^4\ \text{h}}{750\ \text{h/bulb}} = 13.3$

total cost = 13.3(\$0.420) + (1.00×10^3 kWh)(\$0.080/kWh) = \$85.60

Savings with energy-efficient bulb = \$85.60 – \$39.40 = $\boxed{\$46.2}$

7.49 (a) $$\text{fuel needed} = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{\text{useful energy per gallon}} = \frac{\frac{1}{2}mv_f^2 - 0}{\text{eff.} \times (\text{energy content of fuel})}$$

$$= \frac{\frac{1}{2}(900 \text{ kg})(24.6 \text{ m/s})^2}{(0.150)(1.34 \times 10^8 \text{ J/gal})} = \boxed{1.35 \times 10^{-2} \text{ gal}}$$

(b) $\boxed{73.8}$

(c) $$\text{power} = \left(\frac{1 \text{ gal}}{38.0 \text{ mi}}\right)\left(\frac{55.0 \text{ mi}}{1.00 \text{ h}}\right)\left(\frac{1.00 \text{ h}}{3600 \text{ s}}\right)\left(\frac{1.34 \times 10^8 \text{ J}}{1 \text{ gal}}\right)(0.150) = \boxed{8.08 \text{ kW}}$$

7.50 At a speed of 26.8 m/s (60.0 mph), the car described in Table 7.2 delivers a power of $P_1 = 18.3$ kW to the wheels. If an additional load of 350 kg is added to the car, a larger output power of

$$P_2 = P_1 + (\text{power input to move 350 kg at speed } v)$$

will be required. The additional power output needed to move 350 kg at speed v is:

$$\Delta P_{\text{out}} = (\Delta f)v = (\mu_r mg)v$$

Assuming a coefficient of rolling friction of $\mu_r = 0.0160$, the power output now needed from the engine is

$$P_2 = P_1 + (0.0160)(350 \text{ kg})(9.80 \text{ m/s}^2)(26.8 \text{ m/s}) = 18.3 \text{ kW} + 1.47 \text{ kW}$$

With the assumption of constant efficiency of the engine, the input power must increase by the same factor as the output power. Thus, the fuel economy must decrease by this factor:

$$(\text{fuel economy})_2 = \left(\frac{P_1}{P_2}\right)(\text{fuel economy})_1 = \left(\frac{18.3}{18.3 + 1.47}\right)\left(6.40 \frac{\text{km}}{\text{L}}\right)$$

or $$(\text{fuel economy})_2 = \boxed{5.92 \frac{\text{km}}{\text{L}}}$$

7.51 When the car of Table 7.2 is traveling at 26.8 m / s (60.0 mph), the engine delivers a power of $P_1 = 18.3$ kW to the wheels. When the air conditioner is turned on, an additional output power of $\Delta P = 1.54$ kW is needed. The total power output now required is

$$P_2 = P_1 + \Delta P = 18.3 \text{ kW} + 1.54 \text{ kW}$$

Assuming a constant efficiency of the engine, the fuel economy must decrease by the same factor as the power output increases. The expected fuel economy with the air conditioner on is therefore

$$(\text{fuel economy})_2 = \left(\frac{P_1}{P_2}\right)(\text{fuel economy})_1 = \left(\frac{18.3}{18.3 + 1.54}\right)\left(6.40\ \frac{\text{km}}{\text{L}}\right)$$

or $\quad (\text{fuel economy})_2 = \boxed{5.90\ \frac{\text{km}}{\text{L}}}$

7.52 (a) $K = \left(\dfrac{1}{\sqrt{1-(v/c)^2}} - 1\right) mc^2 = \left(\dfrac{1}{\sqrt{1-(0.995)^2}} - 1\right)(9.11 \times 10^{-31})(2.998 \times 10^{8})^2$

$K = \boxed{7.38 \times 10^{-13} \text{ J}}$

(b) Classically,

$$K = \frac{1}{2}\, mv^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})\ [(0.995)(2.998 \times 10^{8} \text{ m/s})]^2 = 4.05 \times 10^{-14} \text{ J}$$

This differs from the relativistic result by

$$\%\ \text{error} = \left(\frac{7.38 \times 10^{-13} \text{ J} - 4.05 \times 10^{-14} \text{ J}}{7.38 \times 10^{-13} \text{ J}}\right) 100\% = \boxed{94.5\%}$$

7.53 $\sum W = K_f - K_i = \left(\dfrac{1}{\sqrt{1-(v_f/c)^2}} - 1\right) mc^2 - \left(\dfrac{1}{\sqrt{1-(v_i/c)^2}} - 1\right) mc^2$

or $\quad \sum W = \left(\dfrac{1}{\sqrt{1-(v_f/c)^2}} - \dfrac{1}{\sqrt{1-(v_i/c)^2}}\right) mc^2$

(a) $\sum W = \left(\dfrac{1}{\sqrt{1-(0.750)^2}} - \dfrac{1}{\sqrt{1-(0.500)^2}}\right)(1.673 \times 10^{-27} \text{ kg})(2.998 \times 10^{8} \text{ m/s})^2$

$\sum W = \boxed{5.37 \times 10^{-11} \text{ J}}$

(b) $\sum W = \left(\dfrac{1}{\sqrt{1-(0.995)^2}} - \dfrac{1}{\sqrt{1-(0.500)^2}}\right)(1.673 \times 10^{-27} \text{ kg})(2.998 \times 10^{8} \text{ m/s})^2$

$\sum W = \boxed{1.33 \times 10^{-9} \text{ J}}$

<u>Goal Solution</u>

G: Since particle accelerators have typical maximum energies on the order of GeV (1eV = 1.60×10^{-19} J), we could expect the work required to be ~10^{-10} J.

O: The work-energy theorem is $W = K_f - K_i$ which for relativistic speeds ($v \sim c$) is:

$$W = \left(\frac{1}{\sqrt{1 - v_f^2/c^2}}\right) mc^2 - \left(\frac{1}{\sqrt{1 - v_i^2/c^2}}\right) mc^2$$

A: (a) $W = \left(\frac{1}{\sqrt{1 - (0.750)^2}} - 1\right)(1.67 \times 10^{-27}\text{ kg})(3.00 \times 10^8\text{ m/s})^2$

$$-\left(\frac{1}{\sqrt{1 - (0.500)^2}} - 1\right)(1.50 \times 10^{-10}\text{ J})$$

$$W = (0.512 - 0.155)(1.50 \;\; 10^{-10}\text{ J}) = 5.37 \;\; 10^{-11}\text{ J} \quad \Diamond$$

(b) $E = \left(\frac{1}{\sqrt{1 - (0.995)^2}} - 1\right)(1.50 \times 10^{-10}\text{ J}) - (1.155 - 1)(1.50 \times 10^{-10}\text{ J})$

$$W = (9.01 - 0.155)(1.50 \;\; 10^{-10}\text{ J}) = 1.33 \;\; 10^{-9}\text{ J} \quad \Diamond$$

L: Even though these energies may seem like small numbers, we must remember that the proton has very small mass, so these input energies are comparable to the rest mass energy of the proton (938 MeV = 1.50×10^{-10} J). To produce a speed higher by 33%, the answer to part (b) is 25 times larger than the answer to part (a). Even with arbitrarily large accelerating energies, the particle will never reach or exceed the speed of light. This is a consequence of special relativity, which will be examined more closely in a later chapter.

***7.54** (a) Using the classical equation,

$$K = \frac{1}{2} mv^2 = \frac{1}{2}(78.0\text{ kg})(1.06 \times 10^5\text{ m/s})^2 = \boxed{4.38 \times 10^{11}\text{ J}}$$

(b) Using the relativistic equation,

$$K = \left(\frac{1}{\sqrt{1 - (v/c)^2}} - 1\right) mc^2$$

$$= \left(\frac{1}{\sqrt{1 - (1.06 \times 10^5 / 2.998 \times 10^8)^2}} - 1\right)(78.0\text{ kg})(2.998 \times 10^8\text{ m/s})^2$$

$$K = \boxed{4.38 \times 10^{11}\text{ J}}$$

When $(v/c) << 1$, the binomial series expansion gives

$$[1-(v/c)^2]^{-1/2} \approx 1 + \frac{1}{2}(v/c)^2$$

Thus, $[1-(v/c)^2]^{-1/2} - 1 \approx (v/c)^2$

and the relativistic expression for kinetic energy becomes $K \approx \frac{1}{2}(v/c)^2\, mc^2 = \frac{1}{2}\, mv^2$. That is, in the limit of speeds much smaller than the speed of light, the relativistic and classical expressions yield the same results.

***7.55** At start, $\mathbf{v} = (40.0 \text{ m/s}) \cos 30.0°\mathbf{i} + (40.0 \text{ m/s}) \sin 30.0°\mathbf{j}$

At apex, $\mathbf{v} = (40.0 \text{ m/s}) \cos 30.0°\mathbf{i} + 0\mathbf{j} = 34.6\mathbf{i}$ m/s

and $K = \frac{1}{2}\, mv^2 = \frac{1}{2}(0.150 \text{ kg})(34.6 \text{ m/s})^2 = \boxed{90.0 \text{ J}}$

***7.56** Concentration of Energy output $= (0.600 \text{ J/kg} \cdot \text{step})(60.0 \text{ kg})\left(\frac{1 \text{ step}}{1.50 \text{ m}}\right) = 24.0\,\frac{\text{J}}{\text{m}}$

$$F = \left(24.0\,\frac{\text{J}}{\text{m}}\right)\left(1\,\frac{\text{N} \cdot \text{m}}{\text{J}}\right) = 24.0 \text{ N}$$

$$P = Fv$$

$$70.0 \text{ W} = (24.0 \text{ N})v$$

$$v = \boxed{2.92 \text{ m/s}}$$

7.57 The work-kinetic energy theorem is

$$K_i + \sum W = K_f$$

The total work is equal to the work by the constant total force:

$$\frac{1}{2}\, mv_i^2 + (\Sigma\mathbf{F}) \cdot (\mathbf{r} - \mathbf{r}_\text{i}) = \frac{1}{2}\, mv_f^2$$

$$\frac{1}{2}\, mv_i^2 + m\mathbf{a} \cdot (\mathbf{r} - \mathbf{r}_\text{i}) = \frac{1}{2}\, mv_f^2$$

$$\boxed{v_i^2 + 2\mathbf{a} \cdot (\mathbf{r} - \mathbf{r}_\text{i}) = v_f^2}$$

7.58 (a) $\mathbf{A}\cdot\mathbf{i} = (A)(1)\cos\alpha$. But also, $\mathbf{A}\cdot\mathbf{i} = A_x$.

Thus, $(A)(1)\cos\alpha = A_x$ or $\boxed{\cos\alpha = \frac{A_x}{A}}$

Similarly, $\boxed{\cos\beta = \frac{A_y}{A}}$

and $\boxed{\cos\gamma = \frac{A_z}{A}}$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

(b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\frac{A_x}{A}\right)^2 + \left(\frac{A_y}{A}\right)^2 + \left(\frac{A_z}{A}\right)^2 = \frac{A^2}{A^2} = 1$

7.59 (a) $x = t + 2.00t^3$

therefore,

$$v = \frac{dx}{dt} = 1 + 6.00t^2$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)(1 + 6.00t^2)^2 = \boxed{(2.00 + 24.0t^2 + 72.0t^4)\ \text{J}}$$

(b) $a = \frac{dv}{dt} = \boxed{(12.0t)\ \text{m/s}^2}$

$F = ma = 4.00(12.0t) = \boxed{(48.0t)\ \text{N}}$

(c) $P = Fv = (48.0t)(1 + 6.00t^2) = \boxed{(48.0t + 288t^3)\ \text{W}}$

(d) $W = \int_0^{2.00} P\,dt = \int_0^{2.00} (48.0t + 288t^3)\,dt = \boxed{1250\ \text{J}}$

***7.60** (a) The work done by the traveler is mgh_sN where N is the number of steps he climbs during the ride.

$$N = (\text{time on escalator})(n)$$

where $(\text{time on escalator}) = \frac{h}{\text{vertical velocity of person}}$, and

$$\text{vertical velocity of person} = v + nh_s$$

Then, $N = \dfrac{nh}{v + nh_s}$ and the work done by the person becomes

$$W_{\text{person}} = \boxed{\dfrac{mgnhh_s}{v + nh_s}}$$

(b) The work done by the escalator is

$$W_e = \text{(power)(time)} = \text{[(force exerted)(speed)](time)} = mgvt$$

where $t = \dfrac{h}{v + nh_s}$ as above. Thus,

$$W_e = \boxed{\dfrac{mgvh}{v + nh_s}}$$

As a check, the total work done on the person's body must add up to *mgh*, the work an elevator would do in lifting him. It does add up as follows:

$$\sum W = W_{\text{person}} + W_e = \frac{mgnhh_s}{v + nh_s} + \frac{mgvh}{v + nh_s} = \frac{mgh(nh_s + v)}{v + nh_s} = mgh$$

7.61 $W = \int_{x_i}^{x_f} F\,dx = \int_0^{x_f} (-kx + \beta x^3)\,dx$

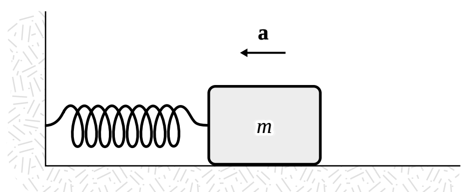
$$W = \frac{-kx^2}{2} + \frac{\beta x^4}{4}\Bigg|_0^{x_f} = \frac{-kx_f^2}{2} + \frac{\beta x_f^4}{4}$$

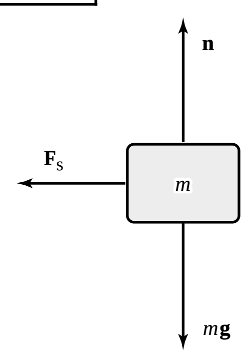
$$W = \frac{(-10.0\text{ N/m})(0.100\text{ m})^2}{2} + \frac{(100\text{ N/m}^3)(0.100\text{ m})^4}{4}$$

$$W = -5.00 \times 10^{-2}\text{ J} + 2.50 \times 10^{-3}\text{ J} = \boxed{-4.75 \times 10^{-2}\text{ J}}$$

***7.62** $\sum F_x = ma_x \Rightarrow kx = ma$

$$k = \frac{ma}{x} = \frac{(4.70 \times 10^{-3}\text{ kg})0.800(9.80\text{ m/s}^2)}{0.500 \times 10^{-2}\text{ m}} = \boxed{7.37\text{ N/m}}$$





7.63 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall.

$$\sum W = \Delta K \Rightarrow W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

so $(mg)(h + d) \cos 0° + (\bar{F})(d) \cos 180° = 0 - 0$

Thus, $\bar{F} = \dfrac{(mg)(h + d)}{d} = \dfrac{(2100 \text{ kg})(9.80 \text{ m/s}^2)(5.12 \text{ m})}{0.120 \text{ m}} = \boxed{8.78 \times 10^5 \text{ N}}$

Goal Solution

G: Anyone who has hit their thumb with a hammer knows that the resulting force is greater than just the weight of the hammer, so we should also expect the force of the pile driver to be greater than its weight: $F > mg$ ~20 kN. The force *on* the pile driver will be directed upwards.

O: The average force stopping the driver can be found from the work that results from the gravitational force starting its motion. The initial and final kinetic energies are zero.

A: Choose the initial point when the mass is elevated and the final point when it comes to rest again 5.12 m below. Two forces do work on the pile driver: gravity and the normal force exerted by the beam on the pile driver.

$$W_{\text{net}} = K_f - K_i \quad \text{so that} \quad mgs_w \cos 0 + ns_n \cos 180 = 0$$

where $m = 2\,100$ kg, $s_w = 5.12$ m, and $s_n = 0.120$ m.

In this situation, the weight vector is in the direction of motion and the beam exerts a force on the pile driver opposite the direction of motion.

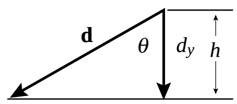
$$(2100 \text{ kg})(9.80 \text{ m/s}^2)(5.12 \text{ m}) - n(0.120 \text{ m}) = 0$$

Solve for n. $n = \dfrac{1.05 \times 10^5 \text{ J}}{0.120 \text{ m}} = 878$ kN (upwards) ◊

L: The normal force is larger than 20 kN as we expected, and is actually about 43 times greater than the weight of the pile driver, which is why this machine is so effective.

Additional Calculation:
Show that the work done by gravity on an object can be represented by mgh, where h is the vertical height that the object falls. Apply your results to the problem above.

By the figure, where **d** is the path of the object, and h is the height that the object falls,



$$h = |d_y| = d\cos\theta$$

Since $F = mg$, $mgh = Fd\cos\theta = \mathbf{F}\cdot\mathbf{d}$

In this problem, $mgh = n(d_n)$, or $(2100\text{ kg})(9.80\text{ m/s}^2)(5.12\text{ m}) = n(0.120\text{ m})$ and $n = 878\text{ kN}$

7.64 Let b represent the proportionality constant of air drag f_a to speed: $|f_a| = bv$

Let f_r represent the other frictional forces.

Take x-axis along each roadway.

For the gentle hill $\sum F_x = ma_x$

$$-bv - f_r + mg\sin 2.00° = 0$$

$$-b(4.00\text{ m/s}) - f_r + 25.7\text{ N} = 0$$

For the steeper hill

$$-b(8.00\text{ m/s}) - f_r + 51.3\text{ N} = 0$$

Subtracting,

$$b(4.00\text{ m/s}) = 25.6\text{ N}$$

$$b = 6.40\text{ N}\cdot\text{s/m}$$

and then $f_r = 0.0313$ N.

Now at 3.00 m/s the vehicle must pull her with force

$$bv + f_r = (6.40\text{ N}\cdot\text{s/m})(3.00\text{ m/s}) + 0.0313\text{ N} = 19.2\text{ N}$$

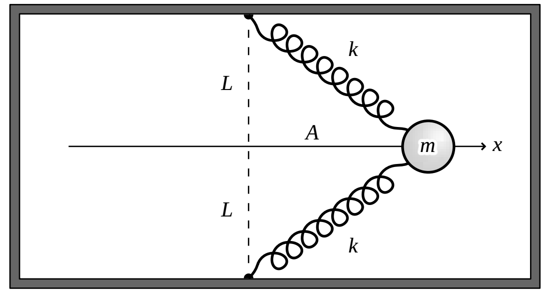
and with power

$$P = \mathbf{F}\cdot\mathbf{v} = 19.2\text{ N}(3.00\text{ m/s})\cos 0° = \boxed{57.7\text{ W}}$$

7.65 (a) $P = Fv = F(v_i + at) = F\left(0 + \frac{F}{m}t\right) = \boxed{\left(\frac{F^2}{m}\right)t}$

(b) $P = \left[\frac{(20.0\text{ N})^2}{5.00\text{ kg}}\right](3.00\text{ s}) = \boxed{240\text{ W}}$

7.66 (a) The new length of each spring is $\sqrt{x^2 + L^2}$, so its extension is $\sqrt{x^2 + L^2} - L$ and the force it exerts is $k\left(\sqrt{x^2 + L^2} - L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to



(top view)

$$\mathbf{F} = -2\mathbf{i}k\left(\sqrt{x^2 + L^2} - L\right)x/\sqrt{x^2 + L^2}$$

$$\mathbf{F} = \boxed{-2kx\mathbf{i}\left(1 - L/\sqrt{x^2 + L^2}\right)}$$

(b) $W = \int_i^f F_x\, dx$

$$W = \int_A^0 -2kx\left(1 - L/\sqrt{x^2 + L^2}\right)dx$$

$$W = -2k\int_A^0 x\, dx + kL\int_A^0 (x^2 + L^2)^{-1/2}\, 2x\, dx$$

$$W = -2k\frac{x^2}{2}\Big|_A^0 + kL\frac{(x^2 + L^2)^{1/2}}{(1/2)}\Big|_A^0$$

$$W = -0 + kA^2 + 2kL^2 - 2kL\sqrt{A^2 + L^2}$$

$$W = \boxed{2kL^2 + kA^2 - 2kL\sqrt{A^2 + L^2}}$$

7.67 (a) $\mathbf{F}_1 = (25.0\text{ N})(\cos 35.0°\,\mathbf{i} + \sin 35.0°\,\mathbf{j}) = \boxed{(20.5\mathbf{i} + 14.3\mathbf{j})\text{ N}}$

$\mathbf{F}_2 = (42.0\text{ N})(\cos 150°\,\mathbf{i} + \sin 150°\,\mathbf{j}) = \boxed{(-36.4\,\mathbf{i} + 21.0\,\mathbf{j})\text{ N}}$

(b) $\sum\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \boxed{(-15.9\mathbf{i} + 35.3\mathbf{j})\text{ N}}$

(c) $\mathbf{a} = \frac{\sum\mathbf{F}}{m} = \boxed{(-3.18\mathbf{i} + 7.07\mathbf{j})\text{ m/s}^2}$

(d) $\mathbf{v} = \mathbf{v}_i + \mathbf{a}t = (4.00\mathbf{i} + 2.50\mathbf{j})\text{ m/s} + (-3.18\mathbf{i} + 7.07\mathbf{j})(\text{m/s}^2)(3.00\text{ s})$

$\mathbf{v} = \boxed{(-5.54\mathbf{i} + 23.7\mathbf{j})\text{ m/s}}$

(e) $\mathbf{r} = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$

$$\mathbf{r} = 0 + (4.00\mathbf{i} + 2.50\mathbf{j})(\text{m/s})(3.00\text{ s}) + \frac{1}{2}(-3.18\mathbf{i} + 7.07\mathbf{j})(\text{m/s}^2)(3.00\text{ s})^2$$

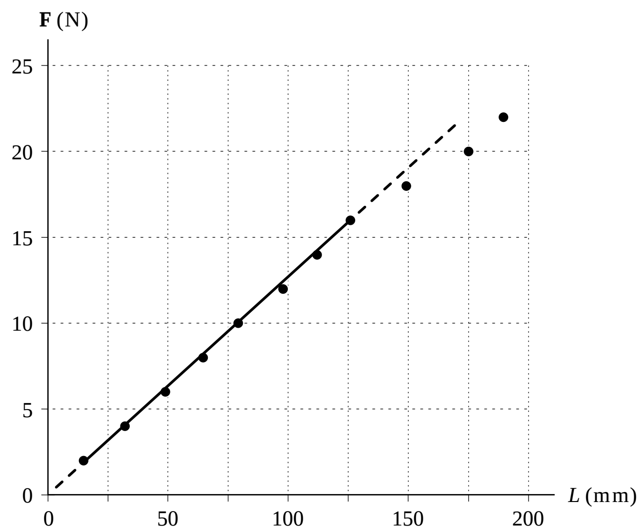
$$\mathbf{d} = \mathbf{r} = \boxed{(-2.30\mathbf{i} + 39.3\mathbf{j})\text{ m}}$$

(f) $K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(5.00\text{ kg})\ [(5.54)^2 + (23.7)^2](\text{m/s})^2 = \boxed{1.48\text{ kJ}}$

(g) $K_f = \frac{1}{2}mv_i^2 + \sum\mathbf{F}\cdot\mathbf{d} = \frac{1}{2}(5.00\text{ kg})\ [(4.00)^2 + (2.50)^2](\text{m/s})^2$

$$+ [(-15.9\text{ N})(-2.30\text{ m}) + (35.3\text{ N})(39.3\text{ m})] = 55.6\text{ J} + 1426\text{ J} = \boxed{1.48\text{ kJ}}$$

7.68 (a)



F (N)	L (mm)	F (N)	L (mm)
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190
12.0	98.0		

(b) A straight line fits the first eight points, and the origin. By least-square fitting, its slope is 0.125 N / mm ± 2% = $\boxed{125\text{ N/m}}$ ± 2%. In $F = kx$, the spring constant is $k = F/x$, the same as the slope of the F-versus-x graph.

(c) $F = kx = (125\text{ N/m})(0.105\text{ m}) = \boxed{13.1\text{ N}}$

7.69 (a) $\sum W = \Delta K$

$W_s + W_g = 0$

$\frac{1}{2}(1.40 \times 10^3 \text{ N/m}) \times (0.100 \text{ m})^2 - (0.200 \text{ kg})(9.80)(\sin 60.0°)x = 0$

$x = \boxed{4.12 \text{ m}}$

(b) $\sum W = \Delta K$

$W_s + W_g + W_f = 0$

$\frac{1}{2}(1.40 \times 10^3 \text{ N/m}) \times (0.100)^2 - [(0.200)(9.80)(\sin 60.0°)$

$+ (0.200)(9.80)(0.400)(\cos 60.0°)]x = 0$

$x = \boxed{3.35 \text{ m}}$

***7.70** (a) $W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2) = \frac{1}{2}(0.400 \text{ kg}) [(6.00)^2 - (8.00)^2] (\text{m/s})^2 = \boxed{-5.60 \text{ J}}$

(b) $W = fd \cos 180° = -\mu_k mg(2\pi r)$

$-5.60 \text{ J} = -\mu_k (0.400 \text{ kg})(9.80 \text{ m/s}^2) 2\pi (1.50 \text{ m})$

Thus, $\mu_k = \boxed{0.152}$

(c) After N revolutions, the mass comes to rest and $K_f = 0$. Thus,

$$W = \Delta K = 0 - K_i = -\frac{1}{2} m v_i^2 \quad \text{or} \quad -\mu_k mg[N(2\pi r)] = -\frac{1}{2} m v_i^2$$

This gives

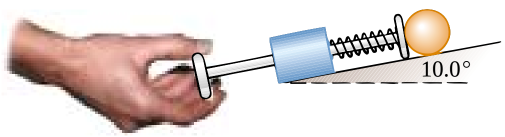
$$N = \frac{\frac{1}{2} m v_i^2}{\mu_k mg(2\pi r)} = \frac{\frac{1}{2}(8.00 \text{ m/s})^2}{(0.152)(9.80 \text{ m/s}^2) 2\pi (1.50 \text{ m})} = \boxed{2.28 \text{ rev}}$$

7.71 $\frac{1}{2}\left(1.20\ \frac{\text{N}}{\text{cm}}\right)(5.00\ \text{cm})(0.0500\ \text{m})$

$= (0.100\ \text{kg})(9.80\ \text{m/s}^2)(0.0500\ \text{m})\sin 10.0° + \frac{1}{2}(0.100\ \text{kg})\ v^2$

$0.150\ \text{J} = 8.51\times 10^{-3}\ \text{J} + (0.0500\ \text{kg})v^2$

$v = \sqrt{\frac{0.141}{0.0500}} = \boxed{1.68\ \text{m/s}}$



7.72 If positive F represents an outward force, (same direction as r), then

$$W = \int_i^f \mathbf{F}\cdot d\mathbf{s} = \int_{r_i}^{r_f}(2F_0\sigma^{13}r^{-13} - F_0\sigma^7 r^{-7})\,dr$$

$$W = \frac{+2F_0\sigma^{13}r^{-12}}{(-12)} - \frac{F_0\sigma^7 r^{-6}}{(-6)}\Bigg|_{r_i}^{r_f}$$

$$W = \frac{-F_0\sigma^{13}(r_f^{-12} - r_i^{-12})}{6} + \frac{F_0\sigma^7(r_f^{-6} - r_i^{-6})}{6} = \frac{F_0\sigma^7}{6}[r_f^{-6} - r_i^{-6}] - \frac{F_0\sigma^{13}}{6}[r_f^{-12} - r_i^{-12}]$$

$$W = 1.03\times 10^{-77}[r_f^{-6} - r_i^{-6}] - 1.89\times 10^{-134}[r_f^{-12} - r_i^{-12}]$$

$$W = 1.03\times 10^{-77}[1.88\times 10^{-6} - 2.44\times 10^{-6}]\ 10^{+60}$$

$$- 1.89\times 10^{-134}[3.54\times 10^{-12} - 5.96\times 10^{-8}]\ 10^{120}$$

$$W = -2.49\times 10^{-21}\ \text{J} + 1.12\times 10^{-21}\ \text{J} = \boxed{-1.37\times 10^{-21}\ \text{J}}$$

7.73 (a) $\sum W = \Delta K$

$$m_2 gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)(v^2 - v_i^2)$$

$$v = \sqrt{\frac{2gh(m_2 - \mu m_1)}{(m_1 + m_2)}}$$

$$= \sqrt{\frac{2(9.80)(20.0)[0.400 - (0.200)(0.250)]}{(0.400 + 0.250)}} = \boxed{14.5\ \text{m/s}}$$

(b) $W_f + W_g = \Delta K = 0$

$$-\mu(\Delta m_1 + m_1)gh + m_2gh = 0$$

$$\mu(\Delta m_1 + m_1) = m_2$$

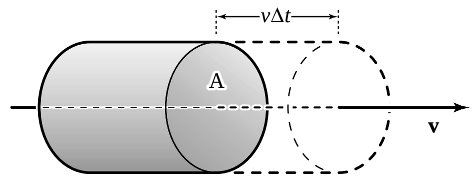
$$\Delta m_1 = \frac{m_2}{\mu} - m_1 = \frac{0.400\text{ kg}}{0.200} - 0.250\text{ kg} = \boxed{1.75\text{ kg}}$$

(c) $W_f + W_g = \Delta K = 0$

$$-\mu m_1gh + (m_2 - \Delta m_2)gh = 0$$

$$\Delta m_2 = m_2 - \mu m_1 = 0.400\text{ kg} - (0.200)(0.250\text{ kg}) = \boxed{0.350\text{ kg}}$$

7.74 $P\,\Delta t = W = \Delta K = \dfrac{(\Delta m)v^2}{2}$



The density is

$$\rho = \frac{\Delta m}{\text{vol}} = \frac{\Delta m}{A\,\Delta x}$$

Substituting this into the first equation and solving for P, since

$$\frac{\Delta x}{\Delta t} = v$$

for a constant speed, we get

$$\boxed{P = \frac{\rho A v^3}{2}}$$

Also, since $P = Fv$,

$$\boxed{F = \frac{\rho A v^2}{2}}$$

7.75 We evaluate $\displaystyle\int_{12.8}^{23.7} \frac{375dx}{x^3 + 3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3 + 3.75(12.8)} + \frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \ldots \frac{375(0.100)}{(23.6)^3 + 3.75(23.6)} = 0.806$$

and $$\frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \frac{375(0.100)}{(13.0)^3 + 3.75(13.0)} + \ldots \frac{375(0.100)}{(23.7)^3 + 3.75(23.7)} = 0.791$$

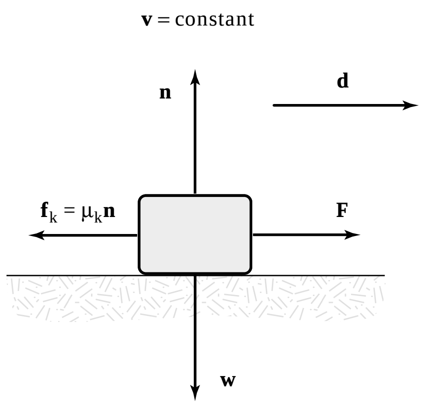
The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799\text{ N}\cdot\text{m}}$.

***7.76** (a) The suggested equation $P\,t = bwd$ implies all of the following cases:

(1) $P\,t = b\left(\frac{w}{2}\right)(2d)$ (2) $P\left(\frac{t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $P\left(\frac{t}{2}\right) = bw\left(\frac{d}{2}\right)$ and (4) $\left(\frac{P}{2}\right)t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.



(b) For one example, consider a horizontal force F pushing an object of weight w at constant velocity across a horizontal floor with which the object has coefficient of friction μ_k.

$\sum \mathbf{F} = m\mathbf{a}$ implies that:

$$+n - w = 0 \quad \text{and} \quad F - \mu_k n = 0$$

so that $F = \mu_k w$

As the object moves a distance d, the agent exerting the force does work

$$W = Fd\cos\theta = Fd\cos 0° = \mu_k wd \quad \text{and puts out power } P = W/t$$

This yields the equation $P\,t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.

Chapter 8 Solutions

***8.1** (a) With our choice for the zero level for potential energy at point B, $\boxed{U_B = 0}$.

At point A, the potential energy is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With

$$135 \text{ ft} = 41.1 \text{ m}$$

this height is found as:

$$y = (41.1 \text{ m}) \sin 40.0° = 26.4 \text{ m}$$

Thus,

$$U_A = (1000 \text{ kg})(9.80 \text{ m/s}^2)(26.4 \text{ m}) =$$

The change in potential energy as it moves from A to B is

$$U_B - U_A = 0 - 2.59 \times 10^5 \text{ J} =$$

(b) With our choice of the zero level at point A, we have $\boxed{U_A = 0}$.

The potential energy at B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number. Thus,

$$U_B = (1000 \text{ kg})(9.80 \text{ m/s}^2)(-26.5 \text{ m}) =$$

The change in potential energy in going from A to B is

$$U_B - U_A = -2.59 \times 10^5 \text{ J} - 0 =$$

***8.2** (a) We take the zero level of potential energy at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (40.0\text{ N})(2.00\text{ m}) = \boxed{80.0\text{ J}}$$

(b) From the sketch, we see that at an angle of 30.0° the ball is at a vertical height of (2.00 m)(1 – cos 30.0°) above the lowest point of the arc. Thus,

$$U_g = mgy = (40.0\text{ N})(2.00\text{ m})(1 - \cos 30.0°) = \boxed{10.7\text{ J}}$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

8.3 $F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = 39.2\text{ N}$

(a) Work along OAC = work along OA + work along AC

$= F_g(\text{OA}) \cos 90.0° + F_g(\text{AC}) \cos 180°$

$= (39.2\text{ N})(5.00\text{ m})(0) + (39.2\text{ N})(5.00\text{ m})(-1)$

$= \boxed{-196\text{ J}}$

(b) W along OBC = W along OB + W along BC

$= (39.2\text{ N})(5.00\text{ m}) \cos 180°$
$\quad + (39.2\text{ N})(5.00\text{ m}) \cos 90.0°$

$= \boxed{-196\text{ J}}$

(c) Work along OC = $F_g(\text{OC}) \cos 135°$

$$= (39.2\text{ N})(5.00 \times \sqrt{2}\text{ m})\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196\text{ J}}$$

The results should all be the same, since gravitational forces are conservative.

8.4 (a) $W =$ and if the force is constant, this can be written as

$$W = \mathbf{F} \cdot \int d\mathbf{s} =$$

(b) $W = \quad = \int (3\mathbf{i} + 4\mathbf{j}) \cdot (dx\,\mathbf{i} + dy\,\mathbf{j}) = (3.00\text{ N}) \int_0^{5.00\text{ m}} dx + (4.00\text{ N}) \int_0^{5.00\text{ m}} dy$

$$W = (3.00\text{ N})\, x \Big|_0^{5.00\text{ m}} + (4.00\text{ N}) y \Big|_0^{5.00\text{ m}} = 15.0\text{ J} + 20.0\text{ J} = \boxed{35.0\text{ J}}$$

The same calculation applies for all paths.

8.5 (a) $W_{OA} = \int_0^{5.00\text{ m}} dx\mathbf{i} \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int_0^{5.00\text{ m}} 2y\, dx$ and since along this path, $y = 0$

$W_{OA} = 0$

$W_{AC} = \int_0^{5.00\text{ m}} dy\,\mathbf{j} \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int_0^{5.00\text{ m}} x^2\, dy$. For $x = 5.00$ m, $W_{AC} = 125$ J

and $W_{OAC} = 0 + 125 = \boxed{125\text{ J}}$

(b) $W_{OB} = \quad \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int_0^{5.00\text{ m}} x^2\, dy$ since along this path, $x = 0$

$W_{OB} = 0$

$W_{BC} = \int_0^{5.00\text{ m}} dx\mathbf{i} \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int_0^{5.00\text{ m}} 2y\, dx$ since $y = 5.00$ m, $W_{BC} = \boxed{50.0\text{ J}}$

and $W_{OBC} = 0 + 50.0 = \boxed{50.0\text{ J}}$

(c) $W_{OC} = \int (dx\mathbf{i} + dy\mathbf{j}) \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int (2y\, dx + x^2\, dy)$

Since $x = y$ along OC,

$$W_{OC} = \int_0^{5.00\text{ m}} (2x + x^2)dx = \boxed{66.7\text{ J}}$$

(d) F is $\boxed{\text{non-conservative}}$ since the work done is path dependent.

8.6 (a) $U_f = K_i - K_f + U_i$

$U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\text{ J}}$

$\boxed{E = 40.0\text{ J}}$

(b) Yes, $\Delta E = \Delta K + \Delta U$; for conservative forces $\Delta K + \Delta U = 0$.

8.7 (a) $W = \vee F_x\, dx = \int_1^{5.00\text{ m}} (2x + 4)dx = \left(\frac{2x^2}{2} + 4x\right)_1^{5.00\text{ m}}$

$$= 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0\text{ J}}$$

(b) $\Delta K + \Delta U = 0$

$\Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \dfrac{mv_1^2}{2}$

$K_f = \Delta K + \dfrac{mv_1^2}{2} = \boxed{62.5\text{ J}}$

8.8 (a) $\mathbf{F} = (3.00\mathbf{i} + 5.00\mathbf{j})$ N $\qquad m = 4.00$ kg

$\mathbf{r} = (2.00\mathbf{i} - 3.00\mathbf{j})$ m

$W = 3.00(2.00) + 5.00(-3.00) = \boxed{-9.00\text{ J}}$

The result does not depend on the path since the force is conservative.

(b) $W = \Delta K$

$$-9.00 = \frac{4.00v^2}{2} - 4.00\left(\frac{(4.00)^2}{2}\right)$$

so $\quad v = \sqrt{\dfrac{32.0 - 9.00}{2.00}} = \boxed{3.39\text{ m/s}}$

(c) $\Delta U = -W = \boxed{9.00\text{ J}}$

8.9 (a) $U = -\int_0^x (-Ax + Bx^2)dx = \boxed{\dfrac{Ax^2}{2} - \dfrac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F\,dx = \dfrac{A[(3.00)^2 - (2.00)^2]}{2} - \dfrac{B[(3.00)^3 - (2.00)^3]}{3} = \boxed{\dfrac{5.00}{2}A - \dfrac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\dfrac{5.00}{2}A + \dfrac{19.0}{3}B\right)}$

8.10 (a) Energy is conserved between point P and the apex of the trajectory.

Since the horizontal component of velocity is constant,

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_{ix}^2 + \frac{1}{2}mv_{iy}^2 = \frac{1}{2}mv_{ix}^2 + mgh$$

$$v_{iy} = \qquad\qquad = \boxed{19.8 \text{ m/s}}$$

(b) $\Delta K \&_{P \varnothing B} = W_g = mg(60.0 \text{ m}) = (0.500 \text{ kg})(9.80 \text{ m/s}^2)(60.0 \text{ m}) = \boxed{294 \text{ J}}$

(c) Now let the final point be point B.

$v_{ix} = v_{fx} = 30.0 \text{ m/s}$

$$\Delta K \&_{P \varnothing B} = \frac{1}{2} m v_{fy}^2 - \frac{1}{2} m v_{iy}^2 = 294 \text{ J}$$

$$v_{fy}^2 = \frac{2}{m}(294) + v_{iy}^2 = 1176 + 392$$

$v_{fy} = -39.6 \text{ m/s}$

$\mathbf{v}_B = \boxed{(30.0 \text{ m/s})\mathbf{i} - (39.6 \text{ m/s})\mathbf{j}}$

8.11 $mgh = \frac{1}{2} kx^2$

$$(3.00 \text{ kg})(9.80 \text{ m/s}^2)(d + 0.200 \text{ m})\sin 30.0° = \frac{1}{2} 400(0.200 \text{ m})^2$$

$14.7d + 2.94 = 8.00$

$d = \boxed{0.344 \text{ m}}$

8.12 Choose the zero point of gravitational potential energy at the level where the mass comes to rest. Then because the incline is frictionless, we have

$$E_B = E_A \Rightarrow K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

or $0 + mg(d + x)\sin\theta + 0 = 0 + 0 + \frac{1}{2} kx^2$

Solving for d gives $d = \boxed{\frac{kx^2}{2mg\sin\theta} - x}$

8.13 (a) $(\Delta K)_{A\varnothing B} = \Sigma W = W_g = mg\Delta h = mg(5.00 - 3.20)$

$$\frac{1}{2}\ mv_B^2 - \frac{1}{2}\ mv_A^2 = m(9.80)(1.80)$$

$v_B = \boxed{5.94\ \text{m/s}}$

Similarly, $v_C = \sqrt{v_A^2 + 2g(5.00 - 2.00)} = \boxed{7.67\ \text{m/s}}$

(b) $W_g\&_{A\,\varnothing\,C} = mg(3.00\ \text{m}) = \boxed{147\ \text{J}}$

***8.14** $K_i + U_i + \Delta E = K_f + U_f$

$$0 + m(9.80\ \text{m/s}^2)(2.00\ \text{m} - 2.00\ \text{m}\cos 25.0°) = \frac{1}{2}\ mv_f^2 + 0$$

$v_f = \sqrt{(2)(9.80\ \text{m/s}^2)(0.187\ \text{m})} = \boxed{1.92\ \text{m/s}}$

8.15 $U_i + K_i = U_f + K_f$

$$mgh + 0 = mg(2R) + \frac{1}{2}\ mv^2$$

$$g(3.50\ R) = 2\ g(R) + \frac{1}{2}\ v^2$$

$$\boxed{v = \sqrt{3.00\ g\ R}}$$

$$\cdot F = m\frac{v^2}{R}$$

$$n + mg = m\frac{v^2}{R}$$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00\ g\ R}{R} - g\right]$$

$n = 2.00\ mg$

$n = 2.00\ (5.00 \infty 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)$

$n = \boxed{0.0980\ \text{N downward}}$

Goal Solution

G: Since the bead is released above the top of the loop, it will have enough potential energy to reach point A and still have excess kinetic energy. The energy of the bead at the top will be proportional to h and g. If it is moving relatively slowly, the track will exert an upward force on the bead, but if it is whipping around fast, the normal force will push it toward the center of the loop.

O: The speed at the top can be found from the conservation of energy, and the normal force can be found from Newton's second law.

A: We define the bottom of the loop as the zero level for the gravitational potential energy.

Since $v_i = 0$,

$$E_i = K_i + U_i = 0 + mgh = mg(3.50R)$$

The total energy of the bead at point A can be written as

$$E_A = K_A + U_A = \frac{1}{2}mv_A^2 + mg(2R)$$

Since mechanical energy is conserved, $E_i = E_A$, and we get

$$\frac{1}{2}mv_A^2 + mg(2R) = mg(3.50R)$$

$$v_A^2 = 3.00gR \quad \text{or} \quad v_A = \sqrt{3.00gR}$$

To find the normal force at the top, we may construct a free-body diagram as shown, where we assume that **n** is downward, like $m\mathbf{g}$. Newton's second law gives $F = ma_c$, where a_c is the centripetal acceleration.

$$n + mg = \frac{mv_A^2}{R} = \frac{m(3.00gR)}{R} = 3.00mg$$

$$n = 3.00mg - mg = 2.00mg$$

$n = 2.00(5.00 \infty 10^{-3}\text{ kg})(9.80\text{ m/s}^2) = 0.0980\text{ N downward}$

L: Our answer represents the speed at point A as proportional to the square root of the product of g and R, but we must not think that simply increasing the diameter of the loop will increase the speed of the bead at the top. In general, the speed will increase with increasing release height, which for this problem was defined in terms of the radius. The normal force may seem small, but it is twice the weight of the bead.

8.16 (a) At the equilibrium position for the mass, the tension in the spring equals the weight of the mass. Thus, elongation of the spring when the mass is at equilibrium is:

$$kx_o = mg \Rightarrow x_o = \frac{mg}{k} = \frac{(0.120)(9.80)}{40.0} = 0.0294\text{ m}$$

The mass moves with maximum speed as it passes through the equilibrium position. Use energy conservation, taking $U_g = 0$ at the initial position of the mass, to find this speed:

$$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$$

$$\frac{1}{2}mv_{\max}^2 + mg(-x_o) + \frac{1}{2}kx_0^2 = 0 + 0 + 0$$

$$v_{\max} = \sqrt{2gx_0 - \frac{kx_0^2}{m}} = \sqrt{2(9.80)(0.0294) - \frac{(40.0)(0.0294)^2}{0.120}} = \boxed{0.537\text{ m/s}}$$

(b) When the mass comes to rest, $K_f = 0$. Therefore,

$$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si} \quad \text{becomes}$$

$$0 + mg(-x) + \frac{1}{2}\,kx^2 = 0 + 0 + 0 \quad \text{which becomes}$$

$$x = \frac{2mg}{k} = 2x_0 = \boxed{0.0588\text{ m}}$$

8.17 From conservation of energy, $U_{gf} = U_{si}$, or

$(0.250\text{ kg})(9.80\text{ m/s}^2)h = (1/2)(5000\text{ N/m})(0.100\text{ m})^2$

This gives a maximum height $h = \boxed{10.2\text{ m}}$

***8.18** From leaving ground to highest point

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}\,m(6.00\text{ m/s})^2 + 0 = 0 + m(9.80\text{ m/s}^2)y$$

The mass makes no difference:

$$\therefore y = \frac{(6.00\text{ m/s})^2}{(2)(9.80\text{ m/s}^2)} = \boxed{1.84\text{ m}}$$

***8.19** (a) $\frac{1}{2}\,mv^2 = mgh$

$$v = \sqrt{2gh} = \boxed{19.8\text{ m/s}}$$

(b) $E = mgh = \boxed{78.4\text{ J}}$

(c) $K_{10} + U_{10} = 78.4\text{ J}$

$$K_{10} = 39.2\text{ J} \quad U_{10} = 39.2\text{ J} \quad \frac{K_{10}}{U_{10}} = \boxed{1.00}$$

8.20 Choose $y = 0$ at the river. Then $y_i = 36.0$ m, $y_f = 4.00$ m, the jumper falls 32.0 m, and the cord stretches 7.00 m. Between balloon and bottom,

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}\ kx_f^2$$

$$(700\text{ N})(36.0\text{ m}) = (700\text{ N})(4.00\text{ m}) + \frac{1}{2}\ k(7.00\text{ m})^2$$

$$k = \frac{22400\text{ J}}{24.5\text{ m}^2} = \boxed{914\text{ N/m}}$$

8.21 Using conservation of energy

(a) $(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00 + 3.00)\ v^2$

$$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$$

(b) $\frac{1}{2}(3.00)\ v^2 = mg\ \Delta y = 3.00g\ \Delta y$

$$\Delta y = 1.00\text{ m}$$

$$y_{\max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

8.22 $m_1 > m_2$

(a) $m_1gh = \frac{1}{2}(m_1 + m_2)\ v^2 + m_2gh$

$$v = \boxed{\sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}}}$$

(b) Since m_2 has kinetic energy $\frac{1}{2}\ m_2v^2$, it will rise an additional height Δh determined from

$$m_2g\ \Delta h = \frac{1}{2}\ m_2v^2$$

or from (a),

$$\Delta h = \frac{v^2}{2g} = \frac{(m_1 - m_2)h}{(m_1 + m_2)}$$

The total height m_2 reaches is $h + \Delta h = \boxed{\frac{2m_1 h}{m_1 + m_2}}$

8.23 (a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

But $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1000 \sin 37.0°)^2}{2(9.80)} =$$

and for the second

$$y_f = \frac{(1000)^2}{2(9.80)} =$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0 \text{ kg})(1000 \text{ m/s})^2 =$$

8.24 In the swing down to the breaking point, energy is conserved:

$$mgr \cos\theta = \frac{1}{2}mv^2$$

at the breaking point consider radial forces

$$\cdot F_r = ma_r$$

$$+T_{\max} - mg\cos\theta = m\frac{v^2}{r}$$

Eliminate $\dfrac{v^2}{r} = 2g\cos\theta$

$$T_{\max} - mg\cos\theta = 2\,mg\cos\theta$$

$$T_{\max} = 3\,mg\cos\theta$$

$$\theta = \text{Arc}\cos\left(\frac{T_{\max}}{3\,mg}\right) = \text{Arc}\cos\left(\frac{44.5 \text{ N}}{3(2.00 \text{ kg})(9.80 \text{ m/s}^2)}\right)$$

$$\theta = \boxed{40.8°}$$

***8.25** (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer.

From the free-body diagram for the performer's body, as shown,

$$F - mg\cos\theta = m\frac{v^2}{l}$$

or $$F = mg\cos\theta + m\frac{v^2}{l}$$

Apply conservation of mechanical energy between the starting point and any later point:

$$mg(l - l\cos\theta_i) = mg(l - l\cos\theta) + \frac{1}{2}mv^2$$

Solve for mv^2/l and substitute into the force equation to obtain

$$F = \boxed{mg(3\cos\theta - 2\cos\theta_i)}$$

(b) At the bottom of the swing, $\theta = 0°$ so $F = mg(3 - 2\cos\theta_i)$.

$F = 2mg = mg(3 - 2\cos\theta_i)$, which gives

$\theta_i = 60.0°$

***8.26** (a) At point 3, $\Sigma F_y = ma_y$ gives $n + mg = m\frac{v_3^2}{R}$.

For apparent weightlessness, $n = 0$. This gives

$$v_3 = \sqrt{Rg} = \sqrt{(20.0)(9.80)} = \boxed{14.0\text{ m/s}}$$

(b) Now, from conservation of energy applied between points 1 and 3,

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_3^2 + mgy_3$$

so $$v_1 = \sqrt{v_3^2 + 2g(y_3 - y_1)} = \sqrt{(14.0)^2 + 2(9.80)(40.0)} = \boxed{31.3\text{ m/s}}$$

(c) The total energy is the same at points 1 and 2:

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2, \text{ which yields}$$

$$v_2 = \sqrt{v_1^2 + 2g(y_1 - y_2)} = \sqrt{(31.3)^2 + 2(9.80)(-20.0)} = \boxed{24.2 \text{ m/s}}$$

(d) Between points 1 and 4:

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_4^2 + mgy_4, \text{ giving}$$

$$H = y_4 - y_1 = \frac{v_1^2 - v_4^2}{2g} = \frac{(31.3)^2 - (10.0)^2}{2(9.80)}$$

$= \boxed{44.9 \text{ m}}$

***8.27** The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_i^2 + 0 = 0 + mg(2L)$$

$$v_i = \sqrt{4gL} = \sqrt{4(9.80)(0.770)} = \boxed{5.49 \text{ m/s}}$$

***8.28** We shall take the zero level of gravitational potential energy to be at the lowest level reached by the diver under the water, and consider the energy change from when the diver started to fall until he came to rest.

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i = f_k\, d \cos 180–$$

$$0 - 0 - mg(y_i - y_f) = -f_k\, d$$

$$f_k = \frac{mg(y_i - y_f)}{d} = \frac{(70.0 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m} + 5.00 \text{ m})}{5.00 \text{ m}} = \boxed{2.06 \text{ kN}}$$

***8.29** $\frac{1}{2}mv^2 = \int_0^x F_x\,dx$ = area under the F_x vs x curve.

for $x = 2.00$ m $\qquad \int_0^{2.00} F_x\,dx = 10.0\ \text{N}\cdot\text{m}$

$$\therefore v\&_{x=2.00\ \text{m}} = \sqrt{\frac{2(10.0)}{5.00}} = \boxed{2.00\ \text{m/s}}$$

Similarly,

$$v\&_{x=4.00\ \text{m}} = \sqrt{\frac{2(19.5)}{5.00}} = \boxed{2.79\ \text{m/s}}$$

$$v\&_{x=6.00\ \text{m}} = \sqrt{\frac{2(25.5)}{5.00}} = \boxed{3.19\ \text{m/s}}$$

***8.30** The distance traveled by the ball from the top of the arc to the bottom is $s = \pi r$. The work done by the non-conservative force, the force exerted by the pitcher, is $\Delta E = Fs\cos 0° = F(\pi R)$.

We shall choose the gravitational potential energy to be zero at the bottom of the arc. Then

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i \text{ becomes}$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R)$$

or $$v_f = \sqrt{v_i^2 + 2gy_i + \frac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \frac{2(30.)\pi(0.600)}{0.250}}$$

$$v_f = \boxed{26.5\ \text{m/s}}$$

8.31 $U_i + K_i + \Delta E = U_f + K_f$

$$m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1gh = \frac{1}{2}(m_1 + m_2)\,v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\ \text{m/s}^2)(1.50\ \text{m})[5.00\ \text{kg} - 0.400(3.00\ \text{kg})]}{8.00\ \text{kg}}} = \boxed{3.74\ \text{m/s}}$$

Goal Solution

G: Assuming that the block does not reach the pulley within the 1.50 m distance, a reasonable speed for the ball might be somewhere between 1 and 10 m/s based on common experience.

O: We could solve this problem by using $\Sigma F = ma$ to give a pair of simultaneous equations in the unknown acceleration and tension; then we would have to solve a motion problem to find the final speed. We may find it easier to solve using the work-energy theorem.

A: For objects A (block) and B (ball), the work-energy theorem is

$$(K_A + K_B + U_A + U_B)_i + W_{\text{app}} - f_k d = (K_A + K_B + U_A + U_B)_f$$

Choose the initial point before release and the final point after each block has moved 1.50 m. For the 3.00-kg block, choose $U_g = 0$ at the tabletop. For the 5.00-kg ball, take the zero level of gravitational energy at the final position. So $K_{Ai} = K_{Bi} = U_{Ai} = U_{Af} = U_{Bf} = 0$. Also, since the only external forces are gravity and friction, $W_{app} = 0$.

We now have $0 + 0 + 0 + m_B g y_{Bi} - f_1 d = \frac{1}{2} m_A v_f^2 + \frac{1}{2} m_B v_f^2 + 0 + 0$

where the frictional force is $f_1 = \mu_1 n = \mu_1 m_A g$ and does negative work since the force opposes the motion. Since all of the variables are known except for v_f, we can substitute and solve for the final speed.

$$(5.00\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m}) - (0.400)(3.00\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m})$$

$$= \frac{1}{2}(3.00\text{ kg})\, v_f^2 + \frac{1}{2}(5.00\text{ kg})\, v_f^2$$

$$73.5\text{ J} - 17.6\text{ J} = \frac{1}{2}(8.00\text{ kg})\, v_f^2 \quad \text{or} \quad v_f = \sqrt{\frac{2(55.9\text{ J})}{8.00\text{ kg}}} = 3.74\text{ m/s}$$

L: The final speed seems reasonable based on our expectation. This speed must also be less than if the rope were cut and the ball simply fell, in which case its final speed would be

$$v_f' = \sqrt{2gy} = \sqrt{2(9.80\text{ m/s}^2)(1.50\text{ m})} = 5.42\text{ m/s}$$

***8.32** The initial vertical height of the car above the zero reference level at the base of the hill is

$$h = (5.00 \text{ m}) \sin 20.0° = 1.71 \text{ m}$$

The energy lost through friction is

$$\Delta E = -fs = -(4000 \text{ N})(5.00 \text{ m}) = -2.00 \infty 10^4 \text{ J}$$

We now use,

$$\Delta E = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 + mgy_f - mgy_i$$

$$-2.00 \times 10^4 \text{ J} = \frac{1}{2}(2000 \text{ kg}) v^2 - 0 + 0 - (2000 \text{ kg})g(1.71 \text{ m})$$

and $v = \boxed{3.68 \text{ m/s}}$

8.33 (a) $\Delta K = \frac{1}{2} m(v^2 - v_i^2) = -\frac{1}{2} mv_i^2 = \boxed{-160 \text{ J}}$

(b) $\Delta U = mg(3.00 \text{ m}) \sin 30.0° = \boxed{73.5 \text{ J}}$

(c) The energy lost to friction is 86.5 J

$$f = \frac{86.5 \text{ J}}{3.00 \text{ m}} = \boxed{28.8 \text{ N}}$$

(d) $f = \mu_k n = \mu_K mg \cos 30.0° = 28.8 \text{ N}$

$$\mu = \frac{28.8 \text{ N}}{(5.00 \text{ kg})(9.80 \text{ m/s}^2) \cos 30.0°} = \boxed{0.679}$$

***8.34** $\Delta E = (K_f - K_i) + (U_{gf} - U_{gi})$

But $\Delta E = W_{\text{app}} + fs \cos 180°$ where W_{app} is the work the boy did pushing forward on the wheels.

Thus, $W_{\text{app}} = (K_f - K_i) + (U_{gf} - U_{gi}) + fs$, or

$$W_{\text{app}} = \frac{1}{2} m(v_f^2 - v_i^2) + mg(-h) + fs$$

$$W_{\text{app}} = \frac{1}{2}(47.0) [(6.20)^2 - (1.40)^2] - (47.0)(9.80)(2.60) + (41.0)(12.4)$$

$$W_{\text{app}} = \boxed{168 \text{ J}}$$

8.35 $\Delta E = mgh_i - \frac{1}{2}\ mv_f^2$

$= (50.0)(9.80)(1000) - \frac{1}{2}(50.0)(5.00)\ ^2$

$\Delta E = \boxed{489\ \text{kJ}}$

8.36 Consider the whole motion: $K_i + U_i + \Delta E = K_f + U_f$

(a) $0 + mgy_i + f_1 d_1 \cos 180° + f_2 d_2 \cos 180° = \frac{1}{2}\ mv_f^2\ + 0$

$(80.0\ \text{kg})(9.80\ \text{m/s}^2)1000\ \text{m} - (50.0\ \text{N})(800\ \text{m}) - (3600\ \text{N})(200\ \text{m}) = \frac{1}{2}(80.0\ \text{kg})\ \ v_f^2$

$784{,}000\ \text{J} - 40{,}000\ \text{J} - 720{,}000\ \text{J} = \frac{1}{2}(80.0\ \text{kg})\ \ v_f^2$

$v_f = \sqrt{\frac{2(24{,}000\ \text{J})}{80.0\ \text{kg}}} = \boxed{24.5\ \text{m/s}}$

(b) $\boxed{\text{Yes}}$, this is too fast for safety.

(c) Now in the same work-energy equation d_2 is unknown, and $d_1 = 1000\ \text{m} - d_2$:

$784{,}000\ \text{J} - (50.0\ \text{N})(1000\ \text{m} - d_2) - (3600\ \text{N})d_2 = \frac{1}{2}(80.0\ \text{kg})(5.00\ \text{m/s})\ ^2$

$784{,}000\ \text{J} - 50{,}000\ \text{J} - (3550\ \text{N})d_2 = 1000\ \text{J}$

$d_2 = \frac{733{,}000\ \text{J}}{3550\ \text{N}} = \boxed{206\ \text{m}}$

(d) Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

***8.37** (a) $(K + U)_i + \Delta E = (K + U)_f$

$0 + \frac{1}{2}\ kx^2 - fd = \frac{1}{2}\ mv^2 + 0$

$^2 - (3.20 \times 10^{-2}\ \text{N})(0.150\ \text{m}) = \qquad v^2$

$v = \qquad = \boxed{1.40\ \text{m/s}}$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|F_s| = kx$, the spring is compressed by

$$= 0.400 \text{ cm}$$

and the ball has moved 5.00 cm – 0.400 cm = $\boxed{\text{4.60 cm from the start.}}$

(c) Between start and maximum speed points,

$$\frac{1}{2} kx_i^2 - fx = \frac{1}{2} mv^2 + \frac{1}{2} kx_f^2$$

$$\frac{1}{2} 8.00(5.00 \times 10^{-2})^2 - (3.20 \times 10^{-2})(4.60 \times 10^{-2})$$

$$= v^2 + \frac{1}{2} 8.00(4.00 \times 10^{-3})^2$$

$$v = \boxed{1.79 \text{ m/s}}$$

8.38 (a) The mass moves down distance 1.20 m + x. Choose $y = 0$ at its lower point.

$$K_i + U_{gi} + U_{si} + \Delta E = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 + 0 = 0 + 0 + \frac{1}{2}\ kx^2$$

$$(1.50 \text{ kg})9.80 \text{ m/s}^2\ (1.20 \text{ m} + x) = \frac{1}{2}(320 \text{ N/m})\ x^2$$

$$0 = (160 \text{ N/m})x^2 - (14.7 \text{ N})x - 17.6 \text{ J}$$

$$x = \frac{14.7 \text{ N} \pm \sqrt{(-14.7 \text{ N})^2 - 4(160 \text{ N/m})(-17.6 \text{ N} \cdot \text{m})}}{2(160 \text{ N/m})}$$

$$x = \frac{14.7 \text{ N} \pm 107 \text{ N}}{320 \text{ N/m}}$$

The negative root tells how high the mass will rebound if it is instantly glued to the spring. We want

$$x = \boxed{0.381 \text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})1.63\text{ m/s}^2\,(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})\,x^2$$

$$0 = 160x^2 - 2.44x - 2.93$$

The positive root is $x = \boxed{0.143\text{ m}}$

(c) The full work-energy theorem has one more term:

$$mgy_i + fy_i\cos 180° = \frac{1}{2}kx^2$$

$$(1.50\text{ kg})\,9.80\text{ m/s}^2\,(1.20\text{ m} + x) - 0.700\text{ N}(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})\,x^2$$

$$17.6\text{ J} + 14.7\text{ N}\,x - 0.840\text{ J} - 0.700\text{ N}\,x = 160\text{ N/m}\,x^2$$

$$160x^2 - 14.0x - 16.8 = 0$$

$$x = \frac{14.0 \pm \sqrt{(14.0)^2 - 4(160)(-16.8)}}{320}$$

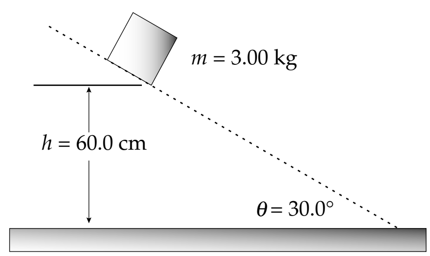
$$x = \boxed{0.371\text{ m}}$$

8.39 Choose $U_g = 0$ at the level of the horizontal surface.

Then $\Delta E = (K_f - K_i) + (U_{gf} - U_{gi})$ becomes:

$$-f_1 s - f_2 x = (0 - 0) + (0 - mgh)$$

or $$-(\mu_k mg\cos 30.0°)\left(\frac{h}{\sin 30.0°}\right) - (\mu_k mg)x = -mgh$$



Thus, the distance the block slides across the horizontal surface before stopping is:

$$x = \frac{h}{\mu_k} - h\cot 30.0° = h\left(\frac{1}{\mu_k} - \cot 30.0°\right) = (0.600\text{ m})\left(\frac{1}{0.200} - \cot 30.0°\right)$$

or $x = \boxed{1.96\text{ m}}$

***8.40** The total mechanical energy of the diver is $E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh$. Since the diver has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv$$

The rate he is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\text{ kg})(9.80\text{ m/s}^2)(60.0\text{ m/s}) = \boxed{44.1\text{ kW}}$$

8.41 $U(r) = \frac{A}{r}$

$$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$$

8.42 $F_x = -\frac{fU}{fx} = -\frac{f(3x^3y - 7x}{fx} = -(9x^2y - 7) = 7 - 9x^2y$

$$F_y = -\frac{fU}{fy} = -\frac{f(3x^3y - 7x}{fy} = -(3x^3 - 0) = -3x^3$$

Thus, the force acting at the point (x, y) is

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} = \boxed{(7 - 9x^2y)\mathbf{i} - 3x^3\,\mathbf{j}}$$

***8.43** (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

At r = 1.5 mm and 3.2 mm, the equilibrium is stable. At r = 2.3 mm, the equilibrium is unstable. A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The particle energy cannot be less than –5.6 J. The particle is bound if $\boxed{-5.6\text{ J} \le E < 1\text{ J}}$.

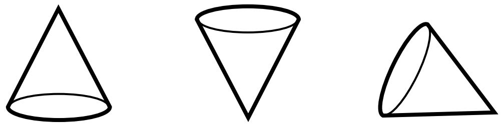
(c) If the particle energy is –3 J, its potential energy must be less than or equal to –3 J. Thus, its position is limited to $\boxed{0.6\text{ mm} \le r \le 3.6\text{ mm}}$.

(d) $K + U = E$. Thus, $K_{\text{max}} = E - U_{\text{min}} = -3.0\text{ J} - (-5.6\text{ J}) = \boxed{2.6\text{ J}}$

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\text{ mm}}$.

(f) $-3\text{ J} + W = 1\text{ J}$. Hence, the binding energy is $W = \boxed{4\text{ J}}$.

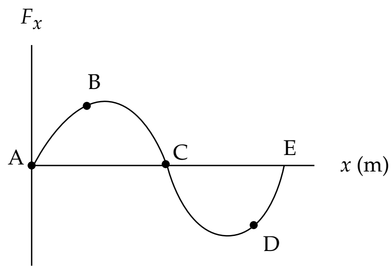
***8.44**



8.45 (a) F_x is zero at points A, C and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

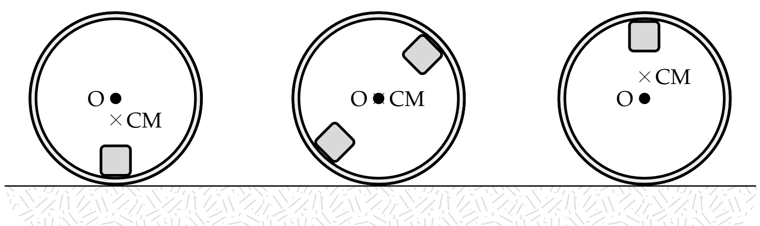
(c)



8.46 (a) As the pipe is rotated, the CM rises, so this is $\boxed{\textit{stable}}$ equilibrium.

(b) As the pipe is rotated, the CM moves horizontally, so this is $\boxed{\textit{neutral}}$ equilibrium.

(c) As the pipe is rotated, the CM falls, so this is $\boxed{\textit{unstable}}$ equilibrium.



8.47 (a) When the mass moves distance x, the length of each spring changes from L to $\sqrt{x^2 + L^2}$, so each exerts force $k(\sqrt{x^2 + L^2} - L)$ toward its fixed end. The y-components cancel out and the x-components add to:

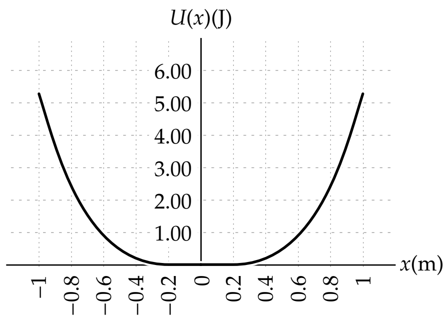
$$F_x = -2k(\sqrt{x^2 + L^2} - L)\left(\frac{x}{\sqrt{x^2 + L^2}}\right) = -2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}$$

Choose $U = 0$ at $x = 0$. Then at any point

$$U(x) = -\int_0^x F_x dx = -\int_0^x \left(-2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}\right) dx = 2k\int_0^x x dx - 2kL\int_0^x \frac{x}{\sqrt{x^2 + L^2}}\, dx$$

$$U(x) = \boxed{kx^2 + 2kL(L - \sqrt{x^2 + L^2})}$$

(b) $U(x) = 40.0x^2 + 96.0(1.20 - \sqrt{x^2 + 1.44}\,)$



x, m	0	0.200	0.400	0.600	0.800	1.00	1.50	2.00	2.50
U, J	0	0.011	0.168	0.802	2.35	5.24	20.8	51.3	99.0

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x = 0}$.

(c) $K_i + U_i + \Delta E = K_f + U_f$

$$0 + 0.400 \text{ J} + 0 = \frac{1}{2}\, m v_f^2 + 0$$

$$v_f = \boxed{\sqrt{\frac{0.800 \text{ J}}{m}}}$$

8.48 (a) $E = mc^2 = (9.11\ \chi\ 10^{-31}\text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = \boxed{8.19 \times 10^{-14}\text{ J}}$

(b) $\boxed{3.60 \times 10^{-8}\text{ J}}$

(c) $\boxed{1.80 \times 10^{14}\text{ J}}$

(d) $\boxed{5.38 \times 10^{41}\text{ J}}$

8.49 (a) Rest energy $= mc^2 = (1.673 \infty 10^{-27}\text{ kg})(2.998 \infty 10^8 \text{ m/s})^2 = \boxed{1.50 \times 10^{-10}\text{ J}}$

(b) $E = \gamma mc^2 = \dfrac{mc^2}{\sqrt{1 - (v/c)^2}} = \dfrac{1.50 \times 10^{-10}\text{ J}}{\sqrt{1 - (.990)^2}} = \boxed{1.07 \times 10^{-9}\text{ J}}$

(c) $K = \gamma mc^2 - mc^2 = 1.07 \times 10^{-9}\text{ J} - 1.50 \times 10^{-10}\text{ J} = \boxed{9.15 \times 10^{-10}\text{ J}}$

8.50 The potential energy of the block is mgh.

An amount of energy $\mu_k mgs \cos\theta$ is lost to friction on the incline.

Therefore the final height $y_{\max}$ is found from

$$mgy_{\max} = mgh - \mu_k mgs \cos\theta$$

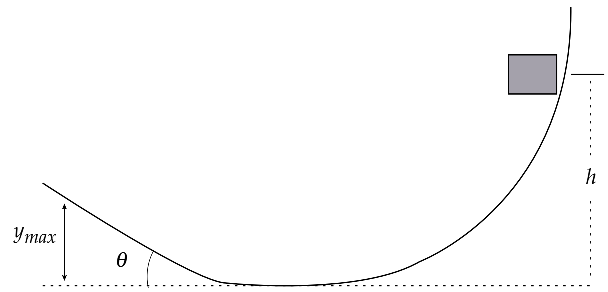
where

$$s = \frac{y_{\max}}{\sin\theta}$$

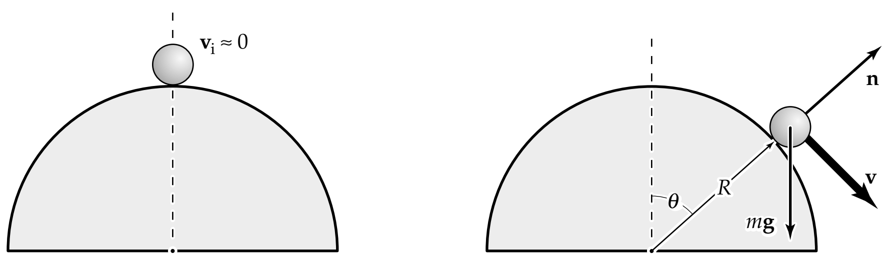
$$\therefore\ mgy_{\max} = mgh - \mu_k mgy_{\max} \cot\theta$$

Solving,

$$y_{\max} = \frac{h}{1 + \mu_k \cot\theta}$$



***8.51** m = mass of pumpkin
R = radius of silo top



$$\Sigma F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$

When the pumpkin is ready to lose contact with the surface, $n = 0$. Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

Choose $U_g = 0$ in the $\theta = 90.0°$plane. Then applying conservation of energy from the starting point to the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

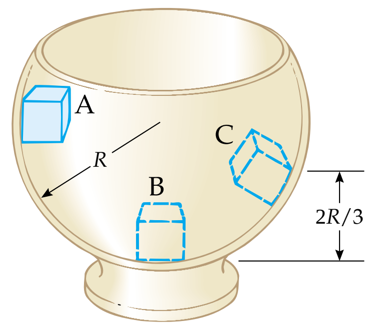
Using the result from the force analysis, this becomes

$$\frac{1}{2}\, mRg \cos\theta + mgR \cos\theta = mgR, \text{ which reduces to}$$

$$\cos\theta = \frac{2}{3}\text{ , and gives } \theta = \cos^{-1}(2/3) = \boxed{48.2\Upsilon}$$

as the angle at which the pumpkin will lose contact with the surface.

8.52 (a) $U_A = mgR = (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0.300 \text{ m}) =$ $\boxed{0.588 \text{ J}}$



(b) $K_A + U_A = K_B + U_B$

$$K_B = K_A + U_A - U_B = mgR = \boxed{0.588 \text{ J}}$$

(c) $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2(0.588 \text{ J})}{0.200 \text{ kg}}} = \boxed{2.42 \text{ m/s}}$

(d) $U_C = mgh_C = (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0.200 \text{ m}) =$ $\boxed{0.392 \text{ J}}$

$$K_C = K_A + U_A - U_C = mg(h_A - h_C)$$

$$K_C = (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0.300 - 0.200) \text{ m} = \boxed{0.196 \text{ J}}$$

8.53 (a) $K_B = \frac{1}{2} mv_B^2 = \frac{1}{2}(0.200 \text{ kg})(1.50 \text{ m/s})^2 = \boxed{0.225 \text{ J}}$

(b) $\Delta E = \Delta K + \Delta U = K_B - K_A + U_B - U_A$

$$= K_B + mg(h_B - h_A)$$

$$= 0.225 \text{ J} + (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0 - 0.300 \text{ m})$$

$$= 0.225 \text{ J} - 0.588 \text{ J} = \boxed{-0.363 \text{ J}}$$

(c) It's possible to find an effective coefficient of friction, but not the actual value of μ since n and f vary with position.

***8.54** $v = 100 \text{ km/h} = 27.8 \text{ m/s}$

The retarding force due to air resistance is

$$R = \frac{1}{2} D\rho A v^2 = \frac{1}{2}(0.330)(1.20 \text{ kg/m}^3)(2.50 \text{ m}^2)(27.8 \text{ m/s})^2 = 382 \text{ N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or $$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where ΔW_e is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + (K_f - K_i) + (U_{gf} - U_{gi})$$

Recognizing that $K_f = K_i$ and dividing by the travel time Δt gives the required power input from the engine as

$$P = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv \sin\theta$$

$$P = (382 \text{ N})(27.8 \text{ m/s}) + (1500 \text{ kg})(9.80 \text{ m/s}^2)(27.8 \text{ m/s})\sin 3.20°$$

$$P = \boxed{33.4 \text{ kW} = 44.8 \text{ hp}}$$

***8.55** At a pace I could keep up for a half-hour exercise period, I climb two stories up, forty steps each 18 cm high, in 20 s. My output work becomes my final gravitational energy,

$$mgy = 85 \text{ kg}(9.80 \text{ m/s}^2)(40 \times 0.18 \text{ m}) = 6000 \text{ J}$$

making my sustainable power

$$\frac{6000 \text{ J}}{20 \text{ s}} = \boxed{\sim 10^2 \text{ W}}$$

8.56 $k = 2.50 \times 10^4$ N/m $\quad m = 25.0$ kg $\quad x_A = -0.100$ m $\quad U_g|_{x=0} = U_s|_{x=0} = 0$

(a) $$E = K_A + U_{gA} + U_{sA} = 0 + mgx_A + \frac{1}{2}kx_A^2$$

$$E = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(-0.100 \text{ m}) + \frac{1}{2}(2.50 \leftarrow 10^4 \text{ N}/\mu)(0.100\mu)^2$$

$$E = -24.5 \text{ J} + 125 \text{ J} = \boxed{100 \text{ J}}$$

(b) Since only conservative forces are involved, the total energy at point C is the same as that at point A.

$$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$$

$$0 + (25.0 \text{ kg})(9.80 \text{ m/s}^2)x_C + 0 = 0 + -24.5 \text{ J} + 125 \text{ J} \Rightarrow x_C = \boxed{0.410 \text{ J}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$

$$\frac{1}{2}(25.0\text{ kg})\,v_B^2 + 0 + 0 = 0 + -24.5\text{ J} + 125\text{ J} \Rightarrow v_B = \boxed{2.84\text{ m/s}}$$

(d) K and v are at a maximum when $a = \dfrac{\Sigma F}{m} = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force). This occurs at $x < 0$ where

$$k|x| = mg \quad \text{or} \quad |x| = \frac{(25.0kg)(9.80m/s^2)}{2.50 \times 10^4\text{ N}/m} = 9.80 \times 10^{-3}\text{ m}$$

Thus, $K = K_{\max}$ at $x = \boxed{-9.80\text{ mm}}$

(e) $K_{\max} = K_A + (U_{gA} - U_g|_{x=-9.80\text{ mm}}) + (U_{sA} - U_s|_{x=-9.80\text{ mm}})$, or

$$\frac{1}{2}(25.0\text{ kg})\,v_{\max}^2 = (25.0\text{ kg})(9.80\text{ m/s}^2)[(-0.100\text{ m}) - (-0.0098\text{ m})]$$

$$+\frac{1}{2}(2.50 \times 10^4\text{ N/m})\,[(-0.100\text{ m})^2 - (-0.0098\text{ m})^2]$$

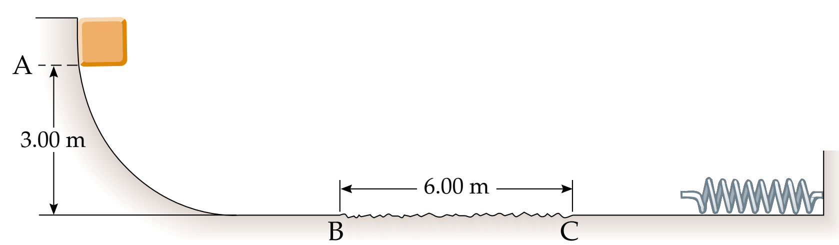
yielding $v_{\max} = \boxed{2.85\text{ m/s}}$

8.57 $\Delta E = W_f$

$E_f - E_i = -f \cdot d_{BC}$

$$\frac{1}{2}\,k\,\Delta x^2 - mgh = -\,\mu mgd$$

$$\mu = \frac{mgh - \frac{1}{2}k \cdot \Delta x^2}{mgd} = \boxed{0.328}$$



Goal Solution

G: We should expect the coefficient of friction to be somewhere between 0 and 1 since this is the range of typical μ_k values. It is possible that μ_k could be greater than 1, but it can never be less than 0.

O: The easiest way to solve this problem is by considering the energy conversion for each part of the motion: gravitational potential to kinetic energy from A to B, loss of kinetic energy due to friction from B to C, and kinetic to elastic potential energy as the block compresses the spring. Choose the gravitational energy to be zero along the flat portion of the track.

A: Putting the energy equation into symbols: $U_{gA} - \left| W \right|_{BC} = U_{sf}$

expanding into specific variables: $mgy_A - f_1 d_{BC} = \frac{1}{2}\, kx_s^2$ where $f_1 = \mu_1 mg$

solving for the unknown variable: $\mu_1 mgd = mgy - \frac{1}{2}\, kx^2$ or $\mu_1 = \frac{y}{d} - \frac{kx^2}{2mgd}$

substituting: $\mu_1 = \frac{3.00 \text{ m}}{6.00 \text{ m}} - \frac{(2250 \text{ N/m})(0.300 \text{ m})^2}{2(10.0 \text{ kg})(9.80 \text{ m/s}^2)(6.00 \text{ m})} = 0.328$

L: Our calculated value seems reasonable based on the friction data in Table 5.2. The most important aspect to solving these energy problems is considering how the energy is transferred from the initial to final energy states and remembering to subtract the energy resulting from any non-conservative forces (like friction).

8.58 The nonconservative work (due to friction) must equal the change in the kinetic energy plus the change in the potential energy.

Therefore,

$$-\mu_k mgx \cos\theta = \Delta K + \frac{1}{2}\, kx^2 - mgx \sin\theta$$

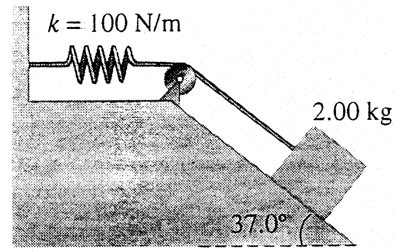
and since $v_i = v_f = 0$, $\Delta K = 0$.

Thus,

$$-\mu_k(2.00)(9.80)(\cos 37.0°)(0.200) = \frac{(100)(0.200)^2}{2} - (2.00)(9.80)(\sin 37.0°)(0.200)$$

and we find $\mu_k = \boxed{0.115}$. Note that in the above we had a *gain* in elastic potential energy for the spring and a *loss* in gravitational potential energy. The net loss in mechanical energy is equal to the energy lost due to friction.

8.59 (a) Since no nonconservative work is done, $\Delta E = 0$



Also $\Delta K = 0$

therefore, $U_i = U_f$

where $U_i = (mg \sin \theta)x$

and $U_f = \frac{1}{2} kx^2$

Substituting values yields $(2.00)(9.80) \sin 37.0° = (100)\frac{x}{2}$ and solving we find

$x = \boxed{0.236 \text{ m}}$

(b) $\Sigma F = ma$. Only gravity and the spring force act on the block, so

$-kx + mg \sin \theta = ma$

For $x = 0.236$ m,

$a = \boxed{-5.90 \text{ m/s}^2}$ The negative sign indicates a is up the incline.

The $\boxed{\text{acceleration depends on position}}$.

(c) U(gravity) decreases monotonically as the height decreases.

U(spring) increases monotonically as the spring is stretched.

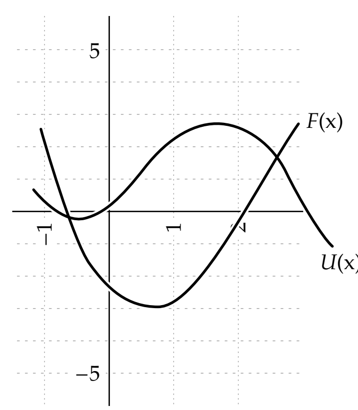
K initially increases, but then goes back to zero.

***8.60** (a) $\boxed{\mathbf{F} = -\frac{d}{dx}(-x^3 + 2x^2 + 3x)\mathbf{i} = (3x^2 - 4x - 3)\mathbf{i}}$

(b) $F = 0$ when $x = \boxed{1.87 \text{ and } -0.535}$

(c) The stable point is at $x = -0.535$ point of minimum $U(x)$

The unstable point is at $x = 1.87$ maximum in $U(x)$



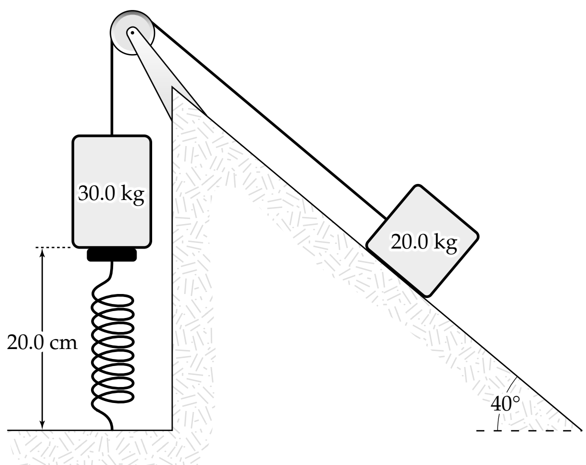
8.61 $(K+U)_i = (K+U)_f$

$$0 + (30.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) + \frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$

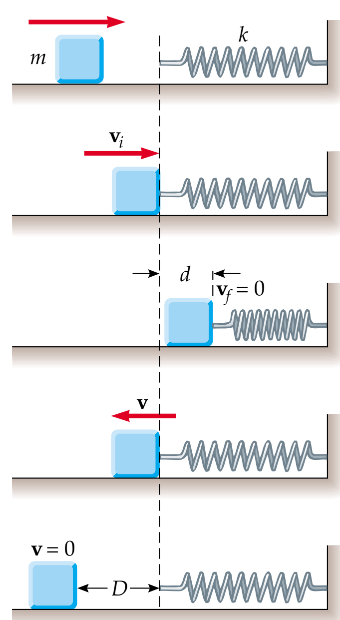
$$= \frac{1}{2}(50.0\text{ kg})\,v^2 + (20.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J} + 5.00\text{ J} = (25.0\text{ kg})v^2 + 25.2\text{ J}$$

$$\boxed{v = 1.24\text{ m/s}}$$



8.62 (a) Between the second and the third picture, $\Delta E = \Delta K + \Delta U$



$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0\text{ N/m})\,d^2 + 0.250(1.00\text{ kg})(9.80\text{ m/s}^2)d - \frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s}^2) = 0$$

$$d = \frac{[-2.45 \pm 21.35]\text{ N}}{50.0\text{ N/m}} = \boxed{0.378\text{ m}}$$

(b) Between picture two and picture four, $\Delta E = \Delta K + \Delta U$

$$-f(2d) = -\frac{1}{2}mv^2 + \frac{1}{2}mv_i^2$$

$$v = \sqrt{(3.00\text{ m/s})^2 - \frac{2}{(1.00\text{ kg})}(2.45\text{ N})(2)(0.378\text{ m})}$$

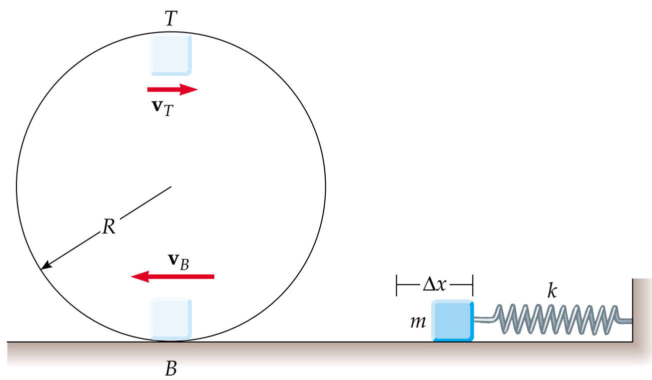
$$= \boxed{2.30\text{ m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E = \Delta K + \Delta U$

$$-f(D + 2d) = -\frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s})^2$$

$$D = \frac{9.00\text{ J}}{2(0.250)(1.00\text{ kg})(9.80\text{ m/s}^2)} - 2(0.378\text{ m}) = \boxed{1.08\text{ m}}$$

8.63 (a)



Initial compression of spring: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$\frac{1}{2}(450\text{ N/m})(\Delta x)^2 = \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$

$$\therefore\ \Delta x = \boxed{0.400\text{ m}}$$

(b) Speed of block at top of track:

$$\Delta E = W_f$$

$$\left(mgh_T + \frac{1}{2}mv_T^2\right) - \left(mgh_B + \frac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) + \frac{1}{2}(0.500\text{ kg})\,v_T^2 - \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$

$$= -(7.00\text{ N})(\pi)(1.00\text{ m})$$

$$0.250v_T^2 = 4.21 \qquad \therefore\ v_T = \boxed{4.10\text{ m/s}}$$

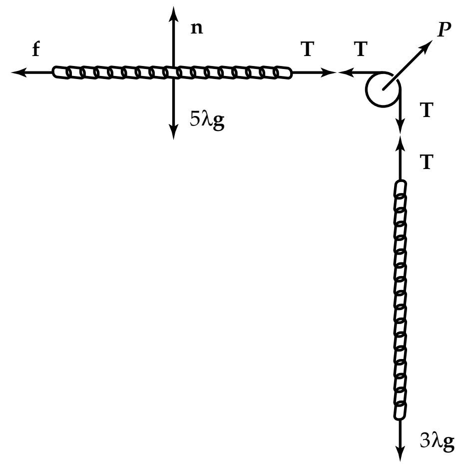
(c) Does block fall off at or before top of track?

Block falls if $a_r < g$

$$a_r = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8\text{ m/s}^2$$

therefore $a_r > g$ and the $\boxed{\text{block stays on the track}}$.

8.64 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless pulley.



(a) For the five meters on the table with motion impending,

$\sum F_y = 0 \qquad +n - 5\lambda g = 0$

$n = 5\lambda g \qquad f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$

$\sum F_x = 0 \qquad +T - f_s = 0 \qquad T = f_s \qquad T \le 3\lambda g$

The maximum value is barely enough to support the hanging segment according to

$\sum F_y = 0 \qquad +T - 3\lambda g = 0 \qquad T = 3\lambda g$

so it is at this point that the chain starts to slide.

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5 - x)$ remains on the table, with now

$\sum F_y = 0 \qquad +n - (5 - x)\lambda g = 0 \qquad n = (5 - x)\lambda g$

$f_k = \mu_k n = 0.4(5 - x)\lambda g = 2\lambda g - 0.4x\lambda g$

Consider energies at the initial moment when the chain starts to slip, and a final moment when $x = 5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8 - \frac{3}{2} = 6.5$ m.

$$K_i + U_i + \Delta E = K_f + U_f$$

$$0 + (m_1 g y_1 + m_2 g y_2)_i + \int_i^f f_k\, dx \cos\theta = \left(\frac{1}{2}mv^2 + mgy\right)_f$$

$$(5\lambda g)8 + (3\lambda g)6.5 + \int_0^5 (2\lambda g - 0.4x\lambda g)\ dx \cos 180°$$

$$= \frac{1}{2}(8\lambda)\, v^2 + (8\lambda g)4$$

$$40.0\,g + 19.5\,g - 2.00\,g\int_0^5 dx + 0.400\,g\int_0^5 x\,dx = 4.00v^2 + 32.0\,g$$

$$27.5\,g - 2.00\,gx\,\Big|_0^5 + 0.400\,g\,\frac{x^2}{2}\Big|_0^5 = 4.00v^2$$

$$27.5\,g - 2.00\,g(5.00) + 0.400\,g(12.5) = 4.00v^2$$

$$22.5\,g = 4.00v^2$$

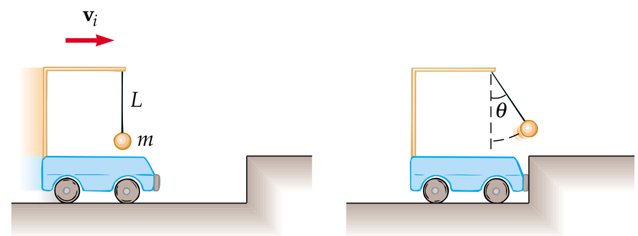
$$v = \sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

8.65 (a) On the upward swing of the mass:

$$K_i + U_\text{i} + \Delta E = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + 0 + 0 = 0 + mgL(1 - \cos\theta)$$

$$v_i = \sqrt{2gL(1 - \cos\theta)}$$

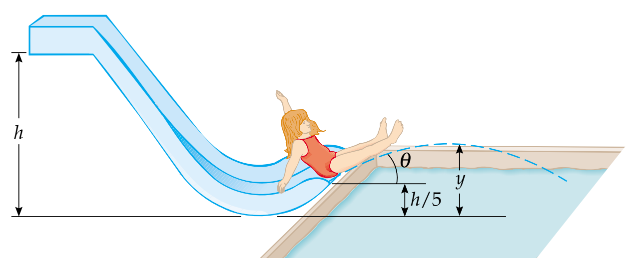


(a) (b)

(b) $v_i = \sqrt{2(9.80\text{ m/s}^2)(1.20\text{ m})(1 - \cos 35.0°)}$

$v_i = \boxed{2.06\text{ m/s}}$

8.66 Launch speed is found from



$$mg\left(\frac{4}{5}h\right) = \frac{1}{2}mv^2$$

$$v = \sqrt{2g\left(\frac{4}{5}\right)h}$$

$$v_y = v\sin\theta$$

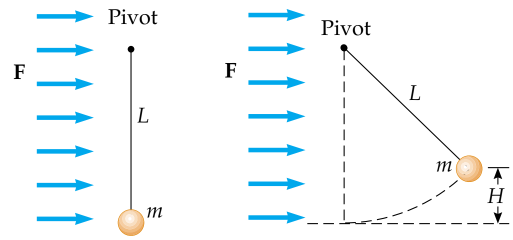
The height y above the water (by conservation of energy) is found from

$$mgy = \frac{1}{2}mv_y^2 + mg\frac{h}{5} \left(\text{since } \frac{1}{2}mv_x^2 \text{ is constant in projectile motion}\right)$$

$$y = \frac{1}{2g}v_y^2 + \frac{h}{5} = \frac{1}{2g}v^2\sin^2\theta + \frac{h}{5}$$

$$y = \frac{1}{2g}\left[2g\left(\frac{4}{5}h\right)\right]\sin^2\theta + \frac{h}{5} = \boxed{\frac{4}{5}h\sin^2\theta + \frac{h}{5}}$$

8.67 (a) Take the original point where the ball is released and the final point where its upward swing stops at height H and horizontal displacement



(a) (b)

$$x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \vee\ \mathbf{F} \cdot d\mathbf{s} = F \vee dx = F\sqrt{2LH - H^2}$$

[The wind force potential energy change would be $-F\sqrt{2LH - H^2}$]

The work-energy theorem can be written:

$$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}, \text{ or}$$

$$0 + 0 + F\sqrt{2LH - H^2} = 0 + mg\,H \quad \text{giving} \quad F^2 2LH - F^2 H^2 = m^2 g^2 H^2$$

Here $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = (F^2 + m^2 g^2)H$. Solving for H yields

$$H = \frac{2LF^2}{F^2 + m^2 g^2} = \boxed{\frac{2L}{1 + (mg/F)^2}}$$

As $F \to 0$, $H \to 0$ as is reasonable.
As $F \to \infty$, $H \to 2L$, which is unreasonable.

(b) $$H = \frac{2(2.00\text{ m})}{1 + [(2.00\text{ kg})(9.80\text{ m/s}^2)/14.7\text{ N}]^2} = \boxed{1.44\text{ m}}$$

(c) Call θ the equilibrium angle with the vertical.

$$\Sigma F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$

$$\Sigma\varpi F_y = 0 \Rightarrow T\cos\theta = mg$$

Dividing: $\tan\theta = \dfrac{F}{mg} = \dfrac{14.7\text{ N}}{19.6\text{ N}} = 0.750$, or $\theta = 36.9°$

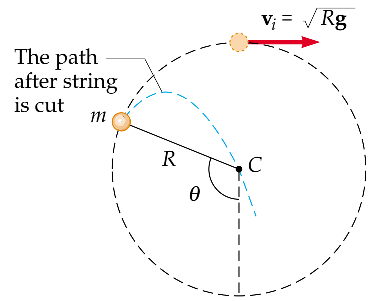
Therefore, $H_{\text{eq}} = L(1 - \cos\theta) = (2.00\text{ m})(1 - \cos 36.9°) = \boxed{0.400\text{ m}}$

(d) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$ and $H_{\text{eq}} \to\varpi\varpi\ L$

A very strong wind pulls the string out horizontal, parallel to the ground. Thus, $\boxed{(H_{\text{eq}})_{\max} = L}$

8.68 Call $\phi = 180° - \theta$ the angle between the upward vertical and the radius to the release point. Call v_r the speed here. By conservation of energy

$$K_i + U_i + \Delta E = K_r + U_r$$



$$\frac{1}{2}mv_i^2 + mgR + 0 = \frac{1}{2}mv_r^2 + mgR\cos\phi$$

$$gR + 2gR = v_r^2 + 2gR\cos\phi$$

$$v_r = \sqrt{3gR - 2gR\cos\phi}$$

The components of velocity at release are

$$v_x = v_r\cos\phi \quad \text{and} \quad v_y = v_r\sin\phi$$

so for the projectile motion we have

$$x = v_x t \qquad R\sin\phi = v_r\cos\phi\, t$$

$$y = v_y t - \frac{1}{2}gt^2 \qquad -R\cos\phi = v_r\sin\phi\, t - \frac{1}{2}gt^2$$

By substitution

$$-R\cos\phi = v_r\sin\phi\frac{R\sin\phi}{v_r\cos\phi} - \frac{g}{2}\frac{R^2\sin^2\phi}{v_r^2\cos^2\phi}$$

with $\sin^2\phi + \cos^2\phi = 1$,

$$gR\sin^2\phi = 2v_r^2\cos\phi = 2\cos\phi(3gR - 2gR\cos\phi)$$

$$\sin^2\phi = 6\cos\phi - 4\cos^2\phi = 1 - \cos^2\phi$$

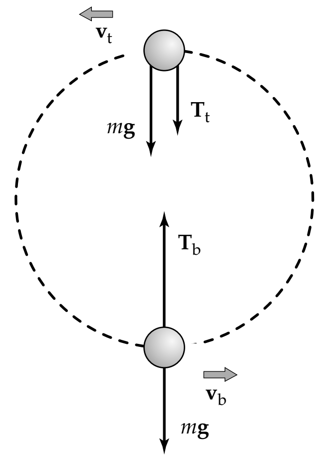
$$3\cos^2\phi - 6\cos\phi + 1 = 0$$

$$\cos\phi = \frac{6 \pm \sqrt{36 - 12}}{6}$$

Only the – sign gives a value for $\cos\phi$ that is less than one:

$$\cos\phi = 0.1835 \qquad \phi = 79.43° \qquad \text{so} \quad \theta = \boxed{100.6°}$$

8.69 Applying Newton's second law at the bottom (b) and top (t) of the circle gives



$$T_b - mg = \frac{mv_b^2}{R} \quad \text{and} \quad -T_t - mg = -\frac{mv_t^2}{R}$$

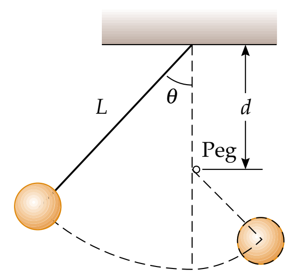
Adding these gives $\quad T_b = T_t + 2mg + \dfrac{m(v_b^2 - v_t^2)}{R}$

Also, energy must be conserved and $\Delta U + \Delta K = 0$

So, $\dfrac{m(v_b^2 - v_t^2)}{2} + (0 - 2mgR) = 0 \quad$ and $\quad \dfrac{m(v_b^2 - v_t^2)}{R} = 4mg$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$

8.70 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.



(b) Relative to the point of suspension,

$$U_i = 0, \quad U_f = -mg[d - (L - d)]$$

From this we find that

$$-mg(2d - L) + \frac{1}{2}\, mv^2 = 0$$

Also for centripetal motion,

$$mg = \frac{mv^2}{R} \quad \text{where } R = L - d.$$

Upon solving, we get $\boxed{d = \dfrac{3L}{5}}$

8.71 (a) The potential energy associated with the wind force is $+Fx$, where x is the horizontal distance traveled, with x positive when swinging into the wind and negative when swinging in the direction the wind is blowing. The initial energy of Jane is, (using the pivot point of the swing as the point of zero gravitational energy),

$$E_i = (K + U_g + U_{\text{wind}})_i = \frac{1}{2}\, mv_i^2 - mgL\cos\theta - FL\sin\theta$$

where m is her mass. At the end of her swing, her energy is

$$E_f = (K + U_g + U_{\text{wind}})_f = 0 - mgL\cos\phi + FL\sin\phi$$

so conservation of energy ($E_i = E_f$) gives

$$\frac{1}{2}\, mv_i^2 - mgL\cos\theta - FL\sin\theta = -mgL\cos\phi + FL\sin\phi$$

This leads to $v_i = \sqrt{2gL(\cos\theta - \cos\phi) + 2\dfrac{FL}{m}(\sin\theta + \sin\phi)}$

But $D = L\sin\phi + L\sin\theta$, so that $\sin\phi = \dfrac{D}{L} - \sin\theta = \dfrac{50.0}{40.0} - \sin 50.0° = 0.484$

which gives $\phi = 28.9°$. Using this, we have $\boxed{v_i = 6.15\text{ m/s}}$.

(b) Here (again using conservation of energy) we have,

$$-MgL\cos\phi + FL\sin\phi + \frac{1}{2}\, Mv^2 = -MgL\cos\theta - FL\sin\theta$$

where M is the combined mass of Jane and Tarzan.

Therefore, $v = \sqrt{2gL(\cos\phi - \cos\theta) - 2\dfrac{FL}{M}(\sin\phi + \sin\theta)}$ which gives $\boxed{v = 9.87\text{ m/s}}$ as the minimum speed needed.

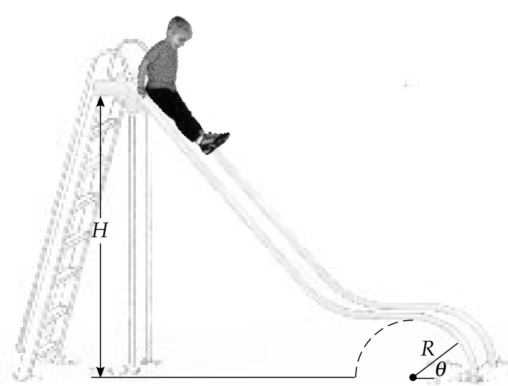
8.72 Find the velocity at the point where the child leaves the slide, height h:

$$(U + K)_i = (U + K)_f$$

$$mgH + 0 = mgh + \frac{1}{2}\, mv^2$$

$$v = \sqrt{2g(H - h)}$$

Use Newton's laws to compare h and H.



(Recall the normal force will be zero):

$$\Sigma\, F_r = ma_r = \frac{mv^2}{R}$$

$$mg \sin \theta - n = \frac{mv^2}{R}$$

$$mg \sin \theta = \frac{m(2\,g)(H - h)}{R}$$

Put θ in terms of R: $\sin \theta = \dfrac{h}{R}$

$$mg\left(\frac{h}{R}\right) = \frac{2\,mg(H - h)}{R}$$

$$\boxed{h = \frac{2}{3}H}$$

Notice if $H \geq \dfrac{3}{2}\ R$, the assumption that the child will leave the slide at a height $\dfrac{2}{3}\ H$ is no longer valid. Then the velocity will be too large for the centripetal force to keep the child on the slide. Thus if $H \geq \dfrac{3}{2}\ R$, the child will leave the track at $h = R$.

8.73 Case I: Surface is frictionless

$$\frac{1}{2}\ mv^2 = \frac{1}{2}\ kx^2$$

$$k = \frac{mv^2}{x^2} = \frac{(5.00\text{ kg})(1.20\text{ m/s})^2}{10^{-2}\text{ m}^2} = 7.20 \infty 10^2\text{ N/m}$$

Case II: Surface is rough, $\mu_k = 0.300$

$$\frac{1}{2}\ mv^2 = \frac{1}{2}\ kx^2 - \mu_k mgx$$

$$\frac{5.00\text{ kg}}{2}\ v^2 = \frac{1}{2}(7.20 \times 10^2\text{ N}/m)(10^{-1}\ m)^2 - (0.300)(5.00\text{ kg})(9.80\text{ m/s}^2)(10^{-1}\text{ m})$$

$$\boxed{v = 0.923\text{ m/s}}$$

8.74 $\Sigma F_y = n - mg \cos 37.0° = 0, \quad \therefore\ n = mg \cos 37.0° = 400\text{ N}$

$f = \mu N = (0.250)(400) = 100\text{ N}$

$W_f = \Delta E$

$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$

$\Delta U_A = m_A g(h_f - h_i) = (50.0)(9.80)(20.0 \sin 37.0°) = 5.90 \infty 10^3$

$$\Delta U_B = m_B g(h_f - h_i) = (100)(9.80)(-20.0) = -1.96 \infty 10^4$$

$$\Delta K_A = \frac{1}{2}\ m_A(v_f^2 - v_i^2)$$

$$\Delta K_B\ \frac{1}{2}\ m_B(v_f^2 - v_i^2) = \frac{m_B}{m_A}\ \Delta K_A = 2\Delta K_A$$

Adding and solving, $\Delta K_A = \boxed{3.92\ \text{kJ}}$

Chapter 9 Solutions

9.1 $m = 3.00$ kg, $\mathbf{v} = (3.00\mathbf{i} - 4.00\mathbf{j})$ m / s

(a) $\mathbf{p} = m\mathbf{v} = (9.00\mathbf{i} - 12.0\mathbf{j})$ kg · m / s

Thus, $\boxed{p_x = 9.00 \text{ kg} \cdot \text{m/s}}$ and $\boxed{p_y = -12.0 \text{ kg} \cdot \text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (-12.0)^2} = \boxed{15.0 \text{ kg} \cdot \text{m/s}}$

$\theta = \tan^{-1}(p_y / p_x) = \tan^{-1}(-1.33) = \boxed{307°}$

***9.2** (a) At maximum height $\mathbf{v} = 0$, so $\mathbf{p} = \boxed{0}$

(b) Its original kinetic energy is its constant total energy,

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2}(0.100) \text{ kg}(15.0 \text{ m/s})^2 = 11.2 \text{ J}$$

At the top all of this energy is gravitational. Halfway up, one-half of it is gravitational and the other half is kinetic:

$$K = 5.62 \text{ J} = \frac{1}{2}(0.100 \text{ kg})\, v^2$$

$$v = \sqrt{\frac{2 \times 5.62 \text{ J}}{0.100 \text{ kg}}} = 10.6 \text{ m/s}$$

Then $\mathbf{p} = m\mathbf{v} = (0.100 \text{ kg})(10.6 \text{ m/s})\mathbf{j}$

$$\mathbf{p} = \boxed{1.06 \text{ kg} \cdot \text{m/s}\ \mathbf{j}}$$

***9.3** The initial momentum = 0. Therefore, the final momentum, p_f, must also be zero.

We have, (taking eastward as the positive direction),

$$p_f = (40.0 \text{ kg})(v_c) + (0.500 \text{ kg})(5.00 \text{ m/s}) = 0$$

$$v_c = \boxed{-6.25 \times 10^{-2} \text{ m/s}}$$

(The child recoils westward.)

***9.4** $p_{\text{baseball}} = p_{\text{bullet}}$

$(0.145 \text{ kg})v = (3.00 \times 10^{-3} \text{ kg})(1500 \text{ m/s}) = 4.50 \text{ kg} \cdot \text{m/s}$

$$v = \frac{4.50 \text{ kg} \cdot \text{m/s}}{0.145 \text{ kg}} = \boxed{31.0 \text{ m/s}}$$

***9.5** I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm.

I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a(x - x_i)$$

$$0 - v_i^2 = 2(-9.80\ \text{m/s}^2)(0.250\ \text{m})$$

$$v_i = 2.20\ \text{m/s}$$

Total momentum is conserved as I push the earth down and myself up:

$$0 = -(5.98 \times 10^{24}\ \text{kg})v_e + (85.0\ \text{kg})(2.20\ \text{m/s})$$

$$v_e\ \boxed{\sim 10^{-23}\ \text{m/s}}$$

9.6 (a) For the system of two blocks $\Delta p = 0$,

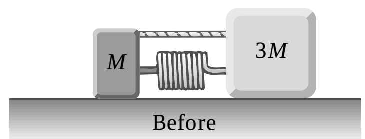
or $p_i = p_f$

Therefore,

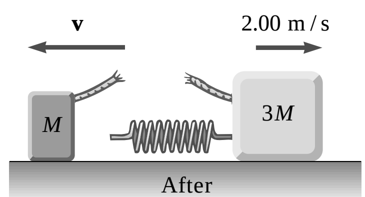
$$0 = Mv_m + (3\,M)(2.00\ \text{m/s})$$

Solving gives

$$v_m = \boxed{-6.00\ \text{m/s}}\ \text{(motion toward the left)}$$



(a)



(b)

(b) $$\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3\,M)\,v_{3M}^2 = \boxed{8.40\ \text{J}}$$

***9.7** (a) The momentum is $p = mv$, so $v = p/m$ and

the kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$

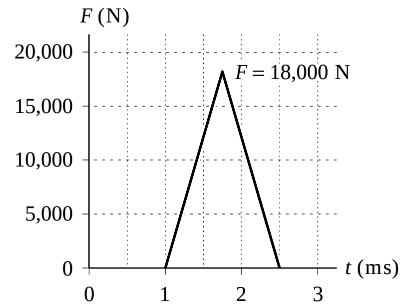
(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{2K/m}$, so

$$p = mv = m\sqrt{2K/m} = \boxed{\sqrt{2mK}}$$

9.8 $I = \Delta p = m\,\Delta v = (70.0\ \text{kg})(5.20\ \text{m/s}) = \boxed{364\ \text{kg}\cdot\text{m/s}}$

$$F = \frac{\Delta p}{\Delta t} = \frac{364}{0.832} = \boxed{438\ \text{N}}$$

9.9 (a) $I = \int F dt$ = area under curve



$$= \frac{1}{2}(1.50 \times 10^{-3}\text{ s})(18000\text{ N}) = \boxed{13.5\text{ N}\cdot\text{s}}$$

(b) $F = \dfrac{13.5\text{ N}\cdot\text{s}}{1.50\times 10^{-3}\text{ s}} = \boxed{9.00\text{ kN}}$

(c) From the graph, we see that $F_{\max} = \boxed{18.0\text{ kN}}$

9.10 Assume the initial direction of the ball in the $-x$ direction.

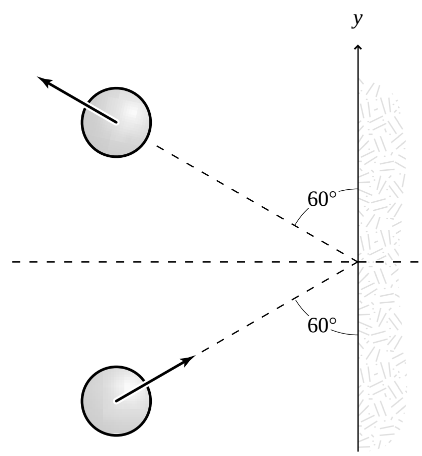
(a) Impulse,

$$\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = (0.0600)(40.0)\mathbf{i} - (0.0600)(50.0)(-\mathbf{i}) = \boxed{5.40\mathbf{i}\text{ N}\cdot\text{s}}$$

(b) Work $= K_f - K_i = \dfrac{1}{2}(0.0600)\ [(40.0)^2 - (50.0)^2] =$

$\boxed{-27.0\text{ J}}$

9.11 $\Delta p = F\,\Delta t$



$$\Delta p_y = m(v_{fy} - v_{iy}) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\Delta p_x = m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°$$

$$= -2(3.00\text{ kg})(10.0\text{ m/s})(0.866)$$

$$= -52.0\text{ kg}\cdot\text{m/s}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\text{ kg}\cdot\text{m/s}}{0.200\text{ s}} = \boxed{-260\text{ N}}$$

<u>Goal Solution</u>

G: If we think about the angle as a variable and consider the limiting cases, then the force should be zero when the angle is 0 (no contact between the ball and the wall). When the angle is 90° the force will be its maximum and can be found from the momentum-impulse equation, so that $F < 300$N, and the force on the ball must be directed to the left.

O: Use the momentum-impulse equation to find the force, and carefully consider the direction of the velocity vectors by defining up and to the right as positive.

A: $\Delta p = F\,\Delta t$

$\Delta p_y = mv_{yf} - mv_{yx} = m(v\cos 60.0° - v\cos 60.0°) = 0$

So the wall does not exert a force on the ball in the y direction.

$$\Delta p_x = mv_{xf} - mv_x = m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°$$

$$\Delta p_x = -2(3.00\text{ kg})(10.0\text{ m/s})(0.866) = -52.0\text{ kg}\cdot\text{m/s}$$

$$\bar{F} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{\Delta p_x \mathbf{i}}{\Delta t} = \frac{-52.0\,\mathbf{i}\text{ kg}\cdot\text{m/s}}{0.200\text{ s}} = -260\,\mathbf{i}\text{ N}$$

L: The force is to the left and has a magnitude less than 300 N as expected.

9.12 Take x-axis toward the pitcher

(a) $p_{ix} + I_x = p_{fx}$

$0.200\text{ kg}(15.0\text{ m/s})(-\cos 45.0°) + I_x = 0.200\text{ kg}(40.0\text{ m/s})\cos 30.0°$

$I_x = 9.05\text{ N}\cdot\text{s}$

$p_{iy} + I_y = p_{fy}$

$0.200\text{ kg}(15.0\text{ m/s})(-\sin 45.0°) + I_y = 0.200\text{ kg}(40.0\text{ m/s})\sin 30.0°$

$\mathbf{I} = \boxed{(9.05\mathbf{i} + 6.12\mathbf{j})\text{ N}\cdot\text{s}}$

(b) $\mathbf{I} = \frac{1}{2}(0 + \mathbf{F}_m)(4.00\text{ ms}) + \mathbf{F}_m(20.0\text{ ms}) + \frac{1}{2}\mathbf{F}_m(4.00\text{ ms})$

$\mathbf{F}_m \times 24.0 \times 10^{-3}\text{ s} = (9.05\mathbf{i} + 6.12\mathbf{j})\text{ N}\cdot\text{s}$

$\mathbf{F}_m = \boxed{(377\mathbf{i} + 255\mathbf{j})\text{ N}}$

9.13 The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\text{ kg})(25.0\text{ m/s}) - 0}{1.00\text{ s}} = \boxed{15.0\text{ N}}$$

According to Newton's 3rd law, the water exerts a force of equal magnitude back on the hose. Thus, the holder must apply a 15.0 N force (in the direction of the velocity of the exiting water steam) to hold the hose stationary.

***9.14** If the diver starts from rest and drops vertically into the water, the velocity just before impact is found from

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv_{\text{impact}}^2 + 0 = 0 + mgh \Rightarrow v_{\text{impact}} = \sqrt{2gh}$$

With the diver at rest after an impact time of Δt, the average force during impact is given by

$$\bar{F} = \frac{m(0 - v_{\text{impact}})}{\Delta t} = \frac{-m\sqrt{2gh}}{\Delta t} \quad \text{or} \quad \bar{F} = \frac{m\sqrt{2gh}}{\Delta t} \quad \text{(directed upward)}$$

Assuming a mass of 55 kg and an impact time of ≈1.0 s, the magnitude of this average force is

$$\left|\bar{F}\right| = \frac{(55\text{ kg})\sqrt{2(9.8\text{ m/s}^2)(10\text{ m})}}{1.0\text{ s}} = 770\text{ N, or } \boxed{\sim 10^3\text{ N}}$$

***9.15** $(200\text{ g})(55.0\text{ m/s}) = (46.0\text{ g})v + (200\text{ g})(40.0\text{ m/s})$

$v = \boxed{65.2\text{ m/s}}$

9.16 For each skater,

$$\bar{F} = \frac{m\,\Delta v}{\Delta t} = \frac{(75.0\text{ kg})(5.00\text{ m/s})}{(0.100\text{ s})} = \boxed{3750\text{ N}}$$

Since $\bar{F} < 4500$ N, there are no broken bones.

9.17 Momentum is conserved

$(10.0 \times 10^{-3}\text{ kg})v = (5.01\text{ kg})(0.600\text{ m/s})$

$$v = \boxed{301\text{ m/s}}$$

Goal Solution

G: A reasonable speed of a bullet should be somewhere between 100 and 1000 m / s.

O: We can find the initial speed of the bullet from conservation of momentum. We are told that the block of wood was originally stationary.

A: Since there is no external force on the block and bullet system, the total momentum of the system is constant so that $\Delta\mathbf{p} = 0$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$(0.0100 \text{ kg})v_{1i} + 0 = (0.0100 \text{ kg})(0.600 \text{ m/s})\mathbf{i} + (5.00 \text{ kg})(0.600 \text{ m/s})\,\mathbf{i}$$

$$v_{1i} = \frac{(5.01 \text{ kg})(0.600 \text{ m/s})\mathbf{i}}{0.0100 \text{ kg}} = 301\,\mathbf{i} \text{ m/s}$$

L: The speed seems reasonable, and is in fact just under the speed of sound in air (343 m / s at 20°C).

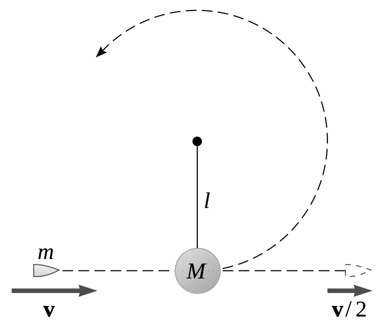
9.18 Energy is conserved for the bob between bottom and top of swing:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}Mv_b^2 + 0 = 0 + Mg\,2l$$

$$v_b^2 = g\,4l$$

$$v_b = 2\sqrt{gl}$$



Momentum is conserved in the collision:

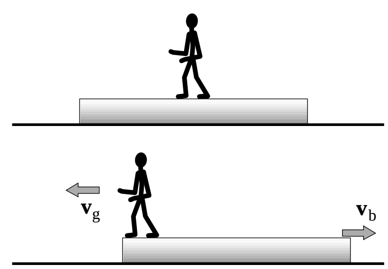
$$mv = m\frac{v}{2} + M \cdot 2\sqrt{gl}$$

$$\boxed{v = \frac{4M}{m}\sqrt{gl}}$$

9.19 (a) and (b) Let v_g and v_p be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that

$$v_g - v_p = 1.50 \quad (1)$$

But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore



$$45.0v_g + 150v_p = 0, \quad \text{or} \quad v_g = -3.33\,v_p$$

Putting this into the equation (1) above gives

$$-3.33\,v_p - v_p = 1.50, \quad \text{or} \quad v_p = \boxed{-0.346 \text{ m/s}}$$

Then $v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$

9.20 Gayle jumps on the sled:

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$(50.0 \text{ kg})(4.00 \text{ m/s}) = (50.0 \text{ kg} + 5.00 \text{ kg})v_2$$

$$v_2 = 3.64 \text{ m/s}$$

They glide down 5.00 m:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(55.0 \text{ kg})(3.64 \text{ m/s})^2 + 55.0 \text{ kg}(9.8 \text{ m/s}^2)5.00 \text{ m} = \frac{1}{2}(55.0 \text{ kg})\,v_3^2$$

$v_3 = 10.5 \text{ m/s}$

Brother jumps on:

$$55.0\ \text{kg}(10.5\ \text{m/s}) + 0 = (85.0\ \text{kg})v_4$$

$$v_4 = 6.82\ \text{m/s}$$

All slide 10.0 m down:

$$\frac{1}{2}(85.0\ \text{kg})(6.82\ \text{m/s})^2 + 85.0\ \text{kg}\ (9.80\ \text{m/s}^2)10.0\ \text{m} = \frac{1}{2}(85.0\ \text{kg})\ v_5^2$$

$$v_5 = \boxed{15.6\ \text{m/s}}$$

9.21 $p_i = p_f$

(a) $m_c v_{ic} + m_T v_{iT} = m_c v_{fc} + m_T v_{fT}$

$$v_{fT} = \left(\frac{1}{m_T}\right)[m_c v_{ic} + m_T v_{iT} - m_c v_{fc}]$$

$$v_{fT} = \left(\frac{1}{9000\ \text{kg}}\right)[(1200\ \text{kg})(25.0\ \text{m/s}) + (9000\ \text{kg})(20.0\ \text{m/s}) - (1200\ \text{kg})(18.0\ \text{m/s})]$$

$$= \boxed{20.9\ \text{m/s}}\quad \text{East}$$

(b) $K_{\text{lost}} = K_i - K_f$

$$= \frac{1}{2}m_c v_{ic}^2 + \frac{1}{2}m_T v_{iT}^2 - \frac{1}{2}m_c v_{fc} - \frac{1}{2}m_T v_{fT}^2$$

$$= \frac{1}{2}[m_c(v_{ic}^2 - v_{fc}^2) + m_T(v_{iT}^2 - v_{fT}^2)]$$

$$= \frac{1}{2}[(1200\ \text{kg})(625 - 324)(\text{m}^2/\text{s}^2) + (9000\ \text{kg})(400 - 438.2)\text{m}^2/\text{s}^2]$$

$$K_{\text{lost}} = \boxed{8.68\ \text{kJ}}\quad \text{becomes internal energy.}$$

(If 20.9 m/s were used to determine the energy lost instead of 20.9333, the answer would be very different. We keep extra significant figures until the problem is complete!)

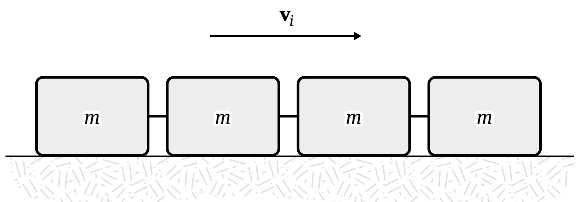
9.22 (a) $mv_{1i} + 3mv_{2i} = 4mv_f$ where $m = 2.50 \times 10^4$ kg

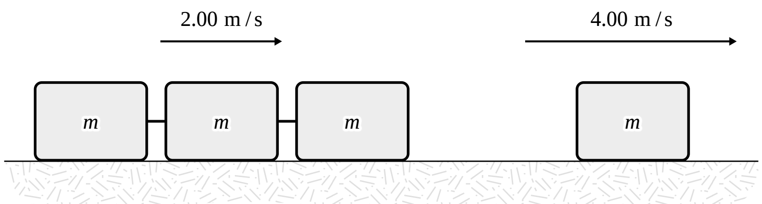
$$v_f = \frac{4.00 + 3(2.00)}{4} = \boxed{2.50\ \text{m/s}}$$

(b) $K_f - K_i = \frac{1}{2}(4m)\ v_f^2 - \left[\frac{1}{2}m\,v_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right]$

$$= 2.50 \times 10^4\ [12.5 - 8.00 - 6.00] = \boxed{-3.75 \times 10^4\ \text{J}}$$

***9.23** (a) The internal forces exerted by the actor do not change total momentum.





$$(4m)v_i = (3m)(2.00\text{ m/s}) + m(4.00\text{ m/s})$$

$$v_i = \frac{6.00\text{ m/s} + 4.00\text{ m/s}}{4} = \boxed{2.50\text{ m/s}}$$

(b) $$W_{\text{actor}} = K_f - K_i = \frac{1}{2}\left[(3m)(2.00\text{ m/s})^2 + m(4.00\text{ m/s})^2\right] - \frac{1}{2}(4m)(2.50\text{ m/s})^2$$

$$W_{\text{actor}} = \frac{(2.50\times 10^4\text{ kg})}{2}[12.0 + 16.0 - 25.0](\text{m/s})^2 = \boxed{37.5\text{ kJ}}$$

(c) The explosion considered here is the time reversal of the perfectly inelastic collision in problem 9.22. The same momentum conservation equation describes both processes.

***9.24** We call the initial speed of the bowling ball v_i and from momentum conservation,

$$(7.00\text{ kg})(v_i) + (2.00\text{ kg})(0) = (7.00\text{ kg})(1.80\text{ m/s}) + (2.00\text{ kg})(3.00\text{ m/s})$$

gives

$$v_i = \boxed{2.66\text{ m/s}}$$

9.25 (a) Following Example 9.8, the fraction of total kinetic energy transferred to the moderator is

$$f_2 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

where m_2 is the moderator nucleus and in this case,

$$m_2 = 12m_1$$

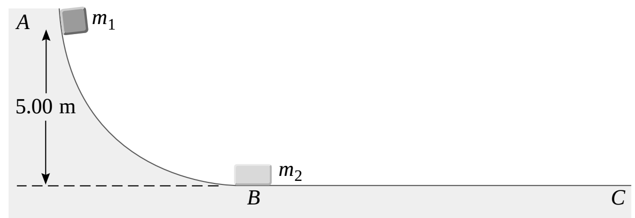
$$f_2 = \frac{4m_1(12m_1)}{(13m_1)^2} = \frac{48}{169} = \boxed{0.284\text{ or }28.4\%}$$

of the neutron energy is transferred to the carbon nucleus.

(b) $K_C = (0.284)(1.6 \times 10^{-13}\ \text{J}) = \boxed{4.54 \times 10^{-14}\ \text{J}}$

$K_n = (0.716)(1.6 \times 10^{-13}\ \text{J}) = \boxed{1.15 \times 10^{-13}\ \text{J}}$

9.26 v_1, speed of m_1 at B before collision.



$$\frac{1}{2}m_1 v_1^2 = m_1 g h$$

$$v_1 = \sqrt{2 \times 9.80 \times 5.00} = 9.90\ \text{m/s}$$

v_{1f}, speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\ v_1 = -\frac{1}{3}(9.90)\ \text{m/s} = -3.30\ \text{m/s}$$

At the highest point (after collision)

$$m_1 g h_{\max} = \frac{1}{2}m_1(-3.30)^2$$

$$h_{\max} = \frac{(-3.30\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{0.556\ \text{m}}$$

9.27 At impact momentum is conserved, so:

$$m_1 v_1 = (m_1 + m_2)v_2$$

After impact the change in kinetic energy is equal to the work done by friction:

$$\frac{1}{2}(m_1 + m_2)\ v_2^2 = f_f d = \mu(m_1 + m_2)gd$$

$$\frac{1}{2}(0.112\ \text{kg})\ v_2^2 = 0.650(0.112\ \text{kg})(9.80\ \text{m/s}^2)(7.50\ \text{m})$$

$$v_2^2 = 95.6\ \text{m}^2/\text{s}^2$$

$$v_2 = 9.77\ \text{m/s}$$

$$(12.0 \times 10^{-3}\ \text{kg})v_1 = (0.112\ \text{kg})(9.77\ \text{m/s})$$

$$v_1 = \boxed{91.2\ \text{m/s}}$$

9.28 We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These equal forces act backwards on the two bullets.

For the first, $K_i + \Delta E = K_f$

$$\frac{1}{2}(7.00 \times 10^{-3}\text{ kg})\, v^2 + F(8.00 \times 10^{-2}\text{ m}) \cos 180° = 0$$

For the second, $p_i = p_f$

$$(7.00 \times 10^{-3}\text{ kg})v = (1.014\text{ kg})v_f$$

$$v_f = \frac{(7.00 \times 10^{-3})v}{1.014}$$

Again, $K_i + \Delta E = K_f$

$$\frac{1}{2}(7.00 \times 10^{-3}\text{ kg})\, v^2 + Fd \cos 180° = \frac{1}{2}(1.014\text{ kg})\, v_f^2$$

Substituting,

$$\frac{1}{2}(7.00 \times 10^{-3}\text{ kg})\, v^2 - Fd = \frac{1}{2}(1.014\text{ kg})\left(\frac{7.00 \times 10^{-3}\, v}{1.014}\right)^2$$

$$Fd = \frac{1}{2}(7.00 \times 10^{-3}\text{ kg})\, v^2 - \frac{1}{2}(7.00 \times 10^{-3}\text{ kg})\frac{7.00 \times 10^{-3}}{1.014}\, v^2$$

Substituting again,

$$Fd = F(8.00 \times 10^{-2}\text{ m})\left(1 - \frac{7.00 \times 10^{-3}}{1.014}\right)$$

$$d = \boxed{7.94\text{ cm}}$$

***9.29** (a) First, we conserve momentum in the x direction (the direction of travel of the fullback).

$$(90.0\text{ kg})(5.00\text{ m/s}) + 0 = (185\text{ kg})V \cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V \cos\theta = 2.43\text{ m/s} \quad (1)$$

Now consider conservation of momentum in the y direction (the direction of travel of the opponent).

$$(95.0\text{ kg})(3.00\text{ m/s}) + 0 = (185\text{ kg})(V \sin\theta)$$

which gives,

$$V \sin\theta = 1.54\text{ m/s} \quad (2)$$

Divide equation (2) by (1)

$$\tan\theta = \frac{1.54}{2.43} = 0.633$$

From which

$$\boxed{\theta = 32.3°}$$

Then, either (1) or (2) gives

$$V = \boxed{2.88 \text{ m/s}}$$

(b) $K_i = \frac{1}{2}(90.0 \text{ kg})(5.00 \text{ m/s})^2 + \frac{1}{2}(95.0 \text{ kg})(3.00 \text{ m/s})^2 = 1.55 \times 10^3 \text{ J}$

$$K_f = \frac{1}{2}(185 \text{ kg})(2.88 \text{ m/s})^2 = 7.67 \times 10^2 \text{ J}$$

Thus, the kinetic energy lost is $\boxed{783 \text{ J}}$ into internal energy.

***9.30** The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0 \text{ m/s})$$

and $v_{Bi} = 8.33 \text{ m/s}$

$$K_i = \frac{1}{2}m(10.0 \text{ m/s})^2 + \frac{1}{2}(1.20m)(8.33 \text{ m/s})^2 = \frac{1}{2}m(183 \text{ m}^2/\text{s}^2)$$

$$K_f = \frac{1}{2}m(v_G)^2 + \frac{1}{2}(1.20m)(v_B)^2 = \frac{1}{2}\left(\frac{1}{2}m(183 \text{ m}^2/\text{s}^2)\right)$$

or $v_G^2 + 1.20v_B^2 = 91.7 \text{ m}^2/\text{s}^2$ (1)

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

or $v_G = 1.20v_B$ (2)

Solving (1) and (2) simultaneously, we find

$\boxed{v_G = 7.07 \text{ m/s}}$ (speed of green puck after collision)

and $\boxed{v_B = 5.89 \text{ m/s}}$ (speed of blue puck after collision)

***9.31** We use conservation of momentum for both northward and eastward components.

For the eastward direction: $M(13.0\text{ m/s}) = 2MV_f \cos 55.0°$

For the northward direction: $MV = 2MV_f \sin 55.0°$

Divide the northward equation by the eastward equation to find:

$$V = (13.0\text{ m/s})\tan 55.0° = 18.6\text{ m/s} = \boxed{41.5\text{ mi/h}}$$

Thus, the driver of the north bound car was untruthful.

***9.32** (a) $\mathbf{p}_i = \mathbf{p}_f$

so $p_{xi} = p_{xf}$

and $p_{yi} = p_{yf}$

$$mv_i = mv\cos\theta + mv\cos\phi \qquad (1)$$

$$0 = mv\sin\theta - mv\sin\phi \qquad (2)$$

From (2),

$$\sin\theta = \sin\phi \text{ so } \theta = \phi$$

Furthermore, energy conservation requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so $\boxed{v = \dfrac{v_i}{\sqrt{2}}}$

(b) Hence, (1) gives

$$v_i = \frac{2v_i\cos\theta}{\sqrt{2}}$$

$$\theta = \boxed{45.0°} \quad \phi = \boxed{45.0°}$$

9.33 By conservation of momentum (with all masses equal),

$$5.00\text{ m/s} + 0 = (4.33\text{ m/s})\cos 30.0° + v_{2fx}$$

$$v_{2fx} = 1.25\text{ m/s}$$

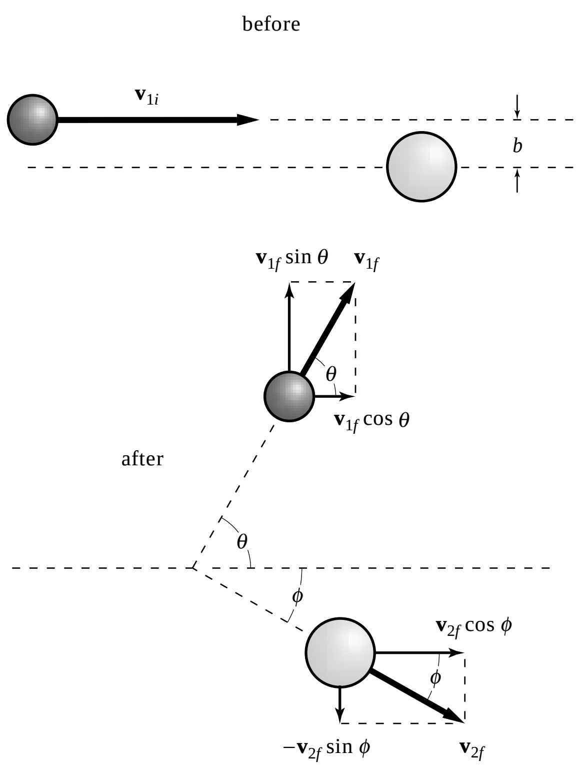
$$0 = (4.33\text{ m/s})\sin 30.0° + v_{2fy}$$

$$v_{2fy} = -2.16\text{ m/s}$$

$$\mathbf{v} = \boxed{2.50\text{ m/s at } -60.0°}$$

Note that we did not need to use the fact that the collision is perfectly elastic.

9.34 (a) Use Equations 9.24 and 9.25 and refer to the figures below.



Let the puck initially at rest be m_2.

$$m_1v_{1i} = m_1v_{1f}\cos\theta + m_2v_{2f}\cos\phi$$

$$0 = m_1v_{1f}\sin\theta - m_2v_{2f}\sin\phi$$

$$(0.200\text{ kg})(2.00\text{ m/s}) = (0.200\text{ kg})(1.00\text{ m/s})\cos 53.0° + (0.300\text{ kg})v_{2f}\cos\phi$$

$$0 = (0.200\text{ kg})(1.00\text{ m/s})\sin 53.0° - (0.300\text{ kg})(v_{2f}\sin\phi)$$

From these equations we find

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{0.160}{0.280} = 0.571, \quad \boxed{\phi = 29.7°}$$

Then

$$v_{2f} = \frac{(0.160 \text{ kg}\cdot\text{m/s})}{(0.300 \text{ kg})(\sin 29.7°)} = \boxed{1.07 \text{ m/s}}$$

(b) $f_{\text{lost}} = \dfrac{\Delta K}{K_i} = \dfrac{K_f - K_i}{K_i} = \boxed{-0.318}$

9.35 $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

$3.00(5.00)\mathbf{i} - 6.00\mathbf{j} = 5.00\mathbf{v}$

$\mathbf{v} = \boxed{(3.00\mathbf{i} - 1.20\mathbf{j}) \text{ m/s}}$

9.36 $p_{xf} = p_{xi}$

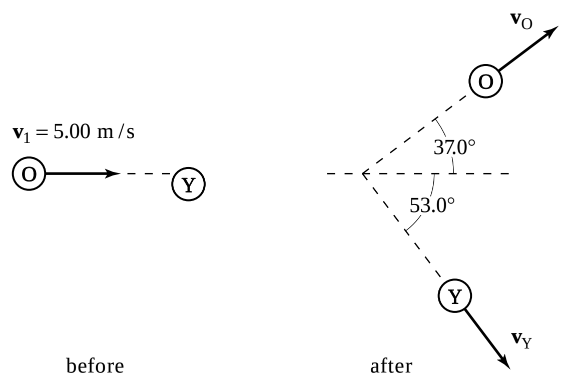
$mv_O \cos 37.0° + mv_Y \cos 53.0° = m(5.00 \text{ m/s})$

$0.799 v_O + 0.602 v_Y = 5.00 \text{ m/s} \qquad (1)$

$p_{yf} = p_{yi}$

$mv_O \sin 37.0° - mv_Y \sin 53.0° = 0$

$0.602 v_O = 0.799 v_Y \ (2)$



Solving (1) and (2) simultaneously,

$\boxed{v_O = 3.99 \text{ m/s}}$ and $\boxed{v_Y = 3.01 \text{ m/s}}$

9.37 $p_{xf} = p_{xi}$

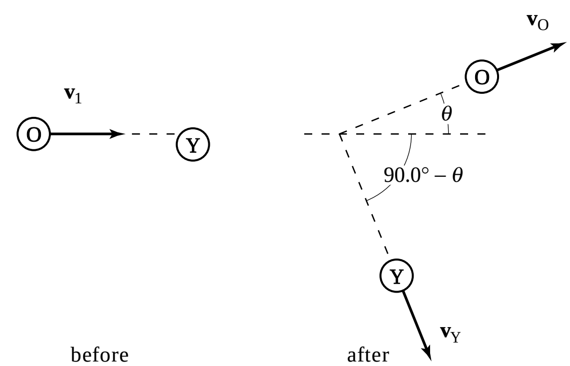
$mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$

$v_O \cos\theta + v_Y \sin\theta = v_i \qquad (1)$

$p_{yf} = p_{yi}$

$mv_O \sin\theta - mv_Y \sin(90.0° - \theta) = 0$

$v_O \sin\theta = v_Y \cos\theta \qquad (2)$



From equation (2),

$v_O = v_Y (\cos\theta / \sin\theta) \qquad (3)$

Substituting into equation (1),

$$v_Y\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_Y \sin\theta = v_i$$

so $v_Y(\cos^2\theta + \sin^2\theta) = v_i \sin\theta$, and $\boxed{v_Y = v_i \sin\theta}$

Then, from equation (3), $\boxed{v_O = v_i \cos\theta}$

9.38 The horizontal and vertical components of momentum are conserved:

$$(5.00\text{ g})(250\text{ m/s})\cos 20.0^\circ - (3.00\text{ g})(280\text{ m/s})\cos 15.0^\circ = (8.00\text{ g})v_{fx}$$
$$v_{fx} = 45.4\text{ m/s}$$

$$(5.00\text{ g})(250\text{ m/s})\sin 20.0^\circ + (3.00\text{ g})(280\text{ m/s})\sin 15.0^\circ = (8.00\text{ g})v_{fy}$$
$$v_{fy} = 80.6\text{ m/s}$$

$$\mathbf{v} = \boxed{45.4\text{ m/s }\mathbf{i} + 80.6\text{ m/s }\mathbf{j}} = 92.5\text{ m/s at }60.6^\circ$$

9.39 $m_0 = 17.0 \times 10^{-27}$ kg $\quad \mathbf{v}_i = 0$ (the parent nucleus)

$m_1 = 5.00 \times 10^{-27}$ kg $\quad \mathbf{v}_1 = 6.00 \times 10^6\ \mathbf{j}$ m/s

$m_2 = 8.40 \times 10^{-27}$ kg $\quad \mathbf{v}_2 = 4.00 \times 10^6\ \mathbf{i}$ m/s

(a) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 = 0$

where $m_3 = m_0 - m_1 - m_2 = 3.60 \times 10^{-27}$ kg

$$(5.00 \times 10^{-27})(6.00 \times 10^6\ \mathbf{j}) + (8.40 \times 10^{-27})(4.00 \times 10^6\ \mathbf{i}) + (3.60 \times 10^{-27})\mathbf{v}_3 = 0$$

$$\mathbf{v}_3 = \boxed{(-9.33 \times 10^6\ \mathbf{i} - 8.33 \times 10^6\ \mathbf{j})\text{ m/s}}$$

(b) $E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2$

$$= \frac{1}{2}\left[(5.00 \times 10^{-27})(6.00 \times 10^6)^2 + (8.40 \times 10^{-27})(4.00 \times 10^6)^2\right.$$
$$\left. + (3.60 \times 10^{-27})(12.5 \times 10^6)^2\right]$$

$$\boxed{E = 4.39 \times 10^{-13}\text{ J}}$$

***9.40** The x-coordinate of the center of mass is

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(0+0+0+0)}{(2.00\text{ kg} + 3.00\text{ kg} + 2.50\text{ kg} + 4.00\text{ kg})}$$

$$\boxed{x_{CM} = 0}$$

and the y-coordinate of the center of mass is

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i}$$

$$y_{CM} = \frac{(2.00\text{ kg})(3.00\text{ m}) + (3.00\text{ kg})(2.50\text{ m}) + (2.50\text{ kg})(0) + (4.00\text{ kg})(-0.500\text{ m})}{2.00\text{ kg} + 3.00\text{ kg} + 2.50\text{ kg} + 4.00\text{ kg}}$$

$$\boxed{y_{CM} = 1.00\text{ m}}$$

9.41 Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.

<u>Goal Solution</u>

G: By inspection, it appears that the center of mass is located at about (12 **i** +13 **j**) cm.

O: Think of the sheet as composed of three sections, and consider the mass of each section to be at the geometric center of that section. Define the mass per unit area to be σ, and number the rectangles as shown. We can then calculate the mass and identify the center of mass of each section.

A: $m_{\text{I}} = (30.0\text{ cm})(10.0\text{ cm})\sigma$ $\quad$ $\text{CM}_{\text{I}} = (15.0\text{ cm}, 5.0\text{ cm})$

$m_{\text{II}} = (10.0\text{ cm})(10.0\text{ cm})\sigma$ $\quad$ $\text{CM}_{\text{II}} = (5.0\text{ cm}, 15.0\text{ cm})$

$m_{\text{III}} = (10.0\text{ cm})(20.0\text{ cm})\sigma$ $\quad$ $\text{CM}_{\text{III}} = (10.0\text{ cm}, 25.0\text{ cm})$

The overall CM is at a point defined by the vector equation $\mathbf{r}_{\text{cm}} \equiv \dfrac{\sum m_i \mathbf{r}_i}{\sum m_i}$

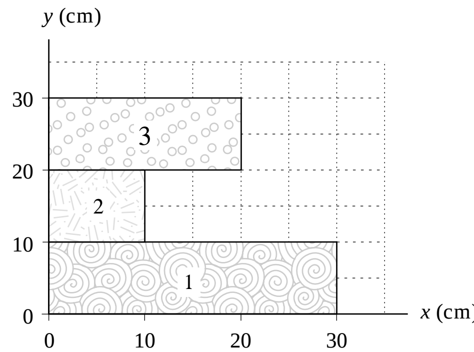
Substituting the appropriate values, $\mathbf{r}_{\text{CM}}$ is calculated to be:

$$\mathbf{r}_{\text{CM}} = \frac{(300\sigma\text{ cm}^3)(15.0\mathbf{i} + 5.0\mathbf{j}) + (100\sigma\text{ cm}^3)(5.0\mathbf{i} + 15.0\mathbf{j}) + (200\sigma\text{ cm}^3)(10.0\mathbf{i} + 25.0\mathbf{j})}{(300 + 200 + 100)\sigma\text{ cm}^2}$$

$$\mathbf{r}_{\text{CM}} = \frac{(45.0\mathbf{i} + 15.0\mathbf{j} + 5.0\mathbf{i} + 15.0\mathbf{j} + 20.0\mathbf{i} + 50.0\mathbf{j})}{6.00}\text{ cm}$$

$$\mathbf{r}_{\text{CM}} = (11.7\mathbf{i} + 13.3\mathbf{j})\text{ cm}$$

L: The coordinates are close to our eyeball estimate. In solving this problem, we could have chosen to divide the original shape some other way, but the answer would be the same. This problem also shows that the center of mass can lie outside the boundary of the object.



$A = A_1 + A_2 + A_3$

$M = M_1 + M_2 + M_3$

$$\frac{M_1}{A_1} = \frac{M}{A}$$

$A_1 = 300\text{ cm}^2$, $A_2 = 100\text{ cm}^2$, $A_3 = 200\text{ cm}^2$

$A = 600\text{ cm}^2$

$$M_1 = M\left(\frac{A_1}{A}\right) = \frac{300\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{2}$$

$$M_2 = M\left(\frac{A_2}{A}\right) = \frac{100\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{6}$$

$$M_3 = M\left(\frac{A_3}{A}\right) = \frac{200\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{3}$$

$$x_{\text{CM}} = \frac{x_1M_1 + x_2M_2 + x_3M_3}{M} = \frac{15.0\text{ cm}\left(\frac{1}{2}M\right) + 5.00\text{ cm}\left(\frac{1}{6}M\right) + 10.0\text{ cm}\left(\frac{1}{3}M\right)}{M}$$

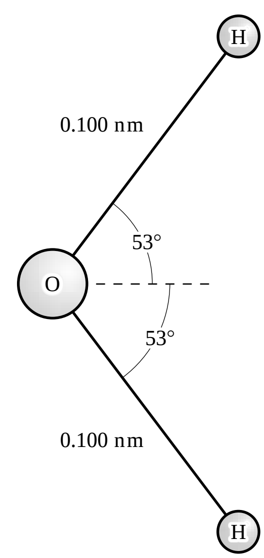
$$\boxed{x_{\text{CM}} = 11.7\text{ cm}}$$

$$y_{\text{CM}} = \frac{\frac{1}{2}M\,(5.00\text{ cm}) + \frac{1}{6}M\,(15.0\text{ cm}) + \left(\frac{1}{3}M\right)(25.0\text{ cm})}{M} = \boxed{13.3\text{ cm}}$$

***9.42** We use, with $x = 0$ at the center of the earth,

$$x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0 + (7.36\times10^{22}\text{ kg})(3.84\times10^{8}\text{ m})}{5.98\times10^{24}\text{ kg} + 7.36\times10^{22}\text{ kg}} = \boxed{4.67\times10^{6}\text{ m}}$$

This is 0.732 (radius of the earth).



9.43 Take x-axis starting from the oxygen nucleus and pointing toward the middle of the **V**.

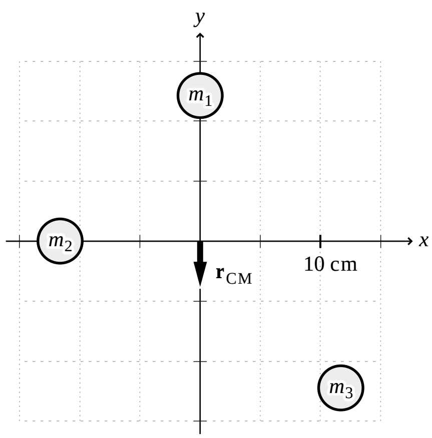
Then $\boxed{y_{\text{CM}} = 0}$

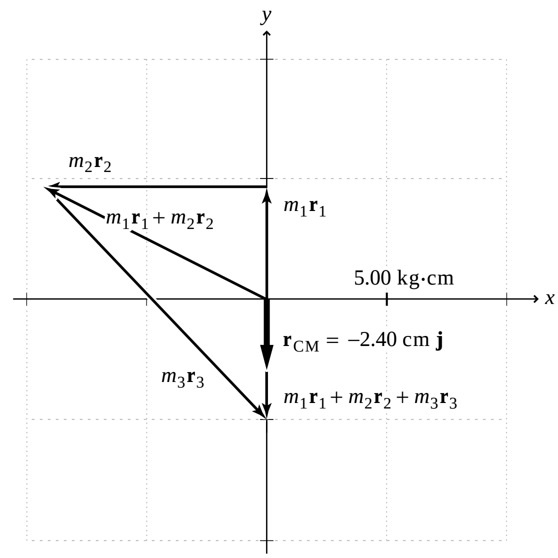
and $x_{\text{CM}} = \dfrac{\sum m_i x_i}{\sum m_i}$

$$x_{\text{CM}} = \frac{0 + 1.008\text{ u}(0.100\text{ nm})\cos 53.0° + 1.008\text{ u}(0.100\text{ nm})\cos 53.0°}{(15.999 + 1.008 + 1.008)\text{u}}$$

$$x_{\text{CM}} = \boxed{0.006\,73\text{ nm from the oxygen nucleus}}$$

***9.44**





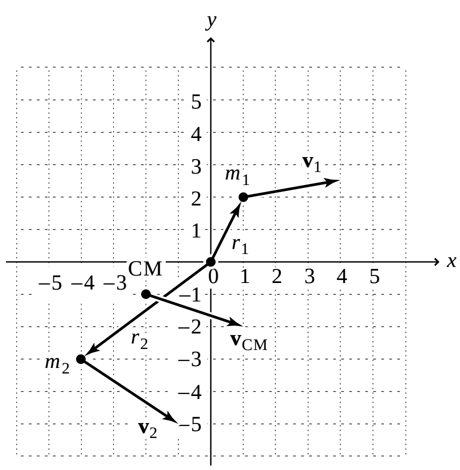
***9.45** (a) $M = \int dm = \int_0^{0.300\text{ m}} \lambda dx = \int_0^{0.300\text{ m}} [50.0\text{ g/m} + 20.0x\text{ g/m}^2]dx$

$$M = [50.0x\text{ g/m} + 10.0x^2\text{ g/m}^2]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b) $x_{CM} = \dfrac{\int_{\text{all mass}} x\,dm}{M} = \dfrac{1}{M}\int_0^{0.300\text{ m}} \lambda\, xdx = \dfrac{1}{M}\int_0^{0.300\text{ m}} [50.0x\text{ g/m} + 20.0x^2\text{ g/m}^2]dx$

$$x_{CM} = \frac{1}{15.9\text{ g}}\left[25.0x^2\text{ g/m} + \frac{20x^3\text{ g/m}^2}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

***9.46** (a)



(b) Using the definition of the position vector at the center of mass,

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} = \frac{2.00\text{ kg} \times (1.00\text{ m}, 2.00\text{ m}) + (3.00\text{ kg})(-4.00\text{ m}, -3.00\text{ m})}{2.00\text{ kg} + 3.00\text{ kg}}$$

$$\mathbf{r}_{CM} = \boxed{(-2.00\mathbf{i} - 1.00\mathbf{j})\text{m}}$$

(c) The velocity of the center of mass is

$$\mathbf{v}_{CM} = \frac{\mathbf{P}}{M} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$$

$$= \frac{2.00\text{ kg} \times (3.00\text{ m/s}, 0.50\text{ m/s}) + (3.00\text{ kg})(3.00\text{ m/s}, -2.00\text{ m/s})}{(2.00\text{ kg} + 3.00\text{ kg})}$$

$$\mathbf{v}_{CM} = \boxed{(3.00\mathbf{i} - 1.00\mathbf{j})\text{m/s}}$$

(d) The total linear momentum of the system can be calculated as $\mathbf{P} = M\mathbf{v}_{CM}$ or as

$$\mathbf{P} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

Either gives

$$\mathbf{P} = \boxed{(15.0\mathbf{i} - 5.00\mathbf{j})\text{ kg}\cdot\text{m/s}}$$

9.47 Let x = distance from shore to center of boat

l = length of boat

x' = distance boat moves as Juliet moves toward Romeo

Before: $$x_{CM} = \frac{\left[M_B x + M_J\left(x - \frac{l}{2}\right) + M_R\left(x + \frac{l}{2}\right)\right]}{(M_B + M_J + M_R)}$$

After: $$x_{CM} = \frac{\left[M_B(x - x') + M_J\left(x + \frac{l}{2} - x'\right) + M_R\left(x + \frac{l}{2} - x'\right)\right]}{(M_B + M_J + M_R)}$$

$$l\left(-\frac{55.0}{2} + \frac{77.0}{2}\right) = x'(-80.0 - 55.0 - 77.0) + \frac{l}{2}(55.0 + 77.0)$$

$$x' = \frac{55.0l}{212} = \frac{55.0(2.70)}{212} = \boxed{0.700\text{ m}}$$

***9.48** $M\mathbf{r}_{CM} = m_1\mathbf{r}_1 + m_2\mathbf{r}_2$

(a) $M\mathbf{v}_{CM} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

$$(0.900\text{ kg})\mathbf{v}_{CM} = (0.600\text{ kg})[(-0.800\text{ m/s})\cos 45.0°\,\mathbf{i} + (0.800\text{ m/s})\sin 45.0°\,\mathbf{j}]$$

$$+ (0.300\text{ kg})[(0.800\text{ m/s})\cos 45.0°\,\mathbf{i} + (0.800\text{ m/s})\sin 45.0°\,\mathbf{j}]$$

$$\mathbf{v}_{CM} = \frac{-1.70\text{ kg}\cdot\text{m/s}\,\mathbf{i} + 5.09\text{ kg}\cdot\text{m/s}\,\mathbf{j}}{0.900\text{ kg}} = \boxed{(-0.189\mathbf{i} + 0.566\mathbf{j})\text{ m/s}}$$

(b) $\mathbf{v}_{CM} = [(0.189\text{ m/s})^2 + (0.566\text{ m/s})^2]^{1/2}$ at $\tan^{-1}\left(\dfrac{0.566}{-0.189}\right)$

$$\mathbf{v}_{CM} = \boxed{0.596\text{ m/s at }\theta = 108°}$$

(c) The center of mass starts from the origin at $t = 0$. At any later time, its location is

$$\mathbf{r}_{CM} = \mathbf{v}_{CM}t = \boxed{(-0.189\mathbf{i} + 0.566\mathbf{j})t\text{ m}}$$

9.49 (a) $\mathbf{v}_{CM} = \dfrac{\sum m_i\mathbf{v}_i}{M} = \dfrac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{M}$

$$\mathbf{v}_{CM} = \frac{(2.00\text{ kg})(2.00\mathbf{i} - 3.00\mathbf{j})\text{ m/s} + (3.00\text{ kg})(1.00\mathbf{i} + 6.00\mathbf{j})\text{ m/s}}{5.00\text{ kg}}$$

$$\mathbf{v}_{CM} = \boxed{(1.40\mathbf{i} + 2.40\mathbf{j})\text{ m/s}}$$

(b) $\mathbf{p} = M\mathbf{v}_{CM} = (5.00\text{ kg})(1.40\mathbf{i} + 2.40\mathbf{j})\text{ m/s} = \boxed{(7.00\mathbf{i} + 12.0\mathbf{j})\text{ kg}\cdot\text{m/s}}$

***9.50** (a) Conservation of momentum:

$$2.00\text{ kg}(1.50\text{ m/s}) + 0.300\text{ kg}(-0.400\text{ m/s}) = 0.200\text{ kg }v_{1f} + 0.300\text{ kg }v_{2f}$$

Relative velocity equation:

$$v_{2f} - v_{1f} = 1.90\text{ m/s}$$

Then

$$0.300 - 0.120 = 0.200v_{1f} + 0.300(1.90 + v_{1f})$$

$$\boxed{v_{1f} = -0.780\text{ m/s}}$$

$$\boxed{v_{2f} = 1.12\text{ m/s}}$$

(b) Before,

$$\mathbf{v}_{\text{CM}} = \frac{0.200 \text{ kg}(1.50 \text{ m/s})\mathbf{i} + 0.300 \text{ kg}(-0.400 \text{ m/s})\mathbf{i}}{0.500 \text{ kg}}$$

$$\boxed{\mathbf{v}_{\text{CM}} = (0.360 \text{ m/s})\mathbf{i}}$$

Afterwards, the center of mass must move at the same velocity, as momentum is conserved.

9.51 (a) Thrust $= \left| v_e \dfrac{dM}{dt} \right| = (2.60 \times 10^3 \text{ m/s})(1.50 \times 10^4 \text{ kg/s}) = \boxed{3.90 \times 10^7 \text{ N}}$

(b) $\Sigma F_y = \text{Thrust} - Mg = Ma$

$3.90 \times 10^7 - (3.00 \times 10^6)(9.80) = (3.00 \times 10^6)a$

$a = \boxed{3.20 \text{ m/s}^2}$

Goal Solution

G: The thrust must be at least equal to the weight of the rocket (30 MN); otherwise the launch will not be successful! However, since a Saturn V rocket accelerates rather slowly compared to the acceleration of falling objects, the thrust should be less than about twice the rocket's weight so that $0<a<g$.

O: Use Newton's second law to find the force and acceleration from the changing momentum.

(a) The impulse due to the thrust, F, is equal to the change in momentum as fuel is exhausted from the rocket.

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv_e)$$

$$F = v_e\left(\frac{dm}{dt}\right)$$

Since v_e is a constant exhaust velocity, $\dfrac{dm}{dt}$ is the fuel comsumption rate, so

$$F = (2.60 \times 10^3 \text{ m/s})(1.50 \times 10^4 \text{ kg/s}) = 39.0 \text{ MN}$$

(b) Applying $\Sigma F = ma$, $(3.90 \times 10^7 \text{ N}) - (3.00 \times 10^6 \text{ kg})(9.80 \text{ m/s}^2) = (3.00 \times 10^6 \text{ kg})a$

$$a = \frac{(3.90 \times 10^7 \text{ N}) - (29.4 \times 10^6 \text{ N})}{3.00 \times 10^6 \text{ kg}} = 3.20 \text{ m/s}^2 \text{ up}$$

L: As expected, the thrust is slightly greater than the weight of the rocket, and the acceleration is about 0.3 g, so the answers appear to be reasonable. This kind of rocket science is not so complicated after all!

***9.52** (a) From equation 9.42, the thrust

$$T = 2.40 \times 10^7 \text{ N} = v_e \frac{dM}{dt}$$

If $v_e = 3000$ m/s, then

$$\frac{dM}{dt} = \boxed{8000 \text{ kg/s}}$$

(b) From Equation 9.41,

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

$$v_f - 0 = (3000 \text{ m/s}) \ln\left(\frac{M_i}{0.100M_i}\right)$$

$$v_f = \boxed{6.91 \text{ km/s}}$$

9.53 $v = v_r \ln \dfrac{M_i}{M_f}$

(a) $M_i = e^{v/v_r} M_f = e^5(3.00 \times 10^3 \text{ kg}) = 4.45 \times 10^5 \text{ kg}$

The mass of fuel and oxidizer is

$$\Delta M = M_i - M_f = (445 - 3.00) \times 10^3 \text{ kg} = \boxed{442 \text{ metric tons}}$$

(b) $\Delta M = e^2 (3.00 \text{ metric tons}) - 3.00 \text{ metric tons} = \boxed{19.2 \text{ metric tons}}$

Because of the exponential, a relatively small increase in fuel and/or engine efficiency causes a large change in the amount of fuel and oxidizer required.

***9.54** (a) $v_f - 0 = v_e \ln\left(\dfrac{M_i}{M_f}\right)$

$$\ln\left(\frac{M_i}{M_f}\right) = \frac{v_f}{v_e}, \text{ so } \frac{M_i}{M_f} = \exp\left(\frac{v_f}{v_e}\right)$$

or $$M_f = M_i \exp\left(-\frac{v_f}{v_e}\right) = (5000 \text{ kg}) \exp\left(-\frac{225}{2500}\right) = 4.57 \times 10^3 \text{ kg}$$

$$M_{\text{fuel}} = M_i - M_f = \boxed{430 \text{ kg}}$$

(b) $430 \text{ kg} = (30.0 \text{ kg/s})t \Rightarrow t = \boxed{14.3 \text{ s}}$

9.55 (a) $(60.0\text{ kg})\,4.00\text{ m/s} = (120 + 60.0)\text{ kg}\; v_f$

$$\mathbf{v}_f = \boxed{1.33\text{ m/s }\mathbf{i}}$$

(b) $\sum F_y = 0$

$$n - (60.0\text{ kg})\,9.80\text{ m/s}^2 = 0$$

$$f_k = \mu_k n = 0.400(588\text{ N}) = \boxed{235\text{ N}}$$

(c) For the person $p_i + I = p_f$

$$mv_i + Ft = mv_f$$

$$(60.0\text{ kg})\,4.00\text{ m/s} - (235\text{ N})t = (60.0\text{ kg})\,1.33\text{ m/s}$$

$$t = \boxed{0.680\text{ s}}$$

(d) person: $m\mathbf{v}_f - m\mathbf{v}_i = 60.0\text{ kg}\,(1.33 - 4.00)\text{ m/s} = \boxed{-160\text{ N}\cdot\text{s }\mathbf{i}}$

cart: $120\text{ kg}\,(1.33\text{ m/s}) - 0 = \boxed{+160\text{ N}\cdot\text{s }\mathbf{i}}$

(e) $x - x_i = \frac{1}{2}(v_i + v)\,t$

$$= \frac{1}{2}\left[(4.00 + 1.33)\text{ m/s}\right]\,0.680\text{ s} = \boxed{1.81\text{ m}}$$

(f) $x - x_i = \frac{1}{2}(v_i + v)\,t$

$$= \frac{1}{2}(0 + 1.33\text{ m/s})\;0.680\text{ s} = \boxed{0.454\text{ m}}$$

(g) $\frac{1}{2}\,mv_f^2 - \frac{1}{2}\,mv_i^2 = \frac{1}{2}\;60.0\text{ kg}\,(1.33\text{ m/s})^2 - 60.0\text{ kg}\,(4.00\text{ m/s})^2 = \boxed{-427\text{ J}}$

(h) $\frac{1}{2}\,mv_f^2 - \frac{1}{2}\,mv_i^2 = \frac{1}{2}\,120\text{ kg}\,(1.33\text{ m/s})^2 - 0 = \boxed{107\text{ J}}$

(i) Equal friction forces act through different distances on person and cart, to do different amounts of work on them. The total work on both together, –320 J, becomes +320 J of internal energy in this perfectly inelastic collision.

Chapter 10 Solutions

10.1 (a) $\alpha = \dfrac{\omega - \omega_i}{t} = \dfrac{12.0\ \text{rad/s}}{3.00\ \text{s}} = \boxed{4.00\ \text{rad/s}^2}$

(b) $\theta = \omega_i t + \dfrac{1}{2}\alpha t^2 = \dfrac{1}{2}(4.00\ \text{rad/s}^2)(3.00\ \text{s})^2 = \boxed{18.0\ \text{rad}}$

10.2 (a) $\omega = \dfrac{2\pi\ \text{rad}}{365\ \text{days}}\dfrac{1\ \text{day}}{24\ \text{h}}\dfrac{1\ \text{h}}{3600\ \text{s}} = \boxed{1.99 \times 10^{-7}\ \text{rad/s}}$

(b) $\omega = \dfrac{2\pi\ \text{rad}}{27.3\ \text{days}}\dfrac{1\ \text{day}}{24\ \text{h}}\dfrac{1\ \text{h}}{3600\ \text{s}} = \boxed{2.65 \times 10^{-6}\ \text{rad/s}}$

***10.3** $\omega_i = 2000\ \text{rad/s}$

$\alpha = -80.0\ \text{rad/s}^2$

(a) $\omega = \omega_i + \alpha t = [2000 - (80.0)(10.0)] = \boxed{1200\ \text{rad/s}}$

(b) $0 = \omega_i + \alpha t$

$t = \dfrac{\omega_i}{-\alpha} = \dfrac{2000}{80.0} = \boxed{25.0\ \text{s}}$

10.4 (a) Let ω_h and ω_m be the angular speeds of the hour hand and minute hand, so that

$$\omega_h = \frac{2\pi\ \text{rad}}{12\ \text{h}} = \frac{\pi}{6}\ \text{rad/h} \quad \text{and} \quad \omega_m = 2\pi\ \text{rad/h}$$

Then if θ_h and θ_m are the angular positions of the hour hand and minute hand with respect to the 12 o'clock position, we have

$$\theta_h = \omega_h t \quad \text{and} \quad \theta_m = \omega_m t$$

For the two hands to coincide, we need $\theta_m = \theta_h + 2\pi n$, where n is a positive integer. Therefore, we may write $\omega_m t - \omega_h t = 2\pi n$, or

$$t_n = \frac{2\pi n}{\omega_m - \omega_h} = \frac{2\pi n}{2\pi - \frac{\pi}{6}} = \boxed{\frac{12n}{11}\ \text{h}}$$

Construct the following table:

n	t_n (h)	time (h:min:s)
0	0.00	12:00:00
1	1.09	1:05:27
2	2.18	2:10:55
3	3.27	3:16:22
4	4.36	4:21:49
5	5.45	5:27:16
6	6.55	6:32:44
7	7.64	7:38:11
8	8.73	8:43:38
9	9.82	9:49:05
10	10.91	10:54:33

(b) Let θ_s and ω_s be the angular position and angular speed of the second hand, then

$$\omega_s = 2\pi \text{ rad/min} = 120\pi \text{ rad/h} \quad \text{and} \quad \theta_s = \omega_s t$$

For all three hands to coincide, we need $\theta_s = \theta_m + 2\pi k$ (k is any positive integer) at any of the times given above. That is, we need

$$\omega_s t_n - \omega_m t_n = 2\pi k, \text{ or}$$

$$k = \frac{\omega_s - \omega_m}{2\pi} t_n = \frac{118}{2\pi}\frac{12}{11} n = \frac{(3)(4)(59)n}{11}$$

to be an integer. This is possible only for $n = 0$ or 11. Therefore, all three hands coincide $\boxed{\text{only when straight up at 12 o'clock}}$.

10.5 $\omega_i = \left(\frac{100 \text{ rev}}{1.00 \text{ min}}\right)\left(\frac{1.00 \text{ min}}{60.0 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \frac{10\pi}{3} \text{ rad/s}, \; \omega_f = 0$

(a) $t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 10\pi/3}{-2.00} \text{ s} = \boxed{5.24 \text{ s}}$

(b) $\theta = \bar{\omega} t = \left(\frac{\omega_f + \omega_i}{2}\right) t = \left(\frac{10\pi}{6} \text{ rad/s}\right)\left(\frac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$

***10.6** $\omega_i = 3600 \text{ rev/min} = 3.77 \times 10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14 \times 10^2 \text{ rad}$ and $\omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = (3.77 \times 10^2 \text{ rad/s})^2 + 2\alpha(3.14 \times 10^2 \text{ rad})$

$\alpha = \boxed{-2.26 \times 10^2 \text{ rad/s}^2}$

10.7 (a) $\theta\big|_{t=0} = \boxed{5.00\text{ rad}}$

$$\omega\big|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0\text{ rad/s}}$$

$$\alpha_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00\text{ rad/s}^2}$$

(b) $\theta\big|_{t=3.00\text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0\text{ rad}}$

$$\omega\big|_{t=3.00\text{ s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00\text{s}} = 10.0 + 4.00t\big|_{t=3.00\text{ s}} = \boxed{22.0\text{ rad/s}}$$

$$\alpha\big|_{t=3.00\text{ s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00\text{s}} = \boxed{4.00\text{ rad/s}^2}$$

***10.8** $\omega = 5.00\text{ rev/s} = 10.0\pi\text{ rad/s}$

We will break the motion into two stages: (1) an acceleration period and (2) a deceleration period.

While speeding up,

$$\theta_1 = \bar{\omega}t = \frac{0 + 10.0\pi\text{ rad/s}}{2}(8.00\text{ s}) = 40.0\pi\text{ rad}$$

While slowing down,

$$\theta_2 = \bar{\omega}t = \frac{10.0\pi\text{ rad/s} + 0}{2}(12.0\text{ s}) = 60.0\pi\text{ rad}$$

So, $\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi\text{ rad} = \boxed{50.0\text{ rev}}$

10.9 $\theta - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega = \omega_i + \alpha t$

are two equations in two unknowns ω_i and α.

$$\omega_i = \omega - \alpha t$$

$$\theta - \theta_i = (\omega - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega t - \frac{1}{2}\alpha t^2$$

$$37.0\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = (98.0\text{ rad/s})(3.00\text{ s}) - \frac{1}{2}\alpha(3.00\text{ s})^2$$

$$232\text{ rad} = 294\text{ rad} - (4.50\text{ s}^2)\alpha$$

$$\alpha = \frac{61.5 \text{ rad}}{4.50 \text{ s}^2} = \boxed{13.7 \text{ rad/s}^2}$$

***10.10** (a) $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{1\text{ rev}}{1\text{ day}} = \dfrac{2\pi\text{ rad}}{86400\text{ s}} = \boxed{7.27\times10^{-5}\text{ rad/s}}$

(b) $\Delta t = \dfrac{\Delta\theta}{\omega} = \dfrac{107°}{7.27\times10^{-5}\text{ rad/s}}\left(\dfrac{2\pi\text{ rad}}{360°}\right) = \boxed{2.57\times10^{4}\text{ s}}$ (428 min)

***10.11** Estimate the tire's radius at 0.250 m and miles driven as 10,000 per year.

$$\theta = \frac{s}{r} = \frac{1.00\times10^{4}\text{ mi}}{0.250\text{ m}}\left(\frac{1609\text{ m}}{1\text{ mi}}\right) = 6.44\times10^{7}\,\frac{\text{rad}}{\text{yr}}$$

$$\theta = 6.44\times10^{7}\,\frac{\text{rad}}{\text{yr}}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 1.02\times10^{7}\,\frac{\text{rev}}{\text{yr}}$$

or $\boxed{\sim10^{7}\,\dfrac{\text{rev}}{\text{yr}}}$

***10.12** Main Rotor:

$$v = r\omega = (3.80\text{ m})\left(450\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{179\text{ m/s}}$$

$$v = \left(179\,\frac{\text{m}}{\text{s}}\right)\left(\frac{v_{\text{sound}}}{343\text{ m/s}}\right) = \boxed{0.522 v_{\text{sound}}}$$

Tail Rotor:

$$v = r\omega = (0.510\text{ m})\left(4138\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{221\text{ m/s}}$$

$$v = \left(221\,\frac{\text{m}}{\text{s}}\right)\left(\frac{v_{\text{sound}}}{343\text{ m/s}}\right) = \boxed{0.644 v_{\text{sound}}}$$

10.13 (a) $v = r\omega$; $\omega = \dfrac{v}{r} = \dfrac{45.0\text{ m/s}}{250\text{ m}} = \boxed{0.180\text{ rad/s}}$

(b) $a_r = \dfrac{v^2}{r} = \dfrac{(45.0\text{ m/s})^2}{250\text{ m}} = \boxed{8.10\text{ m/s}^2\text{ toward the center of track}}$

10.14 $v = 36.0\,\dfrac{\text{km}}{\text{h}}\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)\left(\dfrac{10^{3}\text{ m}}{1\text{ km}}\right) = 10.0\text{ m/s}$

$$\omega = \frac{v}{r} = \frac{10.0\text{ m/s}}{0.250\text{ m}} = \boxed{40.0\text{ rad/s}}$$

10.15 Given $r = 1.00$ m, $\alpha = 4.00\ \text{rad/s}^2$, $\omega_i = 0$, and $\theta_i = 57.3° = 1.00$ rad

(a) $\omega = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00$ s, $\omega = (4.00\ \text{rad/s}^2)(2.00\ \text{s}) = \boxed{8.00\ \text{rad/s}}$

(b) $v = r\omega = (1.00\ \text{m})(8.00\ \text{rad/s}) = \boxed{8.00\ \text{m/s}}$

$a_r = r\omega^2 = (1.00\ \text{m})(8.00\ \text{rad/s})^2 = \boxed{64.0\ \text{m/s}^2}$

$a_t = r\alpha = (1.00\ \text{m})(4.00\ \text{rad/s}^2) = \boxed{4.00\ \text{m/s}^2}$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(64.0\ \text{m/s}^2)^2 + (4.00\ \text{m/s}^2)^2} = 64.1\ \text{m/s}^2$$

The direction of the total acceleration vector makes an angle ϕ with respect to the radius to point P:

$$\phi = \tan^{-1}\left(\frac{a_t}{a_r}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = 3.58°$$

(c) $\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00\ \text{rad}) + \frac{1}{2}(4.00\ \text{rad/s}^2)(2.00\ \text{s})^2 = \boxed{9.00\ \text{rad}}$

10.16 (a) $\omega = \dfrac{v}{r} = \dfrac{25.0\ \text{m/s}}{1.00\ \text{m}} = \boxed{25.0\ \text{rad/s}}$

(b) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0\ \text{rad/s})^2 - 0}{2[(1.25\ \text{rev})(2\pi\ \text{rad/rev})]} = \boxed{39.8\ \text{rad/s}^2}$$

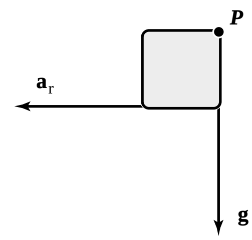
(c) $\Delta t = \dfrac{\Delta\omega}{\alpha} = \dfrac{25.0\ \text{rad/s}}{39.8\ \text{rad/s}^2} = \boxed{0.628\ \text{s}}$

10.17 (a) $s = \bar{v}t = (11.0\ \text{m/s})(9.00\ \text{s}) = 99.0$ m

$$\theta = \frac{s}{r} = \frac{99.0\ \text{m}}{0.290\ \text{m}} = 341\ \text{rad} = \boxed{54.3\ \text{rev}}$$

(b) $\omega = \dfrac{v}{r} = \dfrac{22.0\ \text{m/s}}{0.290\ \text{m}} = 75.9\ \text{rad/s} = \boxed{12.1\ \text{rev/s}}$

10.18 $K_A + U_A = K_p + U_p$



$$6.00 \times 9.80 \times 5.00 = \frac{1}{2}(6.00)\, v_p^2 + 6.00 \times 9.80 \times 2.00$$

$$v_p^2 = 58.8\ \text{m}^2/\text{s}^2$$

Radial acceleration at P,

$$a_r = \frac{v_p^2}{R} = \boxed{29.4\ \text{m}/\text{s}^2}$$

Tangential acceleration at P,

$$a_t = g = \boxed{9.80\ \text{m}/\text{s}^2}$$

10.19 (a) $\omega = 2\pi f = \dfrac{2\pi\ \text{rad}}{\text{rev}}\dfrac{1200}{60.0}\ \text{rev}/\text{s} = \boxed{126\ \text{rad}/\text{s}}$

(b) $v = \omega r = (126\ \text{rad}/\text{s})(3.00 \times 10^{-2}\ \text{m}) = \boxed{3.77\ \text{m}/\text{s}}$

(c) $a_r = \omega^2 r = (126)^2(8.00 \times 10^{-2}) = \boxed{1260\ \text{m}/\text{s}^2} = \boxed{1.26\ \text{km}/\text{s}^2}$

(d) $s = \theta r = \omega\, t\, r = (126\ \text{rad}/\text{s})(2.00\ \text{s})(8.00 \times 10^{-2}\ \text{m}) = \boxed{20.1\ \text{m}}$

10.20 Just before it starts to skid,

$$\sum F_r = m a_r$$

$$f = \frac{mv^2}{r} = \mu_s n = \mu_s mg$$

$$\mu_s = \frac{v^2}{rg} = \frac{\omega^2 r}{g} = \frac{(\omega^2 - \omega_i^2) r}{g} = \frac{2\alpha\theta r}{g} = \frac{2a_t\theta}{g}$$

$$\mu_s = \frac{2(1.70\ \text{m}/\text{s}^2)(\pi/2)}{9.80\ \text{m}/\text{s}^2} = \boxed{0.545}$$

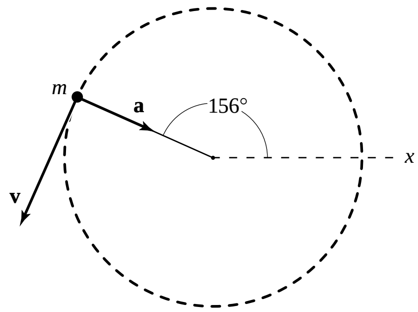
***10.21** (a) $x = r\cos\theta = (3.00\ \text{m})\cos(9.00\ \text{rad}) = (3.00\ \text{m})\cos 516° = -2.73\ \text{m}$

$y = r\sin\theta = (3.00\ \text{m})\sin(9.00\ \text{rad}) = (3.00\ \text{m})\sin 516° = 1.24\ \text{m}$

$\mathbf{r} = x\mathbf{i} + y\mathbf{j} = \boxed{(-2.73\mathbf{i} + 1.24\mathbf{j})\ \text{m}}$

(b) $516° - 360° = 156°$. This is between 90.0° and 180°, so the object is in the $\boxed{\text{second quadrant}}$. The vector $\mathbf{r}$ makes an angle of $\boxed{156°}$ with the positive x-axis or 24.3° with the negative x-axis.

(d) The direction of motion (i.e., the direction of the velocity vector is at $156° + 90.0° = \boxed{246°}$ from the positive x axis. The direction of the acceleration vector is at $156° + 180° = 336°$ from the positive x axis.



(c) $\mathbf{v} = [(4.50 \text{ m/s}) \cos 246°]\mathbf{i} + [(4.50 \text{ m/s}) \sin 246°]\mathbf{j}$

$$= \boxed{(-1.85\mathbf{i} - 4.10\mathbf{j}) \text{ m/s}}$$

(e) $a = \dfrac{v^2}{r} = \dfrac{(4.50 \text{ m/s})^2}{3.00 \text{ m}} = 6.75 \text{ m/s}^2$ directed toward the center or at 336°

$$\mathbf{a} = (6.75 \text{ m/s}^2)(\mathbf{i} \cos 336° + \mathbf{j} \sin 336°) = \boxed{(6.15\mathbf{i} - 2.78\mathbf{j}) \text{ m/s}^2}$$

(f) $\sum \mathbf{F} = m\mathbf{a} = (4.00 \text{ kg})[(6.15\mathbf{i} - 2.78\mathbf{j}) \text{ m/s}^2] = \boxed{(24.6\mathbf{i} - 11.1\mathbf{j}) \text{ N}}$

***10.22** When completely rewound, the tape is a hollow cylinder with a difference between the inner and outer radii of ~1 cm. Let N represent the number of revolutions through which the driving spindle turns in 30 minutes (and hence the number of layers of tape on the spool). We can determine N from:

$$N = \frac{\Delta\theta}{2\pi} = \frac{\omega(\Delta t)}{2\pi} = \frac{(1 \text{ rad/s})(30 \text{ min})(60 \text{ s/min})}{2\pi \text{ rad/rev}} = 286 \text{ rev}$$

Then, *thickness* $\sim \dfrac{1 \text{ cm}}{N} \boxed{\sim 10^{-2} \text{ cm}}$

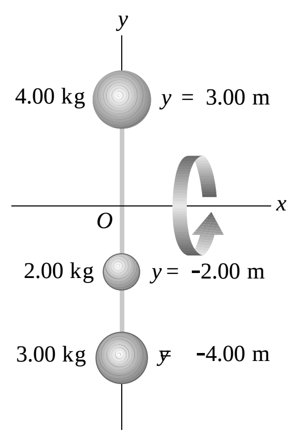
10.23 $m_1 = 4.00$ kg, $r_1 = y_1 = 3.00$ m; $m_2 = 2.00$ kg, $r_2 = |y_2| = 2.00$ m;

$m_3 = 3.00$ kg, $r_3 = |y_3| = 4.00$ m; $\omega = 2.00$ rad/s about the x-axis

(a) $I_x = m_1 r_1^2 + m_2 r_2^2 + m_1 r_2^2 = (4.00)(3.00)^2 + (2.00)(2.00)^2 + (3.00)(4.00)^2$

$I_x = \boxed{92.0 \text{ kg} \cdot \text{m}^2}$

$K_R = \dfrac{1}{2} I_x \omega^2 = \dfrac{1}{2}(92.0)(2.00)^2 = \boxed{184 \text{ J}}$



(b) $v_1 = r_1\omega = (3.00)(2.00) = \boxed{6.00 \text{ m/s}}$

$v_2 = r_2\omega = (2.00)(2.00) = \boxed{4.00 \text{ m/s}}$

$v_3 = r_3\omega = (4.00)(2.00) = \boxed{8.00 \text{ m/s}}$

$$K_1 = \frac{1}{2}\ m_1 v_1^2\ = \frac{1}{2}(4.00)(6.00)^2 = 72.0\ \text{J}$$

$$K_2 = \frac{1}{2}\ m_2 v_2^2\ = \frac{1}{2}(2.00)(4.00)^2 = 16.0\ \text{J}$$

$$K_3 = \frac{1}{2}\ m_3 v_3^2\ = \frac{1}{2}(3.00)(8.00)^2 = 96.0\ \text{J}$$

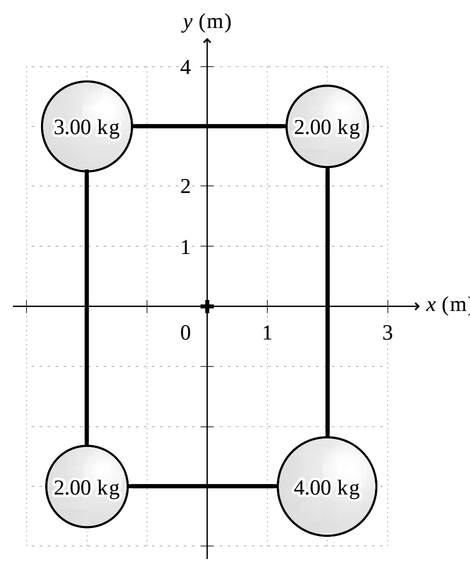
$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184\ \text{J}} = \frac{1}{2}\ I_x \omega^2$$

10.24 $v = 38.0\ \text{m/s}$ $\omega = 125\ \text{rad/s}$

$$\text{RATIO} = \frac{\frac{1}{2} I\omega^2}{\frac{1}{2} m v^2} = \frac{\frac{1}{2}\left(\frac{2}{5} m r^2\right)\omega^2}{\frac{1}{2} m v^2}$$

$$\text{RATIO} = \frac{\frac{2}{5}(3.80 \times 10^{-2})^2\ (125)^2}{(38.0)^2} = \boxed{\frac{1}{160}}$$

10.25 (a) $I = \sum_j m_j r_j^2$



In this case,

$$r_1 = r_2 = r_3 = r_4$$

$$r = \sqrt{(3.00\ \text{m})^2 + (2.00\ \text{m})^2} = \sqrt{13.0}\ \text{m}$$

$$I = [\sqrt{13.0}\ \text{m}]^2\ [3.00 + 2.00 + 2.00 + 4.00]\text{kg}$$

$$= \boxed{143\ \text{kg} \cdot \text{m}^2}$$

(b) $K_R = \frac{1}{2}\ I\omega^2 = \frac{1}{2}(143\ \text{kg} \cdot \text{m}^2)(6.00\ \text{rad/s})^2$

$$= \boxed{2.57 \times 10^3\ \text{J}}$$

10.26 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}\ ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}\ I_h \omega^2{}_h + \frac{1}{2}\ I_m \omega^2{}_m\text{, with}$$

$$I_h = \frac{m_h L_h^2}{3} = \frac{(60.0\ \text{kg})(2.70\ \text{m})^2}{3} = 146\ \text{kg m}^2\text{, and}$$

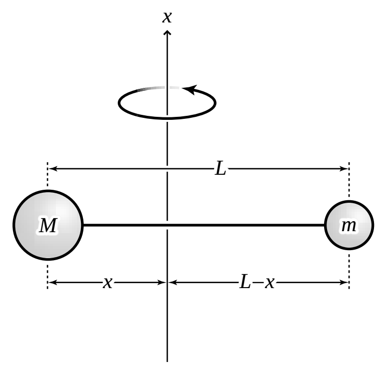
$$I_m = \frac{m_m L_m^2}{3} = \frac{(100\ \text{kg})(4.50\ \text{m})^2}{3} = 675\ \text{kg m}^2$$

In addition, $\omega_h = \frac{(2\pi \text{ rad})}{(12 \text{ h})}\frac{1 \text{ h}}{3600 \text{ s}} = 1.45 \times 10^{-4} \text{ rad/s}$, while

$$\omega_m = \frac{(2\pi \text{ rad})}{(1 \text{ h})}\frac{1 \text{ h}}{3600 \text{ s}} = 1.75 \times 10^{-3} \text{ rad/s}.$$ Therefore,

$$K_R = \frac{1}{2}(146)(1.45 \times 10^{-4})^2 + \frac{1}{2}(675)(1.75 \times 10^{-3})^2 = \boxed{1.04 \times 10^{-3} \text{ J}}$$

10.27 $I = Mx^2 + m(L - x)^2$



$$\frac{dI}{dx} = 2Mx - 2m(L - x) = 0 \text{ (for an extremum)}$$

$$\therefore x = \frac{mL}{M + m}$$

$\frac{d^2I}{dx^2} = 2m + 2M$; therefore I is minimum when the axis of rotation passes through $x = \frac{mL}{M + m}$

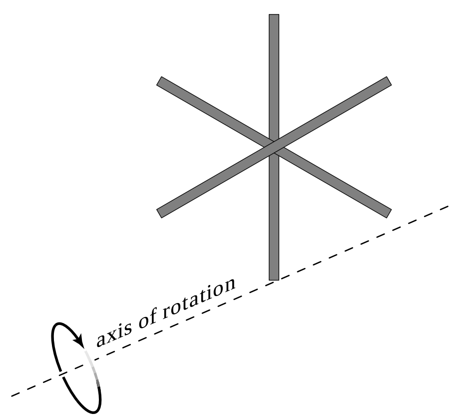
which is also the center of mass of the system. The moment of inertia about an axis passing through x is

$$I_{\text{CM}} = M\left[\frac{mL}{M + m}\right]^2 + m\left[1 - \frac{m}{M + m}\right]^2 L^2$$

$$= \frac{Mm}{M + m}L^2 = \mu L^2$$

where $\mu = \frac{Mm}{M + m}$

10.28 We assume the rods are thin, with radius much less than L. Call the junction of the rods the origin of coordinates, and the axis of rotation the z-axis.



For the rod along the y-axis, $I = \frac{1}{3}mL^2$ from the table.

For the rod parallel to the z-axis, the parallel-axis theorem gives

$$I = \frac{1}{2}mr^2 + m\left(\frac{L}{2}\right)^2 \cong \frac{1}{4}mL^2$$

In the rod along the x-axis, the bit of material between x and $x + dx$ has mass $(m/L)dx$ and is at distance $r = \sqrt{x^2 + (L/2)^2}$ from the axis of rotation. The total rotational inertia is:

$$I_{\text{total}} = \frac{1}{3}mL^2 + \frac{1}{4}mL^2 + \int_{-L/2}^{L/2}(x^2 + L^2/4)(m/L)dx$$

$$= \frac{7}{12}mL^2 + \left(\frac{m}{L}\right)\frac{x^3}{3}\Bigg|_{-L/2}^{L/2} + \frac{mL}{4}x\Bigg|_{-L/2}^{L/2}$$

$$= \frac{7}{12}mL^2 + \frac{mL^2}{12} + \frac{mL^2}{4} = \boxed{\frac{11\,mL^2}{12}}$$

***10.29** Treat the tire as consisting of three parts. The two sidewalls are each treated as a hollow cylinder of inner radius 16.5 cm, outer radius 30.5 cm, and height 0.635 cm. The tread region is treated as a hollow cylinder of inner radius 30.5 cm, outer radius 33.0 cm, and height 20.0 cm.

Use $I = \frac{1}{2}m(R_1^2 + R_2^2)$ for the moment of inertia of a hollow cylinder.

Sidewall:

$$m = \pi[(0.305\text{ m})^2 - (0.165\text{ m})^2](6.35\times10^{-3}\text{ m})(1.10\times10^3\text{ kg/m}^3) = 1.44\text{ kg}$$

$$I_{\text{side}} = \frac{1}{2}(1.44\text{ kg})\ [(0.165\text{ m})^2 + (0.305\text{ m})^2] = 8.68\times10^{-2}\text{ kg}\cdot\text{m}^2$$

Tread:

$$m = \pi[(0.330\text{ m})^2 - (0.305\text{ m})^2](0.200\text{ m})(1.10\times10^3\text{ kg/m}^3) = 11.0\text{ kg}$$

$$I_{\text{tread}} = \frac{1}{2}(11.0\text{ kg})\ [(0.330\text{ m})^2 + (0.305\text{ m})^2] = 1.11\text{ kg}\cdot\text{m}^2$$

Entire Tire:

$$I_{\text{total}} = 2I_{\text{side}} + I_{\text{tread}} = 2(8.68\times10^{-2}\text{ kg}\cdot\text{m}^2) + 1.11\text{ kg}\cdot\text{m}^2 = \boxed{1.28\text{ kg}\cdot\text{m}^2}$$

10.30 (a) $I = I_{\text{CM}} + MD^2 = \frac{1}{2}MR^2 + MR^2 = \boxed{\frac{3}{2}MR^2}$

(b) $I = I_{\text{CM}} + MD^2 = \frac{2}{5}MR^2 + MR^2 = \boxed{\frac{7}{5}MR^2}$

***10.31** Model your body as a cylinder of mass 60.0 kg and circumference 75.0 cm. Then its radius is

$$\frac{0.750\text{ m}}{2\pi} = 0.120\text{ m}$$

and its moment of inertia is

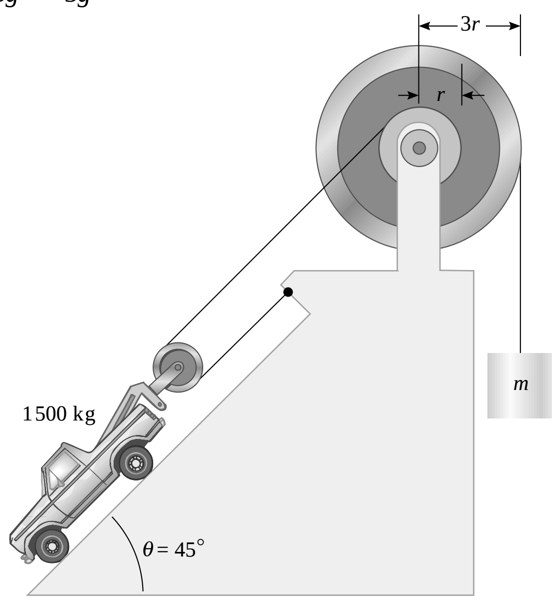
$$\frac{1}{2}MR^2 = \frac{1}{2}(60.0\text{ kg})(0.120\text{ m})^2 = 0.432\text{ kg m}^2 \sim \boxed{10^0\text{ kg}\cdot\text{m}^2 = 1\text{ kg}\cdot\text{m}^2}$$

10.32 $\sum\tau = 0 = mg(3r) - Tr$

$2T - Mg\sin 45.0° = 0$

$$T = \frac{Mg\sin 45.0°}{2} = \frac{1500\text{ kg}(g)\sin 45.0°}{2} = (530)(9.80)\text{ N}$$

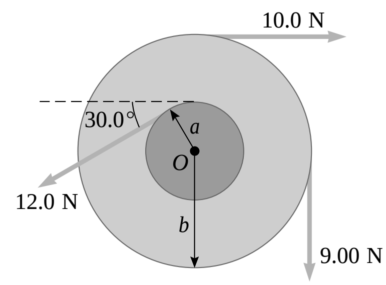
$$m = \frac{T}{3g} = \frac{530g}{3g} = \boxed{177\text{ kg}}$$



10.33 $\sum \tau$ = (0.100 m)(12.0 N) – (0.250 m)(9.00 N) – (0.250 m)(10.0 N)

= $\boxed{-3.55 \text{ N} \cdot \text{m}}$

The thirty-degree angle is unnecessary information.



Goal Solution

G: By simply examining the magnitudes of the forces and their respective lever arms, it appears that the wheel will rotate clockwise, and the net torque appears to be about 5 Nm.

O: To find the net torque, we simply add the individual torques, remembering to apply the convention that a torque producing clockwise rotation is negative and a counterclockwise torque is positive.

A: $\sum \tau = \sum Fd$

$\sum \tau$ = (12.0 N)(0.100 m) – (10.0 N)(0.250 m) – (9.00 N)(0.250 m)

$\sum \tau$ = –3.55 N · m

The minus sign means perpendicularly into the plane of the paper, or it means clockwise.

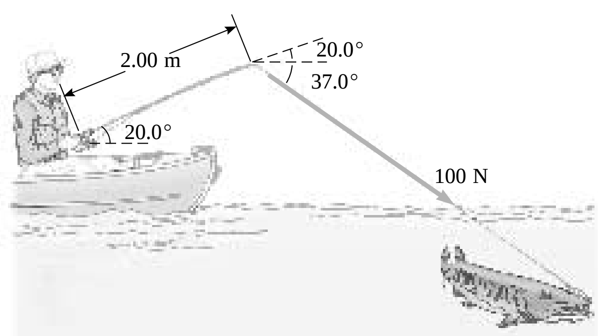
L: The resulting torque has a reasonable magnitude and produces clockwise rotation as expected. Note that the 30° angle was not required for the solution since each force acted perpendicular to its lever arm. The 10-N force is to the right, but its torque is negative – that is, clockwise, just like the torque of the downward 9-N force.

10.34 Resolve the 100 N force into components perpendicular to and parallel to the rod, as

$$F_{\text{par}} = (100 \text{ N}) \cos 57.0° = 54.5 \text{ N}$$

and

$$F_{\text{perp}} = (100 \text{ N}) \sin 57.0° = 83.9 \text{ N}$$



Torque of F_{par} = 0 since its line of action passes through the pivot point.

Torque of F_{perp} is

$$\tau = (83.9 \text{ N})(2.00 \text{ m}) = \boxed{168 \text{ N} \cdot \text{m}} \text{ (clockwise)}$$

***10.35** The normal force exerted by the ground on each wheel is

$$n = \frac{mg}{4} = \frac{(1500\text{ kg})(9.80\text{ m/s}^2)}{4} = 3680\text{ N}$$

The torque of friction can be as large as

$$\tau_{\text{max}} = f_{\text{max}}r = (\mu_s n)r = (0.800)(3680\text{ N})(0.300\text{ m}) = \boxed{882\text{ N}\cdot\text{m}}$$

The torque of the axle on the wheel can be equally as large as the light wheel starts to turn without slipping.

***10.36** We calculated the maximum torque that can be applied without skidding in Problem 35 to be 882 N · m. This same torque is to be applied by the frictional force, f, between the brake pad and the rotor for this wheel. Since the wheel is slipping against the brake pad, we use the coefficient of kinetic friction to calculate the normal force.

$$\tau = fr = (\mu_k n)r\text{, so } n = \frac{\tau}{\mu_k r} = \frac{882\text{ N}\cdot\text{m}}{(0.500)(0.220\text{ m})} = 8.02\times10^3\text{ N} = \boxed{8.02\text{ kN}}$$

10.37 $m = 0.750\text{ kg}$ $F = 0.800\text{ N}$

(a) $\tau = rF = (30.0\text{ m})(0.800\text{ N}) = \boxed{24.0\text{ N}\cdot\text{m}}$

(b) $\alpha = \frac{\tau}{I} = \frac{rF}{mr^2} = \frac{24.0}{(0.750)(30.0)^2} = \boxed{0.0356\text{ rad/s}^2}$

(c) $a_T = \alpha r = (0.0356)(30.0) = \boxed{1.07\text{ m/s}^2}$

***10.38** $\tau = 36.0\text{ N}\cdot\text{m} = I\alpha$ $\omega_f = \omega_i + \alpha t$

$10.0\text{ rad/s} = 0 + \alpha(6.00\text{ s})$

$$\alpha = \frac{10.00}{6.00}\text{ rad/s}^2 = 1.67\text{ rad/s}^2$$

(a) $I = \frac{\tau}{\alpha} = \frac{36.0\text{ N}\cdot\text{m}}{1.67\text{ rad/s}^2} = \boxed{21.6\text{ kg}\cdot\text{m}^2}$

(b) $\omega_f = \omega_i + \alpha t$

$0 = 10.0 + \alpha(60.0)$

$\alpha = -0.167\text{ rad/s}^2$

$\tau = I\alpha = (21.6\text{ kg}\cdot\text{m})(0.167\text{ rad/s}^2) = \boxed{3.60\text{ N}\cdot\text{m}}$

(c) Number of revolutions

$$\theta = \theta_i + \omega_i t + \frac{1}{2}\ \alpha\, t^2$$

During first 6.00 s

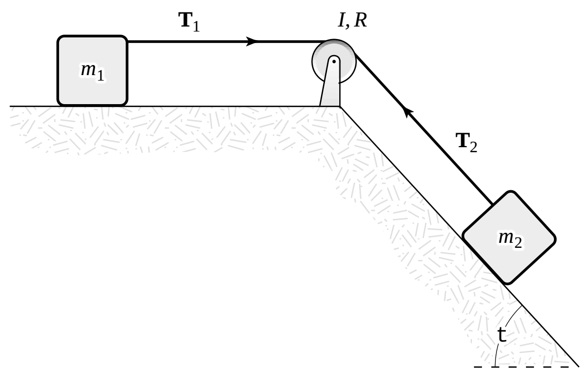
$$\theta = \frac{1}{2}(1.67)(6.00)^2 = 30.1 \text{ rad}$$

During next 60.0 s

$$\theta = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299 \text{ rad}$$

$$\theta_{\text{total}} = (329 \text{ rad})\frac{\text{rev}}{2\pi \text{ rad}} = \boxed{52.4 \text{ rev}}$$

10.39 For m_1: $\sum F_y = ma_y$ $\quad +n - m_1 g = 0$



$n = m_1 g = 19.6$ N

$f_k = \mu_k n = 7.06$ N

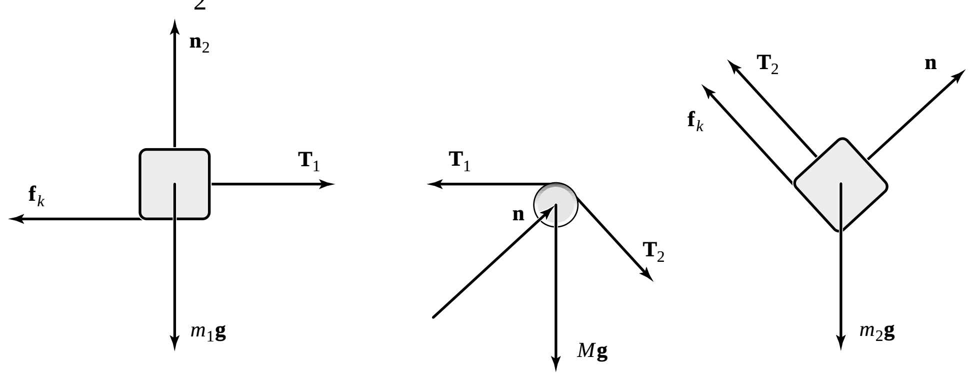
$\sum F_x = ma_x \qquad -7.06 \text{ N} + T_1 = (2.00 \text{ kg})a \quad (1)$

For the pulley

$\sum \tau = I\alpha$

$$-T_1 R + T_2 R = \frac{1}{2}\ MR^2\left(\frac{a}{R}\right)$$

$$-T_1 + T_2 = \frac{1}{2}(10.0 \text{ kg})\, a \qquad -T_1 + T_2 = (5.00 \text{ kg})a \quad (2)$$



For m_2: $+n - m_2 g \cos\theta = 0$

$$n = 6.00\text{ kg}(9.80\text{ m/s}^2)\cos 30.0° = 50.9\text{ N}$$

$$f_k = \mu_k n = 18.3\text{ N}$$

$$-18.3\text{ N} - T_2 + m_2 g \sin\theta = m_2 a$$

$$-18.3\text{ N} - T_2 + 29.4\text{ N} = (6.00\text{ kg})a \qquad (3)$$

(a) Add equations (1) (2) and (3):

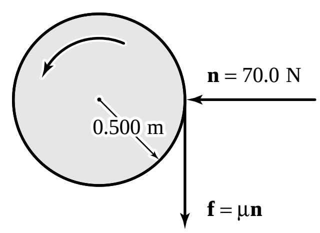
$$-7.06\text{ N} - 18.3\text{ N} + 29.4\text{ N} = (13.0\text{ kg})a$$

$$a = \frac{4.01\text{ N}}{13.0\text{ kg}} = \boxed{0.309\text{ m/s}^2}$$

(b) $T_1 = 2.00\text{ kg}(0.309\text{ m/s}^2) + 7.06\text{ N} = \boxed{7.67\text{ N}}$

$$T_2 = 7.67\text{ N} + 5.00\text{ kg}(0.309\text{ m/s}^2) = \boxed{9.22\text{ N}}$$

10.40 $I = \frac{1}{2}mR^2 = \frac{1}{2}(100\text{ kg})(0.500\text{ m})^2 = 12.5\text{ kg}\cdot\text{m}^2$



$$\omega_i = 50.0\text{ rev/min} = 5.24\text{ rad/s}$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 5.24\text{ rad/s}}{6.00\text{ s}} = -0.873\text{ rad/s}^2$$

$$\tau = I\alpha = (12.5\text{ kg}\cdot\text{m}^2)(-0.873\text{ rad/s}^2) = -10.9\text{ N}\cdot\text{m}$$

The magnitude of the torque is given by $fR = 10.9\text{ N}\cdot\text{m}$, where f is the force of friction.

Therefore, $f = \frac{10.9\text{ N}\cdot\text{m}}{0.500\text{ m}}$, and

$f = \mu_k n$ yields $\mu_k = \frac{f}{n} = \frac{21.8\text{ N}}{70.0\text{ N}} = \boxed{0.312}$

10.41 $I = MR^2 = 1.80\text{ kg}(0.320\text{ m})^2 = 0.184\text{ kg}\cdot\text{m}^2$

$$\sum\tau = I\alpha$$

(a) $F_a(4.50\times10^{-2}\text{ m}) - 120\text{ N}(0.320\text{ m}) = 0.184\text{ kg}\cdot\text{m}^2(4.50\text{ rad/s}^2)$

$$F_a = \frac{(0.829\text{ N}\cdot\text{m} + 38.4\text{ N}\cdot\text{m})}{4.50\times10^{-2}\text{ m}} = \boxed{872\text{ N}}$$

(b) $F_b(2.80\times10^{-2}\text{ m}) - 38.4\text{ N}\cdot\text{m} = 0.829\text{ N}\cdot\text{m}$

$$F_b = \boxed{1.40\text{ kN}}$$

10.42 We assume the rod is thin. For the compound object

$$I = \frac{1}{3}\ M_{\text{rod}}L^2 + \left[\frac{2}{5}M_{\text{ball}}R^2 + M_{\text{ball}}D^2\right]$$

$$I = \frac{1}{3}\ 1.20\text{ kg }(0.240\text{ m})^2 + \frac{2}{5}\ 20.0\text{ kg}(4.00 \times 10^{-2}\text{ m})^2 + 20.0\text{ kg}(0.280\text{ m})^2$$

$$I = 1.60\text{ kg}\cdot\text{m}^2$$

(a) $K_f + U_f = K_i + U_i + \Delta E$

$$\frac{1}{2}\ I\omega^2 + 0 = 0 + M_{\text{rod}}g(L/2) + M_{\text{ball}}g(L + R) + 0$$

$$\frac{1}{2}(1.60\text{ kg}\cdot\text{m}^2)\ \omega^2 = 1.20\text{ kg}(9.80\text{ m/s}^2)(0.120\text{ m}) + 20.0\text{ kg}(9.80\text{ m/s}^2)(0.280\text{ m})$$

$$\frac{1}{2}(1.60\text{ kg}\cdot\text{m}^2)\ \omega^2 = \boxed{56.3\text{ J}}$$

(b) $\omega = \boxed{8.38\text{ rad/s}}$

(c) $v = r\omega = (0.280\text{ m})8.38\text{ rad/s} = \boxed{2.35\text{ m/s}}$

(d) $v^2 = v_i^2\ + 2a(y - y_i)$

$$v = \sqrt{0 + 2(9.80\text{ m/s}^2)(0.280\text{ m})} = 2.34\text{ m/s}$$

The speed it attains in swinging is greater by $2.35/2.34 = \boxed{1.00140\text{ times}}$

10.43 Choose the zero gravitational potential energy at the level where the masses pass.

$$K_f + U_{gf} = K_i + U_{gi} + \Delta E$$

$$\frac{1}{2}\ m_1v^2 + \frac{1}{2}\ m_2v^2 + \frac{1}{2}\ I\omega^2 = 0 + m_1gh_{1i} + m_2gh_{2i} + 0$$

$$\frac{1}{2}(15.0 + 10.0)\ v^2 + \frac{1}{2}\left[\frac{1}{2}(3.00)R^2\right]\left(\frac{v}{R}\right)^2 = 15.0(9.80)(1.50) + 10.0(9.80)(-1.50)$$

$$\frac{1}{2}(26.5\text{ kg})\ v^2 = 73.5\text{ J} \Rightarrow v = \boxed{2.36\text{ m/s}}$$

10.44 Choose the zero gravitational potential energy at the level where the masses pass.

$$K_f + U_{gf} = K_i + U_{gi} + \Delta E$$

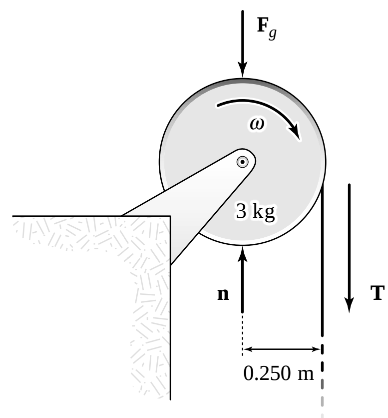
$$\frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I\omega^2 = 0 + m_1 g h_{1i} + m_2 g h_{2i} + 0$$

$$\frac{1}{2}(m_1 + m_2)\, v^2 + \frac{1}{2}\left[\frac{1}{2} MR^2\right]\left(\frac{v}{R}\right)^2 = m_1 g\left(\frac{d}{2}\right) + m_2 g\left(-\frac{d}{2}\right)$$

$$\frac{1}{2}\left(m_1 + m_2 + \frac{1}{2} M\right) v^2 = (m_1 - m_2) g\left(\frac{d}{2}\right)$$

$$v = \boxed{\sqrt{\frac{(m_1 - m_2) g d}{m_1 + m_2 + \frac{1}{2} M}}}$$

10.45 (a) $50.0 - T = \dfrac{50.0}{9.80}\, a$



$$TR = I\alpha = I\frac{a}{R}$$

$$I = \frac{1}{2} MR^2 = 0.0938 \text{ kg} \cdot \text{m}^2$$

$$50.0 - T = 5.10\left(\frac{TR^2}{I}\right)$$

$$T = \boxed{11.4 \text{ N}}$$

$$a = \frac{50.0 - 11.4}{5.10} = \boxed{7.57 \text{ m/s}^2}$$

$$v = \sqrt{2a(y_i - 0)} = \boxed{9.53 \text{ m/s}}$$

(b) Use conservation of energy:

$$(K+U)_i = (K+U)_f$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2mgh = mv^2 + I\left(\frac{v^2}{R^2}\right)$$

$$= v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}} = \sqrt{\frac{2(50.0\text{ N})(6.00\text{ m})}{5.10\text{ kg} + \frac{0.0938}{(0.250)^2}}} = \boxed{9.53\text{ m/s}}$$

Goal Solution

G: Since the rotational inertia of the reel will slow the fall of the weight, we should expect the downward acceleration to be less than g. If the reel did not rotate, the tension in the string would be equal to the weight of the object; and if the reel disappeared, the tension would be zero. Therefore, $T < mg$ for the given problem. With similar reasoning, the final speed must be less than if the weight were to fall freely: $v_f < \sqrt{2gy} \approx 11\text{ m/s}$

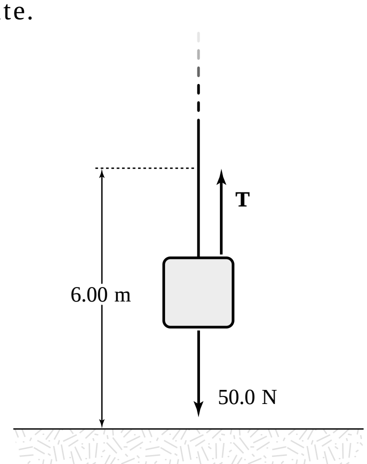
O: We can find the acceleration and tension using the rotational form of Newton's second law. The final speed can be found from the kinematics equation stated above and from conservation of energy. Free-body diagrams will greatly assist in analyzing the forces.

A: (a) Use $\sum\tau = I\alpha$ to find T and a.

First find I for the reel, which we assume to be a uniform disk:

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\,3.00\text{ kg}\,(0.250\text{ m})^2 = 0.0938\text{ kg}\cdot\text{m}^2$$

The forces on the reel are shown, including a normal force exerted by its axle. From the diagram, we can see that the tension is the only unbalanced force causing the reel to rotate.



$\sum\tau = I\alpha$ becomes

$$n(0) + F_g(0) + T(0.250\text{ m}) = (0.0938\text{ kg}\cdot\text{m}^2)(a/0.250\text{ m}) \qquad (1)$$

where we have applied $a_t = r\alpha$ to the point of contact between string and reel.

The falling weight has mass

$$m = \frac{F_g}{g} = \frac{50.0\text{ N}}{9.80\text{ m/s}^2} = 5.10\text{ kg}$$

For this mass, $\sum F_y = ma_y$ becomes

$$+T - 50.0\text{ N} = (5.10\text{ kg})(-a) \qquad (2)$$

Note that since we have defined upwards to be positive, the minus sign shows that its acceleration is downward. We now have our two equations in the unknowns T and a for the two linked objects. Substituting T from equation (2) into equation (1), we have:

$$[50.0\text{ N} - (5.10\text{ kg})a](0.250\text{ m}) = 0.0938\text{ kg}\cdot\text{m}^2\,\frac{a}{0.250\text{ m}}$$

$$12.5\text{ N}\cdot\text{m} - (1.28\text{ kg}\cdot\text{m})a = (0.375\text{ kg}\cdot\text{m})a$$

$$12.5\text{ N}\cdot\text{m} = a(1.65\text{ kg}\cdot\text{m}) \quad\text{or}\quad a = 7.57\text{ m/s}^2$$

and $T = 50.0\text{ N} - 5.10\text{ kg}(7.57\text{ m/s}^2) = 11.4\text{ N}$

For the motion of the weight,

$$v_f^2 = v_i^2 + 2a(y_f - y_i) = 0^2 + 2(7.57\text{ m/s}^2)(6.00\text{ m})$$

$$v_f = 9.53\text{ m/s}$$

(b) The work-energy theorem can take account of multiple objects more easily than Newton's second law. Like your bratty cousins, the work-energy theorem keeps growing between visits. Now it reads:

$$(K_1 + K_{2,\text{rot}} + U_{g1} + U_{g2})_\text{i} = (K_1 + K_{2,\text{rot}} + U_{g1} + U_{g2})_f$$

$$0 + 0 + m_1 g y_{1i} + 0 = \frac{1}{2}\,m_1 v_{1f}^2 + \frac{1}{2}\,I_2\omega_{2f}^2 + 0 + 0$$

Now note that $\omega = \dfrac{v}{r}$ as the string unwinds from the reel. Making substitutions:

$$50.0\text{ N}(6.00\text{ m}) = \frac{1}{2}(5.10\text{ kg})\,v_f^2 + \frac{1}{2}(0.0938\text{ kg}\cdot\text{m}^2)\left(\frac{v_f}{0.250\text{ m}}\right)^2$$

$$300\text{ N}\cdot\text{m} = \frac{1}{2}(5.10\text{ kg})\,v_f^2 + \frac{1}{2}(1.50\text{ kg})\,v_f^2$$

$$v_f = \sqrt{\frac{2(300\text{ N}\cdot\text{m})}{6.60\text{ kg}}} = 9.53\text{ m/s}$$

L: As we should expect, both methods give the same final speed for the falling object. The acceleration is less than g, and the tension is less than the object's weight as we predicted. Now that we understand the effect of the reel's moment of inertia, this problem solution could be applied to solve other real-world pulley systems with masses that should not be ignored.

10.46 $\tau \cdot \theta = \frac{1}{2} I\omega^2$

$$(25.0 \text{ N} \cdot \text{m})(15.0 \cdot 2\pi) = \frac{1}{2}(0.130 \text{ kg} \cdot \text{m}^2)\, \omega^2$$

$$\omega = 190 \text{ rad/s} = \boxed{30.3 \text{ rev/s}}$$

10.47 From conservation of energy,

$$\frac{1}{2} I\left(\frac{v}{r}\right)^2 + \frac{1}{2} mv^2 = mgh$$

$$I\frac{v^2}{r^2} = 2mgh - mv^2$$

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

10.48 $E = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2} MR^2\right)\left(\frac{3000 \times 2\pi}{60.0}\right)^2 = 6.17 \times 10^6 \text{ J}$

$$P = \frac{\Delta E}{\Delta t} = 1.00 \times 10^4 \text{ J/s}$$

$$\Delta t = \frac{\Delta E}{P} = \frac{6.17 \times 10^6 \text{ J}}{1.00 \times 10^4 \text{ J/s}} = 617 \text{ s} = \boxed{10.3 \text{ min}}$$

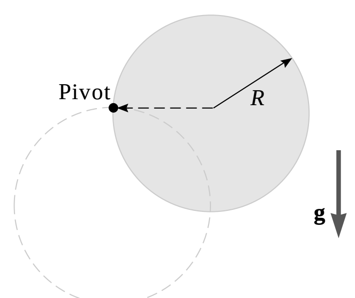
10.49 (a) Find the velocity of the CM

$$(K + U)_i = (K + U)_f$$

$$0 + mgR = \frac{1}{2} I\omega^2$$

$$\omega = \sqrt{\frac{2mgR}{I}} = \sqrt{\frac{2mgR}{\frac{3}{2}mR^2}}$$

$$v_{\text{CM}} = R\sqrt{\frac{4g}{3R}} = \boxed{2\sqrt{\frac{Rg}{3}}}$$



(b) $v_L = 2v_{\text{CM}} = \boxed{4\sqrt{\frac{Rg}{3}}}$

(c) $v_{\text{CM}} = \sqrt{\frac{2mgR}{2m}} = \boxed{\sqrt{Rg}}$

***10.50** The moment of inertia of the cylinder is

$$I = \frac{1}{2}\,mr^2 = \frac{1}{2}(81.6\text{ kg})(1.50\text{ m})^2 = 91.8\text{ kg}\cdot\text{m}^2$$

and the angular acceleration of the merry-go-round is found as

$$\alpha = \frac{\tau}{I} = \frac{(Fr)}{I} = \frac{(50.0\text{ N})(1.50\text{ m})}{(91.8\text{ kg}\cdot\text{m}^2)} = 0.817\text{ rad/s}^2$$

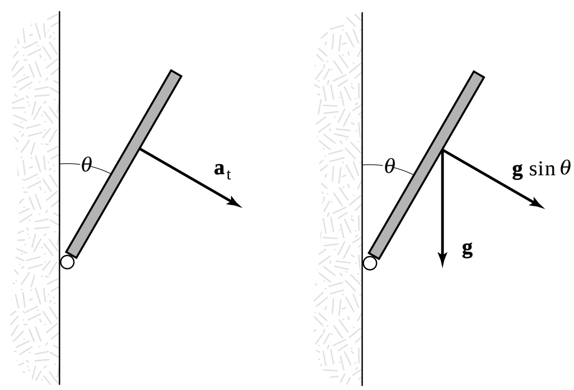
At $t = 3.00$ s, we find the angular velocity

$$\omega = \omega_i + \alpha t$$

$$\omega = 0 + (0.817\text{ rad/s}^2)(3.00\text{ s}) = 2.45\text{ rad/s}$$

and $K = \frac{1}{2}\,I\omega^2 = \frac{1}{2}(91.8\text{ kg}\cdot\text{m}^2)(2.45\text{ rad/s})^2 = \boxed{276\text{ J}}$

10.51 $mg\frac{\ell}{2}\sin\theta = \frac{1}{3}m\ell^2\alpha$



$$\alpha = \frac{3}{2}\frac{g}{\ell}\sin\theta$$

$$a_t = \left(\frac{3}{2}\frac{g}{\ell}\sin\theta\right)r$$

Then $\left(\frac{3}{2}\frac{g}{\ell}\right)r > g\sin\theta$

for $r > \frac{2}{3}\ell$

$\therefore$ About $\boxed{\text{1/3 the length of the chimney}}$ will have a tangential acceleration greater than $g\sin\theta$.

***10.52** The resistive force on each ball is $R = D\rho Av^2$. Here $v = r\omega$, where r is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three ball system is $\tau_{\text{total}} = 3rR$. The power required to maintain a constant rotation rate is $P = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$P = \tau_{\text{total}}\omega = 3r\,[D\rho A(r\omega)^2]\omega = (3r^3DA\omega^3)\rho$$

With $\omega = \left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(10^3\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = \left(\frac{1000\pi}{30.0}\right)\text{ rad/s}$,

$$P = 3(0.100\text{ m})^3(0.600)(4.00\times10^{-4}\text{ m}^2)(1000\pi/30.0\text{ s})^3\rho$$

or $P = (0.827\text{ m}^5/\text{s}^3)\rho$ where ρ is the density of the resisting medium.

(a) In air, $\rho = 1.20\ \mathrm{kg/m^3}$, and

$$P = (0.827\ \mathrm{m^5/s^3})(1.20\ \mathrm{kg/m^3}) = 0.992\ \mathrm{N \cdot m/s} = \boxed{0.992\ \mathrm{W}}$$

(b) In water, $\rho = 1000\ \mathrm{kg/m^3}$ and $P = \boxed{827\ \mathrm{W}}$

10.53 (a) $I = \frac{1}{2} MR^2 = \frac{1}{2}(2.00\ \mathrm{kg})(7.00 \times 10^{-2}\ \mathrm{m})^2 = 4.90 \times 10^{-3}\ \mathrm{kg \cdot m^2}$

$$\alpha = \frac{\tau}{I} = \frac{0.600}{4.90 \times 10^{-3}} = 122\ \mathrm{rad/s^2}$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1200\left(\frac{2\pi}{60}\right)}{122} = \boxed{1.03\ \mathrm{s}}$$

(b) $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122\ \mathrm{rad/s})(1.03\ \mathrm{s})^2 = 64.7\ \mathrm{rad} = \boxed{10.3\ \mathrm{rev}}$

10.54 For a spherical shell $\frac{2}{3}\, dm\, r^2 = \frac{2}{3}\, [(4\pi r^2 dr)\rho] r^2$

$$I \int dI = \int \frac{2}{3}(4\pi r^2)\, r^2 \rho(r) dr$$

$$I = \int_0^R \frac{2}{3}(4\pi r^4)\left(14.2 - 11.6\,\frac{r}{R}\right)\left(10^3\,\frac{\mathrm{kg}}{\mathrm{m^3}}\right) dr$$

$$= \left(\frac{2}{3}\right) 4\pi(14.2 \times 10^3)\frac{R^5}{5} - \left(\frac{2}{3}\right) 4\pi(11.6 \times 10^3)\frac{R^5}{6}$$

$$I = \frac{8\pi}{3}(10^3)\, R^5\left(\frac{14.2}{5} - \frac{11.6}{6}\right)$$

$$M = \int dm = \int_0^R 4\pi r^2\left(14.2 - 11.6\,\frac{r}{R}\right) 10^3\, dr$$

$$= 4\pi \times 10^3\left(\frac{14.2}{3} - \frac{11.6}{4}\right) R^3$$

$$\frac{I}{MR^2} = \frac{\frac{8\pi}{3}(10^3)\, R^5\left(\frac{14.2}{5} - \frac{11.6}{6}\right)}{4\pi \times 10^3 R^3 R^2\left(\frac{14.2}{3} - \frac{11.6}{4}\right)} = \frac{2}{3}\left(\frac{.907}{1.83}\right) = 0.330$$

$$\therefore I = \boxed{0.330MR^2}$$

10.55 (a) $W = \Delta K = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_i^2$

$= \frac{1}{2}I(\omega^2 - \omega_i^2)$ where $I = \frac{1}{2}mR^2$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1.00\text{ kg})(0.500\text{ m})^2\left[\left(8.00\ \frac{\text{rad}}{\text{s}}\right)^2 - 0\right] = \boxed{4.00\text{ J}}$

(b) $t = \frac{\omega - 0}{\alpha} = \frac{\omega r}{a} = \frac{(8.00\text{ rad/s})(0.500\text{ m})}{2.50\text{ m/s}^2} = \boxed{1.60\text{ s}}$

(c) $\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$; $\theta_i = 0$; $\omega_i = 0$

$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{2.50\text{ m/s}^2}{0.500\text{ m}}\right)(1.60\text{ s})^2 = 6.40\text{ rad}$

$s = r\theta = (0.500\text{ m})(6.40\text{ rad}) = \boxed{3.20\text{ m} < 4.00\text{ m Yes}}$

10.56 (a) $I = \frac{1}{2}mr^2 = \frac{1}{2}(200\text{ kg})(0.300\text{ m})^2 = \boxed{9.00\text{ kg}\cdot\text{m}^2}$

(b) $\omega = (1000\text{ rev/min})(1\text{ min/}60\text{ s})(2\pi\text{ rad})/1\text{ rev} = 105\text{ rad/s}$

$W = K = \frac{1}{2}I\omega^2 = \frac{1}{2}(9.00)(104.7)^2 = \boxed{49.3\text{ kJ}}$

(c) $\omega_f = 500\text{ rev/min} \times 1\text{ min/}60\text{ s}(2\pi\text{ rad/}1\text{ rev}) = 52.4\text{ rad/s}$

$K_f = \frac{1}{2}I\omega^2 = 12.3\text{ kJ}$

$W = \Delta K = 12.3\text{ kJ} - 49.3\text{ kJ} = \boxed{-37.0\text{ kJ}}$

***10.57** $\alpha = -10.0\text{ rad/s}^2 - (5.00\text{ rad/s}^3)t = d\omega/dt$

$\int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t]dt = -10.0t - 2.50t^2 = \omega - 65.0\text{ rad/s}$

$\omega = \frac{d\theta}{dt} = 65.0\text{ rad/s} - (10.0\text{ rad/s}^2)t - (2.50\text{ rad/s}^3)t^2$

(a) At $t = 3.00$ s,

$\omega = 65.0\text{ rad/s} - (10.0\text{ rad/s}^2)(3.00\text{ s}) - (2.50\text{ rad/s}^3)(9.00\text{ s}^2) = \boxed{12.5\text{ rad/s}}$

(b) $\int_0^{\theta} d\theta = \int_0^{t} \omega\, dt = \int_0^{t} [65.0 \text{ rad/s} - (10.0 \text{ rad/s}^2)t - (2.50 \text{ rad/s}^3)t^2]dt$

$\theta = (65.0 \text{ rad/s})t - (5.00 \text{ rad/s}^2)t^2 - (0.833 \text{ rad/s}^3)t^3$

At $t = 3.00$ s,

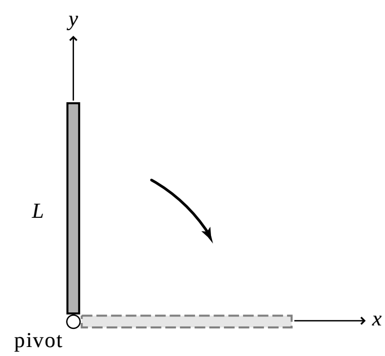
$\theta = (65.0 \text{ rad/s})(3.00 \text{ s}) - (5.00 \text{ rad/s}^2)9.00 \text{ s}^2 - (0.833 \text{ rad/s}^3)27.0 \text{ s}^3$

$\theta = \boxed{128 \text{ rad}}$

10.58 (a) $MK^2 = \dfrac{MR^2}{2}$, $\quad K = \dfrac{R}{\sqrt{2}}$

(b) $MK^2 = \dfrac{ML^2}{12}$, $\quad K = \dfrac{L\sqrt{3}}{6}$

(c) $MK^2 = \dfrac{2}{5} MR^2$, $\quad K = R\sqrt{\dfrac{2}{5}}$



10.59 (a) Since only conservative forces act, $\Delta E = 0$, so

$K_f + U_f = K_i + U_i$

$\dfrac{1}{2} I\omega^2 + 0 = 0 + mg\left(\dfrac{L}{2}\right)$, where $I = \dfrac{1}{3} mL^2$

$\omega = \boxed{\sqrt{3g/L}}$

(b) $\tau = I\alpha$ so that in the horizontal, $mg\left(\dfrac{L}{2}\right) = \dfrac{mL^2}{3}\alpha \quad \alpha = \boxed{\dfrac{3g}{2L}}$

(c) $a_x = a_r = r\omega^2 = \left(\dfrac{L}{2}\right)\omega^2 = \boxed{-\dfrac{3g}{2}} \qquad a_y = -a_t = -r\alpha = -\alpha\left(\dfrac{L}{2}\right) = \boxed{-\dfrac{3g}{4}}$

(d) Using Newton's second law, we have $R_x = ma_x = \boxed{-\dfrac{3}{2} mg}$

$R_y - mg = -ma_y \quad$ or $\quad R_y = \boxed{\dfrac{1}{4} mg}$

10.60 The first drop has a velocity leaving the wheel given by $\frac{1}{2}mv_i^2 = mgh_1$, so

$$v_1 = \sqrt{2gh_1} = \sqrt{2(9.80\text{ m/s}^2)(0.540\text{ m})} = 3.25\text{ m/s}$$

The second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2(9.80\text{ m/s}^2)(0.510\text{ m})} = 3.16\text{ m/s}$$

From $\omega = \frac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25\text{ m/s}}{0.381\text{ m}} = 8.53\text{ rad/s} \quad\text{and}\quad \omega_2 = \frac{v_2}{r} = \frac{3.16\text{ m/s}}{0.381\text{ m}} = 8.29\text{ rad/s}$$

or $$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{(8.29\text{ rad/s})^2 - (8.53\text{ rad/s})^2}{4\pi} = \boxed{-0.322\text{ rad/s}^2}$$

10.61 At the instant it comes off the wheel, the first drop has a velocity v_1, directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \quad\text{or}\quad v_1 = \sqrt{2gh_1}$$

and the angular velocity of the wheel at the instant the first drop leaves is

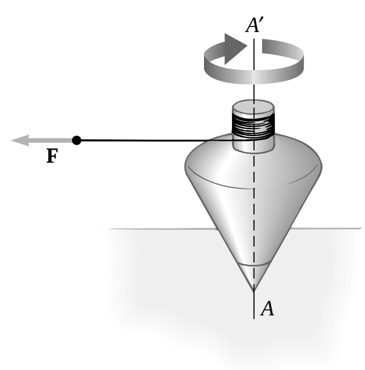
$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$

The angular acceleration of the wheel is then

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{2gh_2/R^2 - 2gh_1/R^2}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}$$

10.62 Work done = $Fs = (5.57\text{ N})(0.800\text{ m}) = 4.46\text{ J}$



and Work $= \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

(The last term is zero because the top starts from rest.)

Thus, $4.46\text{ J} = \frac{1}{2}(4.00 \times 10^{-4}\text{ kg}\cdot\text{m}^2)\,\omega_f^2$

and from this, $\omega_f = \boxed{149 \text{ rad/s}}$

10.63 $K_f = \frac{1}{2}\,Mv_f^2 + \frac{1}{2}\,I\omega_f^2$: $U_f = Mgh_f = 0$; $K_i = \frac{1}{2}\,Mv_i^2 + \frac{1}{2}\,I\omega_i^2 = 0$

$U_i = (Mgh)_i$: $f = \mu N = \mu Mg\cos\theta$; $\omega = \frac{v}{r}$; $h = d\sin\theta$ and $I = \frac{1}{2}\,mr^2$

(a) $\Delta E = E_f - E_i$ or $-fd = K_f + U_f - K_i - U_i$

$$-fd = \frac{1}{2}\,Mv_f^2 + \frac{1}{2}\,I\omega_f^2 - Mgh$$

$$-(\mu Mg\cos\theta)d = \frac{1}{2}\,Mv^2 + (mr^2/2)(v^2/r^2)/2 - Mgd\sin\theta$$

$$\frac{1}{2}\left[M + \frac{m}{2}\right]v^2 = Mgd\sin\theta - (\mu Mg\cos\theta)d \text{ or}$$

$$v^2 = 2Mgd\,\frac{(\sin\theta - \mu\cos\theta)}{(m/2) + M}$$

$$v_d = \left[4gd\,\frac{M}{(m + 2M)}(\sin\theta - \mu\cos\theta)\right]^{1/2}$$

(b) $v^2 = v_i^2 - 2as$, $v_d^2 = 2ad$

$$a = \frac{v_d^2}{2d} = \boxed{2g\left(\frac{M}{m + 2M}\right)(\sin\theta - \mu\cos\theta)}$$

10.64 (a) $E = \frac{1}{2}\left(\frac{2}{5}MR^2\right)(\omega^2)$

$$E = \frac{1}{2}\cdot\frac{2}{5}(5.98\times 10^{24})(6.37\times 10^6)^2\left(\frac{2\pi}{86400}\right)^2 = \boxed{2.57\times 10^{29}\text{ J}}$$

(b) $\frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)^2\right]$

$$= \frac{1}{5}\,MR^2(2\pi)^2(-2T^{-3})\,\frac{dT}{dt}$$

$$= \frac{1}{5}\,MR^2\left(\frac{2\pi}{T}\right)^2\left(\frac{-2}{T}\right)\frac{dT}{dt}$$

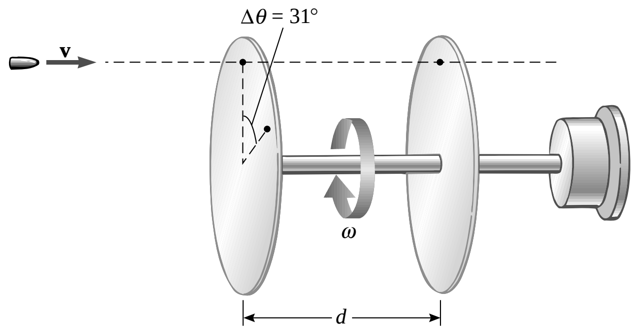
$$= (2.57\times 10^{29}\text{ J})\left(\frac{-2}{86400\text{ s}}\right)\left(\frac{10\times 10^{-6}\text{ s}}{3.16\times 10^7\text{ s}}\right)(86400\text{ s/day})$$

$$\frac{dE}{dt} = \boxed{-1.63\times 10^{17}\text{ J/day}}$$

10.65 $\Delta\theta = \omega t$

$$t = \frac{\Delta\theta}{\omega} = \frac{(31.0°/360°)\text{ rev}}{900\text{ rev}/60\text{ s}} = 0.00574\text{ s}$$

$$v = \frac{0.800\text{ m}}{0.00574\text{ s}} = \boxed{139\text{ m/s}}$$



10.66 (a) Each spoke counts as a thin rod pivoted at one end.

$$I = \boxed{MR^2 + n\frac{mR^2}{3}}$$

(b) By the parallel-axis theorem,

$$I = MR^2 + \frac{nmR^2}{3} + (M + nm)R^2$$

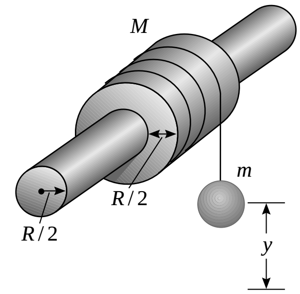
$$= \boxed{2MR^2 + \frac{4\,nmR^2}{3}}$$

***10.67** Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(23.0\text{ kg})(0.870\text{ m})^2 = \boxed{5.80\text{ kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the door is unnecessary}}$ data.

10.68 τ_f will oppose the torque causing the motion:



$$\sum\tau = I\alpha = TR - \tau_f \Rightarrow \tau_f = TR - I\alpha \qquad (1)$$

Now find T, I and α in given or known terms and substitute into equation (1)

$$\sum F_y = T - mg = -ma \text{ then } T = m(g - a) \qquad (2)$$

also $\Delta y = v_i t + \dfrac{at^2}{2} \Rightarrow a = \dfrac{2y}{t^2} \qquad (3)$

and $\alpha = \dfrac{a}{R} = \dfrac{2y}{Rt^2} \qquad (4)$

$$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \qquad (5)$$

Substituting (2), (3), (4) and (5) into (1) we find

$$\tau_f = m\left(g - \frac{2y}{t^2}\right)R - \frac{5}{8}\frac{MR^2 2y}{(Rt^2)} = \boxed{R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]}$$

10.69 (a) While decelerating,

$$\tau_f = I\alpha' = (20\,000 \text{ kg}\cdot\text{m}^2)\left(\frac{2.00 \text{ rev/min }(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{10.0 \text{ s}}\right)$$

$$\tau_f = 419 \text{ N}\cdot\text{m}$$

While accelerating,

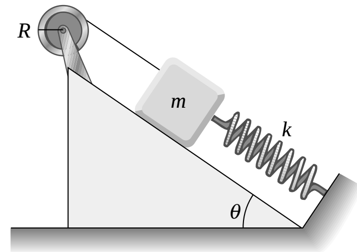
$$\sum\tau = I\alpha \quad \text{or} \quad \tau - \tau_f = I(\Delta\omega/\Delta t)$$

$$\tau = 419 \text{ N}\cdot\text{m} + (20\,000 \text{ kg}\cdot\text{m}^2)\left(\frac{10.00 \text{ rev/min }(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{12.0 \text{ s}}\right)$$

$$\tau = \boxed{2.16\times10^3 \text{ N}\cdot\text{m}}$$

(b) $P = \tau_f\cdot\omega = (419 \text{ N}\cdot\text{m})\left[10.0\,\dfrac{\text{rev}}{\text{min}}\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\right] = \boxed{439 \text{ W}} \qquad (\approx 0.6 \text{ hp})$

10.70 (a) $W = \Delta K + \Delta U$



$$W = K_f - K_i + U_f - U_i$$

$$0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgd\sin\theta - \frac{1}{2}kd^2$$

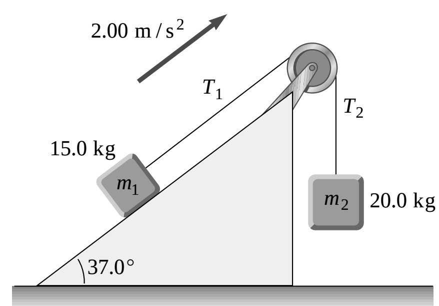
$$\frac{1}{2}\omega^2(I + mR^2) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

(b) $$\omega = \sqrt{\frac{2(0.500\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})(\sin 37.0°) + (50.0\text{ N/m})(0.200\text{ m})^2}{1.00\text{ kg}\cdot\text{m}^2 + (0.500\text{ kg})(0.300\text{ m})^2}}$$

$$\omega = \sqrt{\frac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74\text{ rad/s}}$$

10.71 (a) $m_2 g - T_2 = m_2 a$



$$T_2 = m_2(g - a) = (20.0\text{ kg})(9.80 - 2.00)\text{m/s}^2 = \boxed{156\text{ N}}$$

$$T_1 = m_1 g\sin 37.0° + m_1 a$$

$$T_1 = (15.0\text{ kg})(9.80\sin 37.0° + 2.00)\text{m/s}^2 = \boxed{118\text{ N}}$$

(b) $$(T_2 - T_1)R = I\alpha = I\left(\frac{a}{R}\right)$$

$$I = \frac{(T_2 - T_1)R^2}{a} = \frac{(156 - 118)\text{N}(0.250\text{ m})^2}{2.00\text{ m/s}^2} = \boxed{1.17\text{ kg}\cdot\text{m}^2}$$

Goal Solution

G: In earlier problems, we assumed that the tension in a string was the same on either side of a pulley. Here we see that the moment of inertia changes that assumption, but we should still expect the tensions to be similar in magnitude (about the weight of each mass ~150 N), and $T_2 > T_1$ for the pulley to rotate clockwise as shown.

If we knew the mass of the pulley, we could calculate its moment of inertia, but since we only know the acceleration, it is difficult to estimate I. We at least know that I must have units of kgm^2, and a 50-cm disk probably has a mass less than 10 kg, so I is probably less than 0.3 kgm^2.

O: For each block, we know its mass and acceleration, so we can use Newton's second law to find the net force, and from it the tension. The difference in the two tensions causes the pulley to rotate, so this net torque and the resulting angular acceleration can be used to find the pulley's moment of inertia.

A: (a) Apply $\sum F = ma$ to each block to find each string tension.

The forces acting on the 15-kg block are its weight, the normal support force from the incline, and T_1. Taking the positive x axis as directed up the incline, $\sum F_x = ma_x$ yields:

$$-(m_1g)_x + T_1 = m_1(+a)$$

$$-(15.0\text{ kg})(9.80\text{ m/s}^2)\sin 37° + T_1 = (15.0\text{ kg})(2.00\text{ m/s}^2)$$

$$T_1 = 118\text{ N}$$

Similarly for the counterweight, we have $\sum F_y = ma_y$, or $T_2 - m_2g = m_2(-a)$

$$T_2 - (20.0\text{ kg})(9.80\text{ m/s}^2) = (20.0\text{ kg})(-2.00\text{ m/s}^2)$$

So, $T_2 = 156$ N

(b) Now for the pulley, $\sum \tau = r(T_2 - T_1) = I\alpha$. We may choose to call clockwise positive. The angular acceleration is

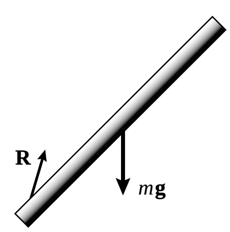
$$\alpha = \frac{a}{r} = \frac{2.00\text{ m/s}^2}{0.250\text{ m}} = 8.00\text{ rad/s}^2$$

$$\sum \tau = I\alpha \quad \text{or} \quad (0.250\text{ m})(156\text{ N} - 118\text{ N}) = I(8.00\text{ rad/s}^2)$$

$$I = \frac{9.38\text{ N}\cdot\text{m}}{8.00\text{ rad/s}^2} = 1.17\text{ kg}\cdot\text{m}^2$$

L: The tensions are close to the weight of each mass and $T_2 > T_1$ as expected. However, the moment of inertia for the pulley is about 4 times greater than expected. Unless we made a mistake in solving this problem, our result means that the pulley has a mass of 37.4 kg (about 80 lb), which means that the pulley is probably made of a dense material, like steel. This is certainly not a problem where the mass of the pulley can be ignored since the pulley has more mass than the combination of the two blocks!

10.72 For the board just starting to move,



$$\sum \tau = I\alpha$$

$$mg\frac{\ell}{2}\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$$

The tangential acceleration of the end is

$$a_t = \ell\alpha = \frac{3}{2}g\cos\theta$$

Its vertical component is $a_y = a_t\cos\theta = \frac{3}{2}g\cos^2\theta$

If this is greater than g, the board will pull ahead of the ball in falling:

(a) $\frac{3}{2}g\cos^2\theta \ge g \Rightarrow \cos^2\theta \ge \frac{2}{3}$

so $\cos\theta \ge \sqrt{\frac{2}{3}}$ and $\boxed{\theta \le 35.3^\circ}$

(b) When $\theta = 35.3^\circ$ ($\Rightarrow \cos^2\theta = 2/3$), the cup will land underneath the release-point of the ball if

$$r_c = \ell\cos\theta = \frac{\ell\cos^2\theta}{\cos\theta} = \boxed{\frac{2\ell}{3\cos\theta}}$$

(c) When $\ell = 1.00$ m, and $\theta = 35.3^\circ$

$$r_c = \frac{2(1.00\text{ m})}{3\sqrt{2/3}} = 0.816\text{ m}$$

which is (1.00 m – 0.816 m) = $\boxed{\text{0.184 m from the moving end}}$

10.73 At $t = 0$, $\omega = 3.50$ rad/s $= \omega_0 e^0$. Thus, $\omega_0 = 3.50$ rad/s

At $t = 9.30$ s, $\omega = 2.00$ rad/s $= \omega_0 e^{-\sigma(9.30\text{ s})}$, yielding $\sigma = 6.02 \times 10^{-2}\text{ s}^{-1}$

(a) $\alpha = \frac{d\omega}{dt} = \frac{d(\omega_0 e^{-\sigma t})}{dt} = \omega_0(-\sigma)e^{-\sigma t}$

At $t = 3.00$ s,

$$\alpha = (3.50\text{ rad/s})(-6.02 \times 10^{-2}\text{ s}^{-1})e^{-3.00(6.02 \times 10^{-2})} = \boxed{-0.176\text{ rad/s}^2}$$

(b) $\theta = \int_0^t \omega_0 e^{-\sigma t}\,dt = \frac{\omega_0}{-\sigma}[e^{-\sigma t} - 1] = \frac{\omega_0}{\sigma}[1 - e^{-\sigma t}]$

At $t = 2.50$ s,

$$\theta = \frac{3.50\text{ rad/s}}{(6.02 \times 10^{-2})1/\text{s}}[1 - e^{-(6.02 \times 10^{-2})(2.50)}] = 8.12\text{ rad} = \boxed{1.29\text{ rev}}$$

(c) As $t \to \infty$, $\theta \to \frac{\omega_0}{\sigma}(1 - e^{-\infty}) = \frac{3.50 \text{ rad/s}}{6.02 \times 10^{-2} \text{ s}^{-1}} = 58.2 \text{ rad} = \boxed{9.26 \text{ rev}}$

10.74 Consider the total weight of each hand to act at the center of gravity (mid-point) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m = \frac{-g}{2}(m_h L_h \sin\theta_h + m_m L_m \sin\theta_m)$$

If we take $t = 0$ at 12 o'clock, then the angular positions of the hands at time t are

$$\theta_h = \omega_h t,\ \text{where}\ \omega_h = \frac{\pi}{6}\ \text{rad/h and}\ \theta_m = \omega_m t,\ \text{where}\ \omega_m = 2\pi\ \text{rad/h}$$

Therefore,

$$\tau = -\left(4.90\ \frac{\text{m}}{\text{s}^2}\right)[(60.0\ \text{kg})(2.70\ \text{m})\sin(\pi t/6) + (100\ \text{kg})(4.50\ \text{m})\sin 2\pi t]$$

or $\tau = -794\ \text{N}\cdot\text{m}[\sin(\pi t/6) + 2.78\sin 2\pi t]$, where t is in hours.

(a) (i) At 3:00, $t = 3.00$ h, so

$$\tau = -794\ \text{N}\cdot\text{m}\ [\sin(\pi/2) + 2.78\sin 6\pi] = \boxed{-794\ \text{N}\cdot\text{m}}$$

(ii) At 5:15, $t = 5\ \text{h} + \frac{15}{60}\ \text{h} = 5.25\ \text{h}$, and substitution gives:

$$\tau = \boxed{-2510\ \text{N}\cdot\text{m}}$$

(iii) At 6:00, $\tau = \boxed{0\ \text{N}\cdot\text{m}}$

(iv) At 8:20, $\tau = \boxed{-1160\ \text{N}\cdot\text{m}}$

(v) At 9:45, $\tau = \boxed{-2940\ \text{N}\cdot\text{m}}$

(b) The total torque is zero at those times when

$$\sin(\pi t/6) + 2.78\sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.5152955, ..., corresponding to the times

12:00:00	12:30:55	12:58:19	1:32:31	1:57:01
2:33:25	2:56:29	3:33:22	3:56:55	4:32:24
4:58:14	5:30:52	6:00:00	6:29:08	7:01:46
7:27:36	8:03:05	8:26:38	9:03:31	9:26:35
10:02:59	10:27:29	11:01:41	11:29:05	

Chapter 11 Solutions

11.1 (a) $K_{\text{trans}} = \frac{1}{2}\,mv^2 = \frac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $K_{\text{rot}} = \frac{1}{2}\,I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)\left(\frac{v^2}{r^2}\right) = \frac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\text{ J}}$

11.2 $K = \frac{1}{2}\,I\omega^2 + \frac{1}{2}\,mv^2$

$$K = \frac{1}{2}(1.60 \times 10^{-2}\text{ kg}\cdot\text{m}^2)\left(\frac{4.00\text{ m/s}}{0.100\text{ m}}\right)^2 + \frac{1}{2}(4.00\text{ kg})(4.00\text{ m/s})^2$$

$= 12.8 + 32.0 = \boxed{44.8\text{ J}}$

11.3 $W = K_f - K_i = (K_{\text{trans}} + K_{\text{rot}})_f - (K_{\text{trans}} + K_{\text{rot}})_i$

$$W = \frac{1}{2}\,Mv^2 + \frac{1}{2}\,I\omega^2 - 0 - 0 = \frac{1}{2}\,Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

or $W = \boxed{(7/10)Mv^2}$

11.4 $K = \frac{1}{2}\,mv^2 + \frac{1}{2}\,I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping.

Also, $U_i = mgh$, $U_f = 0$, and $v_i = 0$

Therefore, $\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \dfrac{2gh}{[1 + (I/mR^2)]}$

For a disk, $I = \frac{1}{2}\,mR^2$, so

$v^2 = \dfrac{2gh}{[1 + (1/2)]}$ or $\boxed{v_{\text{disk}} = \sqrt{\dfrac{4gh}{3}}}$

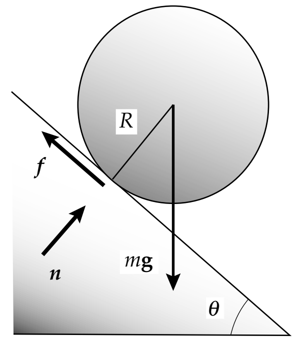
For a ring, $I = mR^2$ so $v^2 = \dfrac{2gh}{2}$ or $\boxed{v_{\text{ring}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

11.5 (a) $\tau = I\alpha$

$$mg\,R\sin\theta = (I_{CM} + mR^2)\alpha$$

$$a = \frac{mg\,R^2\sin\theta}{I_{CM} + mR^2}$$



$$a_{hoop} = \frac{mg\,R^2\sin\theta}{2mR^2} = \boxed{\frac{1}{2}g\sin\theta}$$

$$a_{disk} = \frac{mg\,R^2\sin\theta}{\frac{3}{2}mR^2} = \boxed{\frac{2}{3}g\sin\theta}$$

The disk moves with $\frac{4}{3}$ the acceleration of the hoop.

(b) $Rf = I\alpha$

$$f = \mu n = \mu mg\cos\theta$$

$$\mu = \frac{f}{mg\cos\theta} = \frac{I\alpha/R}{mg\cos\theta} = \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \boxed{\frac{1}{3}\tan\theta}$$

Goal Solution

G: The acceleration of the disk will depend on the angle of the incline. In fact, it should be proportional to $g\sin\theta$ since the disk should not accelerate when the incline angle is zero, and since $a = g$ when the angle is 90° and the disk can fall freely. The acceleration of the disk should also be greater than a hoop since the mass of the disk is closer to its center, giving it less rotational inertia so that it can roll faster than the hoop. The required coefficient of friction is difficult to predict, but is probably between 0 and 1 since this is a typical range for μ.

O: We can find the acceleration by applying Newton's second law and considering both the linear and rotational motion. A free-body diagram will greatly assist in defining our variables and seeing how the forces are related.

A: $\Sigma F_x = mg\sin\theta - f = ma_{CM}$ (1)

$\Sigma F_y = n - mg\cos\theta = 0$ (2)

$\tau = fr = I_{CM}\alpha = \dfrac{I_{CM}a_{CM}}{r}$ (3)

(a) For a disk, $(I_{CM})_{disk} = \frac{1}{2}\, mr^2$. From (3) we find $f = \frac{\left[\frac{1}{2} mr^2\right] a_{CM}}{r^2} = \frac{1}{2}\, ma_{CM}$.

Substituting this into (1) gives

$$mg \sin \theta - \frac{1}{2}\, ma_{CM} \quad \text{so that} \quad (a_{CM})_{disk} = \frac{2}{3}\, g \sin \theta$$

For a hoop, $(I_{CM})_{hoop} = mr^2$

From (3), $f = \frac{mr^2 a_{CM}}{r^2} = ma_{CM}$

Substituting this into (1) gives

$$mg \sin \theta - ma_{CM} \quad \text{so} \quad (a_{CM})_{hoop} = \frac{1}{2}\, g \sin \theta$$

Therefore, $\frac{a_{CM\ disk}}{a_{CM\ hoop}} = \frac{\frac{2}{3} g \sin \theta}{\frac{1}{2} g \sin \theta} = \frac{4}{3}$

(b) From (2) we find $n = mg \cos \theta$, and $f = \mu n = \mu mg \cos \theta$

Likewise, from equation (1), $f = mg \sin \theta - ma_{CM}$

Setting these two equations equal,

$$\mu mg \cos \theta = mg \sin \theta - \frac{2}{3}\, mg \sin \theta$$

so $\quad \mu = \frac{1}{3}\left(\frac{\sin \theta}{\cos \theta}\right) = \frac{1}{3} \tan \theta$

L: As expected, the acceleration of the disk is proportional to $g \sin \theta$ and is slightly greater than the acceleration of the hoop. The coefficient of friction result is similar to the result found for a block on an incline plane, where $\mu = \tan \theta$ (see Example 5.12). However, μ is not always between 0 and 1 as predicted. For angles greater than 72° the coefficient of friction must be larger than 1. For angles greater than 80° it must be extremely large to make the disk roll without slipping.

11.6 $I = \frac{1}{2}\,M(R_1^2 + R_2^2) = \frac{1}{2}\,M\left(\left(\frac{3R_2}{4}\right)^2 + R_2^2\right) = \frac{25MR_2^2}{32}$

Energy is conserved between $x = 2.00$ m and $x + \Delta x$.

$$\frac{1}{2}\,Mv_i^2 + \frac{1}{2}\,I\omega_i^2 = Mgy_f$$

$$\frac{1}{2}\,Mv_i^2 + \frac{1}{2}\frac{25}{32}\,MR_2^2\,(v_i/R_2)^2 = Mg\,\Delta x \sin\theta$$

$$\frac{57}{64}\,v_i^2 = g\,\Delta x \sin\theta$$

$$\Delta x = \frac{57(2.80\text{ m/s})^2}{64(9.80\text{ m/s}^2)(\sin 36.9°)} = 1.19\text{ m}$$

So the final position is 2.00 m + 1.19 m = $\boxed{3.19\text{ m}}$

11.7 $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{3.00\text{ m}}{1.50\text{ s}} = 2.00\text{ m/s} = \frac{1}{2}(0 + v_f)$

$v_f = 4.00$ m/s and $\omega_f = \frac{v_f}{r} = \frac{4.00\text{ m/s}}{(6.38\times10^{-2}\text{ m})/2} = \frac{8.00}{6.38\times10^{-2}}$ rad/s

We ignore internal friction and suppose the can rolls without slipping.

$$(K_{\text{trans}} + K_{\text{rot}} + U_g)_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}} + U_g)_f$$

$$(0 + 0 + mgy_i) + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0\right)$$

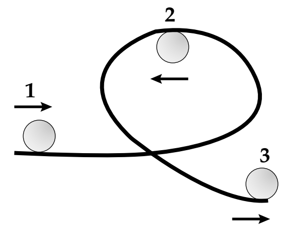
$$(0.215\text{ kg})(9.80\text{ m/s}^2)[(3.00\text{ m})\sin 25.0°] = \frac{1}{2}(0.215\text{ kg})(4.00\text{ m/s})^2 + \frac{1}{2}\,I\left(\frac{8.00}{6.38\times10^{-2}}\text{rad/s}\right)^2$$

$$2.67\text{ J} = 1.72\text{ J} + (7860/\text{s}^2)I$$

$$I = \frac{(0.951\text{ kg}\cdot\text{m}^2/\text{s}^2)}{7860/\text{s}^2} = \boxed{1.21\times10^{-4}\text{ kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the can}}$ is unnecessary data.

***11.8** (a) Energy conservation between the horizontal section and top of loop:



$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2} = \sqrt{(4.03\text{ m/s})^2 - \frac{6}{5}(9.80\text{ m/s}^2)(0.900\text{ m})} = \boxed{2.38\text{ m/s}}$$

The centripetal acceleration is $\dfrac{v_2^2}{r} = \dfrac{(2.38\text{ m/s})^2}{0.450\text{ m}} = 12.6\text{ m/s}^2 > g$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(b) $$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3} = \sqrt{(4.03\text{ m/s})^2 - \frac{6}{5}(9.80\text{ m/s}^2)(-0.200\text{ m})} = \boxed{4.31\text{ m/s}}$$

(c) $$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.900\text{ m})} = \sqrt{-1.40\text{ m}^2/\text{s}^2}$$

This result is imaginary. In the case where the ball does not roll, the ball starts with less energy than in part (a) and $\boxed{\text{never makes it to the top}}$ of the loop.

***11.9** $$\mathbf{M}\times\mathbf{N} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 2 & -1 \\ 2 & -1 & -3 \end{vmatrix} = \boxed{-7.00\mathbf{i} + 16.0\mathbf{j} - 10.0\mathbf{k}}$$

***11.10** (a) area $= |\mathbf{A}\times\mathbf{B}| = AB\sin\theta = (42.0\text{ cm})(23.0\text{ cm})\sin(65.0° - 15.0°) = \boxed{740\text{ cm}^2}$

(b) $\mathbf{A}+\mathbf{B} = [(42.0\text{ cm})\cos 15.0° + (23.0\text{ cm})\cos 65.0°]\mathbf{i} + [(42.0\text{ cm})\sin 15.0°$

$+ (23.0\text{ cm})\sin 65.0°]\mathbf{j}$

$\mathbf{A}+\mathbf{B} = (50.3\text{ cm})\mathbf{i} + (31.7\text{ cm})\mathbf{j}$

length $= |\mathbf{A}+\mathbf{B}| = \sqrt{(50.3\text{ cm})^2 + (31.7\text{ cm})^2} = \boxed{59.5\text{ cm}}$

11.11 (a) $\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \boxed{-17.0\mathbf{k}}$

(b) $|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin\theta$

$17 = 5\sqrt{13}\ \sin\theta$

$\theta = \arc\sin\left(\frac{17}{5\sqrt{13}}\right) = \boxed{70.6°}$

***11.12** $\mathbf{A}\cdot\mathbf{B} = (-3.00)(6.00) + (7.00)(-10.0) + (-4.00)(9.00) = -124$

$AB = \sqrt{(-3.00)^2 + (7.00)^2 + (-4.00)^2}\sqrt{(6.00)^2 + (-10.0)^2 + (9.00)^2} = 127$

(a) $\cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}(-0.979) = \boxed{168°}$

(b) $\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3.00 & 7.00 & -4.00 \\ 6.00 & -10.0 & 9.00 \end{vmatrix} = 23.0\mathbf{i} + 3.00\mathbf{j} - 12.0\mathbf{k}$

$|\mathbf{A}\times\mathbf{B}| = \sqrt{(23.0)^2 + (3.00)^2 + (-12.0)^2} = 26.1$

$\sin^{-1}\left(\frac{|\mathbf{A}\times\mathbf{B}|}{AB}\right) = \sin^{-1}(0.206) = \boxed{11.9°}$ or 168°

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously.

11.13 (a) $\tau = \mathbf{r}\times\mathbf{F} = (4.00\mathbf{i} + 5.00\mathbf{j})\times(2.00\mathbf{i} + 3.00\mathbf{j})$

$\tau = |12.00\mathbf{k} - 10.0\mathbf{k}| = |2.00\mathbf{k}| = \boxed{2.00\ \text{N}\cdot\text{m}}$

(b) The torque vector is in the direction of the unit vector **k**, or in the $\boxed{+z\text{ direction}}$.

11.14 The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero:

Does $(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})\cdot(4\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = 0$?

$8 - 9 - 4 = -5 \neq 0$

$\boxed{\text{No}}$ The cross product could not work out that way.

11.15 (a) in the $\boxed{\text{negative } z \text{ direction}}$ given by the right-hand rule

(b) in the $\boxed{\text{positive } z \text{ direction}}$ given by the right-hand rule

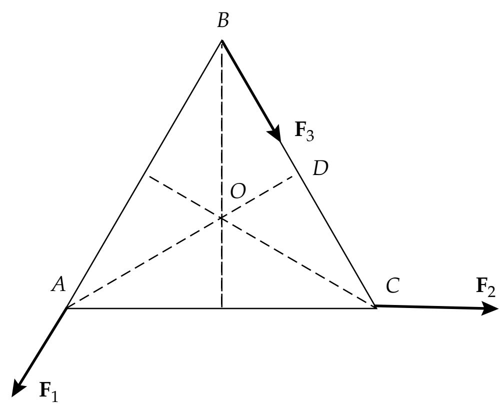
11.16 (a) $\tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \mathbf{i}(0-0) - \mathbf{j}(0-0) + \mathbf{k}(2-9) = \boxed{(-7.00 \text{ N} \cdot \text{m})\mathbf{k}}$

(b) $\tau = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \boxed{(11.0 \text{ N} \cdot \text{m})\mathbf{k}}$

11.17 $|\mathbf{A} \times \mathbf{B}| = \mathbf{A} \cdot \mathbf{B} \Rightarrow AB \sin\theta = AB\cos\theta \Rightarrow \tan\theta = 1$ or

$\theta = \boxed{45.0°}$

11.18 $\boxed{|\mathbf{F}_3| = |\mathbf{F}_1| + |\mathbf{F}_2|}$



The torque produced by $\mathbf{F}_3$ depends on the perpendicular distance **OD**, therefore translating the point of application of $\mathbf{F}_3$ to any other point along **BC** $\boxed{\text{will not change the net torque}}$.

11.19 $L = \sum m_i v_i r_i$

$= (4.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m}) + (3.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m})$

$L = 17.5 \text{ kg} \cdot \text{m}^2/\text{s}$, and

$\boxed{\mathbf{L} = (17.5 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}}$

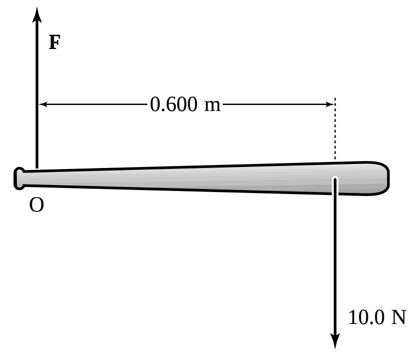
***11.20** $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$\mathbf{L} = (1.50\mathbf{i} + 2.20\mathbf{j})\text{m} \times (1.50 \text{ kg})(4.20\mathbf{i} - 3.60\mathbf{j}) \text{ m/s}$

$\mathbf{L} = (-8.10\mathbf{k} - 13.9\mathbf{k}) \text{ kg} \cdot \text{m}^2/\text{s} = \boxed{(-22.0 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}}$

Chapter 12 Solutions

12.1 To hold the bat in equilibrium, the player must exert both a force and a torque on the bat to make



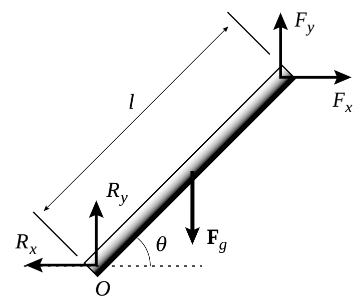
$$\sum F_x = \sum F_y = 0 \quad \text{and} \quad \sum \tau = 0$$

$\sum \boldsymbol{F_y} = 0 \Rightarrow F - 10.0\text{ N} = 0$, or the player must exert a net upward force of $F = \boxed{10.0\text{ N}}$

To satisfy the second condition of equilibrium, the player must exert an applied torque τ_a to make $\sum \tau = \tau_a - (0.600\text{ m})(10.0\text{ N}) = 0$. Thus, the required torque is

$$\tau_a = +6.00\text{ N} \cdot \text{m} \quad \text{or} \quad \boxed{6.00\text{ N} \cdot \text{m counterclockwise}}$$

12.2 Use distances, angles, and forces as shown. The conditions of equilibrium are:

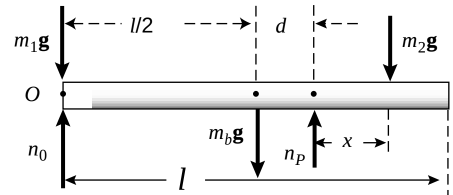


$$\sum F_y = 0 \Rightarrow \boxed{F_y + R_y - F_g = 0}$$

$$\sum F_x = 0 \Rightarrow \boxed{F_x - R_x = 0}$$

$$\sum \tau = 0 \Rightarrow \boxed{F_y\, \ell \cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x\, \ell \sin\theta = 0}$$

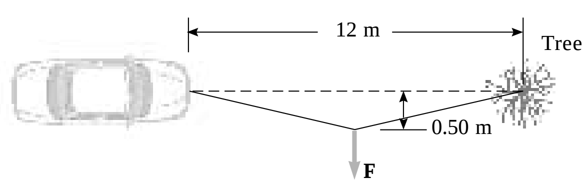
12.3 Take torques about P.



$$\sum \tau_p = -n_0\left[\frac{\ell}{2} + d\right] + m_1 g\left[\frac{\ell}{2} + d\right] + m_b g d - m_2 g x = 0$$

We want to find x for which $n_0 = 0$.

$$x = \frac{(m_1 g + m_b g)d + m_1 g \ell/2}{m_2 g} = \boxed{\frac{(m_1 + m_b)d + m_1 \ell/2}{m_2}}$$



12.4 $\tan\alpha = \dfrac{0.500}{6.00}$

$\alpha = 4.76°$

$F = 2T \sin\alpha$

$$T = \frac{F}{2 \sin \alpha} = \boxed{3.01 \text{ kN}}$$

***12.5** The location of the center of gravity is defined as

$$x_{\mathrm{CG}} \equiv \frac{\sum_{i=1}^{n} m_i g_i x_i}{\sum_{i=1}^{n} m_i g_i}$$

If the system is in a uniform gravitational field, this reduces to

$$x_{\mathrm{CG}} \equiv \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

Thus, for the given two particle system:

$$x_{\mathrm{CG}} = \frac{(3.00 \text{ kg})(-5.00 \text{ m}) + (4.00 \text{ kg})(3.00 \text{ m})}{3.00 \text{ kg} + 4.00 \text{ kg}} = \boxed{-0.429 \text{ m}}$$

12.6 The hole we can count as negative mass

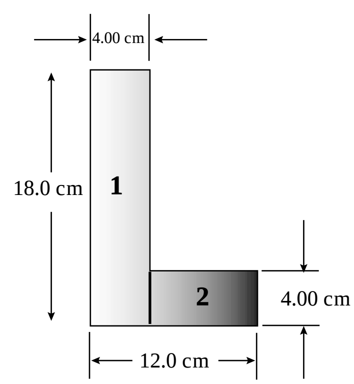
$$x_{\mathrm{CG}} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Call σ the mass of each unit of pizza area.

$$x_{\mathrm{CG}} = \frac{\sigma \pi R^2\, 0 - \sigma\pi (R/2)^2\, (-R/2)}{\sigma\pi R^2 - \sigma\pi(-R/2)^2}$$

$$x_{\mathrm{CG}} = \frac{R/8}{3/4} = \boxed{\frac{R}{6}}$$

12.7 The coordinates of the center of gravity of piece 1 are



$$x_1 = 2.00 \text{ cm} \quad \text{and} \quad y_1 = 9.00 \text{ cm}$$

The coordinates for piece 2 are

$$x_2 = 8.00 \text{ cm} \quad \text{and} \quad y_2 = 2.00 \text{ cm}$$

The area of each piece is

$$A_1 = 72.0 \text{ cm}^2 \quad \text{and} \quad A_2 = 32.0 \text{ cm}^2$$

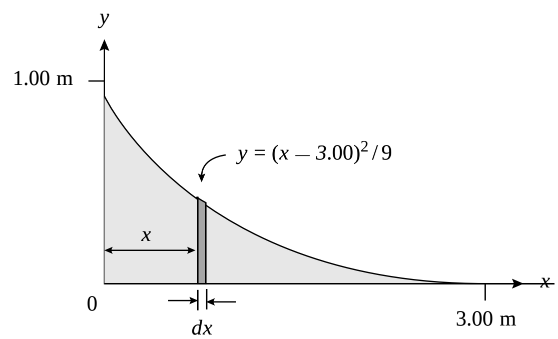
And the mass of each piece is proportional to the area. Thus,

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(72.0\text{ cm}^2)(2.00\text{ cm}) + (32.0\text{ cm}^2)(8.00\text{ cm})}{72.0\text{ cm}^2 + 32.0\text{ cm}^2} = \boxed{3.85\text{ cm}}$$

and

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(72.0\text{ cm}^2)(9.00\text{ cm}) + (32.0\text{ cm}^2)(2.00\text{ cm})}{104\text{ cm}^2} = \boxed{6.85\text{ cm}}$$

12.8 Let σ represent the mass-per-face area. A vertical strip at position x, with width dx and height $(x - 3.00)^2/9$ has mass



$$dm = \sigma(x - 3.00)^2 dx/9$$

The total mass is

$$M = \int dm = \int_{x=0}^{3.00} \sigma(x-3)^2 dx/9$$

$$M = (\sigma/9)\int_0^{3.00} (x^2 - 6x + 9)dx$$

$$M = \left(\frac{\sigma}{9}\right)\left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_0^{3.00} = \sigma$$

The x-coordinate of the center of gravity is

$$x_{CG} = \frac{\int x\,dm}{M} = \frac{1}{9\sigma}\int_0^{3.00} \sigma\, x(x-3)^2\,dx = \frac{\sigma}{9\sigma}\int_0^{3.00} (x^3 - 6x^2 + 9x)\,dx$$

$$x_{CG} = \frac{1}{9}\left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2}\right]_0^{3.00} = \frac{6.75\text{ m}}{9.00} = \boxed{0.750\text{ m}}$$

12.9 Let the fourth mass (8.00 kg) be placed at (x, y), then

$$x_{CG} = 0 = \frac{(3.00)(4.00) + m_4(x)}{12.0 + m_4}$$

$$x = -\frac{12.0}{8.00} = \boxed{-1.50\text{ m}}$$

Similarly, $y_{CG} = 0 = \dfrac{(3.00)(4.00) + 8.00(y)}{12.0 + 8.00}$

$$y = \boxed{-1.50\text{ m}}$$

***12.10** In a uniform gravitational field, the center of mass and center of gravity of an object coincide. Thus, the center of gravity of the triangle is located at $x = 6.67$ m, $y = 2.33$ m (see Example 9.14).

The coordinates of the three-object system are then:

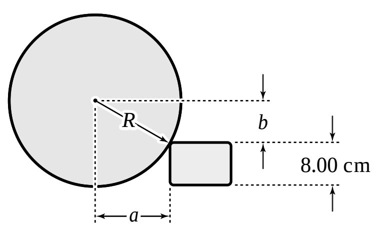
$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6.00\text{ kg})(5.50\text{ m}) + (3.00\text{ kg})(6.67\text{ m}) + (5.00\text{ kg})(-3.50\text{ m})}{(6.00 + 3.00 + 5.00)\text{ kg}}$$

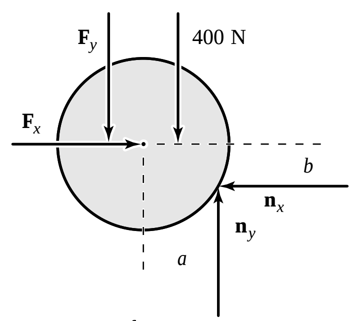
$$x_{CG} = \frac{35.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}} = \boxed{2.54\text{ m}} \quad \text{and}$$

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6.00\text{ kg})(7.00\text{ m}) + (3.00\text{ kg})(2.33\text{ m}) + (5.00\text{ kg})(+3.50\text{ m})}{14.0\text{ kg}}$$

$$y_{CG} = \frac{69.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}} = \boxed{4.75\text{ m}}$$

12.11 Call the required force F, with components $F_x = F\cos 15.0°$ and $F_y = -F\sin 15.0°$, transmitted to the center of the wheel by the handles.





Just as the wheel leaves the ground, the ground exerts no force on it.

$\sum F_x = 0$: $F\cos 15.0° - n_x$ (1)

$\sum F_y = 0$: $-F\sin 15.0° - 400\text{ N} + n_y = 0$ (2)

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b = R - 8.00\text{ cm} = (20.0 - 8.00)\text{ cm} = 12.0\text{ cm}$$

$$a = \sqrt{R^2 - b^2} = 16.0\text{ cm}$$

(a) $\sum \tau = 0$: $-F_x b + F_y a + (400\text{ N})a = 0$, or

$$F[-(12.0\text{ cm})\cos 15.0° + (16.0\text{ cm})\sin 15.0°] + (400\text{ N})(16.0\text{ cm}) = 0$$

so $$F = \frac{6400\text{ N}\cdot\text{cm}}{7.45\text{ cm}} = \boxed{859\text{ N}}$$

(b) Then, using Equations (1) and (2),

$n_x = (859\text{ N})\cos 15.0° = 830\text{ N}$ and

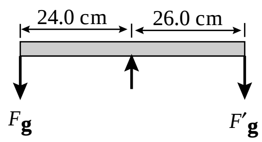
$n_y = 400\text{ N} + (859\text{ N})\sin 15.0° = 622\text{ N}$

$$n = \sqrt{n_x^2 + n_y^2} = \boxed{1.04\text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{n_y}{n_x}\right) = \tan^{-1}(0.749) = \boxed{36.9° \text{ to the left and upward}}$$

12.12 $F_g \rightarrow$ standard weight

$F'_g \rightarrow$ weight of goods sold



$F_g(0.240) = F'_g(0.260)$

$$F_g = F'_g\left(\frac{13}{12}\right)$$

$$\left(\frac{F_g - F'_g}{F'_g}\right)100 = \left(\frac{13}{12} - 1\right)\times 100 = \boxed{8.33\%}$$

12.13 (a) $\sum F_x = f - n_w = 0$

$\sum F_y = n_g - 800\text{ N} - 500\text{ N} = 0$

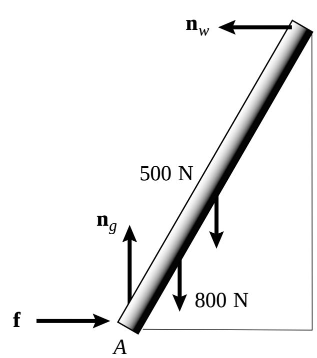
Taking torques about an axis at the foot of the ladder,

$$(800\text{ N})(4.00\text{ m})\sin 30.0° + (500\text{ N})(7.50\text{ m})\sin 30.0° - n_w(15.0\text{ m})\cos 30.0° = 0$$

Solving the torque equation,

$$n_w = \frac{[(4.00\text{ m})(800\text{ N}) + (7.50\text{ m})(500\text{ N})]\tan 30.0°}{15.0\text{ m}} =$$

268 N



Next substitute this value into the F_x equation to find

$f = n_w = \boxed{268\text{ N}}$ in the positive x direction

Solving the equation $\sum F_y = 0$,

$n_g = \boxed{1300\text{ N}}$ in the positive y direction

(b) In this case, the torque equation $\sum \tau_A = 0$ gives:

$$(9.00 \text{ m})(800 \text{ N}) \sin 30.0° + (7.50 \text{ m})(500 \text{ N}) \sin 30.0° - (15.0 \text{ m})(n_w) \sin 60.0° = 0$$

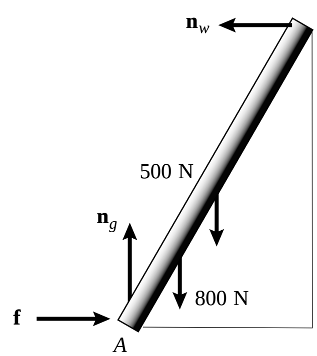
or $n_w = 421$ N

Since $f = n_w = 421$ N and $f = f_{\max} = \mu n_g$, we find

$$\mu = \frac{f_{\max}}{n_g} = \frac{421 \text{ N}}{1300 \text{ N}} = \boxed{0.324}$$

Goal Solution

G: Since the wall is frictionless, only the ground exerts an upward force on the ladder to oppose the combined weight of the ladder and firefighter, so $n_g = 1300$ N. Based on the angle of the ladder, $f < 1300$ N. The coefficient of friction is probably somewhere between 0 and 1.



O: Draw a free-body diagram, apply Newton's second law, and sum torques to find the unknown forces. Since this is a statics problem (no motion), both the net force and net torque are zero.

A: (a) $\sum F_x = f - n_w = 0$

$\sum F_y = n_g - 800 \text{ N} - 500 \text{ N} = 0$ so that $n_g = 1300$ N (upwards)

Taking torques about an axis at the foot of the ladder, $\sum \tau_A = 0$

$$-(800 \text{ N})(4.00 \text{ m}) \sin 30° - (500 \text{ N})(7.50 \text{ m}) \sin 30° + n_w(15.0 \text{ m}) \cos 30° = 0$$

Solving the torque equation for n_w,

$$n_w = \frac{[(4.00 \text{ m})(800 \text{ N}) + (7.50 \text{ m})(500 \text{ N})]}{15.0 \text{ m}} = 267.5 \text{ N}$$

Next substitute this value into the F_x equation to find

$f = n_w = 268$ N (**f** is directed toward the wall)

(b) When the firefighter is 9.00 m up the ladder, the torque equation $\sum\tau_A = 0$ gives

$$-(800\text{ N})(9.00\text{ m})\sin 30° - (500\text{ N})(7.50\text{ m})\sin 30° + n_w(15.0\text{ m})\sin 60° = 0$$

or $n_w = 421$ N

Since $f = n_w = 421$ N and $f = f_{\max} = \mu_s n_g$,

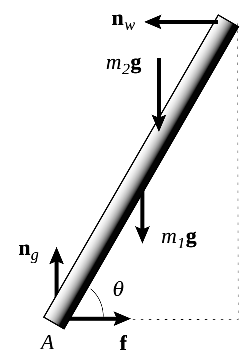
$$\mu_s = \frac{f_{\max}}{n_g} = \frac{421\text{ N}}{1300\text{ N}} = 0.324$$

L: The calculated answers seem reasonable since they agree with our predictions. This problem would be more realistic if the wall were not frictionless, in which case an additional vertical force would be added. This more complicated problem could be solved if we knew at least one of the coefficients of friction.

12.14 (a) $\sum F_x = f - n_w = 0$ (1)

$\sum F_y = n_g - m_1 g - m_2 g = 0$ (2)

$$\sum\tau_A = -m_1\left(\frac{L}{2}\right)\cos\theta - m_2\, gx\cos\theta + n_w L\sin\theta = 0$$



From the torque equation,

$$n_w = \left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta$$

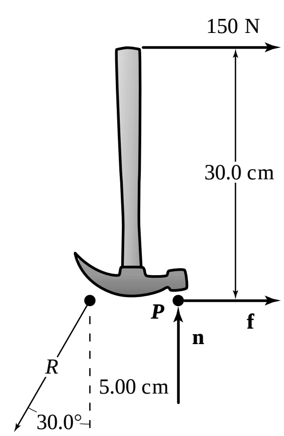
Then, from Equation (1): $f = n_w = \boxed{\left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta}$

and from Equation (2): $n_g = \boxed{(m_1 + m_2)g}$

(b) If the ladder is on the verge of slipping when $x = d$, then

$$\mu = \frac{f\,|_{x=d}}{n_g} = \boxed{\frac{\left[\frac{1}{2}m_1 g + \left(\frac{d}{L}\right)m_2 g\right]\cot\theta}{(m_1 + m_2)g}}$$

12.15 (a) Taking moments about P,



$$(R\sin 30.0°)0 + (R\cos 30.0°)(5.00\text{ cm}) - (150\text{ N})(30.0\text{ cm}) = 0$$

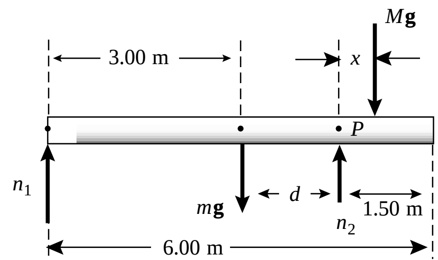
$$R = 1039.2\text{ N} = \boxed{1.04\text{ kN}}$$

(b) $f = R\sin 30.0° - 150\text{ N} = 370\text{ N}$

$n = R\cos 30.0° = 900\text{ N}$

$\boxed{\mathbf{F}_{\text{surface}} = (370\text{ N})\mathbf{i} + (900\text{ N})\mathbf{j}}$

12.16 See the free-body diagram at the right.



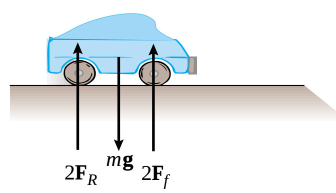
When the plank is on the verge of tipping about point P, the normal force n_1 goes to zero. Then, summing torques about point P gives

$$\Sigma\tau_p = -mgd + Mgx = 0 \quad\text{or}\quad x = \left(\frac{m}{M}\right)d$$

From the dimensions given on the free-body diagram, observe that $d = 1.50$ m. Thus, when the plank is about to tip,

$$x = \left(\frac{30.0\text{ kg}}{70.0\text{ kg}}\right)(1.50\text{ m}) = \boxed{0.643\text{ m}}$$

12.17 Torque about the front wheel is zero.



$$0 = (1.20\text{ m})(mg) - (3.00\text{ m})(2F_r)$$

Thus, the force at each rear wheel is $F_r = 0.200mg = \boxed{2.94\text{ kN}}$

The force at each front wheel is then $F_f = \dfrac{mg - 2F_r}{2} = \boxed{4.41\text{ kN}}$

Goal Solution

G: Since the center of mass lies in the front half of the car, there should be more force on the front wheels than the rear ones, and the sum of the wheel forces must equal the weight of the car.

O: Draw a free-body diagram, apply Newton's second law, and sum torques to find the unknown forces for this statics problem.

A: The car's weight is $F_g = mg = (1500\text{ kg})(9.80\text{ m/s}^2) = 14700\text{ N}$

Call F the force of the ground on each of the front wheels and R the normal force on each of the rear wheels.

If we take torques around the front axle, the equations are as follows:

$\sum F_x = 0 \qquad 0 = 0$

$\sum F_y = 0 \qquad 2R - 14700\text{ N} + 2F = 0$

$\sum \tau = 0 \qquad -2R(3.00\text{ m}) + (14700\text{ N})(1.20\text{ m}) + 2F(0) = 0$

The torque equation gives :

$$R = \frac{17\ 640\text{ N}\cdot\text{m}}{6.00\text{ m}} = 2940\text{ N} = 2.94\text{ kN}$$

Then, from the second force equation,

$$2(2.94\text{ kN}) - 14.7\text{ kN} + 2F = 0$$

and $F = 4.41\text{ kN}$

L: As expected, the front wheels experience a greater force wheels (about 50% more) than the rear wheels. Since the frictional force between the tires and road is proportional to this normal force, it makes sense that most cars today are built with front wheel drive so that the wheels under power are the ones with more traction (friction).

***12.18** $\sum F_x = F_b - F_t + 5.50\text{ N} = 0$ (1)

$\sum F_y = n - mg = 0$

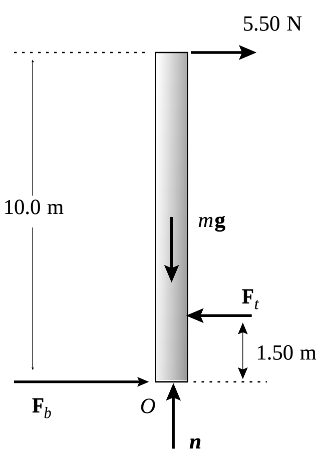
Summing torques about point O,

$$\sum \tau_O = F_t(1.50\text{ m}) - (5.50\text{ m})(10.0\text{ m}) = 0$$

which yields $F_t = \boxed{36.7\text{ N to the left}}$

Then, from Equation (1),

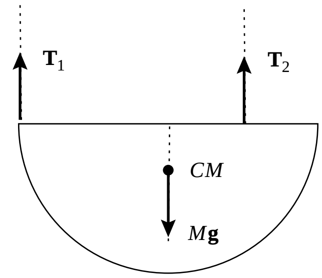
$$F_b = 36.7\text{ N} - 5.50\text{ N} = \boxed{31.2\text{ N to the right}}$$



12.19 (a) $T_e \sin 42.0° = 20.0\text{ N}$ $\boxed{T_e = 29.9\text{ N}}$

(b) $T_e \cos 42.0° = T_m$ $\boxed{T_m = 22.2\text{ N}}$

12.20 We call the tension in the cord at the left end of the sign, T_1, and the tension in the cord near the middle of the sign, T_2; and we choose our pivot point at the point where T_1 is attached.



$$\sum \tau_{\text{pivot}} = 0 = (-Mg)(0.500\text{ m}) + T_2(0.750\text{ m}) = 0,$$

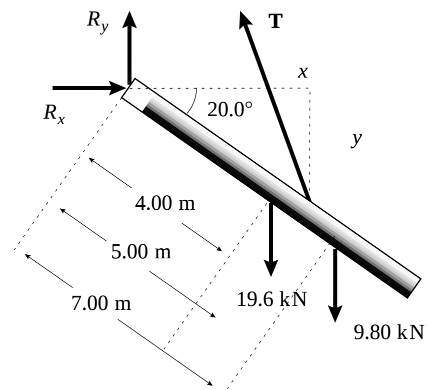
so, $T_2 = \boxed{\frac{2}{3}Mg}$

From $\sum F_y = 0$, $T_1 + T_2 - Mg = 0$

Substituting the expression for T_2 and solving, we find

$$T_1 = \boxed{\frac{1}{3}Mg}$$

12.21 Relative to the hinge end of the bridge, the cable is attached horizontally out a distance $x = (5.00\text{ m})\cos 20.0° =$ 4.70 m and vertically down a distance $y = (5.00\text{ m})\sin 20.0° = 1.71$ m. The cable then makes the following angle with the horizontal:



$$\theta = \tan^{-1}\left[\frac{(12.0 + 1.71)\text{ m}}{4.70\text{ m}}\right] = 71.1°$$

(a) Take torques about the hinge end of the bridge:

$$R_x(0) + R_y(0) - 19.6\text{ kN}(4.00\text{ m})\cos 20.0° - T\cos 71.1°(1.71\text{ m})$$

$$+\, T\sin 71.1°\,(4.70\text{ m}) - 9.80\text{ kN}(7.00\text{ m})\cos 20.0° = 0$$

which yields $T = \boxed{35.5\text{ kN}}$

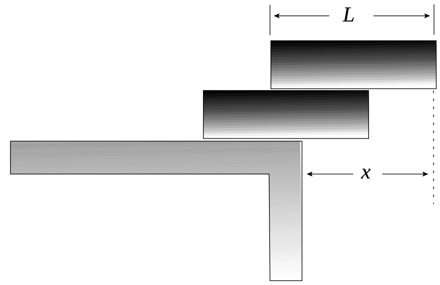
(b) $\sum F_x = 0 \Rightarrow R_x - T\cos 71.7° = 0$

or $R_x = (35.5\text{ kN})\cos 71.7° = \boxed{11.5\text{ kN (right)}}$

(c) $\sum F_y = 0 \Rightarrow R_y - 19.6\text{ kN} + T\sin 71.7° - 9.80\text{ kN} = 0$

Thus, $R_y = 29.4\text{ kN} - (35.5\text{ kN})\sin 71.7° = -4.19\text{ kN} = \boxed{4.19\text{ kN down}}$

12.22 $x = \boxed{\dfrac{3L}{4}}$



If the CM of the two bricks does not lie over the edge, then the bricks balance.

If the lower brick is placed $\dfrac{L}{4}$ over the edge, then the second brick may be placed so that its end protrudes $\dfrac{3L}{4}$ over the edge.

12.23 To find U, measure distances and forces from point A. Then, balancing torques,

$$(0.750)U = 29.4(2.25) \qquad \boxed{U = 88.2\text{ N}}$$

To find D, measure distances and forces from point B. Then, balancing torques,

$$(0.750)D = (1.50)(29.4) \qquad \boxed{D = 58.8\text{ N}}$$

Also, notice that $U = D + F_g$, so $\sum F_y = 0$

12.24 (a) stress $= F/A = F/\pi r^2$

$$F = (\text{stress})\pi(d/2)^2$$

$$F = (1.50 \times 10^8\text{ N/m}^2)\pi(2.50 \times 10^{-2}\text{ m}/2)^2$$

$$F = \boxed{73.6\text{ kN}}$$

(b) stress $= \Upsilon(\text{strain}) = \Upsilon \Delta L/L_i$

$$\Delta L = \frac{(\text{stress})L_i}{\Upsilon} = \frac{(1.50 \times 10^8\text{ N/m}^2)(0.250\text{ m})}{1.50 \times 10^{10}\text{ N/m}^2} = \boxed{2.50\text{ mm}}$$

12.25 $\dfrac{F}{A} = Y\dfrac{\Delta L}{L_i}$

$$\Delta L = \frac{FL_i}{AY} = \frac{(200)(9.80)(4.00)}{(0.200 \times 10^{-4})(8.00 \times 10^{10})} = \boxed{4.90\text{ mm}}$$

Goal Solution

G: Since metal wire does not stretch very much, the length will probably not change by more than 1% (<4 cm in this case) unless it is stretched beyond its elastic limit.

O: Apply the Young's Modulus strain equation to find the increase in length.

A: Young's Modulus is: $\Upsilon = \dfrac{F/A}{\Delta L/L_i}$

The load force is $F = (200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$

so $$\Delta L = \frac{FL_0}{A\Upsilon} = \frac{(1960\text{ N})(4.00\text{ m})(1000\text{ mm/m})}{(0.200 \times 10^{-4}\text{ m}^2)(8.00 \times 10^{10}\text{ N/m}^2)} = 4.90\text{ mm}$$

L: The wire only stretched about 0.1% of its length, so this seems like a reasonable result.

***12.26** Count the wires. If they are wrapped together so that all support nearly equal stress, the number should be

$$\frac{20.0\text{ kN}}{0.200\text{ kN}} = 100$$

Since cross-sectional area is proportional to diameter squared, the diameter of the cable will be

$$(1\text{ mm})\sqrt{100}\ \boxed{\sim 1\text{ cm}}$$

***12.27** From the defining equation for the shear modulus, we find Δx as

$$\Delta x = \frac{hf}{SA} = \frac{(5.00\times 10^{-3}\text{ m})(20.0\text{ N})}{(3.0\times 10^{6}\text{ N/m}^2)(14.0\times 10^{-4}\text{ m}^2)} = 2.38\times 10^{-5}\text{ m}$$

or $\Delta x = \boxed{2.38\times 10^{-2}\text{ mm}}$

***12.28** The force acting on the hammer changes its momentum according to

$$mv_i + \bar{F}(\Delta t) = mv_f \quad \text{so} \quad \left|\bar{F}\right| = \frac{m\left|v_f - v_i\right|}{\Delta t}$$

Hence, $\left|\bar{F}\right| = \dfrac{30.0\text{ kg}\left|-10.0\text{ m/s} - 20.0\text{ m/s}\right|}{0.110\text{ s}} = 8.18\times 10^{3}\text{ N}$

By Newton's third law, this is also the magnitude of the average force exerted on the spike by the hammer during the blow. Thus, the stress in the spike is:

$$\text{stress} = \frac{F}{A} = \frac{8.18\times 10^{3}\text{ N}}{\pi(0.0230\text{ m})^2/4} = 1.97\times 10^{7}\text{ N/m}^2$$

and the strain is: $\text{strain} = \dfrac{\text{stress}}{Y} = \dfrac{1.97\times 10^{7}\text{ N/m}^2}{20.0\times 10^{10}\text{ N/m}^2} = \boxed{9.85\times 10^{-5}}$

12.29 In this problem, $F = mg = 10.0(9.80) = 98.0\text{ N}$, $A = \pi d^2/4$,

and the maximum stress $= \dfrac{F}{A} = 1.50\times 10^{8}\text{ N/m}^2$

$$A = \frac{\pi d^2}{4} = \frac{F}{\text{Stress}} = \frac{98.0\text{ N}}{1.50\times 10^{8}\text{ N/m}^2} = 6.53\times 10^{-7}\text{ m}^2$$

$$d^2 = \frac{4(6.53\times 10^{-7}\text{ m}^2)}{\pi}$$

$$d = 9.12\times 10^{-4}\text{ m} = \boxed{0.912\text{ mm}}$$

12.30 Let the 3.00 kg mass be mass #1, with the 5.00 kg mass, mass # 2. Applying Newton's second law to each mass gives:

$$m_1 a = T - m_1 g \quad (1) \qquad \text{and} \qquad m_2 a = m_2 g - T \quad (2)$$

where T is the tension in the wire.

Solving equation (1) for the acceleration gives: $a = \frac{T}{m_1} - g$,

and substituting this into equation (2) yields: $\frac{m_2}{m_1} T - m_2 g = m_2 g - T$

Solving for the tension T gives

$$T = \frac{2m_1 m_2 g}{m_2 + m_1} = \frac{2(3.00\text{ kg})(5.00\text{ kg})(9.80\text{ m/s}^2)}{8.00\text{ kg}} = 36.8\text{ N}$$

From the definition of Young's modulus, $Y = \frac{FL_i}{A(\Delta L)}$, the elongation of the wire is:

$$\Delta L = \frac{TL_i}{YA} = \frac{(36.8\text{ N})(2.00\text{ m})}{(2.00\times 10^{11}\text{ N/m}^2)\,\pi\,(2.00\times 10^{-3}\text{ m})^2} = \boxed{0.0293\text{ mm}}$$

12.31 Assume that $m_2 > m_1$. Then, application of Newton’s second law to each mass yields the following equations of motion:

$$T - m_1 g = m_1 a \quad (1) \qquad \text{and} \qquad m_2 g - T = m_2 a \quad (2)$$

Solving Equation (1) for the acceleration gives $a = \frac{T}{m_1} - g$

and substitution into Equation (2) yields $m_2 g - T = \left(\frac{m_2}{m_1}\right) T - m_2 g$

The tension in the wire is then: $T = \frac{2m_2 g}{(m_1 + m_2)/m_1} = \frac{2m_1 m_2 g}{m_1 + m_2}$

From the definition of Young’s modulus, $Y = \frac{FL_i}{A(\Delta L)}$, the elongation of the wire is found to be:

$$\Delta L = \frac{TL_i}{AY} = \frac{[2m_1 m_2 g/(m_1 + m_2)]L_i}{(\pi d^2/4)Y} = \boxed{\frac{8m_1 m_2 g L_i}{\pi d^2 Y(m_1 + m_2)}}$$

12.32 At the surface 1030 kg of water fills 1.00 m^3. A kilometer down its volume has shrunk by ΔV in

$$\Delta V = \frac{-(\Delta P)V_i}{B} = \frac{-(10^7 \text{ N/m}^2)(1.00 \text{ m}^3)}{0.210 \times 10^{10} \text{ N/m}^2} = -4.76 \times 10^{-3} \text{ m}^3$$

so the new volume is $V = 1.00 \text{ m}^3 - 4.76 \times 10^{-3} \text{ m}^3 = 0.99524 \text{ m}^3$

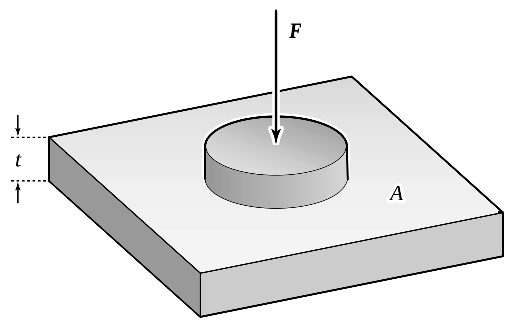
$$\therefore \text{ its density is } \rho = \frac{m}{V} = \frac{1030 \text{ kg}}{0.99524 \text{ m}^3} = \boxed{1.035 \times 10^3 \text{ kg/m}^3}$$

12.33 (a) $F = (A)(\text{stress}) = \pi(5.00 \times 10^{-3} \text{ m})^2(4.00 \times 10^8 \text{ N/m}^2) = \boxed{3.14 \times 10^4 \text{ N}}$

(b) The area over which the shear occurs is equal to the circumference of the hole times its thickness. Thus,

$$A = (2\pi r)t = 2\pi(5.00 \times 10^{-3} \text{ m})(5.00 \times 10^{-3} \text{ m}) = 1.57 \times 10^{-4} \text{ m}^2$$

So, $F = (A)\text{Stress} = (1.57 \times 10^{-4} \text{ m}^2)(4.00 \times 10^8 \text{ N/m}^2) = \boxed{6.28 \times 10^4 \text{ N}}$



12.34 (a) Using $Y = \dfrac{FL_i}{A(\Delta L)}$, we get $A = \dfrac{FL_i}{Y(\Delta L)} = \pi(d/2)^2$

So, $$d = \sqrt{\frac{4mgL_i}{\pi Y(\Delta L)}} = \sqrt{\frac{4(380 \text{ kg})(9.80 \text{ m/s}^2)(18.0 \text{ m})}{\pi(2.00 \times 10^{11} \text{ N/m}^2)(9.00 \times 10^{-3} \text{ m})}} = \boxed{6.89 \text{ mm}}$$

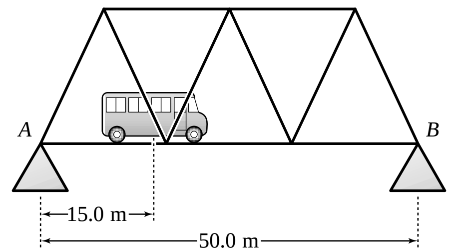
(b) $A = 3.72 \times 10^{-5} \text{ m}^2$ $\qquad F/A = 1.00 \times 10^8 \text{ N/m}^2$ $\boxed{\text{No}}$

12.35 $$\Delta P = -B\left(\frac{\Delta V}{V_i}\right) = -\left(2.00 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)(-0.090) = \boxed{1.80 \times 10^8 \text{ N/m}^2} \approx 1800 \text{ atm}$$

12.36 Using $Y = \dfrac{FL_i}{A(\Delta L)}$ with $A = \pi(d/2)^2$ and $F = mg$, we get

$$Y = \frac{4mgL_i}{\pi d^2(\Delta L)} = \frac{4(90.0 \text{ kg})(9.80 \text{ m/s}^2)(50.0 \text{ m})}{\pi(0.0100 \text{ m})^2(1.60 \text{ m})} = \boxed{3.51 \times 10^8 \text{ N/m}^2}$$

12.37 Let n_A and n_B be the normal forces at the points of support.



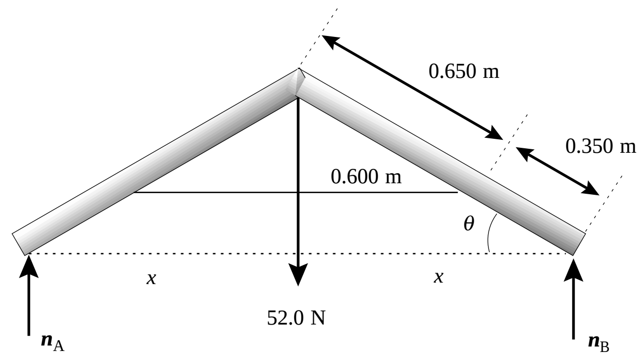
Choosing the origin at point A with $\sum F_y = 0$ and $\sum \tau = 0$, we find:

$$n_A + n_B - (8.00 \times 10^4)g - (3.00 \times 10^4)g = 0 \quad \text{and}$$

$$-(3.00 \times 10^4)(g)15.0 - (8.00 \times 10^4)(g)25.0 + n_B(50.0) = 0$$

The equations combine to give $n_A = \boxed{5.98 \times 10^5 \text{ N}}$ and $n_B = \boxed{4.80 \times 10^5 \text{ N}}$

12.38 Using similar triangles in the first figure at the right, the horizontal extent of each bar is found as

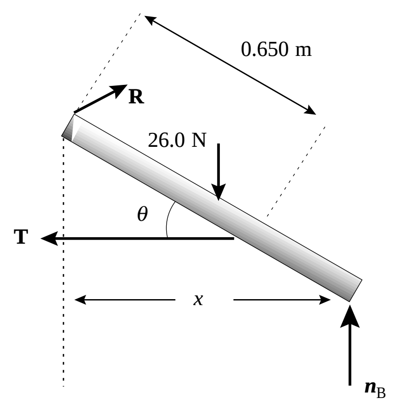


$$\frac{x}{(0.650 + 0.350) \text{ m}} = \frac{0.600 \text{ m}}{0.650 \text{ m}}$$

or $x = 0.923$ m. The angle each bar makes with the horizontal is

$$\theta = \cos^{-1}\left(\frac{x}{1.00 \text{ m}}\right) = \cos^{-1}(0.923)$$

or $\quad \theta = 22.6°$



choose the whole frame as object and take torques about point A, its left contact with the ground:

$$-(52.0 \text{ N})x + n_B(2x) = 0$$

giving $n_B = 26.0$ N

Isolate the right-side bar and take torques about its upper end:

$$R(0) - (26.0 \text{ N})[(0.500 \text{ m})\cos\theta] - (T\sin\theta)(0.650 \text{ m}) + n_B x = 0$$

so $$T = \frac{(26.0 \text{ N})(0.923 \text{ m}) - (26.0 \text{ N})(0.500 \text{ m})\cos 22.6°}{(0.650 \text{ m})\sin 22.6°} = \boxed{48.0 \text{ N}}$$

***12.39** When the concrete has cured and the pre-stressing tension has been released, the rod presses in on the concrete and with equal force, T_2, the concrete produces tension in the rod.

(a) In the concrete: stress $= 8.00 \times 10^6 \text{ N/m}^2 = Y \cdot (\text{strain}) = Y(\Delta L / L_i)$

$$\text{Thus, } \Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(8.00 \times 10^6 \text{ N/m}^2)(1.50 \text{ m})}{30.0 \times 10^9 \text{ N/m}^2}$$

or $\quad \Delta L = 4.00 \times 10^{-4} \text{ m} = \boxed{0.400 \text{ mm}}$

(b) In the concrete: stress $= \dfrac{T_2}{A_c} = 8.00 \times 10^6\ \text{N/m}^2$, so

$$T_2 = (8.00 \times 10^6\ \text{N/m}^2)(50.0 \times 10^{-4}\ \text{m}^2) = \boxed{40.0\ \text{kN}}$$

(c) For the rod: $\dfrac{T_2}{A_R} = \left(\dfrac{\Delta L}{L_i}\right) Y_{\text{steel}}$ so $\Delta L = \dfrac{T_2 L_i}{A_R Y_{\text{steel}}}$

$$\Delta L = \frac{(4.00 \times 10^4\ \text{N})(1.50\ \text{m})}{(1.50 \times 10^{-4}\ \text{m}^2)(20.0 \times 10^{10}\ \text{N/m}^2)} = 2.00 \times 10^{-3}\ \text{m} = \boxed{2.00\ \text{mm}}$$

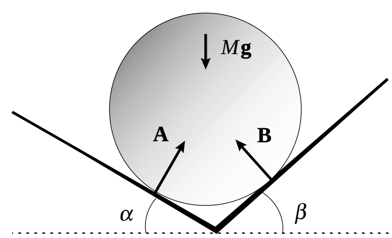
(d) The rod in the finished concrete is 2.00 mm longer than its unstretched length. To remove stress from the concrete, one must stretch the rod 0.400 mm farther, by a total of $\boxed{2.40\ \text{mm}}$.

(e) For the stretched rod around which the concrete is poured:

$$\frac{T_1}{A_R} = \left(\frac{\Delta L_{\text{total}}}{L_i}\right) Y_{\text{steel}} \quad \text{or} \quad T_1 = \left(\frac{\Delta L_{\text{total}}}{L_i}\right) A_R Y_{\text{steel}}$$

$$T_1 = \left(\frac{2.40 \times 10^{-3}\ \text{m}}{1.50\ \text{m}}\right)(1.50 \times 10^{-4}\ \text{m}^2)(20.0 \times 10^{10}\ \text{N/m}^2) = \boxed{48.0\ \text{kN}}$$

12.40 Call the normal forces A and B. They make angles α and β with the vertical.



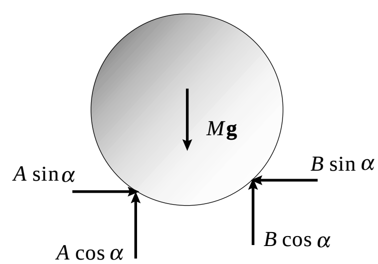
$$\sum F_x = 0: \quad A \sin\alpha - B \sin\beta = 0$$

$$\sum F_y = 0: \quad A \cos\alpha - Mg + B \cos\beta = 0$$

Substitute $B = A \sin\alpha / \sin\beta$

$$A \cos\alpha + A \cos\beta \sin\alpha / \sin\beta = Mg$$

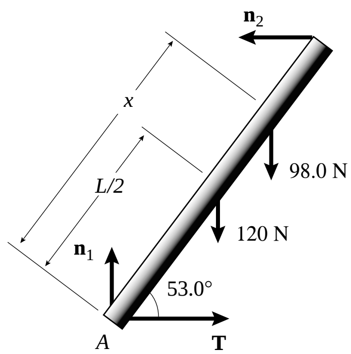
$$A(\cos\alpha \sin\beta + \sin\alpha \cos\beta) = Mg \sin\beta$$



$$A = \boxed{Mg \frac{\sin\beta}{\sin(\alpha + \beta)}}$$

$$B = \boxed{Mg \frac{\sin\alpha}{\sin(\alpha + \beta)}}$$

12.41 (a) See figure.



(b) Using $\sum F_x = \sum F_y = \sum \tau = 0$, we have (with A the bottom of the ladder):

$$\sum F_x = T - n_2 = 0$$

$$\sum F_y = n_2 - 218 \text{ N} = 0$$

$$\sum \tau_A = 98.0 \cos 53.0° + 120\left(\frac{L}{2}\right) \cos 53.0° - n_2 L \sin 53.0° = 0$$

where x is the distance of the monkey from the bottom of the ladder. When $x = L/3$, the above equation gives

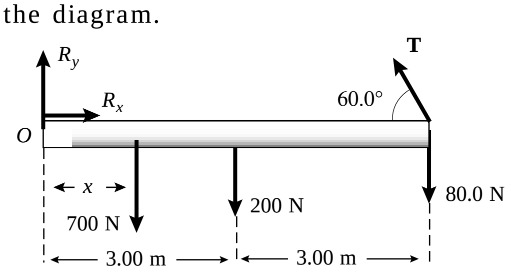
$$T = \frac{(18.7 + 36.1)}{0.800} = \boxed{69.8 \text{ N}}$$

(c) The rope breaks when $T = 110 \text{ N} = n_2$

$$\sum \tau_A = 10.0(9.80)x \cos 53.0° + 120(L/2) \cos 53.0° - 110L \sin 53.0° = 0$$

$$x = \frac{100L \sin 53.0° - 60.0L (\cos 53.0°)}{10.0(9.80) \cos 53.0°} = \boxed{0.877L}$$

12.42 (a) See the diagram.



(b) If $x = 1.00$ m, then

$$\sum \tau_O = (-700 \text{ N})(1.00 \text{ m}) - (200 \text{ N})(3.00 \text{ m}) - (80.0 \text{ N})(6.00 \text{ m})$$

$$+ (T \sin 60.0°)(6.00 \text{ m}) = 0$$

Solving for the tension gives: $T = \boxed{343 \text{ N}}$

From $\sum F_x = 0$, $R_x = T\cos 60.0° = \boxed{171 \text{ N}}$

From $\sum F_y = 0$, $R_y = 980 \text{ N} - T\sin 60.0° = \boxed{683 \text{ N}}$

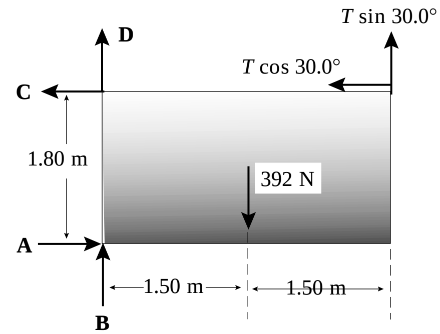
(c) If $T = 900$ N:

$$\sum \tau_O = (-700 \text{ N})x - (200 \text{ N})(3.00 \text{ m}) - (80.0 \text{ N})(6.00 \text{ m})$$

$$+ [(900 \text{ N})\sin 60.0°](6.00 \text{ m}) = 0$$

Solving for x gives: $x = \boxed{5.13 \text{ m}}$

12.43 (a) Sum the torques about top hinge:



$\sum \tau = 0$:

$$C(0) + D(0) + 200 \text{ N}\cos 30.0° \,(0)$$

$$+ 200 \text{ N}\sin 30.0°(3.00 \text{ m})$$

$$- 392 \text{ N}(1.50 \text{ m}) + A(1.80 \text{ m})$$

$$+ B(0) = 0$$

Giving $A = \boxed{160 \text{ N (right)}}$

(b) $\sum F_x = 0$:

$$-C - 200 \text{ N}\cos 30.0° + A = 0$$

$$C = 160 \text{ N} - 173 \text{ N} = -13.2 \text{ N}$$

In our diagram, this means $\boxed{13.2 \text{ N to the right}}$

(c) $\sum F_y = 0$: $+B + D - 392 \text{ N} + 200 \text{ N}\sin 30.0° = 0$

$B + D = 392 \text{ N} - 100 \text{ N} = \boxed{292 \text{ N(up)}}$

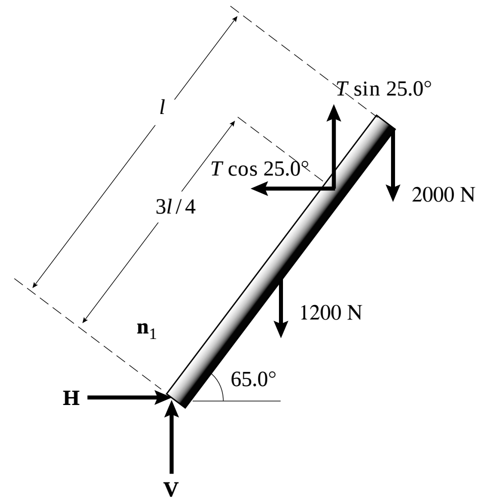
(d) Given $C = 0$: Take torques about bottom hinge to obtain

$$A(0) + B(0) + 0(1.80 \text{ m}) + D(0) - 392 \text{ N}(1.50 \text{ m})$$

$$+ T\sin 30.0°(3.00 \text{ m}) + T\cos 30.0°(1.80 \text{ m}) = 0$$

so $$T = \frac{588 \text{ N}\cdot\text{m}}{(1.50 \text{ m} + 1.56 \text{ m})} = \boxed{192 \text{ N}}$$

12.44 $\sum \tau_{\text{point 0}} = 0$ gives



$$(T\cos 25.0°)\left(\frac{3l}{4}\sin 65.0°\right) + (T\sin 25.0°)\left(\frac{3l}{4}\cos 65.0°\right)$$

$$= (2000\text{ N})(l\cos 65.0°) + (1200\text{ N})\left(\frac{l}{2}\cos 65.0°\right)$$

From which, $T = 1465\text{ N} = \boxed{1.46\text{ kN}}$

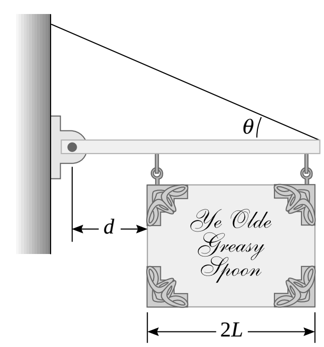
From $\sum F_x = 0$,

$H = T\cos 25.0° = 1328\text{ N}$ (toward right) = $\boxed{1.33\text{ kN}}$

From $\sum F_y = 0$,

$V = 3200\text{ N} - T\sin 25.0° = 2581\text{ N}$ (upward) = $\boxed{2.58\text{ kN}}$

12.45 Using $\sum F_x = \sum F_y = \sum \tau = 0$, choosing the origin at the left end of the beam, we have (neglecting the weight of the beam)



$\sum F_x = R_x - T\cos\theta = 0,$

$\sum F_y = R_y + T\sin\theta - F_g = 0,$ and

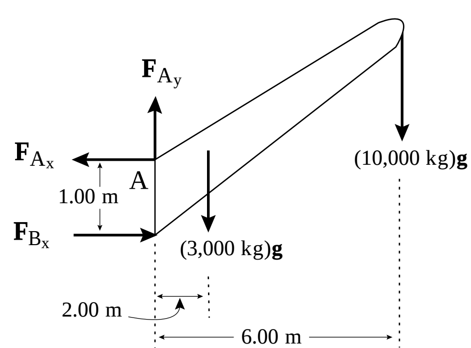
$\sum \tau = -F_g(L + d) + T\sin\theta\,(2L + d) = 0$

Solving these equations, we find:

(a) $T = \boxed{\dfrac{F_g(L+d)}{\sin\theta(2L+d)}}$ and

(b) $R_x = \boxed{\dfrac{F_g(L+d)\cot\theta}{2L+d}}$ $\quad R_y = \boxed{\dfrac{F_g L}{2L+d}}$

12.46 At point B since the support is *smooth* the reaction force is in the x direction. If we choose point A as the origin, then we have



$\sum F_x = F_{Bx} - F_{Ax} = 0$

$F_{Ay} - (3000 + 10000)g = 0$

and $\sum \tau = -(3000g)(2.00) - (10000g)(6.00) + F_{Bx}(1.00) = 0$

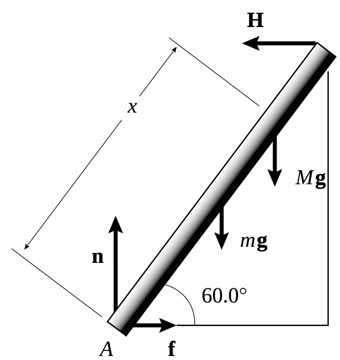
These equations combine to give

$$F_{Ax} = F_{Bx} = \boxed{6.47 \times 10^5 \text{ N}}$$

and $F_{By} = \boxed{0}$

$$\boxed{F_{Ay} = 1.27 \times 10^5 \text{ N}}$$

12.47 $n = (M + m)g \quad H = f$



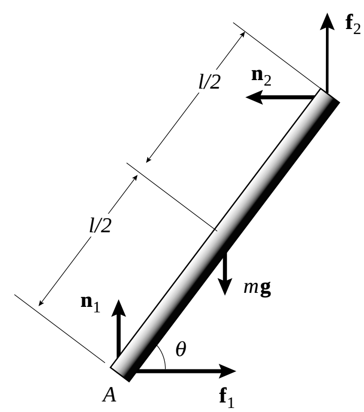
$H_{\max} = f_{\max} = \mu_s(m + M)g$

$$\sum\tau_A = 0 = \frac{mgL}{2}\cos 60.0° + Mgx\cos 60.0° - HL\sin 60.0°$$

$$\frac{x}{L} = \frac{H\tan 60.0°}{Mg} - \frac{m}{2M} = \frac{\mu_s(m + M)\tan 60.0°}{M} - \frac{m}{2M}$$

$$= \frac{3}{2}\mu_s \tan 60.0° - \frac{1}{4} = \boxed{0.789}$$

12.48 Since the ladder is about to slip, $f = (f_s)_{\max} = \mu_s n$ at each contact point. Because the ladder is still (barely) in equilibrium: $\sum F_x = 0$, which gives



$$f_1 - n_2 = 0 \quad \text{or} \quad \mu_s n_1 = n_2$$

giving $\quad n_1 = \dfrac{n_2}{\mu_s}$

Since $\sum F_y = 0$, use $n_1 - mg + f_2$ to eliminate n_1,

$$n_2 = \frac{\mu_s mg}{1 + \mu_s^2} \quad (1)$$

$$\sum\tau_{\text{lower end}} = 0 \quad \text{gives} \quad -mg\frac{L}{2}\cos\theta + n_2 L\sin\theta + f_2 L\cos\theta = 0$$

which can be written as $-\dfrac{mg}{2} + n_2\tan\theta + \mu_s n_2 = 0$, or

$$mg = 2n_2(\tan\theta + \mu_s) \qquad (2)$$

Substituting equation (1) into equation (2) gives

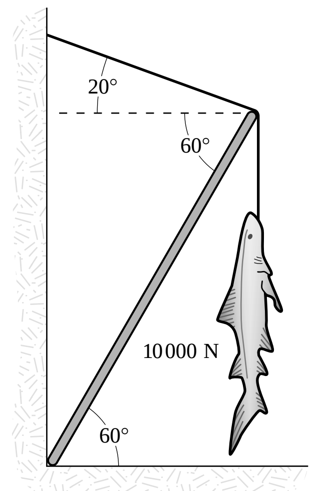
$$mg = \frac{2\mu_s mg(\tan\theta + \mu_s)}{1+\mu_s^2} \text{ which reduces to}$$

$$1+\mu_s^2 = 2\mu_s \tan\theta + 2\mu_s^2 \quad \text{or} \quad \mu_s^2 + (2\tan\theta)\mu_s - 1 = 0$$

With $\theta = 60.0°$, this becomes $\mu_s^2 + 3.646\mu_s - 1 = 0$,

which has one positive solution: $\mu_s = \boxed{0.268}$

12.49 Summing torques around the base of the rod,



$$\sum\tau = -(4.00\text{ m})(10000\text{ N})\cos 60.0° + T(4.00\text{ m})\sin 80.0° = 0$$

$$T = \frac{(10000\text{ N})\cos 60.0°}{\sin 80.0°} = \boxed{5.08\times10^3\text{ N}}$$

Since $F_H - T\cos 20.0° = 0$, $F_H = \boxed{4.77\text{ kN}}$

$$F_V + T\sin 20.0° - 10.0\text{ kN} = 0, \quad F_V = \boxed{8.26\text{ kN}}$$

<u>Goal Solution</u>

G: Since the rod helps support the weight of the shark by exerting a vertical force, the tension in the upper portion of the cable must be less than 10 000 N. Likewise, the vertical and horizontal forces on the base of the rod should also be less than 10 kN.

O: This is another statics problem where the sum of the forces and torques must be zero. To find the unknown forces, draw a free-body diagram, apply Newton's second law, and sum torques.

A: From the free-body diagram, the angle **T** makes with the rod is

$$\theta = 60.0° + 20.0° = 80.0°$$

and the perpendicular component of **T** is $T\sin 80.0°$.

Summing torques around the base of the rod,

$$\sum\tau = 0: \qquad -(4.00\text{ m})(10000\text{ N})\cos 60° + T(4.00\text{ m})\sin 80° = 0$$

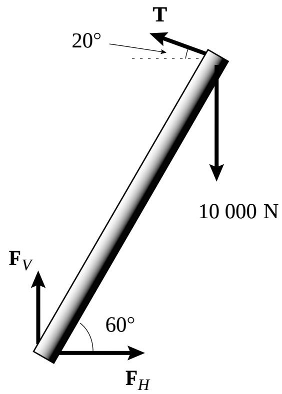
$$T = \frac{(10000\text{ N})\cos 60.0°}{\sin 80.0°} = 5.08\times10^3\text{ N}$$

$\sum F_x = 0$: $F_H - T\cos 20.0° = 0$

$F_H = T\cos 20.0° = 4.77 \times 10^3$ N

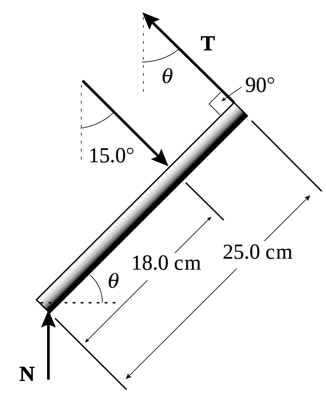
$\sum F_y = 0$: $F_V + T\sin 20.0° - 10000\text{ N} = 0$

and $F_V = (10000\text{ N}) - T\sin 20.0° = 8.26 \times 10^3$ N



L: The forces calculated are indeed less than 10 kN as predicted. That shark sure is a big catch; it weighs about a ton!

12.50 Choosing the origin at **R**,



(1) $\sum F_x = +R\sin 15.0° - T\sin\theta = 0$

(2) $\sum F_y = 700 - R\cos 15.0° + T\cos\theta = 0$

(3) $\sum\tau = -700\cos\theta(0.180) + T(0.0700) = 0$

Solve the equations for θ

from (3), $T = 1800\cos\theta$ from (1), $R = \dfrac{1800\sin\theta\cos\theta}{\sin 15.0°}$

Then (2) gives $700 - \dfrac{1800\sin\theta\cos\theta\cos 15.0°}{\sin 15.0°} + 1800\cos^2\theta = 0$

or $\cos^2\theta + 0.3889 - 3.732\sin\theta\cos\theta = 0$

Squaring, $\cos^4\theta - 0.8809\cos^2\theta + 0.01013 = 0$

Let $u = \cos^2\theta$ then using the quadratic equation,

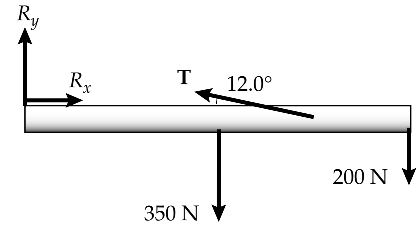
$u = 0.01165$ or 0.8693

Only the second root is physically possible,

$\therefore\theta = \cos^{-1}\sqrt{0.8693} = \boxed{21.2°}$

$\therefore T = \boxed{1.68 \times 10^3\text{ N}}$ and $R = \boxed{2.34 \times 10^3\text{ N}}$

12.51 Choosing torques about **R**, with $\sum \tau = 0$,



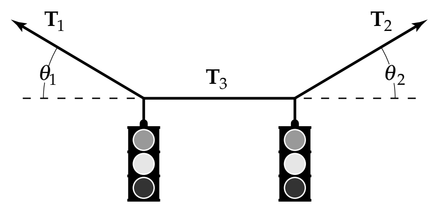
$$-\frac{L}{2}(350\text{ N}) + (T\sin 12.0°)\left(\frac{2L}{3}\right) - (200\text{ N})L = 0$$

From which, $T = \boxed{2.71\text{ kN}}$

Let R_x = compression force along spine, and from $\sum F_x = 0$,

$$R_x = T_x = T\cos 12.0° = \boxed{2.65\text{ kN}}$$

12.52 (a) Using the first diagram, $\sum F_x = 0$ gives



$$-T_1\cos\theta_1 + T_2\cos\theta_2 = 0$$

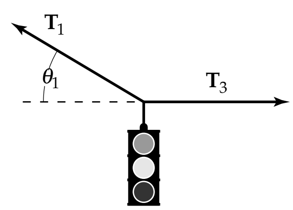
or $$T_2 = \left(\frac{\cos\theta_1}{\cos\theta_2}\right)T_1$$

If $\theta_1 = \theta_2$,

then $\boxed{T_2 = T_1}$

(b) Since $\theta_1 = \theta_2$, $T_2 = T_1$

Using the second diagram and $\sum F_y$ gives:



$$T_1\sin 8.00° - mg = 0 \quad \text{so}$$

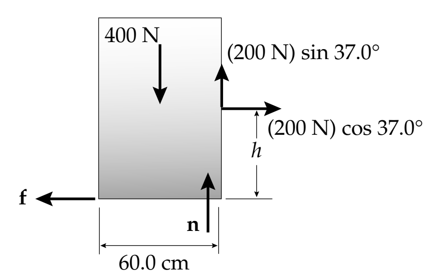
$$T_1 = \frac{200\text{ N}}{\sin 8.00°} = 1.44\text{ kN}$$

Then, $\boxed{T_2 = T_1 = 1.44\text{ kN}}$

Also, $\sum F_x = 0$ gives $-T_1\cos 8.00° + T_3 = 0$, or

$$T_3 = (1.44\text{ kN})\cos 8.00° = \boxed{1.42\text{ kN}}$$

12.53 (a) Locate the origin at the bottom left corner of the cabinet and let x = distance between the *resultant normal force* and the *front of the cabinet*. Then we have



(1) $\sum F_x = 200\cos(37.0°) - \mu n = 0$

(2) $\sum F_y = 200\sin(37.0°) + n - 400 = 0$, and

(3) $\sum \tau = n(0.600 - x) - 400(0.300) + 200\sin 37.0°(0.600) - 200\cos 37.0°(0.400) = 0$

From (2), $n = 400 - 200 \sin 37.0° = 280\text{ N}$

From (3), $x = \dfrac{[72.2 - 120 + 260(0.600) - 64.0]}{280} = \boxed{20.1\text{ cm}}$ to the left of the front edge.

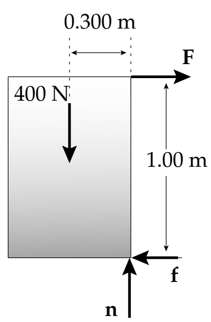
Then from (1), $\mu_k = \dfrac{200 \cos 37.0°}{280} = \boxed{0.571}$

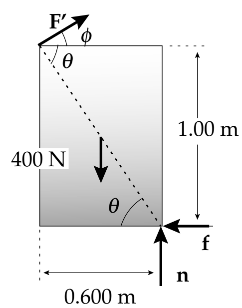
(b) In this case, locate the origin $x = 0$ at the bottom right corner of the cabinet. Since the cabinet is about to tip, we can use $\sum\tau = 0$ to find h:

$$\sum\tau = 400(0.300) - 300 \cos 37.0°(h) = 0$$

$$h = \frac{120}{300 \cos 37.0°} = \boxed{0.501\text{ m}}$$

12.54 (a) & (b) Use the first diagram and sum the torques about the lower front corner of the cabinet.





$$\sum\tau = 0 \Rightarrow -F(1.00\text{ m}) + (400\text{ N})(0.300\text{ m}) = 0$$

yielding $F = \dfrac{(400\text{ N})(0.300\text{ m})}{1.00\text{ m}} = \boxed{120\text{ N}}$

$\sum F_x = 0 \Rightarrow -f + 120\text{ N} = 0,$ or $f = 120\text{ N}$

$\sum F_y = 0 \Rightarrow -400\text{ N} + n = 0,$ so $n = 400\text{ N}$

Thus, $\mu_s = \dfrac{f}{n} = \dfrac{120\text{ N}}{400\text{ N}} = \boxed{0.300}$

(c) Apply F' at the upper rear corner and directed so $\theta + \phi = 90.0°$ to obtain the largest possible lever arm.

$$\theta = \tan^{-1}\left(\frac{1.00\text{ m}}{0.600\text{ m}}\right) = 59.0°$$

Thus, $\phi = 90.0° - 59.0° = 31.0°$

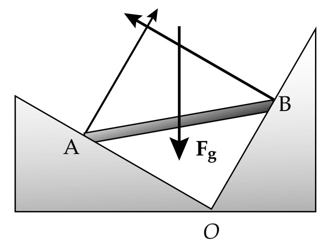
Sum the torques about the lower front corner of the cabinet:

$$-F'\sqrt{(1.00\text{ m})^2 + (0.600\text{ m})^2} + (400\text{ N})(0.300\text{ m}) = 0$$

so $$F' = \frac{120\text{ N}\cdot\text{m}}{1.17\text{ m}} = 103\text{ N}$$

Therefore, the minimum force required to tip the cabinet is
$\boxed{\text{103 N applied at 31.0° above the horizontal at the upper left corner}}$.

12.55 (a) Just three forces act on the rod: forces perpendicular to the sides of the trough at A and B, and its weight. The lines of action of A and B will intersect at a point above the rod. They will have no torque about this point. The rod's weight will cause a torque about the point of intersection as in Figure 1, and the rod will not be in equilibrium unless the center of the rod lies vertically below the intersection point, as in Figure 2. All three forces must be concurrent. Then the line of action of the weight is a diagonal of the rectangle formed by the trough and the normal forces, and the rod's center of gravity is vertically above the bottom of the trough.

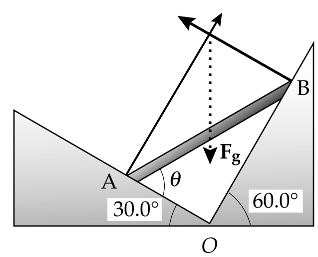


(b) In Figure 2, $\overline{AO}\cos 30.0° = \overline{BO}\cos 60.0°$ and

$$L^2 = \overline{AO}^{\,2} + \overline{BO}^{\,2} = \overline{AO}^{\,2} + \overline{AO}^{\,2}\left(\frac{\cos^2 30.0°}{\cos^2 60.0°}\right)$$

$$\overline{AO} = L/\sqrt{1 + \cos^2 30.0°/\cos^2 60.0°} = L/2$$

So $\cos\theta = \overline{AO}/L = 1/2$ and $\theta = \boxed{60.0°}$

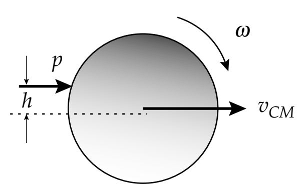


12.56 (1) $ph = I\omega$

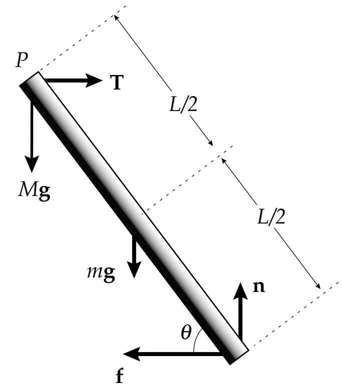
(2) $p = Mv_{\text{CM}}$

If the ball rolls without slipping, $R\omega = v_{\text{CM}}$

So, $$h = \frac{I\omega}{p} = \frac{I\omega}{Mv_{\text{CM}}} = \frac{I}{MR} = \boxed{\frac{2}{5}R}$$



12.57 (a) We can use $\Sigma F_x = \Sigma F_y = 0$ and $\Sigma\tau = 0$ with pivot point at the contact on the floor.



Then $\Sigma F_x = T - \mu_s n = 0$,

$\Sigma F_y = n - Mg - mg = 0$, and

$$\Sigma\tau = Mg(L\cos\theta) + mg\left(\frac{L}{2}\cos\theta\right) - T(L\sin\theta) = 0$$

Solving the above equations gives

$$M = \boxed{\frac{m}{2}\left(\frac{2\mu_s\sin\theta - \cos\theta}{\cos\theta - \mu_s\sin\theta}\right)}$$

(b) At the floor, we have the normal force in the y-direction and frictional force in the x-direction. The reaction force then is

$$R = \sqrt{n + (\mu_s n)^2} = \boxed{(M+m)g\sqrt{1+\mu_s^2}}$$

At point P, the force of the beam on the rope is

$$F = \sqrt{T^2 + (Mg)^2} = \boxed{g\sqrt{M^2 + \mu_s^2(M+m)^2}}$$

Goal Solution

G: The solution to this problem is not as obvious as some other problems because there are three independent variables that affect the maximum mass M. We could at least expect that more mass can be supported for higher coefficients of friction (μ_s), larger angles (θ), and a more massive beam (m).

O: Draw a free-body diagram, apply Newton's second law, and sum torques to find the unknown forces for this statics problem.

A: (a) Use $\Sigma F_x = \Sigma F_y = \Sigma\tau = 0$ and choose the origin at the point of contact on the floor to simplify the torque analysis.

On the verge of slipping, the friction $f = \mu_s n$, and

$\Sigma F_x = 0$: $T - \mu_s n = 0$

$\Sigma F_y = 0$: $n - Mg - mg = 0$

Solving these two equations, $T = \mu_s g(M+m)$

From $\sum \tau = 0$, $Mg(\cos\theta)L + mg(\cos\theta)\dfrac{L}{2} - T(\sin\theta)L = 0$

where we have used L for the length of the beam.

Substituting for T, we get $M = \dfrac{m}{2}\left[\dfrac{2\mu_s \sin\theta - \cos\theta}{\cos\theta - \mu_s \sin\theta}\right]$

Notice that this result does not depend on L, which is reasonable since the center of mass of the beam is proportional to the length of the beam.

(b) At the floor, we see that the normal force is in the y direction and frictional force is in the x direction. The reaction force of the floor on the beam opposes these two forces and is

$$R = \sqrt{n^2 + (\mu_s n)^2} = g(M+m)\sqrt{1+\mu_s^2}$$

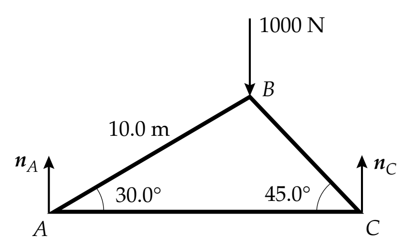
At point P, the force of the beam on the rope is

$$F = \sqrt{T^2 + (Mg)^2} = g\sqrt{M^2 + \mu_s^2 (M+m)^2}$$

L: The answer to this problem is certainly more complex than most problems. We can see that the maximum mass M that can be supported is proportional to m, but it is not clear from the solution that M increases proportional to μ_s and θ as predicted. To further examine the solution to part (a), we could graph or calculate the ratio M/m as a function of for several reasonable values of μ_s ranging from 0.5 to 1.0. Since the mass values must be positive, we find that only angles from about 40 to 60 are possible for this scenario (which explains why we don't encounter this precarious configuration very often!).

***12.58** (a) The height of pin B is

$$(10.0\text{ m})\sin 30.0° = 5.00\text{ m}$$



The length of bar BC is then

$$\overline{\text{BC}} = 5.00\text{ m}/\sin 45.0° = 7.07\text{ m}$$

Consider the entire truss:

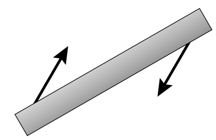
$$\sum F_y = n_A - 1000\text{ N} + n_C = 0$$

$$\sum \tau_A = -(1000\text{ N})10.0\cos 30.0° + n_C[10.0\cos 30.0° + 7.07\cos 45.0°] = 0$$

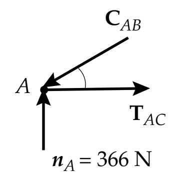
Which gives $\boxed{n_C = 634\text{ N}}$

Then, $\boxed{n_A = 1000\text{ N} - n_C = 366\text{ N}}$

(b) Suppose that a bar exerts on a pin a force not along the length of the bar. Then, the pin exerts on the bar a force with a component perpendicular to the bar. The only other force on the bar is the pin force on the other end. For $\sum \mathbf{F} = 0$, this force must also have a component perpendicular to the bar. Then, the total torque on the bar is not zero. The contradiction proves that the bar can only exert forces along its length.



(c) Joint A:



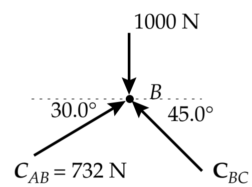
$$\sum F_y = 0: \; -C_{AB} \sin 30.0° + 366 \text{ N} = 0,$$

so $C_{AB} = \boxed{732 \text{ N}}$

$$\sum F_x = 0: \; -C_{AB} \cos 30.0° + T_{AC} = 0$$

$$T_{AC} = (732 \text{ N}) \cos 30.0° = \boxed{634 \text{ N}}$$

Joint B:

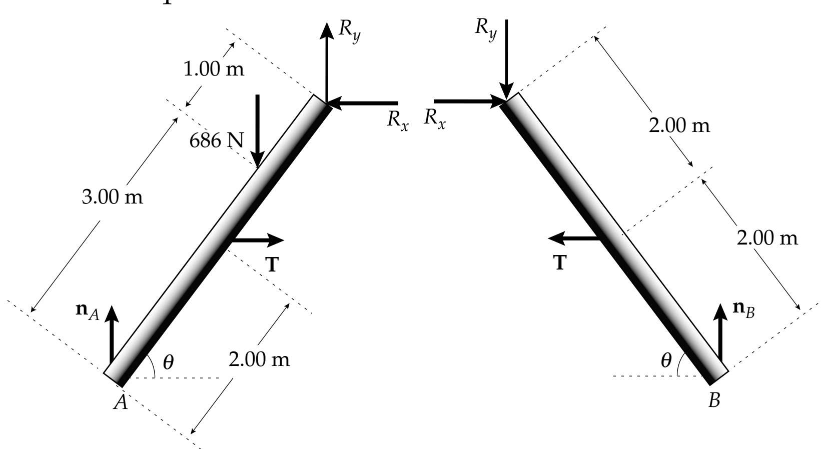


$$\sum F_x = 0: \; (732 \text{ N}) \cos 30.0° - C_{BC} \cos 45.0° = 0$$

$$C_{BC} = \frac{(732 \text{ N}) \cos 30.0°}{\cos 45.0°} = \boxed{897 \text{ N}}$$

12.59 From geometry, observe that

$$\cos \theta = \frac{1}{4} \quad \text{and} \quad \theta = 75.5°$$



For the left half of the ladder, we have

(1) $\sum F_x = T - R_x = 0$

(2) $\sum F_y = R_y + n_A - 686 \text{ N} = 0$

(3) $\sum \tau_{\text{top}} = (686 \text{ N})(1.00 \cos 75.5°) + T(2.00 \sin 75.5°) - n_A(4.00 \cos 75.5°) = 0$

For the right half of the ladder we have

$$\Sigma F_x = R_x - T = 0$$

(4) $\Sigma F_y = n_B - R_y = 0$

(5) $\Sigma \tau_{\text{top}} = n_B\,(4.00 \cos 75.5°) - T(2.00 \sin 75.5°) = 0$

Solving Equations 1 through 5 simultaneously yields:

(a) $\boxed{T = 133 \text{ N}}$

(b) $\boxed{n_A = 429 \text{ N}}$ and $\boxed{n_B = 257 \text{ N}}$

(c) $\boxed{R_x = 133 \text{ N}}$ and $\boxed{R_y = 257 \text{ N}}$

12.60 (a) $x_{\text{CG}} = \dfrac{\Sigma m_i x_i}{\Sigma m_i}$

$$= \frac{(1000\text{ kg})10.0\text{ m} + (125\text{ kg})0 + (125\text{ kg})0 + (125\text{ kg})20.0\text{ m}}{1375\text{ kg}} = \boxed{9.09\text{ m}}$$

$$y_{\text{CG}} = \frac{(1000\text{ kg})10.0\text{ m} + (125\text{ kg})20.0\text{ m} + (125\text{ kg})20.0\text{ m} + (125\text{ kg})0}{1375\text{ kg}}$$

$$= \boxed{10.9\text{ m}}$$

(b) By symmetry, $x_{\text{CG}} = \boxed{10.0\text{ m}}$

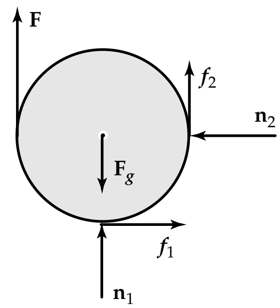
There is no change in $y_{\text{CG}} = \boxed{10.9\text{ m}}$

(c) $v_{\text{CG}} = \left(\dfrac{10.0\text{ m} - 9.09\text{ m}}{8.00\text{ s}}\right) = \boxed{0.114\text{ m/s}}$

12.61 Considering the torques about the point at the bottom of the bracket yields:

$(0.0500\text{ m})(80.0\text{ N}) - F(0.0600\text{ m}) = 0$ so $\boxed{F = 66.7\text{ N}}$

12.62 $f_1 = n_2 = \mu_s n_1$ and $f_2 = \mu_s n_2$



$F + n_1 + f_2 = F_g$ and $F = f_1 + f_2$

As F grows so do f_1 and f_2

Therefore, since $\mu_s = \dfrac{1}{2}$,

$$f_1 = \frac{n_1}{2} \quad \text{and} \quad f_2 = \frac{n_2}{2} = \frac{n_1}{4}$$

$$F + n_1 + \frac{n_1}{4} = F_g \ (1) \quad \text{and} \quad F = \frac{n_1}{2} + \frac{n_1}{4} = \frac{3}{4} n_1 \ (2)$$

$$F + \frac{5}{4} n_1 = F_g \quad \text{becomes} \quad F + \frac{5}{4}\left(\frac{4}{3} F\right) = F_g \quad \text{or} \quad \frac{8}{3} F = F_g$$

Therefore, $F = \boxed{\dfrac{3}{8} F_g}$

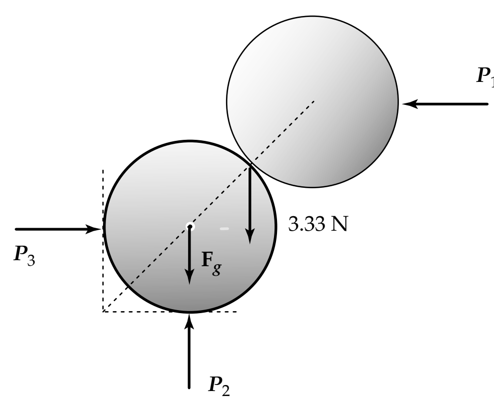
12.63 (a) $|F| = k(\Delta L)$, Young's modulus is $Y = \left(\dfrac{F}{A}\right) \Big/ \left(\dfrac{\Delta L}{L_i}\right) = \dfrac{F L_i}{A(\Delta L)}$

Thus, $Y = \dfrac{kL_i}{A}$ and $k = \boxed{\dfrac{YA}{L_i}}$

(b) $W = -\int_0^{\Delta L} F dx = -\int_0^{\Delta L} (-kx) dx = \dfrac{YA}{L_i} \int_0^{\Delta L} x\, dx =$

$\boxed{YA \dfrac{(\Delta L)^2}{2L_i}}$

12.64 (a) Take both balls together. Their weight is 3.33 N and their CG is at their contact point.



$\sum F_x = 0$: $+P_3 - P_1 = 0$

$\sum F_y = 0$: $+P_2 - 3.33 \text{ N} = 0$ $\quad P_2 = \boxed{3.33 \text{ N}}$

$\sum \tau_A = 0$: $-P_3 R + P_2 R - 3.33 \text{ N}(R + R \cos 45.0°)$

$+ P_1(R + 2R \cos 45.0°) = 0$

Substituting,

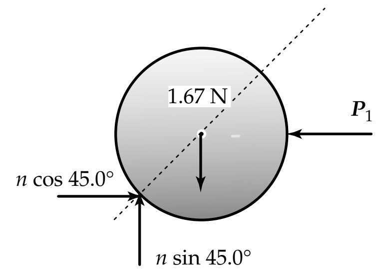
$-P_1 R + (3.33 \text{ N})R - (3.33 \text{ N})R(1 + \cos 45.0°)$

$+ P_1 R(1 + 2 \cos 45.0°) = 0$

$(3.33 \text{ N}) \cos 45.0° = 2P_1 \cos 45.0°$

$P_1 = \boxed{1.67\text{ N}}$ so $P_3 = \boxed{1.67\text{ N}}$

(b) Take the upper ball. The lines of action of its weight, of P_1, and of the normal force n exerted by the lower ball all go through its center, so for rotational equilibrium there can be no frictional force.

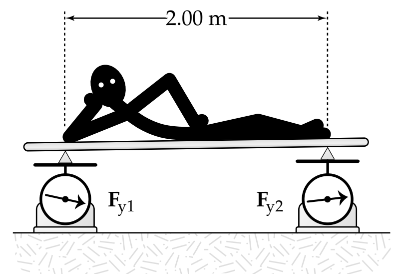


$$\Sigma F_x = 0:\ n\cos 45.0° - P_1 = 0$$

$$n = 1.67\text{ N}/\cos 45.0° = \boxed{2.36\text{ N}}$$

$\Sigma F_y = 0:\ n\sin 45.0° - 1.67\text{ N} = 0$ gives the same result

12.65 $\Sigma F_y = 0: \quad +380\text{ N} - F_g + 320\text{ N} = 0$



$$F_g = 700\text{ N}$$

Take torques about her feet:

$\Sigma\tau = 0: \quad -380\text{ N}(2.00\text{ m}) + 700\text{ N}(x) + 320\text{ N}(0) = 0$

$$x = \boxed{1.09\text{ m}}$$

12.66 The tension in this cable is not uniform, so this becomes a fairly difficult problem.

$$\frac{dL}{L} = \frac{F}{YA}$$

At any point in the cable, F is the weight of cable below that point. Thus, $F = \mu g y$ where μ is the mass per unit length of the cable.

Then, $\Delta y = \int_0^{L_i}\left(\frac{dL}{L}\right)dy = \frac{\mu g}{YA}\int_0^{L_i} y\,dy = \frac{1}{2}\frac{\mu g L_i^2}{YA}$

$$\Delta y = \frac{1}{2}\frac{(2.40)(9.80)(500)^2}{(2.00\times 10^{11})(3.00\times 10^{-4})} = 0.0490\text{ m} = \boxed{4.90\text{ cm}}$$

12.67 (a) $F = m\left(\frac{\Delta v}{\Delta t}\right) = (1.00\text{ kg})\frac{(10.0 - 1.00)\text{ m/s}}{0.002\text{ s}} = \boxed{4500\text{ N}}$

(b) $\text{stress} = \frac{F}{A} = \frac{4500\text{ N}}{(0.010\text{ m})(0.100\text{ m})} = \boxed{4.50\times 10^6\text{ N/m}^2}$

(c) $\boxed{\text{Yes}}$ This is more than sufficient to break the board.

12.68 The CG lies above the center of the bottom. Consider a disk of water at height y above the bottom. Its radius is

$$25.0\text{ cm} + (35.0 - 25.0\text{ cm})\left(\frac{y}{30.0\text{ cm}}\right) = 25.0\text{ cm} + \frac{y}{3}$$

Its area is $\pi(25.0\text{ cm} + y/3)^2$. Its volume is $\pi(25.0\text{ cm} + y/3)^2 dy$ and its mass is $\pi\rho(25.0\text{ cm} + y/3)^2 dy$. The whole mass of the water is

$$M = \int_{y=0}^{30.0\text{ cm}} dm = \int_0^{30.0\text{ cm}} \pi\rho\,(625 + 50.0y/3 + y^2/9)\,dy$$

$$M = \pi\rho\,[625y + 50.0y^2/6 + y^3/27]_0^{30.0}$$

$$M = \pi\rho\,[625(30.0) + 50.0(30.0)^2/6 + (30.0)^3/27]$$

$$M = \pi(10^{-3}\text{ kg/cm}^3)(27250\text{ cm}^3) = 85.6\text{ kg}$$

The height of the center of gravity is

$$y_{CG} = \int_{y=0}^{30.0\text{ cm}} y\,dm/M$$

$$= \pi\rho\int_0^{30.0\text{ cm}} (625y + 50.0y^2/3 + y^3/9)dy/M$$

$$= \frac{\pi\rho}{M}\,[625y^2/2 + 50.0y^3/9 + y^4/36]_0^{30.0\text{ cm}}$$

$$= \frac{\pi\rho}{M}\,[625(30.0)^2/2 + 50.0(30.0)^3/9 + (30.0)^4/36]$$

$$= \frac{\pi(10^{-3}\text{ kg/cm}^3)}{M}\,[453750\text{ cm}^4]$$

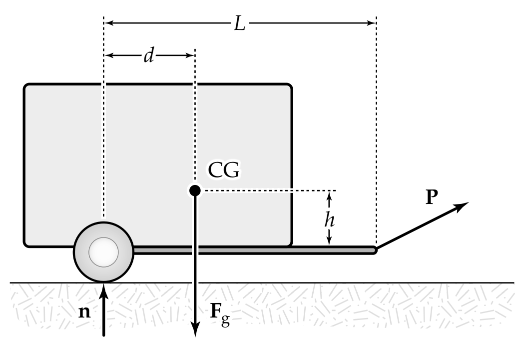
$$y_{CG} = \frac{1.43\times10^3\text{ kg}\cdot\text{cm}}{85.6\text{ kg}} = \boxed{16.7\text{ cm}}$$

12.69 (a) If the acceleration is a, we have $P_x = ma$ and $P_y + n - F_g = 0$. Taking the origin at the center of gravity, the torque equation gives

$$P_y(L-d) + P_x h - nd = 0$$

Solving these equations, we find

$$P_y = \boxed{\frac{F_g}{L}\left(d - \frac{ah}{g}\right)}$$



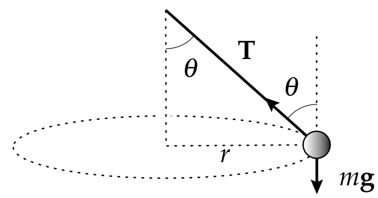
(b) If $P_y = 0$, then $d = \dfrac{ah}{g} = \dfrac{(2.00\text{ m/s}^2)(1.50\text{ m})}{9.80\text{ m/s}^2} = \boxed{0.306\text{ m}}$

(c) Using the given data, $P_x = -306$ N and $P_y = 553$ N

Thus, $\boxed{\mathbf{P} = (-306\mathbf{i} + 553\mathbf{j})\text{ N}}$

***12.70** Let θ represent the angle of the wire with the vertical. The radius of the circle of motion is $r = (0.850\text{ m})\sin\theta$.

For the mass:



$$\sum F_r = ma_r = m\frac{v^2}{r} = mr\omega^2$$

$$T\sin\theta = m\,[(0.850\text{ m})\sin\theta]\omega^2$$

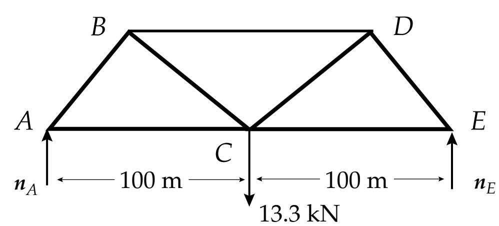
Further, $\dfrac{T}{A} = Y\cdot(\text{strain})$ or $T = AY\cdot(\text{strain})$

Thus, $AY\cdot(\text{strain}) = m(0.850\text{ m})\omega^2$, giving

$$\omega = \sqrt{\frac{AY\cdot(\text{strain})}{m(0.850\text{ m})}} = \sqrt{\frac{\pi(3.90\times10^{-4}\text{ m})^2(7.00\times10^{10}\text{ N/m}^2)(1.00\times10^{-3})}{(1.20\text{ kg})(0.850\text{ m})}}$$

or $\omega = \boxed{5.73\text{ rad/s}}$

***12.71** For the bridge as a whole:



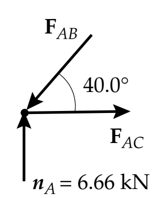
$$\sum\tau_A = n_A(0) - (13.3\text{ kN})(100\text{ m}) + n_E(200\text{ m}) = 0$$

so $n_E = \dfrac{(13.3\text{ kN})(100\text{ m})}{200\text{ m}} = \boxed{6.66\text{ kN}}$

$\sum F_y = n_A - 13.3\text{ kN} + n_E = 0$ gives

$n_A = 13.3\text{ kN} - n_E = \boxed{6.66\text{ kN}}$

At Pin A:



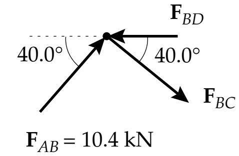
$\sum F_y = -F_{AB}\sin 40.0° + 6.66\text{ kN} = 0$ or

$$F_{AB} = \frac{6.66\text{ kN}}{\sin 40.0°} = \boxed{10.4\text{ kN (compression)}}$$

$\sum F_x = F_{AC} - (10.4\text{ kN})\cos 40.0° = 0$ so

$$F_{AC} = (10.4\text{ kN})\cos 40.0° = \boxed{7.94\text{ kN (tension)}}$$

At Pin B:



$$\Sigma F_y = (10.4 \text{ kN}) \sin 40.0° - F_{BC} \sin 40.0° = 0$$

Thus, $F_{BC} = \boxed{10.4 \text{ kN (tension)}}$

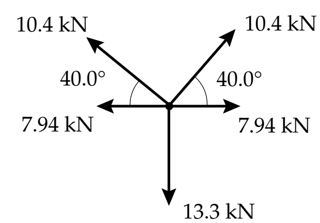
$$\Sigma F_x = F_{AB} \cos 40.0° + F_{BC} \cos 40.0° - F_{BD} = 0$$

$$F_{BD} = 2(10.4 \text{ kN}) \cos 40.0° = \boxed{15.9 \text{ kN (compression)}}$$

By symmetry: $\boxed{F_{DE} = F_{AB} = 10.4 \text{ kN (compression)}}$

$\boxed{F_{DC} = F_{BC} = 10.4 \text{ kN (tension)}}$

and $\boxed{F_{EC} = F_{AC} = 7.94 \text{ kN (tension)}}$



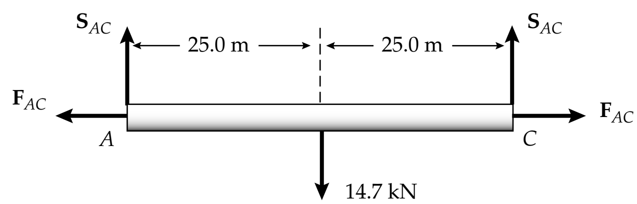
We can check by analyzing Pin C:

$$\Sigma F_x = +7.94 \text{ kN} - 7.94 \text{ kN} = 0 \quad \text{or} \quad 0 = 0$$

$$\Sigma F_y = 2(10.4 \text{ kN}) \sin 40.0° - 13.3 \text{ kN} = 0$$

which yields 0 = 0

***12.72** Member AC is not in pure compression or tension. It also has shear forces present. It exerts a downward force S_{AC} and a tension force F_{AC} on Pin A and on Pin C. Still, this member is in equilibrium.



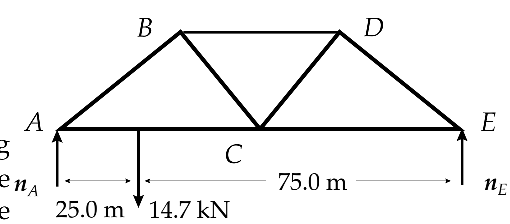
$$\Sigma F_x = F_{AC} - F'_{AC} = 0 \Rightarrow F_{AC} = F'_{AC}$$

$$\Sigma \tau_A = 0:$$

$$-(14.7 \text{ kN})(25.0 \text{ m}) + S'_{AC}(50.0 \text{ m}) = 0$$

or $S'_{AC} = 7.35 \text{ kN}$

$$\Sigma F_y = S_{AC} - 14.7 \text{ kN} + 7.35 \text{ kN} = 0 \Rightarrow S_{AC} = 7.35$$
kN



Then $S_{AC} = S'_{AC}$ and we have proved that the loading by the car is equivalent to one-half the weight of the car pulling down on each of pins A and C, so far as the rest of the truss is concerned.

For the Bridge as a whole: $\sum \tau_A = 0$:

$$-(14.7\text{ kN})(25.0\text{ m}) + n_E(100\text{ m}) = 0$$

$$\boxed{n_E = 3.67\text{ kN}}$$

$$\sum F_y = n_A - 14.7\text{ kN} + 3.67\text{ kN} = 0$$

$$\boxed{n_A = 11.0\text{ kN}}$$

At Pin A:

$$\sum F_y = -7.35\text{ kN} + 11.0\text{ kN} - F_{AB}\sin 30.0° = 0$$

$$\boxed{F_{AB} = 7.35\text{ kN (compression)}}$$

$$\sum F_x = F_{AC} - (7.35\text{ kN})\cos 30.0° = 0$$

$$\boxed{F_{AC} = 6.37\text{ kN (tension)}}$$

At Pin B:

$$\sum F_y = -(7.35\text{ kN})\sin 30.0° - F_{BC}\sin 60.0° = 0$$

$$\boxed{F_{BC} = 4.24\text{ kN (tension)}}$$

$$\sum F_x = (7.35\text{ kN})\cos 30.0° + (4.24\text{ kN})\cos 60.0° - F_{BD} = 0$$

$$\boxed{F_{BD} = 8.49\text{ kN (compression)}}$$

At Pin C:

$$\sum F_y = (4.24\text{ kN})\sin 60.0° + F_{CD}\sin 60.0° - 7.35\text{ kN} = 0$$

$$\boxed{F_{CD} = 4.24\text{ kN (tension)}}$$

$$\sum F_x = -6.37\text{ kN} - (4.24\text{ kN})\cos 60.0° + (4.24\text{ kN})\cos 60.0° + F_{CE} = 0$$

$$\boxed{F_{CE} = 6.37\text{ kN (tension)}}$$

At Pin E:

$$\sum F_y = -F_{DE}\sin 30.0° + 3.67\text{ kN} = 0$$

$$\boxed{F_{DE} = 7.35\text{ kN (compression)}}$$

or $\quad \sum F_x = -6.37\text{ kN} - F_{DE}\cos 30.0° = 0$

which gives $F_{DE} = 7.35$ kN as before.

Chapter 13 Solutions

13.1 $x = (4.00 \text{ m}) \cos (3.00\pi t + \pi)$

Compare this with $x = A \cos (\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$

or $\boxed{f = 1.50 \text{ Hz}}$ $\qquad T = \dfrac{1}{f} = \boxed{0.667 \text{ s}}$

(b) $A = \boxed{4.00 \text{ m}}$

(c) $\phi = \boxed{\pi \text{ rad}}$

(d) $x(t = 0.250 \text{ s}) = (4.00 \text{ m}) \cos (1.75\pi) = \boxed{2.83 \text{ m}}$

13.2 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$.

(b) To determine the period, we use: $x = \dfrac{1}{2} gt^2$

The time for the ball to hit the ground is

$$t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.909 \text{ s}$$

This equals one-half the period, so $T = 2(0.909 \text{ s}) = \boxed{1.82 \text{ s}}$

(c) $\boxed{\text{No}}$ The net force acting on the mass is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

13.3 (a) $\boxed{20.0 \text{ cm}}$

(b) $v_{\text{max}} = \omega A = 2\pi f A = \boxed{94.2 \text{ cm/s}}$

This occurs as the particle passes through equilibrium.

(c) $a_{\text{max}} = \omega^2 A = (2\pi f)^2 A = \boxed{17.8 \text{ m/s}^2}$

This occurs at maximum excursion from equilibrium.

***13.4** (a) $x = (5.00\text{ cm})\cos\left(2t + \frac{\pi}{6}\right)$

At $t = 0$, $x = (5.00\text{ cm})\cos\left(\frac{\pi}{6}\right) = \boxed{4.33\text{ cm}}$

(b) $v = \frac{dx}{dt} = -(10.0\text{ cm/s})\sin\left(2t + \frac{\pi}{6}\right)$

At $t = 0$, $v = \boxed{-5.00\text{ cm/s}}$

(c) $a = \frac{dv}{dt} = -(20.0\text{ cm/s}^2)\cos\left(2t + \frac{\pi}{6}\right)$

At $t = 0$, $a = \boxed{-17.3\text{ cm/s}^2}$

(d) $A = \boxed{5.00\text{ cm}}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14\text{ s}}$

13.5 (a) At $t = 0$, $x = 0$ and v is positive (to the right). Therefore, this situation corresponds to

$x = A\sin\omega t$ and $v = v_i\cos\omega t$

Since $f = 1.50\text{ Hz}$, $\omega = 2\pi f = 3.00\pi$

Also, $A = 2.00\text{ cm}$, so that $\boxed{x = (2.00\text{ cm})\sin 3.00\pi t}$

(b) $v_{\text{max}} = v_i = A\omega = (2.00)(3.00\pi) = \boxed{6.00\pi\text{ cm/s}}$

The particle has this speed at $t = 0$ and next at $t = \frac{T}{2} = \boxed{\frac{1}{3}\text{ s}}$

(c) $a_{\text{max}} = A\omega^2 = 2(3.00\pi)^2 = \boxed{18.0\pi^2\text{ cm/s}^2}$

The acceleration has this positive value for the first time at

$t = \frac{3T}{4} = \boxed{0.500\text{ s}}$

(d) Since $T = \frac{2}{3}$ s and $A = 2.00$ cm, the particle will travel 8.00 cm in this time.

Hence, in $1.00\text{ s}\left(= \frac{3T}{2}\right)$, the particle will travel

$8.00\text{ cm} + 4.00\text{ cm} = \boxed{12.0\text{ cm}}$

13.6 The proposed solution $x(t) = x_i \cos \omega t + \left(\frac{v_i}{\omega}\right) \sin \omega t$

implies velocity $v = \frac{dx}{dt} = -x_i\omega \sin \omega t + v_i \cos \omega t$

and acceleration $a = \frac{dv}{dt} = -x_i\omega^2 \cos \omega t - v_i\omega \sin \omega t$

$$= -\omega^2\left(x_i \cos \omega t + \left(\frac{v_i}{\omega}\right)\sin \omega t\right) = -\omega^2 x$$

(a) The acceleration being a negative constant times position means we do have SHM, and its angular frequency is ω. At $t = 0$ the equations reduce to

$x = x_i$ and $v = v_i$

so they satisfy all the requirements.

(b) $v^2 - ax = (-x_i\omega \sin \omega t + v_i \cos \omega t)^2$

$-(-x_i\omega^2 \cos \omega t - v_i\omega \sin \omega t)\left(x_i \cos \omega t + \left(\frac{v_i}{\omega}\right)\sin \omega t\right)$

$= x_i^2\ \omega^2 \sin^2 \omega t - 2x_i v_i\omega \sin \omega t \cos \omega t + v_i^2\ \cos^2 \omega t$

$+ x_i^2\ \omega^2 \cos^2\omega t + x_i v_i\omega \cos \omega t \sin \omega t + x_i v_i\omega \sin \omega t \cos \omega t$

$+ v_i^2\ \sin^2 \omega t = x_i^2\ \omega^2 + v_i^2$

So this expression is constant in time. On one hand, it must keep its original value

$v_i^2 - a_i x_i$

On the other hand, if we evaluate it at a turning point where $v = 0$ and $x = A$, it is

$A^2\omega^2 + 0^2 = A^2\omega^2$

Thus it is proved.

13.7 $k = \frac{F}{x} = \frac{(10.0 \times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{3.90 \times 10^{-2}\text{ m}} = 2.51\text{ N/m}$ and

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{25.0 \times 10^{-3}\text{ kg}}{2.51\text{ N/m}}} = \boxed{0.627\text{ s}}$

13.8 (a) $T = \dfrac{12.0\text{ s}}{5} = \boxed{2.40\text{ s}}$

(b) $f = \dfrac{1}{T} = \dfrac{1}{2.40} = \boxed{0.417\text{ Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62\text{ rad/s}}$

13.9 (a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{8.00\text{ N/m}}{0.500\text{ kg}}} = 4.00\text{ s}^{-1}$

Therefore, position is given by $x = 10.0\sin(4.00t)$ cm

From this we find that

$v = 40.0\cos(4.00t)$ cm/s $\quad v_{\max} = \boxed{40.0\text{ cm/s}}$

$a = -160\sin(4.00t)$ cm/s^2 $\quad a_{\max} = \boxed{160\text{ cm/s}^2}$

(b) $t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$

and when $x = 6.00$ cm, $t = 0.161$ s, and we find

$v = 40.0\cos[4.00(0.161)] = \boxed{32.0\text{ cm/s}}$

$a = -160\sin[4.00(0.161)] = \boxed{-96.0\text{ cm/s}^2}$

(c) Using $\quad t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$

when $x = 0$, $t = 0$ and when $x = 8.00$ cm, $t = 0.232$ s

Therefore, $\Delta t = \boxed{0.232\text{ s}}$

13.10 $m = 1.00$ kg, $k = 25.0$ N/m, and $A = 3.00$ cm

At $t = 0$, $x = -3.00$ cm

(a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{25.0}{1.00}} = 5.00$ rad/s

so that, $\quad T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5.00} = \boxed{1.26\text{ s}}$

(b) $v_{\max} = A\omega = (3.00 \times 10^{-2}\text{ m})(5.00\text{ rad/s}) = \boxed{0.150\text{ m/s}}$

$a_{\max} = A\omega^2 = (3.00 \times 10^{-2}\text{ m})(5.00\text{ rad/s})^2 = \boxed{0.750\text{ m/s}^2}$

(c) Because $x = -3.00$ cm and $v = 0$ at $t = 0$, the required solution is

$$x = -A\cos\omega t$$

or $\boxed{x = -3.00\cos(5.00t)\text{ cm}}$

$$v = \frac{dx}{dt} = \boxed{15.0\sin(5.00t)\text{ cm/s}}$$

$$a = \frac{dv}{dt} = \boxed{75.0\cos(5.00t)\text{ cm/s}^2}$$

13.11 $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ or $T = \dfrac{1}{f} = 2\pi\sqrt{\dfrac{m}{k}}$

Solving for k, $k = \dfrac{4\pi^2 m}{T^2} = \dfrac{(4\pi)^2(7.00\text{ kg})}{(2.60\text{ s})^2} = \boxed{40.9\text{ N/m}}$

13.12 (a) Energy is conserved between the maximum-displacement and the half-maximum points:

$$(K + U)_i = (K + U)_f$$

$$0 + \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}mx^2$$

$$\frac{1}{2}(6.50\text{ N/m})(0.100\text{ m})^2 = \frac{1}{2}m(0.300\text{ m/s})^2 + \frac{1}{2}(6.50\text{ N/m})(5.00 \times 10^{-2}\text{ m})^2$$

$$32.5\text{ mJ} = \frac{1}{2}m(0.300\text{ m/s})^2 + 8.12\text{ mJ}$$

$$m = \frac{2(24.4\text{ mJ})}{9.00 \times 10^{-2}\text{ m}^2/\text{s}^2} = \boxed{0.542\text{ kg}}$$

(b) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{6.50\text{ N/m}}{0.542\text{ kg}}} = 3.46\text{ rad/s}$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{3.46/\text{s}} = \boxed{1.81\text{ s}}$$

(c) $a_{\max} = \omega^2 A = (3.46/\text{s})^2(0.100\text{ m}) = \boxed{1.20\text{ m/s}^2}$

13.13 (a) $v_{\max} = \omega A$

$$A = \frac{v_{\max}}{\omega} = \frac{1.50 \text{ m/s}}{2.00 \text{ rad/s}} = \boxed{0.750 \text{ m}}$$

(b) $x = \boxed{-(0.750 \text{ m}) \sin 2.00t}$

13.14 (a) $v_{\max} = \omega A$

$$A = \frac{v_{\max}}{\omega} = \boxed{\frac{v}{\omega}}$$

(b) $x = -A \sin \omega t = \boxed{-\left(\frac{v}{\omega}\right) \sin \omega t}$

13.15 The 0.500 s must elapse between one turning point and the other. Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28/\text{s}$$

and $v_{\max} = \omega A = (6.28/\text{s})(0.100 \text{ m}) = \boxed{0.628 \text{ m/s}}$

***13.16** $m = 200$ g, $T = 0.250$ s, $E = 2.00$ J; $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.250} = 25.1$ rad/s

(a) $k = m\omega^2 = (0.200 \text{ kg})(25.1 \text{ rad/s})^2 = \boxed{126 \text{ N/m}}$

(b) $E = \frac{kA^2}{2} \Rightarrow A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(2.00)}{126}} = \boxed{0.178 \text{ m}}$

13.17 By conservation of energy, $\frac{1}{2} mv^2 = \frac{1}{2} kx^2$

$$v = \sqrt{\frac{k}{m}}\, x = \sqrt{\frac{5.00 \times 10^6}{10^3}}\,(3.16 \times 10^{-2} \text{ m}) = \boxed{2.23 \text{ m/s}}$$

<u>Goal Solution</u>

G: If the bumper is only compressed 3 cm, the car is probably not permanently damaged, so v is most likely less than 10 mph (< 5 m/s).

O: Assuming no energy is lost during impact with the wall, the initial energy (kinetic) equals the final energy (elastic potential):

A: $K_i = U_f$ or $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$v = x\sqrt{\frac{k}{m}} = (3.16 \times 10^{-2}\text{ m})\sqrt{\frac{5.00 \times 10^6\text{ N/m}}{1000\text{ kg}}}$$

$$v = 2.23\text{ m/s}$$

L: The speed is less than 5 m/s as predicted, so the answer seems reasonable. If the speed of the car were sufficient to compress the bumper beyond its elastic limit, then some of the initial kinetic energy would be lost to deforming the front of the car. In this case, some other procedure would have to be used to estimate the car's initial speed.

13.18 (a) $E = \frac{kA^2}{2} = \frac{(250\text{ N/m})(3.50 \times 10^{-2}\text{ m})^2}{2} = \boxed{0.153\text{ J}}$

(b) $v_{\text{max}} = A\omega$

where $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250}{0.500}} = 22.4\text{ s}^{-1}$

$v_{\text{max}} = \boxed{0.784\text{ m/s}}$

(c) $a_{\text{max}} = A\omega^2 = (3.50 \times 10^{-2}\text{ m})(22.4\text{ s}^{-1})^2 = \boxed{17.5\text{ m/s}^2}$

13.19 (a) $E = \frac{1}{2}kA^2 = \frac{1}{2}(35.0\text{ N/m})(4.00 \times 10^{-2}\text{ m})^2 = \boxed{28.0\text{ mJ}}$

(b) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$

$|v| = \sqrt{\frac{35.0}{50.0 \times 10^{-3}}}\sqrt{(4.00 \times 10^{-2})^2 - (1.00 \times 10^{-2})^2} = \boxed{1.02\text{ m/s}}$

(c) $\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}(35.0)\ [(4.00 \times 10^{-2})^2 - (3.00 \times 10^{-2})^2] = \boxed{12.2\text{ mJ}}$

(d) $\frac{1}{2}kx^2 = E - \frac{1}{2}mv^2 = \boxed{15.8\text{ mJ}}$

13.20 (a) $k = \dfrac{F}{x} = \dfrac{20.0 \text{ N}}{0.200 \text{ m}} = \boxed{100 \text{ N/m}}$

(b) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{50.0} \text{ rad/s}$

$f = \dfrac{\omega}{2\pi} = \boxed{1.13 \text{ Hz}}$

(c) $v_{\text{max}} = \omega A = \sqrt{50.0}\,(0.200) = \boxed{1.41 \text{ m/s}}$ at $x = 0$

(d) $a_{\text{max}} = \omega^2 A = 50.0(0.200) = \boxed{10.0 \text{ m/s}^2}$ at $x = \pm A$

(e) $E = \dfrac{1}{2} kA^2 = \dfrac{1}{2}(100)(0.200)^2 = \boxed{2.00 \text{ J}}$

(f) $v = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\dfrac{8}{9}(0.200)^2} = \boxed{1.33 \text{ m/s}}$

(g) $a = \omega^2 x = (50.0)\left(\dfrac{0.200}{3}\right) = \boxed{3.33 \text{ m/s}^2}$

13.21 (a) In the presence of non-conservative forces, we use

$$\Delta E = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 + mgy_f - mgy_i + \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$$

$$(20.0 \text{ N})(0.300 \text{ m}) = \frac{1}{2}(1.50 \text{ kg})\, v_f^2 - 0 + 0 - 0 + \frac{1}{2}(19.6 \text{ N/m})(0.300 \text{ m})^2 - 0$$

$$v_f = \boxed{2.61 \text{ m/s}}$$

(b) $f_k = \mu_k n = 0.200(14.7 \text{ N}) = 2.94 \text{ N}$

$$(K + U)_i + \Delta E = (K + U)_f$$

$$0 + 0 + Fd\cos 0° + fd\cos 180° = \frac{1}{2} mv_f^2 + \frac{1}{2} kx^2$$

$$6.00 \text{ J} + (2.94 \text{ N})(0.300 \text{ m})\cos 180° = \frac{1}{2}(1.50 \text{ kg})\, v_f^2 + 0.882 \text{ J}$$

$$v_f = \sqrt{2(6.00 \text{ J} - 0.882 \text{ J} - 0.882 \text{ J})/1.50 \text{ kg}} = \boxed{2.38 \text{ m/s}}$$

13.22 (a) $E = \frac{1}{2}kA^2$, so if $A' = 2A$, $E' = \frac{1}{2}k(A')^2 = \frac{1}{2}k(2A)^2 = 4E$

Therefore $\boxed{E \text{ increases by factor of } 4}$.

(b) $v_{\max} = \sqrt{\frac{k}{m}}\,A$, so if A is doubled, $\boxed{v_{\max} \text{ is doubled}}$.

(c) $a_{\max} = \frac{k}{m}A$, so if A is doubled, $\boxed{a_{\max} \text{ also doubles}}$.

(d) $T = 2\pi\sqrt{\frac{m}{k}}$ is independent of A, so $\boxed{\text{the period is unchanged}}$.

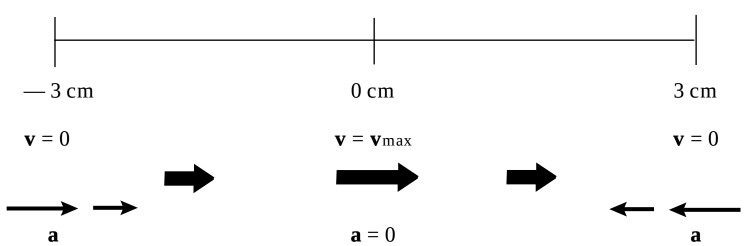
13.23 From energy considerations, $v^2 + \omega^2 x^2 = \omega^2 A^2$

$v_{\max} = \omega A$ and $v = \frac{\omega A}{2}$ so $\left(\frac{w\,A}{2}\right)^2 + \omega^2 x^2 = \omega^2 A^2$

From this we find $x^2 = \frac{3A^2}{4}$ and $x = \frac{A\sqrt{3}}{2} = \boxed{\pm 2.60 \text{ cm}}$ where $A = 3.00$ cm

Goal Solution

G: If we consider the speed of the particle along its path as shown in the sketch, we can see that the particle is at rest momentarily at one endpoint while being accelerated toward the middle by an elastic force that decreases as the particle approaches the equilibrium position. When it reaches the midpoint, the direction of acceleration changes so that the particle slows down until it stops momentarily at the opposite endpoint. From this analysis, we can estimate that $v = v_{\max}/2$ somewhere in the outer half of the travel: $1.5 < x < 3$.



O: We can analyze this problem in more detail by examining the energy of the system, which should be constant since we are told that the motion is SHM (no damping).

A: From energy considerations (Eq. 13.23), $v^2 + \omega^2 x^2 = \omega^2 A^2$. The speed v will be maximum when x is zero. Thus, $v_{\max} = \omega A$ and

$$v_{1/2} = \frac{v_{\max}}{2} = \frac{\omega A}{2}$$

Substituting $v_{1/2}$ in for v, $\frac{1}{4}\ \omega^2 A^2 + \omega^2 x^2 = \omega^2 A^2$

Solving for x we find that $x^2 = \frac{3A^2}{4}$

Given that $A = 3.00$ cm, $x = \pm\frac{A\sqrt{3}}{2} = \pm\frac{(3.00\text{ cm})\sqrt{3}}{2} = \pm 2.60\text{ cm}$ ◊

L: The calculated position is in the outer half of the travel as predicted, and is in fact very close to the endpoints. This means that the speed of the particle is mostly constant until it reaches the ends of its travel, where it experiences the maximum restoring force of the spring, which is proportional to x.

***13.24** The potential energy is

$$U_s = \frac{1}{2}\ kx^2 = \frac{1}{2}\ kA^2 \cos^2(\omega t)$$

The rate of change of potential energy is

$$\frac{dU_s}{dt} = \frac{1}{2}\ kA^2\ 2\cos(\omega t)[-\omega \sin(\omega t)] = -\frac{1}{2}\ kA^2\ \omega \sin 2\omega t$$

(a) This rate of change is maximal and negative at

$$2\omega t = \frac{\pi}{2},\ 2\omega t = 2\pi + \frac{\pi}{2},\ \text{or in general, } 2\omega t = 2n\pi + \frac{\pi}{2} \text{ for integer } n.$$

Then, $t = \frac{\pi}{4\omega}(4n+1) = \frac{\pi(4n+1)}{4(3.60\text{ s}^{-1})}$

For $n = 0$, this gives $t = \boxed{0.218\text{ s}}$ while $n = 1$ gives $t = \boxed{1.09\text{ s}}$.

All other values of n yield times outside the specified range.

(b) $$\left|\frac{dU_s}{dt}\right|_{\max} = \frac{1}{2}\ kA^2\omega = \frac{1}{2}(3.24\text{ N/m})(5.00\times 10^{-2}\text{ m})^2(3.60\text{ s}^{-2}) = \boxed{14.6\text{ mW}}$$

***13.25** (a) $T = 2\pi\sqrt{\dfrac{L}{g}}$

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80 \text{ m/s}^2)(12.0 \text{ s})^2}{4\pi^2} = \boxed{35.7 \text{ m}}$$

(b) $T_{\text{moon}} = 2\pi\sqrt{\dfrac{L}{g_{\text{moon}}}} = 2\pi\sqrt{\dfrac{35.7 \text{ m}}{1.67 \text{ m/s}^2}} = \boxed{29.1 \text{ s}}$

***13.26** The period in Tokyo is $T_t = 2\pi\sqrt{\dfrac{L_t}{g_t}}$

and the period in Cambridge is $T_c = 2\pi\sqrt{\dfrac{L_c}{g_c}}$

We know $T_t = T_c = 2.00 \text{ s}$

For which, we see $\dfrac{L_t}{g_t} = \dfrac{L_c}{g_c}$

or $\dfrac{g_c}{g_t} = \dfrac{L_c}{L_t} = \dfrac{0.9942}{0.9927} = \boxed{1.0015}$

***13.27** The swinging box is a physical pendulum with period $T = 2\pi\sqrt{\dfrac{I}{mgd}}$.
The moment of inertia is given approximately by

$I = \dfrac{1}{3}mL^2$ (treating the box as a rod suspended from one end).

Then, with $L \approx 1.0 \text{ m}$ and $d \approx L/2$,

$$T \approx 2\pi\sqrt{\frac{(1/3)mL^2}{mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(1.0 \text{ m})}{3(9.8 \text{ m/s}^2)}} = 1.6 \text{ s} \quad \text{or} \quad T \sim \boxed{10^0 \text{ s}}$$

13.28 $\omega = \dfrac{2\pi}{T}$ $\qquad T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.43} = \boxed{1.42 \text{ s}}$

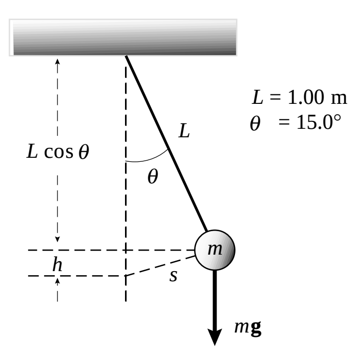
$\omega = \sqrt{\dfrac{g}{L}}$ $\qquad L = \dfrac{g}{\omega^2} = \dfrac{9.80}{(4.43)^2} = \boxed{0.499 \text{ m}}$

13.29 (a) $mgh = \dfrac{1}{2}mv^2$

$h = L(1 - \cos\theta)$

$\therefore v_{\text{max}} = \sqrt{2gL(1 - \cos\theta)}$

$v_{\text{max}} = \boxed{0.817 \text{ m/s}}$



(b) $I\alpha = mgL \sin\theta$

$$\alpha_{\max} = \frac{mgL\sin\theta}{mL^2} = \frac{g}{L}\sin\theta_i = \boxed{2.54\ \mathrm{rad/s^2}}$$

(c) $F_{\max} = mg\sin\theta_i = (0.250)(9.80)(\sin 15.0°) = \boxed{0.634\ \mathrm{N}}$

13.30 (a) The string tension must support the weight of the bob, accelerate it upward, and also provide the restoring force, just as if the elevator were at rest in a gravity field (9.80 + 5.00) $\mathrm{m/s^2}$

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{5.00\ \mathrm{m}}{14.8\ \mathrm{m/s^2}}}$$

$$T = \boxed{3.65\ \mathrm{s}}$$

(b) $$T = 2\pi\sqrt{\frac{5.00\ \mathrm{m}}{(9.80\ \mathrm{m/s^2} - 5.00\ \mathrm{m/s^2})}} = \boxed{6.41\ \mathrm{s}}$$

(c) $$g_{\text{eff}} = \sqrt{(9.80\ \mathrm{m/s^2})^2 + (5.00\ \mathrm{m/s^2})^2} = 11.0\ \mathrm{m/s^2}$$

$$T = 2\pi = \sqrt{\frac{5.00\ \mathrm{m}}{11.0\ \mathrm{m/s^2}}} = \boxed{4.24\ \mathrm{s}}$$

13.31 Referring to the sketch we have

$$F = -mg\sin\theta$$

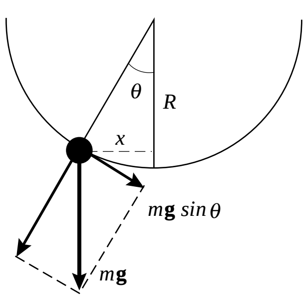
and $\tan\theta \approx \dfrac{x}{R}$

For small displacements,

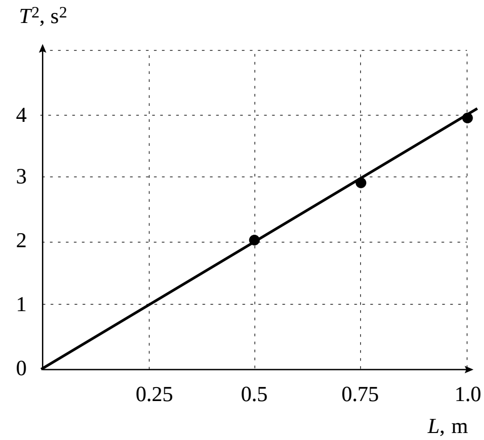
$$\tan\theta \approx \sin\theta$$

and $F = -\dfrac{mg}{R}\,x = -kx$

and $\boxed{\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{g}{R}}}$



13.32 (a) $T = \dfrac{\text{total measured time}}{50}$



The measured periods are:

Length, L (m)	1.000	0.750	0.500
Period, T (s)	1.996	1.732	1.422

(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ so $g = \dfrac{4\pi^2 L}{T^2}$

The calculated values for g are:

Period, T (s)	1.996	1.732	1.422
g (m/s^2)	9.91	9.87	9.76

Thus, $g_{\text{ave}} = \boxed{9.85 \text{ m/s}^2}$ this agrees with the accepted value of $g = 9.80$ m/s^2 within 0.5%.

(c) Slope of T^2 versus L graph $= 4\pi^2/g = 4.01$ s^2/m

Thus, $g = \dfrac{4\pi^2}{\text{slope}} = 9.85$ m/s^2. You should find that $\boxed{\%\text{ difference} \approx 0.5\%}$.

13.33 $f = 0.450$ Hz, $d = 0.350$ m, and $m = 2.20$ kg

$$T = \frac{1}{f}\ ; \qquad T = 2\pi\sqrt{\frac{I}{mgd}}\ ; \qquad T^2 = 4\pi^2\left(\frac{I}{mgd}\right)$$

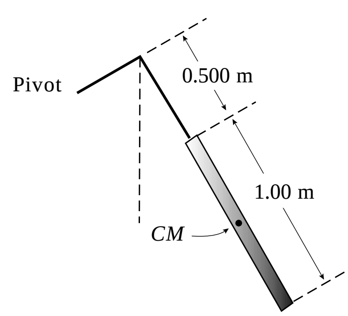
$$I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} = \frac{(2.20)(9.80)(0.350)}{(0.450 \text{ s}^{-1})^2(4\pi^2)}$$

$$I = \boxed{0.944 \text{ kg}\cdot\text{m}^2}$$

13.34 (a) The parallel-axis theorem:

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2$$

$$= \frac{1}{12}M(1.00 \text{ m})^2 + M(1.00 \text{ m})^2 = M\left(\frac{13}{12}\text{ m}^2\right)$$



$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M(13\text{ m}^2)}{12Mg(1.00\text{ m})}}$$

$$T = 2\pi\sqrt{\frac{13\text{ m}}{12(9.80\text{ m/s}^2)}} = \boxed{2.09\text{ s}}$$

(b) For the simple pendulum

$$T = 2\pi\sqrt{\frac{1.00\text{ m}}{9.80\text{ m/s}^2}} = 2.01\text{ s}$$

$$\text{difference} = \frac{2.09\text{ s} - 2.01\text{ s}}{2.01\text{ s}} = \boxed{4.08\%}$$

13.35 (a) The parallel axis theorem says directly $I = I_{\text{CM}} + md^2$

so $$T = 2\pi\sqrt{\frac{I}{mgd}} = \boxed{2\pi\sqrt{\frac{(I_{\text{CM}} + md^2)}{mgd}}}$$

(b) When d is very large $\quad T \to 2\pi\sqrt{\frac{d}{g}}\quad$ gets large.

When d is very small $\quad T \to 2\pi\sqrt{\frac{I_{\text{CM}}}{mgd}}\quad$ gets large.

So there must be a minimum, found by

$$\frac{dT}{dd} = 0 = \frac{d}{dd}\; 2\pi\,(I_{\text{CM}} + md^2)^{1/2}(mgd)^{-1/2}$$

$$= 2\pi\,(I_{\text{CM}} + md^2)^{1/2}\left(-\frac{1}{2}\right)(mgd)^{-3/2}\,mg$$

$$+\; 2\pi(mgd)^{-1/2}\left(\frac{1}{2}\right)(I_{\text{CM}} + md^2)^{-1/2}\,2md$$

= **Error!** mg,$(I_{\text{CM}} + md^2)^{1/2}\,(mgd)^{3/2}$) + **Error!** = 0

(b) This requires

$$-I_{\text{CM}} - md^2 + 2md^2 = 0$$

or $\boxed{I_{\text{CM}} = md^2}$

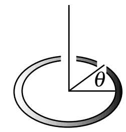
13.36 We suppose the stick moves in a horizontal plane. Then,

$$I = \frac{1}{12}\,mL^2 = \frac{1}{12}(2.00\text{ kg})(1.00\text{ m})^2 = 0.167\text{ kg}\cdot\text{m}^2$$

$$T = 2\pi\sqrt{I/\kappa}$$

$$\kappa = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2(0.167\text{ kg}\cdot\text{m}^2)}{(180\text{ s})^2} = \boxed{203\ \mu\text{N}\cdot\text{m}}$$

13.37 $T = 0.250$ s; $I = mr^2 = (20.0 \times 10^{-3}\ \text{kg})(5.00 \times 10^{-3}\ \text{m})^2$



(a) $I = \boxed{5.00 \times 10^{-7}\ \text{kg} \cdot \text{m}^2}$

(b) $I\dfrac{d^2\theta}{dt^2} = -\kappa\theta;\quad \sqrt{\dfrac{\kappa}{I}} = \omega = \dfrac{2\pi}{T}$

$$\kappa = I\omega^2 = (5.00 \times 10^{-7})\left(\frac{2\pi}{0.250}\right)^2 = \boxed{3.16 \times 10^{-4}\ \frac{\text{N} \cdot \text{m}}{\text{rad}}}$$

13.38 (a) The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the motion of the boss projected in a plane perpendicular to the tire.

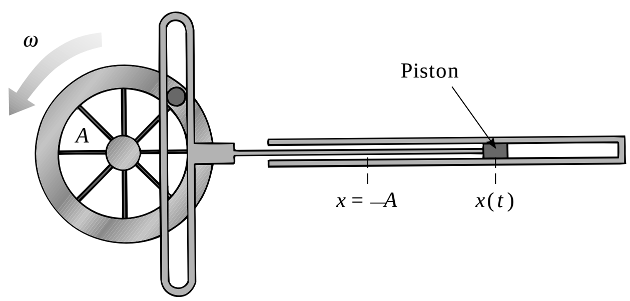
(b) Since the car is moving with speed $v = 3.00$ m/s, and its radius is 0.300 m, we have:

$$\omega = \frac{3.00\ \text{m/s}}{0.300\ \text{m}} = 10.0\ \text{rad/s}$$

Therefore, the period of the motion is:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(10.0\ \text{rad/s})} = \boxed{0.628\ \text{s}}$$

13.39 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is $\boxed{x = A\cos\theta = A\cos\omega t}$ where A is the distance from the center of the wheel to the crank pin. This is of the form $x = A\cos(\omega t + \phi)$, so the yoke and piston rod move with simple harmonic motion.



13.40 $E = \dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2$

$$\frac{dE}{dt} = mv\frac{d^2x}{dt^2} + kxv$$

Use Equation 13.32:

$$\frac{md^2x}{dt^2} = -kx - bv$$

$$\frac{dE}{dt} = v(-kx - bv) + kvx$$

$$\boxed{\frac{dE}{dt} = -bv^2 < 0}$$

13.41 $\theta_i = 15.0°$ $\theta(t = 1000) = 5.50°$

$$x = Ae^{-bt/2m}$$

$$\frac{x_{1000}}{x_i} = \frac{Ae^{-bt/2m}}{A} = \frac{5.50}{15.0} = e^{-b(1000)/2m} \approx e^{-1}$$

$$\therefore \frac{b}{2m} = \boxed{1.00 \times 10^{-3}\ \text{s}^{-1}}$$

***13.42** Show that $x = Ae^{-bt/2m}\cos(\omega t + \phi)$ is a solution of

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2} \qquad (1)$$

and $$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \qquad (2)$$

$$x = Ae^{-bt/2m}\cos(\omega t + \phi) \qquad (3)$$

$$\frac{dx}{dt} = Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\,\omega\,(\sin(\omega t + \phi)) \qquad (4)$$

$$\frac{d^2x}{dt^2} = -\frac{b}{2m}\left\{Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\,\omega\sin(\omega t + \phi)\right\}$$

$$-\left\{Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\omega\sin(\omega t + \phi) + Ae^{-bt/2m}\,\omega^2\cos(\omega t + \phi)\right\} \qquad (5)$$

Substitute (3), (4) into the left side of (1) and (5) into the right side of (1);

$$-kAe^{-bt/2m}\cos(\omega t + \phi) + \frac{b^2}{2m}Ae^{-bt/2m}\cos(\omega t + \phi)$$

$$+ b\omega Ae^{-bt/2m}\sin(\omega t + \phi)$$

$$= -\frac{b}{2}\left\{Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\,\omega\sin(\omega t + \phi)\right\}$$

$$+\frac{b}{2}Ae^{-bt/2m}\,\omega\sin(\omega t + \phi) - m\omega^2 Ae^{-bt/2m}\cos(\omega t + \phi)$$

Compare the coefficients of $Ae^{-bt/2m}\cos(\omega t + \phi)$ and $Ae^{-bt/2m}\sin(\omega t + \phi)$:

$$\text{cosine-term: } -k + \frac{b^2}{2m} = -\frac{b}{2}\left(-\frac{b}{2m}\right) - m\omega^2 = \frac{b^2}{4m} - (m)\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)$$

$$= -k + \frac{b^2}{2m}$$

$$\text{sine-term: } b\omega = +\frac{b}{2}(\omega) + \frac{b}{2}\omega = b\omega$$

Since the coefficients are equal,

$x = Ae^{-bt/2m}\cos(\omega t + \phi)$ is a solution of the equation.

***13.43** (a) For resonance, her frequency must match

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.30 \times 10^3 \text{ N/m}}{12.5 \text{ kg}}} = \boxed{2.95 \text{ Hz}}$$

(b) From $x = A\cos\omega t$, $v = dx/dt = -A\omega\sin\omega t$, and $a = dv/dt = -A\omega^2\cos\omega t$, the maximum acceleration is $A\omega^2$. When this becomes equal to the acceleration of gravity, the normal force exerted on her by the mattress will drop to zero at one point in the cycle:

$$A\omega^2 = g \quad \text{or} \quad A = \frac{g}{\omega^2} = \frac{g}{k/m} = \frac{gm}{k}$$

$$A = \frac{(9.80 \text{ m/s}^2)(12.5 \text{ kg})}{4.30 \times 10^3 \text{ N/m}} = \boxed{2.85 \text{ cm}}$$

13.44 $F = 3.00\cos(2\pi t)$ N and $k = 20.0$ N/m

(a) $\omega = \dfrac{2\pi}{T} = 2\pi \text{ rad/s}$ so $T = \boxed{1.00 \text{ s}}$

(b) In this case, $\omega_0 = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{20.0}{2.00}} = 3.16 \text{ rad/s}$

Taking $b = 0$ in Equation 13.37 gives

$$A = \left(\frac{F_0}{m}\right)(\omega^2 - \omega_0^2)^{-1} = \frac{3}{2}\left[4\pi^2 - (3.16)^2\right]^{-1}$$

$$A = 0.0509 \text{ m} = \boxed{5.09 \text{ cm}}$$

***13.45** $F_0 \cos(\omega t) - kx = m\dfrac{d^2x}{dt^2}$ $\qquad \omega_0 = \sqrt{\dfrac{k}{m}}$ $\qquad$ (1)

$x = A\cos(\omega t + \phi)$ $\qquad$ (2)

$\dfrac{dx}{dt} = -A\omega \sin(\omega t + \phi)$ $\qquad$ (3)

$\dfrac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \phi)$ $\qquad$ (4)

Substitute (2) and (4) into (1):

$$F_0 \cos(\omega t) - kA\cos(\omega t + \phi) = m(-A\omega^2)\cos(\omega t + \phi)$$

Solve for the amplitude: $(kA - mA\omega^2)\cos(\omega t + \phi) = F_0 \cos \omega t$

These will be equal, provided only that ϕ must be zero and $(kA - mA\omega^2) = F_0$

Thus, $\qquad A = \dfrac{F_0/m}{\dfrac{k}{m} - \omega^2}$

13.46 From Equation 13.37 with no damping,

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2}}$$

$\omega = 2\pi f = (20.0\pi\ \text{s}^{-1})$ $\qquad \omega_0^2 = \dfrac{k}{m} = \dfrac{200}{(40.0/9.80)} = 49.0\ \text{s}^{-2}$

$$F_0 = mA(\omega^2 - \omega_0^2)$$

$$F_0 = \left(\frac{40.0}{9.80}\right)(2.00 \times 10^{-2})(3950 - 49.0) = \boxed{318\ \text{N}}$$

***13.47** $A = \dfrac{F_{\text{ext}}/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}$

With $b = 0$, $A = \dfrac{F_{\text{ext}}/m}{\sqrt{(\omega^2 - \omega_0^2)^2}} = \dfrac{F_{\text{ext}}/m}{\pm(\omega^2 - \omega_0^2)} = \pm\dfrac{F_{\text{ext}}/m}{\omega^2 - \omega_0^2}$

Thus, $\omega^2 = \omega_0^2 \pm \dfrac{F_{\text{ext}}/m}{A} = \dfrac{k}{m} \pm \dfrac{F_{\text{ext}}}{mA} = \dfrac{6.30\ \text{N/m}}{0.150\ \text{kg}} \pm \dfrac{1.70\ \text{N}}{(0.150\ \text{kg})(0.440\ \text{m})}$

This yields $\omega = 8.23$ rad/s or $\omega = 4.03$ rad/s

Then, $f = \dfrac{\omega}{2\pi}$ gives either $f = \boxed{1.31\ \text{Hz}}$ or $f = \boxed{0.641\ \text{Hz}}$

***13.48** The beeper must resonate at the frequency of a simple pendulum of length 8.21 cm:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80 \text{ m/s}^2}{0.0821 \text{ m}}} = \boxed{1.74 \text{ Hz}}$$

13.49 Assume that each spring supports an equal portion of the car's mass, i.e. $\frac{m}{4}$.

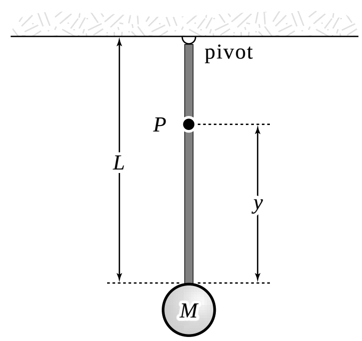
Then $T = 2\pi\sqrt{\frac{m}{4k}}$ and $k = \frac{4\pi^2 m}{4T^2} = \frac{4\pi^2 1500}{(4)(1.50)^2} = \boxed{6580 \text{ N/m}}$

13.50 $$\frac{T_1}{T_0} = \frac{2\pi/\omega_1}{2\pi/\omega_0} = \frac{2\pi/\sqrt{k/m_1}}{2\pi/\sqrt{k/m_0}}$$

$$\frac{T_1}{T_0} = \sqrt{\frac{m_1}{m_0}} = \sqrt{\frac{1650}{1500}} = \sqrt{1.10}$$

$$T_1 = \sqrt{1.10} \times 1.50 = \boxed{1.57 \text{ s}}$$

13.51 Let F represent the tension in the rod.



(a) At the pivot, $F = Mg + Mg = \boxed{2Mg}$

A fraction of the rod's weight $Mg\left(\frac{y}{L}\right)$ as well as the weight of the ball pulls down on point P. Thus, the tension in the rod at point P is

$$F = Mg\left(\frac{y}{L}\right) + Mg = \boxed{Mg\left(1 + \frac{y}{L}\right)}$$

(b) Relative to the pivot, $I = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$

For the physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}}$ where $m = 2M$ and d is the distance from the pivot to the center of mass of the rod and ball combination. Therefore,

$$d = \frac{M(L/2) + ML}{M + M} = \frac{3L}{4} \quad \text{and} \quad T = 2\pi\sqrt{\frac{(4/3)ML^2}{(2M)g(3L/4)}} = \boxed{\frac{4\pi}{3}\sqrt{\frac{2L}{g}}}$$

For $L = 2.00$ m, $T = \frac{4\pi}{3}\sqrt{\frac{2(2.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{2.68 \text{ s}}$

Goal Solution

G: The tension in the rod at the pivot = weight of rod + weight of $M = 2\,Mg$. The tension at point P should be slightly less since the portion of the rod between P and the pivot does not contribute to the tension.

The period should be slightly less than for a simple pendulum since the mass of the rod effectively shortens the length of the simple pendulum (massless rod) by moving the center of mass closer to the pivot, so that $T < 2\pi\sqrt{\frac{L}{g}}$

O: The tension can be found from applying Newton's Second Law to the static case. The period of oscillation can be found by analyzing the components of this physical pendulum and using Equation 13.28.

A: (a) At the pivot, the net downward force is: $T = Mg + Mg = 2Mg$ ◊

At P, a fraction of the rod's mass (y/L) pulls down along with the ball.

Therefore, $T = Mg\left(\frac{y}{L}\right) + Mg = Mg\left(1 + \frac{y}{L}\right)$ ◊

(b) Relative to the pivot, $I_{\text{total}} = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$

For a physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}}$

In this case, $m = 2M$ and d is the distance from the pivot to the center of mass.

$$d = \frac{\left(\frac{ML}{2} + ML\right)}{(M + M)} = \frac{3L}{4}$$

so we have, $T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{(4ML^2)4}{3(2M)g(3L)}} = \frac{4\pi}{3}\sqrt{\frac{2L}{g}}$ ◊

For $L = 2.00$ m, $T = \frac{4\pi}{3}\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m}/\text{s}^2}} = 2.68\text{ s}$ ◊

L: In part (a), the tensions agree with the initial predictions. In part (b) we found that the period is indeed slightly less (by about 6%) than a simple pendulum of length L. It is interesting to note that we were able to calculate a value for the period despite not knowing the mass value. This is because the period of any pendulum depends on the *location* of the center of mass and not on the *size* of the mass.

13.52 (a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100\text{ N/m})(0.200\text{ m})^2 = 2.00\text{ J}$

At equilibrium, the total energy is:

$$\frac{1}{2}(m_1 + m_2)\,v^2 = \frac{1}{2}(16.0\text{ kg})\,v^2 = (8.00\text{ kg})v^2$$

Therefore, $(8.00\text{ kg})v^2 = 2.00\text{ J}$, and $v = \boxed{0.500\text{ m/s}}$

This is the speed of m_1 and m_2 at the equilibrium point. Beyond this point, the mass m_2 moves with the constant speed of 0.500 m/s while mass m_1 starts to slow down due to the restoring force of the spring.

(b) The energy of the m_1-spring at equilibrium is:

$$\frac{1}{2}m_1v^2 = \frac{1}{2}(9.00\text{ kg})(0.500\text{ m/s})^2 = 1.125\text{ J}$$

This is also equal to $\frac{1}{2}k(A')^2$, where A' is the amplitude of the m_1-spring system.

Therefore, $\frac{1}{2}(100)(A')^2 = 1.125$ or $A' = 0.150\text{ m}$

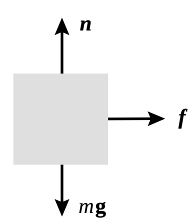
The period of the m_1-spring system is:

$$T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885\text{ s}$$

and it takes $\frac{T}{4} = 0.471\text{ s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating m_1 and m_2 at this time is:

$$D = v\left(\frac{T}{4}\right) - A' = (0.500\text{ m/s})(0.471\text{ s}) - 0.150\text{ m} = 0.0856 = \boxed{8.56\text{ cm}}$$

13.53 $\left(\frac{d^2x}{dt^2}\right)_{\max} = A\omega^2$ $\qquad f_{\max} = \mu_s n = \mu_s mg = mA\omega^2$



$$A = \frac{\mu_s g}{\omega^2} = \boxed{6.62\text{ cm}}$$

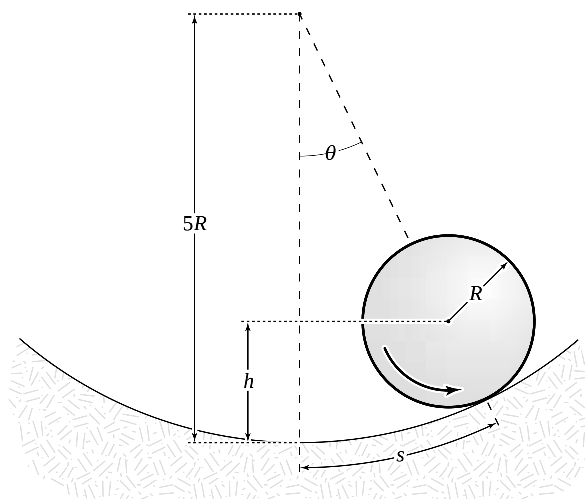
13.54 The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block B is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m(4\pi^2 Af^2) \quad \text{or} \quad A = \boxed{\frac{\mu_s g}{4\pi^2 f^2}}$$

13.55 $M_{D_2} = 2M_{H_2}$

$$\frac{\omega_D}{\omega_H} = \frac{\sqrt{k/M_D}}{\sqrt{k/M_H}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}}$$

$$f_{D_2} = \frac{f_{H_2}}{\sqrt{2}} = \boxed{0.919 \times 10^{14}\ \text{Hz}}$$



13.56 The kinetic energy of the ball is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\Omega^2$, where Ω is the rotation rate of the ball about its center of mass. Since the center of the ball moves along a circle of radius $4R$, its displacement from equilibrium is $s = (4R)\theta$ and its speed is $v = \frac{ds}{dt} = 4R\left(\frac{d\theta}{dt}\right)$. Also, since the ball rolls without slipping,

$$v = \frac{ds}{dt} = R\Omega \quad \text{so} \quad \Omega = \frac{v}{R} = 4\left(\frac{d\theta}{dt}\right)$$

The kinetic energy is then

$$K = \frac{1}{2}m\left(4R\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(4\frac{d\theta}{dt}\right)^2 = \frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2$$

When the ball has an angular displacement θ, its center is distance $h = 4R(1 - \cos\theta)$ higher than when at the equilibrium position. Thus, the potential energy is $U_g = mgh = 4mgR(1 - \cos\theta)$. For small angles, $(1 - \cos\theta) \approx \frac{\theta^2}{2}$ (see Appendix B). Hence, $U_g \approx 2mgR\theta^2$, and the total energy is

$$E = K + U_g = \frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2 + 2mgR\theta^2$$

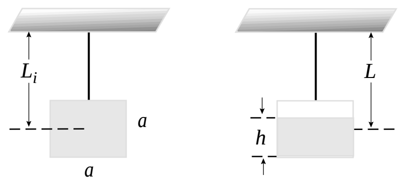
Since E = constant in time, $\frac{dE}{dt} = 0 = \frac{112mR^2}{5}\left(\frac{d\theta}{dt}\right)\frac{d^2\theta}{dt^2} + 4mgR\theta\left(\frac{d\theta}{dt}\right)$

This reduces to $\frac{28R}{5}\frac{d^2\theta}{dt^2} + g\theta = 0$, or $\frac{d^2\theta}{dt} = -\left(\frac{5g}{28R}\right)\theta$

This is in the classical form of a simple harmonic motion equation with $\omega = \sqrt{\dfrac{5g}{28R}}$.

The period of the simple harmonic motion is then $T = \dfrac{2\pi}{\omega} = \boxed{2\pi\sqrt{\dfrac{28R}{5g}}}$

13.57 (a)



(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ $\qquad \dfrac{dT}{dt} = \dfrac{\pi}{\sqrt{g}}\dfrac{1}{\sqrt{L}}\dfrac{dL}{dt}$ $\qquad$ (1)

We need to find $L(t)$ and $\dfrac{dL}{dt}$. From the diagram in (a),

$$L = L_i + \frac{a}{2} - \frac{h}{2}\ ; \quad \frac{dL}{dt} = -\left(\frac{1}{2}\right)\frac{dh}{dt}$$

But $\dfrac{dM}{dt} = \rho\dfrac{dV}{dt} = -\rho A\dfrac{dh}{dt}$. $\qquad$ Therefore,

$$\frac{dh}{dt} = -\frac{1}{\rho A}\frac{dM}{dt}\ ; \quad \frac{dL}{dt} = \left(\frac{1}{2\rho A}\right)\frac{dM}{dt} \quad (2)$$

Also, $\displaystyle\int_{L_i}^{L} dL = \left(\frac{1}{2\rho A}\right)\left(\frac{dM}{dt}\right) t = L - L_i$ $\qquad$ (3)

Substituting Equation (2) and Equation (3) into Equation (1):

$$\frac{dT}{dt} = \boxed{\frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\frac{1}{\sqrt{L_i + \dfrac{1}{2\rho a^2}\left(\dfrac{dM}{dt}\right)t}}}$$

(c) Substitute Equation (3) into the equation for the period.

$$T = \boxed{\frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

Or one can obtain T by integrating (b):

$$\int_T^{T_i} dT = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\int_0^t \frac{dt}{\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

$$T - T_i = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\left[\frac{2}{\frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)}\right]\left[\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t} - \sqrt{L_i}\right]$$

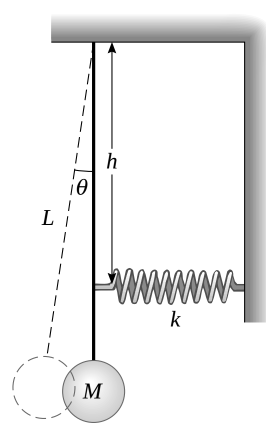
But $T_i = 2\pi\sqrt{\frac{L_\text{i}}{g}}$, so $T = \frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}$

13.58 $\omega_0 = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$

(a) $k = \omega_0^2\, m = \boxed{\frac{4\pi^2 m}{T^2}}$

(b) $m' = \frac{k(T')^2}{4\pi^2} = \boxed{m\left(\frac{T'}{T}\right)^2}$

13.59 For the pendulum (see sketch) we have



$\tau = I\alpha$ and $\frac{d^2\theta}{dt^2} = -\alpha$

$\tau = MgL\sin\theta + kxh\cos\theta = -I\frac{d^2\theta}{dt^2}$

For small amplitude vibrations, use the approximations:

$\sin\theta \approx \theta \qquad \cos\theta \approx 1$

and $x \approx s = h\theta$

Therefore,

$$\frac{d^2\theta}{dt^2} = -\left(\frac{MgL + kh^2}{I}\right)\theta = -\omega^2\theta$$

$$\omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$$

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}}}$$

Goal Solution

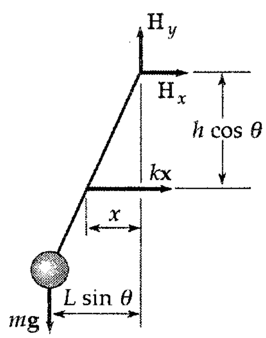
G: The frequency of vibration should be greater than that of a simple pendulum since the spring adds an additional restoring force: $f > \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

O: We can find the frequency of oscillation from the angular frequency, which is found in the equation for angular SHM: $\frac{d^2\theta}{dt^2} = -\omega^2\theta$. The angular acceleration can be found from analyzing the torques acting on the pendulum.

A: For the pendulum (see sketch), we have

$$\sum\tau = I\alpha \quad \text{and} \quad \frac{d^2\theta}{dt^2} = -\alpha$$

The negative sign appears because positive θ is measured clockwise in the picture. We take torque around the point of suspension:



$$\sum\tau = MgL\sin\theta + kxh\cos\theta = I\alpha$$

For small amplitude vibrations, use the approximations:

$$\sin\theta \approx \theta,\ \cos\theta \approx 1, \text{ and } x \approx s = h\theta$$

Therefore, with $I = mL^2$,

$$\frac{d^2\theta}{dt^2} = -\left[\frac{MgL + kh^2}{I}\right]\theta = -\left[\frac{MgL + kh^2}{ML^2}\right]\theta$$

This is of the form $\frac{d^2\theta}{dt^2} = -\omega^2\theta$ required for SHM,

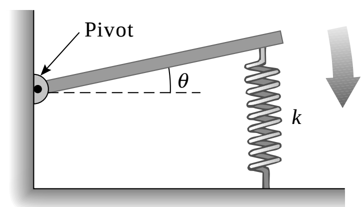
with angular frequency, $\omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$

The ordinary frequency is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}}$

L: The frequency is greater than for a simple pendulum as we expected. In fact, the additional portion resembles the frequency of a mass on a spring scaled by *h/L* since the spring is connected to the rod and not directly to the mass. So we can think of the solution as:

$$f^2 = \frac{1}{4\pi^2}\frac{MgL + kh^2}{ML^2} = \frac{1}{4\pi^2}\frac{g}{L} + \frac{h^2}{L^2}\frac{1}{4\pi^2}\frac{k}{M} = f^2_{\text{pendulum}} + \frac{h^2}{L^2}f^2_{\text{spring}}$$

***13.60** (a) At equilibrium, we have



$$\sum\tau = 0 = -mg\frac{L}{2} + kx_0L$$

where x_0 is the equilibrium compression.

After displacement by a small angle,

$$\sum\tau = -mg\frac{L}{2} + kxL$$

$$= -mg\frac{L}{2} + k(x_0 - L\theta)L$$

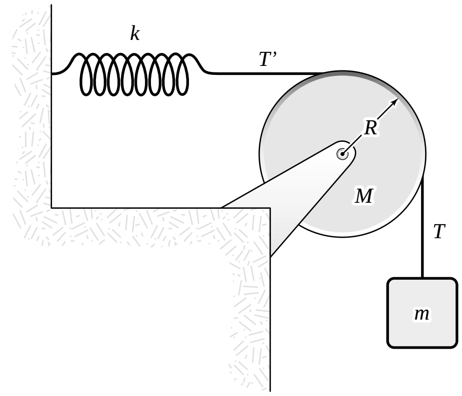
$$= -k\theta L^2 = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

So $\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\theta$

The angular acceleration is opposite in direction and proportional to the displacement, so we have simple harmonic motion with $\boxed{\omega^2 = \frac{3k}{m}}$.

(b) $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3(100\text{ N/m})}{5.00\text{ kg}}} = \boxed{1.23\text{ Hz}}$

***13.61** One can write the following equations of motion:



$$mg - T = ma = m\frac{d^2x}{dt^2} \quad \text{(for the mass)}$$

$$T' - kx = 0 \quad \text{(describes the spring)}$$

$$R(T - T') = I\frac{d^2\theta}{dt^2} = \frac{I}{R}\frac{d^2x}{dt^2} \quad \text{(for the pulley)}$$

with $I = \frac{1}{2}MR^2$.

Combining these equations gives the equation of motion

$$\left(m + \frac{1}{2}M\right)\frac{d^2x}{dt^2} + kx = mg$$

The solution is $x(t) = A \sin \omega t +$
Error! , with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{1}{2}M}} = \frac{1}{2\pi}\sqrt{\frac{100 \text{ N/m}}{0.200 \text{ kg} + \frac{1}{2}M}}$$

(a) For $M = 0$, $f = \boxed{3.56 \text{ Hz}}$

(b) For $M = 0.250$ kg, $f = \boxed{2.79 \text{ Hz}}$

(c) For $M = 0.750$ kg, $f = \boxed{2.10 \text{ Hz}}$

13.62 (a) $\omega_0 = \sqrt{\frac{k}{m}} = \boxed{15.8 \text{ rad/s}}$

(b) $F_s - mg = ma = m\left(\frac{1}{3}g\right)$

$$F_s = \frac{4}{3}\,mg = 26.1 \text{ N}$$

$$x_s = \frac{F_s}{k} = \boxed{5.23 \text{ cm}}$$

(c) When the acceleration of the car is zero, the new equilibrium position can be found as follows:

$$F'_s = mg = 19.6 \text{ N} = kx'_s \qquad x'_s = 3.92 \text{ cm}$$

Thus, $A = \left|x'_s - x_s\right| = \boxed{1.31 \text{ cm}}$

The phase constant is $\boxed{\pi \text{ rad}}$

13.63 (a) $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{L}{g}} = \boxed{3.00\text{ s}}$

(b) $E = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(6.74)(2.06)^2 = \boxed{14.3\text{ J}}$

(c) At maximum angular displacement,

$$mgh = \frac{1}{2}mv^2$$

$$h = \frac{v^2}{2g} = 0.217\text{ m}$$

$$h = L - L\cos\theta = L(1 - \cos\theta)$$

$$\cos\theta = 1 - \frac{h}{L}$$

$$\boxed{\theta = 25.5°}$$

***13.64** Suppose a 100-kg biker compresses the suspension 2.00 cm. Then,

$$k = \frac{F}{x} = \frac{980\text{ N}}{2.00 \times 10^{-2}\text{ m}} = 4.90 \times 10^4\text{ N/m}$$

If total mass of motorcycle and biker is 500 kg, the frequency of free vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.90 \times 10^4\text{ N/m}}{500\text{ kg}}} = 1.58\text{ Hz}$$

If he encounters washboard bumps at the same frequency, resonance will make the motorcycle bounce a lot. Assuming a speed of 20.0 m/s, these ridges are separated by

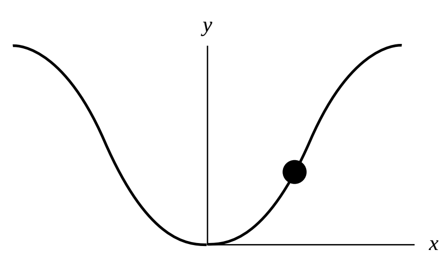
$$\frac{(20.0\text{ m/s})}{1.58/\text{s}} = 12.7\text{ m}\ \boxed{\sim 10^1\text{ m}}$$

In addition to this vibration mode of bouncing up and down as one unit, the motorcycle can also vibrate at higher frequencies by rocking back and forth between front and rear wheels, by having just the front wheel bounce inside its fork, or by doing other things. Other spacings of bumps will excite all of these other resonances.

***13.65** $y = (20.0\text{ cm})\,[1 - \cos(0.160\text{ m}^{-1}x)]$

$$\approx (20.0\text{ cm})\left[1 - 1 + \frac{1}{2}(0.160\text{ m}^{-1}x)^2\right]$$

or $\quad y \approx (10.0\text{ cm})(0.160\text{ m}^{-1}x)^2$



The geometric slope of the wire is

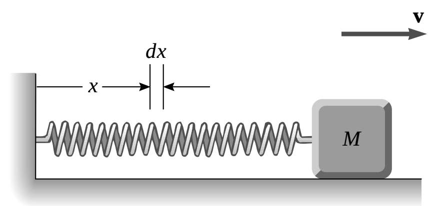
$$\frac{dy}{dx} = (0.100\text{ m})(0.160\text{ m}^{-1})^2(2x) = (5.12 \times 10^{-3}\text{ m}^{-1})x$$

If m is the mass of the bead, the component of the bead's weight that acts as a restoring force is

$$\Sigma F = -mg\frac{dy}{dx} = -m(9.80\text{ m/s}^2)(5.12 \times 10^{-3}\text{ m}^{-1})x = ma$$

Thus, $a = -(0.0502\text{ s}^{-2})x = -\omega^2 x$. Since the acceleration of the bead is opposite the displacement from equilibrium and is proportional to the displacement, the motion is simple harmonic with $\omega^2 = 0.0502\text{ s}^{-2}$, or $\omega = \boxed{0.224\text{ rad/s}}$.

***13.66** (a) For each segment of the spring



$$dK = \frac{1}{2}(dm)\,v_x^2$$

Also, $v_x = \frac{x}{l}\,v$

and $dm = \frac{m}{l}\,dx$

Therefore, the total kinetic energy is

$$K = \frac{1}{2}\,Mv^2 + \frac{1}{2}\int_0^l \left(\frac{x^2v^2}{l^2}\right)\frac{m}{l}\,dx = \boxed{\frac{1}{2}\left(M + \frac{m}{3}\right)v^2}$$

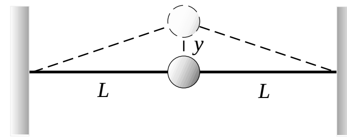
(b) $\omega = \sqrt{\frac{k}{m_{\text{eff}}}}$

and $\frac{1}{2}\,m_{\text{eff}}v^2 = \frac{1}{2}\left(M + \frac{m}{3}\right)v^2$

Therefore, $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M + \frac{m}{3}}{k}}}$

13.67 (a) $\sum \mathbf{F} = -2T \sin\theta \,\mathbf{j}$

where $\theta = \tan^{-1}\left(\dfrac{y}{L}\right)$



Therefore, for a small displacement

$$\sin\theta \approx \tan\theta = \frac{y}{L}$$

and $\boxed{\sum \mathbf{F} = \dfrac{-2Ty}{L}\mathbf{j}}$

(b) For a spring system, $\sum \mathbf{F} = -k\mathbf{x}$ becomes $\sum \mathbf{F} = -\dfrac{2T}{L}\,\mathbf{y}$

Therefore, $\boxed{\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2T}{mL}}}$

13.68 (a) Assuming a Hooke's Law type spring,

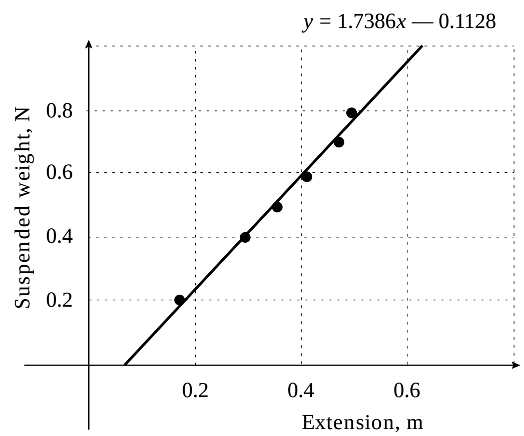
$$F = Mg = kx$$

and empirically $Mg = 1.74x - 0.113$

so $\boxed{k \approx 1.74 \text{ N/m} \pm 6\%}$

M, kg	x, m	Mg, N
0.0200	0.17	0.196
0.0400	0.293	0.392
0.0500	0.353	0.49
0.0600	0.413	0.588
0.0700	0.471	0.686
0.0800	0.493	0.784

Static stretching of a spring



(b) We may write the equation as theoretically

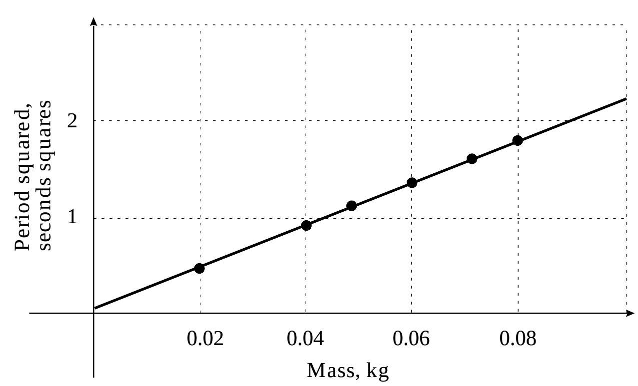
$$T^2 = \frac{4\pi^2}{k} M + \frac{4\pi^2}{3k} m_s$$

and empirically $T^2 = 21.7\,M + 0.0589$

so $k = \dfrac{4\pi^2}{21.7}$ $\boxed{\approx 1.82 \text{ N/m} \pm 3\%}$

Time, s	T, s	M, kg	T^2, s^2
7.03	0.703	0.0200	0.494
9.62	0.962	0.0400	0.925
10.67	1.067	0.0500	1.138
11.67	1.167	0.0600	1.362
12.52	1.252	0.0700	1.568
13.41	1.341	0.0800	1.798

Squared period as a function of the mass of an object bouncing on a spring



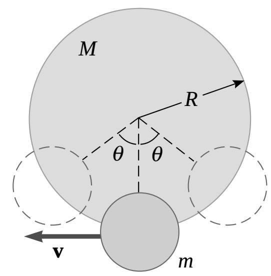
The k values 1.74 N / m ± 6% and 1.82 N / m ± 3% differ by 4%, so they agree.

(c) Utilizing the axis-crossing point,

$$m_s = 3\left(\frac{0.0589}{21.7}\right)\text{ kg} \approx \boxed{8\text{ grams} \pm 12\%}$$

in agreement with 7.4 grams.

13.69 (a) $\Delta K + \Delta U = 0$



Thus, $K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}}$

where $K_{\text{top}} = U_{\text{bot}} = 0$

Therefore, $mgh = \frac{1}{2} I\omega^2$, but

$$h = R - R\cos\theta = R(1 - \cos\theta)$$

$$\omega = \frac{v}{R}$$

and $I = \frac{MR^2}{2} + \frac{mr^2}{2} + mR^2$

Substituting we find

$$mgR(1 - \cos\theta) = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{mr^2}{2} + mR^2\right)\frac{v^2}{R^2}$$

$$mgR(1 - \cos\theta) = \left[\frac{M}{4} + \frac{mr^2}{4R^2} + \frac{m}{2}\right] v^2$$

and $v^2 = 4gR \dfrac{(1 - \cos\theta)}{\left(\frac{M}{m} + \frac{r^2}{R^2} + 2\right)}$

so $\boxed{v = 2\sqrt{\dfrac{Rg(1 - \cos\theta)}{\frac{M}{m} + \frac{r^2}{R^2} + 2}}}$

(b) $T = 2\pi\sqrt{\dfrac{I}{m_T g d_{\text{CM}}}}$

$m_T = m + M$ $\quad d_{\text{CM}} = \dfrac{mR + M(0)}{m + M}$

$T = \boxed{2\pi\sqrt{\dfrac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2}{mgR}}}$

13.70 (a) We require

$$Ae^{-bt/2m} = \frac{A}{2}$$

$$e^{+bt/2m} = 2$$

or $\dfrac{bt}{2m} = \ln 2$ or $\dfrac{(0.100\text{ kg/s})}{2(0.375\text{ kg})}t = 0.693$

$\therefore t = \boxed{5.20\text{ s}}$ The spring constant is irrelevant.

(b) We can evaluate the energy at successive turning points, where

$\cos(\omega t + \phi) = \pm 1$ and the energy is $\frac{1}{2}kx^2 = \frac{1}{2}kA^2e^{-bt/m}$

We require $\frac{1}{2}kA^2e^{-bt/m} = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$

or $e^{+bt/m} = 2$

$$\therefore t = \frac{m\ln 2}{b} = \frac{0.375\text{ kg}(0.693)}{0.100\text{ kg/s}} = \boxed{2.60\text{ s}}$$

(c) From $E = \frac{1}{2}kA^2$,

the fractional rate of change of energy over time is

$$\frac{\frac{dE}{dt}}{E} = \frac{\frac{d}{dt}\frac{1}{2}kA^2}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}k2A\frac{dA}{dt}}{\frac{1}{2}kA^2} = 2\frac{\frac{dA}{dt}}{A}$$

two times faster than the fractional rate of change in amplitude.

13.71 (a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2. By Newton's third law, we expect $k_1x_1 = k_2x_2$. When this is combined with the requirement that $x = x_1 + x_2$, we find

$$x_1 = \left[\frac{k_2}{(k_1 + k_2)}\right]x$$

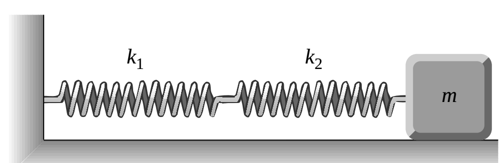
The force on either spring is given by

$$F_1 = \left[\frac{k_1k_2}{(k_1 + k_2)}\right]x = ma$$

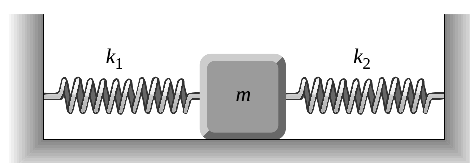
where a is the acceleration of the mass m.

This is in the form $F = k_{\text{eff}}x = ma$

and $T = 2\pi\sqrt{\dfrac{m}{k_{\text{eff}}}} = \boxed{2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1k_2}}}$



(a)



(b)

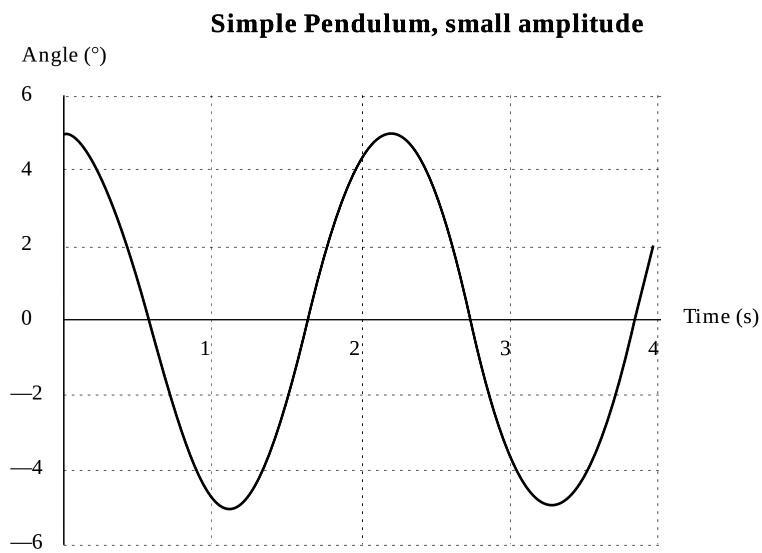
(b) In this case each spring is stretched by the distance x which the mass is displaced. Therefore, the restoring force is

$$F = -(k_1 + k_2)x \qquad \text{and} \qquad k_{\text{eff}} = (k_1 + k_2)$$

so that $T = \boxed{2\pi\sqrt{\dfrac{m}{(k_1 + k_2)}}}$

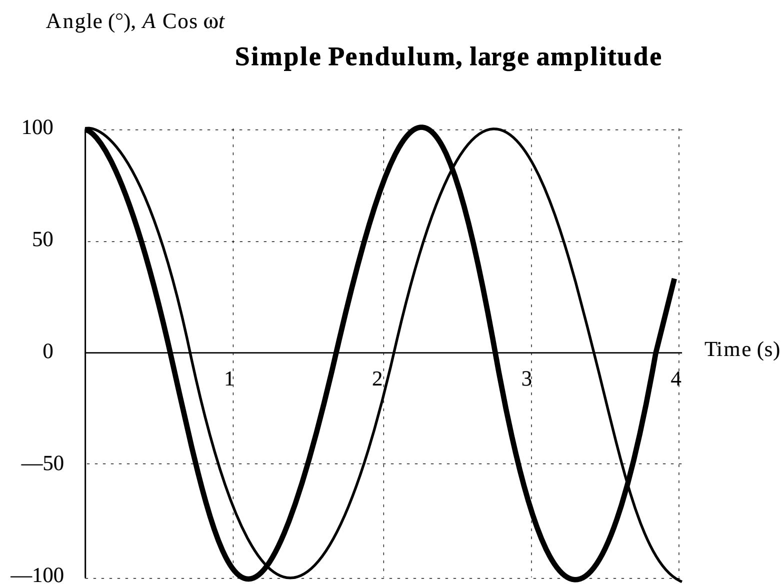
13.72 For $\theta_{\max} = 5.00°$, the motion calculated by the Euler method agrees quite precisely with the prediction of $\theta_{\max} \cos \omega t$. The period is $T = 2.20$ s.

Time, t (s)	Angle, θ (°)	Ang. speed (°/s)	Ang. Accel. (°/s^2)	$\theta_{\max} \cos \omega t$
0.000	5.0000	0.0000	–40.7815	5.0000
0.004	4.9993	–0.1631	–40.7762	4.9997
0.008	4.9980	–0.3262	–40.7656	4.9987
…				
0.544	0.0560	–14.2823	–0.4576	0.0810
0.548	–0.0011	–14.2842	0.0090	0.0239
0.552	–0.0582	–14.2841	0.4756	–0.0333
…				
1.092	–4.9994	–0.3199	40.7765	–4.9989
1.096	–5.0000	–0.1568	40.7816	–4.9998
1.100	–5.0000	0.0063	40.7814	–5.0000
1.104	–4.9993	0.1694	40.7759	–4.9996
…				
1.644	–0.0638	14.2824	0.4397	–0.0716
1.648	0.0033	14.2842	–0.0270	–0.0145
1.652	0.0604	14.2841	–0.4936	0.0427
…				
2.192	4.9994	0.3137	–40.7768	4.9991
2.196	5.0000	0.1506	–40.7817	4.9999
2.200	5.0000	–0.0126	–40.7813	5.0000
2.204	4.9993	–0.1757	–40.7756	4.9994


Simple Pendulum, small amplitude
Angle (°)
6
4
2
0
—2
—4
—6
Time (s)
1
2
3
4

For $\theta_{max} = 100°$, the simple harmonic motion approximation $\theta_{max} \cos \omega t$ diverges greatly from the Euler calculation. The period is $T = 2.71$ s, larger than the small–angle period by 23%.

Time, t (s)	Angle, θ (˚)	Ang. speed (˚/s)	Ang. Accel. (°/s^2)	$\theta_{max} \cos \omega t$
0.000	100.0000	0.0000	–460.6066	100.0000
0.004	99.9926	–1.8432	–460.8173	99.9935
0.008	99.9776	–3.6865	–460.8382	99.9739
…				
1.096	–84.7449	–120.1910	465.9488	–99.9954
1.100	–85.2182	–118.3272	466.2869	–99.9998
1.104	–85.6840	–116.4620	466.5886	–99.9911
…				
1.348	–99.9960	–3.0533	460.8125	–75.7979
1.352	–100.0008	–1.2100	460.8057	–75.0474
1.356	–99.9983	0.6332	460.8093	–74.2870
…				
2.196	40.1509	224.8677	–301.7132	99.9971
2.200	41.0455	223.6609	–307.2607	99.9993
2204	41.9353	222.4318	–312.7035	99.9885
…				
2.704	99.9985	2.4200	–460.8090	12.6422
2.708	100.0008	0.5768	–460.8057	11.5075
2.712	99.9957	–1.2664	–460.8129	10.3712



Chapter 14 Solutions

*14.1 For two 70.0-kg persons, modeled as spheres,

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(70.0\ \text{kg})(70.0\ \text{kg})}{(2.00\ \text{m})^2} = \boxed{\sim 10^{-7}\ \text{N}}$$

14.2 (a) At the midpoint between the two masses, the forces exerted by the 200-kg and 500-kg masses are oppositely directed, and from $F_g = \frac{Gm_1m_2}{r^2}$ we have

$$\Sigma F = \frac{G(50.0\ \text{kg})(500\ \text{kg} - 200\ \text{kg})}{(0.200\ \text{m})^2} = \boxed{2.50 \times 10^{-5}\ \text{N}} \text{ toward the 500-kg mass}$$

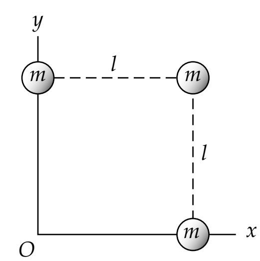
(b) At a point between the two masses at a distance d from the 500-kg mass, the net force will be zero when

$$\frac{G(50.0\ \text{kg})(200\ \text{kg})}{(0.400\ \text{m} - d)^2} = \frac{G(50.0\ \text{kg})(500\ \text{kg})}{d^2} \quad \text{or} \quad d = \boxed{0.245\ \text{m}}$$

14.3 $$\mathbf{g} = \frac{Gm}{l^2}\,\mathbf{i} + \frac{Gm}{l^2}\,\mathbf{j} + \frac{Gm}{2l^2}(\cos 45.0°\ \mathbf{i} + \sin 45.0°\ \mathbf{j})$$

so $$\mathbf{g} = \frac{GM}{l^2}\left(1 + \frac{1}{2\sqrt{2}}\right)(\mathbf{i} + \mathbf{j}) \quad \text{or}$$

$$\mathbf{g} = \boxed{\frac{Gm}{l^2}\left(\sqrt{2} + \frac{1}{2}\right) \text{toward the opposite corner}}$$



14.4 $m_1 + m_2 = 5.00\ \text{kg}\ m_2 = 5.00\ \text{kg} - m_1$

$$F = G\frac{m_1m_2}{r^2} \Rightarrow 1.00 \times 10^{-8}\ \text{N} = \left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{m_1(5.00\ \text{kg} - m_1)}{(0.200\ \text{m})^2}$$

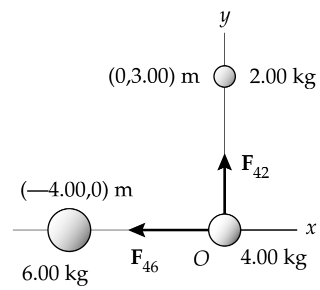
$$(5.00\ \text{kg})m_1 - m_1^2 = \frac{(1.00 \times 10^{-8}\ \text{N})(0.0400\ \text{m}^2)}{6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2} = 6.00\ \text{kg}^2$$

Thus, $m_1^2 - (5.00\ \text{kg})m_1 + 6.00\ \text{kg} = 0$

or $(m_1 - 3.00\ \text{kg})(m_1 - 2.00\ \text{kg}) = 0$

giving $\boxed{m_1 = 3.00\ \text{kg, so } m_2 = 2.00\ \text{kg}}$. The answer $m_1 = 2.00$ kg and $m_2 = 3.00$ kg is physically equivalent.

14.5 The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by



$$\mathbf{F}_{42} = G\frac{m_4 m_2}{r_{42}^2}\mathbf{j}$$

$$= \left(6.67\times10^{-11}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\frac{(4.00\ \mathrm{kg})(2.00\ \mathrm{kg})}{(3.00\ \mathrm{m})^2}\ \mathbf{j}$$

$$= 5.93\times10^{-11}\ \mathbf{j}\ \mathrm{N}$$

The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left:

$$\mathbf{F}_{46} = G\frac{m_4 m_6}{r_{46}^2}(-\mathbf{i})$$

$$= \left(-6.67\times10^{-11}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\frac{(4.00\ \mathrm{kg})(6.00\ \mathrm{kg})}{(4.00\ \mathrm{m})^2}\ \mathbf{i} = -10.0\times10^{-11}\ \mathbf{i}\ \mathrm{N}$$

Therefore, the resultant force on the 4.00-kg mass is

$$\mathbf{F}_4 = \mathbf{F}_{42} + \mathbf{F}_{46} = \boxed{(-10.0\mathbf{i} + 5.93\mathbf{j})\times10^{-11}\ \mathrm{N}}$$

14.6 $$g = \frac{GM}{R^2} = \frac{G\rho(4\pi R^3/3)}{R^2} = \frac{4}{3}\pi G\rho R$$

If $\dfrac{g_{\mathrm{M}}}{g_{\mathrm{E}}} = \dfrac{1}{6} = \dfrac{4\pi G\rho_M R_M/3}{4\pi G\rho_E R_E/3}$ then $\dfrac{\rho_M}{\rho_E} = \left(\dfrac{g_M}{g_E}\right)\left(\dfrac{R_E}{R_M}\right) = \left(\dfrac{1}{6}\right)(4) = \boxed{\dfrac{2}{3}}$

14.7 (a) The Sun-Earth distance is 1.496×10^{11} m, and the Earth-Moon distance is 3.84×10^{8} m, so the distance from the Sun to the Moon during a solar eclipse is 1.496×10^{11} m $- 3.84\times10^{8}$ m $= 1.492\times10^{11}$ m.

The mass of the Sun, Earth , and Moon are $M_s = 1.99\times10^{30}$ kg, $M_E = 5.98\times10^{24}$ kg, and $M_M = 7.36\times10^{22}$ kg. We have

$$F_{SM} = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})(1.99\times10^{30})(7.36\times10^{22})}{(1.492\times10^{11})^2} = \boxed{4.39\times10^{20}\ \mathrm{N}}$$

(b) $$F_{EM} = \frac{(6.67\times10^{-11}\ \mathrm{Nm^2/kg^2})(5.98\times10^{24})(7.36\times10^{22})}{(3.84\times10^{8})^2} = \boxed{1.99\times10^{20}\ \mathrm{N}}$$

(c) $$F_{SE} = \frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(1.99\times10^{30})(5.98\times10^{24})}{(1.496\times10^{11})^2} = \boxed{3.55\times10^{22}\ \mathrm{N}}$$

14.8 (a) $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(384\ 400) \times 10^3\ \text{m}}{27.3 \times (86\ 400\ \text{s})} = \boxed{1.02 \times 10^3\ \text{m/s}}$

(b) In one second, the Moon falls a distance

$$x = \frac{1}{2}at^2 = \frac{1}{2}\frac{v^2}{r}t^2 = \frac{1}{2}\frac{(1.02 \times 10^3)^2}{(3.844 \times 10^8)} \times (1.00)^2 = 1.35 \times 10^{-3}\ \text{m} = \boxed{1.35\ \text{mm}}$$

The Moon only moves inward 1.35 mm for every 1020 meters it moves along a straight-line path.

14.9 $a = \dfrac{MG}{(4R_E)^2} = \dfrac{9.80\ \text{m/s}^2}{16.0} = \boxed{0.613\ \text{m/s}^2}$ toward the Earth

***14.10** $F = m_1 g = \dfrac{Gm_1 m_2}{r^2}$

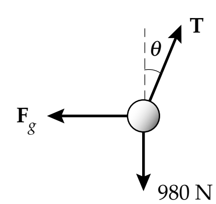
$$g = \frac{Gm_2}{r^2} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(4.00 \times 10^4 \times 10^3\ \text{kg})}{(100\ \text{m})^2} = \boxed{2.67 \times 10^{-7}\ \text{m/s}^2}$$

14.11 The separation is nearly 1.000 m, so one ball attracts the other with force

$$F_g = \frac{Gm_1 m_2}{r^2} = \frac{6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2(100\ \text{kg})^2}{\text{kg}^2(1.000\ \text{m})^2} = 6.67 \times 10^{-7}\ \text{N}$$

Call θ the angle of each cable from the vertical and T its tension.

Each ball is in equilibrium, with



$$T\cos\theta = mg = 980\ \text{N}$$

$$T\sin\theta = 6.67 \times 10^{-7}\ \text{N}$$

$$\tan\theta = \frac{6.67 \times 10^{-7}\ \text{N}}{980\ \text{N}} = 6.81 \times 10^{-10}$$

Each ball scrunches in by

$$(45.0\ \text{m})\sin\theta = 45.0\ \text{m}(6.81 \times 10^{-10}) = 3.06 \times 10^{-8}\ \text{m}$$

so their separation is $\boxed{1.000\ \text{m} - 61.3\ \text{nm}}$

14.12 (a) At the zero-total field point, $\dfrac{GmM_E}{r_E^2} = \dfrac{GmM_M}{r_M^2}$ so

$$r_M = r_E\sqrt{\frac{M_M}{M_E}} = r_E\sqrt{\frac{7.36\times10^{22}}{5.98\times10^{24}}} = \frac{r_E}{9.01}$$

$$r_E + r_M = 3.84\times10^8\text{ m} = r_E + \frac{r_E}{9.01}$$

$$r_E = \frac{3.84\times10^8\text{ m}}{1.11} = \boxed{3.46\times10^8\text{ m}}$$

(b) At this distance the acceleration due to the Earth's gravity is

$$g_E = \frac{GM_E}{r_E^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{(3.46\times10^8\text{ m})^2}$$

$$g_E = \boxed{3.34\times10^{-3}\text{ m/s}^2\text{ directed toward the Earth}}$$

14.13 Since speed is constant, the distance traveled between t_1 and t_2 is equal to the distance traveled between t_3 and t_4. The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

So $$\frac{1}{2}\,bv(t_2 - t_1) = \frac{1}{2}\,bv(t_4 - t_3)$$

states that the particle's radius vector sweeps out equal areas in equal times.

***14.14** (a) For the geosynchronous satellite, $\Sigma F_r = \dfrac{GmM_E}{r^2} = ma_r = \dfrac{mv^2}{r}$ becomes

$$\frac{GM_E}{r} = \left(\frac{2\pi r}{T}\right)^2 \quad\text{or}\quad r^3 = \frac{GM_ET^2}{4\pi^2}$$

Thus, the radius of the satellite orbit is

$$r = \left[\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})(86\,400\text{ s})^2}{4\pi^2}\right]^{1/3} = \boxed{4.23\times10^7\text{ m}}$$

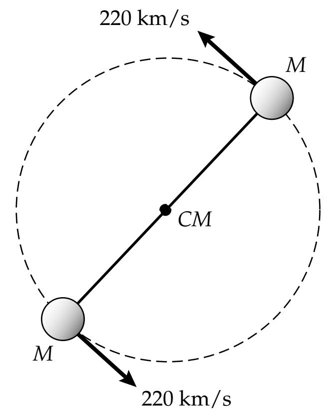
(b) The satellite is so far out that its distance from the north pole,

$$d = \sqrt{(6.37\times10^6\text{ m})^2 + (4.23\times10^7\text{ m})^2} = 4.27\times10^7\text{ m}$$

is nearly the same as its orbital radius. The travel time for the radio signal is

$$t = \frac{2d}{c} = \frac{2(4.27\times10^7\text{ m})}{3.00\times10^8\text{ m/s}} = \boxed{0.285\text{ s}}$$

14.15 Centripetal force = Gravitational force between stars' centers



$$\frac{Mv^2}{r} = \frac{GMM}{(2r)^2}$$

For a circular orbit, $v = \frac{2\pi r}{T}$ for each star.

Solving,

$$M = \boxed{12.6 \times 10^{31} \text{ kg} = 63.3 \text{ solar masses}}$$

for each star.

The two blue giant stars comprising Plaskett's binary system are among the most massive known.

Goal Solution

G: From the given data, it is difficult to estimate a reasonable answer to this problem without actually working through the details to actually solve it. A reasonable guess might be that each star has a mass equal to or larger than our Sun since our Sun happens to be less massive than many stars in the universe.

O: The only force acting on the two stars is the central gravitational force of attraction which results in a centripetal acceleration. When we solve Newton's 2nd law, we can find the unknown mass in terms of the variables given in the problem.

A: Applying Newton's 2nd Law, $\sum F = ma$ yields $F_g = ma_c$ for each star:

$$\frac{GMM}{(2r)^2} = \frac{Mv^2}{r} \quad \text{or} \quad M = \frac{4v^2 r}{G}$$

We can write r in terms of the period, T, by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit , so $2\pi r = vT$. Therefore

$$M = \frac{4v^2 r}{G} = \frac{4v^2}{G}\left(\frac{vT}{2\pi}\right)$$

so, $$M = \frac{2v^3 T}{\pi G} = \frac{2(220 \times 10^3 \text{ m/s})^3 (14.4 \text{ d})(86\,400 \text{ s/d})}{\pi(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 1.26 \times 10^{32} \text{ kg}$$

L: The mass of each star is about 63 solar masses, much more than our initial guess! A quick check in an astronomy book reveals that stars over 8 solar masses are considered to be *heavyweight* stars, and astronomers estimate that the maximum theoretical limit is about 100 solar masses before a star becomes unstable. So these 2 stars are exceptionally massive!

14.16 Centripetal force = Gravitational force between stars' centers

$$\frac{Mv^2}{r} = \frac{GMM}{(2r)^2} \quad \text{which reduces to} \quad M = \frac{4rv^2}{G}$$

For a circular orbit, $v = \dfrac{2\pi r}{T}$ for each star. Hence, the radius of the orbit is given by $r = \dfrac{vT}{2\pi}$, and the mass of each star is then $M = \boxed{\dfrac{2v^3T}{\pi G}}$

14.17 By conservation of angular momentum,

$$r_p v_p = r_a v_a$$

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{2289 \text{ km} + 6.37 \times 10^3 \text{ km}}{459 \text{ km} + 6.37 \times 10^3 \text{ km}} = \frac{8659 \text{ km}}{6829 \text{ km}} = \boxed{1.27}$$

We do not need to know the period.

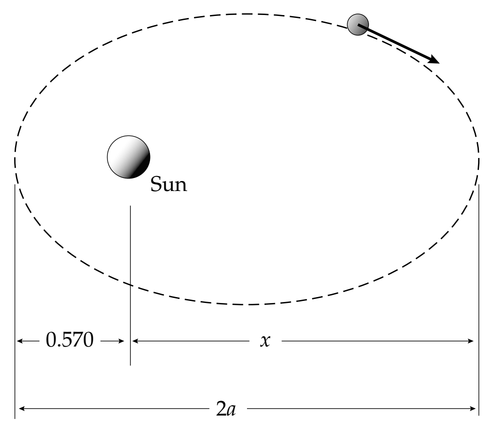
14.18 By Kepler's Third Law,

$$T^2 = ka^3 \quad (a = \text{semi-major axis})$$

For any object orbiting the Sun, with T in years and a in A.U., $k = 1.00$

Therefore, for Comet Halley

$$(75.6)^2 = (1.00)\left(\frac{0.570 + x}{2}\right)^3$$



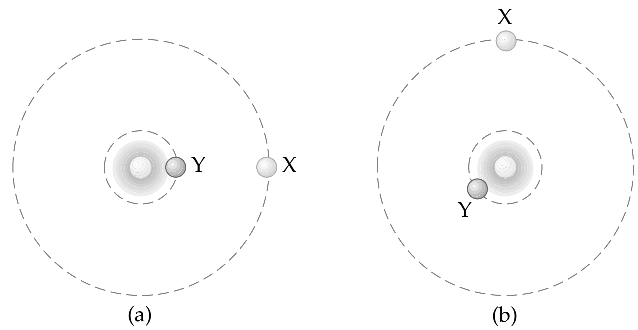
The farthest distance the comet gets from the Sun is

$$x = 2(75.6)^{2/3} - 0.570 = \boxed{35.2 \text{ A.U.}} \quad \text{(out around the orbit of Pluto)}$$

14.19 $T^2 = \dfrac{4\pi^2 d^2}{GM}$ (Kepler's Third Law with $m << M$)

$$M = \frac{4\pi^2 d^3}{GT^2} = \boxed{1.90 \times 10^{27} \text{ kg}} \quad \text{(approximately 316 Earth masses)}$$

14.20 $\sum F = ma$



$$\frac{Gm_{\text{planet}}M_{\text{star}}}{r^2} = \frac{m_{\text{planet}}v^2}{r}$$

$$\frac{GM_{\text{star}}}{r} = v^2 = r^2\omega^2$$

$$GM_{\text{star}} = r^3\omega^2 = r_x^3\ \omega_x^2 = r_y^3\ \omega_y^2$$

$$\omega_y = \omega_x\left(\frac{r_x}{r_y}\right)^{3/2}$$

$$\omega_y = \left(\frac{90.0°}{5.00\text{ yr}}\right)3^{3/2} = \frac{468°}{5.00\text{ yr}}$$

So $\boxed{\text{planet } Y \text{ has turned through 1.30 revolutions}}$

14.21 $$\frac{GM_J}{(R_J+d)^2} = \frac{4\pi^2(R_J+d)}{T^2}$$

$$GM_JT^2 = 4\pi^2(R_J+d)^3$$

$$(6.67\times10^{-11})\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}(1.90\times10^{27}\text{ kg})(9.84\times3600)^2 = 4\pi^2(6.99\times10^7+d)^3$$

$d = \boxed{8.92\times10^7\text{ m}} = \boxed{89\,200\text{ km}}$ above the planet

***14.22** The gravitational force on a small parcel of material at the star's equator supplies the necessary centripetal force:

$$\frac{GM_sm}{R_s^2} = \frac{mv^2}{R_s} = mR_s\omega^2$$

so $$\omega = \sqrt{\frac{GM_s}{R_s^3}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)[2(1.99\times10^{30}\text{ kg})]}{(10.0\times10^3\text{ m})^3}}$$

$$\omega = \boxed{1.63\times10^4\text{ rad/s}}$$

***14.23** Let m represent the mass of the spacecraft, r_E the radius of the Earth's orbit, and x the distance from Earth to the spacecraft.

The Sun exerts a radial inward force of $F_S = \dfrac{GM_S m}{(r_E - x)^2}$ on the spacecraft while the Earth exerts a radial outward force of $F_E = \dfrac{GM_E m}{x^2}$ on it. The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year. Thus,

$$F_S - F_E = \frac{GM_S m}{(r_E - x)^2} - \frac{GM_E m}{x^2} = \frac{mv^2}{(r_E - x)} = \frac{m}{(r_E - x)}\left[\frac{2\pi(r_E - x)}{T}\right]^2$$

which reduces to

$$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2} \qquad (1)$$

This equation is fifth degree in x, so we do not solve it algebraically. We may test the assertion that x is between 1.47×10^9 m and 1.48×10^9 m by substituting both of these as trial solutions, along with the following data: $M_S = 1.991 \times 10^{30}$ kg, $M_E = 5.983 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000$ yr $= 3.156 \times 10^7$ s.

With $x = 1.47 \times 10^9$ m substituted into Equation (1), we obtain

$$6.052 \times 10^{-3} \text{ m/s}^2 - 1.85 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

or $\quad 5.868 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$

With $x = 1.48 \times 10^9$ m substituted into the same equation, the result is

$$6.053 \times 10^{-3} \text{ m/s}^2 - 1.82 \times 10^{-3} \text{ m/s}^2 \approx 5.8708 \times 10^{-3} \text{ m/s}^2$$

or $\quad 5.8709 \times 10^{-3} \text{ m/s}^2 \approx 5.808 \times 10^{-3} \text{ m/s}^2$

Since the first trial solution makes the left-hand side of Equation (1) slightly less than the right-hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three-digit precision, it is 1.48×10^9 m.

14.24 (a) $F = \dfrac{GMm}{r^2} = \dfrac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)[100(1.99 \times 10^{30} \text{ kg})(10^3 \text{ kg})]}{(1.00 \times 10^4 \text{ m} + 50.0 \text{ m})^2}$

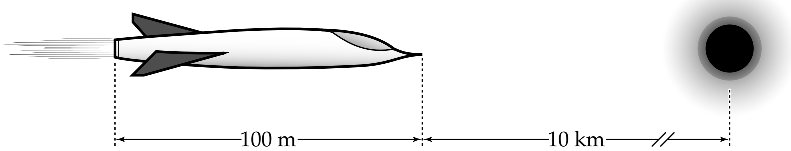
$F = \boxed{1.31 \times 10^{17} \text{ N}}$

(b) $\Delta F = \dfrac{GMm}{r_{\text{front}}^2} - \dfrac{GMm}{r_{\text{back}}^2}$

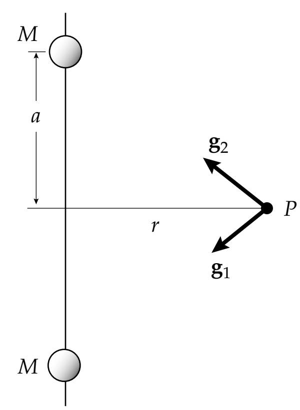
$$\Delta g = \frac{\Delta F}{m} = \frac{GM(r_{\text{back}}^2 - r_{\text{front}}^2)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

$$= \frac{(6.67\times10^{-11})[100(1.99\times10^{30})][(1.01\times10^4\text{ m})^2 - (1.00\times10^4\text{ m})^2]}{(1.00\times10^4\text{ m})^2(1.01\times10^4\text{ m})^2}$$

$$\Delta g = \boxed{2.62\times10^{12}\text{ N/kg}}$$



14.25 $g_1 = g_2 = \dfrac{MG}{r^2 + a^2}$



$g_{1y} = -g_{2y}$

$g_y = g_{1y} + g_{2y} = 0$

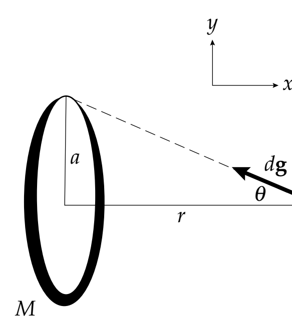
$g_{1x} = g_{2x} = g_2 \cos\theta$

$\cos\theta = \dfrac{r}{(a^2 + r^2)^{1/2}}$

$\mathbf{g} = 2g_{2x}(-\mathbf{i})$ or

$$\mathbf{g} = \boxed{\frac{2MGr}{(r^2 + a^2)^{3/2}}\text{ toward the center of mass}}$$

14.26 $\cos\theta = \dfrac{r}{(a^2 + r^2)^{1/2}}$



$dg_x = dg\cos\theta \qquad g_y = 0$

$$\int dg_x = \int \frac{GdM}{a^2 + r^2}\cos\theta$$

$$g_x = \int \frac{GdM}{(a^2 + r^2)}\frac{r}{(a^2 + r^2)^{1/2}}$$

$$g_x = \boxed{\frac{GMr}{(a^2 + r^2)^{3/2}}\text{ inward along } r}$$

14.27 (a) $U = -\dfrac{GM_E m}{r}$

$$U = -\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ g})(100\text{ kg})}{(6.37+2.00)\times10^6\text{ m}} = \boxed{-4.77\times10^9\text{ J}}$$

(b) and (c) Planet and satellite exert forces of equal magnitude on each other, directed downward on the satellite and upward on the planet.

$$F = \frac{GM_E m}{r^2}$$

$$F = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})(100\text{ kg})}{(8.37\times10^6\text{ m})^2} = \boxed{569\text{ N}}$$

14.28 $U = -G\dfrac{Mm}{r}$ and $g = \dfrac{GM_E}{R_E^2}$

so that $\Delta U = -GMm\left(\dfrac{1}{3R_E} - \dfrac{1}{R_E}\right) = \dfrac{2}{3}\,mgR_E$

$$\Delta U = \frac{2}{3}(1000\text{ kg})(9.80\text{ m/s}^2)(6.37\times10^6\text{ m}) = \boxed{4.17\times10^{10}\text{ J}}$$

14.29 (a) $\rho = \dfrac{M_S}{\frac{4}{3}\pi r_E^3} = \dfrac{3(1.99\times10^{30}\text{ kg})}{4\pi(6.37\times10^6\text{ m})^3} = \boxed{1.84\times10^9\text{ kg/m}^3}$

(b) $g = \dfrac{GM_S}{r_E^2} = \dfrac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}{(6.37\times10^6\text{ m})^2} = \boxed{3.27\times10^6\text{ m/s}^2}$

(c) $U_g = -\dfrac{GM_S m}{r_E} = -\dfrac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})(1.00\text{ kg})}{6.37\times10^6\text{ m}}$

$$U_g = \boxed{-2.08\times10^{13}\text{ J}}$$

Goal Solution

(a) $\rho = \dfrac{M_s}{V} = \dfrac{M_s}{\left(\frac{4}{3}\right)\pi R_E^3} = \dfrac{1.99\times10^{30}\text{ kg}}{\left(\frac{4}{3}\right)\pi\,(6.37\times10^6\text{ m})^3} = 1.84\times10^9\text{ kg/m}^3$

(This white dwarf is on the order of 1 million times more dense than concrete!)

(b) For an object of mass m on its surface, $mg = GM_s m/R_E^2$. Thus,

$$G = \frac{GM_S}{R_E^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}{(6.37\times10^6\text{ m})^2} = 3.27\times10^6\text{ m/s}^2$$

(This acceleration is about 1 million times more than g earth!)

(c) $U_g = \frac{-GM_s m}{R_E} = \frac{(-6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})(1 \text{ kg})}{(6.37 \times 10^6 \text{ m})} = -2.08 \times 10^{13} \text{ J}$

(Such a large loss of potential energy could yield a big gain in kinetic energy. For example, dropping the 1.00-kg object from a height of 1.00 m would result in a final velocity of 2 560 m/s!).

***14.30** The height attained is not small compared to the radius of the Earth, so $U = mgy$ does not apply; $U = -\frac{GM_1M_2}{r}$ does. From launch to apogee at height h,

$$K_i + U_i + \Delta E = K_f + U_f$$

$$\frac{1}{2} M_p v_i^2 - \frac{GM_E M_p}{R_E} + 0 = 0 - \frac{GM_E M_p}{R_E + h}$$

$$\frac{1}{2}(10.0 \times 10^3 \text{ m/s})^2 - (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)\left(\frac{5.98 \times 10^{24} \text{ kg}}{6.37 \times 10^6 \text{ m}}\right)$$

$$= -(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)\left(\frac{5.98 \times 10^{24} \text{ kg}}{6.37 \times 10^6 \text{ m} + h}\right)$$

$$(5.00 \times 10^7 \text{ m}^2/\text{s}^2) - (6.26 \times 10^7 \text{ m}^2/\text{s}^2) = \frac{-3.99 \times 10^{14} \text{ m}^3/\text{s}^2}{6.37 \times 10^6 \text{ m} + h}$$

$$6.37 \times 10^6 \text{ m} + h = \frac{3.99 \times 10^{14} \text{ m}^3/\text{s}^2}{1.26 \times 10^7 \text{ m}^2/\text{s}^2} = 3.16 \times 10^7 \text{ m}$$

$$\boxed{h = 2.52 \times 10^7 \text{ m}}$$

***14.31** (a) $U_{\text{Tot}} = U_{12} + U_{13} + U_{23} = 3U_{12} = 3\left(-\frac{Gm_1m_2}{r_{12}}\right)$

$$U_{\text{Tot}} = -\frac{3(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.00 \times 10^{-3} \text{ kg})^2}{0.300 \text{ m}} = \boxed{-1.67 \times 10^{-14} \text{ J}}$$

(b) $\boxed{\text{At the center}}$ of the equilateral triangle

14.32 $W = -\Delta U = -\left(\frac{-Gm_1m_2}{r} - 0\right)$

$$W = \frac{+\,6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2(7.36 \times 10^{22} \text{ kg})(1.00 \times 10^3 \text{ kg})}{\text{kg}^2\,(1.74 \times 10^6 \text{ m})} = \boxed{2.82 \times 10^9 \text{ J}}$$

14.33 $\dfrac{v_i^2}{R_E + h} = \dfrac{GM_E}{(R_E + h)^2}$

$K_i = \dfrac{1}{2}\, mv_i^2 = \dfrac{1}{2}\dfrac{GM_E m}{R_E + h}$

$= \dfrac{1}{2}\dfrac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.98 \times 10^{24}\ \mathrm{kg})(500\ \mathrm{kg})}{(6.37 \times 10^6\ \mathrm{m}) + (0.500 \times 10^6\ \mathrm{m})} = 1.45 \times 10^{10}\ \mathrm{J}$

The change in gravitational potential energy is

$$\Delta U = \frac{GM_E m}{R_i} - \frac{GM_E m}{R_f} = GM_E m\left(\frac{1}{R_i} - \frac{1}{R_f}\right)$$

$$= (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.98 \times 10^{24}\ \mathrm{kg})(500\ \mathrm{kg})(-1.14 \times 10^{-8}\ \mathrm{m^{-1}})$$

$$= -2.27 \times 10^9\ \mathrm{J}$$

Also, $K_f = \dfrac{1}{2}\, mv_f^2 = \dfrac{1}{2}(500\ \mathrm{kg})(2.00 \times 10^3\ \mathrm{m/s})^2 = 1.00 \times 10^9\ \mathrm{J}$

The energy lost to friction is

$$E_f = K_i - K_f - \Delta U = (14.5 - 1.00 + 2.27) \times 10^9\ \mathrm{J} = \boxed{1.58 \times 10^{10}\ \mathrm{J}}$$

14.34 (a) $v_{\text{solar escape}} = \sqrt{\dfrac{2M_{\text{Sun}}G}{R_{E \cdot \text{Sun}}}} = \boxed{42.1\ \mathrm{km/s}}$

(b) $v = \sqrt{\dfrac{2M_{\text{Sun}}G}{R_{E \cdot S}\,x}} = \dfrac{42.1}{\sqrt{x}}$

If $v = \dfrac{125\,000\ \mathrm{km}}{3\,600\ \mathrm{s}}$, then $x = 1.47$ A.U. $= \boxed{2.20 \times 10^{11}\ \mathrm{m}}$

(at or beyond the orbit of Mars, 125 000 km/h is sufficient for escape)

14.35 $F_c = F_G$ gives $\dfrac{mv^2}{r} = \dfrac{GmM_E}{r^2}$

which reduces to $v = \sqrt{\dfrac{GM_E}{r}}$

and period $= \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM_E}}$

(a) $r = R_E + 200\text{ km} = 6370\text{ km} + 200\text{ km} = 6570\text{ km}$

Thus,

$$\text{period} = 2\pi(6.57\times10^6\text{ m})\sqrt{\frac{(6.57\times10^6\text{ m})}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}}$$

$$T = 5.30\times10^3\text{ s} = 88.3\text{ min} = \boxed{1.47\text{ h}}$$

(b) $$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{(6.57\times10^6\text{ m})}} = \boxed{7.79\text{ km/s}}$$

(c) $K_f + U_f = K_i + U_i +$ energy input, gives

$$\text{input} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \left(\frac{-GM_Em}{r_f}\right) - \left(\frac{-GM_Em}{r_i}\right) \quad (1)$$

$$r_i = R_E = 6.37\times10^6\text{ m}$$

$$v_i = \frac{2\pi R_E}{86\,400\text{ s}} = 4.63\times10^2\text{ m/s}$$

Substituting the appropriate values into (1) yields the

$$\text{minimum energy input} = \boxed{6.43\times10^9\text{ J}}$$

14.36 The gravitational force supplies the needed centripetal acceleration. Thus,

$$\frac{GM_Em}{(R_E+h)^2} = \frac{mv^2}{(R_E+h)} \quad\text{or}\quad v^2 = \frac{GM_E}{R_E+h}$$

(a) $$T = \frac{2\pi r}{v} = \frac{2\pi(R_E+h)}{\sqrt{GM_E/(R_E+h)}} = \boxed{2\pi\sqrt{\frac{(R_E+h)^3}{GM_E}}}$$

(b) $$v = \boxed{\sqrt{\frac{GM_E}{R_E+h}}}$$

(c) minimum energy input $= \Delta E_{\min} = (K_f + U_{gf}) - (K_i + U_{gi})$

where $K_i = \dfrac{1}{2}mv_i^2$ with $v_i = \dfrac{2\pi R_E}{1.00\text{ day}} = \dfrac{2\pi R_E}{86\,400\text{ s}}$

and $U_{gi} = -\dfrac{GM_Em}{R_E}$

Thus,

$$\Delta E_{\min} = \frac{1}{2}\,m\left(\frac{GM_E}{R_E+h}\right) - \frac{GM_E m}{R_E+h} - \frac{1}{2}\,m\left[\frac{4\pi^2 R_E^2}{(86\ 400\ \text{s})^2}\right] + \frac{GM_E m}{R_E}$$

or $$\Delta E_{\min} = \boxed{GM_E m\left[\frac{R_E+2h}{2R_E(R_E+h)}\right] - \frac{2\pi^2 R_E^2 m}{(86\ 400\ \text{s})^2}}$$

14.37 $$\frac{mv_1^2}{2} - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{\max}} + \frac{mv_f^2}{2}$$

or $$v_f^2 = v_1^2 - \frac{2GM_E}{R_E}$$

and $$v_f = \left(v_1^2 - \frac{2GM_E}{R_E}\right)^{1/2}$$

$$v_f = [(2.00\times10^4)^2 - 1.25\times10^8]^{1/2} = \boxed{1.66\times10^4\ \text{m/s}}$$

<u>Goal Solution</u>
Energy is conserved between surface and the distant point:

$$(K+U_g)_i + \Delta E = (K+U_g)_f$$

$$\frac{1}{2}\,mv_i^2 - \frac{GM_E m}{R_E} + 0 = \frac{1}{2}\,mv_f^2 - \frac{GM_E m}{\infty}$$

$$v_f^2 = v_i^2 - \frac{2GM_E}{R_E} \qquad \left(\text{Note: } \frac{2GM_E}{R_E} \text{ is simply } v_{\text{esc}}^2\right)$$

$$v_f^2 = (2.00\times10^4\ \text{m/s})^2 - \frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{(6.37\times10^6\ \text{m})}$$

$$v_f^2 = 4.00\times10^8\ \text{m}^2/\text{s}^2 - 1.25\times10^8\ \text{m}^2/\text{s}^2 = 2.75\times10^8\ \text{m}^2/\text{s}^2$$

$$v_f = 1.66\times10^4\ \text{m/s}$$

14.38 $E_{\text{tot}} = -\dfrac{GMm}{2r}$

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{2}\frac{10^3\text{ kg}}{10^3\text{ m}}\left(\frac{1}{6370 + 100} - \frac{1}{6370 + 200}\right)$$

$\Delta E = 4.69 \times 10^8\text{ J} = \boxed{469\text{ MJ}}$

14.39 To obtain the orbital velocity, we use

$$\Sigma F = \frac{mMG}{R^2} = \frac{mv^2}{R}$$

or $\quad v = \sqrt{\dfrac{MG}{R}}$

We can obtain the escape velocity from

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{mMG}{R}$$

or $\quad v_{\text{esc}} = \sqrt{\dfrac{2MG}{R}} = \boxed{\sqrt{2}\; v}$

***14.40** $g_E = \dfrac{Gm_E}{r_E^2} \qquad g_U = \dfrac{Gm_U}{r_U^2}$

(a) $\dfrac{g_U}{g_E} = \dfrac{m_U r_E^2}{m_E r_U^2} = 14.0\left(\dfrac{1}{3.70}\right)^2 = 1.02$

$g_U = (1.02)(9.80\text{ m/s}^2) = \boxed{10.0\text{ m/s}^2}$

(b) $v_{\text{esc},E} = \sqrt{\dfrac{2Gm_E}{r_E}} \qquad v_{\text{esc},U} = \sqrt{\dfrac{2Gm_U}{r_U}}$

$$\frac{v_{\text{esc},U}}{v_{\text{esc},E}} = \sqrt{\frac{m_U r_E}{m_E r_U}} = \sqrt{\frac{14.0}{3.70}} = 1.95$$

For the Earth,

$v_{\text{esc},\,E} = 11.2\text{ km/s}$ (from Table 14.3)

$\therefore\; v_{\text{esc},\,U} = (1.95)(11.2\text{ km/s}) = \boxed{21.8\text{ km/s}}$

14.41 The rocket is in a potential well at Ganymede's surface with energy

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2\, m_2\,(1.495 \times 10^{23}\ \text{kg})}{\text{kg}^2\,(2.64 \times 10^6\ \text{m})}$$

$$U_1 = -3.78 \times 10^6\, m_2\ \text{m}^2/\text{s}^2$$

The potential well from Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2\, m_2\,(1.90 \times 10^{27}\ \text{kg})}{\text{kg}^2\,(1.071 \times 10^9\ \text{m})}$$

$$U_2 = -1.18 \times 10^8\, m_2\ \text{m}^2/\text{s}^2$$

To escape from both requires

$$\frac{1}{2}m_2 v_{\text{esc}}^2 = +(3.78 \times 10^6 + 1.18 \times 10^8)m_2\ \text{m}^2/\text{s}^2$$

$$v_{\text{esc}} = \sqrt{2 \times 1.22 \times 10^8\ \text{m}^2/\text{s}^2} = \boxed{15.6\ \text{km/s}}$$

14.42 We interpret "lunar escape speed" to be the escape speed from the surface of a stationary moon alone in the Universe:

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{GM_m m}{R_m}$$

$$v_{\text{esc}} = \sqrt{\frac{2GM_m}{R_m}}$$

$$v_{\text{launch}} = 2\sqrt{\frac{2GM_m}{R_m}}$$

Now for the flight from moon to Earth

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}mv_{\text{launch}}^2 - \frac{GmM_m}{R_m} - \frac{GmM_E}{r_{\text{el}}} = \frac{1}{2}mv_{\text{impact}}^2 - \frac{GmM_m}{r_{m2}} - \frac{GmM_E}{R_E}$$

$$\frac{4GM_m}{R_m} - \frac{GM_m}{R_m} - \frac{GM_E}{r_{\text{el}}} = \frac{1}{2}v_{\text{impact}}^2 - \frac{GM_m}{r_{m2}} - \frac{GM_E}{R_E}$$

$$v_{\text{impact}} = \left[2G\left(\frac{3M_m}{R_m} + \frac{M_m}{r_{m2}} + \frac{M_E}{R_E} - \frac{M_E}{r_{\text{el}}}\right)\right]^{1/2}$$

$$= \left[2G\left(\frac{3\times 7.36\times 10^{22}\text{ kg}}{1.74\times 10^{6}\text{ m}} + \frac{7.36\times 10^{22}\text{ kg}}{3.84\times 10^{8}\text{ m}}\right.\right.$$

$$\left.\left. + \frac{5.98\times 10^{24}\text{ kg}}{6.37\times 10^{6}\text{ m}} - \frac{5.98\times 10^{24}\text{ kg}}{3.84\times 10^{8}\text{ m}}\right)\right]^{1/2}$$

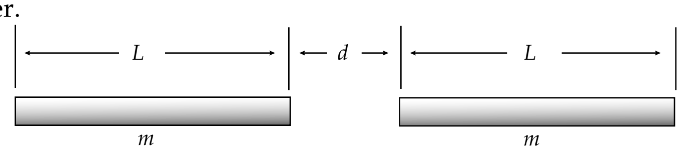
$$= [2G(1.27\times 10^{17} + 1.92\times 10^{14} + 9.39\times 10^{17} - 1.56\times 10^{16})\text{ kg/m}]^{1/2}$$

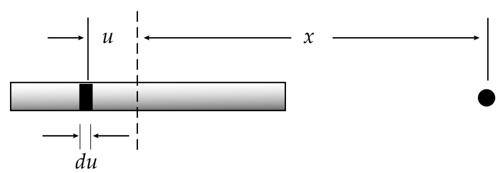
$$= \left[2\times 6.67\times 10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\,10.5\times 10^{17}\text{ kg/m}\right]^{1/2} = \boxed{11.8\text{ km/s}}$$

14.43 In a circular orbit of radius r, the total energy of an Earth satellite is $E = -\dfrac{GM_E}{2r}$. Thus, in changing from a circular orbit of radius $r = 2R_E$ to one of radius $r = 3R_E$, the required work is

$$W = \Delta E = -\frac{GM_Em}{2r_f} + \frac{GM_Em}{2r_i} = GM_Em\left[\frac{1}{4R_E} - \frac{1}{6R_E}\right] = \boxed{\frac{GM_Em}{12R_E}}$$

14.44 First find the acceleration of gravity created by the left-hand rod at a point distant x from its center.





A bit of the left-hand rod of width du has mass $(m/L)du$ and creates field

$$dg = \frac{Gdm}{r^2} = \frac{Gmdu}{L(x+u)^2}$$

Then, $g = \displaystyle\int_{-L/2}^{L/2} \frac{Gmdu}{L(x+u)^2} = \frac{Gm}{L}\left.\frac{(x+u)^{-1}}{-1}\right|_{-L/2}^{L/2}$

$$g = \frac{Gm}{L}\left(\frac{-1}{x+L/2} - \frac{-1}{x-L/2}\right) = \frac{Gm}{L}\frac{L}{x^2 - L^2/4}$$

$$g = \frac{Gm}{x^2 - L^2/4}$$

Now the force on the right-hand rod is the summation of bits $dF = gdm = gmdx/L$.

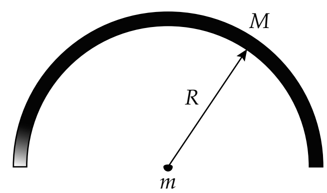
Thus: $F = \int_{x=d+L/2}^{d+3L/2} \frac{Gm\ mdx}{(x^2 - L^2/4)L} = \frac{Gm^2}{L}\int_{x=d+L/2}^{d+3L/2} \frac{dx}{x^2 - L^2/4}$. Use the table of integrals in the Appendix of the textbook.

$$F = \frac{Gm^2}{L}\frac{1}{L}\ln\left(\frac{x - L/2}{x + L/2}\right)\Bigg|_{d+L/2}^{d+3L/2}$$

$$= \frac{Gm^2}{L^2}\left[\ln\left(\frac{d+L}{d+2L}\right) - \ln\left(\frac{d}{d+L}\right)\right]$$

$$= \frac{Gm^2}{L^2}\left[\ln\left(\frac{d+L}{d+2L}\frac{d+L}{d}\right)\right] = \boxed{\frac{Gm^2}{L^2}\ln\left[\frac{(d+L)^2}{d(d+2L)}\right]}$$

14.45 By symmetry, **F** is in the y direction.



$$dM = \left(\frac{M}{\pi R}\right) Rd\theta = \left(\frac{M}{\pi}\right) d\theta \quad \text{and} \quad dF = \frac{GmdM}{R^2}$$

$$dF_y = \frac{GmdM\cos\theta}{R^2} = \frac{[Gm(\frac{M}{\pi})\ d\theta\cos\theta]}{R^2}$$

$$F_y = \int_{-\pi/2}^{\pi/2} \frac{GmM}{\pi R^2}\cos\theta\, d\theta = \frac{GMm}{\pi R^2}\sin\theta\ \Big|_{-\pi/2}^{\pi/2} = \frac{GMm}{\pi R^2}[1-(-1)] = \boxed{\frac{2GmM}{\pi R^2}}$$

Goal Solution

G: If the rod completely encircled the point mass, the net force on m would be zero (by symmetry), and if all the mass M would be concentrated at the middle of the rod, the force would be $F = \frac{GmM}{R^2}$ directed upwards. Since the given configuration is somewhere between these two extreme cases, we can expect the net force on m to be upwards and $F < \frac{GmM}{R^2}$

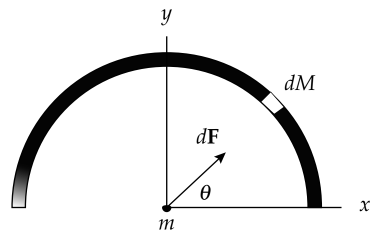
O: The net force on the point mass can be found by integrating the contributions from each small piece of the semicircular rod.

A: If we consider a segment of the curved rod, dM, to act like a point mass, we can apply Eq. 14.1 to find the force exerted on m due to dM as:

$$d\mathbf{F} = \frac{Gm(dM)}{R^2}\ \text{(directed toward } dM) .$$

If we could integrate this differential force, we could find the total force on the point mass, m, but the differential mass element must first be written in terms of a variable that we can integrate easily, like the angle, θ, which ranges from 0 to 180° for this semicircular rod. One segment of the arc, dM, subtends an angle $d\theta$ and has length $R d\theta$. Since the whole rod has length πR and mass M, this incremental element has mass

$$dM = \left(\frac{M}{\pi R}\right) R d\theta = \frac{M d\theta}{\pi}$$



This mass element exerts a force $d\mathbf{F}$ on the point mass at the center.

$$d\mathbf{F} = \frac{GmdM}{R^2}\hat{\mathbf{r}} = \frac{Gm}{R^2}\left(\frac{M}{\pi}d\theta\right)\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is directed at an angle θ above the x-axis

or $\quad d\mathbf{F} = \dfrac{GmM}{\pi R^2}\, d\theta\,(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$

To find the net force on the point mass, we integrate the contributions for all mass elements from $\theta = 0°$ to 180°:

$$\mathbf{F} = \int_{\text{all } m} d\mathbf{F} = \int_{\theta=0}^{180°}\left(\frac{GmMd\theta}{\pi R^2}\right)(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

$$\mathbf{F} = \left(\frac{GmM\,\mathbf{i}}{\pi R^2}\right)\int_0^{180°}\cos\theta d\theta + \left(\frac{GmM\mathbf{j}}{\pi R^2}\right)\int_0^{180°}\sin\theta d\theta$$

$$\mathbf{F} = \left(\frac{GmM\,\mathbf{i}}{\pi R^2}\right)[\sin\theta]_0^{180°} + \left(\frac{GmM\mathbf{j}}{\pi R^2}\right)[-\cos\theta]_0^{180°}$$

$$\mathbf{F} = \left(\frac{GmM\,\mathbf{i}}{\pi R^2}\right)(0-0) + \left(\frac{GmM\mathbf{j}}{\pi R^2}\right)[-(-1)+1]$$

$$\mathbf{F} = 0\mathbf{i} + \left(\frac{2GmM}{\pi R^2}\right)\mathbf{j}$$

L: As predicted, the direction of the force on the point mass at the center is vertically upward. Also, the net force has the same algebraic form as for two point masses, reduced by a factor of $2/\pi$, so this answer agrees with our prediction.

14.46 (a) From Example 14.10, $T = 2\pi\sqrt{\dfrac{R_E^3}{GM_E}}$

At the surface $g = GM_E/R_E^2$ so indeed $\boxed{T = 2\pi(\sqrt{R_E/g})}$

(b) $T = 2\pi\sqrt{\dfrac{R_M^3}{GM_M}}$

$$T = 2\pi\sqrt{\frac{(1.74 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(7.36 \times 10^{22} \text{ kg})}}$$

$T = 6.51 \times 10^3 \text{ s} = \boxed{1.81 \text{ h}}$

(c) The Moon may be hot down deep inside, but $\boxed{\text{it is not molten}}$.

14.47 (a) $F = \dfrac{GmM}{r^2} = \dfrac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(0.0500 \text{ kg})(500 \text{ kg})}{(1.500 \text{ m})^2} = \boxed{7.41 \times 10^{-10} \text{ N}}$

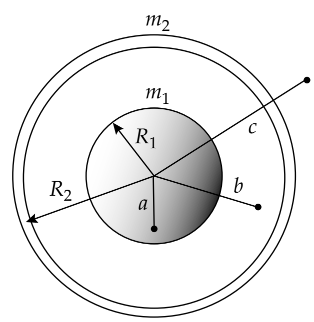
(b) $F = \dfrac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(0.0500 \text{ kg})(500 \text{ kg})}{(0.400 \text{ m})^2} = \boxed{1.04 \times 10^{-8} \text{ N}}$

(c) In this case the mass m is a distance r from a sphere of mass,

$M = (500 \text{ kg})\left(\dfrac{0.200 \text{ m}}{0.400 \text{ m}}\right)^3 = 62.5 \text{ kg}$ and

$$F = \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(0.0500 \text{ kg})(62.5 \text{ kg})}{(0.200 \text{ m})^2} = \boxed{5.21 \times 10^{-9} \text{ N}}$$

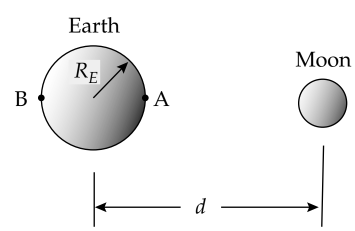
14.48 (a) $F = \boxed{\dfrac{Gmm_1a}{R_1^3}}$ toward the center of the sphere



(b) $F = \boxed{\dfrac{Gmm_1}{b^2}}$ toward the center of the sphere

(c) $F = \boxed{\dfrac{Gm(m_1 + m_2)}{c^2}}$ toward the center of the sphere

14.49 The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is given by $a = G\dfrac{M_{\text{Moon}}}{d^2}$



At the point nearest the Moon, $a_+ = G\dfrac{M_M}{(d-r)^2}$

At the point farthest from the Moon, $a_- = G\dfrac{M_M}{(d+r)^2}$

$$\Delta a = a_+ - a = GM_M\left[\frac{1}{(d-r)^2} - \frac{1}{d^2}\right]$$

For $d >> r$, $\Delta a = \dfrac{2GM_M r}{d^3} = 1.11 \times 10^{-6}\ \text{m/s}^2$

Across the planet, $\dfrac{\Delta g}{g} = \dfrac{2\Delta a}{g} = \dfrac{2.22 \times 10^{-6}\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \boxed{2.26 \times 10^{-7}}$

14.50 Momentum is conserved:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

$$0 = M\mathbf{v}_{1f} + 2M\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f} = -\frac{1}{2}\,\mathbf{v}_{1f}$$

Energy is conserved:

$$(K+U)_i + \Delta E = (K+U)_f$$

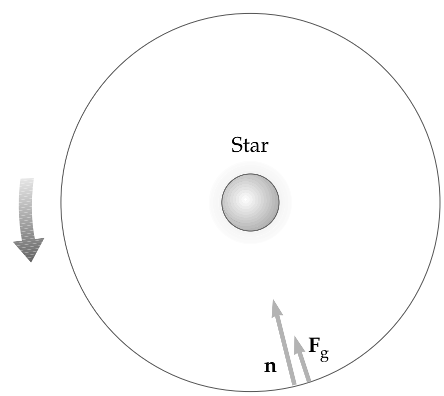
$$0 - \frac{Gm_1m_2}{r_i} + 0 = \frac{1}{2}\,m_1v_{2f}^2 - \frac{Gm_1m_2}{r_f}$$

$$-\frac{GM(2M)}{12R} = \frac{1}{2}\,Mv_{1f}^2 + \frac{1}{2}(2M)\left(\frac{1}{2}\,v_{1f}\right)^2 - \frac{GM(2M)}{4R}$$

$$v_{1f} = \boxed{\frac{2}{3}\sqrt{\frac{GM}{R}}} \qquad v_{2f} = \frac{1}{2}\,v_{1f} = \boxed{\frac{1}{3}\sqrt{\frac{GM}{R}}}$$

14.51 (a) $a_r = \dfrac{v^2}{r}$

$$a_r = \frac{(1.25 \times 10^6\ \text{m/s})^2}{1.53 \times 10^{11}\ \text{m}} = \boxed{10.2\ \text{m/s}^2}$$



(b) diff $= 10.2 - 9.90 = 0.312\ \text{m/s}^2 = \dfrac{GM}{r^2}$

$$M = \frac{(0.312\ \text{m/s}^2)(1.53 \times 10^{11}\ \text{m})^2}{6.67 \times 10^{-11}\ \text{N} \cdot \text{m/kg}^2} = \boxed{1.10 \times 10^{32}\ \text{kg}}$$

***14.52** (a) The free-fall acceleration produced by the Earth is

$$g = \frac{GM_E}{r^2} = GM_E r^{-2} \quad \text{(directed downward)}$$

Its rate of change is $\dfrac{dg}{dr} = GM_E(-2)r^{-3} = -2GM_E r^{-3}$. The minus sign indicates that g decreases with increasing height.

At the Earth's surface, $\boxed{\dfrac{dg}{dr} = -\dfrac{2GM_E}{R_E^3}}$

(b) For small differences,

$$\frac{|\Delta g|}{\Delta r} = \frac{|\Delta g|}{h} = \frac{2GM_E}{R_E^3} \qquad \text{Thus, } \boxed{|\Delta g| = \frac{2GM_E h}{R_E^3}}$$

(c) $$|\Delta g| = \frac{2(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24}\ \text{kg})(6.00\ \text{m})}{(6.37\times 10^{6}\ \text{m})^3} = \boxed{1.85\times 10^{-5}\ \text{m/s}^2}$$

14.53 (a) Initially take the particle from infinity and move it to the sphere's surface. Then,

$$U = \int_\infty^R \frac{GmM}{r^2}\,dr = -\frac{GmM}{R}$$

Now move it to a position r from the center of the sphere. The force in this case is a function of the mass enclosed by r at any point.

Since $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$ we have

$$U = \int_R^r \frac{Gm(4\pi r^3)\rho}{3r^2}\,dr = \frac{GmM}{R^3}\left(\frac{r^2 - R^2}{2}\right)$$

and the total gravitational potential energy is

$$U = \boxed{\left(\frac{GmM}{2R^3}\right)r^2 - \frac{3GmM}{2R}}$$

(b) $U(R) = -\dfrac{GMm}{R}$ and $U(0) = -\dfrac{3GmM}{2R}$

so $W_g = -[U(0) - U(R)] = \boxed{\dfrac{GMm}{2R}}$

14.54 To approximate the height of the sulfur, set

$$\frac{mv^2}{2} = mgh \qquad h = 70\,000 \text{ m} \qquad g_{\text{Io}} = \frac{GM}{r^2} = 1.79 \text{ m/s}^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2(1.79)(70\,000)} \approx 500 \text{ m/s (over 1000 mph)}$$

A more precise answer is given by

$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{r_2}$$

$$\frac{1}{2}v^2 = (6.67 \times 10^{-11})(8.90 \times 10^{22})\left(\frac{1}{1.82 \times 10^6} - \frac{1}{1.89 \times 10^6}\right)$$

$$v = \boxed{492 \text{ m/s}}$$

***14.55** From the walk, $2\pi r = 25\,000$ m. Thus, the radius of the planet is

$$r = \frac{25\,000 \text{ m}}{2\pi} = 3.98 \times 10^3 \text{ m}$$

From the drop: $\Delta y = \frac{1}{2}gt^2 = \frac{1}{2}g(29.2 \text{ s})^2 = 1.40 \text{ m}$

so, $g = \dfrac{2(1.40 \text{ m})}{(29.2 \text{ s})^2} = 3.28 \times 10^{-3} \text{ m/s}^2 = \dfrac{MG}{r^2}$

$$\therefore M = \boxed{7.79 \times 10^{14} \text{ kg}}$$

14.56 For a 6.00 km diameter cylinder, $r = 3000$ m and to simulate 1 $g = 9.80 \text{ m/s}^2$

$$g = \frac{v^2}{r} = \omega^2 r$$

$$\omega = \sqrt{\frac{g}{r}} = \boxed{0.0572 \text{ rad/s}}$$

The required rotation rate of the cylinder is $\boxed{1 \text{ rev/110 s}}$

(For a description of proposed cities in space, see Gerard K. O'Neill in *Physics Today*, Sept. 1974.)

14.57 $F = \dfrac{GMm}{r^2} = \left(6.67 \times 10^{-11} \dfrac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\dfrac{(1.50 \text{ kg})(15.0 \times 10^3 \text{ kg})}{(4.50 \times 10^{-2} \text{ m})^2} = \boxed{7.41 \times 10^{-10} \text{ N}}$

***14.58** (a) G has units $\dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2} = \dfrac{\text{kg}\cdot\text{m}\cdot\text{m}^2}{\text{s}^2\cdot\text{kg}^2} = \dfrac{\text{m}^3}{\text{s}^2\cdot\text{kg}}$

and dimensions $[G] = \dfrac{\text{L}^3}{\text{T}^2\cdot\text{M}}$

The speed of light has dimensions of $[c] = \dfrac{\text{L}}{\text{T}}$, and Planck's constant has the same dimensions as angular momentum or $[h] = \dfrac{\text{M}\cdot\text{L}^2}{\text{T}}$.

We require $[G^p c^q h^r] = \text{L}$, or $\text{L}^{3p}\text{T}^{-2p}\text{M}^{-p}\text{L}^q\text{T}^{-q}\text{M}^r\text{L}^{2r}\text{T}^{-r} = \text{L}^1\text{M}^0\text{T}^0$.

Thus, $3p + q + 2r = 1$

$-2p - q - r = 0$

$-p + r = 0$

which reduces (using $r = p$) to $3p + q + 2p = 1$

$-2p - q - p = 0$

These equations simplify to $5p + q = 1$ and $q = -3p$

Then, $5p - 3p = 1$, yielding $p = \dfrac{1}{2}$, $q = -\dfrac{3}{2}$, and $r = \dfrac{1}{2}$

Therefore, Planck length = $\boxed{G^{1/2}c^{-3/2}h^{1/2}}$

(b) $(6.67\times10^{-11})^{1/2}(3\times10^{8})^{-3/2}(6.63\times10^{-34})^{1/2} = (1.64\times10^{-69})^{1/2} = 4.05\times10^{-35}\text{ m}$ $\boxed{\sim10^{-34}\text{ m}}$

***14.59** $\dfrac{1}{2}m_0 v_{\text{esc}}^2 = \dfrac{Gm_p m_0}{R}$

$$v_{\text{esc}} = \sqrt{\frac{2Gm_p}{R}}$$

With $m_p = \rho\dfrac{4}{3}\pi R^3$, we have

$$v_{\text{esc}} = \sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}}$$

$$= \boxed{\sqrt{\frac{8\pi G\rho}{3}}\,R}$$

So, $v_{\text{esc}} \propto R$

***14.60** (a) To see this, let $M(r)$ be the mass up to radius r, $\rho(r)$ be the density at radius r, and $\rho_{av}(r)$ be the average density up to radius r.

Then, $\dfrac{dM(r)}{dr} = 4\pi r^2\rho(r)$ and $M(r) = \dfrac{4}{3}\pi r^3\rho_{av}(r)$

The gravitational acceleration at radius r is $g(r) = \dfrac{GM(r)}{r^2}$; its derivative with respect to r is

$$\frac{dg}{dr} = \frac{-2GM(r)}{r^3} + \frac{G}{r^2}\frac{dM(r)}{dr} = \frac{-2GM(r)}{r^3} + 4\pi G\rho(r)$$

or $$\boxed{\frac{dg}{dr} = 4\pi G\left[\rho(r) - \frac{2}{3}\rho_{av}(r)\right]}$$

This is positive only if $\rho(r) > \dfrac{2}{3}\rho_{av}(r)$; if $\rho(r) < \dfrac{2}{3}\rho_{av}(r)$, g will actually decrease with increasing r.

(b) From the numbers given, it is clear that at the surface, the average density is less than 2/3 of the average density of the whole Earth. Clearly, then, $\boxed{\text{the value of } g \text{ increases as one descends into the Earth}}$. Geophysical evidence shows that the maximum value of g inside the Earth is greater than 10.0 N/kg, and occurs about halfway between the center and the surface.

14.61 (a) At infinite separation $U = 0$ and at rest $K = 0$. Since energy is conserved we have,

$$0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} \qquad (1)$$

The initial momentum is zero and momentum is conserved.

Therefore, $0 = m_1v_1 - m_2v_2$ (2)

Combine Equations (1) and (2) to find

$$\boxed{v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}} \quad \text{and} \quad \boxed{v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}}$$

Relative velocity $v_r = v_1 - (-v_2) = \boxed{\sqrt{\dfrac{2G(m_1+m_2)}{d}}}$

(b) Substitute given numerical values into the equation found for v_1 and v_2 in part (a) to find

$$v_1 = 1.03 \times 10^4 \text{ m/s} \quad \text{and} \quad v_2 = 2.58 \times 10^3 \text{ m/s}$$

Therefore, $K_1 = \frac{1}{2} m_1 v_1^2 = \boxed{1.07 \times 10^{32}\text{ J}}$

and $K_2 = \frac{1}{2} m_2 v_2^2 = \boxed{2.67 \times 10^{31}\text{ J}}$

14.62 (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum is conserved;

$$m r_a v_a = m r_p v_p$$

and $v_a = v_p \left(\frac{r_p}{r_a}\right) = (3.027 \times 10^4 \text{ m/s})\left(\frac{1.471}{1.521}\right) = \boxed{2.93 \times 10^4 \text{ m/s}}$

(b) $K_p = \frac{1}{2} m v_p^2 = \frac{1}{2}(5.98 \times 10^{24})(3.027 \times 10^4)^2 = \boxed{2.74 \times 10^{33}\text{ J}}$

$$U_p = -\frac{GmM}{r_p} = -\frac{(6.673 \times 10^{-11})(5.98 \times 10^{24})(1.99 \times 10^{30})}{1.471 \times 10^{11}} = \boxed{-5.40 \times 10^{33}\text{ J}}$$

(c) Using the same form as in part (b), we find

$$K_a = \boxed{2.57 \times 10^{33}\text{ J}} \quad \text{and} \quad U_a = \boxed{-5.22 \times 10^{33}\text{ J}}$$

Compare $K_p + U_p = \boxed{-2.66 \times 10^{33}\text{ J}}$

and $K_a + U_a = \boxed{-2.65 \times 10^{33}\text{ J}}$ They agree.

14.63 (a) If we consider a hollow shell in the sphere with radius r and thickness dr, then $dM = \rho dV = \rho(4\pi r^2 dr)$. The total mass is then

$$M = \int_0^R \rho dV \int_0^R (Ar)(4\pi r^2 dr) = \pi A R^4$$

and $A = \boxed{\frac{M}{\pi R^4}}$

(b) The total mass of the sphere acts as if it were at the center of the sphere and

$F = \boxed{\frac{GmM}{r^2}}$ directed toward the center of the sphere.

(c) Inside the sphere at the distance r from the center, $dF = \left(\frac{Gm}{r^2}\right) dM$ where dM is just the mass of a shell enclosed within the radius r.

Therefore, $F = \frac{Gm}{r^2}\int_0^r dM = \frac{Gm}{r^2}\int_0^r Ar\,4\pi r^2 dr$

$$F = \frac{Gm}{r^2}\frac{M}{\pi R^4}\frac{4\pi r^4}{4} = \boxed{\frac{GmMr^2}{R^4}}$$

***14.64** (a) The work must provide the increase in gravitational energy

$$W = \Delta U_g = U_{gf} - U_{gi}$$

$$= -\frac{GM_EM_p}{r_f} + \frac{GM_EM_p}{r_i}$$

$$= -\frac{GM_EM_p}{R_E + y} + \frac{GM_EM_p}{R_E}$$

$$= GM_EM_p\left(\frac{1}{R_E} - \frac{1}{R_E + y}\right)$$

$$= \left(\frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98\times10^{24}\text{ kg})(100\text{ kg})\left(\frac{1}{6.37\times10^6\text{ m}} - \frac{1}{7.37\times10^6\text{ m}}\right)$$

$$W = \boxed{850\text{ MJ}}$$

(b) In a circular orbit, gravity supplies the centripetal force:

$$\frac{GM_EM_p}{(R_E + y)^2} = \frac{M_pv^2}{(R_E + y)}$$

Then, $\frac{1}{2}M_pv^2 = \frac{1}{2}\frac{GM_EM_p}{(R_E + y)}$

So, additional work = kinetic energy required

$$= \frac{1}{2}\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2)(5.98\times10^{24}\text{ kg})(100\text{ kg})}{(\text{kg}^2)(7.37\times10^6\text{ m})}$$

$$\Delta W = \boxed{2.71\times10^9\text{ J}}$$

14.65 Centripetal acceleration comes from gravitational acceleration.

$$\frac{v^2}{r} = \frac{M_c G}{r^2} = \frac{4\pi^2 r^2}{T^2 r}$$

$$GM_c T^2 = 4\pi^2 r^3$$

$$(6.67 \times 10^{-11})(20)(1.99 \times 10^{30})(5.00 \times 10^{-3})^2 = 4\pi^2 r^3$$

$$r_{\text{orbit}} = \boxed{119\text{ km}}$$

14.66 (a) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(30\,000 \times 9.46 \times 10^{15}\text{ m})}{2.50 \times 10^5\text{ m/s}} = 7.13 \times 10^{15}\text{ s} = \boxed{2.26 \times 10^8\text{ yr}}$

(b) $M = \dfrac{4\pi^2 a^3}{GT^2} = \dfrac{4\pi^2\,(30\,000 \times 9.46 \times 10^{15}\text{ m})^3}{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.13 \times 10^{15}\text{ s})^2} = 2.66 \times 10^{41}\text{ kg}$

$M = 1.34 \times 10^{11}$ solar masses $\boxed{\sim 10^{11}\text{ solar masses}}$

The number of stars is $\boxed{\text{on the order of }10^{11}}$

***14.67** (a) From the data about perigee, the energy is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_E m}{r_p} = \frac{1}{2}(1.60)(8.23 \times 10^3)^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.60)}{7.02 \times 10^6}$$

or $E = \boxed{-3.67 \times 10^7\text{ J}}$

(b) $L = mvr\sin\theta = mv_p r_p \sin 90.0°$

$= (1.60\text{ kg})(8.23 \times 10^3\text{ m/s})(7.02 \times 10^6\text{ m}) = \boxed{9.24 \times 10^{10}\text{ kg}\cdot\text{m}^2/\text{s}}$

(c) At apogee, we must have $\dfrac{1}{2}mv_a^2 - \dfrac{GMm}{r_a} = E$, and $mv_a r_a \sin 90.0° = L$ since both energy and angular momentum are conserved. Thus,

$$\frac{1}{2}(1.60)\,v_a^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.60)}{r_a} = -3.67 \times 10^7\text{ J}$$

and $(1.60\text{ kg})v_a r_a = 9.24 \times 10^{10}\text{ kg}\cdot\text{m}^2/\text{s}$

Solving simultaneously,

$$\frac{1}{2}(1.60)\,v_a^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.60)(1.60)v_a}{9.24 \times 10^{10}} = -3.67 \times 10^7$$

which reduces to $0.800v_a^2 - 11046v_a + 3.6723 \times 10^7 = 0$

so $$v_a = \frac{11046 \pm \sqrt{(11046)^2 - 4(0.800)(3.6723 \times 10^7)}}{2(0.800)}$$

This gives $v_a = 8230$ m/s or $\boxed{5580 \text{ m/s}}$. The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus, $r_a = \dfrac{L}{mv_a} = \dfrac{9.24 \times 10^{10} \text{ kg} \cdot \text{m}^2/\text{s}}{(1.60 \text{ kg})(5.58 \times 10^3 \text{ m/s})} = \boxed{1.04 \times 10^7 \text{ m}}$

(d) The major axis is $2a = r_p + r_a$, so the semi-major axis is

$$a = \frac{1}{2}(7.02 \times 10^6 \text{ m} + 1.04 \times 10^7 \text{ m}) = \boxed{8.69 \times 10^6 \text{ m}}$$

(e) $T = \sqrt{\dfrac{4\pi^2 a^3}{GM_E}} = \sqrt{\dfrac{4\pi^2 (8.69 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}}$

$T = 8060 \text{ s} = \boxed{134 \text{ min}}$

***14.68** $v_i = 2\sqrt{Rg}$ $\qquad g = \dfrac{MG}{R^2}$

Utilizing conservation of energy,

$$\frac{mv^2}{2} - \frac{mGM}{r} = \frac{mv_i^2}{2} - \frac{mGM}{R}$$

$$\frac{mv^2}{2} = \frac{mv_i^2}{2} - \frac{mGM}{R} + \frac{mGM}{r}$$

$$v^2 = v_i^2 - 2MG\left(\frac{1}{R} - \frac{1}{r}\right)$$

$$v = \sqrt{v_i^2 + 2MG\left(\frac{1}{r} - \frac{1}{R}\right)}$$

$$v = \sqrt{4Rg + 2MG\left(\frac{1}{r} - \frac{1}{R}\right)}$$

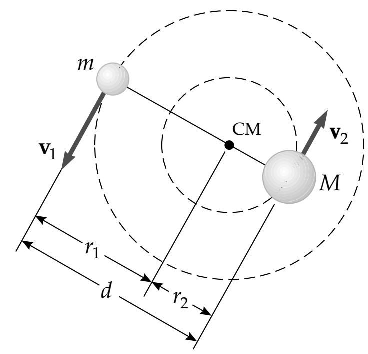
$$v = \sqrt{\frac{4MG}{R} - \frac{2MG}{R} + \frac{2MG}{r}}$$

$$v = \boxed{\sqrt{2MG\left(\frac{1}{R} + \frac{1}{r}\right)} = \sqrt{2R^2 g\left(\frac{1}{R} + \frac{1}{r}\right)}}$$

14.69 If we choose the coordinate of the center of mass at the origin, then

$$0 = \frac{(Mr_2 - mr_1)}{M + m} \qquad \text{and} \qquad Mr_2 = mr_1$$

(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass $F = ma$ and



$$mr_1\omega_1^2 = \frac{MGm}{d^2}$$

and $$Mr_2\omega_2^2 = \frac{MGm}{d^2}$$

Combining these two equations and using $d = r_1 + r_2$ gives

$$(r_1 + r_2)\omega^2 = \frac{(M + m)G}{d^2}$$

with $\omega_1 = \omega_2 = \omega$ and $T = \dfrac{2\pi}{\omega}$, we find $\boxed{T^2 = \dfrac{4\pi^2 d^3}{G(M + m)}}$

<u>Goal Solution</u>

For the star of mass M and orbital radius r_2,

$\Sigma F = ma$ gives

$$\frac{GMm}{d^2} = \frac{Mv_2^2}{r_2} = \frac{M}{r_2}\left(\frac{2\pi r_2}{T}\right)^2$$

For the star of mass m, $\Sigma F = ma$ gives

$$\frac{GMm}{d^2} = \frac{mv_1^2}{r_1} = \frac{m}{r_1}\left(\frac{2\pi r_1}{T}\right)^2$$

Cross-multiplying, we then obtain simultaneous equations:

$$GmT^2 = 4\pi^2 d^2 r_2$$

$$GMT^2 = 4\pi^2 d^2 r_1$$

Adding, we find

$$G(M + m)T^2 = 4\pi^2 d^2(r_1 + r_2) = 4\pi^2 d^3$$

$$T^2 = \frac{4\pi^2 d^3}{G(M + m)}$$

In a visual binary star system T, d, r_1, and r_2 can be measured, so the mass of each component can be computed.

***14.70** (a) The gravitational force exerted on m_2 by the Earth (mass m_1) accelerates m_2 according to: $m_2 g_2 = \frac{Gm_1 m_2}{r^2}$. The equal magnitude force exerted on the Earth by m_2 produces negligible acceleration of the Earth. The acceleration of relative approach is then

$$g_2 = \frac{Gm_1}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{(1.20\times10^{7}\ \text{m})^2} = \boxed{2.77\ \text{m/s}^2}$$

(b) Again, m_2 accelerates toward the center of mass with $g_2 = 2.77\ \text{m/s}^2$. Now the Earth accelerates toward m_2 with an acceleration given as

$$m_1 g_1 = \frac{Gm_1 m_2}{r^2}$$

$$g_1 = \frac{Gm_2}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.00\times10^{24}\ \text{kg})}{(1.20\times10^{7}\ \text{m})^2} = 0.926\ \text{m/s}^2$$

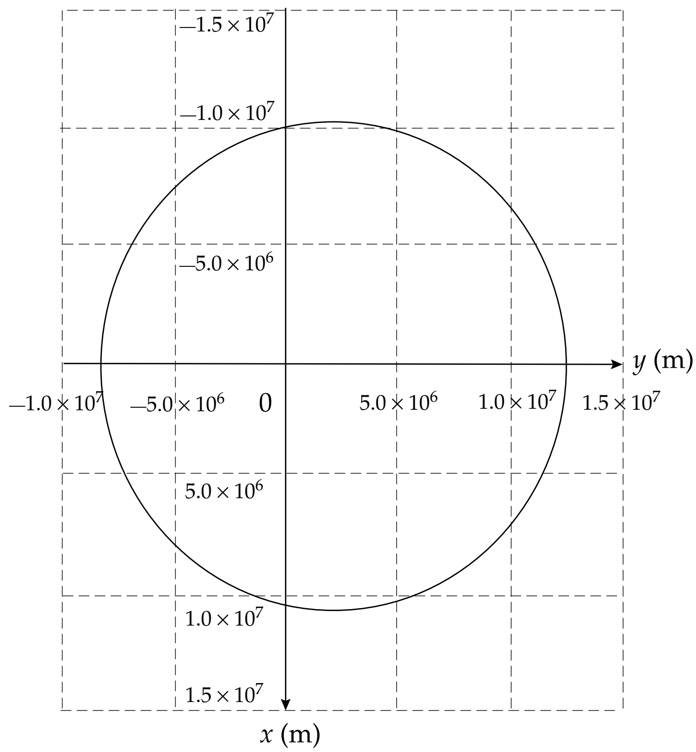
The distance between the masses closes with relative acceleration of

$$g_{\text{rel}} = g_1 + g_2 = 0.926\ \text{m/s}^2 + 2.77\ \text{m/s}^2 = \boxed{3.70\ \text{m/s}^2}$$

14.71 Initial Conditions and Constants:

Mass of planet: 5.98×10^{24} kg
Radius of planet: 6.37×10^{6} m
Initial x: 0.0 planet radii
Initial y: 2.0 planet radii
Initial v_x: +5000 m/s
Initial v_y: 0.0 m/s
Time interval: 10.9 s

t (s)	x (m)	y (m)	r (m)	v_x (m/s)	v_y (m/s)	a_x (m/s)	a_y (m/s^2)
0.0	0.0	12,740,000.0	12,740,000.0	5,000.0	0.0	0.0000	–2.4575
10.9	54,315.3	12,740,000.0	12,740,115.8	4,999.9	–26.7	–0.0100	–2.4574
21.7	108,629.4	12,739,710.0	12,740,173.1	4,999.7	–53.4	–0.0210	–2.4573
32.6	162,941.1	12,739,130.0	12,740,172.1	4,999.3	–80.1	–0.0310	–2.4572
…							
5,431.6	112,843.8	–8,466,816.0	8,467,567.9	–7,523.0	–39.9	–0.0740	5.5625
5,442.4	31,121.4	–8,467,249.7	8,467,306.9	–7,523.2	20.5	–0.0200	5.5633
5,453.3	–50,603.4	–8,467,026.9	8,467,178.2	–7,522.8	80.9	0.0330	5.5634
5,464.1	–132,324.3	–8,466,147.7	8,467,181.7	–7,521.9	141.4	0.0870	5.5628
…							
10,841.3	–108,629.0	12,739,134.4	12,739,597.5	4,999.9	53.3	0.0210	–2.4575
10,852.2	–54,314.9	12,739,713.4	12,739,829.2	5,000.0	26.6	0.0100	–2.4575
10,863.1	0.4	12,740,002.4	12,740,002.4	5,000.0	–0.1	0.0000	–2.4575



The $\boxed{\text{object does not hit the Earth}}$; its minimum radius is $\boxed{1.33R_E}$.

Its period is $\boxed{1.09 \times 10^4 \text{ s}}$. A circular orbit would require a speed of $\boxed{5.60 \text{ km/s}}$.

Chapter 15 Solutions

15.1 $M = \rho_{\text{iron}} V = (7860 \text{ kg/m}^3)\left[\frac{4}{3}\pi\,(0.0150 \text{ m})^3\right]$

$M = \boxed{0.111 \text{ kg}}$

15.2 The density of the nucleus is of the same order of magnitude as that of one proton, according to the assumption of close packing:

$$\rho = \frac{m}{V} \sim \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3}\pi(10^{-15} \text{ m})^3} \boxed{\sim 10^{18} \text{ kg/m}^3}$$

15.3 $P = \dfrac{F}{A} = \dfrac{50.0(9.80)}{\pi\,(0.500 \times 10^{-2})^2} = \boxed{6.24 \times 10^6 \text{ N/m}^2}$

***15.4** Let F_g be its weight. Then each tire supports $F_g/4$, so $P = \dfrac{F}{A} = \dfrac{F_g}{4A}$

yielding $F_g = 4AP = 4(0.0240 \text{ m}^2)(200 \times 10^3 \text{ N/m}^2) = \boxed{1.92 \times 10^4 \text{ N}}$

15.5 The Earth's surface area is $4\pi R^2$. The force pushing inward over this area amounts to

$$F = P_0 A = P_0 4\pi R^2$$

This force is the weight of the air:

$$F_g = mg = P_0 4\pi R^2$$

so the mass of the air is

$$m = \frac{P_0 4\pi R^2}{g} = \frac{(1.013 \times 10^5 \text{ N/m}^2)\, 4\pi (6.37 \times 10^6 \text{ m})^2}{9.80 \text{ m/s}^2} = \boxed{5.27 \times 10^{18} \text{ kg}}$$

15.6 (a) $P = P_0 + \rho g h = 1.013 \times 10^5 \text{ Pa} + (1024 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1000 \text{ m})$

$P = \boxed{1.01 \times 10^7 \text{ Pa}}$

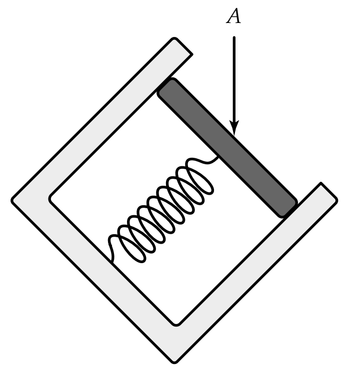
(b) The gauge pressure is the difference in pressure between the water outside and the air inside the submarine, which we suppose is at 1.00 atmosphere.

$$P_{\text{gauge}} = P - P_0 = \rho g h = 1.00 \times 10^7 \text{ Pa}$$

The resultant inward force on the porthole is then

$$F = P_{\text{gauge}} A = (1.00 \times 10^7 \text{ Pa})\,[\pi(0.150 \text{ m})^2] = \boxed{7.09 \times 10^5 \text{ N}}$$

15.7 $F_{el} = F_{fluid}$ or $kx = \rho g h A$



and $h = \dfrac{kx}{\rho g A}$

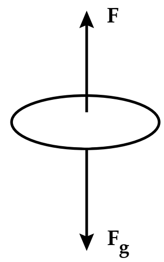
$$h = \frac{(1000\ \mathrm{N/m^2})(5.00 \times 10^{-3}\ \mathrm{m})}{(10^3\ \mathrm{kg/m^3})(9.80\ \mathrm{m/s^2})\pi(1.00 \times 10^{-2}\ \mathrm{m})^2} = \boxed{1.62\ \mathrm{m}}$$

15.8 Since the pressure is the same on both sides, $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

In this case, $\dfrac{15\,000}{200} = \dfrac{F_2}{3.00}$

or $F_2 = \boxed{225\ \mathrm{N}}$

15.9 $F_g = (80.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 784\ \mathrm{N}$



$F_g = F = PA = (1.013 \times 10^5\ \mathrm{Pa})(A)$

$A = \dfrac{F_g}{P} = \dfrac{784}{1.013 \times 10^5} = \boxed{7.74 \times 10^{-3}\ \mathrm{m^2}}$

Goal Solution

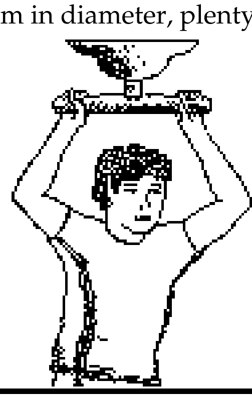
G: The suction cups used by burglars seen in movies are about 10 cm in diameter, and it seems reasonable that one of these might be able to support the weight of an 80-kg student. The area of a 10-cm cup is approximately: $\pi(0.05\ \mathrm{m})^2 \cong 8 \times 10^{-3}\ \mathrm{m}^2$

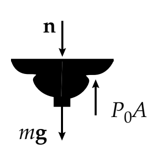
O: "Suction" is not a new kind of force. Familiar forces hold the cup in equilibrium, one of which is the atmospheric pressure acting over the area of the cup. This problem is simply one situation where Newton's 2nd law can be applied.

A: The vacuum between cup and ceiling exerts no force on either. The atmospheric pressure of the air below the cup pushes up on it with a force. If the cup barely supports the student's weight, then the normal force of the ceiling is approximately zero, and $+P_0A - mg = 0$

Therefore, $A = \dfrac{mg}{P_o} = \dfrac{80\ \mathrm{kg}\ (9.8\ \mathrm{m/s^2})}{1.013 \times 10^5\ \mathrm{Pa}} = 7.74 \times 10^{-3}\ \mathrm{m}^2$ ◊

L: This calculated area agrees with our prediction and corresponds to a suction cup that is 9.93 cm in diameter (Our 10 cm estimate was right on—a lucky guess considering that a burglar would probably use at least two suction cups, not one.) Also, the suction cup in the drawing appears to be about 30 cm in diameter, plenty big enough to support the weight of the student.





***15.10** (a) Suppose the "vacuum cleaner" functions as a high–vacuum pump. The air below the brick will exert on it a lifting force

$$F = PA = (1.013 \times 10^3 \text{ Pa})[\pi(1.43 \times 10^{-2} \text{ m})^2] = \boxed{65.1 \text{ N}}$$

(b) The octopus can pull the bottom away from the top shell with a force that could be no larger than

$$F = PA = (P_0 + \rho g h)A$$

$$= [1.013 \times 10^3 \text{ Pa} + (1030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(32.3 \text{ m})][\pi(1.43 \times 10^{-2} \text{ m})^2]$$

$$F = \boxed{275 \text{ N}}$$

***15.11** The excess water pressure (over air pressure) halfway down is

$$P_{\text{gauge}} = \rho g h = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.20 \text{ m}) = 1.18 \times 10^4 \text{ Pa}$$

The force on the wall due to the water is

$$F = P_{\text{gauge}}A = (1.18 \times 10^4 \text{ Pa})(2.40 \text{ m})(9.60 \text{ m}) = \boxed{2.71 \times 10^5 \text{ N}}$$

horizontally toward the back of the hole.

15.12 The pressure on the bottom due to the water is

$$P_b = \rho g z = 1.96 \times 10^4 \text{ Pa}$$

So, $F_b = P_b A = \boxed{5.88 \times 10^6 \text{ N}}$

On each end,

$$F = \bar{P} A = (9.80 \times 10^3 \text{ Pa})(20.0 \text{ m}^2) = \boxed{196 \text{ kN}}$$

On the side

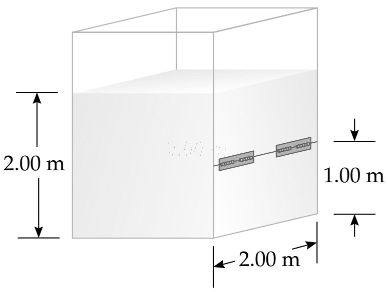
$$F = \bar{P} A = (9.80 \times 10^3 \text{ Pa})(60.0 \text{ m}^2) = \boxed{588 \text{ kN}}$$

***15.13** In the reference frame of the fluid, the cart's acceleration causes a fictitious force to act backward, as if the acceleration of gravity were $\sqrt{g^2 + a^2}$ directed downward and backward at $\theta = \tan^{-1}(a/g)$ from the vertical. The center of the spherical shell is at depth $d/2$ below the air bubble and the pressure there is

$$P = P_0 + \rho g_{\text{eff}} h = \boxed{P_0 + \frac{1}{2}\rho d \sqrt{g^2 + a^2}}$$

15.14 The air outside and water inside both exert atmospheric pressure, so only the excess water pressure $\rho g h$ counts for the net force. Take a strip of hatch between depth h and $h + dh$. It feels force

$$dF = PdA = \rho g h(2.00\text{ m})dh$$



(a) The total force is

$$F = \int dF = \int_{h=1.00\text{ m}}^{2.00\text{ m}} \rho g h\,(2.00\text{ m})dh$$

$$F = \rho g(2.00\text{ m})\left.\frac{h^2}{2}\right|_{1.00\text{ m}}^{2.00\text{ m}}$$

$$= (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)\frac{(2.00\text{ m})}{2}\left[(2.00\text{ m})^2 - (1.00\text{ m})^2\right]$$

$$F = \boxed{29.4\text{ kN (to the right)}}$$

(b) The lever arm of dF is the distance $(h - 1.00\text{ m})$ from hinge to strip:

$$\tau = \int d\tau = \int_{h=1.00\text{ m}}^{2.00\text{ m}} \rho g h\,(2.00\text{ m})(h - 1.00\text{ m})dh$$

$$\tau = \rho g(2.00\text{ m})\left[\frac{h^3}{3} - (1.00\text{ m})\frac{h^2}{2}\right]_{1.00\text{ m}}^{2.00\text{ m}}$$

$$\tau = 1000\frac{\text{kg}}{\text{m}^3}(9.80\text{ m/s}^2)(2.00\text{ m})\left(\frac{7.00\text{ m}^3}{3} - \frac{3.00\text{ m}^3}{2}\right)$$

$$\tau = \boxed{16.3\text{ kN}\cdot\text{m counterclockwise}}$$

15.15 The pressure on the ball is given by: $P = P_{atm} + \rho_w g h$ so the change in pressure on the ball from when it is on the surface of the ocean to when it is at the bottom of the ocean is $\Delta P = \rho_w g h$.

In addition:

$$\Delta V = \frac{-V\,\Delta P}{B} = -\frac{\rho_w g h V}{B} = -\frac{4\pi\rho_w g h r^3}{3B}\text{, where } B \text{ is the Bulk Modulus.}$$

$$\Delta V = -\frac{4\pi(1030\text{ kg/m}^3)(9.80\text{ m/s}^2)(10\,000\text{ m})(1.50\text{ m})^3}{(3)(14.0\times 10^{10}\text{ Pa})} = -0.0102\text{ m}^3$$

Therefore, the volume of the ball at the bottom of the ocean is

$$V - \Delta V = \frac{4}{3}\pi(1.50\text{ m})^3 - 0.0102\text{ m}^3 = 14.137\text{ m}^3 - 0.0102\text{ m}^3 = 14.127\text{ m}^3$$

This gives a radius of 1.49964 m and a new diameter of 2.9993 m. Therefore the diameter decreases by $\boxed{0.722\text{ mm}}$.

15.16 $\Delta P_0 = \rho g \Delta h = -2.66 \times 10^3$ Pa

$$P = P_0 + \Delta P_0 = (1.013 - 0.0266) \times 10^5 \text{ Pa} = \boxed{0.986 \times 10^5 \text{ Pa}}$$

15.17 $P_0 = \rho g h$

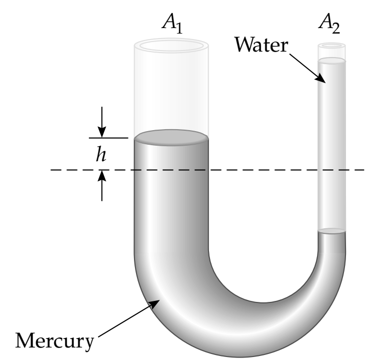
$$h = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5 \text{ Pa}}{(0.984 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{10.5 \text{ m}}$$

Some alcohol and water will evaporate.

15.18 (a) Using the definition of density, we have

$$h_w = \frac{m_{\text{water}}}{A_2 \rho_{\text{water}}} = \frac{100 \text{ g}}{(5.00 \text{ cm}^2)(1.00 \text{ g/cm}^3)} = \boxed{20.0 \text{ cm}}$$

(b) The sketch at the right represents the situation after the water is added. A volume ($A_2 h_2$) of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is $A_1 h$. Since the total volume of mercury has not changed,



(b)

$$A_2 h_2 = A_1 h$$

or $$h_2 = \frac{A_1}{A_2} h \quad (1)$$

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as:

$$P = P_0 + \rho_{\text{water}} g h_w$$

The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{\text{Hg}} g (h + h_2) = P_0 + \rho_{\text{water}} g h_w$$

which, using equation (1) above, reduces to

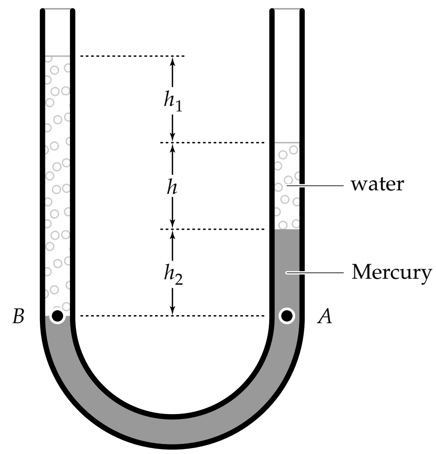
$$\rho_{\text{Hg}} h \left[1 + \frac{A_1}{A_2}\right] = \rho_{\text{water}} h_w$$

or $$h = \frac{\rho_{\text{water}} h_w}{\rho_{\text{Hg}} \left(1 + \frac{A_1}{A_2}\right)}$$

Thus, the level of mercury has risen a distance of

$$h = \frac{(1.00\ \mathrm{g/cm^3})(20.0\ \mathrm{cm})}{(13.6\ \mathrm{g/cm^3})\left(1 + \dfrac{10.0}{5.00}\right)} = \boxed{0.490\ \mathrm{cm}}\quad \text{above the original level.}$$

15.19 Let h be the height of the water column added to the right side of the U–tube. Then when equilibrium is reached, the situation is as shown in the sketch at right. Now consider two points, A and B shown in the sketch, at the level of the water–mercury interface. By Pascal's Principle, the absolute pressure at B is the same as that at A. But,



$$P_A = P_0 + \rho_w g h + \rho_{\mathrm{Hg}} g h_2 \quad \text{and}$$

$$P_B = P_0 + \rho_w g(h_1 + h + h_2)$$

Thus, from $P_A = P_B$, $\rho_w h_1 + \rho_w h + \rho_w h_2 = \rho_w h + \rho_{\mathrm{Hg}} h_2$, or

$$h_1 = \left[\frac{\rho_{\mathrm{Hg}}}{\rho_w} - 1\right] h_2 = (13.6 - 1)(1.00\ \mathrm{cm}) = \boxed{12.6\ \mathrm{cm}}$$

***15.20** (a) The balloon is nearly in equilibrium:

$$\Sigma F_y = ma_y \Rightarrow B - (F_g)_{\mathrm{helium}} - (F_g)_{\mathrm{payload}} = 0$$

or $\quad \rho_{\mathrm{air}} gV - \rho_{\mathrm{helium}} gV - m_{\mathrm{payload}} g = 0$

This reduces to

$$m_{\mathrm{payload}} = (\rho_{\mathrm{air}} - \rho_{\mathrm{helium}})V$$

$$= (1.29\ \mathrm{kg/m^3} - 0.179\ \mathrm{kg/m^3})(400\ \mathrm{m^3})$$

$$m_{\mathrm{payload}} = \boxed{444\ \mathrm{kg}}$$

(b) Similarly,

$$m_{\mathrm{payload}} = (\rho_{\mathrm{air}} - \rho_{\mathrm{hydrogen}})V$$

$$= (1.29\ \mathrm{kg/m^3} - 0.0899\ \mathrm{kg/m^3})(400\ \mathrm{m^3})$$

$$m_{\mathrm{payload}} = \boxed{480\ \mathrm{kg}}$$

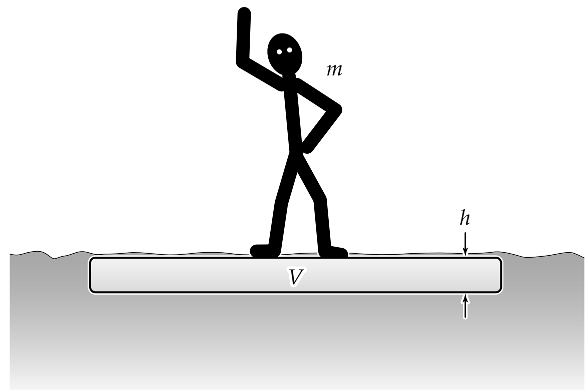
The air does the lifting, nearly the same for the two balloons.

15. 21 The total weight supported is:

$$F_g = (m + \rho_s V)g$$

$$F_g = \left[75.0 \text{ kg} + \left(300 \frac{\text{kg}}{\text{m}^3}\right)(0.100 \text{ m})A\right]\left(9.80 \frac{\text{m}}{\text{s}^2}\right)$$

$$F_g = 735 \text{ N} + (294 \text{ N/m}^2)A$$



The buoyancy force is:

$$F_b = \rho_w g V = \left(10^3 \frac{\text{kg}}{\text{m}^3}\right)\left(9.80 \frac{\text{m}}{\text{s}^2}\right)[(0.100 \text{ m})A]$$

$$F_b = (980 \text{ N/m}^2)A$$

Since $F_b = F_g$,

$$(980 \text{ N/m}^2)A = 735 \text{ N} + (294 \text{ N/m}^2)A$$

or $A = \dfrac{735 \text{ N}}{(980 - 294) \text{ N/m}^2} = \boxed{1.07 \text{ m}^2}$

15.22 $F_g = (m + \rho_s V)g = F_b = \rho_w V g$ (see the figure with 15.21)

Since $V = Ah$, $m + \rho_s Ah = \rho_w Ah$,

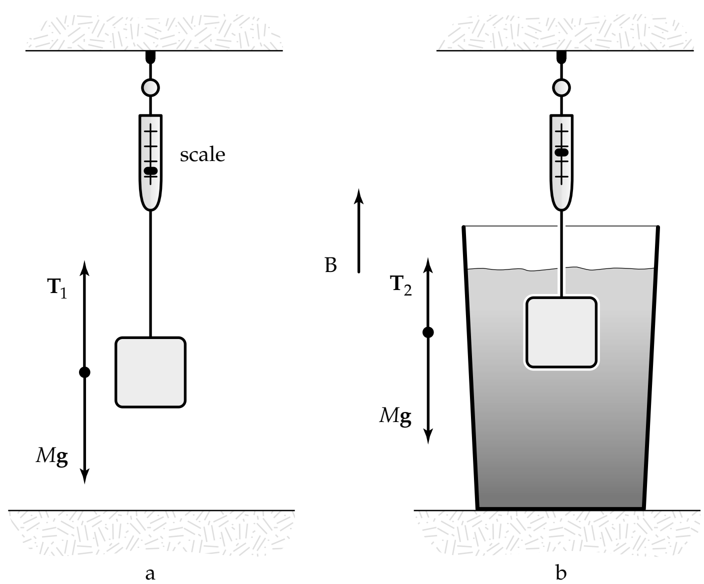
and $A = \boxed{\dfrac{m}{(\rho_w - \rho_s)h}}$

15.23 (a) Before the metal is immersed:

$\sum F_y = T_1 - Mg = 0$ or

$T_1 = Mg = (1.00 \text{ kg})\left(9.80 \dfrac{\text{m}}{\text{s}^2}\right)$

$= \boxed{9.80 \text{ N}}$



(b) After the metal is immersed:

$\Sigma F_y = T_2 + B - Mg = 0$ or

$T_2 = Mg - B = Mg - (\rho_w V)g$

$V = \dfrac{M}{\rho} = \dfrac{1.00\text{ kg}}{2700\text{ kg/m}^3}$

Thus,

$$T_2 = Mg - B = 9.80\text{ N} - (1000\text{ kg/m}^3)\left(\frac{1.00\text{ kg}}{2700\text{ kg/m}^3}\right)\left(9.80\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{6.17\text{ N}}$$

15.24 (a) $P = P_0 + \rho g h$

Taking $P_0 = 1.0130 \times 10^3\text{ N/m}^2$ and $h = 5.00\text{ cm}$

we find $P_{\text{top}} = 1.0179 \times 10^5\text{ N/m}^2$

For $h = 17.0$ cm, we get $P_{\text{bot}} = 1.0297 \times 10^5\text{ N/m}^2$

Since the areas of the top and bottom are

$A = (0.100\text{ m} \times 0.100\text{ m}) = 10^{-2}\text{ m}^2$

we find $F_{\text{top}} = P_{\text{top}}A = \boxed{1.0179 \times 10^3\text{ N}}$

and $F_{\text{bot}} = \boxed{1.0297 \times 10^3\text{ N}}$

(b) $T + B - Mg = 0$ where

$B = \rho_w V g = (10^3\text{ kg/m}^3)(1.20 \times 10^{-3}\text{ m}^3)(9.80\text{ m/s}^2) = 11.8\text{ N}$

and $Mg = 10.0(9.80) = 98.0$ N

Therefore, $T = Mg - B = 98.0 - 11.8 = \boxed{86.2\text{ N}}$

(c) $F_{\text{bot}} - F_{\text{top}} = (1.0297 - 1.0179) \times 10^3\text{ N} = \boxed{11.8\text{ N}}$

which is equal to B found in part (b).

15.25 (a) According to Archimedes,

$$B = \rho_{\text{water}} Vg = (1.00 \text{ g/cm}^3)[20.0 \times 20.0 \times (20.0 - h)]g$$

But

$$B = \text{Weight of Block} = Mg = \rho_{\text{wood}} V_{\text{wood}}\, g = (0.650 \text{ g/cm}^3)(20.0 \text{ cm})^3 g$$

$$(0.650)(20.0)^3 g = (1.00)(20.0)(20.0)(20.0 - h)g$$

$$20.0 - h = 20.0(0.650)$$

$$h = 20.0(1 - 0.650) = \boxed{7.00 \text{ cm}}$$

(b) $B = F_g + Mg$ where M = mass of lead

$$(1.00)(20.0)^3 g = (0.650)(20.0)^3 g + Mg$$

$$M = (20.0)^3(1.00 - 0.650) = (20.0)^3(0.350) = 2800 \text{ g} = \boxed{2.80 \text{ kg}}$$

***15.26** Consider spherical balloons of radius 12.5 cm containing helium at STP and immersed in air at 0°C and 1 atm. If the rubber envelope has mass 5.00 g, the upward force on each is

$$B - F_{g,\text{He}} - F_{g,\text{env}} = \rho_{\text{air}} Vg - \rho_{\text{He}} Vg - m_{\text{env}}\, g$$

$$F_{\text{up}} = (\rho_{\text{air}} - \rho_{\text{He}}) \frac{4}{3} \pi r^3 g - m_{\text{env}}\, g$$

$$F_{\text{up}} = (1.29 - 0.179) \frac{\text{kg}}{\text{m}^3} \frac{4}{3} \pi (0.125 \text{ m})^3 (9.80 \text{ m/s}^2) - 5.00 \times 10^{-3} \text{ kg}(9.80 \text{ m/s}^2)$$

$$F_{\text{up}} = 0.0401 \text{ N}$$

If your weight (including harness, strings, and submarine sandwich) is

$$(70.0 \text{ kg})(9.80 \text{ m/s}^2) = 686 \text{ N}$$

you need this many balloons:

$$\frac{686 \text{ N}}{0.0401 \text{ N}} = 17\,000 \sim \boxed{10^4}$$

15.27 $\rho_{H_2O}\, g \dfrac{V}{2} = \rho_{\text{sphere}}\, g V$

$\rho_{\text{sphere}} = \dfrac{1}{2}\rho_{H_2O} = \boxed{500\ \text{kg/m}^3}$

$\rho_{\text{oil}}\, g \dfrac{4}{10} V - \rho_{\text{sphere}}\, g V = 0$

$\rho_{\text{oil}} = \dfrac{10}{4}(500\ \text{kg/m}^3) = \boxed{1250\ \text{kg/m}^3}$

15.28 Since the frog floats, the buoyant force = the weight of the frog. Also, the weight of the displaced water = weight of the frog, so

$$\rho_{\text{ooze}} V g = m_{\text{frog}}\, g$$

or $\quad m_{\text{frog}} = \rho_{\text{ooze}} V = \rho_{\text{ooze}} \dfrac{1}{2}\left(\dfrac{4}{3}\pi r^3\right) = (1.35\times 10^3\ \text{kg/m}^3)\dfrac{2\pi}{3}(6.00\times 10^{-2}\ \text{m})^3$

Hence, $m_{\text{frog}} = \boxed{0.611\ \text{kg}}$

15.29 The balloon stops rising when $(\rho_{\text{air}} - \rho_{\text{He}}) g V = Mg$

or, when $\quad (\rho_{\text{air}} - \rho_{\text{He}}) V = M$

Therefore, $V = \dfrac{M}{(\rho_{\text{air}} - \rho_{\text{He}})} = \dfrac{400}{(1.25e^{-1} - 0.180)} = \boxed{1430\ \text{m}^3}$

15.30 Let ℓ represent the length below water at equilibrium and M the tube's mass:

$$\Sigma F_y = 0 \Rightarrow -Mg + \rho\pi r^2 \ell g = 0$$

Now with any excursion x from equilibrium

$$-Mg + \rho\pi r^2(\ell - x)g = Ma$$

Subtracting the equilibrium equation gives

$$-\rho\pi r^2 g x = Ma$$

$$a = -(\rho\pi r^2 g/M)x = -\omega^2 x$$

The opposite direction and direct proportionality of a to x imply SHM with angular frequency

$$\omega = \sqrt{\rho\pi r^2 g/M}$$

$$T = \frac{2\pi}{\omega} = \boxed{\left(\frac{2}{r}\right)\sqrt{\frac{\pi M}{\rho g}}}$$

***15.31** Constant velocity implies zero acceleration, which means that the submersible is in equilibrium under the gravitational force, the upward buoyant force, and the upward resistance force:

$$\sum F_y = ma_y = 0$$

$$-(1.20\times 10^4 \text{ kg} + m)g + \rho_w gV + 1100 \text{ N} = 0$$

where m is the mass of the added water and V is the sphere's volume.

$$(1.20\times 10^4 \text{ kg} + m) = (1.03\times 10^3)\left(\frac{4\pi}{3}\right)(1.50)^3 + \left(\frac{1100 \text{ N}}{9.80 \text{ m/s}^2}\right)$$

so $\boxed{m = 2.67\times 10^3 \text{ kg}}$

***15.32** By Archimedes's principle, the weight of the fifty planes is equal to the weight of a horizontal slice of water 11.0 cm thick and circumscribed by the water line:

$$\Delta B = \rho_{\text{water}} g(\Delta V)$$

$$50(2.90\times 10^4 \text{ kg})g = (1030 \text{ kg/m}^3)g(0.110 \text{ m})A$$

giving $A = \boxed{1.28\times 10^4 \text{ m}^2}$ The acceleration of gravity does not affect the answer.

***15.33** Volume flow rate $= A_1v_1 = A_2v_2$

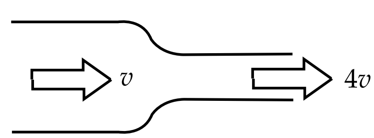
$$\frac{20.0 \text{ L}}{60.0 \text{ s}}\left(\frac{1000 \text{ cm}^3}{1.00 \text{ L}}\right) = \pi(1.00 \text{ cm})^2 v_{\text{hose}} = \pi(0.500 \text{ cm})^2 v_{\text{nozzle}}$$

(a) $v_{\text{hose}} = \dfrac{333 \text{ cm}^3/\text{s}}{3.14 \text{ cm}^2} = \boxed{106 \text{ cm/s}}$

(b) $v_{\text{nozzle}} = \dfrac{333 \text{ cm}^3/\text{s}}{0.785 \text{ cm}^2} = \boxed{424 \text{ cm/s}}$

15.34 By Bernoulli's equation,

$$8.00\times 10^4 \frac{\text{N}}{\text{m}^2} + \frac{1}{2}\,1000v^2 = 6.00\times 10^4 \frac{\text{N}}{\text{m}^2} + \frac{1}{2}\,1000(16v^2)$$



$$2.00\times 10^4 \frac{\text{N}}{\text{m}^2} = \frac{1}{2}\,1000(15v^2)$$

$$v = 1.63 \text{ m/s}$$

$$\frac{dm}{dt} = \rho Av = 1000\pi(5.00\times 10^{-2})^2(1.63 \text{ m/s}) = \boxed{12.8 \text{ kg/s}}$$

15.35 Assuming the top is open to the atmosphere, then

$$P_1 = P_0$$

Note $P_2 = P_0$

Flow rate $= 2.50 \times 10^{-3}\ \text{m}^3/\text{min} = 4.17 \times 10^{-5}\ \text{m}^3/\text{s}$

(a) $A_1 >> A_2$ so $v_1 << v_2$

Assuming $v_1 \approx 0$,

$$P_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = P_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$

$$v_2 = (2gy_1)^{1/2} = [2(9.80)(16.0)]^{1/2} = \boxed{17.7\ \text{m/s}}$$

(b) Flow rate $= A_2 v_2 = \left(\frac{\pi d^2}{4}\right)(17.7) = 4.17 \times 10^{-5}\ \text{m}^3/\text{s}$

$$d = \boxed{1.73 \times 10^{-3}\ \text{m}} = 1.73\ \text{mm}$$

15.36 (a) Suppose the flow is very slow

$$\left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{river}} = \left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{rim}}$$

$$P + 0 + \rho g(564\ \text{m}) = 1\ \text{atm} + 0 + \rho g(2096\ \text{m})$$

$$P = 1\ \text{atm} + 1000\ \text{kg/m}^3(9.80\ \text{m/s}^2)1532\ \text{m}$$

$$P = \boxed{1\ \text{atm} + 15.0\ \text{MPa}}$$

(b) The volume flow rate is

$$\frac{4500\ \text{m}^3}{\text{d}} = Av = \frac{\pi d^2 v}{4}$$

$$v = \frac{4500\ \text{m}^3}{\text{d}}\left(\frac{1\ \text{d}}{86400\ \text{s}}\right)\frac{4}{\pi(0.150\ \text{m})^2}$$

$$v = \boxed{2.98\ \text{m/s}}$$

(c) Imagine the pressure as applied to stationary water at the bottom of the pipe:

$$\left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{bottom}} = \left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{top}}$$

$$P + 0 = 1 \text{ atm} + \frac{1}{2}(1000 \text{ kg/m}^3)(2.98 \text{ m/s})^2 + (1000 \text{ kg})9.80 \text{ m/s}^2(1532 \text{ m})$$

$$P = 1 \text{ atm} + 15.0 \text{ MPa} + 4.45 \text{ kPa}$$

The additional pressure is $\boxed{4.45 \text{ kPa}}$

15.37 Flow rate $Q = 0.0120 \text{ m}^3/\text{s} = v_1 A_1 = v_2 A_2$

$$v_2 = \frac{Q}{A_2} = \frac{0.0120}{A_2} = \boxed{31.6 \text{ m/s}}$$

***15.38** (a) For upward flight of a water–drop projectile from geyser vent to fountain–top:

$$v_{yf}^2 = v_{yi}^2 + 2a_y(\Delta y)$$

$$0 = v_i^2 + 2(-9.80 \text{ m/s}^2)(+40.0 \text{ m})$$

$$v_i = \boxed{28.0 \text{ m/s}}$$

(b) Between geyser vent and fountain–top:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Air is so low in density that very nearly $P_1 = P_2 = 1$ atm. Then,

$$\frac{1}{2} v_1^2 + 0 = 0 + (9.80 \text{ m/s}^2)(40.0 \text{ m}) \qquad v_1 = \boxed{28.0 \text{ m/s}}$$

(c) Between the chamber and the fountain–top:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_1 + 0 + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(-175 \text{ m})$$

$$= P_0 + 0 + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(+40.0 \text{ m})$$

$$P_1 - P_0 = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(215 \text{ m}) = \boxed{2.11 \text{ MPa}}$$

15.39 $Mg = (P_1 - P_2)A$ for a balanced condition

$$\frac{(16000)(9.80)}{A} = 7.00 \times 10^4 - P_2$$

where $A = 80.0\text{ m}^2$,

$$\therefore\ P_2 = 7.00 \times 10^4 - 0.196 \times 10^4 = \boxed{6.80 \times 10^4\text{ Pa}}$$

15.40 $P_1 + \dfrac{\rho v_1^2}{2} = P_2 + \dfrac{\rho v_2^2}{2}$ (Bernoulli equation)

$v_1 A_1 = v_2 A_2$ where $\dfrac{A_1}{A_2} = 4$

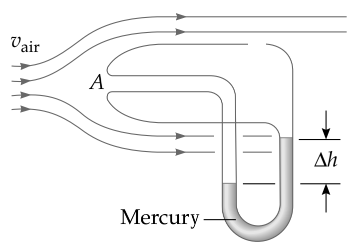
$$\Delta P = P_1 - P_2 = \frac{\rho}{2}(v_2^2 - v_1^2) = \frac{\rho}{2}\, v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$\Delta P = \frac{\rho v_1^2}{2}(15) = 21\,000\text{ Pa}$$

$v_1 = 2.00\text{ m/s}$

$v_2 = 4v_1 = 8.00\text{ m/s}$

and $Q = v_1 A_1 = \boxed{2.51 \times 10^{-3}\text{ m}^3\text{/s}}$



15.41 $\rho_{\text{Air}}\dfrac{v^2}{2} = \Delta P = \rho_{\text{Hg}}\, g\, \Delta h$

$$v = \sqrt{\frac{2\rho_{\text{Hg}}\, g \Delta h}{\rho_{\text{Air}}}} = \boxed{103\text{ m/s}}$$

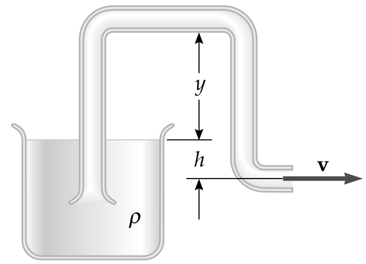
***15.42** The assumption of incompressibility is surely unrealistic, but allows an estimate of the speed:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$1.00\text{ atm} + 0 + 0 = 0.287\text{ atm} + 0 + \frac{1}{2}(1.20\text{ kg/m}^3)\, v_2^2$$

$$v_2 = \sqrt{\frac{2(1.00 - 0.287)(1.03 \times 10^5\text{ N/m}^2)}{1.20\text{ kg/m}^3}} = \boxed{347\text{ m/s}}$$

15.43 (a) $P_0 + \rho g h + 0 = P_0 + 0 + \frac{1}{2}\rho v_3^2$



$v_3 = \sqrt{2gh}$

If $h = 1.00$ m,

$$v_3 = \boxed{4.43\ \text{m/s}}$$

(b) $P + \rho g y + \frac{1}{2}\rho v_2^2 = P_0 + 0 + \frac{1}{2}\rho v_3^2$

Since $v_2 = v_3$,

$$P = P_0 - \rho g y$$

Since $P \ge 0$,

$$y \le \frac{P_0}{\rho g} = \frac{(1.013 \times 10^5\ \text{Pa})}{(10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = \boxed{10.3\ \text{m}}$$

***15.44** In the reservoir, the gauge pressure is

$$\Delta P = \frac{2.00\ \text{N}}{2.50 \times 10^{-5}\ \text{m}^2} = 8.00 \times 10^4\ \text{Pa}$$

From the equation of continuity:

$$A_1 v_1 = A_2 v_2$$

$$(2.50 \times 10^{-5}\ \text{m}^2)v_1 = (1.00 \times 10^{-8}\ \text{m}^2)v_2$$

$$v_1 = (4.00 \times 10^{-4})v_2$$

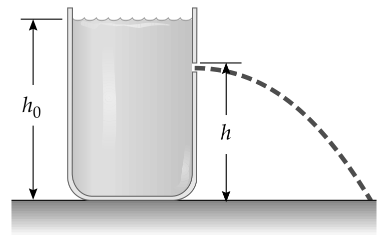
Thus, v_1^2 is negligible in comparison to v_2^2. Then, from Bernoulli's equation:

$$(P_1 - P_2) + \rho g y_1 + \frac{1}{2}\rho v_1^2 = \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$8.00 \times 10^4\ \text{Pa} + 0 + 0 = 0 + \frac{1}{2}(1000\ \text{kg/m}^3)\, v_2^2$$

$$v_2 = \sqrt{\frac{2(8.00 \times 10^4\ \text{Pa})}{1000\ \text{kg/m}^3}} = \boxed{12.6\ \text{m/s}}$$

15.45 Apply Bernoulli's equation between the top surface and the exiting stream.



$$P_0 + 0 + \rho g h_0 = P_0 + \frac{1}{2}\rho v_x^2 + \rho g h$$

$$v_x^2 = 2g(h_0 - h) \qquad \therefore v_x = \sqrt{2g(h_0 - h)}$$

$$x = v_x t$$

$$y = h = \frac{1}{2} g t^2$$

$$\therefore t = \sqrt{\frac{2h}{g}}$$

and $x = v_x\sqrt{\frac{2h}{g}} = \sqrt{2g(h_0 - h)}\sqrt{\frac{2h}{g}}$

$$x = \boxed{2\sqrt{h(h_0 - h)}}$$

15.46 $x = v_x t \qquad h = \frac{1}{2} g t^2$

$v_x = \sqrt{2g(h_0 - h)}$ from Problem 45

and $t = \sqrt{\frac{2h}{g}}$

$$x = \sqrt{2g(h_0 - h)}\sqrt{\frac{2h}{g}} = \sqrt{4h(h_0 - h)}$$

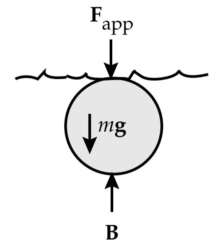
(a) Maximize x with respect to h:

$$\frac{dx}{dh} = 0$$

$$\frac{dx}{dh} = \frac{\frac{1}{2}(4h_0 - 8h)}{\sqrt{4h(h_0 - h)}} = 0 \quad \text{when } \boxed{h = \frac{h_0}{2}}$$

(b) For $h = \frac{h_0}{2}$, $\quad v_x = \sqrt{g h_0}$, $\quad t = \sqrt{\frac{h_0}{g}}$, $\quad$ then $\quad x = v_x t = \boxed{h_0}$

15.47 At equilibrium $\sum F = 0$



or $\mathbf{F}_{app} + m\mathbf{g} = \mathbf{B}$ where $\mathbf{B}$ is the buoyant force

The applied force,

$$\mathbf{F}_{app} = \mathbf{B} - m\mathbf{g} \quad \text{where } \mathbf{B} \equiv (\text{Vol})(\rho_{water})g$$

and $m = (\text{Vol})\rho_{ball}$

So, $F_{app} = (\text{Vol})g(\rho_{water} - \rho_{ball}) = \frac{4}{3}\pi r^3 g(\rho_{water} - \rho_{ball})$

$$F_{app} = \frac{4}{3}\pi(1.90 \times 10^{-2}\ \text{m})^3(9.80\ \text{m/s}^2)(10^3\ \text{kg/m}^3 - 84.0\ \text{kg/m}^3)$$

$$\boxed{F_{app} = 0.258\ \text{N}}$$

Goal Solution

G: According to Archimedes's Principle, the buoyant force acting on the submerged the ball will be equal to the weight of the water the ball will displace. The ball has a volume of about 30 cm^3, so the weight of this water is approximately:

$$B = F_g = \rho V g \approx (1\ \text{g/cm}^3)(30\ \text{cm}^3)(10\ \text{m/s}^2) = 0.3\ \text{N}$$

Since the ball is much less dense than the water, the applied force will approximately equal this buoyant force.

O: Apply Newton's 2nd law to find the applied force.

A: At equilibrium, $\sum F = 0$ or $F_{app} + mg - B = 0$

Where the buoyant force is $B = \rho_w V g$ and $\rho_w = 1000\ \text{kg/m}^3$

The applied force is then, $F_{app} = \rho_w V g - mg$

Using $m = \rho_{ball} V$ to eliminate the unknown mass of the ball, this becomes

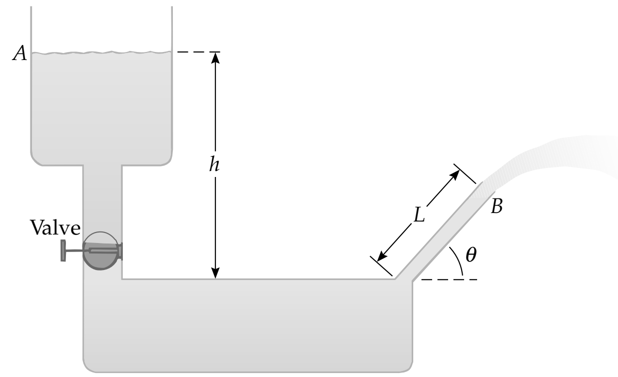
$$F_{app} = Vg(\rho_w - \rho_{ball}) = \frac{4}{3}\pi r^3 g(\rho_w - \rho_{ball})$$

$$F_{app} = \frac{4}{3}\pi(1.90 \times 10^{-2}\ \text{m})^3(9.80\ \text{m/s}^2)(10^3\ \text{kg/m}^3 - 84\ \text{kg/m}^3)$$

$$F_{app} = 0.258\ \text{N}$$

L: The force is approximately what we expected, so our result is reasonable. If the applied force would be greater than 0.258 N, the ball would sink until it hit the bottom (which would then provide a normal force directed upwards).

15.48 Consider the diagram and apply Bernoulli's equation to points A and B, taking $y = 0$ at the level of point B, and recognizing that v_A is approximately zero.



This gives:

$$P_A + \frac{1}{2}\rho_w(0)^2 + \rho_w g(h - L\sin\theta)$$

$$= P_B + \frac{1}{2}\rho_w v_B^2 + \rho_w g(0)$$

Now, recognize that $P_A = P_B = P_{\text{atmosphere}}$ since both points are open to the atmosphere (neglecting variation of atmospheric pressure with altitude). Thus, we obtain

$$v_B = \sqrt{2g(h - L\sin\theta)} = \sqrt{2(9.80\text{ m/s}^2)[10.0\text{ m} - (2.00\text{ m})\sin 30.0°]}$$

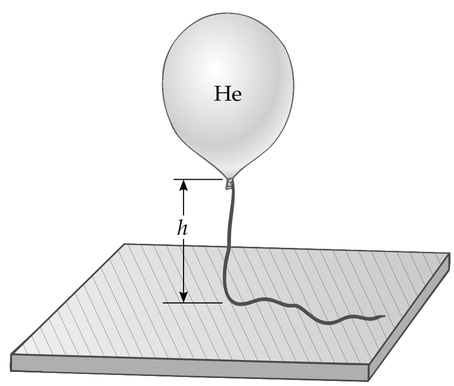
$$v_B = 13.3\text{ m/s}$$

Now the problem reduces to one of projectile motion with $v_{yi} = v_B \sin 30.0° = 6.64$ m/s.. Then, $v_y^2 = v_{yi}^2 + 2a(\Delta y)$ gives at the top of the arc (where $y = y_{max}$ and $v_y = 0$)

$$0 = (6.64\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(y_{max} - 0)$$

or $y_{max} =$
Error!

15.49 When the balloon comes into equilibrium, we must have



$$\Sigma F_y = B - F_{g,\text{ balloon}} - F_{g,\text{He}} - F_{g,\text{string}} = 0$$

$F_{g,\text{ string}}$ is the weight of the string above the ground, and B is the buoyant force. Now

$$F_{g,\text{ balloon}} = m_{\text{balloon}}\, g$$

$$F_{g,\text{ He}} = \rho_{\text{He}}\, Vg$$

$$B = \rho_{\text{air}}\, Vg$$

and $F_{g,\text{ string}} = m_{\text{string}}\dfrac{h}{L}\, g$

Therefore, we have

$$\rho_{\text{air}}\, Vg - m_{\text{balloon}}\, g - \rho_{\text{He}}\, Vg - m_{\text{string}} \frac{h}{L}\, g = 0$$

or $$h = \frac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{m_{\text{string}}}\, L$$

giving,

$$h = \frac{(1.29 - 0.179)\ \text{kg/m}^3 \left(\frac{4\pi(0.400\ \text{m})^3}{3}\right) - 0.250\ \text{kg}}{0.0500\ \text{kg}}(2.00\ \text{m}) = \boxed{1.91\ \text{m}}$$

15.50 Assume $v_{\text{inside}} \approx 0$

$$P + 0 + 0 = 1\ \text{atm} + \frac{1}{2}(1000)(30.0\ \text{m/s})^2 + 1000(9.80)(0.500)$$

$$P_{\text{gauge}} = P - 1\ \text{atm} = 4.50 \times 10^5 + 4.90 \times 10^3 = \boxed{455\ \text{kPa}}$$

15.51 The "balanced" condition is one in which the apparent weight of the body equals the apparent weight of the weights. This condition can be written as:

$$F_g - B = F_g' - B'$$

where B and B' are the buoyant forces on the body and weights respectively. The buoyant force experienced by an object of volume V in air equals:

Buoyant force = (Volume of object)$\rho_{\text{air}}g$

so we have $B = V\rho_{\text{air}}g$ and $B' = \left(\frac{F_g'}{\rho g}\right)\rho_{\text{air}}g$

Therefore, $F_g = \boxed{F_g' + \left(V - \frac{F_g'}{\rho g}\right)\rho_{\text{air}}g}$

Goal Solution

The "balanced" condition is one in which the net torque on the balance is zero. Since the balance has lever arms of equal length, the total force on each pan is equal. Applying $\sum\tau = 0$ around the pivot leads to

$$F_g - B = F_g' - B'$$

where B and B' are the buoyant forces on the body and weights respectively. The buoyant force experienced by an object of volume V in air is

$$B = V\rho_{\text{air}}g \quad \text{and} \quad B' = V'\rho_{\text{air}}g$$

Since the volume of the weights is not given explicitly, we must use the density equation to eliminate it:

$$V' = \frac{m'}{\rho} = \frac{m'g}{\rho g} = \frac{F_g'}{\rho g}$$

With this substitution, the buoyant force on the weights becomes

$$B' = \left(\frac{F_g'}{\rho g}\right) \rho_{\text{air}} g$$

Therefore, $F_g = F_g' + \left(V - \frac{F_g'}{\rho g}\right) \rho_{\text{air}} g$

Side note: We can now answer the popular riddle: Which weighs more, a pound of feathers or a pound of bricks?

Answer: Like in the problem above, the feathers have a greater buoyant force than the bricks , so if they "weigh" the same on a scale as a pound of bricks, then the feathers must have more mass and therefore a greater "true weight."

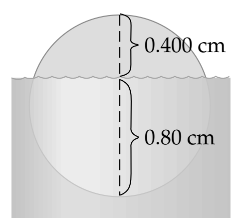
15.52 $P = \rho g h$

$1.013 \times 10^5 = (1.29)(9.80)h$

$h = \boxed{8.01 \text{ km}}$

For Mt. Everest, 29 300 ft = 8.88 km $\boxed{\text{Yes}}$

15.53 The cross–sectional area above water is



$$\frac{2.46 \text{ rad}}{2\pi} \pi (0.600 \text{ cm})^2 - (0.200 \text{ cm})(0.566 \text{ cm}) = 0.330 \text{ cm}^2$$

$A_{\text{all}} = \pi (0.600)^2 = 1.13 \text{ cm}^2$

$\rho_{\text{water}} g A_{\text{under}} = \rho_{\text{wood}} A_{\text{all}} g$

$$\rho_{\text{wood}} = \frac{1.13 - 0.330}{1.13} = 0.709 \text{ g/cm}^3 = \boxed{709 \text{ kg/m}^3}$$

15.54 At equilibrium, $\sum F_y = 0$

or $B - F_{\text{spring}} - F_{g,\,\text{He}} - F_{g,\,\text{balloon}} = 0$

giving $F_{\text{spring}} = kL = B - (m_{\text{He}} + m_{\text{balloon}})g$

But B = weight of displaced air = $\rho_{\text{air}} Vg$ and $m_{\text{He}} = \rho_{\text{He}} V$

Therefore, we have:

$$kL = \rho_{\text{air}} Vg - \rho_{\text{He}} Vg - m_{\text{balloon}} g$$

or $$L = \frac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{k} g$$

From the given data, this gives

$$L = \frac{(1.29 \text{ kg/m}^3 - 0.180 \text{ kg/m}^3)(5.00 \text{ m}^3) - 2.00 \times 10^{-3} \text{ kg}}{90.0 \text{ N/m}}(9.80) = \boxed{0.604 \text{ m}}$$

15.55 Looking first at the top scale and the iron block, we have:

$$T_1 + B = F_{g,\,\text{iron}}$$

where T_1 is the tension in the spring scale, B is the buoyant force, and $F_{g,\,\text{iron}}$ is the weight of the iron block. Now if m_{iron} is the mass of the iron block, we have

$$m_{\text{iron}} = \rho_{\text{iron}} V \quad \text{so} \quad V = \frac{m_{\text{iron}}}{\rho_{\text{iron}}} = V_{\text{displaced oil}}$$

Then, $B = \rho_{\text{oil}} V_{\text{iron}} g$

Therefore, $T_1 = F_{g,\,\text{iron}} - \rho_{\text{oil}} V_{\text{iron}} g = m_{\text{iron}} g - \rho_{\text{oil}} \dfrac{m_{\text{iron}}}{\rho_{\text{iron}}} g$

or $$T_1 = \left(1 - \frac{\rho_{\text{oil}}}{\rho_{\text{iron}}}\right) m_{\text{iron}} g = \left(1 - \frac{916}{7860}\right)(2.00)(9.80) = \boxed{17.3 \text{ N}}$$

Next, we look at the bottom scale which reads T_2 (i.e., exerts an upward force T_2 on the system). Consider the external vertical forces acting on the beaker–oil–iron combination.

$$\sum F_y = 0 \quad \text{gives}$$

$$T_1 + T_2 - F_{g,\,\text{beaker}} - F_{g,\,\text{oil}} - F_{g,\,\text{iron}} = 0$$

or $$T_2 = (m_{\text{beaker}} + m_{\text{oil}} + m_{\text{iron}})g - T_1 = (5.00 \text{ kg})(9.80 \text{ m/s}^2) - 17.3 \text{ N}$$

Thus, $T_2 = \boxed{31.7 \text{ N}}$ is the lower scale reading.

15.56 Looking at the top scale and the iron block:

$$T_1 + B = F_{g,\mathrm{Fe}} \quad \text{where} \quad B = \rho_0 V_{\mathrm{Fe}} g = \rho_0 \left(\frac{m_{\mathrm{Fe}}}{\rho_{\mathrm{Fe}}}\right) g$$

is the buoyant force exerted on the iron block by the oil. Thus,

$$T_1 = F_{g,\mathrm{Fe}} - B = m_{\mathrm{Fe}} g - \rho_0 \left(\frac{m_{\mathrm{Fe}}}{\rho_{\mathrm{Fe}}}\right) g$$

or $T_1 = \boxed{\left(1 - \frac{\rho_0}{\rho_{\mathrm{Fe}}}\right) m_{\mathrm{Fe}} g}$ is the reading on the top scale.

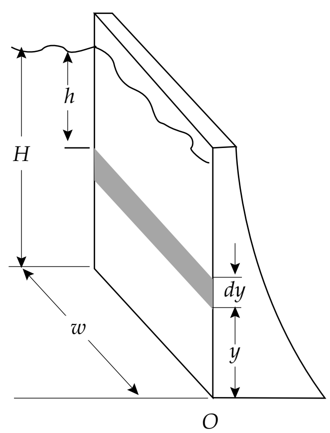
Now, consider the bottom scale which exerts an upward force of T_2 on the beaker–oil–iron combination.

$$\sum F_y = 0 \Rightarrow T_1 + T_2 - F_{g,\mathrm{beaker}} - F_{g,\mathrm{oil}} - F_{g,\mathrm{Fe}} = 0$$

$$T_2 = F_{g,\mathrm{beaker}} + F_{g,\mathrm{oil}} + F_{g,\mathrm{Fe}} - T_1 = (m_b + m_0 + m_{\mathrm{Fe}})g - \left(1 - \frac{\rho_0}{\rho_{\mathrm{Fe}}}\right) m_{\mathrm{Fe}} g$$

or $T_2 = \boxed{\left[m_b + m_0 + \left(\frac{\rho_0}{\rho_{\mathrm{Fe}}}\right) m_{\mathrm{Fe}}\right] g}$ is the reading on the bottom scale.

15.57 The torque is



$$\tau = \int d\tau = \int r\,dF$$

From the figure,

$$\tau = \int_0^H y[\rho g(H - y) w\,dy] = \boxed{\frac{1}{6}\rho g w H^3}$$

The total force is given as

$$\frac{1}{2}\rho g w H^2$$

If this were applied at a height y_{eff} such that the torque remains unchanged, we have

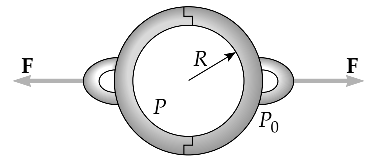
$$\frac{1}{6}\rho g w H^3 = y_{\mathrm{eff}}\left[\frac{1}{2}\rho g w H^2\right]$$

and $y_{\mathrm{eff}} = \boxed{\frac{1}{3} H}$

15.58 (a) The pressure on the surface of the two hemispheres is constant at all points, and the force on each element of surface area is directed along the radius of the hemispheres. The applied force along the axis must balance the force on the "effective" area which is the projection of the actual surface onto a plane perpendicular to the x axis,

$$A = \pi R^2$$

Therefore,



$$F = \boxed{(P_0 - P)\pi R^2}$$

(b) For the values given

$$F = (P_0 - 0.100P_0)\pi(0.300\text{ m})^2 = 0.254P_0 = \boxed{2.58 \times 10^4\text{ N}}$$

15.59 $\rho_{\text{Cu}} V = 3.083\text{ g}$

$\rho_{\text{Zn}}(xV) + \rho_{\text{Cu}}(1-x)V = 2.517\text{ g}$

$$\rho_{\text{Zn}}\left(\frac{3.083}{\rho_{\text{Cu}}}\right) x + 3.083(1-x) = 2.517$$

$$\left(1 - \frac{7.133}{8.960}\right) x = \left(1 - \frac{2.517}{3.083}\right)$$

$x = 0.9004$

$\%\text{Zn} = \boxed{90.04\%}$

15.60 (a) From

$$\Sigma F = ma$$

$$B - m_{\text{shell}}\, g - m_{\text{He}}\, g = m_{\text{total}}\, a = (m_{\text{shell}} + m_{\text{He}})a \qquad (1)$$

Where $B = \rho_{\text{water}} V g$ and $m_{\text{He}} = \rho_{\text{He}} V$

Also, $V = \frac{4}{3}\pi r^3 = \frac{\pi d^3}{6}$

Putting these into equation (1) above,

$$\left(m_{\text{shell}} + \rho_{\text{He}}\frac{\pi d^3}{6}\right) a = \left(\rho_{\text{water}}\frac{\pi d^3}{6} - m_{\text{shell}} - \rho_{\text{He}}\frac{\pi d^3}{6}\right) g$$

which gives

$$a = \frac{(\rho_{\text{water}} - \rho_{\text{He}})\frac{\pi d^3}{6} - m_{\text{shell}}}{m_{\text{shell}} + \rho_{\text{He}}\frac{\pi d^3}{6}}\, g$$

or $$a = \frac{(1000 - 0.180)\left(\frac{\text{kg}}{\text{m}^3}\right)\frac{\pi(0.200\text{ m})^3}{6} - 4.00\text{ kg}}{4.00\text{ kg} + \left(0.180\,\frac{\text{kg}}{\text{m}^3}\right)\frac{\pi(0.200\text{ m})^3}{6}}\, 9.80\text{ m/s}^2 = \boxed{0.461\text{ m/s}^2}$$

(b) $$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(h-d)}{a}} = \sqrt{\frac{2(4.00\text{ m} - 0.200\text{ m})}{0.461\text{ m/s}^2}} = \boxed{4.06\text{ s}}$$

15.61 Energy is conserved

$$(K+U)_i + \Delta E = (K+U)_f$$

$$0 + \frac{mgL}{2} + 0 = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{gL} = \sqrt{2.00\text{ m}(9.80\text{ m/s}^2)} = \boxed{4.43\text{ m/s}}$$

***15.62** Inertia of the disk: $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(0.250\text{ m})^2 = 0.312\text{ kg}\cdot\text{m}^2$

Angular acceleration: $\omega_f = \omega_i + \alpha t$

$$\alpha = \left(\frac{0 - 300\text{ rev/min}}{60.0\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = -0.524\text{ rad/s}^2$$

Braking torque: $\sum\tau = I\alpha \Rightarrow -fd = I\alpha$, so $f = \frac{-I\alpha}{d}$

Friction force: $f = \frac{(0.312\text{ kg}\cdot\text{m}^2)(0.524\text{ rad/s}^2)}{0.220\text{ m}} = 0.744\text{ N}$

Normal force: $f = \mu_k n \Rightarrow n = \frac{f}{\mu_k} = \frac{0.744\text{ N}}{0.500} = 1.49\text{ N}$

gauge pressure: $P = \frac{n}{A} = \frac{1.49\text{ N}}{\pi(2.50\times 10^{-2}\text{ m})^2} = \boxed{758\text{ Pa}}$

***15.63** (a) We imagine the superhero to produce a perfect vacuum in the straw. Take point 1 at the water surface in the basin and point 2 at the water surface in the straw:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$1.013 \times 10^5 \text{ N/m}^2 + 0 + 0 = 0 + 0 + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2) y_2$$

$$y_2 = \boxed{10.3 \text{ m}}$$

(b) No atmosphere can lift the water in the straw through $\boxed{\text{zero}}$ height difference.

15.64 Differentiating $P = \rho g y$ we have

$$\frac{dP}{dy} = -\rho g$$

Also at any given height the density of air is proportional to pressure, or

$$\frac{P}{\rho} = \frac{P_0}{\rho_0}$$

Combining these two equations we have

$$\int_{P_0}^{P} \frac{dP}{P} = -g\frac{\rho_0}{P_0}\int_0^h dy$$

and integrating gives

$$P = P_0 e^{-\alpha h}$$

15.65 Let s stand for the edge of the cube, h for the depth of immersion, ρ_{ice} stand for the density of the ice, ρ_w stand for density of water, and ρ_a stand for density of the alcohol.

(a) According to Archimedes's principle, at equilibrium we have

$$\rho_{\text{ice}} g s^3 = \rho_w g h s^2 \Rightarrow h = s\frac{\rho_{\text{ice}}}{\rho_w}$$

With

$$\rho_{\text{ice}} = 0.917 \times 10^3 \text{ kg/m}^3$$

$$\rho_w = 1.00 \times 10^3 \text{ kg/m}^3$$

and $s = 20.0 \text{ mm}$

we get $h = 20.0(0.917) = 18.34 \text{ mm} \approx \boxed{18.3 \text{ mm}}$

(b) We assume that the top of the cube is still above the alcohol surface. Letting h_a stand for the thickness of the alcohol layer, we have

$$\rho_a g s^2 h_a + \rho_w g s^2 h_w = \rho_{\text{ice}} g s^3 \Rightarrow h_w = \left(\frac{\rho_{\text{ice}}}{\rho_w}\right) s - \left(\frac{\rho_a}{\rho_w}\right) h_a$$

With $\rho_a = 0.806 \times 10^3 \text{ kg/m}^3$ and $h_a = 5.00 \text{ mm}$

we obtain $h_w = 18.34 - (0.806)(5.00) = 14.31 \text{ mm} \approx \boxed{14.3 \text{ mm}}$

(c) Here $h'_w = s - h'_a$, so Archimedes's principle gives . . .

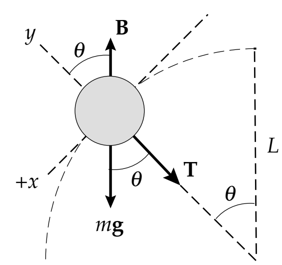
$$\rho_a g s^2 h'_a + \rho_w g s^2 (s - h'_a) = \rho_{\text{ice}} g s^3 \Rightarrow \rho_a h'_a + \rho_w (s - h'_a) = \rho_{\text{ice}} s$$

$$\Rightarrow h'_a = \frac{s(\rho_w - \rho_{\text{ice}})}{(\rho_w - \rho_a)} = 20.0 \frac{(1.000 - 0.917)}{(1.000 - 0.806)} = 8.557 \approx \boxed{8.56 \text{ mm}}$$

15.66 (a) A study of the forces on the balloon shows that the tangential restoring force is given as:

$$F_x = -B \sin\theta + mg \sin\theta = -(B - mg) \sin\theta$$

But $B = \rho_{\text{air}} V g$ and $m = \rho_{\text{He}} V$



Also,

$$\sin\theta \approx \theta \text{ (for small } \theta)$$

so $F_x \approx -(\rho_{\text{air}} V g - \rho_{\text{He}} V g)\theta = -(\rho_{\text{air}} - \rho_{\text{He}}) V g \theta$

But $\theta = \dfrac{s}{L}$

and $\boxed{F_x = -(\rho_{\text{air}} - \rho_{\text{He}}) V g \dfrac{s}{L} = -ks}$

with $k = (\rho_{\text{air}} - \rho_{\text{He}}) \dfrac{Vg}{L}$

(b) Then

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\rho_{\text{He}} V}{(\rho_{\text{air}} - \rho_{\text{He}}) \frac{Vg}{L}}} = 2\pi\sqrt{\frac{\rho_{\text{He}} L}{(\rho_{\text{air}} - \rho_{\text{He}}) g}}$$

giving

$$T = 2\pi\sqrt{\frac{(0.180)(3.00 \text{ m})}{(1.29 - 0.180)(9.80 \text{ m/s}^2)}} = \boxed{1.40 \text{ s}}$$

15.67 (a) The flow rate, Av, as given may be expressed as follows:

$$25.0 \text{ liters}/30.0 \text{ s} = 0.833 \text{ liters/s} = 833 \text{ cm}^3/\text{s}$$

The area of the faucet tap is π cm^2, so we can find the velocity as

$$v = \frac{\text{flow rate}}{A} = \frac{833 \text{ cm}^3/\text{s}}{\pi \text{ cm}^2} = 265 \text{ cm/s} = \boxed{2.65 \text{ m/s}}$$

(b) We choose point 1 to be in the entrance pipe and point 2 to be at the faucet tap. $A_1v_1 = A_2v_2$ gives $v_1 = 0.295$ m/s. Bernoulli's equation is:

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1), \text{ and gives}$$

$$P_1 - P_2 = \frac{1}{2}(10^3 \text{ kg/m}^3)\ [(2.65 \text{ m/s})^2 - (0.295 \text{ m/s})^2]$$

$$+ (10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^3)(2.00 \text{ m})$$

or $P_{\text{gauge}} = P_1 - P_2 = \boxed{2.31 \times 10^4 \text{ Pa}}$

15.68 (a) Since the upward buoyant force is balanced by the weight of the sphere,

$$m_1 g = \rho V g = \rho\left(\frac{4}{3}\pi R^3\right) g$$

In this problem, $\rho = 0.789\,45$ g/cm^3 at 20.0°C, and $R = 1.00$ cm, so we find:

$$m_1 = \rho\left(\frac{4}{3}\pi R^3\right) = (0.789\,45 \text{ g/cm}^3)\left(\frac{4}{3}\pi R^3\right)(1.00 \text{ cm})^3 = \boxed{3.307 \text{ g}}$$

(b) Following the same procedure as in part (a), with $\rho' = 0.780\,97$ g/cm^3 at 30.0°C, we find:

$$m_2 = \rho'\left(\frac{4}{3}\pi R^3\right) = (0.780\,97 \text{ g/cm}^3)\left(\frac{4}{3}\pi R^3\right)(1.00 \text{ cm})^3$$

or $m_2 = \boxed{3.271 \text{ g}}$

(c) When the first sphere is resting on the bottom of the tube,

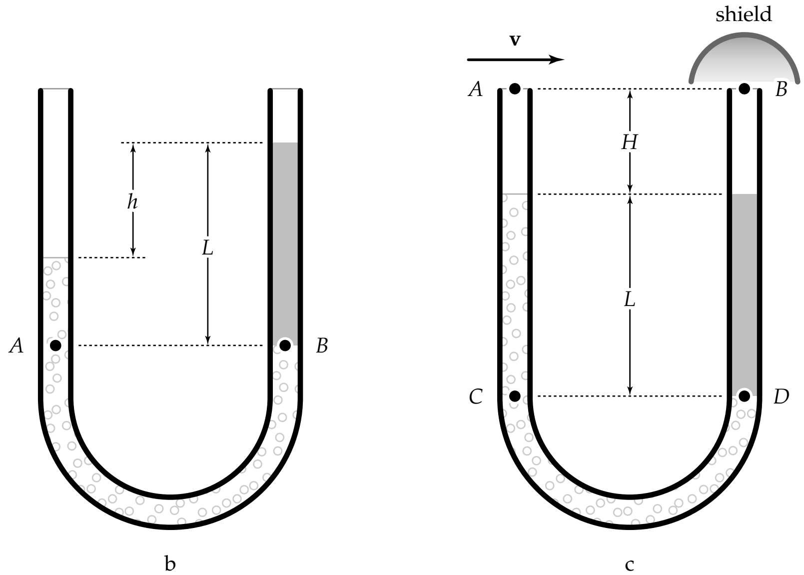
$n + B = F_{g1} = m_1 g$, where n is the normal force.

Since $B = \rho' V g$,

$$n = m_1 g - \rho' V g = \left[3.307 \text{ g} - \left(0.780\,97 \frac{\text{g}}{\text{cm}^3}\right)(1.00 \text{ cm}^3)\right]\left(980 \frac{\text{cm}}{\text{s}^2}\right)$$

$$n = 34.8 \text{ g} \cdot \text{cm/s}^2 = \boxed{3.48 \times 10^{-4} \text{ N}}$$

15.69 **Note**: Variation of atmospheric pressure with altitude is included in this solution. Because of the small distances involved, this effect is unimportant in the final answers.



(a) Consider the pressure at points A and B in part (b) of the figure:

Using the left tube: $P_A = P_{\text{atm}} + \rho_a gh + \rho_w g(L - h)$, where the second term is due to the variation of air pressure with altitude.

Using the right tube: $P_B = P_{\text{atm}} + \rho_0 gL$

But Pascal's principle says that $P_A = P_B$. Therefore,

$$P_{\text{atm}} + \rho_0 gL = P_{\text{atm}} + \rho_a gh + \rho_w g(L - h) \quad \text{or}$$

$$(\rho_w - \rho_a)h = (\rho_w - \rho_0)L, \text{ giving}$$

$$h = \left(\frac{\rho_w - \rho_0}{\rho_w - \rho_a}\right) L = \left(\frac{1000 - 750}{1000 - 1.29}\right)(5.00 \text{ cm}) = \boxed{1.25 \text{ cm}}$$

(b) Consider part (c) of the diagram showing the situation when the air flow over the left tube equalizes the fluid levels in the two tubes. First, apply Bernoulli's equation to points A and B ($y_A = y_B$, $v_A = v$, and $v_B = 0$).

This gives: $P_A + \frac{1}{2}\rho_a v^2 + \rho_a g y_A = P_B + \frac{1}{2}\rho_a (0)^2 + \rho_a g y_B$

and since $y_A = y_B$, this reduces to: $P_B - P_A = \frac{1}{2}\rho_a v^2 \quad (1)$

Now consider points C and D, both at the level of the oil–water interface in the right tube. Using the variation of pressure with depth in static fluids, we have:

$$P_C = P_A + \rho_a gH + \rho_w gL \quad \text{and} \quad P_D = P_B + \rho_a gH + \rho_0 gL$$

But Pascal's principle says that $P_C = P_D$. Equating these two gives:

$$P_B + \rho_a gH + \rho_0 gL = P_A + \rho_a gH + \rho_w gL \quad \text{or}$$

$$P_B - P_A = (\rho_w - \rho_0)gL \ (2)$$

Substitute equation (1) for $P_B - P_A$ into (2) to obtain

$$\frac{1}{2}\rho_a v^2 = (\rho_w - \rho_0)gL \quad \text{or}$$

$$v = \sqrt{\frac{2gL(\rho_w - \rho_0)}{\rho_a}} = \sqrt{2(9.80\ \text{m/s}^2)(0.0500\ \text{m})\left(\frac{1000 - 750}{1.29}\right)}$$

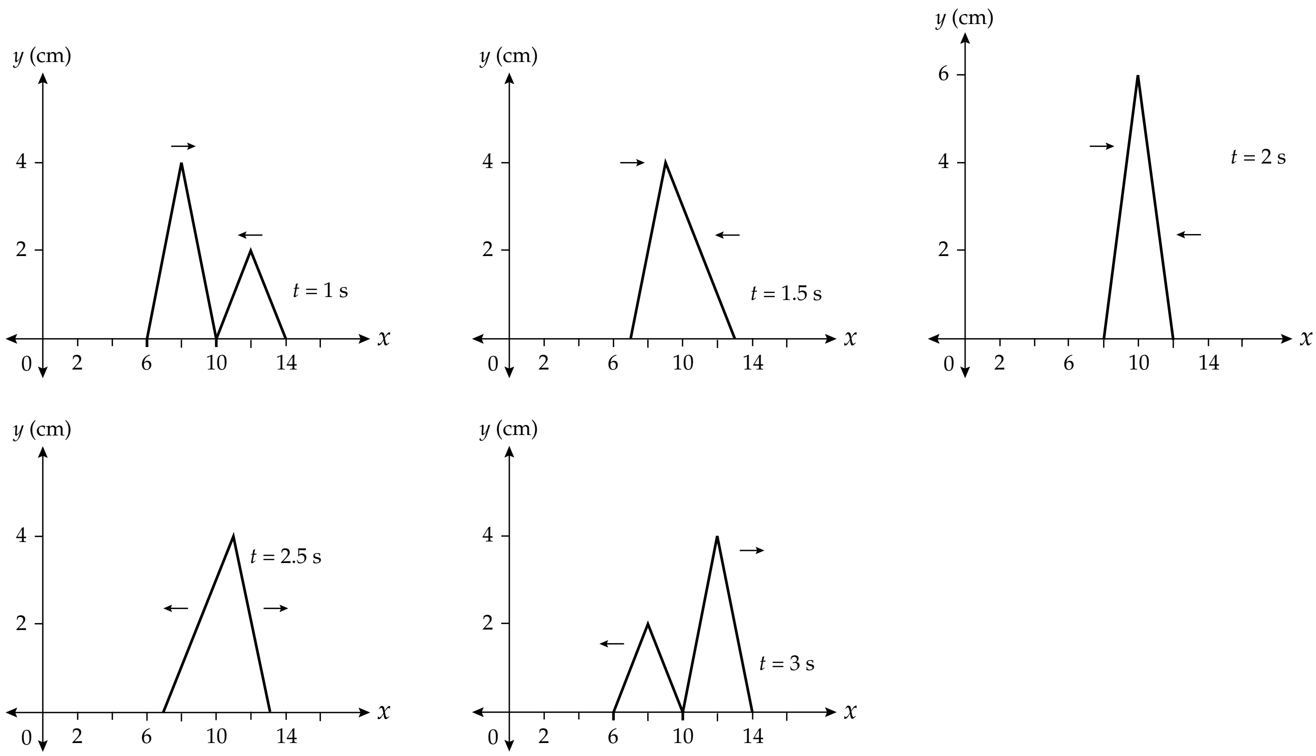
$$v = \boxed{13.8\ \text{m/s}}$$

Chapter 16 Solutions

16.1 Replace x by $x - vt = x - 4.5t$

to get $\boxed{y = \dfrac{6}{[(x - 4.5t)^2 + 3]}}$

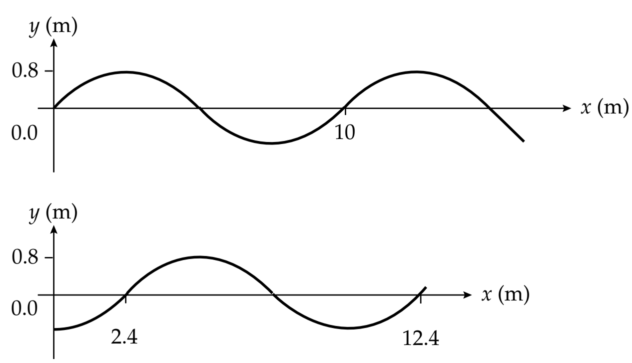
16.2



16.3 $5.00e^{-(x + 5t)^2}$ is of the form $f(x + vt)$

so it describes a wave moving to the $\boxed{\text{left}}$ at $v = \boxed{5.00\text{ m/s}}$

16.4



16.5 (a) The $\boxed{\text{longitudinal}}$ wave travels a shorter distance and is moving faster, so it will arrive at point B first.

(b) The wave that travels through the Earth must travel a distance of $2R\sin 30.0° = 2(6.37\times10^6\text{ m})\sin 30.0° = 6.37\times10^6$ m at a speed of 7800 m/s.

Therefore, it takes $\dfrac{6.37\times10^6\text{ m}}{7800\text{ m/s}} = 817\text{ s}$

The wave that travels along the Earth's surface must travel a distance of

$$S = R\theta = R\left(\frac{\pi}{3}\text{rad}\right) = 6.67\times10^6\text{ m at a speed of 4500 m/s}$$

Therefore, it takes $\dfrac{6.67\times10^6}{4500} = 1482\text{ s}$

The time difference is $\boxed{665\text{ s}} = 11.1$ min.

***16.6** The distance the waves have traveled is

$$d = (7.80\text{ km/s})t = (4.50\text{ km/s})(t + 17.3\text{ s})$$

where t is the travel time for the faster wave.

Then, $(7.80 - 4.50)(\text{km/s})t = (4.50\text{ km/s})(17.3\text{ s})$

or $\quad t = \dfrac{(4.50\text{ km/s})(17.3\text{ s})}{(7.80-4.50)(\text{km/s})} = 23.6\text{ s}$, and

the distance is $d = (7.80\text{ km/s})(23.6\text{ s}) = \boxed{184\text{ km}}$

16.7 (a) $\phi_1 = (20.0\text{ rad/cm})(5.00\text{ cm}) - (32.0\text{ rad/s})(2.00\text{ s}) = 36.0\text{ rad}$

$\phi_1 = (25.0\text{ rad/cm})(5.00\text{ cm}) - (40.0\text{ rad/s})(2.00\text{ s}) = 45.0\text{ rad}$

$\Delta\phi = 9.00\text{ radians} = 516° = \boxed{156°}$

(b) $\Delta\phi = \left|20.0x - 32.0t - [25.0x - 40.0t]\right| = \left|-5.00x + 8.00t\right|$

At $t = 2.00$ s, the requirement is

$$\Delta\phi = \left|-5.00x + 8.00\,(2.00)\right| = (2n+1)\pi \text{ for any integer } n.$$

For $x < 3.20$, $-5.00x + 16.0$ is positive, so we have

$$-5.00x + 16.0 = (2n+1)\pi, \text{ or}$$

$$x = 3.20 - \frac{(2n+1)\pi}{5.00}$$

The smallest positive value of x occurs for $n = 2$ and is

$$x = 3.20 - \frac{(4+1)\pi}{5.00} = 3.20 - \pi = \boxed{0.0584\text{ cm}}$$

16.8 $y = y_1 + y_2 = 3.00 \cos(4.00x - 1.60t) + 4.00 \sin(5.00x - 2.00t)$ evaluated at the given x values.

(a) $x = 1.00,\ t = 1.00$

$y = 3.00 \cos(2.40 \text{ rad}) + 4.00 \sin(+3.00 \text{ rad}) = \boxed{-1.65}$

(b) $x = 1.00,\ t = 0.500$

$y = 3.00 \cos(+3.20 \text{ rad}) + 4.00 \sin(+4.00 \text{ rad}) = \boxed{-6.02}$

(c) $x = 0.500,\ t = 0$

$y = 3.00 \cos(+2.00 \text{ rad}) + 4.00 \sin(+2.50 \text{ rad}) = \boxed{1.15}$

16.9 (a) $y_1 = f(x - vt)$, so wave 1 travels in the $\boxed{+x \text{ direction}}$.

$y_2 = f(x + vt)$, so wave 2 travels in the $\boxed{-x \text{ direction}}$.

(b) To cancel, $y_1 + y_2 = 0$:

$$\frac{5}{(3x - 4t)^2 + 2} = \frac{+5}{(3x + 4t - 6)^2 + 2}$$

$$(3x - 4t)^2 = (3x + 4t - 6)^2$$

$$3x - 4t = \pm(3x + 4t - 6)$$

$$+\text{root} \rightarrow 8t = 6 \rightarrow \boxed{t = 0.750 \text{ s}}$$

(at $t = 0.750$ s, the waves cancel everywhere)

(c) $-\text{root} \rightarrow 6x = 6 \rightarrow \boxed{x = 1.00 \text{ m}}$ (at $x = 1.00$ m, the waves cancel always)

***16.10** The down and back distance is 4.00 m + 4.00 m = 8.00 m.

The speed is then $v = \dfrac{d_{\text{total}}}{t} = \dfrac{4(8.00 \text{ m})}{0.800 \text{ s}} = 40.0 \text{ m/s} = \sqrt{T/\mu}$

Now, $\mu = \dfrac{0.200 \text{ kg}}{4.00 \text{ m}} = 5.00 \times 10^{-2} \text{ kg/m}$, so

$T = \mu v^2 = (5.00 \times 10^{-2} \text{ kg/m})(40.0 \text{ m/s})^2 = \boxed{80.0 \text{ N}}$

16.11 The mass per unit length is: $\mu = \dfrac{0.0600 \text{ kg}}{5.00 \text{ m}} = 1.20 \times 10^{-2} \text{ kg/m}$

The required tension is: $T = \mu v^2 = (0.0120 \text{ kg/m})(50.0 \text{ m/s})^2 = \boxed{30.0 \text{ N}}$

16.12 $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{1350\text{ kg}\cdot\text{m/s}^2}{5.00\times10^{-3}\text{ kg/m}}} = \boxed{520\text{ m/s}}$

16.13 $T = Mg$ is the tension

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{(m/L)}} = \sqrt{\frac{MgL}{m}} = \frac{L}{t} \quad \text{is the wave speed}$$

Then, $\dfrac{MgL}{m} = \dfrac{L^2}{t^2}$

and $g = \dfrac{Lm}{Mt^2} = \dfrac{1.60\text{ m }(4.00\times10^{-3}\text{ kg})}{3.00\text{ kg }(3.61\times10^{-3}\text{ s})^2} = \boxed{1.64\text{ m/s}^2}$

***16.14** $v = \sqrt{\dfrac{T}{\mu}}$

$T = \mu v^2 = \rho A v^2 = \rho\pi r^2 v^2$

$T = (8920\text{ kg/m}^3)(\pi)(7.50\times10^{-4}\text{ m})^2(200\text{ m/s})^2$

$T = \boxed{631\text{ N}}$

16.15 Since μ is constant, $\mu = \dfrac{T_2}{v_2^2} = \dfrac{T_1}{v_1^2}$ and

$$T_2 = \left(\frac{v_2}{v_1}\right)^2 T_1 = \left(\frac{30.0\text{ m/s}}{20.0\text{ m/s}}\right)^2 (6.00\text{ N}) = \boxed{13.5\text{ N}}$$

Goal Solution

G: Since $v \propto \sqrt{F}$, the new tension must be about twice as much as the original to achieve a 50% increase in the wave speed.

O: The equation for the speed of a transverse wave on a string under tension can be used if we assume that the linear density of the string is constant. Then the ratio of the two wave speeds can be used to find the new tension.

A: The 2 wave speeds can be written as: $v_1 = \sqrt{\dfrac{F_1}{\mu}}$ and $v_2 = \sqrt{\dfrac{F_2}{\mu}}$

Dividing, $\dfrac{v_2}{v_1} = \sqrt{\dfrac{F_2}{F_1}}$

so $F_2 = \left(\dfrac{v_2}{v_1}\right)^2 F_1 = \left(\dfrac{30.0\text{ m/s}}{20.0\text{ m/s}}\right)^2 (6.00\text{ N}) = 13.5\text{ N}$

L: The new tension is slightly more than twice the original, so the result agrees with our initial prediction and is therefore reasonable.

16.16 The period of the pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$

Let F represent the tension in the string (to avoid confusion with the period) when the pendulum is vertical and stationary. The speed of waves in the string is then:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{Mg}{(m/L)}} = \sqrt{\frac{MgL}{m}}$$

Since it might be difficult to measure L precisely, we eliminate $\sqrt{L} = \frac{T\sqrt{g}}{2\pi}$

so $$v = \sqrt{\frac{Mg}{m}}\frac{T\sqrt{g}}{2\pi} = \boxed{\frac{Tg}{2\pi}\sqrt{\frac{M}{m}}}$$

16.17 If the tension in the wire is T, the tensile stress is

$$\text{Stress} = T/A \quad \text{so} \quad T = A(\text{stress})$$

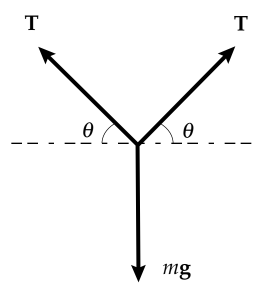
The speed of transverse waves in the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{A(\text{Stress})}{m/L}} = \sqrt{\frac{\text{Stress}}{m/AL}} = \sqrt{\frac{\text{Stress}}{m/(\text{Volume})}} = \sqrt{\frac{\text{Stress}}{\rho}}$$

where ρ is the density. The maximum velocity occurs when the stress is a maximum:

$$v_{\text{max}} = \sqrt{\frac{2.70 \times 10^9 \text{ Pa}}{7860 \text{ kg/m}^3}} = \boxed{586 \text{ m/s}}$$

16.18 From the free-body diagram,



$$mg = 2T\sin\theta$$

$$T = \frac{mg}{2\sin\theta}$$

The angle θ is found from

$$\cos\theta = \frac{3L/8}{L/2} = \frac{3}{4}$$

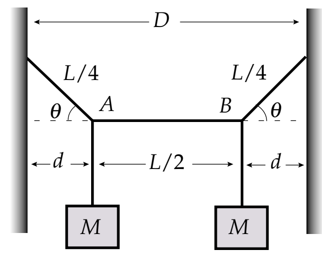
$$\therefore \theta = 41.4°$$

(a) $$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{2\mu\sin 41.4°}} = \left(\sqrt{\frac{9.80 \text{ m/s}^2}{2(8.00 \times 10^{-3} \text{ kg/m})\sin 41.4°}}\right)\sqrt{m}$$

or $$v = \boxed{\left(30.4\frac{\text{m/s}}{\sqrt{\text{kg}}}\right)\sqrt{m}}$$

(b) $v = 60.0 = 30.4\sqrt{m}$ and $\boxed{m = 3.89 \text{ kg}}$

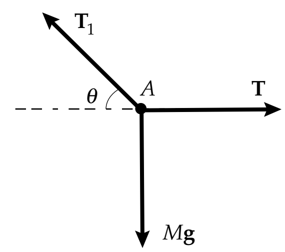
16.19 First, observe from the geometry shown in the figure that $2d + \frac{L}{2} = D$, or



$$d = \frac{D}{2} - \frac{L}{4} = 1.00 \text{ m} - 0.750 \text{ m} = 0.250 \text{ m}$$

Thus, $\cos\theta = \frac{0.250 \text{ m}}{0.750 \text{ m}} = \frac{1}{3}$, and $\theta = 70.5°$

Now, consider a free body diagram of point A:



$\Sigma F_x = 0$ becomes $T = T_1 \cos\theta$, and

$\Sigma F_y = 0$ becomes $Mg = T_1 \sin\theta$

Dividing the second of these equations by the first gives:

$$\frac{Mg}{T} = \tan\theta \quad \text{or} \quad T = \frac{19.6 \text{ N}}{\tan 70.5°} = 6.94 \text{ N}$$

The linear density of the string is: $\mu = \frac{m}{L} = \frac{0.0100 \text{ kg}}{3.00 \text{ m}} = 3.33 \times 10^{-3} \text{ kg/m}$

so the speed of transverse waves in the string between points A and B is:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{6.94 \text{ N}}{3.33 \times 10^{-3} \text{ kg/m}}} = 45.6 \text{ m/s}$$

The time for the pulse to travel 1.50 m from A to B is:

$$t = \frac{1.50 \text{ m}}{45.6 \text{ m/s}} = 0.0329 \text{ s} = \boxed{32.9 \text{ ms}}$$

16.20 Refer to the diagrams given in the solution for Problem 19 above. From the free-body diagram of point A:

$$\Sigma F_y = 0 \Rightarrow T_1 \sin\theta = Mg \quad \text{and} \quad \Sigma F_x = 0 \Rightarrow T_1 \cos\theta = T$$

Combining these equations to eliminate T_1 gives the tension in the string connecting points A and B as: $T = \frac{Mg}{\tan\theta}$

The speed of transverse waves in this segment of string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg/\tan\theta}{m/L}} = \sqrt{\frac{MgL}{m\tan\theta}}$$

and the time for a pulse to travel from A to B is

$$t = \frac{L/2}{v} = \sqrt{\frac{mL\tan\theta}{4Mg}}$$

To evaluate tan θ, refer to the geometry shown in the first diagram:

$$\tan\theta = \frac{\sqrt{(L/4)^2 - d^2}}{d} = \sqrt{\left(\frac{L}{4d}\right)^2 - 1}$$

Also, $2d = D - L/2$, which gives $4d = 2D - L$

Thus, $\tan\theta = \sqrt{\left(\frac{L}{2D - L}\right)^2 - 1}$

The travel time for the pulse going from A to B is then

$$t = \boxed{\sqrt{\frac{mL\tan\theta}{4Mg}} \text{ where } \tan\theta = \sqrt{\left(\frac{L}{2D-L}\right)^2 - 1}}$$

16.21 The total time is the sum of the two times.

In each wire $t = \frac{L}{v} = L\sqrt{\frac{\mu}{T}}$

where $\mu = \rho A = \frac{\pi\rho d^2}{4}$

Thus, $t = L\left(\frac{\pi\rho d^2}{4T}\right)^{1/2}$

For copper, $t = (20.0)\left[\frac{(\pi)(8920)(1.00 \times 10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.137 \text{ s}$

For steel, $t = (30.0)\left[\frac{(\pi)(7860)(1.00 \times 10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.192 \text{ s}$

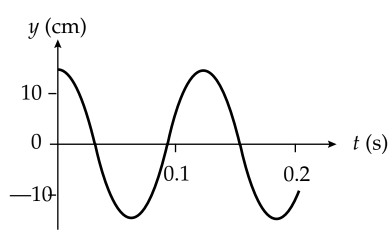
The total time is $0.137 + 0.192 = \boxed{0.329 \text{ s}}$

***16.22** (a) If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is $\boxed{\text{zero}}$.

(b) If the end is free, there is no inversion on reflection. When they meet, the amplitude is $2A = 2(0.150 \text{ m}) = \boxed{0.300 \text{ m}}$.

16.23 (a) See figure at right.

(b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{50.3} = \boxed{0.125 \text{ s}}$



16.24 Using data from the observations, we have $\lambda = 1.20$ m and $f = \dfrac{8.00}{12.0 \text{ s}}$. Therefore,

$$v = \lambda f = (1.20 \text{ m})\left(\frac{8.00}{12.0 \text{ s}}\right) = \boxed{0.800 \text{ m/s}}$$

16.25 $f = \dfrac{40.0 \text{ vibrations}}{30.0 \text{ s}} = \dfrac{4}{3}$ Hz

$$v = \frac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$$

$$\lambda = \frac{v}{f} = \frac{42.5 \text{ cm/s}}{\left(\frac{4}{3}\text{ Hz}\right)} = 31.9 \text{ cm} = \boxed{0.319 \text{ m}}$$

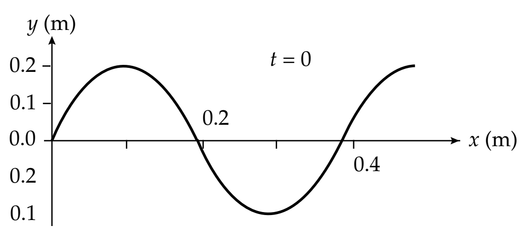
***16.26** At time t, the phase of $y = (15.0 \text{ cm})\cos(0.157x - 50.3t)$ at coordinate x is $\phi = (0.157 \text{ rad/cm})x - (50.3 \text{ rad/s})t$. Since $60.0° = \dfrac{\pi}{3}$ rad, the requirement for point B is that $\phi_B = \phi_A \pm \dfrac{\pi}{3}$ rad, or (since $x_A = 0$),

$$(0.157 \text{ rad/cm})x_B - (50.3 \text{ rad/s})t = 0 - (50.3 \text{ rad/s})t \pm \frac{\pi}{3} \text{ rad}$$

This reduces to $x_B = \dfrac{\pm\pi \text{ rad}}{3(0.157 \text{ rad/cm})} = \boxed{\pm 6.67 \text{ cm}}$

16.27 $v = f\lambda = (4.00 \text{ Hz})(60.0 \text{ cm}) = 240 \text{ cm/s} = \boxed{2.40 \text{ m/s}}$

16.28 (a)



(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.350 \text{ m}} = \boxed{18.0 \text{ rad/m}}$

$$T = \frac{1}{f} = \frac{1}{12.0/\text{s}} = \boxed{0.0833 \text{ s}}$$

$$\omega = 2\pi f = 2\pi\, 12.0/\text{s} = \boxed{75.4 \text{ rad/s}}$$

$$|v| = f\lambda = (12.0/\text{s})(0.350 \text{ m}) = \boxed{4.20 \text{ m/s}}$$

(c) $y = A\sin(kx + \omega t + \phi)$ specializes to

$y = 0.200\text{ m}\sin(18.0\,x/\text{m} + 75.4\,t/\text{s} + \phi)$

at $x = 0$, $t = 0$ we require

$$-3.00 \times 10^{-2}\text{ m} = 0.200\text{ m}\sin(+\phi)$$

$$\phi = -8.63° = -0.151\text{ rad}$$

so $y(x, t) = \boxed{(0.200\text{ m})\sin(18.0\,x/\text{m} + 75.4\,t/\text{s} - 0.151\text{ rad})}$

16.29 $y = 0.250\sin(0.300x - 40.0t)$ m

Compare this with the general expression $y = A\sin(kx - \omega t)$

(a) $A = \boxed{0.250\text{ m}}$ (b) $\omega = \boxed{40.0\text{ rad/s}}$ (c) $k = \boxed{0.300\text{ rad/m}}$

(d) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{0.300\text{ rad/m}} = \boxed{20.9\text{ m}}$

(e) $v = f\lambda = \left(\dfrac{\omega}{2\pi}\right)\lambda = \left(\dfrac{40.0\text{ rad/s}}{2\pi}\right)(20.9\text{ m}) = \boxed{133\text{ m/s}}$

(f) The wave moves to the right, $\boxed{\text{in }+x\text{ direction}}$.

16.30 $y = (0.120\text{ m})\sin\left(\dfrac{\pi}{8}x + 4\pi\,t\right)$

(a) $v = \dfrac{dy}{dt} = (0.120)(4\pi)\cos\left(\dfrac{\pi}{8}x + 4\pi\,t\right)$

$v(0.200\text{ s}, 1.60\text{ m}) = \boxed{-1.51\text{ m/s}}$

$a = \dfrac{dv}{dt} = (-\,0.120\text{ m})(4\pi)^2\sin\left(\dfrac{\pi}{8}x + 4\pi\,t\right)$

$a = (0.200\text{ s}, 1.60\text{ m}) = \boxed{0}$

(b) $k = \dfrac{\pi}{8} = \dfrac{2\pi}{\lambda}$ $\lambda = \boxed{16.0\text{ m}}$

$\omega = 4\pi = \dfrac{2\pi}{T}$ $T = \boxed{0.500\text{ s}}$

$v = \dfrac{\lambda}{T} = \dfrac{16.0\text{ m}}{0.500\text{ s}} = \boxed{32.0\text{ m/s}}$

16.31 (a) $A = y_{\max} = 8.00 \text{ cm} = 0.0800 \text{ m}$: $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{(0.0800 \text{ m})} = 7.85 \text{ m}^{-1}$

$\omega = 2\pi f = 2\pi(3.00) = 6.00\pi \text{ rad/s}$

Therefore,

$$y = A\sin(kx + \omega t)$$

or $\boxed{y = (0.0800)\sin(7.85x + 6\pi t)\text{m}}$ [where $y(0, t) = 0$]

(b) In general,

$$y = 0.0800\sin(7.85x + 6\pi t + \phi)$$

Assuming $y(x, 0) = 0$ at $x = 0.100$ m, then we require that

$$0 = 0.0800\sin(0.785 + \phi)$$

or $\phi = -0.785$

Therefore,

$$\boxed{y = 0.0800\sin(7.85x + 6\pi t - 0.785)\text{ m}}$$

16.32 (a) Let us write the wave function as $y(x, t) = A\sin(kx + \omega t + \phi)$

$$y(0, 0) = A\sin\phi = 0.0200 \text{ m}$$

$$\left.\frac{dy}{dt}\right|_{0,0} = A\omega\cos\phi = -2.00 \text{ m/s}$$

Also, $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.0250 \text{ s}} = 80.0\,\pi/\text{s}$

$$A^2 = x_i^2 + (v_i/\omega)^2 = (0.0200 \text{ m})^2 + \left(\frac{2.00 \text{ m/s}}{80.0\pi/\text{s}}\right)^2$$

$$A = \boxed{0.0215 \text{ m}}$$

(b) $\dfrac{A\sin\phi}{A\cos\phi} = \dfrac{0.0200}{-2/80.0\pi} = -2.51 = \tan\phi$

Your calculator's answer $\tan^{-1}(-2.51) = -1.19$ rad has a negative sine and positive cosine, just the reverse of what is required. You must look beyond your calculator to find

$$\phi = \pi - 1.19 \text{ rad} = \boxed{1.95 \text{ rad}}$$

(c) $v_{y,\max} = A\omega = 0.0215 \text{ m } (80.0\pi/\text{s}) = \boxed{5.41 \text{ m/s}}$

(d) $\lambda = v_x T = (30.0 \text{ m/s})(0.0250 \text{ s}) = 0.750 \text{ m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.750 \text{ m}} = 8.38/\text{m}$$

$\omega = 80.0\pi/\text{s}$

$$\boxed{y(x, t) = (0.0215 \text{ m}) \sin(8.38x \text{ rad/m} + 80.0\pi t \text{ rad/s} + 1.95 \text{ rad})}$$

16.33 (a) $f = \dfrac{v}{\lambda} = \dfrac{(1.00 \text{ m/s})}{2.00 \text{ m}} = \boxed{0.500 \text{ Hz}}$

$\omega = 2\pi f = 2\pi(0.500/\text{s}) = \boxed{3.14 \text{ rad/s}}$

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{2.00 \text{ m}} = \boxed{3.14 \text{ rad/m}}$

(c) $y = A \sin(kx - \omega t + \phi)$ becomes

$y = \boxed{(0.100 \text{ m}) \sin(3.14\, x/\text{m} - 3.14\, t/\text{s} + 0)}$

(d) For $x = 0$ the wave function requires

$$\boxed{y = (0.100 \text{ m}) \sin(-3.14\, t/\text{s})}$$

(e) $\boxed{y = (0.100 \text{ m}) \sin(4.71 \text{ rad} - 3.14\, t/\text{s})}$

(f) $v_y = \dfrac{\partial y}{\partial t} = 0.100 \text{ m } (-3.14/\text{s}) \cos(3.14\, x/\text{m} - 3.14\, t/\text{s})$

The cosine varies between +1 and –1, so

$$v_y \le \boxed{0.314 \text{ m/s}}$$

16.34 $y = (0.150 \text{ m}) \sin(3.10x - 9.30t)$ SI units

$$v = \frac{\omega}{k} = \frac{9.30}{3.10} = 3.00 \text{ m/s}$$

$s = vt = \boxed{30.0 \text{ m in positive } x\text{-direction}}$

16.35 $y = (0.0200\text{ m})\sin(2.11x - 3.62t)$ SI units

$A = \boxed{2.00\text{ cm}}$ $\quad k = 2.11\text{ rad/m}$

$$\lambda = \frac{2\pi}{k} = \boxed{2.98\text{ m}}$$

$\omega = 3.62\text{ rad/s}$

$$f = \frac{\omega}{2\pi} = \boxed{0.576\text{ Hz}}$$

$$v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k} = \frac{3.62}{2.11} = \boxed{1.72\text{ m/s}}$$

16.36 (a) $\omega = 2\pi f = 2\pi(500) = 3140\text{ rad/s}$, $k = \omega/v = (3140)/(196) = 16.0\text{ rad/m}$

$$\boxed{y = (2.00 \times 10^{-4}\text{ m})\sin(16.0x - 3140t)}$$

(b) $v = 196\text{ m/s} = \sqrt{\dfrac{T}{4.10 \times 10^{-3}\text{ kg/m}}}$

$T = \boxed{158\text{ N}}$

16.37 (a) at $x = 2.00$ m, $y = \boxed{(0.100\text{ m})\sin(1.00\text{ rad} - 20.0t)}$

(b) $y = (0.100\text{ m})\sin(0.500x - 20.0t) = A\sin(kx - \omega t)$

so $\omega = 20.0\text{ rad/s}$ and $f = \dfrac{\omega}{2\pi} = \boxed{3.18\text{ Hz}}$

16.38 $f = \dfrac{v}{\lambda} = \dfrac{30.0}{0.500} = 60.0\text{ Hz}$ $\quad \omega = 2\pi f = 120\pi\text{ rad/s}$

$$\wp = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\left(\frac{0.180}{3.60}\right)(120\pi)^2(0.100)^2(30.0) = \boxed{1.07\text{ kW}}$$

16.39 Suppose that no energy is absorbed or carried down into the water. Then a fixed amount of power is spread thinner farther away from the source, spread over the circumference $2\pi r$ of an expanding circle. The power-per-width across the wave front

$$\frac{\wp}{2\pi r}$$

is proportional to amplitude squared so amplitude is proportional to

$$\sqrt{\frac{\wp}{2\pi r}}$$

16.40 T = constant; $v = \sqrt{\frac{T}{\mu}}$; $\wp = \frac{1}{2}\mu\omega^2 A^2$

(a) If L is doubled, v remains constant and $\boxed{\wp \text{ is constant}}$.

(b) If A is doubled and ω is halved, $\wp \propto \omega^2 A^2$ $\boxed{\text{remains constant}}$.

(c) If λ and A are doubled, the product $\omega^2 A^2 \propto A^2/\lambda^2$ remains constant, so $\boxed{\wp \text{ remains constant}}$.

(d) If L and λ are halved, then $\omega^2 \propto 1/\lambda^2$ is quadrupled, so $\boxed{\wp \text{ is quadrupled}}$. (Changing L doesn't affect $\wp$).

16.41 $A = 5.00 \times 10^{-2}$ m $\mu = 4.00 \times 10^{-2}$ kg/m $\wp = 300$ W $T = 100$ N

Therefore,

$$v = \sqrt{\frac{T}{\mu}} = 50.0 \text{ m/s}$$

$$\wp = \frac{1}{2}\mu\omega^2 A^2 v$$

$$\omega^2 = \frac{2\wp}{\mu A^2 v} = \frac{2(300)}{(4.00 \times 10^{-2})(5.00 \times 10^{-2})^2(50.0)}$$

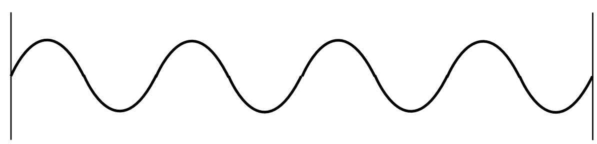
$$\omega = 346 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \boxed{55.1 \text{ Hz}}$$

***16.42** $\mu = 30.0$ g/m $= 30.0 \times 10^{-3}$ kg/m $\lambda = 1.50$ m

$f = 50.0$ Hz $\omega = 2\pi f = 314 \text{ s}^{-1}$

$2A = 0.150$ m $A = 7.50 \times 10^{-2}$ m



(a) $y = A \sin\left(\frac{2\pi}{\lambda}x - \omega t\right)$

$$\boxed{y = (7.50 \times 10^{-2} \text{ m}) \sin(4.19x - 314t)}$$

(b) $\wp = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}(30.0 \times 10^{-3})(314)^2(7.50 \times 10^{-2})^2\left(\frac{314}{4.19}\right)$ W

$$\boxed{\wp = 625 \text{ W}}$$

16.43 (a) $v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k} = \frac{50.0}{0.800}\text{ m/s} = \boxed{62.5\text{ m/s}}$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.800}\text{ m} = \boxed{7.85\text{ m}}$

(c) $f = \frac{50.0}{2\pi} = \boxed{7.96\text{ Hz}}$

(d) $\wp = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}(12.0\times 10^{-3})(50.0)^2(0.150)^2(62.5)\text{ W} = \boxed{21.1\text{ W}}$

16.44 Originally,

$$\wp_o = \frac{1}{2}\mu\omega^2A^2v$$

$$\wp_o = \frac{1}{2}\mu\omega^2A^2\sqrt{\frac{T}{\mu}}$$

$$\wp_o = \frac{1}{2}\omega^2A^2\sqrt{T\mu}$$

The doubled string will have doubled mass-per-length. Presuming that we hold tension constant, it can carry power larger by $\sqrt{2}$ times.

$$\boxed{\sqrt{2}\,\wp_o} = \frac{1}{2}\omega^2A^2\sqrt{T\,2\mu}$$

***16.45** (a) $A = (7.00 + 3.00)4.00$ yields $\boxed{A = 40.0}$

(b) In order for two vectors to be equal, they must have the same magnitude and the same direction in three-dimensional space. All of their components must be equal. Thus, $7.00\mathbf{i} + 0\mathbf{j} + 3.00\mathbf{k} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$ requires $\boxed{A = 7.00,\ B = 0,\text{ and }C = 3.00}$.

(c) In order for two functions to be identically equal, they must be equal for every value of every variable. They must have the same graphs. In $A + B\cos(Cx + Dt + E) = 0 + 7.00\text{ mm}\cos(3.00x + 4.00t + 2.00)$, the equality of average values requires that $\boxed{A = 0}$. The equality of maximum values requires $\boxed{B = 7.00\text{ mm}}$. The equality for the wavelength or periodicity as a function of x requires $\boxed{C = 3.00\text{ rad/m}}$. The equality of period requires $\boxed{D = 4.00\text{ rad/s}}$, and the equality of zero-crossings requires $\boxed{E = 2.00\text{ rad}}$.

16.46 Equation 16.26, with $v = \sqrt{T/\mu}$ is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

If $y = e^{b(x - vt)}$,

then $\frac{\partial y}{\partial t} = -bve^{b(x-vt)}$ and $\frac{\partial y}{\partial x} = be^{b(x-vt)}$

$$\frac{\partial^2 y}{\partial t^2} = b^2v^2e^{b(x-vt)} \quad \text{and} \quad \frac{\partial^2 y}{\partial x^2} = b^2e^{b(x-vt)}$$

Therefore, $\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$, and $e^{b(x-vt)}$ is a solution.

16.47 From Equation 16.25, $\left(\frac{\mu}{T}\right)\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$

To verify that $y = \ln[b(x - vt)]$ is a solution, find the first and second derivatives of y with respect to x and t and substitute into Equation 16.25:

$$\frac{\partial y}{\partial t} = [b(x - vt)]^{-1}(-bv); \frac{\partial^2 y}{\partial t^2} = -\frac{v^2}{(x - vt)^2}$$

$$\frac{\partial y}{\partial x} = [b(x - vt)]^{-1}(b); \frac{\partial^2 y}{\partial t^2} = -\frac{1}{(x - vt)^2}$$

Substituting into Equation 16.25 we have $\frac{\mu}{T}\left[-\frac{v^2}{(x - vt)^2}\right] = -\frac{1}{(x - vt)^2}$

But $v^2 = \frac{T}{\mu}$, therefore the given function is a solution.

16.48 (a) From $y = x^2 + v^2t^2$,

evaluate $\frac{\partial y}{\partial x} = 2x$ $\quad \frac{\partial^2 y}{\partial x^2} = 2$

$\frac{\partial y}{\partial t} = v^2 2t$ $\quad \frac{\partial^2 y}{\partial t^2} = 2v^2$

Does $\frac{\partial^2 y}{\partial t} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$?

By substitution: $2 = \frac{1}{v^2}\,2v^2$ and this is true, so the wave function does satisfy the wave equation.

(b) Note $\frac{1}{2}(x+vt)^2+\frac{1}{2}(x-vt)^2$

$$=\frac{1}{2}x^2+xvt+\frac{1}{2}v^2t^2+\frac{1}{2}x^2-xvt+\frac{1}{2}v^2t^2$$

$$=x^2+v^2t^2 \text{ as required}$$

So $\boxed{f(x+vt)=\frac{1}{2}(x+vt)^2}$ and $\boxed{g(x-vt)=\frac{1}{2}(x-vt)^2}$

(c) $y=\sin x\cos vt$ makes

$$\frac{\partial y}{\partial x}=\cos x\cos vt \qquad \frac{\partial^2 y}{\partial x^2}=-\sin x\cos vt$$

$$\frac{\partial y}{\partial t}=-v\sin x\sin vt \qquad \frac{\partial^2 y}{\partial t^2}=-v^2\sin x\cos vt$$

Then $\frac{\partial^2 y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

becomes $-\sin x\cos vt=\frac{-1}{v^2}v^2\sin x\cos vt$ which is true as required.

Note $\sin(x+vt)=\sin x\cos vt+\cos x\sin vt$

$\sin(x-vt)=\sin x\cos vt-\cos x\sin vt$

So $\sin x\cos vt=f(x+vt)+g(x-vt)$ with

$\boxed{f(x+vt)=\frac{1}{2}\sin(x+vt)}$ and $\boxed{g(x-vt)=\frac{1}{2}\sin(x-vt)}$

***16.49** Assume a typical distance between adjacent people ~1 m. Then the wave speed is

$$v=\frac{x}{t}\sim\frac{1\text{ m}}{0.1\text{ s}}\sim 10\text{ m/s}$$

Model the stadium as a circle with a radius of order 100 m. Then, the time for one circuit around the stadium is

$$T=\frac{2\pi r}{v}\sim\frac{2\pi(10^2\text{ m})}{10\text{ m/s}}=63\text{ s}\ \boxed{\sim 1\text{ min}}$$

16.50 Compare the given wave function $y = 4.00 \sin(2.00x - 3.00t)$ cm to the general form $y = A \sin(kx - \omega t)$ to find

(a) amplitude $A = 4.00 \text{ cm} = \boxed{0.0400 \text{ m}}$

(b) $k = \dfrac{2\pi}{\lambda} = 2.00 \text{ cm}^{-1}$ and $\lambda = \pi \text{ cm} = \boxed{0.0314 \text{ m}}$

(c) $\omega = 2\pi f = 3.00 \text{ s}^{-1}$ and $f = \boxed{0.477 \text{ Hz}}$

(d) $T = \dfrac{1}{f} = \boxed{2.09 \text{ s}}$

(e) The minus sign indicates that the wave is traveling in the $\boxed{\text{positive } x\text{-direction}}$.

16.51 (a) Let $u = 10\pi t - 3\pi x + \dfrac{\pi}{4}$

$$\frac{du}{dt} = 10\pi - 3\pi\frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = \frac{10}{3} = \boxed{3.33 \text{ m/s}}$$

The velocity is in the $\boxed{\text{positive } x\text{-direction}}$.

(b) $y(0.100, 0) = (0.350 \text{ m}) \sin\left(-0.300\pi + \dfrac{\pi}{4}\right) = -0.0548 \text{ m} = \boxed{-5.48 \text{ cm}}$

(c) $k = \dfrac{2\pi}{\lambda} = 3\pi$

$\lambda = \boxed{0.667 \text{ m}}$

$\omega = 2\pi f = 10\pi$

$f = \boxed{5.00 \text{ Hz}}$

(d) $v_y = \dfrac{\partial y}{\partial t} = (0.350)(10\pi) \cos\left(10\pi t - 3\pi x + \dfrac{\pi}{4}\right)$

$v_{y,\max} = (10\pi)(0.350) = \boxed{11.0 \text{ m/s}}$

***16.52** The equation $v = \lambda f$ is a special case of

speed = (cycle length)(repetition rate)

Thus, $v = \left(19.0 \times 10^{-3} \dfrac{\text{m}}{\text{frame}}\right)\left(24.0 \dfrac{\text{frames}}{\text{s}}\right) = \boxed{0.456 \text{ m/s}}$

16.53 Assuming the incline to be frictionless and taking the positive x-direction to be up the incline:

$$\Sigma F_x = T - Mg \sin \theta = 0$$

or the tension in the string is $T = Mg \sin \theta$

The speed of transverse waves in the string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg \sin \theta}{m/L}} = \sqrt{\frac{MgL \sin \theta}{m}}$$

and the time for a pulse to travel the length of the string is

$$t = \frac{L}{v} = L\sqrt{\frac{m}{MgL \sin \theta}} = \boxed{\sqrt{\frac{mL}{Mg \sin \theta}}}$$

16.54 (a) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{80.0 \text{ N}}{(5.00 \times 10^{-3} \text{ kg}/2.00 \text{ m})}} = \boxed{179 \text{ m/s}}$

(b) From Equation 16.21, $\wp = \dfrac{1}{2} \mu v \omega^2 A^2$ and $\omega = 2\pi\left(\dfrac{v}{\lambda}\right)$

$$\wp = \frac{1}{2} \mu v A^2 \left(\frac{2\pi v}{\lambda}\right)^2 = \frac{2\pi^2 \mu A^2 v^3}{\lambda^2}$$

$$\wp = \frac{2\pi^2 (5.00 \times 10^{-3} \text{ kg}/2.00 \text{ m})(0.0400 \text{ m})^2 (179 \text{ m/s})^3}{(0.160 \text{ m})^2}$$

$$\wp = 1.77 \times 10^4 \text{ W} = \boxed{17.7 \text{ kW}}$$

16.55 Energy is conserved as the block moves down distance x:

$$(K + U_g + U_s)_{\text{top}} + \Delta E = (K + U_g + U_s)_{\text{bottom}}$$

$$0 + Mgx + 0 + 0 = 0 + 0 + \frac{1}{2} kx^2$$

$$x = \frac{2Mg}{k}$$

(a) $T = kx = 2Mg = 2(2.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{39.2 \text{ N}}$

(b) $L = L_0 + x = L_0 + \dfrac{2Mg}{k}$

$L = 0.500\text{ m} + \dfrac{39.2\text{ N}}{100\text{ N/m}} = \boxed{0.892\text{ m}}$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}}$

$v = \sqrt{\dfrac{39.2\text{ N} \times 0.892\text{ m}}{5.00 \times 10^{-3}\text{ kg}}}$

$v = \boxed{83.6\text{ m/s}}$

16.56 $Mgx = \dfrac{1}{2}kx^2$

(a) $T = kx = \boxed{2Mg}$

(b) $L = L_0 + x = \boxed{L_0 + \dfrac{2Mg}{k}}$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}} = \boxed{\sqrt{\dfrac{2Mg}{m}\left(L_0 + \dfrac{2Mg}{k}\right)}}$

16.57 $v = \sqrt{\dfrac{T}{\mu}}$ and in this case $T = mg$; therefore, $m = \dfrac{\mu v^2}{g}$

From Equation 16.13, $v = \omega/k$ so that

$$m = \frac{\mu}{g}\left(\frac{\omega}{k}\right)^2 = \frac{0.250\text{ kg/m}}{9.80\text{ m/s}^2}\left[\frac{18\pi\text{ s}^{-1}}{0.750\pi\text{ m}^{-1}}\right]^2 = \boxed{14.7\text{ kg}}$$

16.58 (a) $\mu = \dfrac{dm}{dL} = \rho A\dfrac{dx}{dx} = \rho A$

$v = \sqrt{T/\mu} = \sqrt{T/\rho A} = \sqrt{T/[\rho(ax + b)]}$

$= \sqrt{T/[\rho(10^{-3}x + 10^{-2})\text{cm}^2]}$

With all SI units, $\boxed{v = \sqrt{T/[\rho(10^{-3}x + 10^{-2})10^{-4}]}\text{ m/s}}$

(b) $v\big|_{x=0} = \sqrt{24.0/[(2700)(0 + 10^{-2})(10^{-4})]} = \boxed{94.3\text{ m/s}}$

$v\big|_{x=10.0} = \sqrt{24.0/[(2700)(10^{-2} + 10^{-2})(10^{-4})]} = \boxed{66.7\text{ m/s}}$

16.59 $v = \sqrt{\dfrac{T}{\mu}}$ where $T = \mu x g$, the weight of a length x, of rope.

Therefore, $v = \sqrt{gx}$

But $v = \dfrac{dx}{dt}$ so that

$$dt = \frac{dx}{\sqrt{gx}}$$

and $t = \displaystyle\int_0^L \frac{dx}{\sqrt{gx}} = \boxed{2\sqrt{\frac{L}{g}}}$

16.60 At distance x from the bottom, the tension is $T = (mxg/L) + Mg$, so the wave speed is:

$$v = \sqrt{T/\mu} = \sqrt{TL/m} = \sqrt{xg + (MgL/m)} = \frac{dx}{dt}$$

Then

(a) $t = \displaystyle\int_0^t dt = \int_0^L [xg + (MgL/m)]^{-1/2}\,dx$

$$t = \frac{1}{g}\left.\frac{[xg + (MgL/m)]^{1/2}}{\frac{1}{2}}\right|_{x=0}^{x=L}$$

$$t = \frac{2}{g}\left[(Lg + MgL/m)^{1/2} - (MgL/m)^{1/2}\right]$$

$$\boxed{t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m+M} - \sqrt{M}}{\sqrt{m}}\right)}$$

(b) When $M = 0$, $t = 2\sqrt{\dfrac{L}{g}}\left(\dfrac{\sqrt{m} - 0}{\sqrt{m}}\right) = \boxed{2\sqrt{\dfrac{L}{g}}}$ as in Problem 59.

(c) As $m \to 0$ we expand

$$\sqrt{m+M} = \sqrt{M}\,(1+m/M)^{1/2} = \sqrt{M}\left(1+\frac{1}{2}m/M - \frac{1}{8}m^2/M^2 + \ldots\right)$$

to obtain

$$t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{M}+\frac{1}{2}m/\sqrt{M}-\frac{1}{8}m^2/M^{3/2}+\ldots-\sqrt{M}}{\sqrt{m}}\right)$$

$$t \approx 2\sqrt{\frac{L}{g}}\left(\frac{1}{2}\sqrt{\frac{m}{M}}\right) = \boxed{\sqrt{\frac{mL}{Mg}}}$$

16.61 (a) The speed in the lower half of a rope of length L is the same function of distance (from the bottom end) as the speed along the entire length of a rope of length $(L/2)$.

Thus, the time required = $2\sqrt{\frac{L'}{g}}$ with $L' = \frac{L}{2}$

and the time required = $2\sqrt{\frac{L}{2g}} = \boxed{0.707\left(2\sqrt{\frac{L}{g}}\right)}$

It takes the pulse more that 70% of the total time to cover 50% of the distance.

(b) By the same reasoning applied in part (a), the distance climbed in τ is given by

$$d = \frac{g\tau^2}{4}$$

For $\tau = \frac{t}{2} = \sqrt{\frac{L}{g}}$, we find the *distance climbed* = $\boxed{\frac{L}{4}}$

In half the total trip time, the pulse has climbed $\frac{1}{4}$ of the total length.

Goal Solution

G: The wave pulse travels faster as it goes up the rope because the tension higher in the rope is greater (to support the weight of the rope below it). Therefore it should take more than half the total time t for the wave to travel halfway up the rope. Likewise, the pulse should travel less than halfway up the rope in time $t/2$.

O: By using the time relationship given in the problem and making suitable substitutions, we can find the required time and distance.

A: (a) From the equation given, the time for a pulse to travel any distance, d, up from the bottom of a rope is $t_d = 2\sqrt{\frac{d}{g}}$.

So the time for a pulse to travel a distance $L/2$ from the bottom is

$$t_{L/2} = 2\sqrt{\frac{L}{2g}} = 0.707\left(2\sqrt{\frac{L}{g}}\right)$$

(b) Likewise, the distance a pulse travels from the bottom of a rope in a time t_d is $d = \frac{g t_d^2}{4}$.

So the distance traveled by a pulse after a time $t_d = \sqrt{L/g}$ is

$$d = \frac{g(L/g)}{4} = \frac{L}{4}$$

L: As expected, it takes the pulse more than 70% of the total time to cover 50% of the distance. In half the total trip time, the pulse has climbed only 1/4 of the total length.

16.62 (a) $v = \frac{\omega}{k} = \frac{15.0}{3.00} = \boxed{5.00 \text{ m/s in positive } x\text{-direction}}$

(b) $v = \frac{15.0}{3.00} = \boxed{5.00 \text{ m/s in negative } x\text{-direction}}$

(c) $v = \frac{15.0}{2.00} = \boxed{7.50 \text{ m/s in negative } x\text{-direction}}$

(d) $v = \frac{12.0}{1/2} = \boxed{24.0 \text{ m/s in positive } x\text{-direction}}$

16.63 Young's modulus for the wire may be written as $Y = \frac{T/A}{\Delta L/L}$, where T is the tension maintained in the wire and ΔL is the elongation produced by this tension. Also, the mass density of the wire may be expressed as $\rho = \frac{\mu}{A}$.

The speed of transverse waves in the wire is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T/A}{\mu/A}} = \sqrt{\frac{Y(\Delta L/L)}{\rho}}$$

and the strain in the wire is $\frac{\Delta L}{L} = \frac{\rho v^2}{Y}$

If the wire is aluminum and $v = 100$ m/s, the strain is

$$\frac{\Delta L}{L} = \frac{(2.70 \times 10^3 \text{ kg/m}^3)(100 \text{ m/s})^2}{7.00 \times 10^{10} \text{ N/m}^2} = \boxed{3.86 \times 10^{-4}}$$

16.64 (a) For an increment of spring length dx and mass dm, $F = ma$ becomes

$$k\,dx = a\,dm, \quad \text{or} \quad \frac{k}{(dm/dx)} = a$$

But $\dfrac{dm}{dx} = \mu$ so $a = \dfrac{k}{\mu}$

Also, $a = \dfrac{dv}{dt} = \dfrac{v}{t}$ when $v_i = 0$. But $L = vt$, so $a = \dfrac{v^2}{L}$.

Equating the two expressions for a, we have $\dfrac{k}{\mu} = \dfrac{v^2}{L}$ or

$$\boxed{v = \sqrt{\frac{kL}{\mu}}}$$

(b) Using the expression from part (a)

$$v = \sqrt{\frac{kL}{\mu}} = \sqrt{\frac{kL^2}{m}} = \sqrt{\frac{(100\text{ N/m})(2.00\text{ m})^2}{0.400\text{ kg}}} = \boxed{31.6\text{ m/s}}$$

16.65 (a) $v = (T/\mu)^{1/2} = (2T_0/\mu_o)^{1/2} = \boxed{v_0\sqrt{2}}$ where $v_0 \equiv (T_0/\mu_0)^{1/2}$

$$v' = (T/\mu')^{1/2} = (2T_0/3\mu_0)^{1/2} = \boxed{v_0\sqrt{2/3}}$$

(b) $t_{\text{left}} = \dfrac{L/2}{v} = \dfrac{L}{2v_0\sqrt{2}} = \dfrac{t_0}{2\sqrt{2}} = 0.354t_0$ where $t_0 \equiv \dfrac{L}{v_0}$

$$t_{\text{right}} = \frac{L/2}{v'} = \frac{L}{2v_0\sqrt{2/3}} = \frac{t_0}{2\sqrt{2/3}} = 0.612t_0$$

$$t_{\text{left}} + t_{\text{right}} = \boxed{0.966t_0}$$

16.66 (a) $\wp(x) = \dfrac{1}{2}\mu\omega^2A^2v = \dfrac{1}{2}\mu\omega^2A_0^2\,e^{-2bx}\left(\dfrac{\omega}{k}\right) = \boxed{\dfrac{\mu\omega^2}{2k}A_0^2\,e^{-2bx}}$

(b) $\wp(0) = \boxed{\dfrac{\mu\omega^2}{2k}A_0^2}$

(c) $\dfrac{\wp(x)}{\wp(0)} = \boxed{e^{-2bx}}$

16.67 $v = \dfrac{4450\text{ km}}{9.50\text{ h}} = 468\text{ km/h} = \boxed{130\text{ m/s}}$

$$\bar{d} = \frac{v^2}{g} = \frac{(130\text{ m/s})^2}{(9.80\text{ m/s}^2)} = \boxed{1730\text{ m}}$$

Chapter 17 Solutions

17.1 Since $v_{\text{light}} >> v_{\text{sound}}$, $d \approx (343 \text{ m/s})(16.2 \text{ s}) = \boxed{5.56 \text{ km}}$

<u>Goal Solution</u>

G: There is a common rule of thumb that lightning is about a mile away for every 5 seconds of delay between the flash and thunder (or ~3 s/km). Therefore, this lightning strike is about 3 miles (~5 km) away.

O: The distance can be found from the speed of sound and the elapsed time. The time for the light to travel to the observer will be much less than the sound delay, so the speed of light can be ignored.

A: Assuming that the speed of sound is constant through the air between the lightning strike and the observer,

$$v_s = \frac{d}{\Delta t} \quad \text{or} \quad d = v_s\,\Delta t = (343 \text{ m/s})(16.2 \text{ s}) = 5.56 \text{ km}$$

L: Our calculated answer is consistent with our initial estimate, but we should check the validity of our assumption that the speed of light could be ignored. The time delay for the light is

$$t_{\text{light}} = \frac{d}{c} = \frac{5560 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 1.85 \times 10^{-5} \text{ s}$$

and $\Delta t = t_{\text{sound}} - t_{\text{light}} = 16.2 \text{ s} - 1.85 \times 10^{-5} \text{ s} \approx 16.2 \text{ s}$ (when properly rounded)

Since the travel time for the light is much smaller than the uncertainty in the time of 16.2 s, t_{light} can be ignored without affecting the distance calculation. However, our assumption of a constant speed of sound in air is probably not valid due to local variations in air temperature during a storm. We must assume that the given speed of sound in air is an accurate *average* value for the conditions described.

17.2 $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{2.80 \times 10^{10}}{13.6 \times 10^3}} = \boxed{1.43 \text{ km/s}}$

17.3 Sound takes this time to reach the man:

$$\frac{(20.0 \text{ m} - 1.75 \text{ m})}{343 \text{ m/s}} = 5.32 \times 10^{-2} \text{ s}$$

so the warning should be shouted no later than

$$0.300 \text{ s} + 5.32 \times 10^{-2} \text{ s} = 0.353 \text{ s} \quad \text{before the pot strikes.}$$

Since the whole time of fall is given by

$$y = \frac{1}{2}\, gt^2 \qquad 18.25\text{ m} = \frac{1}{2}(9.80\text{ m/s}^2)\, t^2$$

$$t = 1.93\text{ s}$$

the warning needs to come 1.93 s – 0.353 s = 1.58 s

into the fall, when the pot has fallen $\frac{1}{2}(9.80\text{ m/s}^2)(1.58\text{ s})^2 = 12.2\text{ m}$

to be above the ground by 20.0 m – 12.2 m = $\boxed{7.82\text{ m}}$

17.4 $v(\text{air}) = 343\text{ m/s}$ and $v(\text{salt water}) = 1533\text{ m/s}$

Let d = width of inlet $t = \dfrac{d}{v_w}$; $t + 4.50 = \dfrac{d}{v_a}$, so $\dfrac{d}{v_w} + 4.50 = \dfrac{d}{v_a}$

$$d = \frac{4.50 v_w v_a}{v_w - v_a} = \frac{(4.50)(1533)(343)}{1533 - 343} = \boxed{1.99\text{ km}}$$

17.5 (a) At 9000 m, $\Delta T = \left(\dfrac{9000}{150}\right)(-1.00°\text{C}) = -60.0°\text{C}$ so $T = -30.0°\text{C}$

Using the chain rule:

$$\frac{dv}{dt} = \frac{dv}{dT}\frac{dT}{dx}\frac{dx}{dt} = v\frac{dv}{dT}\frac{dT}{dx} = v(0.607)\left(\frac{1}{150}\right) = \frac{v}{247}\text{, so } dt = (247\text{ s})\frac{dv}{v}$$

$$\int_0^t dt = (247\text{ s})\int_{v_i}^{v_f}\frac{dv}{v}$$

$$t = (247\text{ s})\ln\left(\frac{v_f}{v_i}\right) = (247\text{ s})\ln\left[\frac{331.5 + 0.607(30.0)}{331.5 + 0.607(-30.0)}\right]$$

$t = \boxed{27.2\text{ s}}$ for sound to reach ground

(b) $$t = \frac{h}{v} = \frac{9000}{[331.5 + 0.607(30.0)]} = \boxed{25.7\text{ s}}$$

It takes longer when the air cools off than if it were at a uniform temperature.

17.6 From $\lambda = \dfrac{v}{f}$, we get: $\lambda = \dfrac{340\text{ m/s}}{60.0 \times 10^3\text{ s}^{-1}} = \boxed{5.67\text{ mm}}$

17.7 It is easiest to solve part (b) first:

(b) The distance the sound travels to the plane is $d_s = \sqrt{h^2 + (h/2)^2} = h\sqrt{5}/2$

The sound travels this distance in 2.00 s, so

$$d_s = \frac{h\sqrt{5}}{2} = (343 \text{ m/s})(2.00 \text{ s}) = 686 \text{ m}$$

giving the altitude of the plane as $h = \dfrac{2(686 \text{ m})}{\sqrt{5}} = \boxed{614 \text{ m}}$

(a) The distance the plane has traveled in 2.00 s is $v(2.00 \text{ s}) = h/2 = 307$ m

Thus, the speed of the plane is: $v = \dfrac{307 \text{ m}}{2.00 \text{ s}} = \boxed{153 \text{ m/s}}$

17.8 $\Delta P_{\text{max}} = \rho v \omega s_{\text{max}}$

$$s_{\text{max}} = \frac{\Delta P_{\text{max}}}{\rho v \omega} = \frac{(4.00 \times 10^{-3} \text{ N/m}^2)}{(1.20 \text{ kg/m}^3)(343 \text{ m/s})(2\pi)(10.0 \times 10^3 \text{ s}^{-1})} = \boxed{1.55 \times 10^{-10} \text{ m}}$$

17.9 (a) $A = \boxed{2.00\ \mu\text{m}}$ $\lambda = \dfrac{2\pi}{15.7} = 0.400 \text{ m} = \boxed{40.0 \text{ cm}}$

$$v = \frac{\omega}{k} = \frac{858}{157} = \boxed{54.6 \text{ m/s}}$$

(b) $s = 2.00 \cos[(15.7)(0.0500) - (858)(3.00 \times 10^{-3})] = \boxed{-0.433\mu\text{m}}$

(c) $v_{\text{max}} = A\omega = (2.00\ \mu\text{m})(858 \text{ s}^{-1}) = \boxed{1.72 \text{ mm/s}}$

17.10 (a) $\Delta P = (1.27 \text{ Pa})\sin(\pi x/\text{m} - 340\pi t/\text{s})$ (SI units)

The pressure amplitude is: $\Delta P_{\text{max}} = \boxed{1.27 \text{ Pa}}$

(b) $\omega = 2\pi f = 340\pi/\text{s}$, so $f = \boxed{170 \text{ Hz}}$

(c) $k = \dfrac{2\pi}{\lambda} = \pi/\text{m}$, giving $\lambda = \boxed{2.00 \text{ m}}$

(d) $v = \lambda f = (2.00 \text{ m})(170 \text{ Hz}) = \boxed{340 \text{ m/s}}$

17.11 $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{(0.100\text{ m})} = 62.8\text{ m}^{-1}$

$$\omega = \frac{2\pi v}{\lambda} = \frac{2\pi(343\text{ m/s})}{(0.100\text{ m})} = 2.16 \times 10^4\text{ s}^{-1}$$

Therefore, $\boxed{\Delta P = (0.200\text{ Pa})\sin[62.8x/\text{m} - 2.16 \times 10^4 t/\text{s}]}$

17.12 $\omega = 2\pi f = \dfrac{2\pi v}{\lambda} = \dfrac{2\pi(343\text{ m/s})}{(0.100\text{ m})} = 2.16 \times 10^4\text{ rad/s}$

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v\omega} = \frac{(0.200\text{ Pa})}{(1.20\text{ kg/m}^3)(343\text{ m/s})(2.16 \times 10^4\text{ s}^{-1})} = 2.25 \times 10^{-8}\text{ m}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.100\text{ m})} = 62.8\text{ m}^{-1}$$

Therefore, $\boxed{s = s_{\max}\cos(kx - \omega t) = (2.25 \times 10^{-8}\text{ m})\cos(62.8x/\text{m} - 2.16 \times 10^4 t/\text{s})}$

17.13 (a) The sound "pressure" is extra tensile stress for one-half of each cycle. When it becomes $(0.500\%)(13.0 \times 10^{10}\text{ Pa}) = 6.50 \times 10^8\text{ Pa}$, the rod will break.

Then, $\Delta P_{\max} = \rho v \omega s_{\max}$

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v\omega} = \frac{6.50 \times 10^8\text{ N/m}^2}{(8.92 \times 10^3\text{ kg/m}^3)(3560\text{ m/s})(2\pi 500/\text{s})} = \boxed{6.52\text{ mm}}$$

(b) From $s = s_{\max}\cos(kx - \omega t)$

$$v = \frac{\partial s}{\partial t} = -\omega s_{\max}\sin(kx - \omega t)$$

$$v_{\max} = \omega s_{\max} = (2\pi\, 500/\text{s})(6.52\text{ mm}) = \boxed{20.5\text{ m/s}}$$

17.14 $\Delta P_{\max} = \rho\omega v s_{\max} = (1.20\text{ kg/m}^3)[2\pi(2000\text{ s}^{-1})](343\text{ m/s})(2.00 \times 10^{-8}\text{ m})$

$$\Delta P_{\max} = \boxed{0.103\text{ Pa}}$$

17.15 $\Delta P_{\max} = \rho v\omega s_{\max} = \rho v\left(\dfrac{2\pi v}{\lambda}\right) s_{\max}$

$$\lambda = \frac{2\pi\rho v^2 s_{\max}}{\Delta P_{\max}}$$

$$\lambda = \frac{2\pi(1.20)(343)^2(5.50 \times 10^{-6})}{0.840} = \boxed{5.81\text{ m}}$$

17.16 (a) $\Delta P = \Delta P_{\max} \sin[kx - \omega t + \phi]$ with $\Delta P_{\max} = 4.00$ Pa

$\Delta P(0, 0) = \Delta P_{\max} \sin\phi = 0 \Rightarrow \phi = 0$

$\omega = 2\pi f = 2\pi(5000\ \text{s}^{-1}) = \pi \times 10^4\ \text{s}^{-1}$

Therefore, $\Delta P = (4.00\ \text{Pa}) \sin(kx - \pi \times 10^4\ t/\text{s})$

At $x = 0$, $t = 2.00 \times 10^{-4}$ s, $\Delta P = (4.00\ \text{Pa}) \sin(0 - 2.00\pi) = \boxed{0}$

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{v} = \dfrac{\pi \times 10^4\ \text{s}^{-1}}{343\ \text{m/s}} = 91.5\ \text{m}^{-1}$

At $x = 0.0200$ m, $t = 0$, $\Delta P = (4.00\ \text{Pa}) \sin[(91.5\ \text{m}^{-1})(0.0200\ \text{m}) - 0]$

$\Delta P = \boxed{3.87\ \text{Pa}}$

17.17 $\beta = 10 \log\left(\dfrac{I}{I_0}\right) = 10 \log\left(\dfrac{4.00 \times 10^{-6}}{1.00 \times 10^{-12}}\right) = \boxed{66.0\ \text{dB}}$

17.18 (a) $70.0\ \text{dB} = 10 \log\left(\dfrac{I}{1.00 \times 10^{-12}\ \text{W/m}^{-2}}\right)$

Therefore, $I = (1.00 \times 10^{-12}\ \text{W/m}^{-2})10^{(70.0/10)} = \boxed{1.00 \times 10^{-5}\ \text{W/m}^2}$

(b) $I = \Delta P_{\max}^2 / 2\rho v$, so

$\Delta P_{\max} = \sqrt{2\rho v I} = \sqrt{2(1.20\ \text{kg/m}^3)(343\ \text{m/s})(1.00 \times 10^{-5}\ \text{W/m}^2)}$

$\Delta P_{\max} = \boxed{90.7\ \text{mPa}}$

17.19 $I = \dfrac{1}{2} \rho\omega^2 s_{\max}^2 v$

(a) At $f = 2500$ Hz, the frequency is increased by a factor of 2.50, so the intensity (at constant $s_{\max}$) increases by $(2.50)^2 = 6.25$.

Therefore, $6.25(0.600) = \boxed{3.75\ \text{W/m}^2}$

(b) $\boxed{0.600\ \text{W/m}^2}$

17.20 The original intensity is $I_1 = \frac{1}{2}\rho\omega^2 s_{max}^2 v = 2\pi^2\rho v f^2 s_{max}^2$

(a) If the frequency is increased to $f\,'$ while a constant displacement amplitude is maintained, the new intensity is

$$I_2 = 2\pi^2\rho v(f\,')^2 s_{max}^2 \quad \text{so} \quad \frac{I_2}{I_1} = \frac{2\pi^2\rho v(f\,')s_{max}^2}{2\pi^2\rho v f^2 s_{max}^2} = \left(\frac{f\,'}{f}\right)^2 \quad \text{or} \quad \boxed{I_2 = \left(\frac{f\,'}{f}\right)^2 I_1}$$

(b) If the frequency is reduced to $f\,' = f/2$ while the displacement amplitude is doubled, the new intensity is

$$I_2 = 2\pi^2\rho v\left(\frac{f}{2}\right)^2(2s_{max})^2 = 2\pi^2\rho v f^2 s_{max}^2 = I_1$$

or the $\boxed{\text{intensity is unchanged}}$.

17.21 (a) $I_1 = (1.00 \times 10^{-12}\ \text{W/m}^2)10^{(\beta_1/10)} = (1.00 \times 10^{-12}\ \text{W/m}^2)10^{80.0/10}$

or $I_1 = 1.00 \times 10^{-4}\ \text{W/m}^2$

$I_2 = (1.00 \times 10^{-12}\ \text{W/m}^2)10^{(\beta_2/10)} = (1.00 \times 10^{-12}\ \text{W/m}^2)10^{75.0/10}$

or $I_2 = 1.00 \times 10^{-4.5}\ \text{W/m}^2 = 31.6 \times 10^{-5}\ \text{W/m}^2$

When both sounds are present, the total intensity is

$$I = I_1 + I_2 = 1.00 \times 10^{-4}\ \text{W/m}^2 + 31.6 \times 10^{-5}\ \text{W/m}^2 = \boxed{1.32 \times 10^{-4}\ \text{W/m}^2}$$

(b) The decibel level for the combined sounds is

$$\beta = 10\log\left(\frac{1.32 \times 10^{-4}\ \text{W/m}^2}{1.00 \times 10^{-12}\ \text{W/m}^2}\right) = 10\log(1.32 \times 10^8) = \boxed{81.2\ \text{db}}$$

17.22 We begin with $\beta_2 = 10\log\left(\frac{I_2}{I_0}\right)$, and $\beta_1 = 10\log\left(\frac{I_1}{I_0}\right)$, so

$$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$$

Also, $I_2 = \frac{\wp}{4\pi r_2^2}$, and $I_1 = \frac{\wp}{4\pi r_1^2}$, giving $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$

Then, $\beta_2 - \beta_1 = 10\log\left(\frac{r_1}{r_2}\right)^2 = \boxed{20\log\left(\frac{r_1}{r_2}\right)}$

17.23 Since intensity is inversely proportional to the square of the distance,

$$I_4 = \frac{1}{100} I_{0.4} \quad \text{and} \quad I_{0.4} = \frac{\Delta P_{\max}^2}{2\rho v} = \frac{(10.0)^2}{2(1.20)(343)} = 0.121 \text{ W/m}^2$$

The difference in sound intensity level is

$$\Delta\beta = 10 \log\left(\frac{I_{4\text{ km}}}{I_{0.4\text{ km}}}\right) = 10(-200) = -20.0 \text{ dB}$$

At 0.400 km,

$$\beta_{0.4} = 10 \log\left(\frac{0.121 \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right) = 110.8 \text{ db}$$

At 4.00 km,

$$\beta_4 = \beta_{0.4} + \Delta\beta = (110.8 - 20.0) \text{ dB} = 90.8 \text{ dB}$$

Allowing for absorption of the wave over the distance traveled,

$$\beta_4' = \beta_4 - (7.00 \text{ dB/km})(3.60 \text{ km}) = \boxed{65.6 \text{ dB}}$$

This is equivalent to the sound intensity level of heavy traffic.

Goal Solution

G: At a distance of 4 km, an explosion should be audible, but probably not extremely loud. So based on the data in Table 17.2, we might expect the sound level to be somewhere between 50 and 100 dB.

O: From the sound pressure data given in the problem, we can find the intensity, which is used to find the sound level in dB. The sound intensity will decrease with increased distance from the source and from the absorption of the sound by the air.

A: At a distance of 400 m from the explosion, $\Delta P_{\max} = 10$ Pa.

At this point, $I_{\max} = \dfrac{(10 \text{ N/m}^2)^2}{2(1.20 \text{ kg/m}^3)(343 \text{ m/s})} = 0.121 \text{ W/m}^2$

Therefore, the maximum sound level is

$$\beta_{\max} = 10 \log\left(\frac{I_{\max}}{I_0}\right) = 10 \log\left(\frac{0.121 \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2}\right) = 111 \text{ dB}$$

From equations 17.8 and 17.7, we can calculate the intensity and decibel level (due to distance alone) 4 km away:

$$I' = \frac{I(400\text{ m})^2}{(4000\text{ m})^2} = 1.21 \times 10^{-3}\text{ W/m}^2 \quad \text{and} \quad \beta = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{1.21\times10^{-3}}{1.00\times10^{-12}}\right) = 90.8\text{ dB}$$

At a distance of 4 km from the explosion, absorption from the air will have decreased the sound level by an additional $\Delta\beta = (7\text{ dB/km})(3.6\text{ km}) = 25.2\text{ dB}$

So at 4 km, the sound level will be $\beta_f = \beta - \Delta\beta = 90.8\text{ dB} - 25.2\text{ dB} = \boxed{65.6\text{ dB}}$

L: This sound level falls within our expected range. Evidently, this explosion is rather loud (about the same as a vacuum cleaner) even at a distance of 4 km from the source. It is interesting to note that the distance and absorption effects each reduce the sound level by about the same amount (~20 dB). If the explosion were at ground level, the sound level would be further reduced by reflection and absorption from obstacles between the source and observer, and the calculation would be much more complicated (if not impossible).

17.24 Let r_1 and r_2 be the distance from the speaker to the observer that hears 60.0 dB and 80.0 dB, respectively. Use the result of problem 22,

$$\beta_2 - \beta_1 = 20\log\left(\frac{r_1}{r_2}\right), \quad \text{to obtain} \quad 80.0 - 60.0 = 20\log\left(\frac{r_1}{r_2}\right)$$

Thus, $\log\left(\frac{r_1}{r_2}\right) = 1$, so $r_1 = 10.0r_2$. Also: $r_1 + r_2 = 110\text{ m}$, so

$10.0r_2 + r_2 = 110\text{ m}$ giving $\boxed{r_2 = 10.0\text{ m}}$, and $\boxed{r_1 = 100\text{ m}}$

17.25 We presume the speakers broadcast equally in all directions.

(a) $r_{AC} = \sqrt{3.00^2 + 4.00^2}\text{ m} = 5.00\text{ m}$

$I = \wp/4\pi r^2 = 1.00\times10^{-3}\text{ W}/4\pi(5.00\text{ m})^2 = 3.18\times10^{-6}\text{ W/m}^2$

$\beta = 10\text{ dB}\log(3.18\times10^{-6}\text{ W/m}^2/10^{-12}\text{ W/m}^2)$

$\beta = 10\text{ dB } 6.50 = \boxed{65.0\text{ dB}}$

(b) $r_{BC} = 4.47\text{ m}$

$I = 1.50\times10^{-3}\text{ W}/4\pi(4.47\text{ m})^2 = 5.97\times10^{-6}\text{ W/m}^2$

$\beta = 10\text{ dB}\log(5.97\times10^{-6}/10^{-12})$

$\beta = \boxed{67.8\text{ dB}}$

(c) $I = 3.18\ \mu\text{W/m}^2 + 5.97\ \mu\text{W/m}^2$

$\beta = 10\text{ dB}\log(9.15\times10^{-6}/10^{-12}) = \boxed{69.6\text{ dB}}$

17.26 $I = \dfrac{\wp}{4\pi r^2}$, where $I = 1.20\text{ W/m}^2$

$\wp = 4\pi r^2 I = 4\pi(4.00)^2(1.20) = \boxed{241\text{ W}}$

17.27 $40.0\text{ dB} = 10\text{ dB}\log\left(\dfrac{I}{10^{-12}\text{ W/m}^2}\right)$

$4.00 = \log\dfrac{I}{10^{-12}}$

$I = 10^{-12}(1.00\times10^4) = 1.00\times10^{-8}\ \text{W/m}^2$

$\wp = 4\pi r^2 I = (4\pi)(9.00)(1.00\times10^{-8}) = \boxed{1.13\ \mu\text{W}}$

***17.28** In $I = \dfrac{\wp}{4\pi r^2}$, intensity I is proportional to $\dfrac{1}{r^2}$,

so between locations 1 and 2: $\dfrac{I_2}{I_1} = \dfrac{r_1^2}{r_2^2}$

In $I = \dfrac{1}{2}\rho v(\omega s_{\max})^2$, intensity is proportional to $s_{\max}^2$, so $\dfrac{I_2}{I_1} = \dfrac{s_2^2}{s_1^2}$

Then, $\left(\dfrac{s_2}{s_1}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^2$ or $\left(\dfrac{1}{2}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^2$, giving $r_2 = 2r_1 = 2(50.0\text{ m}) = 100\text{ m}$

But, $r_2 = \sqrt{(50.0\text{ m})^2 + d^2}$ yields $d = \boxed{86.6\text{ m}}$

17.29 $\beta = 10\log\left(\dfrac{I}{10^{-12}}\right)$ $\qquad I = [10^{(\beta/10)}](10^{-12})\text{W/m}^2$

$I_{(120\text{ dB})} = 1.00\text{ W/m}^2$; $I_{(100\text{ dB})} = 1.00\times10^{-2}\text{ W/m}^2$; $I_{(10\text{ dB})} = 1.00\times10^{-11}\text{ W/m}^2$

(a) $\wp = 4\pi r^2 I$ so that $r_1^2 I_1 = r_2^2 I_2$

$r_2 = r_1(I_1/I_2)^{1/2} = (3.00\text{ m})\sqrt{\dfrac{1.00}{1.00\times10^{-2}}} = \boxed{30.0\text{ m}}$

(b) $r_2 = r_1(I_1/I_2)^{1/2} = (3.00\text{ m})\sqrt{\dfrac{1.00}{1.00\times10^{-11}}} = \boxed{9.49\times10^5\text{ m}}$

17.30 (a) $E = \wp t = 4\pi r^2 It = 4\pi(100\text{ m})^2(7.00 \times 10^{-2}\text{ W/m}^2)(0.200\text{ s}) = \boxed{1.76\text{ kJ}}$

(b) $\beta = 10\log\left(\dfrac{7.00 \times 10^{-2}}{1.00 \times 10^{-12}}\right) = \boxed{108\text{ dB}}$

***17.31** (a) The sound intensity inside the church is given by

$$\beta = 10\ln(I/I_0)$$

$$101\text{ dB} = (10\text{ dB})\ln(I/10^{-12}\text{ W/m}^2)$$

$$I = 10^{10.1}(10^{-12}\text{ W/m}^2) = 10^{-1.90}\text{ W/m}^2 = 0.0126\text{ W/m}^2$$

We suppose that sound comes perpendicularly out through the windows and doors. Then, the radiated power is

$$\wp = IA = (0.0126\text{ W/m}^2)(22.0\text{ m}^2) = 0.277\text{ W}$$

Are you surprised by how small this is? The energy radiated in 20.0 minutes is

$$E = \wp t = (0.277\text{ J/s})(20.0\text{ min})(60.0\text{ s}/1.00\text{ min}) = \boxed{332\text{ J}}$$

(b) If the ground reflects all sound energy headed downward, the sound power, $\wp = 0.277$ W, covers the area of a hemisphere. One kilometer away, this area is $A = 2\pi r^2 = 2\pi(1000\text{ m})^2 = 2\pi \times 10^6\text{ m}^2$.

The intensity at this distance is

$$I = \frac{\wp}{A} = \frac{0.277\text{ W}}{2\pi \times 10^6\text{ m}^2} = 4.41 \times 10^{-8}\text{ W/m}^2$$

and the sound intensity level is

$$\beta = (10\text{ dB})\ln\left(\frac{4.41 \times 10^{-8}\text{ W/m}^2}{1.00 \times 10^{-12}\text{ W/m}^2}\right) = \boxed{46.4\text{ dB}}$$

17.32 (a) $\Delta P_{\text{max}} = \dfrac{25.0}{r} = \dfrac{25.0}{4.00} = \boxed{6.25\text{ Pa}}$

(b) $v = \dfrac{\omega}{k} = \dfrac{1870}{1.25} = 1496\text{ m/s}$ so the material is $\boxed{\text{water}}$

(c) $I = \dfrac{\Delta P_{\text{max}}^2}{2\rho v} = \dfrac{(6.25)^2}{(2)(1000)(1496)} = 1.31 \times 10^{-5}\text{ W/m}^2$

$$\beta = 10\log\left(\frac{1.31 \times 10^{-5}}{1.00 \times 10^{-12}}\right) = \boxed{71.2\text{ dB}}$$

(d) $\Delta P = \left(\frac{25.0}{5.00}\right) \sin\,[(1.25)(5.00) - (1870)(0.0800)] = \boxed{4.59 \text{ Pa}}$

***17.33** (a) $f' = f\frac{v}{(v \pm v_S)}$

Approach: $f' = 320\frac{(343)}{(343 - 40.0)} = 362.2 \text{ Hz}$

Receding: $f' = 320\frac{(343)}{(343 + 40.0)} = 286.5 \text{ Hz}$

The *change* in frequency observed = 362 – 287 = $\boxed{75.7 \text{ Hz}}$

(b) $\lambda = \frac{v}{f'} = \frac{343 \text{ m/s}}{362 \text{ Hz}} = \boxed{0.948 \text{ m}}$

17.34 (a) $f' = \frac{f(v + v_O)}{(v + v_S)}$ where from observer to source is positive.

$f' = 2500\frac{(343 + 25.0)}{(343 - 40.0)} = \boxed{3.04 \text{ kHz}}$

(b) $f' = 2500\frac{(343 - 25.0)}{(343 + 40.0)} = \boxed{2.08 \text{ kHz}}$

(c) $f' = 2500\frac{(343 - 25.0)}{(343 - 40.0)} = \boxed{2.62 \text{ kHz}}$ while police car overtakes

$f' = 2500\frac{(343 + 25.0)}{(343 + 40.0)} = \boxed{2.40 \text{ kHz}}$ after police car passes

17.35 Approaching car

$$f' = \frac{f}{\left(1 - \frac{v_S}{v}\right)} \qquad \text{(Equation 17.14)}$$

Departing car

$$f'' = \frac{f}{\left(1 + \frac{v_S}{v}\right)} \qquad \text{(Equation 17.15)}$$

Since $f' = 560$ Hz and $f'' = 480$ Hz,

$$560\left(1 - \frac{v_S}{v}\right) = 480\left(1 + \frac{v_S}{v}\right)$$

$$1040\frac{v_S}{v} = 80.0$$

$$v_S = \frac{80.0(343)}{1040}\text{ m/s} = \boxed{26.4\text{ m/s}}$$

Goal Solution

G: We can assume that a police car with its siren on is in a hurry to get somewhere, and is probably traveling between 20 and 100 mph (~10 to 50 m/s), depending on the driving conditions.

O: We can use the equation for the Doppler effect to find the speed of the car.

A: Approaching car: $f' = \dfrac{f}{\left(\dfrac{v_s}{v}\right)}$ (Eq. 17.14)

Departing car: $f'' = \dfrac{f}{\left(1 + \dfrac{v_s}{v}\right)}$ (Eq. 17.15)

Where $f' = 560$ Hz and $f'' = 480$ Hz. Solving the two equations above for f and setting them equal gives:

$$f'\left(1 - \frac{v_s}{v}\right) = f''\left(1 + \frac{v_s}{v}\right) \quad \text{or} \quad f' - f'' = \frac{v_s}{v}(f' + f'')$$

so the speed of the source is $v_s = \dfrac{v(f' - f'')}{f' + f''} = \dfrac{(343\text{ m/s})(560\text{ Hz} - 480\text{ Hz})}{(560\text{ Hz} + 480\text{ Hz})} = \boxed{26.4\text{ m/s}}$

L: This seems like a reasonable speed (about 50 mph) for a police car, unless the street is crowded or the car is traveling on an open highway.

***17.36** (a) $\omega = 2\pi f = 2\pi\left(\dfrac{115/\text{min}}{60.0\text{ s/min}}\right) = 12.0\text{ rad/s}$

$$v_{\max} = \omega A = (12.0\text{ rad/s})(1.80 \times 10^{-3}\text{ m}) = \boxed{0.0217\text{ m/s}}$$

(b) The heart wall is a moving observer.

$$f' = f\left(\frac{v + v_O}{v}\right) = (2{,}000{,}000\text{ Hz})\left(\frac{1500 + 0.0217}{1500}\right) = \boxed{2\ 000\ 028.9\text{ Hz}}$$

(c) Now the heart wall is a moving source.

$$f'' = f'\left(\frac{v}{v - v_s}\right) = (2{,}000{,}029\text{ Hz})\left(\frac{1500}{1500 - 0.0217}\right) = \boxed{2\ 000\ 057.8\text{ Hz}}$$

17.37 $f' = f\left(\dfrac{v}{v + v_S}\right)$

$$485 = 512\left(\frac{340}{340 + 9.80t_{\text{fall}}}\right)$$

$$485(340) + (485)(9.80t_f) = (512)(340)$$

$$t_f = \left(\frac{512 - 485}{485}\right)\frac{340}{9.80} = 1.93\text{ s}$$

$$d_1 = \frac{1}{2}\,g t_f^2 = 18.3\text{ m}$$

$t_{\text{return}} = \dfrac{18.3}{340} = 0.0538\text{ s}$ The fork continues to fall while the sound returns.

$$t_{\text{total fall}} = t_f + t_{\text{return}} = 1.93\text{ s} + 0.0538\text{ s} = 1.985\text{ s}$$

$$d_{\text{total}} = \frac{1}{2}\,g t_{\text{total fall}}^2 = \boxed{19.3\text{ m}}$$

17.38 (a) The maximum speed of the speaker is described by

$$\frac{1}{2}\,m v_{\max}^2 = \frac{1}{2}\,kA^2$$

$$v_{\max} = \sqrt{k/m}\,A = \sqrt{\frac{20.0\text{ N/m}}{5.00\text{ kg}}}(0.500\text{ m}) = 1.00\text{ m/s}$$

The frequencies heard by the stationary observer range from

$$f'_{\min} = f\left(\frac{v}{v + v_{\max}}\right) \quad \text{to} \quad f'_{\max} = f\left(\frac{v}{v - v_{\max}}\right)$$

where v is the speed of sound.

$$f'_{\min} = 440\text{ Hz}\left(\frac{343\text{ m/s}}{343\text{ m/s} + 1.00\text{ m/s}}\right) = \boxed{439\text{ Hz}}$$

$$f'_{\max} = 440\text{ Hz}\left(\frac{343\text{ m/s}}{343\text{ m/s} - 1.00\text{ m/s}}\right) = \boxed{441\text{ Hz}}$$

(b) $\beta = 10\text{ dB}\log(I/I_0) = 10\text{ dB}\log\left(\dfrac{\wp/4\pi r^2}{I_0}\right)$

The maximum intensity level (of 60.0 dB) occurs at $r = r_{\min} = 1.00$ m. The minimum intensity level occurs when the speaker is farthest from the listener (i.e., when $r = r_{\max} = r_{\min} + 2A = 2.00$ m).

Thus, $\beta_{\max} - \beta_{\min} = 10\text{ dB}\log\left(\dfrac{\wp}{4\pi I_0 r_{\min}^2}\right) - 10\text{ dB}\log\left(\dfrac{\wp}{4\pi I_0 r_{\max}^2}\right)$

or $\quad \beta_{\max} - \beta_{\min} = 10\text{ dB}\log\left(\dfrac{\wp}{4\pi I_0 r_{\min}^2}\,\dfrac{4\pi I_0 r_{\max}^2}{\wp}\right) = 10\text{ dB}\log\left(\dfrac{r_{\max}^2}{r_{\min}^2}\right)$

This gives: 60.0 dB $- \beta_{\min} = 10\text{ dB}\log(4.00) = 6.02$ dB, and $\beta_{\min} = \boxed{54.0\text{ dB}}$

17.39 $f' = f\dfrac{(v \pm v_O)}{(v \pm v_S)}$

(a) $f' = 320\dfrac{(343 + 40.0)}{(343 + 20.0)} = \boxed{338\text{ Hz}}$

(b) $f' = 510\dfrac{(343 + 20.0)}{(343 + 40.0)} = \boxed{483\text{ Hz}}$

***17.40** (a) $v = (331\text{ m/s})\sqrt{1 + \dfrac{T}{273°\text{C}}} = (331\text{ m/s})\sqrt{1 + \dfrac{-10.0°\text{C}}{273°\text{C}}} = \boxed{325\text{ m/s}}$

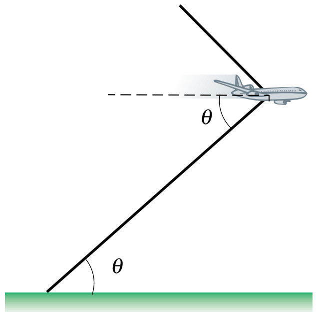
(b) Approaching the bell, the athlete hear a frequency of $f' = f\left(\dfrac{v + v_O}{v}\right)$

After passing the bell, she hears a lower frequency of $f'' = f\left(\dfrac{v - v_O}{v}\right)$

The ratio is $\dfrac{f''}{f'} = \dfrac{v - v_O}{v + v_O} = \dfrac{5}{6}$, which gives $6v - 6v_O = 5v + 5v_O$

or $\quad v_O = \dfrac{v}{11} = \dfrac{325\text{ m/s}}{11} = \boxed{29.5\text{ m/s}}$

17.41 $\sin\theta = \dfrac{v_{\text{sound}}}{v_{\text{jet}}} = \dfrac{v_{\text{sound}}}{1.20 v_{\text{sound}}} = \dfrac{1}{1.20}$ $\qquad \theta = \boxed{56.4°}$



17.42 The *half angle* of the shock wave cone is given by $\sin\theta = \dfrac{v_{\text{light}}}{v_S}$

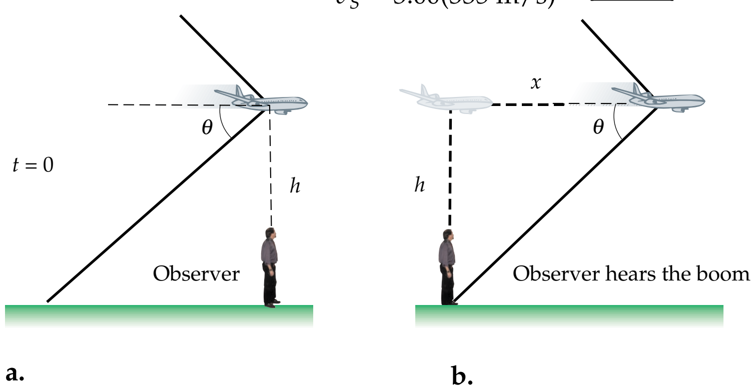
$$v_S = \frac{v_{\text{light}}}{\sin\theta} = \frac{2.25\times10^8\text{ m/s}}{\sin(53.0°)} = \boxed{2.82\times10^8\text{ m/s}}$$

17.43 (b) $\sin\theta = \dfrac{v}{v_S} = \dfrac{1}{3.00}$ $\qquad \theta = 19.5°$

$$\tan\theta = \frac{h}{x} \qquad x = \frac{h}{\tan\theta}$$

$$x = \frac{20000\text{ m}}{\tan 19.5°} = 5.66\times10^4\text{ m} = \boxed{56.6\text{ km}}$$

(a) It takes the plane $t = \dfrac{x}{v_S} = \dfrac{5.66\times10^4\text{ m}}{3.00(335\text{ m/s})} = \boxed{56.3\text{ s}}$ to travel this distance.



17.44 $\theta = \sin^{-1}\dfrac{v}{v_S} = \sin^{-1}\dfrac{1}{1.38} = \boxed{46.4°}$

17.45 Let d be the distance the stone drops.

$$t = \frac{d}{v_S} + \sqrt{\frac{2d}{g}}$$

$$d + \left(\sqrt{\frac{2}{g}}\; v_S\right)\sqrt{d} - v_S t = 0$$

$$\sqrt{d} = -\frac{1}{2}\left(\sqrt{\frac{2}{g}}\; v_S + \sqrt{\frac{2v_S^2}{g} + 4v_S t}\right)$$

$$\sqrt{d} = \frac{1}{2}(-155.0 \pm \sqrt{38\,000})$$

Choose the positive root so that $\sqrt{d} > 0$

$$\sqrt{d} = 20.0$$

$$d = \boxed{400 \text{ m}}$$

If the speed of sound is ignored,

$$t = \sqrt{\frac{2\,d'}{g}}$$

$$d = \frac{1}{2}\,gt^2 = 510 \text{ m}$$

The percentage error is given by

$$\frac{d' - d}{d} = 0.275 = \boxed{27.5\%}$$

***17.46** Model your loud, sharp sound impulse as a single narrow peak in a graph of air pressure versus time. It is a noise with no pitch, no frequency, wavelength, or period. It radiates away from you in all directions and some of it is incident on each one of the solid vertical risers of the bleachers. Suppose that, at the ambient temperature, sound moves at 340 m/s; and suppose that the horizontal width of each row of seats is 60 cm. Then there is a time delay of

$$\frac{0.6 \text{ m}}{(340 \text{ m/s})} \approx 0.002 \text{ s}$$

between your sound impulse reaching each riser and the next. Whatever its material, each will reflect much of the sound that reaches it. The reflected wave sounds very different from the sharp pop you made. If there are twenty rows of seats, you hear from the bleachers a tone with twenty crests, each separated from the next in time by

$$\frac{2(0.6 \text{ m})}{(340 \text{ m/s})} \approx 0.004 \text{ s}$$

This is the extra time for it to cross the width of one seat twice, once as an incident pulse and once again after its reflection. Thus, you hear a sound of definite pitch, with period about 0.004 s, frequency

$$\frac{1}{0.0035\text{ s}}\ \boxed{\sim 300\text{ Hz}}$$

wavelength

$$\lambda = \frac{v}{f} = \frac{(340\text{ m/s})}{(300/\text{s})} = 1.2\text{ m} \sim \boxed{10^0\text{ m}}$$

and duration

$$20(0.004\text{ s}) \sim \boxed{10^{-1}\text{ s}}$$

***17.47** (a) $\lambda = \dfrac{v}{f} = \dfrac{343\text{ m/s}}{1480\text{ s}^{-1}} = \boxed{0.232\text{ m}}$

(b) $\beta = 81.0\text{ dB} = 10\text{ dB}\log\left[I/(10^{-12}\text{ W/m}^2)\right]$

$$I = (10^{-12}\text{ W/m}^2)10^{8.10} = 10^{-3.90}\text{ W/m}^2 = 1.26\times10^{-4}\text{ W/m}^2 = \frac{1}{2}\rho v\omega^2 s_{\max}^2$$

$$s_{\max} = \sqrt{\frac{2I}{\rho v\omega^2}} = \sqrt{\frac{2(1.26\times10^{-4}\text{ W/m}^2)}{(1.20\text{ kg/m}^3)(343\text{ m/s})4\pi^2(1480\text{ s}^{-1})^2}} = \boxed{8.41\times10^{-8}\text{ m}}$$

(c) $\lambda' = \dfrac{v}{f'} = \dfrac{343\text{ m/s}}{1397\text{ s}^{-1}} = 0.246\text{ m}$ $\qquad \Delta\lambda = \lambda' - \lambda = \boxed{13.8\text{ mm}}$

17.48 Since $\cos^2\theta + \sin^2\theta = 1$ $\qquad \sin\theta = \pm(1-\cos^2\theta)^{1/2}$

each sign applying half the time.

$$\Delta P = \Delta P_{\max}\sin(kx-\omega t)$$

$$= \pm\rho v\omega s_{\max}\left[1-\cos^2(kx-\omega t)\right]^{1/2}$$

$$= \pm\rho v\omega\left[s_{\max}^2 - s_{\max}^2\cos^2(kx-\omega t)\right]^{1/2}$$

$$= \pm\rho v\omega(s_{\max}^2 - s^2)^{1/2}$$

***17.49** The trucks form a train analogous to a wave train of crests with speed $v = 19.7$ m/s and unshifted frequency $f = \dfrac{2}{3.00\text{ min}} = 0.667/\text{min}$.

(a) The cyclist as observer measures a lower Doppler-shifted frequency:

$$f' = f\left(\frac{v - v_O}{v}\right) = (0.667/\text{min})\left(\frac{19.7-4.47}{19.7}\right) = \boxed{0.515/\text{min}}$$

(b) $f'' = f\left(\dfrac{v - v_O'}{v}\right) = (0.667/\text{min})\left(\dfrac{19.7 - 1.56}{19.7}\right) = \boxed{0.614/\text{min}}$

The cyclist's speed has decreased very significantly, but there is only a modest increase in the frequency of trucks passing him.

***17.50** $v = \dfrac{2d}{t}$

$$d = \frac{vt}{2} = \frac{1}{2}(6.50 \times 10^3 \text{ m/s})(1.85 \text{ s}) = \boxed{6.01 \text{ km}}$$

***17.51** Call d the distance to the reflection point. We have

$$2d = (6.20 \text{ km/s})t$$

and $2d = (3.20 \text{ km/s})(t + 2.40 \text{ s})$

To solve for d we eliminate t by substitution:

$$t = \frac{2d}{6.20 \text{ km/s}} \qquad 2d = (3.20 \text{ km/s})\left(\frac{2d \text{ s}}{6.20 \text{ km}} + 2.40 \text{ s}\right)$$

$$2d = 1.03d + 7.68 \text{ km}$$

$$d = \frac{7.68 \text{ km}}{0.968} = \boxed{7.94 \text{ km}}$$

***17.52** (a) From the equation given in Example 17.1, the speed of a compression wave in a bar is

$$v = \sqrt{Y/\rho} = \sqrt{(20.0 \times 10^{10} \text{ N/m}^2)/(7860 \text{ kg/m}^3)} = \boxed{5.04 \times 10^3 \text{ m/s}}$$

(b) The signal to stop passes between layers of atoms as a sound wave, reaching the back end of the bar in time

$$t = L/v = (0.800 \text{ m})/(5.04 \times 10^3 \text{ m/s}) = \boxed{1.59 \times 10^{-4} \text{ s}}$$

(c) As described by Newton's first law, the rearmost layer of steel has continued to move forward with its original speed v_i for this time, compressing the bar by

$$\Delta L = v_i t = (12.0 \text{ m/s})(1.59 \times 10^{-4} \text{ s}) = 1.90 \times 10^{-3} \text{ m} = \boxed{1.90 \text{ mm}}$$

(d) The strain in the rod is: $\Delta L/L = (1.90 \times 10^{-3} \text{ m})/(0.800 \text{ m}) = \boxed{2.38 \times 10^{-3}}$

(e) The stress in the rod is:

$$\sigma = Y(\Delta L/L) = (20.0 \times 10^{10} \text{ N/m}^2)(2.38 \times 10^{-3}) = \boxed{476 \text{ MPa}}$$

Since $\sigma > 400$ MPa, the rod will be permanently distorted.

(f) We go through the same steps as in parts (a) through (e), but use algebraic expressions rather than numbers:

The speed of sound in the rod is $v = \sqrt{Y/\rho}$.

The back end of the rod continues to move forward at speed v_i for a time of $t = L/v = L\sqrt{\rho/Y}$, traveling distance $\Delta L = v_i t$ after the front end hits the wall.

The strain in the rod is: $\Delta L/L = v_i t/L = v_i\sqrt{\rho/Y}$

The stress is then: $\sigma = Y(\Delta L/L) = Yv_i\sqrt{\rho/Y} = v_i\sqrt{\rho Y}$

For this to be less than the yield stress, σ_y, it is necessary that

$$v_i\sqrt{\rho Y} < \sigma_y \quad \text{or} \quad \boxed{v_i < \frac{\sigma_y}{\sqrt{\rho Y}}}$$

With the given numbers, this speed is 10.1 m/s. The fact that the length of the rod divides out means that the steel will start to bend right away at the front end of the rod. There it will yield enough so that eventually the remainder of the rod will experience only stress within the elastic range. You can see this effect when sledgehammer blows give a mushroom top to a rod used as a tent stake.

17.53 (a) $f' = f\dfrac{v}{(v - v_{\text{diver}})}$

so $1 - \dfrac{v_{\text{diver}}}{v} = \dfrac{f}{f'}$

$$\Rightarrow v_{\text{diver}} = v\left(1 - \frac{f}{f'}\right)$$

with $v = 343$ m/s, $f' = 1800$ Hz and $f = 2150$ Hz

we find

$$v_{\text{diver}} = 343\left(1 - \frac{1800}{2150}\right) = \boxed{55.8 \text{ m/s}}$$

(b) If the waves are reflected, and the skydiver is moving into them, we have

$$f'' = f'\frac{(v + v_{\text{diver}})}{v} \Rightarrow f'' = f\left[\frac{v}{(v - v_{\text{diver}})}\right]\frac{(v + v_{\text{diver}})}{v}$$

so $f'' = 1800\dfrac{(343 + 55.8)}{(343 - 55.8)} = \boxed{2500 \text{ Hz}}$

Goal Solution

G: Sky divers typically reach a terminal speed of about 150 mph (~75 m/s), so this sky diver should also fall near this rate. Since her friend receives a higher frequency as a result of the Doppler shift, the sky diver should detect a frequency with twice the Doppler shift: $f' = 1800\text{ Hz} + 2(2150 - 1800)\text{ Hz} = 2500\text{ Hz}$.

O: We can use the equation for the Doppler effect to answer both parts of this problem.

A: Call $f_e = 1800$ Hz the emitted frequency; v_e , the speed of the sky diver; and $f_g = 2150$ Hz, the frequency of the wave crests reaching the ground.

(a) The sky diver source is moving toward the stationary ground, so we use the equation

$$f_g = f_s\left(\frac{v}{v - v_s}\right)$$

and $$v_e = v\left(1 - \frac{f_e}{f_g}\right) = (343\text{ m/s})\left(1 - \frac{1800\text{ Hz}}{2150\text{ Hz}}\right) = 55.8\text{ m/s}$$

(b) The ground now becomes a stationary source, reflecting crests with the 2150-Hz frequency at which they reach the ground, and sending them to a moving observer:

$$f_{e2} = f_g\left(\frac{v + v_e}{v}\right) = 2150\left(\frac{343\text{ m/s} + 55.8\text{ m/s}}{343\text{ m/s}}\right) = 2500\text{ Hz}$$

L: The answers appear to be consistent with our predictions, although the sky diver is falling somewhat slower than expected. The Doppler effect can be used to find the speed of many different types of moving objects, like raindrops (Doppler radar) and cars (police radar).

17.54 (a) $f' = \dfrac{fv}{v - u}$ $\qquad f'' = \dfrac{fv}{v + u}$

$$f' - f'' = fv\left(\frac{1}{v - u} - \frac{1}{v + u}\right)$$

$$\Delta f = \frac{fv(v + u - v + u)}{v^2 - u^2} = \frac{2uvf}{v^2\left(1 - \frac{u^2}{v^2}\right)}$$

$$\Delta f = \boxed{\frac{2(u/v)}{1 - (u^2/v^2)}f}$$

(b) $130\text{ km/h} = 36.1\text{ m/s}$

$$\therefore \Delta f = \frac{2(36.1)(400)}{340\left[1 - \frac{(36.1)^2}{340^2}\right]} = \boxed{85.9\text{ Hz}}$$

17.55 Use the Doppler formula, and remember that the bat is a moving source.

If the velocity of the insect is v_x ,

$$40.4 = 40.0\frac{(340 + 5.00)(340 - v_x)}{(340 - 5.00)(340 + v_x)}$$

Solving,

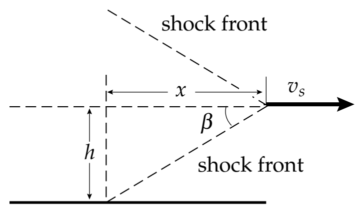
$$v_x = 3.31 \text{ m/s}$$

Therefore, $\boxed{\text{the bat is gaining on its prey at 1.69 m/s}}$

17.56 $\sin\beta = \frac{v}{v_S} = \frac{1}{N_M}$

$h = v(12.8 \text{ s})$

$x = v_S(10.0 \text{ s})$



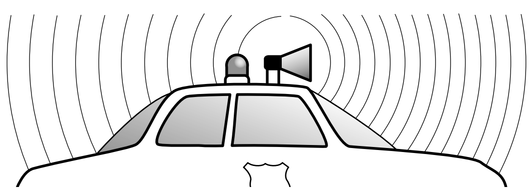
$\tan\beta = \frac{h}{x} = 1.28\frac{v}{v_S} = \frac{1.28}{N_M}$

$\cos\beta = \frac{\sin\beta}{\tan\beta} = \frac{1}{1.28}$

$\beta = 38.6°$

$N_M = \frac{1}{\sin\beta} = \boxed{1.60}$

***17.57** (a)



(b) $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{1000 \text{ s}^{-1}} = \boxed{0.343 \text{ m}}$

(c) $\lambda' = \frac{v}{f'} = \frac{v}{f}\left(\frac{v - v_S}{v}\right) = \frac{(343 - 40.0) \text{ m/s}}{1000 \text{ s}^{-1}} = \boxed{0.303 \text{ m}}$

(d) $\lambda'' = \frac{v}{f''} = = \frac{(343 + 40.0) \text{ m/s}}{1000 \text{ s}^{-1}} = \boxed{0.383 \text{ m}}$

(e) $f' = f\left(\frac{v - v_O}{v - v_S}\right) = (1000 \text{ Hz})\frac{(343 - 30.0) \text{ m/s}}{(343 - 40.0) \text{ m/s}} = \boxed{1.03 \text{ kHz}}$

17.58 $\Delta t = L\left(\frac{1}{v_{\text{air}}} - \frac{1}{v_{\text{cu}}}\right) = L\frac{v_{\text{cu}} - v_{\text{air}}}{v_{\text{air}}v_{\text{cu}}}$

$$L = \frac{v_{\text{air}}v_{\text{cu}}}{v_{\text{cu}} - v_{\text{air}}}\,\Delta t = \frac{(331\text{ m/s})(3.56\times 10^{3}\text{ m/s})}{(3560 - 331)\text{ m/s}}(6.40\times 10^{-3}\text{ s})$$

$$\boxed{L = 2.34\text{ m}}$$

17.59 (a) $120\text{ dB} = 10\text{ dB}\log\left[I/(10^{-12}\text{ W/m}^2)\right]$

$I = 1.00\text{ W/m}^2 = \wp/4\pi r^2$

$$r = \sqrt{\frac{\wp}{4\pi I}} = \sqrt{\frac{6.00\text{ W}}{4\pi(1.00\text{ W/m}^2)}} = \boxed{0.691\text{ m}}$$

We have assumed the speaker is an isotropic point source.

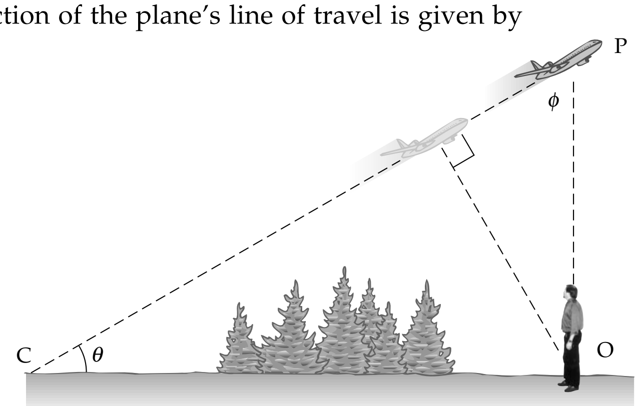
(b) $0\text{ dB} = 10\text{ dB}\log\left(I/10^{-12}\text{ W/m}^2\right)$

$I = 1.00\times 10^{-12}\text{ W/m}^2$

$$r = \sqrt{\frac{\wp}{4\pi I}} = \sqrt{\frac{6.00\text{ W}}{4\pi(1.00\times 10^{-12}\text{ W/m}^2)}} = \boxed{691\text{ km}}$$

We have assumed a uniform medium that absorbs no energy.

17.60 The shock wavefront connects all observers first hearing the plane, including our observer O and the plane P, so here it is vertical. The angle ϕ that the shock wavefront makes with the direction of the plane's line of travel is given by



$$\sin\phi = \frac{v}{v_S} = \frac{340\text{ m/s}}{1963\text{ m/s}} = 0.173$$

so $\phi = 9.97°$

Using the right triangle CPO, the angle θ is seen to be

$$\theta = 90.0° - \phi = 90.0° - 9.97° = \boxed{80.0°}$$

17.61 When observer is moving in front of and in the same direction as the source, $f' = f\dfrac{v - v_O}{v - v_S}$ where v_O and v_S are measured relative to the medium in which the sound is propagated. In this case the ocean current is opposite the direction of travel of the ships and

$$v_O = 45.0\text{ km/h} - (-10.0\text{ km/h}) = 55.0\text{ km/h} = 15.3\text{ m/s}, \quad \text{and}$$

$$v_S = 64.0\text{ km/h} - (-10.0\text{ km/h}) = 74.0\text{ km/h} = 20.55\text{ m/s}$$

Therefore, $f' = (1200.0\text{ Hz})\dfrac{1520\text{ m/s} - 15.3\text{ m/s}}{1520\text{ m/s} - 20.55\text{ m/s}} = \boxed{1204.2\text{ Hz}}$

17.62 We suppose the sound level is uniform over the outer surface of area

$$2(0.400\text{ m})(0.400\text{ m}) + 4(0.400\text{ m})(0.500\text{ m}) = 1.12\text{ m}^2$$

The intensity is given by

$$40.0\text{ dB} = 10\text{ dB}\log(I/10^{-12}\text{ W/m}^2)$$

$$I = 10^{-12+4}\text{ W/m}^2 = 10^{-8}\text{ W/m}^2$$

The sound power is

$$\wp = IA = 1.12 \times 10^{-8}\text{ W}$$

So the oven's energy efficiency as a sound source is

$$\wp/\wp_{\text{input}} = (1.12 \times 10^{-8}\text{ W})/(1.00 \times 10^3\text{ W}) = \boxed{1.12 \times 10^{-11}}$$

17.63 (a) $\theta = \sin^{-1}\left(\dfrac{v_{\text{sound}}}{v_{\text{obj}}}\right) = \sin^{-1}\left(\dfrac{331}{20.0 \times 10^3}\right) = \boxed{0.948°}$

(b) $\theta' = \sin^{-1}\left(\dfrac{1533}{20.0 \times 10^3}\right) = \boxed{4.40°}$

17.64 $\Delta P_{\max} = \rho\omega v s_{\max} = \rho\left(\dfrac{2\pi v}{\lambda}\right) v s_{\max}$

Also, ΔP and s are 90° out of phase.

Therefore, $\Delta P = -\left(\dfrac{2\pi\rho v^2 s_{\max}}{\lambda}\right)\cos(kx - \omega t)$

17.65 For the longitudinal wave $v_L = (Y/\rho)^{1/2}$

For the transverse wave $v_T = (T/\mu)^{1/2}$

If we require $\dfrac{v_L}{v_T} = 8.00$, we have $T = \dfrac{\mu Y}{64.0\rho}$ where $\mu = \dfrac{m}{L}$ and

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\pi r^2 L}$$

This gives $T = \dfrac{\pi r^2 Y}{64.0} = \dfrac{\pi(2.00 \times 10^{-3}\text{ m})^2(6.80 \times 10^{10}\text{ N/m}^2)}{64.0} = \boxed{1.34 \times 10^4\text{ N}}$

17.66 $\wp_2 = \dfrac{1}{20.0}\wp_1 \qquad \beta_1 - \beta_2 = 10\log\dfrac{\wp_1}{\wp_2}$

$80.0 - \beta_2 = 10\log 20.0 = +13.0$

$\beta_2 = \boxed{67.0\text{ dB}}$

17.67 $t = \dfrac{0.300 \times 10^3\text{ J}}{4\pi(500\text{ m})^2(10^{-12}\text{ W/m}^2)(10^6)} = \boxed{95.5\text{ s}}$

17.68 The total output sound energy is $eE = \wp t$, where $\wp$ is the power radiated.

Thus, $t = \dfrac{eE}{\wp} = \dfrac{eE}{IA} = \dfrac{eE}{(4\pi r^2)I}$

But, $\beta = 10\log\left(\dfrac{I}{I_0}\right)$. Therefore, $I = I_0(10^{\beta/10})$ and $t = \boxed{\dfrac{eE}{4\pi r^2 I_0(10^{\beta/10})}}$

17.69 (a) If the source and the observer are moving away from each other, we have: $\theta_S = \theta_O = 180°$, and since $\cos 180° = -1$, we get Equation 17.17 with the lower signs.

(b) If $v_O = 0$ m/s, then $f' = \dfrac{v}{v - v_S\cos\theta_S}f$

Also, when the train is 40.0 m from the intersection, and the car is 30.0 m from the intersection,

$\cos\theta_S = \dfrac{4}{5}$ so

$f' = \dfrac{343\text{ m/s}}{343\text{ m/s} - 0.800(25.0\text{ m/s})}(500\text{ Hz})$ or $f' = \boxed{531\text{ Hz}}$

Note that as the train approaches, passes, and departs from the intersection, θ_S varies from 0° to 180° and the frequency heard by the observer varies from:

$$f'_{\max} = \frac{v}{v - v_S \cos 0°}\, f = \frac{343 \text{ m/s}}{343 \text{ m/s} - 25.0 \text{ m/s}}(500 \text{ Hz}) = 539 \text{ Hz}$$

$$f'_{\min} = \frac{v}{v - v_S \cos 180°}\, f = \frac{343 \text{ m/s}}{343 \text{ m/s} + 25.0 \text{ m/s}}(500 \text{ Hz}) = 466 \text{ Hz}$$

***17.70** Let T represent the period of the source vibration, and E be the energy put into each wavefront. Then $\wp_{\text{av}} = E/T$. When the observer is at distance r in front of the source, he is receiving a spherical wavefront of radius vt, where t is the time since this energy was radiated, given by $vt - v_S t = r$. Then,

$$t = \frac{r}{v - v_S}$$

The area of the sphere is $4\pi(vt)^2 = \dfrac{4\pi v^2 r^2}{(v - v_S)^2}$. The energy per unit area over the spherical wavefront is uniform with the value $\dfrac{E}{A} = \dfrac{\wp_{\text{av}} T(v - v_S)^2}{4\pi v^2 r^2}$.

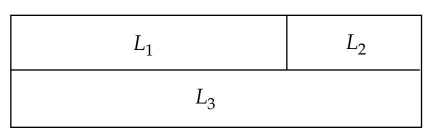
The observer receives parcels of energy with the Doppler shifted frequency $f' = f\left(\dfrac{v}{v - v_S}\right) = \dfrac{v}{T(v - v_S)}$, so the observer receives a wave with intensity

$$I = \left(\frac{E}{A}\right) f' = \left(\frac{\wp_{\text{av}} T(v - v_S)^2}{4\pi v^2 r^2}\right)\left(\frac{v}{T(v - v_S)}\right) = \boxed{\frac{\wp_{\text{av}}}{4\pi r^2}\left(\frac{v - v_S}{v}\right)}$$

17.71 (a) The time required for a sound pulse to travel distance L at speed v is given by $t = \dfrac{L}{v} = \dfrac{L}{\sqrt{Y/\rho}}$. Using this expression we find

$$t_1 = \frac{L_1}{\sqrt{Y_1/\rho_1}} = \frac{L_1}{\sqrt{(7.00 \times 10^{10} \text{ N/m}^2)/(2700 \text{ kg/m}^3)}} = (1.96 \times 10^{-4}\, L_1) \text{ s}$$

$$t_2 = \frac{1.50 \text{ m} - L_1}{\sqrt{Y_2/\rho_2}} = \frac{1.50 \text{ m} - L_1}{\sqrt{(1.60 \times 10^{10} \text{ N/m}^2)/(11.3 \times 10^3 \text{ kg/m}^3)}}$$



or $t_2 = (1.26 \times 10^{-3} - 8.40 \times 10^{-4}\, L_1)$ s

$$t_3 = \frac{1.50 \text{ m}}{\sqrt{(11.0 \times 10^{10} \text{ N/m}^3)/(8800 \text{ kg/m}^3)}}$$

$t_3 = 4.24 \times 10^{-4}$ s

We require $t_1 + t_2 = t_3$, or

$$1.96 \times 10^{-4}\, L_1 + 1.26 \times 10^{-3} - 8.40 \times 10^{-4}\, L_1 = 4.24 \times 10^{-4}$$

This gives $L_1 = 1.30$ m and $L_2 = 1.50 - 1.30 = 0.201$ m

The ratio of lengths is then $\dfrac{L_1}{L_2} = \boxed{6.45}$

(b) The ratio of lengths L_1/L_2 is adjusted in part (a) so that $t_1 + t_2 = t_3$. Sound travels the two paths in equal time and the phase difference, $\boxed{\Delta\phi = 0}$.

17.72 Let $\theta = \theta_0 \log R$, $I = kR$:

$$\beta = 10 \log\left(\frac{I}{I_0}\right) = 10 \log I - 10 \log I_0 = 10 \log kR - 10 \log I_0$$

or $$\beta = 10 \log R + 10(\log k - \log I_0) = 10\left(\frac{\theta}{\theta_0}\right) + 10(\log k - \log I_0)$$

$$\boxed{\beta = \left(\frac{10}{\theta_0}\right)\theta + 10 \log\left(\frac{k}{I_0}\right)} \leftarrow \text{the equation of a straight line. } [y = mx + b]$$

***17.73** To find the separation of adjacent molecules, use a model where each molecule occupies a sphere of radius r given by

$$\rho_{\text{air}} = \frac{\text{average mass per molecule}}{\frac{4}{3}\pi r^3}$$

or $$1.20 \text{ kg/m}^3 = \frac{4.82 \times 10^{-26} \text{ kg}}{\frac{4}{3}\pi r^3}, \quad r = \left[\frac{3(4.82 \times 10^{-26} \text{ kg})}{4\pi(1.20 \text{ kg/m}^3)}\right]^{1/3} = 2.12 \times 10^{-9} \text{ m}$$

Intermolecular separation is $2r = 4.25 \times 10^{-9}$ m, so the highest possible frequency sound wave is

$$f_{\max} = \frac{v}{\lambda_{\min}} = \frac{v}{2r} = \frac{343 \text{ m/s}}{4.25 \times 10^{-9} \text{ m}} = 8.03 \times 10^{10} \text{ Hz } \boxed{\sim 10^{11} \text{ Hz}}$$

Chapter 18 Solutions

18.1 The resultant wave function has the form

$$y = 2A_0 \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

(a) $A = 2A_0 \cos\left(\frac{\phi}{2}\right) = 2(5.00) \cos\left[\frac{-(\pi/4)}{2}\right] = \boxed{9.24 \text{ m}}$

(b) $f = \frac{\omega}{2\pi} = \frac{1200\pi}{2\pi} = \boxed{600 \text{ Hz}}$

***18.2** We write the second wave function as

$$y_2 = A \sin(kx - \omega t + \phi)$$

$$y_2 = (0.0800 \text{ m}) \sin[2\pi(0.100x - 80.0t) + \phi]$$

Then

$$y_1 + y_2 = (0.0800 \text{ m}) \sin[2\pi(0.100x - 80.0t)]$$

$$+ (0.0800 \text{ m}) \sin[2\pi(0.100x - 80.0t) + \phi]$$

$$= 2(0.0800 \text{ m}) \cos\frac{\phi}{2} \sin\left[2\pi(0.100x - 80.0t) + \frac{\phi}{2}\right]$$

We require $2(0.0800 \text{ m}) \cos\frac{\phi}{2} = 0.0800\sqrt{3}$

$$\cos\frac{\phi}{2} = \frac{\sqrt{3}}{2}$$

$$\phi = 60.0° = \frac{\pi}{3}$$

Then the second wave function is

$$y_2 = (0.0800 \text{ m}) \sin\left[2\pi\left(0.100x - 80.0t + \frac{1}{6}\right)\right]$$

$$y_2 = \boxed{(0.0800 \text{ m}) \sin[2\pi(0.100x - 80.0t + 0.167)]}$$

18.3 Suppose the waves are sinusoidal. The sum is

$$(4.00 \text{ cm}) \sin(kx - \omega t) + (4.00 \text{ cm}) \sin(kx - \omega t + 90.0°)$$

$$2(4.00 \text{ cm}) \sin(kx - \omega t + 45.0°) \cos 45.0°$$

So the amplitude is $(8.00 \text{ cm}) \cos 45.0° = \boxed{5.66 \text{ cm}}$

18.4 $2A_0 \cos\left(\dfrac{\phi}{2}\right) = A_0$, so $\dfrac{\phi}{2} = \cos^{-1}\left(\dfrac{1}{2}\right) = 60.0° = \dfrac{\pi}{3}$

Thus, the phase difference is $\phi = 120° = \dfrac{2\pi}{3}$

This phase difference results if the time delay is $\dfrac{T}{3} = \dfrac{1}{3f} = \dfrac{\lambda}{3v}$

Time delay $= \dfrac{3.00 \text{ m}}{3(2.00 \text{ m/s})} = \boxed{0.500 \text{ s}}$

18.5 Waves reflecting from the near end travel 28.0 m (14.0 m down and 14.0 m back), while waves reflecting from the far end travel 66.0 m. The path difference for the two waves is:

$$\Delta r = 66.0 \text{ m} - 28.0 \text{ m} = 38.0 \text{ m}$$

Since $\lambda = \dfrac{v}{f}$, then $\dfrac{\Delta r}{\lambda} = \dfrac{(\Delta r)f}{v} = \dfrac{(38.0 \text{ m})(246 \text{ Hz})}{343 \text{ m/s}} = 27.254$

or, $\Delta r = 27.254\lambda$

The phase difference between the two reflected waves is then

$$\phi = 0.254(1 \text{ cycle}) = 0.254(2\pi \text{ rad}) = \boxed{91.3°}$$

18.6 (a) First we calculate the wavelength: $\lambda = \dfrac{v}{f} = \dfrac{344 \text{ m/s}}{21.5 \text{ Hz}} = 16.0 \text{ m}$

Then we note that the path difference equals $9.00 \text{ m} - 1.00 \text{ m} = \boxed{\dfrac{1}{2}\lambda}$

Therefore, the receiver will record a minimum in sound intensity.

(b) If the receiver is located at point (x, y), then we must solve:

$$\sqrt{(x+5.00)^2+y^2}-\sqrt{(x-5.00)^2+y^2}=\frac{1}{2}\lambda$$

Then, $\sqrt{(x+5.00)^2+y^2}=\sqrt{(x-5.00)^2+y^2}+\frac{1}{2}\lambda$

Square both sides and simplify to get: $20.0x-\frac{\lambda^2}{4}=\lambda\sqrt{(x-5.00)^2+y^2}$

Upon squaring again, this reduces to:

$$400x^2-10.0\lambda^2x+\frac{\lambda^4}{16.0}=\lambda^2(x-5.00)^2+\lambda^2y^2$$

Substituting $\lambda = 16.0$ m, and reducing, we have:

$$\boxed{9.00x^2-16.0y^2=144} \quad \text{or} \quad \frac{x^2}{16.0}-\frac{y^2}{9.00}=1$$

(When plotted this yields a curve called a hyperbola.)

18.7 We suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\sqrt{L^2+d^2}-L$.

He hears a minimum when this is $(2n-1)\lambda/2$ with $n = 1, 2, 3, \ldots$

Then, $\sqrt{L^2+d^2}-L=(n-1/2)v/f$

$$\sqrt{L^2+d^2}=(n-1/2)v/f+L$$

$$L^2+d^2=(n-1/2)^2v^2/f^2+L^2+2(n-1/2)vL/f$$

$$L=\frac{d^2-(n-1/2)^2v^2/f^2}{2(n-1/2)v/f} \qquad n=1,2,3,\ldots$$

This will give us the answer to (b). The path difference starts from nearly zero when the man is very far away and increases to d when $L = 0$. The number of minima he hears is the greatest integer solution to $d \geq (n-1/2)\, v/f$

n = greatest integer $\leq df/v + 1/2$

(a) $df/v+\frac{1}{2}=(4.00\text{ m})(200/\text{s})/330\text{ m/s}+\frac{1}{2}=2.92$

He hears $\boxed{\text{two}}$ minima.

(b) With $n = 1$,

$$L = \frac{d^2 - (1/2)^2 v^2/f^2}{2(1/2)v/f} = \frac{(4.00\text{ m})^2 - (330\text{ m/s})^2/4(200/\text{s})^2}{(330\text{ m/s})/200/\text{s}}$$

$$L = \boxed{9.28\text{ m}}$$

with $n = 2$

$$L = \frac{d^2 - (3/2)^2 v^2/f^2}{2(3/2)v/f} = \boxed{1.99\text{ m}}$$

18.8 Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r = \sqrt{L^2 + d^2} - L$.

He hears a minimum when

$$\Delta r = (2n-1)\left(\frac{\lambda}{2}\right) \text{ with } n = 1, 2, 3, \ldots$$

Then, $\sqrt{L^2 + d^2} - L = (n - 1/2)(v/f)$

$$\sqrt{L^2 + d^2} = (n - 1/2)(v/f) + L$$

$$L^2 + d^2 = (n - 1/2)^2 (v/f)^2 + 2(n - 1/2)(v/f)L + L^2 \qquad (1)$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference Δr starts from nearly zero when the man is very far away and increases to d when $L = 0$.

(a) The number of minima he hears is the greatest integer value for which $L \geq 0$. This is the same as the greatest integer solution to $d \geq (n - 1/2)(v/f)$, or

$$\boxed{\text{number of minima heard} = n_{\text{max}} = \text{greatest integer} \leq d(f/v) + 1/2}$$

(b) From Equation 1, the distances at which minima occur are given by

$$\boxed{L_n = \frac{d^2 - (n - 1/2)^2 (v/f)^2}{2(n - 1/2)(v/f)} \text{ where } n = 1, 2, \ldots, n_{\text{max}}}$$

18.9 $y = (1.50\text{ m})\sin(0.400x)\cos(200t) = 2A_0 \sin kx \cos \omega t$

Therefore,

$$k = \frac{2\pi}{\lambda} = 0.400\frac{\text{rad}}{\text{m}} \qquad \lambda = \frac{2\pi}{0.400\text{ rad/m}} = \boxed{15.7\text{ m}}$$

and $\omega = 2\pi f$, so $f = \dfrac{\omega}{2\pi} = \dfrac{200\text{ rad/s}}{2\pi\text{ rad}} = \boxed{31.8\text{ Hz}}$

The speed of waves in the medium is

$$v = \lambda f = \frac{\lambda}{2\pi}\, 2\pi f = \frac{\omega}{k} = \frac{200 \text{ rad/s}}{0.400 \text{ rad/m}} = \boxed{500 \text{ m/s}}$$

18.10 $y = 0.0300 \text{ m} \cos\left(\frac{x}{2}\right) \cos(40t)$

(a) nodes occur where $y = 0$:

$$\frac{x}{2} = (2n+1)\frac{\pi}{2}$$

so $x = \boxed{(2n+1)\pi = \pi, 3\pi, 5\pi, \ldots}$

(b) $y_{\text{max}} = 0.0300 \text{ m} \cos\left(\frac{0.400}{2}\right) = \boxed{0.0294 \text{ m}}$

18.11 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{\text{NN}} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343 \text{ m/s}}{2(800 \text{ s}^{-1})} = 0.214 \text{ m}$$

If the speakers vibrate in phase, the point halfway between them is an antinode, at $\frac{1.25 \text{ m}}{2} = 0.625$ m from either speaker.

Then there is a node at

$0.625 \text{ m} - \frac{0.214 \text{ m}}{2} = \boxed{0.518 \text{ m}}$, a node at

$0.518 \text{ m} - 0.214 \text{ m} = \boxed{0.303 \text{ m}}$, a node at

$0.303 \text{ m} - 0.214 \text{ m} = \boxed{0.0891 \text{ m}}$, a node at

$0.518 \text{ m} + 0.214 \text{ m} = \boxed{0.732 \text{ m}}$, a node at

$0.732 \text{ m} + 0.214 \text{ m} = \boxed{0.947 \text{ m}}$, and a node at

$0.947 \text{ m} + 0.214 \text{ m} = \boxed{1.16 \text{ m}}$ from either speaker

***18.12** (a) The resultant wave is

$$y = 2A \sin\left(kx + \frac{\phi}{2}\right) \cos\left(\omega t - \frac{\phi}{2}\right)$$

The nodes are located at

$$kx + \frac{\phi}{2} = n\pi$$

so $x = \dfrac{n\pi}{k} - \dfrac{\phi}{2k}$

which means that each node is shifted $\dfrac{\phi}{2k}$ to the left.

(b) The separation of nodes is

$$\Delta x = \left[(n+1)\frac{\pi}{k} - \frac{\phi}{2k}\right] - \left[\frac{n\pi}{k} - \frac{\phi}{2k}\right]$$

$$\Delta x = \frac{\pi}{k} = \frac{\lambda}{2}$$

The nodes are still separated by half a wavelength.

18.13 $y_1 = 3.00 \sin[\pi(x + 0.600t)]$ cm $\qquad y_2 = 3.00 \sin[\pi(x - 0.600t)]$ cm

$y = y_1 + y_2 = [3.00 \sin(\pi x)\cos(0.600\,\pi t) + 3.00 \sin(\pi x)\cos(0.600\pi t)]$ cm

$= (6.00 \text{ cm}) \sin(\pi x)\cos(0.600\pi t)$

(a) We can take $\cos(0.600\pi t) = 1$ to get the maximum y.

At $x = 0.250$ cm, $y_{\max} = (6.00 \text{ cm})\sin(0.250\pi) = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500$ cm, $y_{\max} = (6.00 \text{ cm})\sin(0.500\pi) = \boxed{6.00 \text{ cm}}$

(c) Now take $\cos(0.600\pi t) = -1$ to get $y_{\max}$:

At $x = 1.50$ cm, $y_{max} = (6.00 \text{ cm})\sin(1.50\pi)(-1) = \boxed{6.00 \text{ cm}}$

(d) The antinodes occur when $x = n\lambda/4$ $(n = 1, 3, 5, \ldots)$. But

$k = 2\pi/\lambda = \pi$, so $\lambda = 2.00$ cm, and

$x_1 = \lambda/4 = \boxed{0.500 \text{ cm}}$ as in (b)

$x_2 = 3\lambda/4 = \boxed{1.50 \text{ cm}}$ as in (c)

$x_3 = 5\lambda/4 = \boxed{2.50 \text{ cm}}$

18.14 (a) Using the given parameters, the wave function is

$$y = 2\pi \sin\left(\frac{\pi x}{2}\right) \cos(10\pi t)$$

We need to find values of x for which $\left|\sin\left(\frac{\pi x}{2}\right)\right| = 1$

This condition requires that $\frac{\pi x}{2} = \pi\left(n + \frac{1}{2}\right)$; $n = 0, 1, 2, \ldots$

For $n = 0$, $x = 1.00$ cm and for $n = 1$, $x = 3.00$ cm

Therefore, the distance between antinodes, $\Delta x = \boxed{2.00 \text{ cm}}$

(b) $A = 2\pi \sin\left(\frac{\pi x}{2}\right)$; when $x = 0.250$ cm, $A = \boxed{2.40 \text{ cm}}$

18.15 $y = 2A_0 \sin kx \cos \omega t$

$$\frac{\partial^2 y}{\partial x^2} = -2A_0 k^2 \sin kx \cos \omega t$$

$$\frac{\partial^2 y}{\partial t^2} = -2A_0 \omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives

$$-2A_0 k^2 \sin kx \cos \omega t = \left(\frac{1}{v^2}\right)(-2A_0 \omega^2 \sin kx \cos \omega t)$$

This is satisfied, provided that $v = \frac{\omega}{k}$

18.16 $\mu = \frac{0.100 \text{ kg}}{2.00 \text{ m}} = 0.0500 \text{ kg/m}$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(20.0 \text{ kg} \cdot \text{m/s}^2)}{0.0500 \text{ kg/m}}} = 20.0 \text{ m/s}$$

For the simplest vibration possibility, NAN,

$$d_{NN} = 2.00 \text{ m} = \frac{\lambda}{2} \qquad \lambda = 4.00 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{(20.0 \text{ m/s})}{4.00 \text{ m}} = \boxed{5.00 \text{ Hz}}$$

For the second state NANAN,

$$d_{NN} = 1.00 \text{ m} \qquad \lambda = 2.00 \text{ m}$$

$$f = \frac{(20.0 \text{ m/s})}{2.00 \text{ m}} = \boxed{10.0 \text{ Hz}}$$

For the third resonance, NANANAN,

$$d_{NN} = \frac{2.00 \text{ m}}{3} \qquad \lambda = 1.33 \text{ m} \qquad f = \boxed{15.0 \text{ Hz}}$$

The mode mentioned in the problem has

$$d_{NN} = 0.400 \text{ m} \qquad \lambda = 0.800 \text{ m} \qquad f = \boxed{25.0 \text{ Hz}}$$

It is the $\boxed{\text{fifth allowed state}}$.

18.17 $L = 30.0 \text{ m} \qquad \mu = 9.00 \times 10^{-3} \text{ kg/m} \qquad T = 20.0 \text{ N}$

$$f_1 = \frac{v}{2L}$$

where $v = \left(\frac{T}{\mu}\right)^{1/2} = 47.1 \text{ m/s}$

so $\quad f_1 = \dfrac{47.1}{60.0} = \boxed{0.786 \text{ Hz}}$

$$f_2 = 2f_1 = \boxed{1.57 \text{ Hz}} \qquad f_3 = 3f_1 = \boxed{2.36 \text{ Hz}} \qquad f_4 = 4f_1 = \boxed{3.14 \text{ Hz}}$$

Goal Solution

G: The string described in the problem is very long, loose, and somewhat massive, so it should have a very low fundamental frequency, maybe only a few vibrations per second.

O: The tension and linear density of the string can be used to find the wave speed, which can then be used along with the required wavelength to find the fundamental frequency.

A: The wave speed is $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{20 \text{ N}}{9.0 \times 10^{-3} \text{ kg/m}}} = 47.1 \text{ m/s}$

For a vibrating string of length L fixed at both ends, the wavelength of the fundamental frequency is $\lambda = 2L = 60.0$ m; and the frequency is

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L} = \frac{47.1 \text{ m/s}}{60 \text{ m}} = \boxed{0.786 \text{ Hz}}$$

The next three harmonics are

$$f_2 = 2f_1 = \boxed{1.57\text{ Hz}}$$

$$f_3 = 3f_1 = \boxed{2.36\text{ Hz}}$$

$$f_4 = 4f_1 = \boxed{3.14\text{ Hz}}$$

L: The fundamental frequency is even lower than expected, less than 1 Hz. In fact, all 4 of the lowest resonant frequencies are below the normal human hearing range (20 to 17 000 Hz), so these harmonics are not even audible.

18.18 $L = 120\text{ cm}$ $f = 120\text{ Hz}$

(a) For four segments,

$$L = 2\lambda$$

or $\lambda = 60.0\text{ cm} = \boxed{0.600\text{ m}}$

(b) $v = \lambda f = 72.0\text{ m/s}$

$$f_1 = \frac{v}{2L} = \frac{72.0}{2(1.20)} = \boxed{30.0\text{ Hz}}$$

18.19 $d_{NN} = 0.700\text{ m}$

$\lambda = 1.40\text{ m}$

$$f\lambda = v = 308\text{ m/s} = \sqrt{\frac{T}{(1.20 \times 10^{-3})/(0.700)}}$$

(a) $T = \boxed{163\text{ N}}$

(b) $f_3 = \boxed{660\text{ Hz}}$

Goal Solution

G: The tension should be less than 100 lbs. (~500 N) since excessive force on the 4 cello strings would break the neck of the instrument. If the string vibrates in three segments, there will be three antinodes (instead of one for the fundamental mode), so the frequency should be three times greater than the fundamental.

O: The length of the string can be used to find the wavelength, which can be used with the fundamental frequency to find the wave speed. The tension can then be found from the wave speed and linear mass density of the string.

A: When the string vibrates in the lowest frequency mode, the length of string forms a standing wave where $L = \lambda/2$ (see Figure 18.2b), so the fundamental harmonic wavelength is

$$\lambda = 2L = 2(0.700\text{ m}) = 1.40\text{ m}$$

and the velocity is $v = f\lambda = (220\text{ s}^{-1})(1.40\text{ m}) = 308\text{ m/s}$

From the tension equation $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}}$ we get

(a) $$T = \frac{v^2 m}{L} = \frac{(308\text{ m/s})^2(1.20 \times 10^{-3}\text{ kg})}{0.700\text{ m}} = 163\text{ N}$$

(b) For the third harmonic, the tension, linear density, and speed are the same. However, the string vibrates in three segments so that the wavelength is one third as long as in the fundamental (see Figure 18.2d).

$$\lambda_3 = \lambda/3$$

From the equation $\lambda f = v$, we find that the frequency is three times as high:

$$f_3 = \frac{v}{\lambda_3} = 3\frac{v}{\lambda} = 3f = 660\text{ Hz}$$

L: The tension seems reasonable, and the third harmonic is three times the fundamental frequency as expected. Related to part (b), some stringed instrument players use a technique to double the frequency of a note by "cutting" a vibrating string in half. When the string is suddenly held at its midpoint to form a node, the second harmonic is formed, and the resulting note is one octave higher (twice the original fundamental frequency).

***18.20** $f_1 = \dfrac{v}{2L}$, where $v = \left(\dfrac{T}{\mu}\right)^{1/2}$

(a) If L is doubled, then $f_1 \propto L^{-1}$ will be reduced by a factor $\boxed{\dfrac{1}{2}}$.

(b) If μ is doubled, then $f_1 \propto \mu^{-1/2}$ will be reduced by a factor $\boxed{\dfrac{1}{\sqrt{2}}}$.

(c) If T is doubled, then $f_1 \propto \sqrt{T}$ will increase by a factor of $\boxed{\sqrt{2}}$.

18.21 $L = 60.0\text{ cm} = 0.600\text{ m}$ $T = 50.0\text{ N}$ $\mu = 0.100\text{ g/cm} = 0.0100\text{ kg/m}$

$$f_n = \frac{nv}{2L}$$

where

$$v = \left(\frac{T}{\mu}\right)^{1/2} = 70.7\text{ m/s}$$

$$f_n = n\left(\frac{70.7}{1.20}\right) = 58.9n = 20{,}000\text{ Hz}$$

Largest $n = 339 \Rightarrow f = \boxed{19.976\text{ kHz}}$

18.22 $f = \dfrac{v}{\lambda} = \sqrt{\dfrac{T}{\mu}}\;\dfrac{1}{\lambda} = \sqrt{\dfrac{T4}{\rho\pi d^2}}\;\dfrac{2}{L}$

since $\mu = \dfrac{M}{L} = \dfrac{\rho V}{L} = \rho\dfrac{AL}{L}$

$$f_{\text{new}} = \sqrt{\frac{4T_{\text{old}}4}{\rho_{\text{old}}\pi(2d_{\text{old}})^2}}\;\frac{2}{L_{\text{old}}/2}$$

$$= \sqrt{\frac{T_{\text{old}}4}{\rho_{\text{old}}\,\pi\,d_{\text{old}}^2}}\;\frac{2}{L_{\text{old}}} \times 2 = 2f_{\text{old}} = \boxed{800\text{ Hz}}$$

18.23 $\lambda_G = 2(0.350\text{ m}) = \dfrac{v}{f_G}$

$$\lambda_A = 2L_A = \frac{v}{f_A}$$

$L_G - L_A = L_G - \left(\dfrac{f_G}{f_A}\right) L_G = L_G$ **Error!**) $= (0.350\text{ m})$ **Error!** $= 0.0382\text{ m}$

Thus, $L_A = L_G - 0.0382\text{ m} = 0.350\text{ m} - 0.0382\text{ m} = 0.312\text{ m}$, or the finger should be placed $\boxed{\text{31.2 cm from the bridge}}$.

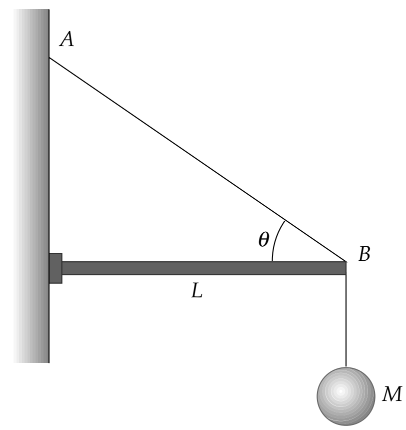
$$L_A = \frac{v}{2f_A} = \frac{1}{2f_A}\sqrt{\frac{T}{\mu}}$$

$$dL_A = \frac{dT}{4f_A\sqrt{T\mu}}$$

$$\frac{dL_A}{L_A} = \frac{1}{2}\frac{dT}{T}$$

$$\frac{dT}{T} = 2\frac{dL_A}{L_A} = 2\frac{0.600\text{ cm}}{(35.0 - 3.82)\text{ cm}} = \boxed{3.84\%}$$

18.24 In the fundamental mode, the string above the rod has only two nodes, at A and B, with an anti-node halfway between A and B. Thus,

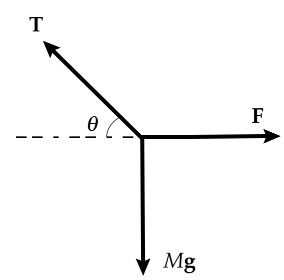


$$\frac{\lambda}{2} = \overline{AB} = \frac{L}{\cos\theta} \quad \text{or} \quad \lambda = \frac{2L}{\cos\theta}$$

Since the fundamental frequency is f, the wave speed in this segment of string is

$$v = \lambda f = \frac{2Lf}{\cos\theta}$$

Also, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{T}{m/\overline{AB}}} = \sqrt{\dfrac{TL}{m\cos\theta}}$



where T is the tension in this part of the string. Thus,

$$\frac{2Lf}{\cos\theta} = \sqrt{\frac{TL}{m\cos\theta}} \quad \text{or} \quad \frac{4L^2f^2}{\cos^2\theta} = \frac{TL}{m\cos\theta}$$

and the mass of string above the rod is:

$$m = \frac{T\cos\theta}{4Lf^2} \qquad \text{[Equation 1]}$$

Now, consider the tension in the string. The light rod would rotate about point P if the string exerted any vertical force on it. Therefore, recalling Newton's third law, the rod must exert only a horizontal force on the string. Consider a free-body diagram of the string segment in contact with the end of the rod.

$$\Sigma F_y = T\sin\theta - Mg = 0 \Rightarrow T = \frac{Mg}{\sin\theta}$$

Then, from Equation 1, the mass of string above the rod is

$$m = \left(\frac{Mg}{\sin\theta}\right)\frac{\cos\theta}{4Lf^2} = \boxed{\frac{Mg}{4Lf^2\tan\theta}}$$

18.25 (a) Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n+1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = 2L/n$, and the frequency is $f = v/\lambda$.

Thus, $f = \dfrac{n}{2L}\sqrt{\dfrac{T_n}{\mu}}$, and also $f = \dfrac{n+1}{2L}\sqrt{\dfrac{T_{n+1}}{\mu}}$

Thus, $\dfrac{n+1}{n} = \sqrt{\dfrac{T_n}{T_{n+1}}} = \sqrt{\dfrac{(25.0\text{ kg})g}{(16.0\text{ kg})g}} = \dfrac{5}{4}$

Therefore, $4n + 4 = 5n$, or $n = 4$

Then, $f = \dfrac{4}{2(2.00\text{ m})}\sqrt{\dfrac{(25.0\text{ kg})(9.80\text{ m/s}^2)}{0.00200\text{ kg/m}}} = \boxed{350\text{ Hz}}$

(b) The largest mass will correspond to a standing wave of 1 loop

$(n = 1)$, so $350\text{ Hz} = \dfrac{1}{2(2.00\text{ m})}\sqrt{\dfrac{m(9.80\text{ m/s}^2)}{0.00200\text{ kg/m}}}$

yielding $m = \boxed{400\text{ kg}}$

***18.26** Using the frets does not change the speed of the wave. Therefore, if d_{NN} is the distance between adjacent nodes,

$\lambda_1 f_1 = \lambda_2 f_2 = 2d_{NN1}f_1 = 2d_{NN2}f_2$ or

$$d_{NN2} = d_{NN1}\left(\frac{f_1}{f_2}\right) = 21.4\text{ cm}\left(\frac{2349\text{ Hz}}{2217\text{ Hz}}\right) = 22.7\text{ cm}$$

Thus, the distance between frets is

$$d_{NN2} - d_{NN1} = 22.7\text{ cm} - 21.4\text{ cm} = \boxed{1.27\text{ cm}}$$

***18.27** The natural frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{2.00\text{ m}}} = 0.352\text{ Hz}$$

The big brother must push at this same frequency of $\boxed{0.352\text{ Hz}}$ to produce resonance.

***18.28** The distance between adjacent nodes is one-quarter of the circumference.

$$d_{NN} = d_{AA} = \frac{\lambda}{2} = \frac{20.0\text{ cm}}{4} = 5.00\text{ cm}$$

so $\lambda = 10.0$ cm and $f = \frac{v}{\lambda} = \frac{900\text{ m/s}}{0.100\text{ m}} = 9000\text{ Hz} = \boxed{9.00\text{ kHz}}$

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

***18.29** (a) The wave speed is $v = \frac{9.15\text{ m}}{2.50\text{ s}} = \boxed{3.66\text{ m/s}}$

(b) From Figure P18.29, there are antinodes at both ends, so the distance between adjacent antinodes is

$$d_{AA} = \frac{\lambda}{2} = 9.15\text{ m, and the wavelength is } \lambda = 18.3\text{ m}$$

The frequency is then $f = \frac{v}{\lambda} = \frac{3.66\text{ m/s}}{18.3\text{ m}} = \boxed{0.200\text{ Hz}}$

We have assumed the wave speed is the same for all wavelengths.

***18.30** The wave speed is $v = \sqrt{gd} = \sqrt{(9.80\text{ m/s}^2)(36.1\text{ m})} = 18.8\text{ m/s}$

The bay has one end open and one end closed, so its simplest resonance is with a node (of velocity, antinode of displacement) at the head of the bay and an antinode (of velocity, node of displacement) at the mouth. Then,

$$d_{NA} = 210 \times 10^3\text{ m} = \frac{\lambda}{4} \quad \text{and} \quad \lambda = 840 \times 10^3\text{ m}$$

Therefore, the period is

$$T = \frac{1}{f} = \frac{\lambda}{v} = \frac{840 \times 10^3\text{ m}}{18.8\text{ m/s}} = 4.47 \times 10^4\text{ s} = \boxed{12\text{ h }24\text{ min}}$$

$\boxed{\text{This agrees precisely with the period of the lunar excitation}}$, so we identify the extra-high tides as amplified by resonance.

18.31 (a) For the fundamental mode in a closed pipe, $\lambda = 4L$. (see Figure 18.3b)

But $v = f\lambda$, therefore $L = \frac{v}{4f}$

So, $L = \frac{343\text{ m/s}}{4(240\text{/s})} = \boxed{0.357\text{ m}}$

(b) For an open pipe, $\lambda = 2L$. (see Figure 18.3a)

So, $L = \frac{v}{2f} = \frac{343\text{ m/s}}{2(240\text{/s})} = \boxed{0.715\text{ m}}$

***18.32** $\frac{\lambda}{2} = d_{AA} = \frac{L}{n}$ or $L = \frac{n\lambda}{2}$ for $n = 1, 2, 3, \ldots$

Since $\lambda = \frac{v}{f}$, $L = n\left(\frac{v}{2f}\right)$ for $n = 1, 2, 3, \ldots$

With $v = 343$ m/s, and $f = 680$ Hz,

$$L = n\left(\frac{343 \text{ m/s}}{2(680 \text{ Hz})}\right) = n(0.252 \text{ m}) \text{ for } n = 1, 2, 3, \ldots$$

Possible lengths for resonance are:

$$L = \boxed{0.252 \text{ m}, 0.504 \text{ m}, 0.757 \text{ m}, \ldots, n(0.252) \text{ m}}$$

18.33 $d_{AA} = 0.320$ m $\qquad \lambda = 0.640$ m

(a) $f = \frac{v}{\lambda} = \boxed{531 \text{ Hz}}$

(b) $\lambda = 0.0850$ m $\qquad d_{AA} = \boxed{42.5 \text{ mm}}$

***18.34** The wavelength is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{261.6/\text{s}} = 1.31 \text{ m}$$

so the length of the open pipe vibrating in its simplest (A-N-A) mode is

$$d_{A \text{ to } A} = \frac{1}{2}\lambda = \boxed{0.656 \text{ m}}$$

A closed pipe has (N-A) for its simplest resonance, (N-A-N-A) for the second, and (N-A-N-A-N-A) for the third. Here, the pipe length is

$$5d_{N \text{ to } A} = \frac{5\lambda}{4} = \frac{5}{4}(1.31 \text{ m}) = \boxed{1.64 \text{ m}}$$

***18.35** The air in the auditory canal, about 3 cm long, can vibrate with a node at the closed end and antinode at the open end, with

$$d_{N \text{ to } A} = 3 \text{ cm} = \frac{\lambda}{4}$$

so $\quad \lambda = 0.12$ m

and $\quad f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{0.12 \text{ m}} \approx \boxed{3 \text{ kHz}}$

A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

18.36 $\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{440\text{/s}} = 0.780\text{ m}$

$d_{N\text{ to }A} = \frac{\lambda}{4} = 0.195\text{ m}$ = length of resonant air column

Water height = 0.400 m – 0.195 m = 0.205 m

$m = \rho V = \rho A h = (1000\text{ kg/m}^3)(0.100\text{ m}^2)(0.205\text{ m}) = \boxed{20.5\text{ kg}}$

18.37 For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $L = \frac{n\lambda}{2}$, $(n = 1, 2, 3, \ldots)$.

i.e., $L = \frac{n\lambda}{2} = \frac{nv}{2f}$

and $f = \frac{nv}{2L}$

Therefore, with $L = 0.860$ m and $L' = 2.10$ m, the resonant frequencies are

$f_n = \boxed{n(206\text{ Hz})}$ for $L = 0.860$ m for each n from 1 to 9

and $f'_n = \boxed{n(84.5\text{ Hz})}$ for $L' = 2.10$ m for each n from 2 to 23

18.38 We suppose these are the lowest resonances of the enclosed air columns.

For one,

$\lambda = \frac{v}{f} = \frac{(343\text{ m/s})}{256\text{/s}} = 1.34\text{ m}$

length $= d_{AA} = \frac{\lambda}{2} = 0.670\text{ m}$

For the other,

$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{440\text{/s}} = 0.780\text{ m}$

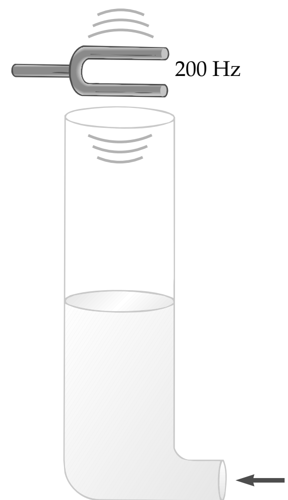
length = 0.390 m

So,

(b) original length = $\boxed{1.06\text{ m}}$

$\lambda = 2d_{AA} = 2.12\text{ m}$

(a) $f = \frac{(343\text{ m/s})}{2.12\text{ m}} = \boxed{162\text{ Hz}}$



18.39 The fork radiates sound with $\lambda = \dfrac{v}{f}$

The distance between successive water levels at resonance is

$$d_{NN} = \frac{v}{2f}$$

So $$Rt = \frac{\pi r^2 v}{2f}$$

$$t = \frac{\pi r^2 v}{2Rf}$$

$$t = \frac{\pi(4.00 \times 10^{-2}\text{ m})^2(343\text{ m/s})}{2(18.0 \times 10^{-6}\text{ m}^3/\text{s})(200/\text{s})} = \boxed{239\text{ s}}$$

18.40 The wavelength of sound is $\lambda = \dfrac{v}{f}$

The distance between water levels at resonance is

$$d = \frac{v}{2f}$$

$$\therefore Rt = \pi r^2 d = \frac{\pi r^2 v}{2f}$$

and $$t = \boxed{\frac{\pi r^2 v}{2Rf}}$$

18.41 The length corresponding to the fundamental satisfies $f = \dfrac{v}{4L}$, giving

$$L_1 = \frac{v}{4f} = \frac{343}{4(512)} = 0.167\text{ m}$$

Since $L > 20.0$ cm, the *next* two modes will be observed, corresponding to

$$f = \frac{3v}{4L_2} \quad \text{and} \quad f = \frac{5v}{4L_3}$$

or $$L_2 = \frac{3v}{4f} = \boxed{0.502\text{ m}} \quad \text{and} \quad L_3 \frac{5v}{4f} = \boxed{0.837\text{ m}}$$

18.42 Call L the depth of the well and v the speed of sound. Then for some integer n

$$L = (2n-1)\frac{\lambda_1}{4} = (2n-1)\frac{v}{4f_1} = \frac{(2n-1)(343\text{ m/s})}{4(51.5\text{ s}^{-1})}$$

and for the next resonance

$$L = [2(n+1)-1]\frac{\lambda_2}{4} = (2n+1)\frac{v}{4f_2} = \frac{(2n+1)(343\text{ m/s})}{4(60.0\text{ s}^{-1})}$$

Thus, $\dfrac{(2n-1)(343\text{ m/s})}{4(51.5\text{ s}^{-1})} = \dfrac{(2n+1)(343\text{ m/s})}{4(60.0\text{ s}^{-1})}$,

and we require an *integer* solution to $\dfrac{2n+1}{60.0} = \dfrac{2n-1}{51.5}$

The equation gives $n = \dfrac{111.5}{17} = 6.56$, so the best fitting integer is $n = 7$.

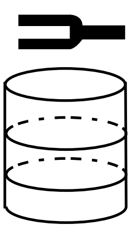
Then $L = \dfrac{[2(7)-1](343\text{ m/s})}{4(51.5\text{ s}^{-1})} = 21.6\text{ m}$

and $L = \dfrac{[2(7)+1](343\text{ m/s})}{4(60.0\text{ s}^{-1})} = 21.4\text{ m}$

suggest the best value for the depth of the well is $\boxed{21.5\text{ m}}$.

18.43 For resonance in a tube open at one end,

$$f = n\frac{v}{4L} \quad (n = 1, 3, 5, \ldots) \qquad \text{Equation 18.12}$$



(a) Assuming $n = 1$ and $n = 3$,

$$384 = \frac{v}{4(0.228)} \quad \text{and} \quad 384 = \frac{3v}{4(0.683)}$$

In either case, $v = \boxed{350\text{ m/s}}$

(b) For the next resonance, $n = 5$, and

$$L = \frac{5v}{4f} = \frac{5(350\text{ m/s})}{4(384\text{ s}^{-1})} = \boxed{1.14\text{ m}}$$

18.44 (a) $f_1 = \frac{v}{\lambda} = \frac{v}{4L} = \frac{331.5\text{ m/s}}{4(4.88\text{ m})} = \boxed{17.0\text{ Hz}}$

(b) $f_1 = \frac{v}{\lambda} = \frac{v}{2L} = \boxed{34.0\text{ Hz}}$

(c) For the closed pipe, $f = \frac{v(20.0°\text{C})}{v(0°\text{C})} f_1 = \sqrt{1 + \frac{20.0}{273}}\ f_1 = \boxed{17.6\text{ Hz}}$

For the open pipe, $f = \sqrt{1 + \frac{20.0}{273}}\ f_1 = \boxed{35.2\text{ Hz}}$

***18.45** (a) For the fundamental mode of an open tube,

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{2(880\text{ s}^{-1})} = \boxed{0.195\text{ m}}$$

(b) $v = 331\text{ m/s}\sqrt{1 + \frac{(-5.00)}{273}} = 328\text{ m/s}$

We ignore the thermal expansion of the metal.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{328\text{ m/s}}{2(0.195\text{ m})} = \boxed{841\text{ Hz}}$$

The flute is flat by a semitone.

18.46 When the rod is clamped at one-quarter of its length, the fundamental frequency corresponds to a mode of vibration in which $L = \lambda$.

Therefore, $L = \frac{v}{f} = \frac{5100\text{ m/s}}{4400\text{ Hz}} = \boxed{1.16\text{ m}}$

18.47 (a) $f = \frac{v}{2L} = \frac{5100}{(2)(1.60)} = \boxed{1.59\text{ kHz}}$

(b) Since it is held in the center, there must be a node in the center as well as antinodes at the ends. The even harmonics have an antinode at the center so only $\boxed{\text{the odd harmonics}}$ are present.

(c) $f = \frac{v'}{2L} = \frac{3560}{(2)(1.60)} = \boxed{1.11\text{ kHz}}$

18.48 $v = 4500\text{ m/s}$ $\qquad \lambda_1 = 4L = 240\text{ cm} = 2.40\text{ m}$

so, $f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = \frac{4500}{2.40} = \boxed{1.88\text{ kHz}}$

18.49 $f \propto v \propto \sqrt{T}$

$$f_{\text{new}} = 110\sqrt{\frac{540}{600}} = 104.4 \text{ Hz}$$

$$\Delta f = \boxed{5.64 \text{ beats/s}}$$

Goal Solution

G: Beat frequencies are usually only a few Hertz, so we should not expect a frequency much greater than this.

O: As in previous problems, the two wave speed equations can be used together to find the frequency of vibration that corresponds to a certain tension. The beat frequency is then just the difference in the two resulting frequencies from the two strings with different tensions.

A: Combining the velocity equation $v = f/\lambda$ and the tension equation $v = \sqrt{\frac{T}{\mu}}$ we find that

$$f = \sqrt{\frac{T}{\mu\lambda^2}}$$

and since μ and λ are constant, we can divide to get $\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}}$

With $f_1 = 110$ Hz, $T_1 = 600$ N, and $T_2 = 540$ N: $f_2 = (110 \text{ Hz})\sqrt{\frac{540 \text{ N}}{600 \text{ N}}} = 104.4 \text{ Hz}$

The beat frequency is: $f_b = \left|f_1 - f_2\right| = 110 \text{ Hz} - 104.4 \text{ Hz} = 5.64 \text{ Hz}$

L: As expected, the beat frequency is only a few cycles per second. This result from the interference of the two sound waves with slightly different frequencies has a tone that varies in amplitude over time, similar to the sound made by saying "wa-wa-wa…"
Note: The beat frequency above is written with three significant figures on the assumption that the data and known precisely enough to warrant them. This assumption implies that the original frequency is known more precisely than to the three significant digits quoted in "110 Hz." For example, if the original frequency of the strings were 109.6 Hz, the beat frequency would be 5.62 Hz.

***18.50** (a) The string could be tuned to either $\boxed{521 \text{ Hz or } 525 \text{ Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down. Instead, the frequency must have started at 525 Hz to become $\boxed{526 \text{ Hz}}$.

(c) From $f = \dfrac{v}{\lambda} = \dfrac{\sqrt{T/\mu}}{2L} = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$,

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \quad \text{and} \quad T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1 = \left(\frac{523\text{ Hz}}{526\text{ Hz}}\right)^2 T_1 = 0.989T_1$$

The fractional change that should be made in the tension is then

$$\text{fractional change} = \frac{T_1 - T_2}{T_1} = 1 - 0.989 = 0.0114 = 1.14\%\text{ lower}$$

The tension should be $\boxed{\text{reduced by } 1.14\%}$.

18.51 For an echo $f\,' = f\dfrac{(v + v_s)}{(v - v_s)}$

the beat frequency is $f_b = \boxed{|f\,' - f|}$

Solving for f_b gives $f_b = f\dfrac{(2v_s)}{(v - v_s)}$ when approaching wall.

(a) $f_b = (256)\dfrac{(2)(1.33)}{(343 - 1.33)} = \boxed{1.99\text{ Hz}}$ beat frequency

(b) When moving away from wall, v_s changes sign. Solving for v_s gives

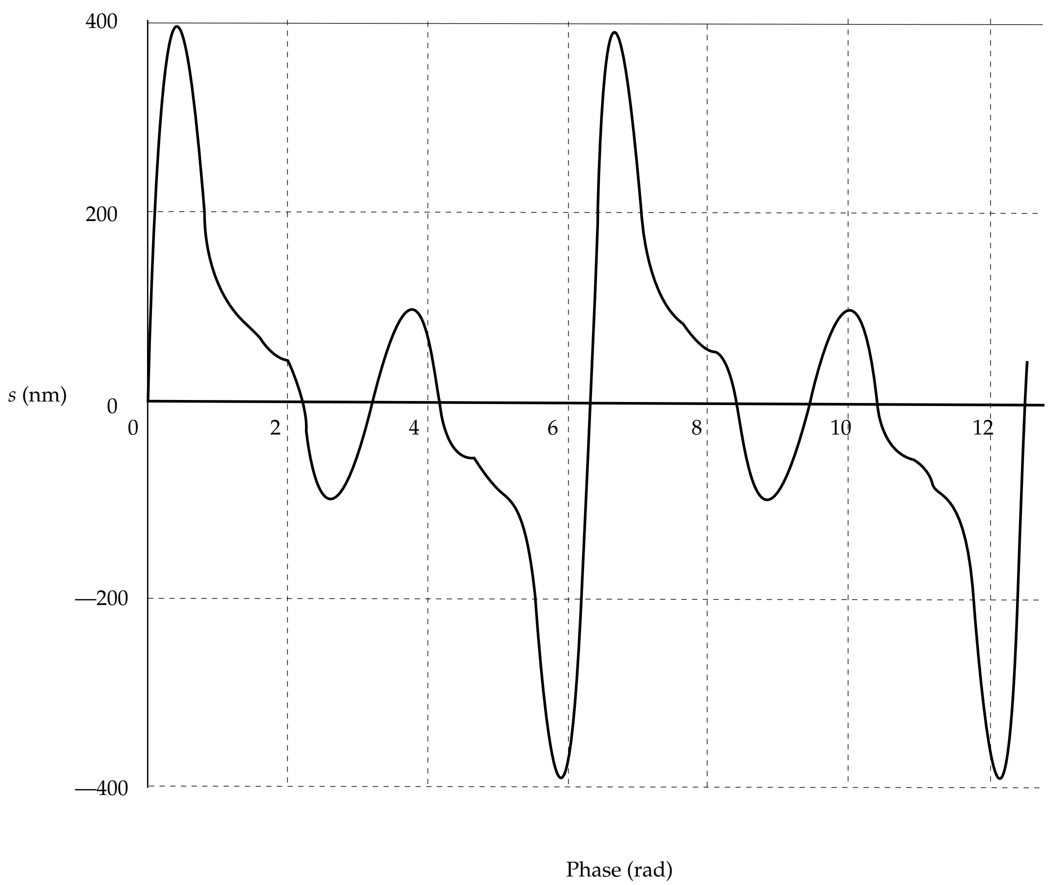
$$v_s = \frac{f_b v}{2f - f_b} = \frac{(5)(343)}{(2)(256) - 5} = \boxed{3.38\text{ m/s}}$$

***18.52** We evaluate

$$s = 100 \sin \theta + 157 \sin 2\theta + 62.9 \sin 3\theta + 105 \sin 4\theta + 51.9 \sin 5\theta + 29.5 \sin 6\theta + 25.3 \sin 7\theta$$

where s represents particle displacement in nanometers and θ represents the phase of the wave in radians. As θ advances by 2π, time advances by (1/523) s. Here is the result:

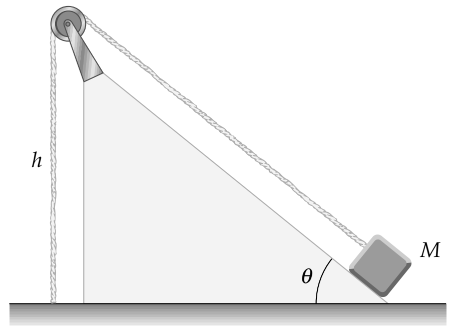
Flute Waveform



***18.53** We list the frequencies of the harmonics of each note in Hz:

	Harmonic				
Note	1	2	3	4	5
A	440.00	880.00	1320.0	1760.0	2200.0
C#	554.37	1108.7	1663.1	2217.5	2771.9
E	659.26	1318.5	1977.8	2637.0	3296.3

The second harmonic of E is close to the third harmonic of A, and the fourth harmonic of C# is close to the fifth harmonic of A.



18.54 (a) For the block:

$$\Sigma F_x = T - Mg \sin 30.0° = 0$$

so $T = Mg \sin 30.0° = \boxed{\frac{1}{2}Mg}$

(b) The length of the section of string parallel to the incline is $h/\sin 30.0° = 2h$. The total length of the string is then $\boxed{3h}$.

(c) The mass per unit length of the string is $\mu = \boxed{m/3h}$.

(d) The speed of waves in the string is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\left(\frac{Mg}{2}\right)\left(\frac{3h}{m}\right)} = \boxed{\sqrt{\frac{3Mgh}{2m}}}$

(e) In the fundamental mode, the segment of length h vibrates as one loop. The distance between adjacent nodes is then $d_{NN} = \lambda/2 = h$, so the wavelength is $\lambda = 2h$.

The frequency is $f = \frac{v}{\lambda} = \frac{1}{2h}\sqrt{\frac{3Mgh}{2m}} = \boxed{\sqrt{\frac{3Mg}{8mh}}}$

(g) When the vertical segment of string vibrates with 2 loops (i.e., 3 nodes), then $h = 2\left(\frac{\lambda}{2}\right)$ and the wavelength is $\lambda = \boxed{h}$.

(f) The period of the standing wave of 3 nodes (or two loops) is

$$T = \frac{1}{f} = \frac{\lambda}{v} = h\sqrt{\frac{2m}{3Mgh}} = \boxed{\sqrt{\frac{2mh}{3Mg}}}$$

(h) $f_b = 1.02f - f = (2.00 \times 10^{-2})f = \boxed{(2.00 \times 10^{-2})\sqrt{\frac{3Mg}{8mh}}}$

18.55 (a) $\Delta x = \sqrt{(9.00 + 4.00)} - 3.00 = \sqrt{13.0} - 3.00 = 0.606 \text{ m}$

The wavelength is $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{300 \text{ Hz}} = 1.14 \text{ m}$

Thus, $\frac{\Delta x}{\lambda} = \frac{0.606}{1.14} = 0.530$

of a wave, or

$$\Delta \phi = 2\pi(0.530) = \boxed{3.33 \text{ rad}}$$

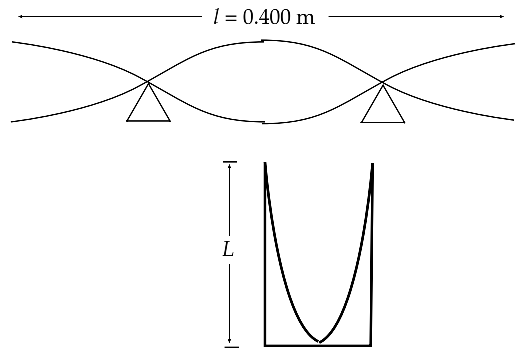
(b) For destructive interference, we want

$$\frac{\Delta x}{\lambda} = 0.500 = f\frac{\Delta x}{v}$$

where Δx is a constant in this set up.

$$f = \frac{v}{2\,\Delta x} = \frac{343}{(2)(0.606)} = \boxed{283\text{ Hz}}$$

18.56 $f = 87.0$ Hz



speed of sound in air: $v_a = 340$ m/s

(a) $\lambda_b = l$

$$v = f\lambda_b = (87.0\text{ s}^{-1})(0.400\text{ m})$$

$$v = \boxed{34.8\text{ m/s}}$$

(b) $$\left.\begin{aligned}\lambda_a &= 4L\\ v_a &= \lambda_a f\end{aligned}\right\}\; L = \frac{v_a}{4f} = \frac{340\text{ m/s}}{4(87.0\text{ s}^{-1})} = \boxed{0.977\text{ m}}$$

18.57 Moving away from station, frequency is depressed:

$$f' = 180 - 2.00 = 178\text{ Hz}$$

$$178 = 180\frac{(343)}{(343 + v)}$$

Solving for v gives

$$v = \frac{(2.00)(343)}{178}$$

Therefore,

$$v = \boxed{3.85\text{ m/s away from station}}$$

Moving towards the station, the frequency is enhanced:

$$f' = 180 + 2.00 = 182\text{ Hz}$$

$$182 = 180\frac{(343)}{(343 - v)}$$

Solving for v gives

$$v = \frac{(2.00)(343)}{182}$$

Therefore,

$$v = \boxed{3.77 \text{ m/s towards the station}}$$

***18.58** Use the Doppler formula

$$f' = f\frac{(v \pm v_0)}{(v \mp v_s)}$$

With f_1' = frequency of the speaker in front of student and

f_2' = frequency of the speaker behind the student.

$$f_1' = (456 \text{ Hz})\frac{(343 \text{ m/s} + 1.50 \text{ m/s})}{(343 \text{ m/s} - 0)} = 458 \text{ Hz}$$

$$f_2' = (456 \text{ Hz})\frac{(343 \text{ m/s} - 1.50 \text{ m/s})}{(343 \text{ m/s} + 0)} = 454 \text{ Hz}$$

Therefore, $f_b = f_1' - f_1' = \boxed{3.99 \text{ Hz}}$

18.59 From the leading train she hears

$$f_1' = f\left(\frac{v + 0}{v + v_s}\right) = f\left(\frac{343 \text{ m/s}}{343 \text{ m/s} + 8.00 \text{ m/s}}\right)$$

From the still-approaching train,

$$f_2' = f\left(\frac{343}{343 - 8.00}\right)$$

Then, $4.00 \text{ Hz} = f_2' - f_1' = 1.0239f - 0.9772f$

$$f = \frac{4.00 \text{ Hz}}{0.0467} = \boxed{85.7 \text{ Hz}}$$

18.60 $v = \sqrt{\dfrac{(48.0)(2.00)}{4.80 \times 10^{-3}}} = 141 \text{ m/s}$

$d_{NN} = 1.00 \text{ m}$ $\lambda = 2.00 \text{ m}$ $f = \dfrac{v}{\lambda} = 70.7 \text{ Hz}$

$$\lambda_a = \frac{v_a}{f} = \frac{343 \text{ m/s}}{70.7 \text{ Hz}} = \boxed{4.85 \text{ m}}$$

***18.61** The second standing wave mode of the air in the pipe reads ANAN, with

$$d_{NA} = \frac{\lambda}{4} = \frac{1.75\text{ m}}{3} \quad \text{so} \quad \lambda = 2.33\text{ m} \quad \text{and}$$

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{2.33\text{ m}} = 147\text{ Hz}$$

For the string, λ and v are different but f is the same.

$$\frac{\lambda}{2} = d_{NN} = \frac{0.400\text{ m}}{2} \quad \text{so} \quad \lambda = 0.400\text{ m}$$

$$v = \lambda f = (0.400\text{ m})(147\text{ Hz}) = 58.8\text{ m/s} = \sqrt{T/\mu}$$

$$T = \mu v^2 = (9.00 \times 10^{-3}\text{ kg/m})(58.8\text{ m/s})^2 = \boxed{31.1\text{ N}}$$

18.62 (a) $L = \dfrac{v}{4f}$ so $\dfrac{L'}{L} = \dfrac{f}{f'}$

Letting the longest L be 1, the ratio is $1 : \frac{4}{5} : \frac{2}{3} : \frac{1}{2}$

or in integers $\boxed{30 : 24 : 20 : 15}$

(b) $L = \boxed{\dfrac{343}{(4)(256)} = 33.5\text{ cm}}$

This is the longest pipe, so using the ratios the lengths are:

$\boxed{33.5, 26.8, 22.3, 16.7\text{ cm}}$

(c) The frequencies are using the ratio $\boxed{256, 320, 384, \text{and } 512\text{ Hz}}$. These represent notes C, E, G, and C' on the physical pitch scale.

18.63 (a) Since the first node is at the weld, the wavelength in the thin wire is $2L$ or 80.0 cm. The frequency and tension are the same in both sections, so

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(0.400)}\sqrt{\frac{4.60}{2.00 \times 10^{-3}}} = \boxed{59.9\text{ Hz}}$$

(b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire.

$$\mu' = 8.00 \text{ g/m}$$

so $$L' = \frac{1}{2f}\sqrt{\frac{T}{\mu'}}$$

$$L' = \left[\frac{1}{(2)(59.9)}\right]\sqrt{\left[\frac{(4.60)}{(8.00 \times 10^{-3})}\right]}$$

$$= \boxed{20.0 \text{ cm}}$$ half the length of the thin wire

18.64 $f_B = f_A \qquad \lambda_B = \frac{1}{3}\lambda_A \qquad v_B = \frac{1}{3} v_A$

$$v_B^2 = \frac{1}{9} v_A^2$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\frac{T_B}{T_A} = \frac{v_B^2}{v_A^2} = \boxed{0.111}$$

18.65 (a) $f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

so $$\frac{f'}{f} = \frac{L}{L'} = \frac{L}{2L} = \frac{1}{2}$$

The $\boxed{\text{frequency should be halved}}$ to get the same number of antinodes for twice the length.

(b) $\frac{n'}{n} = \sqrt{\frac{T}{T'}}$

so $$\frac{T'}{T} = \left(\frac{n}{n'}\right)^2 = \left[\frac{n}{(n+1)}\right]^2$$

The tension must be

$$T' = \boxed{\left[\frac{n}{(n+1)}\right]^2 T}$$

(c) $\dfrac{f'}{f} = \dfrac{n'}{n}\dfrac{L}{L'}\sqrt{\dfrac{T'}{T}}$

so $\dfrac{T'}{T} = \left(\dfrac{n}{n'}\dfrac{f'L'}{fL}\right)^2$

$\dfrac{T'}{T} = \left(\dfrac{3}{2 \cdot 2}\right)^2$

$\boxed{\dfrac{T'}{T} = \dfrac{9}{16}}$ to get twice as many antinodes.

18.66 For the wire,

$$\mu = \frac{0.0100 \text{ kg}}{2.00 \text{ m}} = 5.00 \times 10^{-3} \text{ kg/m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(200 \text{ kg} \cdot \text{m/s}^2)}{5.00 \times 10^{-3} \text{ kg/m}}}$$

$$v = 200 \text{ m/s}$$

If it vibrates in its simplest state,

$$d_{NN} = 2.00 \text{ m} = \frac{\lambda}{2}$$

$$f = \frac{v}{\lambda} = \frac{(200 \text{ m/s})}{4.00 \text{ m}} = 50.0 \text{ Hz}$$

(a) The tuning fork can have frequencies

$\boxed{45.0 \text{ Hz} \quad \text{or} \quad 55.0 \text{ Hz}}$

(b) If $f = 45.0$ Hz,

$$v = f\lambda = (45.0/\text{s})\ 4.00 \text{ m} = 180 \text{ m/s}$$

Then,

$$T = v^2\mu = (180 \text{ m/s})^2(5.00 \times 10^{-3} \text{ kg/m}) = \boxed{162 \text{ N}}$$

$\boxed{\text{or}}$ if $f = 55.0$ Hz

$$T = v^2\mu = f^2\lambda^2\mu = (55.0/\text{s})^2(4.00 \text{ m})^2(5.00 \times 10^{-3} \text{ kg/m}) = \boxed{242 \text{ N}}$$

18.67 The odd-numbered harmonics of the organ-pipe vibration are:

650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50.0 Hz

Closed $f_1 = \boxed{50.0 \text{ Hz}}$ $\lambda = 6.80 \text{ m}$ $\boxed{L = 1.70 \text{ m}}$

18.68 We look for a solution of the form

$$5.00 \sin(2.00x - 10.0t) + 10.0 \cos(2.00x - 10.0t)$$

$$= A \sin(2.00x - 10.0t + \phi)$$

$$= A \sin(2.00x - 10.0t)\cos\phi + A\cos(2.00x - 10.0t)\sin\phi$$

This will be true if both $5.00 = A\cos\phi$ and $10.0 = A\sin\phi$,

requiring $(5.00)^2 + (10.0)^2 = A^2$ $A = 11.2$ and $\phi = 63.4°$

The resultant wave $\boxed{11.2 \sin(2.00x - 10.0t + 63.4°)}$ is sinusoidal.

***18.69** (a) With $k =$ **Error!** and $\omega = 2\pi f =$ **Error!**

$$y(x, t) = 2A \sin kx \cos \omega t = \boxed{2A \sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi v t}{\lambda}\right)}$$

(b) For the fundamental vibration,

$$\lambda_1 = 2L$$

so $y_1(x, t) = \boxed{2A \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi v t}{L}\right)}$

(c) For the second harmonic

$$\lambda_2 = L$$

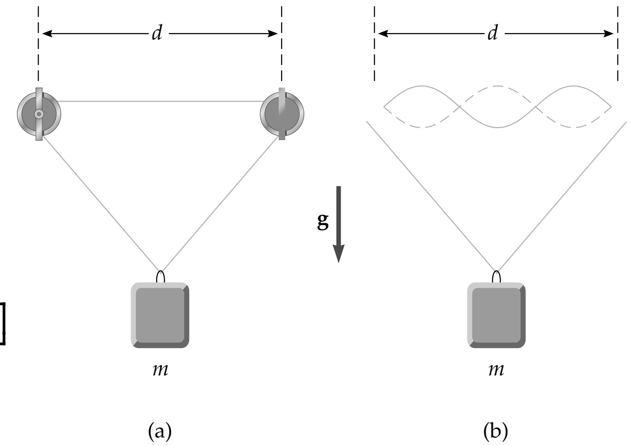
and

$$y_2(x, t) = \boxed{2A \sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi v t}{L}\right)}$$

(d) In general,

$$\lambda_n = \frac{2L}{n} \quad \text{and} \quad y_n(x, t) = \boxed{2A \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi v t}{L}\right)}$$

18.70 (a) In the diagram, observe that:



$$\sin\theta = \frac{1.00\text{ m}}{1.50\text{ m}} = \frac{2}{3} \quad \text{or} \quad \theta = 41.8°$$

Considering the mass,

$$\Sigma F_y = 0 \quad \text{gives} \quad 2T\cos\theta = mg$$

or $$T = \frac{(12.0\text{ kg})(9.80\text{ m/s}^2)}{2\cos 41.8°} = \boxed{78.9\text{ N}}$$

(b) The speed of transverse waves in the string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{78.9\text{ N}}{0.00100\text{ kg/m}}} = 281\text{ m/s}$$

For the standing wave pattern shown (3 loops), $d = \frac{3}{2}\lambda$, or

$$\lambda = \frac{2(2.00\text{ m})}{3} = 1.33\text{ m}$$

Thus, the required frequency is

$$f = \frac{v}{\lambda} = \frac{281\text{ m/s}}{1.33\text{ m}} = \boxed{211\text{ Hz}}$$

Chapter 19 Solutions

***19.1** (a) To convert from Fahrenheit to Celsius, we use

$$T_C = \frac{5}{9}(T_F - 32.0) = \frac{5}{9}(98.6 - 32.0) = \boxed{37.0°\text{C}}$$

and the Kelvin temperature is found as

$$T = T_C + 273 = \boxed{310 \text{ K}}$$

(b) In a fashion identical to that used in (a), we find

$$T_C = \boxed{-20.6°\text{C}} \quad \text{and} \quad T = \boxed{253 \text{ K}}$$

19.2 $P_1V = nRT_1$ and $P_2V = nRT_2$

imply that $\dfrac{P_2}{P_1} = \dfrac{T_2}{T_1}$

(a) $P_2 = \dfrac{P_1T_2}{T_1} = \dfrac{(0.980 \text{ atm})(273 + 45.0)\text{K}}{(273 + 20.0)\text{K}} = \boxed{1.06 \text{ atm}}$

(b) $T_3 = \dfrac{T_1P_3}{P_1} = \dfrac{(293 \text{ K})(0.500 \text{ atm})}{(0.980 \text{ atm})} = 149 \text{ K} = \boxed{-124°\text{C}}$

19.3 Since we have a linear graph, the pressure is related to the temperature as $P = A + BT$, where A and B are constants. To find A and B, we use the data

$$0.900 \text{ atm} = A + (-80.0°\text{C})B \qquad (1)$$

$$1.635 \text{ atm} = A + (78.0°\text{C})B \qquad (2)$$

Solving (1) and (2) simultaneously, we find

$$A = 1.272 \text{ atm}$$

and $B = 4.652 \times 10^{-3} \text{ atm}/°\text{C}$

Therefore, $P = 1.272 \text{ atm} + (4.652 \times 10^{-3} \text{ atm}/°\text{C})T$

(a) At absolute zero

$$P = 0 = 1.272 \text{ atm} + (4.652 \times 10^{-3} \text{ atm}/°\text{C})T$$

which gives $\boxed{T = -274°\text{C}}$

(b) At the freezing point of water

$$P = 1.272 \text{ atm} + 0 = \boxed{1.27 \text{ atm}}$$

(c) and at the boiling point

$$P = 1.272 \text{ atm} + (4.652 \times 10^{-3} \text{ atm}/°\text{C})(100°\text{C}) = \boxed{1.74 \text{ atm}}$$

19.4 Let us use $T_C = \frac{5}{9}(T_F - 32.0)$ with $T_F = -40.0°\text{C}$. We find

$$T_C = \frac{5}{9}(-40.0 - 32.0) = -40.0°\text{C}$$

19.5 (a) $T_F = \frac{9}{5}T_C + 32.0°\text{F} = \frac{9}{5}(-195.81) + 32.0 = \boxed{-320°\text{F}}$

(b) $T = T_C + 273.15 = -195.81 + 273.15 = \boxed{77.3 \text{ K}}$

19.6 Require $0.00°\text{C} = a(-15.0°\text{S}) + b$

$100°\text{C} = a(60.0°\text{S}) + b$

Subtracting, $100°\text{C} = a(75.0°\text{S})$

$a = 1.33\ \text{C}°/\text{S}°$

Then $0.00°\text{C} = 1.33(-15.0°\text{S})\text{C}° + b$

$b = 20.0°\text{C}$

So the conversion is $\boxed{T_C = (1.33\ \text{C}°/\text{S}°)T_S + 20.0°\text{C}}$

19.7 (a) $\Delta T = 450\ \text{C}° = 450\ \text{C}°\left(\frac{212°\text{F} - 32.0°\text{F}}{100°\text{C} - 0.00°\text{C}}\right) = \boxed{810\ \text{F}°}$

(b) $\Delta T = 450\ \text{C}° = \boxed{450 \text{ K}}$

19.8 (a) $T = 1064 + 273 = \boxed{1337 \text{ K}}$ melting point

$T = 2660 + 273 = \boxed{2933 \text{ K}}$ boiling point

(b) $\Delta T = \boxed{1596\ \text{C}°} = \boxed{1596 \text{ K}}$ The differences are the same.

19.9 The wire is 35.0 m long when $T_C = -20.0°C$

$$\Delta L = L_i\,\bar{\alpha}\,(T - T_i)$$

$$\bar{\alpha} \approx \alpha(20.0°C) = 1.70 \times 10^{-5}\ (C°)^{-1} \text{ for Cu.}$$

$$\Delta L = (35.0\text{ m})(1.70 \times 10^{-5}\ (C°)^{-1})\big(35.0°C - (-20.0°C)\big) = \boxed{+3.27\text{ cm}}$$

Goal Solution

G: Based on everyday observations of telephone wires, we might expect the wire to expand by less than a meter since the change in length of these wires is generally not noticeable.

O: The change in length can be found from the linear expansion of copper wire (we will assume that the insulation around the copper wire can stretch more easily than the wire itself). From Table 19.2, the coefficient of linear expansion for copper is $17 \times 10^{-6}\ (°C)^{-1}$.

A: The change in length between cold and hot conditions is

$$\Delta L = \alpha L_0 \Delta T = [17 \times 10^{-6}\ (°C)^{-1}](35.0\text{ m})(35.0°C - (-20.0°C))$$

$$\Delta L = 3.27 \times 10^{-2}\text{ m} \quad \text{or} \quad \Delta L = 3.27\text{ cm}$$

L: This expansion is well under our expected limit of a meter. From ΔL, we can find that the wire sags 0.757 m at its midpoint on the hot summer day, which also seems reasonable based on everyday observations.

19.10 $\Delta L = L_i \alpha\ \Delta T = (25.0\text{ m})(12.0 \times 10^{-6}/C°)(40.0\text{ C°}) = \boxed{1.20\text{ cm}}$

19.11 (a) $\Delta L = \alpha L_i\,\Delta T = 24.0 \times 10^{-6}(C°)^{-1}(3.0000\text{ m})(80.0°C) = 0.00576\text{ m}$

$$L_f = \boxed{3.0058\text{ m}}$$

(b) $\Delta L = 24.0 \times 10^{-6}(C°)^{-1}(3.0000\text{ m})(-20.0°C) = -0.0014$

$$\boxed{L_f = 2.9986\text{ m}}$$

19.12 (a) $L_{Al}(1 + \alpha_{Al}\,\Delta T) = L_{Brass}\,(1 + \alpha_{Brass}\,\Delta T)$

$$\Delta T = \frac{L_{Al} - L_{Brass}}{L_{Brass}\,\alpha_{Brass} - L_{Al}\,\alpha_{Al}}$$

$$\Delta T = \frac{(10.01 - 10.00)}{(10.00)(19.0 \times 10^{-6}) - (10.01)(24.0 \times 10^{-6})}$$

$$\Delta T = -199\text{ C°} \quad \text{so } T = \boxed{-179°C \quad \text{This is attainable.}}$$

(b) $\Delta T = \dfrac{(10.02 - 10.00)}{(10.00)(19.0 \times 10^{-6}) - (10.02)(24.0 \times 10^{-6})}$

$\Delta T = -396\text{ C°}$ so $T = \boxed{-376\text{°C which is below 0 K so it cannot be reached}}$

19.13 For the dimensions to increase,

$$\Delta L = \alpha L_i \Delta T$$

$1.00 \times 10^{-2}\text{ cm} = (1.30 \times 10^{-4}/\text{°C})(2.20\text{ cm})(T - 20.0\text{°C})$

$$T = \boxed{55.0\text{°C}}$$

19.14 $\alpha = 1.10 \times 10^{-5}\text{ deg}^{-1}$ for steel

$\Delta L = (518\text{ m})(1.10 \times 10^{-5}\text{ deg}^{-1})\left[35.0\text{°C} - (-20.0\text{°C})\right] = \boxed{0.313\text{ m}}$

***19.15** (a) $\Delta A = 2\alpha A_i(\Delta T)$

$\Delta A = 2(17.0 \times 10^{-6}/\text{°C})(0.0800\text{ m})^2(50.0\text{°C})$

$\Delta A = 1.09 \times 10^{-5}\text{ m}^2 = \boxed{0.109\text{ cm}^2}$

(b) The length of each side of the hole has increased. Thus, this represents an $\boxed{\text{increase}}$ in the area of the hole.

19.16 $\Delta V = (\beta - 3\alpha)V_i \Delta T$

$= [(5.81 \times 10^{-4} - 3(11.0 \times 10^{-6})](50.0\text{ gal})(20.0)$

$= \boxed{0.548\text{ gal}}$

19.17 (a) $\Delta L = \alpha L_i \Delta T = 9.00 \times 10^{-6}(\text{C°})^{-1}(30.0\text{ cm})(65.0\text{°C}) = \boxed{0.176\text{ mm}}$

(b) $\Delta L = 9.00 \times 10^{-6}(\text{C°})^{-1}(1.50\text{ cm})(65.0\text{°C}) = \boxed{8.78 \times 10^{-4}\text{ cm}}$

(c) $\Delta V = 3\alpha V_i\ \Delta T = 3(9.00 \times 10^{-6}/\text{°C})\dfrac{(30.0)(\pi)(1.50)^2}{4}\ \text{cm}^3(65.0\text{°C}) = \boxed{0.0930\text{ cm}^3}$

19.18 (a) $V_f = V_i(1 + \beta\,\Delta T) = 100[1 + 1.50 \times 10^{-4}(-15.0)] = \boxed{99.8 \text{ mL}}$

(b) $\Delta V_{\text{acetone}} = (\beta V_i\,\Delta T)_{\text{acetone}}$

$\Delta V_{\text{flask}} = (\beta V_i\,\Delta T)_{\text{Pyrex}} = (3\alpha V_i\,\Delta T)_{\text{Pyrex}}$

for same V_i, ΔT,

$$\frac{\Delta V_{\text{acetone}}}{\Delta V_{\text{flask}}} = \frac{\beta_{\text{acetone}}}{\beta_{\text{flask}}} = \frac{1.50 \times 10^{-4}}{3(3.20 \times 10^{-6})} = \frac{1}{6.40 \times 10^{-2}}$$

The volume change of flask is

$\boxed{\text{about 6\% of the change in the acetone's volume}}$.

19.19 (a) and (b) The material would expand by $\Delta L = \alpha L_i\,\Delta T$,

$\frac{\Delta L}{L_i} = \alpha\,\Delta T$, but instead feels stress

$$\frac{F}{A} = \frac{Y\,\Delta L}{L_i} = Y\alpha\,\Delta T = (7.00 \times 10^9 \text{ N/m}^2)\ 12.0 \times 10^{-6}\,(\text{C°})^{-1}(30.0\text{ C°})$$

$= \boxed{2.52 \times 10^6 \text{ N/m}^2}$ This will $\boxed{\text{not break}}$ concrete.

19.20 (a) and (b) The gap width is a linear dimension, so it $\boxed{\text{increases}}$ in "thermal enlargement" by

$\Delta L = \alpha L_i\,\Delta T = (11.0 \times 10^{-6}/\text{C°})(1.60 \text{ cm})(160\text{ C°}) = 2.82 \times 10^{-3}$ cm

so $L_f = \boxed{1.603 \text{ cm}}$

19.21 In $\frac{F}{A} = \frac{Y\,\Delta L}{L_i}$ require $\Delta L = \alpha L_i\,\Delta T$

$$\frac{F}{A} = Y\alpha\,\Delta T$$

$$\Delta T = \frac{F}{AY\alpha} = \frac{500 \text{ N}}{(2.00 \times 10^{-4} \text{ m}^2)(20.0 \times 10^{-10} \text{ N/m}^2)(11.0 \times 10^{-6}/\text{C°})}$$

$\Delta T = \boxed{1.14\text{°C}}$

19.22 $\Delta L = \alpha L_i(\Delta T)$ and $\dfrac{F}{A} = Y\dfrac{\Delta L}{L_i}$

$F = AY\dfrac{\Delta L}{L_i} = AY\alpha\,(\Delta T)$

$= \pi(0.0200\text{ m})^2\left(20.6\times 10^{10}\ \dfrac{\text{N}}{\text{m}^2}\right)(11.0\times 10^{-6}/\text{C°})(70.0°\text{C})$

$F = \boxed{199\text{ kN}}$

19.23 (a) $\Delta V = V_t\beta_t\,\Delta T - V_{\text{Al}}\beta_{\text{Al}}\Delta T = (\beta_t - 3\alpha_{\text{Al}})V_i\,\Delta T$

$= (9.00\times 10^{-4} - 0.720\times 10^{-4})(°\text{C})^{-1}(2000\text{ cm}^3)(60.0°\text{C})$

$\Delta V = \boxed{99.4\text{ cm}^3}$ overflows

(b) The whole new volume of turpentine is

$2000\text{ cm}^3 + (9.00\times 10^{-4}/°\text{C})(2000\text{ cm}^3)(60.0°\text{C}) = 2108\text{ cm}^3$

so the fraction lost is $\dfrac{99.4\text{ cm}^3}{2108\text{ cm}^3} = 4.71\times 10^{-2}$

and this fraction of the cylinder's depth will be empty upon cooling:

$(4.71\times 10^{-2})(20.0\text{ cm}) = \boxed{0.943\text{ cm}}$

19.24 (a) $L = L_i\,(1 + \alpha\Delta T)$

$5.050\text{ cm} = (5.000\text{ cm})\,[1 + 24.0\times 10^{-6}\ °\text{C}^{-1}(T - 20.0°\text{C})]$

$T = \boxed{437°\text{C}}$

(b) We must get $L_{\text{Al}} = L_{\text{Brass}}$ for some ΔT, or

$L_{i,\text{Al}}(1 + \alpha_{\text{Al}}\,\Delta T) = L_{i,\text{brass}}(1 + \alpha_{\text{brass}}\Delta T)$

$(5.000\text{ cm})[1 + (24.0\times 10^{-6}\ °\text{C}^{-1})\Delta T] = (5.050\text{ cm})[1 + (19.0\times 10^{-6}\ °\text{C}^{-1})\,\Delta T]$

Solving for ΔT gives $\Delta T = 2080°\text{C}$, so $\boxed{T = 3000°\text{C}}$

This will not work because $\boxed{\text{aluminum melts at } 660°\text{C}}$

***19.25** (a) $n = \dfrac{PV}{RT} = \dfrac{(9.00\text{ atm})(1.013\times 10^5\text{ Pa/atm})(8.00\times 10^{-3}\text{ m}^3)}{(8.315\text{ N}\cdot\text{mol K})(293\text{ K})} = \boxed{3.00\text{ mol}}$

(b) $N = nN_{\text{A}} = (3.00\text{ mol})(6.02\times 10^{23}\text{ molecules/mol}) = \boxed{1.80\times 10^{24}\text{ molecules}}$

19.26 $PV = NP'V' = \frac{4}{3}\pi r^3 NP'$

$$N = \frac{3PV}{4\pi r^3 P'} = \frac{(3)(150)(0.100)}{(4\pi)(0.150)^3(1.20)} = \boxed{884 \text{ balloons}}$$

19.27 $(1.01 \times 10^5)(6000) = n(8.315)(293)$

$n = 2.49 \times 10^5$ mol

$N = \boxed{1.50 \times 10^{29} \text{ molecules}}$

Goal Solution

G: The given room conditions are close to Standard Temperature and Pressure (STP is 0°C and 101.3 kPa), so we can use the estimate that one mole of an ideal gas at STP occupies a volume of about 22 L. The volume of the auditorium is 6000 m^3 and 1 m^3 = 1000 L , so we can estimate the number of molecules to be:

$$N \approx (6 \times 10^3 \text{ m}^3)\left(\frac{1000 \text{ L}}{1 \text{ m}^3}\right)\left(\frac{1 \text{ mol}}{22 \text{ L}}\right)\left(\frac{6.02 \times 10^{23} \text{ molecules}}{1 \text{ mol}}\right) \approx 1.6 \times 10^{29} \text{ molecules of air}$$

O: The number of molecules can be found more precisely by applying $PV = nRT$.

A: The equation of state of an ideal gas is $PV = nRT$ so we need to solve for the number of moles to find N.

$$n = \frac{PV}{RT} = \frac{(1.01 \times 10^5 \text{ N/m}^2)[(10.0 \text{ m})(20.0 \text{ m})(30.0 \text{ m})]}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 2.49 \times 10^5 \text{ mol}$$

$$N = n(N_A) = (2.49 \times 10^5 \text{ mol})\left(6.022 \times 10^{23} \frac{\text{molecules}}{\text{mol}}\right) = \boxed{1.50 \times 10^{29} \text{ molecules}}$$

L: This result agrees quite well with our initial estimate. The numbers would match even better if the temperature of the auditorium was 0°C.

19.28 $$P = \frac{nRT}{V} = \left(\frac{9.00 \text{ g}}{18.0 \text{ g/mol}}\right)\left(\frac{8.315 \text{ J}}{\text{mol K}}\right)\left(\frac{773 \text{ K}}{2.00 \times 10^{-3} \text{ m}^3}\right) = \boxed{1.61 \text{ MPa}} = 15.9 \text{ atm}$$

19.29 $\rho_{out}\, gV - \rho_{in}\, gV - (200\text{ kg})g = 0$

$(\rho_{out} - \rho_{in})(400\text{ m}^3) = 200\text{ kg}$

$$\left(1.25\ \frac{\text{kg}}{\text{m}^3}\right)\left(1 - \frac{283\text{ K}}{T_{in}}\right)(400\text{ m}^3) = 200\text{ kg}$$

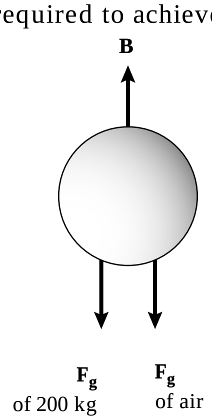
$$1 - \frac{283}{T_{in}} = 0.400$$

$$0.600 = \frac{283}{T_{in}} \qquad T_{in} = \boxed{472\text{ K}}$$

Goal Solution

G: The air inside the balloon must be significantly hotter than the outside air in order for the balloon to have a net upward force, but the temperature must also be less than the melting point of the nylon used for the balloon's envelope (rip-stop nylon melts around 200°C), otherwise the results could be disastrous!

O: The density of the air inside the balloon must be sufficiently low so that the buoyant force is greater than the weight of the balloon, its cargo, and the air inside. The temperature of the air required to achieve this density can be found from the equation of state of an ideal gas.



A: The buoyant force equals the weight of the air at 10.0°C displaced by the balloon:

$$B = m_{air}g = \rho_a Vg = (1.25\text{ kg/m}^3)(400\text{ m}^3)(9.8\text{ m/s}^2) = 4900\text{ N}$$

The weight of the balloon and its cargo is

$$F_g = m_b g = (200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$$

Since $B > F_g$, the balloon has a chance of lifting off as long as the weight of the air inside the balloon is less than the difference in these forces:

$$F_{g(\text{air})} < B - F_{g(\text{balloon})} = 4900\text{ N} - 1960\text{ N} = 2940\text{ N}$$

The mass of this air is $m_{air} = \dfrac{F_{g(\text{air})}}{g} = \dfrac{2940\text{ N}}{9.80\text{ m/s}^2} = 300\text{ kg}$

To find the required temperature of this air from $PV = nRT$, we must find the corresponding number of moles of air. Dry air is approximately 20% O_2, and 80% N_2. Using data from a periodic table, we can calculate the molar mass of the air to be approximately

$$M = 0.80(28\text{ g/mol}) + 0.20(32\text{ g/mol}) = 29\text{ g/mol}$$

so the number of moles is $n = \dfrac{m}{M} = \dfrac{300\text{ kg}}{29\text{ g/mol}}\left(\dfrac{10^3\text{ g}}{1\text{ kg}}\right) = 1.0 \times 10^4\text{ mol}$

The pressure of this air is the ambient pressure; from $PV = nRT$, we can now find the minimum temperature required for lift off:

$$T = \frac{PV}{nR} = \frac{(1.013 \times 10^5\text{ N/m}^2)(400\text{ m}^3)}{(1.0 \times 10^4\text{ mol})(8.315\text{ J/(mol K)})} = 471\text{ K} = 198°\text{C}$$

L: The average temperature of the air inside the balloon required for lift off appears to be close to the melting point of the nylon fabric, so this seems like a dangerous situation! A larger balloon would be better suited for the given weight of the balloon. (A quick check on the internet reveals that this balloon is only about 1/10 the size of most sport balloons, which have a volume of about 3000 m^3).
If the buoyant force were less than the weight of the balloon and its cargo, the balloon would not lift off no matter how hot the air inside might be! If this were the case, then either the weight would have to be reduced or a bigger balloon would be required.
Even though our result for T is shown with 3 significant figures, the answer should probably be rounded to 2 significant figures to reflect the approximate value of the molar mass of the air.

***19.30** (a) $T_2 = T_1 \dfrac{P_2}{P_1} = (300\text{ K})(3) = \boxed{900\text{ K}}$

(b) $T_2 = T_1 \dfrac{P_2 V_2}{P_1 V_1} = 300(2)(2) = \boxed{1200\text{ K}}$

***19.31** (a) $PV = nRT$

$$n = \frac{PV}{RT} = \frac{(1.013 \times 10^5\text{ Pa})(1.00\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})} = \boxed{41.6\text{ mol}}$$

(b) $m = nM = (41.6\text{ mol})(28.9\text{ g/mol}) = \boxed{1.20\text{ kg}}$, in agreement with the tabulated density of 1.20 kg/m^3 at 20.0°C.

***19.32** (a) $PV = nRT$ $\qquad n = \dfrac{PV}{RT}$

$$m = nM = \frac{PVM}{RT} = \frac{(1.013 \times 10^5\text{ Pa})(0.100\text{ m})^3(28.9 \times 10^{-3}\text{ kg/mol})}{(8.315\text{ J/mol}\cdot\text{K})(300\text{ K})}$$

$$m = \boxed{1.17 \times 10^{-3}\text{ kg}}$$

(b) $F_g = mg = (1.17 \times 10^{-3}\text{ kg})(9.80\text{ m/s}^2) = \boxed{11.5\text{ mN}}$

(c) $F = PA = (1.013 \times 10^5 \text{ N/m}^2)(0.100 \text{ m})^2 = \boxed{1.01 \text{ kN}}$

(d) The $\boxed{\text{molecules must be moving very fast}}$ to hit the walls hard.

19.33 (a) Initially, $P_i V_i = n_i R T_i$

$(1.00 \text{ atm})V_i = n_i R(10.0 + 273.15)\text{K}$

Finally, $P_f V_f = n_f R T_f$ $\quad P_f(0.280 V_i) = n_i R(40.0 + 273.15)\text{K}$

Dividing these equations,

$$\frac{0.280 \times P_f}{1.00 \text{ atm}} = \frac{313.15 \text{ K}}{283.15 \text{ K}} \quad \text{giving } P_f = 3.95 \text{ atm or}$$

$$P_f = \boxed{4.00 \times 10^5 \text{ Pa(abs.)}}$$

(b) After being driven

$$P_d(1.02)(0.280 V_i) = n_i R(85.0 + 273.15)\text{K}$$

$$P_d = 1.121 P_f = \boxed{4.49 \times 10^5 \text{ Pa}}$$

***19.34** Let us use $V = \frac{4}{3}\pi r^3$ as the volume of the balloon, and the ideal gas law in the form $\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}$

to give, $r_i^3 = \frac{300 \text{ K}}{200 \text{ K}} \frac{0.0300 \text{ atm}}{1.00 \text{ atm}} (20.0 \text{ m})^3$

$\boxed{r_i = 7.11 \text{ m}}$

19.35 $P_1 V_1 = n_1 R T_1$

$P_2 V_2 = n_2 R T_2$

$$n_1 - n_2 = \frac{PV}{RT_1} - \frac{PV}{RT_2}$$

$$n_1 - n_2 = \frac{(101 \times 10^3 \text{ Pa})80.0 \text{ m}^3}{8.315 \text{ J/mol K}}\left(\frac{1}{291 \text{ K}} - \frac{1}{298 \text{ K}}\right)$$

$$n_1 - n_2 = 78.4 \text{ mol}$$

$$\Delta m = \Delta n M = 78.4 \text{ mol}(28.9 \text{ g/mol}) = \boxed{2.27 \text{ kg}}$$

19.36 $P_0V = n_1RT_1 = (m_1 / M)RT_1$

$P_0V = n_2RT_2 = (m_2 / M)RT_2$

$$\boxed{m_1 - m_2 = \frac{P_0VM}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

19.37 At depth, $P = P_0 + \rho gh$

and $PV_i = nRT_i$

at the surface, $P_0V_f = nRT_f$

$$\frac{P_0V_f}{(P_0 + \rho gh)V_i} = \frac{T_f}{T_i}$$

$$V_f = V_i \frac{T_f}{T_i}\left(\frac{P_0 + \rho gh}{P_0}\right)$$

$$V_f = 1.00\text{ cm}^3 \frac{293\text{ K}}{278\text{ K}}\left(\frac{1.013 \times 10^5\text{ Pa} + 1025\text{ kg/m}^3\,(9.80\text{ m/s}^2)25.0\text{ m}}{1.013 \times 10^5\text{ Pa}}\right)$$

$$V_f = \boxed{3.67\text{ cm}^3}$$

***19.38** My bedroom is 4.00 m long, 4.00 m wide, and 2.40 m high, enclosing air at 100 kPa and 20.0°C = 293 K. Think of the air as 80.0% N_2 and 20.0% O_2.

Avogadro's number of molecules has mass

$$0.800 \times 28.0\text{ g/mol} + 0.200 \times 32.0\text{ g/mol} = 0.0288\text{ kg/mol}$$

Then $PV = nRT = (m/M)RT$ gives

$$m = \frac{PVM}{RT} = \frac{(1.00 \times 10^5\text{ N/m}^2)(38.4\text{ m}^3)(0.0288\text{ kg/mol})}{(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})}$$

$$m = 45.4\text{ kg} \quad \boxed{\sim 10^2\text{ kg}}$$

19.39 $PV = nRT$

$$\frac{m_f}{m_i} = \frac{n_f}{n_i} = \frac{P_fV_f}{RT_f}\frac{RT_i}{P_iV_i} = \frac{P_f}{P_i}\text{, so } m_f = m_i\left(\frac{P_f}{P_i}\right)$$

$$\Delta m = m_i - m_f = m_i\left(\frac{P_i - P_f}{P_i}\right) = (12.0\text{ kg})\left(\frac{41.0\text{ atm} - 26.0\text{ atm}}{41.0\text{ atm}}\right) = \boxed{4.39\text{ kg}}$$

19.40 $$N = \frac{PVN_A}{RT} = \frac{(10^{-9}\ \text{Pa})(1.00\ \text{m}^3)(6.02\times 10^{23})\ \dfrac{\text{molecule}}{\text{mol}}}{\left(8.315\ \dfrac{\text{J}}{\text{K}\cdot\text{mol}}\right)(300\ \text{K})} = \boxed{2.41\times 10^{11}\ \text{molecules}}$$

19.41 $PV = nRT$

$$V = \frac{nRT}{P} = \frac{(1.00\ \text{mol})(8.315\ \text{J/mol}\cdot\text{K})(273\ \text{K})}{1.013\times 10^5\ \text{N/m}^2}\left(\frac{10^3\ \text{L}}{1.00\ \text{m}^3}\right) = \boxed{22.4\ \text{L}}$$

19.42 (a) Initially the air in the bell satisfies $P_0 V_{\text{bell}} = nRT_i$

or $P_0[(2.50\ \text{m})A] = nRT_i$ (1)

When the bell is lowered, the air in the bell satisfies

$P_{\text{bell}}(2.50\ \text{m} - x)A = nRT_f$ (2)

where x is the height the water rises in the bell. Also, the pressure in the bell, once it is lowered, is equal to the sea water pressure at the depth of the water level in the bell.

$P_{\text{bell}} = P_0 + \rho g\,(82.3\ \text{m} - x) \approx P_0 + \rho g(82.3\ \text{m})$ (3)

The approximation is good, as $x < 2.50$ m. Substituting (3) into (2) and substituting nR from (1) into (2),

$$[P_0 + \rho g\,(82.3\ \text{m})](2.50\ \text{m} - x)A = P_0 V_{\text{bell}}\frac{T_f}{T_i}$$

Using $P_0 = 1\ \text{atm} = 1.013\times 10^5$ Pa and $\rho = 1.025\times 10^3\ \dfrac{\text{kg}}{\text{m}^3}$

$$x = (2.50\ \text{m})\left[1 - \frac{T_f}{T_0}\left(1 + \frac{\rho g(82.3\ \text{m})}{P_0}\right)^{-1}\right]$$

$$= (2.50\ \text{m})\left[1 - \frac{277.15\ \text{K}}{293.15\ \text{K}}\left(1 + \frac{(1.025\times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(82.3\ \text{m})}{1.013\times 10^5\ \text{N/m}^2}\right)^{-1}\right]$$

$x = \boxed{2.24\ \text{m}}$

(b) If the water in the bell is to be expelled, the air pressure in the bell must be raised to the water pressure at the bottom of the bell. That is,

$$P_{\text{bell}} = P_0 + \rho g(82.3 \text{ m})$$

$$= 1.013 \times 10^5 \text{ Pa} + \left(1.025 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right)\left(9.80 \frac{\text{m}}{\text{s}^2}\right)(82.3 \text{ m})$$

$$P_{\text{bell}} = 9.28 \times 10^5 \text{ Pa} = \boxed{9.16 \text{ atm}}$$

19.43 The excess expansion of the brass is

$$\Delta L_{\text{rod}} - \Delta L_{\text{tape}} = (\alpha_{\text{brass}} - \alpha_{\text{steel}})L_i \,\Delta T$$

$$\Delta(\Delta L) = (19.0 - 11.0)10^{-6} \,(\text{C}°)^{-1}\; 0.950 \text{ m}(35.0 \text{ C}°)$$

$$\Delta(\Delta L) = 2.66 \times 10^{-4} \text{ m}$$

(a) The rod contracts more than tape to

a length reading $0.9500 \text{ m} - 0.000266 \text{ m} = \boxed{0.9497 \text{ m}}$

(b) $0.9500 \text{ m} + 0.000266 \text{ m} = \boxed{0.9503 \text{ m}}$

***19.44** At 0°C, 10.0 gallons of gasoline has mass, from $\rho = m / V$

$$m = \rho V = \left(730 \frac{\text{kg}}{\text{m}^3}\right)(10.0 \text{ gal})\left(\frac{0.00380 \text{ m}^3}{1.00 \text{ gal}}\right) = 27.7 \text{ kg}$$

The gasoline will expand in volume by

$$\Delta V = \beta V_i \,\Delta T = (9.60 \times 10^{-4} / \text{C}°)(10.0 \text{ gal})(20.0°\text{C} - 0.0°\text{C}) = 0.192 \text{ gal}$$

At 20.0°C, we have 10.192 gal = 27.7 kg

$$10.0 \text{ gal} = \left(\frac{10.0 \text{ gal}}{10.192 \text{ gal}}\right)(27.7 \text{ kg}) = 27.2 \text{ kg}$$

The extra mass contained in 10.0 gallons at 0.0°C is

$$27.7 \text{ kg} - 27.2 \text{ kg} = \boxed{0.523 \text{ kg}}$$

19.45 $R_B + \alpha_B R_B (T - 20.0) = R_s + \alpha_s R_s (T - 20.0)$

$$3.994 \text{ cm} + (19.0 \times 10^{-6} / \text{C}°)(3.994 \text{ cm})(T - 20.0°\text{C})$$

$$= 4.000 \text{ cm} + (11.0 \times 10^{-6})(\text{C}°)^{-1}(4.000 \text{ cm})(T - 20.0°\text{C})$$

$3.189 \times 10^{-5}\,T = 0.006638$ $\boxed{T = 208°\text{C}}$

***19.46** The frequency played by the cold-walled flute is $f_i = \dfrac{v}{\lambda_i} = \dfrac{v}{2L_i}$.

When the instrument warms up

$$f_f = \frac{v}{\lambda_f} = \frac{v}{2L_f} = \frac{v}{2L_i(1 + \alpha\,\Delta T)} = \frac{f_i}{1 + \alpha\,\Delta T}$$

The final frequency is lower. The change in frequency is

$$\Delta f = f_i - f_f = f_i\left(1 - \frac{1}{1 + \alpha\,\Delta T}\right)$$

$$\Delta f = \frac{v}{2L_i}\left(\frac{\alpha\,\Delta T}{1 + \alpha\,\Delta T}\right) \approx \frac{v}{2L_i}(\alpha\,\Delta T)$$

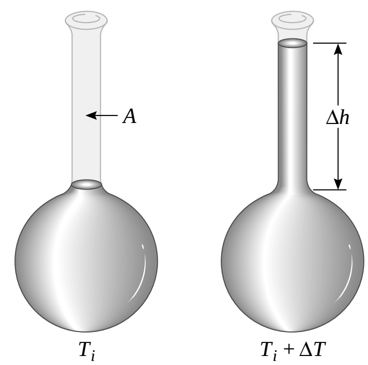
$$\Delta f \approx \frac{(343\text{ m/s})(24.0 \times 10^{-6}/\text{C°})(15.0\text{ C°})}{2(0.655\text{ m})} = \boxed{0.0943\text{ Hz}}$$

This change in frequency is imperceptibly small.

19.47 Neglecting the expansion of the glass,

$$\Delta h = \frac{V}{A}\beta\Delta T$$

$$\Delta h = \frac{\frac{4}{3}\pi\left(\frac{0.250\text{ cm}}{2}\right)^3}{\pi(2.00 \times 10^{-3}\text{ cm})^2}(1.82 \times 10^{-4}/\text{C°})(30.0°\text{C}) = \boxed{3.55\text{ cm}}$$



19.48 (a) The volume of the liquid increases as $\Delta V_1 = V_i\beta\Delta T$. The volume of the flask increases as $\Delta V_g = 3\alpha V_i\Delta T$. Therefore, the overflow in the capillary is $V_c = V_i\Delta T(\beta - 3\alpha)$; and in the capillary $V_c = A\,\Delta h$.

Therefore, $\boxed{\Delta h = \dfrac{V_i}{A}(\beta - 3\alpha)\,\Delta T}$

(b) For a mercury thermometer $\beta(\text{Hg}) = 1.82 \times 10^{-4}/°\text{C}$ and for glass, $3\alpha = 3 \times 3.20 \times 10^{-6}/°\text{C}$. Thus $\beta - 3\alpha \approx \beta$, or $\boxed{\alpha \ll \beta}$

19.49 (a) $\rho = \dfrac{m}{V}$ and $d\rho = -\dfrac{m}{V^2}\,dV$

For very small changes in V and ρ, this can be expressed as

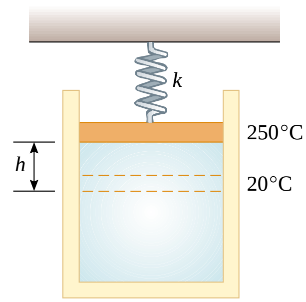
$$\Delta\rho = -\frac{m}{V}\frac{\Delta V}{V} = -\rho\beta\Delta T$$

The negative sign means that any increase in temperature causes the density to decrease and vice versa.

(b) For water we have

$$\beta = \left|\frac{\Delta\rho}{\rho\,\Delta T}\right| = \left|\frac{(1.0000\text{ g/cm}^3 - 0.9997\text{ g/cm}^3)}{(1.0000\text{ g/cm}^3)(10.0 - 4.00)\text{C}°}\right| = \boxed{5 \times 10^{-5}/°\text{C}}$$

19.50 (a) $\dfrac{P_0V}{T} = \dfrac{P'V'}{T'}$



$V' = V + Ah$

$P' = P_0 + \dfrac{kh}{A}$

$$\left(P_0 + \frac{kh}{A}\right)(V + Ah) = P_0V\left(\frac{T'}{T}\right)$$

$$\left(1.013 \times 10^5\ \frac{\text{N}}{\text{m}^2} + 2.00 \times 10^5\ \frac{\text{N}}{\text{m}^3}h\right)(5.00 \times 10^{-3}\text{ m}^3 + (0.0100\text{ m}^2)h)$$

$$= \left(1.013 \times 10^5\ \frac{\text{N}}{\text{m}^2}\right)(5.00 \times 10^{-3}\text{ m}^3)\left(\frac{523\text{ K}}{293\text{ K}}\right)$$

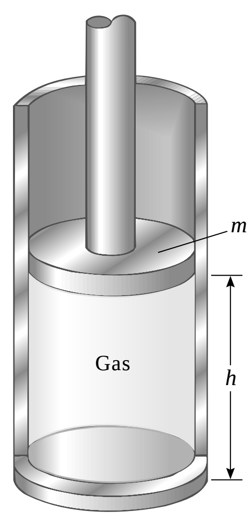
$2000h^2 + 2013h - 397 = 0$

$$h = \frac{-2013 \pm 2689}{4000} = \boxed{0.169\text{ m}}$$

(b) $P' = P + \dfrac{kh}{A} = 1.013 \times 10^5\text{ Pa} + \dfrac{(2.00 \times 10^3\text{ N/m})(0.169)}{0.0100\text{ m}^2}$

$P' = \boxed{1.35 \times 10^5\text{ Pa}}$

19.51 (a) We assume that air at atmospheric pressure is above the piston.



In equilibrium $P_{\text{gas}} = \dfrac{mg}{A} + P_0$. Therefore,

$\dfrac{nRT}{hA} = \dfrac{mg}{A} + P_0$ or $\boxed{h = \dfrac{nRT}{(mg + P_0A)}}$ where

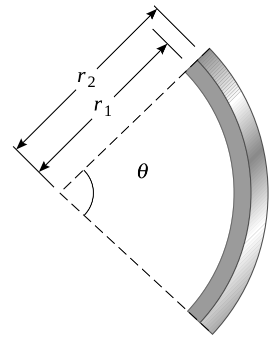
we have used $V = hA$ as the volume of the gas.

(b) From the data given,

$$h = \frac{(0.200\text{ mol})(8.315\text{ J/K}\cdot\text{mol})(400\text{ K})}{(20.0\text{ kg})(9.80\text{ m/s}^2) + (1.013 \times 10^5\text{ N/m}^2)(0.00800\text{ m}^2)}$$

$$= \boxed{0.661\text{ m}}$$

19.52 (a) $L_1 = r_1\theta = L_i(1 + \alpha_1 \Delta T)$ $L_2 = r_2\theta = L_i(1 + \alpha_2 \Delta T)$



$$\Delta r = r_2 - r_1 = \frac{L_i(\alpha_2 - \alpha_1)\Delta T}{\theta}$$

$$\therefore r_2 - r_1 = \frac{1}{\theta}\,[L_i + L_i\alpha_2\ \Delta T - L_i - L_i\alpha_1\ \Delta T]$$

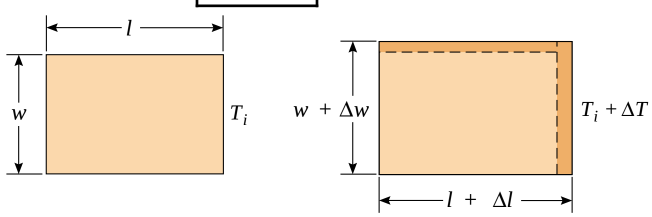
$$\boxed{\theta = L_i(\alpha_2 - \alpha_1)\frac{\Delta T}{(r_2 - r_1)}}$$

(b) $\boxed{\theta \to 0 \text{ as } \Delta T \to 0}$ $\boxed{\theta \to 0 \text{ as } \alpha_1 \to \alpha_2}$

$\theta < 0$ when $\Delta T < 0$ cooling means temperature is decreasing.

(c) $\boxed{\text{It bends the other way.}}$

19.53 From the diagram we see that the change in area is $\Delta A = l\,\Delta w + w\,\Delta l + \Delta w\,\Delta l$. Since Δl and Δw are each small quantities, the product $\Delta w\,\Delta l$ will be very small. Therefore, we assume $\Delta w\,\Delta l \approx 0$. Since $\Delta w = w\alpha\,\Delta T$ *and* $\Delta l = l\alpha\,\Delta T$, we then have $\Delta A = lw\alpha\,\Delta T + wl\alpha\,\Delta T$ and since $A = lw$, we have $\boxed{\Delta A = 2\alpha A\,\Delta T}$. The approximation assumes $\Delta w\Delta l \approx 0$, or $\alpha\,\Delta T \approx 0$. Another way of stating this is $\boxed{\alpha\,\Delta T \ll 1}$.



19.54 (a) $R = R_0(1 + AT_C$

$50.0\ \Omega = R_0(1 + 0) \Rightarrow R_0 = 50.0\ \Omega$

$71.5\ \Omega = (50.0\ \Omega)\,[1 + (231.97°C)A]$

$\boxed{A = 1.85 \times 10^{-3}(C°)^{-1}}$ $\boxed{R_0 = 50.0\Omega}$

(b) $$T = \frac{1}{A}\left(\frac{R}{R_0} - 1\right) = \frac{1}{1.85 \times 10^{-3}\ (C°)^{-1}}\left(\frac{89.0}{50.0} - 1\right) = \boxed{422°C}$$

19.55 (a) $T_i = 2\pi\sqrt{\dfrac{L_i}{g}}$

$$L_i = \frac{T_i^2 g}{4\pi^2} = \frac{(1.000\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2} = 0.2482\text{ m}$$

$$\Delta L = \alpha L_i\,\Delta T = (19.0\times10^{-6}/\text{C°})(0.2483\text{ m})(10.0\text{°C}) = 4.72\times10^{-5}\text{ m}$$

$$T_f = 2\pi\sqrt{\frac{L_i+\Delta L}{g}} = 2\pi\sqrt{\frac{0.2483\text{ m}}{9.80\text{ m/s}^2}} = 1.0000949\text{ s}$$

$$\Delta T = \boxed{9.49\times10^{-5}\text{ s}}$$

(b) In one week, the time lost is

time lost = (1 week)(9.49×10^{-5} s lost per second)

$$= \left(7.00\,\frac{\text{d}}{\text{week}}\right)\left(\frac{86\,400\text{ s}}{1.00\text{ d}}\right)\left(9.49\times10^{-5}\,\frac{\text{s lost}}{\text{s}}\right) = \boxed{57.4\text{ s lost}}$$

19.56 $I = \int r^2\,dm$ and since $r(T) = r(T_i)(1+\alpha\,\Delta T)$, for $\alpha\,\Delta T << 1$ we find

$$\frac{I(T)}{I(T_i)} = (1+\alpha\,\Delta T)^2,\ \text{thus}\ \frac{I(T)-I(T_i)}{I(T_i)} \approx 2\alpha\,\Delta T$$

(a) With $\alpha = 17.0\times10^{-6}/\text{°C}$ and $\Delta T = 100\text{°C}$, we find for Cu:

$$\frac{\Delta I}{I} = 2(17.0\times10^{-6}/\text{°C})(100\text{°C}) = \boxed{0.340\%}$$

(b) With $\alpha = 24.0\times10^{-6}/\text{°C}$ and $\Delta T = 100\text{°C}$, we find for Al:

$$\frac{\Delta I}{I} = 2(24.0\times10^{-6}/\text{°C})(100\text{°C}) = \boxed{0.480\%}$$

19.57 (a) $B = \rho g V'$ $\qquad P' = P_0 + \rho g d$ $\qquad P'V' = P_0 V_i$

$$B = \frac{\rho g P_0 V_i}{P'} = \boxed{\frac{\rho g P_0 V_i}{(P_0+\rho g d)}}$$

(b) Since d is in the denominator, B must $\boxed{\text{decrease}}$ as the depth increases. (The volume of the balloon becomes smaller with increasing pressure.)

(c) $$\frac{1}{2} = \frac{B(d)}{B(0)} = \frac{\rho g P_0 V_i/(P_0 + \rho g d)}{\rho g P_0 V_i / P_0} = \frac{P_0}{P_0 + \rho g d}$$

$$P_0 + \rho g d = 2P_0$$

$$d = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{10.3 \text{ m}}$$

19.58 (a) Let m represent the sample mass. Then the number of moles is $n = m/M$ and the density is $\rho = m/V$. So $PV = nRT$ becomes

$$PV = \frac{m}{M}RT \quad \text{or} \quad PM = \frac{m}{V}RT$$

Then, $\rho = \frac{m}{V} = \boxed{\frac{PM}{RT}}$

(b) $$\rho = \frac{PM}{RT} = \frac{(1.013 \times 10^5 \text{ N/m}^2)(0.0320 \text{ kg/mol})}{(8.315 \text{ J/mol}\cdot\text{K})(293 \text{ K})} = \boxed{1.33 \text{ kg/m}^3}$$

19.59 For each gas alone, $P_1 = \frac{N_1 kT}{V}$ and $P_2 = \frac{N_2 kT}{V}$ and $P_3 = \frac{N_3 kR}{V}$, etc.

For all gases

$$P_1V_1 + P_2V_2 + P_3V_3 \ldots = (N_1 + N_2 + N_3 \ldots)kT \quad \text{and}$$

$$(N_1 + N_2 + N_3 \ldots)kT = PV$$

Also, $V_1 = V_2 = V_3 = \ldots = V$, therefore $\boxed{P = P_1 + P_2 + P_3 \ldots}$

19.60 (a) Using the Periodic Table, we find the molecular masses of the air components to be

$$M(N_2) = 28.01 \text{ u},\ M(O_2) = 32.00 \text{ u},\ M(\text{Ar}) = 39.95 \text{ u}$$

and $M(CO_2) = 44.01$ u

Thus, the number of moles of each gas in the sample is

$$n(N_2) = \frac{75.52 \text{ g}}{28.01 \text{ g/mol}} = 2.696 \text{ mol}$$

$$n(O_2) = \frac{23.15 \text{ g}}{32.00 \text{ g/mol}} = 0.7234 \text{ mol}$$

$$n(\text{Ar}) = \frac{1.28 \text{ g}}{39.95 \text{ g/mol}} = 0.0320 \text{ mol}$$

$$n(CO_2) = \frac{0.05 \text{ g}}{44.01 \text{ g/mol}} = 0.0011 \text{ mol}$$

The total number of moles is $n_0 = \sum n_i = 3.453$ mol. Then, the partial pressure of N_2 is

$$P(N_2) = \frac{2.696 \text{ mol}}{3.453 \text{ mol}} \cdot (1.013 \times 10^5 \text{ Pa}) = \boxed{79.1 \text{ kPa}}$$

Similarly,

$$P(O_2) = \boxed{21.2 \text{ kPa}} \qquad P(\text{Ar}) = \boxed{940 \text{ Pa}} \qquad P(CO_2) = \boxed{33.3 \text{ Pa}}$$

(b) Solving the ideal gas law equation for V and using $T = 273.15 + 15.00 = 288.15$ K, we find

$$V = \frac{n_0 RT}{P} = \frac{(3.453 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(288.15 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 8.167 \times 10^{-2} \text{ m}^3$$

Then, $\rho = \dfrac{m}{V} = \dfrac{100 \times 10^{-3} \text{kg}}{8.167 \times 10^{-2} \text{ m}^3} = \boxed{1.22 \text{ kg/m}^3}$

(c) The 100 g sample must have an appropriate molar mass to yield n_0 moles of gas: that is

$$M(\text{air}) = \frac{100 \text{ g}}{3.453 \text{ mol}} = \boxed{29.0 \text{ g/mol}}$$

19.61 In any one section of concrete, length L_i expands by

$$\Delta L = \alpha L_i \, \Delta T$$

$$= (12.0 \times 10^{-6} / \text{C}°) L_i \, (25.0 \text{ C}°) = 3.00 \times 10^{-4} L_i$$

The unstressed length of that rail increases by

$$(11.0 \times 10^{-6} / \text{C}°) L_i \, (25.0 \text{ C}°) = 2.75 \times 10^{-4} L_i$$

(a) So the rail is stretched elastically by the extra

$$3.00 \times 10^{-4} L_i - 2.75 \times 10^{-4} L_i = 2.50 \times 10^{-5} L_i$$

in $\dfrac{F}{A} = Y \dfrac{\Delta L}{L_i} = (20.0 \times 10^{10} \text{ N/m})(2.50 \times 10^{-5}) = \boxed{5.00 \times 10^6 \text{ N/m}^2}$

(b) Fraction of yield strength $= \dfrac{5.00 \times 10^6 \text{ N/m}^2}{52.2 \times 10^7 \text{ N/m}^2} = \boxed{9.58 \times 10^{-3}}$

19.62 (a) From $PV = nRT$, the volume is: $V = \left(\frac{nR}{P}\right) T$

Therefore, when pressure is held constant,

$$\frac{dV}{dT} = \frac{nR}{P} = \frac{V}{T}$$

Thus, $\beta \equiv \left(\frac{1}{V}\right)\frac{dV}{dT} = \left(\frac{1}{V}\right)\frac{V}{T}$, or $\beta = \boxed{\frac{1}{T}}$

(b) At $T = 0°C = 273$ K, this predicts $\beta = \frac{1}{273\text{ K}} = \boxed{3.66 \times 10^{-3}/\text{K}}$

Experimental values are: $\beta_{\text{He}} = 3.665 \times 10^{-3}/\text{K}$ and $\beta_{\text{air}} = 3.67 \times 10^{-3}/\text{K}$

19.63 After expansion, the length of one of the spans is

$$L_f = L_i(1 + \alpha\,\Delta T) = (125\text{ m})[1 + (12 \times 10^{-6}/\text{C°})(20.0\text{ C°})] = 125.03\text{ m}$$

L_f, y, and the original 125 m length of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives:

$$(125.03\text{ m})^2 = y^2 + (125\text{ m})^2 \quad \text{yielding} \quad y = \boxed{2.74\text{ m}}$$

19.64 After expansion, the length of one of the spans is $L_f = L(1 + \alpha\,\Delta T)$. L_f, y, and the original length L of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives $L_f^2 = L^2 + y^2$, or

$$y = \sqrt{L_f^2 - L^2} = L\sqrt{(1 + \alpha\,\Delta T)^2 - 1} = L\sqrt{2\alpha\,\Delta T + (\alpha\,\Delta T)^2}$$

Since $\alpha\,\Delta T << 1$, $y \approx \boxed{L\sqrt{2\alpha\,\Delta T}}$

19.65 For $\Delta L = L_s - L_c$ to be constant, the rods must expand by equal amounts:

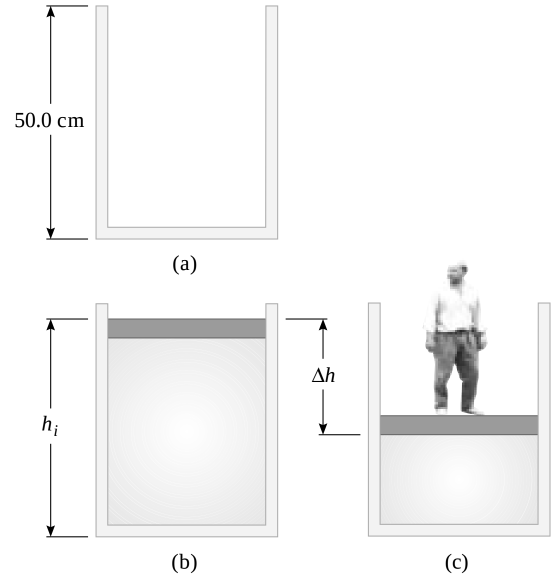
$$\alpha_c L_c\,\Delta T = \alpha_s L_s\,\Delta T$$

$$L_s = \frac{\alpha_c L_c}{\alpha_s}$$

$$\Delta L = \frac{\alpha_c L_c}{\alpha_s} - L_c$$

$$\therefore \quad L_c = \frac{\Delta L\,\alpha_s}{(\alpha_c - \alpha_s)} = \frac{5.00\text{ cm}(11.0 \times 10^{-6}/\text{C°})}{(17.0 \times 10^{-6}/\text{C°} - 11.0 \times 10^{-6}/\text{C°})} = \boxed{9.17\text{ cm}}$$

$$\text{and} \quad L_s = \frac{\Delta L\,\alpha_c}{(\alpha_c - \alpha_s)} = 5.00\text{ cm}\left(\frac{17.0}{6.00}\right) = \boxed{14.2\text{ cm}}$$



19.66 (a) With piston alone:

T = constant, so $PV = P_0 V_i$

or $P(Ah_0) = P_0(Ah_i)$

With A = constant, $P = P_0\left(\frac{h_i}{h_0}\right)$

But, $P = P_0 + \frac{m_p g}{A}$, where m_p is the mass of the piston.

Thus, $P_0 + \frac{m_p g}{A} = P_0\left(\frac{h_i}{h_0}\right)$, which reduces to

$$h_0 = \frac{h_i}{1 + \frac{m_p g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{(20.0 \text{ kg})(9.80 \text{ m/s}^2)}{(1.013 \times 10^5 \text{ Pa})\pi(0.400 \text{ m})^2}} = 49.81 \text{ cm}$$

With the man of mass M on the piston, a very similar calculation (replacing m_p by $m_p + M$) gives:

$$h' = \frac{h_i}{1 + \frac{(m_p + M)g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{(95.0 \text{ kg})(9.80 \text{ m/s}^2)}{(1.013 \times 10^5 \text{ Pa})\pi(0.400 \text{ m})^2}} = 49.10 \text{ cm}$$

Thus, when the man steps on the piston, it moves downward by

$$\Delta h = h_0 - h' = 49.81 \text{ cm} - 49.10 \text{ cm} = 0.710 \text{ cm} = \boxed{7.10 \text{ mm}}$$

(b) P = const, so $\frac{V}{T} = \frac{V'}{T_i}$ or $\frac{Ah_0}{T} = \frac{Ah'}{T_i}$, giving

$$T = T_i\left(\frac{h_0}{h'}\right) = (293 \text{ K})\left(\frac{49.81}{49.10}\right) = \boxed{297 \text{ K}} \text{ (or } 24°\text{C)}$$

19.67 (a) $\dfrac{dL}{L} = \alpha dT$

$$\int_{T_i}^{T_i} \alpha dT = \int_{L_i}^{L_i} \frac{dL}{L} \Rightarrow \ln\left(\frac{L_f}{L_i}\right) = \alpha \Delta T \Rightarrow \boxed{L_f = L_i e^{\alpha \Delta T}}$$

(b) $L_f = (1.00 \text{ m})e^{[2.00 \times 10^{-5}(\text{C}°)^{-1}](100°\text{C})} = 1.002002 \text{ m}$

$L_f' = (1.00 \text{ m})\,[1 + (2.00 \times 10^{-5}/°\text{C})(100°\text{C})] = 1.002000 \text{ m}$

$$\frac{L_f - L_f'}{L_f} = 2.00 \times 10^{-6} = \boxed{2.00 \times 10^{-4}\%}$$

$L_f = (1.00 \text{ m})e^{[2.00 \times 10^{-2}(°\text{C})^{-1}](100°\text{C})} = 7.389 \text{ m}$

$L_f' = (1.00 \text{ m})[1 + (0.0200/°\text{C})(100°\text{C})] = 3.000 \text{ m}$

$$\frac{L_f - L_f'}{L_f} = \boxed{59.4\%}$$

19.68 At 20.0˚C, the unstretched lengths of the steel and copper wires are

$$L_s(20.0°\text{C}) = (2.000 \text{ m})[1 + 11.0 \times 10^{-6})(\text{C}°)^{-1}(-20.0°\text{C})] = 1.99956 \text{ m}$$

$$L_c(20.0°\text{C}) = (2.000 \text{ m})[1 + 17.0 \times 10^{-6})(\text{C}°)^{-1}(-20.0°\text{C})] = 1.99932 \text{ m}$$

Under a tension F, the length of the steel and copper wires are

$$L_s' = L_s\left[1 + \frac{F}{YA}\right]_s \qquad L_c' = L_c\left[1 + \frac{F}{YA}\right]_c \qquad \text{where } L_s' + L_c' = 4.000 \text{ m}$$

Since the tension, F, must be the same in each wire, solve for F:

$$F = \frac{(L_s' + L_c') - (L_s + L_c)}{\dfrac{L_s}{Y_s A_s} + \dfrac{L_c}{Y_c A_c}}$$

When the wires are stretched, their areas become

$$A_s = \pi(1.000 \times 10^{-3} \text{ m})^2[1 + (11.0 \times 10^{-6})(-20.0)]^2 = 3.140 \times 10^{-6} \text{ m}^2$$

$$A_c = \pi(1.000 \times 10^{-3} \text{ m})^2[1 + (17.0 \times 10^{-6})(-20.0)]^2 = 3.139 \times 10^{-6} \text{ m}^2$$

Recall $Y_s = 20.0 \times 10^{10}$ Pa and $Y_c = 11.0 \times 10^{10}$ Pa. Substituting into the equation for F, we obtain

$$F = \frac{4.000 \text{ m} - (1.99956 \text{ m} + 1.99932 \text{ m})}{\dfrac{1.99956 \text{ m}}{(20.0 \times 10^{10} \text{ Pa})(3.140 \times 10^{-6}) \text{ m}^2} + \dfrac{1.99932 \text{ m}}{(11.0 \times 10^{10} \text{ Pa})(3.139 \times 10^{-6}) \text{ m}^2}}$$

$$F = \boxed{125 \text{ N}}$$

To find the x-coordinate of the junction,

$$L_s' = (1.99956\text{ m})\left[1 + \frac{125\text{ N}}{(20.0\times10^{10}\text{ N/m}^2)(3.140\times10^{-6}\text{ m}^2)}\right] = 1.999958\text{ m}$$

Thus the x-coordinate is $-2.000 + 1.999958 = \boxed{-4.20\times10^{-5}\text{ m}}$

19.69 (a) $\mu = \pi r^2\rho = \pi(5.00\times10^{-4}\text{ m})^2(7.86\times10^3\text{ kg/m}^3) = \boxed{6.17\times10^{-3}\text{ kg/m}}$

(b) $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{T}{\mu}}$ so $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

Therefore, $T = \mu(2Lf_1)^2 = (6.17\times10^{-3})(2\times0.800\times200)^2 = \boxed{632\text{ N}}$

(c) First find the unstressed length of the string at 0°C:

$$L = L_{\text{natural}}\left(1 + \frac{T}{AY}\right) \quad\text{so}\quad L_{\text{natural}} = \frac{L}{1 + T/AY}$$

$$A = \pi(5.00\times10^{-4}\text{ m})^2 = 7.854\times10^{-7}\text{ m}^2 \quad\text{and}\quad Y = 20.0\times10^{10}\text{ Pa}$$

Therefore, $\dfrac{T}{AY} = \dfrac{632}{(7.854\times10^{-7})(20.0\times10^{10})} = 4.02\times10^{-3}$, and

$$L_{\text{natural}} = \frac{(0.800\text{ m})}{(1 + 4.02\times10^{-3})} = 0.7968\text{ m}$$

The unstressed length at 30.0˚C is $L_{30°\text{C}} = L_{\text{natural}}[1 + \alpha(30.0°\text{C} - 0.0°\text{C})]$,

or $L_{30°\text{C}} = (0.7968\text{ m})[1 + (11.0\times10^{-6})(30.0)] = 0.79706\text{ m}$

Since $L = L_{30°\text{C}}\left[1 + \dfrac{T'}{AY}\right]$, where T' is the tension in the string at 30.0˚C,

$$T' = AY\left[\frac{L}{L_{30°\text{C}}} - 1\right] = (7.854\times10^{-7})(20.0\times10^{10})\left[\frac{0.800}{0.79706} - 1\right] = 580\text{ N}$$

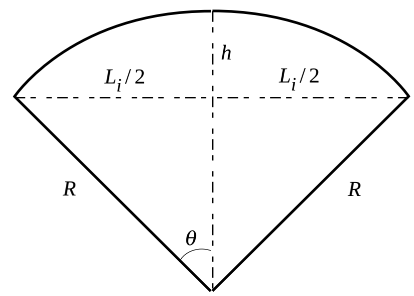
To find the frequency at 30.0˚C, realize that

$$\frac{f_1'}{f_1} = \sqrt{\frac{T'}{T}} \quad\text{so}\quad f_1' = (200\text{ Hz})\sqrt{\frac{580\text{ N}}{632\text{ N}}} = \boxed{192\text{ Hz}}$$

19.70 Let 2θ represent the angle the curved rail subtends. We have

$$L_i + \Delta L = 2\theta R = L_i(1 + \alpha\,\Delta T) \quad \text{and}$$

$$\sin\theta = \frac{L_i/2}{R} = \frac{L_i}{2R}$$



Thus, $\theta = \frac{L_i}{2R}(1 + \alpha\,\Delta T) = (1 + \alpha\,\Delta T)\sin\theta$

and we must solve the transcendental equation

$$\theta = (1 + \alpha\,\Delta T)\sin\theta = (1.000\,0055)\sin\theta$$

Homing in on the non-zero solution gives, to four digits,

$$\theta = 0.01816 \text{ rad} = 1.0405°$$

Now, $h = R - R\cos\theta = \frac{L_i(1-\cos\theta)}{2\sin\theta}$

This yields $\boxed{h = 4.54 \text{ m}}$, a remarkably large value compared to $\Delta L = 5.50$ cm.

Chapter 20 Solutions

20.1 Taking $m = 1.00$ kg, we have

$$\Delta U_g = mgh = (1.00 \text{ kg})(9.80 \text{ m/s}^2)(50.0 \text{ m}) = 490 \text{ J}$$

But $\Delta U_g = Q = mc\Delta T = (1.00 \text{ kg})(4186 \text{ J/kg}\cdot{}^\circ\text{C})\Delta T = 490 \text{ J}$ so $\Delta T = 0.117 \text{ C}^\circ$

$$T_f = T_i + \Delta T = \boxed{(10.0 + 0.117)^\circ\text{C}}$$

Goal Solution

G: Water has a high specific heat, so the difference in water temperature between the top and bottom of the falls is probably less than 1°C. (Besides, if the difference was significantly large, we might have heard about this phenomenon at some point.)

O: The temperature change can be found from the potential energy that is converted to thermal energy. The final temperature is this change added to the initial temperature of the water.

A: The change in potential energy is $\Delta U = mgy$ and the change in internal energy is $\Delta E_{\text{int}} = mc\Delta T$ so $mgy = mc\Delta T$

Therefore, $$\Delta T = \frac{gy}{c} = \frac{(9.80 \text{ m/s}^2)(50.0 \text{ m})}{4.186 \times 10^3 \text{ J/kg}\cdot{}^\circ\text{C}} = 0.117^\circ\text{C}$$

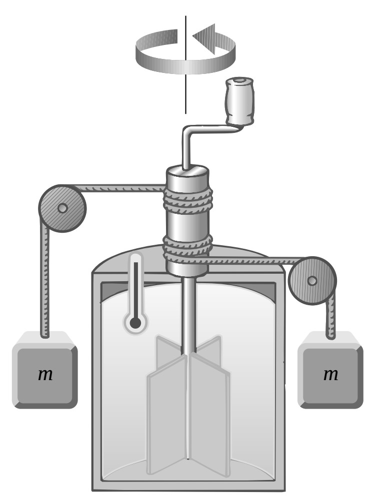
$$T_f = T_1 + \Delta T = 10.0^\circ\text{C} + 0.117^\circ\text{C} = \boxed{10.1^\circ\text{C}}$$

L: The water temperature rose less than 1°C as expected; however, the final temperature might be less than we calculated since this solution does not account for cooling of the water due to evaporation as it falls. It is interesting to note that the change in temperature is independent of the amount of water.

20.2 The container is thermally insulated, so no heat flows: $Q = 0$ and $\Delta E_{\text{int}} = Q - W_{\text{output}} = 0 - W_{\text{output}} = +W_{\text{input}} = 2mgh$. For convenience of calculation, we imagine setting the water on a stove and putting in this same amount of heat. Then we would have $2mgh = \Delta E_{\text{int}} = Q = m_{\text{water}}\, c\, \Delta T$.

$$\Delta T = \frac{2mg\,h}{m_{\text{water}}\, c} = \frac{2 \times 1.50 \text{ kg}(9.80 \text{ m/s}^2)3.00 \text{ m}}{(0.200 \text{ kg})(4186 \text{ J/kg}\cdot\text{C}^\circ)} = \frac{88.2 \text{ J}}{837 \text{ J/C}^\circ}$$

$$= \boxed{0.105 \text{ C}^\circ}$$



Thermal insulator

20.3 $\Delta Q = mc_{\text{silver}}\,\Delta T$

$1.23\text{ kJ} = (0.525\text{ kg})c_{\text{silver}}\,(10.0°\text{C})$

$c_{\text{silver}} = \boxed{0.234\text{ kJ/kg}\cdot°\text{C}}$

***20.4** From $Q = mc\,\Delta T$, we find

$$\Delta T = \frac{Q}{mc} = \frac{1200\text{ J}}{(0.0500\text{ kg})(387\text{ J/kg}\cdot°\text{C})} = 62.0°\text{C}$$

Thus, the final temperature is $\boxed{87.0°\text{C}}$

20.5 $Q_{\text{cold}} = -Q_{\text{hot}}$

$(mc\Delta T)_{\text{water}} = -(mc\Delta T)_{\text{iron}}$

$(20.0\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(T_f - 25.0°\text{C}) = -(1.50\text{ kg})(448\text{ J/kg}\cdot°\text{C})(T_f - 600°\text{C})$

$T_f = \boxed{29.6°\text{C}}$

<u>Goal Solution</u>

G: Even though the horseshoe is much hotter than the water, the mass of the water is significantly greater, so we might expect the water temperature to rise less than 10°C.

O: The heat lost by the iron will be gained by the water, and from this heat transfer, the change in water temperature can be found.

A: $\Delta Q_{\text{iron}} = -\Delta Q_{\text{water}}$ or $(mc\Delta T)_{\text{iron}} = -(mc\Delta T)_{\text{water}}$

$(1.50\text{ kg})(448\text{ J/kg}\cdot°\text{C})(T - 600°\text{C}) = -(20.0\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(T - 25.0°\text{C})$

$T = \boxed{29.6\ °\text{C}}$

L: The temperature only rose about 5°C, so our answer seems reasonable. The specific heat of the water is about 10 times greater than the iron, so this effect also reduces the change in water temperature. In this problem, we assumed that a negligible amount of water boiled away, but in reality, the final temperature of the water would be less than what we calculated since some of the heat energy would be used to vaporize the water.

***20.6** Let us find the energy transferred in one minute.

$$Q = [m_{\text{cup}}c_{\text{cup}} + m_{\text{water}}c_{\text{water}}]\Delta T$$

$$Q = \left[(0.200\text{ kg})\left(900\,\frac{\text{J}}{\text{kg}\cdot°\text{C}}\right) + (0.800\text{ kg})\left(4186\,\frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)\right](-1.50\ \text{C}°) = -5290\text{ J}$$

If this much energy is removed from the system each minute, the rate of removal is

$$\wp = \frac{|Q|}{\Delta t} = \frac{5290\text{ J}}{60.0\text{ s}} = 88.2\,\frac{\text{J}}{\text{s}} = \boxed{88.2\text{ W}}$$

***20.7** (a) $Q_{\text{cold}} = -Q_{\text{hot}}$

$$(m_w c_w + m_c c_c)(T_f - T_c) = -m_{\text{Cu}} c_{\text{Cu}}(T_f - T_{\text{Cu}}) - m_{\text{unk}} c_{\text{unk}}(T_f - T_{\text{unk}})$$

where w is for water, c the calorimeter, Cu the copper sample, and *unk* the unknown.

$$\left[(250\text{ g})\left(100\,\frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right) + (100\text{ g})\left(0.215\,\frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)\right](20.0 - 10.0)\,{}^\circ\text{C}$$

$$= -(50.0\text{ g})\left(0.0924\,\frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)(20.0 - 80.0)\,{}^\circ\text{C} - (70.0\text{ g})c_{\text{unk}}(20.0 - 100){}^\circ\text{C}$$

$$2.44 \times 10^3\text{ cal} = (5.60 \times 10^3\text{ g}\cdot{}^\circ\text{C})c_{\text{unk}} \quad \text{or} \quad c_{\text{unk}} = \boxed{0.435\text{ cal/g}\cdot{}^\circ\text{C}}$$

(b) The material of the sample is $\boxed{\text{beryllium}}$.

20.8 $m = (4.00 \times 10^{11}\text{ m}^3)(1000\text{ kg/m}^3)$

(a) $\Delta Q = mc\,\Delta T = Pt = (4.00 \times 10^{14}\text{ kg})(4186\text{ J/kg}\cdot{}^\circ\text{C})(1.00{}^\circ\text{C})$

$$\Delta Q = \boxed{1.68 \times 10^{18}\text{ J}} = Pt$$

(b) $t = \dfrac{1.68 \times 10^{18}\text{ J}}{1.00 \times 10^9\text{ J/s}} = 1.68 \times 10^9\text{ s} = \boxed{53.1\text{ yr}}$

20.9 (a) $(f)(mgh) = mc\,\Delta T$

$$\frac{(0.600)(3.00 \times 10^{-3}\text{ kg})(9.80\text{ m/s})(50.0\text{ m})}{4.186\text{ J/cal}} = (3.00\text{ g})(0.0924\text{ cal/g}\cdot\text{C}^\circ)(\Delta T)$$

$$\Delta T = 0.760{}^\circ\text{C}; \quad \boxed{T = 25.8{}^\circ\text{C}}$$

(b) $\boxed{\text{No}}$ Both the change in potential energy and the heat absorbed are proportional to the mass; hence, the mass cancels in the energy relation.

20.10 $Q_{\text{cold}} = -Q_{\text{hot}}$

$m_{\text{Al}}c_{\text{Al}}(T_f - T_c) + m_c c_w(T_f - T_c) = -m_h c_w(T_f - T_h)$

$(m_{\text{Al}}c_{\text{Al}} + m_c w_w)T_f - (m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c = -m_h c_w T_f + m_h c_w T_h$

$(m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w)T_f = (m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c + m_h c_w T_h$

$$T_f = \boxed{\frac{(m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c + m_h c_w T_h}{(m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w)}}$$

20.11 The rate of collection of heat = $\wp$ = (550 W / m^2)(6.00 m^2) = 3300 W. The amount of heat required to raise the temperature of 1000 kg of water by 40.0°C is:

$$Q = mc\Delta T = (1000\text{ kg})(4186\text{ J/kg}\cdot\text{C°})(40.0\text{°C}) = 1.67 \times 10^8\text{ J}$$

Thus, $\wp\Delta t = 1.67 \times 10^8$ J

or $$\Delta t = \frac{1.67 \times 10^8\text{ J}}{3300\text{ W}} = \boxed{50.7\text{ ks}} = 14.1\text{ h}$$

***20.12** The heat needed is the sum of the following terms:

Q_{needed} = (heat to reach melting point) + (heat to melt)

+ (heat to reach boiling point)

+ (heat to vaporize) + (heat to reach 110°C)

Thus, we have

$Q_{\text{needed}} = 0.0400\text{ kg}[(2090\text{ J/kg}\cdot\text{°C})(10.0\text{°C}) + (3.33 \times 10^5\text{ J/kg})$

$+ (4186\text{ J/kg}\cdot\text{°C})(100\text{°C}) + (2.26 \times 10^6\text{ J/kg})$

$+ (2010\text{ J/kg}\cdot\text{°C})(10.0\text{°C})]$

$Q_{\text{needed}} = \boxed{1.22 \times 10^5\text{ J}}$

20.13 The bullet will not melt all the ice, so its final temperature is 0°C. Then

$$\left(\frac{1}{2}mv^2 + mc\,\Delta T\right)_{\text{bullet}} = m_w L_f$$

where m_w is the meltwater mass

$$m_w = \frac{0.500(3.00 \times 10^{-3}\text{ kg})(240\text{ m/s})^2 + (3.00 \times 10^{-3}\text{ kg})(128\text{ J/kg}\cdot\text{C°})(30.0\text{ C°})}{3.33 \times 10^5\text{ J/kg}}$$

$$m_w = \frac{86.4\text{ J} + 11.5\text{ J}}{333\,000\text{ J/kg}} = \boxed{0.294\text{ g}}$$

Goal Solution

G: The amount of ice that melts is probably small, maybe only a few grams based on the size, speed, and initial temperature of the bullet.

O: We will assume that all of the initial kinetic and excess internal energy of the bullet goes into internal energy to melt the ice, the mass of which can be found from the latent heat of fusion.

A: At thermal equilibrium, the energy lost by the bullet equals the energy gained by the ice:

$$\Delta K_{\text{bullet}} + \Delta Q_{\text{bullet}} = \Delta Q_{\text{ice}}$$

$$\frac{1}{2}\,m_b v^2 + m_b c_{\text{lead}}\Delta T = m_{\text{ice}} L_f$$

$$\frac{1}{2}(3\times10^{-3}\text{ kg})(240\text{ m/s})^2 + (3\times10^{-3}\text{ kg})(128\text{ J/kg}\cdot{}^\circ\text{C})(30.0^\circ\text{C}) = m_{\text{ice}}(3.33\times10^5\text{ J/kg})$$

$$m_{\text{ice}} = \frac{86.4\text{ J} + 11.5\text{ J}}{3.33\times10^5\text{ J/kg}} = 2.94\times10^{-4}\text{ kg} = 0.294\text{ g}$$

L: The amount of ice that melted is less than a gram, which agrees with our prediction. It appears that most of the energy used to melt the ice comes from the kinetic energy of the bullet (88%), while the excess internal energy of the bullet only contributes 12% to melt the ice. Small chips of ice probably fly off when the bullet makes impact. So some of the energy is transferred to their kinetic energy, so in reality, the amount of ice that would melt should be less than what we calculated. If the block of ice were colder than 0°C (as is often the case), then the melted ice would refreeze.

***20.14** (a) Q_1 = heat to melt all the ice $= (50.0\times10^{-3}\text{ kg})(3.33\times10^5\text{ J/kg}) = 1.67\times10^4\text{ J}$

Q_2 = (heat to raise temp of ice to 100°C)

$= (50.0\times10^{-3}\text{ kg})(4186\text{ J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C}) = 2.09\times10^4\text{ J}$

Thus, the total heat to melt ice and raise temp to 100°C $= 3.76\times10^4$ J

$$Q_3 = \begin{matrix}\text{heat available}\\ \text{as steam condenses}\end{matrix} = (10.0\times10^{-3}\text{ kg})(2.26\times10^6\text{ J/kg}) = 2.26\times10^4\text{ J}$$

Thus, we see that $Q_3 > Q_1$, but $Q_3 < Q_1 + Q_2$.

Therefore, all the ice melts but $T_f < 100°\text{C}$. Let us now find T_f

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$(50.0 \times 10^{-3}\text{ kg})(3.33 \times 10^5\text{ J/kg}) + (50.0 \times 10^{-3}\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(T_f - 0°\text{C})$$

$$= -(10.0 \times 10^{-3}\text{ kg})(-2.26 \times 10^6\text{ J/kg})$$

$$-(10.0 \times 10^{-3}\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(T_f - 100°\text{C})$$

From which, $\boxed{T_f = 40.4°\text{C}}$

(b) Q_1 = heat to melt all ice $= 1.67 \times 10^4$ J [See part (a)]

$$Q_2 = \begin{matrix}\text{heat given up}\\ \text{as steam condenses}\end{matrix} = (10^{-3}\text{ kg})(2.26 \times 10^6\text{ J/kg}) = 2.26 \times 10^3\text{ J}$$

$$Q_3 = \begin{matrix}\text{heat given up as condensed}\\ \text{steam cools to }0°\text{C}\end{matrix} = (10^{-3}\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(100°\text{C}) = 419\text{ J}$$

Note that $Q_2 + Q_3 < Q_1$. Therefore, the final temperature will be 0°C with some ice remaining. Let us find the mass of ice which must melt to condense the steam and cool the condensate to 0°C.

$$mL_f = Q_2 + Q_3 = 2.68 \times 10^3\text{ J}$$

Thus, $m = \dfrac{2.68 \times 10^3\text{ J}}{3.33 \times 10^5\text{ J/kg}} = 8.04 \times 10^{-3}\text{ kg} = 8.04\text{ g}$

Therefore, there is $\boxed{\text{42.0 g of ice left over}}$

20.15 $\Delta Q = m_{\text{Cu}}\, c_{\text{Cu}}\, \Delta T = m_{N2}\,(L_{\text{vap}})_{N2}$

$$(1.00\text{ kg})\left(0.0920\,\frac{\text{cal}}{\text{g}\cdot\text{C}°}\right)(293 - 77.3)\text{ C}° = m\left(48.0\,\frac{\text{cal}}{\text{g}}\right)$$

$m = \boxed{0.414\text{ kg}}$

***20.16** $Q_{\text{cold}} = -Q_{\text{hot}}$

$$[m_w c_w + m_c c_c](T_f - T_i) = -m_s[-L_v + c_w(T_f - 100)]$$

$$\left[(0.250\text{ kg})\left(4186\,\frac{\text{J}}{\text{kg}\cdot\text{C}°}\right) + (0.0500\text{ kg})\left(387\,\frac{\text{J}}{\text{kg}\cdot\text{C}°}\right)\right](50.0°\text{C} - 20.0°\text{C})$$

$$= -m_s\left[-2.26 \times 10^6\,\frac{\text{J}}{\text{kg}} + \left(4186\,\frac{\text{J}}{\text{kg}\cdot\text{C}°}\right)(50.0°\text{C} - 100°\text{C})\right]$$

$$m_s = \frac{3.20 \times 10^4 \text{ J}}{2.47 \times 10^6 \text{ J/kg}} = 0.0129 \text{ kg} = \boxed{12.9 \text{ g steam}}$$

20.17 (a) Since the heat required to melt 250 g of ice at 0°C *exceeds* the heat required to cool 600 g of water from 18°C to 0°C, the final temperature of the system (water + ice) must be $\boxed{0°\text{C}}$.

(b) Let m represent the mass of ice that melts before the system reaches equilibrium at 0°C.

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$mL_f = -m_w c_w(0°\text{C} - T_i)$$

$$m(3.33 \times 10^5 \text{ J/kg}) = -(0.600 \text{ kg})(4186 \text{ J/kg} \cdot °\text{C})(0°\text{C} - 18.0°\text{C})$$

$m = 136$ g, so the ice remaining = 250 g – 136 g = $\boxed{114 \text{ g}}$

20.18 The original kinetic energy all becomes thermal energy:

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = 2\left(\frac{1}{2}\right)(5.00 \times 10^{-3} \text{ kg})(500 \text{ m/s})^2 = 1.25 \text{ kJ}$$

Raising the temperature to the melting point requires

$$Q = mc\,\Delta T = 10.0 \times 10^{-3} \text{ kg}(128 \text{ J/kg} \cdot \text{C°})(327°\text{C} - 20.0°\text{C}) = 393 \text{ J}$$

Since 1250 J > 393 J, the lead starts to melt. Melting it all requires

$$Q = mL = (10.0 \times 10^{-3} \text{ kg})\,(2.45 \times 10^4 \text{ J/kg}) = 245 \text{ J}$$

Since 1250 J > 393 + 245 J, it all melts. If we assume $\boxed{\text{liquid lead}}$ has the same specific heat as solid lead, the final temperature is given by

$$1.25 \times 10^3 \text{ J} = 393 \text{ J} + 245 \text{ J} + 10.0 \times 10^{-3} \text{ kg}(128 \text{ J/kg} \cdot \text{C°})(T_f - 327°\text{C})$$

$$\boxed{T_f = 805°\text{C}}$$

20.19 $Q_{\text{cold}} = -Q_{\text{hot}}$

$$m_{\text{Fe}}c_{\text{Fe}}(\Delta T)_{\text{Fe}} = -m_{\text{Pb}}[-L_f + c\Delta T)]_{\text{Pb}}$$

$$(0.300 \text{ kg})\left(448 \frac{\text{J}}{\text{kg} \cdot °\text{C}}\right)(T_f - 20.0°\text{C})$$

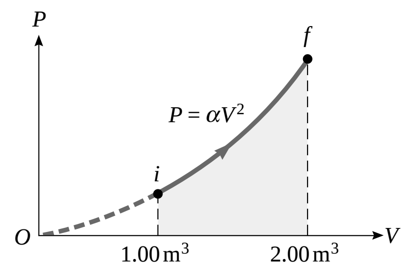
$$= -0.0900 \text{ kg}\left[-2.45 \times 10^4 \frac{\text{J}}{\text{kg}} + \left(128 \frac{\text{J}}{\text{kg} \cdot °\text{C}}\right)(T_f - 327.3°\text{C})\right]$$

and $T = \boxed{59.4°\text{C}}$

20.20 (a) $W = \int P\,dV = P\,\Delta V = (1.50\text{ atm})(4.00\text{ m}^3) = \boxed{6.08 \times 10^5\text{ J}}$

(b) $W = \int P\,dV = P\,\Delta V = (1.50\text{ atm})(1.00 - 4.00)\text{m}^3 = \boxed{-4.56 \times 10^5\text{ J}}$

20.21 $W_{if} = \int_i^f P\,dV$



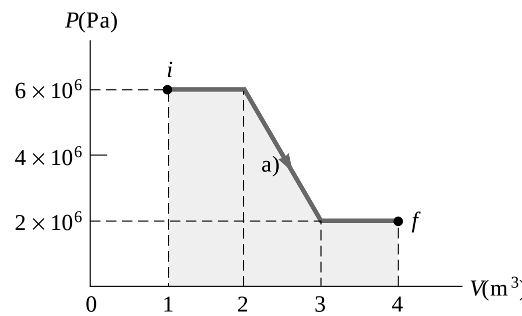
The work done by the gas is just the area under the curve $P = \alpha V^2$ between V_i and V_f.

$$W_{if} = \int_i^f \alpha V^2 dV = \frac{1}{3}\alpha\left(V_f^3 - V_i^3\right)$$

$$V_f = 2V_i = 2(1.00\text{ m}^3) = 2.00\text{ m}^3$$

$$W_{if} = \frac{1}{3}\left(5.00\,\frac{\text{atm}}{\text{m}^6} \times 1.013 \times 10^5\,\frac{\text{Pa}}{\text{atm}}\right)\left[(2.00\text{ m}^3)^3 - (1.00\text{ m}^3)^3\right] = \boxed{1.18\text{ MJ}}$$

***20.22** (a) $W = \int P\,dV$



$$= (6.00 \times 10^6\text{ Pa})(2.00 - 1.00)\text{m}^3$$

$$+ (4.00 \times 10^6\text{ Pa})(3.00 - 2.00)\text{m}^3$$

$$+ (2.00 \times 10^6\text{ Pa})(4.00 - 3.00)\text{m}^3$$

$$W_{i \to f} = \boxed{+12.0\text{ MJ}}$$

(b) $W_{f \to i} = \boxed{-12.0\text{ MJ}}$

20.23 During the heating process $P = (P_i / V_i)V$.

(a) $W = \int_i^f P\,dV = \int_{V_i}^{3V_i} (P_i / V_i)\,V\,dV$

$$W = (P_i / V_i)\left.\frac{V^2}{2}\right|_{V_i}^{3V_i} = \frac{P_i}{2V_i}\left(9V_i^2 - V_i^2\right) = \boxed{4P_iV_i}$$

(b) $PV = nRT$

$[(P_i / V_i)V]V = nRT$

$\boxed{T = (P_i / nRV_i)V^2}$

Temperature must be proportional to the square of volume, rising to nine times its original value.

20.24 $W = \int_i^f P\,dV = P\int_i^f dV = PV_f - PV_i$

$W = nRT_f - nRT_i$

$$n = \frac{W}{R(\Delta T)} = \frac{20.0\text{ J}}{(8.315\text{ J/mol}\cdot\text{K})(100\text{ K})} = 0.0241\text{ mol}$$

$$m = nM = (0.0241\text{ mol})\left(\frac{4.00\text{ g}}{\text{mol}}\right) = \boxed{0.0962\text{ g}}$$

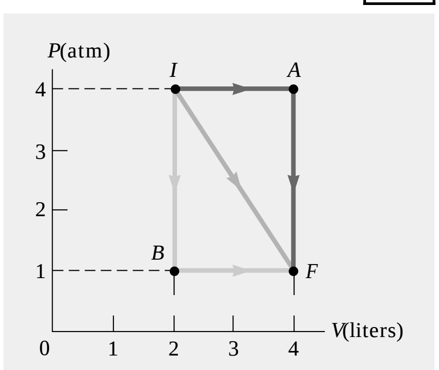
20.25 $W = P\,\Delta V = P\left(\frac{nR}{P}\right)(T_f - T_i) = nR\,\Delta T = 0.200(8.315)(280) = \boxed{466\text{ J}}$

20.26 $W = \int_i^f P\,dV = P\int_i^f dV = P(\Delta V) = nR(\Delta T) = \boxed{nR(T_2 - T_1)}$

20.27 (a) Along *IAF*, $W = (4.00\text{ atm})(2.00\text{ liter}) = 8.00\text{ L}\cdot\text{atm} = \boxed{810\text{ J}}$

(b) Along *IF*, $W = 5.00\text{ L}\cdot\text{atm} = \boxed{506\text{ J}}$

(c) Along *IBF*, $W = 2.00\text{ L}\cdot\text{atm} = \boxed{203\text{ J}}$



20.28 (a) $W = P\,\Delta V = (0.800\text{ atm})(-7.00\text{ L}) = \boxed{-567\text{ J}}$

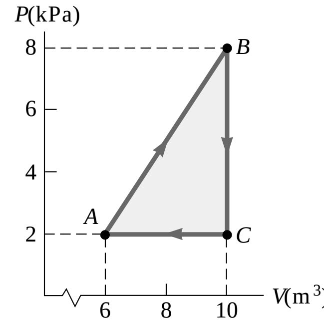
(b) $\Delta E_{\text{int}} = Q - W = -400\text{ J} + 567\text{ J} = \boxed{167\text{ J}}$

20.29 $\Delta E_{\text{int}} = Q - W$

$\underline{Q} = \Delta E_{\text{int}} + W = -500\text{ J} - 220\text{ J} = \boxed{-720\text{ J}}$

Positive heat is transferred *from* the system.

20.30 (a) $Q = W = \text{Area of triangle} = \frac{1}{2}(4.00 \text{ m}^3)(6.00 \text{ kPa}) = \boxed{12.0 \text{ kJ}}$

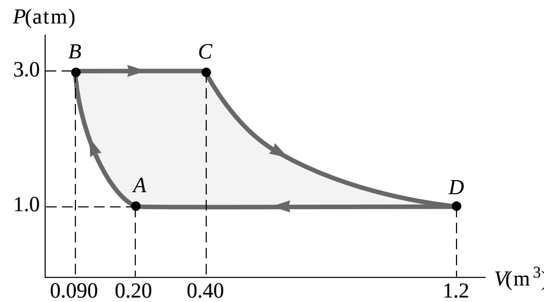


(b) $Q = W = \boxed{-12.0 \text{ kJ}}$

***20.31**

	Q	W	ΔE_{int}	
BC	–	0	–	($Q = \Delta E_{\text{int}}$ since $W_{BC} = 0$)
CA	–	–	–	($\Delta E_{\text{int}} < 0$ and $W < 0$, so $Q < 0$)
AB	+	+	+	($W > 0$, $\Delta E_{\text{int}} > 0$ since $\Delta E_{\text{int}} < 0$ for $B \to C \to A$; so $Q > 0$)

20.32 $W_{\text{BC}} = P_{\text{B}}(V_{\text{C}} - V_{\text{B}}) = 3.00 \text{ atm}(0.400 - 0.0900) \text{ m}^3$
$= 94.2 \text{ kJ}$



$\Delta E_{\text{int}} = Q - W$

$E_{\text{int,C}} - E_{\text{int,B}} = (100 - 94.2) \text{ kJ}$

$E_{\text{int,C}} - E_{\text{int,B}} = 5.79 \text{ kJ}$

Since T is constant,

$$E_{\text{int,D}} - E_{\text{int,C}} = 0$$

$$W_{\text{DA}} = P_{\text{D}}(V_A - V_D) = 1.00 \text{ atm}(0.200 - 1.20) \text{ m}^3 = -101 \text{ kJ}$$

$$E_{\text{int,A}} - E_{\text{int,D}} = -150 \text{ kJ} - (-101 \text{ kJ}) = -48.7 \text{ kJ}$$

Now, $E_{\text{int,B}} - E_{\text{int,A}} = -[(E_{\text{int,C}} - E_{\text{int,B}}) + (E_{\text{int,D}} - E_{\text{int,C}}) + (E_{\text{int,A}} - E_{\text{int,D}})]$

$$E_{\text{int,B}} - E_{\text{int,A}} = -[5.79 \text{ kJ} + 0 - 48.7 \text{ kJ}] = \boxed{+42.9 \text{ kJ}}$$

20.33 (a) $\Delta E_{\text{int}} = Q - P\,\Delta V = 12.5 \text{ kJ} - (2.50 \text{ kPa})(3.00 - 1.00)\text{m}^3 = \boxed{7.50 \text{ kJ}}$

(b) $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$; $T_2 = \dfrac{V_2}{V_1} T_1 = \dfrac{3.00}{1.00}(300 \text{ K}) = \boxed{900 \text{ K}}$

20.34 (a) $W = nRT \ln\left(\dfrac{V_f}{V_i}\right) = P_f V_f \ln\dfrac{V_f}{V_i}$ so $V_i = V_f \exp\left(-\dfrac{W}{P_f V_f}\right)$

$$V_i = (0.0250) \exp\left[-\frac{3000}{(0.0250)(1.013 \times 10^5)}\right] = \boxed{0.00765 \text{ m}^3}$$

(b) $T_f = \dfrac{P_f V_f}{nR} = \dfrac{(1.013 \times 10^5 \text{ N/m}^2)(0.0250 \text{ m}^3)}{(1.00 \text{ mol})(8.315 \text{ J/K} \cdot \text{mol})} = \boxed{305 \text{ K}}$

20.35 $W = P\,\Delta V = P(V_s - V_w) = \dfrac{P(nRT)}{P} - P\left[\dfrac{(18.0\text{ g})}{(1.00\text{ g/cm}^3)(10^6\text{ cm}^3/\text{m}^3)}\right]$

$$W = (1.00\text{ mol})(8.315\text{ J/K}\cdot\text{mol})(373\text{ K}) - (1.013\times 10^5\text{ N/m}^2)\left(\frac{18.0\text{ g}}{10^6\text{ g/m}^3}\right)$$

$W = \boxed{3.10\text{ kJ}}$

$Q = mL_v = (0.0180\text{ kg})(2.26\times 10^6\text{ J/kg}) = 40.7\text{ kJ}$

$\Delta E_{\text{int}} = Q - W = \boxed{37.6\text{ kJ}}$

20.36 (a) $W = P\Delta V = P\,[3\alpha V\,\Delta T]$

$$= (1.013\times 10^5\text{ N}\cdot\text{m}^2)(3)(24.0\times 10^{-6})(\text{C°})^{-1}\left(\frac{1.00\text{ kg}}{2.70\times 10^3\text{ kg/m}^3}\right)(18.0\text{°C})$$

$W = \boxed{48.6\text{ mJ}}$

(b) $Q = cm\,\Delta T = (900\text{ J/kg}\cdot\text{°C})(1.00\text{ kg})(18.0\text{°C}) = \boxed{16.2\text{ kJ}}$

(c) $\Delta E_{\text{int}} = Q - W = 16.2\text{ kJ} - 48.6\text{ mJ} = \boxed{16.2\text{ kJ}}$

20.37 (a) $P_i V_i = P_f V_f = nRT = (2.00\text{ mol})(8.315\text{ J/K}\cdot\text{mol})(300\text{ K}) = 4.99\times 10^3\text{ J}$

$$V_i = \frac{nRT}{P_i} = \frac{4.99\times 10^3\text{ J}}{0.400\text{ atm}}$$

$$V_f = \frac{nRT}{P_f} = \frac{4.99\times 10^3\text{ J}}{1.20\text{ atm}} = \frac{1}{3}V_i = \boxed{0.0410\text{ m}^3}$$

(b) $W = \int P\,dV = nRT\ln\left(\dfrac{V_f}{V_i}\right) = (4.99\times 10^3)\ln\left(\dfrac{1}{3}\right) = \boxed{-5.48\text{ kJ}}$

(c) $\Delta E_{\text{int}} = 0 = Q - W$

$Q = \boxed{-5.48\text{ kJ}}$

20.38 The condensing and cooling water loses heat

$$mL_v + mc\,\Delta T = 0.0180\text{ kg}\left[2.26 \times 10^6\text{ J/kg} + (4186\text{ J/kg}\cdot\text{C°})90.0\text{ C°}\right]$$

$$Q = 4.75 \times 10^4\text{ J}$$

From the First Law,

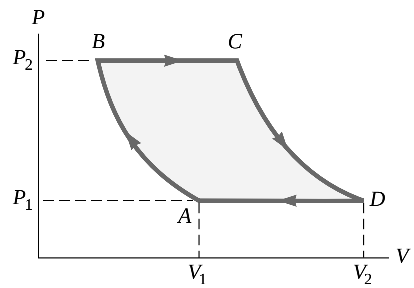
$$Q = \Delta E_{\text{int}} + W = 0 + nRT\ln(V_f/V_i)$$

$$4.75 \times 10^4\text{ J} = 10.0\text{ mol }(8.315\text{ J/mol}\cdot\text{K})(273\text{ K})\ln(20.0\text{ L}/V_i)$$

$$2.09 = \ln(20.0\text{ L}/V_i)$$

$$V_i = \boxed{2.47\text{ L}}$$

20.39 $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$



$$W = \int_A^B P\,dV + \int_B^C P\,dV + \int_C^D P\,dV + \int_D^A P\,dV$$

$$W = nRT_1\int_A^B \frac{dV}{V} + P_2\int_B^C dV$$

$$+\, nRT_2\int_C^D \frac{dV}{V} + P_1\int_D^A dV$$

$$W = nRT_1\ln\left(\frac{V_B}{V_1}\right) + P_2(V_C - V_B) + nRT_2\ln\left(\frac{V_2}{V_C}\right) + P_1(V_A - V_D)$$

Now $P_1V_A = P_2V_B$ and $P_2V_C = P_1V_D$, so only the logarithmic terms do not cancel out:

Also, $\dfrac{V_B}{V_1} = \dfrac{P_1}{P_2}$ and $\dfrac{V_2}{V_C} = \dfrac{P_2}{P_1}$

$$\Sigma W = nRT_1\ln\left(\frac{P_1}{P_2}\right) + nRT_2\ln\left(\frac{P_2}{P_1}\right)$$

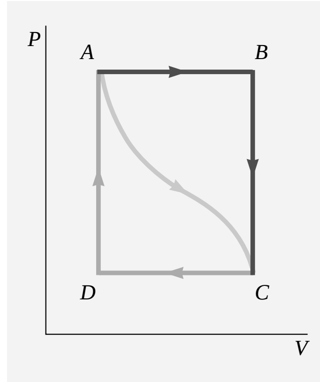
$$= -nRT_1\ln\left(\frac{P_2}{P_1}\right) + nRT_2\ln\left(\frac{P_2}{P_1}\right)$$

$$= nR(T_2 - T_1)\ln\left(\frac{P_2}{P_1}\right)$$

Moreover $P_1V_2 = nRT_2$ and $P_1V_1 = nRT_1$

$$\Sigma W = \boxed{P_1(V_2 - V_1)\ln\left(\frac{P_2}{P_1}\right)}$$

20.40 $\Delta E_{\text{int},ABC} = \Delta E_{\text{int},AC}$ (conservation of energy)



(a) $\Delta E_{\text{int},ABC} = Q_{ABC} - W_{ABC}$ (First Law)

$Q_{ABC} = 800\text{ J} + 500\text{ J} = \boxed{1300\text{ J}}$

(b) $W_{CD} = P_C \Delta V_{CD}$, $\Delta V_{AB} = \Delta V_{CD}$, and $P_A = 5P_C$

Then, $W_{CD} = \dfrac{1}{5} P_A\, \Delta V_{AB} = \dfrac{1}{5}\, W_{AB} = \boxed{-100\text{ J}}$

(– means that work is done on the system)

(c) $W_{CDA} = W_{CD}$ so that $Q_{CA} = \Delta E_{\text{int},CA} + W_{CDA} = -800\text{ J} - 100\text{ J} = \boxed{-900\text{ J}}$

(– means that heat must be removed from the system)

(d) $\Delta E_{\text{int},CD} = \Delta E_{\text{int},CDA} - \Delta E_{\text{int},DA} = -800\text{ J} - 500\text{ J} = -1300\text{ J}$

and $Q_{CD} = \Delta E_{\text{int},CD} + W_{CD} = -1300\text{ J} - 100\text{ J} = \boxed{-1400\text{ J}}$

20.41 $\wp = \dfrac{kA(T_2 - T_1)}{L}$

$$\wp = \frac{(0.200\text{ cal/s}\cdot\text{cm}\cdot\text{C°})(20.0\text{ cm})(5000\text{ cm})(180\text{ C°})}{1.50\text{ cm}} = \boxed{2.40\times 10^{6}\text{ cal/s}}$$

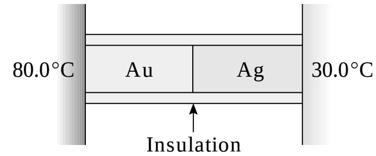
20.42 $\wp = kA\dfrac{\Delta T}{L}$

$$k = \frac{\wp L}{A\,\Delta T} = \frac{(10.0\text{ W})(0.0400\text{ m})}{(1.20\text{ m}^2)(15.0°\text{C})} = \boxed{2.22\times 10^{-2}\text{ W/m}\cdot\text{C°}}$$

20.43 $$\wp = \frac{kA\,\Delta T}{L} = \frac{(0.800\text{ W/m}\cdot°\text{C})(3.00\text{ m}^2)(25.0°\text{C})}{6.00\times 10^{-3}\text{ m}} = 1.00\times 10^{4}\text{ W} = \boxed{10.0\text{ kW}}$$

20.44 $$\wp = \frac{A\,\Delta T}{\sum_i \dfrac{L_i}{k_i}} = \frac{(6.00\text{ m}^2)(50.0°\text{C})}{\dfrac{2(4.00\times 10^{-3}\text{ m})}{0.800\text{ W/m}\cdot°\text{C}} + \dfrac{5.00\times 10^{-3}\text{ m}}{0.0234\text{ W/m}\cdot°\text{C}}} = \boxed{1.34\text{ kW}}$$

20.45 In the steady state condition, $\wp_{\text{Au}} = \wp_{\text{Ag}}$ so that



$$k_{\text{Au}} A_{\text{Au}} \left(\frac{\Delta T}{\Delta x}\right)_{\text{Au}} = k_{\text{Ag}} A_{\text{Ag}} \left(\frac{\Delta T}{\Delta x}\right)_{\text{Ag}}$$

In this case $A_{\text{Au}} = A_{\text{Ag}}$, $\Delta x_{\text{Au}} = \Delta x_{\text{Ag}}$, $\Delta T_{\text{Au}} = (80.0 - T)$

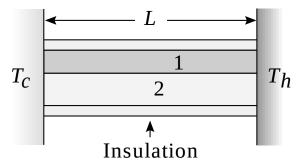
and $\Delta T_{\text{Ag}} = (T - 30.0)$

where T is the temperature of the junction. Therefore,

$$k_{\text{Au}}(80.0 - T) = k_{\text{Ag}}(T - 30.0) \quad \text{and} \quad \boxed{T = 51.2°\text{C}}$$

20.46 Two rods: $\wp = (k_1 A_1 + k_2 A_2)\dfrac{\Delta T}{L}$

$$\wp = \boxed{(k_1 A_1 + k_2 A_2)\frac{(T_h - T_c)}{L}}$$



In general:

$$\wp = (\Sigma k_i A_i)\frac{\Delta T}{L} = \boxed{(\Sigma k_i A_i)\frac{(T_h - T_c)}{L}}$$

20.47 From Table 20.4,

(a) $R = \boxed{0.890 \text{ ft}^2 \cdot °\text{F} \cdot \text{h}/\text{Btu}}$

(b) The insulating glass in the table must have sheets of glass less than 1/8 inch thick. So we estimate the R-value of a 0.250-inch air space as (0.250/3.50) times that of the thicker air space. Then for the double glazing

$$R_b = \left[0.890 + \left(\frac{0.250}{3.50}\right)1.01 + 0.890\right]\frac{\text{ft}^2 \cdot \text{F}° \cdot \text{h}}{\text{Btu}} = \boxed{1.85 \frac{\text{ft}^2 \cdot °\text{F} \cdot \text{h}}{\text{Btu}}}$$

(c) Since A and $(T_2 - T_1)$ are constants, heat flow is reduced by a factor of

$$\frac{1.85}{0.890} = \boxed{2.08}$$

20.48 $\wp = \sigma A e T^4 = (5.6696 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)\,[4\pi(6.96 \times 10^8 \text{ m})^2](0.965)(5800 \text{ K})^4$

$\wp = \boxed{3.77 \times 10^{26} \text{ W}}$

***20.49** Suppose the pizza is 70 cm in diameter and l = 2.0 cm thick, sizzling at 100˚C. It cannot lose heat by conduction or convection. It radiates according to $\wp = \sigma AeT^4$. Here, A is its surface area,

$$A = 2\pi r^2 + 2\pi r l = 2\pi(0.35\text{ m})^2 + 2\pi(0.35\text{ m})(0.02\text{ m}) = 0.81\text{ m}^2$$

Suppose it is dark in the infrared, with emissivity about 0.8. Then

$$\wp = (5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(0.81\text{ m}^2)(0.80)(373\text{ K})^4 = 710\text{ W}\ \boxed{\sim 10^3\text{ W}}$$

If the density of the pizza is half that of water, its mass is

$$m = \rho V = \rho\pi r^2 l = (500\text{ kg/m}^3)\pi(0.35\text{ m})^2(0.02\text{ m}) \approx 4\text{ kg}$$

Suppose its specific heat is c = 0.6 cal/g · C°. The drop in temperature of the pizza is described by:

$$Q = mc(T_f - T_i)$$

$$\wp = \frac{dQ}{dt} = mc\frac{dT_f}{dt} - 0$$

$$\frac{dT_f}{dt} = \frac{\wp}{mc} = \frac{710\text{ J/s}}{(4\text{ kg})(0.6\cdot 4186\text{ J/kg}\cdot\text{C}°)} = 0.07\text{ C}°/\text{s}\ \boxed{\sim 10^{-1}\text{ K/s}}$$

***20.50** $\wp = \sigma AeT^4$

$$2.00\text{ W} = (5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(0.250\times10^{-6}\text{ m}^2)(0.950)T^4$$

$$T = [1.49\times10^{14}\text{ K}^4]^{1/4} = \boxed{3.49\times10^3\text{ K}}$$

***20.51** We suppose the earth below is an insulator. The square meter must radiate in the infrared as much energy as it absorbs, $\wp = \sigma AeT^4$. Assuming that e = 1.00 for blackbody blacktop:

$$1000\text{ W} = (5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1.00\text{ m}^2)(1.00)T^4$$

$$T = (1.76\times10^{10}\text{ K}^4)^{1/4} = \boxed{364\text{ K}}$$ (You can cook an egg on it.)

***20.52** The sphere of radius R absorbs sunlight over the area of its day hemisphere, projected as a flat circle perpendicular to the light: πR^2. It radiates in all directions, over area $4\pi R^2$. Then, in steady state,

$$\wp_{\text{in}} = \wp_{\text{out}}$$

$$e(1340\text{ W/m}^2)\pi R^2 = e\sigma(4\pi R^2)T^4$$

The emissivity e, the radius R, and π all cancel. Therefore,

$$T = \left[\frac{1340\text{ W/m}^2}{4(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \boxed{277\text{ K}} = 4°\text{C}$$

***20.53** 77.3 K = –195.8°C is the boiling point of nitrogen. It gains no heat to warm as a liquid, but gains heat to vaporize:

$$Q = mL_v = (0.100\text{ kg})(2.01 \times 10^5\text{ J/kg}) = 2.01 \times 10^4\text{ J}$$

The water first loses heat by cooling. Before it starts to freeze, it can lose

$$Q = mc\,\Delta T = (0.200\text{ kg})(4186\text{ J/kg}\cdot\text{C°})(5.00\text{ C°}) = 4.19 \times 10^3\text{ J}$$

The remaining $(2.01 \times 10^4 - 4.19 \times 10^3)\text{J} = 1.59 \times 10^4$ J that is removed from the water can freeze a mass x of water:

$$Q = mL_f$$

$$1.59 \times 10^4\text{ J} = x(3.33 \times 10^5\text{ J/kg})$$

$x = 0.0477\text{ kg} = \boxed{47.7\text{ g}}$ of water can be frozen

***20.54** The energy required to melt 1.00 kg of snow is

$$Q = (1.00\text{ kg})(3.33 \times 10^5\text{ J/kg}) = 3.33 \times 10^5\text{ J}$$

The force of friction is

$$f = \mu n = \mu mg = (0.200)(75.0\text{ kg})(9.80\text{ m/s}^2) = 147\text{ N}$$

Therefore, the work done is

$$W = fs = (147\text{ N})s = 3.33 \times 10^5\text{ J}$$

and $s = \boxed{2.27 \times 10^3\text{ m}}$

20.55 (a) Before conduction has time to become important, the heat energy lost by the rod equals the heat energy gained by the helium. Therefore,

$$(mL_v)_{\text{He}} = (mc\Delta T)_{\text{Al}} \quad \text{or} \quad (\rho VL_v)_{\text{He}} = (\rho Vc\,\Delta T)_{\text{Al}} \quad \text{so} \quad V_{\text{He}} = \frac{(\rho Vc\,\Delta T)_{\text{Al}}}{(\rho L_v)_{\text{He}}}$$

$$V_{\text{He}} = \frac{(2.70\text{ g/cm}^3)(62.5\text{ cm}^3)(0.210\text{ cal/g}\cdot\text{°C})(295.8\text{°C})}{(0.125\text{ g/cm}^3)(2.09 \times 10^4\text{ J/kg})(1.00\text{ cal/4.186 J})(1.00\text{ kg/1000 g})}$$

$$= 1.68 \times 10^4\text{ cm}^3 = \boxed{16.8\text{ liters}}$$

(b) The rate at which energy is supplied to the rod in order to maintain constant temperatures is given by

$$\wp = kA\frac{dT}{dx} = (31.0\text{ J/s}\cdot\text{cm}\cdot\text{K})(2.50\text{ cm}^2)\left[\frac{295.8\text{ K}}{25.0\text{ cm}}\right] = 917\text{ W}$$

This power supplied to the helium will produce a "boil-off" rate of

$$\frac{\wp}{\rho L_v} = \frac{917\ \text{W}}{(0.125\ \text{g}/\text{cm}^3)(2.09\times 10^4\ \text{J}/\text{kg})(10^{-3}\ \text{kg}/\text{g})} = 351\,\frac{\text{cm}^3}{\text{s}} = \boxed{0.351\,\frac{\text{L}}{\text{s}}}$$

Goal Solution

G: Demonstrations with liquid nitrogen give us some indication of the phenomenon described. Since the rod is much hotter than the liquid helium and of significant size (almost 2 cm in diameter), a substantial volume (maybe as much as a liter) of helium will boil off before thermal equilibrium is reached. Likewise, since aluminum conducts rather well, a significant amount of helium will continue to boil off as long as the upper end of the rod is maintained at 300 K.

O: Until thermal equilibrium is reached, the excess heat energy of the rod will be used to vaporize the liquid helium, which is already at its boiling point (so there is no change in the temperature of the helium).

A: As you solve this problem, be careful not to confuse L (the *conduction length* of the rod) with L_v (the *heat of vaporization* of the helium).

(a) Before heat conduction has time to become important, we suppose the heat energy lost by half the rod equals the heat energy gained by the helium. Therefore,

$$(mL_v)_{\text{He}} = (mc\Delta T)_{\text{Al}} \quad \text{or} \quad (\rho V L_v)_{\text{He}} = (\rho V c\Delta T)_{\text{Al}}$$

so that $v_{\text{He}} = \dfrac{(\rho V c\Delta T)_{\text{Al}}}{(\rho L_v)_{\text{He}}} = \dfrac{(2.7\text{ g/cm}^3)(62.5\text{ cm}^3)(0.21\text{ cal/g °C})(295.8\text{°C})}{(0.125\text{ g/cm}^3)(4.99\text{ cal/g})}$

and $v_{\text{He}} = 1.68 \times 10^4\text{ cm}^3 = 16.8\text{ liters}$

(b) Heat energy will be conducted along the rod at a rate of $\dfrac{dQ}{dt} = P = \dfrac{kA\Delta T}{L}$.

During any time interval, this will boil a mass of helium according to

$$Q = mL_v \quad \text{or} \quad \frac{dQ}{dt} = \left(\frac{dm}{dt}\right) L_v$$

Combining these two equations gives us the "boil-off" rate: $\dfrac{dm}{dt} = \dfrac{kA\Delta T}{L \cdot L_v}$

Set the conduction length $L = 25\text{ cm}$, and use $k = 31\text{ J/s cm} \cdot \text{K} = 7.41\text{ cal/s cm} \cdot \text{K}$:

$$\frac{dm}{dt} = \frac{(7.41\text{ cal/s} \cdot \text{cm} \cdot \text{K})(2.5\text{ cm}^2)(295.8\text{ K})}{(25\text{ cm})(4.99\text{ cal/g})} = 43.9\text{ g/s}$$

or $\dfrac{dm}{dt} = \dfrac{43.9\text{ g/s}}{0.125\text{ g/cm}^3} = 351\text{ cm}^3\text{/s} = 0.351\text{ liter/s}$

L: The volume of helium boiled off initially is much more than expected. If our calculations are correct, that sure is a lot of liquid helium that is wasted! Since liquid helium is much more expensive than liquid nitrogen, most low-temperature equipment is designed to avoid unnecessary loss of liquid helium by surrounding the liquid with a container of liquid nitrogen.

***20.56** (a) The heat thus far gained by the copper equals the heat loss by the silver. Your down parka is an excellent insulator.

$$Q_{\text{cold}} = -Q_{\text{hot}} \quad \text{or} \quad m_{\text{Cu}}c_{\text{Cu}}(T_f - T_i)_{\text{Cu}} = -m_{\text{Ag}}c_{\text{Ag}}(T_f - T_i)_{\text{Ag}}$$

$$(9.00\text{ g})(387\text{ J/kg}\cdot\text{C}°)(16.0°\text{C}) = -(14.0\text{ g})(234\text{ J/kg}\cdot\text{C}°)(T_f - 30.0°\text{C})_{\text{Ag}}$$

$$(T_f - 30.0°\text{C})_{\text{Ag}} = -17.0°\text{C} \quad \text{so} \quad T_{f,\text{Ag}} = \boxed{13.0°\text{C}}$$

(b) For heat flow: $m_{\text{Ag}}c_{\text{Ag}}\left(\dfrac{dT}{dt}\right)_{\text{Ag}} = -m_{\text{Cu}}c_{\text{Cu}}\left(\dfrac{dT}{dt}\right)_{\text{Cu}}$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = -\frac{m_{\text{Cu}}c_{\text{Cu}}}{m_{\text{Ag}}c_{\text{Ag}}}\left(\frac{dT}{dt}\right)_{\text{Cu}} = -\frac{(9.00\text{ g})(387\text{ J/kg}\cdot\text{C}°)}{(14.0\text{ g})(234\text{ J/kg}\cdot\text{C}°)}(+0.500\text{ C}°/\text{s})$$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = \boxed{-0.532\text{ C}°/\text{s}} \quad \text{(negative sign} \Rightarrow \text{decreasing temperature)}$$

20.57 $Q = cm\,\Delta T$; $m = \rho V$; $\dfrac{dQ}{dt} = \rho c\,\Delta T\left(\dfrac{dV}{dt}\right)$

$$c = \frac{dQ/dt}{\rho\,\Delta T(dV/dt)} = \frac{(30.0\text{ J/s})}{(0.780\text{ g/cm}^2)(4.80°\text{C})(4.00\text{ cm}^3/\text{s})} = 2.00\text{ J/g}\cdot\text{C}°$$

$$c = \boxed{2.00\text{ kJ/kg}\cdot\text{C}°}$$

20.58 $Q = mc\,\Delta T = (\rho V)c\,\Delta T$

Thus, when a constant temperature difference ΔT is maintained, the rate of adding heat to the liquid is

$$\wp = \frac{dQ}{dt} = \rho\left(\frac{dV}{dt}\right)c\,\Delta T = \rho R c\,\Delta T$$

and the specific heat of the liquid is $c = \boxed{\dfrac{\wp}{\rho R\,\Delta T}}$

20.59 Call the initial pressure P_1. In the constant volume process $1 \to 2$ the work is zero.

$$P_1V_1 = nRT_1$$

$$P_2V_2 = nRT_2$$

so $\dfrac{P_2V_2}{P_1V_1} = \dfrac{T_2}{T_1}$ $\qquad T_2 = 300\text{ K}\left(\dfrac{1}{4}\right)(1) = 75.0\text{ K}$

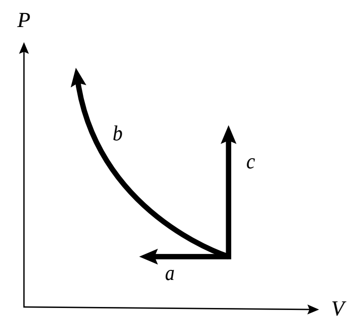
Now in $2 \rightarrow 3$

$$W = \int_2^3 P\,dV = P_2(V_3 - V_2) = P_3V_3 - P_2V_2$$

$$W = nRT_3 - nRT_2 = (1.00 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(300 \text{ K} - 75.0 \text{ K})$$

$$W = \boxed{1.87 \text{ kJ}}$$

***20.60** (a) Work done by the gas is the area under the PV curve



$$W = P_i\left(\frac{V_i}{2} - V_i\right) = \boxed{-\frac{P_i V_i}{2}}$$

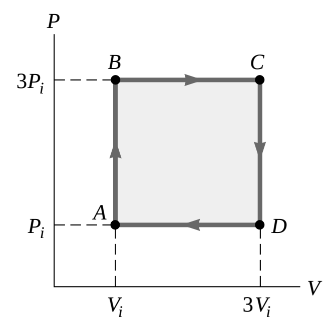
(b) In this case the area under the curve is $W = \int P\,dV$. Since the process is isothermal,

$$PV = P_iV_i = 4P_i\left(\frac{V_i}{4}\right) = nRT_i \quad \text{and}$$

$$W = \int_{V_i}^{V_i/4}\left(\frac{dV}{V}\right)(P_iV_i) = P_iV_i \ln\left(\frac{V_i/4}{V_i}\right) = -P_iV_i \ln 4 = \boxed{-1.39P_iV_i}$$

(c) The area under the curve is 0 and $\boxed{W = 0}$

20.61 (a) The work done during each step of the cycle equals the area under that segment of the PV curve



$$W = W_{DA} + W_{AB} + W_{BC} + W_{CD}$$

$$W = P_i(V_i - 3V_i) + 0 + 3P_i(3V_i - V_i) + 0 = \boxed{4P_iV_i}$$

(b) The initial and final values of T for the system are equal.

Therefore, $\Delta E_{\text{int}} = 0$, and $Q = W = \boxed{4P_i V_i}$

(c) $W = 4P_iV_i = 4nRT_i = 4(1.00)(8.315)(273) = \boxed{9.08 \text{ kJ}}$

20.62 (a) $Fv = (50.0 \text{ N})(40.0 \text{ m/s}) = \boxed{2000 \text{ W}}$

(b) Energy received by each object is $(1000 \text{ W})(10 \text{ s}) = 10^4 \text{ J} = 2389$ cal. The specific heat of iron is 0.107 cal/g · °C, so the heat capacity of each object is $5.00 \times 10^3 \times 0.107 =$ 535.0 cal/°C.

$$\Delta T = \frac{2389 \text{ cal}}{535.0 \text{ cal/°C}} = \boxed{4.47\text{°C}}$$

20.63 The power incident on the solar collector is

$$\wp_i = IA = (600\ \mathrm{W/m^2})\pi(0.300\ \mathrm{m})^2 = 170\ \mathrm{W}$$

For a 40.0% reflector, the collected power is $\wp_c = 67.9\ \mathrm{W}$

The total energy required to increase the temperature of the water to the boiling point and to evaporate it is

$$Q = cm\,\Delta T + mL_V$$

$$= (0.500\ \mathrm{kg})[(4186\ \mathrm{J/kg\cdot C^\circ})(80.0\ \mathrm{C^\circ}) + 2.26\times 10^6\ \mathrm{J/kg}]$$

$$Q = 1.30\times 10^6\ \mathrm{J}$$

The time required is $t = \dfrac{Q}{\wp_c} = \dfrac{1.30\times 10^6\ \mathrm{J}}{67.9\ \mathrm{W}} = \boxed{5.31\ \mathrm{h}}$

20.64 From $Q = mL_V$ the rate of boiling is described by

$$\wp = \frac{Q}{t} = \frac{L_V m}{t} \qquad \therefore \frac{m}{t} = \frac{\wp}{L_V}$$

Model the water vapor as an ideal gas

$$P_0 V = nRT = \left(\frac{m}{M}\right)RT$$

$$\frac{P_0 V}{t} = \frac{m}{t}\left(\frac{RT}{M}\right)$$

$$P_0 A v = \frac{\wp}{L_V}\left(\frac{RT}{M}\right)$$

$$v = \frac{\wp RT}{ML_V P_0 A}$$

$$= \frac{(1000\ \mathrm{W})(8.315\ \mathrm{J/mol\cdot K})(373\ \mathrm{K})}{(0.0180\ \mathrm{kg/mol})(2.26\times 10^6\ \mathrm{J/kg})(1.013\times 10^5\ \mathrm{N/m^2})(2.00\times 10^{-4}\ \mathrm{m^2})}$$

$$v = \boxed{3.76\ \mathrm{m/s}}$$

***20.65** To vaporize water requires an addition of 2.26×10^6 J/kg of energy, while water gives up 3.33×10^5 J/kg as it freezes. The heat to vaporize part of the water must come from the heat of fusion as some water freezes. Thus, if x kilograms of water freezes while a mass of (1.00 kg – x) is vaporized,

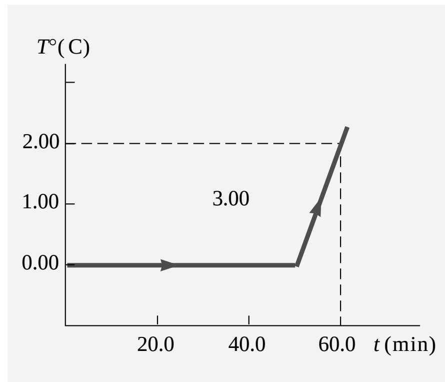
$$(3.33 \times 10^5 \text{ J/kg})x = (2.26 \times 10^6 \text{ J/kg})(1.00 \text{ kg} - x)$$

or $\quad x = 6.79 \text{ kg} - 6.79x$

This yields, $7.79x = 6.79$ kg, and

$$x = 0.872 \text{ kg} = \boxed{872 \text{ g}} \text{ of water freezes.}$$

20.66 Energy goes in at a constant rate $\wp$. For the period from



50.0 min to 60.0 min, $Q = mc\,\Delta T$

$\wp(10.0 \text{ min}) = (10 \text{ kg} + m_i)(4186 \text{ J/kg} \cdot \text{C°})(2.00\text{ °C} - 0\text{°C})$

$\wp(10.0 \text{ min}) = 83.7 \text{ kJ} + (8.37 \text{ kJ/kg})m_i \qquad (1)$

For the period from 0 to 50.0 min, $Q = m_i L_f$

$\wp(50.0 \text{ min}) = m_i(3.33 \times 10^5 \text{ J/kg})$

Substitute $\wp = m_i(3.33 \times 10^5 \text{ J/kg})/50.0$ min into Equation (1) to find

$$m_i(3.33 \times 10^5 \text{ J/kg})/5.00 = 83.7 \text{ kJ} + (8.37 \text{ kJ/kg})m_i$$

$$m_i = \frac{83.7 \text{ kJ}}{(66.6 - 8.37) \text{ kJ/kg}} = \boxed{1.44 \text{ kg}}$$

***20.67** (a) The block starts with $K_i = \frac{1}{2} m v_i^2 = \frac{1}{2}(1.60 \text{ kg})(2.50 \text{ m/s})^2 = 5.00$ J. All this becomes extra internal energy in the ice, melting some according to "Q" $= m_{\text{ice}} L_f$. Thus, the mass of ice that melts is

$$m_{\text{ice}} = \frac{\text{"}Q\text{"}}{L_f} = \frac{K_i}{L_f} = \frac{5.00 \text{ J}}{3.33 \times 10^5 \text{ J/kg}} = 1.50 \times 10^{-5} \text{ kg} = \boxed{15.0 \text{ mg}}$$

For the block: $Q = 0$ (no heat can flow since there is no temperature difference), $W = +5.00$ J, $\Delta E_{\text{int}} = 0$ (no temperature change), and $\Delta K = -5.00$ J. For the ice, $Q = 0$, $W = -5.00$ J, $\Delta E_{\text{int}} = +5.00$ J, and $\Delta K = 0$.

(b) Again, $K_i = 5.00$ J and $m_{\text{ice}} = \boxed{15.0 \text{ mg}}$.
For the block of ice: $Q = 0$, $\Delta E_{\text{int}} = +5.00$ J, $\Delta K = -5.00$ J, so $W = 0$.
For the copper, nothing happens: $Q = \Delta E_{\text{int}} = \Delta K = W = 0$.

(c) Again, $K_i = 5.00$ J. Both blocks must rise equally in temperature:

$$"Q" = mc\,\Delta T = \frac{"Q"}{mc} = \frac{5.00\text{ J}}{2(1.60\text{ kg})(387\text{ J/kg}\cdot\text{C°})} = \boxed{4.04\times10^{-3}\text{ C°}}$$

At any instant, the two blocks are at the same temperature, so for both $Q = 0$. For the moving block: $\Delta K = -5.00$ J and $\Delta E_{\text{int}} = +2.50$ J, so $W = +2.50$ J. For the stationary block: $\Delta K = 0$, $\Delta E_{\text{int}} = +2.50$ J, so $W = -2.50$ J.

20.68 $A = A_{\text{end walls}} + A_{\text{ends of attic}} + A_{\text{side walls}} + A_{\text{roof}}$

$$A = 2(8.00\text{ m}\times5.00\text{ m}) + 2\left[2\times\frac{1}{2}\times(4.00\text{ m})\times(4.00\text{ m}\tan 37.0°)\right]$$

$$+\,2(10.0\text{ m}\times5.00\text{ m}) + 2\left(10.0\text{ m}\times\frac{4.00\text{ m}}{\cos 37.0°}\right)$$

$A = 304\text{ m}^2$

$$\wp = \frac{kA\,\Delta T}{L} = \frac{(4.80\times10^{-4}\text{ kW/m}\cdot\text{°C})(304\text{ m}^2)(25.0\text{°C})}{0.210\text{ m}} = 17.4\text{ kW} = 4.15\,\frac{\text{kcal}}{\text{s}}$$

Thus, the heat lost per day = $(4.15\text{ kcal/s})(86\,400\text{ s}) = 3.59\times10^5$ kcal/day.

The gas needed to replace this loss $= \dfrac{3.59\times10^5\text{ kcal/day}}{9300\text{ kcal/m}^3} = \boxed{38.6\text{ m}^3\text{/day}}$

20.69 $\dfrac{L\rho A\,dx}{dt} = kA\dfrac{\Delta T}{x}$ $\qquad L\rho\displaystyle\int_{4.00}^{8.00} x\,dx = k\,\Delta T\int_0^t dt$

$$L\rho\left.\frac{x^2}{2}\right|_{4.00}^{8.00} = k\,\Delta T\,t$$

$$(3.33\times10^5\text{ J/kg})(917\text{ kg/m}^3)\left(\frac{(0.0800\text{ m})^2-(0.0400\text{ m})^2}{2}\right) = \left(2.00\,\frac{\text{W}}{\text{m}\cdot\text{°C}}\right)(10.0\text{ C°})\,t$$

$t = 3.66\times10^4\text{ s} = \boxed{10.2\text{ h}}$

20.70 For a cylindrical shell of radius r, height L, and

thickness dr, Equation 20.14, $\dfrac{dQ}{dt} = -kA\dfrac{dT}{dx}$,

becomes $\dfrac{dQ}{dt} = -k(2\pi rL)\dfrac{dT}{dr}$

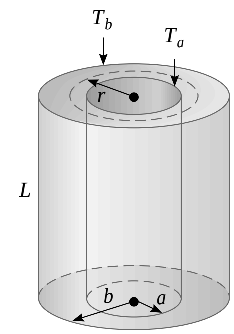
Under equilibrium conditions, $\dfrac{dQ}{dt}$

is constant; therefore, $dT = -\dfrac{dQ}{dt}\left(\dfrac{1}{2\pi kL}\right)\dfrac{dr}{r}$

and $\quad T_b - T_a = -\dfrac{dQ}{dt}\left(\dfrac{1}{2\pi kL}\right)\ln\left(\dfrac{b}{a}\right) \quad$ but $T_a > T_b$.

Therefore, $\boxed{\dfrac{dQ}{dt} = \dfrac{2\pi kL(T_a - T_b)}{\ln(b/a)}}$

20.71 From problem 70, the rate of heat flow through the wall is



$$\frac{dQ}{dt} = \frac{2\pi kL(T_a - T_b)}{\ln(b/a)}$$

$$\frac{dQ}{dt} = \frac{2\pi(4.00\times10^{-5}\ \text{cal/s}\cdot\text{cm}\cdot\text{C}^\circ)(3500\ \text{cm})(60.0\ \text{C}^\circ)}{\ln(256\ \text{cm}/250\ \text{cm})}$$

$$\frac{dQ}{dt} = 2.23\times10^{3}\ \text{cal/s} = \boxed{9.32\ \text{kW}}$$

This is the rate of heat loss from the plane, and consequently the rate at which energy must be supplied in order to maintain a constant temperature.

20.72 $Q_{\text{cold}} = -Q_{\text{hot}} \quad$ or $\quad Q_{\text{Al}} = -(Q_{\text{water}} + Q_{\text{calo}})$

$m_{\text{Al}}c_{\text{Al}}(T_f - T_i)_{\text{Al}} = -(m_w c_w + m_c c_c)(T_f - T_i)_w$

$(0.200\ \text{kg})c_{\text{Al}}(+39.3\ \text{C}^\circ)$

$$= -\left[(0.400\ \text{kg})\left(4186\ \frac{\text{J}}{\text{kg}\cdot\text{C}^\circ}\right) + (0.0400\ \text{kg})\left(630\ \frac{\text{J}}{\text{kg}\cdot\text{C}^\circ}\right)\right](-3.70\ \text{C}^\circ)$$

$c_{\text{Al}} = \dfrac{6.29\times10^{3}\ \text{J}}{7.86\ \text{kg}\cdot\text{C}^\circ} = \boxed{800\ \text{J/kg}\cdot\text{C}^\circ}$

Chapter 21 Solutions

***21.1** One mole of helium contains Avogadro's number of molecules and has a mass of 4.00 g. Let us call m the mass of one atom, and we have

$$N_A m = 4.00 \text{ g/mol}$$

or $$m = \frac{4.00 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.64 \times 10^{-24} \text{ g/molecule}$$

$$m = \boxed{6.64 \times 10^{-27} \text{ kg}}$$

***21.2** We first find the pressure exerted by the gas on the wall of the container.

$$P = \frac{NkT}{V} = \frac{3N_A k_B T}{V} = \frac{3RT}{V} = \frac{3(8.315 \text{ N}\cdot\text{m/mol}\cdot\text{K})(293 \text{ K})}{8.00 \times 10^{-3} \text{ m}^3} = 9.13 \times 10^5 \text{ Pa}$$

Thus, the force on one of the walls of the cubical container is

$$F = PA = (9.13 \times 10^5 \text{ Pa})(4.00 \times 10^{-2} \text{ m}^2) = \boxed{3.65 \times 10^4 \text{ N}}$$

21.3 $$\overline{F} = Nm\frac{\Delta v}{\Delta t} = 500(5.00 \times 10^{-3} \text{ kg})\frac{[8.00 \sin 45.0° - (-8.00 \sin 45.0°)]\text{m/s}}{30.0 \text{ s}} = \boxed{0.943 \text{ N}}$$

$$P = \frac{\overline{F}}{A} = 1.57 \text{ N/m}^2 = \boxed{1.57 \text{ Pa}}$$

21.4 Consider the x axis to be perpendicular to the plane of the window. Then, the average force exerted on the window by the hail stones is

$$\overline{F} = Nm\frac{\Delta v}{\Delta t} = Nm\frac{[v_{xf} - v_{xi}]}{t} = Nm\frac{[v \sin\theta - (-v \sin\theta)]}{t} = \boxed{Nm\frac{2v \sin\theta}{t}}$$

Thus, the pressure on the window pane is

$$P = \frac{\overline{F}}{A} = \boxed{Nm\frac{2v \sin\theta}{At}}$$

***21.5** $$\overline{F} = \frac{(5.00 \times 10^{23})(2 \times 4.68 \times 10^{-26} \text{ kg} \times 300 \text{ m/s})}{1.00 \text{ s}} = 14.0 \text{ N}$$

and $$P = \frac{\overline{F}}{A} = \frac{14.0 \text{ N}}{8.00 \times 10^{-4} \text{ m}^2} = \boxed{17.6 \text{ kPa}}$$

21.6 Use Equation 21.2, $P = \frac{2N}{3V}\left(\frac{mv^2}{2}\right)$, so that

$$K_{av} = \frac{mv^2}{2} = \frac{3PV}{2N} \quad \text{where } N = nN_A = 2N_A$$

$$K_{av} = \frac{3PV}{2(2N_A)} = \frac{3(8.00\text{ atm})(1.013\times10^5\text{ Pa/atm})(5.00\times10^{-3}\text{ m}^3)}{2(2\text{ mol})(6.02\times10^{23}\text{ molecules/mol})}$$

$$K_{av} = \boxed{5.05\times10^{-21}\text{ J/molecule}}$$

21.7 $P = \frac{2}{3}\frac{N}{V}(\overline{KE})$ Equation 21.2

$$N = \frac{3}{2}\frac{PV}{(\overline{KE})} = \frac{3}{2}\frac{(1.20\times10^5)(4.00\times10^{-3})}{(3.60\times10^{-22})} = 2.00\times10^{24}\text{ molecules}$$

$$n = \frac{N}{N_A} = \frac{2.00\times10^{24}\text{ molecules}}{6.02\times10^{23}\text{ molecules/mol}} = \boxed{3.32\text{ mol}}$$

Goal Solution

G: The balloon has a volume of 4.00 L and a diameter of about 20 cm, which seems like a reasonable size for a typical helium balloon. The pressure of the balloon is only slightly more than 1 atm, and if the temperature is anywhere close to room temperature, we can use the estimate of 22 L/mol for an ideal gas at STP conditions. If this is valid, the balloon should contain about 0.2 moles of helium.

O: The average kinetic energy can be used to find the temperature of the gas, which can be used with $PV=nRT$ to find the number of moles.

A: The gas temperature must be that implied by $\frac{1}{2}mv^2 = \frac{3}{2}k_BT$ for a monatomic gas like He.

$$T = \frac{2}{3}\left(\frac{\frac{1}{2}mv^2}{k_a}\right) = \frac{2}{3}\left(\frac{3.6\times10^{-22}\text{ J}}{1.38\times10^{-23}\text{ J/K}}\right) = 17.4\text{ K}$$

Now $PV = nRT$ gives

$$n = \frac{PV}{RT} = \frac{(1.20\times10^5\text{ N/m}^2)(4.00\times10^{-3}\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(17.4\text{ K})}$$

$$n = 3.32\text{ mol}$$

L: This result is more than ten times the number of moles we predicted, primarily because the temperature of the helium is *much colder* than room temperature! In fact, T is only slightly above the temperature at which the helium would liquify (4.2 K at 1 atm). We should hope this balloon is not being held by a child; not only would the balloon sink in the air, it is cold enough to cause frostbite!

21.8 $v = \sqrt{\dfrac{3k_BT}{m}}$

$$\frac{v_{\text{O}}}{v_{\text{He}}} = \sqrt{\frac{M_{\text{He}}}{M_O}} = \sqrt{\frac{4.00}{32.0}} = \sqrt{\frac{1}{8.00}}$$

$$v_O = \frac{1350 \text{ m/s}}{\sqrt{8.00}} = \boxed{477 \text{ m/s}}$$

21.9 (a) $PV = Nk_BT$

$$N = \frac{PV}{k_BT} = \frac{(1.013 \times 10^5 \text{ Pa})\frac{4}{3}\pi(0.150 \text{ m})^3}{(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})} = \boxed{3.53 \times 10^{23} \text{ atoms}}$$

(b) $\overline{K} = \frac{3}{2}k_BT = \frac{3}{2}(1.38 \times 10^{-23})(293) \text{ J} = \boxed{6.07 \times 10^{-21} \text{ J}}$

(c) $\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT \quad \therefore v_{\text{rms}} = \sqrt{\dfrac{3k_BT}{m}} = \boxed{1.35 \text{ km/s}}$

21.10 (a) $PV = nRT = \dfrac{Nmv^2}{3} \qquad K = \dfrac{Nmv^2}{2} = E_{\text{trans}}$

$$E_{\text{trans}} = \frac{3PV}{2} = \frac{3}{2}(3.00 \times 1.013 \times 10^5)(5.00 \times 10^{-3}) = \boxed{2.28 \text{ kJ}}$$

(b) $\dfrac{mv^2}{2} = \dfrac{3k_BT}{2} = \dfrac{3RT}{2N_A} = \dfrac{3}{2}\dfrac{(8.315)(300)}{(6.02 \times 10^{23})} = \boxed{6.22 \times 10^{-21} \text{ J}}$

21.11 (a) $\overline{K} = \frac{3}{2}k_{\text{B}}T = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(423 \text{ K}) = \boxed{8.76 \times 10^{-21} \text{ J}}$

(b) $\overline{K} = \frac{1}{2}mv_{\text{rms}}^2 = 8.76 \times 10^{-21} \text{ J}$

so $v_{\text{rms}} = \sqrt{\dfrac{1.75 \times 10^{-20} \text{ J}}{m}} \qquad (1)$

For helium,

$$m = \frac{4.00 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.64 \times 10^{-24} \text{ g/molecule}$$

$$m = 6.64 \times 10^{-27} \text{ kg/molecule}$$

Similarly for argon,

$$m = \frac{39.9 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.63 \times 10^{-23} \text{ g/molecule}$$

$$m = 6.63 \times 10^{-26} \text{ kg/molecule}$$

Substituting in (1) above, we find

for helium, $\boxed{v_{\text{rms}} = 1.62 \text{ km/s}}$; and for argon, $\boxed{v_{\text{rms}} = 514 \text{ m/s}}$

***21.12** (a) $1 \text{ Pa} = (1 \text{ Pa})\left(\frac{1 \text{ N/m}^2}{1 \text{ Pa}}\right)\left(\frac{1 \text{ J}}{1 \text{ N} \cdot \text{m}}\right) = \boxed{1 \frac{\text{J}}{\text{m}^3}}$

(b) For a monatomic ideal gas, $E_{\text{int}} = \frac{3}{2} nRT$

For any ideal gas, the energy of molecular translation is the same,

$$E_{\text{trans}} = \frac{3}{2} nRT = \frac{3}{2} PV$$

Thus, the energy per volume is $\frac{E_{\text{trans}}}{V} = \boxed{\frac{3}{2} P}$

21.13 $E_{\text{int}} = \frac{3}{2} nRT$

$$\Delta E_{\text{int}} = \frac{3}{2} nR\,\Delta T = \frac{3}{2}(3.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(2.00 \text{ K}) = \boxed{75.0 \text{ J}}$$

21.14 The piston moves to keep pressure constant. Since $V = \frac{nRT}{P}$, then

$\Delta V = \frac{nR\,\Delta T}{P}$ for a constant pressure process.

$$Q = nC_P\,\Delta T = n(C_V + R)\Delta T \quad \text{so} \quad \Delta T = \frac{Q}{n(C_V + R)} = \frac{Q}{n(5R/2 + R)} = \frac{2Q}{7nR}$$

and $\Delta V = \frac{nR}{P}\left(\frac{2Q}{7nR}\right) = \frac{2Q}{7P} = \frac{2}{7}\frac{QV}{nRT}$

$$\Delta V = \frac{2}{7}\frac{(4.40 \times 10^3 \text{ J})(5.00 \text{ L})}{(1.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 2.52 \text{ L}$$

Thus, $V_f = V_i + \Delta V = 5.00 \text{ L} + 2.52 \text{ L} = \boxed{7.52 \text{ L}}$

21.15 Use C_P and C_V from Table 21.2.

(a) $Q = nC_P\,\Delta T = (1.00\text{ mol})(28.8\text{ J/mol}\cdot\text{K})(420 - 300)\text{ K} = \boxed{3.46\text{ kJ}}$

(b) $\Delta E_{\text{int}} = nC_V\,\Delta T = (1.00\text{ mol})(20.4\text{ J/mol}\cdot\text{K})(120\text{ K}) = \boxed{2.45\text{ kJ}}$

(c) $W = Q - \Delta E_{\text{int}} = 3.46\text{ kJ} - 2.45\text{ kJ} = \boxed{1.01\text{ kJ}}$

21.16 $n = 1.00\text{ mol}$, $T_i = 300\text{ K}$

(b) Since V = constant, $W = \boxed{0}$

(a) $\Delta E_{\text{int}} = Q - \text{W} = 209\text{ J} - 0 = \boxed{209\text{ J}}$

(c) $\Delta E_{\text{int}} = nC_V\,\Delta T = n\left(\frac{3}{2}R\right)\Delta T$

so $$\Delta T = \frac{2(\Delta E_{\text{int}})}{3nR} = \frac{2(209\text{ J})}{3(1.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = 16.8\text{ K}$$

$$T = T_i + \Delta T = 300\text{ K} + 16.8\text{ K} = \boxed{317\text{ K}}$$

21.17 (a) Consider heating it at constant pressure. Oxygen and nitrogen are diatomic, so $C_P = 7R/2$

$$Q = nC_P\,\Delta T = \frac{7}{2}\,nR\,\Delta T = \frac{7}{2}\left(\frac{PV}{T}\right)\Delta T$$

$$Q = \frac{7}{2}\frac{(1.013\times 10^5\text{ N/m}^2)(100\text{ m}^3)}{300\text{ K}}(1.00\text{ K}) = \boxed{118\text{ kJ}}$$

(b) $U_g = mgy$

$$m = \frac{U_g}{gy} = \frac{1.18\times 10^5\text{ J}}{(9.80\text{ m/s}^2)2.00\text{ m}} = \boxed{6.03\times 10^3\text{ kg}}$$

***21.18** (a) $C_V = \frac{5}{2}\,R = \frac{5}{2}\left(8.315\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\left(\frac{1.00\text{ mol}}{0.0289\text{ kg}}\right) = 719\,\frac{\text{J}}{\text{kg}\cdot\text{K}} = \boxed{0.719\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}}$

(b) $m = nM = M\left(\frac{PV}{RT}\right)$

$$m = \left(0.0289\,\frac{\text{kg}}{\text{mol}}\right)\frac{(200\times 10^3\text{ Pa})(0.350\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(300\text{ K})} = \boxed{0.811\text{ kg}}$$

(c) We consider a constant volume process where no work is done.

$$Q = mC_v\,\Delta T = (0.811\text{ kg})\left(0.719\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(700\text{ K} - 300\text{ K}) = \boxed{233\text{ kJ}}$$

(d) We now consider a constant pressure process where the internal energy of the gas is increased and work is done.

$$Q = mC_P\,\Delta T = m(C_V + R)\Delta T = m(7R/2)\Delta T = m(7C_V/5)\Delta T$$

$$Q = (0.811\text{ kg})\left[\frac{7}{5}\left(0.719\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\right](400\text{ K}) = \boxed{327\text{ kJ}}$$

***21.19** Consider 800 cm^3 of (flavored) water at 90.0˚C mixing with 200 cm^3 of diatomic ideal gas at 20.0˚C:

$$Q_{\text{cold}} = -Q_{\text{hot}} \quad \text{or} \quad m_{\text{air}}C_{P,\text{air}}(T_f - T_{i,\text{air}}) = -m_w c_w(\Delta T)_w$$

$$(\Delta T)_w = -\frac{m_{\text{air}}C_{P,\text{air}}(T_f - T_{i,\text{air}})}{m_w c_w} = \frac{[\rho V]_{\text{air}}\,C_{P,\text{air}}(90.0°\text{C} - 20.0°\text{C})}{(\rho_w V_w)c_w}$$

where we have anticipated that the final temperature of the mixture will be close to 90.0˚C.

$$C_{P,\text{air}} = \frac{7}{2}\,R = \frac{7}{2}\left(8.315\,\frac{\text{J}}{\text{mol}\cdot\text{C°}}\right)\left(\frac{1.00\text{ mol}}{28.9\text{ g}}\right) = 1.01\text{ J/g}\cdot\text{C°}$$

$$(\Delta T)_w = -\frac{[(1.20\times10^{-3}\text{ g/cm}^3)(200\text{ cm}^3)](1.01\text{ J/g}\cdot\text{C°})(70.0\text{ C°})}{[(1.00\text{ g/cm}^3)(800\text{ cm}^3)](4.186\text{ J/g}\cdot\text{C°})}$$

or $\quad (\Delta T)_w \approx 5.05\times10^{-3}\text{ C°}$

The change of temperature for the water is $\boxed{\text{between }10^{-3}\text{ °C and }10^{-2}\text{ °C}}$

21.20 $Q = (nC_P\,\Delta T)_{\text{isobaric}} + (nC_V\,\Delta T)_{\text{isovolmetric}}$

In the isobaric process, V doubles so T must double, to $2T_i$.
In the isovolumetric process, P triples so T changes from $2T_i$ to $6T_i$.

$$Q = n\left(\frac{7}{2}R\right)(2T_i - T_i) + n\left(\frac{5}{2}R\right)(6T_i - 2T_i)$$

$$Q = 13.5nRT_i = \boxed{13.5PV}$$

21.21 In the isovolumetric process $A \to B$, $W = 0$ and $Q = nC_V\,\Delta T = 500\text{ J}$

$$500\text{ J} = n(3R/2)(T_B - T_A) \quad \text{or} \quad T_B = T_A + \frac{2(500\text{ J})}{3nR}$$

$$T_B = 300\text{ K} + \frac{2(500\text{ J})}{3(1.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = 340\text{ K}$$

In the isobaric process $B \to C$,

$$Q = nC_P\,\Delta T = \frac{5nR}{2}(T_C - T_B) = -500\text{ J}$$

Thus,

(a) $$T_C = T_B - \frac{2(500\text{ J})}{5nR} = 340\text{ K} - \frac{1000\text{ J}}{5(1.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = \boxed{316\text{ K}}$$

(b) The work done by the gas during the isobaric process is

$$W_{BC} = P_B\,\Delta V = nR(T_C - T_B) = (1.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(316\text{ K} - 340\text{ J})$$

or $W_{BC} = -200\text{ J}$

The total work done <u>on</u> the gas is then

$$W_{\text{on gas}} = -W_{\text{by gas}} = -(W_{AB} + W_{BC}) = -(0 - 200\text{ J})$$

or $W_{\text{on gas}} = \boxed{+200\text{ J}}$

21.22 (a) The heat required to produce a temperature change is

$$Q = n_1C_1\,\Delta T + n_2C_2\,\Delta T$$

The number of molecules is $N_1 + N_2$, so the number of "moles of the mixture" is $n_1 + n_2$ and $Q = (n_1 + n_2)C\,\Delta T$,

so $$C = \boxed{\frac{n_1C_1 + n_2C_2}{n_1 + n_2}}$$

(b) $$Q = \sum_{i=1}^{m} n_iC_i\,\Delta T = \left(\sum_{i=1}^{m} n_i\right)C\,\Delta T$$

$$C = \boxed{\sum_{i=1}^{m} n_iC_i \Big/ \sum_{i=1}^{m} n_i}$$

21.23 The rms speed of the gas molecules is $v = \sqrt{\dfrac{3RT}{M}}$. Thus, to double the rms speed, the temperature must increase by a factor of 4: $T_f = 4T_i$.

Since the pressure is proportional to volume in this process,

$$\frac{P}{V} = \frac{P_i}{V_i} = \text{constant} \quad \text{or} \quad P = \left(\frac{P_i}{V_i}\right) V$$

Then, $PV = nRT$ becomes

$$\left(\frac{P_i}{V_i}\right) V^2 = nRT \quad \text{or} \quad V^2 = \left(\frac{V_i}{P_i}\right) nRT$$

Therefore, $\dfrac{V_f^2}{V_i^2} = \dfrac{T_f}{T_i} = 4$ or $V_f = 2V_i$.

The work done by the gas is then

$$W = \int_{V_i}^{V_f} P\,dV = \frac{P_i}{V_i}\int_{V_i}^{2V_i} V\,dV = \frac{3}{2}\,P_iV_i$$

The change in the internal energy of the gas is

$$\Delta E_{\text{int}} = nC_V\,\Delta T = n\left(\frac{5}{2}R\right)(4T_i - T_i) = \frac{15}{2}\,nRT_i = \frac{15}{2}\,P_iV_i$$

and the energy transferred to the gas as heat is

$$Q = \Delta E_{\text{int}} + W = \frac{15}{2}\,P_iV_i + \frac{3}{2}\,P_iV_i = \boxed{9P_iV_i}$$

***21.24** (a) $P_iV_i^\gamma = P_fV_f^\gamma$ so $\dfrac{V_f}{V_i} = \left(\dfrac{P_i}{P_f}\right)^{1/\gamma} = \left(\dfrac{1.00}{20.0}\right)^{5/7} = \boxed{0.118}$

(b) $\dfrac{T_f}{T_i} = \dfrac{P_fV_f}{P_iV_i} = \left(\dfrac{P_f}{P_i}\right)\left(\dfrac{V_f}{V_i}\right) = (20.0)(0.118) = \boxed{2.35}$

(c) Since the process is adiabatic, $\boxed{Q = 0}$

Since $\gamma = 1.40 = \dfrac{C_P}{C_V} = \dfrac{R + C_V}{C_V}$, $C_V = \dfrac{5}{2}\,R$, and $\Delta T = 2.35T_i - T_i = 1.35T_i$

$$\Delta E_{\text{int}} = nC_V\,\Delta T = (0.0160\text{ mol})\left(\frac{5}{2}\right)\left(8.315\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)[1.35(300\text{ K})] = \boxed{135\text{ J}}$$

and $W = Q - \Delta E_{\text{int}} = 0 - 135\text{ J} = \boxed{-135\text{ J}}$

***21.25** (a) $P_i V_i^{\gamma} = P_f V_f^{\gamma}$

$$P_f = \left(\frac{V_i}{V_f}\right)^{\gamma} P_i = (5.00 \text{ atm})\left(\frac{12.0}{30.0}\right)^{1.40} = \boxed{1.39 \text{ atm}}$$

(b) $$T_i = \frac{P_i V_i}{nR} = \frac{(5.00)(1.013 \times 10^5 \text{ Pa})(12.0 \times 10^{-3} \text{ m}^3)}{(2.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = \boxed{366 \text{ K}}$$

$$T_f = \frac{P_f V_f}{nR} = \frac{(1.39)(1.013 \times 10^5 \text{ Pa})(30.0 \times 10^{-3} \text{ m}^3)}{(2.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = \boxed{253 \text{ K}}$$

(c) The process is adiabatic: $\boxed{Q = 0}$

$$\gamma = 1.40 = \frac{C_P}{C_V} = \frac{R + C_V}{C_V} \qquad C_V = \frac{5}{2} R$$

$$\Delta E_{\text{int}} = nC_V \, \Delta T = (2.00 \text{ mol})\left(\frac{5}{2}(8.315 \text{ J/mol} \cdot \text{K})\right)(366 - 253) \text{ K} = \boxed{4.66 \text{ kJ}}$$

$$W = Q - \Delta E_{\text{int}} = 0 - 4.66 \text{ kJ} = \boxed{-4.66 \text{ kJ}}$$

21.26 $V_i = \pi(2.50 \times 10^{-2} \text{ m}/2)^2 \, 0.500 \text{ m} = 2.45 \times 10^{-4} \text{ m}^3$

The quantity of air we find from $P_i V_i = nRT_i$

$$n = \frac{P_i V_i}{RT_i} = \frac{(1.013 \times 10^5 \text{ Pa})(2.45 \times 10^{-4} \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(300 \text{ K})}$$

$$n = 9.97 \times 10^{-3} \text{ mol}$$

Adiabatic compression: $P_f = 101.3 \text{ kPa} + 800 \text{ kPa} = 901.3 \text{ kPa}$

(a) $P_i V_f^{\gamma} = P_f V_f^{\gamma}$

$$V_f = V_i(P_i/P_f)^{1/\gamma} = 2.45 \times 10^{-4} \text{ m}^3(101.3/901.3)^{5/7}$$

$$V_f = \boxed{5.15 \times 10^{-5} \text{ m}^3}$$

(b) $P_f V_f = nRT_f$

$$T_f = T_i \frac{P_f V_f}{P_i V_i} = T_i \frac{P_f}{P_i}\left(\frac{P_i}{P_f}\right)^{1/\gamma} = T_i(P_i/P_f)^{(1/\gamma - 1)}$$

$$T_f = 300 \text{ K}(101.3/901.3)^{(5/7 - 1)} = \boxed{560 \text{ K}}$$

(c) The work put into the gas in compressing it is $\Delta E_{\text{int}} = nC_V\,\Delta T$

$$W = (9.97\times10^{-3}\text{ mol})\frac{5}{2}(8.315\text{ J/mol}\cdot\text{K})(560-300)\text{ K}$$

$$W = 53.9\text{ J}$$

Now imagine this energy being shared with the inner wall as the gas is held at constant volume. The pump wall has outer diameter 25.0 mm + 2.00 mm + 2.00 mm = 29.0 mm, and volume

$$[\pi(14.5\times10^{-3}\text{ m})^2-\pi(12.5\times10^{-3}\text{ m})^2]4.00\times10^{-2}\text{ m} = 6.79\times10^{-6}\text{ m}^3$$

and mass $\rho V = (7.86\times10^3\text{ kg/m}^3)(6.79\times10^{-6}\text{ m}^3) = 53.3\text{ g}$

The overall warming process is described by

$$53.9\text{ J} = nC_V\,\Delta T + mc\,\Delta T$$

$$53.9\text{ J} = (9.97\times10^{-3}\text{ mol})\frac{5}{2}(8.315\text{ J/mol}\cdot\text{K})(T_{ff}-300\text{ K})$$

$$+ (53.3\times10^{-3}\text{ kg})(448\text{ J/kg}\cdot\text{K})(T_{ff}-300\text{ K})$$

$$53.9\text{ J} = (0.207\text{ J/K} + 23.9\text{ J/K})(T_{ff}-300\text{ K})$$

$$T_{ff}-300\text{ K} = \boxed{2.24\text{ K}}$$

21.27 $$\frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} = \left(\frac{1}{2}\right)^{0.400}$$

If $T_i = 300$ K, then $T_f = \boxed{227\text{ K}}$

Goal Solution

G: The air should cool as it expands, so we should expect $T_f < 300$ K.

O: The air expands adiabatically, losing no heat but dropping in temperature as it does work on the air around it, so we assume that PV^γ = constant (where $\gamma = 1.40$ for an ideal gas).

A: Combine $P_1V_1^\gamma = P_2V_2^\gamma$ and $P_1 = \dfrac{nRT_1}{V_1}$ with $P_2 = \dfrac{nRT_2}{V_2}$

to find $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\text{ K}\left(\frac{1}{2}\right)^{(1.40-1)} = 227\text{ K}$$

L: The air does cool, but the rate is not linear with the change in volume (the temperature drops only 24% while the volume doubles)

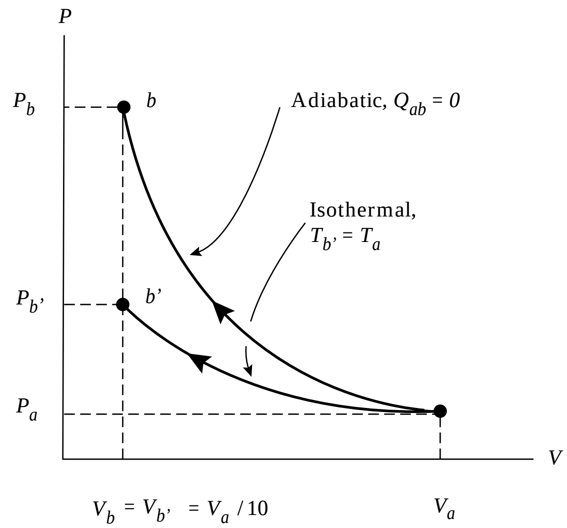
21.28 (a) The work done *on* the gas is

$$-W_{ab} = -\int_{V_a}^{V_b} P\,dV$$

For the isothermal process,

$$-W_{ab'} = -nRT_a \int_{V_a}^{V_{b'}} \left(\frac{1}{V}\right) dV$$

$$= -nRT_a \ln\left(\frac{V_{b'}}{V_a}\right) = nRT_a \ln\left(\frac{V_a}{V_{b'}}\right)$$



Thus, $-W_{ab'} = (5.00\text{ mol})\left(8.315\,\dfrac{\text{J}}{\text{mol}\cdot\text{K}}\right)(293\text{ K})\ \ln(10.0)$

$$= \boxed{28.0\text{ kJ}}$$

(b) For the adiabatic process, we must first find the final temperature, T_b. Since air consists primarily of diatomic molecules, we shall use

$$\gamma_{\text{air}} = 1.40 \quad\text{and}\quad C_{V,\text{air}} = 5R/2 = 5(8.315)/2 = 20.8\text{ J/mol}\cdot\text{K}$$

Then, from Equation 21.20,

$$T_b = T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = (293\text{ K})(10.0)^{0.400} = 736\text{ K}$$

Thus, the work done on the gas during the adiabatic process is

$$-W_{ab} = -(Q - \Delta E_{\text{int}})_{ab} = -(0 - nC_V\,\Delta T)_{ab} = nC_V(T_b - T_a)$$

or $\quad -W_{ab} = (5.00\text{ mol})(20.8\text{ J/mol}\cdot\text{K})(736 - 293)\text{ K} = \boxed{46.1\text{ kJ}}$

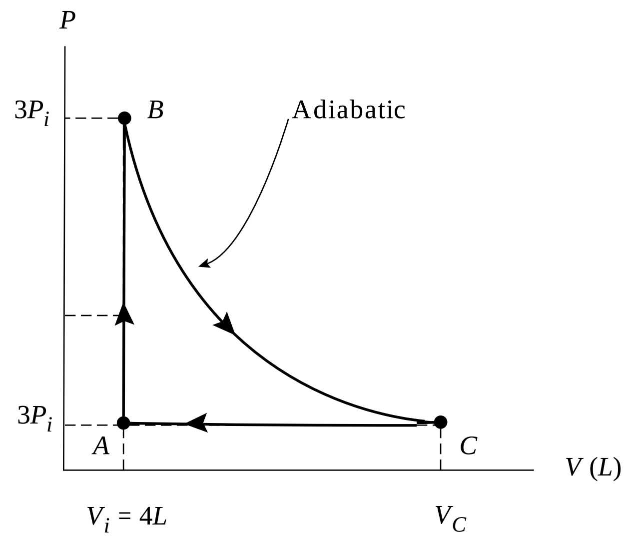
(c) For the isothermal process, we have $P_{b'}V_{b'} = P_aV_a$

Thus, $P_{b'} = P_a\left(\dfrac{V_{a'}}{V_{b'}}\right) = (1.00\text{ atm})(10.0) = \boxed{10.0\text{ atm}}$

For the adiabatic process, we have $P_{b'}V_b^{\gamma} = P_aV_a^{\gamma}$

Thus, $P_b = P_a\left(\dfrac{V_a}{V_b}\right)^{\gamma} = (1.00\text{ atm})(10.0)^{1.40} = \boxed{25.1\text{ atm}}$

21.29 (a) See the diagram at the right.



(b) $P_B V_B^\gamma = P_C V_C^\gamma$

$3P_i V_i^\gamma = P_i V_C^\gamma$

$V_C = 3^{1/\gamma} V_i = 3^{5/7} V_i = 2.19 V_i$

$V_C = 2.19(4.00\text{ L}) = \boxed{8.79\text{ L}}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$T_B = 3T_i = 3(300\text{ K}) = \boxed{900\text{ K}}$

(d) After one whole cycle, $T_A = T_i = \boxed{300\text{ K}}$

(e) In AB, $Q_{AB} = nC_V\,\Delta T = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$

$Q_{BC} = 0$ as this process is abiabatic

$P_C V_C = nRT_C = P_i(2.19V_i) = 2.19nRT_i$ so $T_C = 2.19T_i$

$Q_{CA} = nC_P\,\Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = -4.17nRT_i$

For the whole cycle,

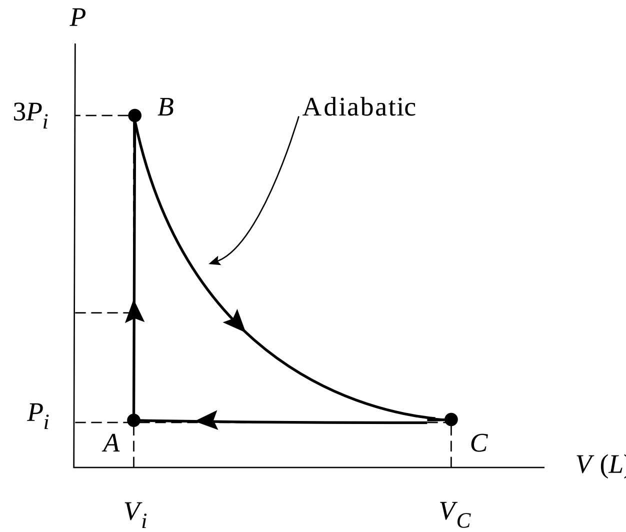
$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = 0.830nRT_i$

$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} - W_{ABCA}$

$W_{BACA} = Q_{ABCA} = 0.830nRT_i = 0.830P_i V_i$

$W_{ABCA} = 0.830(1.013 \times 10^5\text{ Pa})(4.00 \times 10^{-3}\text{ m}^3) = \boxed{336\text{ J}}$

21.30 (a) See the diagram at the right.



(b) $P_B V_B^{\gamma} = P_C V_C^{\gamma}$

$3P_i V_i^{\gamma} = P_i V_C^{\gamma}$

$V_C = 3^{1/\gamma} V_i = 3^{5/7} V_i = \boxed{2.19 V_i}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$T_B = \boxed{3T_i}$

(d) After one whole cycle, $T_A = \boxed{T_i}$

(e) In AB, $Q_{AB} = nC_V \, \Delta T = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$

$Q_{BC} = 0$ as this process is abiabatic

$P_C V_C = nRT_C = P_i(2.19V_i) = 2.19nRT_i$ so $T_C = 2.19T_i$

$Q_{CA} = nC_P \, \Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = -4.17nRT_i$

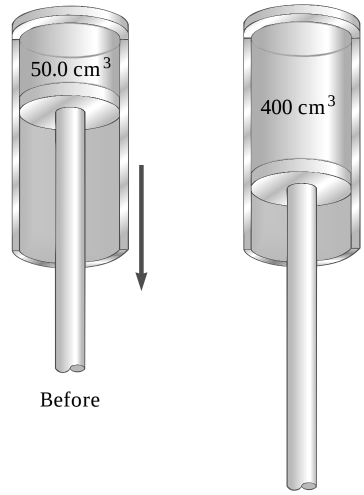
For the whole cycle,

$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = 0.830nRT_i$

$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} - W_{ABCA}$

$W_{ABCA} = Q_{ABCA} = 0.830nRT_i = \boxed{0.830P_i V_i}$

21.31 We suppose the air plus burnt gasoline behaves likes a diatomic ideal gas. We find its final absolute pressure:



Before After

$$21.0\text{ atm}(50.0\text{ cm}^3)^{7/5} = P_f(400\text{ cm}^3)^{7/5}$$

$$P_f = 21.0\text{ atm}\,(1/8)^{7/5} = 1.14\text{ atm}$$

Now $Q = 0$, and $W = -\Delta E_{\text{int}} = -nC_V(T_f - T_i)$

$$\therefore\ W = -n\frac{5}{2}\,RT_f + \frac{5}{2}\,nRT_i = \frac{5}{2}(-P_fV_f + P_iV_i)$$

$$= \frac{5}{2}\,[-(1.14\text{ atm})(400\text{ cm}^3) + (21.0\text{ atm})(50.0\text{ cm}^3)]\left(\frac{1.013\times10^5\text{ N/m}^2}{1\text{ atm}}\right)\left(\frac{10^{-6}\text{ m}^3}{\text{cm}^3}\right)$$

$$W = 150\text{ J}$$

The time for this stroke is $\frac{1}{4}\frac{1\text{ min}}{2500}\left(\frac{60\text{ s}}{1\text{ min}}\right) = 6.00\times10^{-3}\text{ s}$

So $\wp = \frac{W}{t} = \frac{150\text{ J}}{6.00\times10^{-3}\text{ s}} = \boxed{25.0\text{ kW}}$

21.32 (1) $E_{\text{int}} = Nf\frac{k_BT}{2} = f\frac{nRT}{2}$

(2) $C_V = \frac{1}{n}\left(\frac{dE_{\text{int}}}{dT}\right) = \frac{1}{2}\,fR$

(3) $C_P = C_V + R = \frac{1}{2}(f+2)\,R$

(4) $\gamma = \frac{C_P}{C_V} = \frac{(f+2)}{f}$

21.33 (a) $C'_V = \frac{5}{2}\,nR = \boxed{9.95\text{ cal/K}}$ $\qquad C'_P = \frac{7}{2}\,nR = \boxed{13.9\text{ cal/K}}$

(b) $C'_V = \frac{7}{2}\,nR = \boxed{13.9\text{ cal/K}}$ $\qquad C'_P = \frac{9}{2}\,nR = \boxed{17.9\text{ cal/K}}$

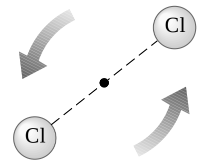
21.34 A more massive diatomic or polyatomic molecule will generally have a lower frequency of vibration. At room temperature, vibration has more chance of being excited than in a less massive molecule. Absorbing energy into vibration shows up in higher specific heats.

21.35 Rotational Kinetic Energy $= \frac{1}{2} I\omega^2$

$I = 2mr^2$, $m = 35.0 \times 1.67 \times 10^{-27}$ kg, $r = 10^{-10}$ m

$I = 1.17 \times 10^{-45}$ kg · m^2 $\quad \omega = 2.00 \times 10^{12}$ s^{-1}

$$\therefore K_{\text{rot}} = \frac{1}{2} I\omega^2 = \boxed{2.33 \times 10^{-21}\ \text{J}}$$



21.36 The ratio of the number at higher energy to the number at lower energy is $e^{-\Delta E / k_B T}$ where ΔE is the energy difference. Here,

$$\Delta E = (10.2\ \text{eV})(1.60 \times 10^{-19}\ \text{J} / 1\ \text{eV}) = 1.63 \times 10^{-18}\ \text{J}$$

and at 0°C,

$$k_B T = (1.38 \times 10^{-23}\ \text{J} / \text{K})(273\ \text{K}) = 3.77 \times 10^{-21}\ \text{J}$$

Since this is much less than the excitation energy, nearly all the atoms will be in the ground state and the number excited is

$$(2.70 \times 10^{25})\exp(-1.63 \times 10^{-18}\ \text{J} / 3.77 \times 10^{-21}\ \text{J}) = (2.70 \times 10^{25})e^{-433}$$

This number is much less than one, so almost all of the time $\boxed{\text{no atom is excited}}$.

At 10000°C,

$$k_B T = (1.38 \times 10^{-23}\ \text{J} / \text{K})10273\ \text{K} = 1.42 \times 10^{-19}\ \text{J}$$

The number excited is

$$(2.70 \times 10^{25})\exp(-1.63 \times 10^{-18}\ \text{J} / 1.42 \times 10^{-19}\ \text{J})$$

$$= (2.70 \times 10^{25})e^{-11.5} = \boxed{2.70 \times 10^{20}}$$

21.37 Call n_{00} the sea-level number density of oxygen molecules, $n_{\text{N}0}$ the sea-level number of nitrogen per volume, and n_0 and n_{N} their respective densities at $y = 10.0$ km.

Then, $n_0 = n_{00} \exp\left(-m_0 g y / k_B T\right)$

$$n_{\text{N}} = n_{\text{NO}} \exp\left(-m_{\text{N}} g y / k_B T\right)$$

and $\dfrac{n_0}{n_{\text{N}}} = \dfrac{n_{00}}{n_{\text{N}0}} \exp\left(-m_0 g y / k_B T + m_{\text{N}} g y / k_B T\right)$

So $$\frac{(n_0/n_{\rm N})}{(n_{00}/n_{\rm N0})} = \exp\left[-(m_0 - m_{\rm N})gy/k_BT\right]$$

$$= \exp\left(-\frac{(32.0-28.0)\text{ u }(1.66\times10^{-27}\text{ kg/u})(9.80\text{ m/s}^2)10^4\text{ m}}{(1.38\times10^{-23}\text{ J/K})300\text{ K}}\right)$$

$$= 0.855$$

$\boxed{\text{The ratio of oxygen to nitrogen molecules decreases to 85.5\% of its sea-level value.}}$

21.38 (a) $$\frac{V_{\rm rms,35}}{V_{\rm rms,37}} = \frac{\sqrt{3RT/M_{35}}}{\sqrt{3RT/M_{37}}} = \left(\frac{37.0\text{ g/mol}}{35.0\text{ g/mol}}\right)^{1/2} = \boxed{1.03}$$

(b) The lighter atom, $\boxed{^{35}\text{Cl}}$, moves faster.

21.39 (a) $$v_{\rm av} = \frac{\sum n_i v_i}{N} = \frac{1}{15}\,[1(2)+2(3)+3(5)+4(7)+3(9)+2(12)] = \boxed{6.80\text{ m/s}}$$

(b) $$(v^2)_{\rm av} = \frac{\sum n_i v_i^2}{N} = 54.9\text{ m}^2/\text{s}^2$$

so $$v_{\rm rms} = \sqrt{(v^2)_{\rm av}} = \sqrt{54.9} = \boxed{7.41\text{ m/s}}$$

(c) $v_{\rm mp} = \boxed{7.00\text{ m/s}}$

21.40 Following Equation 21.29,

$$v_{\rm mp} = \sqrt{\frac{2k_BT}{m}} = \sqrt{\frac{2(1.38\times10^{-23}\text{ J/K})(4.20\text{ K})}{6.64\times10^{-27}\text{ kg}}} = \boxed{132\text{ m/s}}$$

21.41 Use Equation 21.26.

Take $$\frac{dN_v}{dv} = 0:\ 4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2}\exp\left(-\frac{mv^2}{2k_BT}\right)\left(2v - \frac{2mv^3}{2k_BT}\right) = 0$$

and solve for $v_{\rm mp}$ to get Equation 21.29. Reject the solutions $v = 0$ and $v = \infty$.
Retain only $2 - \dfrac{mv^2}{k_BT} = 0$.

21.42 (a) From $v_{\rm rms} = \sqrt{\dfrac{3k_BT}{m}}$, we find the temperature as

$$T = \frac{(6.64\times10^{-27}\text{ kg})(1.12\times10^4\text{ m/s})^2}{3(1.38\times10^{-23}\text{ J/mol}\cdot\text{K})} = \boxed{2.01\times10^4\text{ K}}$$

(b) $$T = \frac{(6.64\times10^{-27}\text{ kg})(2.37\times10^3\text{ m/s})^2}{3(1.38\times10^{-23}\text{ J/mol}\cdot\text{K})} = \boxed{9.01\times10^2\text{ K}}$$

21.43 At 0°C, $\frac{1}{2}\ m v_{\text{rms}0}^2 = \frac{3}{2}\ k_B T_0$

At the higher temperature, $\frac{1}{2}\ m(2v_{\text{rms}0})^2 = \frac{3}{2}\ k_B T$

$$T = 4T_0 = 4(273\text{ K}) = 1092\text{ K} = \boxed{819°\text{C}}$$

21.44 Visualize the molecules in liquid water at 20°C jostling about randomly. One happens to get kinetic energy corresponding to 2430 J/g, and happens to be at the surface and headed upward. Then this molecule can break out of the liquid.

(a) $$2430\text{ J/g} = \frac{2430\text{ J}}{\text{g}}\left(\frac{18.0\text{ g}}{1\text{ mol}}\right)\left(\frac{1\text{ mol}}{6.02\times 10^{23}\text{ molecules}}\right)$$

$$= \boxed{7.27\times 10^{-20}\text{ J/molecule}}$$

(b) $$7.27\times 10^{-20}\text{ J} = \frac{1}{2}\ mv^2$$

$$v = \sqrt{\frac{2(7.27\times 10^{-20}\text{ J})}{18.0\text{ u}(1.66\times 10^{-27}\text{ kg/1 u})}} = \boxed{2.21\text{ km/s}}$$

(c) If these were typical molecules in an ideal gas instead of exceptional molecules in liquid water,

$$\frac{1}{2}\ mv^2 = \frac{3}{2}\ k_B T$$

$$T = \frac{2}{3}\frac{7.27\times 10^{-20}\text{ J}}{1.38\times 10^{-23}\text{ J/K}} = \boxed{3510\text{ K}}$$

These molecules got to be fast-moving in collisions that made other molecules slow-moving; the average molecular energy is unaffected.

21.45 (a) $PV = \left(\frac{N}{N_A}\right) RT$ and $N = \frac{PVN_A}{RT}$ so that

$$N = \frac{(1.00\times 10^{-10})(133)(1.00)(6.02\times 10^{23})}{(8.315)(300)} = \boxed{3.21\times 10^{12}\text{ molecules}}$$

(b) $$l = \frac{1}{n_V \pi d^2 2^{1/2}} = \frac{V}{N\pi d^2 2^{1/2}} = \frac{1.00\text{ m}^3}{(3.21\times 10^{12}\text{ molecules})\pi(3.00\times 10^{-10}\text{ m})^2(2)^{1/2}}$$

$$l = \boxed{778\text{ km}}$$

(c) $$f = \frac{v}{l} = \boxed{6.42\times 10^{-4}\text{ s}^{-1}}$$

Goal Solution

G: Since high vacuum means low pressure as a result of a low molecular density, we should expect a relatively low number of molecules, a long free path, and a low collision frequency compared with the values found in Example 21.7 for normal air. Since the ultrahigh vacuum is 13 orders of magnitude lower than atmospheric pressure, we might expect corresponding values of $N \sim 10^{12}$ molecules / m^3, $l \sim 10^6$ m, and $f \sim 0.0001$ / s.

O: The equation of state for an ideal gas can be used with the given information to find the number of molecules in a specific volume. The mean free path can be found directly from equation 21.30, and this result can be used with the average speed to find the collision frequency.

A: (a) $PV = \left(\frac{N}{N_A}\right) RT$ and $N = \frac{PVN_A}{RT}$ so that

$$N = \frac{(1.00 \times 10^{-10}\ \text{torr})(133\ \text{Pa/torr})(6.02 \times 10^{23}\ \text{molecules/mol})}{(8.315\ \text{J/mol} \cdot \text{K})(300\ \text{K})} = 3.21 \times 10^{12}\ \text{molecules}$$

(b) $$l = \frac{1}{\sqrt{2}\ \pi d^2 n_v} = \frac{V}{\sqrt{2}\ N \pi d^2} = \frac{1.00\ \text{m}^3}{\sqrt{2}\ (3.21 \times 10^{12}\ \text{molecules})\pi(3.00 \times 10^{-10}\ \text{m})^2}$$

$$l = 7.78 \times 10^5\ \text{m} = 778\ \text{km}$$

(c) $$f = \frac{v}{l} = \frac{500\ \text{m/s}}{7.78 \times 10^5\ \text{m}} = 6.42 \times 10^{-4}\ \text{s}^{-1}$$

L: The pressure and the calculated results differ from the results in Example 21.7 by about 13 orders of magnitude as we expected. This ultrahigh vacuum provides conditions that are extremely different from normal atmosphere, and these conditions provide a "clean" environment for a variety of experiments and manufacturing processes that would otherwise be impossible.

21.46 The average molecular speed is

$$v = \sqrt{8k_BT/\pi m} = \sqrt{8k_BN_AT/\pi N_A m}$$

$$v = \sqrt{8RT/\pi M}$$

$$v = \sqrt{8(8.315\ \text{J/mol} \cdot \text{K})3.00\ \text{K}/\pi(2.016 \times 10^{-3}\ \text{kg/mol})}$$

$$v = 178\ \text{m/s}$$

(a) The mean free path is

$$l = \frac{1}{\sqrt{2}\pi d^2 n_V} = \frac{1}{\sqrt{2}\pi(0.200 \times 10^{-9}\ \text{m})^2\ 1/\text{m}^3}$$

$$l = \boxed{5.63 \times 10^{18}\ \text{m}}$$

The mean free time is

$$l / v = 5.63 \times 10^{18}\ \text{m} / 178\ \text{m/s} = 3.17 \times 10^{16}\ \text{s} = \boxed{1.00 \times 10^{9}\ \text{yr}}$$

(b) Now n_V is 10^6 times larger, to make l smaller by 10^6 times:

$$l = \boxed{5.63 \times 10^{12}\ \text{m}}$$

Thus, $l / v = 3.17 \times 10^{10}\ \text{s} = \boxed{1.00 \times 10^{3}\ \text{yr}}$

21.47 From Equation 21.30, $l = \dfrac{1}{\sqrt{2}\ \pi d^2 n_V}$

For an ideal gas, $n_V = \dfrac{N}{V} = \dfrac{P}{k_B T}$

Therefore, $l = \dfrac{k_B T}{\sqrt{2}\pi d^2 P}$, as required.

21.48 $l = [\sqrt{2}\ \pi d^2 n_V]^{-1}$ $\qquad n_V = \dfrac{P}{k_B T}$

$d = 3.60 \times 10^{-10}\ \text{m}$ $\qquad n_V = \dfrac{1.013 \times 10^5}{(1.38 \times 10^{-23})(293)} = 2.51 \times 10^{25}/\text{m}^3$

$\therefore l = 6.93 \times 10^{-8}$ m, or about $\boxed{\text{193 molecular diameters}}$

21.49 Using $P = n_V k_B T$, Equation 21.30 becomes $l = \dfrac{k_B T}{\sqrt{2}\ \pi P d^2}$ (1)

(a) $$l = \frac{(1.38 \times 10^{-23}\ \text{J/K})(293\ \text{K})}{\sqrt{2}\ \pi(1.0113 \times 10^5\ \text{Pa})(3.10 \times 10^{-10}\ \text{m})^2} = \boxed{9.36 \times 10^{-8}\ \text{m}}$$

(b) Equation (1) shows that $P_1 l_1 = P_2 l_2$. Taking $P_1 l_1$ from (a) and with $l_2 = 1.00$ m, we find

$$P_2 = \frac{(1.00\ \text{atm})(9.36 \times 10^{-8}\text{m})}{1.00\ \text{m}} = \boxed{9.36 \times 10^{-8}\ \text{atm}}$$

(c) For $l_3 = 3.10 \times 10^{-10}$ m, we have

$$P_3 = \frac{(1.00 \text{ atm})(9.36 \times 10^{-8} \text{ m})}{3.10 \times 10^{-10} \text{ m}} = \boxed{302 \text{ atm}}$$

***21.50** (a) $n = \dfrac{PV}{RT} = \dfrac{(1.013 \times 10^5 \text{ Pa})(4.20 \text{ m} \times 3.00 \text{ m} \times 2.50 \text{ m})}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 1.31 \times 10^3 \text{ mol}$

$N = nN_A = (1.31 \times 10^3 \text{ mol})(6.02 \times 10^{23} \text{ molecules/mol})$

$N = \boxed{7.88 \times 10^{26} \text{ molecules}}$

(b) $m = nM = (1.31 \times 10^3 \text{ mol})(0.0289 \text{ kg/mol}) = \boxed{37.9 \text{ kg}}$

(c) $\dfrac{1}{2} m_0 v^2 = \dfrac{3}{2} k_B T = \dfrac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) = \boxed{6.07 \times 10^{-21} \text{ J/molecule}}$

(d) For one molecule,

$$m_0 = \frac{M}{N_A} = \frac{0.0289 \text{ kg/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 4.80 \times 10^{-26} \text{ kg/molecule}$$

$$v_{\text{rms}} = \sqrt{\frac{2(6.07 \times 10^{-21} \text{ J/molecule})}{4.80 \times 10^{-26} \text{ kg/molecule}}} = \boxed{503 \text{ m/s}}$$

(e) and (f)

$$E_{\text{int}} = nC_V T = n\left(\frac{5}{2} R\right) T = \frac{5}{2} PV$$

$$E_{\text{int}} = \frac{5}{2}(1.013 \times 10^5 \text{ Pa})(31.5 \text{ m}^3) = \boxed{7.98 \text{ MJ}}$$

***21.51** (a) $P_f = \boxed{100 \text{ kPa}}$ $\quad$ $T_f = \boxed{400 \text{ K}}$

$V_f = \dfrac{nRT_f}{P_f} = \dfrac{(2.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(400 \text{ K})}{100 \times 10^3 \text{ Pa}} = 0.0665 \text{ m}^3 = \boxed{66.5 \text{ L}}$

$\Delta E_{\text{int}} = 3.50 nR\, \Delta T = 3.50(2.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(100 \text{ K}) = \boxed{5.82 \text{ kJ}}$

$W = P\, \Delta V = nR\, \Delta T = (2.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(100 \text{ K}) = \boxed{1.66 \text{ kJ}}$

$Q = \Delta E_{\text{int}} + W = 5.82 \text{ kJ} + 1.66 \text{ kJ} = \boxed{7.48 \text{ kJ}}$

(b) $T_f = \boxed{400 \text{ K}}$

$$V_f = V_i = \frac{nRT_i}{P_i} = \frac{(2.00 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(300 \text{ K})}{100 \times 10^3 \text{ Pa}} = 0.0499 \text{ m}^3 = \boxed{49.9 \text{ L}}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right) = (100 \text{ kPa})\left(\frac{400 \text{ K}}{300 \text{ K}}\right) = \boxed{133 \text{ kPa}}$$

$W = \int P\,dV = \boxed{0}$ since V = constant

$\Delta E_{\text{int}} = \boxed{5.82 \text{ kJ}}$ as in (a)

$$Q = \Delta E_{\text{int}} + W = 5.82 \text{ kJ} + 0 = \boxed{5.82 \text{ kJ}}$$

(c) $T_f = \boxed{300 \text{ K}}$ $\quad P_f = \boxed{120 \text{ kPa}}$

$$V_f = V_i\left(\frac{P_i}{P_f}\right) = (49.9 \text{ L})\left(\frac{100 \text{ kPa}}{120 \text{ kPa}}\right) = \boxed{41.6 \text{ L}}$$

$\Delta E_{\text{int}} = 3.50nR\,\Delta T = \boxed{0}$ since T = constant

$$W = \int P\,dV = nRT_i\int_{V_i}^{V_f}\frac{dV}{V} = nRT_i\ln\left(\frac{V_f}{V_i}\right) = nRT_i\ln\left(\frac{P_i}{P_f}\right)$$

$$W = (2.00 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(300 \text{ K})\ln\left(\frac{100 \text{ kPa}}{120 \text{ kPa}}\right) = \boxed{-910 \text{ J}}$$

$$Q = \Delta E_{\text{int}} + W = 0 - 910 \text{ J} = \boxed{-910 \text{ J}}$$

(d) $P_f = \boxed{120 \text{ kPa}}$ $\quad \gamma = \dfrac{C_P}{C_V} = \dfrac{C_V + R}{C_V} = \dfrac{3.50R + R}{3.50R} = \dfrac{4.50}{3.50R} = \dfrac{4.50}{3.50} = \dfrac{9}{7}$

$$P_f V_f^\gamma = P_i V_i^\gamma \quad \text{so} \quad V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = (49.9 \text{ L})\left(\frac{100 \text{ kPa}}{120 \text{ kPa}}\right)^{7/9} = \boxed{43.3 \text{ L}}$$

$$T_f = T_i\left(\frac{P_f V_f}{P_i V_i}\right) = (300 \text{ K})\left(\frac{120 \text{ kPa}}{100 \text{ kPa}}\right)\left(\frac{43.3 \text{ L}}{49.9 \text{ L}}\right) = \boxed{312 \text{ K}}$$

$$\Delta E_{\text{int}} = 3.50nR\,\Delta T = 3.50(2.00 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(12.4 \text{ K}) = \boxed{722 \text{ J}}$$

$Q = \boxed{0}$ (abiabatic process)

$$W = Q - \Delta E_{\text{int}} = 0 - 722 \text{ J} = \boxed{-722 \text{ J}}$$

21.52 (a) The average speed v_{av} is just the weighted average of all the speeds.

$$v_{av} = \frac{[2(v) + 3(2v) + 5(3v) + 4(4v) + 3(5v) + 2(6v) + 1(7v)]}{(2 + 3 + 5 + 4 + 3 + 2 + 1)} = \boxed{3.65v}$$

(b) First find the average of the square of the speeds,

$$v_{av}^2 = \frac{[2(v)^2 + 3(2v)^2 + 5(3v)^2 + 4(4v)^2 + 3(5v)^2 + 2(6v)^2 + 1(7v)^2]}{2 + 3 + 5 + 4 + 3 + 2 + 1} = 15.95v^2$$

The root-mean square speed is then $v_{rms} = \sqrt{v_{av}^2} = \boxed{3.99v}$

(c) The most probable speed is the one that most of the particles have; i.e., five particles have speed $\boxed{3.00v}$

(d) $PV = \frac{1}{3}\, Nmv_{av}^2$

Therefore, $P = \frac{20}{3}\frac{[m(15.95)v^2]}{V} = \boxed{106\left(\frac{mv^2}{V}\right)}$

(e) The average kinetic energy for each particle is

$$\overline{K} = \frac{1}{2}\, mv_{av}^2 = \frac{1}{2}\, m(15.95v^2) = \boxed{7.98mv^2}$$

21.53 (a) $PV^\gamma = k$. So, $W = \int_i^f P\, dV = k\int_i^f \frac{dV}{V^\gamma} = \frac{P_iV_i - P_fV_f}{\gamma - 1}$

(b) $dE_{int} = dQ - dW$ and $dQ = 0$ for an adiabatic process.

Therefore, $W = -\Delta E_{int} = -\frac{3}{2}\, nR\,\Delta T = nC_V(T_i - T_f)$

To show consistency between these 2 equations, consider that $\gamma = C_P / C_V$ and $C_P - C_V = R$. Therefore, $1/(\gamma - 1) = C_V / R$.

Using this, the result found in part (a) becomes

$$W = (P_iV_i - P_fV_f)\frac{C_V}{R}$$

Also, for an ideal gas $\frac{PV}{R} = nT$ so that $W = nC_V(T_i - T_f)$

21.54 (a) Maxwell's speed distribution function is

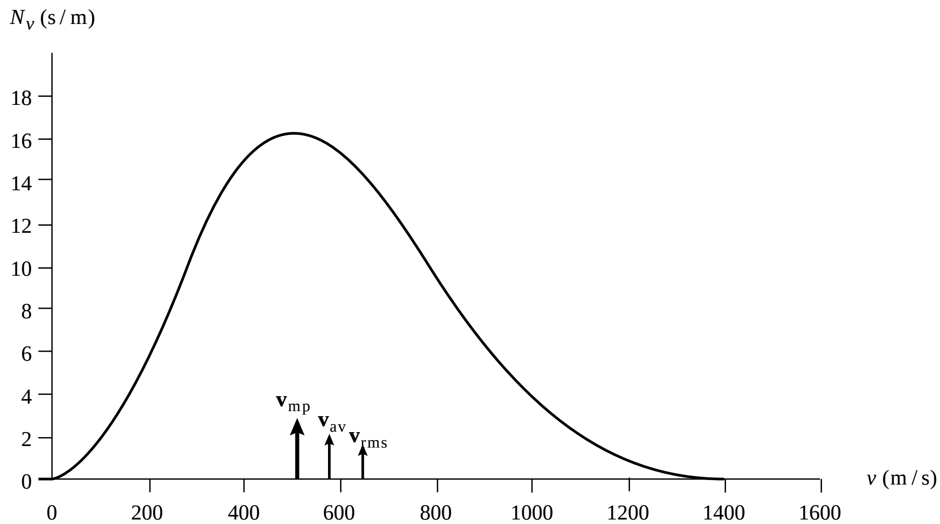
$$N_v = 4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT}$$

With $N = 1.00 \times 10^4$, $m = \dfrac{M}{N_A} = \dfrac{0.032\text{ kg}}{6.02 \times 10^{23}} = 5.32 \times 10^{-26}\text{ kg}$

$T = 500$ K, and $k_B = 1.38 \times 10^{-23}$ J/molecule · K; this becomes

$$N_v = (1.71 \times 10^{-4})v^2 e^{-(3.85 \times 10^{-6})v^2}$$

The following is a plot of this function for the range $0 \le v \le 1400$ m/s.



(b) The most probable speed occurs where N_v is a maximum.

From the graph, $\boxed{v_{mp} \approx 510\text{ m/s}}$

(c) $v_{av} = \sqrt{\dfrac{8k_BT}{\pi m}} = \sqrt{\dfrac{8(1.38 \times 10^{-23})(500)}{\pi(5.32 \times 10^{-26})}} = \boxed{575\text{ m/s}}$

Also, $v_{rms} = \sqrt{\dfrac{3k_BT}{m}} = \sqrt{\dfrac{3(1.38 \times 10^{-23})(500)}{5.32 \times 10^{-26}}} = \boxed{624\text{ m/s}}$

(d) The fraction of particles in the range $300 \le v \le 600$ m/s is $\dfrac{\int_{300}^{600} N_v dv}{N}$ where $N = 10^4$ and the integral of N_v is read from the graph as the area under the curve. This is approximately 4400 and the fraction is 0.44 or $\boxed{44\%}$.

21.55 The pressure of the gas in the lungs of the diver will be the same as the absolute pressure of the water at this depth of 50.0 meters. This is:

$$P = P_0 + \rho g h = 1.00 \text{ atm} + (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(50.0 \text{ m})$$

or $$P = 1.00 \text{ atm} + (4.90 \times 10^5 \text{ Pa})\left(\frac{1.00 \text{ atm}}{1.013 \times 10^5 \text{ Pa}}\right) = 5.84 \text{ atm}$$

If the partial pressure due to the oxygen in the gas mixture is to be 1.00 atmosphere or less (or approximately one-sixth of the total pressure), oxygen molecules should make up only about one-sixth of the total number of molecules. This will be true if 1.00 mole of oxygen is used for every 5.00 moles of helium. The ratio by weight is therefore,

$$\frac{(5.00 \text{ mol He})g}{(1.00 \text{ mol O}_2)g} = \frac{(20.0 \text{ g})g}{(32.0 \text{ g})g} = \boxed{0.625}$$

***21.56** $n = \dfrac{m}{M} = \dfrac{1.20 \text{ kg}}{0.0289 \text{ kg/mol}} = 41.5 \text{ mol}$

(a) $$V_i = \frac{nRT_i}{P_i} = \frac{(41.5 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(298 \text{ K})}{200 \times 10^3 \text{ Pa}} = \boxed{0.514 \text{ m}^3}$$

(b) $$\frac{P_f}{P_i} = \frac{\sqrt{V_f}}{\sqrt{V_i}} \quad \text{so} \quad V_f = V_i\left(\frac{P_f}{P_i}\right)^2 = (0.514 \text{ m}^3)\left(\frac{400}{200}\right)^2 = \boxed{2.06 \text{ m}^3}$$

(c) $$T_f = \frac{P_f V_f}{nR} = \frac{(400 \times 10^3 \text{ Pa})(2.06 \text{ m}^3)}{(41.5 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})} = \boxed{2.38 \times 10^3 \text{ K}}$$

(d) $$W = \int_{V_i}^{V_f} P\,dV = C\int_{V_i}^{V_f} V^{1/2} dV = \left(\frac{P_i}{V_i^{1/2}}\right)\frac{2V^{3/2}}{3}\Bigg|_{V_i}^{V_f} = \frac{2}{3}\left(\frac{P_i}{V_i^{1/2}}\right)\left(V_f^{3/2} - V_i^{3/2}\right)$$

$$W = \frac{2}{3}\left(\frac{200 \times 10^3 \text{ Pa}}{\sqrt{0.514 \text{ m}}}\right)\left[(2.06 \text{ m}^3)^{3/2} - (0.514 \text{ m})^{3/2}\right] = \boxed{4.80 \times 10^5 \text{ J}}$$

(e) $$\Delta E_{\text{int}} = nC_V\,\Delta T = (41.5 \text{ mol})\left[\frac{5}{2}(8.315 \text{ J/mol}\cdot\text{K})\right](2.38 \times 10^3 - 298) \text{ K}$$

$$\Delta E_{\text{int}} = 1.80 \times 10^6 \text{ J}$$

$$Q = \Delta E_{\text{int}} + W = 1.80 \times 10^6 \text{ J} + 4.80 \times 10^5 \text{ J} = 2.28 \times 10^6 \text{ J} = \boxed{2.28 \text{ MJ}}$$

21.57 (a) Since $\boxed{\text{pressure increases as volume decreases}}$ (and vice versa) ,

$$\frac{dV}{dP} < 0 \quad \text{and} \quad -\frac{1}{V}\left[\frac{dV}{dP}\right] > 0$$

(b) For an ideal gas, $V = \dfrac{nRT}{P}$ and $\kappa_1 = -\dfrac{1}{V}\dfrac{d}{dP}\left(\dfrac{nRT}{P}\right)$

If the compression is isothermal, T is constant and

$$\kappa_1 = -\frac{nRT}{V}\left(-\frac{1}{P^2}\right) = \frac{1}{P}$$

(c) For an adiabatic compression, $PV^{\gamma} = C$ (where C is a constant) and

$$\kappa_2 = -\frac{1}{V}\frac{d}{dP}\left(\frac{C}{P}\right)^{1/\gamma} = \frac{1}{V}\left(\frac{1}{\gamma}\right)\frac{C^{1/\gamma}}{P^{(1/\gamma)+1}} = \frac{P^{1/\gamma}}{\gamma P^{1/\gamma+1}} = \frac{1}{\gamma P}$$

(d) $\kappa_1 = \dfrac{1}{P} = \dfrac{1}{(2.00\text{ atm})} = \boxed{0.500\text{ atm}^{-1}}$

$\gamma = \dfrac{C_P}{C_V}$ and for a monatomic ideal gas, $\gamma = 5/3$, so that

$$\kappa_2 = \frac{1}{\gamma P} = \frac{1}{(5/3)(2.00\text{ atm})} = \boxed{0.300\text{ atm}^{-1}}$$

***21.58** (a) The speed of sound is $v = \sqrt{\dfrac{B}{\rho}}$ where $B = -V\dfrac{dP}{dV}$.

According to Problem 57, in an adiabatic process, this is $B = \dfrac{1}{\kappa_2} = \gamma P$.

Also, $\rho = \dfrac{m_s}{V} = \dfrac{nM}{V} = \dfrac{(nRT)M}{V(RT)} = \dfrac{PM}{RT}$ where m_s is the sample mass. Then, the speed of sound in the ideal gas is $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\gamma P\left(\dfrac{RT}{PM}\right)} = \boxed{\sqrt{\dfrac{\gamma RT}{M}}}$

(b) $v = \sqrt{\dfrac{1.40(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})}{0.0289\text{ kg/mol}}} = \boxed{344\text{ m/s}}$

This nearly agrees with the 343 m/s listed in Table 17.1.

(c) We use $k_B = \dfrac{R}{N_A}$ and $M = mN_A$: $v = \sqrt{\dfrac{\gamma R T}{M}} = \sqrt{\dfrac{\gamma k_B N_A T}{m N_A}} = \sqrt{\dfrac{\gamma k_B T}{m}}$

The most probable molecular speed is $\sqrt{\dfrac{2k_B T}{m}}$,

the average speed is $\sqrt{\dfrac{8k_B T}{\pi m}}$, and the rms speed is $\sqrt{\dfrac{3k_B T}{m}}$.

$\boxed{\text{All are somewhat larger}}$ than the speed of sound.

21.59 $N_v(v) = 4\pi N \left(\dfrac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp(-mv^2/2k_B T)$

Note that $v_{\text{mp}} = (2k_B T/m)^{1/2}$

Thus, $N_v(v) = 4\pi N \left(\dfrac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{(-v^2/v_{\text{mp}}^2)}$

and $\dfrac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\dfrac{v}{v_{\text{mp}}}\right)^2 e^{(1 - v^2/v_{\text{mp}}^2)}$

For $v = v_{\text{mp}}/50$, $\dfrac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\dfrac{1}{50}\right)^2 e^{[1-(1/50)^2]} = 1.09 \times 10^{-3}$

The other values are computed similarly, with the following results:

$\dfrac{v}{v_{\text{mp}}}$	$\dfrac{N_v(v)}{N_v(v_{\text{mp}})}$
1/50	1.09×10^{-3}
1/10	2.69×10^{-2}
1/2	0.529
1	1.00
2	0.199
10	1.01×10^{-41}
50	1.25×10^{-1082}

To find the last value, note:

$$(50)^2 e^{1-2500} = 2500 e^{-2499}$$

$$= 10^{\log 2500} e^{(\ln 10)(-2499/\ln 10)} = 10^{\log 2500} 10^{-2499/\ln 10}$$

$$= 10^{\log 2500 - 2499/\ln 10} = 10^{-1081.904}$$

***21.60** The ball loses energy

$$\frac{1}{2}\,mv_i^2 - \frac{1}{2}\,mv_f^2 = \frac{1}{2}(0.142\text{ kg})\,[(47.2)^2 - (42.5)^2]\text{m}^2/\text{s}^2 = 29.9\text{ J}$$

The air volume is $V = \pi(0.0370\text{ m})^2(19.4\text{ m}) = 0.0834\text{ m}^3$,

and its quantity is $n = \dfrac{PV}{RT} = \dfrac{(1.013\times 10^5\text{ Pa})(0.0834\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})} = 3.47\text{ mol}$

The air absorbs energy according to $Q = nC_P\,\Delta T$, so

$$\Delta T = \frac{Q}{nC_P} = \frac{29.9\text{ J}}{(3.47\text{ mol})(7/2)(8.315\text{ J/mol}\cdot\text{K})} = \boxed{0.296\text{ C}^\circ}$$

21.61 (a) The effect of high angular speed is like the effect of a very high gravitational field on an atmosphere. The result is:

$\boxed{\text{The larger-mass molecules settle to the outside}}$ while the region at smaller r has a higher concentration of low-mass molecules.

(b) Consider a single kind of molecules, all of mass m. To supply the centripetal force on the molecules between r and $r + dr$, the pressure must increase outward according to $\sum F_r = ma_r$. Thus,

$$PA - (P + dP)A = -(nmA\ dr)(r\omega^2)$$

where n is the number of molecules per unit volume and A is the area of any cylindrical surface. This reduces to $dP = nm\omega^2 r dr$.

But also $P = nk_BT$, so $dP = k_BT\,dn$. Therefore, the equation becomes

$$\frac{dn}{n} = \frac{m\omega^2}{k_BT}\,r\,dr \quad \text{giving} \quad \int_{n_0}^{n}\frac{dn}{n} = \frac{m\omega^2}{k_BT}\int_0^r r\,dr \quad \text{or}$$

$$\ln(n)\Big|_{n_0}^{n} = \frac{m\omega^2}{k_BT}\left(\frac{r^2}{2}\right)\Bigg|_0^r$$

$$\ln\left(\frac{n}{n_0}\right) = \frac{m\omega^2}{2k_BT}\,r^2 \text{ and solving for } n\text{: } \boxed{n = n_0 e^{mr^2\omega^2/2k_BT}}$$

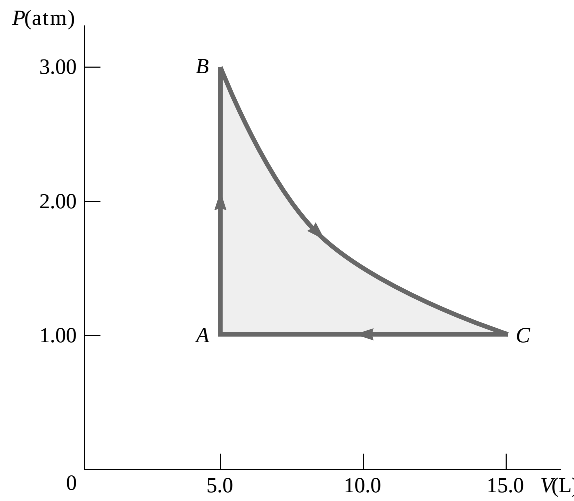
21.62 First find v_{av}^2 as $v_{\text{av}}^2 = \frac{1}{N}\int_0^\infty v^2 N_v dv$. Let $a = \frac{m}{2k_BT}$.

Then, $v_{\text{av}}^2 = \frac{[4N\pi^{-1/2}a^{3/2}]}{N}\int_0^\infty v^4 e^{-av^2}dv = [4a^{3/2}\pi^{-1/2}]\frac{3}{8a^2}\sqrt{\frac{\pi}{a}} = \frac{3k_BT}{m}$

The root-mean square speed is then $v_{\text{rms}} = \sqrt{v_{\text{av}}^2} = \boxed{\sqrt{\frac{3k_BT}{m}}}$

To find the average speed, we have

$$v_{\text{av}} = \frac{1}{N}\int_0^\infty vN_v dv = \frac{(4Na^{3/2}\pi^{-1/2})}{N}\int_0^\infty v^3 e^{-av^2}dv = \frac{4a^{3/2}\pi^{-1/2}}{2a^2} = \boxed{\sqrt{\frac{8k_BT}{\pi m}}}$$



***21.63** (a) $n = \frac{PV}{RT} = \frac{(1.013\times10^5\text{ Pa})(5.00\times10^{-3}\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(300\text{ K})}$

$n = \boxed{0.203\text{ mol}}$

(b) $T_B = T_A\left(\frac{P_B}{P_A}\right) = (300\text{ K})\left(\frac{3.00}{1.00}\right) = \boxed{900\text{ K}}$

$T_C = T_B = \boxed{900\text{ K}}$

$V_C = V_A\left(\frac{T_C}{T_A}\right) = (5.00\text{ L})\left(\frac{900}{300}\right) = \boxed{15.0\text{ L}}$

(c) $E_{\text{int},A} = \frac{3}{2}nRT_A = \frac{3}{2}(0.203\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(300\text{ K}) = \boxed{760\text{ J}}$

$E_{\text{int},C} = E_{\text{int},B} = \frac{3}{2}nRT_B = \frac{3}{2}(0.203\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(900\text{ K}) = \boxed{2.28\text{ kJ}}$

(d)

	P(atm)	V(L)	T(K)	E_{int} (kJ)
A	1.00	5.00	300	0.760
B	3.00	5.00	900	2.28
C	1.00	15.00	900	2.28

(e) For the process AB, lock the piston in place and put the cylinder into an oven at 900 K. For BC, keep the sample in the oven while gradually letting the gas expand to lift a load on the piston as far as it can. For *CA*, carry the cylinder back into the room at 300 K and let the gas cool without touching the piston.

(f) For AB: $W = \boxed{0}$, $\Delta E_{\text{int}} = E_{\text{int},B} - E_{\text{int},A} = (2.28 - 0.769)\text{ kJ} = \boxed{1.52\text{ kJ}}$

$Q = \Delta E_{\text{int}} + W = \boxed{1.52\text{ kJ}}$

For BC: $\Delta E_{\text{int}} = \boxed{0}$, $W = nRT_B \ln(V_C / V_B)$

$W = (0.203\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(900\text{ K})\ln(3.00) = \boxed{1.67\text{ kJ}}$

$Q = \Delta E_{\text{int}} + W = \boxed{1.67\text{ kJ}}$

For CA: $\Delta E_{\text{int}} = E_{\text{int},A} - E_{\text{int},C} = (0.760 - 2.28)\text{ kJ} = \boxed{-1.52\text{ kJ}}$

$W = P\,\Delta V = nR\,\Delta T$

$W = (0.203\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(-600\text{ K}) = \boxed{-1.01\text{ kJ}}$

$Q = \Delta E_{\text{int}} + W = -1.52\text{ kJ} - 1.01\text{ kJ} = \boxed{-2.53\text{ kJ}}$

(g) We add the amounts of energy for each process to find them for the whole cycle.

$Q_{ABCA} = +1.52\text{ kJ} + 1.67\text{ kJ} - 2.53\text{ kJ} = \boxed{0.656\text{ kJ}}$

$W_{ABCA} = 0 + 1.67\text{ kJ} - 1.01\text{ kJ} = \boxed{0.656\text{ kJ}}$

$(\Delta E_{\text{int}})_{ABCA} = +1.52\text{ kJ} + 0 - 1.52\text{ kJ} = \boxed{0}$

21.64 With number-per-volume $n_0 e^{(-mgy/k_BT)}$, the number of molecules above unit ground area is $\int_0^\infty n(y)dy$, and the number below altitude h is $\int_0^h n(y)dy$. So,

(a) $$f = \frac{\int_0^h n(y)dy}{\int_0^\infty n(y)dy} = \frac{n_0\int_0^h e^{(-mgy/k_BT)}dy}{n_0\int_0^\infty e^{(-mgy/k_BT)}dy} = \frac{-k_BT/mg\int_0^h e^{(-mgy/k_BT)}(-mg\,dy/k_BT)}{-k_BT/mg\int_0^\infty e^{(-mgy/k_BT)}(-mg\,dy/k_BT)}$$

$$= \frac{e^{(-mgy/k_BT)}\big|_0^h}{e^{(-mgy/k_BT)}\big|_0^\infty} = \frac{e^{(-mgh/k_BT)}-1}{0-1} = \boxed{1-e^{(-mgh/k_BT)}}$$

(b) $$\frac{1}{2} = 1 - e^{(-mgh'/k_BT)}$$

$$e^{(-mgh'/k_BT)} = \frac{1}{2} \quad \text{or} \quad e^{(+mgh'/k_BT)} = 2$$

$$mgh'/k_BT = \ln 2 \quad \text{so}$$

$$h' = \frac{k_BT\ln 2}{mg} = \frac{(1.38\times 10^{-23}\text{ J/K})(270\text{ K})(\ln 2)}{(28.9\text{ u})(1.66\times 10^{-27}\text{ kg/u})(9.80\text{ m/s}^2)} = \boxed{5.47\text{ km}}$$

***21.65** (a) $$(10\,000\text{ g})\left(\frac{1.00\text{ mol}}{18.0\text{ g}}\right)\left(\frac{6.02\times 10^{23}\text{ molecules}}{1.00\text{ mol}}\right) = \boxed{3.34\times 10^{26}\text{ molecules}}$$

(b) After one day, 10^{-1} of the original molecules would remain. After two days, the fraction would be 10^{-2}, and so on. After 26 days, only 3 of the original molecules would likely remain, and after $\boxed{27\text{ days}}$, likely none.

(c) The soup is this fraction of the hydrosphere: $\left(\dfrac{10.0\text{ kg}}{1.32\times 10^{21}\text{ kg}}\right)$

Therefore, today's soup likely contains this fraction of the original molecules. The number of original molecules likely in the pot again today is:

$$\left(\frac{10.0\text{ kg}}{1.32\times 10^{21}\text{ kg}}\right)(3.34\times 10^{26}\text{ molecules}) = \boxed{2.53\times 10^{6}\text{ molecules}}$$

21.66 (a) For escape, $\frac{1}{2}mv^2 = \frac{GmM}{R}$. Since the free-fall acceleration at the surface is $g = \frac{GM}{R^2}$, this can also be written as: $\frac{1}{2}mv^2 = \frac{GmM}{R} = \boxed{mgR}$

(b) For O_2, the mass of one molecule is

$$m = \frac{0.0320\text{ kg/mol}}{6.02\times 10^{23}\text{ molecules/mol}} = 5.32\times 10^{-26}\text{ kg/molecule}$$

Then, if $mgR = 10(3k_BT/2)$, the temperature is

$$T = \frac{mgR}{15k_B} = \frac{(5.32 \times 10^{-26}\ \text{kg})(9.80\ \text{m/s}^2)(6.37 \times 10^6\ \text{m})}{15(1.38 \times 10^{-23}\ \text{J/mol}\cdot\text{K})} = \boxed{1.60 \times 10^4\ \text{K}}$$

21.67 (a) For sodium atoms (with a molar mass $M = 32.0$ g/mol)

$$\frac{1}{2}\,mv^2 = \frac{3}{2}\,k_BT$$

$$\frac{1}{2}\left(\frac{M}{N_A}\right)v^2 = \frac{3}{2}\,k_BT$$

(a) $$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.315\ \text{J/mol}\cdot\text{K})(2.40 \times 10^{-4}\ \text{K})}{23.0 \times 10^{-3}\ \text{kg}}} = \boxed{0.510\ \text{m/s}}$$

(b) $$t = \frac{d}{v_{\text{rms}}} = \frac{0.010\ \text{m}}{0.510\ \text{m/s}} \approx \boxed{20\ \text{ms}}$$

Chapter 22 Solutions

22.1 (a) $e = \dfrac{W}{Q_h} = \dfrac{25.0 \text{ J}}{360 \text{ J}} = \boxed{0.0694}$ or $\boxed{6.94\%}$

(b) $Q_c = Q_h - W = 360 \text{ J} - 25.0 \text{ J} = \boxed{335 \text{ J}}$

22.2 (a) $e = \dfrac{W}{Q_h} = \dfrac{W}{3W} = \dfrac{1}{3} = \boxed{0.333}$ or $\boxed{33.3\%}$

(b) $Q_c = Q_h - W = 3W - W = 2W$

Therefore, $\dfrac{Q_c}{Q_h} = \dfrac{2W}{3W} = \boxed{\dfrac{2}{3}}$

22.3 (a) We have $e = \dfrac{W}{Q_h} = \dfrac{Q_h - Q_c}{Q_h} = 1 - \dfrac{Q_c}{Q_h} = 0.250,$

with $Q_c = 8000$ J, we have $Q_h = \boxed{10.7 \text{ kJ}}$

(b) $W = Q_h - Q_c = 2667 \text{ J}$

and from $\wp = \dfrac{W}{t}$, we have $t = \dfrac{W}{\wp} = \dfrac{2667 \text{ J}}{5000 \text{ J/s}} = \boxed{0.533 \text{ s}}$

22.4 $W = Q_h - Q_c = 200 \text{ J}$ (1)

$e = \dfrac{W}{Q_h} = 1 - \dfrac{Q_c}{Q_h} = 0.300$ (2)

From (2), $Q_c = 0.700 Q_h$(3)

Solving (3) and (1) simultaneously, we have

$\boxed{Q_h = 667 \text{ J}}$ and $\boxed{Q_c = 467 \text{ J}}$

22.5 It is easiest to solve part (b) first:

(b) $\Delta E_{\text{int}} = nC_V\,\Delta T$ and since the temperature is held constant during the compression, $\Delta E_{\text{int}} = \boxed{0}$.

(a) From the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$.

Since $\Delta E_{\text{int}} = 0$, this gives: $W = Q = 1000 \text{ J} = \boxed{1.00 \text{ kJ}}$

22.6 Q_c = heat to melt 15.0 g of Hg $= mL_f = (15.0 \times 10^{-3}\text{ kg})(1.18 \times 10^4\text{ J/kg}) = 177\text{ J}$

Q_h = heat absorbed to freeze 1.00 g of aluminum

$$= mL_f = (10^{-3}\text{ kg})(3.97 \times 10^5\text{ J/kg}) = 397\text{ J}$$

and the work output $= W = Q_h - Q_c = 220\text{ J}$

$$e = \frac{W}{Q_h} = \frac{220\text{ J}}{397\text{ J}} = 0.554\text{, or } \boxed{55.4\%}$$

$$\left[\text{Theoretical Eff (Carnot)} = \left(\frac{T_h}{T_h - T_c}\right) = \frac{933\text{ K} - 243.1\text{ K}}{933\text{ K}} = 0.749 = 74.9\%\right]$$

22.7 $T_c = 703\text{ K}$, $T_h = 2143\text{ K}$

(a) $e_C = \dfrac{\Delta T}{T_h} = \dfrac{1440}{2143} = \boxed{67.2\%}$

(b) $Q_h = 1.40 \times 10^5\text{ J}$, $W = 0.420Q_h$

$$\wp = \frac{W}{t} = \frac{5.88 \times 10^4\text{ J}}{1\text{ s}} = \boxed{58.8\text{ kW}}$$

***22.8** The Carnot efficiency of the engine is

$$e_C = \frac{\Delta T}{T_h} = \frac{120\text{ K}}{473\text{ K}} = 0.253$$

At 20.0% of this maximum efficiency,

$$e = (0.200)(0.253) = 0.0506$$

From Equation 22.2,

$$W = Q_h e \quad \text{and} \quad Q_h = \frac{W}{e} = \frac{10.0\text{ kJ}}{0.0506} = \boxed{197\text{ kJ}}$$

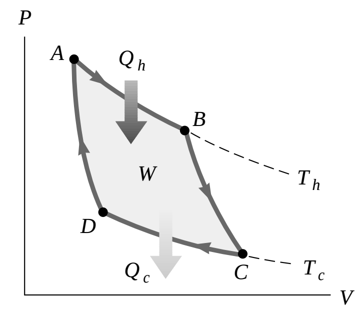
22.9 When $e = e_C$, $1 - \dfrac{T_c}{T_h} = \dfrac{W}{Q_h}$, and $\dfrac{\left(\frac{W}{t}\right)}{\left(\frac{Q_h}{t}\right)} = 1 - \dfrac{T_c}{T_h}$

(a) $Q_h = \dfrac{(W/t)t}{1 - (T_c/T_h)} = \dfrac{(1.50 \times 10^5\text{ W})(3600\text{ s})}{1 - (293/773)}$

$$Q_h = 8.69 \times 10^8\text{ J} = \boxed{869\text{ MJ}}$$

(c) $Q_c = Q_h - \left(\dfrac{W}{t}\right)t = 8.69 \times 10^8 - (1.50 \times 10^5)(3600) = 3.30 \times 10^8\text{ J} = \boxed{330\text{ MJ}}$

22.10 From Equation 22.4,



(a) $e_C = \frac{\Delta T}{T_h} = \frac{100}{373} = 0.268 = \boxed{26.8\%}$

(b) $e_C = \frac{\Delta T}{T_h} = \frac{200}{473} = 0.423 = \boxed{42.3\%}$

***22.11** Isothermal expansion at $T_h = 523$ K

Isothermal compression at $T_c = 323$ K

Gas absorbs 1200 J during expansion.

(a) $Q_c = Q_h \frac{T_c}{T_h} = (1200\text{ J})\left(\frac{323}{523}\right) = \boxed{741\text{ J}}$

(b) $W = Q_h - Q_c = (1200 - 741)\text{ J} = \boxed{459\text{ J}}$

***22.12** We use $e_C = 1 - \frac{T_c}{T_h}$

as, $0.300 = 1 - \frac{573\text{ K}}{T_h}$

From which, $T_h = 819\text{ K} = \boxed{546°\text{C}}$

22.13 The Carnot summer efficiency is

$$e_{C,s} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273 + 20)\text{K}}{(273 + 350)\text{K}} = 0.530$$

And in winter, $e_{C,w} = 1 - \frac{283}{623} = 0.546$

Then the actual winter efficiency is

$$0.320\left(\frac{0.546}{0.530}\right) = \boxed{0.330} \quad \text{or} \quad \boxed{33.0\%}$$

***22.14** (a) In an adiabatic process, $P_f V_f^{\gamma} = P_i V_i^{\gamma}$. Also, $\left(\dfrac{P_f V_f}{T_f}\right)^{\gamma} = \left(\dfrac{P_i V_i}{T_i}\right)^{\gamma}$

Dividing the second equation by the first yields $T_f = T_i \left(\dfrac{P_f}{P_i}\right)^{(\gamma-1)/\gamma}$

Since $\gamma = \dfrac{5}{3}$ for Argon, $\dfrac{\gamma - 1}{\gamma} = \dfrac{2}{5} = 0.400$ and we have

$$T_f = (1073\text{ K})\left(\frac{300 \times 10^3\text{ Pa}}{1.50 \times 10^6\text{ Pa}}\right)^{0.400} = \boxed{564\text{ K}}$$

(b) $\Delta E_{\text{int}} = nC_V\,\Delta T = Q - W = 0 - W$, so $W = -nC_V\,\Delta T$,

and the power output is

$$\wp = \frac{W}{t} = \frac{-nC_V\,\Delta T}{t} \quad \text{or}$$

$$= \frac{(-80.0\text{ kg})(1.00\text{ mol}/0.0399\text{ kg})(3/2)(8.315\text{ J/mol}\cdot\text{K})(564 - 1073)\text{ K}}{60.0\text{ s}}$$

$$\wp = 2.12 \times 10^5\text{ W} = \boxed{212\text{ kW}}$$

(c) $e_C = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{564\text{ K}}{1073\text{ K}} = 0.475$ or $\boxed{47.5\%}$

22.15 (a) $e_{\max} = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{278}{293} = 5.12 \times 10^{-2} = \boxed{5.12\%}$

(b) $\wp = \dfrac{W}{t} = 75.0 \times 10^6\text{ J/s}$

Therefore, $W = (75.0 \times 10^6\text{ J/s})(3600\text{ s/h}) = 2.70 \times 10^{11}\text{ J/h}$

From $e = \dfrac{W}{Q_h}$, we find

$$Q_h = \frac{W}{e} = \frac{2.70 \times 10^{11}\text{ J/h}}{5.12 \times 10^{-2}} = 5.27 \times 10^{12}\text{ J/h} = \boxed{5.27\text{ TJ/h}}$$

(c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

22.16 The work output is $W = \frac{1}{2} m_{\text{train}} (5.00 \text{ m/s})^2$

We are told $e = \frac{W}{Q_h}$

$$0.200 = \frac{1}{2} m_t (5.00 \text{ m/s})^2 / Q_h$$

and $e_C = 1 - \frac{300 \text{ K}}{T_h} = \frac{1}{2} m_t (6.50 \text{ m/s})^2 / Q_h$

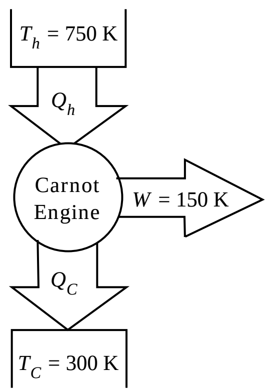
Substitute $Q_h = \frac{1}{2} m_t (5.00 \text{ m/s})^2 / 0.200$

Then, $1 - \frac{300 \text{ K}}{T_h} = 0.200 \left(\frac{\frac{1}{2} m_t (6.50 \text{ m/s})^2}{\frac{1}{2} m_t (5.00 \text{ m/s})^2} \right)$

$$1 - \frac{300 \text{ K}}{T_h} = 0.338$$

$$T_h = \frac{300 \text{ K}}{0.662} = \boxed{453 \text{ K}}$$

***22.17** For the Carnot engine,



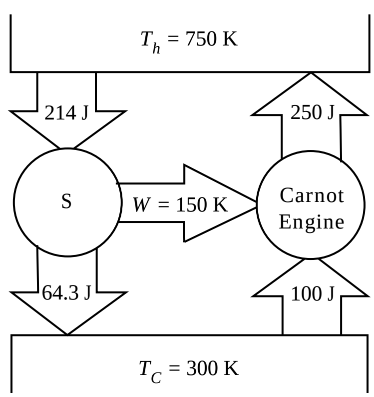
$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{300 \text{ K}}{750 \text{ K}} = 0.600$$

Also, $e_C = \frac{W}{Q_h}$, so $Q_h = \frac{W}{e_C} = \frac{150 \text{ J}}{0.600} = 250 \text{ J}$

and $Q_c = Q_h - W = 250 \text{ J} - 150 \text{ J} = 100 \text{ J}$

(a) $Q_h = \frac{W}{e_S} = \frac{150 \text{ J}}{0.700} = \boxed{214 \text{ J}}$

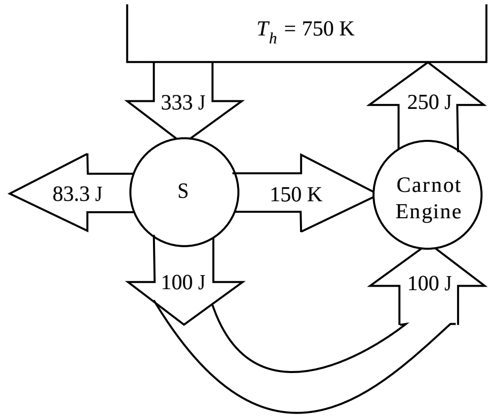
$$Q_c = Q_h - W = 214 \text{ J} - 150 \text{ J} = \boxed{64.3 \text{ J}}$$



(b) $Q_{h,\text{net}} = 214 \text{ J} - 250 \text{ J} = \boxed{-35.7 \text{ J}}$

$$Q_{c,\text{net}} = 64.3 \text{ J} - 100 \text{ J} = \boxed{-35.7 \text{ J}}$$

The flow of net heat from the cold to the hot reservoir, without work input, is impossible.



(c) For engine S: $Q_c = Q_h - W = \dfrac{W}{e_s} - W$, so

$$W = \frac{Q_c}{1/e_s - 1} = \frac{100 \text{ J}}{(1/0.700) - 1} = \boxed{233 \text{ J}}$$

and $Q_h = Q_c + W = 233 \text{ J} + 100 \text{ J} = \boxed{333 \text{ J}}$

(d) $Q_{h,\text{net}} = 333 \text{ J} - 250 \text{ J} = \boxed{83.3 \text{ J}}$

$W_{\text{net}} = 233 \text{ J} - 150 \text{ J} = \boxed{83.3 \text{ J}}$

$Q_{c,\text{ net}} = \boxed{0}$

The conversion of 83.3 J of heat entirely into work, without heat exhaust, is impossible.

(e) Both engines operate in cycles, so

$$\Delta S_s = \Delta S_{\text{Carnot}} = 0$$

For the reservoirs, $\Delta S_h = -\dfrac{Q_h}{T_h}$ and $\Delta S_c = +\dfrac{Q_c}{T_c}$

Thus,

$$\Delta S_{\text{total}} = \Delta S_s + \Delta S_{\text{Carnot}} + \Delta S_h + \Delta S_c = 0 + 0 - \frac{83.3 \text{ J}}{750 \text{ K}} + \frac{0}{300 \text{ K}} = \boxed{-0.111 \text{ J/K}}$$

A decrease in total entropy is impossible.

22.18 (a) First, consider the adiabatic process $D \to A$:

$$P_D V_D^\gamma = P_A V_A^\gamma \quad \text{so} \quad P_D = P_A \left(\frac{V_A}{V_D}\right)^\gamma = 1400 \text{ kPa} \left(\frac{10.0 \text{ L}}{15.0 \text{ L}}\right)^{5/3} = \boxed{712 \text{ kPa}}$$

Also, $\left(\dfrac{nRT_D}{V_D}\right) V_D^\gamma = \left(\dfrac{nRT_A}{V_A}\right) V_A^\gamma$, or

$$T_D = T_A \left(\frac{V_A}{V_D}\right)^{\gamma - 1} = (720 \text{ K}) \left(\frac{10.0}{15.0}\right)^{2/3} = \boxed{549 \text{ K}}$$

Now, consider the isothermal process $C \to D$:

$$T_C = T_D = \boxed{549 \text{ K}}$$

$$P_C = P_D\left(\frac{V_D}{V_C}\right) = \left[P_A\left(\frac{V_A}{V_D}\right)^{\gamma}\right]\left(\frac{V_D}{V_C}\right) = \frac{P_A V_A^{\gamma}}{V_C V_D^{\gamma-1}}$$

$$P_C = \frac{(1400 \text{ kPa})(10.0 \text{ L})^{5/3}}{(24.0 \text{ L})(15.0 \text{ L})^{2/3}} = \boxed{445 \text{ kPa}}$$

Next, consider the adiabatic process $B \to C$:

$$P_B V_B^{\gamma} = P_C V_C^{\gamma}$$

But, $P_C = \dfrac{P_A V_A^{\gamma}}{V_C V_D^{\gamma-1}}$ from above. Also, considering the isothermal

process, $P_B = P_A\left(\dfrac{V_A}{V_B}\right)$. Hence, $P_A\left(\dfrac{V_A}{V_B}\right) V_B^{\gamma} = \left(\dfrac{P_A V_A^{\gamma}}{V_C V_D^{\gamma-1}}\right) V_C^{\gamma}$ which

reduces to $V_B = \dfrac{V_A V_C}{V_D} = \dfrac{(10.0 \text{ L})(24.0 \text{ L})}{15.0 \text{ L}} = \boxed{16.0 \text{ L}}$

Finally, $P_B = P_A\left(\dfrac{V_A}{V_B}\right) = (1400 \text{ kPa})\left(\dfrac{10.0 \text{ L}}{16.0 \text{ L}}\right) = \boxed{875 \text{ kPa}}$

State	P(kPa)	V(L)	T(K)
A	1400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

(b) For the isothermal process $A \to B$:

$$\Delta E_{\text{int}} = nC_V \,\Delta T = \boxed{0}$$

so $$Q = W = nRT \ln\left(\frac{V_B}{V_A}\right) = (2.34 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(720 \text{ K}) \ln\left(\frac{16.0}{10.0}\right)$$

$$Q = W = \boxed{+6.58 \text{ kJ}}$$

For the adiabatic process $B \to C$:

$$Q = \boxed{0}$$

$$\Delta E_{\text{int}} = nC_V(T_C - T_B) = (2.34\text{ mol})\left[\frac{3}{2}\left(8.315\ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\right](549 - 720) = \boxed{-4.99\text{ kJ}}$$

and $W = Q - \Delta E_{\text{int}} = 0 - (-4.99\text{ kJ}) = \boxed{+4.99\text{ kJ}}$

For the isothermal process $C \to D$:

$$\Delta E_{\text{int}} = nC_V\,\Delta T = \boxed{0}$$

and $Q = W = nRT\ln\left(\frac{V_D}{V_C}\right) = (2.34\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(549\text{ K})\ln\left(\frac{15.0}{24.0}\right)$

$$Q = W = \boxed{-5.02\text{ kJ}}$$

Finally, for the adiabatic process $D \to A$:

$$Q = \boxed{0}$$

$$\Delta E_{\text{int}} = nC_V(T_A - T_D) = (234\text{ mol})\left[\frac{3}{2}\left(8.315\ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\right](720 - 549) = \boxed{+4.99\text{ kJ}}$$

and $W = Q - \Delta E_{\text{int}} = 0 - (+4.99\text{ kJ}) = \boxed{-4.99\text{ kJ}}$

Process	Q(kJ)	W(kJ)	ΔE_{int}(kJ)
$A \to B$	+6.58	+6.58	0
$B \to C$	0	+4.99	−4.99
$C \to D$	−5.02	−5.02	0
$D \to A$	0	−4.99	+4.99
$ABCDA$	+1.56	+1.56	0

(c) $e = \frac{W_{\text{net}}}{Q_h} = \frac{W_{ABCDA}}{Q_{A\to B}} = \frac{1.56\text{ kJ}}{6.58\text{ kJ}} = 0.237$ or $\boxed{23.7\%}$

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{549}{720} = 0.237 \quad\text{or}\quad \boxed{23.7\%}$$

22.19 (a) $P_iV_i^\gamma = P_fV_f^\gamma$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = (3.00\times 10^6\text{ Pa})\left(\frac{50.0\text{ cm}^3}{300\text{ cm}^3}\right)^{1.40} = \boxed{244\text{ kPa}}$$

(b) $W = \int_{V_i}^{V_f} P\, dV \qquad P = P_i\left(\frac{V_i}{V}\right)^{\gamma}$

Integrating,

$$W = \left(\frac{1}{\gamma - 1}\right) P_i V_i \left[1 - \left(\frac{V_i}{V_f}\right)^{\gamma - 1}\right]$$

$$= (2.50)(3.00 \times 10^6 \text{ Pa})(5.00 \times 10^{-5} \text{ m}^3)\left[1 - \left(\frac{50.0 \text{ cm}^3}{300 \text{ cm}^3}\right)^{0.400}\right] \text{J} = \boxed{192 \text{ J}}$$

Goal Solution

G: The pressure will decrease as the volume increases, so a reasonable estimate of the final pressure might be $P_f \approx \frac{50}{300}(3 \times 10^6 \text{ Pa}) = 5 \times 10^5$ Pa. As the gas expands, it does work on the piston, so $W > 0$, and the amount of work can be estimated from the average pressure and volume: $W \sim (10^6 \text{ N/m}^2)(100 \text{ cm}^3)(1 \text{ m}^3/10^6 \text{ cm}^3) = 100$ J

O: The gas expands adiabatically (there is not enough time for significant heat transfer), so equation 21.18 can be applied to find the final pressure. With $Q = 0$, the amount of work can be found from the change in internal energy.

A: (a) For adiabatic expansion, $P_i V_i^{\gamma} = P_f V_f^{\gamma}$

Therefore, $P_f = P_i\left(\frac{V_i}{V_f}\right)^{\gamma} = 3.0 \times 10^6 \text{ Pa}\left(\frac{50 \text{ cm}^3}{300 \text{ cm}^3}\right)^{1.40} = 2.44 \times 10^5$ Pa

(b) Since $Q = 0$, we have $W = Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}} = -nC_V\Delta T = -nC_V(T_f - T_i)$

From $\gamma = \frac{C_P}{C_v} = \frac{C_V + R}{C_v}$, we get $(\gamma - 1)C_V = R$

So that $C_V = \frac{R}{1.40 - 1} = 2.5R$

Therefore, $W = n(2.5R)(T_i - T_f) = 2.5P_iV_i - 2.5P_fV_f$

$$W = 2.5(3 \times 10^6 \text{ Pa})(50 \times 10^{-6} \text{ m}^3) - (2.5)(2.44 \times 10^5 \text{ Pa})(300 \times 10^{-6} \text{ m}^3) = 192 \text{ J}$$

L: The final pressure is about half what we predicted because we assumed a linear proportionality ($\gamma = 1$) in our initial estimate, when in fact $\gamma = 1.40$. The work done is about twice what we predicted, and the difference is again because our estimate assumed a linear relationship when this is not the case.
From the work done by the gas in part (a), the average power (horsepower) of the engine could be calculated if the time for one cycle was known. Adiabatic expansion is the power stroke of our industrial civilization!

22.20 Compression ratio = 6.00, $\gamma = 1.40$

(a) Efficiency of an Otto-engine $e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$

$$e = 1 - \left(\frac{1}{6.00}\right)^{0.400} = \boxed{51.2\%}$$

(b) If actual efficiency $e' = 15.0\%$ losses in system are $e - e' = \boxed{36.2\%}$

22.21 $e_{\text{Otto}} = 1 - \frac{1}{(V_1 / V_2)^{\gamma - 1}} = 1 - \frac{1}{(6.20)^{(7/5 - 1)}} = 1 - \frac{1}{(6.20)^{0.400}}$

$e_{\text{Otto}} = 0.518$

We have assumed the fuel-air mixture to behave like a diatomic gas.

Now $e = W / Q_h = (W / t) / (Q_h / t)$

$$Q_h / t = (W / t) / e = 102 \text{ hp } (746 \text{ W} / 1 \text{ hp}) / 0.518$$

$$Q_h / t = \boxed{146 \text{ kW}}$$

$$Q_h = W + Q_c$$

$$Q_c / t = Q_h / t - W / t$$

$$Q_h / t = 146 \times 10^3 \text{ W} - 102 \text{ hp}(746 \text{ W} / 1 \text{ hp}) = \boxed{70.8 \text{ kW}}$$

***22.22** (a) and (b) The quantity of gas is

$$n = \frac{P_A V_A}{R T_A} = \frac{(100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)}{(8.315 \text{ J} / \text{mol} \cdot \text{K})(293 \text{ K})} = 0.0205 \text{ mol}$$

$$E_{\text{int},A} = \frac{5}{2} nRT_A = \frac{5}{2} P_A V_A = \frac{5}{2}(100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3) = \boxed{125 \text{ J}}$$

In process AB, $P_B = P_A \left(\frac{V_A}{V_B}\right)^{\gamma} = (100 \times 10^3 \text{ Pa})(8.00)^{1.40} = \boxed{1.84 \times 10^6 \text{ Pa}}$

$$T_B = \frac{P_B V_B}{nR} = \frac{(100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3 / 8.00)}{(0.0205 \text{ mol})(8.315 \text{ J} / \text{mol} \cdot \text{K})} = \boxed{673 \text{ K}}$$

$$E_{\text{int},B} = \frac{5}{2} nRT_B = \frac{5}{2}(0.0205 \text{ mol})(8.315 \text{ J} / \text{mol} \cdot \text{K})(673 \text{ K}) = \boxed{287 \text{ J}}$$

so $\Delta E_{\text{int},AB} = 287 \text{ J} - 125 \text{ J} = \boxed{162 \text{ J}} = Q - W = 0 - W \qquad W_{AB} = \boxed{-162 \text{ J}}$

Process BC takes us to:

$$P_C = \frac{nRT_C}{V_C} = \frac{(0.0205\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(1023\text{ K})}{62.5\times10^{-6}\text{ m}^3} = \boxed{2.79\times10^6\text{ Pa}}$$

$$E_{\text{int},C} = \frac{5}{2}nRT_C = \frac{5}{2}(0.0205\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(1023\text{ K}) = \boxed{436\text{ J}}$$

$$\Delta E_{\text{int},BC} = 436\text{ J} - 287\text{ J} = \boxed{149\text{ J}} = Q - W = Q - 0 \qquad Q_{BC} = \boxed{149\text{ J}}$$

In process CD:

$$P_D = P_C\left(\frac{V_C}{V_D}\right)^{\gamma} = (2.79\times10^6\text{ Pa})\left(\frac{1}{8.00}\right)^{1.40} = \boxed{1.52\times10^5\text{ Pa}}$$

$$T_D = \frac{P_DV_D}{nR} = \frac{(1.52\times10^5\text{ Pa})(500\times10^{-6}\text{ m}^3)}{(0.0205\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = \boxed{445\text{ K}}$$

$$E_{\text{int},D} = \frac{5}{2}nRT_D = \frac{5}{2}(0.0205\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(445\text{ K}) = \boxed{190\text{ J}}$$

$$\Delta E_{\text{int},CD} = 190\text{ J} - 436\text{ J} = \boxed{-246\text{ J}} = Q - W = 0 - W \qquad W_{CD} = \boxed{246\text{ J}}$$

and $\Delta E_{\text{int},DA} = E_{\text{int},A} - E_{\text{int},D} = 125\text{ J} - 190\text{ J} = \boxed{-65.0\text{ J}} = Q - W = Q - 0$

$$Q_{DA} = \boxed{-65.0\text{ J}}$$

For the entire cycle, $\Delta E_{\text{int, net}} = 162\text{ J} + 149 - 246 - 65.0 = \boxed{0}$

$$W_{\text{net}} = -162\text{ J} + 0 + 246\text{ J} + 0 = \boxed{84.3\text{ J}}$$

$$Q_{\text{net}} = 0 + 149\text{ J} + 0 - 65.0\text{ J} = \boxed{84.3\text{ J}}$$

The tables look like:

State	T(K)	P(kPa)	V(cm^3)	E_{int} (J)
A	293	100	500	125
B	673	1840	62.5	287
C	1023	2790	62.5	436
D	445	152	500	190
A	293	100	500	125

Process	Q(J)	W(J)	ΔE_{int}(J)
AB	0	–162	162
BC	149	0	149
CD	0	246	–246
DA	–65.0	0	–65.0

ABCDA	84.3	84.3	0

(c) The input energy is $Q_h = \boxed{149 \text{ J}}$, the waste is $Q_c = \boxed{65.0 \text{ J}}$, and $W_{\text{net}} = \boxed{84.3 \text{ J}}$

(d) The efficiency is: $e = \dfrac{W_{\text{net}}}{Q_h} = \dfrac{84.3 \text{ J}}{149 \text{ J}} = \boxed{0.565}$

(e) Let f represent the angular speed of the crankshaft. Then $f/2$ is the frequency at which we obtain work in the amount of 84.3 J/cycle:

$$1000 \text{ J/s} = (f/2)(84.3 \text{ J/cycle})$$

$$f = \frac{2000 \text{ J/s}}{84.3 \text{ J/cycle}} = 23.7 \text{ rev/s} = \boxed{1.42 \times 10^3 \text{ rev/min}}$$

***22.23** $(\text{COP})_{\text{refrig}} = \dfrac{T_c}{\Delta T} = \dfrac{270}{30.0} = \boxed{9.00}$

22.24 $(\text{COP})_{\text{heat pump}} = \dfrac{Q_c + W}{W} = \dfrac{T_h}{\Delta T} = \dfrac{295}{25} = \boxed{11.8}$

22.25 (a) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $W = Q_h - Q_c = Q_c\left[\dfrac{Q_h}{Q_c} - 1\right]$

We have already shown that for a Carnot cycle (and only for a Carnot cycle)

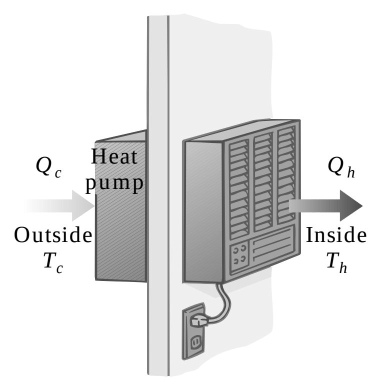
$$\frac{Q_h}{Q_c} = \frac{T_h}{T_c}$$

Therefore, $W = \boxed{Q_c\left[\dfrac{T_h - T_c}{T_c}\right]}$

(b) We have from Equation 22.7, $\text{COP} = \dfrac{Q_c}{W}$.

Using the result from part (a), this becomes

$$\text{COP} = \boxed{\frac{T_c}{T_h - T_c}}$$



***22.26** $\text{COP} = 0.100\ \text{COP}_{\text{Carnot Cycle}}$ or

$$\frac{Q_h}{W} = 0.100\left(\frac{Q_h}{W}\right)_{\text{Carnot Cycle}} = 0.100\left(\frac{1}{\text{Carnot efficiency}}\right)$$

$$= 0.100\left(\frac{T_h}{T_h - T_c}\right) = 0.100\left(\frac{293 \text{ K}}{293 \text{ K} - 268 \text{ K}}\right) = 1.17$$

Thus, $\boxed{\text{1.17 Joules of heat are delivered for each joule of work done.}}$

22.27 $(\text{COP})_{\text{Carnot refrig}} = \dfrac{T_c}{\Delta T} = \dfrac{4.00}{289}$ $\quad \therefore W = \boxed{72.2\ \text{J}}$ per 1 J heat removed.

22.28 $(\text{COP})_{\text{Carnot refrig}} = \dfrac{Q_c}{W} = \dfrac{T_c}{T_h - T_c}$ $\qquad$ Thus, $W = \boxed{Q\left(\dfrac{T_h - T_c}{T_c}\right)}$

22.29 $\text{COP(refrigerator)} = \dfrac{Q_c}{W}$

(a) If $Q_c = 120$ J and COP = 5.00, then $\boxed{W = 24.0\ \text{J}}$

(b) Heat expelled = Heat removed + Work done.

$$Q_h = Q_c + W = 120\ \text{J} + 24\ \text{J} = \boxed{144\ \text{J}}$$

***22.30** A Carnot refrigerator runs on minimum power.

For it: $\dfrac{Q_h}{T_h} = \dfrac{Q_c}{T_c}$ $\quad$ so $\quad \dfrac{Q_h/t}{T_h} = \dfrac{Q_c/t}{T_c}$

Solving part (b) first:

(b) $\dfrac{Q_h}{t} = \dfrac{Q_c}{t}\left(\dfrac{T_h}{T_c}\right) = (8.00\ \text{MJ/h})\left(\dfrac{298\ \text{K}}{273\ \text{K}}\right) = \left(8.73 \times 10^6\ \dfrac{\text{J}}{\text{h}}\right)\left(\dfrac{1\ \text{h}}{3600\ \text{s}}\right) = \boxed{2.43\ \text{kW}}$

(a) $\dfrac{W}{t} = \dfrac{Q_h}{t} - \dfrac{Q_c}{t} = 2.43\ \text{kW} - \dfrac{8.00 \times 10^6\ \text{J/h}}{3600\ \text{s/h}} = \boxed{204\ \text{W}}$

22.31 For a freezing process,

$$\Delta S = \frac{\Delta Q}{T} = \frac{-(0.500\ \text{kg})(3.33 \times 10^5\ \text{J/kg})}{273\ \text{K}} = \boxed{-610\ \text{J/K}}$$

22.32 At a constant temperature of 4.20 K,

$$\Delta S = \frac{\Delta Q}{T} = \frac{L_v}{4.20\ \text{K}} = \frac{20.5\ \text{kJ/kg}}{4.20\ \text{K}}$$

$$\Delta S = \boxed{4.88\ \text{kJ/kg}\cdot\text{K}}$$

22.33 $\Delta S = \displaystyle\int_i^f \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc \ln \frac{T_f}{T_i}$

$$\Delta S = (250\ \text{g})(1.00\ \text{cal/g}\cdot\text{C°}) \ln\left(\frac{353}{293}\right) = 46.6\ \text{cal/K} = \boxed{195\ \text{J/K}}$$

22.34 From Equation 22.12,

(a) $\Delta S = nC_V \ln(T_f / T_i) + nR \ln(V_f / V_i)$

$$= n\left(\frac{5}{2}\right) R \ln(255\text{ K}/298\text{ K}) + 0$$

$$= -0.390(2.50\text{ mol})(8.315\text{ J/mol}\cdot\text{K}) = \boxed{-8.10\text{ J/K}}$$

(b) The volume now decreases with $V_f / V_i = T_f / T_i$

$$\Delta S = n\left(\frac{5}{2}\right) R \ln(0.856) + nR\ln(0.856)$$

$$= n\left(\frac{7}{2}\right) R \ln(0.856) = nC_P \ln(T_f / T_i)$$

$$= -0.545(2.50\text{ mol})(8.315\text{ J/mol}\cdot\text{K}) = \boxed{-11.3\text{ J/K}}$$

22.35 $\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = \left(\frac{1000}{290} - \frac{1000}{5700}\right)\text{ J/K} = \boxed{3.27\text{ J/K}}$

22.36 $c_{\text{iron}} = 448\text{ J/kg °C}\, c_{\text{water}} = 4186\text{ J/kg °C}$

$Q_{\text{cold}} = -Q_{\text{hot}}$

$$(4.00\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{C°}}\right)(T_f - 10.0\text{°C}) = -(1.00\text{ kg})\left(448\frac{\text{J}}{\text{kg}\cdot\text{C°}}\right)(T_f - 900\text{°C})$$

which yields $T_f = 33.2\text{°C} = 306.2\text{ K}$

$$\therefore \quad \Delta S = \int_{283\text{ K}}^{306.2} \frac{c_{\text{water}} m_{\text{water}} dT}{T} + \int_{T = 1173\text{ K}}^{306.2} \frac{c_{\text{iron}} m_{\text{iron}} dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\frac{306.2}{283} + c_{\text{iron}} m_{\text{iron}} \ln\frac{306.2}{1173}$$

$$= (4186\text{ J/kg}\cdot\text{K})(4.00\text{ kg})(0.0788) + (448\text{ J/kg}\cdot\text{K})(1.00\text{ kg})(-1.34)$$

$$\Delta S = \boxed{718\text{ J/K}}$$

***22.37** $\Delta S = \frac{\frac{1}{2}mv^2}{T} = \frac{750(20.0)^2}{293}\text{ J/K} = \boxed{1.02\text{ kJ/K}}$

***22.38** Sitting here writing, I convert chemical energy, in ordered molecules in food, into heat that I put out to the room-temperature surroundings. My heating power is my metabolic rate,

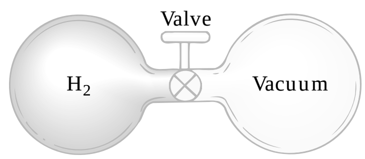
$$2500\ \text{kcal/d} = \frac{2500\times 10^3\ \text{cal}}{86\,400\ \text{s}}\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right) = 120\ \text{W}$$

My body is in steady state, changing little in entropy, as the environment increases in entropy at the rate

$$\frac{\Delta S}{t} = \frac{Q/T}{t} = \frac{Q/t}{T} = \frac{120\ \text{W}}{293\ \text{K}} = 0.4\ \text{W/K} \sim \boxed{1\ \text{W/K}}$$

When using powerful appliances or an automobile, my personal contribution to entropy production is much greater than the above estimate, based only on metabolism.

22.39 $\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = R\ln 2 = \boxed{5.76\ \text{J/K}}$



There is $\boxed{\text{no change in temperature}}$.

***22.40** (a) $V = \frac{nRT_i}{P_i} = \frac{(40.0\ \text{g})(8.315\ \text{J/mol}\cdot\text{K})(473\ \text{K})}{(39.9\ \text{g/mol})(100\times 10^3\ \text{Pa})} = 39.4\times 10^{-3}\ \text{m}^3 = \boxed{39.4\ \text{L}}$

(b) $\Delta E_{\text{int}} = nC_V\Delta T = \left(\frac{40.0\ \text{gm}}{39.9\ \text{g/mol}}\right)\left[\frac{3}{2}\left(8.315\ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\right](-200\ \text{C}°) = \boxed{-2.50\ \text{kJ}}$

(c) $W = 0$ so $Q = \Delta E_{\text{int}} = \boxed{-2.50\ \text{kJ}}$

(d) $\Delta S_{\text{argon}} = \int_i^f \frac{dQ}{T} = nC_V\ln\left(\frac{T_f}{T_i}\right)$

$$= \left(\frac{40.0\ \text{g}}{39.9\ \text{g/mol}}\right)\left[\frac{3}{2}(8.315\ \text{J/mol}\cdot\text{K})\right]\ln\left(\frac{273}{473}\right) = \boxed{-6.87\ \text{J/K}}$$

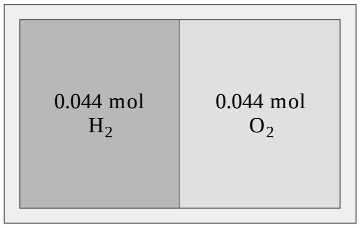
(e) $\Delta S_{\text{bath}} = \frac{2.50\ \text{kJ}}{273\ \text{K}} = \boxed{+9.16\ \text{J/K}}$

The total change in entropy is

$$\Delta S_{\text{total}} = \Delta S_{\text{argon}} + \Delta S_{\text{bath}} = -6.87\ \text{J/K} + 9.16\ \text{J/K} = +2.29\ \text{J/K}$$

$\Delta S_{\text{total}} > 0$ for this irreversible process.

22.41 $\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = (0.0440)(2)\,R\ln 2 = (0.0880)(8.315)\ln 2 = \boxed{0.507\ \mathrm{J/K}}$



22.42 $\Delta S = \int_{T_i = 268\ \mathrm{K}}^{T_f = 273\ \mathrm{K}} \frac{mc_{\mathrm{ice}}\,dT}{T} + \frac{mL_{\mathrm{ice}}}{273} = mc_{\mathrm{ice}}\ln\left(\frac{273}{268}\right) + \frac{mL_{\mathrm{ice}}}{273}$

$$\Delta S = (10^5\ \mathrm{kg})\left(2010\ \frac{\mathrm{J}}{\mathrm{kg\ °C}}\right)\ln\left(\frac{273}{268}\right) + \frac{(10^5)(3.33\times 10^5)}{273}\ \mathrm{J/K} = \boxed{1.26\times 10^8\ \mathrm{J/K}}$$

22.43 From Equation 22.12, $\Delta S = nC_V\ln\frac{T_f}{T_i} + nR\ln\frac{V_f}{V_i}$

and from the ideal gas law, $\frac{T_f}{T_i} = \frac{P_fV_f}{P_iV_i}$

$$\text{Thus, } \Delta S = (1.00\ \mathrm{mol})\left[\frac{3}{2}\left(8.315\ \frac{\mathrm{J}}{\mathrm{mole\cdot K}}\right)\right]\ln\left(\frac{(2.00)(0.0400)}{(1.00)(0.0250)}\right)$$

$$+ (1.00\ \mathrm{mol})\left(8.315\ \frac{\mathrm{J}}{\mathrm{mole\cdot K}}\right)\ln\left(\frac{0.0400}{0.0250}\right)$$

$$\Delta S = \boxed{18.4\ \mathrm{J/K}}$$

22.44 $\Delta S = nC_V\ln(T_f/T_i) + nR\ln(V_f/V_i)$

$$= (1.00\ \mathrm{mol})\left[\frac{5}{2}(8.315\ \mathrm{J/mol\cdot K})\right]\ln\left(\frac{2P\cdot 2V}{PV}\right)$$

$$+ (1.00\ \mathrm{mol})\left(8.315\ \frac{\mathrm{J}}{\mathrm{mol\cdot K}}\right)\ln\left(\frac{2V}{V}\right)$$

$$\Delta S = \boxed{34.6\ \mathrm{J/K}}$$

***22.45** (a) A 12 can only be obtained $\boxed{\text{one}}$ way 6 + 6

(b) A 7 can be obtained $\boxed{\text{six}}$ ways: 6 + 1, 5 + 2, 4 + 3, 3 + 4, 2 + 5, 1 + 6

***22.46** (a) The table is shown below. On the basis of the table, the most probable result of a toss is $\boxed{\text{2 heads and 2 tails}}$.

(b) The most ordered state is the least likely state. Thus, on the basis of the table this is $\boxed{\text{either all heads or all tails}}$.

(c) The most disordered is the most likely state. Thus, this is $\boxed{\text{2 heads and 2 tails}}$.

Result	Possible Combinations	Total
All heads	HHHH	1
3H, 1T	THHH, HTHH, HHTH, HHHT	4
2H, 2T	TTHH, THTH, THHT, HTTH HTHT, HHTT	6
1H, 3T	HTTT, THTT, TTHT, TTTH	4
All tails	TTTT	1

***22.47** (a)

Result	Possible Combinations	Total
All red	RRR	1
2R, 1G	RRG, RGR, GRR	3
1R, 2G	RGG, GRG, GGR	3
All green	GGG	1

(b)

Result	Possible Combinations	Total
All red	RRRRR	1
4R, 1G	RRRRG, RRRGR, RRGRR, RGRRR, GRRRR	5
3R, 2G	RRRGG, RRGRG, RGRRG, GRRRG, RRGGR, RGRGR, GRRGR, RGGRR, GRGRR, GGRRR	10
2R, 3G	GGGRR, GGRGR, GRGGR, RGGGR, GGRRG, GRGRG, RGGRG, GRRGG, RGRGG, RRGGG	10
1R, 4G	RGGGG, GRGGG, GGRGG, GGGRG, GGGGR	5
All green	GGGGG	1

***22.48** The conversion of gravitational potential energy into kinetic energy as the water falls is reversible. But the subsequent conversion into internal energy is not. We imagine arriving at the same final state by adding heat, in amount *mgy*, to the water from a stove at a temperature infinitesimally above 20.0°C. Then,

$$\Delta S = \int \frac{dQ}{T} = \frac{Q}{T} = \frac{mgy}{T}$$

$$= \frac{(5000\ \text{m}^3)(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(50.0\ \text{m})}{293\ \text{K}} = \boxed{8.36 \times 10^6\ \text{J/K}}$$

22.49 $e = \dfrac{W}{Q_h} = 0.350 \qquad W = 0.350Q_h$

$Q_h = W + Q_c \qquad Q_c = 0.650Q_h$

$$\text{COP(refrigerator)} = \frac{Q_c}{W} = \frac{0.650Q_h}{0.350Q_h} = \boxed{1.86}$$

22.50 $Q_c = mc\,\Delta T + mL + mc\,\Delta T$

$$= 0.500\text{ kg}(4186\text{ J/kg}\cdot\text{C°})10.0\text{ C°} + 0.500\text{ kg}(3.33\times 10^5\text{ J/kg})$$

$$+ 0.500\text{ kg}(2090\text{ J/kg}\cdot\text{C°})20.0\text{ C°} = 2.08\times 10^5\text{ J}$$

$$\frac{Q_c}{W} = \text{COP}_c\text{ (refrigerator)} = \frac{T_c}{T_h - T_c}$$

$$W = \frac{Q_c\,(T_h - T_c)}{T_c} = \frac{(2.08\times 10^5\text{ J})\,[20.0\text{°C} - (-20.0\text{°C})]}{(273 - 20.0)\text{K}} = \boxed{32.9\text{ kJ}}$$

22.51 $\dfrac{dQ}{dt} = 5000\text{ W} \qquad T_h = 295\text{ K} \qquad T_c = 268\text{ K}$

(a) If $\dfrac{\Delta Q}{\Delta t} = \dfrac{\Delta E}{\Delta t}$ then $\boxed{\wp_{\text{El}} = 5.00\text{ kW}}$

(b) For a heat pump,

$$(\text{COP})_{\text{Carnot}} = \frac{T_h}{\Delta T} = \frac{295}{27.0} = 10.9$$

Actual COP = (0.600)(10.9) = 6.55

Therefore, to bring 5000 W of heat into the house only requires $\boxed{763\text{ W}}$

Goal Solution

G: The electric heater should be 100% efficient, so P = 5 kW. The heat pump is only 60% efficient, so we might expect P = 9 kW.

O: Power is the change of energy per unit of time, so we can find the power for both cases by examining the change in heat energy.

A: (a) $P_{\text{electric}} = \dfrac{\Delta E}{\Delta t}$ so if all of the electricity is converted into internal energy, $\Delta E = \Delta Q$.

Therefore, $P_{\text{electric}} = \dfrac{\Delta Q}{\Delta t} = 5000\text{ W}$

(b) For a heat pump, $(\text{COP})_{\text{Carnot}} = \dfrac{T_h}{\Delta T} = \dfrac{295\text{ K}}{27\text{ K}} = 10.92$

Actual COP = (0.6)(10.92) = 6.55 $= \dfrac{Q_h}{W} = \dfrac{Q_h/t}{W/t}$

Therefore, to bring 5000 W of heat into the house only requires input power

$$P_{\text{heat pump}} = \frac{W}{t} = \frac{Q_h/t}{\text{COP}} = \frac{5000\text{ W}}{6.56} = 763\text{ W}$$

L: The result for the electric heater's power is consistent with our prediction, but the heat pump actually requires *less* power than we expected. Since both types of heaters use electricity to operate, we can now see why it is more cost effective to use a heat pump even though it is less than 100% efficient!

***22.52** $\Delta S_{\text{hot}} = \dfrac{-1000}{600}\dfrac{\text{J}}{\text{K}}$

$\Delta S_{\text{cold}} = \dfrac{+750}{350}\dfrac{\text{J}}{\text{K}}$

(a) $\Delta S_U = \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = \boxed{0.476 \text{ J/K}}$

(b) $e_C = 1 - \dfrac{T_1}{T_2} = 0.417$

$W = Q_h e_C = (1000 \text{ J})(0.417) = \boxed{417 \text{ J}}$

(c) $W_{\text{net}} = 417 \text{ J} - 250 \text{ J} = 167 \text{ J}$

$T_1\, \Delta S_U = (350 \text{ K})(0.476 \text{ J/K}) = \boxed{167 \text{ J}}$

22.53 (a) For an isothermal process, $Q = nRT \ln\left(\dfrac{V_2}{V_1}\right)$

Therefore, $Q_1 = nR\,(3T_i) \ln 2$ and $Q_3 = nR(T_i) \ln (1/2)$

For the constant volume processes, we have

$Q_2 = \Delta E_{\text{int},2} = \dfrac{3}{2}\, nR\,(T_i - 3T_i)$ and $Q_4 = \Delta E_{\text{int},4} = \dfrac{3}{2}\, nR\,(3T_i - T_i)$

The net heat transferred is then

$Q = Q_1 + Q_2 + Q_3 + Q_4$ or $Q = \boxed{2nRT_i \ln 2}$

(b) Heat > 0 is the heat added to the system. Therefore,

$Q_h = Q_1 + Q_4 = 3nRT_i\,(1 + \ln 2)$

Since the change in temperature for the complete cycle is zero,

$\Delta E_{\text{int}} = 0$ and $W = Q$

Therefore, the efficiency is

$$e_C = \frac{W}{Q_h} = \frac{Q}{Q_h} = \frac{2 \ln 2}{3(1 + \ln 2)} = \boxed{0.273}$$

***22.54** $\text{COP} = 3.00 = \dfrac{Q_c}{W}$ Therefore, $W = \dfrac{Q_c}{3.00}$

The heat removed each minute is

$$\frac{Q_C}{t} = (0.0300\text{ kg})(4186\text{ J/kg°C})(22.0°\text{C}) + (0.0300\text{ kg})(3.33\times10^5\text{ J/kg})$$

$$+ (0.0300\text{ kg})(2090\text{ J/kg°C})(20.0°\text{C}) = 1.40\times10^4\text{ J/min}$$

or, $\dfrac{Q_c}{t} = 233\text{ J/s}$

Thus, the work done per sec $= \wp = \dfrac{233\text{ J/s}}{3.00} = \boxed{77.8\text{ W}}$

22.55 $\dfrac{Q_c}{W} = \text{COP}_C\text{ (refrigerator)} = \dfrac{T_c}{T_h - T_c} = \dfrac{Q_c/t}{W/t}$

$$\frac{0.150\text{ W}}{W/t} = \frac{260\text{ K}}{40.0\text{ K}}$$

$$\wp = \frac{W}{t} = (0.150\text{ W})\frac{40.0\text{ K}}{260\text{ K}} = \boxed{23.1\text{ mW}}$$

22.56 (a) $\dfrac{W}{t} = 1.50\times10^8\text{ W}_{\text{(electrical)}}$, $Q = mL = \left[\dfrac{W/t}{0.150}\right]\Delta t$,

and $L = 33.0\text{ kJ/g} = 33.0\times10^6\text{ J/kg}$

$$m = \left[\frac{W/t}{0.150}\right]\Delta t/L$$

$$m = \frac{(1.50\times10^8\text{ W})(86{,}400\text{ s/day})}{0.150(33.0\times10^6\text{ J/kg})(10^3\text{ kg/metric ton})} = \boxed{2620\text{ metric tons/day}}$$

(b) Cost = (\$8.00 / metric ton)(2618 metric tons / day)(365 days / y)

Cost = $\boxed{\$7.65\text{ million/year}}$

(c) First find the rate at which heat energy is discharged into the water. If the plant is 15.0% efficient in producing electrical energy then the rate of heat production is

$$\frac{Q_c}{t} = \left(\frac{W}{t}\right)\left(\frac{1}{e} - 1\right) = (1.50\times10^8\text{ W})\left(\frac{1}{0.150} - 1\right) = 8.50\times10^8\text{ W}$$

Then, $\dfrac{Q_c}{t} = \dfrac{mc\,\Delta T}{t}$ and

$$\frac{m}{t} = \frac{Q_c / t}{c\,\Delta T} = \frac{8.50 \times 10^8 \text{ J/s}}{(4186 \text{ J/kg} \cdot \text{C°})(5.00 \text{ C°})} = \boxed{4.06 \times 10^4 \text{ kg/s}}$$

***22.57** $e_C = 1 - \frac{T_c}{T_h} = \frac{W}{Q_h} = \frac{(W/t)}{(Q_h/t)}$

$$\frac{Q_h}{t} = \frac{\wp}{\left(1 - \frac{T_c}{T_h}\right)} = \frac{\wp T_h}{(T_h - T_c)}$$

$$Q_h = W + Q_c$$

$$\frac{Q_c}{t} = \frac{Q_h}{t} - \frac{W}{t}$$

$$\frac{Q_c}{t} = \frac{\wp T_h}{(T_h - T_c)} - \wp = \frac{\wp T_c}{(T_h - T_c)}$$

$$Q_c = mc\,\Delta T$$

$$\frac{Q_c}{t} = \left(\frac{m}{t}\right) c\,\Delta T = \frac{\wp T_c}{(T_h - T_c)}$$

$$\frac{m}{t} = \frac{\wp T_c}{(T_h - T_c)\, c\,\Delta T}$$

$$\frac{m}{t} = \frac{(1.00 \times 10^9\ \text{W})(300\ \text{K})}{(200\ \text{K})(4186\ \text{J/kg}\cdot\text{C}°)(6.00\ \text{C}°)} = \boxed{5.97 \times 10^4\ \text{kg/s}}$$

22.58 $e_C = 1 - \frac{T_c}{T_h} = \frac{W}{Q_h} = \frac{\left(\frac{W}{t}\right)}{\left(\frac{Q_h}{t}\right)}$

$$\left(\frac{Q_h}{t}\right) = \frac{\wp}{\left(1 - \frac{T_c}{T_h}\right)} = \frac{\wp T_h}{(T_h - T_c)}$$

$$\left(\frac{Q_c}{t}\right) = \left(\frac{Q_h}{t}\right) - \wp = \frac{\wp T_c}{(T_h - T_c)}$$

$Q_c = mc\,\Delta T$, where c = the specific heat of water.

$$\therefore \left(\frac{Q_c}{t}\right) = \left(\frac{m}{t}\right) c\,\Delta T = \frac{\wp T_c}{(T_h - T_c)}$$

$$\therefore \frac{m}{t} = \boxed{\frac{\wp T_c}{(T_h - T_c)\, c\,\Delta T}}$$

***22.59** (a) $35.0°\text{F} = \frac{5}{9}(35.0 - 32.0)\ °\text{C} = (1.67 + 273.15)\text{K} = 274.82\ \text{K}$

$98.6°\text{F} = \frac{5}{9}(98.6 - 32.0)\ °\text{C} = (37.0 + 273.15)\text{K} = 310.15\ \text{K}$

$$\Delta S_{\text{ice water}} = \int \frac{dQ}{T} = (453.6\ \text{g})(1.00\ \text{cal/g}\cdot\text{K}) \times \int_{274.82}^{310.15} \frac{dT}{T}$$

$$= 453.6 \ln\left(\frac{310.15}{274.82}\right) = 54.86\ \text{cal/K}$$

$$\Delta S_{\text{body}} = -\frac{|Q|}{T_{\text{body}}} = -(453.6)(1.00)\frac{(310.15 - 274.82)}{310.15} = -51.67\ \text{cal/K}$$

$$\Delta S_{\text{system}} = 54.86 - 51.67 = \boxed{3.19\ \text{cal/K}}$$

(b) $(453.6)(1)(T_F - 274.82) = (70.0 \times 10^3)(1)(310.15 - T_F)$

Thus,

$$(70.0 + 0.4536) \times 10^3\ T_F = [(70.0)(310.15) + (0.4536)(274.82)] \times 10^3$$

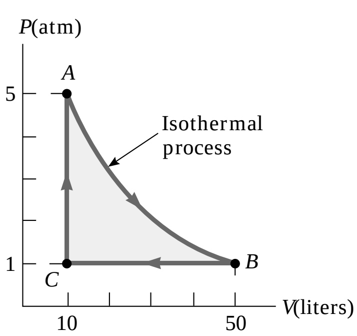
and $T_F = 309.92\ \text{K} = 36.77°\text{C} = \boxed{98.19°\text{F}}$

$$\Delta S'_{\text{ice water}} = 453.6 \ln\left(\frac{309.92}{274.82}\right) = 54.52\ \text{cal/K}$$

$$\Delta S'_{\text{body}} = -(70.0 \times 10^3) \ln\left(\frac{310.15}{309.92}\right) = -51.93\ \text{cal/K}$$

$\Delta S'_{\text{sys}} = 54.52 - 51.93 = \boxed{2.59\ \text{cal/K}}$ which is less than the estimate in part (a).

22.60 (a) For the isothermal process AB,



$$W_{AB} = P_A V_A \ln\left(\frac{V_B}{V_A}\right)$$

$$W_{AB} = (5)(1.013 \times 10^5\ \text{Pa})(10.0 \times 10^{-3}\ \text{m}^3) \ln\left(\frac{50.0}{10.0}\right)$$

$$W_{AB} = 8.15 \times 10^3\ \text{J}$$

where we have used 1.00 atm = 1.013×10^5 Pa

and 1.00 L = 1.00×10^{-3} m^3.

$$W_{BC} = P_B\,\Delta V = (1.013 \times 10^3 \text{ Pa})[(10.0 - 50.0) \times 10^{-3}]\text{m}^3 = -4.05 \times 10^3 \text{ J}$$

$$W_{CA} = 0 \quad \text{and} \quad W = W_{AB} + W_{BC} = 4.11 \times 10^3 \text{ J} = \boxed{4.11 \text{ kJ}}$$

(b) Since AB is an isothermal process, $\Delta E_{\text{int},AB} = 0$, and

$$Q_{AB} = W_{AB} = 8.15 \times 10^3 \text{ J}$$

For an ideal monatomic gas, $C_V = 3R/2$ and $C_P = 5R/2$.

$$T_B = T_A = P_B V_B / nR = (1.013 \times 10^5)(50.0 \times 10^{-3})/R = 5.05 \times 10^3/R$$

Also, $T_C = P_C V_C / nR = (1.013 \times 10^5)(10.0 \times 10^{-3})/R = 1.01 \times 10^3/R$

$$Q_{\text{CA}} = nC_V\,\Delta T = (1.00)\left(\frac{3}{2}R\right)\left(\frac{5.05 \times 10^3 - 1.01 \times 10^3}{R}\right) = 6.08 \text{ kJ}$$

so the total heat absorbed is $Q_{AB} + Q_{CA} = 8.15 \text{ kJ} + 6.08 \text{ kJ} = \boxed{14.2 \text{ kJ}}$

(c) $Q_{BC} = nC_P\,\Delta T = \frac{5}{2}(nR\,\Delta T) = \frac{5}{2} P_B\,\Delta V_{BC}$

$$Q_{BC} = \frac{5}{2}(1.013 \times 10^5)\,[(10.0 - 50.0) \times 10^{-3}] = -1.01 \times 10^4 \text{ J} = \boxed{-10.1 \text{ kJ}}$$

(d) $e = \frac{W}{Q_h} = \frac{W}{Q_{AB} + Q_{CA}} = \frac{4.11 \times 10^3 \text{ J}}{1.42 \times 10^4 \text{ J}} = 0.289$ or $\boxed{28.9\%}$

22.61 Define T_1 = Temp Cream = 5.00°C = 278 K

Define T_2 = Temp Coffee = 60.0°C = 333 K

The final temperature of the mixture is:

$$T_f = \frac{(20.0 \text{ g})T_1 + (200 \text{ g})T_2}{220 \text{ g}} = 55.0°\text{C} = 328 \text{ K}$$

The entropy change due to this mixing is

$$\Delta S = (20.0\text{ g})\int_{T_1}^{T_f}\frac{c_V dT}{T} + (200\text{ g})\int_{T_2}^{T_f}\frac{c_V dT}{T}$$

$$\Delta S = (84.0\text{ J/K})\ln\left(\frac{T_f}{T_1}\right) + (840\text{ J/K})\ln\left(\frac{T_f}{T_2}\right)$$

$$= (84.0\text{ J/K})\ln\left(\frac{328}{278}\right) + (840\text{ J/K})\ln\left(\frac{328}{333}\right)$$

$$\Delta S = \boxed{+1.18\text{ J/K}}$$

***22.62** (a) $10.0\,\frac{\text{Btu}}{\text{h}\cdot\text{W}}\left(\frac{1055\text{ J}}{1\text{ Btu}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1\text{ W}}{1\text{ J/s}}\right) = \boxed{2.93}$

(b) Coefficient of performance for a refrigerator: $\boxed{(\text{COP})_{\text{refrigerator}}}$

(c) With EER 5 $= \frac{10{,}000\text{ Btu/h}}{\wp}$ $\wp = \frac{10{,}000\text{ W}}{5} = 2000\text{ W} = 2.00\text{ kW}$

Energy purchased $= \wp t = (2.00\text{ kW})(1500\text{ h}) = 3.00\times10^3\text{ kWh}$

Cost $= (3.00\times10^3\text{ kWh})\left(0.100\,\frac{\$}{\text{kWh}}\right) = \$300$

With EER 10 $= \frac{10{,}000\text{ Btu/h}}{\wp}$ $\wp = \frac{10{,}000\text{ W}}{10} = 1000\text{ W} = 1.00\text{ kW}$

Energy purchased $= \wp t = (1.00\text{ kW})(1500\text{ h}) = 1.50\times10^3\text{ kWh}$

Cost $= (1.50\times10^3\text{ kWh})\left(0.100\,\frac{\$}{\text{kWh}}\right) = \$150$

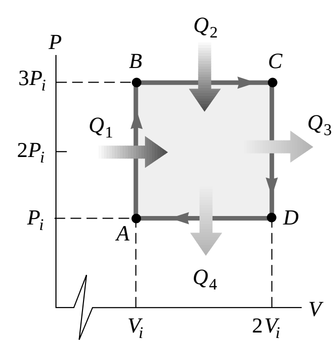
Thus, the cost for air conditioning is
$\boxed{\text{half as much with EER 10}}$

22.63 At point A, $P_iV_i = nRT_i$ and $n = 1.00$ mol

At point B, $3P_iV_i = nRT_B$ so $T_B = 3T_i$

At point C, $(3P_i)(2V_i) = nRT_C$, and $T_C = 6T_i$

At point D, $P_i(2V_i) = nRT_D$, so $T_D = 2T_i$



The heat transfer for each step in the cycle is found using $C_V = 3R/2$ and $C_P = 5R/2$.

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$

$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i \quad \text{and} \quad Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$

$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$. Therefore,

(a) $Q_{(\text{entering})} = Q_h = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$

(b) $Q_{(\text{leaving})} = Q_c = \left|Q_{CD} + Q_{DA}\right| = \boxed{8.50nRT_i}$

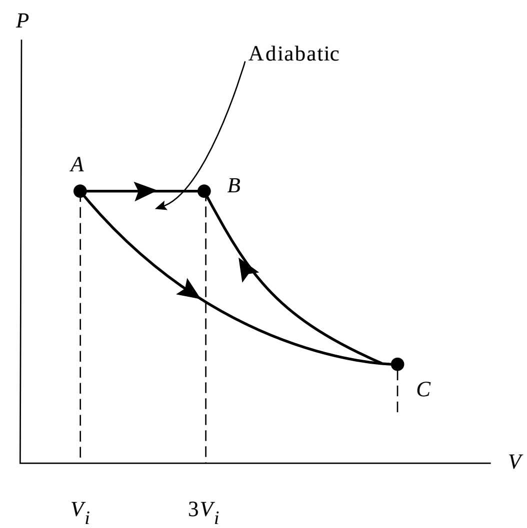
(c) Actual efficiency, $e = \dfrac{Q_h - Q_c}{Q_h} = \boxed{0.190}$

(d) Carnot efficiency, $e_C = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{T_i}{6T_i} = \boxed{0.833}$

22.64 (a) $W = \int_{V_i}^{V_f} P\,dV = nRT\int_{V_i}^{2V_i} \dfrac{dV}{V} = (1.00)RT\ln\left(\dfrac{2V_i}{V_i}\right) = \boxed{RT\ln 2}$

(b) $\boxed{\text{The second law refers to cycles.}}$

22.65 The isobaric process AB is shown along with an isotherm AC and an adiabat CB in the PV diagram. Since the change in entropy is path independent, $\Delta S_{AB} = \Delta S_{AC} + \Delta S_{CB}$ and $\Delta S_{CB} = 0$ for an adiabatic process. Since $T_C = T_A$, Equation 22.12 gives



$$\Delta S_{AC} = nR\ln\left(\frac{V_C}{V_A}\right)$$

For isotherm AC, $P_A V_A = P_C V_C$ and for adiabat CB, $P_C V_C^\gamma = P_B V_B^\gamma$

Combining gives: $V_C = \left[\dfrac{P_B V_B^\gamma}{P_A V_A}\right]^{1/(\gamma-1)}$

$$= \left[\left(\frac{P_A}{P_A}\right)\frac{(3V_i)^\gamma}{V_i}\right]^{1/(\gamma-1)} = 3^{\gamma/(\gamma-1)}V_i$$

Thus, $\Delta S_{AB} = 0 + \Delta S_{AC} = nR\ln[3^{\gamma/(\gamma-1)}] = nR\left(\dfrac{\gamma}{\gamma-1}\right)\ln 3$

But, $\dfrac{\gamma}{\gamma-1} = \dfrac{C_P/C_V}{C_P/C_V - 1} = \dfrac{C_P}{C_P - C_V} = \dfrac{C_P}{R}$

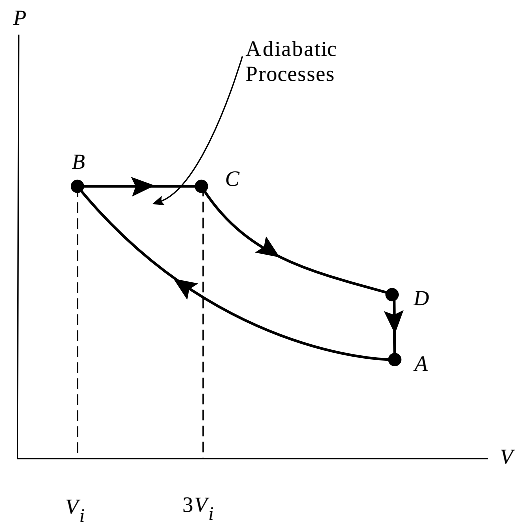
$\therefore \Delta S = \boxed{nC_P \ln 3}$

22.66 Simply evaluate the maximum (Carnot) efficiency.

$$e_C = \frac{\Delta T}{T_h} = \frac{4.00\text{ K}}{277\text{ K}} = \boxed{0.0144}$$

The proposal does not merit serious consideration.

22.67 The heat transfer over the paths CD and BA is zero since they are adiabats.



Over path BC: $Q_{BC} = nC_P(T_C - T_B) > 0$

Over path DA: $Q_{DA} = mC_V(T_A - T_D) < 0$

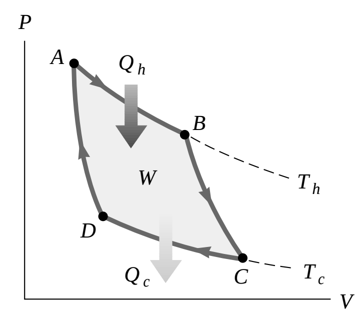
Therefore, $Q_c = \left|Q_{DA}\right|$ and $Q_h = Q_{BC}$

The efficiency is then

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{(T_D - T_A)}{(T_C - T_B)}\frac{C_V}{C_P}$$

$$e = 1 - \frac{1}{\gamma}\left[\frac{(T_D - T_A)}{T_C - T_B}\right]$$

22.68 (a) Use the equation of state for an ideal gas



$$V = nRT/P$$

$$V_A = \frac{(1.00)(8.315)(600)}{(25.0)(1.013 \times 10^5)} = \boxed{1.97 \times 10^{-3}\text{ m}^3}$$

$$V_C = \frac{(1.00)(8.315)(400\text{ K})}{1.013 \times 10^5} = \boxed{32.8 \times 10^{-3}\text{ m}^3}$$

Since AB is isothermal, $P_A V_A = P_B V_B$, and since BC is adiabatic, $P_B V_B^\gamma = P_C V_C^\gamma$.

Combining these expressions, $V_B = \left[\left(\frac{P_C}{P_A}\right)\frac{V_C^\gamma}{V_A}\right]^{1/(\gamma-1)}$

$$V_B = [(1.00/25.0)(32.8 \times 10^{-3}\text{ m}^3)^{1.40}/(1.97 \times 10^{-3}\text{ m}^3)]^{(1/0.400)} = \boxed{11.9 \times 10^{-3}\text{ m}^3}$$

Similarly, $V_D = [(P_A/P_C)(V_A^\gamma/V_C)]^{1/(\gamma-1)}$, or

$$V_D = [(25.0/1.00)(1.97 \times 10^{-3}\text{ m}^3)^{1.40}/(32.8 \times 10^{-3}\text{ m}^3)]^{(1/0.400)} = \boxed{5.44 \times 10^{-3}\text{ m}^3}$$

Since AB is isothermal, $P_AV_A = P_BV_B$ and

$$P_B = P_A\left(\frac{V_A}{V_B}\right) = (25.0\text{ atm})\left(\frac{1.97\times10^{-3}\text{ m}^3}{11.9\times10^{-3}\text{ m}^3}\right) = \boxed{4.14\text{ atm}}$$

Also, CD is isothermal and

$$P_D = P_C\left(\frac{V_C}{V_D}\right) = (1.00\text{ atm})\left(\frac{32.8\times10^{-3}\text{ m}^3}{5.44\times10^{-3}\text{ m}^3}\right) = \boxed{6.03\text{ atm}}$$

Solving part (c) before part (b):

(c) For this Carnot cycle, $e_C = 1-\dfrac{T_c}{T_h} = 1-\dfrac{400\text{ K}}{600\text{ K}} = \boxed{0.333}$

(b) Heat energy is added to the gas during the process AB. For this isothermal process, $\Delta E_{\text{int}} = 0$, and the first law gives

$$Q_{AB} = W_{AB} = nRT_h\ln\left(\frac{V_B}{V_A}\right) \quad \text{or}$$

$$Q_h = Q_{AB} = (1.00\text{ mol})\left(8.315\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(600\text{ K})\ \ln\left(\frac{11.9}{1.97}\right) = 8.97\text{ kJ}$$

Then, from $e = W_{\text{net}}/Q_h$, the net work done per cycle is

$$W_{\text{net}} = e_CQ_h = 0.333(8.97\text{ kJ}) = \boxed{2.99\text{ kJ}}$$

22.69 (a) $\dfrac{dQ}{dt} = (2.00\times10^6\text{ cal}\times4.186\text{ J/cal})/(24.0\text{ h}\times3600\text{ s/h})$

$$= \boxed{96.9\text{ W}} = \boxed{8.33\times10^4\text{ cal/h}}$$

(b) $\dfrac{dT}{dt} = \dfrac{dQ/dt}{mc} = \dfrac{8.33\times10^4\text{ cal/h}}{(70.0\times10^3\text{ g})(1.00\text{ cal/g}\cdot{}^\circ\text{C})}$

$$= \boxed{1.19^\circ\text{C/h}} = \boxed{2.14^\circ\text{F/h}}$$

22.70 (a) $\boxed{20.0°\text{C}}$

(b) $\Delta S = mc \ln \frac{T_f}{T_1} + mc \ln \frac{T_f}{T_2}$

$$= (1.00 \text{ kg})(4.19 \text{ kJ/kg} \cdot \text{K})\left[\ln \frac{T_f}{T_1} + \ln \frac{T_f}{T_2}\right]$$

$$\Delta S = (4.19 \text{ kJ/K}) \ln \left(\frac{293}{283} \cdot \frac{293}{303}\right)$$

(c) $\Delta S = \boxed{+4.88 \text{ J/K}}$

(d) $\boxed{\text{Yes}}$ Entropy has increased.

Chapter 23 Solutions

23.1 (a) $N = \left(\dfrac{10.0 \text{ grams}}{107.87 \text{ grams/mol}}\right)\left(6.02 \times 10^{23} \dfrac{\text{atoms}}{\text{mol}}\right)\left(47.0 \dfrac{\text{electrons}}{\text{atom}}\right) = \boxed{2.62 \times 10^{24}}$

(b) $\# \text{ electrons added} = \dfrac{Q}{e} = \dfrac{1.00 \times 10^{-3} \text{ C}}{1.60 \times 10^{-19} \text{ C/electron}} = 6.25 \times 10^{15}$

or $\boxed{2.38 \text{ electrons for every } 10^9 \text{ already present}}$

23.2 (a) $F_e = \dfrac{k_e q_1 q_2}{r^2} = \dfrac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{(3.80 \times 10^{-10} \text{ m})^2} = \boxed{1.59 \times 10^{-9} \text{ N}}$ (repulsion)

(b) $F_g = \dfrac{G m_1 m_2}{r^2} = \dfrac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)(1.67 \times 10^{-27} \text{ kg})^2}{(3.80 \times 10^{-10} \text{ m})^2} = \boxed{1.29 \times 10^{-45} \text{ N}}$

The electric force is $\boxed{\text{larger by } 1.24 \times 10^{36} \text{ times}}$

(c) If $k_e \dfrac{q_1 q_2}{r^2} = G \dfrac{m_1 m_2}{r^2}$ with $q_1 = q_2 = q$ and $m_1 = m_2 = m$, then

$$\frac{q}{m} = \sqrt{\frac{G}{k_e}} = \sqrt{\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}} = \boxed{8.61 \times 10^{-11} \text{ C / kg}}$$

23.3 If each person has a mass of ≈ 70 kg and is (almost) composed of water, then each person contains

$$N \approx \left(\frac{70{,}000 \text{ grams}}{18 \text{ grams/mol}}\right)\left(6.02 \times 10^{23} \frac{\text{molecules}}{\text{mol}}\right)\left(10 \frac{\text{protons}}{\text{molecule}}\right) \approx 2.3 \times 10^{28} \text{ protons}$$

With an excess of 1% electrons over protons, each person has a charge

$$q = (0.01)(1.6 \times 10^{-19} \text{ C})(2.3 \times 10^{28}) = 3.7 \times 10^7 \text{ C}$$

So $F = k_e \dfrac{q_1 q_2}{r^2} = (9 \times 10^9)\dfrac{(3.7 \times 10^7)^2}{0.6^2} \text{ N} = 4 \times 10^{25} \text{ N} \quad \boxed{\sim 10^{26} \text{ N}}$

This force is almost enough to lift a "weight" equal to that of the Earth:

$$Mg = (6 \times 10^{24} \text{ kg})(9.8 \text{ m/s}^2) = 6 \times 10^{25} \text{ N} \sim 10^{26} \text{ N}$$

23.4 We find the equal-magnitude charges on both spheres:

$$F = k_e \frac{q_1 q_2}{r^2} = k_e \frac{q^2}{r^2} \quad \text{so} \quad q = r\sqrt{\frac{F}{k_e}} = (1.00 \text{ m})\sqrt{\frac{1.00 \times 10^4 \text{ N}}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 1.05 \times 10^{-3} \text{ C}$$

The number of electron transferred is then

$$N_{xfer} = \left(1.05 \times 10^{-3} \text{ C}\right) \Big/ \left(1.60 \times 10^{-19} \text{ C / e}^-\right) = 6.59 \times 10^{15} \text{ electrons}$$

The whole number of electrons in each sphere is

$$N_{tot} = \left(\frac{10.0 \text{ g}}{107.87 \text{ g / mol}}\right)\left(6.02 \times 10^{23} \text{ atoms / mol}\right)\left(47 \text{ e}^- \text{ / atom}\right) = 2.62 \times 10^{24} \text{ e}^-$$

The fraction transferred is then

$$f = \frac{N_{xfer}}{N_{tot}} = \left(\frac{6.59 \times 10^{15}}{2.62 \times 10^{24}}\right) = \boxed{2.51 \times 10^{-9}} = 2.51 \text{ charges in every billion}$$

23.5

$$F = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2 \left(6.02 \times 10^{23}\right)^2}{\left[2(6.37 \times 10^6 \text{ m})\right]^2} = \boxed{514 \text{ kN}}$$

***23.6** (a) The force is one of attraction. The distance r in Coulomb's law is the distance between centers. The magnitude of the force is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{\left(12.0 \times 10^{-9} \text{ C}\right)\left(18.0 \times 10^{-9} \text{ C}\right)}{(0.300 \text{ m})^2} = \boxed{2.16 \times 10^{-5} \text{ N}}$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of repulsion, and its magnitude is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{\left(3.00 \times 10^{-9} \text{ C}\right)\left(3.00 \times 10^{-9} \text{ C}\right)}{(0.300 \text{ m})^2} = \boxed{8.99 \times 10^{-7} \text{ N}}$$

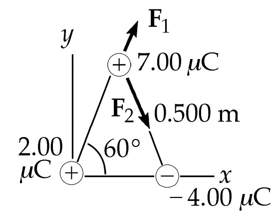
23.7 $$F_1 = k_e \frac{q_1 q_2}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(2.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} = 0.503\ \mathrm{N}$$

$$F_2 = k_e \frac{q_1 q_2}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(4.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} = 1.01\ \mathrm{N}$$

$$F_x = (0.503 + 1.01)\cos 60.0° = 0.755\ \mathrm{N}$$

$$F_y = (0.503 - 1.01)\sin 60.0° = -0.436\ \mathrm{N}$$

$$\mathbf{F} = (0.755\ \mathrm{N})\mathbf{i} - (0.436\ \mathrm{N})\mathbf{j} = \boxed{0.872\ \mathrm{N\ at\ an\ angle\ of\ 330°}}$$



Goal Solution

Three point charges are located at the corners of an equilateral triangle as shown in Figure P23.7. Calculate the net electric force on the 7.00–μC charge.

G: **G**ather Information: The 7.00–μC charge experiences a repulsive force $\mathbf{F}_1$ due to the 2.00–μC charge, and an attractive force $\mathbf{F}_2$ due to the –4.00–μC charge, where $F_2 = 2F_1$. If we sketch these force vectors, we find that the resultant appears to be about the same magnitude as F_2 and is directed to the right about 30.0° below the horizontal.

O: **O**rganize: We can find the net electric force by adding the two separate forces acting on the 7.00–μC charge. These individual forces can be found by applying Coulomb's law to each pair of charges.

A: **A**nalyze: The force on the 7.00–μC charge by the 2.00–μC charge is $\mathbf{F}_1 = k_e \dfrac{q_1 q_2}{r^2}\hat{\mathbf{r}}$

$$\mathbf{F}_1 = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(7.00\times10^{-6}\ \mathrm{C}\right)\left(2.00\times10^{-6}\ \mathrm{C}\right)}{(0.500\ \mathrm{m})^2}\left(\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}\right) = \mathbf{F}_1 = \left(0.252\mathbf{i} + 0.436\mathbf{j}\right)\ \mathrm{N}$$

Similarly, the force on the 7.00–μC by the –4.00–μC charge is $\mathbf{F}_2 = k_e \dfrac{q_1 q_3}{r^2}\hat{\mathbf{r}}$

$$\mathbf{F}_2 = -\left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(7.00\times10^{-6}\ \mathrm{C}\right)\left(-4.00\times10^{-6}\ \mathrm{C}\right)}{(0.500\ \mathrm{m})^2}\left(\cos 60°\,\mathbf{i} - \sin 60°\,\mathbf{j}\right) = \left(0.503\mathbf{i} - 0.872\mathbf{j}\right)\ \mathrm{N}$$

Thus, the total force on the 7.00–μC, expressed as a set of components, is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \left(0.755\ \mathbf{i} - 0.436\ \mathbf{j}\right)\ \mathrm{N} = 0.872\ \mathrm{N\ at\ 30.0°\ below\ the\ } +x \mathrm{\ axis}$$

L: **L**earn: Our calculated answer agrees with our initial estimate. An equivalent approach to this problem would be to find the net electric field due to the two lower charges and apply **F**=q**E** to find the force on the upper charge in this electric field.

***23.8** Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\mathbf{F} = \frac{k_e(3q)Q}{x^2}\mathbf{i} + \frac{k_e(q)Q}{(d-x)^2}(-\mathbf{i})$$

The net force will be zero if $\frac{3}{x^2} = \frac{1}{(d-x)^2}$, or $d - x = \frac{x}{\sqrt{3}}$

This gives an equilibrium position of the third bead of $x = \boxed{0.634d}$

The equilibrium is $\boxed{\text{stable if the third bead has positive charge}}$.

***23.9** (a) $F = \frac{k_e e^2}{r^2} = (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\frac{(1.60 \times 10^{-19}\ \text{C})^2}{(0.529 \times 10^{-10}\ \text{m})^2} = \boxed{8.22 \times 10^{-8}\ \text{N}}$

(b) We have $F = \frac{mv^2}{r}$ from which $v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{(8.22 \times 10^{-8}\ \text{N})(0.529 \times 10^{-10}\ \text{m})}{9.11 \times 10^{-31}\ \text{kg}}} = \boxed{2.19 \times 10^6\ \text{m/s}}$

23.10 The top charge exerts a force on the negative charge $\frac{k_e qQ}{(d/2)^2 + x^2}$ which is directed upward and to the left, at an angle of $\tan^{-1}(d/2x)$ to the x-axis. The two positive charges together exert force

$$\left(\frac{2k_e qQ}{(d^2/4 + x^2)}\right)\left(\frac{(-x)\mathbf{i}}{(d^2/4 + x^2)^{1/2}}\right) = m\mathbf{a} \qquad \text{or for } x \ll d/2, \qquad \mathbf{a} \approx \frac{-2k_e qQ}{md^3/8}\mathbf{x}$$

(a) The acceleration is equal to a negative constant times the excursion from equilibrium, as in $\mathbf{a} = -\omega^2\mathbf{x}$, so we have Simple Harmonic Motion with $\omega^2 = \frac{16k_e qQ}{md^3}$.

$T = \frac{2\pi}{\omega} = \boxed{\frac{\pi}{2}\sqrt{\frac{md^3}{k_e qQ}}}$, where m is the mass of the object with charge $-Q$.

(b) $v_{\max} = \omega A = \boxed{4a\sqrt{\frac{k_e qQ}{md^3}}}$

23.11 For equilibrium, $\mathbf{F}_e = -\mathbf{F}_g$, or $q\mathbf{E} = -mg(-\mathbf{j})$. Thus, $\mathbf{E} = \frac{mg}{q}\mathbf{j}$.

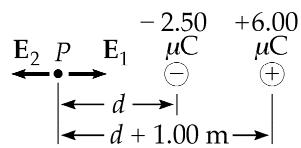
(a) $$\mathbf{E} = \frac{mg}{q}\mathbf{j} = \frac{(9.11\times10^{-31}\text{ kg})(9.80\text{ m/s}^2)}{\left(-1.60\times10^{-19}\text{ C}\right)}\mathbf{j} = \boxed{-\left(5.58\times10^{-11}\text{ N/C}\right)\mathbf{j}}$$

(b) $$\mathbf{E} = \frac{mg}{q}\mathbf{j} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)}\mathbf{j} = \boxed{\left(1.02\times10^{-7}\text{ N/C}\right)\mathbf{j}}$$

23.12 $\sum F_y = 0$: $\quad QE\mathbf{j} + mg(-\mathbf{j}) = 0$

$$\therefore \qquad m = \frac{QE}{g} = \frac{(24.0\times10^{-6}\text{ C})(610\text{ N / C})}{9.80\text{ m / s}^2} = \boxed{1.49\text{ grams}}$$

***23.13** The point is designated in the sketch. The magnitudes of the electric fields, E_1, (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge) are



$$E_1 = \frac{k_e q}{r^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-6}\text{ C})}{d^2} \qquad (1)$$

$$E_2 = \frac{k_e q}{r^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-6}\text{ C})}{(d+1.00\text{ m})^2} \qquad (2)$$

Equate the right sides of (1) and (2) to get $\qquad (d+1.00\text{ m})^2 = 2.40d^2$

or $\qquad d + 1.00\text{ m} = \pm1.55d$

which yields $\qquad d = 1.82$ m $\quad$ or $\quad d = -0.392$ m

The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction. Thus, $\boxed{d = 1.82\text{ m to the left of the -2.50 }\mu\text{C charge.}}$

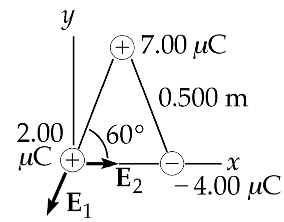
23.14 If we treat the concentrations as point charges,

$$\mathbf{E}_+ = k_e\frac{q}{r^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(40.0\text{ C})}{(1000\text{ m})^2}(-\mathbf{j}) = 3.60\times10^5\text{ N / C}\,(-\mathbf{j})\text{ (downward)}$$

$$\mathbf{E}_- = k_e\frac{q}{r^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(40.0\text{ C})}{(1000\text{ m})^2}(-\mathbf{j}) = 3.60\times10^5\text{ N / C}\,(-\mathbf{j})\text{ (downward)}$$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \boxed{7.20\times10^5\text{ N / C downward}}$$

***23.15** (a) $E_1 = \dfrac{k_e q}{r^2} = \dfrac{\left(8.99\times10^9\right)\left(7.00\times10^{-6}\right)}{(0.500)^2} = 2.52\times10^5 \text{ N/C}$



$E_2 = \dfrac{k_e q}{r^2} = \dfrac{\left(8.99\times10^9\right)\left(4.00\times10^{-6}\right)}{(0.500)^2} = 1.44\times10^5 \text{ N/C}$

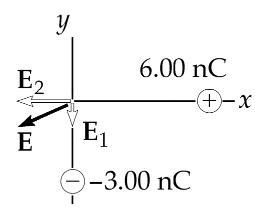
$E_x = E_2 - E_1 \cos 60° = 1.44\times10^5 - 2.52\times10^5 \cos 60.0° = 18.0\times10^3 \text{ N/C}$

$E_y = -E_1 \sin 60.0° = -2.52\times10^5 \sin 60.0° = -218\times10^3 \text{ N/C}$

$\mathbf{E} = [18.0\mathbf{i} - 218\mathbf{j}]\times10^3 \text{ N/C} = \boxed{[18.0\mathbf{i} - 218\mathbf{j}] \text{ kN/C}}$

(b) $\mathbf{F} = q\mathbf{E} = \left(2.00\times10^{-6}\text{ C}\right)\left(18.0\mathbf{i} - 218\mathbf{j}\right)\times10^3 \text{ N/C} = \left(36.0\mathbf{i} - 436\mathbf{j}\right)\times10^{-3} \text{ N} = \boxed{\left(36.0\mathbf{i} - 436\mathbf{j}\right)\text{mN}}$

***23.16** (a) $\mathbf{E}_1 = \dfrac{k_e|q_1|}{r_1^2}(-\mathbf{j}) = \dfrac{\left(8.99\times10^9\right)\left(3.00\times10^{-9}\right)}{(0.100)^2}(-\mathbf{j}) = -\left(2.70\times10^3 \text{ N/C}\right)\mathbf{j}$



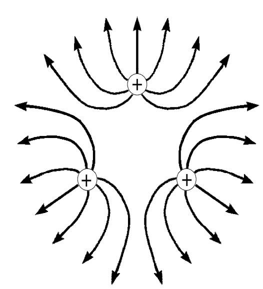
$\mathbf{E}_2 = \dfrac{k_e|q_2|}{r_2^2}(-\mathbf{i}) = \dfrac{\left(8.99\times10^9\right)\left(6.00\times10^{-9}\right)}{(0.300)^2}(-\mathbf{i}) = -\left(5.99\times10^2 \text{ N/C}\right)\mathbf{i}$

$\mathbf{E} = \mathbf{E}_2 + \mathbf{E}_1 = \boxed{-\left(5.99\times10^2 \text{ N/C}\right)\mathbf{i} - \left(2.70\times10^3 \text{ N/C}\right)\mathbf{j}}$

(b) $\mathbf{F} = q\mathbf{E} = \left(5.00\times10^{-9}\text{ C}\right)\left(-599\mathbf{i} - 2700\mathbf{j}\right)\text{N/C}$

$\mathbf{F} = \left(-3.00\times10^{-6}\,\mathbf{i} - 13.5\times10^{-6}\,\mathbf{j}\right)\text{N} = \boxed{\left(-3.00\mathbf{i} - 13.5\mathbf{j}\right)\ \mu\text{N}}$

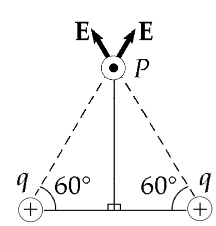
23.17 (a) The electric field has the general appearance shown. It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.



(b) You may need to review vector addition in Chapter Three.

$\mathbf{E} = k_e \sum_i \dfrac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i$

The magnitude of the field at point P due to each of the charges along the base of the triangle is $E = k_e\, q/a^2$. The direction of the field in each case is along the line joining the charge in question to point P as shown in the diagram at the right. The x components add to zero, leaving



$\mathbf{E} = \dfrac{k_e q}{a^2}(\sin 60.0°)\mathbf{j} + \dfrac{k_e q}{a^2}(\sin 60.0°)\mathbf{j} = \boxed{\sqrt{3}\,\dfrac{k_e q}{a^2}\,\mathbf{j}}$

Goal Solution

Three equal positive charges q are at the corners of an equilateral triangle of side a, as shown in Figure P23.17. (a) Assume that the three charges together create an electric field. Find the location of a point (other than ∞) where the electric field is zero. (**Hint:** Sketch the field lines in the plane of the charges.) (b) What are the magnitude and direction of the electric field at P due to the two charges at the base?

G: The electric field has the general appearance shown by the black arrows in the figure to the right. This drawing indicates that $\mathbf{E} = 0$ at the center of the triangle, since a small positive charge placed at the center of this triangle will be pushed away from each corner equally strongly. This fact could be verified by vector addition as in part (b) below.

The electric field at point P should be directed upwards and about twice the magnitude of the electric field due to just one of the lower charges as shown in Figure P23.17. For part (b), we must ignore the effect of the charge at point P, because a charge cannot exert a force on itself.

O: The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

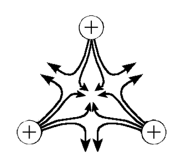
A: (b) The electric field from a point charge is $\mathbf{E} = k_e \dfrac{q}{r^2}\hat{\mathbf{r}}$

As shown in the solution figure above, $\mathbf{E}_1 = k_e \dfrac{q}{a^2}$ to the right and upward at 60°

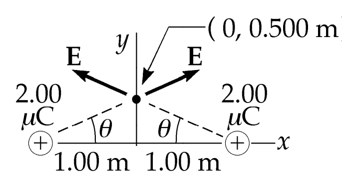
$\mathbf{E}_2 = k_e \dfrac{q}{a^2}$ to the left and upward at 60°

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = k_e \frac{q}{a^2}\left[(\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}) + (-\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j})\right] = k_e \frac{q}{a^2}\left[2(\sin 60°\,\mathbf{j})\right] = 1.73 k_e \frac{q}{a^2}\mathbf{j}$$

L: The net electric field at point P is indeed nearly twice the magnitude due to a single charge and is entirely vertical as expected from the symmetry of the configuration. In addition to the center of the triangle, the electric field lines in the figure to the right indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.



23.18 (a) $E = \dfrac{k_e q}{r^2} = \dfrac{(8.99 \times 10^9)(2.00 \times 10^{-6})}{(1.12)^2} = 14,400$ N/C



$E_x = 0$ and $E_y = 2(14,400)\sin 26.6° = 1.29 \times 10^4$ N/C

so $\boxed{\mathbf{E} = 1.29 \times 10^4\,\mathbf{j}\ \text{N/C}}$

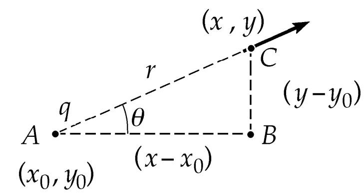
(b) $\mathbf{F} = \mathbf{E}q = (1.29 \times 10^4\,\mathbf{j})(-3.00 \times 10^{-6}) = \boxed{-3.86 \times 10^{-2}\,\mathbf{j}\ \text{N}}$

23.19 (a) $\mathbf{E} = \dfrac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \dfrac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \dfrac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3 = \dfrac{k_e(2q)}{a^2}\mathbf{i} + \dfrac{k_e(3q)}{2a^2}(\mathbf{i}\cos 45.0° + \mathbf{j}\sin 45.0°) + \dfrac{k_e(4q)}{a^2}\mathbf{j}$

$$\mathbf{E} = 3.06\frac{k_e q}{a^2}\mathbf{i} + 5.06\frac{k_e q}{a^2}\mathbf{j} = \boxed{5.91\frac{k_e q}{a^2} \text{ at } 58.8°}$$

(b) $\mathbf{F} = q\mathbf{E} = \boxed{5.91\dfrac{k_e q^2}{a^2} \text{ at } 58.8°}$

23.20 The magnitude of the field at (x,y) due to charge q at (x_0,y_0) is given by $E = k_e q/r^2$ where r is the distance from (x_0,y_0) to (x,y). Observe the geometry in the diagram at the right. From triangle ABC, $r^2 = (x-x_0)^2 + (y-y_0)^2$, or



$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2}, \quad \sin\theta = \frac{(y-y_0)}{r}, \quad \text{and} \quad \cos\theta = \frac{(x-x_0)}{r}$$

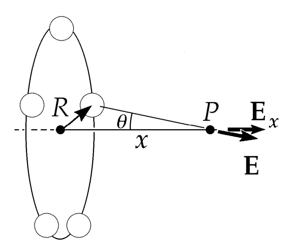
Thus, $E_x = E\cos\theta = \dfrac{k_e q}{r^2}\dfrac{(x-x_0)}{r} = \boxed{\dfrac{k_e q(x-x_0)}{[(x-x_0)^2 + (y-y_0)^2]^{3/2}}}$

and $E_y = E\sin\theta = \dfrac{k_e q}{r^2}\dfrac{(y-y_0)}{r} = \boxed{\dfrac{k_e q(y-y_0)}{[(x-x_0)^2 + (y-y_0)^2]^{3/2}}}$

23.21 The electric field at any point x is $E = \dfrac{k_e q}{(x-a)^2} - \dfrac{k_e q}{(x-(-a))^2} = \dfrac{k_e q(4ax)}{(x^2-a^2)^2}$

When x is much, much greater than a, we find $E \approx \boxed{\dfrac{(4a)(k_e q)}{x^3}}$

23.22 (a) One of the charges creates at P a field $\mathbf{E} = \dfrac{k_e Q/n}{R^2 + x^2}$



at an angle θ to the x-axis as shown.

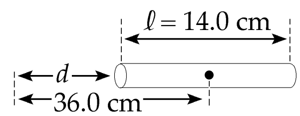
When all the charges produce field, for $n > 1$, the components perpendicular to the x-axis add to zero.

The total field is $\dfrac{nk_e(Q/n)\mathbf{i}}{R^2 + x^2}\cos\theta = \boxed{\dfrac{k_e Qx\mathbf{i}}{(R^2 + x^2)^{3/2}}}$

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Smearing the charge around the circle does not change its amount or its distance from the field point, so it $\boxed{\text{does not change the field.}}$.

23.23 $$\mathbf{E} = \sum \frac{k_e q}{r^2} \sim= \frac{k_e q}{a^2}(-\mathbf{i}) + \frac{k_e q}{(2a)^2}(-\mathbf{i}) + \frac{k_e q}{(3a)^2}(-\mathbf{i}) + \ \ldots\ = \frac{-k_e q\,\mathbf{i}}{a^2}\left(1 + \frac{1}{2^2} + \frac{1}{3^3} + \ \ldots\right) = \boxed{-\frac{\pi^2 k_e q}{6a^2}\,\mathbf{i}}$$

23.24 $$E = \frac{k_e \lambda \ell}{d(\ell + d)} = \frac{k_e (Q/\ell)\ell}{d(\ell + d)} = \frac{k_e Q}{d(\ell + d)} = \frac{(8.99 \times 10^9)(22.0 \times 10^{-6})}{(0.290)(0.140 + 0.290)}$$



$\mathbf{E} = \boxed{1.59 \times 10^6\ \mathrm{N/C}}$, $\boxed{\text{directed toward the rod}}$.

23.25 $E = \int \frac{k_e\, dq}{x^2}$ where $dq = \lambda_0\, dx$

$$E = k_e\, \lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} = k_e \left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} = \boxed{\frac{k_e \lambda_0}{x_0}}$$ $\boxed{\text{The direction is } -\mathbf{i} \text{ or left for } \lambda_0 > 0}$

23.26 $$\mathbf{E} = \int d\mathbf{E} = \int_{x_0}^{\infty} \left[\frac{k_e \lambda_0 x_0\, dx(-\mathbf{i})}{x^3}\right] = -k_e \lambda_0 x_0\, \mathbf{i} \int_{x_0}^{\infty} x^{-3}\, dx = -k_e \lambda_0 x_0\, \mathbf{i}\left(-\frac{1}{2x^2}\Bigg|_{x_0}^{\infty}\right) = \boxed{\frac{k_e \lambda_0}{2x_0}(-\mathbf{i})}$$

23.27 $$E = \frac{k_e x Q}{(x^2 + a^2)^{3/2}} = \frac{(8.99 \times 10^9)(75.0 \times 10^{-6})x}{(x^2 + 0.100^2)^{3/2}} = \frac{6.74 \times 10^5\, x}{(x^2 + 0.0100)^{3/2}}$$

(a) At $x = 0.0100$ m, $\mathbf{E} = 6.64 \times 10^6\, \mathbf{i}$ N/C $= \boxed{6.64\, \mathbf{i}\ \mathrm{MN/C}}$

(b) At $x = 0.0500$ m, $\mathbf{E} = 2.41 \times 10^7\, \mathbf{i}$ N/C $= \boxed{24.1\, \mathbf{i}\ \mathrm{MN/C}}$

(c) At $x = 0.300$ m, $\mathbf{E} = 6.40 \times 10^6\, \mathbf{i}$ N/C $= \boxed{6.40\, \mathbf{i}\ \mathrm{MN/C}}$

(d) At $x = 1.00$ m, $\mathbf{E} = 6.64 \times 10^5\, \mathbf{i}$ N/C $= \boxed{0.664\, \mathbf{i}\ \mathrm{MN/C}}$

23.28 $$E = \frac{k_e Q x}{(x^2 + a^2)^{3/2}}$$

For a maximum, $$\frac{dE}{dx} = Qk_e\left[\frac{1}{(x^2+a^2)^{3/2}} - \frac{3x^2}{(x^2+a^2)^{5/2}}\right] = 0$$

$x^2 + a^2 - 3x^2 = 0$ or $x = \dfrac{a}{\sqrt{2}}$

Substituting into the expression for E gives

$$E = \frac{k_e Qa}{\sqrt{2}\left(\frac{3}{2}a^2\right)^{3/2}} = \frac{k_e Q}{3\frac{\sqrt{3}}{2}a^2} = \boxed{\frac{2k_e Q}{3\sqrt{3}\,a^2}} = \boxed{\frac{Q}{6\sqrt{3}\pi e_0 a^2}}$$

23.29 $$E = 2\pi k_e \sigma\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

$$E = 2\pi\left(8.99\times10^9\right)\left(7.90\times10^{-3}\right)\left(1 - \frac{x}{\sqrt{x^2+(0.350)^2}}\right) = 4.46\times10^8\left(1 - \frac{x}{\sqrt{x^2+0.123}}\right)$$

(a) At x = 0.0500 m, $E = 3.83\times10^8$ N/C = $\boxed{383 \text{ MN/C}}$

(b) At x = 0.100 m, $E = 3.24\times10^8$ N/C = $\boxed{324 \text{ MN/C}}$

(c) At x = 0.500 m, $E = 8.07\times10^7$ N/C = $\boxed{80.7 \text{ MN/C}}$

(d) At x = 2.00 m, $E = 6.68\times10^8$ N/C = $\boxed{6.68 \text{ MN/C}}$

23.30 (a) From Example 23.9: $E = 2\pi k_e \sigma\left(1 - \dfrac{x}{\sqrt{x^2+R^2}}\right)$

$$\sigma = \frac{Q}{\pi R^2} = 1.84\times10^{-3}\ \text{C/m}^2$$

$$E = (1.04\times10^8\ \text{N/C})(0.900) = 9.36\times10^7\ \text{N/C} = \boxed{93.6\ \text{MN/C}}$$

appx: $E = 2\pi k_e \sigma = \boxed{104\ \text{MN/C (about 11\% high)}}$

(b) $$E = (1.04\times10^8\ \text{N / C})\left(1 - \frac{30.0\ \text{cm}}{\sqrt{30.0^2 + 3.00^2}\ \text{cm}}\right) = (1.04\times10^8\ \text{N/C})(0.00496) = \boxed{0.516\ \text{MN/C}}$$

appx: $E = k_e \dfrac{Q}{r^2} = (8.99\times10^9)\dfrac{5.20\times10^{-6}}{(0.30)^2} = \boxed{0.519\ \text{MN/C (about 0.6\% high)}}$

23.31 The electric field at a distance x is $E_x = 2\pi k_e \sigma\left[1 - \dfrac{x}{\sqrt{x^2 + R^2}}\right]$

This is equivalent to $E_x = 2\pi k_e \sigma\left[1 - \dfrac{1}{\sqrt{1 + R^2/x^2}}\right]$

For large x, $R^2/x^2 << 1$ and $\sqrt{1 + \dfrac{R^2}{x^2}} \approx 1 + \dfrac{R^2}{2x^2}$

so $E_x = 2\pi k_e \sigma\left(1 - \dfrac{1}{\left[1 + R^2/(2x^2)\right]}\right) = 2\pi k_e \sigma \dfrac{\left(1 + R^2/(2x^2) - 1\right)}{\left[1 + R^2/(2x^2)\right]}$

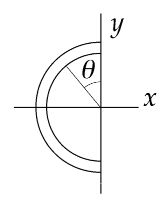
Substitute $\sigma = Q/\pi R^2$, $E_x = \dfrac{k_e Q\left(1/x^2\right)}{\left[1 + R^2/(2x^2)\right]} = k_e Q\left(x^2 + \dfrac{R^2}{2}\right)$

But for $x >> R$, $\dfrac{1}{x^2 + R^2/2} \approx \dfrac{1}{x^2}$, so $\boxed{E_x \approx \dfrac{k_e Q}{x^2} \text{ for a disk at large distances}}$

23.32 The sheet must have negative charge to repel the negative charge on the Styrofoam. The magnitude of the upward electric force must equal the magnitude of the downward gravitational force for the Styrofoam to "float" (i.e., $F_e = F_g$).

Thus, $-qE = mg$, or $-q\left(\dfrac{\sigma}{2\mathrm{e}_0}\right) = mg$ which gives $\sigma = \boxed{-\dfrac{2\mathrm{e}_0 mg}{q}}$

23.33 Due to symmetry $E_y = \int dE_y = 0$, and $E_x = \int dE \sin\theta = k_e \int \dfrac{dq \sin\theta}{r^2}$



where $dq = \lambda ds = \lambda r\, d\theta$, so that, $E_x = \dfrac{k_e \lambda}{r}\int_0^\pi \sin\theta\, d\theta = \dfrac{k_e\lambda}{r}(-\cos\theta)\Big|_0^\pi = \dfrac{2k_e\lambda}{r}$

where $\lambda = \dfrac{q}{L}$ and $r = \dfrac{L}{\pi}$. Thus, $E_x = \dfrac{2k_e q\pi}{L^2} = \dfrac{2(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(7.50 \times 10^{-6} \text{ C})\pi}{(0.140 \text{ m})^2}$

Solving, $E = E_x = 2.16 \times 10^7 \text{ N/C}$

Since the rod has a negative charge, $\mathbf{E} = (-2.16 \times 10^7\, \mathbf{i}) \text{ N/C} = \boxed{-21.6\, \mathbf{i} \text{ MN/C}}$

23.34 (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx, has charge Qdx/h and produces, at the chosen point, a field

$$d\mathbf{E} = \frac{k_e x}{(x^2 + R^2)^{3/2}} \frac{Q\,dx}{h}\,\mathbf{i}$$

The total field is

$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_d^{d+h} \frac{k_e Q\,x\,dx}{h(x^2 + R^2)^{3/2}}\,\mathbf{i} = \frac{k_e Q\,\mathbf{i}}{2h}\int_{x=d}^{d+h} (x^2 + R^2)^{-3/2}\,2x\,dx$$

$$\mathbf{E} = \frac{k_e Q\,\mathbf{i}}{2h}\frac{(x^2 + R^2)^{-1/2}}{(-1/2)}\Bigg|_{x=d}^{d+h} = \boxed{\frac{k_e Q\,\mathbf{i}}{h}\left[\frac{1}{(d^2 + R^2)^{1/2}} - \frac{1}{\left((d+h)^2 + R^2\right)^{1/2}}\right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness dx, charge $Q\,dx/h$, and charge-per-area $\sigma = Q\,dx/\pi R^2 h$. One disk produces a field

$$d\mathbf{E} = \frac{2\pi k_e Q\,dx}{\pi R^2 h}\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)\mathbf{i}$$

So, $$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_{x=d}^{d+h} \frac{2k_e Q\,dx}{R^2 h}\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)\mathbf{i}$$

$$\mathbf{E} = \frac{2k_e Q\,\mathbf{i}}{R^2 h}\left[\int_d^{d+h} dx - \frac{1}{2}\int_{x=d}^{d+h}(x^2 + R^2)^{-1/2}\,2x\,dx\right] = \frac{2k_e Q\,\mathbf{i}}{R^2 h}\left[x\Big|_d^{d+h} - \frac{1}{2}\frac{(x^2 + R^2)^{1/2}}{1/2}\Bigg|_d^{d+h}\right]$$

$$\mathbf{E} = \frac{2k_e Q\,\mathbf{i}}{R^2 h}\left[d + h - d - \left((d+h)^2 + R^2\right)^{1/2} + (d^2 + R^2)^{1/2}\right]$$

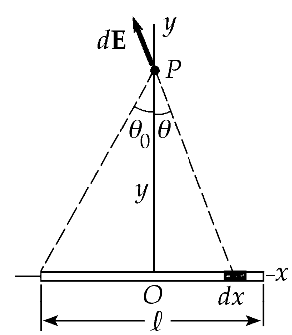
$$\mathbf{E} = \boxed{\frac{2k_e Q\,\mathbf{i}}{R^2 h}\left[h + (d^2 + R^2)^{1/2} - \left((d+h)^2 + R^2\right)^{1/2}\right]}$$

23.35 (a) The electric field at point P due to each element of length dx, is $dE = \dfrac{k_e dq}{(x^2 + y^2)}$ and is directed along the line joining the element of length to point P. By symmetry,

$E_x = \int dE_x = 0$ and since $dq = \lambda\,dx$,

$$E = E_y = \int dE_y = \int dE \cos\theta \quad \text{where} \quad \cos\theta = \frac{y}{(x^2 + y^2)^{1/2}}$$

Therefore, $$E = 2k_e\lambda y\int_0^{\ell/2}\frac{dx}{(x^2 + y^2)^{3/2}} = \boxed{\frac{2k_e\lambda\sin\theta_0}{y}}$$



(b) For a bar of infinite length, $\theta \to 90°$ and $E_y = \boxed{\dfrac{2k_e\lambda}{y}}$

***23.36** (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r+L)$.

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)2\pi(0.0250\ \mathrm{m})[0.0250\ \mathrm{m} + 0.0600\ \mathrm{m}] = \boxed{2.00\times10^{-10}\ \mathrm{C}}$$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)2\pi(0.0250\ \mathrm{m})(0.0600\ \mathrm{m}) = \boxed{1.41\times10^{-10}\ \mathrm{C}}$$

(c) $Q = \rho V = \rho\pi r^2 L = \left(500\times10^{-9}\ \mathrm{C/m^3}\right)\pi(0.0250\ \mathrm{m})^2(0.0600\ \mathrm{m}) = \boxed{5.89\times10^{-11}\ \mathrm{C}}$

***23.37** (a) Every object has the same volume, $V = 8(0.0300\ \mathrm{m})^3 = 2.16\times10^{-4}\ \mathrm{m^3}$.

For each, $Q = \rho V = \left(400\times10^{-9}\ \mathrm{C/m^3}\right)\left(2.16\times10^{-4}\ \mathrm{m^3}\right) = \boxed{8.64\times10^{-11}\ \mathrm{C}}$

(b) We must count the $9.00\ \mathrm{cm^2}$ squares painted with charge:

(i) $6\times4 = 24$ squares

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)24.0\left(9.00\times10^{-4}\ \mathrm{m^2}\right) = \boxed{3.24\times10^{-10}\ \mathrm{C}}$$

(ii) 34 squares exposed

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)34.0\left(9.00\times10^{-4}\ \mathrm{m^2}\right) = \boxed{4.59\times10^{-10}\ \mathrm{C}}$$

(iii) 34 squares

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)34.0\left(9.00\times10^{-4}\ \mathrm{m^2}\right) = \boxed{4.59\times10^{-10}\ \mathrm{C}}$$

(iv) 32 squares

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)32.0\left(9.00\times10^{-4}\ \mathrm{m^2}\right) = \boxed{4.32\times10^{-10}\ \mathrm{C}}$$

(c) (i) total edge length: $\ell = 24\times(0.0300\ \mathrm{m})$

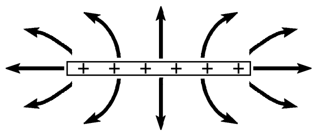
$$Q = \lambda\ell = \left(80.0\times10^{-12}\ \mathrm{C/m}\right)24\times(0.0300\ \mathrm{m}) = \boxed{5.76\times10^{-11}\ \mathrm{C}}$$

(ii) $Q = \lambda\ell = \left(80.0\times10^{-12}\ \mathrm{C/m}\right)44\times(0.0300\ \mathrm{m}) = \boxed{1.06\times10^{-10}\ \mathrm{C}}$

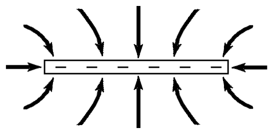
(iii) $Q = \lambda\ell = \left(80.0\times10^{-12}\ \mathrm{C/m}\right)64\times(0.0300\ \mathrm{m}) = \boxed{1.54\times10^{-10}\ \mathrm{C}}$

(iv) $Q = \lambda \ell = \left(80.0 \times 10^{-12} \text{ C/m}\right) 40 \times (0.0300 \text{ m}) = \boxed{0.960 \times 10^{-10} \text{ C}}$

22.38



22.39



23.40 (a) $\frac{q_1}{q_2} = \frac{-6}{18} = \boxed{-\frac{1}{3}}$

(b) $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$

23.41 $F = qE = ma \quad a = \frac{qE}{m}$

$v = v_i + at \quad v = \frac{qEt}{m}$

electron: $v_e = \frac{(1.602\times10^{-19})(520)(48.0\times10^{-9})}{9.11\times10^{-31}} = \boxed{4.39\times10^6 \text{ m/s}}$

in a direction opposite to the field

proton: $v_p = \frac{(1.602\times10^{-19})(520)(48.0\times10^{-9})}{1.67\times10^{-27}} = \boxed{2.39\times10^3 \text{ m/s}}$

in the same direction as the field

23.42 (a) $|a| = \frac{qE}{m} = \frac{(1.602\times10^{-19})(6.00\times10^5)}{(1.67\times10^{-27})} = 5.76\times10^{13} \text{ m/s}$ so $\mathbf{a} = \boxed{-5.76\times10^{13}\,\mathbf{i} \text{ m/s}^2}$

(b) $v = v_i + 2a(x - x_i)$

$0 = v_i^2 + 2(-5.76\times10^{13})(0.0700) \quad \boxed{\mathbf{v}_i = 2.84\times10^6\,\mathbf{i} \text{ m/s}}$

(c) $v = v_i + at$

$0 = 2.84\times10^6 + (-5.76\times10^{13})t \quad t = \boxed{4.93\times10^{-8} \text{ s}}$

23.43 (a) $a = \dfrac{qE}{m} = \dfrac{\left(1.602\times10^{-19}\right)(640)}{\left(1.67\times10^{-27}\right)} = \boxed{6.14\times10^{10}\ \text{m/s}^2}$

(b) $v = v_i + at$

$1.20\times10^{6} = (6.14\times10^{10})t$

$\boxed{t = 1.95\times10^{-5}\ \text{s}}$

(c) $x - x_i = \frac{1}{2}\left(v_i + v\right)t$

$x = \frac{1}{2}\left(1.20\times10^{6}\right)\left(1.95\times10^{-5}\right) = \boxed{11.7\ \text{m}}$

(d) $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\ \text{kg})(1.20\times10^{6}\ \text{m/s})^2 = \boxed{1.20\times10^{-15}\ \text{J}}$

23.44 The required electric field will be $\boxed{\text{in the direction of motion}}$. We know that Work = ΔK

So, $-Fd = -\frac{1}{2}\ m v_i^2$ (since the final velocity = 0)

This becomes $Eed = \dfrac{1}{2}mv_i^2$ or $E = \dfrac{\frac{1}{2}mv_i^2}{ed}$

$E = \dfrac{1.60\times10^{-17}\ \text{J}}{(1.60\times10^{-19}\ \text{C})(0.100\ \text{m})} = \boxed{1.00\times10^{3}\ \text{N/C}}$ (in direction of electron's motion)

23.45 The required electric field will be $\boxed{\text{in the direction of motion}}$.

Work done = ΔK so, $-Fd = -\frac{1}{2}\ m v_i^2$ (since the final velocity = 0)

which becomes $eEd = K$ and $\boxed{E = \dfrac{K}{ed}}$

Goal Solution

The electrons in a particle beam each have a kinetic energy K. What are the magnitude and direction of the electric field that stops these electrons in a distance of d?

G: We should expect that a larger electric field would be required to stop electrons with greater kinetic energy. Likewise, **E** must be greater for a shorter stopping distance, d. The electric field should be in the same direction as the motion of the negatively charged electrons in order to exert an opposing force that will slow them down.

O: The electrons will experience an electrostatic force $\mathbf{F} = q\mathbf{E}$. Therefore, the work done by the electric field can be equated with the initial kinetic energy since energy should be conserved.

A: The work done on the charge is $\quad W = \mathbf{F}\cdot\mathbf{d} = q\mathbf{E}\cdot\mathbf{d}$

and $\quad K_i + W = K_f = 0$

Assuming **v** is in the $+x$ direction, $\quad K + (-e)\mathbf{E}\cdot d\mathbf{i} = 0$

$$e\mathbf{E}\cdot(d\mathbf{i}) = K$$

E is therefore in the direction of the electron beam: $\quad \mathbf{E} = \dfrac{K}{ed}\mathbf{i}$

L: As expected, the electric field is proportional to K, and inversely proportional to d. The direction of the electric field is important; if it were otherwise the electron would speed up instead of slowing down! If the particles were protons instead of electrons, the electric field would need to be directed opposite to **v** in order for the particles to slow down.

23.46 The acceleration is given by $\quad v^2 = v_i^2 + 2a(x - x_i)$ or $\quad v^2 = 0 + 2a(-h)$

Solving, $\quad a = -\dfrac{v^2}{2h}$

Now $\sum \mathbf{F} = m\mathbf{a}$: $\quad -mg\mathbf{j} + q\mathbf{E} = -\dfrac{mv^2\,\mathbf{j}}{2h}$

Therefore $\quad q\mathbf{E} = \left(-\dfrac{mv^2}{2h} + mg\right)\mathbf{j}$

(a) Gravity alone would give the bead downward impact velocity

$$\sqrt{2\left(9.80 \text{ m / s}^2\right)(5.00 \text{ m})} = 9.90 \text{ m / s}$$

To change this to 21.0 m/s down, a $\boxed{\text{downward}}$ electric field must exert a downward electric force.

(b) $$q = \frac{m}{E}\left(\frac{v^2}{2h} - g\right) = \frac{1.00\times10^{-3}\text{ kg}}{1.00\times10^{4}\text{ N/C}}\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{(21.0\text{ m/s})^2}{2(5.00\text{ m})} - 9.80\text{ m/s}^2\right) = \boxed{3.43\ \mu\text{C}}$$

23.47 (a) $t = \dfrac{x}{v} = \dfrac{0.0500}{4.50 \times 10^5} = 1.11 \times 10^{-7}\text{ s} = \boxed{111\text{ ns}}$

(b) $a_y = \dfrac{qE}{m} = \dfrac{(1.602 \times 10^{-19})(9.60 \times 10^3)}{(1.67 \times 10^{-27})} = 9.21 \times 10^{11}\text{ m / s}^2$

$y - y_i = v_{yi}t + \frac{1}{2}a_y t^2$

$y = \frac{1}{2}(9.21 \times 10^{11})(1.11 \times 10^{-7})^2 = 5.67 \times 10^{-3}\text{ m} = \boxed{5.67\text{ mm}}$

(c) $v_x = \boxed{4.50 \times 10^5\text{ m / s}}$

$v_y = v_{y\,i} + a_y = (9.21 \times 10^{11})(1.11 \times 10^{-7}) = \boxed{1.02 \times 10^5\text{ m / s}}$

23.48 $a_y = \dfrac{qE}{m} = \dfrac{(1.602 \times 10^{-19})(390)}{(9.11 \times 10^{-31})} = 6.86 \times 10^{13}\text{ m / s}^2$

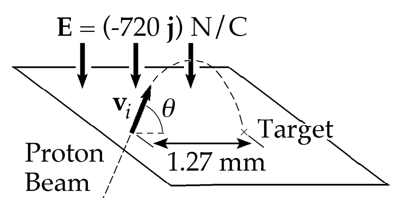
(a) $t = \dfrac{2v_i \sin\theta}{a_y}$ from projectile motion equations

$t = \dfrac{2(8.20 \times 10^5)\sin 30.0°}{6.86 \times 10^{13}} = 1.20 \times 10^{-8}\text{ s}) = \boxed{12.0\text{ ns}}$

(b) $h = \dfrac{v_i^2 \sin^2\theta}{2a_y} = \dfrac{(8.20 \times 10^5)^2 \sin^2 30.0°}{2(6.86 \times 10^{13})} = \boxed{1.23\text{ mm}}$

(c) $R = \dfrac{v_i^2 \sin 2\theta}{2a_y} = \dfrac{(8.20 \times 10^5)^2 \sin 60.0°}{2(6.86 \times 10^{13})} = \boxed{4.24\text{ mm}}$

23.49 $v_i = 9.55 \times 10^3\text{ m / s}$



(a) $a_y = \dfrac{eE}{m} = \dfrac{(1.60 \times 10^{-19})(720)}{(1.67 \times 10^{-27})} = 6.90 \times 10^{10}\text{ m/s}^2$

$R = \dfrac{v_i^2 \sin 2\theta}{a_y} = 1.27 \times 10^{-3}\text{ m}$ so that $\dfrac{(9.55 \times 10^3)^2 \sin 2\theta}{6.90 \times 10^{10}} = 1.27 \times 10^{-3}$

$\sin 2\theta = 0.961$ $\theta = \boxed{36.9°}$ $90.0° - \theta = \boxed{53.1°}$

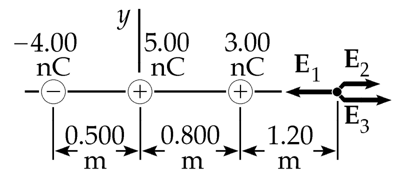
(b) $t = \dfrac{R}{v_{ix}} = \dfrac{R}{v_i \cos\theta}$ If $\theta = 36.9°$, $t = \boxed{167\text{ ns}}$ If $\theta = 53.1°$, $t = \boxed{221\text{ ns}}$

***23.50** (a) The field, E_1, due to the 4.00×10^{-9} C charge is in the $-x$ direction.

$$\mathbf{E}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(-4.00\times10^{-9}\ \mathrm{C})}{(2.50\ \mathrm{m})^2}\mathbf{i} = -5.75\mathbf{i}\ \mathrm{N/C}$$

Likewise, E_2 and E_3, due to the 5.00×10^{-9} C charge and the 3.00×10^{-9} C charge are

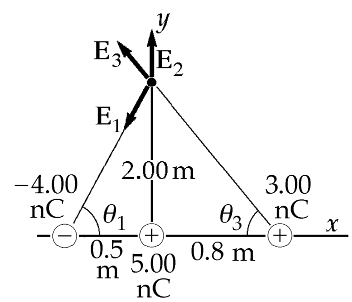
$$\mathbf{E}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(5.00\times10^{-9}\ \mathrm{C})}{(2.00\ \mathrm{m})^2}\mathbf{i} = 11.2\ \mathrm{N/C}$$



$$\mathbf{E}_3 = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.00\times10^{-9}\ \mathrm{C})}{(1.20\ \mathrm{m})^2}\mathbf{i} = 18.7\ \mathrm{N/C}$$

$$\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = \boxed{24.2\ \mathrm{N/C}}\ \text{ in } +x \text{ direction.}$$

(b) $$\mathbf{E}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (-8.46\ \mathrm{N/C})(0.243\mathbf{i} + 0.970\mathbf{j})$$



$$\mathbf{E}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (11.2\ \mathrm{N/C})(+\mathbf{j})$$

$$\mathbf{E}_3 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (5.81\ \mathrm{N/C})(-0.371\mathbf{i} + 0.928\mathbf{j})$$

$E_x = E_{1x} + E_{3x} = -4.21\mathbf{i}$ N/C $\qquad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\mathbf{j}$ N/C

$E_R = \boxed{9.42\ \mathrm{N/C}} \qquad \theta = \boxed{63.4° \text{ above } -x \text{ axis}}$

23.51 The proton moves with acceleration $$|a_p| = \frac{qE}{m} = \frac{(1.60\times10^{-19}\ \mathrm{C})(640\ \mathrm{N/C})}{1.67\times10^{-27}\ \mathrm{kg}} = 6.13\times10^{10}\ \mathrm{m/s^2}$$

while the e⁻ has acceleration $$|a_e| = \frac{(1.60\times10^{-19}\ \mathrm{C})(640\ \mathrm{N/C})}{9.11\times10^{-31}\ \mathrm{kg}} = 1.12\times10^{14}\ \mathrm{m/s^2} = 1836\,a_p$$

(a) We want to find the distance traveled by the proton (i.e., $d = \frac{1}{2}a_p t^2$), knowing:

$$4.00\ \mathrm{cm} = \tfrac{1}{2}a_p t^2 + \tfrac{1}{2}a_e t^2 = 1837\left(\tfrac{1}{2}a_p t^2\right)$$

Thus, $d = \frac{1}{2}a_p t^2 = \dfrac{4.00\ \mathrm{cm}}{1837} = \boxed{21.8\ \mu\mathrm{m}}$

(b) The distance from the positive plate to where the meeting occurs equals the distance the sodium ion travels (i.e., $d_{Na} = \frac{1}{2}a_{Na}t^2$). This is found from:

$$4.00\text{ cm} = \frac{1}{2}a_{Na}t^2 + \frac{1}{2}a_{Cl}t^2:\ 4.00\text{ cm} = \frac{1}{2}\left(\frac{eE}{22.99\text{ u}}\right)t^2 + \frac{1}{2}\left(\frac{eE}{35.45\text{ u}}\right)t^2$$

This may be written as $$4.00\text{ cm} = \frac{1}{2}a_{Na}t^2 + \frac{1}{2}\left(0.649a_{Na}\right)t^2 = 1.65\left(\frac{1}{2}a_{Na}t^2\right)$$

so $$d_{Na} = \frac{1}{2}a_{Na}t^2 = \frac{4.00\text{ cm}}{1.65} = \boxed{2.43\text{ cm}}$$

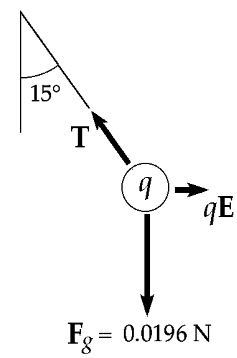
23.52 From the free-body diagram shown, $\sum F_y = 0$

and $T\cos 15.0° = 1.96 \times 10^{-2}$ N

So $T = 2.03 \times 10^{-2}$ N

From $\sum F_x = 0$, we have $qE = T\sin 15.0°$

or $$q = \frac{T\sin 15.0°}{E} = \frac{(2.03 \times 10^{-2}\text{ N})\sin 15.0°}{1.00 \times 10^3\text{ N/C}} = 5.25 \times 10^{-6}\text{ C} = \boxed{5.25\ \mu\text{C}}$$

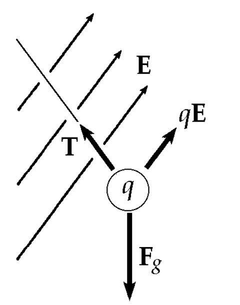


23.53 (a) Let us sum force components to find

$\sum F_x = qE_x - T\sin\theta = 0$, and $\sum F_y = qE_y + T\cos\theta - mg = 0$

Combining these two equations, we get

$$q = \frac{mg}{(E_x\cot\theta + E_y)} = \frac{(1.00 \times 10^{-3})(9.80)}{(3.00\cot 37.0° + 5.00) \times 10^5} = 1.09 \times 10^{-8}\text{ C} = \boxed{10.9\text{ nC}}$$



Free Body Diagram for Goal Solution

(b) From the two equations for $\sum F_x$ and $\sum F_y$ we also find

$$T = \frac{qE_x}{\sin 37.0°} = 5.44 \times 10^{-3}\text{ N} = \boxed{5.44\text{ mN}}$$

Goal Solution

A charged cork ball of mass 1.00 g is suspended on a light string in the presence of a uniform electric field, as shown in Fig. P23.53. When $\mathbf{E} = (3.00\mathbf{i} + 5.00\mathbf{j}) \times 10^5$ N / C, the ball is in equilibrium at $\theta = 37.0°$. Find (a) the charge on the ball and (b) the tension in the string.

G: (a) Since the electric force must be in the same direction as **E**, the ball must be positively charged. If we examine the free body diagram that shows the three forces acting on the ball, the sum of which must be zero, we can see that the tension is about half the magnitude of the weight.

O: The tension can be found from applying Newton's second law to this statics problem (electrostatics, in this case!). Since the force vectors are in two dimensions, we must apply $\Sigma\mathbf{F} = m\mathbf{a}$ to both the x and y directions.

A: Applying Newton's Second Law in the x and y directions, and noting that $\Sigma\mathbf{F} = \mathbf{T} + q\mathbf{E} + \mathbf{F}_g = 0$,

$$\Sigma F_x = qE_x - T\sin 37.0° = 0 \qquad (1)$$

$$\Sigma F_y = qE_y + T\cos 37.0° - mg = 0 \qquad (2)$$

We are given $E_x = 3.00 \times 10^5$ N / C and $E_y = 5.00 \times 10^5$ N / C; substituting T from (1) into (2):

$$q = \frac{mg}{\left(E_y + \dfrac{E_x}{\tan 37.0°}\right)} = \frac{(1.00 \times 10^{-3}\text{ kg})(9.80\text{ m / s}^2)}{\left(5.00 + \dfrac{3.00}{\tan 37.0°}\right) \times 10^5\text{ N / C}} = 1.09 \times 10^{-8}\text{ C}$$

(b) Using this result for q in Equation (1), we find that the tension is $T = \dfrac{qE_x}{\sin 37.0°} = 5.44 \times 10^{-3}$ N

L: The tension is slightly more than half the weight of the ball ($F_g = 9.80 \times 10^{-3}$ N) so our result seems reasonable based on our initial prediction.

23.54 (a) Applying the first condition of equilibrium to the ball gives:

$$\Sigma F_x = qE_x - T\sin\theta = 0 \qquad \text{or} \qquad T = \frac{qE_x}{\sin\theta} = \frac{qA}{\sin\theta}$$

and $$\Sigma F_y = qE_y + T\cos\theta - mg = 0 \qquad \text{or} \qquad qB + T\cos\theta = mg$$

Substituting from the first equation into the second gives:

$$q(A\cot\theta + B) = mg\,, \quad \text{or} \quad q = \boxed{\frac{mg}{(A\cot\theta + B)}}$$

(b) Substituting the charge into the equation obtained from ΣF_x yields

$$T = \frac{mg}{(A\cot\theta + B)}\left(\frac{A}{\sin\theta}\right) = \boxed{\frac{mgA}{A\cos\theta + B\sin\theta}}$$

Goal Solution

A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field, as shown in Figure P23.53. When $\mathbf{E} = (A\mathbf{i} + B\mathbf{j})$ N / C, where A and B are positive numbers, the ball is in equilibrium at the angle θ. Find (a) the charge on the ball and (b) the tension in the string.

G: This is the general version of the preceding problem. The known quantities are A, B, m, g, and θ. The unknowns are q and T.

O: The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 53.

A: Again, Newton's second law: $-T\sin\theta + qA = 0$ (1)

and $+T\cos\theta + qB - mg = 0$ (2)

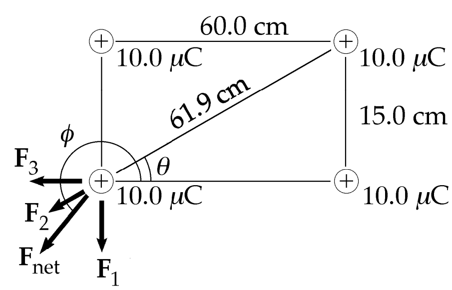
(a) Substituting $T = \dfrac{qA}{\sin\theta}$, into Eq. (2), $\dfrac{qA\cos\theta}{\sin\theta} + qB = mg$

Isolating q on the left, $q = \dfrac{mg}{(A\cot\theta + B)}$

(b) Substituting this value into Eq. (1), $T = \dfrac{mgA}{(A\cos\theta + B\sin\theta)}$

L: If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for q and T to find the numerical results needed for problem 53. If you find this problem more difficult than problem 53, the little list at the Gather step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the Analysis step, and for recognizing when we have an answer.

23.55 $F = \dfrac{k_e q_1 q_2}{r^2}$ $\quad \tan\theta = \dfrac{15.0}{60.0}$ $\quad \theta = 14.0°$



$$F_1 = \frac{(8.99 \times 10^9)(10.0 \times 10^{-6})^2}{(0.150)^2} = 40.0 \text{ N}$$

$$F_3 = \frac{(8.99 \times 10^9)(10.0 \times 10^{-6})^2}{(0.600)^2} = 2.50 \text{ N}$$

$$F_2 = \frac{(8.99 \times 10^9)(10.0 \times 10^{-6})^2}{(0.619)^2} = 2.35 \text{ N}$$

$$F_x = -F_3 - F_2 \cos 14.0° = -2.50 - 2.35 \cos 14.0° = -4.78 \text{ N}$$

$$F_y = -F_1 - F_2 \sin 14.0° = -40.0 - 2.35 \sin 14.0° = -40.6 \text{ N}$$

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{(-4.78)^2 + (-40.6)^2} = \boxed{40.9 \text{ N}}$$

$$\tan\phi = \frac{F_y}{F_x} = \frac{-40.6}{-4.78} \qquad \phi = \boxed{263°}$$

23.56 From Fig. A: $d\cos 30.0° = 15.0\text{ cm}$, or $d = \dfrac{15.0\text{ cm}}{\cos 30.0°}$

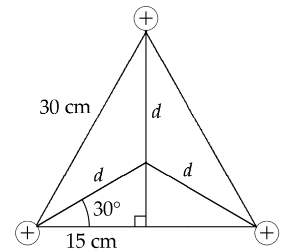


Figure A

From Fig. B: $\theta = \sin^{-1}\left(\dfrac{d}{50.0\text{ cm}}\right) = \sin^{-1}\left(\dfrac{15.0\text{ cm}}{50.0\text{ cm}(\cos 30.0°)}\right) = 20.3°$

$$\frac{F_q}{mg} = \tan\theta$$

or $F_q = mg\tan 20.3°$ (1)

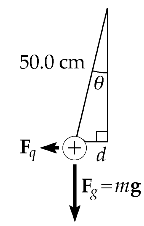


Figure B

From Fig. C: $F_q = 2F\cos 30.0° = 2\left[\dfrac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0°$ (2)

Equating equations (1) and (2), $2\left[\dfrac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0° = mg\tan 20.3°$

$$q^2 = \frac{mg(0.300\text{ m})^2\tan 20.3°}{2k_e\cos 30.0°}$$

$$q^2 = \frac{(2.00\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m})^2\tan 20.3°}{2(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\cos 30.0°}$$

$$q = \sqrt{4.20\times10^{-14}\text{ C}^2} = 2.05\times10^{-7}\text{ C} = \boxed{0.205\ \mu\text{C}}$$

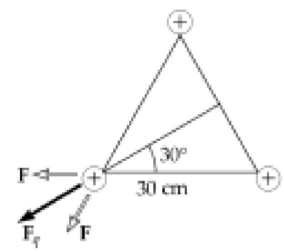


Figure C

23.57 Charge $Q/2$ resides on each block, which repel as point charges:

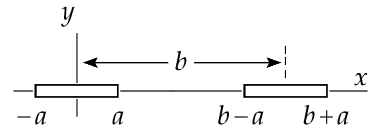
$$F = \frac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$$

$$Q = 2L\sqrt{\frac{k(L-L_i)}{k_e}} = 2(0.400\text{ m})\sqrt{\frac{(100\text{ N/m})(0.100\text{ m})}{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}} = \boxed{26.7\ \mu\text{C}}$$

23.58 Charge $Q/2$ resides on each block, which repel as point charges: $F = \dfrac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$

Solving for Q, $Q = \boxed{2L\sqrt{\dfrac{k(L-L_i)}{k_e}}}$

***23.59** According to the result of Example 23.7, the lefthand rod creates this field at a distance d from its righthand end:



$$E = \frac{k_e Q}{d(2a+d)}$$

$$dF = \frac{k_e Q Q}{2a}\frac{dx}{d(d+2a)}$$

$$F = \frac{k_e Q^2}{2a}\int_{x=b-2a}^{b}\frac{dx}{x(x+2a)} = \frac{k_e Q^2}{2a}\left(-\frac{1}{2a}\ln\frac{2a+x}{x}\right)_{b-2a}^{b}$$

$$F = \frac{+k_e Q^2}{4a^2}\left(-\ln\frac{2a+b}{b} + \ln\frac{b}{b-2a}\right) = \frac{k_e Q^2}{4a^2}\ln\frac{b^2}{(b-2a)(b+2a)} = \boxed{\left(\frac{k_e Q^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2-4a^2}\right)}$$

***23.60** The charge moves with acceleration of magnitude a given by $\sum F = ma = |q|E$

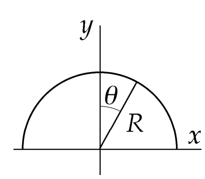
(a) $a = \dfrac{|q|E}{m} = \dfrac{1.60\times10^{-19}\text{ C}(1.00\text{ N/C})}{9.11\times10^{-31}\text{ kg}} = 1.76\times10^{11}\text{ m/s}^2$

Then $v = v_i + at = 0 + at$ gives $t = \dfrac{v}{a} = \dfrac{3.00\times10^{7}\text{ m/s}}{1.76\times10^{11}\text{ m/s}^2} = \boxed{171\ \mu\text{s}}$

(b) $t = \dfrac{v}{a} = \dfrac{vm}{qE} = \dfrac{(3.00\times10^{7}\text{ m/s})(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(1.00\text{ N/C})} = \boxed{0.313\text{ s}}$

(c) From $t = \dfrac{vm}{qE}$, as E increases, t gets $\boxed{\text{shorter}}$ in inverse proportion.

23.61 $Q = \int \lambda\, d\ell = \int_{-90.0°}^{90.0°}\lambda_0\cos\theta\, R\,d\theta = \lambda_0 R\sin\theta\Big|_{-90.0°}^{90.0°} = \lambda_0 R\,[1-(-1)] = 2\lambda_0 R$



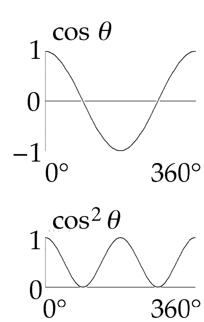
$Q = 12.0\ \mu\text{C} = (2\lambda_0)(0.600)\text{ m} = 12.0\ \mu\text{C}$ so $\lambda_0 = 10.0\ \mu\text{C/m}$

$$dF_y = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})(\lambda\,d\ell)}{R^2}\right)\cos\theta = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})\left(\lambda_0\cos^2\theta R\,d\theta\right)}{R^2}\right)$$

$$F_y = \int_{-90.0°}^{90.0°}\left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(3.00\times10^{-6}\text{ C})(10.0\times10^{-6}\text{ C/m})}{(0.600\text{ m})}\cos^2\theta\,d\theta$$

$$F_y = \frac{8.99(30.0)}{0.600}\left(10^{-3}\text{ N}\right)\int_{-\pi/2}^{\pi/2}\left(\tfrac{1}{2}+\tfrac{1}{2}\cos 2\theta\right)d\theta$$

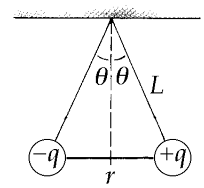
$$F_y = (0.450\text{ N})\left(\tfrac{1}{2}\pi + \tfrac{1}{4}\sin 2\theta\Big|_{-\pi/2}^{\pi/2}\right) = \boxed{0.707\text{ N}}\quad\text{Downward.}$$



Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

23.62 At equilibrium, the distance between the charges is $r = 2(0.100\text{ m})\sin 10.0° = 3.47 \times 10^{-2}\text{ m}$

Now consider the forces on the sphere with charge $+q$, and use $\Sigma F_y = 0$:

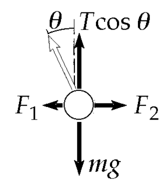


$\Sigma F_y = 0$: $\quad T\cos 10.0° = mg, \quad \text{or} \quad T = \dfrac{mg}{\cos 10.0°}$ (1)

$\Sigma F_x = 0$: $\quad F_{\text{net}} = F_2 - F_1 = T\sin 10.0°$ (2)

F_{net} is the net electrical force on the charged sphere. Eliminate T from (2) by use of (1).

$$F_{\text{net}} = \frac{mg\sin 10.0°}{\cos 10.0°} = mg\tan 10.0° = \left(2.00 \times 10^{-3}\text{ kg}\right)\left(9.80\text{ m / s}^2\right)\tan 10.0° = 3.46 \times 10^{-3}\text{ N}$$



F_{net} is the resultant of two forces, F_1 and F_2. F_1 is the attractive force on $+q$ exerted by $-q$, and F_2 is the force exerted on $+q$ by the external electric field.

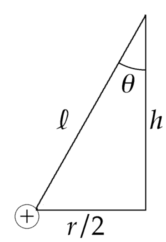
$F_{\text{net}} = F_2 - F_1$ or $F_2 = F_{\text{net}} + F_1$

$$F_1 = \left(8.99 \times 10^9\text{ N}\cdot\text{m}^2\text{ / C}^2\right)\frac{\left(5.00 \times 10^{-8}\text{ C}\right)\left(5.00 \times 10^{-8}\text{ C}\right)}{\left(3.47 \times 10^{-3}\text{ m}\right)^2} = 1.87 \times 10^{-2}\text{ N}$$

Thus, $F_2 = F_{\text{net}} + F_1$ yields $F_2 = 3.46 \times 10^{-3}\text{ N} + 1.87 \times 10^{-2}\text{ N} = 2.21 \times 10^{-2}\text{ N}$

and $F_2 = qE$, or $\quad E = \dfrac{F_2}{q} = \dfrac{2.21 \times 10^{-2}\text{ N}}{5.00 \times 10^{-8}\text{ C}} = 4.43 \times 10^5\text{ N/C} = \boxed{443\text{ kN/C}}$

23.63 (a) From the $2Q$ charge we have $\quad F_e - T_2\sin\theta_2 = 0 \quad$ and $\quad mg - T_2\cos\theta_2 = 0$



Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$

From the Q charge we have $\quad F_e - T_1\sin\theta_1 = 0 \quad$ and $\quad mg - T_1\cos\theta_1 = 0$

Combining these we find $\quad \dfrac{F_e}{mg} = \dfrac{T_1\sin\theta_1}{T_1\cos\theta_1} = \tan\theta_1 \quad$ or $\quad \boxed{\theta_2 = \theta_1}$

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_eQ^2}{r^2}$

If we assume θ is small then $\tan\theta \approx \dfrac{(r/2)}{\ell}$. Substitute expressions for F_e and $\tan\theta$ into either equation found in part (a) and solve for r.

$\dfrac{F_e}{mg} = \tan\theta \quad$ then $\quad \dfrac{2k_eQ^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell} \quad$ and solving for r we find $\quad r = \left[\dfrac{4k_eQ^2\ell}{mg}\right]^{1/3}$

23.64 At an equilibrium position, the net force on the charge Q is zero. The equilibrium position can be located by determining the angle θ corresponding to equilibrium. In terms of lengths s, $\frac{1}{2}a\sqrt{3}$, and r, shown in Figure P23.64, the charge at the origin exerts an attractive force $k_eQq/(s+\frac{1}{2}a\sqrt{3})^2$. The other two charges exert equal repulsive forces of magnitude k_eQq/r^2. The horizontal components of the two repulsive forces add, balancing the attractive force,

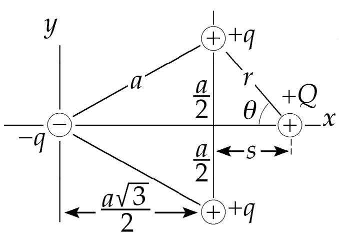
$$F_{\text{net}} = k_eQq\left\{\frac{2\cos\theta}{r^2} - \frac{1}{(s+\frac{1}{2}a\sqrt{3})^2}\right\} = 0$$

From Figure P23.64, $r = \dfrac{\frac{1}{2}a}{\sin\theta}$ $\qquad s = \frac{1}{2}a\cot\theta$

The equilibrium condition, in terms of θ, is $F_{\text{net}} = \left(\dfrac{4}{a^2}\right)k_eQq\left(2\cos\theta\sin^2\theta - \dfrac{1}{(\sqrt{3}+\cot\theta)^2}\right) = 0$

Thus the equilibrium value of θ is $\quad 2\cos\theta\sin^2\theta(\sqrt{3}+\cot\theta)^2 = 1.$

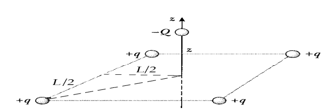
One method for solving for θ is to tabulate the left side. To three significant figures the value of θ corresponding to equilibrium is 81.7°. The distance from the origin to the equilibrium position is $x = \frac{1}{2}a(\sqrt{3}+\cot 81.7°) = \boxed{0.939a}$



θ	$2\cos\theta\sin^2\theta(\sqrt{3}+\cot\theta)^2$
60°	4
70°	2.654
80°	1.226
90°	0
81°	1.091
81.5°	1.024
81.7°	0.997

23.65 (a) The distance from each corner to the center of the square is

$$\sqrt{(L/2)^2+(L/2)^2} = L/\sqrt{2}$$



The distance from each positive charge to $-Q$ is then $\sqrt{z^2+L^2/2}$. Each positive charge exerts a force directed along the line joining q and $-Q$, of magnitude

$$\frac{k_eQq}{z^2+L^2/2}$$

The line of force makes an angle with the z-axis whose cosine is $\dfrac{z}{\sqrt{z^2+L^2/2}}$

The four charges together exert forces whose x and y components add to zero, while the z-components add to

$$\mathbf{F} = \boxed{-\frac{4k_eQqz}{\left(z^2+L^2/2\right)^{3/2}}\mathbf{k}}$$

(b) For $z << L$, the magnitude of this force is $F_z \approx -\dfrac{4k_e Qqz}{\left(L^2/2\right)^{3/2}} = -\left(\dfrac{4(2)^{3/2} k_e Qq}{L^3}\right) z = ma_z$

Therefore, the object's vertical acceleration is of the form $\boxed{a_z = -\omega^2 z}$

with $\omega^2 = \dfrac{4(2)^{3/2} k_e Qq}{mL^3} = \dfrac{k_e Qq\sqrt{128}}{mL^3}$

Since the acceleration of the object is always oppositely directed to its excursion from equilibrium and in magnitude proportional to it, the object will execute simple harmonic motion with a period given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(128)^{1/4}}\sqrt{\frac{mL^3}{k_e Qq}} = \boxed{\frac{\pi}{(8)^{1/4}}\sqrt{\frac{mL^3}{k_e Qq}}}$$

23.66 (a) The total non-contact force on the cork ball is: $F = qE + mg = m\left(g + \dfrac{qE}{m}\right)$,

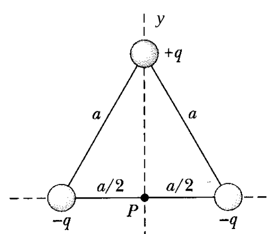
which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$T = 2\pi\sqrt{\frac{L}{g + \dfrac{qE}{m}}} = 2\pi\sqrt{\frac{0.500\text{ m}}{9.80\text{ m / s}^2 + \dfrac{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N / C}\right)}{1.00\times10^{-3}\text{ kg}}}} = \boxed{0.307\text{ s}}$$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get $T = 2\pi\sqrt{\dfrac{L}{qE/m}}$

$$T = 2\pi\sqrt{\frac{0.500\text{ m}}{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N / C}\right)/1.00\times10^{-3}\text{ kg}}} = 0.314\text{ s} \quad \text{(a 2.28\% difference)}.$$

23.67 (a) Due to symmetry the field contribution from each negative charge is equal and opposite to each other. Therefore, their contribution to the net field is zero. The field contribution of the $+q$ charge is



$$E = \frac{k_e q}{r^2} = \frac{k_e q}{\left(3a^2/4\right)} = \frac{4k_e q}{3a^2}$$

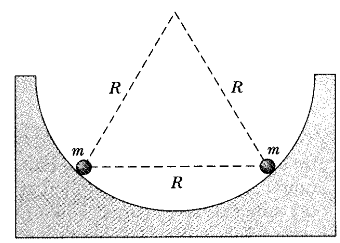
in the negative y direction, i.e., $\mathbf{E} = \boxed{-\dfrac{4k_e q}{3a^2}\mathbf{j}}$

(b) If $F_e = 0$, then E at P must equal zero. In order for the field to cancel at P, the $-4q$ must be above $+q$ on the y-axis.

Then, $E = 0 = -\dfrac{k_e q}{(1.00\text{ m})^2} + \dfrac{k_e(4q)}{y^2}$, which reduces to $y^2 = 4.00\text{ m}^2$.

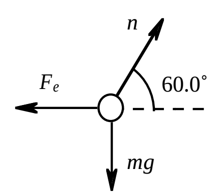
Thus, $y = \pm 2.00\text{ m}$. Only the positive answer is acceptable since the $-4q$ must be located above $+q$. Therefore, the $-4q$ must be placed $\boxed{\text{2.00 meters above point } P \text{ along the } +y-\text{axis}}$.

23.68 The bowl exerts a normal force on each bead, directed along the radius line or at 60.0° above the horizontal. Consider the free-body diagram of the bead on the left:



$\Sigma F_y = n\sin 60.0° - mg = 0$,

or $\quad n = \dfrac{mg}{\sin 60.0°}$

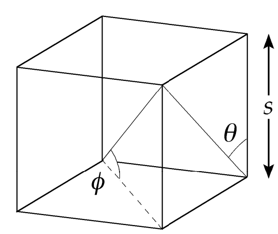


Also, $\quad \Sigma F_x = -F_e + n\cos 60.0° = 0$,

or $\quad \dfrac{k_e q^2}{R^2} = n\cos 60.0° = \dfrac{mg}{\tan 60.0°} = \dfrac{mg}{\sqrt{3}}$

Thus, $\quad q = \boxed{R\left(\dfrac{mg}{k_e\sqrt{3}}\right)^{1/2}}$

23.69 (a) There are 7 terms which contribute:



3 are s away (along sides)

3 are $\sqrt{2}\ s$ away (face diagonals) and $\sin\theta = \dfrac{1}{\sqrt{2}} = \cos\theta$

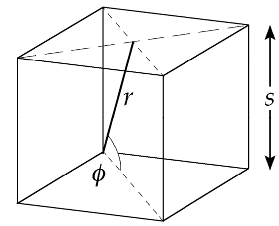
1 is $\sqrt{3}\ s$ away (body diagonal) and $\sin\phi = \dfrac{1}{\sqrt{3}}$

The component in each direction is the same by symmetry.

$$\mathbf{F} = \frac{k_e q^2}{s^2}\left[1 + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right](\mathbf{i}+\mathbf{j}+\mathbf{k}) = \boxed{\frac{k_e q^2}{s^2}(1.90)(\mathbf{i}+\mathbf{j}+\mathbf{k})}$$

(b) $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\dfrac{k_e q^2}{s^2} \text{ away from the origin}}$

23.70 (a) Zero contribution from the same face due to symmetry, opposite face contributes



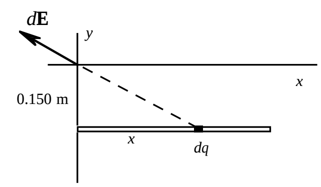
$\sin\phi = s/r$

$$4\left(\frac{k_e q}{r^2}\sin\phi\right) \quad \text{where} \quad r = \sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2 + s^2} = \sqrt{1.5}\, s = 1.22\, s$$

$$E = 4\frac{k_e q s}{r^3} = \frac{4}{(1.22)^3}\frac{k_e q}{s^2} = \boxed{2.18\frac{k_e q}{s^2}}$$

(b) $\boxed{\text{The direction is the } \mathbf{k} \text{ direction.}}$

***23.71**



$$d\mathbf{E} = \frac{k_e\, dq}{x^2 + (0.150\text{ m})^2}\left(\frac{-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right) = \frac{k_e\lambda(-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j})dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$

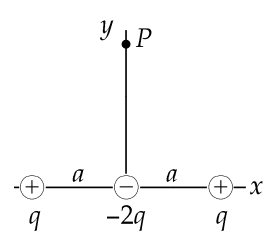
$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = k_e\lambda\int_{x=0}^{0.400\text{ m}} \frac{(-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j})dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$

$$\mathbf{E} = k_e\lambda\left[\left.\frac{+\mathbf{i}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}} + \left.\frac{(0.150\text{ m})\,\mathbf{j}\,x}{(0.150\text{ m})^2\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

$$\mathbf{E} = \left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(35.0\times 10^{-9}\ \frac{\text{C}}{\text{m}}\right)\left[\mathbf{i}(2.34 - 6.67)/\text{m} + \mathbf{j}(6.24 - 0)/\text{m}\right]$$

$$\mathbf{E} = (-1.36\,\mathbf{i} + 1.96\,\mathbf{j})\times 10^3\ \text{N/C} = \boxed{(-1.36\,\mathbf{i} + 1.96\,\mathbf{j})\ \text{kN/C}}$$

23.72 By symmetry $\sum E_x = 0$. Using the distances as labeled,



$$\sum E_y = k_e\left[\frac{q}{(a^2 + y^2)}\sin\theta + \frac{q}{(a^2 + y^2)}\sin\theta - \frac{2q}{y^2}\right]$$

But $\sin\theta = \dfrac{y}{\sqrt{(a^2 + y^2)}}$, so $E = \sum E_y = 2k_e q\left[\dfrac{y}{(a^2 + y^2)^{3/2}} - \dfrac{1}{y^2}\right]$

Expand $(a^2 + y^2)^{-3/2}$ as $(a^2 + y^2)^{-3/2} = y^{-3} - (3/2)a^2 y^{-5} + \ldots$

Therefore, for $a << y$, we can ignore terms in powers higher than 2,

and we have $E = 2k_e q\left[\dfrac{1}{y^2} - \left(\dfrac{3}{2}\right)\dfrac{a^2}{y^4} - \dfrac{1}{y^2}\right]$ or $\boxed{\mathbf{E} = \left[-\dfrac{k_e 3qa^2}{y^4}\right]\mathbf{j}}$

23.73 The field on the axis of the ring is calculated in Example 23.8, $E = E_x = \dfrac{k_e x Q}{(x^2 + a^2)^{3/2}}$

The force experienced by a charge $-q$ placed along the axis of the ring is

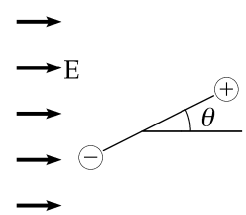
$F = -k_e Qq\left[\dfrac{x}{(x^2 + a^2)^{3/2}}\right]$ and when $x << a$, this becomes $F = \left(\dfrac{k_e Qq}{a^3}\right)x$

This expression for the force is in the form of Hooke's law,

with an effective spring constant of $k = k_e Qq/a^3$

Since $\omega = 2\pi f = \sqrt{k/m}$, we have $f = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e Qq}{ma^3}}}$

23.74 The electrostatic forces exerted on the two charges result in a net torque $\tau = -2Fa \sin\theta = -2Eqa \sin\theta$.



For small θ, $\sin\theta \approx \theta$ and using $p = 2qa$, we have $\tau = -Ep\theta$.

The torque produces an angular acceleration given by $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$

Combining these two expressions for torque, we have $\dfrac{d^2\theta}{dt^2} + \left(\dfrac{Ep}{I}\right)\theta = 0$

This equation can be written in the form $\dfrac{d^2\theta}{dt^2} = -\,\omega^2\theta$ where $\omega^2 = \dfrac{Ep}{I}$

This is the same form as Equation 13.17 and the frequency of oscillation is found by comparison with Equation 13.19, or $f = \dfrac{1}{2\pi}\sqrt{\dfrac{pE}{I}} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{2qaE}{I}}}$

Chapter 24 Solutions

24.1 (a) $\Phi_E = EA\cos\theta = (3.50\times10^3)(0.350\times0.700)\cos 0° = \boxed{858\ \text{N}\cdot\text{m}^2/\text{C}}$

(b) $\theta = 90.0°$ $\boxed{\Phi_E = 0}$

(c) $\Phi_E = (3.50\times10^3)(0.350\times0.700)\cos 40.0° = \boxed{657\ \text{N}\cdot\text{m}^2/\text{C}}$

24.2 $\Phi_E = EA\cos\theta = (2.00\times10^4\ \text{N/C})(18.0\ \text{m}^2)\cos 10.0° = \boxed{355\ \text{kN}\cdot\text{m}^2/\text{C}}$

24.3 $\Phi_E = EA\cos\theta$

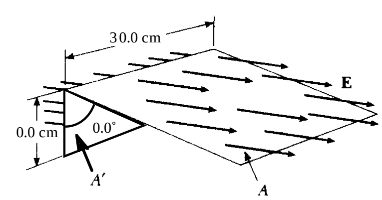
$A = \pi r^2 = \pi(0.200)^2 = 0.126\ \text{m}^2$

$5.20\times10^5 = E(0.126)\cos 0°$

$E = 4.14\times10^6\ \text{N/C} = \boxed{4.14\ \text{MN/C}}$

24.4 The uniform field enters the shell on one side and exits on the other so the total flux is $\boxed{\text{zero}}$.

24.5 (a) $A' = (10.0\ \text{cm})(30.0\ \text{cm})$



$A' = 300\ \text{cm}^2 = 0.0300\ \text{m}^2$

$\Phi_{E,\,A'} = EA'\cos\theta$

$\Phi_{E,\,A'} = (7.80\times10^4)(0.0300)\cos 180°$

$\Phi_{E,\,A'} = \boxed{-2.34\ \text{kN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_{E,\,A} = EA\cos\theta = (7.80\times10^4)(A)\cos 60.0°$

$$A = (30.0\ \text{cm})(w) = (30.0\ \text{cm})\left(\frac{10.0\ \text{cm}}{\cos 60.0°}\right) = 600\ \text{cm}^2 = 0.0600\ \text{m}^2$$

$\Phi_{E,\,A} = (7.80\times10^4)(0.0600)\cos 60° = \boxed{+\,2.34\ \text{kN}\cdot\text{m}^2/\text{C}}$

(c) The bottom and the two triangular sides all lie *parallel* to **E**, so $\Phi_E = 0$ for each of these. Thus,

$$\Phi_{E,\ \text{total}} = -2.34\ \text{kN}\cdot\text{m}^2/\text{C} + 2.34\ \text{kN}\cdot\text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}$$

24.6 (a) $\Phi_E = \mathbf{E}\cdot\mathbf{A} = (a\mathbf{i}+b\mathbf{j})\cdot A\mathbf{i} = \boxed{aA}$

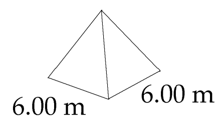
(b) $\Phi_E = (a\mathbf{i}+b\mathbf{j})\cdot A\mathbf{j} = \boxed{bA}$

(c) $\Phi_E = (a\mathbf{i}+b\mathbf{j})\cdot A\mathbf{k} = \boxed{0}$

24.7 Only the charge inside radius R contributes to the total flux.

$\Phi_E = \boxed{q/\epsilon_0}$

24.8 $\Phi_E = EA\cos\theta$ through the base



$\Phi_E = (52.0)(36.0)\cos 180° = -1.87\ \text{kN}\cdot\text{m}^2/\text{C}$

Note the same number of electric field lines go through the base as go through the pyramid's surface (not counting the base).

For the slanting surfaces, $\boxed{\Phi_E = +1.87\ \text{kN}\cdot\text{m}^2/\text{C}}$

24.9 The flux entering the closed surface equals the flux exiting the surface. The flux entering the left side of the cone is $\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = \boxed{ERh}$. This is the same as the flux that exits the right side of the cone. Note that for a uniform field only the cross sectional area matters, not shape.

***24.10** (a) $E = \dfrac{k_e Q}{r^2}$

$8.90\times 10^2 = \dfrac{(8.99\times 10^9)Q}{(0.750)^2}$, But Q is negative since **E** points inward.

$Q = -5.56\times 10^{-8}\ \text{C} = \boxed{-55.6\ \text{nC}}$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution.

24.11 (a) $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{(+5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C})}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -6.89\times 10^6\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{-6.89\ \text{MN}\cdot\text{m}^2/\text{C}}$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

24.12 $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0}$

Through S_1 $\Phi_E = \dfrac{-2Q+Q}{\epsilon_0} = \boxed{-\dfrac{Q}{\epsilon_0}}$

Through S_2 $\Phi_E = \dfrac{+Q-Q}{\epsilon_0} = \boxed{0}$

Through S_3 $\Phi_E = \dfrac{-2Q+Q-Q}{\epsilon_0} = \boxed{-\dfrac{2Q}{\epsilon_0}}$

Through S_4 $\Phi_E = \boxed{0}$

24.13 (a) One-half of the total flux created by the charge q goes through the plane. Thus,

$$\Phi_{E,\ \text{plane}} = \frac{1}{2}\Phi_{E,\ \text{total}} = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \boxed{\frac{q}{2\epsilon_0}}$$

(b) The square looks like an infinite plane to a charge *very close* to the surface. Hence,

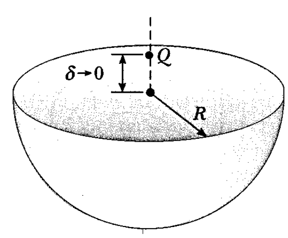
$$\Phi_{E,\ \text{square}} \approx \Phi_{E,\ \text{plane}} = \boxed{\frac{q}{2\epsilon_0}}$$

(c) $\boxed{\text{The plane and the square look the same to the charge.}}$

24.14 The flux through the curved surface is equal to the flux through the flat circle, $\boxed{E_0\,\pi r^2}$.

24.15 (a) $\boxed{\dfrac{+Q}{2\epsilon_0}}$ Simply consider half of a closed sphere.

(b) $\boxed{\dfrac{-Q}{2\epsilon_0}}$ (from $\Phi_{E,\ \text{total}} = \Phi_{E,\ \text{dome}} + \Phi_{E,\ \text{flat}} = 0$)



Goal Solution

A point charge Q is located just above the center of the flat face of a hemisphere of radius R, as shown in Figure P24.15. What is the electric flux (a) through the curved surface and (b) through the flat face?

G: From Gauss's law, the flux through a sphere with a point charge in it should be Q/ϵ_0, so we should expect the electric flux through a hemisphere to be half this value: $\Phi_{\text{curved}} = Q/2\epsilon_0$. Since the flat section appears like an infinite plane to a point just above its surface so that half of all the field lines from the point charge are intercepted by the flat surface, the flux through this section should also equal $Q/2\epsilon_0$.

O: We can apply the definition of electric flux directly for part (a) and then use Gauss's law to find the flux for part (b).

A: (a) With δ very small, all points on the hemisphere are nearly at distance R from the charge, so the field everywhere on the curved surface is $k_e Q / R^2$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:

$$\Phi_{\text{curved}} = \int \mathbf{E} \cdot d\mathbf{A} = E_{\text{local}} A_{\text{hemisphere}} = \left(k_e \frac{Q}{R^2}\right)\left(\tfrac{1}{2}\right)\left(4\pi R^2\right) = \frac{1}{4\pi\epsilon_0} Q(2\pi) = \frac{Q}{2\epsilon_0}$$

(b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \qquad \text{or} \qquad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \frac{-Q}{2\epsilon_0}$$

L: The direct calculations of the electric flux agree with our predictions, except for the negative sign in part (b), which comes from the fact that the area unit vector is defined as pointing outward from an enclosed surface, and in this case, the electric field has a component in the opposite direction (down).

24.16 (a) $\Phi_{E,\text{ shell}} = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{12.0 \times 10^{-6}}{8.85 \times 10^{-12}} = 1.36 \times 10^6 \text{ N} \cdot \text{m}^2 / \text{C} = \boxed{1.36 \text{ MN} \cdot \text{m}^2/\text{C}}$

(b) $\Phi_{E,\text{ half shell}} = \tfrac{1}{2}(1.36 \times 10^6 \text{ N} \cdot \text{m}^2 / \text{C}) = 6.78 \times 10^5 \text{ N} \cdot \text{m}^2 / \text{C} = \boxed{678 \text{ kN} \cdot \text{m}^2/\text{C}}$

(c) $\boxed{\text{No,}}$ the same number of field lines will pass through each surface, no matter how the radius changes.

24.17 From Gauss's Law, $\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \dfrac{q_{in}}{\epsilon_0}$.

Thus, $\Phi_E = \dfrac{Q}{\epsilon_0} = \dfrac{0.0462 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = \boxed{5.22 \text{ kN} \cdot \text{m}^2/\text{C}}$

24.18 If $R \le d$, the sphere encloses no charge and $\Phi_E = q_{\text{in}} / \epsilon_0 = \boxed{0}$

If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$

so $\Phi_E = \boxed{2\lambda \sqrt{R^2 - d^2}\big/\epsilon_0}$

24.19 The total charge is $Q - 6|q|$. The total outward flux from the cube is $\left(Q - 6|q|\right)/\epsilon_0$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q - 6|q|}{6\epsilon_0}}$$

$$(\Phi_E)_{\text{one face}} = \frac{Q - 6|q|}{6\epsilon_0} = \frac{(5.00 - 6.00)\times 10^{-6}\ \text{C}\cdot\text{N}\cdot\text{m}^2}{6\times 8.85\times 10^{-12}\ \text{C}^2} = \boxed{-\,18.8\ \text{kN}\cdot\text{m}^2/\text{C}}$$

24.20 The total charge is $Q - 6|q|$. The total outward flux from the cube is $\left(Q - 6|q|\right)/\epsilon_0$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q - 6|q|}{6\epsilon_0}}$$

24.21 When $R < d$, the cylinder contains no charge and $\Phi_E = \boxed{0}$.

When $R > d$, $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \boxed{\dfrac{\lambda L}{\epsilon_0}}$

24.22 $$\Phi_{E,\ \text{hole}} = \mathbf{E}\cdot\mathbf{A}_{\text{hole}} = \left(\frac{k_e Q}{R^2}\right)\left(\pi r^2\right) = \left(\frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10.0\times 10^{-6}\ \text{C}\right)}{(0.100\ \text{m})^2}\right)\pi\left(1.00\times 10^{-3}\ \text{m}\right)^2$$

$$\Phi_{E,\ \text{hole}} = \boxed{28.2\ \text{N}\cdot\text{m}^2/\text{C}}$$

24.23 $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{170 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = 1.92 \times 10^7\ \ \text{N} \cdot \text{m}^2/\text{C}$

(a) $(\Phi_E)_{\text{one face}} = \frac{1}{6}\Phi_E = \dfrac{1.92 \times 10^7\ \ \text{N} \cdot \text{m}^2/\text{C}}{6}$

$(\Phi_E)_{\text{one face}} = \boxed{3.20\ \ \text{MN} \cdot \text{m}^2/\text{C}}$

(b) $\Phi_E = \boxed{19.2\ \ \text{MN} \cdot \text{m}^2/\text{C}}$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an unsymmetrical charge distribution. The sides of the cube nearer the charge would have more flux and the ones farther away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.

24.24 (a) $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0}$

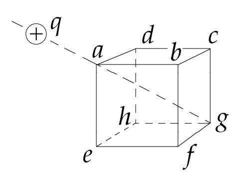
$8.60 \times 10^4 = \dfrac{q_{in}}{8.85 \times 10^{-12}}$

$q_{in} = 7.61 \times 10^{-7}\ \text{C} = \boxed{761\ \text{nC}}$

(b) Since the net flux is positive, $\boxed{\text{the net charge must be positive}}$. It can have any distribution.

(c) $\boxed{\text{The net charge would have the same magnitude but be negative.}}$

24.25 No charge is inside the cube. The net flux through the cube is zero. Positive flux comes out through the three faces meeting at g. These three faces together fill solid angle equal to one-eighth of a sphere as seen from q, and together pass flux $\frac{1}{8}(q/\epsilon_0)$. Each face containing a intercepts equal flux going into the cube:



$0 = \Phi_{E,\ \text{net}} = 3\Phi_{E,\ \text{abcd}} + q / 8\epsilon_0$

$\Phi_{E,\ \text{abcd}} = \boxed{-q / 24\epsilon_0}$

24.26 The charge distributed through the nucleus creates a field at the surface equal to that of a point charge at its center: $E = k_e q / r^2$

$$E = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(82 \times 1.60 \times 10^{-19} \text{ C})}{\left[(208)^{1/3}\ 1.20 \times 10^{-15} \text{ m}\right]^2}$$

$$E = \boxed{2.33 \times 10^{21} \text{ N/C}} \quad \text{away from the nucleus}$$

24.27 (a) $E = \dfrac{k_e Qr}{a^3} = \boxed{0}$

(b) $E = \dfrac{k_e Qr}{a^3} = \dfrac{(8.99 \times 10^9)(26.0 \times 10^{-6})(0.100)}{(0.400)^3} = \boxed{365 \text{ kN/C}}$

(c) $E = \dfrac{k_e Q}{r^2} = \dfrac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.400)^2} = \boxed{1.46 \text{ MN/C}}$

(d) $E = \dfrac{k_e Q}{r^2} = \dfrac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.600)^2} = \boxed{649 \text{ kN/C}}$

The direction for each electric field is radially outward.

***24.28** (a) $E = \dfrac{2k_e \lambda}{r}$

$$3.60 \times 10^4 = \frac{2(8.99 \times 10^9)(Q/2.40)}{(0.190)}$$

$$Q = +9.13 \times 10^{-7} \text{ C} = \boxed{+913 \text{ nC}}$$

(b) $\boxed{E = 0}$

24.29 $\oint \mathbf{E} \cdot d\mathbf{A} = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{\int \rho\, dV}{\epsilon_0} = \dfrac{\rho}{\epsilon_0}\, l \pi r^2$

$$E 2\pi r l = \frac{\rho}{\epsilon_0}\, l \pi r^2$$

$$\boxed{\mathbf{E} = \frac{\rho}{2\epsilon_0} r \text{ away from the axis}}$$

Goal Solution

Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where $r < R$.

G: According to Gauss's law, only the charge enclosed within the gaussian surface of radius r needs to be considered. The amount of charge within the gaussian surface will certainly increase as ρ and r increase, but the area of this gaussian surface will also increase, so it is difficult to predict which of these two competing factors will more strongly affect the electric field strength.

O: We can find the general equation for E from Gauss's law.

A: If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho\pi r^2 L$. The circular end caps have no electric flux through them; there $\mathbf{E} \cdot d\mathbf{A} = EdA\cos 90.0° = 0$. The curved surface has $\mathbf{E} \cdot d\mathbf{A} = EdA\cos 0°$, and E must be the same strength everywhere over the curved surface.

Gauss's law, $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$, becomes $E \int_{\substack{\text{Curved}\\\text{Surface}}} dA = \frac{\rho\pi r^2 L}{\epsilon_0}$

Now the lateral surface area of the cylinder is $2\pi rL$: $E(2\pi r)L = \frac{\rho\pi r^2 L}{\epsilon_0}$

Thus, $\mathbf{E} = \frac{\rho r}{2\epsilon_0}$ radially away from the cylinder axis

L: As we expected, the electric field will increase as ρ increases, and we can now see that E is also proportional to r. For the region outside the cylinder ($r > R$), we should expect the electric field to decrease as r increases, just like for a line of charge.

24.30 $\sigma = \left(8.60 \times 10^{-6} \text{ C / cm}^2\right)\left(\frac{100 \text{ cm}}{\text{m}}\right)^2 = 8.60 \times 10^{-2} \text{ C / m}^2$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{8.60 \times 10^{-2}}{2\left(8.85 \times 10^{-12}\right)} = \boxed{4.86 \times 10^9 \text{ N / C}}$$

The field is essentially uniform as long as the distance from the center of the wall to the field point is much less than the dimensions of the wall.

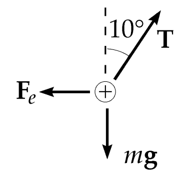
24.31 (a) $\boxed{E = 0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(32.0 \times 10^{-6})}{(0.200)^2} = \boxed{7.19\ \mathrm{MN/C}}$$

24.32 The distance between centers is $2 \times 5.90 \times 10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{(46)^2 (1.60 \times 10^{-19} \text{ C})^2}{(2 \times 5.90 \times 10^{-15} \text{ m})^2} = 3.50 \times 10^3 \text{ N} = \boxed{3.50 \text{ kN}}$$

***24.33** Consider two balloons of diameter 0.2 m, each with mass 1 g, hanging apart with a 0.05 m separation on the ends of strings making angles of 10° with the vertical.



(a) $\Sigma F_y = T\cos 10° - mg = 0 \ \Rightarrow \ T = \dfrac{mg}{\cos 10°}$

$\Sigma F_x = T\sin 10° - F_e = 0 \ \Rightarrow \ F_e = T\sin 10°$, so

$$F_e = \left(\frac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10° = (0.001 \text{ kg})\left(9.8 \ \text{m/s}^2\right)\tan 10°$$

$F_e \approx 2 \times 10^{-3}$ N $\boxed{\sim 10^{-3} \text{ N or } 1 \text{ mN}}$

(b) $F_e = \dfrac{k_e q^2}{r^2}$

$$2 \times 10^{-3} \text{ N} \approx \frac{\left(8.99 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2\right) q^2}{(0.25 \text{ m})^2}$$

$q \approx 1.2 \times 10^{-7}$ C $\boxed{\sim 10^{-7} \text{ C or } 100 \text{ nC}}$

(c) $$E = \frac{k_e q}{r^2} \approx \frac{\left(8.99 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2\right)\left(1.2 \times 10^{-7} \text{ C}\right)}{(0.25 \text{ m})^2} \approx 1.7 \times 10^4 \ \text{N/C} \quad \boxed{\sim 10 \ \text{kN/C}}$$

(d) $$\Phi_E = \frac{q}{\epsilon_0} \approx \frac{1.2 \times 10^{-7} \text{ C}}{8.85 \times 10^{-12} \ \text{C}^2/\text{N} \cdot \text{m}^2} = 1.4 \times 10^4 \ \text{N} \cdot \text{m}^2/\text{C} \quad \boxed{\sim 10 \ \text{kN} \cdot \text{m}^2/\text{C}}$$

24.34 (a) $$\rho = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{5.70 \times 10^{-6}}{\frac{4}{3}\pi(0.0400)^3} = 2.13 \times 10^{-2} \text{ C / m}^3$$

$$q_{\text{in}} = \rho\left(\tfrac{4}{3}\pi r^3\right) = \left(2.13 \times 10^{-2}\right)\left(\tfrac{4}{3}\pi\right)(0.0200)^3 = 7.13 \times 10^{-7} \text{ C} = \boxed{713 \text{ nC}}$$

(b) $$q_{\text{in}} = \rho\left(\tfrac{4}{3}\pi r^3\right) = \left(2.13 \times 10^{-2}\right)\left(\tfrac{4}{3}\pi\right)(0.0400)^3 = \boxed{5.70 \ \mu\text{C}}$$

24.35 (a) $E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\left(2.00\times 10^{-6}\ \text{C}\right)/7.00\ \text{m}\right]}{0.100\ \text{m}}$

$E = \boxed{51.4\ \text{kN/C, radially outward}}$

(b) $\Phi_E = EA\cos\theta = E(2\pi r\ell)\cos 0^\circ$

$\Phi_E = \left(5.14\times 10^4\ \text{N/C}\right)2\pi(0.100\ \text{m})(0.0200\ \text{m})(1.00) = \boxed{646\ \text{N}\cdot\text{m}^2/\text{C}}$

24.36 Note that the electric field in each case is directed radially inward, toward the filament.

(a) $E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times 10^{-6}\ \text{C}\right)}{0.100\ \text{m}} = \boxed{16.2\ \text{MN/C}}$

(b) $E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times 10^{-6}\ \text{C}\right)}{0.200\ \text{m}} = \boxed{8.09\ \text{MN/C}}$

(c) $E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times 10^{-6}\ \text{C}\right)}{1.00\ \text{m}} = \boxed{1.62\ \text{MN/C}}$

24.37 $E = \frac{\sigma}{2\epsilon_0} = \frac{9.00\times 10^{-6}\ \text{C/m}^2}{2(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{508\ \text{kN/C, upward}}$

24.38 From Gauss's Law, $EA = \frac{Q}{\epsilon_0}$

$\sigma = \frac{Q}{A} = \epsilon_0 E = (8.85\times 10^{-12})(130) = 1.15\times 10^{-9}\ \text{C/m}^2 = \boxed{1.15\ \text{nC/m}^2}$

24.39 $\oint EdA = E(2\pi rl) = \frac{q_{\text{in}}}{\epsilon_0}$ $\quad E = \frac{q_{\text{in}}/l}{2\pi\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0 r}$

(a) $r = 3.00$ cm $\boxed{E = 0}$ inside the conductor

(b) $r = 10.0$ cm $\quad E = \frac{30.0\times 10^{-9}}{2\pi(8.85\times 10^{-12})(0.100)} = \boxed{5400\ \text{N/C, outward}}$

(c) $r = 100$ cm $\quad E = \frac{30.0\times 10^{-9}}{2\pi(8.85\times 10^{-12})(1.00)} = \boxed{540\ \text{N/C, outward}}$

***24.40** Just above the aluminum plate (a conductor), the electric field is $E = \sigma'/\epsilon_0$ where the charge Q is divided equally between the upper and lower surfaces of the plate:

Thus $\sigma' = \frac{(Q/2)}{A} = \frac{Q}{2A}$ and $E = \frac{Q}{2\epsilon_0 A}$

For the glass plate (an insulator), $E = \sigma / 2\epsilon_0$ where $\sigma = Q / A$ since the entire charge Q is on the upper surface.

Therefore, $E = \frac{Q}{2\epsilon_0 A}$

The electric field at a point just above the center of the upper surface is the same for each of the plates.

$\boxed{E = \frac{Q}{2\epsilon_0 A}, \text{ vertically upward in each case (assuming } Q > 0)}$

***24.41** (a) $E = \sigma/\epsilon_0$ $\sigma = (8.00 \times 10^4)(8.85 \times 10^{-12}) = 7.08 \times 10^{-7}\ \mathrm{C/m^2}$

$\boxed{\sigma = 708\ \mathrm{nC/m^2}}$, positive on one face and negative on the other.

(b) $\sigma = \frac{Q}{A}$ $Q = \sigma A = (7.08 \times 10^{-7})\,(0.500)^2\ \mathrm{C}$

$Q = 1.77 \times 10^{-7}\ \mathrm{C} = \boxed{177\ \mathrm{nC}}$, positive on one face and negative on the other.

24.42 Use Gauss's Law to evaluate the electric field in each region, recalling that the electric field is zero everywhere within conducting materials. The results are:

$\boxed{E = 0 \text{ inside the sphere and inside the shell}}$

$\boxed{E = k_e \frac{Q}{r^2} \text{ between sphere and shell, directed radially inward}}$

$\boxed{E = k_e \frac{2Q}{r^2} \text{ outside the shell, directed radially inward}}$

Charge $\boxed{-Q \text{ is on the outer surface of the sphere}}$.

Charge $\boxed{+Q \text{ is on the inner surface of the shell}}$, and $\boxed{+2Q \text{ is on the outer surface of the shell.}}$

24.43 The charge divides equally between the identical spheres, with charge $Q/2$ on each. Then they repel like point charges at their centers:

$$F = \frac{k_e(Q/2)(Q/2)}{(L+R+R)^2} = \frac{k_e Q^2}{4(L+2R)^2} = \frac{8.99\times 10^9\ \text{N}\cdot\text{m}^2(60.0\times 10^{-6}\ \text{C})^2}{4\ \text{C}^2(2.01\ \text{m})^2} = \boxed{2.00\ \text{N}}$$

***24.44** The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vise-versa. The local charge density and the electric field intensity are related by

$$E = \frac{\sigma}{e_0} \quad \text{or} \quad \sigma = e_0 E$$

(a) Where the radius of curvature is the greatest,

$$\sigma = e_0 E_{\text{min}} = \left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.80\times 10^4\ \text{N/C}\right) = \boxed{248\ \text{nC/m}^2}$$

(b) Where the radius of curvature is the smallest,

$$\sigma = e_0 E_{\text{max}} = \left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(5.60\times 10^4\ \text{N/C}\right) = \boxed{496\ \text{nC/m}^2}$$

24.45 (a) Inside surface: consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero, so the inside charge/length $= -\lambda$.

$$0 = \lambda\ell + q_{\text{in}} \Rightarrow \frac{q_{\text{in}}}{\square} = \boxed{-\lambda}$$

Outside surface: The total charge on the metal cylinder is $2\lambda l = q_{in} + q_{out}$.

$$q_{out} = 2\lambda l + \lambda l$$

so the outside charge/length $= \boxed{3\lambda}$

(b) $$E = \frac{2k_e(3\lambda)}{r} = \frac{6k_e\lambda}{r} = \boxed{\frac{3\lambda}{2\pi e_0 r}}$$

24.46 (a) $E = \dfrac{k_e Q}{r^2} = \dfrac{\left(8.99\times 10^9\right)\left(6.40\times 10^{-6}\right)}{(0.150)^2} = \boxed{2.56\text{ MN/C, radially inward}}$

(b) $\boxed{E = 0}$

24.47 (a) The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma = \frac{1}{2}\left(\frac{q}{A}\right) = \frac{1}{2}\,\frac{(4.00\times10^{-8}\ \text{C})}{(0.500\ \text{m})^2} = 8.00\times10^{-8}\ \text{C / m}^2 = \boxed{80.0\ \text{nC / m}^2}$$

(b) $$\mathbf{E} = \left(\frac{\sigma}{\epsilon_0}\right)\mathbf{k} = \left(\frac{8.00\times10^{-8}\ \text{C / m}^2}{8.85\times10^{-12}\ \text{C}^2\ /\ \text{N}\cdot\text{m}^2}\right)\mathbf{k} = \boxed{(9.04\ \text{kN / C})\mathbf{k}}$$

(c) $$\mathbf{E} = \boxed{(-9.04\ \text{kN / C})\mathbf{k}}$$

24. 48 (a) The charge $+q$ at the center induces charge $-q$ on the inner surface of the conductor, where its surface density is:

$$\sigma_a = \boxed{\frac{-q}{4\pi a^2}}$$

(b) The outer surface carries charge $Q+q$ with density

$$\sigma_b = \boxed{\frac{Q+q}{4\pi b^2}}$$

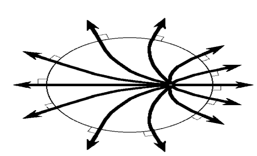
24.49 (a) $\boxed{E=0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times10^9\right)\left(8.00\times10^{-6}\right)}{(0.0300)^2} = 7.99\times10^7\ \text{N / C} = \boxed{79.9\ \text{MN/C}}$$

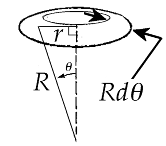
(c) $\boxed{E=0}$

(d) $$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times10^9\right)\left(4.00\times10^{-6}\right)}{(0.0700)^2} = 7.34\times10^6\ \text{N / C} = \boxed{7.34\ \text{MN/C}}$$

24.50 An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.



24.51 (a) Uniform **E**, pointing radially outward, so $\Phi_E = EA$. The arc length is $ds = R\,d\theta$, and the circumference is $2\pi r = 2\pi R \sin\theta$



$$A = \int 2\pi r\,ds = \int_0^\theta (2\pi R\sin\theta) R\,d\theta = 2\pi R^2 \int_0^\theta \sin\theta\,d\theta = 2\pi R^2(-\cos\theta)\Big|_0^\theta = 2\pi R^2(1-\cos\theta)$$

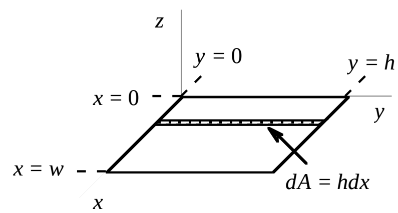
$$\Phi_E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\cdot 2\pi R^2(1-\cos\theta) = \boxed{\frac{Q}{2\epsilon_0}(1-\cos\theta)}$$ [independent of R!]

(b) For $\theta = 90.0°$ (hemisphere): $\Phi_E = \frac{Q}{2\epsilon_0}(1-\cos 90°) = \boxed{\frac{Q}{2\epsilon_0}}$

(c) For $\theta = 180°$ (entire sphere): $\Phi_E = \frac{Q}{2\epsilon_0}(1-\cos 180°) = \boxed{\frac{Q}{\epsilon_0}}$ [Gauss's Law]

***24.52** In general, $\mathbf{E} = ay\,\mathbf{i} + bz\,\mathbf{j} + cx\,\mathbf{k}$

In the xy plane, $z = 0$ and $\mathbf{E} = ay\,\mathbf{i} + cx\,\mathbf{k}$



$$\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = \int (ay\,\mathbf{i} + cx\,\mathbf{k})\cdot\mathbf{k}\,dA$$

$$\Phi_E = ch\int_{x=0}^{w} x\,dx = ch\frac{x^2}{2}\Big|_{x=0}^{w} = \boxed{\frac{chw^2}{2}}$$

***24.53** (a) $q_{\text{in}} = +3Q - Q = \boxed{+2Q}$

(b) The charge distribution is spherically symmetric and $q_{\text{in}} > 0$. Thus, the field is directed $\boxed{\text{radially outward}}$.

(c) $E = \frac{k_e q_{\text{in}}}{r^2} = \boxed{\frac{2k_eQ}{r^2}}$ for $r \ge c$

(d) Since all points within this region are located inside conducting material, $\boxed{E = 0}$ for $b < r < c$.

(e) $\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = 0 \Rightarrow q_{\text{in}} = \epsilon_0\Phi_E = \boxed{0}$

(f) $q_{\text{in}} = \boxed{+3Q}$

(g) $E = \frac{k_e q_{\text{in}}}{r^2} = \boxed{\frac{3k_eQ}{r^2}}$ (radially outward) for $a \le r < b$

(h) $q_{\text{in}} = \rho V = \left(\dfrac{+3Q}{\frac{4}{3}\pi a^3}\right)\left(\dfrac{4}{3}\pi r^3\right) = \boxed{+3Q\dfrac{r^3}{a^3}}$

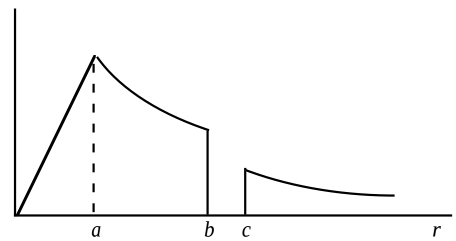
(i) $E = \dfrac{k_e q_{\text{in}}}{r^2} = \dfrac{k_e}{r^2}\left(+3Q\dfrac{r^3}{a^3}\right) = \boxed{3k_e Q\dfrac{r}{a^3}}$ (radially outward) for $0 \le r \le a$

(j) From part (d), $E = 0$ for $b < r < c$. Thus, for a spherical gaussian surface with $b < r < c$, $q_{\text{in}} = +3Q + q_{\text{inner}} = 0$ where q_{inner} is the charge on the inner surface of the conducting shell. This yields $q_{\text{inner}} = \boxed{-3Q}$

(k) Since the total charge on the conducting shell is $q_{\text{net}} = q_{\text{outer}} + q_{\text{inner}} = -Q$, we have

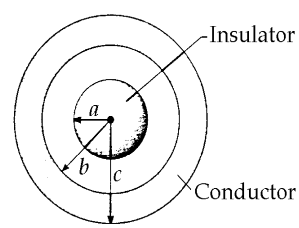
$q_{\text{outer}} = -Q - q_{\text{inner}} = -Q - (-3Q) = \boxed{+2Q}$

(l) This is shown in the figure to the right.



24.54 The sphere with large charge creates a strong field to polarize the other sphere. That means it pushes the excess charge over to the far side, leaving charge of the opposite sign on the near side. This patch of opposite charge is smaller in amount but located in a stronger external field, so it can feel a force of attraction that is larger than the repelling force felt by the larger charge in the weaker field on the other side.

24.55 (a) $\oint \mathbf{E} \cdot d\mathbf{A} = E\left(4\pi r^2\right) = q_{in}/e_0$



For $r < a$, $q_{in} = \rho\left(\frac{4}{3}\pi r^3\right)$ so $\boxed{E = \dfrac{pr}{3e_0}}$

For $a < r < b$ and $c < r$, $q_{\text{in}} = Q$ so that $\boxed{E = \dfrac{Q}{4\pi r^2 e_0}}$

For $b \le r \le c$, $E = 0$, since $\boxed{E = 0}$ inside a conductor.

(b) Let q_1 = induced charge on the inner surface of the hollow sphere. Since $E = 0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b \le r \le c$ must be zero.

Therefore, $q_1 + Q = 0$ and $\sigma_1 = \dfrac{q_1}{4\pi b^2} = \boxed{\dfrac{-Q}{4\pi b^2}}$

Let q_2 = induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require $q_1 + q_2 = 0$

and $\sigma_2 = \dfrac{q_1}{4\pi c^2} = \boxed{\dfrac{Q}{4\pi c^2}}$

24.56 $\oint \mathbf{E} \cdot d\mathbf{A} = E\left(4\pi r^2\right) = \dfrac{q_{\text{in}}}{\epsilon_0}$

(a) $\left(-3.60 \times 10^3 \text{ N/C}\right) 4\pi (0.100 \text{ m})^2 = \dfrac{Q}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2}$ $\quad (a < r < b)$

$Q = -4.00 \times 10^{-9} \text{ C} = \boxed{-4.00 \text{ nC}}$

(b) We take Q' to be the net charge on the hollow sphere. Outside c,

$\left(+2.00 \times 10^2 \text{ N/C}\right) 4\pi (0.500 \text{ m})^2 = \dfrac{Q + Q'}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2}$ $\quad (r > c)$

$Q + Q' = +5.56 \times 10^{-9} \text{ C}$, so $Q' = +9.56 \times 10^{-9} \text{ C} = \boxed{+9.56 \text{ nC}}$

(c) For $b < r < c$: $E = 0$ and $q_{\text{in}} = Q + Q_1 = 0$ where Q_1 is the total charge on the inner surface of the hollow sphere. Thus, $Q_1 = -Q = \boxed{+4.00 \text{ nC}}$

Then, if Q_2 is the total charge on the outer surface of the hollow sphere, $Q_2 = Q' - Q_1 = 9.56 \text{ nC} - 4.00 \text{ nC} = \boxed{+5.56 \text{ nC}}$

24.57 The field direction is radially outward perpendicular to the axis. The field strength depends on r but not on the other cylindrical coordinates θ or z. Choose a Gaussian cylinder of radius r and length L. If $r < a$,

$\Phi_E = \dfrac{q_{in}}{\epsilon_0}$ and $E(2\pi r L) = \dfrac{\lambda L}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi r \epsilon_0}$ or $\boxed{\mathbf{E} = \dfrac{\lambda}{2\pi r \epsilon_0} \quad (r < a)}$

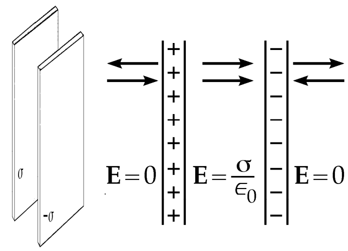
If $a < r < b$, $E(2\pi r L) = \dfrac{\lambda L + \rho\pi\left(r^2 - a^2\right)L}{\epsilon_0}$

$\boxed{\mathbf{E} = \dfrac{\lambda + \rho\pi\left(r^2 - a^2\right)}{2\pi r \epsilon_0} \quad (a < r < b)}$

If $r > b$, $E(2\pi r L) = \dfrac{\lambda L + \rho\pi\left(b^2 - a^2\right)L}{\epsilon_0}$

$$\boxed{\mathbf{E} = \frac{\lambda + \rho\pi\left(b^2 - a^2\right)}{2\pi r \epsilon_0} \qquad (r > b)}$$

24.58 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by Equation 24.8:



$$\left|E_+\right| = \left|E_-\right| = \frac{\sigma}{2\epsilon_0}$$

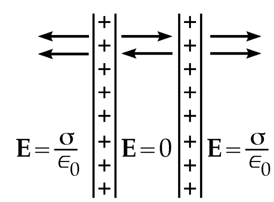
(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\boxed{E = 0}$.

(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\boxed{E = \frac{\sigma}{\epsilon_0} \text{ toward the right}}$$

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\boxed{E = 0}$.

24.59 The magnitude of the field due to each sheet given by Equation 24.8 is



$$E = \frac{\sigma}{2\epsilon_0} \text{ directed perpendicular to the sheet.}$$

(a) In the region to the left of the pair of sheets, both fields are directed toward the left and the net field is

$$E = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the left}}$$

(b) In the region between the sheets, the fields due to the individual sheets are oppositely directed and the net field is

$$E = \boxed{0}$$

(c) In the region to the right of the pair of sheets, both fields are directed toward the right and the net field is

$$E = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

Goal Solution

Repeat the calculations for Problem 58 when both sheets have **positive** uniform charge densities of value σ. **Note:** The new problem statement would be as follows: Two infinite, nonconducting sheets of charge are parallel to each other, as shown in Figure P24.58. Both sheets have positive uniform charge densities σ. Calculate the value of the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets.

G: When both sheets have the same charge density, a positive test charge at a point midway between them will experience the same force in opposite directions from each sheet. Therefore, the electric field here will be zero. (We should ask: can we also conclude that the electron will experience equal and oppositely directed forces *everywhere* in the region between the plates?)

Outside the sheets the electric field will point away and should be twice the strength due to one sheet of charge, so $E = \sigma / \epsilon_0$ in these regions.

O: The principle of superposition can be applied to add the electric field vectors due to each sheet of charge.

A: For each sheet, the electric field at any point is $|\mathbf{E}| = \sigma/(2\epsilon_0)$ directed away from the sheet.

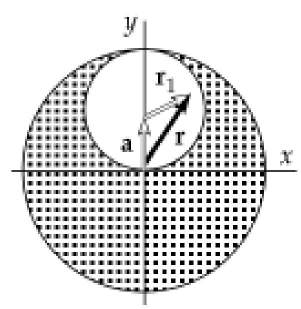
(a) At a point to the left of the two parallel sheets $\quad \mathbf{E} = E_1(-\mathbf{i}) + E_2(-\mathbf{i}) = 2E(-\mathbf{i}) = -\dfrac{\sigma}{\epsilon_0}\mathbf{i}$

(b) At a point between the two sheets $\quad \mathbf{E} = E_1\mathbf{i} + E_2(-\mathbf{i}) = 0$

(c) At a point to the right of the two parallel sheets $\quad \mathbf{E} = E_1\mathbf{i} + E_2\mathbf{i} = 2E\,\mathbf{i} = \dfrac{\sigma}{\epsilon_0}\mathbf{i}$

L: We essentially solved this problem in the Gather information step, so it is no surprise that these results are what we expected. A better check is to confirm that the results are complementary to the case where the plates are oppositely charged (Problem 58).

24.60 The resultant field within the cavity is the superposition of two fields, one $\mathbf{E}_+$ due to a uniform sphere of positive charge of radius $2a$, and the other $\mathbf{E}_-$ due to a sphere of negative charge of radius a centered within the cavity.



$$\frac{4}{3}\frac{\pi r^3 \rho}{\epsilon_0} = 4\pi r^2 E_+ \quad \text{so} \quad \mathbf{E}_+ = \frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}} = \frac{\rho \mathbf{r}}{3\epsilon_0}$$

$$-\frac{4}{3}\frac{\pi r_1^3 \rho}{\epsilon_0} = 4\pi r_1^2 E_- \quad \text{so} \quad \mathbf{E}_- = \frac{\rho r_1}{3\epsilon_0}(-\hat{\mathbf{r}}_1) = \frac{-\rho}{3\epsilon_0}\mathbf{r}_1$$

Since $\mathbf{r} = \mathbf{a} + \mathbf{r}_1$, $\quad \mathbf{E}_- = \dfrac{-\rho(\mathbf{r} - \mathbf{a})}{3\epsilon_0}$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{\rho \mathbf{r}}{3\epsilon_0} - \frac{\rho \mathbf{r}}{3\epsilon_0} + \frac{\rho \mathbf{a}}{3\epsilon_0} = \frac{\rho \mathbf{a}}{3\epsilon_0} = 0\,\mathbf{i} + \frac{\rho a}{3\epsilon_0}\mathbf{j}$$

Thus, $\boxed{E_x = 0}$ and $\boxed{E_y = \dfrac{\rho a}{3\epsilon_0}}$ at all points within the cavity.

24.61 First, consider the field at distance $r < R$ from the center of a uniform sphere of positive charge $(Q = +e)$ with radius R.

$$\left(4\pi r^2\right)E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\rho V}{\epsilon_0} = \left(\frac{+e}{\frac{4}{3}\pi R^3}\right)\frac{\frac{4}{3}\pi r^3}{\epsilon_0} \quad \text{so} \quad E = \left(\frac{e}{4\pi\epsilon_0 R^3}\right)r \text{ directed outward}$$

(a) The force exerted on a point charge $q = -e$ located at distance r from the center is then

$$F = qE = -e\left(\frac{e}{4\pi\epsilon_0 R^3}\right)r = -\left(\frac{e^2}{4\pi\epsilon_0 R^3}\right)r = \boxed{-Kr}$$

(b) $K = \dfrac{e^2}{4\pi\epsilon_0 R^3} = \boxed{\dfrac{k_e e^2}{R^3}}$

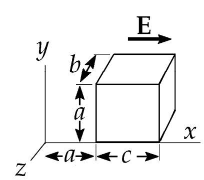
(c) $F_r = m_e a_r = -\left(\dfrac{k_e e^2}{R^3}\right)r$, so $a_r = -\left(\dfrac{k_e e^2}{m_e R^3}\right)r = -\omega^2 r$

Thus, the motion is simple harmonic with frequency $f = \dfrac{\omega}{2\pi} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e R^3}}}$

(d) $$f = 2.47\times10^{15}\text{ Hz} = \frac{1}{2\pi}\sqrt{\frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)R^3}}$$

which yields $R^3 = 1.05\times10^{-30}\text{ m}^3$, or $R = 1.02\times10^{-10}\text{ m} = \boxed{102\text{ pm}}$

24.62 The electric field throughout the region is directed along x; therefore, **E** will be perpendicular to dA over the four faces of the surface which are perpendicular to the yz plane, and E will be parallel to dA over the two faces which are parallel to the yz plane. Therefore,



$$\Phi_E = -\left(E_x\big|_{x=a}\right)A + \left(E_x\big|_{x=a+c}\right)A = -\left(3+2a^2\right)ab + \left(3+2(a+c)^2\right)ab = 2abc(2a+c)$$

Substituting the given values for a, b, and c, we find $\Phi_E = \boxed{0.269\text{ N}\cdot\text{m}^2/\text{C}}$

$Q = \epsilon_0\Phi_E = 2.38\times10^{-12}\text{ C} = \boxed{2.38\text{ pC}}$

24.63 $\oint \mathbf{E}\cdot d\mathbf{A} = E(4\pi r^2) = \dfrac{q_{\text{in}}}{\epsilon_0}$

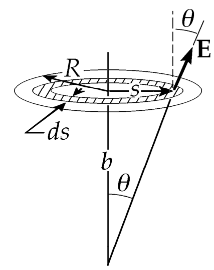
(a) For $r > R$, $\quad q_{\text{in}} = \displaystyle\int_0^R Ar^2(4\pi r^2)dr = 4\pi\frac{AR^5}{5}$ and $E = \boxed{\dfrac{AR^5}{5\epsilon_0 r^2}}$

(b) For $r < R$, $\quad q_{\text{in}} = \displaystyle\int_0^r Ar^2(4\pi r^2)dr = \frac{4\pi Ar^5}{5}$ and $E = \boxed{\dfrac{Ar^3}{5\epsilon_0}}$

24.64 The total flux through a surface enclosing the charge Q is Q/ϵ_0. The flux through the disk is

$$\Phi_{\text{disk}} = \int \mathbf{E} \cdot d\mathbf{A}$$

where the integration covers the area of the disk. We must evaluate this integral and set it equal to $\frac{1}{4}\,Q/\epsilon_0$ to find how b and R are related. In the figure, take $d\mathbf{A}$ to be the area of an annular ring of radius s and width ds. The flux through $d\mathbf{A}$ is



$$\mathbf{E} \cdot d\mathbf{A} = E\,dA\cos\theta = E(2\pi\,s\,ds)\cos\theta$$

The magnitude of the electric field has the same value at all points within the annular ring,

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{s^2+b^2} \qquad \text{and} \qquad \cos\theta = \frac{b}{r} = \frac{b}{(s^2+b^2)^{1/2}}$$

Integrate from $s = 0$ to $s = R$ to get the flux through the entire disk.

$$\Phi_{E,\text{ disk}} = \frac{Qb}{2\epsilon_0}\int_0^R \frac{s\,ds}{(s^2+b^2)^{3/2}} = \frac{Qb}{2\epsilon_0}\left[-(s^2+b^2)^{1/2}\right]\Big|_0^R = \frac{Q}{2\epsilon_0}\left[1-\frac{b}{(R^2+b^2)^{1/2}}\right]$$

The flux through the disk equals $Q/4\epsilon_0$ provided that $\dfrac{b}{(R^2+b^2)^{1/2}} = \dfrac{1}{2}$.

This is satisfied if $\boxed{R = \sqrt{3}\,b}$.

24.65

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{1}{\epsilon_0}\int_0^r \frac{a}{r}4\pi r^2\,dr$$

$$E4\pi r^2 = \frac{4\pi a}{\epsilon_0}\int_0^r r\,dr = \frac{4\pi a}{\epsilon_0}\frac{r^2}{2}$$

$$\boxed{E = \frac{a}{2\epsilon_0}} = \text{constant magnitude}$$

(The direction is radially outward from center for positive a; radially inward for negative a.)

24.66 In this case the charge density is *not uniform*, and Gauss's law is written as $\oint \mathbf{E}\cdot d\mathbf{A} = \frac{1}{\epsilon_0}\int \rho\, dV$.

We use a gaussian surface which is a cylinder of radius r, length ℓ, and is coaxial with the charge distribution.

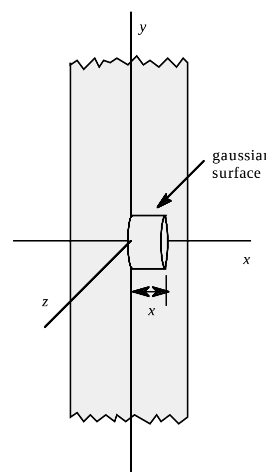
(a) When $r < R$, this becomes $E(2\pi r\ell) = \frac{\rho_0}{\epsilon_0}\int_0^r \left(a - \frac{r}{b}\right) dV$. The element of volume is a cylindrical shell of radius r, length ℓ, and thickness dr so that $dV = 2\pi r\ell\, dr$.

$$E(2\pi r\ell) = \left(\frac{2\pi r^2 \ell \rho_0}{\epsilon_0}\right)\left(\frac{a}{2} - \frac{r}{3b}\right) \quad \text{so inside the cylinder,} \quad E = \boxed{\frac{\rho_0 r}{2\epsilon_0}\left(a - \frac{2r}{3b}\right)}$$

(b) When $r > R$, Gauss's law becomes

$$E(2\pi r\ell) = \frac{\rho_0}{\epsilon_0}\int_0^R \left(a - \frac{r}{b}\right)(2\pi r\ell\, dr) \quad \text{or outside the cylinder,} \quad E = \boxed{\frac{\rho_0 R^2}{2\epsilon_0 r}\left(a - \frac{2R}{3b}\right)}$$

24.67 (a) Consider a cylindrical shaped gaussian surface perpendicular to the yz plane with one end in the yz plane and the other end containing the point x:



Use Gauss's law: $\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$

By symmetry, the electric field is zero in the yz plane and is perpendicular to $d\mathbf{A}$ over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap containing the point x :

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} \quad \text{or} \quad EA = \frac{\rho(Ax)}{\epsilon_0}$$

so that at distance x from the mid-line of the slab, $\boxed{E = \frac{\rho x}{\epsilon_0}}$

(b) $$a = \frac{F}{m_e} = \frac{(-e)E}{m_e} = -\left(\frac{\rho e}{m_e \epsilon_0}\right)x$$

The acceleration of the electron is of the form $a = -\omega^2 x$ with $\omega = \sqrt{\frac{\rho e}{m_e \epsilon_0}}$

Thus, the motion is simple harmonic with frequency $f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{\rho e}{m_e \epsilon_0}}}$

24.68 Consider the gaussian surface described in the solution to problem 67.

(a) For $x > \dfrac{d}{2}$, $\quad dq = \rho\, dV = \rho A\, dx = C\, Ax^2\, dx$

$$\int \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} \int dq$$

$$EA = \frac{CA}{\epsilon_0} \int_0^{d/2} x^2\, dx = \frac{1}{3}\left(\frac{CA}{\epsilon_0}\right)\left(\frac{d^3}{8}\right)$$

$$E = \frac{Cd^3}{24\epsilon_0} \quad \text{or} \quad \boxed{\mathbf{E} = \frac{Cd^3}{24\epsilon_0}\mathbf{i} \ \text{ for } \ x > \frac{d}{2}; \qquad \mathbf{E} = -\frac{Cd^3}{24\epsilon_0}\mathbf{i} \ \text{ for } \ x < -\frac{d}{2}}$$

(b) For $-\dfrac{d}{2} < x < \dfrac{d}{2}$ $\quad \displaystyle\int \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} \int dq = \frac{CA}{\epsilon_0} \int_0^x x^2\, dx = \frac{CAx^3}{3\epsilon_0}$

$$\boxed{\mathbf{E} = \frac{Cx^3}{3\epsilon_0}\mathbf{i} \ \text{ for } \ x > 0; \qquad \mathbf{E} = -\frac{Cx^3}{3\epsilon_0}\mathbf{i} \ \text{ for } \ x < 0}$$

24.69 (a) A point mass m creates a gravitational acceleration $\mathbf{g} = -\dfrac{Gm}{r^2}\,\hat{\mathbf{r}}$ at a distance r.

The flux of this field through a sphere is $\displaystyle\oint \mathbf{g} \cdot d\mathbf{A} = -\frac{Gm}{r^2}\left(4\pi r^2\right) = -4\pi Gm$

Since the r has divided out, we can visualize the field as unbroken field lines. The same flux would go through any other closed surface around the mass. If there are several or no masses inside a closed surface, each creates field to make its own contribution to the net flux according to

$$\boxed{\oint \mathbf{g} \cdot d\mathbf{A} = -4\pi Gm_{\text{in}}}$$

(b) Take a spherical gaussian surface of radius r. The field is inward so

$$\oint \mathbf{g} \cdot d\mathbf{A} = g\,4\pi r^2 \cos 180° = -g\,4\pi r^2$$

and $\quad -4\pi Gm_{\text{in}} = -4G\tfrac{4}{3}\pi r^3 \rho$

Then, $-g\,4\pi r^2 = -4\pi G\tfrac{4}{3}\pi r^3\rho$ and $g = \tfrac{4}{3}\pi r\rho G$

Or, since $\rho = M_E / \tfrac{4}{3}\pi R_E^3$, $\quad g = \dfrac{M_E Gr}{R_E^3}$ or $\boxed{\mathbf{g} = \dfrac{M_E Gr}{R_E^3} \ \text{inward}}$

Chapter 25 Solutions

25.1 $\Delta V = -14.0$ V and

$$Q = -N_A e = -(6.02 \times 10^{23})(1.60 \times 10^{-19}\text{ C}) = -9.63 \times 10^4\text{ C}$$

$$\Delta V = \frac{W}{Q}\ ,\quad \text{so}\quad W = Q(\Delta V) = (-9.63 \times 10^4\text{ C})(-14.0\text{ J/C}) = \boxed{1.35\text{ MJ}}$$

25.2 $\Delta K = q|\Delta V|$ $\qquad 7.37 \times 10^{-17} = q(115)$

$$\boxed{q = 6.41 \times 10^{-19}\text{ C}}$$

25.3 $W = \Delta K = q|\Delta V|$

$$\frac{1}{2}mv^2 = e(120\text{ V}) = 1.92 \times 10^{-17}\text{ J}$$

Thus, $v = \sqrt{\dfrac{3.84 \times 10^{-17}\text{ J}}{m}}$

(a) For a proton, this becomes $\quad v = \sqrt{\dfrac{3.84 \times 10^{-17}\text{ J}}{1.67 \times 10^{-27}\text{ kg}}} = 1.52 \times 10^5\text{ m/s} = \boxed{152\text{ km/s}}$

(b) If an electron, $\quad v = \sqrt{\dfrac{3.84 \times 10^{-17}\text{ J}}{9.11 \times 10^{-31}\text{ kg}}} = 6.49 \times 10^6\text{ m/s} = \boxed{6.49\text{ Mm/s}}$

Goal Solution

(a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V.
(b) Calculate the speed of an electron that is accelerated through the same potential difference.

G: Since 120 V is only a modest potential difference, we might expect that the final speed of the particles will be substantially less than the speed of light. We should also expect the speed of the electron to be significantly greater than the proton because, with $m_e << m_p$, an equal force on both particles will result in a much greater acceleration for the electron.

O: Conservation of energy can be applied to this problem to find the final speed from the kinetic energy of the particles. (Review this work-energy theory of motion from Chapter 8 if necessary.)

A: (a) Energy is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V:

$$K_i + U_i + \Delta E_{nc} = K_f + U_f$$
$$0 + qV + 0 = \tfrac{1}{2} m v_p^{\,2} + 0$$
$$(1.60 \times 10^{-19}\ \text{C})(120\ \text{V})\left(\frac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right) = \tfrac{1}{2}(1.67 \times 10^{-27}\ \text{kg}) v_p^{\,2}$$
$$v_p = 1.52 \times 10^5\ \text{m / s}$$

(b) The electron will gain speed in moving the other way, from $V_i = 0$ to $V_f = 120$ V:

$$K_i + U_i + \Delta E_{nc} = K_f + U_f$$
$$0 + 0 + 0 = \tfrac{1}{2} m v_e^{\,2} + qV$$
$$0 = \tfrac{1}{2}(9.11 \times 10^{-31}\ \text{kg}) v_e^{\,2} + (-1.60 \times 10^{-19}\ \text{C})(120\ \text{J / C})$$
$$v_e = 6.49 \times 10^6\ \text{m / s}$$

L: Both of these speeds are significantly less than the speed of light as expected, which also means that we were justified in not using the relativistic kinetic energy formula. (For precision to three significant digits, the relativistic formula is only needed if *v* is greater than about 0.1 *c*.)

25.4 For speeds larger than one-tenth the speed of light, $\frac{1}{2}\ mv^2$ gives noticeably wrong answers for kinetic energy, so we use

$$K = mc^2\left(\frac{1}{\sqrt{1 - v^2 / c^2}} - 1\right) = \left(9.11 \times 10^{-31}\ \text{kg}\right)\left(3.00 \times 10^8\ \text{m / s}\right)^2\left(\frac{1}{\sqrt{1 - 0.400^2}} - 1\right) = 7.47 \times 10^{-15}\ \text{J}$$

Energy is conserved during acceleration: $K_i + U_i + \Delta E = K_f + U_f$

$$0 + qV_i + 0 = 7.47 \times 10^{-15}\ \text{J} + qV_f$$

The change in potential is $V_f - V_i$: $\quad V_f - V_i = \dfrac{-7.47 \times 10^{-15}\ \text{J}}{q} = \dfrac{-7.47 \times 10^{-15}\ \text{J}}{-1.60 \times 10^{-19}\ \text{C}} = \boxed{+\ 46.7\ \text{kV}}$

The positive answer means that the electron speeds up in moving toward higher potential.

25.5 $W = \Delta K = -\,q\Delta V$

$$0 - \tfrac{1}{2}\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(4.20 \times 10^5\ \text{m / s}\right)^2 = -\left(-\,1.60 \times 10^{-19}\ \text{C}\right)\Delta V$$

From which, $\Delta V = \boxed{-0.502\ \text{V}}$

***25.6** (a) We follow the path from (0, 0) to (20.0 cm, 0) to (20.0 cm, 50.0 cm).

$\Delta U = -$ (work done)

$\Delta U = -(\text{work from origin to } (20.0\text{ cm}, 0)) - (\text{work from } (20.0\text{ cm}, 0) \text{ to } (20.0\text{ cm}, 50.0\text{ cm}))$

Note that the last term is equal to 0 because the force is perpendicular to the displacement.

$$\Delta U = -(qE_x)(\Delta x) = -(12.0 \times 10^{-6}\text{ C})(250\text{ V/m})(0.200\text{ m}) = \boxed{-6.00 \times 10^{-4}\text{ J}}$$

(b) $$\Delta V = \frac{\Delta U}{q} = -\frac{6.00 \times 10^{-4}\text{ J}}{12.0 \times 10^{-6}\text{ C}} = -50.0\text{ J/C} = \boxed{-50.0\text{ V}}$$

***25.7** $$E = \frac{|\Delta V|}{d} = \frac{25.0 \times 10^3\text{ J/C}}{1.50 \times 10^{-2}\text{ m}} = 1.67 \times 10^6\text{ N/C} = \boxed{1.67\text{ MN/C}}$$

***25.8** (a) $|\Delta V| = Ed = (5.90 \times 10^3\text{ V/m})(0.0100\text{ m}) = \boxed{59.0\text{ V}}$

(b) $\frac{1}{2} m v_f^2 = |q(\Delta V)|$; $\frac{1}{2}(9.11 \times 10^{-31})\ v_f^2 = (1.60 \times 10^{-19})(59.0)$

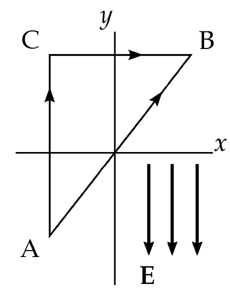
$\boxed{v_f = 4.55 \times 10^6\text{ m/s}}$

25.9 $$\Delta U = -\tfrac{1}{2} m\left(v_f^2 - v_i^2\right) = -\tfrac{1}{2}\left(9.11 \times 10^{-31}\text{ kg}\right)\left[\left(1.40 \times 10^5\text{ m/s}\right)^2 - \left(3.70 \times 10^6\text{ m/s}\right)^2\right] = 6.23 \times 10^{-18}\text{ J}$$

$\Delta U = q\Delta V$: $+6.23 \times 10^{-18} = (-1.60 \times 10^{-19})\Delta V$

$\boxed{\Delta V = -38.9\text{ V}}$ The origin is at higher potential.

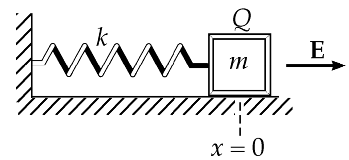
***25.10** $V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\int_A^C \mathbf{E}\cdot d\mathbf{s} - \int_C^B \mathbf{E}\cdot d\mathbf{s}$



$$V_B - V_A = (-E\cos 180°)\int_{-0.300}^{0.500} dy - (E\cos 90.0°)\int_{-0.200}^{0.400} dx$$

$$V_B - V_A = (325)(0.800) = \boxed{+260 \text{ V}}$$

25.11 (a) Arbitrarily choose $V = 0$ at $x = 0$. Then at other points, $V = -Ex$ and $U_e = QV = -QEx$. Between the endpoints of the motion,



$$(K + U_s + U_e)_i = (K + U_s + U_e)_f$$

$$0 + 0 + 0 = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max}$$

so the block comes to rest when the spring is stretched by an amount

$$x_{\max} = \frac{2QE}{k} = \frac{2\left(50.0\times10^{-6}\text{ C}\right)\left(5.00\times10^{5}\text{ V/m}\right)}{100\text{ N/m}} = \boxed{0.500\text{ m}}$$

(b) At equilibrium, $\Sigma F_x = -F_s + F_e = 0$ or $kx = QE$. Thus, the equilibrium position is at

$$x = \frac{QE}{k} = \frac{\left(50.0\times10^{-6}\text{ C}\right)\left(5.00\times10^{5}\text{ N/C}\right)}{100\text{ N/m}} = \boxed{0.250\text{ m}}$$

(c) The equation of motion for the block is $\Sigma F_x = -kx + QE = m\dfrac{d^2x}{dt^2}$. Let $x' = x - \dfrac{QE}{k}$, or $x = x' + \dfrac{QE}{k}$ so the equation of motion becomes:

$$-k\left(x' + \frac{QE}{k}\right) + QE = m\frac{d^2\left(x' + QE/k\right)}{dt^2}, \text{ or } \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'$$

This is the equation for simple harmonic motion $\left(a_{x'} = -\omega^2 x'\right)$, with $\omega = \sqrt{k/m}$. The period of the motion is then

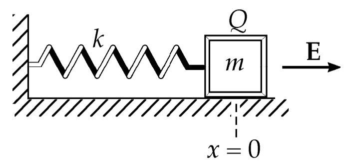
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4.00\text{ kg}}{100\text{ N/m}}} = \boxed{1.26\text{ s}}$$

(d) $(K + U_s + U_e)_i + \Delta E = (K + U_s + U_e)_f$

$$0 + 0 + 0 - \mu_k mgx_{\max} = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max}$$

$$x_{\max} = \frac{2(QE - \mu_k mg)}{k} = \frac{2\left[\left(50.0\times10^{-6}\ \text{C}\right)\left(5.00\times10^{5}\ \text{N/C}\right) - 0.200\left(4.00\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\right]}{100\ \text{N/m}} = \boxed{0.343\ \text{m}}$$

25.12 (a) Arbitrarily choose $V = 0$ at 0. Then at other points $V = -Ex$ and $U_e = QV = -QEx$. Between the endpoints of the motion,



$$(K + U_s + U_e)_i = (K + U_s + U_e)_f$$

$$0 + 0 + 0 = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max} \qquad \text{so} \qquad x_{\max} = \boxed{\frac{2QE}{k}}$$

(b) At equilibrium, $\Sigma F_x = -F_s + F_e = 0$ or $kx = QE$. So the equilibrium position is at $x = \boxed{\dfrac{QE}{k}}$

(c) The block's equation of motion is $\Sigma F_x = -kx + QE = m\dfrac{d^2x}{dt^2}$. Let $x' = x - \dfrac{QE}{k}$, or $x = x' + \dfrac{QE}{k}$, so the equation of motion becomes:

$$-k\left(x' + \frac{QE}{k}\right) + QE = m\frac{d^2\left(x' + QE/k\right)}{dt^2}, \quad \text{or} \quad \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'$$

This is the equation for simple harmonic motion $\left(a_{x'} = -\omega^2 x'\right)$, with $\omega = \sqrt{k/m}$

The period of the motion is then $T = \dfrac{2\pi}{\omega} = \boxed{2\pi\sqrt{\dfrac{m}{k}}}$

(d) $(K + U_s + U_e)_i + \Delta E = (K + U_s + U_e)_f$

$$0 + 0 + 0 - \mu_k mgx_{\max} = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max}$$

$$x_{\max} = \boxed{\frac{2(QE - \mu_k mg)}{k}}$$

25.13 For the entire motion, $y - y_i = v_{yi}t + \frac{1}{2}a_y t^2$

$$0 - 0 = v_i t + \tfrac{1}{2}a_y t^2 \qquad \text{so} \qquad a_y = -\frac{2v_i}{t}$$

$\sum F_y = ma_y$: $\quad -mg - qE = -\dfrac{2mv_i}{t}$

$$E = \frac{m}{q}\left(\frac{2v_i}{t} - g\right) \qquad \text{and} \qquad \mathbf{E} = -\frac{m}{q}\left(\frac{2v_i}{t} - g\right)\mathbf{j}$$

For the upward flight: $v_{yf}^2 = v_{yi}^2 + 2a_y(y - y_i)$

$$0 = v_i^2 + 2\left(-\frac{2v_i}{t}\right)(y_{\max} - 0) \qquad \text{and} \qquad y_{\max} = \tfrac{1}{4}v_i t$$

$$\Delta V = \int_0^{y_{\max}} \mathbf{E} \cdot d\mathbf{y} = +\frac{m}{q}\left(\frac{2v_i}{t} - g\right)y\Big|_0^{y_{\max}} = \frac{m}{q}\left(\frac{2v_i}{t} - g\right)\left(\tfrac{1}{4}v_i t\right)$$

$$\Delta V = \frac{2.00 \text{ kg}}{5.00 \times 10^{-6} \text{ C}}\left(\frac{2(20.1 \text{ m/s})}{4.10 \text{ s}} - 9.80 \text{ m/s}^2\right)\left[\tfrac{1}{4}(20.1 \text{ m/s})(4.10 \text{ s})\right] = \boxed{40.2 \text{ kV}}$$

25.14 Arbitrarily take $V = 0$ at the initial point. Then at distance d downfield, where L is the rod length, $V = -Ed$ and $U_e = -\lambda LEd$

(a) $(K+U)_i = (K+U)_f$

$$0 + 0 = \tfrac{1}{2}\mu L v^2 - \lambda LEd$$

$$v = \sqrt{\frac{2\lambda Ed}{\mu}} = \sqrt{\frac{2(40.0\times10^{-6}\text{ C / m})(100\text{ N / C})(2.00\text{ m})}{(0.100\text{ kg / m})}} = \boxed{0.400\text{ m/s}}$$

(b) $\boxed{\text{The same.}}$

25.15 Arbitrarily take $V = 0$ at point P. Then (from Equation 25.8) the potential at the original position of the charge is $-\mathbf{E}\cdot\mathbf{s} = -EL\cos\theta$. At the final point a, $V = -EL$. Suppose the table is frictionless: $(K+U)_i = (K+U)_f$

$$0 - qEL\cos\theta = \tfrac{1}{2}mv^2 - qEL$$

$$v = = \sqrt{\frac{2qEL(1-\cos\theta)}{\text{m}}} = \sqrt{\frac{2(2.00\times10^{-6}\text{ C})(300\text{ N / C})(1.50\text{ m})(1-\cos 60.0°)}{0.0100\text{ kg}}} = \boxed{0.300\text{ m/s}}$$

***25.16** (a) The potential at 1.00 cm is

$$V_1 = k_e\frac{q}{r} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})}{1.00\times10^{-2}\text{ m}} = \boxed{1.44\times10^{-7}\text{ V}}$$

(b) The potential at 2.00 cm is

$$V_2 = k_e\frac{q}{r} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})}{2.00\times10^{-2}\text{ m}} = 0.719\times10^{-7}\text{ V}$$

Thus, the difference in potential between the two points is

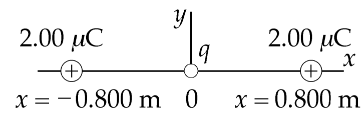
$$\Delta V = V_2 - V_1 = \boxed{-7.19\times10^{-8}\text{ V}}$$

(c) The approach is the same as above except the charge is -1.60×10^{-19} C. This changes the sign of all the answers, with the magnitudes remaining the same.

That is, the potential at 1.00 cm is $\boxed{-1.44\times10^{-7}\text{ V}}$

The potential at 2.00 cm is -0.719×10^{-7} V, so $\Delta V = V_2 - V_1 = \boxed{7.19\times10^{-8}\text{ V}}$.

25.17 (a) Since the charges are equal and placed symmetrically, $\boxed{F = 0}$



(b) Since $F = qE = 0$, $\boxed{E = 0}$

(c) $V = 2k_e \dfrac{q}{r} = 2\left(8.99 \times 10^9 \ \dfrac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\dfrac{2.00 \times 10^{-6}\ \text{C}}{0.800\ \text{m}}\right)$

$V = 4.50 \times 10^4\ \text{V} = \boxed{45.0\ \text{kV}}$

25.18 (a) $E_x = \dfrac{k_e q_1}{x^2} + \dfrac{k_e q_2}{(x - 2.00)^2} = 0$ becomes $E_x = k_e\left(\dfrac{+q}{x^2} + \dfrac{-2q}{(x - 2.00)^2}\right) = 0$

Dividing by k_e, $2qx^2 = q(x - 2.00)^2$

$x^2 + 4.00x - 4.00 = 0$

Therefore $E = 0$ when $x = \dfrac{-4.00 \pm \sqrt{16.0 + 16.0}}{2} = \boxed{-4.83\ \text{m}}$

(Note that the positive root does not correspond to a physically valid situation.)

(b) $V = \dfrac{k_e q_1}{x} + \dfrac{k_e q_2}{(2.00 - x)} = 0$ or $V = k_e\left(\dfrac{+q}{x} - \dfrac{2q}{(2.00 - x)}\right) = 0$

Again solving for x, $2qx = q(2.00 - x)$

For $0 \le x \le 2.00$ $V = 0$ when $x = \boxed{0.667\ \text{m}}$

and $\dfrac{q}{|x|} = \dfrac{-2q}{|2 - x|}$

For $x < 0$ $x = \boxed{-2.00\ \text{m}}$

25.19 (a) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-(8.99 \times 10^9)(1.60 \times 10^{-19})^2}{0.0529 \times 10^{-9}} = -4.35 \times 10^{-18}\ \text{J} = \boxed{-27.2\ \text{eV}}$

(b) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-(8.99 \times 10^9)(1.60 \times 10^{-19})^2}{2^2(0.0529 \times 10^{-9})} = \boxed{-6.80\ \text{eV}}$

(c) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-k_e e^2}{\infty} = \boxed{0}$

<u>Goal Solution</u>

The Bohr model of the hydrogen atom states that the single electron can exist only in certain allowed orbits around the proton. The radius of each Bohr orbit is $r = n^2(0.0529\text{ nm})$ where $n = 1, 2, 3, \ldots$. Calculate the electric potential energy of a hydrogen atom when the electron is in the (a) first allowed orbit, $n = 1$; (b) second allowed orbit, $n = 2$; and (c) when the electron has escaped from the atom ($r = \infty$). Express your answers in electron volts.

G: We may remember from chemistry that the lowest energy level for hydrogen is $E_1 = -13.6\text{ eV}$, and higher energy levels can be found from $E_n = E_1 / n^2$, so that $E_2 = -3.40\text{ eV}$ and $E_\infty = 0\text{ eV}$. (see section 42.2) Since these are the total energies (potential plus kinetic), the electric potential energy alone should be lower (more negative) because the kinetic energy of the electron must be positive.

O: The electric potential energy is given by $U = k_e \dfrac{q_1 q_2}{r}$

A: (a) For the first allowed Bohr orbit,

$$U = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(-1.60\times10^{-19}\text{ C})(1.60\times10^{-19}\text{ C})}{(0.0529\times10^{-9}\text{ m})} = -4.35\times10^{-18}\text{ J} = \frac{-4.35\times10^{-18}\text{ J}}{1.60\times10^{-19}\text{ J / eV}} = -27.2\text{ eV}$$

(b) For the second allowed orbit,

$$U = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-1.60\times10^{-19}\text{ C})(1.60\times10^{-19}\text{ C})}{2^2(0.0529\times10^{-9}\text{ m})} = -1.088\times10^{-18}\text{ J} = -6.80\text{ eV}$$

(c) When the electron is at $r = \infty$,

$$U = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(-1.60\times10^{-19}\text{ C}\right)\left(1.60\times10^{-19}\text{ C}\right)}{\infty} = 0\text{ J}$$

L: The potential energies appear to be twice the magnitude of the total energy values, so apparently the kinetic energy of the electron has the same absolute magnitude as the total energy.

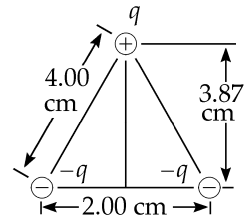
***25.20** (a) $U = \dfrac{qQ}{4\pi\text{e}_0 r} = \dfrac{(5.00\times10^{-9}\text{ C})(-3.00\times10^{-9}\text{ C})(8.99\times10^9\text{ V}\cdot\text{m/C})}{(0.350\text{ m})} = \boxed{-3.86\times10^{-7}\text{ J}}$

The minus sign means it takes 3.86×10^{-7} J to pull the two charges apart from 35 cm to a much larger separation.

(b) $V = \dfrac{Q_1}{4\pi\text{e}_0 r_1} + \dfrac{Q_2}{4\pi\text{e}_0 r_2} = \dfrac{(5.00\times10^{-9}\text{ C})(8.99\times10^9\text{ V}\cdot\text{m / C})}{0.175\text{ m}} + \dfrac{(-3.00\times10^{-9}\text{ C})(8.99\times10^9\text{ V}\cdot\text{m / C})}{0.175\text{ m}}$

$V = \boxed{103\text{ V}}$

25.21 $V = \sum_i k \dfrac{q_i}{r_i}$



$$V = (8.99 \times 10^9)(7.00 \times 10^{-6})\left[\frac{-1}{0.0100} - \frac{1}{0.0100} + \frac{1}{0.0387}\right]$$

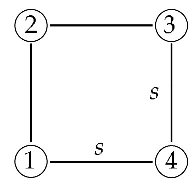
$$V = \boxed{-1.10 \times 10^7\ \text{C} = -11.0\ \text{MV}}$$

***25.22** $U_e = q_4 V_1 + q_4 V_2 + q_4 V_3 = q_4 \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2} + \dfrac{q_3}{r_3}\right)$

$$U_e = \left(10.0\times10^{-6}\ \text{C}\right)^2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{0.600\ \text{m}} + \frac{1}{0.150\ \text{m}} + \frac{1}{\sqrt{(0.600\ \text{m})^2 + (0.150\ \text{m})^2}}\right)$$

$U_e = \boxed{8.95\ \text{J}}$

25.23 $U = U_1 + U_2 + U_3 + U_4$



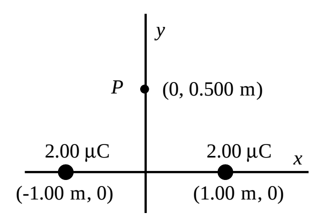
$U = 0 + U_{12} + (U_{13} + U_{23}) + (U_{14} + U_{24} + U_{34})$

$$U = 0 + \frac{k_e Q^2}{s} + \frac{k_e Q^2}{s}\left(\frac{1}{\sqrt{2}} + 1\right) + \frac{k_e Q^2}{s}\left(1 + \frac{1}{\sqrt{2}} + 1\right)$$

$$U = \frac{k_e Q^2}{s}\left(4 + \frac{2}{\sqrt{2}}\right) = \boxed{5.41\,\frac{k_e Q^2}{s}}$$

An alternate way to get the term $\left(4 + 2/\sqrt{2}\right)$ is to recognize that there are 4 side pairs and 2 face diagonal pairs.

***25.24** (a) $V = \dfrac{k_e q_1}{r_1} + \dfrac{k_e q_2}{r_2} = 2\left(\dfrac{k_e q}{r}\right) = 2\left(\dfrac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-6}\ \text{C}\right)}{\sqrt{(1.00\ \text{m})^2 + (0.500\ \text{m})^2}}\right)$



$V = 3.22\times10^4\ \text{V} = \boxed{32.2\ \text{kV}}$

(b) $U = qV = \left(-3.00\times10^{-6}\ \text{C}\right)\left(3.22\times10^4\ \dfrac{\text{J}}{\text{C}}\right) = \boxed{-9.65\times10^{-2}\ \text{J}}$

***25.25** Each charge creates equal potential at the center. The total potential is:

$$V = 5\left[\frac{k_e(-q)}{R}\right] = \boxed{-\frac{5k_e q}{R}}$$

***25.26** (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is $\boxed{\text{no point}}$ located at a finite distance from the charges, where this total potential is zero.

(b) $V = \dfrac{k_e q}{a} + \dfrac{k_e q}{a} = \boxed{\dfrac{2k_e q}{a}}$

25.27 (a) Conservation of momentum: $0 = m_1 v_1 \mathbf{i} + m_2 v_2 (-\mathbf{i})$ or $v_2 = \dfrac{m_1 v_1}{m_2}$

By conservation of energy, $0 + \dfrac{k_e(-q_1)q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dfrac{k_e(-q_1)q_2}{(r_1 + r_2)}$

and $\dfrac{k_e q_1 q_2}{r_1 + r_2} - \dfrac{k_e q_1 q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \dfrac{m_1^2 v_1^2}{m_2}$

$$v_1 = \sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$$

$$v_1 = \sqrt{\frac{2(0.700\text{ kg})(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2\times10^{-6}\text{ C})(3\times10^{-6}\text{ C})}{(0.100\text{ kg})(0.800\text{ kg})}\left(\frac{1}{8\times10^{-3}\text{ m}} - \frac{1}{1.00\text{ m}}\right)} = \boxed{10.8\text{ m/s}}$$

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{(0.100\text{ kg})(10.8\text{ m/s})}{0.700\text{ kg}} = \boxed{1.55\text{ m/s}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

25.28 (a) Conservation of momentum: $0 = m_1 v_1 \mathbf{i} + m_2 v_2(-\mathbf{i})$ or $v_2 = m_1 v_1 / m_2$

By conservation of energy, $0 + \dfrac{k_e(-q_1)q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dfrac{k_e(-q_1)q_2}{(r_1 + r_2)}$

and $\dfrac{k_e q_1 q_2}{r_1 + r_2} - \dfrac{k_e q_1 q_2}{d} = \dfrac{1}{2} m_1 v_1^2 + \dfrac{1}{2} \dfrac{m_1^2 v_1^2}{m_2}$

$$v_1 = \boxed{\sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}} \qquad v_2 = \left(\frac{m_1}{m_2}\right)v_1 = \boxed{\sqrt{\frac{2m_1 k_e q_1 q_2}{m_2(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

25.29 $V = \frac{k_e Q}{r}$ so $r = \frac{k_e Q}{V} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(8.00\times10^{-9}\ \mathrm{C}\right)}{V} = \frac{72.0\ \mathrm{V\cdot m}}{V}$

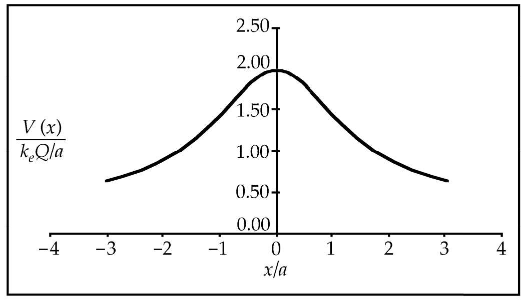
For $V = 100$ V, 50.0 V, and 25.0 V, $\boxed{r = 0.720\ \mathrm{m},\ 1.44\ \mathrm{m},\ \text{and } 2.88\ \mathrm{m}}$

The radii are $\boxed{\text{inversely proportional}}$ to the potential.

25.30 (a) $V(x) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{\sqrt{x^2+a^2}} + \frac{k_e(+Q)}{\sqrt{x^2+(-a)^2}}$

$$V(x) = \frac{2k_e Q}{\sqrt{x^2+a^2}} = \frac{k_e Q}{a}\left(\frac{2}{\sqrt{(x/a)^2+1}}\right)$$

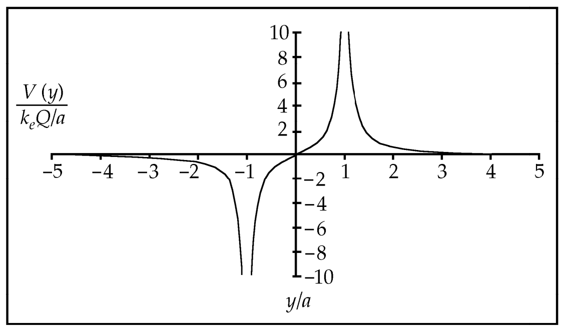
$$\frac{V(x)}{(k_e Q/a)} = \boxed{\frac{2}{\sqrt{(x/a)^2+1}}}$$



(b) $V(y) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{|y-a|} + \frac{k_e(-Q)}{|y+a|}$

$$V(y) = \frac{k_e Q}{a}\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)$$

$$\frac{V(y)}{(k_e Q/a)} = \boxed{\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)}$$



25.31 Using conservation of energy, we have $K_f + U_f = K_i + U_i$.

But $U_i = \frac{k_e q_\alpha q_{\mathrm{gold}}}{r_i}$, and $r_i \approx \infty$. Thus, $U_i = 0$.

Also $K_f = 0$ ($v_f = 0$ at turning point), so $U_f = K_i$, or $\frac{k_e q_\alpha q_{\mathrm{gold}}}{r_{\min}} = \frac{1}{2} m_\alpha v_\alpha^2$

$$r_{\min} = \frac{2k_e q_\alpha q_{\mathrm{gold}}}{m_\alpha v_\alpha^2} = \frac{2(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(2)(79)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{(6.64\times10^{-27}\ \mathrm{kg})(2.00\times10^{7}\ \mathrm{m/s})^2} = 2.74\times10^{-14}\ \mathrm{m} = \boxed{27.4\ \mathrm{fm}}$$

25.32 Using conservation of energy

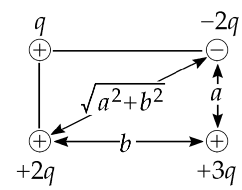
we have: $\frac{k_e eQ}{r_1} = \frac{k_e eQ}{r_2} + \frac{1}{2}mv^2$

which gives: $v = \sqrt{\frac{2k_e eQ}{m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$

or $v = \sqrt{\frac{(2)(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\text{ C})(10^{-9}\text{ C})}{9.11\times10^{-31}\text{ kg}}\left(\frac{1}{0.0300\text{ m}} - \frac{1}{0.0200\text{ m}}\right)}$

Thus, $v = \boxed{7.26\times10^6 \text{ m/s}}$

25.33 $U = \sum \frac{k_e q_i q_j}{r_{ij}}$, summed over all pairs of (i, j) where $i \neq j$



$$U = k_e\left[\frac{q(-2q)}{b} + \frac{(-2q)(3q)}{a} + \frac{(2q)(3q)}{b} + \frac{q(2q)}{a} + \frac{q(3q)}{\sqrt{a^2+b^2}} + \frac{2q(-2q)}{\sqrt{a^2+b^2}}\right]$$

$$U = k_e q^2\left[\frac{-2}{0.400} - \frac{6}{0.200} + \frac{6}{0.400} + \frac{2}{0.200} + \frac{3}{0.447} - \frac{4}{0.447}\right]$$

$$U = \left(8.99\times10^9\right)\left(6.00\times10^{-6}\right)^2\left[\frac{4}{0.400} - \frac{4}{0.200} - \frac{1}{0.447}\right] = \boxed{-3.96 \text{ J}}$$

25.34 Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum(K+U)_i = \sum(K+U)_f$$

$$0 + \frac{4k_e q^2}{L} + \frac{2k_e q^2}{\sqrt{2}L} = 4\left(\frac{1}{2}mv^2\right) + \frac{4k_e q^2}{2L} + \frac{2k_e q^2}{2\sqrt{2}L}$$

$$\left(2 + \frac{1}{\sqrt{2}}\right)\frac{k_e q^2}{L} = 2mv^2$$

$$\boxed{v = \sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{k_e q^2}{mL}}}$$

25.35 A cube has 12 edges and 6 faces. Consequently, there are 12 edge pairs separated by s, $2 \times 6 = 12$ face diagonal pairs separated by $\sqrt{2}\, s$, and 4 interior diagonal pairs separated $\sqrt{3}\, s$.

$$U = \frac{k_e q^2}{s}\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right] = \boxed{22.8\,\frac{k_e q^2}{s}}$$

25.36 $V = a + bx = 10.0 \text{ V} + (-7.00 \text{ V/m})x$

(a) At $x = 0$, $V = \boxed{10.0 \text{ V}}$

At $x = 3.00$ m, $V = \boxed{-11.0 \text{ V}}$

At $x = 6.00$ m, $V = \boxed{-32.0 \text{ V}}$

(b) $E = -\frac{dV}{dx} = -b = -(-7.00 \text{ V/m}) = \boxed{7.00 \text{ N/C in } +x \text{ direction}}$

25.37 $V = 5x - 3x^2y + 2yz^2$ Evaluate E at $(1, 0 - 2)$

$$E_x = -\frac{\partial V}{\partial x} = \boxed{-5 + 6xy} = -5 + 6(1)(0) = -5$$

$$E_y = -\frac{\partial V}{\partial y} = \boxed{+3x^2 - 2z^2} = 3(1)^2 - 2(-2)^2 = -5$$

$$E_z = -\frac{\partial V}{\partial z} = \boxed{-4yz} = -4(0)(-2) = 0$$

$$E = \sqrt{E_x{}^2 + E_y{}^2 + E_z{}^2} = \sqrt{(-5)^2 + (-5)^2 + 0^2} = \boxed{7.07 \text{ N/C}}$$

25.38 (a) For $r < R$ $V = \frac{k_e Q}{R}$

$$E_r = -\frac{dV}{dr} = \boxed{0}$$

(b) For $r \geq R$ $V = \frac{k_e Q}{r}$

$$E_r = -\frac{dV}{dr} = -\left(-\frac{k_e Q}{r^2}\right) = \boxed{\frac{k_e Q}{r^2}}$$

25.39 $E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{k_e Q}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + y^2}}{y}\right)\right]$

$$E_y = \frac{k_e Q}{\ell y}\left[1 - \frac{y^2}{\ell^2 + y^2 + \ell\sqrt{\ell^2 + y^2}}\right] = \boxed{\frac{k_e Q}{y\sqrt{\ell^2 + y^2}}}$$

25.40 Inside the sphere, $E_x = E_y = E_z = 0$.

Outside, $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(V_0 - E_0 z + E_0 a^3 z(x^2 + y^2 + z^2)^{-3/2}\right)$

So $E_x = -\left[0 + 0 + E_0 a^3 z(-3/2)(x^2 + y^2 + z^2)^{-5/2}(2x)\right] = \boxed{3E_0 a^3 xz(x^2 + y^2 + z^2)^{-5/2}}$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(V_0 - E_0 z + E_0 a^3 z(x^2 + y^2 + z^2)^{-3/2}\right)$$

$$E_y = -E_0 a^3 z(-3/2)(x^2 + y^2 + z^2)^{-5/2} 2y = \boxed{3E_0 a^3 yz(x^2 + y^2 + z^2)^{-5/2}}$$

$$E_z = -\frac{\partial V}{\partial z} = E_0 - E_0 a^3 z(-3/2)(x^2 + y^2 + z^2)^{-5/2}(2z) - E_0 a^3 (x^2 + y^2 + z^2)^{-3/2}$$

$$E_z = \boxed{E_0 + E_0 a^3 (2z^2 - x^2 - y^2)(x^2 + y^2 + z^2)^{-5/2}}$$

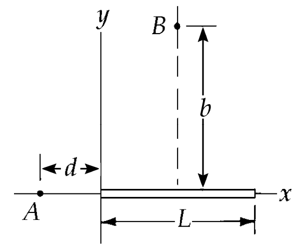
***25.41** $\Delta V = V_{2R} - V_0 = \frac{k_e Q}{\sqrt{R^2 + (2R)^2}} - \frac{k_e Q}{R} = \frac{k_e Q}{R}\left(\frac{1}{\sqrt{5}} - 1\right) = \boxed{-0.553\frac{k_e Q}{R}}$

***25.42** $V = \int dV = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$

All bits of charge are at the same distance from O, so

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \left(8.99\times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-7.50\times 10^{-6}\text{ C}\right)}{(0.140\text{ m}/\pi)} = \boxed{-1.51\text{ MV}}$$

25.43 (a) $[\alpha]=\left[\dfrac{\lambda}{x}\right]=\dfrac{\mathrm{C}}{\mathrm{m}}\cdot\left(\dfrac{1}{\mathrm{m}}\right)=\boxed{\dfrac{\mathrm{C}}{\mathrm{m}^2}}$



(b) $V=k_e\displaystyle\int\frac{dq}{r}=k_e\int\frac{\lambda\,dx}{r}=k_e\alpha\int_0^L\frac{x\,dx}{(d+x)}=\boxed{k_e\alpha\left[L-d\ln\left(1+\frac{L}{d}\right)\right]}$

25.44 $V=\displaystyle\int\frac{k_e\,dq}{r}=k_e\int\frac{\alpha x\,dx}{\sqrt{b^2+(L/2-x)^2}}$

Let $z=\dfrac{L}{2}-x$. Then $x=\dfrac{L}{2}-z$, and $dx=-dz$

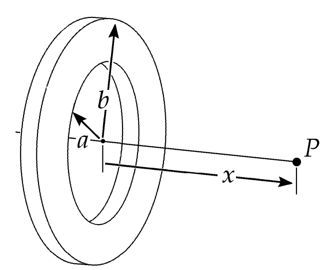
$$V=k_e\alpha\int\frac{(L/2-z)(-dz)}{\sqrt{b^2+z^2}}=-\frac{k_e\alpha L}{2}\int\frac{dz}{\sqrt{b^2+z^2}}+k_e\alpha\int\frac{z\,dz}{\sqrt{b^2+z^2}}=-\frac{k_e\alpha L}{2}\ln\left(z+\sqrt{z^2+b^2}\right)+k_e\alpha\sqrt{z^2+b^2}$$

$$V=-\frac{k_e\alpha L}{2}\ln\left[(L/2-x)+\sqrt{(L/2-x)^2+b^2}\right]\Bigg|_0^L+k_e\alpha\sqrt{(L/2-x)^2+b^2}\,\Bigg|_0^L$$

$$V=-\frac{k_e\alpha L}{2}\ln\left[\frac{L/2-L+\sqrt{(L/2)^2+b^2}}{L/2+\sqrt{(L/2)^2+b^2}}\right]+k_e\alpha\left[\sqrt{(L/2-L)^2+b^2}-\sqrt{(L/2)^2+b^2}\right]$$

$$V=\boxed{-\frac{k_e\alpha L}{2}\ln\left[\frac{\sqrt{b^2+(L^2/4)}-L/2}{\sqrt{b^2+(L^2/4)}+L/2}\right]}$$

25.45 $dV=\dfrac{k_e\,dq}{\sqrt{r^2+x^2}}$ where $dq=\sigma\,dA=\sigma 2\pi r\,dr$



$$V=2\pi\sigma k_e\int_a^b\frac{r\,dr}{\sqrt{r^2+x^2}}=\boxed{2\pi k_e\sigma\left[\sqrt{x^2+b^2}-\sqrt{x^2+a^2}\right]}$$

25.46 $V=k_e\displaystyle\int_{\text{all charge}}\frac{dq}{r}=k_e\int_{-3R}^{-R}\frac{\lambda\,dx}{-x}+k_e\int_{\text{semicircle}}\frac{\lambda\,ds}{R}+k_e\int_R^{3R}\frac{\lambda\,dx}{x}$

$$V=-k_e\lambda\ln(-x)\Big|_{-3R}^{-R}+\frac{k_e\lambda}{R}\pi R+k_e\lambda\ln x\Big|_R^{3R}$$

$$V=k_e\lambda\ln\frac{3R}{R}+k_e\lambda\pi+k_e\lambda\ln 3=\boxed{k_e\lambda(\pi+2\ln 3)}$$

25.47 Substituting given values into $V = \frac{k_e q}{r}$, $7.50 \times 10^3 \text{ V} = \frac{(8.99 \times 10^9 \text{ N·m}^2/\text{C}^2)\, q}{(0.300 \text{ m})}$

Substituting $q = 2.50 \times 10^{-7}$ C, $N = \frac{2.50 \times 10^{-7} \text{ C}}{1.60 \times 10^{-19} \text{ C/e}^-} = \boxed{1.56 \times 10^{12} \text{ electrons}}$

25.48 $q_1 + q_2 = 20.0\ \mu\text{C}$ so $q_1 = 20.0\ \mu\text{C} - q_2$

$\frac{q_1}{q_2} = \frac{r_1}{r_2}$ so $\frac{20.0\ \mu\text{C} - q_2}{q_2} = \frac{4.00 \text{ cm}}{6.00 \text{ cm}}$

Therefore $6.00(20.0\ \mu\text{C} - q_2) = 4.00 q_2$;

Solving, $q_2 = 12.0\ \mu\text{C}$ and $q_1 = 20.0\ \mu\text{C} - 12.0\ \mu\text{C} = 8.00\ \mu\text{C}$

(a) $E_1 = \frac{k_e q_1}{r_1^2} = \frac{\left(8.99 \times 10^9\right)\left(8.00 \times 10^{-6}\right)}{(0.0400)^2} = 4.50 \times 10^7 \text{ V/m} = \boxed{45.0 \text{ MV/m}}$

$E_2 = \frac{k_e q_2}{r_2^2} = \frac{\left(8.99 \times 10^9\right)\left(12.0 \times 10^{-6}\right)}{(0.0600)^2} = 3.00 \times 10^7 \text{ V/m} = \boxed{30.0 \text{ MV/m}}$

(b) $V_1 = V_2 = \frac{k_e q_2}{r_2} = \boxed{1.80 \text{ MV}}$

25.49 (a) $\text{E} = \boxed{0}$; $V = \frac{k_e q}{R} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{0.140} = \boxed{1.67 \text{ MV}}$

(b) $E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.200)^2} = \boxed{5.84 \text{ MN/C}}$ away

$V = \frac{k_e q}{r} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.200)} = \boxed{1.17 \text{ MV}}$

(c) $E = \frac{k_e q}{R^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.140)^2} = \boxed{11.9 \text{ MN/C}}$ away

$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$

25.50 No charge stays on the inner sphere in equilibrium. If there were any, it would create an electric field in the wire to push more charge to the outer sphere. Charge Q is on the outer sphere. Therefore, $\boxed{\text{zero charge is on the inner sphere}}$ and $\boxed{10.0\ \mu\text{C is on the outer sphere}}$.

25.51 (a) $E_{\max} = 3.00\times 10^{6}\text{ V / m} = \dfrac{k_e Q}{r^2} = \dfrac{k_e Q}{r}\dfrac{1}{r} = V_{\max}\dfrac{1}{r}$

$$V_{\max} = E_{\max} r = 3.00\times 10^{6}(0.150) = \boxed{450\text{ kV}}$$

(b) $\dfrac{k_e Q_{\max}}{r^2} = E_{\max}$ $\left\{\text{or } \dfrac{k_e Q_{\max}}{r} = V_{\max}\right\}$

$$Q_{\max} = \frac{E_{\max} r^2}{k_e} = \frac{3.00\times 10^{6}(0.150)^2}{8.99\times 10^{9}} = \boxed{7.51\ \mu\text{C}}$$

Goal Solution

Consider a Van de Graaff generator with a 30.0-cm-diameter dome operating in dry air. (a) What is the maximum potential of the dome? (b) What is the maximum charge on the dome?

G: Van de Graaff generators produce voltages that can make your hair stand on end, somewhere on the order of about 100 kV (see the Puzzler at beginning of Chapter 25). With these high voltages, the maximum charge on the dome is probably more than typical point charge values of about $1\ \mu$C.

The maximum potential and charge will be limited by the electric field strength at which the air surrounding the dome will ionize. This critical value is determined by the **dielectric strength** of air which, from page 789 or from Table 26.1, is $E_{\text{critical}} = 3\times 10^{6}\text{ V / m}$. An electric field stronger than this will cause the air to act like a conductor instead of an insulator. This process is called dielectric breakdown and may be seen as a spark.

O: From the maximum allowed electric field, we can find the charge and potential that would create this situation. Since we are only given the diameter of the dome, we will assume that the conductor is spherical, which allows us to use the electric field and potential equations for a spherical conductor. With these equations, it will be easier to do part (b) first and use the result for part (a).

A: (b) For a spherical conductor with total charge Q, $|\mathbf{E}| = \dfrac{k_e Q}{r^2}$

$$Q = \frac{Er^2}{k_e} = \frac{\left(3.00\times 10^{6}\text{ V / m}\right)(0.150\text{ m})^2}{8.99\times 10^{9}\text{ N}\cdot\text{m}^2\text{ / C}^2}(1\text{ N}\cdot\text{m / V}\cdot\text{C}) = 7.51\ \mu\text{C}$$

(a) $$V = \frac{k_e Q}{r} = \frac{(8.99\times 10^{9}\text{ N}\cdot\text{m}^2\text{ / C}^2)(7.51\times 10^{-6}\text{ C})}{0.150\text{ m}} = 450\text{ kV}$$

L: These calculated results seem reasonable based on our predictions. The voltage is about 4000 times larger than the 120 V found from common electrical outlets, but the charge is similar in magnitude to many of the static charge problems we have solved earlier. This implies that most of these charge configurations would have to be in a vacuum because the electric field near these point charges would be strong enough to cause sparking in air. (Example: A charged ball with $Q = 1\ \mu$C and $r = 1$ mm would have an electric field near its surface of

$$E = \frac{k_e Q}{r^2} = \frac{\left(9\times 10^{9}\text{ N}\cdot\text{m}^2\text{/ C}^2\right)\left(1\times 10^{-6}\text{ C}\right)}{(0.001\text{ m})^2} = 9\times 10^{9}\text{ V / m}$$

which is well beyond the dielectric breakdown of air!)

25.52 $V = \dfrac{k_e q}{r}$ and $E = \dfrac{k_e q}{r^2}$ Since $E = \dfrac{V}{r}$,

(b) $r = \dfrac{V}{E} = \dfrac{6.00 \times 10^5 \text{ V}}{3.00 \times 10^6 \text{ V/m}} = \boxed{0.200 \text{ m}}$ and

(a) $q = \dfrac{Vr}{k_e} = \boxed{13.3\ \mu\text{C}}$

25.53 $U = qV = k_e \dfrac{q_1 q_2}{r_{12}} = (8.99 \times 10^9)\dfrac{(38)(54)(1.60 \times 10^{-19})^2}{(5.50 + 6.20) \times 10^{-15}} = 4.04 \times 10^{-11} \text{ J} = \boxed{253 \text{ MeV}}$

***25.54** (a) To make a spark 5 mm long in dry air between flat metal plates requires potential difference

$$V = Ed = \left(3.0 \times 10^6 \text{ V/m}\right)\left(5.0 \times 10^{-3} \text{ m}\right) = 1.5 \times 10^4 \text{ V} \quad \boxed{\sim 10^4 \text{ V}}$$

(b) Suppose your surface area is like that of a 70-kg cylinder with the density of water and radius 12 cm. Its length would be given by

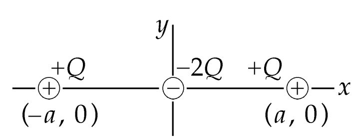
$$70 \times 10^3 \text{ cm}^3 = \pi(12 \text{ cm})^2 \ell \qquad \ell = 1.6 \text{ m}$$

The lateral surface area is $A = 2\pi r \ell = 2\pi(0.12 \text{ m})(1.6 \text{ m}) = 1.2 \text{ m}^2$

The electric field close to your skin is described by $E = \dfrac{\sigma}{e_0} = \dfrac{Q}{A e_0}$, so

$$Q = EA\,\epsilon_0 = \left(3.0 \times 10^6 \ \frac{\text{N}}{\text{C}}\right)\left(1.2 \text{ m}^2\right)\left(8.85 \times 10^{-12} \ \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right) \quad \boxed{\sim 10^{-5} \text{ C}}$$

25.55 (a) $V = k_e Q\left(\dfrac{1}{x+a} - \dfrac{2}{x} + \dfrac{1}{x-a}\right)$



$$V = k_e Q\left[\frac{x(x-a) - 2(x+a)(x-a) + x(x+a)}{x(x+a)(x-a)}\right] = \boxed{\frac{2k_e Q a^2}{x^3 - x a^2}}$$

(b) $V = \boxed{\dfrac{2k_e Q a^2}{x^3}}$ for $\dfrac{a}{x} << 1$

25.56 (a) $E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{2k_eQa^2}{x^3 - xa^2}\right) = \boxed{\frac{(2k_eQa^2)(3x^2 - a^2)}{(x^3 - xa^2)^2}}$ and $\boxed{E_y = E_z = 0}$

(b) $$E_x = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\times10^{-6}\ \text{C})(2\times10^{-3}\ \text{m})^2\left[3(6\times10^{-3}\ \text{m})^2 - (2\times10^{-3}\ \text{m})^2\right]}{\left[(6\times10^{-3}\ \text{m})^3 - (6\times10^{-3}\ \text{m})(2\times10^{-3}\ \text{m})^2\right]^2}$$

$E_x = 609\times10^6\ \text{N/C} = \boxed{609\ \text{MN/C}}$

25.57 (a) $E = \frac{Q}{4\pi\epsilon_0\, r^2}$

$V = \frac{Q}{4\pi\epsilon_0\, r}$

$r = \frac{|V|}{|E|} = \frac{3000\ \text{V}}{500\ \text{V / m}} = \boxed{6.00\ \text{m}}$

(b) $V = -3000\ \text{V} = \frac{Q}{4\pi\epsilon_0\,(6.00\ \text{m})}$

$Q = \frac{-3000\ \text{V}}{(8.99\times10^9\ \text{V}\cdot\text{m / C})}(6.00\ \text{m}) = \boxed{-2.00\ \mu\text{C}}$

25.58 From Example 25.5, the potential created by the ring at the electron's starting point is

$$V_i = \frac{k_eQ}{\sqrt{x_i^2 + a^2}} = \frac{k_e(2\pi\lambda a)}{\sqrt{x_i^2 + a^2}}$$

while at the center, it is $V_f = 2\pi k_e\lambda$. From conservation of energy,

$$0 + (-eV_i) = \tfrac{1}{2}m_e v_f^2 + (-eV_f)$$

$$v_f^2 = \frac{2e}{m_e}(V_f - V_i) = \frac{4\pi e k_e\lambda}{m_e}\left(1 - \frac{a}{\sqrt{x_i^2 + a^2}}\right)$$

$$v_f^2 = \frac{4\pi(1.60\times10^{-19})(8.99\times10^9)(1.00\times10^{-7})}{9.11\times10^{-31}}\left(1 - \frac{0.200}{\sqrt{(0.100)^2 + (0.200)^2}}\right)$$

$v_f = \boxed{1.45\times10^7\ \text{m / s}}$

25.59 (a) Take the origin at the point where we will find the potential. One ring, of width dx, has charge $Q\,dx/h$ and, according to Example 25.5, creates potential

$$dV = \frac{k_e Q\,dx}{h\sqrt{x^2 + R^2}}$$

The whole stack of rings creates potential

$$V = \int_{\text{all charge}} dV = \int_d^{d+h} \frac{k_e Q\,dx}{h\sqrt{x^2+R^2}} = \frac{k_e Q}{h}\ln\left(x + \sqrt{x^2+R^2}\right)\Bigg|_d^{d+h} = \boxed{\frac{k_e Q}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)}$$

(b) A disk of thickness dx has charge $Q\,dx/h$ and charge-per-area $Q\,dx/\pi R^2 h$. According to Example 25.6, it creates potential

$$dV = 2\pi k_e \frac{Q\,dx}{\pi R^2 h}\left(\sqrt{x^2+R^2} - x\right)$$

Integrating,

$$V = \int_d^{d+h} \frac{2k_e Q}{R^2 h}\left(\sqrt{x^2+R^2}\,dx - x\,dx\right) = \frac{2k_e Q}{R^2 h}\left[\frac{1}{2}x\sqrt{x^2+R^2} + \frac{R^2}{2}\ln\left(x+\sqrt{x^2+R^2}\right) - \frac{x^2}{2}\right]_d^{d+h}$$

$$V = \boxed{\frac{k_e Q}{R^2 h}\left[(d+h)\sqrt{(d+h)^2+R^2} - d\sqrt{d^2+R^2} - 2dh - h^2 + R^2\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\right]}$$

25.60 The positive plate by itself creates a field $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{36.0\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.03\ \dfrac{\text{kN}}{\text{C}}$

away from the + plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field 4.07 kN/C in the space between.

(a) Take $V = 0$ at the negative plate. The potential at the positive plate is then

$$V - 0 = -\int_0^{12.0\ \text{cm}} (-4.07\ \text{kN/C})\,dx$$

The potential difference between the plates is $V = (4.07\times10^3\ \text{N/C})(0.120\ \text{m}) = \boxed{488\ \text{V}}$

(b) $\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$

$$qV = (1.60\times10^{-19}\ \text{C})(488\ \text{V}) = \frac{1}{2}mv_f^2 = \boxed{7.81\times10^{-17}\ \text{J}}$$

(c) $v_f = \boxed{306\ \text{km/s}}$

(d) $v_f^2 = v_i^2 + 2a(x - x_i)$

$(3.06 \times 10^5 \text{ m/s})^2 = 0 + 2a(0.120 \text{ m})$

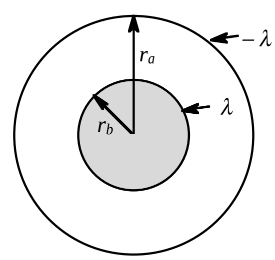
$a = \boxed{3.90 \times 10^{11} \text{ m/s}^2}$

(e) $\sum F = ma = (1.67 \times 10^{-27} \text{ kg})(3.90 \times 10^{11} \text{ m/s}^2) = \boxed{6.51 \times 10^{-16} \text{ N}}$

(f) $E = \dfrac{F}{q} = \dfrac{6.51 \times 10^{-16} \text{ N}}{1.60 \times 10^{-19} \text{ C}} = \boxed{4.07 \text{ kN/C}}$

25.61 $W = \int_0^Q V\,dq$ where $V = \dfrac{k_e q}{R}$; Therefore, $\boxed{W = \dfrac{k_e Q^2}{2R}}$

25.62 (a) $V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$ and the field at distance r from a uniformly charged rod (where $r >$ radius of charged rod) is



$$E = \frac{\lambda}{2\pi \epsilon_0 r} = \frac{2k_e\lambda}{r}$$

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

$$V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e\lambda}{r}\,dr = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right), \quad \text{or} \quad \boxed{\Delta V = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)}$$

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance r from the axis is

$$V = 2k_e\lambda \ln\left(\frac{r_a}{r}\right)$$

The field at r is given by $E = -\dfrac{\partial V}{\partial r} = -2k_e\lambda\left(\dfrac{r}{r_a}\right)\left(-\dfrac{r_a}{r^2}\right) = \dfrac{2k_e\lambda}{r}$

But, from part (a), $2k_e\lambda = \dfrac{\Delta V}{\ln(r_a/r_b)}$. Therefore, $\boxed{E = \dfrac{\Delta V}{\ln(r_a/r_b)}\left(\dfrac{1}{r}\right)}$

25.63 $V_2 - V_1 = -\int_{r_1}^{r_2} \mathbf{E} \cdot d\mathbf{r} = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 r} dr$

$V_2 - V_1 = \boxed{\frac{-\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right)}$

25.64 For the given charge distribution, $V(x, y, z) = \frac{k_e(q)}{r_1} + \frac{k_e(-2q)}{r_2}$

where $r_1 = \sqrt{(x+R)^2 + y^2 + z^2}$ and $r_2 = \sqrt{x^2 + y^2 + z^2}$

The surface on which $V(x, y, z) = 0$

is given by $k_e q\left(\frac{1}{r_1} - \frac{2}{r_2}\right) = 0$, or $2r_1 = r_2$

This gives: $4(x+R)^2 + 4y^2 + 4z^2 = x^2 + y^2 + z^2$

which may be written in the form: $x^2 + y^2 + z^2 + \left(\frac{8}{3}R\right)x + (0)y + (0)z + \left(\frac{4}{3}R^2\right) = 0$ [1]

The general equation for a sphere of radius a centered at (x_0, y_0, z_0) is:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - a^2 = 0$$

or $x^2 + y^2 + z^2 + (-2x_0)x + (-2y_0)y + (-2z_0)z + \left(x_0^2 + y_0^2 + z_0^2 - a^2\right) = 0$ [2]

Comparing equations [1] and [2], it is seen that the equipotential surface for which $V = 0$ is indeed a sphere and that:

$$-2x_0 = \frac{8}{3}R; \quad -2y_0 = 0; \quad -2z_0 = 0; \quad x_0^2 + y_0^2 + z_0^2 - a^2 = \frac{4}{3}R^2$$

Thus, $x_0 = -\frac{4}{3}R$, $y_0 = z_0 = 0$, and $a^2 = \left(\frac{16}{9} - \frac{4}{3}\right)R^2 = \frac{4}{9}R^2$.

The equipotential surface is therefore a sphere centered at $\boxed{\left(-\frac{4}{3}R, 0, 0\right)}$, having a radius $\boxed{\frac{2}{3}R}$

25.65 (a) From Gauss's law, $\boxed{E_A = 0}$ (no charge within)

$$E_B = k_e \frac{q_A}{r^2} = (8.99 \times 10^9)\frac{(1.00 \times 10^{-8})}{r^2} = \boxed{\left(\frac{89.9}{r^2}\right) \text{V / m}}$$

$$E_C = k_e \frac{(q_A + q_B)}{r^2} = (8.99 \times 10^9)\,\frac{(-\,5.00 \times 10^{-9})}{r^2} = \boxed{\left(-\,\frac{45.0}{r^2}\right)\text{V / m}}$$

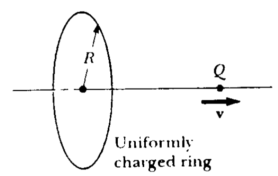
(b) $$V_C = k_e \frac{(q_A + q_B)}{r} = (8.99 \times 10^9)\,\frac{(-\,5.00 \times 10^{-9})}{r} = \boxed{\left(-\,\frac{45.0}{r}\right) \text{V}}$$

$$\therefore \text{ At } r_2, \;\; V = -\,\frac{45.0}{0.300} = -\,150\,\text{V}$$

Inside r_2, $$V_B = -\,150 \text{ V} + \int_{r_2}^{r} \frac{89.9}{r^2}\,dr = -\,150 + 89.9\left(\frac{1}{r} - \frac{1}{0.300}\right) = \boxed{\left(-\,450 + \frac{89.9}{r}\right) \text{V}}$$

$$\therefore \text{ At } r_1, \;\; V = -\,450 + \frac{89.9}{0.150} = +\,150\,\text{V} \quad \text{so} \quad \boxed{V_A = +\,150 \text{ V}}$$

25.66 From Example 25.5, the potential at the center of the ring is $V_i = k_e Q/R$ and the potential at an infinite distance from the ring is $V_f = 0$. Thus, the initial and final potential energies of the point charge are:



$$U_i = QV_i = \frac{k_e Q^2}{R} \quad \text{and} \quad U_f = QV_f = 0$$

From conservation of energy, $K_f + U_f = K_i + U_i$

or $$\tfrac{1}{2} M v_f^2 + 0 = 0 + \frac{k_e Q^2}{R} \qquad \text{giving} \qquad v_f = \boxed{\sqrt{\frac{2 k_e Q^2}{MR}}}$$

25.67 The sheet creates a field $\mathbf{E}_1 = \dfrac{\sigma}{2\,\epsilon_0}\mathbf{i}$ for $x > 0$. Along the $x-$axis, the line of charge creates a field

$$\mathbf{E}_2 = \frac{\lambda}{2\pi r \epsilon_0} \text{ away} = \frac{\lambda}{2\pi\epsilon_0 (3.00 \text{ m} - x)}(-\,\mathbf{i}) \text{ for } x < 3.00 \text{ m}$$

The total field along the $x-$axis in the region $0 < x < 3.00$ m is then

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \left[\frac{\sigma}{2\,\epsilon_0} - \frac{\lambda}{2\pi\epsilon_0\,(3.00 - x)}\right]\mathbf{i}$$

(a) The potential at point x follows from

$$V - V_0 = -\int_0^x \mathbf{E}\cdot\mathbf{i}\,dx = -\int_0^x\left[\frac{\sigma}{2\in_0} - \frac{\lambda}{2\pi\in_0(3.00-x)}\right]dx$$

$$V = V_0 - \frac{\sigma x}{2\in_0} - \frac{\lambda}{2\pi\in_0}\ln\left(1-\frac{x}{3.00}\right)$$

$$V = 1.00\text{ kV} - \frac{(25.0\times10^{-9}\text{ C / m}^2)x}{2(8.85\times10^{-12}\text{ C}^2\text{ / N}\cdot\text{m}^2)} - \frac{80.0\times10^{-9}\text{ C / m}}{2\pi(8.85\times10^{-12}\text{ C}^2\text{ / N}\cdot\text{m}^2)}\ln\left(1-\frac{x}{3.00}\right)$$

$$V = \boxed{1.00\text{ kV} - \left(1.41\ \frac{\text{kV}}{\text{m}}\right)x - (1.44\text{ kV})\ln\left(1.00-\frac{x}{3.00\text{ m}}\right)}$$

(b) At $x = 0.800$ m, $V = 316$ V

and $U = QV = \left(2.00\times10^{-9}\text{ C}\right)\left(316\text{ J/C}\right) = 6.33\times10^{-7}\text{ J} = \boxed{633\text{ nJ}}$

25.68 $$V = k_e\int_a^{a+L}\frac{\lambda\,dx}{\sqrt{x^2+b^2}} = k_e\lambda\ln\left[x+\sqrt{(x^2+b^2)}\right]\Big|_a^{a+L} = \boxed{k_e\lambda\ln\left[\frac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]}$$

25.69 (a) $E_r = -\dfrac{\partial V}{\partial r} = \boxed{\dfrac{2k_e p\cos\theta}{r^3}}$

In spherical coordinates, the θ component of the gradient is $\dfrac{1}{r}\left(\dfrac{\partial}{\partial\theta}\right)$.

Therefore, $E_\theta = -\dfrac{1}{r}\left(\dfrac{\partial V}{\partial\theta}\right) = \boxed{\dfrac{k_e p\sin\theta}{r^3}}$

For $r >> a$, $E_r(0°) = \dfrac{2k_e p}{r^3}$ and $E_r(90°) = 0$, $E_\theta(0°) = 0$ and $E_\theta(90°) = \dfrac{k_e p}{r^3}$

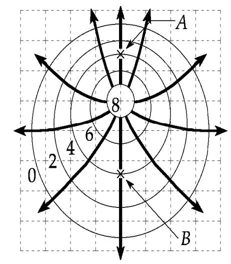
These results are $\boxed{\text{reasonable for } r >> a}$.

However, for $\boxed{r \to 0,\ E(0) \to \infty}$.

(b) $V = \boxed{\dfrac{k_e py}{(x^2+y^2)^{3/2}}}$ and $\boxed{E_x = -\dfrac{\partial V}{\partial x} = \dfrac{3k_e pxy}{(x^2+y^2)^{5/2}}}$

$E_y = -\dfrac{\partial V}{\partial y} = \boxed{\dfrac{k_e p(2y^2-x^2)}{(x^2+y^2)^{5/2}}}$

25.70 (a) $\boxed{E_A > E_B \text{ since } E = \dfrac{\Delta V}{\Delta s}}$



(b) $E_B = -\dfrac{\Delta V}{\Delta s} = -\dfrac{(6-2)\text{ V}}{2\text{ cm}} = \boxed{200\text{ N/C}}$ down

(c) The figure is shown to the right, with sample field lines sketched in.

25.71 For an element of area which is a ring of radius r and width dr, $dV = \dfrac{k_e\,dq}{\sqrt{r^2+x^2}}$

$dq = \sigma\,dA = Cr(2\pi r\,dr)$ and

$$V = C(2\pi k_e)\int_0^R \frac{r^2\,dr}{\sqrt{r^2+x^2}} = \boxed{C(\pi k_e)\left[R\sqrt{R^2+x^2} + x^2\ln\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]}$$

25.72 $dU = V\,dq$ where the potential $V = \dfrac{k_e q}{r}$.

The element of charge in a shell is $dq = \rho$ (volume element) or $dq = \rho(4\pi r^2\,dr)$ and the charge q in a sphere of radius r is

$$q = 4\pi\rho\int_0^r r^2\,dr = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substituting this into the expression for dU, we have

$$dU = \left(\frac{k_e q}{r}\right)dq = k_e\rho\left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right)\rho(4\pi r^2\,dr) = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 r^4\,dr$$

$$U = \int dU = k_e\left(\frac{16\pi^2}{3}\right)\rho^2\int_0^R r^4\,dr = k_e\left(\frac{16\pi^2}{15}\right)\rho^2 R^5$$

But the *total* charge, $Q = \rho\frac{4}{3}\pi R^3$. Therefore, $\boxed{U = \dfrac{3}{5}\dfrac{k_e Q^2}{R}}$

***25.73** (a) From Problem 62,

$$E = \frac{\Delta V}{\ln(r_a/r_b)} \frac{1}{r}$$

We require just outside the central wire

$$5.50 \times 10^6 \ \frac{\text{V}}{\text{m}} = \frac{50.0 \times 10^3 \text{ V}}{\ln\left(\dfrac{0.850 \text{ m}}{r_b}\right)} \left(\frac{1}{r_b}\right)$$

or

$$\left(110 \text{ m}^{-1}\right) r_b \ln\left(\frac{0.850 \text{ m}}{r_b}\right) = 1$$

We solve by homing in on the required value

r_b (m)	0.0100	0.00100	0.00150	0.00145	0.00143	0.00142
$\left(110 \text{ m}^{-1}\right) r_b \ln\left(\dfrac{0.850 \text{ m}}{r_b}\right)$	4.89	0.740	1.05	1.017	1.005	0.999

Thus, to three significant figures, $\boxed{r_b = 1.42 \text{ mm}}$

(b) At r_a, $E = \dfrac{50.0 \text{ kV}}{\ln(0.850 \text{ m}/0.00142 \text{ m})} \left(\dfrac{1}{0.850 \text{ m}}\right) = \boxed{9.20 \text{ kV/m}}$

Chapter 26 Solutions

***26.1** (a) $Q = C(\Delta V) = (4.00 \times 10^{-6}\text{ F})(12.0\text{ V}) = 4.80 \times 10^{-5}\text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C(\Delta V) = (4.00 \times 10^{-6}\text{ F})(1.50\text{ V}) = 6.00 \times 10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

26.2 (a) $C = \dfrac{Q}{\Delta V} = \dfrac{10.0 \times 10^{-6}\text{ C}}{10.0\text{ V}} = 1.00 \times 10^{-6}\text{ F} = \boxed{1.00\ \mu\text{F}}$

(b) $\Delta V = \dfrac{Q}{C} = \dfrac{100 \times 10^{-6}\text{ C}}{1.00 \times 10^{-6}\text{ F}} = \boxed{100\text{ V}}$

26.3 $E = \dfrac{k_e q}{r^2}$: $\quad q = \dfrac{(4.90 \times 10^4\text{ N / C})(0.210\text{ m})^2}{8.99 \times 10^9\text{ N} \cdot \text{m}^2 / \text{C}^2} = 0.240\ \mu\text{C}$

(a) $\sigma = \dfrac{q}{A} = \dfrac{0.240 \times 10^{-6}}{4\pi(0.120)^2} = \boxed{1.33\ \mu\text{C} / \text{m}^2}$

(b) $C = 4\pi \text{e}_0 r = 4\pi(8.85 \times 10^{-12})(0.120) = \boxed{13.3\text{ pF}}$

26.4 (a) $C = 4\pi\,\text{e}_0 R$

$R = \dfrac{C}{4\pi\,\text{e}_0} = k_e C = (8.99 \times 10^9\text{ N} \cdot \text{m}^2 / \text{C}^2)(1.00 \times 10^{-12}\text{ F}) = \boxed{8.99\text{ mm}}$

(b) $C = 4\pi\,\text{e}_0 R = \dfrac{4\pi(8.85 \times 10^{-12}\text{ C}^2)(2.00 \times 10^{-3}\text{ m})}{\text{N} \cdot \text{m}^2} = \boxed{0.222\text{ pF}}$

(c) $Q = CV = (2.22 \times 10^{-13}\text{ F})(100\text{ V}) = \boxed{2.22 \times 10^{-11}\text{ C}}$

26.5 (a) $\dfrac{Q_1}{Q_2} = \dfrac{R_1}{R_2}$

$Q_1 + Q_2 = \left(1 + \dfrac{R_1}{R_2}\right) Q_2 = 3.50 Q_2 = 7.00\ \mu\text{C}$

$\boxed{Q_2 = 2.00\ \mu\text{C}}$ $\quad \boxed{Q_1 = 5.00\ \mu\text{C}}$

(b) $V_1 = V_2 = \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} = \dfrac{5.00\ \mu\text{C}}{\left(8.99 \times 10^9\text{ m/F}\right)^{-1}(0.500\text{ m})} = 8.99 \times 10^4\text{ V} = \boxed{89.9\text{ kV}}$

***26.6** $C = \dfrac{\kappa\,\epsilon_0 A}{d} = \dfrac{(1.00)(8.85 \times 10^{-12}\ \text{C}^2)(1.00 \times 10^3\ \text{m})^2}{\text{N} \cdot \text{m}^2(800\ \text{m})} = \boxed{11.1\ \text{nF}}$

The potential between ground and cloud is

$\Delta V = Ed = (3.00 \times 10^6\ \text{N/C})(800\ \text{m}) = 2.40 \times 10^9\ \text{V}$

$Q = C(\Delta V) = (11.1 \times 10^{-9}\ \text{C/V})(2.40 \times 10^9\ \text{V}) = \boxed{26.6\ \text{C}}$

26.7 (a) $\Delta V = Ed$

$E = \dfrac{20.0\ \text{V}}{1.80 \times 10^{-3}\ \text{m}} = \boxed{11.1\ \text{kV/m}}$

(b) $E = \dfrac{\sigma}{\epsilon_0}$

$\sigma = (1.11 \times 10^4\ \text{N/C})(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2) = \boxed{98.3\ \text{nC/m}^2}$

(c) $C = \dfrac{\epsilon_0 A}{d} = \dfrac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.60\ \text{cm}^2\right)\left(1.00\ \text{m}/100\ \text{cm}\right)^2}{1.80\times10^{-3}\ \text{m}} = \boxed{3.74\ \text{pF}}$

(d) $\Delta V = \dfrac{Q}{C}$ $\quad Q = (20.0\ \text{V})(3.74 \times 10^{-12}\ \text{F}) = \boxed{74.7\ \text{pC}}$

26.8 $C = \dfrac{\kappa\epsilon_0 A}{d} = 60.0\times10^{-15}\ \text{F}$

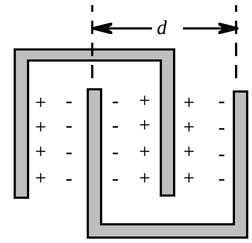
$d = \dfrac{\kappa\epsilon_0 A}{C} = \dfrac{(1)\left(8.85\times10^{-12}\right)\left(21.0\times10^{-12}\right)}{60.0\times10^{-15}}$

$d = 3.10\times10^{-9}\ \text{m} = \boxed{3.10\ \text{nm}}$

26.9 $Q = \dfrac{\epsilon_0 A}{d}(\Delta V)$ $\quad \dfrac{Q}{A} = \sigma = \dfrac{\epsilon_0(\Delta V)}{d}$

$d = \dfrac{\epsilon_0(\Delta V)}{\sigma} = \dfrac{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(150\ \text{V})}{\left(30.0 \times 10^{-9}\ \text{C/cm}^2\right)\left(1.00\times10^4\ \text{cm}^2/\text{m}^2\right)} = \boxed{4.42\ \mu\text{m}}$

26.10 With $\theta = \pi$, the plates are out of mesh and the overlap area is zero. With $\theta = 0$, the overlap area is that of a semi-circle, $\pi R^2/2$. By proportion, the effective area of a single sheet of charge is $(\pi - \theta)R^2/2$.



When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are N plates on each comb, the number of parallel capacitors is $2N - 1$ and the total capacitance is

$$C = (2N-1)\frac{\epsilon_0 A_{\text{effective}}}{\text{distance}} = \frac{(2N-1)\epsilon_0(\pi-\theta)R^2/2}{d/2} = \boxed{\frac{(2N-1)\,\epsilon_0\,(\pi-\theta)R^2}{d}}$$

26.11 (a) $$C = \frac{l}{2k_e \ln\left(\frac{b}{a}\right)} = \frac{50.0}{2(8.99\times 10^9)\ln\left(\frac{7.27}{2.58}\right)} = \boxed{2.68 \text{ nF}}$$

(b) Method 1: $\Delta V = 2k_e\,\lambda \ln\left(\frac{b}{a}\right)$

$$\lambda = q\ /\ l = \frac{8.10\times 10^{-6}\text{ C}}{50.0\text{ m}} = 1.62\times 10^{-7}\text{ C/m}$$

$$\Delta V = 2(8.99\times 10^9)(1.62\times 10^{-7})\ln\left(\frac{7.27}{2.58}\right) = \boxed{3.02 \text{ kV}}$$

Method 2: $$\Delta V = \frac{Q}{C} = \frac{8.10\times 10^{-6}}{2.68\times 10^{-9}} = \boxed{3.02 \text{ kV}}$$

26.12 Let the radii be b and a with $b = 2a$. Put charge Q on the inner conductor and $-Q$ on the outer. Electric field exists only in the volume between them. The potential of the inner sphere is $V_a = k_eQ/a$; that of the outer is $V_b = k_eQ/b$. Then

$$V_a - V_b = \frac{k_e Q}{a} - \frac{k_e Q}{b} = \frac{Q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right) \quad \text{and} \quad C = \frac{Q}{V_a - V_b} = \frac{4\pi\epsilon_0 ab}{b-a}$$

Here $C = \dfrac{4\pi\epsilon_0\, 2a^2}{a} = 8\pi\epsilon_0\, a \qquad a = \dfrac{C}{8\pi\epsilon_0}$

The intervening volume is $$\text{Volume} = \tfrac{4}{3}\pi b^3 - \tfrac{4}{3}\pi a^3 = 7\left(\tfrac{4}{3}\pi a^3\right) = 7\left(\tfrac{4}{3}\pi\right)\frac{C^3}{8^3\pi^3\epsilon_0^{\,3}} = \frac{7C^3}{384\pi^2\epsilon_0^{\,3}}$$

$$\text{Volume} = \frac{7(20.0\times 10^{-6}\text{ C}^2\ /\ \text{N}\cdot\text{m})^3}{384\,\pi^2(8.85\times 10^{-12}\text{ C}^2\ /\ \text{N}\cdot\text{m}^2)^3} = \boxed{2.13\times 10^{16}\text{ m}^3}$$

The outer sphere is 360 km in diameter.

26.13 $\Sigma F_y = 0$: $T\cos\theta - mg = 0$ $\Sigma F_x = 0$: $T\sin\theta - Eq = 0$

Dividing, $\tan\theta = \dfrac{Eq}{mg}$, so $E = \dfrac{mg}{q}\tan\theta$

$$\Delta V = Ed = \frac{mgd\tan\theta}{q} = \frac{(350\times10^{-6}\text{ kg})(9.80\text{ m / s}^2)(4.00\times10^{-2}\text{ m})\tan 15.0°}{30.0\times10^{-9}\text{ C}} = \boxed{1.23\text{ kV}}$$

26.14 $\Sigma F_y = 0$: $T\cos\theta - mg = 0$ $\Sigma F_x = 0$: $T\sin\theta - Eq = 0$

Dividing, $\tan\theta = \dfrac{Eq}{mg}$, so $E = \dfrac{mg}{q}\tan\theta$ and $\Delta V = Ed = \boxed{\dfrac{mgd\tan\theta}{q}}$

26.15 (a) $C = \dfrac{ab}{k_e(b-a)} = \dfrac{(0.0700)(0.140)}{(8.99\times10^9)(0.140-0.0700)} = \boxed{15.6\text{ pF}}$

(b) $C = \dfrac{Q}{\Delta V}$ $\Delta V = \dfrac{Q}{C} = \dfrac{4.00\times10^{-6}\text{ C}}{15.6\times10^{-12}\text{ F}} = \boxed{256\text{ kV}}$

Goal Solution

An air-filled spherical capacitor is constructed with inner and outer shell radii of 7.00 and 14.0 cm, respectively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres results in a charge of 4.00 μC on the capacitor?

G: Since the separation between the inner and outer shells is much larger than a typical electronic capacitor with $d \sim 0.1$ mm and capacitance in the microfarad range, we might expect the capacitance of this spherical configuration to be on the order of picofarads, (based on a factor of about 700 times larger spacing between the conductors). The potential difference should be sufficiently low to prevent sparking through the air that separates the shells.

O: The capacitance can be found from the equation for spherical shells, and the voltage can be found from $Q = C\Delta V$.

A: (a) For a spherical capacitor with inner radius a and outer radius b,

$$C = \frac{ab}{k(b-a)} = \frac{(0.0700\text{ m})(0.140\text{ m})}{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(0.140-0.0700)\text{ m}} = 1.56\times10^{-11}\text{ F} = 15.6\text{ pF}$$

(b) $$\Delta V = \frac{Q}{C} = \frac{(4.00\times10^{-6}\text{ C})}{1.56\times10^{-11}\text{ F}} = 2.56\times10^5\text{ V} = 256\text{ kV}$$

L: The capacitance agrees with our prediction, but the voltage seems rather high. We can check this voltage by approximating the configuration as the electric field between two charged parallel plates separated by $d = 7.00$ cm, so

$$E \sim \frac{\Delta V}{d} = \frac{2.56\times10^5\text{ V}}{0.0700\text{ m}} = 3.66\times10^6\text{ V / m}$$

This electric field barely exceeds the dielectric breakdown strength of air $\left(3\times10^6\text{ V / m}\right)$, so it may not even be possible to place $4.00\ \mu$C of charge on this capacitor!

26.16 $C = 4\pi \epsilon_0 R = 4\pi\left(8.85\times10^{-12}\ \text{C/N}\cdot\text{m}^2\right)\left(6.37\times10^{6}\ \text{m}\right) = \boxed{7.08\times10^{-4}\ \text{F}}$

***26.17** (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$\Delta V = \boxed{9.00\ \text{V}}$

(c) $Q_5 = C(\Delta V) = (5.00\ \mu\text{F})(9.00\ \text{V}) = \boxed{45.0\ \mu\text{C}}$ and $Q_{12} = C(\Delta V) = (12.0\ \mu\text{F})(9.00\ \text{V}) = \boxed{108\ \mu\text{C}}$

***26.18** (a) In series capacitors add as

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{12.0\ \mu\text{F}} \quad \text{and} \quad C_{eq} = \boxed{3.53\ \mu\text{F}}$$

(c) The charge on the equivalent capacitor is

$Q_{eq} = C_{eq}(\Delta V) = (3.53\ \mu\text{F})(9.00\ \text{V}) = 31.8\ \mu\text{C}$

Each of the series capacitors has this same charge on it. So $Q_1 = Q_2 = \boxed{31.8\ \mu\text{C}}$

(b) The voltage across each is

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}} = \boxed{6.35\ \text{V}} \quad \text{and} \quad \Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}} = \boxed{2.65\ \text{V}}$$

26.19 $C_p = C_1 + C_2$ $\quad \dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

Substitute $C_2 = C_p - C_1$ $\quad \dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_p - C_1} = \dfrac{C_p - C_1 + C_1}{C_1(C_p - C_1)}$

Simplifying, $C_1^2 - C_1C_p + C_pC_s = 0$

$$C_1 = \frac{C_p \pm \sqrt{C_p^{\,2} - 4C_pC_s}}{2} = \tfrac{1}{2}C_p \pm \sqrt{\tfrac{1}{4}C_p^{\,2} - C_pC_s}$$

We choose arbitrarily the + sign. (This choice can be arbitrary, since with the case of the minus sign, we would get the same two answers with their names interchanged.)

$$C_1 = \tfrac{1}{2}C_p + \sqrt{\tfrac{1}{4}C_p^{\,2} - C_pC_s} = \tfrac{1}{2}(9.00\ \text{pF}) + \sqrt{\tfrac{1}{4}(9.00\ \text{pF})^2 - (9.00\ \text{pF})(2.00\ \text{pF})} = \boxed{6.00\ \text{pF}}$$

$$C_2 = C_p - C_1 = \tfrac{1}{2}C_p - \sqrt{\tfrac{1}{4}C_p^{\,2} - C_pC_s} = \frac{1}{2}(9.00\ \text{pF}) - 1.50\ \text{pF} = \boxed{3.00\ \text{pF}}$$

26.20 $C_p = C_1 + C_2$ and $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

Substitute $C_2 = C_p - C_1$: $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_p - C_1} = \dfrac{C_p - C_1 + C_1}{C_1(C_p - C_1)}$

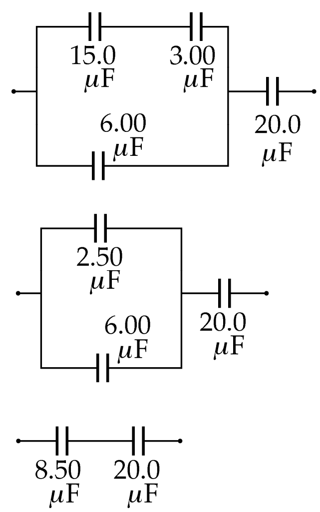
Simplifying, $C_1^2 - C_1C_p + C_pC_s = 0$

and $C_1 = \dfrac{C_p \pm \sqrt{C_p^2 - 4C_pC_s}}{2} = \boxed{\dfrac{1}{2}C_p + \sqrt{\dfrac{1}{4}C_p^2 - C_pC_s}}$

where the positive sign was arbitrarily chosen (choosing the negative sign gives the same values for the capacitances, with the names reversed). Then, from $C_2 = C_p - C_1$

$$C_2 = \boxed{\frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s}}$$

26.21 (a) $\dfrac{1}{C_s} = \dfrac{1}{15.0} + \dfrac{1}{3.00}$ $C_s = 2.50\ \mu\text{F}$



$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$

$C_{eq} = \left(\dfrac{1}{8.50\ \mu\text{F}} + \dfrac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$

(b) $Q = (\Delta V)C = (15.0\ \text{V})(5.96\ \mu\text{F}) = \boxed{89.5\ \mu\text{C}}$ on $20.0\ \mu\text{F}$

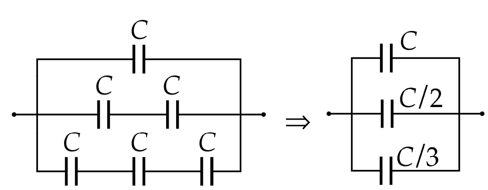
$\Delta V = \dfrac{Q}{C} = \dfrac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$

$15.0 - 4.47 = 10.53\ \text{V}$

$Q = (\Delta V)C = (10.53)(6.00\ \mu\text{F}) = \boxed{63.2\ \mu\text{C}}$ on $6.00\ \mu\text{F}$

$89.5 - 63.2 = \boxed{26.3\ \mu\text{C}}$ on $15.0\ \mu\text{F}$ and $3.00\ \mu\text{F}$

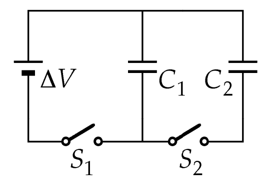
26.22 The circuit reduces first according to the rule for capacitors in series, as shown in the figure, then according to the rule for capacitors in parallel, shown below.



$C_{\text{eq}} = C\left(1 + \dfrac{1}{2} + \dfrac{1}{3}\right) = \dfrac{11}{6}C = \boxed{1.83C}$

26.23 $C = \dfrac{Q}{\Delta V}$ so $6.00 \times 10^{-6} = \dfrac{Q}{20.0}$ and $Q = \boxed{120\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - Q_2$ and $\Delta V = \dfrac{Q}{C}$



$\dfrac{120 - Q_2}{C_1} = \dfrac{Q_2}{C_2}$ or $\dfrac{120 - Q_2}{6.00} = \dfrac{Q_2}{3.00}$

$(3.00)(120 - Q_2) = (6.00)Q_2$

$Q_2 = \dfrac{360}{9.00} = \boxed{40.0\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - 40.0\ \mu\text{C} = \boxed{80.0\ \mu\text{C}}$

***26.24** (a) In $\boxed{\text{series}}$, to reduce the effective capacitance:

$$\frac{1}{32.0\ \mu\text{F}} = \frac{1}{34.8\ \mu\text{F}} + \frac{1}{C_s}$$

$$C_s = \frac{1}{2.51 \times 10^{-3}/\mu\text{F}} = \boxed{398\ \mu\text{F}}$$

(b) In $\boxed{\text{parallel}}$, to increase the total capacitance:

$$29.8\ \mu\text{F} + C_p = 32.0\ \mu\text{F}$$

$$C_p = \boxed{2.20\ \mu\text{F}}$$

26.25 With switch closed, distance $d' = 0.500d$ and capacitance $C' = \dfrac{\epsilon_0 A}{d'} = \dfrac{2\epsilon_0 A}{d} = 2C$

(a) $Q = C'(\Delta V) = 2C(\Delta V) = 2(2.00 \times 10^{-6}\ \text{F})(100\ \text{V}) = \boxed{400\ \mu\text{C}}$

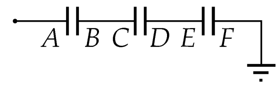
(b) The force stretching out one spring is

$$\mathbf{F} = \frac{Q^2}{2\epsilon_0 A} = \frac{4C^2(\Delta V)^2}{2\epsilon_0 A} = \frac{2C^2(\Delta V)^2}{(\epsilon_0 A/d)d} = \frac{2C(\Delta V)^2}{d}$$

One spring stretches by distance $x = d/4$, so

$$k = \frac{F}{x} = \frac{2C(\Delta V)^2}{d}\left(\frac{4}{d}\right) = \frac{8C(\Delta V)^2}{d^2} = \frac{8(2.00 \times 10^{-6}\ \text{F})(100\ \text{V})^2}{(8.00 \times 10^{-3}\ \text{m})^2} = \boxed{2.50\ \text{kN/m}}$$

26.26 Positive charge on A will induce equal negative charges on B, D, and F, and equal positive charges on C and E. The nesting spheres form three capacitors in series. From Example 26.3,



$$C_{AB} = \frac{ab}{k_e(b-a)} = \frac{R(2R)}{k_e R} = \frac{2R}{k_e}$$

$$C_{CD} = \frac{(3R)(4R)}{k_e R} = \frac{12R}{k_e}$$

$$C_{EF} = \frac{(5R)(6R)}{k_e R} = \frac{30R}{k_e}$$

$$C_{\text{eq}} = \frac{1}{k_e/2R + k_e/12R + k_e/30R} = \boxed{\frac{60R}{37k_e}}$$

26.27 $$nC = \frac{100}{\underbrace{\frac{1}{C}+\frac{1}{C}+\frac{1}{C}+\ . \ . \ .}_{\text{n capacitors}}} = \frac{100}{n/C}$$

$nC = \dfrac{100C}{n}$ so $n^2 = 100$ and $n = \boxed{10}$

Goal Solution

A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 100 times larger than for the series connection. How many capacitors are in the group?

G: Since capacitors in parallel add and ones in series add as inverses, 2 capacitors in parallel would have a capacitance 4 times greater than if they were in series, and 3 capacitors would give a ratio $C_p / C_s = 9$, so maybe $n = \sqrt{C_p / C_s} = \sqrt{100} = 10$.

O: The ratio reasoning above seems like an efficient way to solve this problem, but we should check the answer with a more careful analysis based on the general relationships for series and parallel combinations of capacitors.

A: Call C the capacitance of one capacitor and n the number of capacitors. The equivalent capacitance for n capacitors in parallel is

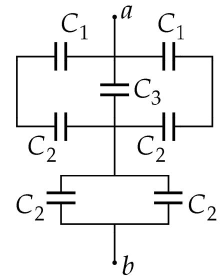
$$C_p = C_1 + C_2 + \ . \ . \ . + C_n = nC$$

The relationship for n capacitors in series is $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ . \ . \ . + \dfrac{1}{C_n} = \dfrac{n}{C}$

Therefore $\dfrac{C_p}{C_s} = \dfrac{nC}{C/n} = n^2$ or $n = \sqrt{\dfrac{C_p}{C_s}} = \sqrt{100} = 10$

L: Our prediction appears to be correct. A qualitative reason that $C_p / C_s = n^2$ is because the amount of charge that can be stored on the capacitors increases according to the area of the plates for a parallel combination, but the total charge remains the same for a series combination.

26.28 $C_s = \left(\frac{1}{5.00} + \frac{1}{10.0}\right)^{-1} = 3.33\ \mu\text{F}$



$C_{p1} = 2(3.33) + 2.00 = 8.66\ \mu\text{F}$

$C_{p2} = 2(10.0) = 20.0\ \mu\text{F}$

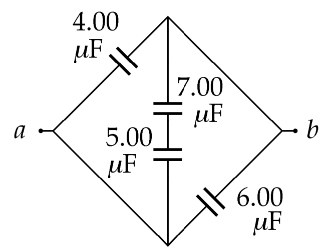
$C_{\text{eq}} = \left(\frac{1}{8.66} + \frac{1}{20.0}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$

26.29 $Q_{\text{eq}} = C_{\text{eq}}(\Delta V) = \left(6.04 \times 10^{-6}\ \text{F}\right)(60.0\ \text{V}) = 3.62 \times 10^{-4}\ \text{C}$

$Q_{p1} = Q_{\text{eq}}$, so $\Delta V_{p1} = \frac{Q_{\text{eq}}}{C_{p1}} = \frac{3.62 \times 10^{-4}\ \text{C}}{8.66 \times 10^{-6}\ \text{F}} = 41.8\ \text{V}$

$Q_3 = C_3\left(\Delta V_{p1}\right) = \left(2.00 \times 10^{-6}\ \text{F}\right)(41.8\ \text{V}) = \boxed{83.6\ \mu\text{C}}$

26.30 $C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$



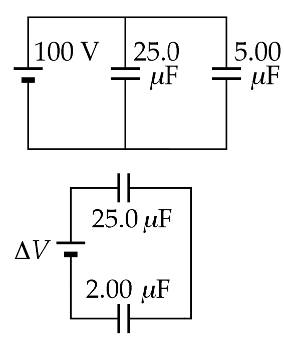
$C_p = 2.92 + 4.00 + 6.00 = \boxed{12.9\ \mu\text{F}}$

***26.31** (a) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(12.0\ \text{V})^2 = \boxed{216\ \mu\text{J}}$

(b) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{54.0\ \mu\text{J}}$

***26.32** $U = \frac{1}{2}C(\Delta V)^2$

The circuit diagram is shown at the right.



(a) $C_p = C_1 + C_2 = 25.0\ \mu\text{F} + 5.00\ \mu\text{F} = 30.0\ \mu\text{F}$

$U = \frac{1}{2}(30.0 \times 10^{-6})(100)^2 = \boxed{0.150\ \text{J}}$

(b) $C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu\text{F}} + \frac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$

$U = \frac{1}{2}C(\Delta V)^2$

$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{(0.150)(2)}{4.17 \times 10^{-6}}} = \boxed{268\ \text{V}}$

***26.33** Use $U = \frac{1}{2}\frac{Q^2}{C}$ and $C = \frac{\epsilon_0 A}{d}$

If $d_2 = 2d_1$, $C_2 = \frac{1}{2} C_1$. Therefore, the $\boxed{\text{stored energy doubles}}$.

26.34 $u = \frac{U}{V} = \frac{1}{2}\epsilon_0 E^2$

$$\frac{1.00 \times 10^{-7}}{V} = \frac{1}{2}(8.85 \times 10^{-12})(3000)^2$$

$$V = \boxed{2.51 \times 10^{-3}\ \text{m}^3} = \left(2.51 \times 10^{-3}\ \text{m}^3\right)\left(\frac{1000\ \text{L}}{\text{m}^3}\right) = \boxed{2.51\ \text{L}}$$

26.35 $W = U = \int F\,dx$ so $F = \frac{dU}{dx} = \frac{d}{dx}\left(\frac{Q^2}{2c}\right) = \frac{d}{dx}\left(\frac{Q^2 x}{2\epsilon_0 A}\right) = \boxed{\frac{Q^2}{2\epsilon_0 A}}$

26.36 Plate a experiences force $-kx\mathbf{i}$ from the spring and force $QE\mathbf{i}$ due to the electric field created by plate b according to $E = \sigma / 2\epsilon_0 = Q / 2A\epsilon_0$. Then,

$$kx = \frac{Q^2}{2A\epsilon_0} \qquad x = \boxed{\frac{Q^2}{2A\epsilon_0 k}}$$

where A is the area of one plate.

26.37 The energy transferred is $W = \frac{1}{2}Q(\Delta V) = \frac{1}{2}(50.0\ \text{C})(1.00 \times 10^8\ \text{V}) = 2.50 \times 10^9\ \text{J}$ and 1% of this (or $W' = 2.50 \times 10^7$ J) is absorbed by the tree. If m is the amount of water boiled away, then

$$W' = m(4186\ \text{J/kg °C})(100\ \text{°C} - 30.0\ \text{°C}) + m(2.26 \times 10^6\ \text{J/kg}) = 2.50 \times 10^7\ \text{J}$$

giving $\boxed{m = 9.79\ \text{kg}}$

26.38 $U = \frac{1}{2}C(\Delta V)^2$ where $C = 4\pi \epsilon_0 R = \frac{R}{k_e}$ and $\Delta V = \frac{k_e Q}{R} - 0 = \frac{k_e Q}{R}$

$$U = \frac{1}{2}\left(\frac{R}{k_e}\right)\left(\frac{k_e Q}{R}\right)^2 = \boxed{\frac{k_e Q^2}{2R}}$$

26.39 $\frac{k_e Q^2}{2R} = mc^2$

$$R = \frac{k_e e^2}{2mc^2} = \frac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C})(1.60\times 10^{-19}\text{ C})^2}{2(9.11\times 10^{-31}\text{ kg})(3.00\times 10^8\text{ m/s})^2} = \boxed{1.40 \text{ fm}}$$

***26.40** $C = \frac{\kappa \epsilon_0 A}{d} = \frac{4.90(8.85\times 10^{-12}\text{ F/m})(5.00\times 10^{-4}\text{ m}^2)}{2.00\times 10^{-3}\text{ m}} = 1.08\times 10^{-11}\text{ F} = \boxed{10.8 \text{ pF}}$

***26.41** (a) $C = \frac{\kappa \epsilon_0 A}{d} = \frac{2.10(8.85\times 10^{-12}\text{ F/m})(1.75\times 10^{-4}\text{ m}^2)}{4.00\times 10^{-5}\text{ m}} = 8.13\times 10^{-11}\text{ F} = \boxed{81.3 \text{ pF}}$

(b) $\Delta V_{\max} = E_{\max} d = (60.0\times 10^6\text{ V/m})(4.00\times 10^{-5}\text{ m}) = \boxed{2.40 \text{ kV}}$

***26.42** $Q_{\max} = C(\Delta V_{\max})$, but $\Delta V_{\max} = E_{\max} d$

Also, $C = \frac{\kappa \epsilon_0 A}{d}$

Thus, $Q_{\max} = \frac{\kappa \epsilon_0 A}{d}(E_{\max} d) = \kappa \epsilon_0 A E_{\max}$

(a) With air between the plates, $\kappa = 1.00$ and $E_{\max} = 3.00\times 10^6$ V/m. Therefore,

$Q_{\max} = \kappa \epsilon_0 A E_{\max} = (8.85\times 10^{-12}\text{ F/m})(5.00\times 10^{-4}\text{ m}^2)(3.00\times 10^6\text{ V/m}) = \boxed{13.3 \text{ nC}}$

(b) With polystyrene between the plates, $\kappa = 2.56$ and $E_{\max} = 24.0\times 10^6$ V/m.

$Q_{\max} = \kappa \epsilon_0 A E_{\max} = 2.56(8.85\times 10^{-12}\text{ F/m})(5.00\times 10^{-4}\text{ m}^2)(24.0\times 10^6\text{ V/m}) = \boxed{272 \text{ nC}}$

26.43 $C = \frac{\kappa \epsilon_0 A}{d}$ or $95.0\times 10^{-9} = \frac{3.70(8.85\times 10^{-12})(0.0700)\ell}{(0.0250\times 10^{-3})}$

= 1.04 m

***26.44** Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them. Suppose the plastic has $\kappa \cong 3$, $E_{\max} \sim 10^7$ V/m and thickness 1 mil = 2.54 cm/1000. Then,

$$C = \frac{\kappa \epsilon_0 A}{d} \sim \frac{3(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(0.4\ \mathrm{m^2})}{2.54 \times 10^{-5}\ \mathrm{m}} \sim \boxed{10^{-6}\ \mathrm{F}}$$

$$\Delta V_{\max} = E_{\max} d \sim \left(10^7\ \frac{\mathrm{V}}{\mathrm{m}}\right)\left(2.54 \times 10^{-5}\ \mathrm{m}\right) \sim \boxed{10^2\ \mathrm{V}}$$

***26.45** (a) With air between theplates, we find $C_0 = \dfrac{Q}{\Delta V} = \dfrac{48.0\ \mu\mathrm{C}}{12.0\ \mathrm{V}} = \boxed{4.00\ \mu\mathrm{F}}$

(b) When Teflon is inserted, the charge remains the same (48.0 μC) because the plates are isolated. However, the capacitance, and hence the voltage, changes. The new capacitance is

$$C' = \kappa C_0 = 2.10(4.00\ \mu\mathrm{F}) = \boxed{8.40\ \mu\mathrm{F}}$$

(c) The voltage on the capacitor now is $\Delta V' = \dfrac{Q}{C'} = \dfrac{48.0\ \mu\mathrm{C}}{8.40\ \mu\mathrm{F}} = \boxed{5.71\ \mathrm{V}}$

and the charge is $\boxed{48.0\ \mu\mathrm{C}}$

26.46 Originally, $C = \epsilon_0 A / d = Q / (\Delta V)_i$

(a) The charge is the same before and after immersion, with value $Q = \epsilon_0 A (\Delta V)_i / d$.

$$Q = \frac{(8.85 \times 10^{-12}\ \mathrm{C^2})(25.0 \times 10^{-4}\ \mathrm{m^2})(250\ \mathrm{V})}{\mathrm{N \cdot m^2}\ (1.50 \times 10^{-2}\ \mathrm{m})} = \boxed{369\ \mathrm{pC}}$$

(b) Finally, $C_f = \kappa \epsilon_0 A / d = Q / (\Delta V)_f$

$$C_f = \frac{80.0(8.85 \times 10^{-12}\ \mathrm{C^2})(25.0 \times 10^{-4}\ \mathrm{m^2})}{\mathrm{N \cdot m^2}\ (1.50 \times 10^{-2}\ \mathrm{m})} = \boxed{118\ \mathrm{pF}}$$

$$(\Delta V)_f = \frac{Qd}{\kappa \epsilon_0 A} = \frac{\epsilon_0 A (\Delta V)_i d}{\kappa \epsilon_0 A d} = \frac{(\Delta V)_i}{\kappa} = \frac{250\ \mathrm{V}}{80.0} = \boxed{3.12\ \mathrm{V}}$$

(c) Originally, $U = \frac{1}{2} C (\Delta V)_i^2 = \dfrac{\epsilon_0 A (\Delta V)_i^2}{2d}$

Finally, $U_f = \frac{1}{2} C_f (\Delta V)_f^2 = \dfrac{\kappa \epsilon_0 A (\Delta V)_i^2}{2d\ \kappa^2} = \dfrac{\epsilon_0 A (\Delta V)_i^2}{2d\kappa}$

So, $\Delta U = U_f - U = -\dfrac{\epsilon_0 A (\Delta V)_i^2 (\kappa - 1)}{2d\ \kappa}$

$$\Delta U = \frac{(-8.85 \times 10^{-12}\ \mathrm{C^2})(25.0 \times 10^{-4}\ \mathrm{m^2})(250\ \mathrm{V})^2 (79.0)}{\mathrm{N \cdot m^2}\ 2(1.50 \times 10^{-2}\ \mathrm{m})80} = \boxed{-45.5\ \mathrm{nJ}}$$

26.47 $$\frac{1}{C} = \frac{1}{\left(\dfrac{\kappa_1 ab}{k_e(b-a)}\right)} + \frac{1}{\left(\dfrac{\kappa_2 bc}{k_e(c-b)}\right)} = \frac{k_e(b-a)}{\kappa_1 ab} + \frac{k_e(c-b)}{\kappa_2 bc}$$

$$C = \frac{1}{\dfrac{k_e(b-a)}{\kappa_1 ab} + \dfrac{k_e(c-b)}{\kappa_2 bc}} = \frac{\kappa_1\kappa_2\, abc}{k_e\,\kappa_2(bc-ac) + k_e\,\kappa_1(ac-ab)} = \boxed{\frac{4\pi\,\kappa_1\kappa_2\, abc\epsilon_0}{\kappa_2 bc - \kappa_1 ab + (\kappa_1 - \kappa_2)ac}}$$

26.48 (a) $$C = \kappa C_0 = \frac{\kappa\epsilon_0 A}{d} = \frac{(173)(8.85\times10^{-12})(1.00\times10^{-4}\ \text{m}^2)}{0.100\times10^{-3}\ \text{m}} = \boxed{1.53\ \text{nF}}$$

(b) The battery delivers the free charge

$$Q = C(\Delta V) = (1.53\times10^{-9}\ \text{F})(12.0\ \text{V}) = \boxed{18.4\ \text{nC}}$$

(c) The surface density of free charge is

$$\sigma = \frac{Q}{A} = \frac{18.4\times10^{-9}\ \text{C}}{1.00\times10^{-4}\ \text{m}^2} = \boxed{1.84\times10^{-4}\ \text{C/m}^2}$$

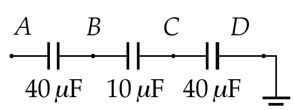
The surface density of polarization charge is

$$\sigma_p = \sigma\left(1-\frac{1}{\kappa}\right) = \sigma\left(1-\frac{1}{173}\right) = \boxed{1.83\times10^{-4}\ \text{C/m}^2}$$

(d) We have $E = E_0/\kappa$ and $E_0 = \Delta V/d$; hence,

$$E = \frac{\Delta V}{\kappa d} = \frac{12.0\ \text{V}}{(173)(1.00\times10^{-4}\ \text{m})} = \boxed{694\ \text{V/m}}$$

26.49 The given combination of capacitors is equivalent to the circuit diagram shown to the right.



Put charge Q on point A. Then,

$$Q = (40.0\ \mu\text{F})\Delta V_{AB} = (10.0\ \mu\text{F})\Delta V_{BC} = (40.0\ \mu\text{F})\Delta V_{CD}$$

So, $\Delta V_{BC} = 4\Delta V_{AB} = 4\Delta V_{CD}$, and the center capacitor will break down first, at $\Delta V_{BC} = 15.0$ V. When this occurs,

$$\Delta V_{AB} = \Delta V_{CD} = \tfrac{1}{4}(\Delta V_{BC}) = 3.75\ \text{V}$$

and $V_{AD} = V_{AB} + V_{BC} + V_{CD} = 3.75\ \text{V} + 15.0\ \text{V} + 3.75\ \text{V} = \boxed{22.5\ \text{V}}$

***26.50** (a) The displacement from negative to positive charge is

$$2\mathbf{a} = (-1.20\,\mathbf{i} + 1.10\,\mathbf{j})\,\text{mm} - (1.40\,\mathbf{i} - 1.30\,\mathbf{j})\,\text{mm} = (-2.60\,\mathbf{i} + 2.40\,\mathbf{j}) \times 10^{-3}\ \text{m}$$

The electric dipole moment is

$$\mathbf{p} = 2\mathbf{a}q = (3.50 \times 10^{-9}\ \text{C})(-2.60\,\mathbf{i} + 2.40\,\mathbf{j}) \times 10^{-3}\ \text{m} = \boxed{(-9.10\,\mathbf{i} + 8.40\,\mathbf{j}) \times 10^{-12}\ \text{C} \cdot \text{m}}$$

(b) $$\tau = \mathbf{p} \times \mathbf{E} = \left[(-9.10\,\mathbf{i} + 8.40\,\mathbf{j}) \times 10^{-12}\ \text{C} \cdot \text{m}\right] \times \left[(7.80\,\mathbf{i} - 4.90\,\mathbf{j}) \times 10^{3}\ \text{N/C}\right]$$

$$\tau = (+44.6\,\mathbf{k} - 65.5\,\mathbf{k}) \times 10^{-9}\ \text{N} \cdot \text{m} = \boxed{-2.09 \times 10^{-8}\ \text{N} \cdot \text{m}\,\mathbf{k}}$$

(c) $$U = -\mathbf{p} \cdot \mathbf{E} = -\left[(-9.10\,\mathbf{i} + 8.40\,\mathbf{j}) \times 10^{-12}\ \text{C} \cdot \text{m}\right] \cdot \left[(7.80\,\mathbf{i} - 4.90\,\mathbf{j}) \times 10^{3}\ \text{N/C}\right]$$

$$U = (71.0 + 41.2) \times 10^{-9}\ \text{J} = \boxed{112\ \text{nJ}}$$

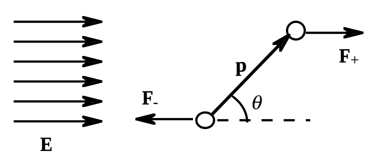
(d) $$|\mathbf{p}| = \sqrt{(9.10)^2 + (8.40)^2} \times 10^{-12}\ \text{C} \cdot \text{m} = 12.4 \times 10^{-12}\ \text{C} \cdot \text{m}$$

$$|\mathbf{E}| = \sqrt{(7.80)^2 + (4.90)^2} \times 10^{3}\ \text{N C} = 9.21 \times 10^{3}\ \text{N/C}$$

$$U_{\max} = |\mathbf{p}|\,|\mathbf{E}| = 114\ \text{nJ}, \qquad U_{\min} = -114\ \text{nJ}$$

$$U_{\max} - U_{\min} = \boxed{228\ \text{nJ}}$$

***26.51** (a) Let x represent the coordinate of the negative charge. Then $x + 2a\cos\theta$ is the coordinate of the positive charge. The force on the negative charge is $\mathbf{F}_{-} = -qE(x)\mathbf{i}$. The force on the positive charge is



$$\mathbf{F}_{+} = +qE(x + 2a\cos\theta)\mathbf{i} \cong qE(x)\mathbf{i} + q\frac{dE}{dx}(2a\cos\theta)\mathbf{i}$$

The force on the dipole is altogether $$\mathbf{F} = \mathbf{F}_{-} + \mathbf{F}_{+} = q\frac{dE}{dx}(2a\cos\theta)\mathbf{i} = \boxed{p\frac{dE}{dx}\cos\theta\,\mathbf{i}}$$

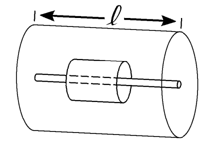
(b) The balloon creates field along the $x-$axis of $\dfrac{k_e q}{x^2}\mathbf{i}$.

Thus, $$\frac{dE}{dx} = \frac{(-2)k_e q}{x^3}$$

At $x = 16.0$ cm, $$\frac{dE}{dx} = \frac{(-2)(8.99 \times 10^9)(2.00 \times 10^{-6})}{(0.160)^3} = \boxed{-8.78\ \frac{\text{MN}}{\text{C} \cdot \text{m}}}$$

$$\mathbf{F} = (6.30 \times 10^{-9}\ \text{C} \cdot \text{m})\left(-8.78 \times 10^{6}\ \frac{\text{N}}{\text{C} \cdot \text{m}}\right)\cos 0^\circ\ \mathbf{i} = \boxed{-55.3\,\mathbf{i}\ \text{mN}}$$

26.52 $2\pi r \ell E = \dfrac{q_{\text{in}}}{\epsilon_0}$ so $E = \dfrac{\lambda}{2\pi r \epsilon_0}$



$$\Delta V = -\int_{r_1}^{r_2} \mathbf{E} \cdot d\mathbf{r} = \int_{r_1}^{r_2} \frac{\lambda}{2\pi r \epsilon_0} dr = \frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{r_1}{r_2}\right)$$

$$\frac{\lambda_{\max}}{2\pi\epsilon_0} = E_{\max} r_{\text{inner}}$$

$$\Delta V = \left(1.20 \times 10^6 \ \frac{\text{V}}{\text{m}}\right)\left(0.100 \times 10^{-3} \ \text{m}\right)\left(\ln \frac{25.0}{0.200}\right) \qquad \Delta V_{\max} = \boxed{579 \text{ V}}$$

***26.53** (a) Consider a gaussian surface in the form of a cylindrical pillbox with ends of area $A' << A$ parallel to the sheet. The side wall of the cylinder passes no flux of electric field since this surface is everywhere parallel to the field. Gauss's law becomes

$$EA' + EA' = \frac{Q}{\in A} A', \quad \text{so} \quad \boxed{E = \frac{Q}{2 \in A}} \quad \text{directed away from the positive sheet.}$$

(b) In the space between the sheets, each creates field $Q/2 \in A$ away from the positive and toward the negative sheet. Together, they create a field of

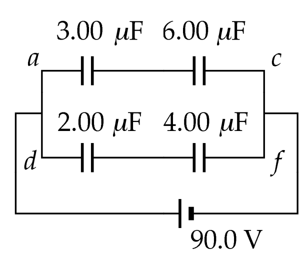
$$\boxed{E = \frac{Q}{\in A}}$$

(c) Assume that the field is in the positive $x-$direction. Then, the potential of the positive plate relative to the negative plate is

$$\Delta V = -\int_{-\text{plate}}^{+\text{plate}} \mathbf{E} \cdot d\mathbf{s} = -\int_{-\text{plate}}^{+\text{plate}} \frac{Q}{\in A} \mathbf{i} \cdot (-\mathbf{i}\, dx) = \boxed{+\frac{Qd}{\in A}}$$

(d) Capacitance is defined by: $C = \dfrac{Q}{\Delta V} = \dfrac{Q}{Qd/\in A} = \boxed{\dfrac{\in A}{d} = \dfrac{\kappa \epsilon_0 A}{d}}$

26.54 (a) $C = \left[\dfrac{1}{3.00} + \dfrac{1}{6.00}\right]^{-1} + \left[\dfrac{1}{2.00} + \dfrac{1}{4.00}\right]^{-1} = \boxed{3.33 \ \mu\text{F}}$



(c) $Q_{ac} = C_{ac}(\Delta V_{ac}) = (2.00 \ \mu\text{F})(90.0 \text{ V}) = 180 \ \mu\text{C}$

Therefore, $Q_3 = Q_6 = \boxed{180 \ \mu\text{C}}$

$$Q_{df} = C_{df}\left(\Delta V_{df}\right) = (1.33 \ \mu F)(90.0 \text{ V}) = \boxed{120 \ \mu\text{C}}$$

(b) $$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$$\Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

(d) $$U_T = \frac{1}{2} C_{\text{eq}} (\Delta V)^2 = \frac{1}{2}(3.33 \times 10^{-6})(90.0\ \text{V})^2 = \boxed{13.4\ \text{mJ}}$$

***26.55** The electric field due to the charge on the positive wire is perpendicular to the wire, radial, and of magnitude

$$E_+ = \frac{\lambda}{2\pi \in_0 r}$$

The potential difference between wires due to the presence of this charge is

$$\Delta V_1 = -\int_{-\text{wire}}^{+\text{wire}} \mathbf{E} \cdot d\mathbf{r} = -\frac{\lambda}{2\pi \in_0} \int_{D-d}^{d} \frac{dr}{r} = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{D-d}{d}\right)$$

The presence of the linear charge density $-\lambda$ on the negative wire makes an identical contribution to the potential difference between the wires. Therefore, the total potential difference is

$$\Delta V = 2(\Delta V_1) = \frac{\lambda}{\pi \in_0} \ln\left(\frac{D-d}{d}\right)$$

and the capacitance of this system of two wires, each of length ℓ, is

$$C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{\Delta V} = \frac{\lambda \ell}{\left(\frac{\lambda}{\pi \in_0}\right) \ln\left(\frac{D-d}{d}\right)} = \frac{\pi \in_0 \ell}{\ln\left(\frac{D-d}{d}\right)}$$

The capacitance per unit length is: $$\boxed{\frac{C}{\ell} = \frac{\pi \in_0}{\ln\left(\frac{D-d}{d}\right)}}$$

26.56 (a) We use Equation 26.11 to find the potential energy. As we will see, the potential difference ΔV changes as the dielectric is withdrawn. The initial and final energies are

$$U_i = \frac{1}{2}\left(\frac{Q^2}{C_i}\right) \quad \text{and} \quad U_f = \frac{1}{2}\left(\frac{Q^2}{C_f}\right)$$

But the initial capacitance (with the dielectric) is $C_i = \kappa C_f$. Therefore, $U_f = \frac{1}{2}\,\kappa\left(\frac{Q^2}{C_i}\right)$

Since the work done by the external force in removing the dielectric equals the change in potential energy, we have

$$W = U_f - U_i = \frac{1}{2}\,\kappa\left(\frac{Q^2}{C_i}\right) - \frac{1}{2}\left(\frac{Q^2}{C_i}\right) = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)(\kappa - 1)$$

To express this relation in terms of potential difference ΔV_i, we substitute $Q = C_i\,(\Delta V_i)$, and evaluate:

$$W = \frac{1}{2}\,C_i\,(\Delta V_i)^2(\kappa - 1) = \frac{1}{2}(2.00 \times 10^{-9}\text{ F})(100\text{ V})^2(5.00 - 1.00) = \boxed{4.00 \times 10^{-5}\text{ J}}$$

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \dfrac{Q}{C_f}$

Substituting $C_f = \dfrac{C_i}{\kappa}$ and $Q = C_i\,(\Delta V_i)$ gives $\Delta V_f = \kappa\,\Delta V_i = (5.00)(100\text{ V}) = \boxed{500\text{ V}}$

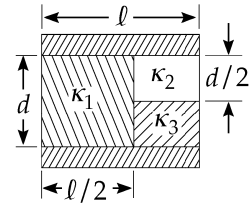
Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

26.57 $\kappa = 3.00$, $E_{\max} = 2.00 \times 10^8\text{ V/m} = \Delta V_{\max}/d$

For $C = \dfrac{\kappa\, e_0\, A}{d} = 0.250 \times 10^{-6}\text{ F}$,

$$A = \frac{Cd}{\kappa\, e_0} = \frac{C(\Delta V_{\max})}{\kappa\, e_0 E_{\max}} = \frac{(0.250 \times 10^{-6})(4000)}{(3.00)(8.85 \times 10^{-12})(2.00 \times 10^8)} = \boxed{0.188\text{ m}^2}$$

26.58 (a) $C_1 = \dfrac{\kappa_1\, e_0\, A/2}{d}$; $C_2 = \dfrac{\kappa_2\, e_0\, A/2}{d/2}$; $C_3 = \dfrac{\kappa_3\, e_0\, A/2}{d/2}$

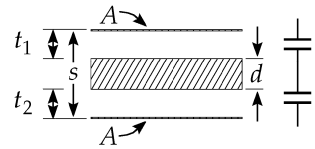


$$\left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \frac{C_2 C_3}{C_2 + C_3} = \frac{e_0\, A}{d}\left(\frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right)$$

$$C = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \boxed{\frac{e_0\, A}{d}\left(\frac{\kappa_1}{2} + \frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right)}$$

(b) Using the given values we find: $C_{\text{total}} = 1.76 \times 10^{-12}\ \text{F} = \boxed{1.76\ \text{pF}}$

26.59 The system may be considered to be two capacitors in series:



$$C_1 = \frac{\epsilon_0 A}{t_1} \quad \text{and} \quad C_2 = \frac{\epsilon_0 A}{t_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{t_1 + t_2}{\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{t_1 + t_2} = \boxed{\frac{\epsilon_0 A}{s - d}}$$

Goal Solution

A conducting slab of a thickness d and area A is inserted into the space between the plates of a parallel-plate capacitor with spacing s and surface area A, as shown in Figure P26.59. The slab is not necessarily halfway between the capacitor plates. What is the capacitance of the system?

G: It is difficult to predict an exact relationship for the capacitance of this system, but we can reason that C should increase if the distance between the slab and plates were decreased (until they touched and formed a short circuit). So maybe $C \propto 1/(s-d)$. Moving the metal slab does not change the amount of charge the system can store, so the capacitance should therefore be independent of the slab position. The slab must have zero net charge, with each face of the plate holding the same magnitude of charge as the outside plates, regardless of where the slab is between the plates.

O: If the capacitor is charged with $+Q$ on the top plate and $-Q$ on the bottom plate, then free charges will move across the conducting slab to neutralize the electric field inside it, with the top face of the slab carrying charge $-Q$ and the bottom face carrying charge $+Q$. Then the capacitor and slab combination is electrically equivalent to two capacitors in series. (We are neglecting the slight fringing effect of the electric field near the edges of the capacitor.) Call x the upper gap, so that $s-d-x$ is the distance between the lower two surfaces.

A: For the upper capacitor, $C_1 = \epsilon_0 A/x$

and the lower has $C_2 = \dfrac{\epsilon_0 A}{s-d-x}$

So the combination has
$$C = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}} = \frac{1}{\dfrac{x}{\epsilon_0 A} + \dfrac{s-d-x}{\epsilon_0 A}} = \frac{\epsilon_0 A}{s-d}$$

L: The equivalent capacitance is inversely proportional to $(s-d)$ as expected, and is also proportional to A. This result is the same as for the special case in Example 26.9 when the slab is just halfway between the plates; the only critical factor is the thickness of the slab relative to the plate spacing.

26.60 (a) Put charge Q on the sphere of radius a and $-Q$ on the other sphere. Relative to $V = 0$ at infinity,

the potential at the surface of a is $\qquad V_a = \dfrac{k_e Q}{a} - \dfrac{k_e Q}{d}$

and the potential of b is $\qquad V_b = \dfrac{-k_e Q}{b} + \dfrac{k_e Q}{d}$

The difference in potential is $\qquad V_a - V_b = \dfrac{k_e Q}{a} + \dfrac{k_e Q}{b} - \dfrac{k_e Q}{d} - \dfrac{k_e Q}{d}$

and $\qquad C = \dfrac{Q}{V_a - V_b} = \boxed{\left(\dfrac{4\pi\epsilon_0}{(1/a)+(1/b)-(2/d)}\right)}$

(b) As $d \to \infty$, $1/d$ becomes negligible compared to $1/a$. Then,

$$C = \frac{4\pi\epsilon_0}{1/a + 1/b} \quad \text{and} \quad \frac{1}{C} = \boxed{\frac{1}{4\pi\epsilon_0 a} + \frac{1}{4\pi\epsilon_0 b}}$$

as for two spheres in series.

26.61 Note that the potential difference between the plates is held constant at ΔV_i by the battery.

$$C_i = \frac{q_0}{\Delta V_i} \quad \text{and} \quad C_f = \frac{q_f}{\Delta V_i} = \frac{q_0 + q}{\Delta V_i}$$

But $C_f = \kappa C_i$, so $\dfrac{q_0 + q}{\Delta V_i} = \kappa\left(\dfrac{q_0}{\Delta V_i}\right)$

Thus, $\kappa = \dfrac{q_0 + q}{q_0}$ or $\kappa = \boxed{1 + \dfrac{q}{q_0}}$

26.62 (a) $C = \dfrac{\epsilon_0}{d}\left[(\ell - x)\ell + \kappa \ell x\right] = \boxed{\dfrac{\epsilon_0}{d}\left[\ell^2 + \ell x(\kappa - 1)\right]}$

(b) $U = \frac{1}{2}C(\Delta V)^2 = \boxed{\dfrac{1}{2}\left(\dfrac{\epsilon_0(\Delta V)^2}{d}\right)\left[\ell^2 + \ell x(\kappa - 1)\right]}$

(c) $|\mathbf{F}| = \left|-\dfrac{dU}{dx}\right| = \boxed{\boxed{} \text{ to the left}}$

(d) $F = \dfrac{(2000)^2(8.85\times10^{-12})(0.0500)(4.50-1)}{2(2.00\times10^{-3})} = \boxed{1.55\times10^{-3}\ \text{N}}$

***26.63** The portion of the capacitor nearly filled by metal has capacitance $\kappa \epsilon_0 (\ell x)/d \to \infty$ and stored energy $Q^2/2C \to 0$. The unfilled portion has capacitance $\epsilon_0 \ell \,(\ell - x)/d$. The charge on this portion is $Q = (\ell - x)Q_0/\ell$.

(a) The stored energy is $U = \dfrac{Q^2}{2C} = \dfrac{\left[(\ell - x)Q_0/\ell\right]^2}{2\epsilon_0 \ell(\ell - x)/d} = \boxed{\dfrac{Q_0^{\,2} d(\ell - x)}{2\epsilon_0 \ell^3}}$

(b) $F = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(\dfrac{Q_0^{\,2} d(\ell - x)}{2\epsilon_0 \ell^3}\right) = +\dfrac{Q_0^{\,2} d}{2\epsilon_0 \ell^3}$

$\mathbf{F} = \boxed{\boxed{\quad\quad} \text{ to the right}}$

(c) $Stress = \dfrac{F}{\ell d} = \boxed{\boxed{\quad\quad}}$

(d) $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\dfrac{\sigma}{\epsilon_0}\right)^2 = \frac{1}{2}\epsilon_0\left(\dfrac{Q_0}{\epsilon_0 \ell^2}\right)^2 = \boxed{\boxed{\quad\quad}}$

26.64 Gasoline: $\left(126\,000\ \dfrac{\text{Btu}}{\text{gal}}\right)\left(1054\ \dfrac{\text{J}}{\text{Btu}}\right)\left(\dfrac{1.00\ \text{gal}}{3.786 \times 10^{-3}\ \text{m}^3}\right)\left(\dfrac{1.00\ \text{m}^3}{670\ \text{kg}}\right) = 5.25 \times 10^7\ \dfrac{\text{J}}{\text{kg}}$

Battery: $\dfrac{(12.0\ \text{J/C})(100\ \text{C/s})(3600\ \text{s})}{16.0\ \text{kg}} = 2.70 \times 10^5\ \text{J/kg}$

Capacitor: $\dfrac{\frac{1}{2}(0.100\ \text{F})(12.0\ \text{V})^2}{0.100\ \text{kg}} = 72.0\ \text{J/kg}$

Gasoline has 194 times the specific energy content of the battery and 727 000 times that of the capacitor

26.65 Call the unknown capacitance C_u

$$Q = C_u(\Delta V_i) = (C_u + C)(\Delta V_f)$$

$$\boxed{C_u = \frac{C(\Delta V_f)}{(\Delta V_i) - (\Delta V_f)}} = \frac{(10.0\ \mu\text{F})(30.0\ \text{V})}{(100\ \text{V} - 30.0\ \text{V})} = \boxed{4.29\ \mu\text{F}}$$

Goal Solution

An isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the charged capacitor is then connected in parallel to an uncharged 10.0-μF capacitor, the voltage across the combination is 30.0 V. Calculate the unknown capacitance.

G: The voltage of the combination will be reduced according to the size of the added capacitance. (Example: If the unknown capacitance were $C = 10.0\ \mu\text{F}$, then $\Delta V_1 = 50.0$ V because the charge is now distributed evenly between the two capacitors.) Since the final voltage is less than half the original, we might guess that the unknown capacitor is about $5.00\ \mu\text{F}$.

O: We can use the relationships for capacitors in parallel to find the unknown capacitance, along with the requirement that the charge on the unknown capacitor must be the same as the total charge on the two capacitors in parallel.

A: We name our ignorance and call the unknown capacitance C_u. The charge originally deposited on **each** plate, + on one, – on the other, is

$$Q = C_u \Delta V = C_u(100\ \text{V})$$

Now in the new connection this same conserved charge redistributes itself between the two capacitors according to $Q = Q_1 + Q_2$.

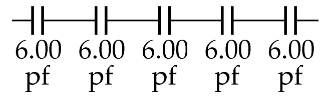
$$Q_1 = C_u(30.0\ \text{V}) \text{ and } \quad Q_2 = (10.0\ \mu\text{F})(30.0\ \text{V}) = 300\ \mu\text{C}$$

We can eliminate Q and Q_1 by substitution:

$$C_u(100\ \text{V}) = C_u(30.0\ \text{V}) + 300\ \mu\text{C} \qquad \text{so} \qquad C_u = \frac{300\ \mu\text{C}}{70.0\ \text{V}} = 4.29\ \mu\text{F}$$

L: The calculated capacitance is close to what we expected, so our result seems reasonable. In this and other capacitance combination problems, it is important not to confuse the charge and voltage of the system with those of the individual components, especially if they have different values. Careful attention must be given to the subscripts to avoid this confusion. It is also important to not confuse the variable "C" for capacitance with the unit of charge, "C" for coulombs.

26.66 $\boxed{\text{Put five 6.00 pF capacitors in series.}}$



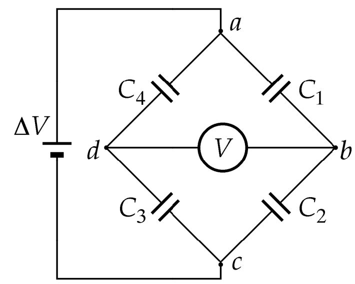
The potential difference across any one of the capacitors will be:

$$\Delta V = \frac{\Delta V_{\max}}{5} = \frac{1000\text{ V}}{5} = 200\text{ V}$$

and the equivalent capacitance is:

$$\frac{1}{C_{\text{eq}}} = 5\left(\frac{1}{6.00\text{ pF}}\right) \quad \text{or} \quad C_{\text{eq}} = \frac{6.00\text{ pF}}{5} = 1.20\text{ pF}$$

26.67 When $\Delta V_{db} = 0$, $\Delta V_{bc} = \Delta V_{dc}$, and $\dfrac{Q_2}{C_2} = \dfrac{Q_3}{C_3}$



Also, $\Delta V_{ba} = \Delta V_{da}$ or $\dfrac{Q_1}{C_1} = \dfrac{Q_4}{C_4}$

From these equations we have $C_2 = \left(\dfrac{C_3}{C_4}\right)\left(\dfrac{Q_2}{Q_1}\right)\left(\dfrac{Q_4}{Q_3}\right)C_1$

However, from the properties of capacitors in series, we have $Q_1 = Q_2$ and $Q_3 = Q_4$

Therefore, $C_2 = \left(\dfrac{C_3}{C_4}\right)C_1 = \dfrac{9.00}{12.0}(4.00\ \mu F) = \boxed{3.00\ \mu\text{F}}$

26.68 Let C = the capacitance of an individual capacitor, and C_s represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$$Q = C\,\Delta V_{\text{chg}} = (5.00 \times 10^{-4}\text{ F})(800\text{ V}) = 0.400\text{ C}$$

While being discharged in series,

$$\Delta V_{\text{disch}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400\text{ C}}{5.00 \times 10^{-5}\text{ F}} = \boxed{8.00\text{ kV}}$$

or 10 times the original voltage.

26.69 (a) $C_0 = \dfrac{\epsilon_0 A}{d} = \dfrac{Q_0}{\Delta V_0}$

When the dielectric is inserted at constant voltage,

$$C = \kappa C_0 = \frac{Q}{\Delta V_0}; \qquad U_0 = \frac{C_0(\Delta V_0)^2}{2}$$

$$U = \frac{C(\Delta V_0)^2}{2} = \frac{\kappa C_0(\Delta V_0)^2}{2} \quad \text{and} \quad \frac{U}{U_0} = \kappa$$

The extra energy comes from (part of the) electrical work done by the battery in separating the extra charge.

(b) $Q_0 = C_0 \Delta V_0$ and $Q = C \Delta V_0 = \kappa C_0 \Delta V_0$ so $\boxed{Q / Q_0 = \kappa}$

26.70 (a) A slice of width (dx) at coordinate x in $0 \le x \le L$ has thickness xd/L filled with dielectric κ_2, and $d - xd/L$ is filled with the material having constant κ_1. This slice has a capacitance given by

$$\frac{1}{dC} = \frac{1}{\left(\dfrac{\kappa_2 \,\mathrm{e}_0 (dx) W}{xd/L}\right)} + \frac{1}{\left(\dfrac{\kappa_1 \,\mathrm{e}_0 (dx) W}{d - xd/L}\right)} = \frac{xd}{\kappa_2 \,\mathrm{e}_0 \, WL(dx)} + \frac{dL - xd}{\kappa_1 \,\mathrm{e}_0 \, WL(dx)} = \frac{\kappa_1 xd + \kappa_2 dL - \kappa_2 xd}{\kappa_1 \kappa_2 \,\mathrm{e}_0 WL(dx)}$$

$$dC = \frac{\kappa_1 \kappa_2 \,\mathrm{e}_0 \, WL(dx)}{\kappa_2 \, dL + (\kappa_1 - \kappa_2) xd}$$

The whole capacitor is all the slices in parallel:

$$C = \int dC = \int_{x=0}^{L} \frac{\kappa_1 \kappa_2 \,\mathrm{e}_0 WL(dx)}{\kappa_2 Ld + (\kappa_1 - \kappa_2) xd} = \frac{\kappa_1 \kappa_2 \,\mathrm{e}_0 \, WL}{(\kappa_1 - \kappa_2) d} \int_{x=0}^{L} \left(\kappa_2 Ld + (\kappa_1 - \kappa_2) xd\right)^{-1} (\kappa_1 - \kappa_2) d(dx)$$

$$C = \frac{\kappa_1 \kappa_2 \,\mathrm{e}_0 \, WL}{(\kappa_1 - \kappa_2) d} \ln\left[\kappa_2 Ld + (\kappa_1 - \kappa_2) xd\right]\Big|_0^L = \frac{\kappa_1 \kappa_2 \,\mathrm{e}_0 WL}{(\kappa_1 - \kappa_2) d} \left[\ln \kappa_1 Ld - \ln \kappa_2 Ld\right] = \boxed{\frac{\kappa_1 \kappa_2 \,\mathrm{e}_0 \, WL}{(\kappa_1 - \kappa_2) d} \ln \frac{\kappa_1}{\kappa_2}}$$

(b) To take the limit $\kappa_1 \to \kappa_2$, write $\kappa_1 = \kappa_2 (1 + x)$ and let $x \to 0$. Then

$$C = \frac{\kappa_2^2 (1 + x) \mathrm{e}_0 \, WL}{(\kappa_2 + \kappa_2 x - \kappa_2) d} \ln (1 + x)$$

Use the expansion of $\ln(1 + x)$ from Appendix B.5.

$$C = \frac{\kappa_2^2 (1 + x) \in_0 WL}{\kappa_2 xd} \left(x - \tfrac{1}{2} x^2 + \tfrac{1}{3} x^3 \ldots\right) = \frac{\kappa_2 (1 + x) \in_0 WL}{d} \left(1 - \tfrac{1}{2} x + \ldots\right)$$

$$\lim_{x \to 0} C = \frac{\kappa_2 \in_0 WL}{d} = \boxed{\frac{\kappa \in_0 A}{d}}$$

26.71 The vertical orientation sets up two capacitors in parallel, with equivalent capacitance

$$C_p = \frac{\in_0 (A/2)}{d} + \frac{\kappa \in_0 (A/2)}{d} = \left(\frac{\kappa + 1}{2}\right) \frac{\in_0 A}{d}$$

where A is the area of either plate and d is the separation of the plates. The horizontal orientation produces two capacitors in series. If f is the fraction of the horizontal capacitor filled with dielectric, the equivalent capacitance is

$$\frac{1}{C_s} = \frac{fd}{\kappa \in_0 A} + \frac{(1 - f) d}{\in_0 A} = \left[\frac{f + \kappa(1 - f)}{\kappa}\right] \frac{d}{\in_0 A}, \quad \text{or} \quad C_s = \left[\frac{\kappa}{f + \kappa(1 - f)}\right] \frac{\in_0 A}{d}$$

Requiring that $C_p = C_s$ gives $\dfrac{\kappa + 1}{2} = \dfrac{\kappa}{f + \kappa(1 - f)}$, or $(\kappa + 1)\left[f + \kappa(1 - f)\right] = 2\kappa$

For $\kappa = 2.00$, this yields $3.00\left[2.00 - (1.00) f\right] = 4.00$, with the solution $f = \boxed{2/3}$.

26.72 Initially (capacitors charged in parallel),

$q_1 = C_1(\Delta V) = (6.00\ \mu\text{F})(250\ \text{V}) = 1500\ \mu\text{C}$

$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(250\ \text{V}) = 500\ \mu\text{C}$

After reconnection (positive plate to negative plate),

$q'_{\text{total}} = q_1 - q_2 = 1000\ \mu\text{C}$ and $\Delta V' = \dfrac{q'_{\text{total}}}{C_{\text{total}}} = \dfrac{1000\ \mu\text{C}}{8.00\ \mu\text{F}} = 125\ \text{V}$

Therefore,

$q'_1 = C_1(\Delta V') = (6.00\ \mu\text{F})(125\ \text{V}) = \boxed{750\ \mu\text{C}}$

$q'_2 = C_2(\Delta V') = (2.00\ \mu\text{F})(125\ \text{V}) = \boxed{250\ \mu\text{C}}$

26.73 $E_{\max}$ occurs at the inner conductor's surface.

$E_{\max} = \dfrac{2k_e\lambda}{a}$ from Equation 24.7.

$\Delta V = 2k_e\lambda \ln\left(\dfrac{b}{a}\right)$ from Example 26.2

$$E_{\max} = \frac{\Delta V}{a\ln(b/a)}$$

$$\Delta V_{\max} = E_{\max} a \ln\left(\frac{b}{a}\right) = (18.0\times 10^{6}\ \text{V/m})(0.800\times 10^{-3}\ \text{m})\ln\left(\frac{3.00}{0.800}\right) = \boxed{19.0\ \text{kV}}$$

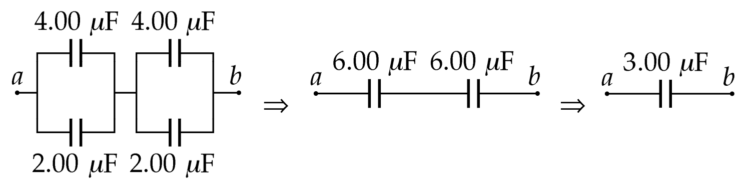
26.74 $E = \dfrac{2\kappa\lambda}{a}$; $\Delta V = 2\kappa\lambda \ln\left(\dfrac{b}{a}\right)$

$$\Delta V_{\max} = E_{\max} a \ln\left(\frac{b}{a}\right)$$

$$\frac{dV_{\max}}{da} = E_{\max}\left[\ln\left(\frac{b}{a}\right) + a\left(\frac{1}{b/a}\right)\left(-\frac{b}{a^2}\right)\right] = 0$$

$\ln\left(\dfrac{b}{a}\right) = 1$ or $\dfrac{b}{a} = e^1$ so $\boxed{a = \dfrac{b}{e}}$

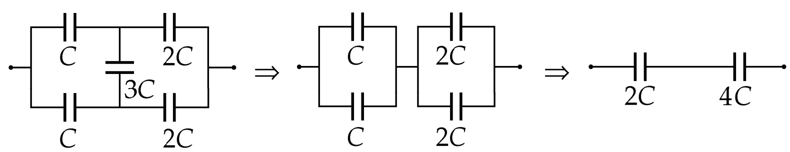
26.75 Assume a potential difference across a and b, and notice that the potential difference across the $8.00\ \mu\text{F}$ capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the following circuit:



$C_{ab} = \boxed{3.00\ \mu\text{F}}$

26.76 By symmetry, the potential difference across $3C$ is zero, so the circuit reduces to

$$C_{\text{eq}} = \frac{(2C)(4C)}{2C + 4C} = \frac{8}{6}C = \boxed{\frac{4}{3}C}$$



Chapter 27 Solutions

27.1 $I = \dfrac{\Delta Q}{\Delta t}$ $\qquad \Delta Q = I\,\Delta t = (30.0 \times 10^{-6}\text{ A})(40.0\text{ s}) = 1.20 \times 10^{-3}\text{ C}$

$$N = \frac{Q}{e} = \frac{1.20 \times 10^{-3}\text{ C}}{1.60 \times 10^{-19}\text{ C/electron}} = \boxed{7.50 \times 10^{15}\text{ electrons}}$$

***27.2** The atomic weight of silver = 107.9, and the volume V is

$V = (\text{area})(\text{thickness}) = (700 \times 10^{-4}\text{ m}^2)(0.133 \times 10^{-3}\text{ m}) = 9.31 \times 10^{-6}\text{ m}^3$

The mass of silver deposited is $m_{\text{Ag}} = \rho V = \left(10.5 \times 10^3\text{ kg / m}^3\right)\left(9.31 \times 10^{-6}\text{ m}^3\right) = 9.78 \times 10^{-2}\text{ kg}$.

and the number of silver atoms deposited is

$$N = (9.78 \times 10^{-2}\text{ kg})\frac{6.02 \times 10^{26}\text{ atoms}}{107.9\text{ kg}} = 5.45 \times 10^{23}$$

$$I = \frac{V}{R} = \frac{12.0\text{ V}}{1.80\ \Omega} = 6.67\text{ A} = 6.67\text{ C/s}$$

$$\Delta t = \frac{\Delta Q}{I} = \frac{Ne}{I} = \frac{(5.45 \times 10^{23})(1.60 \times 10^{-19}\text{ C})}{6.67\text{ C/s}} = 1.31 \times 10^4\text{ s} = \boxed{3.64\text{ h}}$$

27.3 $Q(t) = \int_0^t I\,dt = I_0\tau(1 - e^{-t/\tau})$

(a) $Q(\tau) = I_0\tau(1 - e^{-1}) = \boxed{(0.632)I_0\tau}$

(b) $Q(10\tau) = I_0\tau(1 - e^{-10}) = \boxed{(0.99995)I_0\tau}$

(c) $Q(\infty) = I_0\tau(1 - e^{-\infty}) = \boxed{I_0\tau}$

27.4 (a) Using $\dfrac{k_e e^2}{r^2} = \dfrac{mv^2}{r}$, we get: $\quad v = \sqrt{\dfrac{k_e e^2}{mr}} = \boxed{2.19 \times 10^6\text{ m / s}}$.

(b) The time for the electron to revolve around the proton once is:

$$t = \frac{2\pi r}{v} = \frac{2\pi(5.29 \times 10^{-11}\text{ m})}{(2.19 \times 10^6\text{ m / s})} = 1.52 \times 10^{-16}\text{ s}$$

The total charge flow in this time is 1.60×10^{-19} C, so the current is

$$I = \frac{1.60\times 10^{-19}\ \text{C}}{1.52\times 10^{-16}\ \text{s}} = 1.05\times 10^{-3}\ \text{A} = \boxed{1.05\ \text{mA}}$$

27.5 $\omega = \dfrac{2\pi}{T}$ where T is the period.

$$I = \frac{q}{T} = \frac{q\omega}{2\pi} = \frac{(8.00\times10^{-9}\text{ C})(100\pi\text{ rad / s})}{2\pi} = 4.00\times10^{-7}\text{ A} = \boxed{400\text{ nA}}$$

27.6 The period of revolution for the sphere is $T = \dfrac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I = \dfrac{q}{T} = \boxed{\dfrac{q\omega}{2\pi}}$.

27.7 $q = 4t^3 + 5t + 6$ $\quad A = (2.00\text{ cm}^2)\left(\dfrac{1.00\text{ m}}{100\text{ cm}}\right)^2 = 2.00\times10^{-4}\text{ m}^2$

(a) $I(1.00\text{ s}) = \left.\dfrac{dq}{dt}\right|_{t=1.00\text{ s}} = \left.\left(12t^2 + 5\right)\right|_{t=1.00\text{ s}} = \boxed{17.0\text{ A}}$

(b) $J = \dfrac{I}{A} = \dfrac{17.0\text{ A}}{2.00\times10^{-4}\text{ m}^2} = \boxed{85.0\text{ kA / m}^2}$

27.8 $I = \dfrac{dq}{dt}$

$$q = \int dq = \int I\,dt = \int_0^{1/240\text{ s}} (100\text{ A})\sin(120\pi t\text{ / s})\,dt$$

$$q = \frac{-100\text{ C}}{120\pi}\left[\cos(\pi/2) - \cos 0\right] = \frac{+100\text{ C}}{120\pi} = \boxed{0.265\text{ C}}$$

27.9 (a) $J = \dfrac{I}{A} = \dfrac{5.00\text{ A}}{\pi(4.00\times10^{-3}\text{ m})^2} = \boxed{99.5\text{ kA/m}^2}$

(b) $J_2 = \dfrac{1}{4}J_1$; $\quad \dfrac{I}{A_2} = \dfrac{1}{4}\dfrac{I}{A_1}$

$A_1 = \dfrac{1}{4}A_2$ so $\quad \pi(4.00\times10^{-3})^2 = \dfrac{1}{4}\pi r_2^2$

$r_2 = 2(4.00\times10^{-3}) = 8.00\times10^{-3}\text{ m} = \boxed{8.00\text{ mm}}$

27.10 (a) The speed of each deuteron is given by $K = \frac{1}{2}mv^2$

$(2.00 \times 10^6)(1.60 \times 10^{-19}\text{ J}) = \frac{1}{2}(2 \times 1.67 \times 10^{-27}\text{ kg})\,v^2$ and $v = 1.38 \times 10^7\text{ m/s}$

The time between deuterons passing a stationary point is t in $I = q/t$

$10.0 \times 10^{-6}\text{ C/s} = 1.60 \times 10^{-19}\text{ C}/t$ or $t = 1.60 \times 10^{-14}\text{ s}$

So the distance between them is $vt = \left(1.38 \times 10^7\text{ m/s}\right)\left(1.60 \times 10^{-14}\text{ s}\right) = \boxed{2.21 \times 10^{-7}\text{ m}}$

(b) One nucleus will put its nearest neighbor at potential

$$V = \frac{k_e q}{r} = \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})}{2.21 \times 10^{-7}\text{ m}} = \boxed{6.49 \times 10^{-3}\text{ V}}$$

This is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.

27.11 (a) $J = \frac{I}{A} = \frac{8.00 \times 10^{-6}\text{ A}}{\pi\left(1.00 \times 10^{-3}\text{ m}\right)^2} = \boxed{2.55\text{ A/m}^2}$

(b) From $J = nev_d$, we have $n = \frac{J}{ev_d} = \frac{2.55\text{ A/m}^2}{\left(1.60 \times 10^{-19}\text{ C}\right)\left(3.00 \times 10^8\text{ m/s}\right)} = \boxed{5.31 \times 10^{10}\text{ m}^{-3}}$

(c) From $I = \Delta Q/\Delta t$, we have $\Delta t = \frac{\Delta Q}{I} = \frac{N_A e}{I} = \frac{\left(6.02 \times 10^{23}\right)\left(1.60 \times 10^{-19}\text{ C}\right)}{8.00 \times 10^{-6}\text{ A}} = \boxed{1.20 \times 10^{10}\text{ s}}$

(This is about 381 years!)

***27.12** We use $I = nqAv_d$ where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume). We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molecular weight of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0\text{ g}}{N_A} = \frac{27.0\text{ g}}{6.02 \times 10^{23}} = 4.49 \times 10^{-23}\text{ g/atom}$$

Thus, $n = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70\text{ g/cm}^3}{4.49 \times 10^{-23}\text{ g/atom}}$

$$n = 6.02 \times 10^{22}\ \frac{\text{atoms}}{\text{cm}^3} = 6.02 \times 10^{28}\ \frac{\text{atoms}}{\text{m}^3}$$

Therefore, $v_d = \frac{I}{nqA} = \frac{5.00\text{ A}}{(6.02 \times 10^{28}\text{ m}^{-3})(1.60 \times 10^{-19}\text{ C})(4.00 \times 10^{-6}\text{ m}^2)} = 1.30 \times 10^{-4}\text{ m/s}$

or, $\boxed{v_d = 0.130\text{ mm/s}}$

***27.13** $I = \dfrac{\Delta V}{R} = \dfrac{120\ \text{V}}{240\ \Omega} = 0.500\ \text{A} = \boxed{500\ \text{mA}}$

27.14 (a) Applying its definition, we find the resistance of the rod,

$$R = \frac{\Delta V}{I} = \frac{15.0\ \text{V}}{4.00 \times 10^{-3}\ \text{A}} = 3750\ \Omega = \boxed{3.75\ \text{k}\Omega}$$

(b) The length of the rod is determined from Equation 27.11: $R = \rho\ell / A$. Solving for ℓ and substituting numerical values for R, A, and the values of ρ given for carbon in Table 27.1, we obtain

$$\ell = \frac{RA}{\rho} = \frac{(3.75 \times 10^{3}\ \Omega)(5.00 \times 10^{-6}\ \text{m}^2)}{(3.50 \times 10^{-5}\ \Omega \cdot \text{m})} = \boxed{536\ \text{m}}$$

27.15 $\Delta V = IR$ and $R = \dfrac{\rho l}{A}$: $A = 0.600\ \text{mm}^2 \left(\dfrac{1.00\ \text{m}}{1000\ \text{mm}}\right)^2 = 6.00 \times 10^{-7}\ \text{m}^2$

$\Delta V = \dfrac{I\rho l}{A}$: $I = \dfrac{\Delta V A}{\rho l} = \dfrac{(0.900\ \text{V})(6.00 \times 10^{-7}\ \text{m}^2)}{(5.60 \times 10^{-8}\ \Omega \cdot \text{m})(1.50\ \text{m})}$

$I = \boxed{6.43\ \text{A}}$

27.16 $J = \dfrac{I}{\pi r^2} = \sigma E = \dfrac{3.00\ \text{A}}{\pi (0.0120\ \text{m})^2} = \sigma(120\ \text{N/C})$

$\sigma = 55.3(\Omega \cdot \text{m})^{-1}$ $\rho = \dfrac{1}{\sigma} = \boxed{0.0181\ \Omega \cdot \text{m}}$

27.17 (a) Given $M = \rho_d V = \rho_d A l$ where $\rho_d \equiv$ mass density, we obtain: $A = \dfrac{M}{\rho_d l}$

Taking $\rho_r \equiv$ resistivity, $R = \dfrac{\rho_r l}{A} = \dfrac{\rho_r l}{\left(\dfrac{M}{\rho_d l}\right)} = \dfrac{\rho_r \rho_d l^2}{M}$

Thus, $l = \sqrt{\dfrac{MR}{\rho_r \rho_d}} = \sqrt{\dfrac{\left(1.00 \times 10^{-3}\right)(0.500)}{(1.70 \times 10^{-8})(8.92 \times 10^{3})}} = \boxed{1.82\ \text{m}}$

(b) $V = \dfrac{M}{\rho_d}$, or $\pi r^2 \ell = \dfrac{M}{\rho_d}$

Thus, $r = \sqrt{\dfrac{M}{\pi \rho_d \ell}} = \sqrt{\dfrac{1.00 \times 10^{-3}}{\pi (8.92 \times 10^3)(1.82)}} = 1.40 \times 10^{-4}\ \text{m}$

The diameter is twice this distance: $\boxed{\text{diameter} = 280\ \mu\text{m}}$

***27.18** (a) Suppose the rubber is 10 cm long and 1 mm in diameter.

$$R = \frac{\rho \ell}{A} = \frac{4\rho \ell}{\pi d^2} \sim \frac{4\left(10^{13}\ \Omega \cdot \text{m}\right)\left(10^{-1}\ \text{m}\right)}{\pi \left(10^{-3}\ \text{m}\right)^2} = \boxed{\sim 10^{18}\ \Omega}$$

(b) $R = \dfrac{4\rho \ell}{\pi d^2} \sim \dfrac{4(1.7 \times 10^{-8}\ \Omega \cdot \text{m})(10^{-3}\ \text{m})}{\pi (2 \times 10^{-2}\ \text{m})^2}$ $\boxed{\sim 10^{-7}\ \Omega}$

(c) $I = \dfrac{\Delta V}{R} \sim \dfrac{10^2\ \text{V}}{10^{18}\ \Omega}$ $\boxed{\sim 10^{-16}\ \text{A}}$

$I \sim \dfrac{10^2\ \text{V}}{10^{-7}\ \Omega}$ $\boxed{\sim 10^{9}\ \text{A}}$

27.19 The distance between opposite faces of the cube is $\ell = \left(\dfrac{90.0\ \text{g}}{10.5\ \text{g/cm}^3}\right)^{1/3} = 2.05\ \text{cm}$

(a) $R = \dfrac{\rho \ell}{A} = \dfrac{\rho \ell}{\ell^2} = \dfrac{\rho}{\ell} = \dfrac{1.59 \times 10^{-8}\ \Omega \cdot \text{m}}{2.05 \times 10^{-2}\ \text{m}} = 7.77 \times 10^{-7}\ \Omega = \boxed{777\ \text{n}\Omega}$

(b) $I = \dfrac{\Delta V}{R} = \dfrac{1.00 \times 10^{-5}\ \text{V}}{7.77 \times 10^{-7}\ \Omega} = 12.9\ \text{A}$

$$n = \frac{10.5\ \text{g/cm}^3}{107.87\ \text{g/mol}}\left(6.02 \times 10^{23}\ \frac{\text{electrons}}{\text{mol}}\right)$$

$$n = \left(5.86 \times 10^{22}\ \frac{\text{electrons}}{\text{cm}^3}\right)\left(\frac{1.00 \times 10^6\ \text{cm}^3}{1.00\ \text{m}^3}\right) = 5.86 \times 10^{28}/\text{m}^3$$

$I = nqvA$ and $v = \dfrac{I}{nqA} = \dfrac{12.9\ \text{C/s}}{\left(5.86 \times 10^{28}/\text{m}^3\right)\left(1.60 \times 10^{-19}\ \text{C}\right)(0.0205\ \text{m})^2} = \boxed{3.28\ \mu\text{m/s}}$

27.20 Originally, $R = \dfrac{\rho \ell}{A}$

Finally, $R_f = \dfrac{\rho(\ell / 3)}{3A} = \dfrac{\rho \ell}{9A} = \boxed{\dfrac{R}{9}}$

27.21 The total volume of material present does not change, only its shape. Thus,

$A_f \ell_f = A_f\left(1.25\,\ell_i\right) = A_i \ell_i$ giving $A_f = A_i / 1.25$

The final resistance is then: $R_f = \dfrac{\rho \ell_f}{A_f} = \dfrac{\rho\left(1.25\,\ell_i\right)}{A_i / 1.25} = 1.56\left(\dfrac{\rho \ell_i}{A_i}\right) = \boxed{1.56R}$

27.22 $\dfrac{\rho_{Al}\ell}{\pi\left(r_{Al}\right)^2} = \dfrac{\rho_{Cu}\ell}{\pi\left(r_{Cu}\right)^2}$

$\dfrac{r_{\text{Al}}}{r_{\text{Cu}}} = \sqrt{\dfrac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}} = \sqrt{\dfrac{2.82 \times 10^{-8}}{1.70 \times 10^{-8}}} = \boxed{1.29}$

27.23 $J = \sigma E$ so $\sigma = \dfrac{J}{E} = \dfrac{6.00 \times 10^{-13}\ \text{A / m}^2}{100\ \text{V / m}} = \boxed{6.00 \times 10^{-15}\ (\Omega \cdot \text{m})^{-1}}$

27.24 $R = \dfrac{\rho_1 \ell_1}{A_1} + \dfrac{\rho_2 \ell_2}{A_2} = (\rho_1 \ell_1 + \rho_2 \ell_2) / d^2$

$$R = \frac{(4.00 \times 10^{-3}\ \Omega \cdot \text{m})(0.250\ \text{m}) + (6.00 \times 10^{-3}\ \Omega \cdot \text{m})(0.400\ \text{m})}{(3.00 \times 10^{-3}\ \text{m})^2} = \boxed{378\ \Omega}$$

27.25 $\rho = \dfrac{m}{nq^2 \tau}$ so $\tau = \dfrac{m}{\rho n q^2} = \dfrac{9.11 \times 10^{-31}}{(1.70 \times 10^{-8})(8.49 \times 10^{28})(1.60 \times 10^{19})^2} = 2.47 \times 10^{-14}$ s

$v_d = \dfrac{qE}{m}\,\tau$ so $7.84 \times 10^{-4} = \dfrac{(1.60 \times 10^{-19})E(2.47 \times 10^{-14})}{9.11 \times 10^{-31}}$

Therefore $\boxed{E = 0.181\ \text{V / m}}$

Goal Solution

If the drift velocity of free electrons in a copper wire is 7.84×10^{-4} m / s, what is the electric field in the conductor?

G: For electrostatic cases, we learned that the electric field inside a conductor is always zero. On the other hand, if there is a current, a non-zero electric field must be maintained by a battery or other source to make the charges flow. Therefore, we might expect the electric field to be small, but definitely **not** zero.

O: The drift velocity of the electrons can be used to find the current density, which can be used with Ohm's law to find the electric field inside the conductor.

A: We first need the electron density in copper, which from Example 27.1 is $n = 8.49 \times 10^{28}$ e$^-$ / m^3. The current density in this wire is then

$$J = nqv_d = (8.49 \times 10^{28}\ \text{e}^- / \text{m}^3)(1.60 \times 10^{-19}\ \text{C} / \text{e}^-)(7.84 \times 10^{-4}\ \text{m} / \text{s}) = 1.06 \times 10^7\ \text{A} / \text{m}^2$$

Ohm's law can be stated as $J = \sigma E = E / \rho$ where $\rho = 1.7 \times 10^{-8}\ \Omega \cdot \text{m}$ for copper, so then

$$E = \rho J = (1.70 \times 10^{-8}\ \Omega \cdot \text{m})(1.06 \times 10^7\ \text{A} / \text{m}^2) = 0.181\ \text{V} / \text{m}$$

L: This electric field is certainly smaller than typical static values outside charged objects. The direction of the electric field should be along the length of the conductor, otherwise the electrons would be forced to leave the wire! The reality is that excess charges arrange themselves on the surface of the wire to create an electric field that "steers" the free electrons to flow along the length of the wire from low to high potential (opposite the direction of a positive test charge). It is also interesting to note that when the electric field is being established it travels at the speed of light; but the drift velocity of the electrons is literally at a "snail's pace"!

27.26 (a) n is $\boxed{\text{unaffected}}$

(b) $|J| = \dfrac{I}{A} \propto I$ so it $\boxed{\text{doubles}}$

(c) $J = nev_d$ so v_d $\boxed{\text{doubles}}$

(d) $\tau = \dfrac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as σ does not change due to heating.

27.27 From Equation 27.17,

$$\tau = \frac{m_e}{nq^2\rho} = \frac{9.11 \times 10^{-31}}{\left(8.49 \times 10^{28}\right)\left(1.60 \times 10^{-19}\right)^2\left(1.70 \times 10^{-8}\right)} = 2.47 \times 10^{-14}\ \text{s}$$

$$l = v\tau = \left(8.60 \times 10^5\ \text{m} / \text{s}\right)\left(2.47 \times 10^{-14}\ \text{s}\right) = 2.12 \times 10^{-8}\ \text{m} = \boxed{21.2\ \text{nm}}$$

27.28 At the low temperature T_C we write $R_C = \frac{\Delta V}{I_C} = R_0\left[1+\alpha(T_C - T_0)\right]$ where $T_0 = 20.0°C$

At the high temperature T_h, $R_h = \frac{\Delta V}{I_h} = \frac{\Delta V}{1\text{ A}} = R_0\left[1+\alpha(T_h - T_0)\right]$

Then $\frac{(\Delta V)/(1.00\text{ A})}{(\Delta V)/I_C} = \frac{1 + (3.90\times 10^{-3})(38.0)}{1 + (3.90\times 10^{-3})(-108)}$

and $I_C = (1.00\text{ A})(1.15/0.579) = \boxed{1.98\text{ A}}$

***27.29** $R = R_0\left[1+\alpha(\Delta T)\right]$ gives $140\ \Omega = (19.0\ \Omega)\left[1+(4.50\times 10^{-3}/°\text{C})\Delta T\right]$

Solving, $\Delta T = 1.42\times 10^3\ °\text{C} = T - 20.0\ °\text{C}$

And, the final temperature is $\boxed{T = 1.44\times 10^3\ °\text{C}}$

27.30 $R = R_c + R_n = R_c\left[1+\alpha_c(T-T_0)\right] + R_n\left[1+\alpha_n(T-T_0)\right]$

$0 = R_c\alpha_c\,(T - T_0) + R_n\alpha_n\,(T - T_0)$ so $R_c = -R_n\frac{\alpha_n}{\alpha_c}$

$R = -R_n\frac{\alpha_n}{\alpha_c} + R_n$

$R_n = R(1 - \alpha_n/\alpha_c)^{-1}$ $R_c = R(1 - \alpha_c/\alpha_n)^{-1}$

$R_n = 10.0\text{ k}\Omega\left[1 - \frac{(0.400\times 10^{-3}/\text{C°})}{(-0.500\times 10^{-3}/\text{C°})}\right]^{-1}$

$\boxed{R_n = 5.56\text{ k}\Omega}$ and $\boxed{R_c = 4.44\text{ k}\Omega}$

27.31 (a) $\rho = \rho_0\left[1+\alpha(T-T_0)\right] = (2.82\times 10^{-8}\ \Omega\cdot\text{m})\left[1+3.90\times 10^{-3}(30.0°)\right] = \boxed{3.15\times 10^{-8}\ \Omega\cdot\text{m}}$

(b) $J = \frac{E}{\rho} = \frac{0.200\text{ V/m}}{3.15\times 10^{-8}\ \Omega\cdot\text{m}} = \boxed{6.35\times 10^6\text{ A/m}^2}$

(c) $I = JA = \frac{\pi d^2}{4}J = \frac{\pi(1.00\times 10^{-4}\text{ m})^2}{4}(6.35\times 10^6\text{ A/m}^2) = \boxed{49.9\text{ mA}}$

(d) $n = \frac{6.02\times 10^{23}\text{ electrons}}{\left(\frac{26.98\text{ g}}{2.70\times 10^6\text{ g/m}^3}\right)} = 6.02\times 10^{28}\text{ electrons/m}^3$

$v_d = \frac{J}{ne} = \frac{(6.35\times 10^6\text{ A/m}^2)}{(6.02\times 10^{28}\text{ electrons/m}^3)(1.60\times 10^{-19}\text{ C})} = \boxed{659\ \mu\text{m/s}}$

(e) $\Delta V = E\ell = (0.200\text{ V/m})(2.00\text{ m}) = \boxed{0.400\text{ V}}$

***27.32** For aluminum, $\alpha_E = 3.90 \times 10^{-3}/°\text{C}$ (Table 27.1) $\alpha = 24.0 \times 10^{-6}/°\text{C}$ (Table 19.2)

$$R = \frac{\rho l}{A} = \frac{\rho_0(1+\alpha_E \Delta T)(1+\alpha \Delta T)}{A(1+\alpha \Delta T)^2} = R_0 \frac{(1+\alpha_E \Delta T)}{(1+\alpha \Delta T)} = (1.234\ \Omega)\frac{(1.39)}{(1.0024)} = \boxed{1.71\ \Omega}$$

27.33 $R = R_0[1 + \alpha \Delta T]$

$R - R_0 = R_0 \alpha \Delta T$

$$\frac{R - R_0}{R_0} = \alpha \Delta T = (5.00 \times 10^{-3})25.0 = \boxed{0.125}$$

27.34 Assuming linear change of resistance with temperature, $R = R_0(1 + \alpha \Delta T)$

$$R_{77\,K} = (1.00\ \Omega)\left[1 + \left(3.92 \times 10^{-3}\right)(-216°\text{C})\right] = \boxed{0.153\ \Omega}$$

27.35 $\rho = \rho_0(1 + \alpha \Delta T)$ or $\Delta T_W = \dfrac{1}{\alpha_W}\left(\dfrac{\rho_W}{\rho_{0W}} - 1\right)$

Require that $\rho_W = 4\rho_{0_{\text{Cu}}}$ so that $\Delta T_W = \left(\dfrac{1}{4.50 \times 10^{-3}\ /°\text{C}}\right)\left(\dfrac{4\left(1.70 \times 10^{-8}\right)}{5.60 \times 10^{-8}} - 1\right) = 47.6\ °\text{C}$

Therefore, $T_W = 47.6\ °\text{C} + T_0 = \boxed{67.6°\text{C}}$

27.36 $$\alpha = \frac{1}{R_0}\left(\frac{\Delta R}{\Delta T}\right) = \left(\frac{1}{R_0}\right)\frac{2R_0 - R_0}{T - T_0} = \frac{1}{T - T_0}$$

so, $T = \left(\dfrac{1}{\alpha}\right) + T_0$ and $T = \left(\dfrac{1}{0.400 \times 10^{-3}\ \text{C}°^{-1}}\right) + 20.0\ °\text{C}$ so $T = \boxed{2.52 \times 10^3\ °\text{C}}$

***27.37** $$I = \frac{\mathcal{P}}{\Delta V} = \frac{600\ \text{W}}{120\ \text{V}} = \boxed{5.00\ \text{A}}$$

and $R = \dfrac{\Delta V}{I} = \dfrac{120\ \text{V}}{5.00\ \text{A}} = \boxed{24.0\ \Omega}$

27.38 $\mathcal{P} = 0.800(1500 \text{ hp})(746 \text{ W/hp}) = 8.95 \times 10^5 \text{ W}$

$\mathcal{P} = I(\Delta V)$

$8.95 \times 10^5 = I(2000)$

$\boxed{I = 448 \text{ A}}$

27.39 The heat that must be added to the water is

$Q = mc\,\Delta T = (1.50 \text{ kg})(4186 \text{ J/kg°C})(40.0\text{°C}) = 2.51 \times 10^5 \text{ J}$

Thus, the power supplied by the heater is

$$\mathcal{P} = \frac{W}{t} = \frac{Q}{t} = \frac{2.51 \times 10^5 \text{ J}}{600 \text{ s}} = 419 \text{ W}$$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(110 \text{ V})^2}{419 \text{ W}} = \boxed{28.9\ \Omega}$

27.40 The heat that must be added to the water is $Q = mc(T_2 - T_1)$

Thus, the power supplied by the heat is $\mathcal{P} = \dfrac{W}{\Delta t} = \dfrac{Q}{\Delta t} = \dfrac{mc(T_2 - T_1)}{t}$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \boxed{\dfrac{(\Delta V)^2 t}{mc(T_2 - T_1)}}$

27.41 $$\frac{\mathcal{P}}{\mathcal{P}_0} = \frac{(\Delta V)^2 / R}{(\Delta V_0)^2 / R} = \left(\frac{\Delta V}{\Delta V_0}\right)^2 = \left(\frac{140}{120}\right)^2 = 1.361$$

$$\Delta\% = \left(\frac{\mathcal{P} - \mathcal{P}_0}{\mathcal{P}_0}\right)(100\%) = \left(\frac{\mathcal{P}}{\mathcal{P}_0} - 1\right)(100\%) = (1.361 - 1)100 = \boxed{36.1\%}$$

Goal Solution

Suppose that a voltage surge produces 140 V for a moment. By what percentage does the power output of a 120-V, 100-W light bulb increase? (Assume that its resistance does not change.)

G: The voltage increases by about 20%, but since $\mathcal{P} = (\Delta V)^2 / R$, the power will increase as the square of the voltage:

$$\frac{\mathcal{P}_f}{\mathcal{P}_i} = \frac{(\Delta V_f)^2 / R}{(\Delta V_i)^2 / R} = \frac{(140 \text{ V})^2}{(120 \text{ V})^2} = 1.361 \text{ or a } 36.1\% \text{ increase.}$$

O: We have already found an answer to this problem by reasoning in terms of ratios, but we can also calculate the power explicitly for the bulb and compare with the original power by using Ohm's law and the equation for electrical power. To find the power, we must first find the resistance of the bulb, which should remain relatively constant during the power surge (we can check the validity of this assumption later).

A: From $\mathcal{P} = (\Delta V)^2 / R$, we find that $R = \dfrac{(\Delta V_i)^2}{\mathcal{P}} = \dfrac{(120 \text{ V})^2}{100 \text{ W}} = 144\ \Omega$

The final current is, $I_f = \dfrac{\Delta V_f}{R} = \dfrac{140 \text{ V}}{144\ \Omega} = 0.972 \text{ A}$

The power during the surge is $\mathcal{P}_f = \dfrac{(\Delta V_f)^2}{R} = \dfrac{(140 \text{ V})^2}{144\ \Omega} = 136 \text{ V}$

So the percentage increase is $\dfrac{136 \text{ W} - 100 \text{ W}}{100 \text{ W}} = 0.361 = 36.1\%$

L: Our result tells us that this 100-W light bulb momentarily acts like a 136-W light bulb, which explains why it would suddenly get brighter. Some electronic devices (like computers) are sensitive to voltage surges like this, which is the reason that **surge protectors** are recommended to protect these devices from being damaged.

In solving this problem, we assumed that the resistance of the bulb did not change during the voltage surge, but we should check this assumption. Let us assume that the filament is made of tungsten and that its resistance will change linearly with temperature according to equation 27.21. Let us further assume that the increased voltage lasts for a time long enough so that the filament comes to a new equilibrium temperature. The temperature change can be estimated from the power surge according to Stefan's law (equation 20.18), assuming that all the power loss is due to radiation. By this law, $T \propto \sqrt[4]{\mathcal{P}}$ so that a 36% change in power should correspond to only about a 8% increase in temperature. A typical operating temperature of a white light bulb is about $3000\ °\text{C}$, so $\Delta T \approx 0.08(3273\ °\text{C}) = 260\ °\text{C}$. Then the increased resistance would be roughly

$$R = R_0\left(1 + \alpha\left(T - T_0\right)\right) = (144\ \Omega)\left(1 + 4.5 \times 10^{-3}(260)\right) \cong 310\ \Omega$$

It appears that the resistance could change double from 144 Ω. On the other hand, if the voltage surge lasts only a very short time, the 136 W we calculated originally accurately describes the conversion of electrical into internal energy in the filament.

27.42 $\mathcal{P} = I(\Delta V) = \dfrac{(\Delta V)^2}{R} = 500 \text{ W}$

$R = \dfrac{(110 \text{ V})^2}{(500 \text{ W})} = 24.2\ \Omega$

(a) $R = \dfrac{\rho}{A}\ell$ so $\ell = \dfrac{RA}{\rho} = \dfrac{(24.2\ \Omega)\pi(2.50 \times 10^{-4} \text{ m})^2}{1.50 \times 10^{-6}\ \Omega \cdot \text{m}} = \boxed{3.17 \text{ m}}$

(b) $R = R_0[1 + \alpha\,\Delta T] = 24.2\ \Omega\left[1 + (0.400 \times 10^{-3})(1180)\right] = 35.6\ \Omega$

$\mathcal{P} = \dfrac{(\Delta V)^2}{R} = \dfrac{(110)^2}{35.6} = \boxed{340 \text{ W}}$

27.43 $R = \dfrac{\rho\ell}{A} = \dfrac{(1.50 \times 10^{-6}\ \Omega \cdot \text{m})25.0 \text{ m}}{\pi(0.200 \times 10^{-3} \text{ m})^2} = 298\ \Omega$

$\Delta V = IR = (0.500 \text{ A})(298\ \Omega) = 149 \text{ V}$

(a) $E = \dfrac{\Delta V}{\ell} = \dfrac{149 \text{ V}}{25.0 \text{ m}} = \boxed{5.97 \text{ V/m}}$

(b) $\mathcal{P} = (\Delta V)I = (149 \text{ V})(0.500 \text{ A}) = \boxed{74.6 \text{ W}}$

(c) $R = R_0\left[1 + \alpha(T - T_0)\right] = 298\ \Omega\left[1 + (0.400 \times 10^{-3}\ /\ \text{C°})320\ \text{C°}\right] = 337\ \Omega$

$I = \dfrac{\Delta V}{R} = \dfrac{(149 \text{ V})}{(337\ \Omega)} = 0.443 \text{ A}$

$\mathcal{P} = (\Delta V)I = (149 \text{ V})(0.443 \text{ A}) = \boxed{66.1 \text{ W}}$

27.44 (a) $\Delta U = q(\Delta V) = It(\Delta V) = (55.0 \text{ A} \cdot \text{h})(12.0 \text{ V})\left(\dfrac{1 \text{ C}}{1 \text{ A} \cdot \text{s}}\right)\left(\dfrac{1 \text{ J}}{1 \text{ V} \cdot \text{C}}\right)\left(\dfrac{1 \text{ W} \cdot \text{s}}{1 \text{ J}}\right) = 660 \text{ W} \cdot \text{h} = \boxed{0.660 \text{ kWh}}$

(b) $\text{Cost} = 0.660 \text{ kWh}\left(\dfrac{\$0.0600}{1 \text{ kWh}}\right) = \boxed{3.96¢}$

27.45 $\mathcal{P} = I(\Delta V) \qquad \Delta V = IR$

$\mathcal{P} = \dfrac{(\Delta V)^2}{R} = \dfrac{(10.0)^2}{120} = \boxed{0.833 \text{ W}}$

27.46 The total clock power is $\left(270\times10^{6}\text{ clocks}\right)\left(2.50\ \dfrac{\text{J/s}}{\text{clock}}\right)\left(\dfrac{3600\text{ s}}{1\text{ h}}\right)=2.43\times10^{12}\text{ J/h}$

From $e=\dfrac{W_{\text{out}}}{Q_{\text{in}}}$, the power input to the generating plants must be:

$$\frac{Q_{\text{in}}}{t}=\frac{W_{\text{out}}/t}{e}=\frac{2.43\times10^{12}\text{ J/h}}{0.250}=9.72\times10^{12}\text{ J/h}$$

and the rate of coal consumption is

$$\text{Rate}=\left(9.72\times10^{12}\text{ J/h}\right)\left(\frac{1.00\text{ kg coal}}{33.0\times10^{6}\text{ J}}\right)=2.95\times10^{5}\ \frac{\text{kg coal}}{\text{h}}=\boxed{295\ \frac{\text{metric ton}}{\text{h}}}$$

27.47 $\mathcal{P}=I(\Delta V)=(1.70\text{ A})(110\text{ V})=187\text{ W}$

Energy used in a 24-hour day = (0.187 kW)(24.0 h) = 4.49 kWh

$$\therefore\ \text{cost}=4.49\text{ kWh}\left(\frac{\$0.0600}{\text{kWh}}\right)=\$0.269=\boxed{26.9¢}$$

27.48 $\mathcal{P}=I(\Delta V)=(2.00\text{ A})(120\text{ V})=240\text{ W}$

$\Delta U=(0.500\text{ kg})(4186\text{ J/kg°C})(77.0°\text{C})=161\text{ kJ}$

$$t=\frac{\Delta U}{\mathcal{P}}=\frac{1.61\times10^{5}\text{ J}}{240\text{ W}}=\boxed{672\text{ s}}$$

27.49 At operating temperature,

(a) $\mathcal{P}=I(\Delta V)=(1.53\text{ A})(120\text{ V})=\boxed{184\text{ W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$R=R_0(1+\alpha\,\Delta T)$

$$\frac{120}{1.53}=\frac{120}{1.80}\left[1+(0.400\times10^{-3})\Delta T\right]$$

$\Delta T=441°\text{C}$

$T=20.0°\text{C}+441°\text{C}=\boxed{461°\text{C}}$

Goal Solution

A certain toaster has a heating element made of Nichrome resistance wire. When the toaster is first connected to a 120-V source of potential difference (and the wire is at a temperature of 20.0 °C), the initial current is 1.80 A. However, the current begins to decrease as the resistive element warms up. When the toaster has reached its final operating temperature, the current has dropped to 1.53 A. (a) Find the power the toaster consumes when it is at its operating temperature. (b) What is the final temperature of the heating element?

G: Most toasters are rated at about 1000 W (usually stamped on the bottom of the unit), so we might expect this one to have a similar power rating. The temperature of the heating element should be hot enough to toast bread but low enough that the nickel-chromium alloy element does not melt. (The melting point of nickel is 1455 °C, and chromium melts at 1907 °C.)

O: The power can be calculated directly by multiplying the current and the voltage. The temperature can be found from the linear conductivity equation for Nichrome, with $\alpha = 0.4 \times 10^{-3}\ °\text{C}^{-1}$ from Table 27.1.

A: (a) $\mathcal{P} = (\Delta V)I = (120\ \text{V})(1.53\ \text{A}) = 184\ \text{W}$

(b) The resistance at 20.0 °C is $R_0 = \dfrac{\Delta V}{I} = \dfrac{120\ \text{V}}{1.80\ \text{A}} = 66.7\ \Omega$

At operating temperature, $R = \dfrac{120\ \text{V}}{1.53\ \text{A}} = 78.4\ \Omega$

Neglecting thermal expansion, $R = \dfrac{\rho l}{A} = \dfrac{\rho_0\left(1+\alpha(T-T_0)\right)l}{A} = R_0\left(1+\alpha(T-T_0)\right)$

$$T = T_0 + \frac{R/R_0 - 1}{\alpha} = 20.0\ °\text{C} + \frac{78.4\ \Omega/66.7\ \Omega - 1}{0.4\times 10^{-3}\ °\text{C}^{-1}} = 461\ °\text{C}$$

L: Although this toaster appears to use significantly less power than most, the temperature seems high enough to toast a piece of bread in a reasonable amount of time. In fact, the temperature of a typical 1000-W toaster would only be slightly higher because Stefan's radiation law (Eq. 20.18) tells us that (assuming all power is lost through radiation) $T \propto \sqrt[4]{\mathcal{P}}$, so that the temperature might be about 700 °C. In either case, the operating temperature is well below the melting point of the heating element.

27.50 $\mathcal{P} = (10.0\ \text{W}/\text{ft}^2)(10.0\ \text{ft})(15.0\ \text{ft}) = 1.50\ \text{kW}$

$\text{Energy} = \mathcal{P}\,t = (1.50\ \text{kW})(24.0\ \text{h}) = 36.0\ \text{kWh}$

$\text{Cost} = (36.0\ \text{kWh})(\$0.0800/\text{kWh}) = \boxed{\$2.88}$

***27.51** Consider a 400-W blow dryer used for ten minutes daily for a year. The energy converted is

$$\mathcal{P}\,t = (400\ \text{J/s})(600\ \text{s/d})(365\ \text{d}) \cong 9\times 10^{7}\ \text{J}\left(\frac{1\ \text{kWh}}{3.6\times 10^{6}\ \text{J}}\right) \cong 20\ \text{kWh}$$

We suppose that electrical energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$\text{Cost} \cong (20\ \text{kWh})(\$0.100/\text{kWh}) = \$2$ $\boxed{\sim\$1}$

***27.52** (a) $I = \dfrac{\Delta V}{R}$ so $\mathcal{P} = (\Delta V)I = \dfrac{(\Delta V)^2}{R}$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{25.0\text{ W}} = \boxed{576\ \Omega} \qquad \text{and} \qquad R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$$

(b) $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{25.0\text{ W}}{120\text{ V}} = 0.208\text{ A} = \dfrac{Q}{t} = \dfrac{1.00\text{ C}}{t}$

$t = \dfrac{1.00\text{ C}}{0.208\text{ A}} = \boxed{4.80\text{ s}}$ The charge has $\boxed{\text{lower potential energy}}$.

(c) $\mathcal{P} = 25.0\text{ W} = \dfrac{\Delta U}{t} = \dfrac{1.00\text{ J}}{t}$

$t = \dfrac{1.00\text{ J}}{25.0\text{ W}} = \boxed{0.0400\text{ s}}$ The energy $\boxed{\text{changes from electrical to heat and light}}$.

(d) $\Delta U = \mathcal{P}t = (25.0\text{ J/s})(86400\text{ s/d})(30.0\text{ d}) = 64.8 \times 10^6\text{ J}$

The energy company sells $\boxed{\text{energy}}$.

$$\text{Cost} = 64.8 \times 10^6\text{ J}\left(\frac{\$0.0700}{\text{kWh}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3600\text{ s}}\right) = \boxed{\$1.26}$$

$$\text{Cost per joule} = \frac{\$0.0700}{\text{kWh}}\left(\frac{\text{kWh}}{3.60 \times 10^6\text{ J}}\right) = \boxed{\$1.94 \times 10^{-8}/\text{J}}$$

***27.53** We find the drift velocity from $I = nqv_d A = nqv_d \pi r^2$

$$v_d = \frac{I}{nq\pi r^2} = \frac{1000\text{ A}}{8.00 \times 10^{28}\text{ m}^{-3}(1.60 \times 10^{-19}\text{ C})\pi(10^{-2}\text{ m})^2} = 2.49 \times 10^{-4}\text{ m/s}$$

$$v = \frac{x}{t} \qquad t = \frac{x}{v} = \frac{200 \times 10^3\text{ m}}{2.49 \times 10^{-4}\text{ m/s}} = 8.04 \times 10^8\text{ s} = \boxed{25.5\text{ yr}}$$

***27.54** The resistance of one wire is $\left(\dfrac{0.500\ \Omega}{\text{mi}}\right)(100\text{ mi}) = 50.0\ \Omega$

The whole wire is at nominal 700 kV away from ground potential, but the potential difference between its two ends is

$IR = (1000\text{ A})(50.0\ \Omega) = 50.0\text{ kV}$

Then it radiates as heat power $\mathcal{P} = (\Delta V)I = (50.0 \times 10^3\text{ V})(1000\text{ A}) = \boxed{50.0\text{ MW}}$

27.55 We begin with the differential equation $\alpha = \dfrac{1}{\rho}\dfrac{d\rho}{dT}$

(a) Separating variables, $\displaystyle\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_{T_0}^{T} \alpha\, dT$

$\ln\left(\dfrac{\rho}{\rho_0}\right) = \alpha(T - T_0)$ and $\boxed{\rho = \rho_0 e^{\alpha(T - T_0)}}$

(b) From the series expansion $e^x \cong 1 + x$, $(x << 1)$,

$\boxed{\rho \cong \rho_0\left[1 + \alpha(T - T_0)\right]}$

***27.56** Consider a 1.00-m length of cable. The potential difference between its ends is

$$\Delta V = \frac{\mathcal{P}}{I} = \frac{2.00\text{ W}}{300\text{ A}} = 6.67\text{ mV}$$

The resistance is $R = \dfrac{\Delta V}{I} = \dfrac{6.67 \times 10^{-3}\text{ V}}{300\text{ A}} = 22.2\ \mu\Omega$

Then $R = \dfrac{\rho \ell}{A} = \dfrac{\rho \ell}{\pi r^2}$ gives $r = \sqrt{\dfrac{\rho \ell}{\pi R}} = \sqrt{\dfrac{(1.70 \times 10^{-8}\ \Omega \cdot \text{m})(1.00\text{ m})}{\pi(22.2 \times 10^{-6}\ \Omega)}} = \boxed{1.56\text{ cm}}$

27.57 $\rho = \dfrac{RA}{\ell} = \dfrac{(\Delta V)}{I}\dfrac{A}{\ell}$

ℓ (m)	$R(\Omega)$	$\rho(\Omega \cdot \text{m})$
0.540	10.4	1.41×10^{-6}
1.028	21.1	1.50×10^{-6}
1.543	31.8	1.50×10^{-6}

$\bar{\rho} = \boxed{1.47 \times 10^{-6}\ \Omega \cdot \text{m}}$ (in agreement with tabulated value)

$\rho = \dfrac{RA}{\ell} = \boxed{1.50 \times 10^{-6}\ \Omega \cdot \text{m}}$ (Table 27.1)

27.58 2 wires $\rightarrow \ell = 100$ m

$$R = \frac{0.108\ \Omega}{300\ \text{m}}(100\ \text{m}) = 0.0360\ \Omega$$

(a) $(\Delta V)_{\text{home}} = (\Delta V)_{\text{line}} - IR = 120 - (110)(0.0360) = \boxed{116\ \text{V}}$

(b) $\mathcal{P} = I(\Delta V) = (110\ \text{A})(116\ \text{V}) = \boxed{12.8\ \text{kW}}$

(c) $\mathcal{P}_{\text{wires}} = I^2R = (110\ \text{A})^2(0.0360\ \Omega) = \boxed{436\ \text{W}}$

***27.59** (a) $\mathbf{E} = -\dfrac{dV}{dx}\mathbf{i} = -\dfrac{(0-4.00)\ \text{V}}{(0.500-0)\ \text{m}} = \boxed{8.00\mathbf{i}\ \text{V/m}}$

(b) $R = \dfrac{\rho \ell}{A} = \dfrac{\left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)(0.500\ \text{m})}{\pi\left(1.00\times10^{-4}\ \text{m}\right)^2} = \boxed{0.637\ \Omega}$

(c) $I = \dfrac{\Delta V}{R} = \dfrac{4.00\ \text{V}}{0.637\ \Omega} = \boxed{6.28\ \text{A}}$

(d) $\mathbf{J} = \dfrac{I}{A}\mathbf{i} = \dfrac{6.28\ \text{A}}{\pi\left(1.00\times10^{-4}\ \text{m}\right)^2} = 2.00\times10^{8}\,\mathbf{i}\ \text{A/m}^2 = \boxed{200\mathbf{i}\ \text{MA/m}^2}$

(e) $\rho\mathbf{J} = \left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.00\times10^{8}\,\mathbf{i}\ \text{A/m}^2\right) = 8.00\,\mathbf{i}\ \text{V/m} = \mathbf{E}$

***27.60** (a) $\mathbf{E} = -\dfrac{dV(x)}{dx}\ \mathbf{i} = \boxed{\dfrac{V}{L}\mathbf{i}}$

(b) $R = \dfrac{\rho \ell}{A} = \boxed{\dfrac{4\rho L}{\pi d^2}}$

(c) $I = \dfrac{\Delta V}{R} = \boxed{\dfrac{V\pi d^2}{4\rho L}}$

(d) $\mathbf{J} = \dfrac{I}{A}\ \mathbf{i} = \boxed{\dfrac{V}{\rho L}\mathbf{i}}$

(e) $\rho\,\mathbf{J} = \dfrac{V}{L}\ \mathbf{i} = \boxed{\mathbf{E}}$

27.61 $R = R_0[1+\alpha(T-T_0)]$ so $T = T_0 + \frac{1}{\alpha}\left[\frac{R}{R_0}-1\right] = T_0 + \frac{1}{\alpha}\left[\frac{I_0}{I}-1\right]$

In this case, $I = \frac{I_0}{10}$, so $T = T_0 + \frac{1}{\alpha}(9) = 20° + \frac{9}{0.00450/°\text{C}} = \boxed{2020\ °\text{C}}$

27.62 $R = \frac{\Delta V}{I} = \frac{12.0}{I} = \frac{6.00}{(I-3.00)}$ thus $12.0I - 36.0 = 6.00I$ and $I = 6.00\text{ A}$

Therefore, $R = \frac{12.0\text{ V}}{6.00\text{ A}} = \boxed{2.00\ \Omega}$

27.63 (a) $\mathcal{P} = I(\Delta V)$ so $I = \frac{\mathcal{P}}{\Delta V} = \frac{8.00\times10^3\text{ W}}{12.0\text{ V}} = \boxed{667\text{ A}}$

(b) $t = \frac{\Delta U}{\mathcal{P}} = \frac{2.00\times10^7\text{ J}}{8.00\times10^3\text{ W}} = 2.50\times10^3\text{ s}$ and $d = vt = (20.0\text{ m / s})(2.50\times10^3\text{ s}) = \boxed{50.0\text{ km}}$

27.64 (a) We begin with $R = \frac{\rho l}{A} = \frac{\rho_0[1 + \alpha(T-T_0)]\, l_0[1+\alpha'(T-T_0)]}{A_0[1+2\alpha'(T-T_0)]}$,

which reduces to $R = \boxed{\frac{R_0[1+\alpha(T-T_0)]\,[1+\alpha'(T-T_0)]}{[1+2\alpha'(T-T_0)]}}$

(b) For copper: $\rho_0 = 1.70\times10^{-8}\ \Omega\cdot\text{m}$, $\alpha = 3.90\times10^{-3}\ °\text{C}^{-1}$, and $\alpha' = 17.0\times10^{-6}\ °\text{C}^{-1}$

$$R_0 = \frac{\rho_0 l_0}{A_0} = \frac{(1.70\times10^{-8})(2.00)}{\pi(0.100\times10^{-3})^2} = \boxed{1.08\ \Omega}$$

The simple formula for R gives:

$$R = (1.08\ \Omega)\left[1+(3.90\times10^{-3}\,°\text{C}^{-1})(100°\text{C} - 20.0°\text{C})\right] = \boxed{1.420\ \Omega}$$

while the more complicated formula gives:

$$R = \frac{(1.08\ \Omega)\left[1+(3.90\times10^{-3}\,°\text{C}^{-1})(80.0°\text{C})\right]\left[1+(17.0\times10^{-6}\,°\text{C}^{-1})(80.0°\text{C})\right]}{\left[1+2(17.0\times10^{-6}\,°\text{C}^{-1})(80.0°\text{C})\right]} = \boxed{1.418\ \Omega}$$

27.65 Let α be the temperature coefficient at 20.0°C, and α' be the temperature coefficient at 0 °C. Then $\rho = \rho_0[1+\alpha(T-20.0°\text{C})]$, and $\rho = \rho'[1+\alpha'(T-0°\text{C})]$ must both give the correct resistivity at any temperature T. That is, we must have:

$$\rho_0[1+\alpha(T-20.0°\text{C})] = \rho'[1+\alpha'(T-0°\text{C})] \quad (1)$$

Setting $T = 0$ in equation (1) yields: $\rho' = \rho_0[1-\alpha(20.0°\text{C})]$,

and setting $T = 20.0$°C in equation (1) gives: $\rho_0 = \rho'[1+\alpha'(20.0°\text{C})]$

Put ρ' from the first of these results into the second to obtain:

$$\rho_0 = \rho_0[1-\alpha(20.0°\text{C})][1+\alpha'(20.0°\text{C})]$$

Therefore $1+\alpha'(20.0°\text{C}) = \dfrac{1}{1-\alpha(20.0°\text{C})}$

which simplifies to $\alpha' = \dfrac{\alpha}{[1-\alpha(20.0\ °\text{C})]}$

From this, the temperature coefficient, based on a reference temperature of 0°C, may be computed for any material. For example, using this, $\boxed{\text{Table 27.1 becomes at 0°C}}$:

Material	Temp Coefficients at 0°C
Silver	$4.1\times10^{-3}/°\text{C}$
Copper	$4.2\times10^{-3}/°\text{C}$
Gold	$3.6\times10^{-3}/°\text{C}$
Aluminum	$4.2\times10^{-3}/°\text{C}$
Tungsten	$4.9\times10^{-3}/°\text{C}$
Iron	$5.6\times10^{-3}/°\text{C}$
Platinum	$4.25\times10^{-3}/°\text{C}$
Lead	$4.2\times10^{-3}/°\text{C}$
Nichrome	$0.4\times10^{-3}/°\text{C}$
Carbon	$-0.5\times10^{-3}/°\text{C}$
Germanium	$-24\times10^{-3}/°\text{C}$
Silicon	$-30\times10^{-3}/°\text{C}$

27.66 (a) $R = \dfrac{\rho l}{A} = \boxed{\dfrac{\rho L}{\pi\left(r_b^2 - r_a^2\right)}}$

(b) $R = \dfrac{\left(3.50\times10^5\ \Omega\cdot\text{m}\right)(0.0400\ \text{m})}{\pi\left[(0.0120\ \text{m})^2-(0.00500\ \text{m})^2\right]} = 3.74\times10^7\ \Omega = \boxed{37.4\ \text{M}\Omega}$

(c) $dR = \dfrac{\rho\, d\ell}{A} = \dfrac{\rho\, dr}{(2\pi r)L} = \left(\dfrac{\rho}{2\pi L}\right)\dfrac{dr}{r}$, so $R = \dfrac{\rho}{2\pi L}\displaystyle\int_{r_a}^{r_b}\dfrac{dr}{r} = \boxed{\dfrac{\rho}{2\pi L}\ln\left(\dfrac{r_b}{r_a}\right)}$

(d) $R = \dfrac{\left(3.50\times10^5\ \Omega\cdot\text{m}\right)}{2\pi(0.0400\ \text{m})}\ln\left(\dfrac{1.20}{0.500}\right) = 1.22\times10^6\ \Omega = \boxed{1.22\ \text{M}\Omega}$

27.67 Each speaker receives 60.0 W of power. Using $\mathcal{P} = I^2 R$, we then have

$$I = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{60.0\text{ W}}{4.00\ \Omega}} = 3.87\text{ A}$$

The system is $\boxed{\text{not adequately protected}}$ since the $\boxed{\text{fuse should be set to melt at 3.87 A, or less}}$.

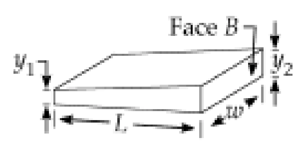
27.68 $\Delta V = -E \cdot \ell$ or $dV = -E \cdot dx$

$\Delta V = -IR = -E \cdot \ell$

$$I = \frac{dq}{dt} = \frac{E \cdot \ell}{R} = \frac{A}{\rho \ell} E \cdot \ell = \frac{A}{\rho} E = -\sigma A \frac{dV}{dx} = \boxed{\sigma A \left| \frac{dV}{dx} \right|}$$

Current flows in the direction of decreasing voltage. Energy flows as heat in the direction of decreasing temperature.

27.69 $R = \int \frac{\rho\, dx}{A} = \int \frac{\rho\, dx}{wy}$ where $y = y_1 + \frac{y_2 - y_1}{L} x$

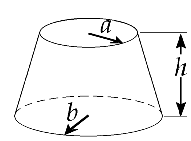


$$R = \frac{\rho}{w} \int_0^L \frac{dx}{y_1 + \frac{y_2 - y_1}{L} x} = \frac{\rho L}{w(y_2 - y_1)} \ln\left[y_1 + \frac{y_2 - y_1}{L} x \right]_0^L$$

$$R = \boxed{\frac{\rho L}{w(y_2 - y_1)} \ln\left(\frac{y_2}{y_1} \right)}$$

27.70 From the geometry of the longitudinal section of the resistor shown in the figure, we see that

$$\frac{(b-r)}{y} = \frac{(b-a)}{h}$$



From this, the radius at a distance y from the base is $r = (a-b)\frac{y}{h} + b$

For a disk-shaped element of volume $dR = \frac{\rho\, dy}{\pi r^2}$: $R = \frac{\rho}{\pi} \int_0^h \frac{dy}{\left[(a-b)(y/h) + b \right]^2}$.

Using the integral formula $\int \frac{du}{(au+b)^2} = -\frac{1}{a(au+b)}$, $\boxed{R = \frac{\rho}{\pi} \frac{h}{ab}}$

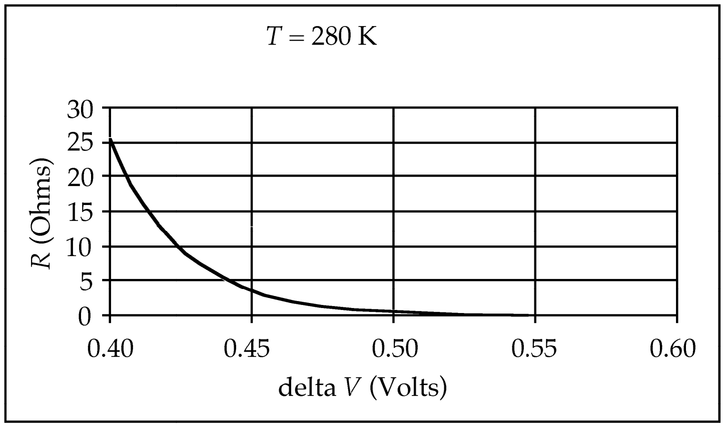
27.71 $I = I_0\left[\exp\left(e\Delta V / k_B T\right) - 1\right]$ and $R = \dfrac{\Delta V}{I}$

with $I_0 = 1.00 \times 10^{-9}$ A, $e = 1.60 \times 10^{-19}$ C, and $k_B = 1.38 \times 10^{-23}$ J/K.

$\boxed{\text{The following includes a partial table of calculated values and a graph for each of the specified temperatures.}}$

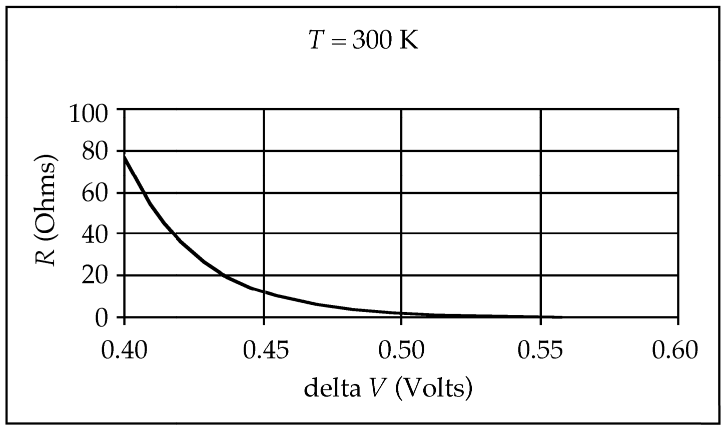
(i) For $T = 280$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.0156	25.6
0.440	0.0818	5.38
0.480	0.429	1.12
0.520	2.25	0.232
0.560	11.8	0.0476
0.600	61.6	0.0097



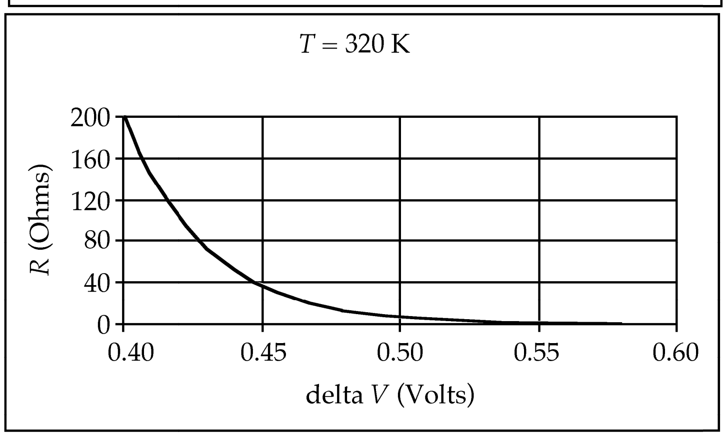
(ii) For $T = 300$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.005	77.3
0.440	0.024	18.1
0.480	0.114	4.22
0.520	0.534	0.973
0.560	2.51	0.223
0.600	11.8	0.051



(iii) For $T = 320$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.0020	203
0.440	0.0084	52.5
0.480	0.0357	13.4
0.520	0.152	3.42
0.560	0.648	0.864
0.600	2.76	0.217



Chapter 28 Solutions

28.1 (a) $\mathcal{P} = \frac{(\Delta V)^2}{R}$ becomes $20.0\text{ W} = \frac{(11.6\text{ V})^2}{R}$ so $R = \boxed{6.73\ \Omega}$

(b) $\Delta V = IR$ so $11.6\text{ V} = I(6.73\ \Omega)$ and $I = 1.72\text{ A}$

$\mathcal{E} = IR + Ir$ so $15.0\text{ V} = 11.6\text{ V} + (1.72\text{ A})r$

$r = \boxed{1.97\ \Omega}$

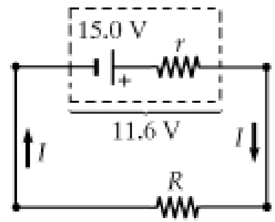


Figure for Goal Solution

<u>Goal Solution</u>

A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance of the battery?

G: The internal resistance of a battery usually is less than $1\ \Omega$, with physically larger batteries having less resistance due to the larger anode and cathode areas. The voltage of this battery drops significantly (23%), when the load resistance is added, so a sizable amount of current must be drawn from the battery. If we assume that the internal resistance is about $1\ \Omega$, then the current must be about 3 A to give the 3.4 V drop across the battery's internal resistance. If this is true, then the load resistance must be about $R \approx 12\text{ V} / 3\text{ A} = 4\ \Omega$.

O: We can find R exactly by using Joule's law for the power delivered to the load resistor when the voltage is 11.6 V. Then we can find the internal resistance of the battery by summing the electric potential differences around the circuit.

A: (a) Combining Joule's law, $\mathcal{P} = \Delta V I$, and the definition of resistance, $\Delta V = IR$, gives

$$R = \frac{\Delta V^2}{\mathcal{P}} = \frac{(11.6\text{ V})^2}{20.0\ W} = 6.73\ \Omega$$

(b) The electromotive force of the battery must equal the voltage drops across the resistances: $\mathcal{E} = IR + Ir$, where $I = \Delta V / R$.

$$r = \frac{\mathcal{E} - IR}{I} = \frac{(\mathcal{E} - \Delta V)R}{\Delta V} = \frac{(15.0\text{ V} - 11.6\text{ V})(6.73\ \Omega)}{11.6\text{ V}} = 1.97\ \Omega$$

L: The resistance of the battery is larger than $1\ \Omega$, but it is reasonable for an old battery or for a battery consisting of several small electric cells in series. The load resistance agrees reasonably well with our prediction, despite the fact that the battery's internal resistance was about twice as large as we assumed. Note that in our initial guess we did not consider the power of the load resistance; however, there is not sufficient information to accurately solve this problem without this data.

28.2 (a) $\Delta V_{\text{term}} = IR$

becomes $10.0\text{ V} = I(5.60\ \Omega)$

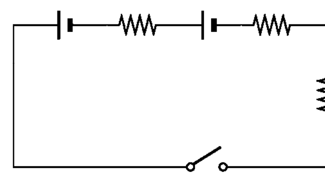
so $I = \boxed{1.79\text{ A}}$

(b) $\Delta V_{\text{term}} = \mathcal{E} - Ir$

becomes $10.0\text{ V} = \mathcal{E} - (1.79\text{ A})(0.200\ \Omega)$

so $\mathcal{E} = \boxed{10.4\text{ V}}$

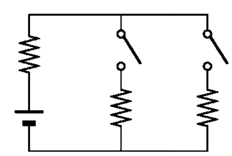
28.3 The total resistance is $R = \dfrac{3.00\text{ V}}{0.600\text{ A}} = 5.00\ \Omega$



(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$

(b) $\dfrac{P_{\text{batteries}}}{P_{\text{total}}} = \dfrac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.0816 = \boxed{8.16\%}$

28.4 (a) Here $\mathcal{E} = I(R + r)$, so $I = \dfrac{\mathcal{E}}{R + r} = \dfrac{12.6\text{ V}}{(5.00\ \Omega + 0.0800\ \Omega)} = 2.48\text{ A}$



Then, $\Delta V = IR = (2.48\text{ A})(5.00\ \Omega) = \boxed{12.4\text{ V}}$

(b) Let I_1 and I_2 be the currents flowing through the battery and the headlights, respectively.

Then, $I_1 = I_2 + 35.0\text{ A}$, and $\mathcal{E} - I_1 r - I_2 R = 0$

so $\mathcal{E} = (I_2 + 35.0\text{ A})(0.0800\ \Omega) + I_2(5.00\ \Omega) = 12.6\text{ V}$

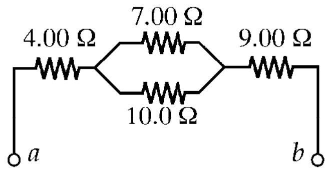
giving $I_2 = 1.93\text{ A}$

Thus, $\Delta V_2 = (1.93\text{ A})(5.00\ \Omega) = \boxed{9.65\text{ V}}$

28.5 $\Delta V = I_1 R_1 = (2.00\text{ A})R_1$ and $\Delta V = I_2(R_1 + R_2) = (1.60\text{ A})(R_1 + 3.00\ \Omega)$

Therefore, $(2.00\text{ A})R_1 = (1.60\text{ A})(R_1 + 3.00\ \Omega)$ or $R_1 = \boxed{12.0\ \Omega}$

28.6 (a) $R_p = \dfrac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$



$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$

(b) $\Delta V = IR$

$34.0\ \text{V} = I(17.1\ \Omega)$

$I = \boxed{1.99\ \text{A}}$ for 4.00 Ω, 9.00 Ω resistors

Applying $\Delta V = IR$, $(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$

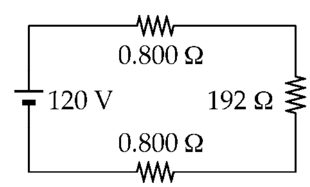
$8.18\ \text{V} = I(7.00\ \Omega)$ so $I = \boxed{1.17\ \text{A}}$ for 7.00 Ω resistor

$8.18\ \text{V} = I(10.0\ \Omega)$ so $I = \boxed{0.818\ \text{A}}$ for 10.0 Ω resistor

***28.7** If $\boxed{\text{all 3 resistors are placed in parallel}}$,

$\dfrac{1}{R} = \dfrac{1}{500} + \dfrac{2}{250} = \dfrac{5}{500}$ and $R = 100\ \Omega$

***28.8** For the bulb in use as intended,



$I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{75.0\ \text{W}}{120\ \text{V}} = 0.625\ \text{A}$ and $R = \dfrac{\Delta V}{I} = \dfrac{120\ \text{V}}{0.625\ \text{A}} = 192\ \Omega$

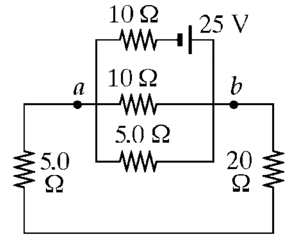
Now, presuming the bulb resistance is unchanged,

$I = \dfrac{120\ \text{V}}{193.6\ \Omega} = 0.620\ \text{A}$

Across the bulb is $\Delta V = IR = 192\ \Omega(0.620\ \text{A}) = 119\ \text{V}$

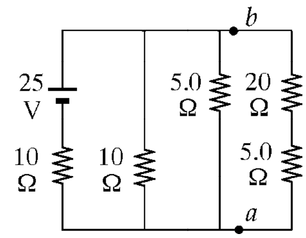
so its power is $\mathcal{P} = (\Delta V)I = 119\ \text{V}(0.620\ \text{A}) = \boxed{73.8\ \text{W}}$

28.9 If we turn the given diagram on its side, we find that it is the same as Figure (a). The 20.0-Ω and 5.00-Ω resistors are in series, so the first reduction is as shown in (b). In addition, since the 10.0-Ω, 5.00-Ω, and 25.0-Ω resistors are then in parallel, we can solve for their equivalent resistance as:



$$R_{eq} = \frac{1}{\left(\frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{25.0\ \Omega}\right)} = 2.94\ \Omega$$

This is shown in Figure (c), which in turn reduces to the circuit shown in (d).

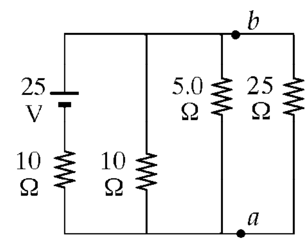


(a)

Next, we work backwards through the diagrams applying $I = \Delta V / R$ and $\Delta V = IR$. The 12.94-Ω resistor is connected across 25.0-V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In Figure (c), this 1.93 A goes through the 2.94-Ω equivalent resistor to give a potential difference of:



(b)

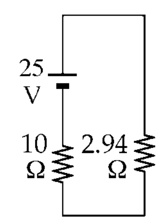
$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$

From Figure (b), we see that this potential difference is the same across V_{ab}, the 10-Ω resistor, and the 5.00-Ω resistor.

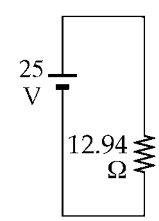
(b) Therefore, $V_{ab} = \boxed{5.68\ \text{V}}$

(a) Since the current through the 20.0-Ω resistor is also the current through the 25.0-Ω line *ab*,

$$I = \frac{V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}$$



(c)

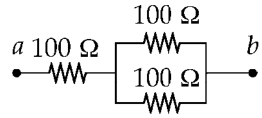


(d)

28.10 $$120\ \text{V} = IR_{eq} = I\left(\frac{\rho\ell}{A_1} + \frac{\rho\ell}{A_2} + \frac{\rho\ell}{A_3} + \frac{\rho\ell}{A_4}\right), \text{ or } \quad I\rho\ell = \frac{(120\ \text{V})}{\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}\right)}$$

$$\Delta V_2 = \frac{I\rho\ell}{A_2} = \frac{(120\ \text{V})}{A_2\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}\right)} = \boxed{29.5\ \text{V}}$$

28.11 (a) Since all the current flowing in the circuit must pass through the series 100-Ω resistor, $P = RI^2$



$$P_{max} = RI_{max}^2 \quad \text{so} \qquad I_{max} = \sqrt{\frac{P}{R}} = \sqrt{\frac{25.0\ \text{W}}{100\ \Omega}} = 0.500\ \text{A}$$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1}\ \Omega = 150\ \Omega$$

$$\Delta V_{max} = R_{eq} I_{max} = \boxed{75.0\ \text{V}}$$

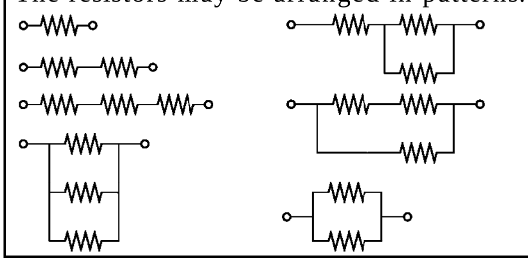
(b) $P = (\Delta V)I = (75.0\ \text{V})(0.500\ \text{A}) = \boxed{37.5\ \text{W}}$ total power

$$P_1 = \boxed{25.0\ \text{W}} \qquad P_2 = P_3 = RI^2 = (100\ \Omega)(0.250\ \text{A})^2 = \boxed{6.25\ \text{W}}$$

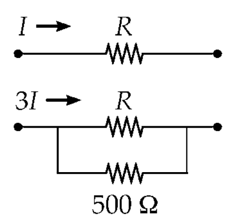
28.12 Using 2.00-Ω, 3.00-Ω, 4.00-Ω resistors, there are 7 series, 4 parallel, and 6 mixed combinations:

Series		Parallel	Mixed
2.00 Ω	6.00 Ω	0.923 Ω	1.56 Ω
3.00 Ω	7.00 Ω	1.20 Ω	2.00 Ω
4.00 Ω	9.00 Ω	1.33 Ω	2.22 Ω
5.00 Ω		1.71 Ω	3.71 Ω
			4.33 Ω
			5.20 Ω

The resistors may be arranged in patterns:



28.13 The potential difference is the same across either combination.



$$\Delta V = IR = 3I\frac{1}{\left(\frac{1}{R} + \frac{1}{500}\right)} \qquad \text{so} \qquad R\left(\frac{1}{R} + \frac{1}{500}\right) = 3$$

$$1 + \frac{R}{500} = 3 \qquad \text{and} \qquad R = 1000\ \Omega = \boxed{1.00\ \text{k}\Omega}$$

28.14 If the switch is open, $I = \mathcal{E} / (R' + R)$ and $P = \mathcal{E}^2 R' / (R' + R)^2$

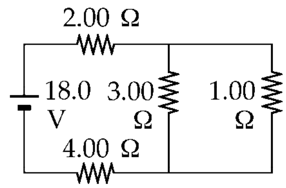
If the switch is closed, $I = \mathcal{E} / (R + R'/2)$ and $P = \mathcal{E}^2 (R'/2) / (R + R'/2)^2$

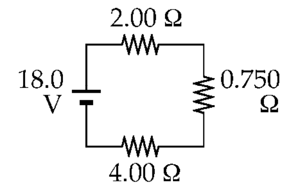
Then,
$$\frac{\mathcal{E}^2 R'}{(R' + R)^2} = \frac{\mathcal{E}^2 R'}{2(R + R'/2)^2}$$

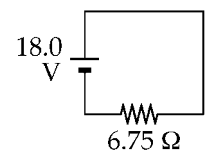
$$2R^2 + 2RR' + R'^2/2 = R'^2 + 2RR' + R^2$$

The condition becomes $R^2 = R'^2 / 2$ so $R' = \sqrt{2}R = \sqrt{2}(1.00\ \Omega) = \boxed{1.41\Omega}$

28.15







$$R_p = \left(\frac{1}{3.00} + \frac{1}{1.00}\right)^{-1} = 0.750\ \Omega$$

$$R_s = (2.00 + 0.750 + 4.00)\ \Omega = 6.75\ \Omega$$

$$I_{\text{battery}} = \frac{\Delta V}{R_s} = \frac{18.0\ \text{V}}{6.75\ \Omega} = 2.67\ \text{A}$$

$$\mathsf{P} = I^2 R: \qquad \mathsf{P}_2 = (2.67\ \text{A})^2 (2.00\ \Omega)$$

$$\mathsf{P}_2 = \boxed{14.2\ \text{W}}\ \text{ in } 2.00\ \Omega$$

$$\mathsf{P}_4 = (2.67\ \text{A})^2 (4.00\ \Omega) = \boxed{28.4\ \text{W}}\ \text{ in } 4.00\ \Omega$$

$$\Delta V_2 = (2.67\ \text{A})(2.00\ \Omega) = 5.33\ \text{V}, \qquad \Delta V_4 = (2.67\ \text{A})(4.00\ \Omega) = 10.67\ \text{V}$$

$$\Delta V_p = 18.0\ \text{V} - \Delta V_2 - \Delta V_4 = 2.00\ \text{V} \qquad (= \Delta V_3 = \Delta V_1)$$

$$\mathsf{P}_3 = \frac{(\Delta V_3)^2}{R_3} = \frac{(2.00\ \text{V})^2}{3.00\ \Omega} = \boxed{1.33\ \text{W}}\ \text{ in } 3.00\ \Omega$$

$$\mathsf{P}_1 = \frac{(\Delta V_1)^2}{R_1} = \frac{(2.00\ \text{V})^2}{1.00\ \Omega} = \boxed{4.00\ \text{W}}\ \text{ in } 1.00\ \Omega$$

28.16 Denoting the two resistors as x and y,

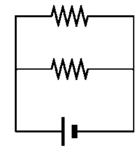
$$x + y = 690, \quad \text{and} \quad \frac{1}{150} = \frac{1}{x} + \frac{1}{y}$$

$$\frac{1}{150} = \frac{1}{x} + \frac{1}{690 - x} = \frac{(690 - x) + x}{x(690 - x)}$$

$$x^2 - 690x + 103{,}500 = 0$$

$$x = \frac{690 \pm \sqrt{(690)^2 - 414{,}000}}{2}$$

$$x = \boxed{470\ \Omega} \qquad y = \boxed{220\ \Omega}$$



28.17 (a) $\Delta V = IR$: $33.0 \text{ V} = I_1(11.0\ \Omega)$ $33.0 \text{ V} = I_2(22.0\ \Omega)$

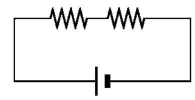
$I_1 = 3.00 \text{ A}$ $I_2 = 1.50 \text{ A}$

$\mathcal{P} = I^2R$: $\mathcal{P}_1 = (3.00 \text{ A})^2(11.0\ \Omega)$ $\mathcal{P}_2 = (1.50 \text{ A})^2(22.0\ \Omega)$

$\mathcal{P}_1 = 99.0 \text{ W}$ $\mathcal{P}_2 = 49.5 \text{ W}$

$\boxed{\text{The 11.0-}\Omega\text{ resistor uses more power.}}$

(b) $\mathcal{P}_1 + \mathcal{P}_2 = \boxed{148 \text{ W}}$ $\mathcal{P} = I(\Delta V) = (4.50)(33.0) = \boxed{148 \text{ W}}$



(c) $R_s = R_1 + R_2 = 11.0\ \Omega + 22.0\ \Omega = 33.0\ \Omega$

$\Delta V = IR$: $33.0 \text{ V} = I(33.0\ \Omega)$, so $I = 1.00 \text{ A}$

$\mathcal{P} = I^2R$: $\mathcal{P}_1 = (1.00 \text{ A})^2(11.0\ \Omega)$ $\mathcal{P}_2 = (1.00 \text{ A})^2(22.0\ \Omega)$

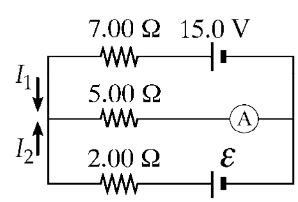
$\mathcal{P}_1 = 11.0 \text{ W}$ $\mathcal{P}_2 = 22.0 \text{ W}$

$\boxed{\text{The 22.0-}\Omega\text{ resistor uses more power.}}$

(d) $\mathcal{P}_1 + \mathcal{P}_2 = I^2(R_1 + R_2) = (1.00 \text{ A})^2(33.0\ \Omega) = \boxed{33.0 \text{ W}}$

$\mathcal{P} = I(\Delta V) = (1.00 \text{ A})(33.0 \text{ V}) = \boxed{33.0 \text{ W}}$

(e) $\boxed{\text{The parallel configuration uses more power.}}$



28.18 $+15.0 - (7.00)I_1 - (2.00)(5.00) = 0$

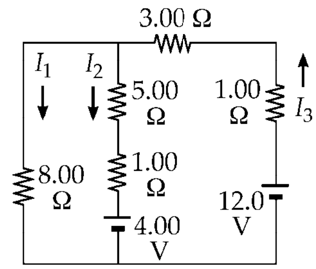
$5.00 = 7.00I_1$ so $\boxed{I_1 = 0.714 \text{ A}}$

$I_3 = I_1 + I_2 = 2.00 \text{ A}$

$0.714 + I_2 = 2.00$ so $\boxed{I_2 = 1.29 \text{ A}}$

$+\mathcal{E} - 2.00(1.29) - (5.00)(2.00) = 0$ $\boxed{\mathcal{E} = 12.6 \text{ V}}$

28.19 We name the currents I_1, I_2, and I_3 as shown.



From Kirchhoff's current rule, $I_3 = I_1 + I_2$

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$12.0\text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00\text{ V} = 0$

$8.00 = (4.00)I_3 + (6.00)I_2$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

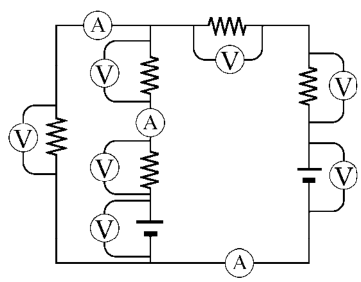
$-(6.00)I_2 - 4.00\text{ V} + (8.00)I_1 = 0$

$(8.00)I_1 = 4.00 + (6.00)I_2$

Solving the above linear systems, $\boxed{I_1 = 846\text{ mA}, \quad I_2 = 462\text{ mA}, \quad I_3 = 1.31\text{ A}}$

All currents flow in the directions indicated by the arrows in the circuit diagram.

***28.20** The solution figure is shown to the right.



***28.21** We use the results of Problem 19.

(a) By the 4.00-V battery: $\Delta U = (\Delta V)It = 4.00\text{ V}(-0.462\text{ A})120\text{ s} = \boxed{-222\text{ J}}$

By the 12.0-V battery: $12.0\text{ V }(1.31\text{ A})\ 120\text{ s} = \boxed{1.88\text{ kJ}}$

(b) By the 8.00 Ω resistor: $I^2 Rt = (0.846\text{ A})^2(8.00\ \Omega)\ 120\text{ s} = \boxed{687\text{ J}}$

By the 5.00 Ω resistor: $(0.462\text{ A})^2(5.00\ \Omega)\ 120\text{ s} = \boxed{128\text{ J}}$

By the 1.00 Ω resistor: $(0.462\text{ A})^2(1.00\ \Omega)\ 120\text{ s} = \boxed{25.6\text{ J}}$

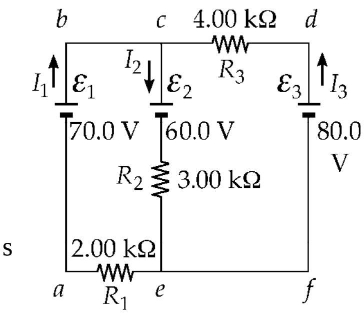
By the 3.00 Ω resistor: $(1.31\text{ A})^2(3.00\ \Omega)\ 120\text{ s} = \boxed{616\text{ J}}$

By the 1.00 Ω resistor: $(1.31\text{ A})^2(1.00\ \Omega)\ 120\text{ s} = \boxed{205\text{ J}}$

(c) $-222\text{ J} + 1.88\text{ kJ} = \boxed{1.66\text{ kJ}}$ from chemical to electrical.

$687\text{ J} + 128\text{ J} + 25.6\text{ J} + 616\text{ J} + 205\text{ J} = 1.66\text{ kJ}$ from electrical to heat.

28.22 We name the currents I_1, I_2, and I_3 as shown.



[1] $70.0 - 60.0 - I_2(3.00\text{ k}\Omega) - I_1(2.00\text{ k}\Omega) = 0$

[2] $80.0 - I_3(4.00\text{ k}\Omega) - 60.0 - I_2(3.00\text{ k}\Omega) = 0$

[3] $I_2 = I_1 + I_3$

(a) Substituting for I_2 and solving the resulting simultaneous equations yields

$I_1 = \boxed{0.385\text{ mA}}$ (through R_1)

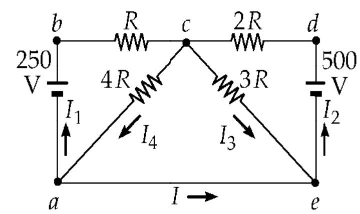
$I_3 = \boxed{2.69\text{ mA}}$ (through R_3)

$I_2 = \boxed{3.08\text{ mA}}$ (through R_2)

(b) $\Delta V_{cf} = -60.0\text{ V} - (3.08\text{ mA})(3.00\text{ k}\Omega) = \boxed{-69.2\text{ V}}$

$\boxed{\text{Point } c \text{ is at higher potential.}}$

28.23 Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.

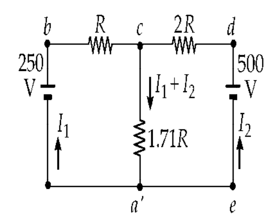


(a)

Apply Kirchhoff's loop rule to both loops in Figure (b) to obtain:

$(2.71R)I_1 + (1.71R)I_2 = 250$ and $(1.71R)I_1 + (3.71R)I_2 = 500$

With $R = 1000\ \Omega$, simultaneous solution of these equations yields:



(b)

$I_1 = 10.0\text{ mA}$ and $I_2 = 130.0\text{ mA}$

From Figure (b), $V_c - V_a = (I_1 + I_2)(1.71R) = 240\text{ V}$

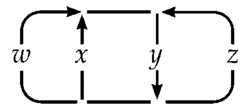
Thus, from Figure (a), $I_4 = \dfrac{V_c - V_a}{4R} = \dfrac{240\text{ V}}{4000\ \Omega} = 60.0\text{ mA}$

Finally, applying Kirchhoff's point rule at point a in Figure (a) gives:

$I = I_4 - I_1 = 60.0\text{ mA} - 10.0\text{ mA} = +50.0\text{ mA}$,

or $I = \boxed{50.0\text{ mA flowing from point } a \text{ to point } e}$.

28.24 Name the currents as shown in the figure to the right. Then $w + x + z = y$. Loop equations are



$$-200w - 40.0 + 80.0x = 0$$

$$-80.0x + 40.0 + 360 - 20.0y = 0$$

$$+360 - 20.0y - 70.0z + 80.0 = 0$$

Eliminate y by substitution.
$$\begin{cases} x = 2.50w + 0.500 \\ 400 - 100x - 20.0w - 20.0z = 0 \\ 440 - 20.0w - 20.0x - 90.0z = 0 \end{cases}$$

Eliminate x:
$$\begin{cases} 350 - 270w - 20.0z = 0 \\ 430 - 70.0w - 90.0z = 0 \end{cases}$$

Eliminate $z = 17.5 - 13.5w$ to obtain $430 - 70.0w - 1575 + 1215w = 0$

$$w = 70.0/70.0 = \boxed{1.00 \text{ A upward in } 200\ \Omega}$$

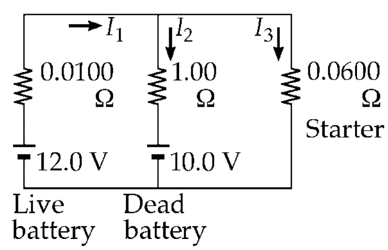
Now
$$z = \boxed{4.00 \text{ A upward in } 70.0\ \Omega}$$

$$x = \boxed{3.00 \text{ A upward in } 80.0\ \Omega}$$

$$y = \boxed{8.00 \text{ A downward in } 20.0\ \Omega}$$

and for the 200 Ω, $\Delta V = IR = (1.00 \text{ A})(200\ \Omega) = \boxed{200 \text{ V}}$

28.25 Using Kirchhoff's rules,



$$12.0 - (0.0100)I_1 - (0.0600)I_3 = 0$$

$$10.0 + (1.00)I_2 - (0.0600)I_3 = 0$$

and $I_1 = I_2 + I_3$

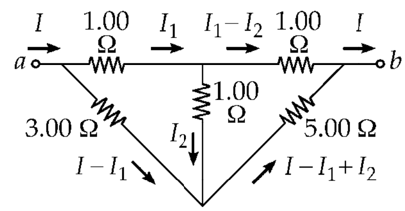
$$12.0 - (0.0100)I_2 - (0.0700)I_3 = 0$$

$$10.0 + (1.00)I_2 - (0.0600)I_3 = 0$$

Solving simultaneously, $I_2 = \boxed{0.283 \text{ A downward}}$ in the dead battery,

and $I_3 = \boxed{171 \text{ A downward}}$ in the starter.

28.26 $V_{ab} = (1.00)I_1 + (1.00)(I_1 - I_2)$



$V_{ab} = (1.00)I_1 + (1.00)I_2 + (5.00)(I - I_1 + I_2)$

$V_{ab} = (3.00)(I - I_1) + (5.00)(I - I_1 + I_2)$

Let $I = 1.00$ A, $I_1 = x$, and $I_2 = y$

Then, the three equations become:

$V_{ab} = 2.00x - y$, or $y = 2.00x - V_{ab}$

$V_{ab} = -4.00x + 6.00y + 5.00$

and $V_{ab} = 8.00 - 8.00x + 5.00y$

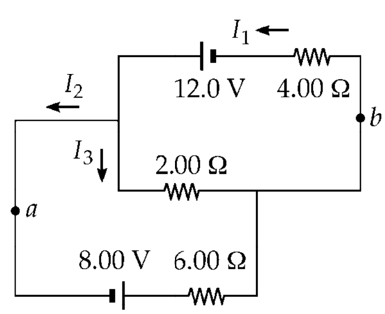
Substituting the first into the last two gives:

$7.00V_{ab} = 8.00x + 5.00$ and $6.00V_{ab} = 2.00x + 8.00$

Solving these simultaneously yields $V_{ab} = \frac{27}{17}$ V

Then, $R_{ab} = \frac{V_{ab}}{I} = \frac{27/17 \text{ V}}{1.00 \text{ A}}$ or $\boxed{R_{ab} = \frac{27}{17}\ \Omega}$

28.27 We name the currents I_1, I_2, and I_3 as shown.



(a) $I_1 = I_2 + I_3$

Counterclockwise around the top loop,

$12.0 \text{ V} - (2.00\ \Omega)I_3 - (4.00\ \Omega)I_1 = 0$

Traversing the bottom loop,

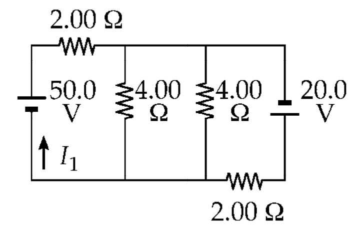
$8.00 \text{ V} - (6.00\ \Omega)I_2 + (2.00\ \Omega)I_3 = 0$

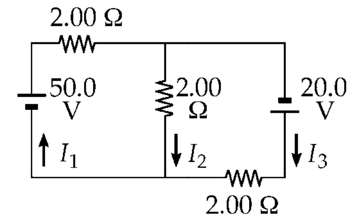
$I_1 = 3.00 - \frac{1}{2} I_3 \quad I_2 = \frac{4}{3} + \frac{1}{3} I_3$ and $\boxed{I_3 = 909 \text{ mA}}$

(b) $V_a - (0.909 \text{ A})(2.00\ \Omega) = V_b$

$V_b - V_a = \boxed{-1.82 \text{ V}}$

28.28 We apply Kirchhoff's rules to the second diagram.





$50.0 - 2.00I_1 - 2.00I_2 = 0$ (1)

$20.0 - 2.00I_3 + 2.00I_2 = 0$ (2)

$I_1 = I_2 + I_3$ (3)

Substitute (3) into (1), and solve for I_1, I_2, and I_3

$I_1 = 20.0$ A; $I_2 = 5.00$ A; $I_3 = 15.0$ A

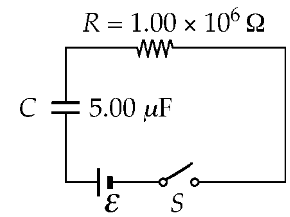
Then apply $\mathcal{P} = I^2R$ to each resistor:

$(2.00\ \Omega)_1$: $\mathcal{P} = I_1^2(2.00\ \Omega) = (20.0\ \text{A})^2(2.00\ \Omega) = \boxed{800\ \text{W}}$

$(4.00\ \Omega)$: $\mathcal{P} = \left(\frac{5.00}{2}\ \text{A}\right)^2(4.00\ \Omega) = \boxed{25.0\ \text{W}}$
(Half of I_2 goes through each)

$(2.00\ \Omega)_3$: $\mathcal{P} = I_3^2(2.00\ \Omega) = (15.0\ \text{A})^2(2.00\ \Omega) = \boxed{450\ \text{W}}$

28.29 (a) $RC = (1.00 \times 10^6\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = \boxed{5.00\ \text{s}}$



(b) $Q = C\mathcal{E} = (5.00 \times 10^{-6}\ \text{C})(30.0\ \text{V}) = \boxed{150\ \mu\text{C}}$

(c) $I(t) = \frac{\mathcal{E}}{R}e^{-t/RC} = \frac{30.0}{1.00 \times 10^6}\exp\left[\frac{-10.0}{(1.00 \times 10^6)(5.00 \times 10^{-6})}\right] = \boxed{4.06\ \mu\text{A}}$

28.30 (a) $I(t) = -I_0 e^{-t/RC}$

$$I_0 = \frac{Q}{RC} = \frac{5.10 \times 10^{-6}\ \text{C}}{(1300\ \Omega)(2.00 \times 10^{-9}\ \text{F})} = 1.96\ \text{A}$$

$$I(t) = -(1.96\ \text{A})\exp\left[\frac{-9.00 \times 10^{-6}\ \text{s}}{(1300\ \Omega)(2.00 \times 10^{-9}\ \text{F})}\right] = \boxed{-61.6\ \text{mA}}$$

(b) $q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\frac{-8.00 \times 10^{-6}\ \text{s}}{(1300\ \Omega)(2.00 \times 10^{-9}\ \text{F})}\right] = \boxed{0.235\ \mu\text{C}}$

(c) The magnitude of the current is $\boxed{I_0 = 1.96\ \text{A}}$

28.31 $U = \frac{1}{2}C(\Delta V)^2$ and $\Delta V = Q/C$

Therefore, $U = Q^2/2C$ and when the charge decreases to half its original value, the stored energy is one-quarter its original value: $\boxed{U_f = \frac{1}{4}U_0}$

28.32 (a) $\tau = RC = (1.50 \times 10^5\ \Omega)(10.0 \times 10^{-6}\ \text{F}) = \boxed{1.50\ \text{s}}$

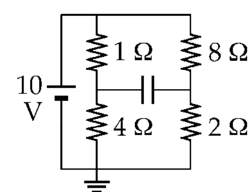
(b) $\tau = (1.00 \times 10^5\ \Omega)(10.0 \times 10^{-6}\ \text{F}) = \boxed{1.00\ \text{s}}$

(c) The battery carries current $\dfrac{10.0\ \text{V}}{50.0 \times 10^3\ \Omega} = 200\ \mu\text{A}$

The 100 kΩ carries current of magnitude $I = I_0 e^{-t/RC} = \left(\dfrac{10.0\ \text{V}}{100 \times 10^3\ \Omega}\right) e^{-t/\ 1.00\ \text{s}}$

So the switch carries downward current $\boxed{200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\ \text{s}}}$

28.33 (a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.



$V_L = 8.00$ V because of voltage divider: $I_L = \dfrac{10.0\ \text{V}}{5.00\ \Omega} = 2.00\ \text{A}$

$V_L = 10.0\ \text{V} - (2.00\ \text{A})(1.00\ \Omega) = 8.00\ \text{V}$

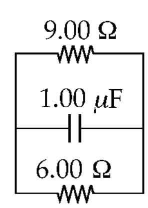
Likewise, $V_R = \left(\dfrac{2.00\ \Omega}{2.00\ \Omega + 8.00\ \Omega}\right) 10.0\ \text{V} = 2.00\ \text{V}$

or $I_R = \dfrac{10.0\ \text{V}}{10.0\ \Omega} = 1.00\ \text{A}$

$V_R = (10.0\ \text{V}) - (8.00\ \Omega)(1.00\ \text{A}) = 2.00\ \text{V}$

Therefore, $\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00\ \text{V}}$

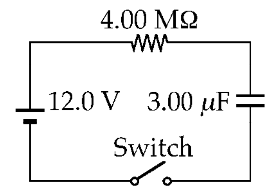
(b) Redraw the circuit $R = \dfrac{1}{(1/9.00\ \Omega) + (1/6.00\ \Omega)} = 3.60\ \Omega$



$RC = 3.60 \times 10^{-6}$ s

and $e^{-t/RC} = \dfrac{1}{10}$ so $t = RC \ln 10 = \boxed{8.29\ \mu\text{s}}$

28.34 (a) $\tau = RC = \left(4.00\times10^{6}\ \Omega\right)\left(3.00\times10^{-6}\ \text{F}\right) = \boxed{12.0\ \text{s}}$



(b) $I = \dfrac{\mathcal{E}}{R}e^{-t/RC} = \dfrac{12.0}{4.00\times10^{6}}\,e^{-t/12.0\ \text{s}}$

$q = C\mathcal{E}\left[1-e^{-t/RC}\right] = 3.00\times10^{-6}(12.0)\left[1-e^{-t/12.0}\right)$

$\boxed{q = 36.0\ \mu\text{C}\left[1-e^{-t/12.0}\right]}$ $\boxed{I = 3.00\ \mu\text{A}e^{-t/12.0}}$

28.35 $\Delta V_0 = \dfrac{Q}{C}$

Then, if $q(t) = Qe^{-t/RC}$ $\Delta V(t) = \Delta V_0 e^{-t/RC}$

$$\frac{\Delta V(t)}{\Delta V_0} = e^{-t/RC}$$

Therefore $\dfrac{1}{2} = \exp\left(-\dfrac{4.00}{R\left(3.60\times10^{-6}\right)}\right)$

$$\ln\left(\frac{1}{2}\right) = -\frac{4.00}{R\left(3.60\times10^{-6}\right)}$$

$$R = \boxed{1.60\ \text{M}\Omega}$$

28.36 $\Delta V_0 = \dfrac{Q}{C}$

Then, if $q(t) = Q\,e^{-t/RC}$ $\Delta V(t) = \left(\Delta V_0\right)e^{-t/RC}$

and $\dfrac{\Delta V(t)}{\left(\Delta V_0\right)} = e^{-t/RC}$

When $\Delta V(t) = \dfrac{1}{2}\left(\Delta V_0\right)$, then $e^{-t/RC} = \dfrac{1}{2}$

$$-\frac{t}{RC} = \ln\left(\frac{1}{2}\right) = -\ln 2$$

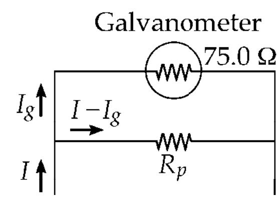
Thus, $\boxed{R = \dfrac{t}{C(\ln 2)}}$

28.37 $q(t) = Q\left[1 - e^{-t/RC}\right]$ so $\dfrac{q(t)}{Q} = 1 - e^{-t/RC}$

$0.600 = 1 - e^{-0.900/RC}$ or $e^{-0.900/RC} = 1 - 0.600 = 0.400$

$\dfrac{-0.900}{RC} = \ln(0.400)$ thus $RC = \dfrac{-0.900}{\ln(0.400)} = \boxed{0.982\ \text{s}}$

28.38 Applying Kirchhoff's loop rule, $-I_g(75.0\ \Omega) + \left(I - I_g\right)R_p = 0$



Therefore, if $I = 1.00$ A when $I_g = 1.50$ mA,

$$R_p = \frac{I_g(75.0\ \Omega)}{\left(I - I_g\right)} = \frac{\left(1.50 \times 10^{-3}\ \text{A}\right)(75.0\ \Omega)}{1.00\ \text{A} - 1.50 \times 10^{-3}\ \text{A}} = \boxed{0.113\ \Omega}$$

28.39 Series Resistor $\rightarrow$ Voltmeter

$\Delta V = IR$: $25.0 = 1.50 \times 10^{-3}(R_s + 75.0)$

Solving, $\boxed{R_s = 16.6\ \text{k}\Omega}$

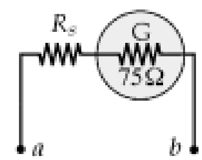


Figure for Goal Solution

Goal Solution

The galvanometer described in the preceding problem can be used to measure voltages. In this case a large resistor is wired in series with the galvanometer in a way similar to that shown in Figure P28.24b This arrangement, in effect, limits the current that flows through the galvanometer when large voltages are applied. Most of the potential drop occurs across the resistor placed in series. Calculate the value of the resistor that enables the galvanometer to measure an applied voltage of 25.0 V at full-scale deflection.

G: The problem states that the value of the resistor must be "large" in order to limit the current through the galvanometer, so we should expect a resistance of kΩ to MΩ.

O: The unknown resistance can be found by applying the definition of resistance to the portion of the circuit shown in Figure 28.24b.

A: $\Delta V_{ab} = 25.0$ V; From Problem 38, $I = 1.50$ mA and $R_g = 75.0\ \Omega$. For the two resistors in series, $R_{eq} = R_s + R_g$ so the definition of resistance gives us: $\Delta V_{ab} = I(R_s + R_g)$

Therefore, $R_s = \dfrac{\Delta V_{ab}}{I} - R_g = \dfrac{25.0\ \text{V}}{1.50 \times 10^{-3}\ \text{A}} - 75.0\ \Omega = 16.6\ \text{k}\Omega$

L: The resistance is relatively large, as expected. It is important to note that some caution would be necessary if this arrangement were used to measure the voltage across a circuit with a comparable resistance. For example, if the circuit resistance was 17 kΩ, the voltmeter in this problem would cause a measurement inaccuracy of about 50%, because the meter would divert about half the current that normally would go through the resistor being measured. Problems 46 and 59 address a similar concern about measurement error when using electrical meters.

28.40 We will use the values required for the 1.00-V voltmeter to obtain the internal resistance of the galvanometer. $\Delta V = I_g(R + r_g)$

Solve for r_g: $r_g = \dfrac{\Delta V}{I_g} - R = \dfrac{1.00 \text{ V}}{1.00 \times 10^{-3} \text{ A}} - 900\ \Omega = 100\ \Omega$

We then obtain the series resistance required for the 50.0-V voltmeter:

$$R = \frac{V}{I_g} - r_g = \frac{50.0 \text{ V}}{1.00 \times 10^{-3} \text{ A}} - 100\ \Omega = \boxed{49.9 \text{ k}\Omega}$$

28.41 $\Delta V = I_g r_g = \left(I - I_g\right)R_p$, or $R_p = \dfrac{I_g r_g}{\left(I - I_g\right)} = \dfrac{I_g(60.0\ \Omega)}{\left(I - I_g\right)}$

Therefore, to have $I = 0.100 \text{ A} = 100 \text{ mA}$ when $I_g = 0.500 \text{ mA}$:

$$R_p = \frac{(0.500 \text{ mA})(60.0\ \Omega)}{99.5 \text{ mA}} = \boxed{0.302\ \Omega}$$

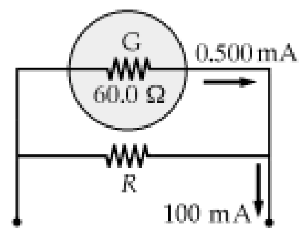


Figure for Goal Solution

Goal Solution

Assume that a galvanometer has an internal resistance of $60.0\ \Omega$ and requires a current of 0.500 mA to produce full-scale deflection. What resistance must be connected in parallel with the galvanometer if the combination is to serve as an ammeter that has a full-scale deflection for a current of 0.100 A?

G: An ammeter reads the flow of current in a portion of a circuit; therefore it must have a low resistance so that it does not significantly alter the current that would exist without the meter. Therefore, the resistance required is probably less than $1\ \Omega$.

O: From the values given for a full-scale reading, we can find the voltage across and the current through the shunt (parallel) resistor, and the resistance value can then be found from the definition of resistance.

A: The voltage across the galvanometer must be the same as the voltage across the shunt resistor in parallel, so when the ammeter reads full scale,

$$\Delta V = (0.500 \text{ mA})(60.0\ \Omega) = 30.0 \text{ mV}$$

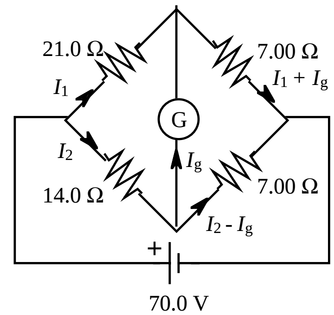
Through the shunt resistor, $I = 100 \text{ mA} - 0.500 \text{ mA} = 99.5 \text{ mA}$

Therefore, $R = \dfrac{\Delta V}{I} = \dfrac{30.0 \text{ mV}}{99.5 \text{ mA}} = 0.302\ \Omega$

L: The shunt resistance is less than $1\ \Omega$ as expected. It is important to note that some caution would be necessary if this meter were used in a circuit that had a low resistance. For example, if the circuit resistance was $3\ \Omega$, adding the ammeter to the circuit would reduce the current by about 10%, so the current displayed by the meter would be lower than without the meter. Problems 46 and 59 address a similar concern about measurement error when using electrical meters.

28.42 $R_x = \dfrac{R_2 R_3}{R_1} = \dfrac{R_2 R_3}{2.50 R_2} = \dfrac{1000\ \Omega}{2.50} = \boxed{400\ \Omega}$

28.43 Using Kirchhoff's rules with $R_g << 1$,



$-(21.0\ \Omega)I_1 + (14.0\ \Omega)I_2 = 0$, so $I_1 = \dfrac{2}{3} I_2$

$70.0 - 21.0 I_1 - 7.00\left(I_1 + I_g\right) = 0$, and

$70.0 - 14.0 I_2 - 7.00\left(I_2 - I_g\right) = 0$

The last two equations simplify to

$10.0 - 4.00\left(\frac{2}{3} I_2\right) = I_g$, and $10.0 - 3.00 I_2 = -I_g$

Solving simultaneously yields: $I_g = \boxed{0.588\ \text{A}}$

28.44 $R = \dfrac{\rho L}{A}$ and $R_i = \dfrac{\rho L_i}{A_i}$

But, $V = AL = A_i L_i$, so $R = \dfrac{\rho L^2}{V}$ and $R_i = \dfrac{\rho L_i^2}{V}$

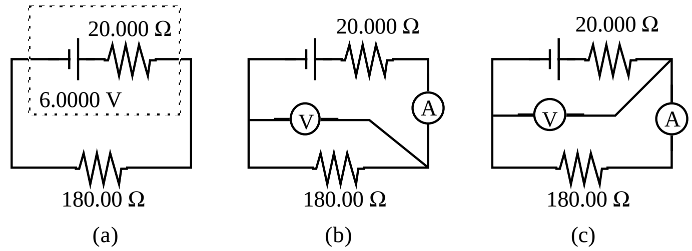
Therefore, $R = \dfrac{\rho\left(L_i + \Delta L\right)^2}{V} = \dfrac{\rho L_i\left[1 + \left(\Delta L / L_i\right)\right]^2}{V} = R_i\left[1 + \alpha\right]^2$ where $\alpha \equiv \dfrac{\Delta L}{L}$

This may be written as: $\boxed{R = R_i(1 + 2\alpha + \alpha^2)}$

28.45 $\dfrac{\mathcal{E}_x}{R_s} = \dfrac{\mathcal{E}_s}{R_s}$; $\mathcal{E}_x = \dfrac{\mathcal{E}_s R_x}{R_s} = \left(\dfrac{48.0\ \Omega}{36.0\ \Omega}\right)(1.0186\ \text{V}) = \boxed{1.36\ \text{V}}$

***28.46** (a) In Figure (a), the emf sees an equivalent resistance of 200.00 Ω.

$$I = \frac{6.000\ 0\ \text{V}}{200.00\ \Omega} = \boxed{0.030\ 000\ \text{A}}$$



The terminal potential difference is $\Delta V = IR = (0.030\ 000\ \text{A})(180.00\ \Omega) = \boxed{5.400\ 0\ \text{V}}$

(b) In Figure (b), $R_{eq} = \left(\frac{1}{180.00\ \Omega} + \frac{1}{20\ 000\ \Omega}\right)^{-1} = 178.39\ \Omega$

The equivalent resistance across the emf is $178.39\ \Omega + 0.500\ 00\ \Omega + 20.000\ \Omega = 198.89\ \Omega$

The ammeter reads $I = \frac{\mathcal{E}}{R} = \frac{6.000\ 0\ \text{V}}{198.89\ \Omega} = \boxed{0.030\ 167\ \text{A}}$

and the voltmeter reads $\Delta V = IR = (0.030\ 167\ \text{A})(178.39\ \Omega) = \boxed{5.381\ 6\ \text{V}}$

(c) In Figure (c), $\left(\frac{1}{180.50\ \Omega} + \frac{1}{20\ 000\ \Omega}\right)^{-1} = 178.89\ \Omega$

Therefore, the emf sends current through $R_{tot} = 178.89\ \Omega + 20.000\ \Omega = 198.89\ \Omega$

The current through the battery is $I = \frac{6.000\ 0\ \text{V}}{198.89\ \Omega} = 0.030\ 168\ \text{A}$

but not all of this goes through the ammeter.

The voltmeter reads $\Delta V = IR = (0.030\ 168\ \text{A})(178.89\ \Omega) = \boxed{5.396\ 6\ \text{V}}$

The ammeter measures current $I = \frac{\Delta V}{R} = \frac{5.396\ 6\ \text{V}}{180.50\ \Omega} = \boxed{0.029\ 898\ \text{A}}$

The connection shown in Figure (c) is better than that shown in Figure (b) for accurate readings.

28.47 (a) $\mathcal{P} = I(\Delta V)$ So for the Heater, $I = \frac{\mathcal{P}}{\Delta V} = \frac{1500\ \text{W}}{120\ \text{V}} = \boxed{12.5\ \text{A}}$

For the Toaster, $I = \frac{750\ \text{W}}{120\ \text{W}} = \boxed{6.25\ \text{A}}$

And for the Grill, $I = \frac{1000\ \text{W}}{120\ \text{V}} = \boxed{8.33\ \text{A}}$ (Grill)

(b) $12.5 + 6.25 + 8.33 = \boxed{27.1\ \text{A}}$ The current draw is greater than 25.0 amps, so this would not be sufficient.

28.48 (a) $\mathcal{P} = I^2R = I^2\left(\frac{\rho l}{A}\right) = \frac{(1.00\text{ A})^2(1.70\times10^{-8}\ \Omega\cdot\text{m})(16.0\text{ ft})(0.3048\text{ m / ft})}{\pi(0.512\times10^{-3}\text{ m})^2} = \boxed{0.101\text{ W}}$

(b) $\mathcal{P} = I^2R = 100(0.101\ \Omega) = \boxed{10.1\text{ W}}$

28.49 $I_{\text{Al}}^2R_{\text{Al}} = I_{\text{Cu}}^2R_{\text{Cu}}$ so $I_{\text{Al}} = \sqrt{\frac{R_{\text{Cu}}}{R_{\text{Al}}}}I_{\text{Cu}} = \sqrt{\frac{\rho_{\text{Cu}}}{\rho_{\text{Al}}}}I_{\text{Cu}} = \sqrt{\frac{1.70}{2.82}}(20.0) = 0.776(20.0) = \boxed{15.5\text{ A}}$

***28.50** (a) Suppose that the insulation between either of your fingers and the conductor adjacent is a chunk of rubber with contact area 4 mm^2 and thickness 1 mm. Its resistance is

$$R = \frac{\rho l}{A} \cong \frac{\left(10^{13}\ \Omega\cdot\text{m}\right)\left(10^{-3}\text{ m}\right)}{4\times10^{-6}\text{ m}^2} \cong 2\times10^{15}\ \Omega$$

The current will be driven by 120 V through total resistance (series)

$$2\times10^{15}\ \Omega + 10^4\ \Omega + 2\times10^{15}\ \Omega \cong 5\times10^{15}\ \Omega$$

It is: $I = \frac{\Delta V}{R} \sim \frac{120\text{ V}}{5\times10^{15}\ \Omega}$ $\boxed{\sim 10^{-14}\text{ A}}$

(b) The resistors form a voltage divider, with the center of your hand at potential $V_h/2$, where V_h is the potential of the "hot" wire. The potential difference between your finger and thumb is $\Delta V = IR \sim \left(10^{-14}\text{ A}\right)\left(10^4\ \Omega\right) \sim 10^{-10}\text{ V}$. So the points where the rubber meets your fingers are at potentials of

$\boxed{\sim \frac{V_h}{2} + 10^{-10}\text{ V}}$ and $\boxed{\sim \frac{V_h}{2} - 10^{-10}\text{ V}}$

***28.51** The set of four batteries boosts the electric potential of each bit of charge that goes through them by $4\times1.50\text{ V} = 6.00\text{ V}$. The chemical energy they store is

$$\Delta U = q\Delta V = (240\text{ C})(6.00\text{ J / C}) = 1440\text{ J}$$

The radio draws current $I = \frac{\Delta V}{R} = \frac{6.00\text{ V}}{200\ \Omega} = 0.0300\text{ A}$

So, its power is $\mathcal{P} = (\Delta V)I = (6.00\text{ V})(0.0300\text{ A}) = 0.180\text{ W} = 0.180\text{ J / s}$

Then for the time the energy lasts, we have $\mathcal{P} = E/t$: $t = \frac{E}{\mathcal{P}} = \frac{1440\text{ J}}{0.180\text{ J / s}} = 8.00\times10^3\text{ s}$

We could also compute this from $I = Q/t$: $t = \frac{Q}{I} = \frac{240\text{ C}}{0.0300\text{ A}} = 8.00\times10^3\text{ s} = \boxed{2.22\text{ h}}$

***28.52** $I = \dfrac{\mathcal{E}}{R+r}$, so $\mathcal{P} = I^2 R = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$ or $(R+r)^2 = \left(\dfrac{\mathcal{E}^2}{\mathcal{P}}\right)R$

Let $x \equiv \dfrac{\mathcal{E}^2}{\mathcal{P}}$, then $(R+r)^2 = xR$ or $R^2 + (2r - x)R - r^2 = 0$

With $r = 1.20\ \Omega$, this becomes $R^2 + (2.40 - x)R - 1.44 = 0$,

which has solutions of $R = \dfrac{-(2.40 - x) \pm \sqrt{(2.40 - x)^2 - 5.76}}{2}$

(a) With $\mathcal{E} = 9.20$ V and $\mathcal{P} = 12.8$ W, $x = 6.61$: $R = \dfrac{+4.21 \pm \sqrt{(4.21)^2 - 5.76}}{2} = \boxed{3.84\ \Omega}$ or $\boxed{0.375\ \Omega}$

(b) For $\mathcal{E} = 9.20$ V and $\mathcal{P} = 21.2$ W, $x \equiv \dfrac{\mathcal{E}^2}{\mathcal{P}} = 3.99$ $R = \dfrac{+1.59 \pm \sqrt{(1.59)^2 - 5.76}}{2} = \dfrac{1.59 \pm \sqrt{-3.22}}{2}$

The equation for the load resistance yields a complex number, so $\boxed{\text{there is no resistance}}$ that will extract 21.2 W from this battery. The maximum power output occurs when $R = r = 1.20\ \Omega$, and that maximum is: $\mathcal{P}_{max} = \mathcal{E}^2/4r = 17.6$ W

28.53 Using Kirchhoff's loop rule for the closed loop, $+12.0 - 2.00I - 4.00I = 0$, so $I = 2.00$ A

$V_b - V_a = +4.00\text{ V} - (2.00\text{ A})(4.00\ \Omega) - (0)(10.0\ \Omega) = -4.00$ V

Thus, $|\Delta V_{ab}| = \boxed{4.00\text{ V}}$ and $\boxed{\text{point } a \text{ is at the higher potential}}$.

28.54 The potential difference across the capacitor $\Delta V(t) = \Delta V_{max}\left[1 - e^{-t/RC}\right]$

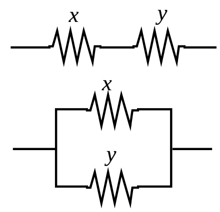
Using $1\text{ Farad} = 1\text{ s}/\Omega$, $4.00\text{ V} = (10.0\text{ V})\left[1 - e^{-(3.00\text{ s})/R(10.0 \times 10^{-6}\text{ s}/\Omega)}\right]$

Therefore, $0.400 = 1.00 - e^{-(3.00 \times 10^5\ \Omega)/R}$ or $e^{-(3.00 \times 10^5\ \Omega)/R} = 0.600$

Taking the natural logarithm of both sides, $-\dfrac{3.00 \times 10^5\ \Omega}{R} = \ln(0.600)$

and $R = -\dfrac{3.00 \times 10^5\ \Omega}{\ln(0.600)} = +5.87 \times 10^5\ \Omega = \boxed{587\text{ k}\Omega}$

28.55 Let the two resistances be x and y.



Then, $R_s = x + y = \frac{P_s}{I^2} = \frac{225\text{ W}}{(5.00\text{ A})^2} = 9.00\ \Omega$ $\qquad y = 9.00\ \Omega - x$

and $R_p = \frac{xy}{x+y} = \frac{P_p}{I^2} = \frac{50.0\text{ W}}{(5.00\text{ A})^2} = 2.00\ \Omega$

so $\frac{x(9.00\ \Omega - x)}{x + (9.00\ \Omega - x)} = 2.00\ \Omega$ $\qquad x^2 - 9.00x + 18.0 = 0$

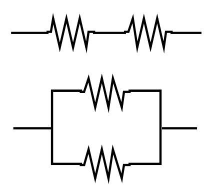
Factoring the second equation, $\qquad (x - 6.00)(x - 3.00) = 0$

so $\qquad x = 6.00\ \Omega$ or $x = 3.00\ \Omega$

Then, $y = 9.00\ \Omega - x$ gives $\qquad y = 3.00\ \Omega$ or $y = 6.00\ \Omega$

The two resistances are found to be $\boxed{6.00\ \Omega}$ and $\boxed{3.00\ \Omega}$.

28.56 Let the two resistances be x and y.



Then, $R_s = x + y = \frac{P_s}{I^2}$ and $R_p = \frac{xy}{x+y} = \frac{P_p}{I^2}$.

From the first equation, $y = \frac{P_s}{I^2} - x$, and the second

becomes $\frac{x\left(P_s/I^2 - x\right)}{x + \left(P_s/I^2 - x\right)} = \frac{P_p}{I^2}$ or $x^2 - \left(\frac{P_s}{I^2}\right)x + \frac{P_s P_p}{I^4} = 0$.

Using the quadratic formula, $x = \frac{P_s \pm \sqrt{P_s^2 - 4P_s P_p}}{2I^2}$.

Then, $y = \frac{P_s}{I^2} - x$ gives $y = \frac{P_s > \sqrt{P_s^2 - 4P_s P_p}}{2I^2}$.

The two resistances are $\boxed{\frac{P_s + \sqrt{P_s^2 - 4P_s P_p}}{2I^2}}$ and $\boxed{\frac{P_s - \sqrt{P_s^2 - 4P_s P_p}}{2I^2}}$

28.57 The current in the simple loop circuit will be $I = \dfrac{\mathcal{E}}{R+r}$

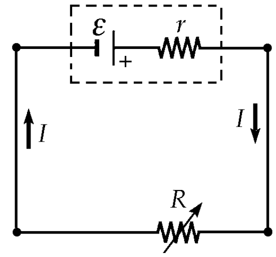


Figure for Goal Solution

(a) $\Delta V_{\text{ter}} = \mathcal{E} - Ir = \dfrac{\mathcal{E}R}{R+r}$ and $\Delta V_{\text{ter}} \to \mathcal{E}$ as $\boxed{R \to \infty}$

(b) $I = \dfrac{\mathcal{E}}{R+r}$ and $I \to \dfrac{\mathcal{E}}{r}$ as $\boxed{R \to 0}$

(c) $\mathcal{P} = I^2R = \mathcal{E}^2 \dfrac{R}{(R+r)^2}$

$$\frac{d\mathcal{P}}{dR} = \mathcal{E}^2R(-2)(R+r)^{-3} + \mathcal{E}^2(R+r)^{-2} = \frac{-2\mathcal{E}^2R}{(R+r)^3} + \frac{\mathcal{E}^2}{(R+r)^2} = 0$$

Then $2R = R + r$ and $\boxed{R = r}$

Goal Solution

A battery has an emf $\mathcal{E}$ and internal resistance r. A variable resistor R is connected across the terminals of the battery. Determine the value of R such that (a) the potential difference across the terminals is a maximum, (b) the current in the circuit is a maximum, (c) the power delivered to the resistor is a maximum.

G: If we consider the limiting cases, we can imagine that the **potential** across the battery will be a maximum when $R = \infty$ (open circuit), the **current** will be a maximum when $R = 0$ (short circuit), and the **power** will be a maximum when R is somewhere between these two extremes, perhaps when $R = r$.

O: We can use the definition of resistance to find the voltage and current as functions of R, and the power equation can be differentiated with respect to R.

A: (a) The battery has a voltage $\Delta V_{\text{terminal}} = \mathcal{E} - Ir = \dfrac{\mathcal{E}R}{R+r}$ or as $R \to \infty$, $\Delta V_{\text{terminal}} \to \mathcal{E}$

(b) The circuit's current is $I = \dfrac{\mathcal{E}}{R+r}$ or as $R \to 0$, $I \to \dfrac{\mathcal{E}}{r}$

(c) The power delivered is $\mathcal{P} = I^2R = \dfrac{\mathcal{E}^2R}{(R+r)^2}$

To maximize the power $\mathcal{P}$ as a function of R, we differentiate with respect to R, and require that $d\mathcal{P}/dR = 0$

$$\frac{d\mathcal{P}}{dR} = \mathcal{E}^2R(-2)(R+r)^{-3} + \mathcal{E}^2(R+r)^{-2} = \frac{-2\mathcal{E}^2R}{(R+r)^3} + \frac{\mathcal{E}^2}{(R+r)^2} = 0$$

Then $2R = R + r$ and $R = r$

L: The results agree with our predictions. Making load resistance equal to the source resistance to maximize power transfer is called impedance matching.

28.58 (a) $\mathcal{E} - I(\Sigma R) - (\mathcal{E}_1 + \mathcal{E}_2) = 0$

$40.0\text{ V} - (4.00\text{ A})[(2.00 + 0.300 + 0.300 + R)\Omega] - (6.00 + 6.00)\text{ V} = 0;$ so $R = \boxed{4.40\ \Omega}$

(b) Inside the supply, $\mathcal{P} = I^2R = (4.00\text{ A})^2(2.00\ \Omega) = \boxed{32.0\text{ W}}$

Inside both batteries together, $\mathcal{P} = I^2R = (4.00\text{ A})^2(0.600\ \Omega) = \boxed{9.60\text{ W}}$

For the limiting resistor, $\mathcal{P} = (4.00\text{ A})^2(4.40\ \Omega) = \boxed{70.4\text{ W}}$

(c) $\mathcal{P} = I(\mathcal{E}_1 + \mathcal{E}_2) = (4.00\text{ A})[(6.00 + 6.00)\text{V}] = \boxed{48.0\text{ W}}$

28.59 Let R_m = measured value, R = actual value,

I_R = current through the resistor R

I = current measured by the ammeter.

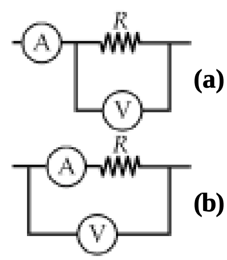


Figure for Goal solution

(a) When using circuit (a), $I_R R = \Delta V = 20\,000(I - I_R)$ or $R = 20\,000\left[\dfrac{I}{I_R} - 1\right]$

But since $I = \dfrac{\Delta V}{R_m}$ and $I_R = \dfrac{\Delta V}{R}$, we have $\dfrac{I}{I_R} = \dfrac{R}{R_m}$

and $R = 20\,000\dfrac{(R - R_m)}{R_m}$ (1)

When $R > R_m$, we require $\dfrac{(R - R_m)}{R} \le 0.0500$

Therefore, $R_m \ge R(1 - 0.0500)$ and from (1) we find $\boxed{R \le 1050\ \Omega}$

(b) When using circuit (b), $I_R R = \Delta V - I_R(0.5\ \Omega)$.

But since $I_R = \dfrac{\Delta V}{R_m}$, $R_m = (0.500 + R)$ (2)

When $R_m > R$, we require $\dfrac{(R_m - R)}{R} \le 0.0500$

From (2) we find $\boxed{R \ge 10.0\ \Omega}$

Goal Solution

The value of a resistor R is to be determined using the ammeter-voltmeter setup shown in Figure P28.59. The ammeter has a resistance of $0.500\ \Omega$, and the voltmeter has a resistance of $20\,000\ \Omega$. Within what range of actual values of R will the measured values be correct to within 5.00% if the measurement is made using (a) the circuit shown in Figure P28.59a (b) the circuit shown in Figure P28.59b?

G: An ideal ammeter has zero resistance, and an ideal voltmeter has infinite resistance, so that adding the meter does not alter the current or voltage of the existing circuit. For the non-ideal meters in this problem, a low values of R will give a large voltage measurement error in circuit (b), while a large value of R will give significant current measurement error in circuit (a). We could hope that these meters yield accurate measurements in either circuit for typical resistance values of $1\ \Omega$ to $1\ \text{M}\Omega$.

O: The definition of resistance can be applied to each circuit to find the minimum and maximum current and voltage allowed within the 5.00% tolerance range.

A: (a) In Figure P28.59a, at least a little current goes through the voltmeter, so less current flows through the resistor than the ammeter reports, and the resistance computed by dividing the voltage by the inflated ammeter reading will be too small. Thus, we require that $\Delta V/I = 0.950R$ where I is the current through the ammeter. Call I_R the current through the resistor; then $I - I_R$ is the current in the voltmeter. Since the resistor and the voltmeter are in parallel, the voltage across the meter equals the voltage across the resistor. Applying the definition of resistance:

$$\Delta V = I_R R = (I - I_R)(20\,000\ \Omega) \quad \text{so} \quad I = \frac{I_R(R + 20\,000\ \Omega)}{20\,000\ \Omega}$$

Our requirement is $$\frac{I_R R}{\left(\dfrac{I_R(R+20\,000\ \Omega)}{20\,000\ \Omega}\right)} \geq 0.95R$$

Solving, $$20\,000\ \Omega \geq 0.95(R + 20\,000\ \Omega) = 0.95R + 19\,000\ \Omega$$

and $$R \leq \frac{1000\ \Omega}{0.95} \quad \text{or} \quad R \leq 1.05\ \text{k}\Omega$$

(b) If R is too small, the resistance of an ammeter in series will significantly reduce the current that would otherwise flow through R. In Figure 28.59b, the voltmeter reading is $I(0.500\ \Omega) + IR$, at least a little larger than the voltage across the resistor. So the resistance computed by dividing the inflated voltmeter reading by the ammeter reading will be too large.

We require $$\frac{V}{I} \leq 1.05R \quad \text{so that} \quad \frac{I\,(0.500\ \Omega) + IR}{I} \leq 1.05R$$

Thus, $$0.500\ \Omega \leq 0.0500R \quad \text{and} \quad R \geq 10.0\ \Omega$$

L: The range of R values seems correct since the ammeter's resistance should be less than 5% of the smallest R value ($0.500\ \Omega \leq 0.05R$ means that R should be greater than $10\ \Omega$), and R should be less than 5% of the voltmeter's internal resistance ($R \leq 0.05 \times 20\ \text{k}\Omega = 1\ \text{k}\Omega$). Only for the restricted range between 10 ohms and 1000 ohms can we indifferently use either of the connections (a) and (b) for a reasonably accurate resistance measurement. For low values of the resistance R, circuit (a) must be used. Only circuit (b) can accurately measure a large value of R.

28.60 The battery supplies energy at a changing rate $\frac{dE}{dt} = \mathcal{P} = \mathcal{E}I = \mathcal{E}\left(\frac{\mathcal{E}}{R}e^{-1/RC}\right)$

Then the total energy put out by the battery is $\int dE = \int_{t=0}^{\infty} \frac{\mathcal{E}^2}{R} \exp\left(-\frac{t}{RC}\right)dt$

$$\int dE = \frac{\mathcal{E}^2}{R}(-RC)\int_0^{\infty} \exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right) = -\mathcal{E}^2 C \exp\left(-\frac{t}{RC}\right)\Big|_0^{\infty} = -\mathcal{E}^2 C[0-1] = \mathcal{E}^2 C$$

The heating power of the resistor is $\frac{dE}{dt} = \mathcal{P} = \Delta V_R I = I^2 R = R\frac{\mathcal{E}^2}{R^2}\exp\left(-\frac{2t}{RC}\right)$

So the total heat is $\int dE = \int_0^{\infty} \frac{\mathcal{E}^2}{R}\exp\left(-\frac{2t}{RC}\right)dt$

$$\int dE = \frac{\mathcal{E}^2}{R}\left(-\frac{RC}{2}\right)\int_0^{\infty} \exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right) = -\frac{\mathcal{E}^2 C}{2}\exp\left(-\frac{2t}{RC}\right)\Big|_0^{\infty} = -\frac{\mathcal{E}^2 C}{2}[0-1] = \frac{\mathcal{E}^2 C}{2}$$

The energy finally stored in the capacitor is $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\mathcal{E}^2$. Thus, energy is conserved: $\mathcal{E}^2 C = \frac{1}{2}\mathcal{E}^2 C + \frac{1}{2}\mathcal{E}^2 C$ and resistor and capacitor share equally in the energy from the battery.

28.61 (a) $q = C(\Delta V)\left[1 - e^{-t/RC}\right]$

$$q = \left(1.00\times10^{-6}\text{ F}\right)(10.0\text{ V})\left[1 - e^{-\frac{10.0}{\left(2.00\times10^{6}\right)\left(1.00\times10^{-6}\right)}}\right] = \boxed{9.93\ \mu\text{C}}$$

(b) $I = \frac{dq}{dt} = \left(\frac{\Delta V}{R}\right)e^{-t/RC}$

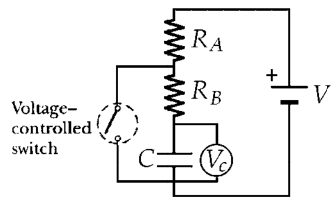
$$I = \left(\frac{10.0\text{ V}}{2.00\times10^{6}\ \Omega}\right)e^{-5.00} = 3.37\times10^{-8}\text{ A} = \boxed{33.7\text{ nA}}$$

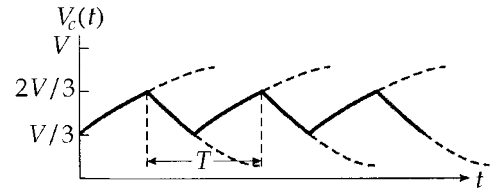
(c) $\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right) = \frac{q}{C}\frac{dq}{dt} = \left(\frac{q}{C}\right)I$

$$\frac{dU}{dt} = \left(\frac{9.93\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ C/V}}\right)\left(3.37\times10^{-8}\text{ A}\right) = 3.34\times10^{-7}\text{ W} = \boxed{334\text{ nW}}$$

(d) $\mathcal{P}_{\text{battery}} = I\mathcal{E} = \left(3.37\times10^{-8}\text{ A}\right)(10.0\text{ V}) = 3.37\times10^{-7}\text{ W} = \boxed{337\text{ nW}}$

28.62 Start at the point when the voltage has just reached $\frac{2}{3}V$ and the switch has just closed. The voltage is $\frac{2}{3}V$ and is decaying towards 0 V with a time constant R_BC.





$$V_C(t) = \left[\frac{2}{3}V\right]e^{-t/R_BC}$$

We want to know when $V_C(t)$ will reach $\frac{1}{3}V$.

Therefore, $\left(\frac{1}{3}\right)V = \left[\frac{2}{3}V\right]e^{-t/R_BC}$ or $e^{-t/R_BC} = \frac{1}{2}$

or $t_1 = R_BC \ln 2$

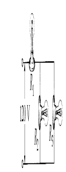
After the switch opens, the voltage is $\frac{1}{3}V$, increasing toward V with time constant $(R_A + R_B)C$:

$$V_C(t) = V - \left[\frac{2}{3}V\right]e^{-t/(R_A+R_B)C}$$

When $V_C(t) = \frac{2}{3}V$, $\frac{2}{3}V = V - \frac{2}{3}Ve^{-t/(R_A+R_B)C}$ or $e^{-t/(R_A+R_B)C} = \frac{1}{2}$

so $t_2 = (R_A + R_B)C \ln 2$ and $T = t_1 + t_2 = \boxed{(R_A + 2R_B)C \ln 2}$

28.63 (a) First determine the resistance of each light bulb: $\mathcal{P} = (\Delta V)^2/R$



$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{60.0\text{ W}} = 240\ \Omega$$

We obtain the equivalent resistance R_{eq} of the network of light bulbs by applying Equations 28.6 and 28.7:

$$R_{eq} = R_1 + \frac{1}{(1/R_2) + (1/R_3)} = 240\ \Omega + 120\ \Omega = 360\ \Omega$$

The total power dissipated in the 360 Ω is $\mathcal{P} = \frac{(\Delta V)^2}{R_{eq}} = \frac{(120\text{ V})^2}{360\ \Omega} = \boxed{40.0\text{ W}}$

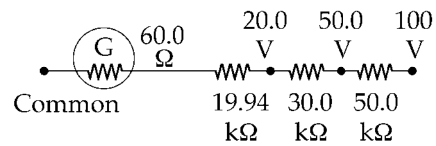
(b) The current through the network is given by $\mathcal{P} = I^2R_{eq}$: $I = \sqrt{\frac{\mathcal{P}}{R_{eq}}} = \sqrt{\frac{40.0\text{ W}}{360\ \Omega}} = \frac{1}{3}\text{ A}$

The potential difference across R_1 is $\Delta V_1 = IR_1 = \left(\frac{1}{3}\text{ A}\right)(240\ \Omega) = \boxed{80.0\text{ V}}$

The potential difference ΔV_{23} across the parallel combination of R_2 and R_3 is

$$\Delta V_{23} = IR_{23} = \left(\frac{1}{3}\text{ A}\right)\left(\frac{1}{(1/240\ \Omega) + (1/240\ \Omega)}\right) = \boxed{40.0\text{ V}}$$

28.64 $\Delta V = IR$



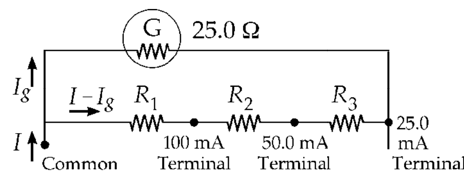
(a) $20.0 \text{ V} = (1.00 \times 10^{-3} \text{ A})(R_1 + 60.0\ \Omega)$

$R_1 = 1.994 \times 10^4\ \Omega = \boxed{19.94 \text{ k}\Omega}$

(b) $50.0 \text{ V} = (1.00 \times 10^{-3} \text{ A})(R_2 + R_1 + 60.0\ \Omega)$ $\qquad R_2 = \boxed{30.0 \text{ k}\Omega}$

(c) $100 \text{ V} = (1.00 \times 10^{-3} \text{ A})(R_3 + R_1 + 60.0\ \Omega)$ $\qquad R_3 = \boxed{50.0 \text{ k}\Omega}$

28.65 Consider the circuit diagram shown, realizing that $I_g = 1.00$ mA. For the 25.0 mA scale:



$$(24.0 \text{ mA})(R_1 + R_2 + R_3) = (1.00 \text{ mA})(25.0\ \Omega)$$

or $$R_1 + R_2 + R_3 = \left(\frac{25.0}{24.0}\right)\Omega \qquad (1)$$

For the 50.0 mA scale: $$(49.0 \text{ mA})(R_1 + R_2) = (1.00 \text{ mA})(25.0\ \Omega + R_3)$$

or $$49.0(R_1 + R_2) = 25.0\ \Omega + R_3 \qquad (2)$$

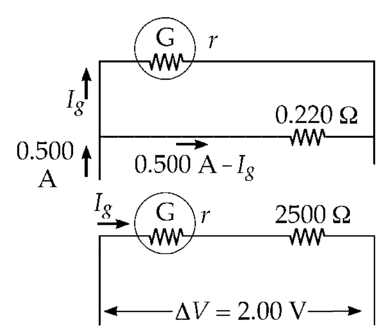
For the 100 mA scale: $$(99.0 \text{ mA})R_1 = (1.00 \text{ mA})(25.0\ \Omega + R_2 + R_3)$$

or $$99.0 R_1 = 25.0\ \Omega + R_2 + R_3 \qquad (3)$$

Solving (1), (2), and (3) simultaneously yields

$$\boxed{R_1 = 0.260\ \Omega, \quad R_2 = 0.261\ \Omega, \quad R_3 = 0.521\ \Omega}$$

28.66 Ammeter: $I_g r = (0.500 \text{ A} - I_g)(0.220\ \Omega)$



or $I_g(r + 0.220\ \Omega) = 0.110 \text{ V}$ (1)

Voltmeter: $2.00 \text{ V} = I_g(r + 2500\ \Omega)$ (2)

Solve (1) and (2) simultaneously to find:

$I_g = \boxed{0.756 \text{ mA}}$ and $r = \boxed{145\ \Omega}$

28.67 (a) After steady-state conditions have been reached, there is no DC current through the capacitor.

Thus, for R_3: $\boxed{I_{R_3} = 0 \quad \text{(steady-state)}}$

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12-kΩ and 15-kΩ resistors in series:

For R_1 and R_2: $$I_{(R_1+R_2)} = \frac{\mathcal{E}}{R_1+R_2} = \frac{9.00\text{ V}}{(12.0\text{ k}\Omega + 15.0\text{ k}\Omega)} = \boxed{333\ \mu\text{A (steady-state)}}$$

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

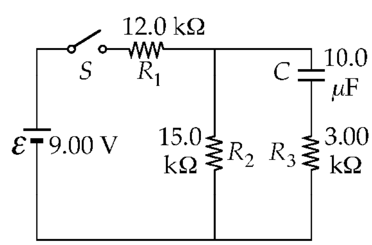
$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10.0\ \mu\text{F})(333\ \mu\text{A})(15.0\text{ k}\Omega) = \boxed{50.0\ \mu\text{C}}$$

(c) When the switch is opened, the branch containing R_1 is no longer part of the circuit. The capacitor discharges through $(R_2 + R_3)$ with a time constant of $(R_2 + R_3)C$ = (15.0 kΩ + 3.00 kΩ)(10.0 μF) = 0.180 s. The initial current I_i in this discharge circuit is determined by the initial potential difference across the capacitor applied to $(R_2 + R_3)$ in series:

$$I_i = \frac{(\Delta V)_C}{(R_2+R_3)} = \frac{IR_2}{(R_2+R_3)} = \frac{(333\ \mu\text{A})(15.0\text{ k}\Omega)}{(15.0\text{ k}\Omega + 3.00\text{ k}\Omega)} = 278\ \mu\text{A}$$

Thus, when the switch is opened, the current through R_2 changes instantaneously from 333 μA (downward) to 278 μA (downward) as shown in the graph. Thereafter, it decays according to

$$I_{R_2} = I_i e^{-t/(R_2+R_3)C} = \boxed{(278\ \mu\text{A})e^{-t/(0.180\text{ s})} \quad (\text{for } t > 0)}$$



(a)

(d) The charge q on the capacitor decays from Q_i to $Q_i/5$ according to

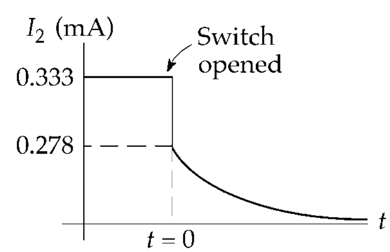
$$q = Q_i e^{-t/(R_2+R_3)C}$$

$$\frac{Q_i}{5} = Q_i e^{(-t/0.180\text{ s})}$$

$$5 = e^{t/0.180\text{ s}}$$

$$\ln 5 = \frac{t}{180\text{ ms}}$$

$$t = (0.180\text{ s})(\ln 5) = \boxed{290\text{ ms}}$$



(b)

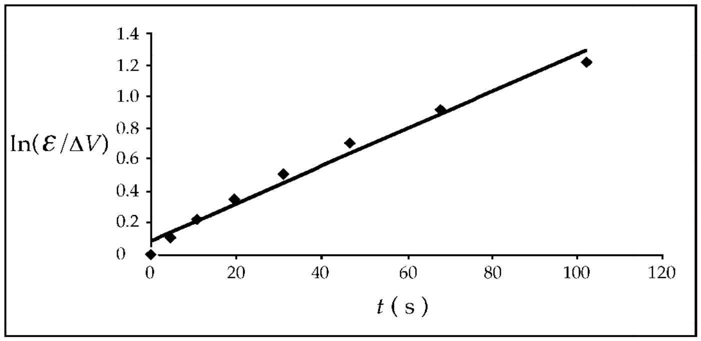
28.68 $\Delta V = \mathcal{E} e^{-t/RC}$ so $\ln\left(\dfrac{\mathcal{E}}{\Delta V}\right) = \left(\dfrac{1}{RC}\right)t$

A plot of $\ln\left(\dfrac{\mathcal{E}}{\Delta V}\right)$ versus t should be a straight line with slope $= \dfrac{1}{RC}$.

Using the given data values:

t (s)	ΔV (V)	$\ln(\mathcal{E}/\Delta V)$
0	6.19	0
4.87	5.55	0.109
11.1	4.93	0.228
19.4	4.34	0.355
30.8	3.72	0.509
46.6	3.09	0.695
67.3	2.47	0.919
102.2	1.83	1.219

(a) A least-square fit to this data yields the graph to the right.



$\Sigma x_i = 282$, $\Sigma x_i^2 = 1.86 \times 10^4$, $\Sigma x_i y_i = 244$, $\Sigma y_i = 4.03$, $N = 8$

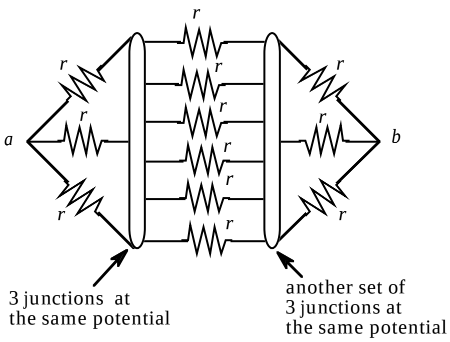
$$\text{Slope} = \frac{N(\Sigma x_i y_i) - (\Sigma x_i)(\Sigma y_i)}{N(\Sigma x_i^2) - (\Sigma x_i)^2} = 0.0118 \qquad \text{Intercept} = \frac{(\Sigma x_i^2)(\Sigma y_i) - (\Sigma x_i)(\Sigma x_i y_i)}{N(\Sigma x_i^2) - (\Sigma x_i)^2} = 0.0882$$

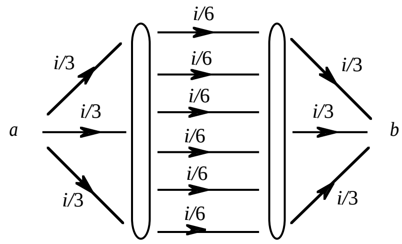
The equation of the best fit line is: $\boxed{\ln\left(\dfrac{\mathcal{E}}{\Delta V}\right) = (0.0118)t + 0.0882}$

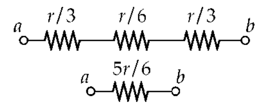
(b) Thus, the time constant is $\tau = RC = \dfrac{1}{\text{slope}} = \dfrac{1}{0.0118} = \boxed{84.7\text{ s}}$

and the capacitance is $C = \dfrac{\tau}{R} = \dfrac{84.7\text{ s}}{10.0 \times 10^6\ \Omega} = \boxed{8.47\ \mu\text{F}}$

28.69







28.70 (a) For the first measurement, the equivalent circuit is as shown in Figure 1.

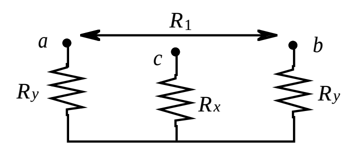


Figure 1

$$R_{ab} = R_1 = R_y + R_y = 2R_y$$

so $$R_y = \frac{1}{2}R_1 \qquad (1)$$

For the second measurement, the equivalent circuit is shown in Figure 2.

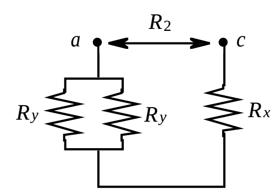


Figure 2

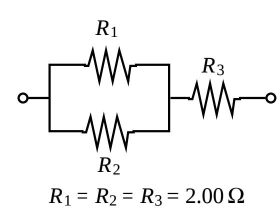
Thus, $$R_{ac} = R_2 = \frac{1}{2}R_y + R_x \qquad (2)$$

Substitute (1) into (2) to obtain: $R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x$, or $\boxed{R_x = R_2 - \frac{1}{4}R_1}$

(b) If $R_1 = 13.0\ \Omega$ and $R_2 = 6.00\ \Omega$, then $\boxed{R_x = 2.75\ \Omega}$

$\boxed{\text{The antenna is inadequately grounded}}$ since this exceeds the limit of $2.00\ \Omega$.

28.71 Since the total current passes through R_3, that resistor will dissipate the most power. When that resistor is operating at its power limit of 32.0 W, the current through it is



$$I_{\text{total}}^2 = \frac{\mathcal{P}}{R} = \frac{32.0\ \text{W}}{2.00\ \Omega} = 16.0\ \text{A}^2,\ \text{or}\ I_{\text{total}} = 4.00\ \text{A}$$

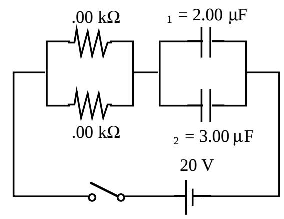
Half of this total current (2.00 A) flows through each of the other two resistors, so the power dissipated in each of them is:

$$\mathcal{P} = \left(\tfrac{1}{2}I_{\text{total}}\right)^2 R = (2.00\ \text{A})^2(2.00\ \Omega) = 8.00\ \text{W}$$

Thus, the total power dissipated in the entire circuit is:

$$\mathsf{P}_{\text{total}} = 32.0\ \text{W} + 8.00\ \text{W} + 8.00\ \text{W} = \boxed{48.0\ \text{W}}$$

28.72 The total resistance between points *b* and *c* is:



$$R = \frac{(2.00\ \text{k}\Omega)(3.00\ \text{k}\Omega)}{2.00\ \text{k}\Omega + 3.00\ \text{k}\Omega} = 1.20\ \text{k}\Omega$$

The total capacitance between points *d* and *e* is:

$$C = 2.00\ \mu\text{F} + 3.00\ \mu\text{F} = 5.00\ \mu\text{F}$$

The potential difference between point *d* and *e* in this series *RC* circuit at any time is:

$$\Delta V = \mathcal{E}\left[1 - e^{-t/RC}\right] = (120.0\ \text{V})\left[1 - e^{-1000t/6}\right]$$

Therefore, the charge on each capacitor between points *d* and *e* is:

$$q_1 = C_1(\Delta V) = (2.00\ \mu\text{F})(120.0\ \text{V})\left[1 - e^{-1000t/6}\right] = \boxed{(240\ \mu\text{C})\left[1 - e^{-1000t/6}\right]}$$

and $q_2 = C_2(\Delta V) = (3.00\ \mu\text{F})(120.0\ \text{V})\left[1 - e^{-1000t/6}\right] = \boxed{(360\ \mu\text{C})\left[1 - e^{-1000t/6}\right]}$

***28.73** (a) $R_{\text{eq}} = 3R$ $\quad I = \dfrac{\mathcal{E}}{3R}$ $\quad \mathsf{P}_{\text{series}} = \mathcal{E}I = \boxed{\mathcal{E}^2/3R}$

(b) $R_{\text{eq}} = \dfrac{1}{(1/R) + (1/R) + (1/R)} = \dfrac{R}{3}$ $\quad I = \dfrac{3\mathcal{E}}{R}$ $\quad \mathsf{P}_{\text{parallel}} = \mathcal{E}I = \boxed{\dfrac{3\mathcal{E}^2}{R}}$

(c) Nine times more power is converted in the $\boxed{\text{parallel}}$ connection.

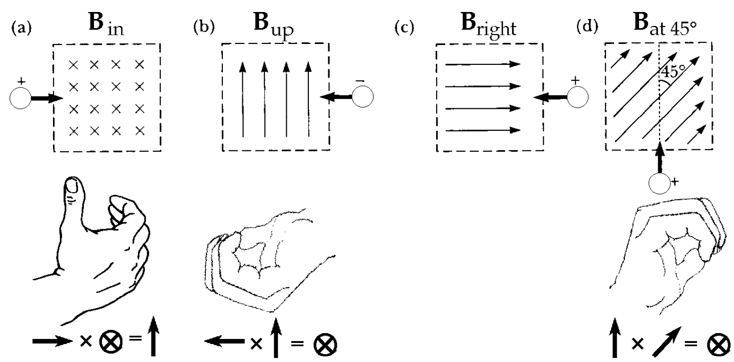
Chapter 29 Solutions

29.1 (a) up

(b) out of the page, since the charge is negative.

(c) no deflection

(d) into the page



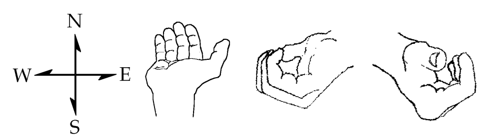
29.2 At the equator, the Earth's magnetic field is horizontally north. Because an electron has negative charge, $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ is opposite in direction to $\mathbf{v} \times \mathbf{B}$. Figures are drawn looking down.

(a) Down × North = East, so the force is directed $\boxed{\text{West}}$

(b) North × North = sin 0° = 0: $\boxed{\text{Zero deflection}}$

(c) West × North = Down, so the force is directed $\boxed{\text{Up}}$

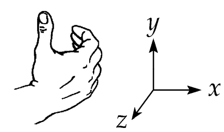
(d) Southeast × North = Up, so the force is $\boxed{\text{Down}}$



(a) (c) (d)

29.3 $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$; $\quad |\mathbf{F}_B|(-\mathbf{j}) = -e\,|\mathbf{v}|\,\mathbf{i} \times \mathbf{B}$

Therefore, $B = |\mathbf{B}|(-\mathbf{k})$ which indicates the $\boxed{\text{negative } z \text{ direction}}$



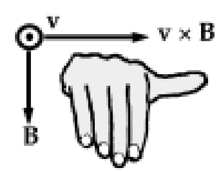
***29.4** (a) $F_B = qvB \sin\theta = (1.60 \times 10^{-19}\text{ C})(3.00 \times 10^{6}\text{ m/s})(3.00 \times 10^{-1}\text{ T}) \sin 37.0°$

$$F_B = \boxed{8.67 \times 10^{-14}\text{ N}}$$

(b) $$a = \frac{F}{m} = \frac{8.67 \times 10^{-14}\text{ N}}{1.67 \times 10^{-27}\text{ kg}} = \boxed{5.19 \times 10^{13}\text{ m/s}^2}$$

29.5 $F = ma = (1.67 \times 10^{-27}\text{ kg})(2.00 \times 10^{13}\text{ m/s}^2) = 3.34 \times 10^{-14}\text{ N} = qvB \sin 90°$

$$B = \frac{F}{qv} = \frac{3.34 \times 10^{-14}\text{ N}}{(1.60 \times 10^{-19}\text{ C})(1.00 \times 10^{7}\text{ m/s})} = \boxed{2.09 \times 10^{-2}\text{ T}}$$



The right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when v is in the z direction.

***29.6** First find the speed of the electron: $\Delta K = \frac{1}{2}mv^2 = e(\Delta V) = \Delta U$

$$v = \sqrt{\frac{2e(\Delta V)}{m}} = \sqrt{\frac{2\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(2400\ \mathrm{J/C}\right)}{\left(9.11\times10^{-31}\ \mathrm{kg}\right)}} = 2.90\times10^{7}\ \mathrm{m/s}$$

(a) $F_{B,\ \max} = qvB = (1.60\times10^{-19}\ \mathrm{C})(2.90\times10^{7}\ \mathrm{m/s})(1.70\ \mathrm{T}) = \boxed{7.90\times10^{-12}\ \mathrm{N}}$

(b) $F_{B,\ \min} = \boxed{0}$ occurs when v is either parallel to or anti-parallel to B

29.7 Gravitational force: $F_g = mg = (9.11\times10^{-31}\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = \boxed{8.93\times10^{-30}\ \mathrm{N\ down}}$

Electric force: $F_e = qE = (-1.60\times10^{-19}\ \mathrm{C})100\ \mathrm{N/C\ down} = \boxed{1.60\times10^{-17}\ \mathrm{N\ up}}$

Magnetic force: $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = \left(-1.60\times10^{-19}\ \mathrm{C}\right)\left(6.00\times10^{6}\ \frac{\mathrm{m}}{\mathrm{s}}\ \mathbf{E}\right)\times\left(50.0\times10^{-6}\ \frac{\mathrm{N\cdot s}}{\mathrm{C\cdot m}}\ \mathbf{N}\right)$

$\mathbf{F}_B = -4.80\times10^{-17}\ \mathrm{N\ up} = \boxed{4.80\times10^{-17}\ \mathrm{N\ down}}$

29.8 We suppose the magnetic force is small compared to gravity. Then its horizontal velocity component stays nearly constant. We call it $v\,\mathbf{i}$.

From $v_y{}^2 = v_{yi}{}^2 + 2a_y(y-y_i)$, the vertical component at impact is $-\sqrt{2gh}\,\mathbf{j}$. Then,

$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = Q\left(v\mathbf{i} - \sqrt{2gh}\ \mathbf{j}\right)\times B\mathbf{k} = QvB(-\mathbf{j}) - Q\sqrt{2gh}\ B\mathbf{i}$$

$\mathbf{F}_B = QvB$ vertical $+\ Q\sqrt{2gh}\ B$ horizontal

$\mathbf{F}_B = 5.00\times10^{-6}\ \mathrm{C}(20.0\ \mathrm{m/s})(0.0100\ \mathrm{T})\ \mathbf{j} + 5.00\times10^{-6}\ \mathrm{C}\sqrt{2(9.80\ \mathrm{m/s^2})(20.0\ \mathrm{m})}\ (0.0100\ \mathrm{T})\ \mathbf{i}$

$\mathbf{F}_B = \boxed{(1.00\times10^{-6}\ \mathrm{N})\ \text{vertical} + (0.990\times10^{-6}\ \mathrm{N})\ \text{horizontal}}$

29.9 $F_B = qvB\sin\theta$ so $8.20\times10^{-13}\ \mathrm{N} = (1.60\times10^{-19}\ \mathrm{C})(4.00\times10^{6}\ \mathrm{m/s})(1.70\ \mathrm{T})\sin\theta$

$\sin\theta = 0.754$ and $\theta = \sin^{-1}(0.754) = \boxed{48.9°\ \text{or}\ 131°}$

29.10 $q\mathbf{E} = \left(-1.60\times10^{-19}\ \text{C}\right)\left(20.0\ \text{N / C}\right)\mathbf{k} = \left(-3.20\times10^{-18}\ \text{N}\right)\mathbf{k}$

$\Sigma\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} = m\mathbf{a}$

$\left(-3.20\times10^{-18}\ \text{N}\right)\mathbf{k} - 1.60\times10^{-19}\ \text{C}(1.20\times10^{4}\ \text{m / s}\ \mathbf{i})\times\mathbf{B} = (9.11\times10^{-31})(2.00\times10^{12}\ \text{m / s}^2)\mathbf{k}$

$-(3.20\times10^{-18}\ \text{N})\mathbf{k} - (1.92\times10^{-15}\ \text{C}\cdot\text{m / s})\mathbf{i}\times\mathbf{B} = (1.82\times10^{-18}\ \text{N})\mathbf{k}$

$(1.92\times10^{-15}\ \text{C}\cdot\text{m / s})\mathbf{i}\times\mathbf{B} = -(5.02\times10^{-18}\ \text{N})\mathbf{k}$

The magnetic field $\boxed{\text{may have any } x\text{-component}}$. $B_z = \boxed{0}$ and $B_y = \boxed{-2.62\ \text{mT}}$

29.11 $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix} = (12-2)\mathbf{i} + (1+6)\mathbf{j} + (4+4)\mathbf{k} = 10\,\mathbf{i} + 7\,\mathbf{j} + 8\,\mathbf{k}$$

$|\mathbf{v}\times\mathbf{B}| = \sqrt{10^2 + 7^2 + 8^2} = 14.6\ \text{T}\cdot\text{m / s}$

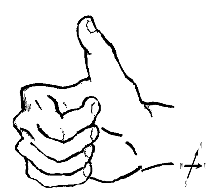
$|\mathbf{F}_B| = q|\mathbf{v}\times\mathbf{B}| = \left(1.60\times10^{-19}\ \text{C}\right)\left(14.6\ \text{T}\cdot\text{m/s}\right) = \boxed{2.34\times10^{-18}\ \text{N}}$

29.12

$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = \left(-1.60\times10^{-19}\right)\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3.70\times10^{5} & 0 \\ 1.40 & 2.10 & 0 \end{vmatrix}$$

$$\mathbf{F}_B = \left(-1.60\times10^{-19}\ \text{C}\right)\left[(0-0)\mathbf{i} + (0-0)\mathbf{j} + \left(0 - (1.40\ \text{T})\left(3.70\times10^{5}\ \text{m/s}\right)\right)\mathbf{k}\right] = \boxed{\left(8.29\times10^{-14}\ \mathbf{k}\right)\text{N}}$$

29.13 $F_B = ILB\sin\theta$ with $F_B = F_g = mg$



$mg = ILB\sin\theta$ so $\dfrac{m}{L}\,g = IB\sin\theta$

$I = 2.00\ \text{A}$ and $\dfrac{m}{L} = \left(0.500\ \text{g / cm}\right)\left(\dfrac{100\ \text{cm / m}}{1000\ \text{g / kg}}\right) = 5.00\times10^{-2}\ \text{kg / m}$

Thus $(5.00\times10^{-2})(9.80) = (2.00)B\sin 90.0°$

$B = \boxed{0.245\ \text{Tesla}}$ with the direction given by right-hand rule: $\boxed{\text{eastward}}$

Goal Solution

A wire having a mass per unit length of 0.500 g/cm carries a 2.00-A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?

G: Since $\mathbf{I} = 2.00$ A south, **B** must be to the east to make **F** upward according to the right-hand rule for currents in a magnetic field.

The magnitude of **B** should be significantly greater than the earth's magnetic field (~ 50 μT), since we do not typically see wires levitating when current flows through them.

O: The force on a current-carrying wire in a magnetic field is $\mathbf{F}_B = I\mathbf{l}\times\mathbf{B}$, from which we can find **B**.

A: With I to the south and **B** to the east, the force on the wire is simply $F_B = IlB\sin 90°$, which must oppose the weight of the wire, mg. So,

$$B = \frac{F_B}{Il} = \frac{mg}{Il} = \frac{g}{I}\left(\frac{m}{l}\right) = \left(\frac{9.80\text{ m / s}^2}{2.00\text{ A}}\right)\left(0.500\ \frac{\text{g}}{\text{cm}}\right)\left(\frac{10^2\text{ cm / m}}{10^3\text{ g / kg}}\right) = 0.245\text{ T}$$

L: The required magnetic field is about 5000 times stronger than the earth's magnetic field. Thus it was reasonable to ignore the earth's magnetic field in this problem. In other situations the earth's field can have a significant effect.

29.14 $\mathbf{F}_B = I\mathbf{L}\times\mathbf{B} = (2.40\text{ A})(0.750\text{ m})\mathbf{i}\times(1.60\text{ T})\mathbf{k} = \boxed{(-2.88\ \mathbf{j})\text{ N}}$

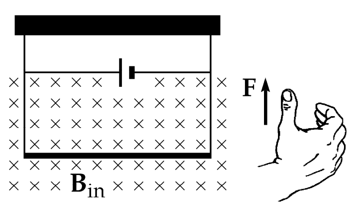
29.15 (a) $F_B = ILB\sin\theta = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 60.0° = \boxed{4.73\text{ N}}$

(b) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 90.0° = \boxed{5.46\text{ N}}$

(c) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 120° = \boxed{4.73\text{ N}}$

29.16 $$\frac{|\mathbf{F}_B|}{L} = \frac{mg}{L} = \frac{I|\mathbf{L}\times\mathbf{B}|}{L}$$

$$I = \frac{mg}{BL} = \frac{(0.0400\text{ kg / m})(9.80\text{ m / s}^2)}{3.60\text{ T}} = \boxed{0.109\text{ A}}$$



The direction of I in the bar is $\boxed{\text{to the right}}$.

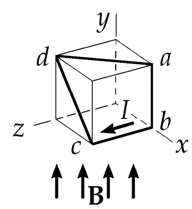
29.17 The magnetic and gravitational forces must balance. Therefore, it is necessary to have $F_B = BIL = mg$, or $I = (mg/BL) = (\lambda g/B)$ [λ is the mass per unit length of the wire].

Thus, $I = \dfrac{(1.00\times 10^{-3}\ \text{kg/m})(9.80\ \text{m/s}^2)}{(5.00\times 10^{-5}\ \text{T})} = \boxed{196\ \text{A}}$ (if $B = 50.0\ \mu\text{T}$)

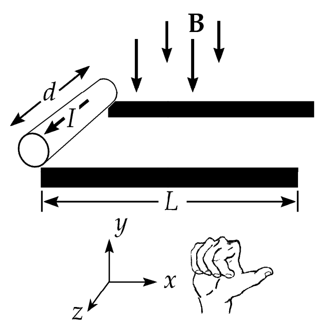
The required direction of the current is $\boxed{\text{eastward}}$, since East × North = Up.

29.18 For each segment, $I = 5.00$ A and $\mathbf{B} = 0.0200\ \text{N/A}\cdot\text{m}\ \mathbf{j}$

Segment	$\mathbf{L}$	$\mathbf{F}_B = I(\mathbf{L}\times\mathbf{B})$
ab	$-0.400\ \text{m}\ \mathbf{j}$	$\boxed{0}$
bc	$0.400\ \text{m}\ \mathbf{k}$	$\boxed{(40.0\ \text{mN})(-\mathbf{i})}$
cd	$-0.400\ \text{m}\ \mathbf{i} + 0.400\ \text{m}\ \mathbf{j}$	$\boxed{(40.0\ \text{mN})(-\mathbf{k})}$
da	$0.400\ \text{m}\ \mathbf{i} - 0.400\ \text{m}\ \mathbf{k}$	$\boxed{(40.0\ \text{mN})(\mathbf{k}+\mathbf{i})}$



29.19 The rod feels force $\mathbf{F}_B = I(\mathbf{d}\times\mathbf{B}) = Id(\mathbf{k})\times B(-\mathbf{j}) = IdB(\mathbf{i})$



The work-energy theorem is $(K_{\text{trans}} + K_{\text{rot}})_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}})_f$

$$0 + 0 + Fs\cos\theta = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$IdBL\cos 0^\circ = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \quad \text{and} \quad IdBL = \tfrac{3}{4}mv^2$$

$$v = \sqrt{\frac{4IdBL}{3m}} = \sqrt{\frac{4(48.0\ \text{A})(0.120\ \text{m})(0.240\ \text{T})(0.450\ \text{m})}{3(0.720\ \text{kg})}} = \boxed{1.07\ \text{m/s}}$$

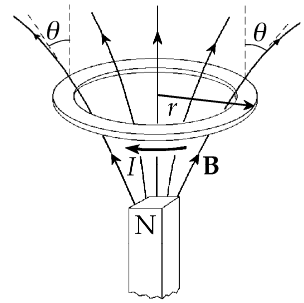
29.20 The rod feels force $\mathbf{F}_B = I(\mathbf{d}\times\mathbf{B}) = Id(\mathbf{k})\times B(-\mathbf{j}) = IdB(\mathbf{i})$

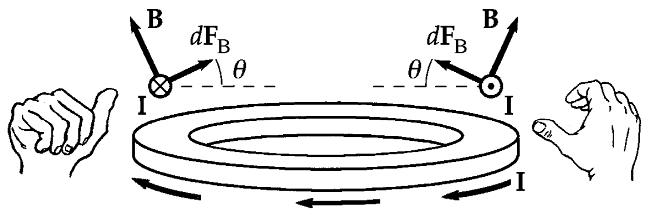
The work-energy theorem is $(K_{\text{trans}} + K_{\text{rot}})_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}})_f$

$$0 + 0 + Fs\cos\theta = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$IdBL\cos 0^\circ = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \quad \text{and} \quad v = \boxed{\sqrt{\frac{4IdBL}{3m}}}$$

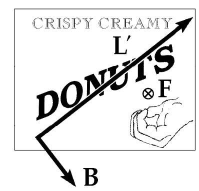
29.21 The magnetic force on each bit of ring is $I\,d\mathbf{s} \times \mathbf{B} = I\,ds\,B$ radially inward and upward, at angle θ above the radial line. The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $I\,ds\,B\sin\theta$ all add to $\boxed{I\,2\pi rB\sin\theta \text{ up}}$.





***29.22** Take the x-axis east, the y-axis up, and the z-axis south. The field is

$$\mathbf{B} = (52.0\ \mu\text{T})\cos 60.0^\circ\,(-\mathbf{k}) + (52.0\ \mu\text{T})\sin 60.0^\circ\,(-\mathbf{j})$$



The current then has equivalent length: $\mathbf{L}' = 1.40\ \text{m}(-\mathbf{k}) + 0.850\ \text{m}(\mathbf{j})$

$$\mathbf{F}_B = I\mathbf{L}' \times \mathbf{B} = (0.0350\ \text{A})(0.850\,\mathbf{j} - 1.40\,\mathbf{k})\,\text{m} \times (-45.0\,\mathbf{j} - 26.0\,\mathbf{k})10^{-6}\ \text{T}$$

$$\mathbf{F}_B = 3.50 \times 10^{-8}\ \text{N}(-22.1\mathbf{i} - 63.0\mathbf{i}) = 2.98 \times 10^{-6}\ \text{N}(-\mathbf{i}) = \boxed{2.98\ \mu\text{N west}}$$

29.23 (a) $2\pi r = 2.00$ m so $r = 0.318$ m

$$\mu = IA = (17.0 \times 10^{-3}\ \text{A})\left[\pi(0.318)^2\ \text{m}^2\right] = \boxed{5.41\ \text{mA}\cdot\text{m}^2}$$

(b) $\tau = \boldsymbol{\mu} \times \mathbf{B}$ so $\tau = (5.41 \times 10^{-3}\ \text{A}\cdot\text{m}^2)(0.800\ \text{T}) = \boxed{4.33\ \text{mN}\cdot\text{m}}$

***29.24** $\tau = \mu B\sin\theta$ so $4.60 \times 10^{-3}\ \text{N}\cdot\text{m} = \mu(0.250)\sin 90.0^\circ$

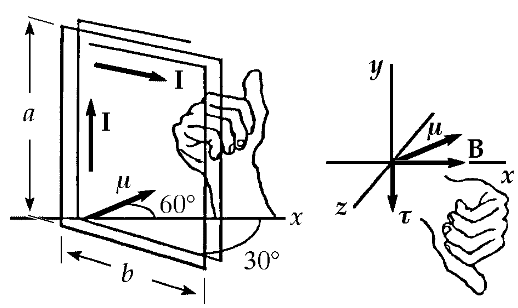
$$\mu = 1.84 \times 10^{-2}\ \text{A}\cdot\text{m}^2 = \boxed{18.4\ \text{mA}\cdot\text{m}^2}$$

29.25 $\tau = NBAI\sin\theta$

$$\tau = 100(0.800\ \text{T})(0.400 \times 0.300\ \text{m}^2)(1.20\ \text{A})\sin 60^\circ$$

$$\tau = \boxed{9.98\ \text{N}\cdot\text{m}}$$

Note that θ is the angle between the magnetic moment and the **B** field. The loop will rotate so as to align the magnetic moment with the **B** field. Looking down along the y-axis, the loop will rotate in a $\boxed{\text{clockwise}}$ direction.



(a) (b)

29.26 (a) Let θ represent the unknown angle; L, the total length of the wire; and d, the length of one side of the square coil. Then, use the right-hand rule to find

$$\mu = NAI = \left(\frac{L}{4d}\right)d^2 I \quad \text{at angle } \theta \text{ with the horizontal.}$$

At equilibrium, $\Sigma\tau = (\boldsymbol{\mu} \times \mathbf{B}) - (\mathbf{r} \times m\mathbf{g}) = 0$

$$\left(\frac{ILBd}{4}\right)\sin(90.0° - \theta) - \left(\frac{mgd}{2}\right)\sin\theta = 0 \quad \text{and} \quad \left(\frac{mgd}{2}\right)\sin\theta = \left(\frac{ILBd}{4}\right)\cos\theta$$

$$\theta = \tan^{-1}\left(\frac{ILB}{2mg}\right) = \tan^{-1}\left(\frac{(3.40\text{ A})(4.00\text{ m})(0.0100\text{ T})}{2(0.100\text{ kg})(9.80\text{ m}/\text{s}^2)}\right) = \boxed{3.97°}$$

(b) $$\tau_m = \left(\frac{ILBd}{4}\right)\cos\theta = \frac{1}{4}(3.40\text{ A})(4.00\text{ m})(0.0100\text{ T})(0.100\text{ m})\cos 3.97° = \boxed{3.39\text{ mN}\cdot\text{m}}$$

29.27 From $\tau = \boldsymbol{\mu} \times \mathbf{B} = I\mathbf{A} \times \mathbf{B}$, the magnitude of the torque is $IAB \sin 90.0°$

(a) Each side of the triangle is 40.0 cm / 3.

Its altitude is $\sqrt{13.3^2 - 6.67^2}$ cm = 11.5 cm and its area is

$$A = \frac{1}{2}(11.5\text{ cm})(13.3\text{ cm}) = 7.70 \times 10^{-3}\text{ m}^2$$

Then $\tau = (20.0\text{ A})(7.70 \times 10^{-3}\text{ m}^2)(0.520\text{ N}\cdot\text{s}/\text{C}\cdot\text{m}) = \boxed{80.1\text{ mN}\cdot\text{m}}$

(b) Each side of the square is 10.0 cm and its area is $100\text{ cm}^2 = 10^{-2}\text{ m}^2$.

$$\tau = (20.0\text{ A})(10^{-2}\text{ m}^2)(0.520\text{ T}) = \boxed{0.104\text{ N}\cdot\text{m}}$$

(c) $r = 0.400\text{ m}/2\pi = 0.0637\text{ m}$

$$A = \pi r^2 = 1.27 \times 10^{-2}\text{ m}^2$$

$$\tau = (20.0\text{ A})(1.27 \times 10^{-2}\text{ m}^2)(0.520) = \boxed{0.132\text{ N}\cdot\text{m}}$$

(d) The circular loop experiences the largest torque.

***29.28** Choose $U = 0$ when the dipole moment is at $\theta = 90.0°$ to the field. The field exerts torque of magnitude $\mu B\sin\theta$ on the dipole, tending to turn the dipole moment in the direction of decreasing θ. Its energy is given by

$$U - 0 = \int_{90.0°}^{\theta} \mu B\sin\theta\, d\theta = \mu B(-\cos\theta)\Big|_{90.0°}^{\theta} = -\mu B\cos\theta + 0 \qquad \text{or} \qquad \boxed{U = -\boldsymbol{\mu}\cdot\mathbf{B}}$$

***29.29** (a) The field exerts torque on the needle tending to align it with the field, so the minimum energy orientation of the needle is:

$\boxed{\text{pointing north at 48.0° below the horizontal}}$

where its energy is $U_{\min} = -\mu B\cos 0^\circ = -\left(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\ \text{T}\right) = -5.34\times10^{-7}\ \text{J}$

It has maximum energy when pointing in the opposite direction,

$\boxed{\text{south at 48.0° above the horizontal}}$

where its energy is $U_{\max} = -\mu B\cos 180^\circ = +\left(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\ \text{T}\right) = +5.34\times10^{-7}\ \text{J}$

(b) $U_{\min} + W = U_{\max}$: $W = U_{\max} - U_{\min} = +5.34\times10^{-7}\ \text{J} - \left(-5.34\times10^{-7}\ \text{J}\right) = \boxed{1.07\ \mu\text{J}}$

29.30 (a) $\tau = \boldsymbol{\mu}\times\mathbf{B}$, so $\tau = \left|\boldsymbol{\mu}\times\mathbf{B}\right| = \mu B\sin\theta = NIAB\sin\theta$

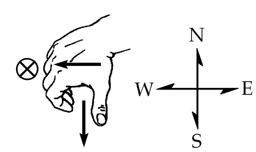
$\tau_{\max} = NIAB\sin 90.0^\circ = 1(5.00\ \text{A})\left[\pi(0.0500\ \text{m})^2\right]\left(3.00\times10^{-3}\ \text{T}\right) = \boxed{118\ \mu\text{N}\cdot\text{m}}$

(b) $U = -\boldsymbol{\mu}\cdot\mathbf{B}$, so $-\mu B \le U \le +\mu B$

Since $\mu B = (NIA)B = 1(5.00\ \text{A})\left[\pi(0.0500\ \text{m})^2\right]\left(3.00\times10^{-3}\ \text{T}\right) = 118\ \mu\text{J}$,

the range of the potential energy is: $\boxed{-118\ \mu\text{J} \le U \le +118\ \mu\text{J}}$

29.31 (a) $B = 50.0\times10^{-6}$ T; $v = 6.20\times10^{6}$ m/s



Direction is given by the right-hand-rule: $\boxed{\text{southward}}$

$F_B = qvB\sin\theta$

$F_B = (1.60\times10^{-19}\ \text{C})(6.20\times10^{6}\ \text{m/s})(50.0\times10^{-6}\ \text{T})\sin 90.0^\circ = \boxed{4.96\times10^{-17}\ \text{N}}$

(b) $F = \dfrac{mv^2}{r}$ so $r = \dfrac{mv^2}{F} = \dfrac{(1.67\times10^{-27}\ \text{kg})(6.20\times10^{6}\ \text{m/s})^2}{4.96\times10^{-17}\ \text{N}} = \boxed{1.29\ \text{km}}$

29.32 (a) $\frac{1}{2}mv^2 = q(\Delta V)$ $\quad \frac{1}{2}(3.20\times10^{-26}\ \text{kg})\,v^2 = (1.60\times10^{-19}\ \text{C})(833\ \text{V})$ $\quad v = 91.3$ km/s

The magnetic force provides the centripetal force: $qvB\sin\theta = \dfrac{mv^2}{r}$

$$r = \frac{mv}{qB\sin 90.0°} = \frac{(3.20 \times 10^{-26}\text{ kg})(9.13 \times 10^{4}\text{ m/s})}{(1.60 \times 10^{-19}\text{ C})(0.920\text{ N}\cdot\text{s/C}\cdot\text{m})} = \boxed{1.98\text{ cm}}$$

29.33 For each electron, $|q|vB \sin 90.0° = \dfrac{mv^2}{r}$ and $v = \dfrac{eBr}{m}$

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic:

$$K = \tfrac{1}{2}mv_{1i}^2 + 0 = \tfrac{1}{2}mv_{1f}^2 + \tfrac{1}{2}mv_{2f}^2$$

$$K = \tfrac{1}{2}m\left(\frac{e^2B^2R_1^2}{m^2}\right) + \tfrac{1}{2}m\left(\frac{e^2B^2R_2^2}{m^2}\right) = \frac{e^2B^2}{2m}\left(R_1^2 + R_2^2\right)$$

$$K = \frac{e(1.60\times10^{-19}\text{ C})(0.0440\text{ N}\cdot\text{s / C}\cdot\text{m})^2}{2(9.11\times10^{-31}\text{ kg})}\left[(0.0100\text{ m})^2 + (0.0240\text{ m})^2\right] = \boxed{115\text{ keV}}$$

29.34 We begin with $qvB = \dfrac{mv^2}{R}$, so $v = \dfrac{qRB}{m}$

The time to complete one revolution is $T = \dfrac{2\pi R}{v} = \dfrac{2\pi R}{\left(\dfrac{qRB}{m}\right)} = \dfrac{2\pi m}{qB}$

Solving for B, $B = \dfrac{2\pi m}{qT} = \boxed{6.56\times10^{-2}\text{ T}}$

29.35 $q(\Delta V) = \tfrac{1}{2}mv^2$ or $v = \sqrt{\dfrac{2q(\Delta V)}{m}}$

Also, $qvB = \dfrac{mv^2}{r}$ so $r = \dfrac{mv}{qB} = \dfrac{m}{qB}\sqrt{\dfrac{2q(\Delta V)}{m}} = \sqrt{\dfrac{2m(\Delta V)}{qB^2}}$

Therefore, $r_p^2 = \dfrac{2m_p(\Delta V)}{eB^2}$

$$r_d^2 = \frac{2m_d(\Delta V)}{q_dB^2} = \frac{2(2m_p)(\Delta V)}{eB^2} = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p^2$$

and $$r_\alpha{}^2 = \frac{2m_\alpha(\Delta V)}{q_\alpha B^2} = \frac{2(4m_p)(\Delta V)}{(2e)B^2} = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p{}^2$$

The conclusion is: $\boxed{r_\alpha = r_d = \sqrt{2}\,r_p}$

Goal Solution

29.35 A proton (charge $+e$, mass m_p), a deuteron (charge $+e$, mass $2m_p$), and an alpha particle, (charge $+2e$, mass $4m_p$) are accelerated through a common potential difference ΔV. The particles enter a uniform magnetic field **B** with a velocity in a direction perpendicular to **B**. The proton moves in a circular path of radius r_p. Determine the values of the radii of the circular orbits for the deuteron r_d and the alpha particle r_α in terms of r_p.

G: In general, particles with greater speed, more mass, and less charge will have larger radii as they move in a circular path due to a constant magnetic force. Since the effects of mass and charge have opposite influences on the path radius, it is somewhat difficult to predict which particle will have the larger radius. However, since the mass and charge ratios of the three particles are all similar in magnitude within a factor of four, we should expect that the radii also fall within a similar range.

O: The radius of each particle's path can be found by applying Newton's second law, where the force causing the centripetal acceleration is the magnetic force: $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$. The speed of the particles can be found from the kinetic energy resulting from the change in electric potential given.

A: An electric field changes the speed of each particle according to $(K+U)_i = (K+U)_f$. Therefore, assuming that the particles start from rest, we can write $q\Delta V = \frac{1}{2}mv^2$.

The magnetic field changes their direction as described by $\Sigma\mathbf{F} = m\mathbf{a}$: $qvB\sin 90° = \dfrac{mv^2}{r}$

thus $r = \dfrac{mv}{qB} = \dfrac{m}{qB}\sqrt{\dfrac{2q\Delta V}{m}} = \dfrac{1}{B}\sqrt{\dfrac{2m\Delta V}{q}}$

For the protons, $r_p = \dfrac{1}{B}\sqrt{\dfrac{2m_p\Delta V}{e}}$

For the deuterons, $r_d = \dfrac{1}{B}\sqrt{\dfrac{2(2m_p)\Delta V}{e}} = \sqrt{2}r_p$

For the alpha particles, $r_\alpha = \dfrac{1}{B}\sqrt{\dfrac{2(4m_p)\Delta V}{2e}} = \sqrt{2}r_p$

L: Somewhat surprisingly, the radii of the deuterons and alpha particles are the same and are only 41% greater than for the protons.

29.36 (a) We begin with $qvB = \dfrac{mv^2}{R}$, or $qRB = mv$. But, $L = mvR = qR^2B$.

Therefore, $R = \sqrt{\dfrac{L}{qB}} = \sqrt{\dfrac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{-3}\text{ T}\right)}} = 0.0500\text{ m} = \boxed{5.00\text{ cm}}$

(b) Thus, $v = \dfrac{L}{mR} = \dfrac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{\left(9.11\times10^{-31}\text{ kg}\right)(0.0500\text{ m})} = \boxed{8.78\times10^{6}\text{ m/s}}$

29.37 $\omega = \dfrac{qB}{m} = \dfrac{(1.60\times10^{-19}\text{ C})(5.20\text{ T})}{1.67\times10^{-27}\text{ kg}} = \boxed{4.98\times10^{8}\text{ rad/s}}$

29.38 $\frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\dfrac{2q(\Delta V)}{m}}$

$r = \dfrac{mv}{qB}$ so $r = \dfrac{m\sqrt{2q(\Delta V)/m}}{qB}$

$r^2 = \dfrac{m}{q}\cdot\dfrac{2(\Delta V)}{B^2}$ and $(r')^2 = \dfrac{m'}{q'}\cdot\dfrac{2(\Delta V)}{B^2}$

$m = \dfrac{qB^2r^2}{2(\Delta V)}$ and $(m') = \dfrac{(q')B^2(r')^2}{2(\Delta V)}$ so $\dfrac{m'}{m} = \dfrac{q'}{q}\cdot\dfrac{(r')^2}{r^2} = \left(\dfrac{2e}{e}\right)\left(\dfrac{2R}{R}\right)^2 = \boxed{8}$

29.39 $E = \frac{1}{2}mv^2 = e(\Delta V)$ and $evB\sin 90° = mv^2/R$

$B = \dfrac{mv}{eR} = \dfrac{m}{eR}\sqrt{\dfrac{2e(\Delta V)}{m}} = \dfrac{1}{R}\sqrt{\dfrac{2m(\Delta V)}{e}}$

$B = \dfrac{1}{5.80\times10^{10}\text{ m}}\sqrt{\dfrac{2(1.67\times10^{-27}\text{ kg})(10.0\times10^{6}\text{ V})}{1.60\times10^{-19}\text{ C}}} = \boxed{7.88\times10^{-12}\text{ T}}$

29.40 $r = \dfrac{mv}{qB}$ so $m = \dfrac{rqB}{v} = \dfrac{(7.94\times10^{-3}\text{ m})(1.60\times10^{-19}\text{ C})(1.80\text{ T})}{4.60\times10^{5}\text{ m/s}}$

$m = 4.97\times10^{-27}\text{ kg}\left(\dfrac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = \boxed{2.99\text{ u}}$

The particle is singly ionized: either a tritium ion, $\boxed{{}^{3}_{1}\text{H}^{+}}$, or a helium ion, $\boxed{{}^{3}_{2}\text{He}^{+}}$.

29.41 $F_B = F_e$ so $qvB = qE$ where $v = \sqrt{2K/m}$. K is kinetic energy of the electrons.

$E = vB = \sqrt{\dfrac{2K}{m}}B = \left(\dfrac{2(750)(1.60\times10^{-19})}{9.11\times10^{-31}}\right)^{1/2}(0.0150) = \boxed{244\text{ kV/m}}$

29.42 $K = \frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\frac{2q(\Delta V)}{m}}$

$$|\mathbf{F}_B| = |q\mathbf{v}\times\mathbf{B}| = \frac{mv^2}{r} \quad r = \frac{mv}{qB} = \frac{m}{q}\frac{\sqrt{2q(\Delta V)/m}}{B} = \frac{1}{B}\sqrt{\frac{2m(\Delta V)}{q}}$$

(a) $r_{238} = \sqrt{\frac{2(238\times1.66\times10^{-27})2000}{1.60\times10^{-19}}}\left(\frac{1}{1.20}\right) = 8.28\times10^{-2}\text{ m} = \boxed{8.28\text{ cm}}$

(b) $r_{235} = \boxed{8.23\text{ cm}}$

$$\frac{r_{238}}{r_{235}} = \sqrt{\frac{m_{238}}{m_{235}}} = \sqrt{\frac{238.05}{235.04}} = 1.0064$$

The ratios of the orbit radius for different ions are independent of ΔV and B.

29.43 In the velocity selector: $v = \frac{E}{B} = \frac{2500\text{ V/m}}{0.0350\text{ T}} = 7.14\times10^4\text{ m/s}$

In the deflection chamber: $r = \frac{mv}{qB} = \frac{(2.18\times10^{-26}\text{ kg})(7.14\times10^4\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.0350\text{ T})} = \boxed{0.278\text{ m}}$

29.44 $K = \frac{1}{2}mv^2$: $(34.0\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV}) = \frac{1}{2}(1.67\times10^{-27}\text{ kg})v^2$

$v = 8.07\times10^7\text{ m/s}$ $r = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\text{ kg})(8.07\times10^7\text{ m/s})}{(1.60\times10^{-19}\text{ C})(5.20\text{ T})} = \boxed{0.162\text{ m}}$

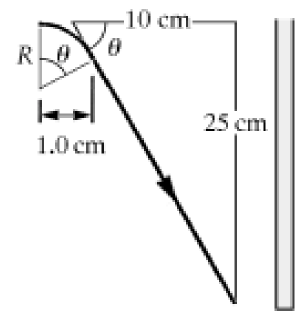
29.45 (a) $F_B = qvB = \frac{mv^2}{R}$

$$\omega = \frac{v}{R} = \frac{qBR}{mR} = \frac{qB}{m} = \frac{(1.60\times10^{-19}\text{ C})(0.450\text{ T})}{1.67\times10^{-27}\text{ kg}} = \boxed{4.31\times10^7\text{ rad/s}}$$

(b) $v = \frac{qBR}{m} = \frac{(1.60\times10^{-19}\text{ C})(0.450\text{ T})(1.20\text{ m})}{1.67\times10^{-27}\text{ kg}} = \boxed{5.17\times10^7\text{ m/s}}$

29.46 $F_B = qvB = \frac{mv^2}{r}$

$$B = \frac{mv}{qr} = \frac{4.80\times10^{-16}\text{ kg}\cdot\text{m/s}}{(1.60\times10^{-19}\text{ C})(1000\text{ m})} = \boxed{3.00\text{ T}}$$



29.47 $\theta = \tan^{-1}\dfrac{25.0}{10.0} = 68.2°$ and $R = \dfrac{1.00\text{ cm}}{\sin 68.2°} = 1.08\text{ cm}$

Ignoring relativistic correction, the kinetic energy of the electrons is

$\frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\dfrac{2q(\Delta V)}{m}} = 1.33\times10^8\text{ m / s}$

From the centripetal force $\dfrac{mv^2}{R} = qvB$, we find the magnetic field

$$B = \frac{mv}{|q|R} = \frac{(9.11\times10^{-31}\text{ kg})(1.33\times10^8\text{ m / s})}{(1.60\times10^{-19}\text{ C})(1.08\times10^{-2}\text{ m})} = \boxed{70.1\text{ mT}}$$

29.48 (a) $R_H \equiv \dfrac{1}{nq}$ so $n = \dfrac{1}{qR_H} = \dfrac{1}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.840\times10^{-10}\text{ m}^3/\text{C}\right)} = \boxed{7.44\times10^{28}\text{ m}^{-3}}$

(b) $\Delta V_H = \dfrac{IB}{nqt}$

$$B = \frac{nqt(\Delta V_H)}{I} = \frac{\left(7.44\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(0.200\times10^{-3}\text{ m}\right)\left(15.0\times10^{-6}\text{ V}\right)}{20.0\text{ A}} = \boxed{1.79\text{ T}}$$

29.49 $$\frac{1}{nq} = \frac{t(\Delta V_H)}{IB} = \frac{(35.0\times10^{-6}\text{ V})(0.400\times10^{-2}\text{ m})}{(21.0\text{ A})(1.80\text{ T})} = \boxed{3.70\times10^{-9}\text{ m}^3/\text{C}}$$

29.50 Since $\Delta V_H = \dfrac{IB}{nqt}$, and given that $I = 50.0$ A, $B = 1.30$ T, and $t = 0.330$ mm, the number of charge carriers per unit volume is

$$n = \frac{IB}{e(\Delta V_H)t} = \boxed{1.28\times10^{29}\text{ m}^{-3}}$$

The number density of atoms we compute from the density:

$$n_0 = \frac{8.92\text{ g}}{\text{cm}^3}\left(\frac{1\text{ mole}}{63.5\text{ g}}\right)\left(\frac{6.02\times10^{23}\text{ atoms}}{\text{mole}}\right)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right) = 8.46\times10^{28}\text{ atom / m}^3$$

So the number of conduction electrons per atom is

$$\frac{n}{n_0} = \frac{1.28\times10^{29}}{8.46\times10^{28}} = \boxed{1.52}$$

29.51 $$B=\frac{nqt(\Delta V_{\rm H})}{I}=\frac{\left(8.48\times 10^{28}\ {\rm m}^{-3}\right)\left(1.60\times 10^{-19}\ {\rm C}\right)\left(5.00\times 10^{-3}\ {\rm m}\right)\left(5.10\times 10^{-12}\ {\rm V}\right)}{8.00\ {\rm A}}$$

$$B=4.32\times 10^{-5}\ {\rm T}=\boxed{43.2\ \mu{\rm T}}$$

Goal Solution

In an experiment designed to measure the Earth's magnetic field using the Hall effect, a copper bar 0.500 cm thick is positioned along an east-west direction. If a current of 8.00 A in the conductor results in a Hall voltage of 5.10 pV, what is the magnitude of the Earth's magnetic field? (Assume that $n = 8.48\times 10^{28}$ electrons/m^3 and that the plane of the bar is rotated to be perpendicular to the direction of **B**.)

G: The Earth's magnetic field is about $50\ \mu$T (see Table 29.1), so we should expect a result of that order of magnitude.

O: The magnetic field can be found from the Hall effect voltage:

$$\Delta V_{\rm H}=\frac{IB}{nqt}\qquad \text{or}\qquad B=\frac{nqt\Delta V_{\rm H}}{I}$$

A: From the Hall voltage,

$$B=\frac{\left(8.48\times 10^{28}\ {\rm e}^-/{\rm m}^3\right)\left(1.60\times 10^{-19}\ {\rm C}/{\rm e}^-\right)(0.00500\ {\rm m})\left(5.10\times 10^{-12}\ {\rm V}\right)}{8.00\ {\rm A}}=4.32\times 10^{-5}\ {\rm T}=43.2\ \mu{\rm T}$$

L: The calculated magnetic field is slightly less than we expected but is reasonable considering that the Earth's local magnetic field varies in both magnitude and direction.

29.52 (a) $\Delta V_{\rm H}=\dfrac{IB}{nqt}$ so $\dfrac{nqt}{I}=\dfrac{B}{\Delta V_{\rm H}}=\dfrac{0.0800\ {\rm T}}{0.700\times 10^{-6}\ {\rm V}}=1.14\times 10^{5}\ \dfrac{\rm T}{\rm V}$

Then, the unknown field is $B=\left(\dfrac{nqt}{I}\right)(\Delta V_{\rm H})$

$$B=\left(1.14\times 10^{5}\ {\rm T/V}\right)\left(0.330\times 10^{-6}\ {\rm V}\right)=0.0377\ {\rm T}=\boxed{37.7\ {\rm mT}}$$

(b) $\dfrac{nqt}{I}=1.14\times 10^{5}\ \dfrac{\rm T}{\rm V}$ so $n=\left(1.14\times 10^{5}\ \dfrac{\rm T}{\rm V}\right)\dfrac{I}{qt}$

$$n=\left(1.14\times 10^{5}\ \frac{\rm T}{\rm V}\right)\frac{0.120\ {\rm A}}{\left(1.60\times 10^{-19}\ {\rm C}\right)\left(2.00\times 10^{-3}\ {\rm m}\right)}=\boxed{4.29\times 10^{25}\ {\rm m}^{-3}}$$

29.53 $|q|\,vB\sin 90° = \dfrac{mv^2}{r} \qquad \therefore\ \omega = \dfrac{v}{r} = \dfrac{eB}{m} = \dfrac{\theta}{t}$

(a) The time it takes the electron to complete π radians is

$$t = \frac{\theta}{\omega} = \frac{\theta m}{eB} = \frac{(\pi\ \text{rad})(9.11\times10^{-31}\ \text{kg})}{(1.60\times10^{-19}\ \text{C})(0.100\ \text{N}\cdot\text{s/C}\cdot\text{m})} = \boxed{1.79\times10^{-10}\ \text{s}}$$

(b) Since $v = \dfrac{|q|\,Br}{m}$,

$$K_e = \tfrac{1}{2}mv^2 = \frac{q^2B^2r^2}{2m} = \frac{e(1.60\times10^{-19}\ \text{C})(0.100\ \text{N}\cdot\text{s / Cm})^2(2.00\times10^{-2}\ \text{m})^2}{2(9.11\times10^{-31}\ \text{kg})} = \boxed{351\ \text{keV}}$$

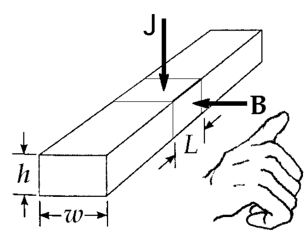
29.54 $\sum F_y = 0: \qquad +n - mg = 0$

$\sum F_x = 0: \qquad -\mu_k n + IBd\sin 90.0° = 0$

$$B = \frac{\mu_k mg}{Id} = \frac{0.100(0.200\ \text{kg})(9.80\ \text{m/s}^2)}{(10.0\ \text{A})(0.500\ \text{m})} = \boxed{39.2\ \text{mT}}$$

29.55 (a) The $\boxed{\text{electric current experiences a magnetic force}}$.

$I(\mathbf{h}\times\mathbf{B})$ in the direction of $\mathbf{L}$.



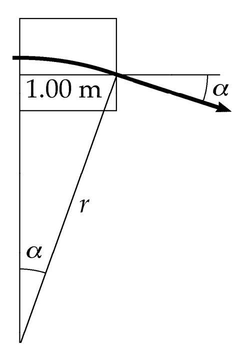
(b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length L, electrons drift upward to constitute downward electric current $\mathbf{J}\times(\text{area}) = \mathbf{J}Lw$.

The current then feels a magnetic force $\quad I|\mathbf{h}\times\mathbf{B}| = JLwhB\sin 90°$

This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}} = \frac{JLwhB}{hw} = \boxed{JLB}$$

29.56 The magnetic force on each proton, $\quad \mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = qvB\sin 90°$



downward perpendicular to velocity, supplies centripetal force, guiding it into a circular path of radius r, with

$$qvB = \frac{mv^2}{r} \qquad \text{and} \qquad r = \frac{mv}{qB}$$

We compute this radius by first finding the proton's speed: $\quad K = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(5.00\times10^6\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J / eV}\right)}{1.67\times10^{-27}\ \text{kg}}} = 3.10\times10^7\ \text{m / s}$$

Now, $r = \dfrac{mv}{qB} = \dfrac{(1.67\times10^{-27}\text{ kg})(3.10\times10^{7}\text{ m/s})(\text{C}\cdot\text{m})}{(1.60\times10^{-19}\text{ C})(0.0500\text{ N}\cdot\text{s})} = 6.46\text{ m}$

(b) From the figure, observe that $\sin\propto = \dfrac{1.00\text{ m}}{r} = \dfrac{1\text{ m}}{6.46\text{ m}}$ $\boxed{\alpha = 8.90°}$

(a) The magnitude of the proton momentum stays constant, and its final y component is

$-(1.67\times10^{-27}\text{ kg})(3.10\times10^{7}\text{ m/s})\sin(8.90°) = \boxed{-8.00\times10^{-21}\text{ kg}\cdot\text{m/s}}$

***29.57** (a) If $\mathbf{B} = B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k}$, $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = e(v_i\,\mathbf{i})\times(B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k}) = 0 + ev_iB_y\,\mathbf{k} - ev_iB_z\,\mathbf{j}$

Since the force actually experienced is $\mathbf{F}_B = F_i\,\mathbf{j}$, observe that

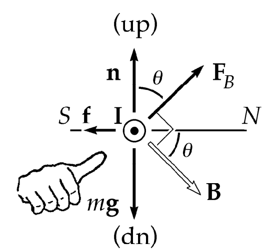
$\boxed{B_x \text{ could have any value}}$, $\boxed{B_y = 0}$, and $\boxed{B_z = -F_i/ev_i}$

(b) If $\mathbf{v} = -v_i\,\mathbf{i}$, then $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = e(-v_i\,\mathbf{i})\times(B_x\,\mathbf{i} + 0\,\mathbf{j} - F_i/ev_i\,\mathbf{k}) = \boxed{-F_i\,\mathbf{j}}$

(c) If $q = -e$ and $\mathbf{v} = v_i\,\mathbf{i}$, then $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = -e(v_i\,\mathbf{i})\times(B_x\,\mathbf{i} + 0\,\mathbf{j} - F_i/ev_i\,\mathbf{k}) = \boxed{-F_i\,\mathbf{j}}$

Reversing either the velocity or the sign of the charge reverses the force.

29.58 A key to solving this problem is that reducing the normal force will reduce the friction force: $F_B = BIL$ or $B = F_B/IL$



When the wire is just able to move, $\Sigma F_y = n + F_B\cos\theta - mg = 0$

so $n = mg - F_B\cos\theta$

and $f = \mu(mg - F_B\cos\theta)$

Also, $\Sigma F_x = F_B\sin\theta - f = 0$

so $F_B\sin\theta = f$: $F_B\sin\theta = \mu(mg - F_B\cos\theta)$ and $F_B = \dfrac{\mu mg}{\sin\theta + \mu\cos\theta}$

We minimize B by minimizing F_B: $\dfrac{dF_B}{d\theta} = (\mu mg)\dfrac{\cos\theta - \mu\sin\theta}{(\sin\theta + \mu\cos\theta)^2} = 0 \Rightarrow \mu\sin\theta = \cos\theta$

Thus, $\theta = \tan^{-1}\left(\dfrac{1}{\mu}\right) = \tan^{-1}(5.00) = 78.7°$ for the smallest field, and

$$B = \frac{F_B}{IL} = \left(\frac{\mu g}{I}\right)\frac{(m/L)}{\sin\theta + \mu\cos\theta}$$

$$B_{\min} = \left[\frac{(0.200)(9.80\text{ m/s}^2)}{1.50\text{ A}}\right]\frac{0.100\text{ kg/m}}{\sin 78.7° + (0.200)\cos 78.7°} = 0.128\text{ T}$$

$\boxed{B_{\min} = 0.128\text{ T pointing north at an angle of } 78.7° \text{ below the horizontal}}$

29.59 (a) The net force is the Lorentz force given by $\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$

$$\mathbf{F} = \left(3.20\times10^{-19}\right)\left[\left(4\,\mathbf{i} - 1\mathbf{j} - 2\,\mathbf{k}\right) + \left(2\,\mathbf{i} + 3\,\mathbf{j} - 1\,\mathbf{k}\right)\times\left(2\,\mathbf{i} + 4\,\mathbf{j} + 1\,\mathbf{k}\right)\right]\,\text{N}$$

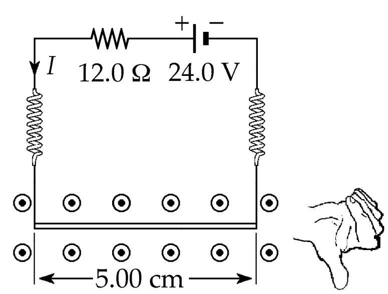
Carrying out the indicated operations, we find: $\mathbf{F} = \boxed{\left(3.52\,\mathbf{i} - 1.60\,\mathbf{j}\right)\times10^{-18}\ \text{N}}$

(b) $\theta = \cos^{-1}\left(\dfrac{F_x}{F}\right) = \cos^{-1}\left(\dfrac{3.52}{\sqrt{(3.52)^2 + (1.60)^2}}\right) = \boxed{24.4°}$

29.60 $r = \dfrac{mv}{qB} = \dfrac{(1.67\times10^{-27})(1.50\times10^{8})}{(1.60\times10^{-19})(5.00\times10^{-5})}\ \text{m} = \boxed{3.13\times10^{4}\ \text{m}} = 31.3\ \text{km}$

No, $\boxed{\text{the proton will not hit the Earth}}$.

29.61 Let Δx_1 be the elongation due to the weight of the wire and let Δx_2 be the additional elongation of the springs when the magnetic field is turned on. Then $F_{\text{magnetic}} = 2k\,\Delta x_2$ where k is the force constant of the spring and can be determined from $k = mg/2\Delta x_1$. (The factor 2 is included in the two previous equations since there are 2 springs in parallel.) Combining these two equations, we find



$$F_{\text{magnetic}} = 2\left(\frac{mg}{2\Delta x_1}\right)\Delta x_2 = \frac{mg\,\Delta x_2}{\Delta x_1};\quad \text{but}\quad \left|\mathbf{F}_B\right| = I\left|\mathbf{L}\times\mathbf{B}\right| = ILB$$

Therefore, where $I = \dfrac{24.0\ \text{V}}{12.0\ \Omega} = 2.00\ \text{A}$, $B = \dfrac{mg\,\Delta x_2}{IL\,\Delta x_1} = \dfrac{(0.0100)(9.80)(3.00\times10^{-3})}{(2.00)(0.0500)(5.00\times10^{-3})} = \boxed{0.588\ \text{T}}$

***29.62** Suppose the input power is $120\ \text{W} = (120\ \text{V})I$: $\boxed{I \sim 1\ \text{A} = 10^{0}\ \text{A}}$

Suppose $\omega = 2000\ \dfrac{\text{rev}}{\text{min}}\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right) \sim 200\ \dfrac{\text{rad}}{\text{s}}$

and the output power is $20\ \text{W} = \tau\omega = \tau\left(200\ \dfrac{\text{rad}}{\text{s}}\right)$ $\boxed{\tau \sim 10^{-1}\ \text{N}\cdot\text{m}}$

Suppose the area is about $(3\ \text{cm})\times(4\ \text{cm})$, or $\boxed{A \sim 10^{-3}\ \text{m}^2}$

From Table 29.1, suppose that the field is $\boxed{B \sim 10^{-1}\ \text{T}}$

Then, the number of turns in the coil may be found from $\tau \cong NIAB$:

$0.1\ \text{N}\cdot\text{m} \sim N\left(1\ \dfrac{\text{C}}{\text{s}}\right)\left(10^{-3}\ \text{m}^2\right)\left(10^{-1}\ \dfrac{\text{N}\cdot\text{s}}{\text{Cm}}\right)$ giving $\boxed{N \sim 10^{3}}$

29.63 Call the length of the rod L and the tension in each wire alone $T/2$. Then, at equilibrium:

$$\Sigma F_x = T\sin\theta - ILB\sin 90.0^0 = 0 \qquad \text{or} \qquad T\sin\theta = ILB$$
$$\Sigma F_y = T\cos\theta - mg = 0, \qquad \text{or} \qquad T\cos\theta = mg$$

Therefore, $\tan\theta = \dfrac{ILB}{mg} = \dfrac{IB}{(m/L)g}$ or $B = \dfrac{(m/L)g}{I}\tan\theta$

$$B = \frac{(0.0100\ \text{kg/m})(9.80\ \text{m/s}^2)}{5.00\ \text{A}}\tan(45.0^\circ) = \boxed{19.6\ \text{mT}}$$

29.64 Call the length of the rod L and the tension in each wire alone $T/2$. Then, at equilibrium:

$$\Sigma F_x = T\sin\theta - ILB\sin 90.0^0 = 0 \qquad \text{or} \qquad T\sin\theta = ILB$$
$$\Sigma F_y = T\cos\theta - mg = 0, \qquad \text{or} \qquad T\cos\theta = mg$$

$$\tan\theta = \frac{ILB}{mg} = \frac{IB}{(m/L)g} \qquad \text{or} \qquad B = \frac{(m/L)g}{I}\tan\theta = \boxed{\frac{\mu g}{I}\tan\theta}$$

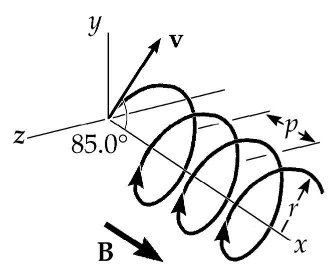
29.65 $\Sigma F = ma$ or $qvB\sin 90.0^\circ = \dfrac{mv^2}{r}$

$\therefore$ the angular frequency for each ion is $\dfrac{v}{r} = \omega = \dfrac{qB}{m} = 2\pi f$ and

$$\Delta f = f_{12} - f_{14} = \frac{qB}{2\pi}\left(\frac{1}{m_{12}} - \frac{1}{m_{14}}\right) = \frac{(1.60\times 10^{-19}\ \text{C})(2.40\ \text{T})}{2\pi(1.66\times 10^{-27}\ \text{kg / u})}\left(\frac{1}{12.0\ \text{u}} - \frac{1}{14.0\ \text{u}}\right)$$

$$\Delta f = f_{12} - f_{14} = 4.38\times 10^5\ \text{s}^{-1} = \boxed{438\ \text{kHz}}$$

29.66 Let v_x and $v_\perp$ be the components of the velocity of the positron parallel to and perpendicular to the direction of the magnetic field.



(a) The pitch of trajectory is the distance moved along x by the positron during each period, T (see Equation 29.15).

$$p = v_x T = (v\cos 85.0^\circ)\left(\frac{2\pi m}{Bq}\right)$$

$$p = \frac{(5.00\times 10^6)(\cos 85.0^\circ)(2\pi)(9.11\times 10^{-31})}{(0.150)(1.60\times 10^{-19})} = \boxed{1.04\times 10^{-4}\ \text{m}}$$

(b) From Equation 29.13, $r = \dfrac{mv_\perp}{Bq} = \dfrac{mv\sin 85.0^\circ}{Bq}$

$$r = \frac{(9.11\times 10^{-31})(5.00\times 10^6)(\sin\ 85.0^\circ)}{(0.150)(1.60\times 10^{-19})} = \boxed{1.89\times 10^{-4}\ \text{m}}$$

29.67 $|\tau| = IAB$ where the effective current due to the orbiting electrons is $I = \dfrac{\Delta q}{\Delta t} = \dfrac{q}{T}$

and the period of the motion is $T = \dfrac{2\pi R}{v}$

The electron's speed in its orbit is found by requiring $\dfrac{k_e q^2}{R^2} = \dfrac{mv^2}{R}$ or $v = q\sqrt{\dfrac{k_e}{mR}}$

Substituting this expression for v into the equation for T, we find $T = 2\pi\sqrt{\dfrac{mR^3}{q^2 k_e}}$

$$T = 2\pi\sqrt{\frac{(9.11\times10^{-31})(5.29\times10^{-11})^3}{(1.60\times10^{-19})^2(8.99\times10^9)}} = 1.52\times10^{-16}\ \text{s}$$

Therefore, $|\tau| = \left(\dfrac{q}{T}\right)AB = \dfrac{1.60\times10^{-19}}{1.52\times10^{-16}}\pi(5.29\times10^{-11})^2(0.400) = \boxed{3.70\times10^{-24}\ \text{N}\cdot\text{m}}$

Goal Solution

Consider an electron orbiting a proton and maintained in a fixed circular path of radius $R = 5.29\times10^{-11}$ m by the Coulomb force. Treating the orbiting charge as a current loop, calculate the resulting torque when the system is in a magnetic field of 0.400 T directed perpendicular to the magnetic moment of the electron.

G: Since the mass of the electron is very small (~10^{-30} kg), we should expect that the torque on the orbiting charge will be very small as well, perhaps ~10^{-30} N·m.

O: The torque on a current loop that is perpendicular to a magnetic field can be found from $|\tau| = IAB\sin\theta$. The magnetic field is given, $\theta = 90°$, the area of the loop can be found from the radius of the circular path, and the current can be found from the centripetal acceleration that results from the Coulomb force that attracts the electron to proton.

A: The area of the loop is $A = \pi r^2 = \pi\left(5.29\times10^{-11}\ \text{m}\right)^2 = 8.79\times10^{-21}\ \text{m}^2$.

If v is the speed of the electron, then the period of its circular motion will be $T = 2\pi R/v$, and the effective current due to the orbiting electron is $I = \Delta Q/\Delta t = e/T$. Applying Newton's second law with the Coulomb force acting as the central force gives

$$\Sigma F = \frac{k_e q^2}{R^2} = \frac{mv^2}{R} \qquad \text{so that} \qquad v = q\sqrt{\frac{k_e}{mR}} \qquad \text{and} \qquad T = 2\pi\sqrt{\frac{mR^3}{q^2 k_e}}$$

$$T = 2\pi\sqrt{\frac{(9.10\times10^{-31}\ \text{kg})(5.29\times10^{-11}\ \text{m})^3}{\left(1.60\times10^{-19}\ \text{C}\right)^2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}} = 1.52\times10^{-16}\ \text{s}$$

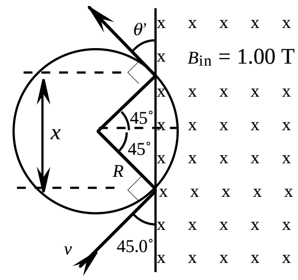
The torque is $|\tau| = \left(\dfrac{q}{T}\right)AB$: $|\tau| = \dfrac{1.60\times10^{19}\ \text{C}}{1.52\times10^{-16}\ \text{s}}(\pi)(5.29\times10^{-11}\ \text{m})^2(0.400\ \text{T}) = 3.70\times10^{-24}\ \text{N}\cdot\text{m}$

L: The torque is certainly small, but a million times larger than we guessed. This torque will cause the atom to precess with a frequency proportional to the applied magnetic field. A similar process on the nuclear, rather than the atomic, level leads to nuclear magnetic resonance (NMR), which is used for magnetic resonance imaging (MRI) scans employed for medical diagnostic testing (see Section 44.2).

29.68 Use the equation for cyclotron frequency $\omega = \frac{qB}{m}$ or $m = \frac{qB}{\omega} = \frac{qB}{2\pi f}$

$$m = \frac{(1.60\times10^{-19}\ \text{C})(5.00\times10^{-2}\ \text{T})}{(2\pi)(5.00\ \text{rev} / 1.50\times10^{-3}\ \text{s})} = \boxed{3.82\times10^{-25}\ \text{kg}}$$

29.69 (a) $K = \frac{1}{2}mv^2 = 6.00\ \text{MeV} = \left(6.00\times10^6\ \text{eV}\right)\left(1.60\times10^{-19}\ \frac{\text{J}}{\text{eV}}\right)$



$K = 9.60\times10^{-13}$ J

$$v = \sqrt{\frac{2\left(9.60\times10^{-13}\ \text{J}\right)}{1.67\times10^{-27}\ \text{kg}}} = 3.39\times10^7\ \text{m s}$$

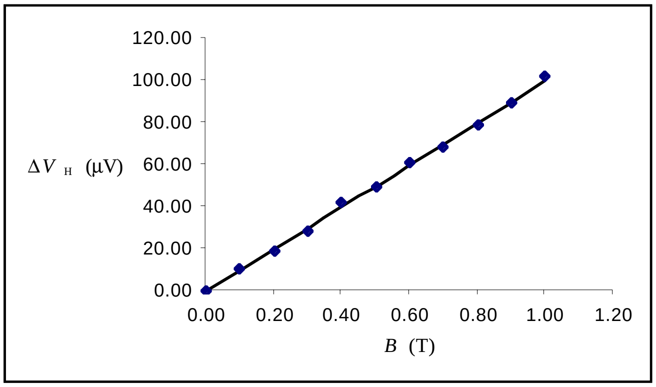
$$F_B = qvB = \frac{mv^2}{R} \quad \text{so} \quad R = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.39\times10^7\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\ \text{T}\right)} = 0.354\ \text{m}$$

Then, from the diagram, $x = 2R\sin 45.0° = 2(0.354\ \text{m})\sin 45.0° = \boxed{0.501\ \text{m}}$

(b) From the diagram, observe that $\theta' = \boxed{45.0°}$.

29.70 (a) See graph to the right. The Hall voltage is directly proportional to the magnetic field. A least-square fit to the data gives the equation of the best fitting line as:

$$\boxed{\Delta V_H = \left(1.00\times10^{-4}\ \text{V/T}\right)B}$$



(b) Comparing the equation of the line which fits the data best to

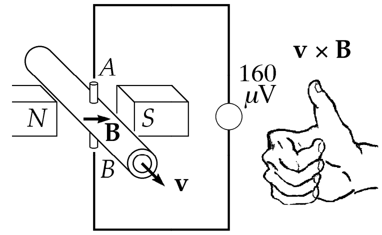
$$\Delta V_H = \left(\frac{I}{nqt}\right)B$$

observe that: $\frac{I}{nqt} = 1.00\times10^{-4}\ \text{V/T}$, or $t = \frac{I}{nq\left(1.00\times10^{-4}\ \text{V/T}\right)}$

Then, if $I = 0.200$ A, $q = 1.60\times10^{-19}$ C, and $n = 1.00\times10^{26}\ \text{m}^{-3}$, the thickness of the sample is

$$t = \frac{0.200\ \text{A}}{\left(1.00\times10^{26}\ \text{m}^{-3}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{-4}\ \text{V/T}\right)} = 1.25\times10^{-4}\ \text{m} = \boxed{0.125\ \text{mm}}$$

***29.71** (a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point A and negative charges toward point B. This separation of charges produces an electric field directed from A toward B. At equilibrium, the electric force caused by this field must balance the magnetic force,

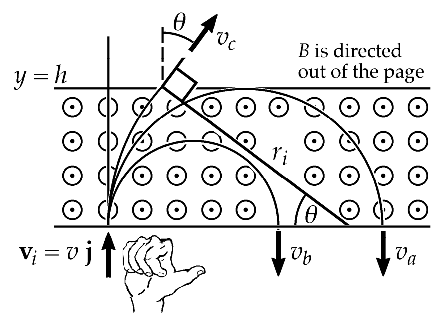


so $$qvB = qE = q\left(\frac{\Delta V}{d}\right)$$

or $$v = \frac{\Delta V}{Bd} = \frac{160 \times 10^{-6}\ \text{V}}{(0.040\ 0\ \text{T})(3.00 \times 10^{-3}\ \text{m})} = \boxed{1.33\ \text{m/s}}$$

(b) $\boxed{\text{No}}$. Negative ions moving in the direction of v would be deflected toward point B, giving A a higher potential than B. Positive ions moving in the direction of v would be deflected toward A, again giving A a higher potential than B. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

***29.72** When in the field, the particles follow a circular path according to $qvB = mv^2/r$, so the radius of the path is: $r = mv / qB$



(a) When $r = h = \dfrac{mv}{qB}$, that is, when $\boxed{v = \dfrac{qBh}{m}}$, the particle will cross the band of field. It will move in a full semicircle of radius h, leaving the field at $(2h, 0, 0)$ with velocity $\boxed{\mathbf{v}_f = -v\mathbf{j}}$.

(b) When $v < \dfrac{qBh}{m}$, the particle will move in a smaller $\boxed{\text{semicircle}}$ of radius $r = \dfrac{mv}{qB} < h$. It will leave the field at $(2r, 0, 0)$ with velocity $\boxed{\mathbf{v}_f = -v\mathbf{j}}$.

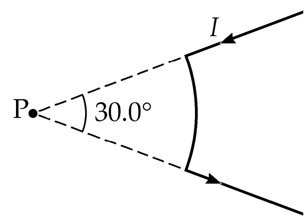
(c) When $v > \dfrac{qBh}{m}$, the particle moves in a $\boxed{\text{circular arc}}$ of radius $r = \dfrac{mv}{qB} > h$, centered at $(r, 0, 0)$. The arc subtends an angle given by $\theta = \sin^{-1}(h/r)$. It will leave the field at the point with coordinates $[r(1 - \cos\theta), h, 0]$ with velocity $\boxed{\mathbf{v}_f = v\sin\theta\,\mathbf{i} + v\cos\theta\,\mathbf{j}}$.

Chapter 30 Solutions

30.1 $B = \dfrac{\mu_0 I}{2R} = \dfrac{\mu_0 q(v/2\pi R)}{2R} = \boxed{12.5\ \text{T}}$

***30.2** We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\mathbf{s}$ is at 0° or 180° to ~, so $d\mathbf{s} \times \sim = 0$. Thus, only the curved section of wire contributes to $\mathbf{B}$ at P. Hence, $d\mathbf{s}$ is tangent to the arc and ~ is radially inward; so $d\mathbf{s} \times \sim = \left|ds\right| \ \ell \sin 90° = \left|ds\right| \ \otimes$. All points along the curve are the same distance $r = 0.600$ m from the field point, so

$$B = \int_{\text{all current}} \left|d\mathbf{B}\right| = \int \frac{\mu_0}{4\pi} \frac{I\left|d\mathbf{s} \times \sim\right|}{r^2} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int \left|ds\right| = \frac{\mu_0}{4\pi} \frac{I}{r^2} s$$



where s is the arclength of the curved wire,

$$s = r\theta = (0.600\ \text{m})30.0°\left(\frac{2\pi}{360°}\right) = 0.314\ \text{m}$$

Then, $B = \left(10^{-7}\ \dfrac{\text{T}\cdot\text{m}}{\text{A}}\right)\dfrac{(3.00\ \text{A})}{(0.600\ \text{m})^2}(0.314\ \text{m})$

$B = \boxed{261\ \text{nT into the page}}$

30.3 (a) $B = \dfrac{4\mu_0 I}{4\pi a}\left(\cos\dfrac{\pi}{4} - \cos\dfrac{3\pi}{4}\right)$ where $a = \dfrac{1}{2}$

is the distance from any side to the center.

$$B = \frac{4.00 \times 10^{-6}}{0.200}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = 2\sqrt{2} \times 10^{-5}\ \text{T} = \boxed{28.3\ \mu\text{T into the paper}}$$

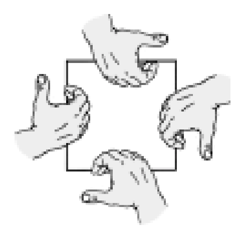


Figure for Goal Solution

(b) For a single circular turn with $4\ell = 2\pi R$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 \pi I}{4\ell} = \frac{(4\pi^2 \times 10^{-7})(10.0)}{4(0.400)} = \boxed{24.7\ \mu\text{T into the paper}}$$

Goal Solution

(a) A conductor in the shape of a square of edge length $l = 0.400$ m carries a current $I = 10.0$ A (Fig. P30.3). Calculate the magnitude and direction of the magnetic field at the center of the square. (b) If this conductor is formed into a single circular turn and carries the same current, what is the value of the magnetic field at the center?

G: As shown in the diagram above, the magnetic field at the center is directed into the page from the clockwise current. If we consider the sides of the square to be sections of four infinite wires, then we could expect the magnetic field at the center of the square to be a little less than four times the strength of the field at a point $l/2$ away from an infinite wire with current I.

$$B < 4\frac{\mu_0 I}{2\pi a} = 4\left(\frac{\left(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}\right)(10.0\ \mathrm{A})}{2\pi(0.200\ \mathrm{m})}\right) = 40.0\ \mu\mathrm{T}$$

Forming the wire into a circle should not significantly change the magnetic field at the center since the average distance of the wire from the center will not be much different.

O: Each side of the square is simply a section of a thin, straight conductor, so the solution derived from the Biot-Savart law in Example 30.1 can be applied to part (a) of this problem. For part (b), the Biot-Savart law can also be used to derive the equation for the magnetic field at the center of a circular current loop as shown in Example 30.3.

A: (a) We use Equation 30.4 for the field created by each side of the square. Each side contributes a field away from you at the center, so together they produce a magnetic field:

$$B = \frac{4\mu_0 I}{4\pi a}\left(\cos\frac{\pi}{4} - \cos\frac{3\pi}{4}\right) = \frac{4\left(4\pi\times 10^{-6}\ \mathrm{T\cdot m/A}\right)(10.0\ \mathrm{A})}{4\pi(0.200\ \mathrm{m})}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)$$

so at the center of the square, $\mathbf{B} = 2.00\sqrt{2}\times 10^{-5}\ \mathrm{T} = 28.3\ \mu\mathrm{T}$ perpendicularly into the page

(b) As in the first part of the problem, the direction of the magnetic field will be into the page. The new radius is found from the length of wire: $4\ell = 2\pi R$, so $R = 2\ell/\pi = 0.255$ m. Equation 30.8 gives the magnetic field at the center of a circular current loop:

$$B = \frac{\mu_0 I}{2R} = \frac{(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A})(10.0\ \mathrm{A})}{2(0.255\ \mathrm{m})} = 2.47\times 10^{-5}\ \mathrm{T} = 24.7\ \mu\mathrm{T}$$

Caution! If you use your calculator, it may not understand the keystrokes: To get the right answer, you may need to use .

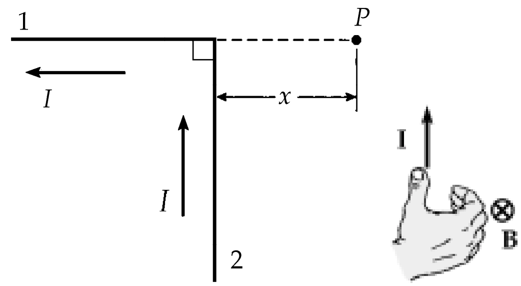
L: The magnetic field in part (a) is less than 40μT as we predicted. Also, the magnetic fields from the square and circular loops are similar in magnitude, with the field from the circular loop being about 15% less than from the square loop.

Quick tip: A simple way to use your right hand to find the magnetic field due to a current loop is to curl the fingers of your right hand in the direction of the current. Your extended thumb will then point in the direction of the magnetic field within the loop or solenoid.

30.4 $$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7}\,(1.00\text{ A})}{2\pi(1.00\text{ m})} = \boxed{2.00 \times 10^{-7}\text{ T}}$$

30.5 For leg 1, $d\mathbf{s} \times \sim = 0$, so there is no contribution to the field from this segment. For leg 2, the wire is only semi-infinite; thus,

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right) = \boxed{\frac{\mu_0 I}{4\pi x}\text{ into the paper}}$$



30.6 $$B = \frac{\mu_0 I}{2R} \qquad R = \frac{\mu_0 I}{2B} = \frac{20.0\pi \times 10^{-7}}{2.00 \times 10^{-5}} = \boxed{31.4\text{ cm}}$$

30.7 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\mu_0 I/2\pi R$ and directed into the page) and the field due to the circular loop (having magnitude $\mu_0 I/2R$ and directed into the page). The resultant magnetic field is:

$$B = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R} = \left(1 + \frac{1}{\pi}\right)\frac{\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m / A}\right)(7.00\text{ A})}{2(0.100\text{ m})} = 5.80 \times 10^{-5}\text{ T}$$

or $\boxed{\mathbf{B} = 58.0\ \mu\text{T}\quad(\text{directed into the page})}$

30.8 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\mu_0 I/2\pi R$ and directed into the page) and the field due to the circular loop (having magnitude $\mu_0 I/2R$ and directed into the page). The resultant magnetic field is:

$$\boxed{\mathbf{B} = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R}\quad(\text{directed into the page})}$$

30.9 For the straight sections $d\mathbf{s} \times \sim = 0$. The quarter circle makes one-fourth the field of a full loop:

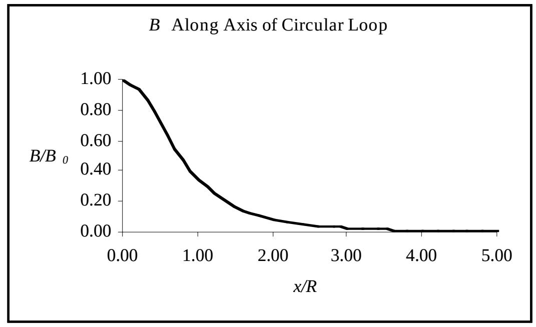
$$\mathbf{B} = \frac{1}{4}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{8R}\text{ into the paper} \qquad \mathbf{B} = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m / A})(5.00\text{ A})}{8(0.0300\text{ m})} = \boxed{26.2\ \mu\text{T into the paper}}$$

30.10 Along the axis of a circular loop of radius R,

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

or $$\frac{B}{B_0} = \left[\frac{1}{(x/R)^2 + 1}\right]^{3/2}$$

where $B_0 \equiv \mu_0 I/2R$.



x/R	B/B_0
0.00	1.00
1.00	0.354
2.00	0.0894
3.00	0.0316
4.00	0.0143
5.00	0.00754

30.11 $$dB = \frac{\mu_0 I}{4\pi} \frac{\left|d\mathbf{l} \times \hat{\mathbf{r}}\right|}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\left(\frac{\frac{1}{6} 2\pi a}{a^2} - \frac{\frac{1}{6} 2\pi b}{b^2}\right)$$

$$\mathbf{B} = \boxed{\frac{\mu_0 I}{12}\left(\frac{1}{a} - \frac{1}{b}\right) \text{ directed out of the paper}}$$

30.12 Apply Equation 30.4 three times:

$$\mathbf{B} = \frac{\mu_0 I}{4\pi a}\left(\cos 0 - \frac{d}{\sqrt{d^2 + a^2}}\right) \text{ toward you}$$

$$+ \frac{\mu_0 I}{4\pi d}\left(\frac{a}{\sqrt{d^2 + a^2}} + \frac{a}{\sqrt{d^2 + a^2}}\right) \text{ away from you}$$

$$+ \frac{\mu_0 I}{4\pi a}\left(\frac{-d}{\sqrt{d^2 + a^2}} - \cos 180°\right) \text{ toward you}$$

$$\mathbf{B} = \boxed{\frac{\mu_0 I\left(a^2 + d^2 - d\sqrt{a^2 + d^2}\right)}{2\pi a d\sqrt{a^2 + d^2}} \text{ away from you}}$$

30.13 The picture requires $L = 2R$

$$\mathbf{B} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right) + \frac{\mu_0 I}{4\pi R}(\cos 90.0° - \cos 135°) + \frac{\mu_0 I}{4\pi R}(\cos 45.0° - \cos 135°)$$

$$+ \frac{\mu_0 I}{4\pi R}(\cos 45.0° - \cos 90.0°) \text{ into the page}$$

$$\mathbf{B} = \frac{\mu_0 I}{R}\left(\frac{1}{4} + \frac{1}{\pi\sqrt{2}}\right) = \boxed{0.475\left(\frac{\mu_0 I}{R}\right) \text{ (into the page)}}$$

30.14 Label the wires 1, 2, and 3 as shown in Figure (a) and let the magnetic field created by the currents in these wires be B_1, B_2, and B_3 respectively.

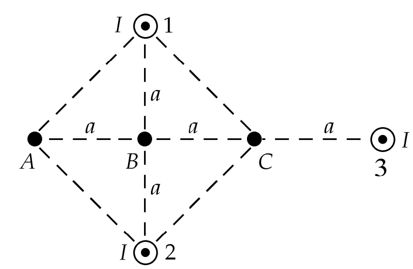


Figure (a)

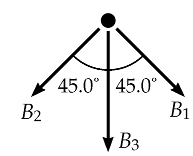


Figure (b)

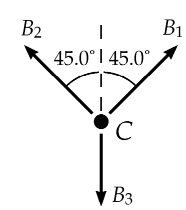


Figure (c)

(a) At Point A: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi(3a)}$.

The directions of these fields are shown in Figure (b). Observe that the horizontal components of B_1 and B_2 cancel while their vertical components both add to B_3.

Therefore, the net field at point A is:

$$B_A = B_1 \cos 45.0° + B_2 \cos 45.0° + B_3 = \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° + \frac{1}{3}\right]$$

$$B_A = \frac{\left(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A}\right)(2.00 \text{ A})}{2\pi\left(1.00\times 10^{-2} \text{ m}\right)}\left[\frac{2}{\sqrt{2}}\cos 45° + \frac{1}{3}\right] = \boxed{53.3\ \mu\text{T}}$$

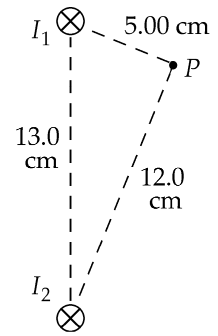
(b) At point B: B_1 and B_2 cancel, leaving $B_B = B_3 = \dfrac{\mu_0 I}{2\pi(2a)}$.

$$B_B = \frac{\left(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A}\right)(2.00 \text{ A})}{2\pi(2)\left(1.00\times 10^{-2} \text{ m}\right)} = \boxed{20.0\ \mu\text{T}}$$

(c) At point C: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi a}$ with the directions shown in Figure (c). Again, the horizontal components of B_1 and B_2 cancel. The vertical components both oppose B_3 giving

$$B_C = 2\left[\frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}\cos 45.0°\right] - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left[\frac{2\cos 45.0°}{\sqrt{2}} - 1\right] = \boxed{0}$$

30.15 Take the x-direction to the right and the y-direction up in the plane of the paper. Current 1 creates at P a field



$$B_1 = \frac{\mu_0 I}{2\pi a} = \frac{\left(2.00\times10^{-7}\ \mathrm{T\cdot m}\right)\left(3.00\ \mathrm{A}\right)}{\mathrm{A}\left(0.0500\ \mathrm{m}\right)}$$

$\mathbf{B}_1 = 12.0\ \mu\mathrm{T}$ downward and leftward, at angle 67.4° below the $-x$ axis.

Current 2 contributes

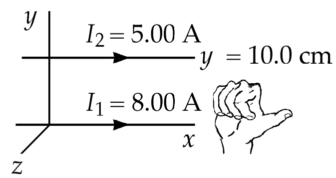
$$B_2 = \frac{\left(2.00\times10^{-7}\ \mathrm{T\cdot m}\right)\left(3.00\ \mathrm{A}\right)}{\mathrm{A}\left(0.120\ \mathrm{m}\right)}$$ clockwise perpendicular to 12.0 cm

$\mathbf{B}_2 = 5.00\ \mu\mathrm{T}$ to the right and down, at angle $-22.6°$

Then, $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = \left(12.0\ \mu\mathrm{T}\right)\left(-\mathbf{i}\cos 67.4° - \mathbf{j}\sin 67.4°\right) + \left(5.00\ \mu\mathrm{T}\right)\left(\mathbf{i}\cos 22.6° - \mathbf{j}\sin 22.6°\right)$

$$\mathbf{B} = \left(-11.1\ \mu\mathrm{T}\right)\mathbf{j} - \left(1.92\ \mu\mathrm{T}\right)\mathbf{j} = \boxed{\left(-13.0\ \mu\mathrm{T}\right)\mathbf{j}}$$

***30.16** Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

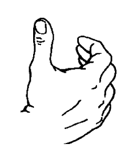


(a) $$\mathbf{B} = \frac{\mu_0 I}{2\pi r}\mathbf{k} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(5.00\ \mathrm{A}\right)}{2\pi\left(0.100\ \mathrm{m}\right)}\mathbf{k}$$

$\mathbf{B} = \boxed{1.00\times10^{-5}\ \mathrm{T}\ \ \text{out of the page}}$

(b) $\mathbf{F}_B = I_2\mathbf{L}\times\mathbf{B} = \left(8.00\ \mathrm{A}\right)\left[\left(1.00\ \mathrm{m}\right)\mathbf{i}\times\left(1.00\times10^{-5}\ \mathrm{T}\right)\mathbf{k}\right] = \left(8.00\times10^{-5}\ \mathrm{N}\right)\left(-\mathbf{j}\right)$

$\boxed{\mathbf{F}_B = 8.00\times10^{-5}\ \mathrm{N}\ \ \text{toward the first wire}}$



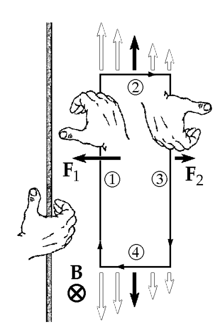
(c) $$\mathbf{B} = \frac{\mu_0 I}{2\pi r}\left(-\mathbf{k}\right) = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(8.00\ \mathrm{A}\right)}{2\pi\left(0.100\ \mathrm{m}\right)}\left(-\mathbf{k}\right) = \left(1.60\times10^{-5}\ \mathrm{T}\right)\left(-\mathbf{k}\right)$$

$\mathbf{B} = \boxed{1.60\times10^{-5}\ \mathrm{T}\ \ \text{into the page}}$

(d) $\mathbf{F}_B = I_1\mathbf{L}\times\mathbf{B} = \left(5.00\ \mathrm{A}\right)\left[\left(1.00\ \mathrm{m}\right)\mathbf{i}\times\left(1.60\times10^{-5}\ \mathrm{T}\right)\left(-\mathbf{k}\right)\right] = \left(8.00\times10^{-5}\ \mathrm{N}\right)\left(+\mathbf{j}\right)$

$\mathbf{F}_B = \boxed{8.00\times10^{-5}\ \mathrm{N}\ \ \text{toward the second wire}}$

30.17 By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (using Equation 30.12)



$$\mathbf{F}_B = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\mathbf{i}$$

Substituting given values $\mathbf{F}_B = -2.70\times10^{-5}\ \mathbf{i}$ N $= \boxed{-27.0\ \mu\mathrm{N}\ \mathbf{i}}$

Goal Solution

In Figure P30.17, the current in the long, straight wire is I_1 = 5.00 A and the wire lies in the plane of the rectangular loop, which carries 10.0 A. The dimensions are c = 0.100 m, a = 0.150 m, and ℓ = 0.450 m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

G: Even though there are forces in opposite directions on the loop, we must remember that the magnetic field is stronger near the wire than it is farther away. By symmetry the forces exerted on sides 2 and 4 (the horizontal segments of length a) are equal and opposite, and therefore cancel. The magnetic field in the plane of the loop is directed into the page to the right of I_1. By the right-hand rule, $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$ is directed toward the **left** for side 1 of the loop and a smaller force is directed toward the **right** for side 3. Therefore, we should expect the net force to be to the left, possibly in the μN range for the currents and distances given.

O: The magnetic force between two parallel wires can be found from Equation 30.11, which can be applied to sides 1 and 3 of the loop to find the net force resulting from these opposing force vectors.

A: $$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\mathbf{i} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\mathbf{i}$$

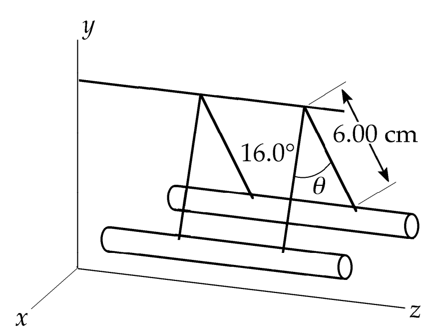
$$\mathbf{F} = \frac{\left(4\pi\times10^{-7}\text{ N / A}^2\right)(5.00\text{ A})(10.0\text{ A})(0.450\text{ m})}{2\pi}\left(\frac{-0.150\text{ m}}{(0.100\text{ m})(0.250\text{ m})}\right)\mathbf{i}$$

$\mathbf{F} = (-2.70\times10^{-5}\mathbf{i})$ N or $\mathbf{F} = 2.70\times10^{-5}$ N toward the left

L: The net force is to the left and in the μN range as we expected. The symbolic representation of the net force on the loop shows that the net force would be zero if either current disappeared, if either dimension of the loop became very small ($a \to 0$ or $\ell \to 0$), or if the magnetic field were uniform ($c \to \infty$).

30.18 The separation between the wires is

a = 2(6.00 cm) sin 8.00° = 1.67 cm.



(a) Because the wires repel, the currents are in

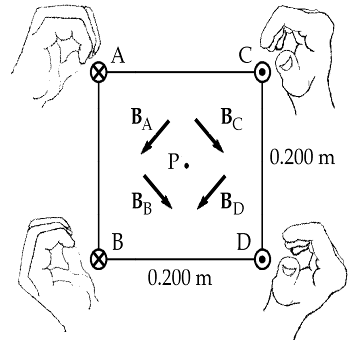
$\boxed{\text{opposite directions}}$.

(b) Because the magnetic force acts horizontally,

$$\frac{F_B}{F_g} = \frac{\mu_0 I^2 \ell}{2\pi a\ mg} = \tan 8.00°$$

$$I^2 = \frac{mg\ 2\pi a}{\ell\,\mu_0}\tan 8.00° \quad \text{so} \quad I = \boxed{67.8\text{ A}}$$

30.19 Each wire is distant from P by (0.200 m) cos 45.0° = 0.141 m



Each wire produces a field at P of equal magnitude:

$$B_A = \frac{\mu_0 I}{2\pi a} = \frac{(2.00\times10^{-7}\ \text{T}\cdot\text{m})(5.00\ \text{A})}{\text{A}(0.141\ \text{m})} = 7.07\ \mu\text{T}$$

Carrying currents into the page, A produces at P a field of 7.07 μT to the left and down at –135°, while B creates a field to the right and down at – 45°. Carrying currents toward you, C produces a field downward and to the right at – 45°, while D's contribution is downward and to the left. The total field is then

$$4\,(7.07\ \mu\text{T})\sin 45.0° = \boxed{20.0\ \mu\text{T}}\ \text{toward the page's bottom}$$

30.20 Let the current I flow to the right. It creates a field $B = \mu_0 I/2\pi d$ at the proton's location. And we have a balance between the weight of the proton and the magnetic force

$$mg(-\mathbf{j}) + qv(-\mathbf{i})\times\frac{\mu_0 I}{2\pi d}(\mathbf{k}) = 0 \quad \text{at a distance } d \text{ from the wire}$$

$$d = \frac{qv\mu_0 I}{2\pi mg} = \frac{(1.60\times10^{-19}\ \text{C})(2.30\times10^{4}\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.20\times10^{-6}\ \text{A})}{2\pi(1.67\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)} = \boxed{5.40\ \text{cm}}$$

30.21 From Ampère's law, the magnetic field at point a is given by $B_a = \mu_0 I_a/2\pi r_a$, where I_a is the net current flowing through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(1.00\ \text{A})}{2\pi(1.00\times10^{-3}\ \text{m})} = \boxed{200\ \mu\text{T toward top of page}}$$

Similarly at point b: $B_b = \dfrac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current flowing through the area of the circle having radius r_b.

Taking out of the page as positive, $I_b = 1.00\ \text{A} - 3.00\ \text{A} = -2.00\ \text{A}$, or $I_b = 2.00$ A into the page. Therefore,

$$B_b = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.00\ \text{A})}{2\pi(3.00\times10^{-3}\ \text{m})} = \boxed{133\ \mu\text{T toward bottom of page}}$$

***30.22** (a) In $B = \dfrac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: $\boxed{400\text{ cm}}$

(b) $\mathbf{B} = \dfrac{\mu_0 I}{2\pi r_1}\,\mathbf{k} + \dfrac{\mu_0 I}{2\pi r_2}(-\mathbf{k})$ so $B = \dfrac{4\pi \times 10^{-7}\text{ T}\cdot\text{m }(2.00\text{ A})}{2\pi\text{ A}}\left(\dfrac{1}{0.3985\text{ m}} - \dfrac{1}{0.4015\text{ m}}\right) = \boxed{7.50\text{ nT}}$

(c) Call r the distance from cord center to field point and $2d = 3.00$ mm the distance between conductors.

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d} - \frac{1}{r+d}\right) = \frac{\mu_0 I}{2\pi}\frac{2d}{r^2 - d^2}$$

$$7.50 \times 10^{-10}\text{ T} = \left(2.00 \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(2.00\text{ A})\,\frac{(3.00 \times 10^{-3}\text{ m})}{r^2 - 2.25 \times 10^{-6}\text{ m}^2} \quad \text{so} \quad r = \boxed{1.26\text{ m}}$$

The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

30.23 (a) $B_{\text{inner}} = \dfrac{\mu_0 N I}{2\pi r} = \boxed{3.60\text{ T}}$

(b) $B_{\text{outer}} = \dfrac{\mu_0 N I}{2\pi r} = \boxed{1.94\text{ T}}$

***30.24** (a) $B = \dfrac{\mu_0 I}{2\pi a^2}\,r$ for $r \le a$ so $B = \dfrac{\mu_0(2.50\text{ A})}{2\pi(0.0250\text{ m})^2}(0.0125\text{ m}) = \boxed{10.0\ \mu\text{T}}$

(b) $r = \dfrac{\mu_0 I}{2\pi B} = \dfrac{\mu_0(2.50\text{ A})}{2\pi(10.0 \times 10^{-6}\text{ T})} = 0.0500\text{ m} =$ $\boxed{\text{2.50 cm beyond the conductor's surface}}$

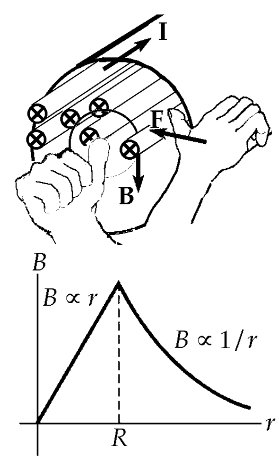
30.25 (a) One wire feels force due to the field of the other ninety-nine.

Within the bundle, $B = \left(\dfrac{\mu_0 I}{2\pi R^2}\right) r = 3.17 \times 10^{-3}$ T.

The force, *acting inward*, is $F_B = I\ell B$, and the force per unit length is

$$\frac{F_B}{\ell} = \boxed{6.34 \times 10^{-3}\text{ N/m inward}}$$

(b) $B \propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is $\boxed{\text{greatest at the outer surface}}$.



Figures for Goal Solution

Goal Solution

A packed bundle of 100 long, straight, insulated wires forms a cylinder of radius R = 0.500 cm. (a) If each wire carries 2.00 A, what are the magnitude and direction of the magnetic force per unit length acting on a wire located 0.200 cm from the center of the bundle? (b) Would a wire on the outer edge of the bundle experience a force greater or less than the value calculated in part (a)?

G: The force **on** one wire comes from its interaction with the magnetic field created **by** the other ninety-nine wires. According to Ampere's law, at a distance r from the center, only the wires enclosed within a radius r contribute to this net magnetic field; the other wires outside the radius produce magnetic field vectors in opposite directions that cancel out at r. Therefore, the magnetic field (and also the force on a given wire at radius r) will be greater for larger radii within the bundle, and will decrease for distances beyond the radius of the bundle, as shown in the graph to the right. Applying $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$, the magnetic force on a single wire will be directed toward the center of the bundle, so that all the wires tend to attract each other.

O: Using Ampere's law, we can find the magnetic field at any radius, so that the magnetic force $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$ on a single wire can then be calculated.

A: (a) Ampere's law is used to derive Equation 30.15, which we can use to find the magnetic field at r = 0.200 cm from the center of the cable:

$$B = \frac{\mu_o I_o r}{2\pi R^2} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m / A}\right)(99)(2.00\ \mathrm{A})\left(0.200\times10^{-2}\ \mathrm{m}\right)}{2\pi\,(0.500\times10^{-2}\ \mathrm{m})^2} = 3.17\times10^{-3}\ \mathrm{T}$$

This field points tangent to a circle of radius 0.200 cm and exerts a force $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$ toward the center of the bundle, on the single hundredth wire:

$$F/l = IB\sin\theta = (2.00\ \mathrm{A})\left(3.17\times10^{-3}\ \mathrm{T}\right)(\sin 90°) = 6.34\ \mathrm{mN / m}$$

(b) As is shown above in Figure 30.12 from the text, the magnetic field increases linearly as a function of r until it reaches a maximum at the outer surface of the cable. Therefore, the force on a single wire at the outer radius $r = 5.00$ cm would be greater than at $r = 2.00$ cm by a factor of $5/2$.

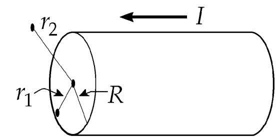
L: We did not estimate the expected magnitude of the force, but 200 amperes is a lot of current. It would be interesting to see if the magnetic force that pulls together the individual wires in the bundle is enough to hold them against their own weight: If we assume that the insulation accounts for about half the volume of the bundle, then a single copper wire in this bundle would have a cross sectional area of about

$$(1/2)(0.01)\pi(0.500\ \mathrm{cm})^2 = 4\times10^{-7}\ \mathrm{m}^2$$

with a weight per unit length of $\rho gA = \left(8920\ \mathrm{kg / m^3}\right)(9.8\ \mathrm{N / kg})\left(4\times10^{-7}\ \mathrm{m}^2\right) = 0.03\ \mathrm{N / m}$

Therefore, the outer wires experience an inward magnetic force that is about half the magnitude of their own weight. If placed on a table, this bundle of wires would form a loosely held mound without the outer sheathing to hold them together.

30.26 From $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$, $I = \dfrac{2\pi r B}{\mu_0} = \dfrac{(2\pi)(1.00\times10^{-3})(0.100)}{4\pi\times10^{-7}} = \boxed{500\ \mathrm{A}}$



30.27 Use Ampère's law, $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I$. For current density $\mathbf{J}$, this becomes

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 \int \mathbf{J}\cdot d\mathbf{A}$$

(a) For $r_1 < R$, this gives

$$2\pi r_1 = \mu_0 \int_0^{r_1} (br)(2\pi r\, dr) \quad \text{and}$$

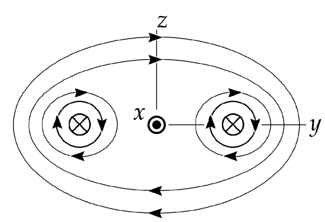
$$B = \boxed{\frac{\mu_0 b r_1^2}{3} \quad (\text{for } r_1 < R \text{ or inside the cylinder})}$$

(b) When $r_2 > R$, Ampère's law yields

$$(2\pi r_2)B = \mu_0 \int_0^{R} (br)(2\pi r\, dr) = 2\pi \mu_0 b R^3/3,$$

or $\quad B = \boxed{\dfrac{\mu_0 b R^3}{3 r_2} \quad (\text{for } r_2 > R \text{ or outside the cylinder})}$

30.28 (a) See Figure (a) to the right.



(Currents are into the paper)

Figure (a)

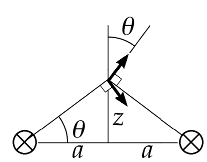
(b) At a point on the z axis, the contribution from each wire has magnitude $B = \dfrac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}$ and is perpendicular to the line from this point to the wire as shown in Figure (b). Combining fields, the vertical components cancel while the horizontal components add, yielding

$$B_y = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}\sin\theta\right) = \frac{\mu_0 I}{\pi\sqrt{a^2+z^2}}\left(\frac{z}{\sqrt{a^2+z^2}}\right) = \frac{\mu_0 I z}{\pi\left(a^2+z^2\right)}$$

The condition for a maximum is:

$$\frac{dB_y}{dz} = \frac{-\mu_0 I z(2z)}{\pi\left(a^2+z^2\right)^2} + \frac{\mu_0 I}{\pi\left(a^2+z^2\right)} = 0, \quad \text{or} \quad \frac{\mu_0 I}{\pi}\frac{\left(a^2-z^2\right)}{\left(a^2+z^2\right)^2} = 0$$

Thus, along the z axis, the field is a maximum at $\boxed{d = a}$.



At a distance z above the plane of the conductors

Figure (b)

30.29 $B = \mu_0 \dfrac{N}{\ell} I$ so $I = \dfrac{B}{\mu_0 n} = \boxed{31.8\ \text{mA}}$

30.30 (a) $I = \dfrac{10.0}{(4\pi \times 10^{-7})(2000)} = \boxed{3.98\ \text{kA}}$

(b) $\dfrac{F_B}{\ell} = IB = \boxed{39.8\ \text{kN/m radially outward}}$

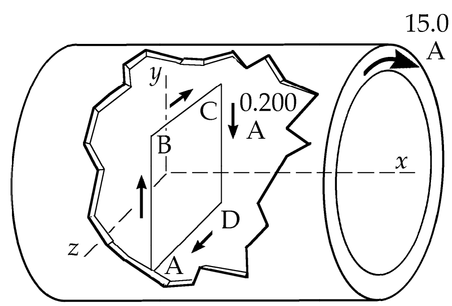
This is the force the windings will have to resist when the magnetic field in the solenoid is 10.0 T.

30.31 The resistance of the wire is $R_e = \dfrac{\rho \ell}{\pi r^2}$, so it carries current $I = \dfrac{\mathcal{E}}{R_e} = \dfrac{\mathcal{E}\pi r^2}{\rho \ell}$.

If there is a single layer of windings, the number of turns per length is the reciprocal of the wire diameter: $n = 1/2r$.

So, $$B = n\mu_0 I = \frac{\mu_0 \mathcal{E}\pi r^2}{\rho \ell\, 2r} = \frac{\mu_0 \mathcal{E}\pi r}{2\rho \ell} = \frac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(20.0\ \text{V})\pi(2.00 \times 10^{-3}\ \text{m})}{2(1.70 \times 10^{-8}\ \Omega \cdot \text{m})(10.0\ \text{m})} = \boxed{464\ \text{mT}}$$

***30.32** The field produced by the solenoid in its interior is given by



$$\mathbf{B} = \mu_0 n I(-\mathbf{i}) = \left(4\pi \times 10^{-7}\ \frac{\text{T} \cdot \text{m}}{\text{A}}\right)\left(\frac{30.0}{10^{-2}\ \text{m}}\right)(15.0\ \text{A})(-\mathbf{i})$$

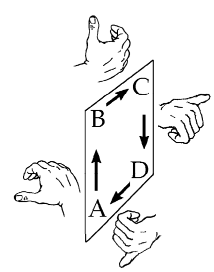
$$\mathbf{B} = -\left(5.65 \times 10^{-2}\ \text{T}\right)\mathbf{i}$$

The force exerted on side AB of the square current loop is

$$\left(\mathbf{F}_B\right)_{AB} = I\mathbf{L} \times \mathbf{B} = (0.200\ \text{A})\left[\left(2.00 \times 10^{-2}\ \text{m}\right)\mathbf{j} \times \left(5.65 \times 10^{-2}\ \text{T}\right)(-\mathbf{i})\right]$$

$$\left(\mathbf{F}_B\right)_{AB} = \left(2.26 \times 10^{-4}\ \text{N}\right)\mathbf{k}$$

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of $\boxed{226\ \mu\text{N directed away from the center}}$. From the above result, it is seen that the net torque exerted on the square loop by the field of the solenoid should be zero. More formally, the magnetic dipole moment of the square loop is given by



$$\mu = I\mathbf{A} = (0.200\ \text{A})\left(2.00 \times 10^{-2}\ \text{m}\right)^2(-\mathbf{i}) = -80.0\ \mu\text{A} \cdot \text{m}^2\,\mathbf{i}$$

The torque exerted on the loop is then $\tau = \mu \times \mathbf{B} = \left(-80.0\ \mu\text{A} \cdot \text{m}^2\,\mathbf{i}\right) \times \left(-5.65 \times 10^{-2}\ \text{T}\mathbf{i}\right) = \boxed{0}$

30.33 (a) $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A} = \mathbf{B} \cdot \mathbf{A} = (5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})\text{T} \cdot (2.50 \times 10^{-2}\text{ m})^2\,\mathbf{i}$

$$\Phi_B = 3.13 \times 10^{-3}\text{ T} \cdot \text{m}^2 = 3.13 \times 10^{-3}\text{ Wb} = \boxed{3.13\text{ mWb}}$$

(b) $(\Phi_B)_{\text{total}} = \oint \mathbf{B} \cdot d\mathbf{A} = \boxed{0}$ for *any* closed surface (Gauss's law for magnetism)

30.34 (a) $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA$ where A is the cross-sectional area of the solenoid.

$$\Phi_B = \left(\frac{\mu_0 NI}{\ell}\right)(\pi r^2) = \boxed{7.40\ \mu\text{Wb}}$$

(b) $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA = \left(\frac{\mu_0 NI}{\ell}\right)\left[\pi(r_2^2 - r_1^2)\right]$

$$\Phi_B = \left[\frac{(4\pi \times 10^{-7}\text{ T} \cdot \text{m/A})(300)(12.0\text{ A})}{(0.300\text{ m})}\right]\pi\left[(8.00)^2 - (4.00)^2\right](10^{-3}\text{ m})^2 = \boxed{2.27\ \mu\text{Wb}}$$

30.35 (a) $(\Phi_B)_{\text{flat}} = \mathbf{B} \cdot \mathbf{A} = B\pi R^2 \cos(180 - \theta) = \boxed{-B\pi R^2 \cos\theta}$

(b) The net flux out of the closed surface is zero: $(\Phi_B)_{\text{flat}} + (\Phi_B)_{\text{curved}} = 0$

$$(\Phi_B)_{\text{curved}} = \boxed{B\pi R^2 \cos\theta}$$

30.36 $\frac{d\Phi_E}{dt} = \frac{d}{dt}(EA) = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0}$

(a) $\frac{dE}{dt} = \frac{I}{\epsilon_0 A} = \boxed{7.19 \times 10^{11}\text{ V/m} \cdot \text{s}}$

(b) $\oint B \cdot ds = \epsilon_0 \mu_0 \frac{\Phi_E}{dt}$ so $2\pi r B = \epsilon_0 \mu_0 \frac{d}{dt}\left[\frac{Q}{\epsilon_0 A} \cdot \pi r^2\right]$

$$B = \frac{\mu_0 Ir}{2A} = \frac{\mu_0(0.200)(5.00 \times 10^{-2})}{2\pi(0.100)^2} = \boxed{2.00 \times 10^{-7}\text{ T}}$$

30.37 (a) $\frac{d\Phi_E}{dt} = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0} = \frac{(0.100\text{ A})}{8.85 \times 10^{-12}\text{ C}^2/\text{N} \cdot \text{m}^2} = \boxed{11.3 \times 10^9\text{ V} \cdot \text{m/s}}$

(b) $I_d = \epsilon_0 \frac{d\Phi_E}{dt} = I = \boxed{0.100\text{ A}}$

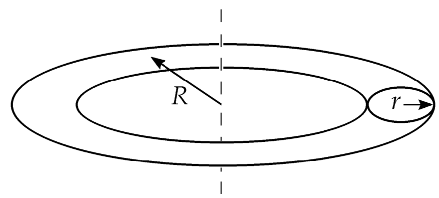
30.38 (a) $I = \dfrac{ev}{2\pi r}$

$$\mu = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \boxed{9.27 \times 10^{-24}\ \text{A} \cdot \text{m}^2}$$

The Bohr model predicts the correct magnetic moment. However, the "planetary model" is seriously deficient in other regards.

(b) Because the electron is (–), its [conventional] current is clockwise, as seen from above, and μ points $\boxed{\text{downward}}$.

30.39 Assuming a uniform B inside the toroid is equivalent to assuming $r \ll R$, then $B_0 \cong \mu_0 \dfrac{NI}{2\pi R}$ and a *tightly* wound solenoid.



$$B_0 = \mu_0 \frac{(630)(3.00)}{2\pi(0.200)} = 0.00189\ \text{T}$$

With the steel, $B = \kappa_m B_0 = (1 + \chi)B_0 =$ (101)(0.00189 T) $\boxed{B = 0.191\ \text{T}}$

30.40 $B = \mu n I = \mu\left(\dfrac{N}{2\pi r}\right)I$ so $I = \dfrac{(2\pi r)B}{\mu N} = \dfrac{2\pi(0.100\ \text{m})(1.30\ \text{T})}{5000(4\pi \times 10^{-7}\ \text{Wb/A} \cdot \text{m})(470)} = \boxed{277\ \text{mA}}$

30.41 $\Phi_B = \mu n I A$

$$B = \mu n I = (750 \times 4\pi \times 10^{-7})\left(\frac{500}{2\pi(0.200)}\right)(0.500) = 0.188\ \text{T}$$

$A = 8.00 \times 10^{-4}\ \text{m}^2$ and $\Phi_B = (0.188\ \text{T})(8.00 \times 10^{-4}\ \text{m}^2) = 1.50 \times 10^{-4}\ \text{T} \cdot \text{m}^2) = \boxed{150\ \mu\text{T} \cdot \text{m}^2}$

30.42 The period is $T = 2\pi / \omega$. The spinning constitutes a current $I = \dfrac{Q}{T} = \dfrac{Q\omega}{2\pi}$.

$\mu = IA = \dfrac{Q\omega}{2\pi}\pi R^2 = \dfrac{Q\omega R^2}{2}$ in the direction of ω

$$\mu = \frac{(6.00 \times 10^{-6}\ \text{C})(4.00\ /\ \text{s})(0.0200\ \text{m})^2}{2} = \boxed{4.80 \times 10^{-9}\ \text{A} \cdot \text{m}^2}$$

30.43 $B = \mu_0(H + M)$ so $H = \dfrac{B}{\mu_0} - M = \boxed{2.62 \times 10^6 \text{ A/m}}$

30.44 $B = \mu_0(H + M)$

If $\mu_0 M = 2.00$ T, then the magnetization of the iron is $M = \dfrac{2.00 \text{ T}}{\mu_0}$.

But $M = xn\mu_B$ where μ_B is the Bohr magneton, n is the number of atoms per unit volume, and x is the number of electrons that contribute per atom. Thus,

$$x = \frac{M}{n\mu_B} = \frac{2.00 \text{ T}}{n\mu_B\mu_0} = \frac{2.00 \text{ T}}{\left(8.50 \times 10^{28} \text{ m}^{-3}\right)\left(9.27 \times 10^{-24} \text{ N}\cdot\text{m/T}\right)\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)} = \boxed{2.02}$$

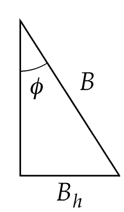
***30.45** (a) Comparing Equations 30.29 and 30.30, we see that the applied field is described by $\mathbf{B}_0 = \mu_0\mathbf{H}$. Then Eq. 30.35 becomes $M = C\dfrac{B_0}{T} = \dfrac{C}{T}\mu_0 H$, and the definition of susceptibility (Eq. 30.32) is

$$\boxed{\chi = \frac{M}{H} = \frac{C}{T}\mu_0}$$

(b) $C = \dfrac{\chi T}{\mu_0} = \dfrac{\left(2.70 \times 10^{-4}\right)(300 \text{ K})}{4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}} = \boxed{6.45 \times 10^4 \ \dfrac{\text{K}\cdot\text{A}}{\text{T}\cdot\text{m}}}$

30.46 (a) $B_h = B_{\text{coil}} = \dfrac{\mu_0 NI}{2R} = \dfrac{(4\pi \times 10^{-7})(5.00)(0.600)}{0.300} = \boxed{12.6 \ \mu\text{T}}$

(b) $B_h = B\sin\phi \rightarrow B = \dfrac{B_h}{\sin\phi} = \dfrac{12.6 \ \mu T}{\sin 13.0°} = \boxed{56.0 \ \mu\text{T}}$



30.47 (a) Number of unpaired electrons $= \dfrac{8.00 \times 10^{22} \text{ A}\cdot\text{m}^2}{9.27 \times 10^{-24} \text{ A}\cdot\text{m}^2} = \boxed{8.63 \times 10^{45}}$

Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2}\left(8.63 \times 10^{45}\right)$.

(b) Mass $= \dfrac{(4.31 \times 10^{45} \text{ atoms})(7900 \text{ kg/m}^3)}{8.50 \times 10^{28} \text{ atoms/m}^3} = \boxed{4.01 \times 10^{20} \text{ kg}}$

Goal Solution

The magnetic moment of the Earth is approximately 8.00×10^{22} A·m². (a) If this were caused by the complete magnetization of a huge iron deposit, how many unpaired electrons would this correspond to? (b) At two unpaired electrons per iron atom, how many kilograms of iron would this correspond to? (Iron has a density of 7900 kg/m³, and approximately 8.50×10^{28} atoms/m³.)

G: We know that most of the Earth is not iron, so if the situation described provides an accurate model, then the iron deposit must certainly be less than the mass of the Earth ($M_{\text{Earth}} = 5.98 \times 10^{24}$ kg). One mole of iron has a mass of 55.8 g and contributes $2(6.02 \times 10^{23})$ unpaired electrons, so we should expect the total unpaired electrons to be less than 10^{50}.

O: The Bohr magneton μ_B is the measured value for the magnetic moment of a single unpaired electron. Therefore, we can find the number of unpaired electrons by dividing the magnetic moment of the Earth by μ_B. We can then use the density of iron to find the mass of the iron atoms that each contribute two electrons.

A: (a) $\mu_B = \left(9.27 \times 10^{-24}\ \frac{\text{J}}{\text{T}}\right)\left(1\ \frac{\text{N}\cdot\text{m}}{\text{J}}\right)\left(\frac{1\ \text{T}}{\text{N}\cdot\text{s/C}\cdot\text{m}}\right)\left(\frac{1\ \text{A}}{\text{C/s}}\right) = 9.27 \times 10^{-24}\ \text{A}\cdot\text{m}^2$

The number of unpaired electrons is $$N = \frac{8.00 \times 10^{22}\ \text{A}\cdot\text{m}^2}{9.27 \times 10^{-24}\ \text{A}\cdot\text{m}^2} = 8.63 \times 10^{45}\ \text{e}^-$$

(b) Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2}N = \frac{1}{2}(8.63 \times 10^{45}) = 4.31 \times 10^{45}$ iron atoms.

Thus, $$M_{Fe} = \frac{(4.31 \times 10^{45}\ \text{atoms})(7900\ \text{kg/m}^3)}{8.50 \times 10^{28}\ \text{atoms/m}^3} = 4.01 \times 10^{20}\ \text{kg}$$

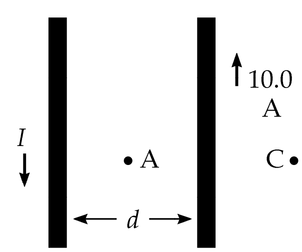
L: The calculated answers seem reasonable based on the limits we expected. From the data in this problem, the iron deposit required to produce the magnetic moment would only be about 1/15 000 the mass of the Earth and would form a sphere 500 km in diameter. Although this is certainly a large amount of iron, it is much smaller than the inner core of the Earth, which is estimated to have a diameter of about 3000 km.

30.48 $B = \dfrac{\mu_0 I}{2\pi R} = 2.00 \times 10^{-5}\ \text{T} = \boxed{20.0\ \mu\text{T}}$

30.49 $B = \dfrac{\mu_0 I R^2}{2(R^2 + R^2)^{3/2}}$ so $\boxed{I = 2.00 \times 10^9\ \text{A}}$ flowing west

30.50 (a) $B_C = \dfrac{\mu_0 I}{2\pi(0.270)} - \dfrac{\mu_0(10.0)}{2\pi(0.0900)} = 0$ so $\boxed{I = 30.0\ \text{A}}$

(b) $B_A = \dfrac{4\mu_0(10.0)}{2\pi(0.0900)} = \boxed{88.9\ \mu\text{T}}$ out of paper



***30.51** Suppose you have two 100-W headlights running from a 12-V battery, with the whole $\frac{200\text{ W}}{12\text{ V}} = 17$ A current going through the switch 60 cm from the compass. Suppose the dashboard contains little iron, so $\mu \cong \mu_0$. Model the current as straight. Then,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})17}{2\pi(0.6)} \boxed{\sim 10^{-5}\text{ T}}$$

If the local geomagnetic field is 5×10^{-5} T, this is $\boxed{\sim 10^{-1} \text{ times as large,}}$ enough to affect the compass noticeably.

30.52 A ring of radius r and width dr has area $dA = 2\pi r\, dr$. The current inside radius r is

$$I = \int_0^r 2\pi J r\, dr = 2\pi J_0 \int_0^r r\, dr - 2\pi\left(J_0/R^2\right)\int_0^r r^3\, dr = 2\pi J_0\, r^2/2 - 2\pi\left(J_0/R^2\right)\left(r^4/4\right)$$

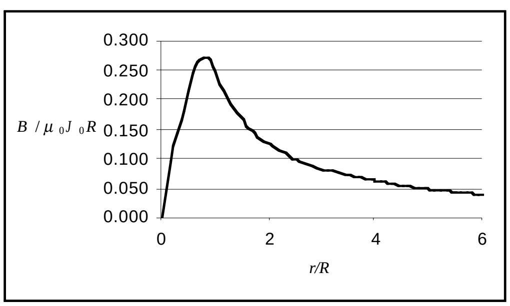
(a) Ampère's law says $B(2\pi r) = \mu_0 I = \mu_0 \pi J_0\left(r^2 - r^4/2R^2\right)$,

or $$\boxed{B = \mu_0 J_0 R\left[\frac{1}{2}\left(\frac{r}{R}\right) - \frac{1}{4}\left(\frac{r}{R}\right)^3\right] \text{ for } r \le R}$$

and $$B(2\pi r) = \mu_0 I_{\text{total}} = \mu_0\left[\pi J_0 R^2 - \pi J_0 R^2/2\right] = \mu_0 \pi J_0 R^2/2$$

or $$\boxed{B = \frac{\mu_0 J_0 R^2}{4r} = \frac{\mu_0 J_0 R}{4(r/R)} \text{ for } r \ge R}$$

(b)

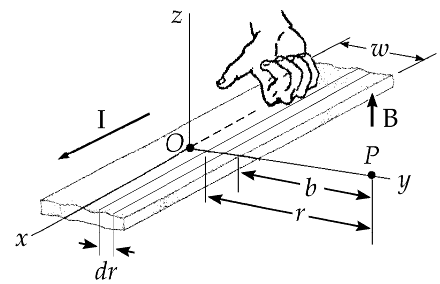


(c) To locate the maximum in the region $r \le R$, require that $\frac{dB}{dr} = \frac{\mu_0 J_0}{2} - 3\frac{\mu_0 J_0 r^2}{4R^2} = 0$

This gives the position of the maximum as $\boxed{r = \sqrt{2/3}\, R}$.

Here $B = \mu_0 J_0 R\left[\frac{1}{2}\left(\frac{2}{3}\right)^{1/2} - \frac{1}{4}\left(\frac{2}{3}\right)^{3/2}\right] = \boxed{0.272\, \mu_0 J_0 R}$

30.53 Consider a longitudinal filament of the strip of width dr as shown in the sketch. The contribution to the field at point P due to the current dI in the element dr is



$$dB = \frac{\mu_0 dI}{2\pi r} \qquad \text{where} \qquad dI = I(dr/w)$$

$$\mathbf{B} = \int d\mathbf{B} = \int_b^{b+w} \frac{\mu_0 I\, dr}{2\pi w r}\,\mathbf{k} = \boxed{\frac{\mu_0 I}{2\pi w}\ln\left(1+\frac{w}{b}\right)\mathbf{k}}$$

30.54 We find the total number of turns: $B = \dfrac{\mu_0 N I}{l}$

$$N = \frac{Bl}{\mu_0 I} = \frac{(0.0300\text{ T})(0.100\text{ m})\text{A}}{(4\pi\times 10^{-7}\text{ T}\cdot\text{m})(1.00\text{ A})} = 2.39\times 10^3$$

Each layer contains $(10.0\text{ cm}/0.0500\text{ cm}) = 200$ closely wound turns

so she needs $(2.39\times 10^3/200) = \boxed{12\text{ layers}}$.

The inner diameter of the innermost layer is 10.0 mm. The outer diameter of the outermost layer is $10.0\text{ mm} + 2\times 12\times 0.500\text{ mm} = 22.0\text{ mm}$. The average diameter is 16.0 mm, so the total length of wire is

$$(2.39\times 10^3)\pi(16.0\times 10^{-3}\text{ m}) = \boxed{120\text{ m}}$$

30.55 On the axis of a current loop, the magnetic field is given by $B = \dfrac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. The magnetic field is directed away from the center, with a strength of

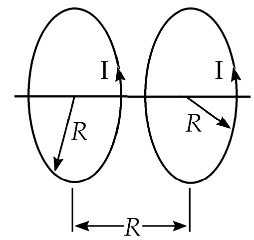
$$B = \frac{\mu_0\omega R^2 q}{4\pi(x^2+R^2)^{3/2}} = \frac{\mu_0(20.0)(0.100)^2(10.0\times 10^{-6})}{4\pi\left[(0.0500)^2+(0.100)^2\right]^{3/2}} = \boxed{1.43\times 10^{-10}\text{ T}}$$

30.56 On the axis of a current loop, the magnetic field is given by $B = \dfrac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. Therefore, $B = \dfrac{\mu_0\omega R^2 q}{4\pi(x^2+R^2)^{3/2}}$

when $x = \dfrac{R}{2}$, then $B = \dfrac{\mu_0\omega q R^2}{4\pi\left(\frac{5}{4}R^2\right)^{3/2}} = \boxed{\dfrac{\mu_0 q\omega}{2.5\sqrt{5}\,\pi R}}$

30.57 (a) Use Equation 30.7 twice: $B_x = \dfrac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$



$$B = B_{x1} + B_{x2} = \frac{\mu_0 I R^2}{2}\left[\frac{1}{(x^2 + R^2)^{3/2}} + \frac{1}{\left((R - x)^2 + R^2\right)^{3/2}}\right]$$

$$\boxed{B = \frac{\mu_0 I R^2}{2}\left[\frac{1}{(x^2 + R^2)^{3/2}} + \frac{1}{(2R^2 + x^2 - 2xR)^{3/2}}\right]}$$

(b) $\dfrac{dB}{dx} = \dfrac{\mu_0 I R^2}{2}\left[-\dfrac{3}{2}(2x)\left(x^2 + R^2\right)^{-5/2} - \dfrac{3}{2}\left(2R^2 + x^2 - 2xR\right)^{-5/2}(2x - 2R)\right]$

Substituting $x = \dfrac{R}{2}$ and cancelling terms, $\boxed{\dfrac{dB}{dx} = 0}$

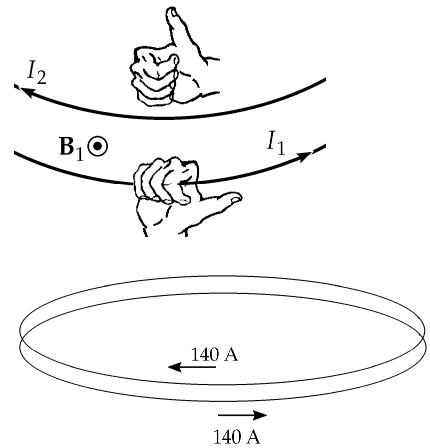
$$\frac{d^2B}{dx^2} = -\frac{3\mu_0 I R^2}{2}\left[(x^2 + R^2)^{-5/2} - 5x^2(x^2 + R^2)^{-7/2} + (2R^2 + x^2 - 2xR)^{-5/2} - 5(x - R)^2(2R^2 + x^2 - 2xR)^{-7/2}\right]$$

Again substituting $x = \dfrac{R}{2}$ and cancelling terms, $\boxed{\dfrac{d^2B}{dx^2} = 0}$

30.58 "Helmholtz pair" $\rightarrow$ separation distance = radius

$$B = \frac{2\mu_0 I R^2}{2\left[(R/2)^2 + R^2\right]^{3/2}} = \frac{\mu_0 I R^2}{\left[\frac{1}{4} + 1\right]^{3/2} R^3} = \frac{\mu_0 I}{1.40R} \quad \text{for 1 turn}$$

For N turns in each coil, $B = \dfrac{\mu_0 N I}{1.40R} = \dfrac{\left(4\pi \times 10^{-7}\right)100(10.0)}{1.40(0.500)} = \boxed{1.80 \times 10^{-3}\ \text{T}}$



30.59 Model the two wires as straight parallel wires (!)

(a) $F_B = \dfrac{\mu_0 I^2 L}{2\pi a}$ (Equation 30.12)

$$F_B = \frac{(4\pi \times 10^{-7})(140)^2 2\pi(0.100)}{2\pi(1.00 \times 10^{-3})} = \boxed{2.46 \text{ N}} \quad \text{upward}$$

(b) $$a_{\text{loop}} = \frac{2.46 \text{ N} - m_{\text{loop}}\, g}{m_{\text{loop}}} = \boxed{107 \text{ m/s}^2} \quad \text{upward}$$

***30.60** (a) In $d\mathbf{B} = \dfrac{\mu_0}{4\pi r^2} I d\mathbf{s} \times \sim$, the moving charge constitutes a bit of current as in $I = nqvA$. For a positive charge the direction of $d\mathbf{s}$ is the direction of $\mathbf{v}$, so $d\mathbf{B} = \dfrac{\mu_0}{4\pi r^2} nqA(ds)\mathbf{v} \times \sim$. Next, $A(ds)$ is the volume occupied by the moving charge, and $nA(ds) = 1$ for just one charge. Then,

$$\boxed{\mathbf{B} = \frac{\mu_0}{4\pi r^2} q\mathbf{v} \times \sim}$$

(b) $$B = \frac{\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)\left(1.60 \times 10^{-19} \text{ C}\right)\left(2.00 \times 10^{7} \text{ m/s}\right)}{4\pi\left(1.00 \times 10^{-3}\right)^2} \sin 90.0° = \boxed{3.20 \times 10^{-13} \text{ T}}$$

(c) $$F_B = q\left|\mathbf{v} \times \mathbf{B}\right| = \left(1.60 \times 10^{-19} \text{ C}\right)\left(2.00 \times 10^{7} \text{ m/s}\right)\left(3.20 \times 10^{-13} \text{ T}\right) \sin 90.0°$$

$$F_B = \boxed{1.02 \times 10^{-24} \text{ N directed away from the first proton}}$$

(d) $$F_e = qE = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99 \times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(1.00 \times 10^{-3}\right)^2}$$

$$F_e = \boxed{2.30 \times 10^{-22} \text{ N directed away from the first proton}}$$

Both forces act together. The electrical force is stronger by two orders of magnitude. It is productive to think about how it would look to an observer in a reference frame moving along with one proton or the other.

***30.61** (a) $$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(24.0 \text{ A})}{2\pi(0.0175 \text{ m})} = \boxed{2.74 \times 10^{-4} \text{ T}}$$

(b) At point C, conductor AB produces a field $\frac{1}{2}\left(2.74 \times 10^{-4} \text{ T}\right)(-\mathbf{j})$,  conductor DE produces a field of $\frac{1}{2}\left(2.74 \times 10^{-4} \text{ T}\right)(-\mathbf{j})$, BD produces no field, and AE produces negligible field. The total field at C is $\boxed{2.74 \times 10^{-4} \text{ T}(-\mathbf{j})}$.

(c) $\mathbf{F}_B = I\mathbf{L}\times\mathbf{B} = (24.0\ \text{A})(0.0350\ \text{m}\,\mathbf{k})\times\left[5\left(2.74\times10^{-4}\ \text{T}\right)(-\mathbf{j})\right] = \boxed{\left(1.15\times10^{-3}\ \text{N}\right)\mathbf{i}}$

(d) $\mathbf{a} = \dfrac{\Sigma\mathbf{F}}{m} = \dfrac{\left(1.15\times10^{-3}\ \text{N}\right)\mathbf{i}}{3.00\times10^{-3}\ \text{kg}} = \boxed{\left(0.384\ \dfrac{\text{m}}{\text{s}^2}\right)\mathbf{i}}$

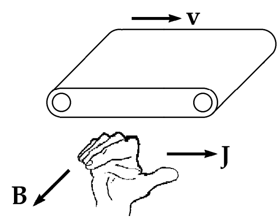
(e) The bar is already so far from AE that it moves through nearly constant magnetic field. The force acting on the bar is constant, and therefore the bar's $\boxed{\text{acceleration is constant}}$.

(f) $v_f^2 = v_i^2 + 2ax = 0 + 2\left(0.384\ \text{m/s}^2\right)(1.30\ \text{m})$, so $\mathbf{v}_f = \boxed{(0.999\ \text{m/s})\mathbf{i}}$

30.62 At equilibrium, $\frac{F_B}{\ell} = \frac{\mu_0 I_A I_B}{2\pi a} = \frac{mg}{\ell}$ or $I_B = \frac{2\pi a(m/\ell)g}{\mu_0 I_A}$

$$I_B = \frac{2\pi(0.0250\ \text{m})(0.0100\ \text{kg/m})(9.80\ \text{m/s}^2)}{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(150\ \text{A})} = \boxed{81.7\ \text{A}}$$

30.63 (a) The magnetic field due to an infinite sheet of charge (or the magnetic field at points near a large sheet of charge) is given by $B = \mu_0 J_s/2$. The current density $J_s = I/\ell$ and in this case the equivalent current of the moving charged belt is



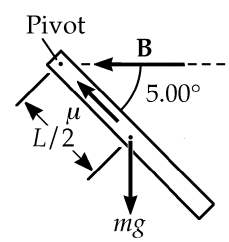
$$I = \frac{dq}{dt} = \frac{d}{dt}(\sigma \ell x) = \sigma \ell v; \qquad v = \frac{dx}{dt}$$

Therefore, $J_s = \sigma v$ and $\boxed{B = \frac{\mu_0 \sigma v}{2}}$

(b) If the sheet is positively charged and moving in the direction shown, the magnetic field is $\boxed{\text{out of the page, parallel to the roller axes.}}$

30.64 $$C = \frac{TM}{B} = \frac{(4.00\ \text{K})(10.0\%)(8.00 \times 10^{27}\ \text{atoms / m}^3)(5.00)(9.27 \times 10^{-24}\ \text{J / T}^2)}{5.00\ \text{T}} = \boxed{2.97 \times 10^4\ \frac{\text{K}\cdot\text{J}}{\text{T}^2\cdot\text{m}^3}}$$

30.65 At equilibrium, $\Sigma\tau = +|\mu \times \mathbf{B}| - mg\left(\frac{L}{2}\cos 5.00°\right) = 0,$



or $\mu B \sin 5.00° = \frac{mgL}{2}\cos 5.00°$

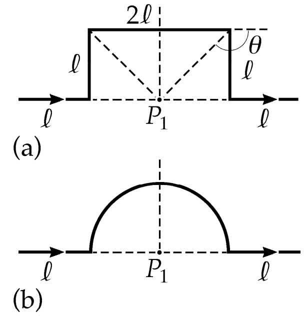
Therefore, $B = \frac{mgL}{2\mu \tan 5.00°} = \frac{(0.0394\ \text{kg})(9.80\ \text{m/s}^2)(0.100\ \text{m})}{2(7.65\ \text{J/T})\tan 5.00°}$

$B = \boxed{28.8\ \text{mT}}$

30.66 The central wire creates field $\mathbf{B} = \mu_0 I_1/2\pi R$ counterclockwise. The curved portions of the loop feels no force since $\boldsymbol{\ell} \times \mathbf{B} = 0$ there. The straight portions both feel $I\boldsymbol{\ell} \times \mathbf{B}$ forces to the right, amounting to

$$\mathbf{F}_B = I_2 2L \frac{\mu_0 I_1}{2\pi R} = \boxed{\frac{\mu_0 I_1 I_2 L}{\pi R}\ \text{to the right}}$$

30.67 When the conductor is in the rectangular shape shown in figure (a), the segments carrying current straight toward or away from point P_1 do not contribute to the magnetic field at P_1. Each of the other four setions of length $\mathtt{l}$ makes an equal contribution to the total field into the page at P_1. To find the contribution of the horizontal section of current in the upper right, we use



$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) \quad \text{with} \quad a = \mathtt{l},\ \theta_1 = 90°,\ \text{and} \quad \theta_2 = 135°$$

So $$B_1 = \frac{4\mu_0 I}{4\pi \mathtt{l}}\left(0 - \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{\sqrt{2}\ \pi \mathtt{l}}$$

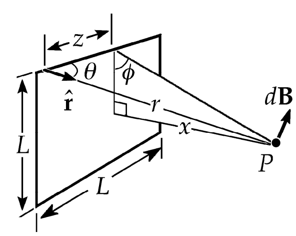
When the conductor is in the shape of a circular arc, the magnitude or the field at the center is given by Equation 30.6, $B = \frac{\mu_0 I}{4\pi R}\theta$. From the geometry in this case, we find $R = \frac{4\mathtt{l}}{\pi}$ and $\theta = \pi$.

Therefore, $B_2 = \frac{\mu_0 I\pi}{4\pi(4\mathtt{l}/\pi)} = \frac{\mu_0 I\pi}{16\mathtt{l}}$; so that $\boxed{\frac{B_1}{B_2} = \frac{8\sqrt{2}}{\pi^2}}$

30.68 $$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(9.00\times10^{3}\right)\left(1.50\times10^{-8}\right)}{4\pi\times10^{-7}} = \boxed{675\ \text{A}}$$

Flow of $\boxed{\text{positive current is downward } \textit{or} \text{ negative charge flows upward}}$.

30.69 By symmetry of the arrangement, the magnitude of the net magnetic field at point P is $B = 8B_{0x}$ where B_0 is the contribution to the field due to current in an edge length equal to $L/2$. In order to calculate B_0, we use the Biot-Savart law and consider the plane of the square to be the yz-plane with point P on the x-axis. The contribution to the magnetic field at point P due to a current element of length dz and located a distance z along the axis is given by Equation 30.3.



$$\mathbf{B}_0 = \frac{\mu_0 I}{4\pi}\int \frac{d\mathtt{1}\times\sim}{r^2}$$

From the figure we see that

$$r = \sqrt{x^2 + (L^2/4) + z^2} \qquad \text{and} \qquad |d\mathtt{1}\times\sim| = dz\sin\theta = dz\sqrt{\frac{L^2/4 + x^2}{L^2/4 + x^2 + z^2}}$$

By symmetry all components of the field $\mathbf{B}$ at P cancel except the components along x (perpendicular to the plane of the square); and

$B_{0x} = B_0 \cos\phi$ where $\cos\phi = \dfrac{L/2}{\sqrt{L^2/4 + x^2}}$.

Therefore, $\mathbf{B}_{0x} = \dfrac{\mu_0 I}{4\pi}\displaystyle\int_0^{L/2} \dfrac{\sin\theta\cos\phi\, dz}{r^2}$ and $B = 8B_{0x}$.

Using the expressions given above for sin θ cos ϕ, and r, we find

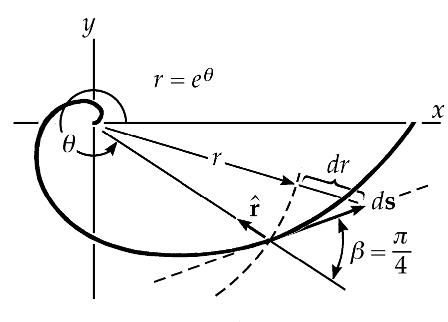
$$B = \frac{\mu_0 I L^2}{2\pi\left(x^2 + \dfrac{L^2}{4}\right)\sqrt{x^2 + \dfrac{L^2}{2}}}$$

30.70 (a) From Equation 30.10, the magnetic field produced by one loop at the center of the second loop is given by $B = \dfrac{\mu_0 I R^2}{2x^3} = \dfrac{\mu_0 I\left(\pi R^2\right)}{2\pi x^3} = \dfrac{\mu_0 \mu}{2\pi x^3}$ where the magnetic moment of either loop is $\mu = I\left(\pi R^2\right)$. Therefore,

$$\left|F_x\right| = \mu\frac{dB}{dx} = \mu\left(\frac{\mu_0\mu}{2\pi}\right)\left(\frac{3}{x^4}\right) = \frac{3\mu_0\left(\pi R^2 I\right)^2}{2\pi x^4} = \boxed{\frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4}}$$

(b) $\left|F_x\right| = \dfrac{3\pi}{2}\dfrac{\mu_0 I^2 R^4}{x^4} = \dfrac{3\pi}{2}\dfrac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(10.0\ \text{A})^2\left(5.00\times10^{-3}\ \text{m}\right)^4}{\left(5.00\times10^{-2}\ \text{m}\right)^4} = \boxed{5.92\times10^{-8}\ \text{N}}$

30.71 There is no contribution from the straight portion of the wire since $d\mathbf{s}\times \sim = 0$. For the field of the spiral,



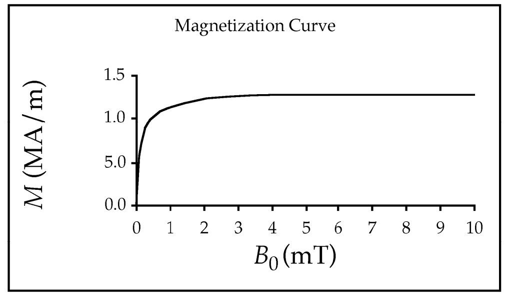
$$d\mathbf{B} = \frac{\mu_0 I}{(4\pi)}\frac{(d\mathbf{s}\times\sim)}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\frac{\left|d\mathbf{s}\right|\sin\theta\left|\sim\right|}{r^2} = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\left(\sqrt{2}\,dr\right)\left[\sin\left(\frac{3\pi}{4}\right)\right]\frac{1}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi} r^{-2}\,dr = -\frac{\mu_0 I}{4\pi}\left(r^{-1}\right)\Big|_{\theta=0}^{2\pi}$$

Substitute $r = e^\theta$: $B = -\dfrac{\mu_0 I}{4\pi}\left[e^{-\theta}\right]_0^{2\pi} = -\dfrac{\mu_0 I}{4\pi}\left[e^{-2\pi} - e^0\right] = \boxed{\dfrac{\mu_0 I}{4\pi}\left(1 - e^{-2\pi}\right)}$ (out of the page)

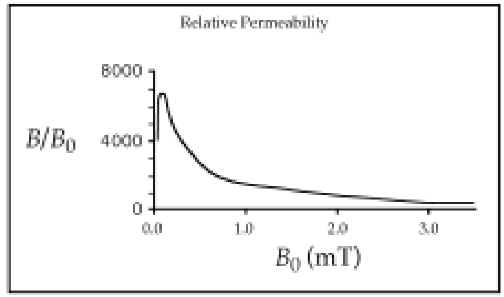
30.72 (a) $\mathbf{B} = \mathbf{B}_0 + \mu_0 \mathbf{M}$



$$\mathbf{M} = \frac{\mathbf{B} - \mathbf{B}_0}{\mu_0} \quad \text{and} \quad M = \frac{|\mathbf{B} - \mathbf{B}_0|}{\mu_0}$$

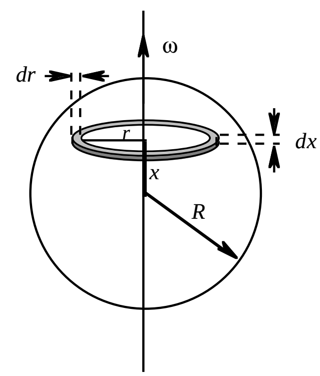
Assuming that **B** and $\mathbf{B}_0$ are parallel, this becomes $M = (B - B_0)/\mu_0$

The magnetization curve gives a plot of M versus B_0.



(b) The second graph is a plot of the relative permeability (B/B_0) as a function of the applied field B_0.

30.73 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The contribution to the field from each ring is



$$dB = \frac{\mu_0 r^2\, dI}{2\left(x^2 + r^2\right)^{3/2}} \qquad \text{where} \qquad dI = \frac{dQ}{t} = \frac{\omega\, dQ}{2\pi}$$

$$dQ = \rho\, dV = \rho(2\pi r\, dr)(dx)$$

$$dB = \frac{\mu_0 \rho \omega r^3\, dr\, dx}{2\left(x^2 + r^2\right)^{3/2}} \quad \text{where} \quad \rho = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$$

$$B = \int_{x=-R}^{+R} \int_{r=0}^{\sqrt{R^2 - x^2}} \frac{\mu_0 \rho \omega}{2} \frac{r^3\, dr\, dx}{\left(x^2 + r^2\right)^{3/2}}$$

Let $v = r^2 + x^2$, $\quad dv = 2r\, dr$, $\quad$ and $\quad r^2 = v - x^2$

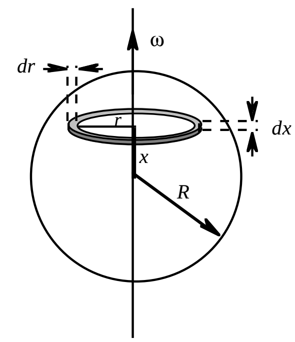
$$B = \int_{x=-R}^{+R} \int_{v=x^2}^{R^2} \frac{\mu_0 \rho \omega}{2} \frac{\left(v - x^2\right) dv}{2 v^{3/2}} dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[\int_{v=x^2}^{R^2} v^{-1/2}\, dv - x^2 \int_{v=x^2}^{R^2} v^{-3/2}\, dv \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[2v^{1/2}\Big|_{x^2}^{R^2} + \left(2x^2\right) v^{-1/2}\Big|_{x^2}^{R^2} \right] dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[2\left(R - |x|\right) + 2x^2 \left(\frac{1}{R} - \frac{1}{|x|} \right) \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{-R}^{R} \left[2\frac{x^2}{R} - 4|x| + 2R \right] dx = \frac{2\mu_0 \rho \omega}{4} \int_{0}^{R} \left[2\frac{x^2}{R} - 4x + 2R \right] dx$$

$$B = \frac{2\mu_0 \rho \omega}{4} \left(\frac{2R^3}{3R} - \frac{4R^2}{2} + 2R^2 \right) = \boxed{\frac{\mu_0 \rho \omega R^2}{3}}$$

30.74 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The current associated with each rotating ring of charge is



$$dI = \frac{dQ}{t} = \frac{\omega}{2\pi}\left[\rho(2\pi r\,dr)(dx)\right]$$

The magnetic moment contributed by this ring is

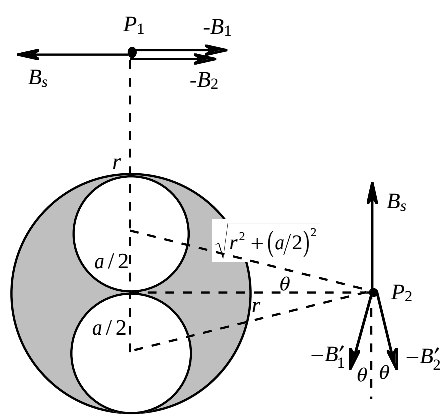
$$d\mu = A(dI) = \pi r^2 \frac{\omega}{2\pi}\left[\rho(2\pi r\,dr)(dx)\right] = \pi\omega\rho r^3\,dr\,dx$$

$$\mu = \pi\omega\rho\int_{x=-R}^{+R}\left[\int_{r=0}^{\sqrt{R^2-x^2}} r^3\,dr\right]dx = \pi\omega\rho\int_{x=-R}^{+R}\frac{\left(\sqrt{R^2-x^2}\right)^4}{4}dx = \pi\omega\rho\int_{x=-R}^{+R}\frac{\left(R^2-x^2\right)^2}{4}dx$$

$$\mu = \frac{\pi\omega\rho}{4}\int_{x=-R}^{+R}\left(R^4-2R^2x^2+x^4\right)dx = \frac{\pi\omega\rho}{4}\left[R^4(2R)-2R^2\left(\frac{2R^2}{3}\right)+\frac{2R^5}{5}\right]$$

$$\mu = \frac{\pi\omega\rho}{4}R^5\left(2-\frac{4}{3}+\frac{2}{5}\right) = \frac{\pi\omega\rho R^5}{4}\left(\frac{16}{15}\right) = \boxed{\frac{4\pi\omega\rho R^5}{15}} \quad \text{up}$$

30.75 Note that the current I exists in the conductor with a current density $J = I/A$, where



$$A = \pi\left[a^2 - a^2/4 - a^2/4\right] = \pi a^2/2$$

Therefore, $J = 2I/\pi a^2$.

To find the field at either point P_1 or P_2, find B_s which would exist if the conductor were solid, using Ampère's law. Next, find B_1 and B_2 that *would* be due to the conductors of radius $a/2$ that *could* occupy the void where the holes exist. Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

(a) At point P_1, $B_s = \dfrac{\mu_0 J(\pi a^2)}{2\pi r}$, $B_1 = \dfrac{\mu_0 J\pi(a/2)^2}{2\pi(r-a/2)}$, and $B_2 = \dfrac{\mu_0 J\pi(a/2)^2}{2\pi(r+a/2)}$.

$$B = B_s - B_1 - B_2 = \frac{\mu_0 J\pi a^2}{2\pi}\left[\frac{1}{r} - \frac{1}{4(r-a/2)} - \frac{1}{4(r+a/2)}\right]$$

$$B = \frac{\mu_0(2I)}{2\pi}\left[\frac{4r^2-a^2-2r^2}{4r\left(r^2-a^2/4\right)}\right] = \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2-a^2}{4r^2-a^2}\right] \text{ directed to the left}}$$

(b) At point P_2, $B_s = \dfrac{\mu_0 J\left(\pi a^2\right)}{2\pi r}$ and $B_1' = B_2' = \dfrac{\mu_0 J\pi\left(a/2\right)^2}{2\pi\sqrt{r^2+\left(a/2\right)^2}}$.

The horizontal components of B_1' and B_2' cancel while their vertical components add.

$$B = B_s - B_1'\cos\theta - B_2'\cos\theta = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r} - 2\left(\frac{\mu_0 J\pi a^2/4}{2\pi\sqrt{r^2+a^2/4}}\right)\frac{r}{\sqrt{r^2+a^2/4}}$$

$$B = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}\left[1-\frac{r^2}{2\left(r^2+a^2/4\right)}\right] = \frac{\mu_0(2I)}{2\pi r}\left[1-\frac{2r^2}{4r^2+a^2}\right] = \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right] \text{ directed toward the top of the page}}$$

Chapter 31 Solutions

31.1 $\mathcal{E} = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = \dfrac{\Delta(NBA)}{\Delta t} = \boxed{500 \text{ mV}}$

31.2 $\mathcal{E} = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = \dfrac{\Delta(\mathbf{B}\cdot\mathbf{A})}{\Delta t} = 1.60 \text{ mV}$ and $I_{\text{loop}} = \dfrac{\mathcal{E}}{R} = \dfrac{1.60 \text{ mV}}{2.00\ \Omega} = \boxed{0.800 \text{ mA}}$

31.3 $\mathcal{E} = -N \dfrac{\Delta BA \cos\theta}{\Delta t} = -NB \pi r^2 \left(\dfrac{\cos\theta_f - \cos\theta_i}{\Delta t}\right)$

$= -25.0\left(50.0\times 10^{-6}\text{ T}\right)\pi\left(0.500\text{ m}\right)^2\left(\dfrac{\cos 180° - \cos 0}{0.200 \text{ s}}\right)$

$\text{E} = \boxed{+9.82 \text{ mV}}$

31.4 (a) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -A\dfrac{dB}{dt} = \boxed{\dfrac{AB_{\max}}{\tau}e^{-t/\tau}}$

(b) $\mathcal{E} = \dfrac{(0.160 \text{ m}^2)(0.350 \text{ T})}{2.00 \text{ s}} e^{-4.00/2.00} = \boxed{3.79 \text{ mV}}$

(c) At $t = 0$, $\mathcal{E} = \boxed{28.0 \text{ mV}}$

31.5 $|\mathcal{E}| = N\dfrac{d\Phi_B}{dt} = \dfrac{\Delta(NBA)}{\Delta t} = 3.20 \text{ kV}$ so $I = \dfrac{\mathcal{E}}{R} = \boxed{160 \text{ A}}$

Goal Solution

A strong electromagnet produces a uniform field of 1.60 T over a cross-sectional area of 0.200 m^2. A coil having 200 turns and a total resistance of 20.0 Ω is placed around the electromagnet. The current in the electromagnet is then smoothly decreased until it reaches zero in 20.0 ms. What is the current induced in the coil?

G: A strong magnetic field turned off in a short time (20.0 ms) will produce a large emf, maybe on the order of 1 kV. With only 20.0 Ω of resistance in the coil, the induced current produced by this emf will probably be larger than 10 A but less than 1000 A.

O: According to Faraday's law, if the magnetic field is reduced uniformly, then a constant emf will be produced. The definition of resistance can be applied to find the induced current from the emf.

A: Noting unit conversions from $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$ and $U = qV$, the induced voltage is

$$\mathcal{E} = -N\frac{d(\mathbf{B}\cdot\mathbf{A})}{dt} = -N\left(\frac{0 - B_i A\cos\theta}{\Delta t}\right) = \frac{+200(1.60\text{ T})(0.200\text{ m}^2)(\cos 0°)}{20.0\times 10^{-3}\text{ s}}\left(\frac{1\text{ N}\cdot\text{s / C}\cdot\text{m}}{\text{T}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{\text{N}\cdot\text{m}}\right) = 3200\text{ V}$$

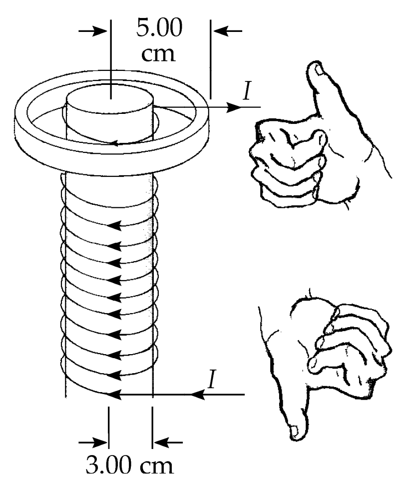
$$I = \frac{\mathcal{E}}{R} = \frac{3200\text{ V}}{20.0\ \Omega} = 160\text{ A}$$

L: This is a large current, as we expected. The positive sign is indicative that the induced electric field is in the positive direction around the loop (as defined by the area vector for the loop).

31.6 $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -\dfrac{N(BA - 0)}{\Delta t}$

$$\Delta t = \frac{NBA}{|\mathcal{E}|} = \frac{NB(\pi r^2)}{\mathcal{E}} = \frac{500(0.200)\pi(5.00\times 10^{-2})^2}{10.0\times 10^3} = \boxed{7.85\times 10^{-5}\text{ s}}$$

31.7 $|\mathcal{E}| = \dfrac{d(BA)}{dt} = 0.500\ \mu_0 nA\dfrac{dI}{dt} = 0.480\times 10^{-3}\text{ V}$



(a) $I_{\text{ring}} = \dfrac{\mathcal{E}}{R} = \dfrac{4.80\times 10^{-4}}{3.00\times 10^{-4}} = \boxed{1.60\text{ A}}$

(b) $B_{\text{ring}} = \dfrac{\mu_0 I}{2r_{\text{ring}}} = \boxed{20.1\ \mu\text{T}}$

(c) Coil's field points downward, and is increasing, so

$\boxed{B_{\text{ring}}\text{ points upward}}$

31.8 $|\mathcal{E}| = \dfrac{d(BA)}{dt} = 0.500\ \mu_0 nA\dfrac{dI}{dt} = 0.500\ \mu_0 n\pi r_2^2\dfrac{\Delta I}{\Delta t}$

(a) $I_{\text{ring}} = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{\mu_0 n \pi r_2^2}{2R} \dfrac{\Delta I}{\Delta t}}$

(b) $B = \dfrac{\mu_0 I}{2r_1} = \boxed{\dfrac{\mu_0^2 n \pi r_2^2}{4 r_1 R} \dfrac{\Delta I}{\Delta t}}$

(c) The coil's field points downward, and is increasing, so $\boxed{B_{\text{ring}} \text{ points upward}}$.

31.9 (a) $d\Phi_B = \mathbf{B} \cdot d\mathbf{A} = \dfrac{\mu_0 I}{2\pi x} L dx$: $\quad \Phi_B = \displaystyle\int_{x=h}^{h+w} \dfrac{\mu_0 IL}{2\pi} \dfrac{dx}{x} = \boxed{\dfrac{\mu_0 IL}{2\pi} \ln\left(\dfrac{h+w}{h}\right)}$

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}\left[\dfrac{\mu_0 IL}{2\pi} \ln\left(\dfrac{h+w}{h}\right)\right] = -\left[\dfrac{\mu_0 L}{2\pi} \ln\left(\dfrac{h+w}{h}\right)\right]\dfrac{dI}{dt}$

$$\mathcal{E} = -\frac{\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(1.00 \text{ m})}{2\pi} \ln\left(\frac{1.00 + 10.0}{1.00}\right)\left(10.0 \ \frac{\text{A}}{\text{s}}\right) = \boxed{-4.80 \ \mu\text{V}}$$

The long wire produces magnetic flux into the page through the rectangle (first figure, above). As it increases, the rectangle wants to produce its own magnetic field out of the page, which it does by carrying $\boxed{\text{counterclockwise}}$ current (second figure, above).

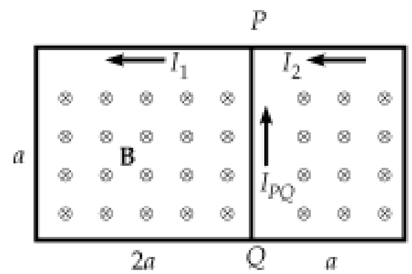
31. 10 $\Phi_B = (\mu_0 n I) A_{\text{solenoid}}$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\frac{dI}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\left(600\ \text{A/s}\right)\cos(120t)$$

$$\mathcal{E} = -15.0\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.00\times10^3/\text{m}\right)\pi(0.0200\ \text{m})^2\left(600\ \text{A/s}\right)\cos(120t)$$

$$\boxed{\mathcal{E} = -14.2\cos(120t)\ \text{mV}}$$

31.11 For a counterclockwise trip around the left-hand loop, with $B = At$



$$\frac{d}{dt}\left[At(2a^2)\cos 0°\right] - I_1(5R) - I_{PQ}R = 0$$

and for the right-hand loop,

$$\frac{d}{dt}\left[Ata^2\right] + I_{PQ}R - I_2(3R) = 0$$

where $I_{PQ} = I_1 - I_2$ is the upward current in QP

Thus, $2Aa^2 - 5R(I_{PQ} + I_2) - I_{PQ}R = 0$

and $\quad Aa^2 + I_{PQ}R = I_2(3R)$

$$2Aa^2 - 6RI_{PQ} - \tfrac{5}{3}(Aa^2 + I_{PQ}R) = 0$$

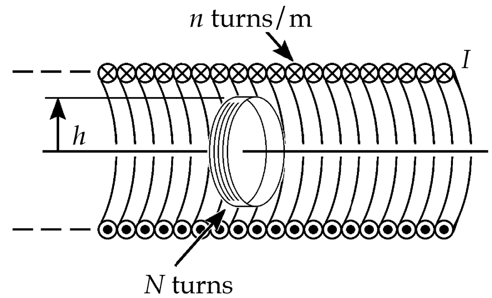
$$I_{PQ} = \frac{Aa^2}{23R}$$ upward, and since $R = (0.100\ \Omega/\text{m})(0.650\ \text{m}) = 0.0650\ \Omega$

$$I_{PQ} = \frac{(1.00\times10^{-3}\ \text{T/s})(0.650\ \text{m})^2}{23(0.0650\ \Omega)} = \boxed{283\ \mu\text{A upward}}$$

31.12 $\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = N\left(\frac{dB}{dt}\right)A = N\left(0.0100 + 0.0800t\right)A$

At $t = 5.00$ s, $\quad \mathcal{E} = 30.0(0.410\ \text{T})\left[\pi\left(0.0400\ \text{m}\right)^2\right] = \boxed{61.8\ \text{mV}}$

31.13



$$B = \mu_0 nI = \mu_0 n(30.0\text{ A})\left(1 - e^{-1.60t}\right)$$

$$\Phi_B = \int B\,dA = \mu_0 n(30.0\text{ A})\left(1 - e^{-1.60t}\right)\int dA$$

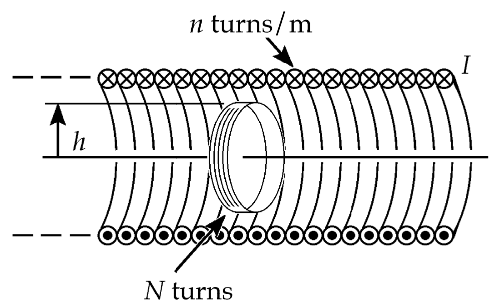
$$\Phi_B = \mu_0 n(30.0\text{ A})\left(1 - e^{-1.60t}\right)\pi R^2$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n(30.0\text{ A})\pi R^2(1.60)e^{-1.60t}$$

$$\mathcal{E} = -(250)(4\pi\times10^{-7}\text{ N/A}^2)(400\text{ m}^{-1})(30.0\text{ A})\left[\pi(0.0600\text{ m})^2\right]1.60\text{ s}^{-1}e^{-1.60t}$$

$$\mathcal{E} = \boxed{(68.2\text{ mV})e^{-1.60t}\ \text{ counterclockwise}}$$

31.14



$$B = \mu_0 nI = \mu_0 nI_{\max}(1 - e^{-\alpha t})$$

$$\Phi_B = \int B\,dA = \mu_0 nI_{\max}(1 - e^{-\alpha t})\int dA$$

$$\Phi_B = \mu_0 nI_{\max}(1 - e^{-\alpha t})\pi R^2$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 nI_{\max}\pi R^2\alpha e^{-\alpha t}$$

$$\mathcal{E} = \boxed{N\mu_0 nI_{\max}\pi R^2\alpha e^{-\alpha t}\ \text{ counterclockwise}}$$

31.15

$$\mathcal{E} = \frac{d}{dt}(NB\ell^2\cos\theta) = \frac{N\ell^2\,\Delta B\cos\theta}{\Delta t}$$

$$\ell = \sqrt{\frac{\mathcal{E}\,\Delta t}{N\,\Delta B\cos\theta}} = \sqrt{\frac{(80.0\times10^{-3}\text{ V})(0.400\text{ s})}{(50)(600\times10^{-6}\text{ T} - 200\times10^{-6}\text{ T})\cos(30.0^\circ)}} = 1.36\text{ m}$$

Length = $4\ell N = 4(1.36\text{ m})(50) = \boxed{272\text{ m}}$

Goal Solution

A coil formed by wrapping 50.0 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 30.0° with the direction of the field. When the magnetic field is increased uniformly from 200 μT to 600 μT in 0.400 s, an emf of 80.0 mV is induced in the coil. What is the total length of the wire?

G: If we assume that this square coil is some reasonable size between 1 cm and 1 m across, then the total length of wire would be between 2 m and 200 m.

O: The changing magnetic field will produce an emf in the coil according to Faraday's law of induction. The constant area of the coil can be found from the change in flux required to produce the emf.

A: By Faraday's law, $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -N\dfrac{d}{dt}(BA\cos\theta) = -NA\cos\theta\dfrac{dB}{dt}$

For magnitudes, $|\mathcal{E}| = NA\cos\theta\left(\dfrac{\Delta B}{\Delta t}\right)$

and the area is $A = \dfrac{|\mathcal{E}|}{N\cos\theta\left(\dfrac{\Delta B}{\Delta t}\right)} = \dfrac{80.0\times10^{-3}\text{ V}}{50(\cos 30.0°)\left(\dfrac{600\times10^{-6}\text{ T} - 200\times10^{-6}\text{ T}}{0.400\text{ s}}\right)} = 1.85\text{ m}^2$

Each side of the coil has length $d = \sqrt{A}$, so the total length of the wire is

$$L = N(4d) = 4N\sqrt{A} = (4)(50)\sqrt{1.85\text{ m}^2} = 272\text{ m}$$

L: The total length of wire is slightly longer than we predicted. With $d = 1.36$ m, a normal person could easily step through this large coil! As a bit of foreshadowing to a future chapter on AC circuits, an even bigger coil with more turns could be hidden in the ground below high-power transmission lines so that a significant amount of power could be "stolen" from the electric utility. There is a story of one man who did this and was arrested when investigators finally found the reason for a large power loss in the transmission lines!

31.16 The average induced emf is given by $\mathcal{E} = -N\left(\dfrac{\Delta\Phi_B}{\Delta t}\right)$

Here $N = 1$, and $\Delta\Phi_B = B(A_{\text{square}} - A_{\text{circle}})$

with $A_{\text{circle}} = \pi r^2 = \pi(0.500\text{ m})^2 = 0.785\text{ m}^2$

Also, the circumference of the circle is $2\pi r = 2\pi(0.500\text{ m}) = 3.14\text{ m}$

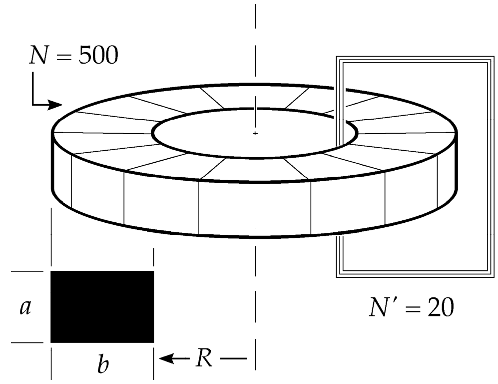
Thus, each side of the square has a length $L = \dfrac{3.14\text{ m}}{4} = 0.785\text{ m}$,

and $A_{\text{square}} = L^2 = 0.617\text{ m}^2$

So $\Delta\Phi_B = (0.400\text{ T})(0.617\text{ m}^2 - 0.785\text{ m}^2) = -0.0672\text{ T}\cdot\text{m}^2$

The average induced emf is therefore: $\mathcal{E} = -\dfrac{-0.0672\text{ T}\cdot\text{m}^2}{0.100\text{ s}} = \boxed{0.672\text{ V}}$

31.17 In a toroid, all the flux is confined to the inside of the toroid.



$$B = \frac{\mu_0 NI}{2\pi r} = \frac{500\,\mu_0 I}{2\pi r}$$

$$\Phi_B = \int B\,dA = \frac{500\,\mu_0 I_{\max}}{2\pi}\sin\omega t \int \frac{dz\,dr}{r}$$

$$\Phi_B = \frac{500\,\mu_0 I_{\max}}{2\pi} a \sin\omega t \ln\left(\frac{b+R}{R}\right)$$

$$\mathcal{E} = N'\frac{d\Phi_B}{dt} = 20\left(\frac{500\,\mu_0 I_{\max}}{2\pi}\right)\omega a \ln\left(\frac{b+R}{R}\right)\cos\omega t$$

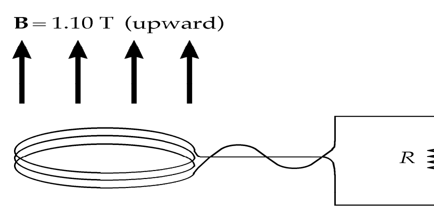
$$\mathcal{E} = \frac{10^4}{2\pi}\left(4\pi\times10^{-7}\,\frac{\text{N}}{\text{A}^2}\right)(50.0\text{ A})\left(377\,\frac{\text{rad}}{\text{s}}\right)(0.0200\text{ m})\ln\left(\frac{(3.00+4.00)\text{ cm}}{4.00\text{ cm}}\right)\cos\omega t = \boxed{(0.422\text{ V})\cos\omega t}$$

31.18 The field inside the solenoid is: $B = \mu_0 nI = \mu_0\left(\frac{N}{\ell}\right)I$

Thus, through the single-turn loop $\Phi_B = BA_{\text{solenoid}} = \mu_0\left(\frac{N}{\ell}\right)\left(\pi r^2\right)I$

and the induced emf in the loop is $\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\mu_0\left(\frac{N}{\ell}\right)\left(\pi r^2\right)\left(\frac{\Delta I}{\Delta t}\right) = \boxed{-\frac{\mu_0 N\pi r^2}{\ell}\left(\frac{I_2 - I_1}{\Delta t}\right)}$

31.19 $\mathcal{E} = -N\frac{d\Phi_B}{dt}$ $\qquad IR = -N\frac{d\Phi_B}{dt}$

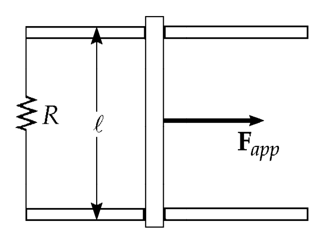


$Idt = -\frac{N}{R}d\Phi_B$ $\qquad \int Idt = -\frac{N}{R}\int d\Phi_B$

$$Q = -\frac{N}{R}\Delta\Phi_B = -\frac{N}{R}A\left(B_f - B_i\right)$$

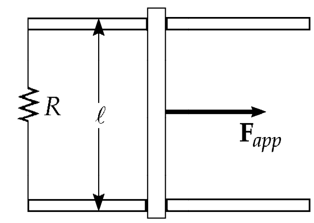
$$Q = -\left(\frac{200}{5.00\ \Omega}\right)(100\times10^{-4}\text{ m}^2)(-1.10-1.10)\text{ T} = \boxed{0.880\text{ C}}$$

31.20 $I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$



$\boxed{v = 1.00\text{ m/s}}$

31.21 (a) $|\mathbf{F}_B| = I|\mathbf{l}\times\mathbf{B}| = IlB.$ When $I = \mathcal{E}/R$ and $\mathcal{E} = Blv$, we get



$$F_B = \frac{Blv}{R}(lB) = \frac{B^2 l^2 v}{R} = \frac{(2.50)^2(1.20)^2(2.00)}{6.00} = 3.00\text{ N}$$

The applied force is $\boxed{3.00\text{ N to the right}}$

(b) $P = I^2 R = \dfrac{B^2 l^2 v^2}{R} = 6.00\text{ W}$ or $P = Fv = \boxed{6.00\text{ W}}$

***31.22** $F_B = IlB$ and $\mathcal{E} = Blv$

$I = \dfrac{\mathcal{E}}{R} = \dfrac{Blv}{R}$ so $B = \dfrac{IR}{lv}$

(a) $F_B = \dfrac{I^2 lR}{lv}$ and $I = \sqrt{\dfrac{F_B v}{R}} = \boxed{0.500\text{ A}}$

(b) $I^2 R = \boxed{2.00\text{ W}}$

(c) For constant force, $P = \mathbf{F}\cdot\mathbf{v} = (1.00\text{ N})(2.00\text{ m/s}) = \boxed{2.00\text{ W}}$

31.23 The downward component of **B**, perpendicular to **v**, is $(50.0\times10^{-6}\text{ T})\sin 58.0° = 4.24\times10^{-5}\text{ T}$

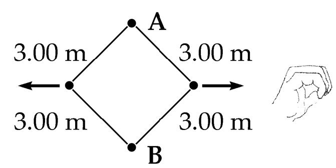
$$\mathcal{E} = Blv = (4.24\times10^{-5}\text{ T})(60.0\text{ m})(300\text{ m/s}) = \boxed{0.763\text{ V}}$$

The $\boxed{\text{left wing tip is positive}}$ relative to the right.

31.24 $\mathcal{E} = -N\dfrac{d}{dt}BA\cos\theta = -NB\cos\theta\left(\dfrac{\Delta A}{\Delta t}\right)$

$$\mathcal{E} = -1(0.100\text{ T})\cos 0°\ \frac{(3.00\text{ m}\times3.00\text{ m}\sin 60.0°) - (3.00\text{ m})^2}{0.100\text{ s}} = 1.21\text{ V}$$

$I = \dfrac{1.21\text{ V}}{10.0\ \Omega} = \boxed{0.121\text{ A}}$



The flux is into the page and decreasing. The loop makes its own magnetic field into the page by carrying $\boxed{\text{clockwise}}$ current.

31.25 $\omega = (2.00\text{ rev/s})(2\pi\text{ rad/rev}) = (4.00)\pi\text{ rad/s}$

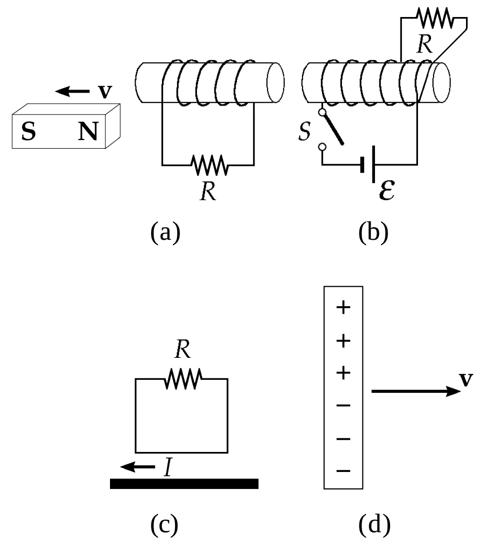
$$\mathcal{E} = \tfrac{1}{2}B\omega l^2 = \boxed{2.83\text{ mV}}$$

31.26 (a) $\mathbf{B}_{\text{ext}} = B_{\text{ext}}\,\mathbf{i}$ and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\,\mathbf{i}$ (to the right). Therefore, the current is $\boxed{\text{to the right}}$ in the resistor.

(b) $\mathbf{B}_{\text{ext}} = B_{\text{ext}}\,(-\mathbf{i})$ increases; therefore, the induced field $\mathbf{B}_0 = B_0\,(+\mathbf{i})$ is to the right, and the current is $\boxed{\text{to the right}}$ in the resistor.

(c) $\mathbf{B}_{\text{ext}} = B_{\text{ext}}\,(-\mathbf{k})$ into the paper and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\,(-\mathbf{k})$ into the paper. Therefore, the current is $\boxed{\text{to the right}}$ in the resistor.

(d) By the Lorentz force law, $F_B = q(\mathbf{v}\times\mathbf{B})$. Therefore, a positive charge will move to the top of the bar if $\mathbf{B}$ is $\boxed{\text{into the paper}}$.



31.27 (a) The force on the side of the coil entering the field (consisting of N wires) is

$$F = N(ILB) = N(IwB)$$

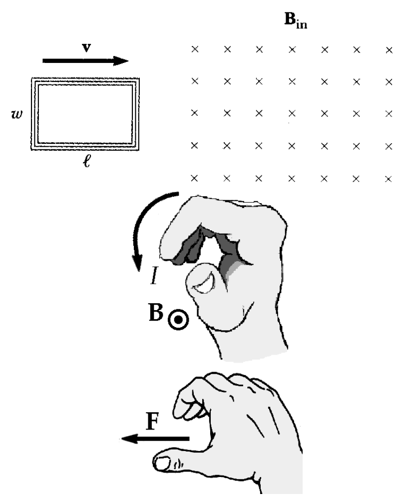
The induced emf in the coil is

$$|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv,$$

so the current is $I = \dfrac{|\mathcal{E}|}{R} = \dfrac{NBwv}{R}$ counterclockwise.

The force on the leading side of the coil is then:

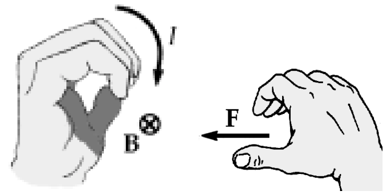
$$F = N\left(\frac{NBwv}{R}\right)wB = \boxed{\frac{N^2B^2w^2v}{R}\ \text{to the left}}$$



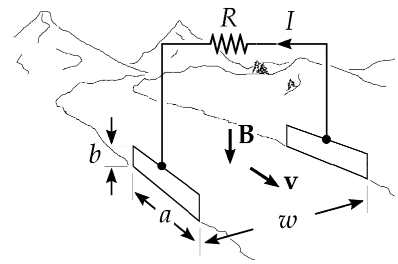
(b) Once the coil is entirely inside the field, $\Phi_B = NBA =$ constant, so $\mathcal{E} = 0$, $I = 0$, and $F = \boxed{0}$.

(c) As the coil starts to leave the field, the flux *decreases* at the rate Bwv, so the magnitude of the current is the same as in part (a), but now the current flows clockwise. Thus, the force exerted on the trailing side of the coil is:

$$F = \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left again}}$$



31.28 (a) Motional emf $\mathcal{E} = Bwv$ appears in the conducting water. Its resistance, if the plates are submerged, is



$$\frac{\rho L}{A} = \frac{\rho w}{ab}$$

Kirchhoff's loop theorem says $\quad Bwv - IR - \dfrac{I\rho w}{ab} = 0$

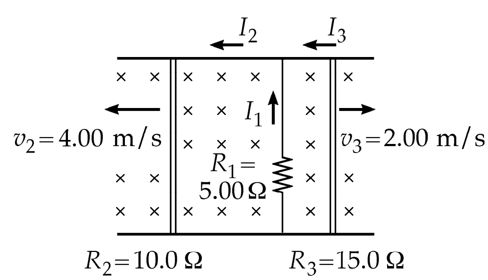
$$I = \frac{Bwv}{R + \frac{\rho w}{ab}} = \frac{abvB}{\rho + \frac{abR}{w}}$$

(b) $I_{sc} = \dfrac{(100\text{ m})(5.00\text{ m})(3.00\text{ m/s})(50.0 \times 10^{-6}\text{ T})}{100\ \Omega \cdot \text{m}} = \boxed{0.750\text{ mA}}$

31.29 Look in the direction of ba. The bar magnet creates a field into the page, and the field increases. The loop will create a field out of the page by carrying a counterclockwise current. Therefore, current must flow from b to a through the resistor. Hence, $V_a - V_b$ will be $\boxed{\text{negative}}$.

31.30 $\mathcal{E} = \frac{1}{2}B\omega l^2 = \boxed{0.259\text{ mV}}$

31.31 Name the currents as shown in the diagram:



Left loop: $\quad +Bdv_2 - I_2R_2 - I_1R_1 = 0$

Right loop: $\quad +Bdv_3 - I_3R_3 + I_1R_1 = 0$

At the junction: $\quad I_2 = I_1 + I_3$

Then, $Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$

$$I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}$$

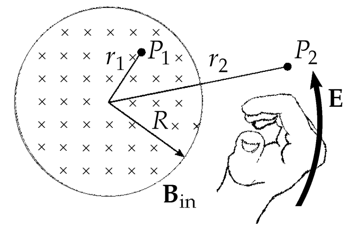
So, $\quad Bdv_2 - I_1(R_1 + R_2) - \dfrac{Bdv_3R_2}{R_3} - \dfrac{I_1R_1R_2}{R_3} = 0$

$$I_1 = Bd\left(\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right) \text{ upward}$$

$$I_1 = (0.0100\text{ T})(0.100\text{ m})\left[\frac{(4.00\text{ m/s})(15.0\ \Omega) - (2.00\text{ m/s})(10.0\ \Omega)}{(5.00\ \Omega)(10.0\ \Omega) + (5.00\ \Omega)(15.0\ \Omega) + (10.0\ \Omega)(15.0\ \Omega)}\right] = \boxed{145\ \mu\text{A}}$$

upward

31.32 (a) $\dfrac{dB}{dt} = 6.00t^2 - 8.00t \quad |\mathcal{E}| = \dfrac{d\Phi_B}{dt}$



At $t = 2.00$ s, $E = \dfrac{\pi R^2(dB/dt)}{2\pi r_2} = \dfrac{8.00\pi(0.0250)^2}{2\pi(0.0500)}$

$F = qE = \boxed{8.00 \times 10^{-21}\text{ N}}$ clockwise for electron

(b) When $6.00t^2 - 8.00t = 0$, $t = \boxed{1.33\text{ s}}$

31.33 $\dfrac{dB}{dt} = 0.0600t \qquad |\mathcal{E}| = \dfrac{d\Phi_B}{dt}$

At $t = 3.00$ s, $E = \pi r_1^2\left(\dfrac{dB}{2\pi r_1\,dt}\right) = \boxed{1.80 \times 10^{-3}\text{ N/C perpendicular to } r_1 \text{ and counterclockwise}}$

***31.34** $\mathcal{E} = \dfrac{d\Phi_B}{dt} = \pi r^2\left(\dfrac{dB}{dt}\right) = \oint \mathbf{E}\cdot d\mathbf{l}$

$E(2\pi R) = \pi r^2\dfrac{dB}{dt}$, or $E = \left(\dfrac{\pi r^2}{2\pi R}\right)\dfrac{dB}{dt}$

$B = \mu_0 nI \qquad \dfrac{dB}{dt} = \mu_0 n\dfrac{dI}{dt}$

$I = 3.00e^{0.200t} \qquad \dfrac{dI}{dt} = 0.600e^{0.200t}$

At $t = 10.0$ s, $E = \dfrac{\pi r^2}{2\pi R}(\mu_0 n)(0.600e^{0.200t})$

becomes $E = \dfrac{(0.0200\text{ m})^2}{2(0.0500\text{ m})}(4\pi \times 10^{-7}\text{ N/A}^2)(1000\text{ turns/m})(0.600)e^{2.00} = \boxed{2.23 \times 10^{-5}\text{ N/C}}$

31.35 (a) $\oint \mathbf{E} \cdot d\mathbf{l} = \left|\frac{d\Phi_B}{dt}\right|$

$2\pi r E = (\pi r^2)\frac{dB}{dt}$ so $E = \boxed{(9.87 \text{ mV/m}) \cos (100\,\pi t)}$

(b) The E field is always opposite to increasing B. $\therefore$ $\boxed{\text{clockwise}}$

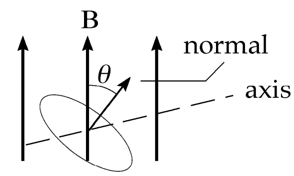
31.36 For the alternator, $\omega = 3000 \dfrac{\text{rev}}{\text{min}}\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 314\ \text{rad/s}$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -250\frac{d}{dt}\left[(2.50\times10^{-4}\ \text{T}\cdot\text{m}^2)\cos(314\,t\,/\,\text{s})\right] = +250(2.50\times10^{-4}\ \text{T}\cdot\text{m}^2)(314/\text{s})\ \sin(314t)$$

(a) $\boxed{\mathcal{E} = (19.6\ \text{V})\sin(314t)}$

(b) $\boxed{\mathcal{E}_{\max} = 19.6\ \text{V}}$

31.37 (a) $\mathcal{E}_{\max} = NAB\omega = (1000)(0.100)(0.200)(120\pi) = \boxed{7.54\ \text{kV}}$



(b) $\mathcal{E}(t) = -NBA\omega\cdot\sin\omega t = -NBA\omega\sin\theta$

$|\mathcal{E}|$ is maximal when $|\sin\theta| = 1$, or $\theta = \pm\dfrac{\pi}{2}$,

so the $\boxed{\text{plane of coil is parallel to } \mathbf{B}}$

31.38 Let θ represent the angle through which the coil turns, starting from $\theta = 0$ at an instant when the horizontal component of the Earth's field is perpendicular to the area. Then,

$$\mathcal{E} = -N\frac{d}{dt}BA\cos\theta = -NBA\frac{d}{dt}\cos\omega t = +NBA\omega\sin\omega t$$

Here $\sin\omega t$ oscillates between +1 and –1, so the spinning coil generates an alternating voltage with amplitude

$$\mathcal{E}_{\max} = NBA\omega = NBA2\pi f = 100(2.00\times10^{-5}\ \text{T})(0.200\ \text{m})^2(1500)\frac{2\pi\ \text{rad}}{60.0\ \text{s}} = \boxed{12.6\ \text{mV}}$$

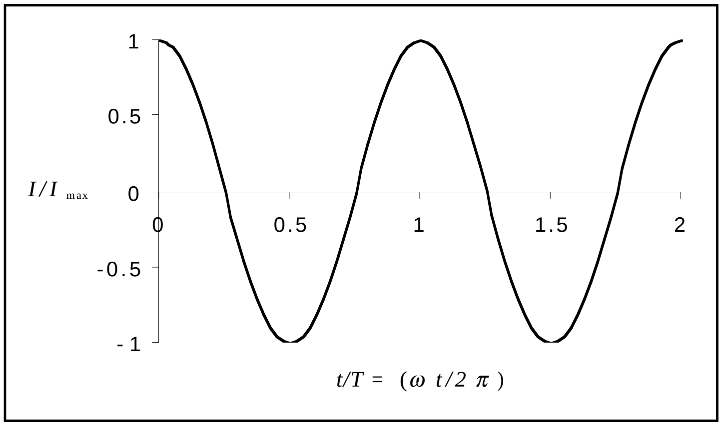
31.39 $B = \mu_0 nI = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(200\ \text{m}^{-1}\right)(15.0\ \text{A}) = 3.77\times10^{-3}\ \text{T}$

For the small coil, $\Phi_B = N\mathbf{B}\cdot\mathbf{A} = NBA\cos\omega t = NB\left(\pi r^2\right)\cos\omega t$

Thus, $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = NB\pi r^2\omega\sin\omega t$

$$\mathcal{E} = (30.0)\left(3.77\times10^{-3}\ \text{T}\right)\pi(0.0800\ \text{m})^2\left(4.00\pi\ \text{s}^{-1}\right)\sin(4.00\pi t) = \boxed{(28.6\ \text{mV})\sin(4.00\pi t)}$$

31.40 As the magnet rotates, the flux through the coil varies sinusoidally in time with $\Phi_B = 0$ at $t = 0$. Choosing the flux as positive when the field passes from left to right through the area of the coil, the flux at any time may be written as $\Phi_B = -\Phi_{\max}\sin\omega t$ so the induced emf is given by



$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \omega\Phi_{\max}\cos\omega t.$$

The current in the coil is then $I = \dfrac{\mathcal{E}}{R} = \dfrac{\omega\Phi_{\max}}{R}\cos\omega t = \boxed{I_{\max}\cos\omega t}$

31.41 (a) $F = NI\ell B$

$\tau_{\max} = 2Fr = NI\ell wB = \boxed{0.640\text{ N}\cdot\text{m}}$

(b) $\mathcal{P} = \tau\omega = (0.640\text{ N}\cdot\text{m})(120\pi\text{ rad/s})$

$\mathcal{P}_{\max} = \boxed{241\text{ W}}$ (about $\frac{1}{3}$ hp)

31.42 (a) $\mathcal{E}_{\max} = BA\omega = B\left(\frac{1}{2}\pi R^2\right)\omega$

$$\mathcal{E}_{\max} = (1.30\text{ T})\frac{\pi}{2}(0.250\text{ m})^2\left(4.00\pi\ \frac{\text{rad}}{\text{s}}\right)$$

$\mathcal{E}_{\max} = \boxed{1.60\text{ V}}$

(b) $\bar{\mathcal{E}} = \displaystyle\int_0^{2\pi}\frac{\mathcal{E}}{2\pi}d\theta = \frac{BA\omega}{2\pi}\int_0^{2\pi}\sin\theta\, d\theta = \boxed{0}$

(c) The maximum and average $\mathcal{E}$ would remain unchanged.

(d) See Figure 1 at the right.

(e) See Figure 2 at the right.

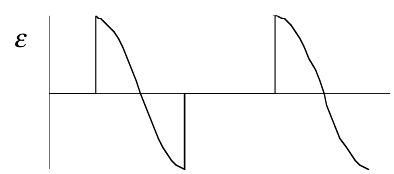


Figure 1

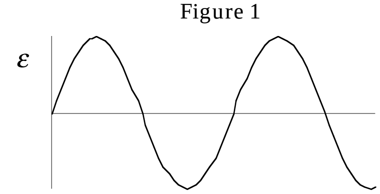


Figure 2

31.43 (a) $\Phi_B = BA\cos\theta = BA\cos\omega t = (0.800\text{ T})(0.0100\text{ m}^2)\cos 2\pi(60.0)t = \boxed{(8.00\text{ mT}\cdot\text{m}^2)\cos(377t)}$

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \boxed{(3.02\text{ V})\sin(377t)}$

(c) $I = \mathcal{E}R = \boxed{(3.02\text{ A})\sin(377t)}$

(d) $\mathcal{P} = I^2R = \boxed{(9.10\text{ W})\sin^2(377t)}$

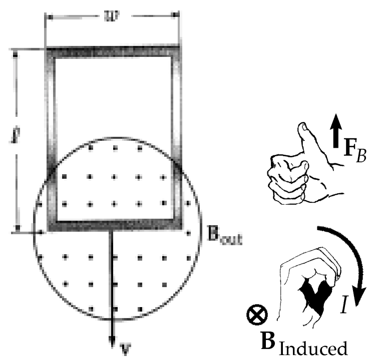
(e) $\mathcal{P} = Fv = \tau\omega$ so $\tau = \dfrac{\mathcal{P}}{\omega} = \boxed{(24.1\text{ mN}\cdot\text{m})\sin^2(377t)}$

31.44 At terminal speed, the upward magnetic force exerted on the lower edge of the loop must equal the weight of the loop. That is,

$$Mg = F_B = IwB = \left(\frac{\mathcal{E}}{R}\right)wB = \left(\frac{Bwv_t}{R}\right)wB = \frac{B^2w^2v_t}{R}$$

Thus,

$$B = \sqrt{\frac{MgR}{w^2v_t}} = \sqrt{\frac{(0.150\text{ kg})(9.80\text{ m/s}^2)(0.750\ \Omega)}{(1.00\text{ m})^2(2.00\text{ m/s})}} = \boxed{0.742\text{ T}}$$



31.45 See the figure above with Problem 31.44.

(a) At terminal speed, $Mg = F_B = IwB = \left(\dfrac{\mathcal{E}}{R}\right)wB = \left(\dfrac{Bwv_t}{R}\right)wB = \dfrac{B^2w^2v_t}{R}$

or $\boxed{v_t = \dfrac{MgR}{B^2w^2}}$

(b) The emf is directly proportional to v_t, but the current is inversely proportional to R. A large R means a small current at a given speed, so the loop must travel faster to get $F_m = mg$.

(c) At given speed, the current is directly proportional to the magnetic field. But the force is proportional to the product of the current and the field. For a small B, the speed must increase to compensate for both the small B and also the current, so $v_t \propto B^2$.

***31.46** The current in the magnet creates an upward magnetic field, so the N and S poles on the solenoid core are shown correctly. On the rail in front of the brake, the upward flux of **B** increases as the coil approaches, so a current is induced here to create a downward magnetic field. This is clockwise current, so the S pole on the rail is shown correctly. On the rail behind the brake, the upward magnetic flux is decreasing. The induced current in the rail will produce upward magnetic field by being counterclockwise as the picture correctly shows.

31.47 $\mathbf{F} = m\mathbf{a} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$

$$\mathbf{a} = \frac{e}{m}[\mathbf{E} + \mathbf{v} \times \mathbf{B}] \quad \text{where} \quad \mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 200 & 0 & 0 \\ 0.200 & 0.300 & 0.400 \end{vmatrix} = -200(0.400)\mathbf{j} + 200(0.300)\mathbf{k}$$

$$\mathbf{a} = \frac{1.60 \times 10^{-19}}{1.67 \times 10^{-27}}[50.0\mathbf{j} - 80.0\mathbf{j} + 60.0\mathbf{k}] = 9.58 \times 10^{7}[-30.0\mathbf{j} + 60.0\mathbf{k}]$$

$$\mathbf{a} = 2.87 \times 10^{9}[-\mathbf{j} + 2\mathbf{k}]\ \text{m/s}^2 = \boxed{(-2.87 \times 10^{9}\,\mathbf{j} + 5.75 \times 10^{9}\,\mathbf{k})\ \text{m/s}^2}$$

31.48 $\mathbf{F} = m\mathbf{a} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$ so $\quad \mathbf{a} = \frac{-e}{m}[\mathbf{E} + \mathbf{v} \times \mathbf{B}] \quad$ where $\quad \mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10.0 & 0 & 0 \\ 0 & 0 & 0.400 \end{vmatrix} = -4.00\mathbf{j}$

$$\mathbf{a} = \frac{\left(-1.60 \times 10^{-19}\right)}{9.11 \times 10^{-31}}[2.50\,\mathbf{i} + 5.00\,\mathbf{j} - 4.00\,\mathbf{j}] = \left(-1.76 \times 10^{11}\right)[2.50\,\mathbf{i} + 1.00\,\mathbf{j}]$$

$$\mathbf{a} = \boxed{\left(-4.39 \times 10^{11}\,\mathbf{i} - 1.76 \times 10^{11}\,\mathbf{j}\right)\ \text{m/s}^2}$$

***31.49** $\mathcal{E} = -N\dfrac{d}{dt}(BA\cos\theta) = -N\left(\pi r^2\right)\cos 0^\circ \dfrac{dB}{dt}$

$$\mathcal{E} = -(30.0)\pi\left(2.70 \times 10^{-3}\ \text{m}\right)^2(1)\frac{d}{dt}\left[50.0\ \text{mT} + (3.20\ \text{mT})\sin\left(2\pi\ 523t/\text{s}\right)\right]$$

$$\mathcal{E} = -(30.0)\pi\left(2.70 \times 10^{-3}\ \text{m}\right)^2\left(3.20 \times 10^{-3}\ \text{T}\right)(2\pi)(523/\text{s})\cos\left(2\pi\ 523t/\text{s}\right)$$

$$\mathcal{E} = \boxed{-\left(7.22 \times 10^{-3}\ \text{V}\right)\cos\left(2\pi\ 523t/\text{s}\right)}$$

***31.50** (a) Doubling the number of turns.

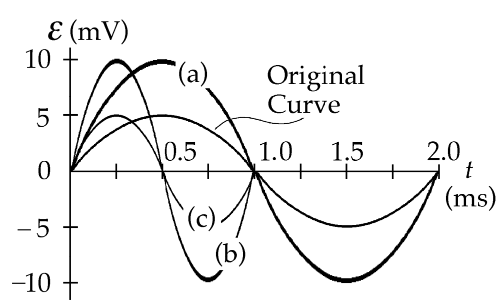
$\boxed{\text{Amplitude doubles: period unchanged}}$

(b) Doubling the angular velocity.

$\boxed{\text{doubles the amplitude: cuts the period in half}}$

(c) Doubling the angular velocity while reducing the number of turns to one half the original value.

$\boxed{\text{Amplitude unchanged: cuts the period in half}}$

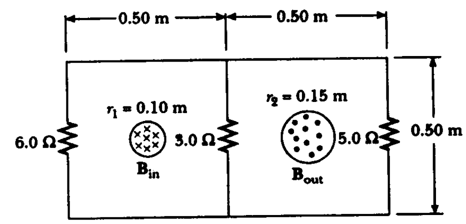


***31.51** $\mathcal{E} = -N\dfrac{\Delta}{\Delta t}(BA\cos\theta) = -N\left(\pi r^2\right)\cos 0° \dfrac{\Delta B}{\Delta t} = -1\left(0.00500 \text{ m}^2\right)(1)\left(\dfrac{1.50 \text{ T} - 5.00 \text{ T}}{20.0\times 10^{-3} \text{ s}}\right) = 0.875 \text{ V}$

(a) $I = \dfrac{\mathcal{E}}{R} = \dfrac{0.875 \text{ V}}{0.0200\ \Omega} = \boxed{43.8 \text{ A}}$

(b) $P = \mathcal{E}I = (0.875 \text{ V})(43.8 \text{ A}) = \boxed{38.3 \text{ W}}$

31.52 In the loop on the left, the induced emf is



$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.100 \text{ m})^2(100 \text{ T/s}) = \pi \text{ V}$$

and it attempts to produce a counterclockwise current in this loop.

In the loop on the right, the induced emf is

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \pi(0.150 \text{ m})^2(100 \text{ T/s}) = 2.25\pi \text{ V}$$

and it attempts to produce a clockwise current. Assume that I_1 flows down through the 6.00-Ω resistor, I_2 flows down through the 5.00-Ω resistor, and that I_3 flows up through the 3.00-Ω resistor.

From Kirchhoff's point rule: $I_3 = I_1 + I_2$ (1)

Using the loop rule on the left loop: $6.00 I_1 + 3.00 I_3 = \pi$ (2)

Using the loop rule on the right loop: $5.00 I_2 + 3.00 I_3 = 2.25\pi$ (3)

Solving these three equations simultaneously,

$I_1 = \boxed{0.0623 \text{ A}}$, $I_2 = \boxed{0.860 \text{ A}}$, and $I_3 = \boxed{0.923 \text{ A}}$

***31.53** The emf induced between the ends of the moving bar is

$$\mathcal{E} = B\ell v = (2.50 \text{ T})(0.350 \text{ m})(8.00 \text{ m/s}) = 7.00 \text{ V}$$

The left-hand loop contains decreasing flux away from you, so the induced current in it will be clockwise, to produce its own field directed away from you. Let I_1 represent the current flowing upward through the 2.00-Ω resistor. The right-hand loop will carry counterclockwise current. Let I_3 be the upward current in the 5.00-Ω resistor.

(a) Kirchhoff's loop rule then gives: $+7.00\text{ V} - I_1(2.00\ \Omega) = 0$ $\quad I_1 = \boxed{3.50\text{ A}}$

and $+7.00\text{ V} - I_3(5.00\ \Omega) = 0$ $\quad I_3 = \boxed{1.40\text{ A}}$

(b) The total power dissipated in the resistors of the circuit is

$$\mathcal{P} = \mathcal{E}I_1 + \mathcal{E}I_3 = \mathcal{E}(I_1 + I_3) = (7.00\text{ V})(3.50\text{ A} + 1.40\text{ A}) = \boxed{34.3\text{ W}}$$

(c) Method 1: The current in the sliding conductor is downward with value $I_2 = 3.50\text{ A} + 1.40\text{ A} = 4.90\text{ A}$. The magnetic field exerts a force of $F_m = I\ell B = (4.90\text{ A})(0.350\text{ m})(2.50\text{ T}) = 4.29\text{ N}$ directed toward the right on this conductor. An outside agent must then exert a force of $\boxed{4.29\text{ N}}$ to the left to keep the bar moving.

Method 2: The agent moving the bar must supply the power according to $\mathcal{P} = \mathbf{F}\cdot\mathbf{v} = Fv\cos 0°$. The force required is then:

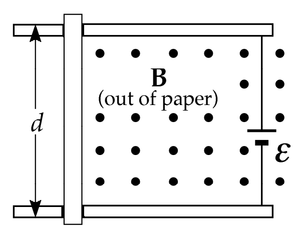
$$F = \frac{\mathcal{P}}{v} = \frac{34.3\text{ W}}{8.00\text{ m/s}} = \boxed{4.29\text{ N}}$$

***31.54** Suppose we wrap twenty turns of wire into a flat compact circular coil of diameter 3 cm. Suppose we use a bar magnet to produce field 10^{-3} T through the coil in one direction along its axis. Suppose we then flip the magnet to reverse the flux in 10^{-1} s. The average induced emf is then

$$\bar{\mathcal{E}} = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta[BA\cos\theta]}{\Delta t} = -NB(\pi r^2)\left(\frac{\cos 180° - \cos 0°}{\Delta t}\right)$$

$$\bar{\mathcal{E}} = -(20)(10^{-3}\text{ T})\pi(0.0150\text{ m})^2\left(\frac{-2}{10^{-1}\text{ s}}\right) \quad \boxed{\sim 10^{-4}\text{ V}}$$

31.55 $I = \dfrac{\mathcal{E} + \mathcal{E}_{\text{Induced}}}{R}$ and $\mathcal{E}_{\text{Induced}} = -\dfrac{d}{dt}(BA)$



$$F = m\frac{dv}{dt} = IBd$$

$$\frac{dv}{dt} = \frac{IBd}{m} = \frac{Bd}{mR}(\mathcal{E} + \mathcal{E}_{\text{Induced}}) = \frac{Bd}{mR}(\mathcal{E} - Bvd)$$

To solve the differential equation, let $u = (\mathcal{E} - Bvd)$, $\quad \dfrac{du}{dt} = -Bd\dfrac{dv}{dt}$.

$-\dfrac{1}{Bd}\dfrac{du}{dt} = \dfrac{Bd}{mR}\,u$ so $\quad \displaystyle\int_{u_0}^{u}\frac{du}{u} = -\int_{t=0}^{t}\frac{(Bd)^2}{mR}dt$

Integrating from $t = 0$ to $t = t$, $\quad \ln\dfrac{u}{u_0} = -\dfrac{(Bd)^2}{mR}t$ or $\dfrac{u}{u_0} = e^{-B^2d^2t/mR}$

Since $v = 0$ when $t = 0$, $\quad u_0 = \mathcal{E}$ and $u = \mathcal{E} - Bvd$

$$\mathcal{E} - Bvd = \mathcal{E}e^{-B^2d^2t/mR} \qquad \text{and} \qquad \boxed{v = \frac{\mathcal{E}}{Bd}\left(1 - e^{-B^2d^2t/mR}\right)}$$

31.56 (a) For maximum induced emf, with positive charge at the top of the antenna,

$\mathbf{F}_+ = q_+ (\mathbf{v} \times \mathbf{B})$, so the auto must move <u>east</u>

(b) $\mathcal{E} = B\ell v = (5.00 \times 10^{-5}\ \text{T})(1.20\ \text{m})\left(\dfrac{65.0 \times 10^3\ \text{m}}{3600\ \text{s}}\right) \cos 65.0° = \boxed{4.58 \times 10^{-4}\ \text{V}}$

31.57 $I = \dfrac{\mathcal{E}}{R} = \dfrac{B}{R}\dfrac{|\Delta A|}{\Delta t}$

so $q = I\,\Delta t = \dfrac{(15.0\ \mu\text{T})(0.200\ \text{m})^2}{0.500\ \Omega} = \boxed{1.20\ \mu\text{C}}$

<u>Goal Solution</u>

The plane of a square loop of wire with edge length $a = 0.200$ m is perpendicular to the Earth's magnetic field at a point where $B = 15.0\ \mu$T, as shown in Figure P31.57. The total resistance of the loop and the wires connecting it to the galvanometer is $0.500\ \Omega$. If the loop is suddenly collapsed by horizontal forces as shown, what total charge passes through the galvanometer?

G: For the situation described, the maximum current is probably less than 1 mA. So if the loop is closed in 0.1 s, then the total charge would be

$$Q = I\Delta t = (1\ \text{mA})(0.1\ \text{s}) = 100\ \mu\text{C}$$

O: We do not know how quickly the loop is collapsed, but we can find the total charge by integrating the change in magnetic flux due to the change in area of the loop ($a^2 \to 0$).

A: $$Q = \int I dt = \int \frac{\mathcal{E}\,dt}{R} = \frac{1}{R}\int -\left(\frac{d\Phi_B}{dt}\right)dt = -\frac{1}{R}\int d\Phi_B = -\frac{1}{R}\int d(BA) = -\frac{B}{R}\int_{A_1=a^2}^{A_2=0} dA$$

$$Q = -\frac{B}{R}A\Big]_{A_1=a^2}^{A_2=0} = \frac{Ba^2}{R} = \frac{(15.0\times 10^{-6}\ \text{T})(0.200\ \text{m})^2}{0.500\ \Omega} = 1.20\times 10^{-6}\ \text{C}$$

L: The total charge is less than the maximum charge we predicted, so the answer seems reasonable. It is interesting that this charge can be calculated without knowing either the current or the time to collapse the loop. **Note**: We ignored the internal resistance of the galvanometer. D'Arsonval galvanometers typically have an internal resistance of 50 to 100 Ω, significantly more than the resistance of the wires given in the problem. A proper solution that includes R_G would reduce the total charge by about 2 orders of magnitude ($Q \sim 0.01\ \mu$C).

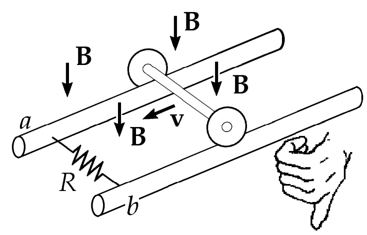
***31.58** (a) $I = \dfrac{dq}{dt} = \dfrac{\mathcal{E}}{R}$ where $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$ so $\int dq = \dfrac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi_B$

and the charge through the circuit will be $|Q| = \dfrac{N}{R}(\Phi_2 - \Phi_1)$

(b) $Q = \dfrac{N}{R}\left[BA\cos 0 - BA\cos\left(\dfrac{\pi}{2}\right)\right] = \dfrac{BAN}{R}$

so $B = \dfrac{RQ}{NA} = \dfrac{(200\ \Omega)(5.00\times 10^{-4}\ \text{C})}{(100)(40.0\times 10^{-4}\ \text{m}^2)} = \boxed{0.250\ \text{T}}$

31.59 (a) $\mathcal{E} = B\ell v = 0.360\ \text{V}$ $\quad I = \dfrac{\mathcal{E}}{R} = \boxed{0.900\ \text{A}}$

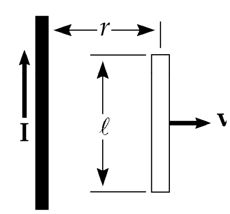


(b) $F_B = I\ell B = \boxed{0.108\ \text{N}}$

(c) Since the magnetic flux $\mathbf{B}\cdot\mathbf{A}$ is in effect decreasing, the induced current flow through R is from b to a. $\boxed{\text{Point } b}$ is at higher potential.

(d) $\boxed{\text{No}}$. Magnetic flux will increase through a loop to the left of ab. Here counterclockwise current will flow to produce upward magnetic field. The in R is still from b to a.

31.60 $\mathcal{E} = B\ell v$ at a distance r from wire



$$|\mathcal{E}| = \left(\frac{\mu_0 I}{2\pi r}\right)\ell v$$

31.61 (a) At time t, the flux through the loop is $\Phi_B = BA\cos\theta = (a+bt)\left(\pi r^2\right)\cos 0^\circ = \pi(a+bt)r^2$

At $t = 0$, $\Phi_B = \boxed{\pi a r^2}$

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -\pi r^2\dfrac{d(a+bt)}{dt} = \boxed{-\pi b r^2}$

(c) $I = \dfrac{\mathcal{E}}{R} = \boxed{-\dfrac{\pi b r^2}{R}}$

(d) $\mathcal{P} = \mathcal{E}I = \left(-\dfrac{\pi b r^2}{R}\right)\left(-\pi b r^2\right) = \boxed{\dfrac{\pi^2 b^2 r^4}{R}}$

31.62 $\mathcal{E} = -\frac{d}{dt}(NBA) = -1\left(\frac{dB}{dt}\right)\pi a^2 = \pi a^2 K$

(a) $Q = C\mathcal{E} = \boxed{C\pi a^2 K}$

(b) **B** into the paper is decreasing; therefore, current will attempt to counteract this. Positive charge will go to $\boxed{\text{upper plate}}$.

(c) The changing magnetic field through the enclosed area $\boxed{\text{induces an electric field}}$, surrounding the **B**-field, and this pushes on charges in the wire.

31.63 The flux through the coil is $\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA\cos\theta = BA\cos\omega t$. The induced emf is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA\frac{d(\cos\omega t)}{dt} = NBA\omega\sin\omega t.$$

(a) $\mathcal{E}_{\max} = NBA\omega = 60.0(1.00\text{ T})\left(0.100\times 0.200\text{ m}^2\right)(30.0\text{ rad/s}) = \boxed{36.0\text{ V}}$

(b) $\frac{d\Phi_B}{dt} = \frac{\mathcal{E}}{N}$, thus $\left|\frac{d\Phi_B}{dt}\right|_{\max} = \frac{\mathcal{E}_{\max}}{N} = \frac{36.0\text{ V}}{60.0} = 0.600\text{ V} = \boxed{0.600\text{ Wb/s}}$

(c) At $t = 0.0500$ s, $\omega t = 1.50$ rad and $\mathcal{E} = \mathcal{E}_{\max}\sin(1.50\text{ rad}) = (36.0\text{ V})\sin(1.50\text{ rad}) = \boxed{35.9\text{ V}}$

(d) The torque on the coil at any time is $\tau = |\mu\times\mathbf{B}| = |NI\mathbf{A}\times\mathbf{B}| = (NAB)I|\sin\omega t| = \left(\frac{\mathcal{E}_{\max}}{\omega}\right)\left(\frac{\mathcal{E}}{R}\right)|\sin\omega t|$

When $\mathcal{E} = \mathcal{E}_{\max}$, $\sin\omega t = 1.00$ and $\tau = \frac{\mathcal{E}_{\max}^2}{\omega R} = \frac{(36.0\text{ V})^2}{(30.0\text{ rad/s})(10.0\ \Omega)} = \boxed{4.32\text{ N}\cdot\text{m}}$

31.64 (a) We use $\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$, with $N = 1$.

Taking $a = 5.00\times 10^{-3}$ m to be the radius of the washer, and $h = 0.500$ m,

$$\Delta\Phi_B = B_2A - B_1A = A(B_2 - B_1) = \pi a^2\left(\frac{\mu_0 I}{2\pi(h+a)} - \frac{\mu_0 I}{2\pi a}\right) = \frac{a^2\mu_0 I}{2}\left(\frac{1}{h+a} - \frac{1}{a}\right) = \frac{-\mu_0 ahI}{2(h+a)}$$

The time for the washer to drop a distance h (from rest) is: $\Delta t = \sqrt{\frac{2h}{g}}$

Therefore, $\mathcal{E} = \frac{\mu_0 ahI}{2(h+a)\Delta t} = \frac{\mu_0 ahI}{2(h+a)}\sqrt{\frac{g}{2h}} = \frac{\mu_0 aI}{2(h+a)}\sqrt{\frac{gh}{2}}$

and $\mathcal{E} = \frac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(5.00\times 10^{-3}\text{ m})(10.0\text{ A})}{2(0.500\text{ m} + 0.00500\text{ m})}\sqrt{\frac{(9.80\text{ m/s}^2)(0.500\text{ m})}{2}} = \boxed{97.4\text{ nV}}$

(b) Since the magnetic flux going through the washer (into the plane of the paper) is decreasing in time, a current will form in the washer so as to oppose that decrease. Therefore, the current will flow in a $\boxed{\text{clockwise direction}}$.

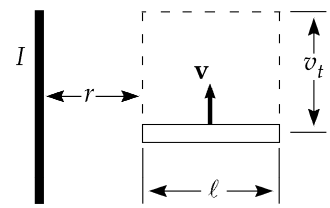
31.65

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BA\cos\theta)$$

$$\mathcal{E} = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right) = -200\left(50.0\times10^{-6}\text{ T}\right)\left(\cos 62.0^\circ\right)\left(\frac{39.0\times10^{-4}\text{ m}^2}{1.80\text{ s}}\right) = \boxed{-10.2\ \mu\text{V}}$$

31.66

Find an expression for the flux through a rectangular area "swept out" by the bar in time t. The magnetic field at a distance x from wire is



$B = \dfrac{\mu_0 I}{2\pi x}$ and $\Phi_B = \int B\,dA$. Therefore,

$\Phi_B = \dfrac{\mu_0 I v t}{2\pi}\displaystyle\int_r^{r+\ell}\dfrac{dx}{x}$ where vt is the distance the bar has moved in time t.

Then, $|\mathcal{E}| = \dfrac{d\Phi_B}{dt} = \boxed{\dfrac{\mu_0 I v}{2\pi}\ln\left(1+\dfrac{\ell}{r}\right)}$

31.67

The magnetic field at a distance x from a long wire is $B = \dfrac{\mu_0 I}{2\pi x}$. Find an expression for the flux through the loop.

$d\Phi_B = \dfrac{\mu_0 I}{2\pi x}(\ell\,dx)$ so $\Phi_B = \dfrac{\mu_0 I \ell}{2\pi}\displaystyle\int_r^{r+w}\dfrac{dx}{x} = \dfrac{\mu_0 I \ell}{2\pi}\ln\left(1+\dfrac{w}{r}\right)$

Therefore, $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 I \ell v}{2\pi r}\dfrac{w}{(r+w)}$ and $I = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{\mu_0 I \ell v}{2\pi R r}\dfrac{w}{(r+w)}}$

31.68

As the wire falls through the magnetic field, a motional emf $\mathcal{E} = B\ell v$ is induced in it. Thus, a counterclockwise induced current of $I = \mathcal{E}/R = B\ell v/R$ flows in the circuit. The falling wire is carrying a current toward the left through the magnetic field. Therefore, it experiences an upward magnetic force given by $F_B = I\ell B = B^2\ell^2 v/R$. The wire will have attained terminal speed when the magnitude of this magnetic force equals the weight of the wire.

Thus, $\dfrac{B^2\ell^2 v_t}{R} = mg$, or the terminal speed is $v_t = \boxed{\dfrac{mgR}{B^2\ell^2}}$

31.69

$\Phi_B = (6.00t^3 - 18.0t^2)\text{ T}\cdot\text{m}^2$ and $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -18.0t^2 + 36.0t$

Maximum $\mathcal{E}$ occurs when $\dfrac{d\mathcal{E}}{dt} = -36.0t + 36.0 = 0$, which gives $t = 1.00$ s.

Therefore, the maximum current (at $t = 1.00$ s) is $I = \dfrac{\mathcal{E}}{R} = \dfrac{(-18.0 + 36.0)\text{V}}{3.00\ \Omega} = \boxed{6.00\text{ A}}$

31.70 For the suspended mass, M: $\Sigma F = Mg - T = Ma$

For the sliding bar, m: $\Sigma F = T - I\ell B = ma$, where $I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$

$$Mg - \frac{B^2\ell^2 v}{R} = (m+M)a \quad \text{or} \quad a = \frac{dv}{dt} = \frac{Mg}{m+M} - \frac{B^2\ell^2 v}{R(M+m)}$$

$$\int_0^v \frac{dv}{(\alpha - \beta v)} = \int_0^t dt \quad \text{where} \quad \alpha = \frac{Mg}{M+m} \quad \text{and} \quad \beta = \frac{B^2\ell^2}{R(M+m)}$$

Therefore, the velocity varies with time as $v = \dfrac{\alpha}{\beta}(1 - e^{-\beta t}) = \boxed{\dfrac{MgR}{B^2\ell^2}\left[1 - e^{-B^2\ell^2 t/R(M+m)}\right]}$

***31.71** (a) $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -NA\dfrac{dB}{dt} = -NA\dfrac{d}{dt}(\mu_0 nI)$

where A = area of coil, N = number of turns in coil, and n = number of turns per unit length in solenoid. Therefore,

$$|\mathcal{E}| = N\mu_0 An\frac{d}{dt}\left[4\sin(120\pi t)\right] = N\mu_0 An(480\pi)\cos(120\pi t)$$

$$|\mathcal{E}| = 40(4\pi \times 10^{-7})\left[\pi(0.0500\text{ m})^2\right](2.00 \times 10^3)(480\pi)\cos(120\pi t) = \boxed{(1.19\text{ V})\cos(120\pi t)}$$

(b) $I = \dfrac{\Delta V}{R}$ and $\mathcal{P} = \Delta VI = \dfrac{(1.19\text{ V})^2\cos^2(120\pi t)}{(8.00\ \Omega)}$

From $\cos^2\theta = \dfrac{1}{2} + \dfrac{1}{2}\cos 2\theta$, the average value of $\cos^2\theta$ is $\dfrac{1}{2}$, so $\bar{\mathcal{P}} = \dfrac{1}{2}\dfrac{(1.19\text{ V})^2}{(8.00\ \Omega)} = \boxed{88.5\text{ mW}}$

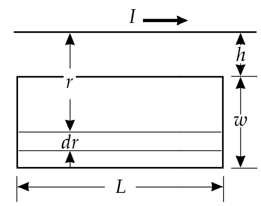
31.72 The induced emf is $\mathcal{E} = B\ell v$ where $B = \dfrac{\mu_0 I}{2\pi y}$, $v = v_i + gt = \left(9.80\text{ m/s}^2\right)t$, and

$y = y_i - \frac{1}{2}gt^2 = 0.800\text{ m} - \left(4.90\text{ m/s}^2\right)t^2$.

$$\mathcal{E} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(200\text{ A})}{2\pi\left[0.800\text{ m} - \left(4.90\text{ m/s}^2\right)t^2\right]}(0.300\text{ m})\left(9.80\text{ m/s}^2\right)t = \boxed{\frac{\left(1.18 \times 10^{-4}\right)t}{\left[0.800 - 4.90t^2\right]}\text{ V}}$$

At $t = 0.300$ s, $\mathcal{E} = \dfrac{\left(1.18 \times 10^{-4}\right)(0.300)}{\left[0.800 - 4.90(0.300)^2\right]}\text{ V} = \boxed{98.3\ \mu\text{V}}$

31.73 The magnetic field produced by the current in the straight wire is perpendicular to the plane of the coil at all points within the coil. The magnitude of the field is $B = \mu_0 I / 2\pi r$. Thus, the flux linkage is



$$N\Phi_B = \frac{\mu_0 NIL}{2\pi}\int_h^{h+w}\frac{dr}{r} = \frac{\mu_0 NI_{\max}L}{2\pi}\ln\left(\frac{h+w}{h}\right)\sin(\omega t + \phi)$$

Finally, the induced emf is $\mathcal{E} = -\frac{\mu_0 NI_{\max}L\omega}{2\pi}\ln\left(1+\frac{w}{h}\right)\cos(\omega t + \phi)$

$$\mathcal{E} = -\frac{\left(4\pi\times10^{-7}\right)(100)(50.0)(0.200\text{ m})(200\pi\text{ s}^{-1})}{2\pi}\ln\left(1+\frac{5.00\text{ cm}}{5.00\text{ cm}}\right)\cos(\omega t + \phi)$$

$$\mathcal{E} = \boxed{-(87.1\text{ mV})\cos(200\pi t + \phi)}$$

The term $\sin(\omega t + \phi)$ in the expression for the current in the straight wire does not change appreciably when ωt changes by 0.100 rad or less. Thus, the current does not change appreciably during a time interval

$$t < \frac{0.100}{(200\pi\text{ s}^{-1})} = 1.60\times10^{-4}\text{ s}.$$

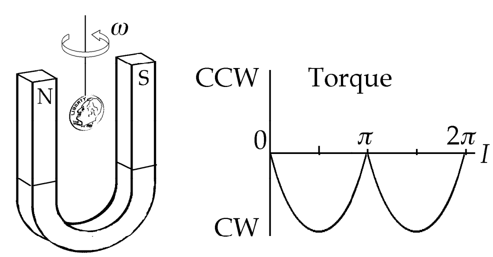
We define a critical length, $ct = (3.00\times10^8\text{ m/s})(1.60\times10^{-4}\text{ s}) = 4.80\times10^4\text{ m}$ equal to the distance to which field changes could be propagated during an interval of 1.60×10^{-4} s. This length is so much larger than any dimension of the coilor its distance from the wire that, although we consider the straight wire to be infinitely long, we can also safely ignore the field propagation effects in the vicinity of the coil. Moreover, the phase angle can be considered to be constant along the wire in the vicinity of the coil.

If the frequency ω were much larger, say, $200\pi\times10^5\text{ s}^{-1}$, the corresponding critical length would be only 48.0 cm. In this situation propagation effects would be important and the above expression for $\mathcal{E}$ would require modification. As a "rule of thumb" we can consider field propagation effects for circuits of laboratory size to be negligible for frequencies, $f = \omega/2\pi$, that are less than about 10^6 Hz.

31.74 $\Phi_B = BA\cos\theta$ $\quad \frac{d\Phi_B}{dt} = -\omega BA\sin\theta$;

$I \propto -\sin\theta$

$\tau \propto IB\sin\theta$ $\boxed{\propto -\sin^2\theta}$



31.75 The area of the tent that is effective in intercepting magnetic field lines is the area perpendicular to the direction of the magnetic field. This is the same as the base of the tent. In the initial configuration, this is

$$A_1 = L(2L\cos\theta) = 2(1.50\text{ m})^2\cos 60.0° = 2.25\text{ m}^2$$

After the tent is flattened, $A_2 = L(2L) = 2L^2 = 2(1.50\text{ m})^2 = 4.50\text{ m}^2$

The average induced emf is: $\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{B(\Delta A)}{\Delta t} = -\frac{(0.300\text{ T})(4.50-2.25)\text{ m}^2}{0.100\text{ s}} = \boxed{-6.75\text{ V}}$

Chapter 32 Solutions

***32.1** $|\bar{\mathcal{E}}| = L\dfrac{\Delta I}{\Delta t} = (3.00\times10^{-3}\ \text{H})\left(\dfrac{1.50\ \text{A} - 0.200\ \text{A}}{0.200\ \text{s}}\right) = 1.95\times10^{-2}\ \text{V} = \boxed{19.5\ \text{mV}}$

32.2 Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(70.0)^2(\pi)(6.50\times10^{-3}\ \text{m})^2}{0.600\ \text{m}} = \boxed{1.36\ \mu\text{H}}$$

32.3 $|\bar{\mathcal{E}}| = +L\left(\dfrac{\Delta I}{\Delta t}\right) = (2.00\ \text{H})\left(\dfrac{0.500\ \text{A}}{0.0100\ \text{s}}\right) = \boxed{100\ \text{V}}$

32.4 $L = \mu_0 n^2 A\ell$ so $n = \sqrt{\dfrac{L}{\mu_0 A\ell}} = \boxed{7.80\times10^3\ \text{turns/m}}$

32.5 $L = \dfrac{N\Phi_B}{I} \to \Phi_B = \dfrac{LI}{N} = \boxed{240\ \text{nT}\cdot\text{m}^2}$ (through each turn)

32.6 $|\mathcal{E}| = L\dfrac{dI}{dt}$ where $L = \dfrac{\mu_0 N^2 A}{\ell}$

Thus, $|\mathcal{E}| = \left(\dfrac{\mu_0 N^2 A}{\ell}\right)\dfrac{dI}{dt} = \dfrac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(300)^2\left(\pi\times10^{-4}\ \text{m}^2\right)}{0.150\ \text{m}}(10.0\ \text{A/s}) = \boxed{2.37\ \text{mV}}$

32.7 $\mathcal{E}_{\text{back}} = -\mathcal{E} = L\dfrac{dI}{dt} = L\dfrac{d}{dt}(I_{\max}\sin\omega t) = L\omega I_{\max}\cos\omega t = (10.0\times10^{-3})(120\pi)(5.00)\cos\omega t$

$\mathcal{E}_{\text{back}} = (6.00\pi)\cos(120\pi t) = \boxed{(18.8\ \text{V})\cos(377t)}$

***32.8** From $|\mathcal{E}| = L\left(\frac{\Delta I}{\Delta t}\right)$, we have $L = \frac{\mathcal{E}}{\left(\frac{\Delta I}{\Delta t}\right)} = \frac{24.0\times10^{-3}\text{ V}}{10.0\text{ A/s}} = 2.40\times10^{-3}\text{ H}$

From $L = \frac{N\Phi_B}{I}$, we have $\Phi_B = \frac{LI}{N} = \frac{(2.40\times10^{-3}\text{ H})(4.00\text{ A})}{500} = \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$

32.9 $L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0(420)^2(3.00\times10^{-4})}{0.160} = 4.16\times10^{-4}\text{ H}$

$\mathcal{E} = -L\frac{dI}{dt} \rightarrow \frac{dI}{dt} = \frac{-\mathcal{E}}{L} = \frac{-175\times10^{-6}\text{ V}}{4.16\times10^{-4}\text{ H}} = \boxed{-0.421\text{ A/s}}$

32.10 The induced emf is $\mathcal{E} = -L\frac{dI}{dt}$, where the self-inductance of a solenoid is given by $L = \frac{\mu_0 N^2 A}{l}$.

Thus, $\frac{dI}{dt} = -\frac{\mathcal{E}}{L} = \boxed{-\frac{\mathcal{E}l}{\mu_0 N^2 A}}$

32.11 $|\mathcal{E}| = L\frac{dI}{dt} = (90.0\times10^{-3})\frac{d}{dt}(t^2 - 6t)\text{ V}$

(a) At $t = 1.00$ s, $\mathcal{E} = \boxed{360\text{ mV}}$

(b) At $t = 4.00$ s, $\mathcal{E} = \boxed{180\text{ mV}}$

(c) $\mathcal{E} = (90.0\times10^{-3})(2t - 6) = 0$ when $\boxed{t = 3.00\text{ s}}$

32.12 (a) $B = \mu_0 nI = \mu_0\left(\frac{450}{0.120}\right)(0.0400\text{ mA}) = \boxed{188\ \mu\text{T}}$

(b) $\Phi_B = BA = \boxed{3.33\times10^{-8}\text{ T}\cdot\text{m}^2}$

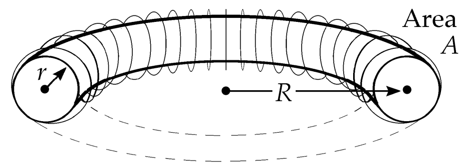
(c) $L = \frac{N\Phi_B}{I} = \boxed{0.375\text{ mH}}$

(d) $\boxed{B \text{ and } \Phi_B \text{ are proportional to current; } L \text{ is independent of current}}$

32.13 (a) $L = \dfrac{\mu_0 N^2 A}{\ell} = \dfrac{\mu_0 (120)^2 \pi (5.00 \times 10^{-3})^2}{0.0900} = \boxed{15.8\ \mu\text{H}}$

(b) $\Phi_B' = \dfrac{\mu_m}{\mu_0}\Phi_B \rightarrow L = \dfrac{\mu_m N^2 A}{\ell} = 800(1.58 \times 10^{-5}\ \text{H}) = \boxed{12.6\ \text{mH}}$

32.14 $L = \dfrac{N\Phi_B}{I} = \dfrac{NBA}{I} \approx \dfrac{NA}{I} \cdot \dfrac{\mu_0 NI}{2\pi R} = \boxed{\dfrac{\mu_0 N^2 A}{2\pi R}}$



32.15 $\mathcal{E} = \mathcal{E}_0 e^{-kt} = -L\dfrac{dI}{dt}$

$dI = -\dfrac{\mathcal{E}_0}{L} e^{-kt}\, dt$

If we require $I \rightarrow 0$ as $t \rightarrow \infty$, the solution is $I = \dfrac{\mathcal{E}_0}{kL} e^{-kt} = \dfrac{dq}{dt}$

$Q = \int I\, dt = \int_0^\infty \dfrac{\mathcal{E}_0}{kL} e^{-kt}\, dt = -\dfrac{\mathcal{E}_0}{k^2 L}$ $\boxed{|Q| = \dfrac{\mathcal{E}_0}{k^2 L}}$

32.16 $I = \dfrac{\mathcal{E}}{R}(1 - e^{-Rt/L})$

$0.900\dfrac{\mathcal{E}}{R} = \dfrac{\mathcal{E}}{R}\left[1 - e^{-R(3.00\ \text{s})/2.50\ \text{H}}\right]$

$\exp\left(-\dfrac{R(3.00\ \text{s})}{2.50\ \text{H}}\right) = 0.100$

$R = \dfrac{2.50\ \text{H}}{3.00\ \text{s}} \ln 10.0 = \boxed{1.92\ \Omega}$

32.17 $\tau = \dfrac{L}{R} = 0.200$ s: $\dfrac{I}{I_{\max}} = 1 - e^{-t/\tau}$

(a) $0.500 = 1 - e^{-t/0.200} \rightarrow t = \tau \ln 2.00 = \boxed{0.139 \text{ s}}$

(b) $0.900 = 1 - e^{-t/0.200} \rightarrow t = \tau \ln 10.0 = \boxed{0.461 \text{ s}}$

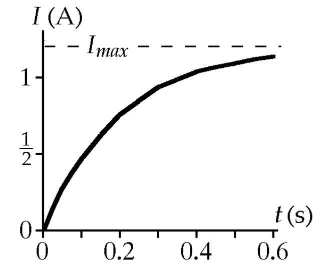


Figure for Goal Solution

Goal Solution

A 12.0-V battery is about to be connected to a series circuit containing a 10.0-Ω resistor and a 2.00-H inductor. How long will it take the current to reach (a) 50.0% and (b) 90.0% of its final value?

G: The time constant for this circuit is $\tau = L/R = 0.2$ s, which means that in 0.2 s, the current will reach $1/e = 63\%$ of its final value, as shown in the graph to the right. We can see from this graph that the time to reach 50% of $I_{\max}$ should be slightly less than the time constant, perhaps about 0.15 s, and the time to reach $0.9I_{\max}$ should be about $2.5\tau = 0.5$ s.

O: The exact times can be found from the equation that describes the rising current in the above graph and gives the current as a function of time for a known emf, resistance, and time constant. We set time $t = 0$ to be the moment the circuit is first connected.

A: At time t, $I(t) = \dfrac{\mathcal{E}(1 - e^{-t/\tau})}{R}$

where, after a long time, $I_{\max} = \dfrac{\mathcal{E}(1 - e^{-\infty})}{R} = \dfrac{\mathcal{E}}{R}$

At $I(t) = 0.500I_{\max}$, $(0.500)\dfrac{\mathcal{E}}{R} = \dfrac{\mathcal{E}(1 - e^{-t/0.200 \text{ s}})}{R}$ so $0.500 = 1 - e^{-t/0.200 \text{ s}}$

Isolating the constants on the right, $\ln\left(e^{-t/2.00 \text{ s}}\right) = \ln(0.500)$

and solving for t, $-\dfrac{t}{0.200 \text{ s}} = -0.693$ or $t = 0.139$ s

(b) Similarly, to reach 90% of $I_{\max}$, $0.900 = 1 - e^{-t/\tau}$ and $t = -\tau \ln(1 - 0.900)$

Thus, $t = -(0.200 \text{ s})\ln(0.100) = 0.461$ s

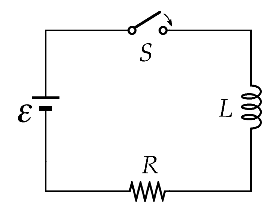
L: The calculated times agree reasonably well with our predictions. We must be careful to avoid confusing the equation for the rising current with the similar equation for the falling current. Checking our answers against predictions is a safe way to prevent such mistakes.

32.18 Taking $\tau = L/R$, $I = I_0 e^{-t/\tau}$: $\dfrac{dI}{dt} = I_0 e^{-t/\tau}\left(-\dfrac{1}{\tau}\right)$

$IR + L\dfrac{dI}{dt} = 0$ will be true if $I_0 R e^{-t/\tau} + L\left(I_0\, e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right) = 0$

Because $\tau = L/R$, we have agreement with 0 = 0

***32.19** (a) $\tau = L/R = 2.00 \times 10^{-3}\text{ s} = \boxed{2.00\text{ ms}}$



(b) $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00\text{ V}}{4.00\ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = \boxed{0.176\text{ A}}$

(c) $I_{\max} = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\text{ V}}{4.00\ \Omega} = \boxed{1.50\text{ A}}$

(d) $0.800 = 1 - e^{-t/2.00\text{ ms}} \rightarrow t = -(2.00\text{ ms})\ln(0.200) = \boxed{3.22\text{ ms}}$

***32.20** $I = \dfrac{\mathcal{E}}{R}\,(1 - e^{-t/\tau}) = \dfrac{120}{9.00}\,(1 - e^{-1.80/7.00}) = 3.02\text{ A}$

$\Delta V_R = IR = (3.02)(9.00) = 27.2\text{ V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = 120 - 27.2 = \boxed{92.8\text{ V}}$

32.21 (a) $\Delta V_R = IR = (8.00\ \Omega)(2.00\text{ A}) = 16.0\text{ V}$ and
$\Delta V_L = \mathcal{E} - \Delta V_R = 36.0\text{ V} - 16.0\text{ V} = 20.0\text{ V}$

Therefore, $\dfrac{\Delta V_R}{\Delta V_L} = \dfrac{16.0\text{ V}}{20.0\text{ V}} = \boxed{0.800}$

(b) $\Delta V_R = IR = (4.50\text{ A})(8.00\ \Omega) = 36.0\text{ V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = \boxed{0}$

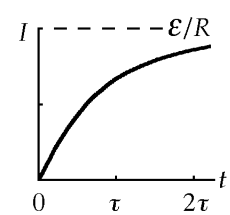


Figure for Goal Solution

Goal Solution

For the *RL* circuit shown in Figure P32.19, let L = 3.00 H, R = 8.00 Ω, and $\mathcal{E}$ = 36.0 V. (a) Calculate the ratio of the potential difference across the resistor to that across the inductor when I = 2.00 A. (b) Calculate the voltage across the inductor when I = 4.50 A.

G: The voltage across the resistor is proportional to the current, $\Delta V_R = IR$, while the voltage across the inductor is proportional to the **rate of change** in the current, $\mathcal{E}_L = -L\,dI/dt$. When the switch is first closed, the voltage across the inductor will be large as it opposes the sudden change in current. As the current approaches its steady state value, the voltage across the resistor increases and the inductor's emf decreases. The maximum current will be $\mathcal{E}/R$ = 4.50 A, so when I = 2.00 A, the resistor and inductor will share similar voltages at this mid-range current, but when I = 4.50 A, the entire circuit voltage will be across the resistor, and the voltage across the inductor will be zero.

O: We can use the definition of resistance to calculate the voltage across the resistor for each current. We will find the voltage across the inductor by using Kirchhoff's loop rule.

A: (a) When $I = 2.00$ A, the voltage across the resistor is $\Delta V_R = IR = (2.00\text{ A})(8.00\ \Omega) = 16.0\text{ V}$

Kirchhoff's loop rule tells us that the sum of the changes in potential around the loop must be zero:

$$\mathcal{E} - \Delta V_R - \mathcal{E}_L = 36.0\text{ V} - 16.0\text{ V} - \mathcal{E}_L = 0 \qquad \text{so} \qquad \mathcal{E}_L = 20.0\text{ V} \quad \text{and} \quad \frac{\Delta V_R}{\mathcal{E}_L} = \frac{16.0\text{ V}}{20.0\text{ V}} = 0.800$$

(b) Similarly, for $I = 4.50$ A, $\Delta V_R = IR = (4.50\text{ A})(8.00\ \Omega) = 36.0\text{ V}$

$$\mathcal{E} - \Delta V_R - \mathcal{E}_L = 36.0\text{ V} - 36.0\text{ V} - \mathcal{E}_L = 0 \qquad \text{so} \qquad \mathcal{E}_L = 0$$

L: We see that when $I = 2.00$ A, $\Delta V_R < \mathcal{E}_L$, but they are similar in magnitude as expected. Also as predicted, the voltage across the inductor goes to zero when the current reaches its maximum value. A worthwhile exercise would be to consider the ratio of these voltages for several different times after the switch is reopened.

***32.22** After a long time, 12.0 V = (0.200 A)R Thus, R = 60.0 Ω. Now, $\tau = \dfrac{L}{R}$ gives

$$L = \tau R = (5.00 \times 10^{-4}\text{ s})(60.0\text{ V/A}) = \boxed{30.0\text{ mH}}$$

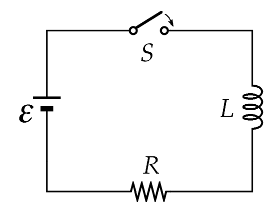
32.23 $I = I_{\max}\left(1 - e^{-t/\tau}\right)$: $\qquad \dfrac{dI}{dt} = -I_{\max}\left(e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right)$

$$\tau = \frac{L}{R} = \frac{15.0\text{ H}}{30.0\ \Omega} = 0.500\text{ s}: \qquad \frac{dI}{dt} = \frac{R}{L} I_{\max} e^{-t/\tau} \quad \text{and} \quad I_{\max} = \frac{\mathcal{E}}{R}$$

(a) $t = 0$: $\dfrac{dI}{dt} = \dfrac{R}{L} I_{\max} e^0 = \dfrac{\mathcal{E}}{L} = \dfrac{100\text{ V}}{15.0\text{ H}} = \boxed{6.67\text{ A/s}}$

(b) $t = 1.50$ s: $\dfrac{dI}{dt} = \dfrac{\mathcal{E}}{L} e^{-t/\tau} = (6.67\text{ A/s})e^{-1.50/(0.500)} = (6.67\text{ A/s})e^{-3.00} = \boxed{0.332\text{ A/s}}$

32.24 $I = I_{\max}\left(1 - e^{-t/\tau}\right)$



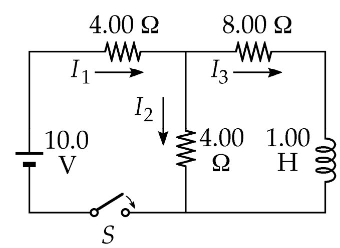
$0.980 = 1 - e^{-3.00\times10^{-3}/\tau}$

$0.0200 = e^{-3.00\times10^{-3}/\tau}$

$$\tau = -\frac{3.00\times10^{-3}}{\ln(0.0200)} = 7.67\times10^{-4}\text{ s}$$

$\tau = L/R$, so $L = \tau R = (7.67\times10^{-4})(10.0) = \boxed{7.67\text{ mH}}$

32.25 Name the currents as shown. By Kirchhoff's laws:



$I_1 = I_2 + I_3$ (1)

$+10.0\text{ V} - 4.00I_1 - 4.00I_2 = 0$ (2)

$+10.0\text{ V} - 4.00I_1 - 8.00I_3 - (1.00)\dfrac{dI_3}{dt} = 0$ (3)

From (1) and (2), $+10.0 - 4.00I_1 - 4.00I_1 + 4.00I_3 = 0$ and $I_1 = 0.500I_3 + 1.25\text{ A}$

Then (3) becomes $10.0\text{ V} - 4.00(0.500I_3 + 1.25\text{ A}) - 8.00I_3 - (1.00)\dfrac{dI_3}{dt} = 0$

$$(1.00\text{ H})(dI_3/dt) + (10.0\ \Omega)I_3 = 5.00\text{ V}$$

We solve the differential equation using Equations 32.6 and 32.7:

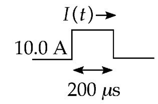
$$I_3(t) = \frac{5.00\text{ V}}{10.0\ \Omega}\left[1 - e^{-(10.0\ \Omega)t/1.00\text{ H}}\right] = \boxed{(0.500\text{ A})\left[1 - e^{-10t/\text{s}}\right]}$$

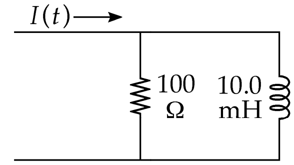
$$I_1 = 1.25 + 0.500I_3 = \boxed{1.50\text{ A} - (0.250\text{ A})e^{-10t/\text{s}}}$$

32.26 (a) Using $\tau = RC = \dfrac{L}{R}$, we get $R = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{3.00\text{ H}}{3.00\times10^{-6}\text{ F}}} = 1.00\times10^{3}\ \Omega = \boxed{1.00\text{ k}\Omega}$

(b) $\tau = RC = \left(1.00\times10^{3}\ \Omega\right)\left(3.00\times10^{-6}\text{ F}\right) = 3.00\times10^{-3}\text{ s} = \boxed{3.00\text{ ms}}$

32.27 For $t \le 0$, the current in the inductor is zero. At $t = 0$, it starts to grow from zero toward 10.0 A with time constant $\tau = L/R = (10.0\ \text{mH})/(100\ \Omega) = 1.00 \times 10^{-4}\ \text{s}$.





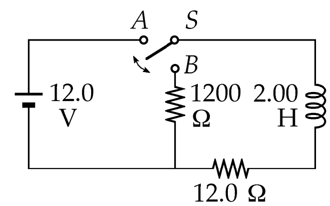
For $0 \le t \le 200\ \mu\text{s}$, $\quad I = I_{\max}\left(1 - e^{-t/\tau}\right) = \boxed{(10.00\ \text{A})\left(1 - e^{-10000\,t/\text{s}}\right)}$

At $t = 200\ \mu\text{s}$, $\quad I = (10.00\ \text{A})\left(1 - e^{-2.00}\right) = 8.65\ \text{A}$

Thereafter, it decays exponentially as $I = I_0 e^{-t'/\tau}$, so for $t \ge 200\ \mu\text{s}$,

$$I = (8.65\ \text{A})e^{-10000(t - 200\ \mu\text{s})/\text{s}} = (8.65\ \text{A})e^{-10000t/\text{s} + 2.00} = \left(8.65e^{2.00}\ \text{A}\right)e^{-10000t/\text{s}} = \boxed{(63.9\ \text{A})e^{-10000t/\text{s}}}$$

32.28 (a) $I = \dfrac{\mathcal{E}}{R} = \dfrac{12.0\ \text{V}}{12.0\ \Omega} = \boxed{1.00\ \text{A}}$



(b) Initial current is 1.00 A, : $\quad \Delta V_{12} = (1.00\ \text{A})(12.00\ \Omega) = \boxed{12.0\ \text{V}}$

$\Delta V_{1200} = (1.00\,\text{A})(1200\,\Omega) = \boxed{1.20\ \text{kV}}$

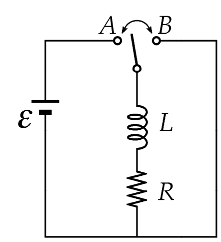
$\Delta V_L = \boxed{1.21\ \text{kV}}$

(c) $I = I_{\max} e^{-Rt/L}$: $\quad \dfrac{dI}{dt} = -I_{\max}\dfrac{R}{L}e^{-Rt/L}$ $\quad$ and $\quad -L\dfrac{dI}{dt} = \Delta V_L = I_{\max} R e^{-Rt/L}$

Solving $\quad 12.0\ \text{V} = (1212\ \text{V})e^{-1212t/2.00}$ $\quad$ so $\quad 9.90 \times 10^{-3} = e^{-606t}$

Thus, $\quad \boxed{t = 7.62\ \text{ms}}$

32.29 $\tau = \dfrac{L}{R} = \dfrac{0.140}{4.90} = 28.6\ \text{ms}; \quad I_{\max} = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\ \text{V}}{4.90\ \Omega} = 1.22\ \text{A}$



(a) $I = I_{\max}\left(1 - e^{-t/\tau}\right)$ $\quad$ so $\quad 0.220 = 1.22\left(1 - e^{-t/\tau}\right)$

$e^{-t/\tau} = 0.820 \qquad t = -\tau \ln(0.820) = \boxed{5.66\ \text{ms}}$

(b) $I = I_{\max}\left(1 - e^{-\frac{10.0}{0.0286}}\right) = (1.22\ \text{A})\left(1 - e^{-350}\right) = \boxed{1.22\ \text{A}}$

(c) $I = I_{\max} e^{-t/\tau}$ $\quad$ and $\quad 0.160 = 1.22 e^{-t/\tau}$ $\quad$ so $\quad t = -\tau \ln(0.131) = \boxed{58.1\ \text{ms}}$

32.30 (a) For a series connection, both inductors carry equal currents at every instant, so dI/dt is the same for both. The voltage across the pair is

$$L_{\text{eq}}\frac{dI}{dt} = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} \qquad \text{so} \qquad \boxed{L_{\text{eq}} = L_1 + L_2}$$

(b) $$L_{\text{eq}}\frac{dI}{dt} = L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} = \Delta V_L \qquad \text{where } I = I_1 + I_2 \quad \text{and} \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

Thus, $$\frac{\Delta V_L}{L_{\text{eq}}} = \frac{\Delta V_L}{L_1} + \frac{\Delta V_L}{L_2} \qquad \text{and} \qquad \boxed{\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

(c) $$L_{\text{eq}}\frac{dI}{dt} + R_{\text{eq}}I = L_1\frac{dI}{dt} + IR_1 + L_2\frac{dI}{dt} + IR_2$$

Now I and dI/dt are separate quantities under our control, so functional equality requires both

$$\boxed{L_{\text{eq}} = L_1 + L_2 \quad \text{and} \quad R_{\text{eq}} = R_1 + R_2}$$

(d) $$\Delta V = L_{\text{eq}}\frac{dI}{dt} + R_{\text{eq}}I = L_1\frac{dI_1}{dt} + R_1I_1 = L_2\frac{dI_2}{dt} + R_2I_2 \qquad \text{where} \qquad I = I_1 + I_2 \quad \text{and} \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

We may choose to keep the currents constant in time. Then, $$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

We may choose to make the current swing through 0. Then, $$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\boxed{\begin{array}{l}\text{This equivalent coil with resistance will be equivalent}\\ \text{to the pair of real inductors for all other currents as well.}\end{array}}$$

32.31 $$L = \frac{N\Phi_B}{I} = \frac{200(3.70\times10^{-4})}{1.75} = 42.3\text{ mH} \qquad \text{so} \qquad U = \frac{1}{2}LI^2 = \frac{1}{2}(0.423\text{ H})(1.75\text{ A})^2 = \boxed{0.0648\text{ J}}$$

32.32 (a) The magnetic energy density is given by

$$u = \frac{B^2}{2\mu_0} = \frac{(4.50\text{ T})^2}{2(1.26\times10^{-6}\text{ T}\cdot\text{m/A})} = \boxed{8.06\times10^6\text{ J/m}^3}$$

(b) The magnetic energy stored in the field equals u times the volume of the solenoid (the volume in which B is non-zero).

$$U = uV = (8.06\times10^6\text{ J/m}^3)\left[(0.260\text{ m})\pi(0.0310\text{ m})^2\right] = \boxed{6.32\text{ kJ}}$$

32.33 $L = \mu_0 \dfrac{N^2 A}{\ell} = \mu_0 \dfrac{(68.0)^2\,\pi(0.600\times 10^{-2})^2}{0.0800} = 8.21\,\mu\text{H}$

$U = \frac{1}{2}LI^2 = \frac{1}{2}(8.21\times 10^{-6}\text{ H})(0.770\text{ A})^2 = \boxed{2.44\,\mu\text{J}}$

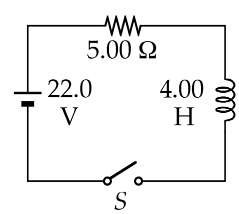
32.34 (a) $U = \frac{1}{2}\,LI^2 = \frac{1}{2}\,L\left(\dfrac{\mathcal{E}}{2R}\right)^2 = \dfrac{L\mathcal{E}^2}{8R^2} = \dfrac{(0.800)(500)^2}{8(30.0)^2} = \boxed{27.8\text{ J}}$

(b) $I = \left(\dfrac{\mathcal{E}}{R}\right)\left[1 - e^{-(R/L)t}\right]$ so $\dfrac{\mathcal{E}}{2R} = \left(\dfrac{\mathcal{E}}{R}\right)\left[1 - e^{-(R/L)t}\right] \to e^{-(R/L)t} = \dfrac{1}{2}$

$\dfrac{R}{L}t = \ln 2$ so $t = \dfrac{L}{R}\ln 2 = \dfrac{0.800}{30.0}\ln 2 = \boxed{18.5\text{ ms}}$

32.35 $u = \epsilon_0 \dfrac{E^2}{2} = \boxed{44.2\text{ nJ/m}^3}$ $u = \dfrac{B^2}{2\mu_0} = \boxed{995\,\mu\text{J/m}^3}$

***32.36** (a) $U = \frac{1}{2}\,LI^2 = \frac{1}{2}(4.00\text{ H})(0.500\text{ A})^2 = \boxed{0.500\text{ J}}$



(b) $\dfrac{dU}{dt} = LI = (4.00\text{ H})(1.00\text{ A}) = 4.00\text{ J/s} = \boxed{4.00\text{ W}}$

(c) $\mathcal{P} = (\Delta V)I = (22.0\text{ V})(0.500\text{ A}) = \boxed{11.0\text{ W}}$

32.37 From Equation 32.7, $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$

(a) The maximum current, after a long time t, is $I = \dfrac{\mathcal{E}}{R} = 2.00\text{ A}$.

At that time, the inductor is fully energized and $\mathcal{P} = I(\Delta V) = (2.00\text{ A})(10.0\text{ V}) = \boxed{20.0\text{ W}}$

(b) $\mathcal{P}_{\text{lost}} = I^2R = (2.00\text{ A})^2(5.00\ \Omega) = \boxed{20.0\text{ W}}$

(c) $\mathcal{P}_{\text{inductor}} = I(\Delta V_{\text{drop}}) = \boxed{0}$

(d) $U = \dfrac{LI^2}{2} = \dfrac{(10.0\text{ H})(2.00\text{ A})^2}{2} = \boxed{20.0\text{ J}}$

32.38 We have $u = \epsilon_0 \dfrac{E^2}{2}$ and $u = \dfrac{B^2}{2\mu_0}$

Therefore $\epsilon_0 \dfrac{E^2}{2} = \dfrac{B^2}{2\mu_0}$ so $B^2 = \epsilon_0 \mu_0 E^2$

$$B = E\sqrt{\epsilon_0\mu_0} = \frac{6.80\times 10^5 \text{ V / m}}{3.00\times 10^8 \text{ m / s}} = \boxed{2.27\times 10^{-3}\text{ T}}$$

32.39 The total magnetic energy is the volume integral of the energy density, $u = \dfrac{B^2}{2\mu_0}$

Because B changes with position, u is not constant. For $B = B_0(R/r)^2$, $u = \left(\dfrac{B_0^{\,2}}{2\mu_0}\right)\left(\dfrac{R}{r}\right)^4$

Next, we set up an expression for the magnetic energy in a spherical shell of radius r and thickness dr. Such a shell has a volume $4\pi r^2\,dr$, so the energy stored in it is

$$dU = u\left(4\pi r^2 dr\right) = \left(\frac{2\pi B_0^{\,2} R^4}{\mu_0}\right)\frac{dr}{r^2}$$

We integrate this expression for $r = R$ to $r = \infty$ to obtain the total magnetic energy outside the sphere. This gives

$$U = \boxed{\frac{2\pi B_0^2 R^3}{\mu_0}} = \frac{2\pi(5.00\times 10^{-5}\text{ T})^2(6.00\times 10^6\text{ m})^3}{(1.26\times 10^{-6}\text{ T}\cdot\text{m / A})} = \boxed{2.70\times 10^{18}\text{ J}}$$

32.40 $I_1(t) = I_{\max}e^{-\alpha t}\sin\omega t$ with $I_{\max} = 5.00$ A, $\alpha = 0.0250\text{ s}^{-1}$, and $\omega = 377$ rad/s.

$$\frac{dI_1}{dt} = I_{\max}e^{-\alpha t}\left(-\alpha\sin\omega t + \omega\cos\omega t\right)$$

At $t = 0.800$ s, $$\frac{dI_1}{dt} = \left(5.00\text{ A/s}\right)e^{-0.0200}\left[-(0.0250)\sin\left(0.800(377)\right) + 377\cos\left(0.800(377)\right)\right]$$

$$\frac{dI_1}{dt} = 1.85\times 10^3\text{ A/s}$$

Thus, $\mathcal{E}_2 = -M\dfrac{dI_1}{dt}$: $M = \dfrac{-\mathcal{E}_2}{dI_1/dt} = \dfrac{+3.20\text{ V}}{1.85\times 10^3\text{ A/s}} = \boxed{1.73\text{ mH}}$

32.41 $$\mathcal{E}_2 = -M\frac{dI_1}{dt} = -(1.00\times10^{-4}\ \text{H})(1.00\times10^{4}\ \text{A / s})\cos(1000t)$$

$$(\mathcal{E}_2)_{\max} = \boxed{1.00\ \text{V}}$$

32.42 $$M = \left|\frac{\mathcal{E}_2}{dI_1/dt}\right| = \frac{96.0\ \text{mV}}{1.20\ \text{A / s}} = \boxed{80.0\ \text{mH}}$$

32.43 (a) $$M = \frac{N_B\Phi_{BA}}{I_A} = \frac{700(90.0\times10^{-6})}{3.50} = \boxed{18.0\ \text{mH}}$$

(b) $$L_A = \frac{\Phi_A}{I_A} = \frac{400(300\times10^{-6})}{3.50} = \boxed{34.3\ \text{mH}}$$

(c) $$\mathcal{E}_B = -M\frac{dI_A}{dt} = -(18.0\ \text{mH})(0.500\ \text{A / s}) = \boxed{-9.00\ \text{mV}}$$

32.44 $$M = \frac{N_2\Phi_{12}}{I_1} = \frac{N_2(B_1A_1)}{I_1} = \frac{N_2\left[(\mu_0 n_1 I_1)A_1\right]}{I_1} = N_2\,\mu_0 n_1 A_1$$

$$M = (1.00)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(\frac{70.0}{0.0500\ \text{m}}\right)\left[\pi\left(5.00\times10^{-3}\ \text{m}\right)^2\right] = \boxed{138\ \text{nH}}$$

32.45 B at center of (larger) loop: $B_1 = \dfrac{\mu_0 I_1}{2R}$

(a) $$M = \frac{\Phi_2}{I_1} = \frac{B_1A_2}{I_1} = \frac{(\mu_0 I_1\ /\ 2R)(\pi r^2)}{I_1} = \boxed{\frac{\mu_0\pi r^2}{2R}}$$

(b) $$M = \frac{\mu_0\pi(0.0200)^2}{2(0.200)} = \boxed{3.95\ \text{nH}}$$

***32.46** Assume the long wire carries current I. Then the magnitude of the magnetic field it generates at distance x from the wire is $B = \mu_0 I / 2\pi x$, and this field passes perpendicularly through the plane of the loop. The flux through the loop is

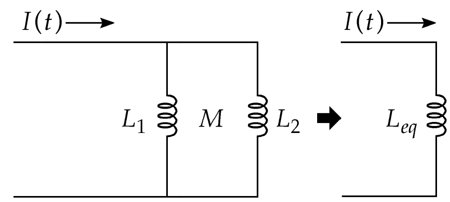
$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A} = \int B\,dA = \int B(\ell\,dx) = \frac{\mu_0 I \ell}{2\pi}\int_{0.400\text{ mm}}^{1.70\text{ mm}} \frac{dx}{x} = \frac{\mu_0 I \ell}{2\pi}\ln\left(\frac{1.70}{0.400}\right)$$

The mutual inductance between the wire and the loop is then

$$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_2 \mu_0 I \ell}{2\pi I}\ln\left(\frac{1.70}{0.400}\right) = \frac{N_2 \mu_0 \ell}{2\pi}(1.45) = \frac{1(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.70\times10^{-3}\text{ m})}{2\pi}(1.45)$$

$$M = 7.81\times10^{-10}\text{ H} = \boxed{781\text{ pH}}$$

32.47 With $I = I_1 + I_2$, the voltage across the pair is:



(a) **(b)**

$$\Delta V = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} = -L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt} = -L_{\text{eq}}\frac{dI}{dt}$$

So, $$-\frac{dI_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1}\frac{dI_2}{dt}$$

and $$-L_2\frac{dI_2}{dt} + \frac{M(\Delta V)}{L_1} + \frac{M^2}{L_1}\frac{dI_2}{dt} = \Delta V$$

$$(-L_1L_2 + M^2)\frac{dI_2}{dt} = \Delta V(L_1 - M) \qquad [1]$$

By substitution, $$-\frac{dI_2}{dt} = \frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{dI_1}{dt}$$

leads to $$(-L_1L_2 + M^2)\frac{dI_1}{dt} = \Delta V(L_2 - M) \qquad [2]$$

Adding [1] to [2], $$(-L_1L_2 + M^2)\frac{dI}{dt} = \Delta V(L_1 + L_2 - 2M)$$

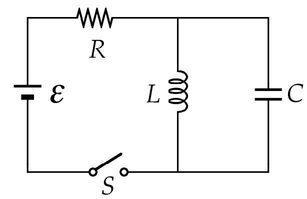
So, $$L_{\text{eq}} = -\frac{\Delta V}{dI/dt} = \boxed{\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

32.48 At different times, $(U_C)_{\max} = (U_L)_{\max}$ so $\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \left(\frac{1}{2}LI^2\right)_{\max}$

$$I_{\max} = \sqrt{\frac{C}{L}}(\Delta V)_{\max} = \sqrt{\frac{1.00\times10^{-6}\text{ F}}{10.0\times10^{-3}\text{ H}}}(40.0\text{ V}) = \boxed{0.400\text{ A}}$$

32.49 $\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \left(\frac{1}{2}LI^2\right)_{\max}$ so $(\Delta V_C)_{\max} = \sqrt{\frac{L}{C}}\, I_{\max} = \sqrt{\frac{20.0\times10^{-3}\text{ H}}{0.500\times10^{-6}\text{ F}}}\,(0.100\text{ A}) = \boxed{20.0\text{ V}}$

32.50 When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_{\max} = \mathcal{E}/R$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the LC loop.



We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV, in $\frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI_{\max}^2$.

Then, $L = \dfrac{C(\Delta V)^2}{I_{\max}^2} = \dfrac{C(\Delta V)^2R^2}{\mathcal{E}^2} = \dfrac{(0.500\times10^{-6}\text{ F})(150\text{ V})^2(250\ \Omega)^2}{(50.0\text{ V})^2} = \boxed{0.281\text{ H}}$

32.51 $C = \dfrac{1}{(2\pi f)^2L} = \dfrac{1}{(2\pi\cdot 6.30\times10^6)^2(1.05\times10^6)} = \boxed{608\text{ pF}}$

Goal Solution

A fixed inductance $L = 1.05\ \mu$H is used in series with a variable capacitor in the tuning section of a radio. What capacitance tunes the circuit to the signal from a station broadcasting at 6.30 MHz?

G: It is difficult to predict a value for the capacitance without doing the calculations, but we might expect a typical value in the μF or pF range.

O: We want the resonance frequency of the circuit to match the broadcasting frequency, and for a simple *RLC* circuit, the resonance frequency only depends on the magnitudes of the inductance and capacitance.

A: The resonance frequency is $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

Thus, $C = \dfrac{1}{(2\pi f_0)^2L} = \dfrac{1}{\left[(2\pi)(6.30\times10^6\text{ Hz})\right]^2(1.05\times10^{-6}\text{ H})} = 608\text{ pF}$

L: This is indeed a typical capacitance, so our calculation appears reasonable. However, you probably would not hear any familiar music on this broadcast frequency. The frequency range for FM radio broadcasting is 88.0 – 108.0 MHz, and AM radio is 535 – 1605 kHz. The 6.30 MHz frequency falls in the Maritime Mobile SSB Radiotelephone range, so you might hear a ship captain instead of Top 40 tunes! This and other information about the radio frequency spectrum can be found on the National Telecommunications and Information Administration (NTIA) website, which at the time of this printing was at http://www.ntia.doc.gov/osmhome/allochrt.html

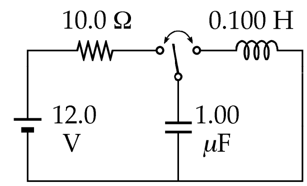
32.52 $f = \dfrac{1}{2\pi\sqrt{LC}}$: $L = \dfrac{1}{(2\pi f)^2 C} = \dfrac{1}{(2\pi \cdot 120)^2(8.00\times 10^{-6})} = \boxed{0.220 \text{ H}}$

32.53 (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.0820 \text{ H})(17.0\times 10^{-6} \text{ F})}} = \boxed{135 \text{ Hz}}$

(b) $Q = Q_{\max}\cos\omega t = (180\ \mu C)\cos(847\times 0.00100) = \boxed{119\ \mu\text{C}}$

(c) $I = \dfrac{dQ}{dt} = -\omega Q_{\max}\sin\omega t = -(847)(180)\sin(0.847) = \boxed{-114 \text{ mA}}$

32.54 (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.100 \text{ H})(1.00\times 10^{-6} \text{ F})}} = \boxed{503 \text{ Hz}}$



(b) $Q = C\mathcal{E} = (1.00\times 10^{-6} \text{ F})(12.0 \text{ V}) = \boxed{12.0\ \mu\text{C}}$

(c) $\frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_{\max}^2$

$$I_{\max} = \mathcal{E}\sqrt{\frac{C}{L}} = 12 \text{ V}\sqrt{\frac{1.00\times 10^{-6} \text{ F}}{0.100 \text{ H}}} = \boxed{37.9 \text{ mA}}$$

(d) At all times $U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}(1.00\times 10^{-6} \text{ F})(12.0 \text{ V})^2 = \boxed{72.0\ \mu\text{J}}$

32.55 $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(3.30 \text{ H})\left(840\times 10^{-12} \text{ F}\right)}} = 1.899\times 10^4 \text{ rad s}$

$$Q = Q_{\max}\cos\omega t, \quad I = \frac{dQ}{dt} = -\omega Q_{\max}\sin\omega t$$

(a) $U_C = \dfrac{Q^2}{2C} = \dfrac{\left(\left[105\times 10^{-6}\right]\cos\left[\left(1.899\times 10^4 \text{ rad/s}\right)\left(2.00\times 10^{-3} \text{ s}\right)\right]\right)^2}{2\left(840\times 10^{-12}\right)} = \boxed{6.03 \text{ J}}$

(b) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\omega^2 Q_{\max}^2\sin^2(\omega t) = \dfrac{Q_{\max}^2\sin^2(\omega t)}{2C}$

$$U_L = \frac{\left(105\times 10^{-6} \text{ C}\right)^2\sin^2\left[\left(1.899\times 10^4 \text{ rad/s}\right)\left(2.00\times 10^{-3} \text{ s}\right)\right]}{2\left(840\times 10^{-12} \text{ F}\right)} = \boxed{0.529 \text{ J}}$$

(c) $U_{\text{total}} = U_C + U_L = \boxed{6.56 \text{ J}}$

32.56 (a) $\omega_d = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2} = \sqrt{\dfrac{1}{\left(2.20\times10^{-3}\right)\left(1.80\times10^{-6}\right)} - \left(\dfrac{7.60}{2\left(2.20\times10^{-3}\right)}\right)^2} = 1.58\times10^4 \text{ rad / s}$

Therefore, $f_d = \dfrac{\omega_d}{2\pi} = \boxed{2.51 \text{ kHz}}$

(b) $R_c = \sqrt{\dfrac{4L}{C}} = \boxed{69.9\ \Omega}$

32.57 (a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.500)(0.100\times10^{-6})}} = \boxed{4.47 \text{ krad / s}}$

(b) $\omega_d = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2} = \boxed{4.36 \text{ krad / s}}$

(c) $\dfrac{\Delta\omega}{\omega_0} = \boxed{2.53\% \text{ lower}}$

32.58 Choose to call positive current clockwise in Figure 32.19. It drains charge from the capacitor according to $I = -dQ/dt$. A clockwise trip around the circuit then gives

$$+\frac{Q}{C} - IR - L\frac{dI}{dt} = 0$$

$+\dfrac{Q}{C} + \dfrac{dQ}{dt}R + L\dfrac{d}{dt}\dfrac{dQ}{dt} = 0,$ identical with Equation 32.29.

32.59 (a) $Q = Q_{\max}e^{-\frac{Rt}{2L}} \cos \omega_d t$ so $I_{\max} \propto e^{-\frac{Rt}{2L}}$

$0.500 = e^{-\frac{Rt}{2L}}$ and $\dfrac{Rt}{2L} = -\ln(0.500)$

$t = -\dfrac{2L}{R}\ln(0.500) = \boxed{0.693\left(\dfrac{2L}{R}\right)}$

(b) $U_0 \propto Q_{\max}^2$ and $U = 0.500U_0$ so $Q = \sqrt{0.500}\, Q_{\max} = 0.707Q_{\max}$

$t = -\dfrac{2L}{R}\ln(0.707) = \boxed{0.347\left(\dfrac{2L}{R}\right)}$ (half as long)

32.60 With $Q = Q_{\max}$ at $t = 0$, the charge on the capacitor at any time is $Q = Q_{\max}\cos\omega t$ where $\omega = 1/\sqrt{LC}$. The energy stored in the capacitor at time t is then

$$U = \frac{Q^2}{2C} = \frac{Q_{\max}^2}{2C}\cos^2\omega t = U_0\cos^2\omega t.$$

When $U = \frac{1}{4}U_0$, $\cos\omega t = \frac{1}{2}$ and $\omega t = \frac{1}{3}\pi$ rad

Therefore, $\dfrac{t}{\sqrt{LC}} = \dfrac{\pi}{3}$ or $\dfrac{t^2}{LC} = \dfrac{\pi^2}{9}$

The inductance is then: $L = \boxed{\dfrac{9t^2}{\pi^2 C}}$

32.61 (a) $\mathcal{E}_L = -L\dfrac{dI}{dt} = -(1.00\text{ mH})\dfrac{d(20.0t)}{dt} = \boxed{-20.0\text{ mV}}$

(b) $Q = \int_0^t I\,dt = \int_0^t (20.0t)\,dt = 10.0t^2$

$$\Delta V_C = \frac{-Q}{C} = \frac{-10.0t^2}{1.00\times10^{-6}\text{ F}} = \boxed{-\left(10.0\text{ MV/s}^2\right)t^2}$$

(c) When $\dfrac{Q^2}{2C} \ge \dfrac{1}{2}LI^2$, or $\dfrac{\left(-10.0t^2\right)^2}{2\left(1.00\times10^{-6}\right)} \ge \dfrac{1}{2}\left(1.00\times10^{-3}\right)(20.0t)^2$,

then $100t^4 \ge \left(400\times10^{-9}\right)t^2$. The earliest time this is true is at $t = \sqrt{4.00\times10^{-9}}\text{ s} = \boxed{63.2\ \mu\text{s}}$

32.62 (a) $\mathcal{E}_L = -L\dfrac{dI}{dt} = -L\dfrac{d}{dt}(Kt) = \boxed{-LK}$

(b) $I = \dfrac{dQ}{dt}$, so $Q = \int_0^t I\,dt = \int_0^t Kt\,dt = \frac{1}{2}Kt^2$

and $\Delta V_C = \dfrac{-Q}{C} = \boxed{-\dfrac{Kt^2}{2C}}$

(c) When $\frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}LI^2$, $\frac{1}{2}C\left(\dfrac{K^2t^4}{4C^2}\right) = \frac{1}{2}L\left(K^2t^2\right)$

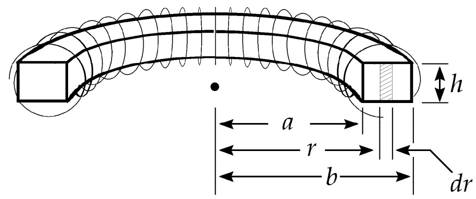
Thus $t = \boxed{2\sqrt{LC}}$

32.63 $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left(\frac{Q}{2}\right)^2 + \frac{1}{2}LI^2$ so $I = \sqrt{\frac{3Q^2}{4CL}}$

The flux through each turn of the coil is $\Phi_B = \frac{LI}{N} = \boxed{\frac{Q}{2N}\sqrt{\frac{3L}{C}}}$

where N is the number of turns.

32.64 Equation 30.16: $B = \frac{\mu_0 NI}{2\pi r}$



(a) $\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 NI}{2\pi r} h\,dr = \frac{\mu_0 NIh}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$

$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)}$

(b) $L = \frac{\mu_0 (500)^2 (0.0100)}{2\pi}\ln\left(\frac{12.0}{10.0}\right) = \boxed{91.2\ \mu\text{H}}$

(c) $L_{\text{appx}} = \frac{\mu_0 N^2}{2\pi}\left(\frac{A}{R}\right) = \frac{\mu_0 (500)^2}{2\pi}\left(\frac{2.00\times 10^{-4}\ \text{m}^2}{0.110}\right) = \boxed{90.9\ \mu\text{H}}$

***32.65** (a) At the center, $B = \frac{N\mu_0 IR^2}{2(R^2+0^2)^{3/2}} = \frac{N\mu_0 I}{2R}$

So the coil creates flux through itself $\Phi_B \approx BA\cos\theta = \frac{N\mu_0 I}{2R}\pi R^2\cos 0° = \frac{\pi}{2}N\mu_0 IR$

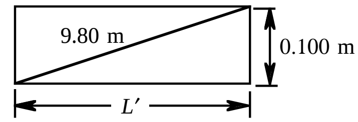
When the current it carries changes, $\mathcal{E}_L = -N\frac{d\Phi_B}{dt} \approx -N\frac{\pi}{2}N\mu_0 R\frac{dI}{dt} = -L\frac{dI}{dt}$

so $\boxed{L \approx \frac{\pi}{2}N^2\mu_0 R}$

(b) $2\pi r \approx 3(0.3\ \text{m})$, so $r \approx 0.14$ m; $L \approx \frac{\pi}{2}\ 1^2\left(4\pi\times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\ 0.14\ \text{m} = 2.8\times 10^{-7}\ \text{H}\ \boxed{\sim 100\ \text{nH}}$

(c) $\frac{L}{R} \approx \frac{2.8\times 10^{-7}\ \text{V}\cdot\text{s/A}}{270\ \text{V/A}} = 1.0\times 10^{-9}\ \text{s}\ \boxed{\sim 1\ \text{ns}}$

32.66 (a) If unrolled, the wire forms the diagonal of a 0.100 m (10.0 cm) rectangle as shown. The length of this rectangle is



$$L' = \sqrt{(9.80\text{ m})^2 - (0.100\text{ m})^2}$$

The mean circumference of each turn is $C = 2\pi r'$, where $r' = \frac{24.0 + 0.644}{2}$ mm is the mean radius of each turn. The number of turns is then:

$$N = \frac{L'}{C} = \frac{\sqrt{(9.80\text{ m})^2 - (0.100\text{ m})^2}}{2\pi\left(\frac{24.0+0.644}{2}\right)\times 10^{-3}\text{ m}} = \boxed{127}$$

(b) $$R = \frac{\rho \ell}{A} = \frac{\left(1.70\times 10^{-8}\ \Omega\cdot\text{m}\right)(10.0\text{ m})}{\pi\left(0.322\times 10^{-3}\text{ m}\right)^2} = \boxed{0.522\ \Omega}$$

(c) $$L = \frac{\mu N^2 A}{\ell'} = \frac{800\mu_0}{\ell'}\left(\frac{L'}{C}\right)^2 \pi (r')^2$$

$$L = \frac{800\left(4\pi\times 10^{-7}\right)}{0.100\text{ m}}\left(\frac{\sqrt{(9.80\text{ m})^2 - (0.100\text{ m})^2}}{\pi(24.0+0.644)\times 10^{-3}\text{ m}}\right)^2 \pi\left[\left(\frac{24.0+0.644}{2}\right)\times 10^{-3}\text{ m}\right]^2$$

$$L = 7.68\times 10^{-2}\text{ H} = \boxed{76.8\text{ mH}}$$

32.67 From Ampere's law, the magnetic field at distance $r \le R$ is found as:

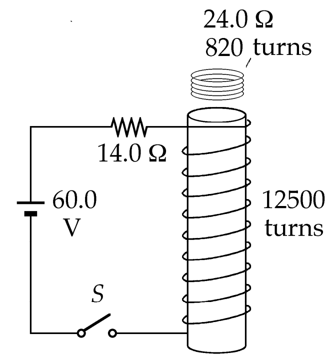
$$B(2\pi r) = \mu_0 J\left(\pi r^2\right) = \mu_0\left(\frac{I}{\pi R^2}\right)\left(\pi r^2\right), \quad \text{or} \quad B = \frac{\mu_0 I r}{2\pi R^2}$$

The magnetic energy per unit length within the wire is then

$$\frac{U}{\ell} = \int_0^R \frac{B^2}{2\mu_0}(2\pi r\,dr) = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3\,dr = \frac{\mu_0 I^2}{4\pi R^4}\left(\frac{R^4}{4}\right) = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

This is independent of the radius of the wire.

32.68 The primary circuit (containing the battery and solenoid) is an RL circuit with $R = 14.0\ \Omega$, and



$$L = \frac{\mu_0 N^2 A}{l} = \frac{(4\pi\times10^{-7})(12\ 500)^2(1.00\times10^{-4})}{0.0700} = 0.280\ \text{H}$$

(a) The time for the current to reach 63.2% of the maximum value is the time constant of the circuit:

$$\tau = \frac{L}{R} = \frac{0.280\ \text{H}}{14.0\ \Omega} = 0.0200\ \text{s} = \boxed{20.0\ \text{ms}}$$

(b) The solenoid's average back emf is $\left|\bar{\mathcal{E}}_L\right| = L\left(\frac{\Delta I}{\Delta t}\right) = L\left(\frac{I_f - 0}{\Delta t}\right)$

where $I_f = 0.632 I_{\max} = 0.632\left(\frac{\Delta V}{R}\right) = 0.632\left(\frac{60.0\ \text{V}}{14.0\ \Omega}\right) = 2.71\ \text{A}$

Thus, $\left|\bar{\mathcal{E}}_L\right| = (0.280\ \text{H})\left(\frac{2.71\ \text{A}}{0.0200\ \text{s}}\right) = \boxed{37.9\ \text{V}}$

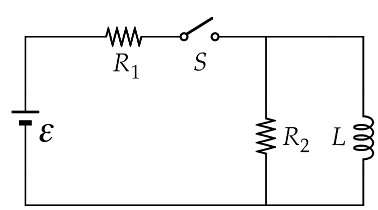
(c) The average rate of change of flux through each turn of the overwrapped concentric coil is the same as that through a turn on the solenoid:

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{\mu_0 n(\Delta I)A}{\Delta t} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(12\,500/0.0700\ \text{m})(2.71\ \text{A})(1.00\times10^{-4}\ \text{m}^2)}{0.0200\ \text{s}} = \boxed{3.04\ \text{mV}}$$

(d) The magnitude of the average induced emf in the coil is $\left|\mathcal{E}_L\right| = N(\Delta\Phi_B/\Delta t)$ and magnitude of the average induced current is

$$I = \frac{\left|\mathcal{E}_L\right|}{R} = \frac{N}{R}\left(\frac{\Delta\Phi_B}{\Delta t}\right) = \frac{820}{24.0\ \Omega}(3.04\times10^{-3}\ \text{V}) = 0.104\ \text{A} = \boxed{104\ \text{mA}}$$

32.69 Left-hand loop: $\mathcal{E} - (I + I_2)R_1 - I_2R_2 = 0$



Outside loop: $\mathcal{E} - (I + I_2)R_1 - L\frac{dI}{dt} = 0$

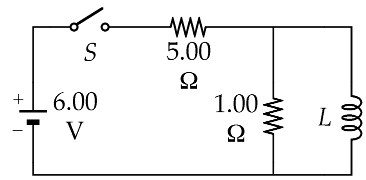
Eliminating I_2 gives $\mathcal{E}' - IR' - L\frac{dI}{dt} = 0$

This is of the same form as Equation 32.6, so its solution is of the same form as Equation 32.7: $I(t) = \frac{\mathcal{E}'}{R'}(1 - e^{-R't/L})$

But $R' = R_1R_2/(R_1 + R_2)$ and $\mathcal{E}' = R_2\mathcal{E}/(R_1 + R_2)$, so $\frac{\mathcal{E}'}{R'} = \frac{\mathcal{E}R_2/(R_1 + R_2)}{R_1R_2/(R_1 + R_2)} = \frac{\mathcal{E}}{R_1}$

Thus $I(t) = \frac{\mathcal{E}}{R_1}(1 - e^{-R't/L})$

32.70 When switch is closed, steady current $I_0 = 1.20$ A. When the switch is opened after being closed a long time, the current in the right loop is



$$I = I_0 e^{-R_2 t/L}$$

so $\quad e^{Rt/L} = \dfrac{I_0}{I} \quad$ and $\quad \dfrac{Rt}{L} = \ln\left(\dfrac{I_0}{I}\right)$

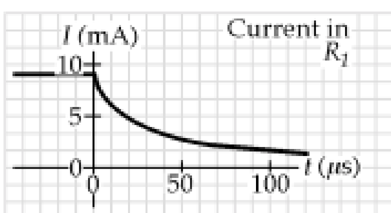
Therefore, $\quad L = \dfrac{R_2 t}{\ln(I_0/I)} = \dfrac{(1.00\ \Omega)(0.150\ \text{s})}{\ln(1.20\ \text{A}/0.250\ \text{A})} = 0.0956\ \text{H} = \boxed{95.6\ \text{mH}}$

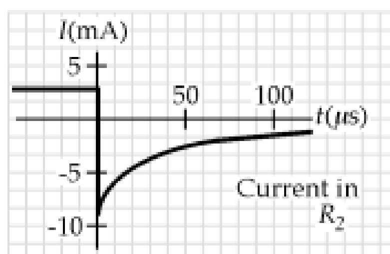
32.71 (a) While steady-state conditions exist, a 9.00 mA flows clockwise around the right loop of the circuit. Immediately after the switch is opened, a 9.00 mA current will flow around the outer loop of the circuit. Applying Kirchhoff's loop rule to this loop gives:

$$+\mathcal{E}_0 - \left[(2.00 + 6.00) \times 10^3\ \Omega\right]\left(9.00 \times 10^{-3}\ \text{A}\right) = 0$$

$$+\mathcal{E}_0 = \boxed{72.0\ \text{V with end } b \text{ at the higher potential}}$$

(b)





(c) After the switch is opened, the current around the outer loop decays as

$I = I_{max} e^{-Rt/L} \quad$ with $\quad I_{max} = 9.00$ mA, $\quad R = 8.00$ kΩ, $\quad$ and $\quad L = 0.400$ H

Thus, when the current has reached a value $I = 2.00$ mA, the elapsed time is:

$$t = \left(\frac{L}{R}\right)\ln\left(\frac{I_{max}}{I}\right) = \left(\frac{0.400\ \text{H}}{8.00 \times 10^3\ \Omega}\right)\ln\left(\frac{9.00}{2.00}\right) = 7.52 \times 10^{-5}\ \text{s} = \boxed{75.2\ \mu\text{s}}$$

32.72 (a) The instant after the switch is closed, the situation is as shown in the circuit diagram of Figure (a). The requested quantities are:

$\boxed{I_L = 0, \quad I_C = \mathcal{E}_0/R, \quad I_R = \mathcal{E}_0/R}$

$\boxed{\Delta V_L = \mathcal{E}_0, \quad \Delta V_C = 0, \quad \Delta V_R = \mathcal{E}_0}$

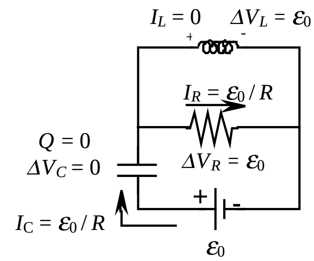


Figure (a)

(b) After the switch has been closed a long time, the steady-state conditions shown in Figure (b) will exist. The currents and voltages are:

$\boxed{I_L = 0, \quad I_C = 0, \quad I_R = 0}$

$\boxed{\Delta V_L = 0, \quad \Delta V_C = \mathcal{E}_0, \quad \Delta V_R = 0}$

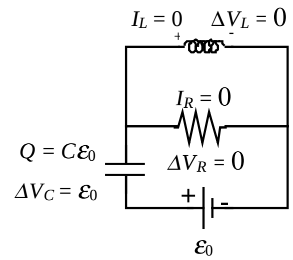


Figure (b)

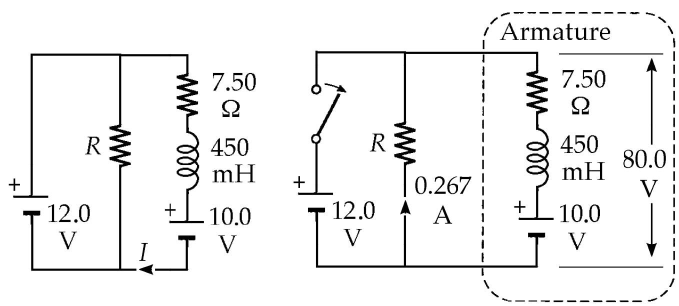
32.73 When the switch is closed, as shown in Figure (a), the current in the inductor is I:

$12.0 - 7.50I - 10.0 = 0 \rightarrow I = 0.267$ A

When the switch is opened, the initial current in the inductor remains at 0.267 A.

$IR = \Delta V$: $(0.267 \text{ A})R \le 80.0$ V

$\boxed{R \le 300\ \Omega}$



(a) **(b)**

Goal Solution
To prevent damage from arcing in an electric motor, a discharge resistor is sometimes placed in parallel with the armature. If the motor is suddenly unplugged while running, this resistor limits the voltage that appears across the armature coils. Consider a 12.0-V dc motor with an armature that has a resistance of 7.50 Ω and an inductance of 450 mH. Assume that the back emf in the armature coils is 10.0 V when the motor is running at normal speed. (The equivalent circuit for the armature is shown in Figure P32.73.) Calculate the maximum resistance R that limits the voltage across the armature to 80.0 V when the motor is unplugged.

G: We should expect R to be significantly greater than the resistance of the armature coil, for otherwise a large portion of the source current would be diverted through R and much of the total power would be wasted on heating this discharge resistor.

O: When the motor is unplugged, the 10-V back emf will still exist for a short while because the motor's inertia will tend to keep it spinning. Now the circuit is reduced to a simple series loop with an emf, inductor, and two resistors. The current that was flowing through the armature coil must now flow through the discharge resistor, which will create a voltage across R that we wish to limit to 80 V. As time passes, the current will be reduced by the opposing back emf, and as the motor slows down, the back emf will be reduced to zero, and the current will stop.

A: The steady-state coil current when the switch is closed is found from applying Kirchhoff's loop rule to the outer loop:

$$+12.0\text{ V} - I(7.50\ \Omega) - 10.0\text{ V} = 0$$

so $$I = \frac{2.00\text{ V}}{7.50\ \Omega} = 0.267\text{ A}$$

We then require that $$\Delta V_R = 80.0\text{ V} = (0.267\text{ A})R$$

so $$R = \frac{\Delta V_R}{I} = \frac{80.0\text{ V}}{0.267\text{ A}} = 300\ \Omega$$

L: As we expected, this discharge resistance is considerably greater than the coil's resistance. Note that while the motor is running, the discharge resistor turns $\mathcal{P} = (12\text{ V})^2/300\ \Omega = 0.48\text{ W}$ of power into heat (or wastes 0.48 W). The source delivers power at the rate of about $\mathcal{P} = IV = [0.267\text{ A} + (12\text{ V}/300\ \Omega)](12\text{ V}) = 3.68\text{ W}$, so the discharge resistor wastes about 13% of the total power. For a sense of perspective, this 4-W motor could lift a 40-N weight at a rate of 0.1 m/s.

32.74 (a) $$L_1 = \frac{\mu_0 N_1^2 A}{\ell_1} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1000)^2(1.00\times10^{-4}\text{ m}^2)}{0.500\text{ m}} = 2.51\times10^{-4}\text{ H} = \boxed{251\ \mu\text{H}}$$

(b) $$M = \frac{N_2\Phi_2}{I_1} = \frac{N_2\Phi_1}{I_1} = \frac{N_2 BA}{I_1} = \frac{N_2[\mu_0(N_1/\ell_1)I_1]A}{I_1} = \frac{\mu_0 N_1 N_2 A}{\ell_1}$$

$$M = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1000)(100)(1.00\times10^{-4}\text{ m}^2)}{0.500\text{ m}} = 2.51\times10^{-5}\text{ H} = \boxed{25.1\ \mu\text{H}}$$

(c) $$\mathcal{E}_1 = -M\frac{dI_2}{dt},\quad\text{or}\quad I_1R_1 = -M\frac{dI_2}{dt}\quad\text{and}\quad I_1 = \frac{dQ_1}{dt} = -\frac{M}{R_1}\frac{dI_2}{dt}$$

$$Q_1 = -\frac{M}{R_1}\int_0^{t_f} dI_2 = -\frac{M}{R_1}(I_{2f} - I_{2i}) = -\frac{M}{R_1}(0 - I_{2i}) = \frac{MI_{2i}}{R_1}$$

$$Q_1 = \frac{(2.51\times10^{-5}\text{ H})(1.00\text{ A})}{1000\ \Omega} = 2.51\times10^{-8}\text{ C} = \boxed{25.1\text{ nC}}$$

32.75 (a) It has a magnetic field, and it stores energy, so $L = \dfrac{2U}{I^2}$ is non-zero.

(b) Every field line goes through the rectangle between the conductors.

(c) $\Phi = LI$ so $L = \dfrac{\Phi}{I} = \dfrac{1}{I}\int_{y=a}^{w-a} B\,da$

$$L = \frac{1}{I}\int_a^{w-a} x\,dy\left(\frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi(w-y)}\right) = \frac{2}{I}\int \frac{\mu_0 I x}{2\pi y}dy = \frac{2\mu_0 x}{2\pi}\ln y\Big|_a^{w-a}$$

Thus $L = \dfrac{\mu_0 x}{\pi}\ln\left(\dfrac{w-a}{a}\right)$

32.76 For an *RL* circuit, $I(t) = I_{\text{max}}e^{-\frac{R}{L}t}$: $\dfrac{I(t)}{I_{\text{max}}} = 1 - 10^{-9} = e^{-\frac{R}{L}t} \cong 1 - \dfrac{R}{L}t$

$\dfrac{R}{L}t = 10^{-9}$ so $R_{\text{max}} = \dfrac{(3.14\times10^{-8})(10^{-9})}{(2.50\text{ yr})(3.16\times10^{7}\text{ s / yr})} = \boxed{3.97\times10^{-25}\ \Omega}$

(If the ring were of purest copper, of diameter 1 cm, and cross-sectional area 1 mm^2, its resistance would be at least $10^{-6}\ \Omega$).

32.77 (a) $U_B = \frac{1}{2}LI^2 = \frac{1}{2}(50.0\text{ H})(50.0\times10^3\text{ A})^2 = \boxed{6.25\times10^{10}\text{ J}}$

(b) Two adjacent turns are parallel wires carrying current in the same direction. Since the loops have such large radius, a one-meter section can be regarded as straight.

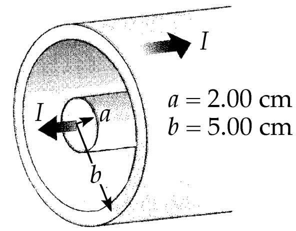
Then one wire creates a field of $B = \dfrac{\mu_0 I}{2\pi r}$

This causes a force on the next wire of $F = I\ell B\sin\theta$

giving $F = I\ell\dfrac{\mu_0 I}{2\pi r}\sin 90° = \dfrac{\mu_0 \ell I^2}{2\pi r}$

Solving for the force, $F = (4\pi\times10^{-7}\text{ N / A}^2)\dfrac{(1.00\text{ m})(50.0\times10^3\text{ A})^2}{(2\pi)(0.250\text{ m})} = \boxed{2000\text{ N}}$

32.78 $\mathcal{P} = I(\Delta V)$



$$I = \frac{\mathcal{P}}{\Delta V} = \frac{1.00\times 10^9 \text{ W}}{200\times 10^3 \text{ V}} = 5.00\times 10^3 \text{ A}$$

From Ampere's law, $B(2\pi r) = \mu_0 I_{\text{enclosed}}$ or $B = \dfrac{\mu_0 I_{\text{enclosed}}}{2\pi r}$

(a) At $r = a = 0.0200$ m, $I_{\text{enclosed}} = 5.00\times 10^3$ A and

$$B = \frac{(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A})(5.00\times 10^3 \text{ A})}{2\pi(0.0200 \text{ m})} = 0.0500 \text{ T} = \boxed{50.0 \text{ mT}}$$

(b) At $r = b = 0.0500$ m, $I_{\text{enclosed}} = I = 5.00\times 10^3$ A and

$$B = \frac{(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A})(5.00\times 10^3 \text{ A})}{2\pi(0.0500 \text{ m})} = 0.0200 \text{ T} = \boxed{20.0 \text{ mT}}$$

(c) $$U = \int u\,dV = \int_{r=a}^{r=b} \frac{[B(r)]^2(2\pi r \ell\, dr)}{2\mu_0} = \frac{\mu_0 I^2 \ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I^2 \ell}{4\pi}\ln\left(\frac{b}{a}\right)$$

$$U = \frac{(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A})(5.00\times 10^3 \text{ A})^2(1000\times 10^3 \text{ m})}{4\pi}\ln\left(\frac{5.00 \text{ cm}}{2.00 \text{ cm}}\right) = 2.29\times 10^6 \text{ J} = \boxed{2.29 \text{ MJ}}$$

(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length ℓ and width w. It carries a current of

$$(5.00\times 10^3 \text{ A})\left(\frac{w}{2\pi(0.0500 \text{ m})}\right)$$

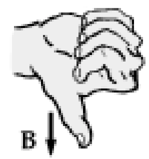
and experiences an outward force $F = I\ell B\sin\theta = \dfrac{(5.00\times 10^3 \text{ A})w}{2\pi(0.0500 \text{ m})}\ell(20.0\times 10^{-3} \text{ T})\sin 90.0°$

The pressure on it is $P = \dfrac{F}{A} = \dfrac{F}{w\ell} = \dfrac{(5.00\times 10^3 \text{ A})(20.0\times 10^{-3} \text{ T})}{2\pi(0.0500 \text{ m})} = \boxed{318 \text{ Pa}}$

***32.79** (a) $B = \dfrac{\mu_0 NI}{\ell} = \dfrac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1400)(2.00\ \text{A})}{1.20\ \text{m}} = \boxed{2.93 \times 10^{-3}\ \text{T (upward)}}$

(b) $u = \dfrac{B^2}{2\mu_0} = \dfrac{\left(2.93 \times 10^{-3}\ \text{T}\right)^2}{2\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)} = 3.42\ \dfrac{\text{J}}{\text{m}^3}\left(\dfrac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right) = 3.42\ \dfrac{\text{N}}{\text{m}^2} = \boxed{3.42\ \text{Pa}}$

(c) To produce a downward magnetic field, the surface of the super conductor must carry a $\boxed{\text{clockwise}}$ current.



(d) The vertical component of the field of the solenoid exerts an inward force on the superconductor. The total horizontal force is zero. Over the top end of the solenoid, its field diverges and has a radially outward horizontal component. This component exerts upward force on the clockwise superconductor current. The total force on the core is $\boxed{\text{upward}}$. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $F = PA = (3.42\ \text{Pa})\left[\pi\left(1.10 \times 10^{-2}\ \text{m}\right)^2\right] = \boxed{1.30 \times 10^{-3}\ \text{N}}$

Note that we have not proven that energy density is pressure. In fact, it is not in some cases; see problem 12 in Chapter 21.

Chapter 33 Solutions

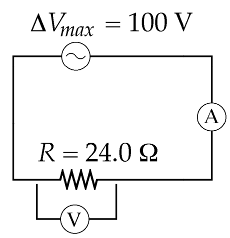
33.1 $\Delta v(t) = \Delta V_{\max} \sin(\omega t) = \sqrt{2}\ \Delta V_{\text{rms}} \sin(\omega t) = 200\sqrt{2}\ \sin[2\pi(100t)] = \boxed{(283\text{ V})\sin(628t)}$

33.2 $$\Delta V_{\text{rms}} = \frac{170\text{ V}}{\sqrt{2}} = 120\text{ V}$$

(a) $$\mathcal{P} = \frac{(\Delta V_{\text{rms}})^2}{R} \rightarrow R = \frac{(120\text{ V})^2}{75.0\text{ W}} = \boxed{193\ \Omega}$$

(b) $$R = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$$

33.3 Each meter reads the rms value.



$$\Delta V_{\text{rms}} = \frac{100\text{ V}}{\sqrt{2}} = \boxed{70.7\text{ V}}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{70.7\text{ V}}{24.0\ \Omega} = \boxed{2.95\text{ A}}$$

33.4 (a) $\Delta v_R = \Delta V_{\max} \sin \omega t$

$\Delta v_R = 0.250(\Delta V_{\max})$, so $\sin \omega t = 0.250$, or $\omega t = \sin^{-1}(0.250)$

The smallest angle for which this is true is $\omega t = 0.253$ rad. Thus, if $t = 0.0100$ s,

$$\omega = \frac{0.253\text{ rad}}{0.0100\text{ s}} = \boxed{25.3\text{ rad/s}}$$

(b) The second time when $\Delta v_R = 0.250(\Delta V_{\max})$, $\omega t = \sin^{-1}(0.250)$ again. For this occurrence, $\omega t = \pi - 0.253\text{ rad} = 2.89\text{ rad}$ (to understand why this is true, recall the identity $\sin(\pi - \theta) = \sin\theta$ from trigonometry). Thus,

$$t = \frac{2.89\text{ rad}}{25.3\text{ rad/s}} = \boxed{0.114\text{ s}}$$

33.5 $i_R = I_{\max} \sin \omega t$ becomes $0.600 = \sin(\omega\ 0.00700)$

Thus, $(0.00700)\omega = \sin^{-1}(0.600) = 0.644$

and $\omega = 91.9\text{ rad/s} = 2\pi f$ so $\boxed{f = 14.6\text{ Hz}}$

33.6 $\mathcal{P} = I_{\text{rms}}(\Delta V_{\text{rms}})$ and $\Delta V_{\text{rms}} = 120\text{ V}$ for each bulb (parallel circuit), so:

$$I_1 = I_2 = \frac{\mathcal{P}_1}{\Delta V_{\text{rms}}} = \frac{150\text{ W}}{120\text{ V}} = \boxed{1.25\text{ A}}, \quad \text{and} \quad R_1 = \frac{\Delta V_{\text{rms}}}{I_1} = \frac{120\text{ V}}{1.25\text{ A}} = \boxed{96.0\ \Omega} = R_2$$

$$I_3 = \frac{\mathcal{P}_3}{\Delta V_{\text{rms}}} = \frac{100\text{ W}}{120\text{ V}} = \boxed{0.833\text{ A}}, \quad \text{and} \quad R_3 = \frac{\Delta V_{\text{rms}}}{I_3} = \frac{120\text{ V}}{0.833\text{ A}} = \boxed{144\ \Omega}$$

33.7 $\Delta V_{\max} = 15.0\text{ V}$ and $R_{\text{total}} = 8.20\ \Omega + 10.4\ \Omega = 18.6\ \Omega$

$$I_{\max} = \frac{\Delta V_{\max}}{R_{\text{total}}} = \frac{15.0\text{ V}}{18.6\ \Omega} = 0.806\text{ A} = \sqrt{2}\ I_{\text{rms}}$$

$$\mathcal{P}_{\text{speaker}} = I_{\text{rms}}^2 R_{\text{speaker}} = \left(\frac{0.806\text{ A}}{\sqrt{2}}\right)^2 (10.4\ \Omega) = \boxed{3.38\text{ W}}$$

33.8 For $I_{\max} = 80.0\text{ mA}$, $I_{\text{rms}} = \dfrac{80.0\text{ mA}}{\sqrt{2}} = 56.6\text{ mA}$

$$(X_L)_{\min} = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{50.0\text{ V}}{0.0566\text{ A}} = 884\ \Omega$$

$$X_L = 2\pi f L \rightarrow L = \frac{X_L}{2\pi f} \geq \frac{884\ \Omega}{2\pi(20.0)} \geq \boxed{7.03\text{ H}}$$

33.9 (a) $X_L = \dfrac{\Delta V_{\max}}{I_{\max}} = \dfrac{100}{7.50} = 13.3\ \Omega$

$$L = \frac{X_L}{\omega} = \frac{13.3}{2\pi(50.0)} = 0.0424\text{ H} = \boxed{42.4\text{ mH}}$$

(b) $X_L = \dfrac{\Delta V_{\max}}{I_{\max}} = \dfrac{100}{2.50} = 40.0\ \Omega$

$$\omega = \frac{X_L}{L} = \frac{40.0}{42.4 \times 10^{-3}} = \boxed{942\text{ rad/s}}$$

33.10 At 50.0 Hz, $X_L = 2\pi(50.0\text{ Hz})L = 2\pi(50.0\text{ Hz})\left(\dfrac{X_L\big|_{60.0\text{ Hz}}}{2\pi(60.0\text{ Hz})}\right) = \dfrac{50.0}{60.0}(54.0\ \Omega) = 45.0\ \Omega$

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} = \frac{\sqrt{2}(\Delta V_{\text{rms}})}{X_L} = \frac{\sqrt{2}(100\text{ V})}{45.0\ \Omega} = \boxed{3.14\text{ A}}$$

33.11 $$i_L(t) = \frac{\Delta V_{\max}}{\omega L}\sin(\omega t - \pi/2) = \frac{(80.0\text{ V})\sin\left[(65.0\pi)(0.0155) - \pi/2\right]}{(65.0\pi\text{ rad/s})(70.0\times10^{-3}\text{ H})}$$

$$i_L(t) = (5.60\text{ A})\sin(1.59\text{ rad}) = \boxed{5.60\text{ A}}$$

33.12 $$\omega = 2\pi f = 2\pi(60.0\text{ / s}) = 377\text{ rad / s}$$

$$X_L = \omega L = (377\text{ / s})(0.0200\text{ V}\cdot\text{s / A}) = 7.54\ \Omega$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{120\text{ V}}{7.54\ \Omega} = 15.9\text{ A}$$

$$I_{\max} = \sqrt{2}\,I_{\text{rms}} = \sqrt{2}\,(15.9\text{ A}) = 22.5\text{ A}$$

$$i(t) = I_{\max}\sin\omega t = (22.5\text{ A})\sin\left(\frac{2\pi(60.0)}{\text{s}}\cdot\frac{1\text{ s}}{180}\right) = (22.5\text{ A})\sin 120° = 19.5\text{ A}$$

$$U = \tfrac{1}{2}Li^2 = \tfrac{1}{2}\left(0.0200\ \frac{\text{V}\cdot\text{s}}{\text{A}}\right)(19.5\text{ A})^2 = \boxed{3.80\text{ J}}$$

33.13 $$L = \frac{N\Phi_B}{I}$$ where Φ_B is the flux through each turn. $$N\Phi_{B,\max} = LI_{B,\ \max} = \frac{X_L}{\omega}\frac{(\Delta V_{L,\ \max})}{X_L}$$

$$N\Phi_{B,\max} = \frac{\sqrt{2}(\Delta V_{L,\text{rms}})}{2\pi f} = \frac{120\text{ V}\cdot\text{s}}{\sqrt{2}\,\pi(60.0)}\left(\frac{\text{T}\cdot\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right) = \boxed{0.450\text{ T}\cdot\text{m}^2}$$

33.14 (a) $$X_C = \frac{1}{2\pi fC}: \qquad \frac{1}{2\pi f(22.0\times10^{-6})} < 175\ \Omega$$

$$\frac{1}{2\pi(22.0\times10^{-6})(175)} < f \qquad \boxed{f > 41.3\text{ Hz}}$$

(b) $$X_C \propto \frac{1}{C},\quad \text{so}\quad X(44) = \tfrac{1}{2}X(22): \quad \boxed{X_C < 87.5\ \Omega}$$

33.15 $$I_{\max} = \sqrt{2}\,I_{\text{rms}} = \frac{\sqrt{2}(\Delta V_{\text{rms}})}{X_C} = \sqrt{2}(\Delta V_{\text{rms}})2\pi fC$$

(a) $$I_{\max} = \sqrt{2}\,(120\text{ V})2\pi(60.0\text{ / s})(2.20\times10^{-6}\text{ C / V}) = \boxed{141\text{ mA}}$$

(b) $$I_{\max} = \sqrt{2}\,(240\text{ V})2\pi(50.0\text{ / s})(2.20\times10^{-6}\text{ F}) = \boxed{235\text{ mA}}$$

33.16 $Q_{\max} = C(\Delta V_{\max}) = C\left[\sqrt{2}(\Delta V_{\text{rms}})\right] = \boxed{\sqrt{2}\,C(\Delta V_{\text{rms}})}$

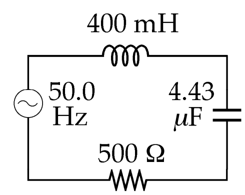
33.17 $I_{\max} = (\Delta V_{\max})\omega C = (48.0\text{ V})(2\pi)(90.0\text{ s}^{-1})(3.70\times10^{-6}\text{ F}) = \boxed{100\text{ mA}}$

33.18 $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi(60.0\text{ / s})(1.00\times10^{-3}\text{ C / V})} = 2.65\ \Omega$

$v_C(t) = \Delta V_{\max}\sin\omega t$, to be zero at $t = 0$

$$i_C = \frac{\Delta V_{\max}}{X_C}\sin(\omega t+\phi) = \frac{\sqrt{2}\ (120\text{ V})}{2.65\ \Omega}\sin\left[2\pi\frac{60\text{ s}^{-1}}{180\text{ s}^{-1}}+90.0°\right] = (64.0\text{ A})\sin(120°+90.0°) = \boxed{-32.0\text{ A}}$$

33.19 (a) $X_L = \omega L = 2\pi(50.0)(400\times10^{-3}) = 126\ \Omega$



$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi(50.0)(4.43\times10^{-6})} = 719\ \Omega$

$Z = \sqrt{R^2+(X_L-X_C)^2} = \sqrt{500^2+(126-719)^2} = 776\ \Omega$

$\Delta V_{\max} = I_{\max}Z = (250\times10^{-3})(776) = \boxed{194\text{ V}}$

(b) $\phi = \tan^{-1}\left(\dfrac{X_L-X_C}{R}\right) = \tan^{-1}\left(\dfrac{126-719}{500}\right) = \boxed{-49.9°}$ Thus, the $\boxed{\text{Current leads the voltage.}}$

33.20 $\omega L = \dfrac{1}{\omega C} \to \omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(57.0\times10^{-6})(57.0\times10^{-6})}} = 1.75\times10^4\text{ rad / s}$

$f = \dfrac{\omega}{2\pi} = \boxed{2.79\text{ kHz}}$

33.21 (a) $X_L = \omega L = 2\pi(50.0\text{ s}^{-1})(250\times10^{-3}\text{ H}) = \boxed{78.5\ \Omega}$

(b) $X_C = \dfrac{1}{\omega C} = \left[2\pi(50.0\text{ s}^{-1})(2.00\times10^{-6}\text{ F})\right]^{-1} = \boxed{1.59\text{ k}\Omega}$

(c) $Z = \sqrt{R^2+(X_L-X_C)^2} = \boxed{1.52\text{ k}\Omega}$

(d) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{210\text{ V}}{1.52\times10^3\ \Omega} = \boxed{138\text{ mA}}$

(e) $\phi = \tan^{-1}\left[\dfrac{X_L-X_C}{R}\right] = \tan^{-1}(-10.1) = \boxed{-84.3°}$

33.22 (a) $Z = \sqrt{R^2 + \left(X_L - X_C\right)} = \sqrt{68.0^2 + (16.0 - 101)^2} = \boxed{109\ \Omega}$

$X_L = \omega L = (100)(0.160) = 16.0\ \Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(100)\left(99.0 \times 10^{-6}\right)} = 101\ \Omega$

(b) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{40.0\ \text{V}}{109\ \Omega} = \boxed{0.367\ \text{A}}$

(c) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{16.0 - 101}{68.0} = -1.25$:

$\phi = -0.896\ \text{rad} = -51.3°$

$\boxed{I_{\max} = 0.367\ \text{A}}$ $\boxed{\omega = 100\ \text{rad/s}}$ $\boxed{\phi = -0.896\ \text{rad} = -51.3°}$

33.23 $X_L = 2\pi f L = 2\pi(60.0)(0.460) = 173\ \Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(60.0)\left(21.0 \times 10^{-6}\right)} = 126\ \Omega$

(a) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{173\ \Omega - 126\ \Omega}{150\ \Omega} = 0.314$

$\phi = 0.304\ \text{rad} = \boxed{17.4°}$

(b) Since $X_L > X_C$, ϕ is positive; so $\boxed{\text{voltage leads the current}}$.

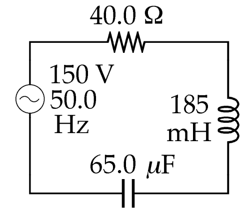
33.24 $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(60.0\ \text{Hz})(20.0 \times 10^{-12}\ \text{F})} = 1.33 \times 10^8\ \Omega$

$Z = \sqrt{(50.0 \times 10^3\ \Omega)^2 + (1.33 \times 10^8\ \Omega)^2} \approx 1.33 \times 10^8\ \Omega$

$I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{5000\ \text{V}}{1.33 \times 10^8\ \Omega} = 3.77 \times 10^{-5}\ \text{A}$

$\left(\Delta V_{\text{rms}}\right)_{\text{body}} = I_{\text{rms}} R_{\text{body}} = (3.77 \times 10^{-5}\ \text{A})(50.0 \times 10^3\ \Omega) = \boxed{1.88\ \text{V}}$

33.25 $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi(50.0)(65.0\times10^{-6})} = 49.0\ \Omega$



$X_L = \omega L = 2\pi(50.0)(185\times10^{-3}) = 58.1\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(40.0)^2 + (58.1 - 49.0)^2} = 41.0\ \Omega$

$I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{150}{41.0} = 3.66\ \text{A}$

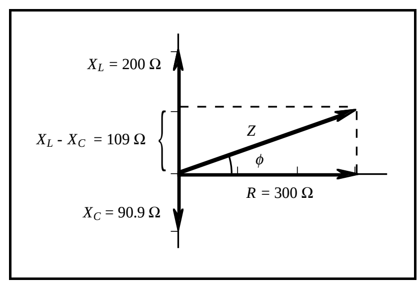
(a) $\Delta V_R = I_{\max} R = (3.66)(40) = \boxed{146\ \text{V}}$

(b) $\Delta V_L = I_{\max} X_L = (3.66)(58.1) = 212.5 = \boxed{212\ \text{V}}$

(c) $\Delta V_C = I_{\max} X_C = (3.66)(49.0) = 179.1\ \text{V} = \boxed{179\ \text{V}}$

(d) $\Delta V_L - \Delta V_C = 212.5 - 179.1 = \boxed{33.4\ \text{V}}$

33.26 $R = 300\ \Omega$

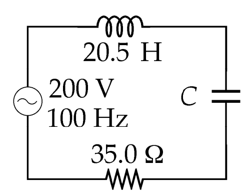


$X_L = \omega L = 2\pi\left(\dfrac{500}{\pi}\ \text{s}^{-1}\right)(0.200\ \text{H}) = 200\ \Omega$

$X_C = \dfrac{1}{\omega C} = \left[2\pi\left(\dfrac{500}{\pi}\ \text{s}^{-1}\right)(11.0\times10^{-6}\ \text{F})\right]^{-1} = 90.9\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 319\ \Omega$ and $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = 20.0°$

33.27 (a) $X_L = 2\pi(100\ \text{Hz})(20.5\ \text{H}) = 1.29\times10^4\ \Omega$



$Z = \dfrac{\Delta V_{\text{rms}}}{I_{\text{rms}}} = \dfrac{200\ \text{V}}{4.00\ \text{A}} = 50.0\ \Omega$

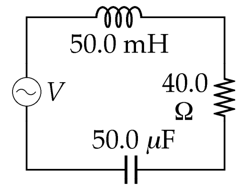
$(X_L - X_C)^2 = Z^2 - R^2 = (50.0\ \Omega)^2 - (35.0\ \Omega)^2$

$X_L - X_C = 1.29\times10^4\ \Omega - \dfrac{1}{2\pi(100\ \text{Hz})C} = \pm 35.7\ \Omega$ $\boxed{C = 123\ \text{nF or } 124\ \text{nF}}$

(b) $\Delta V_{L,\text{rms}} = I_{\text{rms}} X_L = (4.00\ \text{A})(1.29\times10^4\ \Omega) = \boxed{51.5\ \text{kV}}$

Notice that this is a very large voltage!

33.28 $X_L = \omega L = [(1000\ /\ \text{s})(0.0500\ \text{H})] = 50.0\ \Omega$



$X_C = 1/\omega C = \left[(1000\ /\ \text{s})(50.0\times10^{-6}\ \text{F})\right]^{-1} = 20.0\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \sqrt{(40.0)^2 + (50.0 - 20.0)^2} = 50.0\ \Omega$

(a) $I_{\text{rms}} = (\Delta V_{\text{rms}})\ /\ Z = 100\ \text{V}\ /\ 50.0\ \Omega$

$I_{\text{rms}} = \boxed{2.00\ \text{A}}$

$\phi = \text{Arctan}\left(\dfrac{X_L - X_C}{R}\right)$

$\phi = \text{Arctan}\ \dfrac{30.0\ \Omega}{40.0\ \Omega} = 36.9°$

(b) $\mathsf{P} = (\Delta V_{\text{rms}}) I_{\text{rms}} \cos\phi = 100\ \text{V}(2.00\ \text{A}) \cos 36.9° = \boxed{160\ \text{W}}$

(c) $\mathsf{P}_R = I_{\text{rms}}^2 R = (2.00\ \text{A})^2\ 40.0\ \Omega = \boxed{160\ \text{W}}$

33.29 $\omega = 1000\ \text{rad/s},\quad R = 400\ \Omega,\quad C = 5.00\times10^{-6}\ \text{F},\quad L = 0.500\ \text{H}$

$\Delta V_{\max} = 100\ \text{V},\quad \omega L = 500\ \Omega,\quad \left(\dfrac{1}{\omega C}\right) = 200\ \Omega$

$Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2} = \sqrt{400^2 + 300^2} = 500\ \Omega$

$I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{100}{500} = 0.200\ \text{A}$

The average power dissipated in the circuit is $\quad \mathsf{P} = I_{\text{rms}}^2 R = \left(\dfrac{I_{\max}^2}{2}\right) R$

$\mathsf{P} = \dfrac{(0.200\ \text{A})^2}{2}(400\ \Omega) = \boxed{8.00\ \text{W}}$

Goal Solution

An ac voltage of the form $\Delta v = (100\text{ V})\sin(1000\,t)$ is applied to a series *RLC* circuit. If R = 400 Ω, C = 5.00 μF, and L = 0.500 H, what is the average power delivered to the circuit?

G: Comparing $\Delta v = (100\text{ V})\sin(1000\,t)$ with $\Delta v = \Delta V_{\max}\sin\omega t$, we see that

$$\Delta V_{\max} = 100\text{ V} \quad \text{and} \quad \omega = 1000\text{ s}^{-1}$$

Only the resistor takes electric energy out of the circuit, but the capacitor and inductor will impede the current flow and therefore reduce the voltage across the resistor. Because of this impedance, the average power dissipated by the resistor must be less than the maximum power from the source:

$$\mathcal{P}_{\max} = \frac{(\Delta V_{\max})^2}{2R} = \frac{(100\text{ V})^2}{2(400\ \Omega)} = 12.5\text{ W}$$

O: The actual power dissipated by the resistor can be found from $\mathcal{P} = I_{rms}^2 R$, where $I_{rms} = \Delta V_{rms}/Z$.

A: $\Delta V_{rms} = \dfrac{100}{\sqrt{2}} = 70.7\text{ V}$

In order to calculate the impedance, we first need the capacitive and inductive reactances:

$$X_C = \frac{1}{\omega C} = \frac{1}{(1000\text{ s}^{-1})(5.00\times 10^{-6}\text{ F})} = 200\ \Omega \quad \text{and} \quad X_L = \omega L = \left(1000\text{ s}^{-1}\right)(0.500\text{ H}) = 500\ \Omega$$

Then, $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(400\ \Omega)^2 + (500\ \Omega - 200\ \Omega)^2} = 500\ \Omega$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{70.7\text{ V}}{500\ \Omega} = 0.141\text{ A} \quad \text{and} \quad \mathcal{P} = I_{\text{rms}}^2 R = (0.141\text{ A})^2(400\ \Omega) = 8.00\text{ W}$$

L: The power dissipated by the resistor is less than 12.5 W, so our answer appears to be reasonable. As with other *RLC* circuits, the power will be maximized at the resonance frequency where $X_L = X_C$ so that $Z = R$. Then the average power dissipated will simply be the 12.5 W we calculated first.

33.30 $$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{or} \quad (X_L - X_C) = \sqrt{Z^2 - R^2}$$

$$(X_L - X_C) = \sqrt{(75.0\ \Omega)^2 - (45.0\ \Omega)^2} = 60.0\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{60.0\ \Omega}{45.0\ \Omega}\right) = 53.1°$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{210\text{ V}}{75.0\ \Omega} = 2.80\text{ A}$$

$$\mathcal{P} = (\Delta V_{\text{rms}})I_{\text{rms}}\cos\phi = (210\text{ V})(2.80\text{ A})\cos(53.1°) = \boxed{353\text{ W}}$$

33.31 (a) $\mathcal{P} = I_{\text{rms}}(\Delta V_{\text{rms}})\cos\phi = (9.00)(180)\cos(-37.0°) = 1.29\times10^3\text{ W}$

$\mathcal{P} = I_{\text{rms}}^2 R$ so $1.29\times10^3 = (9.00)^2 R$ and $R = \boxed{16.0\ \Omega}$

(b) $\tan\phi = \dfrac{X_L - X_C}{R}$ becomes $\tan(-37.0°) = \dfrac{X_L - X_C}{16}$: so $X_L - X_C = \boxed{-12.0\ \Omega}$

***33.32** $X_L = \omega L = 2\pi(60.0\ /\ \text{s})(0.0250\text{ H}) = 9.42\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20.0)^2 + (9.42)^2}\ \Omega = 22.1\ \Omega$

(a) $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{120\text{ V}}{22.1\ \Omega} = \boxed{5.43\text{ A}}$

(b) $\phi = \tan^{-1}(9.42\ /\ 20.0) = 25.2°$ so power factor $= \cos\phi = \boxed{0.905}$

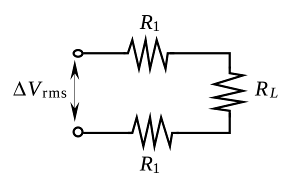
(c) We require $\phi = 0$. Thus, $X_L = X_C$: $9.42\ \Omega = \dfrac{1}{2\pi(60.0\text{ s}^{-1})C}$

and $C = \boxed{281\ \mu\text{F}}$

(d) $\mathcal{P}_b = \mathcal{P}_d$ or $(\Delta V_{\text{rms}})_b(I_{\text{rms}})_b\cos\phi_b = \dfrac{(\Delta V_{\text{rms}})_d^2}{R}$

$(\Delta V_{\text{rms}})_d = \sqrt{R(\Delta V_{\text{rms}})_b(I_{\text{rms}})_b\cos\phi_b} = \sqrt{(20.0\ \Omega)(120\text{ V})(5.43\text{ A})(0.905)} = \boxed{109\text{ V}}$

33.33 Consider a two-wire transmission line:



$I_{\text{rms}} = \dfrac{\mathcal{P}}{\Delta V_{\text{rms}}} = \dfrac{100\times10^6\text{ W}}{50.0\times10^3\text{ V}} = 2.00\times10^3\text{ A}$

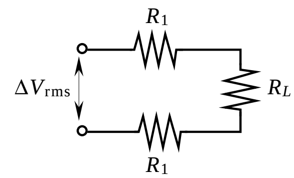
loss $= (0.0100)\mathcal{P} = I_{\text{rms}}^2 R_{\text{line}} = I_{\text{rms}}^2(2R_1)$

Thus, $R_1 = \dfrac{(0.0100)\mathcal{P}}{2I_{\text{max}}^2} = \dfrac{(0.0100)(100\times10^6\text{ W})}{2(2.00\times10^3\text{ A})^2} = 0.125\ \Omega$

But $R_1 = \dfrac{\rho \ell}{A}$ or $A = \dfrac{\pi d^2}{4} = \dfrac{\rho\ell}{R_1}$

Therefore $d = \sqrt{\dfrac{4\rho\ell}{\pi R_1}} = \sqrt{\dfrac{4(1.70\times10^{-8}\ \Omega\cdot\text{m})(100\times10^3\text{ m})}{\pi(0.125\ \Omega)}} = 0.132\text{ m} = \boxed{132\text{ mm}}$

33.34 Consider a two-wire transmission line:



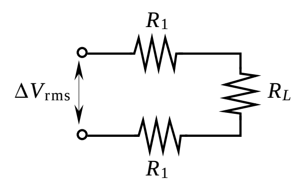
$$I_{\mathrm{rms}} = \frac{\mathcal{P}}{\Delta V_{\mathrm{rms}}} \quad \text{and} \quad \text{power loss} = I_{\mathrm{rms}}^2 R_{\mathrm{line}} = \frac{\mathcal{P}}{100}$$

$$\text{Thus, } \left(\frac{\mathcal{P}}{\Delta V_{\mathrm{rms}}}\right)^2 (2R_1) = \frac{\mathcal{P}}{100} \quad \text{or} \quad R_1 = \frac{(\Delta V_{\mathrm{rms}})^2}{200\mathcal{P}}$$

$$R_1 = \frac{\rho d}{A} = \frac{(\Delta V_{\mathrm{rms}})^2}{200\mathcal{P}} \quad \text{or} \quad A = \frac{\pi (2r)^2}{4} = \frac{200 \rho \mathcal{P} d}{(\Delta V_{\mathrm{rms}})^2}$$

and the diameter is
$$2r = \boxed{\sqrt{\frac{800 \rho \mathcal{P} d}{\pi (\Delta V_{\mathrm{rms}})^2}}}$$

33.35 One-half the time, the left side of the generator is positive, the top diode conducts, and the bottom diode switches off. The power supply sees resistance



$$\left[\frac{1}{2R} + \frac{1}{2R}\right]^{-1} = R \quad \text{and the power is} \quad \frac{(\Delta V_{\mathrm{rms}})^2}{R}$$

The other half of the time the right side of the generator is positive, the upper diode is an open circuit, and the lower diode has zero resistance. The equivalent resistance is then

$$R_{\mathrm{eq}} = R + \left[\frac{1}{3R} + \frac{1}{R}\right]^{-1} = \frac{7R}{4} \quad \text{and} \quad \mathcal{P} = \frac{(\Delta V_{\mathrm{rms}})^2}{R_{\mathrm{eq}}} = \frac{4(\Delta V_{\mathrm{rms}})^2}{7R}$$

The overall time average power is:
$$\frac{\left[(\Delta V_{\mathrm{rms}})^2 / R\right] + \left[4(\Delta V_{\mathrm{rms}})^2 / 7R\right]}{2} = \boxed{\frac{11(\Delta V_{\mathrm{rms}})^2}{14R}}$$

33.36 At resonance,
$$\frac{1}{2\pi f C} = 2\pi f L \quad \text{and} \quad \frac{1}{(2\pi f)^2 L} = C$$

The range of values for C is $\boxed{46.5 \text{ pF to } 419 \text{ pF}}$

33.37
$$\omega_0 = 2\pi(99.7 \times 10^6) = 6.26 \times 10^8 \text{ rad / s} = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(6.26 \times 10^8)^2 (1.40 \times 10^{-6})} = \boxed{1.82 \text{ pF}}$$

33.38 $L = 20.0 \text{ mH},\ C = 1.00\times10^{-7},\ R = 20.0\ \Omega,\ \Delta V_{\max} = 100 \text{ V}$

(a) The resonant frequency for a series $-RLC$ circuit is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}} = \boxed{3.56 \text{ kHz}}$

(b) At resonance, $I_{\max} = \dfrac{\Delta V_{\max}}{R} = \boxed{5.00 \text{ A}}$

(c) From Equation 33.36, $Q = \dfrac{\omega_0 L}{R} = \boxed{22.4}$

(d) $\Delta V_{L,\max} = X_L I_{\max} = \omega_0 L I_{\max} = \boxed{2.24 \text{ kV}}$

33.39 The resonance frequency is $\omega_0 = 1/\sqrt{LC}$. Thus, if $\omega = 2\omega_0$,

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + 2.25(L/C)} \quad \text{so} \quad I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$$

and the energy dissipated in one period is $Q = \mathcal{P}\Delta t$:

$$Q = \frac{\left(\Delta V_{\text{rms}}\right)^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{\left(\Delta V_{\text{rms}}\right)^2 RC}{R^2C + 2.25L}\left(\pi\sqrt{LC}\right) = \frac{4\pi\left(\Delta V_{\text{rms}}\right)^2 RC\sqrt{LC}}{4R^2C + 9.00L}$$

With the values specified for this circuit, this gives:

$$Q = \frac{4\pi(50.0 \text{ V})^2(10.0\ \Omega)\left(100\times10^{-6}\ \text{F}\right)^{3/2}\left(10.0\times10^{-3}\ \text{H}\right)^{1/2}}{4(10.0\ \Omega)^2\left(100\times10^{-6}\ \text{F}\right) + 9.00\left(10.0\times10^{-3}\ \text{H}\right)} = \boxed{242 \text{ mJ}}$$

33.40 The resonance frequency is $\omega_0 = 1/\sqrt{LC}$. Thus, if $\omega = 2\omega_0$,

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

Then $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + 2.25(L/C)}$ so $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$

and the energy dissipated in one period is

$$Q = \mathcal{P}\Delta t = \frac{\left(\Delta V_{\text{rms}}\right)^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{\left(\Delta V_{\text{rms}}\right)^2 RC}{R^2C + 2.25L}\left(\pi\sqrt{LC}\right) = \boxed{\frac{4\pi\left(\Delta V_{\text{rms}}\right)^2 RC\sqrt{LC}}{4R^2C + 9.00L}}$$

***33.41** For the circuit of problem 22, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(160\times10^{-3}\text{ H}\right)\left(99.0\times10^{-6}\text{ F}\right)}} = 251\text{ rad s}$

$$Q = \frac{\omega_0 L}{R} = \frac{\left(251\text{ rad/s}\right)\left(160\times10^{-3}\text{ H}\right)}{68.0\ \Omega} = \boxed{0.591}$$

For the circuit of problem 23, $Q = \frac{\omega_0 L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{150\ \Omega}\sqrt{\frac{460\times10^{-3}\text{ H}}{21.0\times10^{-6}\text{ F}}} = \boxed{0.987}$

The $\boxed{\text{circuit of problem 23}}$ has a sharper resonance.

33.42 (a) $\Delta V_{2,\text{rms}} = \frac{1}{13}\left(120\text{ V}\right) = \boxed{9.23\text{ V}}$

(b) $\Delta V_{1,\text{rms}} I_{1,\text{rms}} = \Delta V_{2,\text{rms}} I_{2,\text{rms}}$

$(120\text{ V})(0.350\text{ A}) = (9.23\text{ V})I_{2,\text{rms}}$

$I_{2,\text{rms}} = \frac{42.0\text{ W}}{9.23\text{ V}} = \boxed{4.55\text{ A}}$ for a transformer with no energy loss

(c) $P = \boxed{42.0\text{ W}}$ from (b)

33.43 $\left(\Delta V_{\text{out}}\right)_{\max} = \frac{N_2}{N_1}\left(\Delta V_{\text{in}}\right)_{\max} = \left(\frac{2000}{350}\right)(170\text{ V}) = 971\text{ V}$

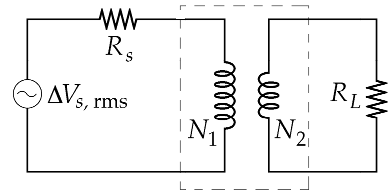
$$\left(\Delta V_{\text{out}}\right)_{\text{rms}} = \frac{(971\text{ V})}{\sqrt{2}} = \boxed{687\text{ V}}$$

33.44 (a) $\left(\Delta V_{2,\text{ rms}}\right) = \frac{N_2}{N_1}\left(\Delta V_{1,\text{ rms}}\right)$ $N_2 = \frac{(2200)(80)}{110} = \boxed{1600\text{ windings}}$

(b) $I_{1,\text{rms}}\left(\Delta V_{1,\text{rms}}\right) = I_{2,\text{rms}}\left(\Delta V_{2,\text{rms}}\right)$ $I_{1,\text{rms}} = \frac{(1.50)(2200)}{110} = \boxed{30.0\text{ A}}$

(c) $0.950\, I_{1,\text{rms}}\left(\Delta V_{1,\text{rms}}\right) = I_{2,\text{rms}}\left(\Delta V_{2,\text{rms}}\right)$ $I_{1,\text{rms}} = \frac{(1.20)(2200)}{110(0.950)} = \boxed{25.3\text{ A}}$

33.45 The rms voltage across the transformer primary is



$$\frac{N_1}{N_2}\left(\Delta V_{2,\text{rms}}\right)$$

so the source voltage is $\Delta V_{s,\text{rms}} = I_{1,\text{rms}} R_s + \dfrac{N_1}{N_2}\left(\Delta V_{2,\text{rms}}\right)$

The secondary current is $\dfrac{\left(\Delta V_{2,\text{rms}}\right)}{R_L}$, so the primary current is $\dfrac{N_2}{N_1}\dfrac{\left(\Delta V_{2,\text{rms}}\right)}{R_L} = I_{1,\text{rms}}$

Then $\Delta V_{s,\text{rms}} = \dfrac{N_2\left(\Delta V_{2,\text{rms}}\right)R_s}{N_1 R_L} + \dfrac{N_1\left(\Delta V_{2,\text{rms}}\right)}{N_2}$

and $R_s = \dfrac{N_1 R_L}{N_2\left(\Delta V_{2,\text{rms}}\right)}\left(\Delta V_{s,\text{rms}} - \dfrac{N_1\left(\Delta V_{2,\text{rms}}\right)}{N_2}\right) = \dfrac{5(50.0\ \Omega)}{2(25.0\ \text{V})}\left(80.0\ \text{V} - \dfrac{5(25.0\ \text{V})}{2}\right) = \boxed{87.5\ \Omega}$

33.46 (a) $\Delta V_{2,\text{rms}} = \dfrac{N_2}{N_1}\left(\Delta V_{1,\text{rms}}\right)$ $\quad \dfrac{N_2}{N_1} = \dfrac{\Delta V_{2,\text{rms}}}{\Delta V_{1,\text{rms}}} = \dfrac{10.0\times 10^3\ \text{V}}{120\ \text{V}} = \boxed{83.3}$

(b) $I_{2,\text{rms}}\left(\Delta V_{2,\text{rms}}\right) = 0.900\, I_{1,\text{rms}}\left(\Delta V_{1,\text{rms}}\right)$

$I_{2,\text{rms}}\left(10.0\times 10^3\ \text{V}\right) = 0.900\left(\dfrac{120\ \text{V}}{24.0\ \Omega}\right)(120\ \text{V})$ $\quad I_{2,\text{rms}} = \boxed{54.0\ \text{mA}}$

(c) $Z_2 = \dfrac{\Delta V_{2,\text{rms}}}{I_{2,\text{rms}}} = \dfrac{10.0\times 10^3\ \text{V}}{0.054\ \text{A}} = \boxed{185\ \text{k}\Omega}$

33.47 (a) $R = (4.50\times 10^{-4}\ \Omega/\text{m})(6.44\times 10^5\ \text{m}) = 290\ \Omega$ and $I_{\text{rms}} = \dfrac{\mathcal{P}}{\Delta V_{\text{rms}}} = \dfrac{5.00\times 10^6\ \text{W}}{5.00\times 10^5\ \text{V}} = 10.0\ \text{A}$

$\mathcal{P}_{\text{loss}} = I_{\text{rms}}^2 R = (10.0\ \text{A})^2(290\ \Omega) = \boxed{29.0\ \text{kW}}$

(b) $\dfrac{\mathcal{P}_{\text{loss}}}{\mathcal{P}} = \dfrac{2.90\times 10^4}{5.00\times 10^6} = \boxed{5.80\times 10^{-3}}$

(c) It is impossible to transmit so much power at such low voltage. Maximum power transfer occurs when load resistance equals the line resistance of 290 Ω, and is

$\dfrac{(4.50\times 10^3\ \text{V})^2}{2\cdot 2(290\ \Omega)} = 17.5\ \text{kW}$, far below the required 5000 kW

33.48 For the filter circuit, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{X_C}{\sqrt{R^2 + X_C^2}}$

(a) At $f = 600\ \text{Hz}$, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(600\ \text{Hz})\left(8.00\times10^{-9}\ \text{F}\right)} = 3.32\times10^4\ \Omega$

and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{3.32\times10^4\ \Omega}{\sqrt{(90.0\ \Omega)^2 + \left(3.32\times10^4\ \Omega\right)^2}} \approx \boxed{1.00}$

(b) At $f = 600\ \text{kHz}$, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi\left(600\times10^3\ \text{Hz}\right)\left(8.00\times10^{-9}\ \text{F}\right)} = 33.2\ \Omega$

and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{33.2\ \Omega}{\sqrt{(90.0\ \Omega)^2 + (33.2\ \Omega)^2}} = \boxed{0.346}$

33.49 For this RC high-pass filter, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{\sqrt{R^2 + X_C^2}}$

(a) When $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = 0.500$,

then $\dfrac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + X_C^2}} = 0.500$ or $X_C = 0.866\ \Omega$

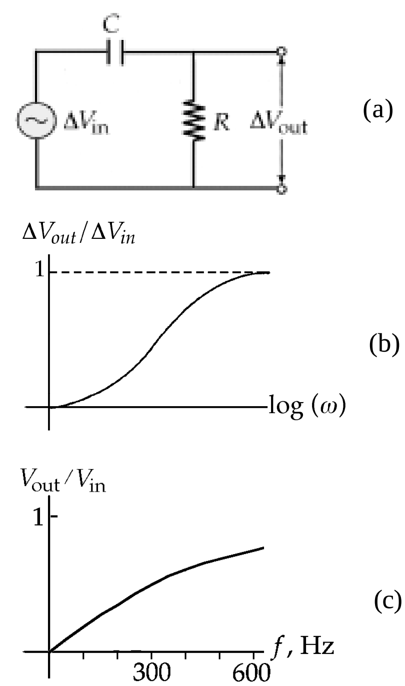
If this occurs at $f = 300\ \text{Hz}$, the capacitance is

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(300\ \text{Hz})(0.866\ \Omega)} = 6.13\times10^{-4}\ \text{F} = \boxed{613\ \mu\text{F}}$$

(b) With this capacitance and a frequency of 600 Hz,

$$X_C = \frac{1}{2\pi(600\ \text{Hz})\left(6.13\times10^{-4}\ \text{F}\right)} = 0.433\ \Omega$$

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + (0.433\ \Omega)^2}} = \boxed{0.756}$$



Figures for Goal Solution

Goal Solution

The RC high-pass filter shown in Figure 33.22 has a resistance $R = 0.500\ \Omega$. (a) What capacitance gives an output signal that has one-half the amplitude of a 300-Hz input signal? (b) What is the gain ($\Delta V_{out} / \Delta V_{in}$) for a 600-Hz signal?

G: It is difficult to estimate the capacitance required without actually calculating it, but we might expect a typical value in the μF to pF range. The nature of a high-pass filter is to yield a larger gain at higher frequencies, so if this circuit is designed to have a gain of 0.5 at 300 Hz, then it should have a higher gain at 600 Hz. We might guess it is near 1.0 based on Figure (b) above.

O: The output voltage of this circuit is taken across the resistor, but the input sees the impedance of the resistor and the capacitor. Therefore, the gain will be the ratio of the resistance to the impedance.

A:
$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}}$$

(a) When $\Delta V_{out} / \Delta V_{in} = 0.500$

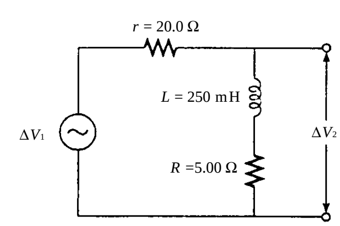
solving for C gives
$$C = \frac{1}{\omega R \sqrt{\left(\frac{\Delta V_{\text{in}}}{\Delta V_{\text{out}}}\right)^2 - 1}} = \frac{1}{(2\pi)(300\text{ Hz})(0.500\ \Omega)\sqrt{(2.00)^2 - 1}} = 613\ \mu\text{F}$$

(b) At 600 Hz, we have $\omega = (2\pi\text{ rad})(600\text{ s}^{-1})$

so
$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + \left(\frac{1}{(1200\pi\ \text{rad / s})(613\ \mu\text{F})}\right)^2}} = 0.756$$

L: The capacitance value seems reasonable, but the gain is considerably less than we expected. Based on our calculation, we can modify the graph in Figure (b) to more transparently represent the characteristics of this high-pass filter, now shown in Figure (c). If this were an audio filter, it would reduce low frequency "humming" sounds while allowing high pitch sounds to pass through. A low pass filter would be needed to reduce high frequency "static" noise.

33.50 $\Delta V_1 = I\sqrt{(r+R)^2 + X_L^2}$, and $\Delta V_2 = I\sqrt{R^2 + X_L^2}$

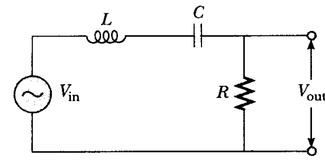


Thus, when $\Delta V_1 = 2\Delta V_2$ $\quad (r+R)^2 + X_L^2 = 4\left(R^2 + X_L^2\right)$

or $\quad (25.0\ \Omega)^2 + X_L^2 = 4(5.00\ \Omega)^2 + 4X_L^2$

which gives $\quad X_L = 2\pi f(0.250\text{ H}) = \sqrt{\frac{625 - 100}{3}}\ \Omega$ and $f = \boxed{8.42\text{ Hz}}$

***33.51** $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$



(a) At 200 Hz: $\dfrac{1}{4} = \dfrac{(8.00\ \Omega)^2}{(8.00\ \Omega)^2 + \left[400\pi L - \dfrac{1}{400\pi C}\right]^2}$

At 4000 Hz: $(8.00\ \Omega)^2 + \left[8000\pi L - \dfrac{1}{8000\pi C}\right]^2 = 4(8.00\ \Omega)^2$

At the low frequency, $X_L - X_C < 0$. This reduces to $400\pi L - \dfrac{1}{400\pi C} = -13.9\ \Omega$ [1]

For the high frequency half-voltage point, $8000\pi L - \dfrac{1}{8000\pi C} = +13.9\ \Omega$ [2]

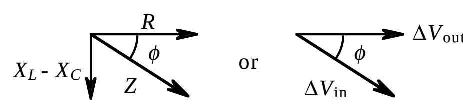
Solving Equations (1) and (2) simultaneously gives $C = \boxed{54.6\ \mu\text{F}}$ and $L = \boxed{580\ \mu\text{H}}$

(b) When $X_L = X_C$, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \left(\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}\right)_{\max} = \boxed{1.00}$

(c) $X_L = X_C$ requires $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(5.80\times 10^{-4}\ \text{H})(5.46\times 10^{-5}\ \text{F})}} = \boxed{894\ \text{Hz}}$

(d) At 200 Hz, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{Z} = \dfrac{1}{2}$ and $X_C > X_L$,

so the phasor diagram is as shown:

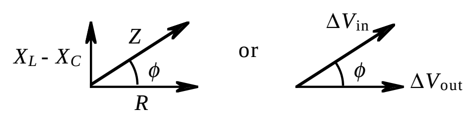


$\phi = -\cos^{-1}\left(\dfrac{R}{Z}\right) = -\cos^{-1}\left(\dfrac{1}{2}\right)$ so $\boxed{\Delta V_{\text{out}} \text{ leads } \Delta V_{\text{in}} \text{ by } 60.0°}$

At f_0, $X_L = X_C$ so $\boxed{\Delta V_{\text{out}} \text{ and } \Delta V_{\text{in}} \text{ have a phase difference of } 0°}$

At 4000 Hz, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{Z} = \dfrac{1}{2}$ and $X_L - X_C > 0$

Thus, $\phi = \cos^{-1}\left(\dfrac{1}{2}\right) = 60.0°$



or $\boxed{\Delta V_{\text{out}} \text{ lags } \Delta V_{\text{in}} \text{ by } 60.0°}$

(e) At 200 Hz and at 4 kHz, $\mathsf{P} = \dfrac{(\Delta V_{\text{out,rms}})^2}{R} = \dfrac{\left(\frac{1}{2}\Delta V_{\text{in,rms}}\right)^2}{R} = \dfrac{\frac{1}{2}\left(\frac{1}{2}\Delta V_{\text{in,max}}\right)^2}{R} = \dfrac{(10.0\ \text{V})^2}{8(8.00\ \Omega)} = \boxed{1.56\ \text{W}}$

At f_0, $\mathsf{P} = \dfrac{(\Delta V_{\text{out,rms}})^2}{R} = \dfrac{(\Delta V_{\text{in,rms}})^2}{R} = \dfrac{\frac{1}{2}(\Delta V_{\text{in,max}})^2}{R} = \dfrac{(10.0\ \text{V})^2}{2(8.00\ \Omega)} = \boxed{6.25\ \text{W}}$

(f) We take: $Q = \dfrac{\omega_0 L}{R} = \dfrac{2\pi f_0 L}{R} = \dfrac{2\pi(894\ \text{Hz})(5.80\times 10^{-4}\ \text{H})}{8.00\ \Omega} = \boxed{0.408}$

33.52 For a high-pass filter,
$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}=\frac{R}{\sqrt{R^2+\left(\dfrac{1}{\omega C}\right)^2}}$$

$$\frac{(\Delta V_{\text{out}})_1}{(\Delta V_{\text{in}})_1}=\frac{R}{\sqrt{R^2+\left(\dfrac{1}{\omega C}\right)^2}}\quad\text{and}\quad\frac{(\Delta V_{\text{out}})_2}{(\Delta V_{\text{in}})_2}=\frac{R}{\sqrt{R^2+\left(\dfrac{1}{\omega C}\right)^2}}$$

Now $(\Delta V_{\text{in}})_2=(\Delta V_{\text{out}})_1$ so
$$\frac{(\Delta V_{\text{out}})_2}{(\Delta V_{\text{in}})_1}=\frac{R^2}{R^2+\left(\dfrac{1}{\omega C}\right)^2}=\boxed{\frac{1}{1+\left(\dfrac{1}{\omega RC}\right)^2}}$$

33.53 Rewrite the circuit in terms of impedance as shown in Fig. (b).

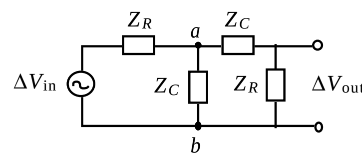


Figure (a)

Find: $$\Delta V_{\text{out}}=\frac{Z_R}{Z_R+Z_C}\Delta V_{ab}\qquad[1]$$

From Figure (c), $$\Delta V_{ab}=\frac{Z_C\parallel(Z_R+Z_C)}{Z_R+Z_C\parallel(Z_R+Z_C)}\Delta V_{\text{in}}$$

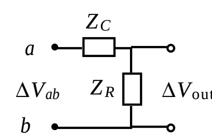


Figure (b)

So Eq. [1] becomes $$\Delta V_{\text{out}}=\frac{Z_R\left[Z_C\parallel(Z_R+Z_C)\right]}{(Z_R+Z_C)\left[Z_R+Z_C\parallel(Z_R+Z_C)\right]}\Delta V_{\text{in}}$$

or $$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}=\frac{Z_R\left[\dfrac{1}{Z_C}+\dfrac{1}{Z_R+Z_C}\right]^{-1}}{(Z_R+Z_C)\left[Z_R+\left(\dfrac{1}{Z_C}+\dfrac{1}{Z_R+Z_C}\right)^{-1}\right]}$$

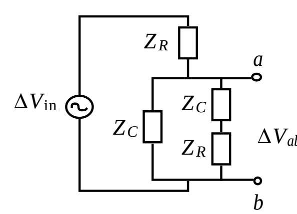
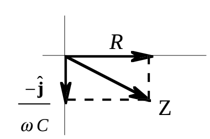


Figure (c)

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}=\frac{Z_RZ_C}{Z_C(Z_C+Z_R)+Z_R(Z_R+2Z_C)}=\frac{Z_R}{3Z_R+Z_C+(Z_R)^2/Z_C}$$

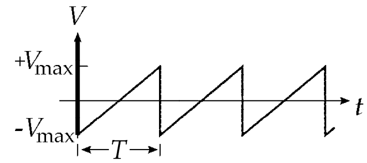
Now, $Z_R=R$ and $Z_C=\dfrac{-\mathrm{j}}{\omega C}$ where $\mathrm{j}=\sqrt{-1}$



$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}=\frac{R}{3R-\left(\dfrac{1}{\omega C}\right)\mathrm{j}+R^2\omega C\,\mathrm{j}}$$ where we used $\dfrac{1}{\mathrm{j}}=-\mathrm{j}$.

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}=\frac{R}{3R-\left(\dfrac{1}{\omega C}-R^2\omega C\right)\mathrm{j}}=\frac{R}{\sqrt{(3R)^2+\left(\dfrac{1}{\omega C}-R^2\omega C\right)^2}}=\frac{1.00\times10^3}{\sqrt{\left(3.00\times10^3\right)^2+(1592-628)^2}}=\boxed{0.317}$$

33.54 The equation for $\Delta v(t)$ during the first period (using $y = mx + b$) is:



$$\Delta v(t) = \frac{2(\Delta V_{\max})t}{T} - \Delta V_{\max}$$

$$\left[(\Delta v)^2\right]_{\text{ave}} = \frac{1}{T}\int_0^T \left[\Delta v(t)\right]^2 dt = \frac{(\Delta V_{\max})^2}{T}\int_0^T\left[\frac{2}{T}t - 1\right]^2 dt$$

$$\left[(\Delta v)^2\right]_{\text{ave}} = \frac{(\Delta V_{\max})^2}{T}\left(\frac{T}{2}\right)\frac{\left[2t/T - 1\right]^3}{3}\Bigg|_{t=0}^{t=T} = \frac{(\Delta V_{\max})^2}{6}\left[(+1)^3 - (-1)^3\right] = \frac{(\Delta V_{\max})^2}{3}$$

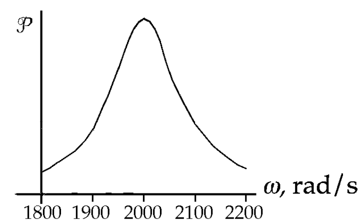
$$\Delta V_{\text{rms}} = \sqrt{\left[(\Delta v)^2\right]_{\text{ave}}} = \sqrt{\frac{(\Delta V_{\max})^2}{3}} = \boxed{\frac{\Delta V_{\max}}{\sqrt{3}}}$$

33.55 $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.0500 \text{ H})(5.00\times 10^{-6} \text{ F})}} = 2000 \text{ s}^{-1}$

so the operating frequency of the circuit is $\omega = \dfrac{\omega_0}{2} = 1000 \text{ s}^{-1}$

Using Equation 33.35, $\mathcal{P} = \dfrac{(\Delta V_{\text{rms}})^2 R\omega^2}{R^2\omega^2 + L^2\left(\omega^2 - \omega_0^2\right)^2}$

$$\mathcal{P} = \frac{(400)^2(8.00)(1000)^2}{(8.00)^2(1000)^2 + (0.0500)^2\left[(1.00 - 4.00)\times 10^6\right]^2} = \boxed{56.7 \text{ W}}$$



$(Q \approx 12.5)$

Figure for Goal Solution

<u>Goal Solution</u>

A series *RLC* circuit consists of an 8.00-Ω resistor, a 5.00-μF capacitor, and a 50.0-mH inductor. A variable frequency source applies an emf of 400 V (rms) across the combination. Determine the power delivered to the circuit when the frequency is equal to one half the resonance frequency.

G: Maximum power is delivered at the resonance frequency, and the power delivered at other frequencies depends on the quality factor, Q. For the relatively small resistance in this circuit, we could expect a high $Q = \omega_0 L/R$. So at half the resonant frequency, the power should be a small fraction of the maximum power, $\mathcal{P}_{av,\ \max} = \Delta V_{rms}^2/R = (400 \text{ V})^2/8\ \Omega = 20$ kW.

O: We must first calculate the resonance frequency in order to find half this frequency. Then the power delivered by the source must equal the power taken out by the resistor. This power can be found from $\mathcal{P}_{av} = I_{rms}^2 R$ where $I_{rms} = \Delta V_{rms} / Z$.

A: The resonance frequency is $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.0500\text{ H})(5.00\times10^{-6}\text{ F})}} = 318\text{ Hz}$

The operating frequency is $f = f_0 / 2 = 159\text{ Hz}$. We can calculate the impedance at this frequency:

$X_L = 2\pi f L = 2\pi(159\text{ Hz})(0.0500\text{ H}) = 50.0\ \Omega$ and $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(159\text{ Hz})(5.00\times10^{-6}\text{ F})} = 200\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{8.00^2 + (50.0 - 200)^2}\ \Omega = 150\ \Omega$

So, $I_{rms} = \dfrac{\Delta V_{rms}}{Z} = \dfrac{400\text{ V}}{150\ \Omega} = 2.66\text{ A}$

The power delivered by the source is the power dissipated by the resistor:

$\mathcal{P}_{av} = I_{rms}{}^2 R = (2.66\text{ A})^2(8.00\ \Omega) = 56.7\text{ W}$

L: This power is only about 0.3% of the 20 kW peak power delivered at the resonance frequency. The significant reduction in power for frequencies away from resonance is a consequence of the relatively high Q-factor of about 12.5 for this circuit. A high Q is beneficial if, for example, you want to listen to your favorite radio station that broadcasts at 101.5 MHz, and you do not want to receive the signal from another local station that broadcasts at 101.9 MHz.

33.56 The resistance of the circuit is $R = \dfrac{\Delta V}{I} = \dfrac{12.0\text{ V}}{0.630\text{ A}} = 19.0\ \Omega$

The impedance of the circuit is $Z = \dfrac{\Delta V_{\text{rms}}}{I_{\text{rms}}} = \dfrac{24.0\text{ V}}{0.570\text{ A}} = 42.1\ \Omega$

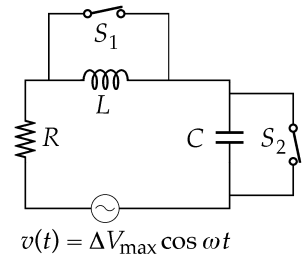
$Z^2 = R^2 + \omega^2 L^2$

$L = \dfrac{1}{\omega}\sqrt{Z^2 - R^2} = \dfrac{1}{377}\sqrt{(42.1)^2 - (19.0)^2} = \boxed{99.6\text{ mH}}$

33.57 (a) When ωL is very large, the bottom branch carries negligible current. Also, $1/\omega C$ will be negligible compared to $200\ \Omega$ and $45.0\text{ V}/200\ \Omega = \boxed{225\text{ mA}}$ flows in the power supply and the top branch.

(b) Now $1/\omega C \to \infty$ and $\omega L \to 0$ so the generator and bottom branch carry $\boxed{450\text{ mA}}$

33.58 (a) With both switches closed, the current goes only through generator and resistor.



$$\boxed{i(t) = \frac{\Delta V_{max}}{R} \cos \omega t}$$

(b) $$\boxed{\mathcal{P} = \frac{1}{2}\frac{(\Delta V_{max})^2}{R}}$$

(c) $$\boxed{i(t) = \frac{\Delta V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \cos\left[\omega t + \text{Arctan}(\omega L / R)\right]}$$

(d) For $$0 = \phi = \text{Arctan}\left(\frac{\omega_0 L - \frac{1}{\omega_0 C}}{R}\right)$$

We require $\omega_0 L = \dfrac{1}{\omega_0 C}$, so $\boxed{C = \dfrac{1}{\omega_0{}^2 L}}$

(e) At this resonance frequency, $Z = \boxed{R}$

(f) $U = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}CI^2 X_C{}^2$

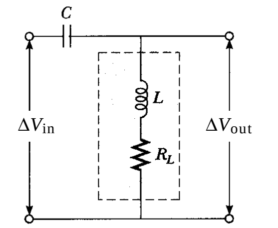
$$U_{max} = \tfrac{1}{2}CI_{max}^2 X_C{}^2 = \tfrac{1}{2}C\frac{(\Delta V_{max})^2}{R^2}\frac{1}{\omega_0{}^2 C^2} = \boxed{\frac{(\Delta V_{max})^2 L}{2R^2}}$$

(g) $$U_{max} = \tfrac{1}{2}LI_{max}^2 = \boxed{\tfrac{1}{2}L\frac{(\Delta V_{max})^2}{R^2}}$$

(h) Now $\omega = 2\omega_0 = \dfrac{2}{\sqrt{LC}}$

So $$\phi = \text{Arctan}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) = \text{Arctan}\left(\frac{2\sqrt{\frac{L}{C}} - \frac{1}{2}\sqrt{\frac{L}{C}}}{R}\right) = \boxed{\text{Arctan}\left(\frac{3}{2R}\sqrt{\frac{L}{C}}\right)}$$

(i) Now $\omega L = \dfrac{1}{2}\dfrac{1}{\omega C}$ $\qquad \omega = \boxed{\dfrac{1}{\sqrt{2LC}}} = \dfrac{\omega_0}{\sqrt{2}}$



Circuit (a)

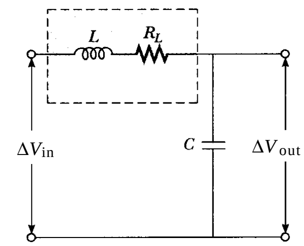
33.59 (a) As shown in part (b), $\boxed{\text{circuit (a) is a high-pass filter}}$

and $\boxed{\text{circuit (b) is a low-pass filter}}$.

(b) For circuit (a), $\dfrac{\Delta V_{out}}{\Delta V_{in}} = \dfrac{\sqrt{R_L^2 + X_L^2}}{\sqrt{R_L^2 + (X_L - X_C)^2}} = \boxed{\dfrac{\sqrt{R_L^2 + (\omega L)^2}}{\sqrt{R_L^2 + (\omega L - 1/\omega C)^2}}}$

As $\omega \to 0$, $\dfrac{\Delta V_{out}}{\Delta V_{in}} \approx \omega R_L C \approx 0$

As $\omega \to \infty$, $\dfrac{\Delta V_{out}}{\Delta V_{in}} \approx 1$ (high-pass filter)



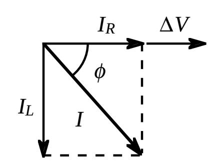
Circuit (b)

For circuit (b), $\dfrac{\Delta V_{out}}{\Delta V_{in}} = \dfrac{X_C}{\sqrt{R_L^2 + (X_L - X_C)^2}} = \boxed{\dfrac{1/\omega C}{\sqrt{R_L^2 + (\omega L - 1/\omega C)^2}}}$

As $\omega \to 0$, $\dfrac{\Delta V_{out}}{\Delta V_{in}} \approx 1$

As $\omega \to \infty$, $\dfrac{\Delta V_{out}}{\Delta V_{in}} \approx \dfrac{1}{\omega^2 LC} \approx 0$ (low-pass filter)

33.60 (a) $I_{R,\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{R} = \dfrac{100 \text{ V}}{80.0\ \Omega} = \boxed{1.25 \text{ A}}$



(b) The total current will $\boxed{\text{lag}}$ the applied voltage as seen in the phasor diagram at the right.

$$I_{L,\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{100 \text{ V}}{2\pi\left(60.0 \text{ s}^{-1}\right)(0.200 \text{ H})} = 1.33 \text{ A}$$

Thus, the phase angle is: $\phi = \tan^{-1}\left(\dfrac{I_{L,\text{rms}}}{I_{R,\text{rms}}}\right) = \tan^{-1}\left(\dfrac{1.33 \text{ A}}{1.25 \text{ A}}\right) = \boxed{46.7°}$

***33.61** Suppose each of the 20 000 people uses an average power of 500 W. (This means 12 kWh per day, or \$36 per 30 days at 10¢ per kWh). Suppose the transmission line is at 20 kV. Then

$$I_{\text{rms}} = \frac{P}{\Delta V_{\text{rms}}} = \frac{(20\,000)(500 \text{ W})}{20\,000 \text{ V}} \ \boxed{\sim 10^3 \text{ A}}$$

If the transmission line had been at 200 kV, the current would be only $\boxed{\sim 10^2 \text{ A}}$.

33.62 $L = 2.00\text{ H},\ C = 10.0\times10^{-6}\text{ F},\ R = 10.0\ \Omega,\ \Delta v(t) = (100\sin\omega t)$

(a) The resonant frequency ω_0 produces the maximum current and thus the maximum power dissipation in the resistor.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.00)(10.0\times10^{-6})}} = \boxed{224\text{ rad/s}}$$

(b) $$\mathcal{P} = \frac{(\Delta V_{\max})^2}{2R} = \frac{(100)^2}{2(10.0)} = \boxed{500\text{ W}}$$

(c) $$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}} \quad\text{and}\quad (I_{\text{rms}})_{\max} = \frac{\Delta V_{\text{rms}}}{R}$$

$$I_{\text{rms}}^2 R = \frac{1}{2}\left(I_{\text{rms}}^2\right)_{\max} R \quad\text{or}\quad \frac{(\Delta V_{\text{rms}})^2}{Z^2}R = \frac{1}{2}\frac{(\Delta V_{\text{rms}})^2}{R^2}R$$

This occurs where $Z^2 = 2R^2$: $$R^2+\left(\omega L-\frac{1}{\omega C}\right)^2 = 2R^2$$

$$\omega^4L^2C^2 - 2L\omega^2C - R^2\omega^2C^2 + 1 = 0 \quad\text{or}\quad L^2C^2\omega^4 - (2LC + R^2C^2)\omega^2 + 1 = 0$$

$$\left[(2.00)^2(10.0\times10^{-6})^2\right]\omega^4 - \left[2(2.00)(10.0\times10^{-6}) + (10.0)^2(10.0\times10^{-6})^2\right]\omega^2 + 1 = 0$$

Solving this quadratic equation, we find that $\omega^2 = 51\,130,\quad 48\,894$

$$\omega_1 = \sqrt{48\,894} = \boxed{221\text{ rad/s}} \quad\text{and}\quad \omega_2 = \sqrt{51\,130} = \boxed{226\text{ rad/s}}$$

33.63 $R = 200\ \Omega,\ L = 663\text{ mH},\ C = 26.5\ \mu\text{F},\ \omega = 377\text{ s}^{-1},\ \Delta V_{\max} = 50.0\text{ V}$

$$\omega L = 250\ \Omega, \quad \left(\frac{1}{\omega C}\right) = 100\ \Omega, \quad Z = \sqrt{R^2+(X_L-X_C)^2} = 250\ \Omega$$

(a) $$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{50.0\text{ V}}{250\ \Omega} = \boxed{0.200\text{ A}}$$

$$\phi = \tan^{-1}\left(\frac{X_L-X_C}{R}\right) = \boxed{36.8^\circ} \quad (\Delta V\text{ leads }I)$$

(b) $\Delta V_{R,\max} = I_{\max}R = \boxed{40.0\text{ V}}$ at $\boxed{\phi = 0^\circ}$

(c) $\Delta V_{C,\max} = \dfrac{I_{\max}}{\omega C} = \boxed{20.0\text{ V}}$ at $\boxed{\phi = -90.0^\circ}$ (I leads ΔV)

(d) $\Delta V_{L,\max} = I_{\max}\omega L = \boxed{50.0\text{ V}}$ at $\boxed{\phi = +90.0^\circ}$ (ΔV leads I)

***33.64** $\mathcal{P} = I_{\text{rms}}^2 R = \left(\frac{\Delta V_{\text{rms}}}{Z}\right)^2 R$, so $250\text{ W} = \frac{(120\text{ V})^2}{Z^2}(40.0\ \Omega)$: $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

$$250 = \frac{(120)^2(40.0)}{(40.0)^2 + \left[2\pi f(0.185) - \frac{1}{2\pi f\left(65.0\times 10^{-6}\right)}\right]^2} \quad \text{and} \quad 250 = \frac{576\,000\,f^2}{1600 f^2 + \left(1.1624 f^2 - 2448.5\right)^2}$$

$$1 = \frac{2304\,f^2}{1600 f^2 + 1.3511 f^4 - 5692.3 f^2 + 5\,995\,300} \quad \text{so} \quad 1.3511 f^4 - 6396.3 f^2 + 5\,995\,300 = 0$$

$$f^2 = \frac{6396.3 \pm \sqrt{(6396.3)^2 - 4(1.3511)(5\,995\,300)}}{2(1.3511)} = 3446.5 \text{ or } 1287.4$$

$f = \boxed{58.7\text{ Hz or } 35.9\text{ Hz}}$

33.65 (a) From Equation 33.39, $\frac{N_1}{N_2} = \frac{\Delta V_1}{\Delta V_2}$

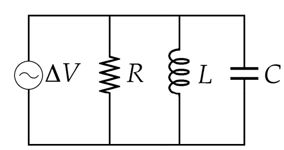
Let output impedance $Z_1 = \frac{\Delta V_1}{I_1}$ and the input impedance $Z_2 = \frac{\Delta V_2}{I_2}$

so that $\frac{N_1}{N_2} = \frac{Z_1 I_1}{Z_2 I_2}$ But from Eq. 33.40, $\frac{I_1}{I_2} = \frac{\Delta V_2}{\Delta V_1} = \frac{N_2}{N_1}$

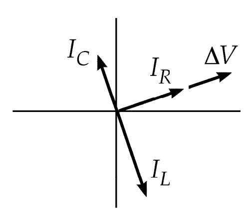
So, combining with the previous result we have $\boxed{\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}}}$

(b) $\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{8000}{8.00}} = \boxed{31.6}$

33.66 $I_R = \frac{\Delta V_{\text{rms}}}{R}$; $I_L = \frac{\Delta V_{\text{rms}}}{\omega_L}$; $I_C = \frac{\Delta V_{\text{rms}}}{(\omega C)^{-1}}$



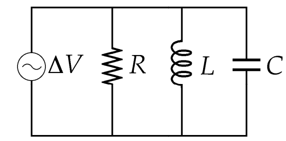
(a) $I_{\text{rms}} = \sqrt{I_R^2 + (I_C - I_L)^2} = \boxed{\Delta V_{\text{rms}}\sqrt{\left(\frac{1}{R^2}\right) + \left(\omega C - \frac{1}{\omega L}\right)^2}}$

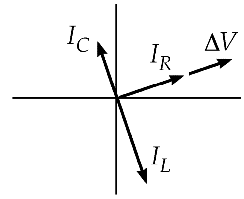


(b) $\tan\phi = \frac{I_C - I_L}{I_R} = \Delta V_{\text{rms}}\left[\frac{1}{X_C} - \frac{1}{X_L}\right]\left(\frac{1}{\Delta V_{\text{rms}} / R}\right)$

$\boxed{\tan\phi = R\left[\frac{1}{X_C} - \frac{1}{X_L}\right]}$

33.67 (a) $I_{\text{rms}} = \Delta V_{\text{rms}}\sqrt{\dfrac{1}{R^2}+\left(\omega C-\dfrac{1}{\omega L}\right)^2}$





$\Delta V_{\text{rms}} \to (\Delta V_{\text{rms}})_{\max}$ when $\omega C = \dfrac{1}{\omega L}$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{200\times10^{-3}\text{ H})(0.150\times10^{-6}\text{ F})}} = \boxed{919 \text{ Hz}}$$

(b) $I_R = \dfrac{\Delta V_{\text{rms}}}{R} = \dfrac{120 \text{ V}}{80.0\ \Omega} = \boxed{1.50 \text{ A}}$

$$I_L = \frac{\Delta V_{\text{rms}}}{\omega L} = \frac{120\text{ V}}{(374\text{ s}^{-1})(0.200\text{ H})} = \boxed{1.60\text{ A}}$$

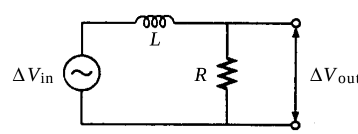
$$I_C = \Delta V_{\text{rms}}(\omega C) = (120\text{ V})(374\text{ s}^{-1})(0.150\times10^{-6}\text{ F}) = \boxed{6.73\text{ mA}}$$

(c) $I_{\text{rms}} = \sqrt{I_R^2+(I_C-I_L)^2} = \sqrt{(1.50)^2+(0.00673-1.60)^2} = \boxed{2.19\text{ A}}$

(d) $\phi = \tan^{-1}\left[\dfrac{I_C-I_L}{I_R}\right] = \tan^{-1}\left[\dfrac{0.00673-1.60}{1.50}\right] = \boxed{-46.7°}$

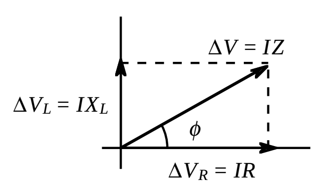
The $\boxed{\text{current is lagging the voltage}}$.

33.68 (a) $\tan\phi = \dfrac{\Delta V_L}{\Delta V_R} = \dfrac{I(\omega L)}{IR} = \dfrac{\omega L}{R}$



Thus, $R = \dfrac{\omega L}{\tan\phi} = \dfrac{\left(200\text{ s}^{-1}\right)(0.500\text{ H})}{\tan(30.0°)} = \boxed{173\ \Omega}$

(b) $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{\Delta V_R}{\Delta V_{\text{in}}} = \cos\phi$



$$\Delta V_{\text{out}} = (\Delta V_{\text{in}})\cos\phi = (10.0\text{ V})\cos 30.0° = \boxed{8.66\text{ V}}$$

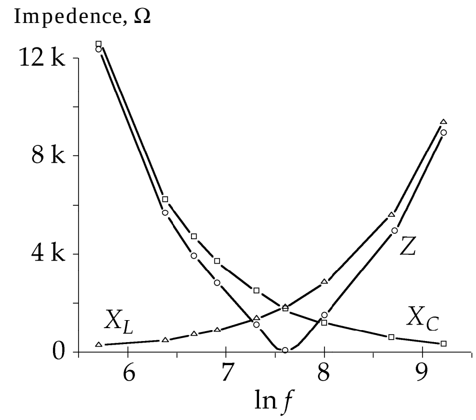
33.69 (a) $X_L = X_C = 1884\ \Omega$ when $f = 2000$ Hz

$$L = \frac{X_L}{2\pi f} = \frac{1884\ \Omega}{4000\pi \text{ rad/s}} = 0.150 \text{ H} \quad \text{and} \quad C = \frac{1}{(2\pi f)X_C} = \frac{1}{(4000\pi \text{ rad/s})(1884\ \Omega)} = 42.2 \text{ nF}$$

$$X_L = 2\pi f(0.150 \text{ H}) \quad X_C = \frac{1}{(2\pi f)(4.22\times 10^{-8} \text{ F})} \quad Z = \sqrt{(40.0\ \Omega)^2 + (X_L - X_C)^2}$$

f (Hz)	X_L (Ω)	X_C (Ω)	Z (Ω)
300	283	12600	12300
600	565	6280	5720
800	754	4710	3960
1000	942	3770	2830
1500	1410	2510	1100
2000	1880	1880	40
3000	2830	1260	1570
4000	3770	942	2830
6000	5650	628	5020
10000	9420	377	9040

(b)



33.70 $$\omega_0 = \frac{1}{\sqrt{LC}} = 1.00\times 10^6 \text{ rad/s}$$

For each angular frequency, we find

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

then $I = (1.00 \text{ V}) / Z$

and $\mathsf{P} = I^2(1.00\ \Omega)$

ω/ω_0	ωL (Ω)	$\frac{1}{\omega C}$ (Ω)	Z (Ω)	$\mathsf{P} = I^2R$ (W)
0.9990	999.0	1001.0	2.24	0.19984
0.9991	999.1	1000.9	2.06	0.23569
0.9993	999.3	1000.7	1.72	0.33768
0.9995	999.5	1000.5	1.41	0.49987
0.9997	999.7	1000.3	1.17	0.73524
0.9999	999.9	1000.1	1.02	0.96153
1.0000	1000	1000.0	1.00	1.00000
1.0001	1000.1	999.9	1.02	0.96154
1.0003	1000.3	999.7	1.17	0.73535
1.0005	1000.5	999.5	1.41	0.50012
1.0007	1000.7	999.3	1.72	0.33799
1.0009	1000.9	999.1	2.06	0.23601
1.0010	1001	999.0	2.24	0.20016

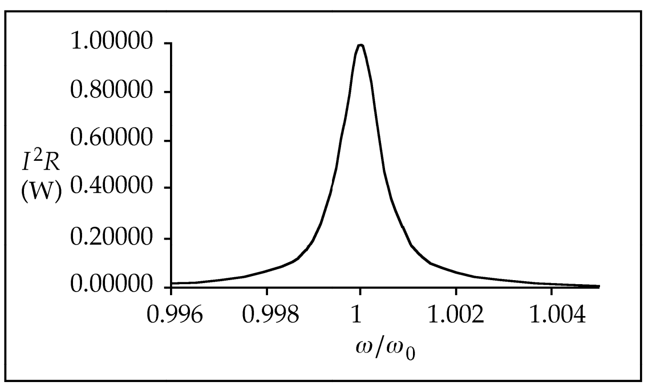
The full width at half maximum is:

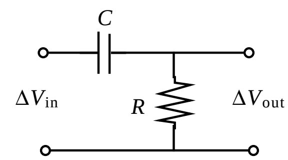
$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{(1.0005 - 0.9995)\omega_0}{2\pi}$$

$$\Delta f = \frac{1.00\times 10^3 \text{ s}^{-1}}{2\pi} = 159 \text{ Hz}$$

while

$$\frac{R}{2\pi L} = \frac{1.00\ \Omega}{2\pi(1.00\times 10^{-3} \text{ H})} = 159 \text{ Hz}$$



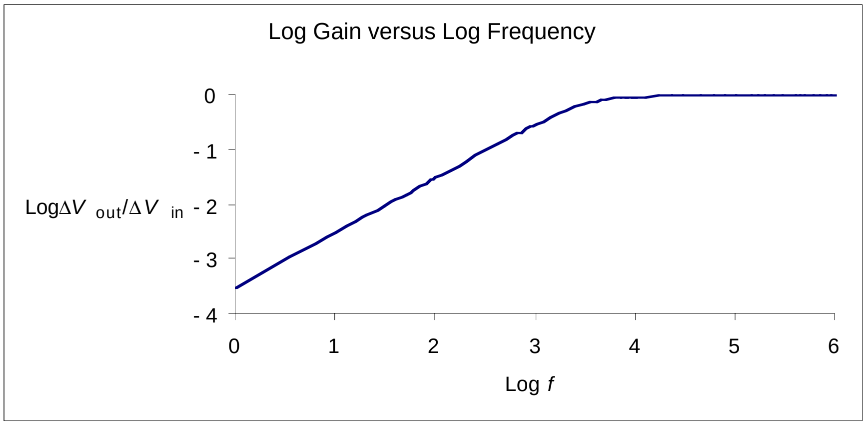


33.71 $$\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{R}{\sqrt{R^2 + (1/2\pi f C)^2}}$$

(a) $\dfrac{\Delta V_{out}}{\Delta V_{in}} = \dfrac{1}{2}$ when $\dfrac{1}{\omega C} = R\sqrt{3}$

Hence, $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi RC\sqrt{3}} = \boxed{1.84 \text{ kHz}}$

(b)



Chapter 34 Solutions

34.1 Since the light from this star travels at 3.00×10^8 m/s, the last bit of light will hit the Earth in $\dfrac{6.44 \times 10^{18}\text{ m}}{3.00 \times 10^8\text{ m/s}} = 2.15 \times 10^{10}$ s = 680 years. Therefore, it will disappear from the sky in the year 1999 + 680 = $\boxed{2.68 \times 10^3 \text{ A.D.}}$

34.2 $$v = \frac{1}{\sqrt{k\mu_0 \epsilon_0}} = \frac{1}{\sqrt{1.78}}c = 0.750c = \boxed{2.25 \times 10^8 \text{ m/s}}$$

34.3 $\dfrac{E}{B} = c$ or $\dfrac{220}{B} = 3.00 \times 10^8$; so $B = 7.33 \times 10^{-7}\text{ T} = \boxed{733 \text{ nT}}$

34.4 $\dfrac{E_{\max}}{B_{\max}} = v$ is the generalized version of Equation 34.13.

$$B_{\max} = \frac{E_{\max}}{v} = \frac{7.60 \times 10^{-3}\text{ V/m}}{(2/3)(3.00 \times 10^8\text{ m/s})}\left(\frac{\text{N}\cdot\text{m}}{\text{V}\cdot\text{C}}\right)\left(\frac{\text{T}\cdot\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}}\right) = 3.80 \times 10^{-11}\text{ T} = \boxed{38.0 \text{ pT}}$$

34.5 (a) $f\lambda = c$ or $f(50.0\text{ m}) = 3.00 \times 10^8$ m/s so $\boxed{f = 6.00 \times 10^6\text{ Hz} = 6.00\text{ MHz}}$

(b) $\dfrac{E}{B} = c$ or $\dfrac{22.0}{B_{\max}} = 3.00 \times 10^8$ so $\mathbf{B}_{\max} = \boxed{(73.3\text{ nT})(-\mathbf{k})}$

(c) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{50.0} = 0.126\text{ m}^{-1}$ and $\omega = 2\pi f = 2\pi(6.00 \times 10^6\text{ s}^{-1}) = 3.77 \times 10^7$ rad/s

$$\mathbf{B} = \mathbf{B}_{\max}\cos(kx - \omega t) = \boxed{(73.3\text{ nT})\cos(0.126x - 3.77 \times 10^7 t)(-\mathbf{k})}$$

34.6 $\omega = 2\pi f = 6.00\pi \times 10^9\text{ s}^{-1} = 1.88 \times 10^{10}\text{ s}^{-1}$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{6.00\pi \times 10^9\text{ s}^{-1}}{3.00 \times 10^8\text{ m/s}} = 20.0\pi = 62.8\text{ m}^{-1} \qquad B_{\max} = \frac{E}{c} = \frac{300\text{ V/m}}{3.00 \times 10^8\text{ m/s}} = 1.00\ \mu\text{T}$$

$$\boxed{E = \left(300\ \frac{\text{V}}{\text{m}}\right)\cos\left(62.8x - 1.88 \times 10^{10}t\right)} \qquad \boxed{B = (1.00\ \mu\text{T})\cos(62.8x - 1.88 \times 10^{10}t)}$$

34.7 (a) $B = \dfrac{E}{c} = \dfrac{100\text{ V/m}}{3.00\times10^{8}\text{ m/s}} = 3.33\times10^{-7}\text{ T} = \boxed{0.333\ \mu\text{T}}$

(b) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{1.00\times10^{7}\text{ m}^{-1}} = \boxed{0.628\ \mu\text{m}}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\text{ m/s}}{6.28\times10^{-7}\text{ m}} = \boxed{4.77\times10^{14}\text{ Hz}}$

34.8 $E = E_{\max}\cos(kx - \omega t)$

$\dfrac{\partial E}{\partial x} = -E_{\max}\sin(kx-\omega t)(k)$ $\qquad$ $\dfrac{\partial E}{\partial t} = -E_{\max}\sin(kx-\omega t)(-\omega)$

$\dfrac{\partial^2 E}{\partial x^2} = -E_{\max}\cos(kx-\omega t)(k^2)$ $\qquad$ $\dfrac{\partial^2 E}{\partial t^2} = -E_{\max}\cos(kx-\omega t)(-\omega)^2$

We must show: $\dfrac{\partial E}{\partial x^2} = \mu_0 e_0 \dfrac{\partial^2 E}{\partial t^2}$

That is, $-\left(k^2\right)E_{\max}\cos\left(kx-\omega t\right) = -\mu_0 e_0(-\omega)^2 E_{\max}\cos\left(kx-\omega t\right)$

But this is true, because $\dfrac{k^2}{\omega^2} = \left(\dfrac{1}{f\lambda}\right)^2 = \dfrac{1}{c^2} = \mu_0 e_0$

The proof for the wave of magnetic field is precisely similar.

***34.9** In the fundamental mode, there is a single loop in the standing wave between the plates. Therefore, the distance between the plates is equal to half a wavelength.

$\lambda = 2L = 2(2.00\text{ m}) = 4.00\text{ m}$

Thus, $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\text{ m/s}}{4.00\text{ m}} = 7.50\times10^{7}\text{ Hz} = \boxed{75.0\text{ MHz}}$

***34.10** $d_{\text{A to A}} = 6\text{ cm} \pm 5\% = \dfrac{\lambda}{2}$

$\lambda = 12\text{ cm} \pm 5\%$

$$v = \lambda f = (0.12 \text{ m } \pm \text{ 5\%})\left(2.45\times 10^{9} \text{ s}^{-1}\right) = \boxed{2.9\times 10^{8} \text{ m/s } \pm \text{ 5\%}}$$

34.11 $S = I = \dfrac{U}{At} = \dfrac{Uc}{V} = uc$

$$\frac{\text{Energy}}{\text{Unit Volume}} = u = \frac{I}{c} = \frac{1000 \text{ W/m}^2}{3.00\times 10^{8} \text{ m/s}} = \boxed{3.33\ \mu\text{J/m}^3}$$

34.12 $S_{\text{av}} = \dfrac{\overline{\mathcal{P}}}{4\pi r^2} = \dfrac{4.00\times 10^3 \text{ W}}{4\pi(4.00\times 1609 \text{ m})^2} = 7.68\ \mu\text{W/m}^2$

$$E_{\max} = \sqrt{2\mu_0 c S_{av}} = 0.0761 \text{ V/m}$$

$$\Delta V_{\max} = E_{\max}\cdot L = (76.1 \text{ mV/m})(0.650 \text{ m}) = \boxed{49.5 \text{ mV (amplitude)}} \quad \text{or} \quad 35.0 \text{ mV (rms)}$$

34.13 $r = (5.00 \text{ mi})(1609 \text{ m/mi}) = 8.04\times 10^3 \text{ m}$

$$S = \frac{\overline{\mathcal{P}}}{4\pi r^2} = \frac{250\times 10^3 \text{ W}}{4\pi(8.04\times 10^3 \text{ m})^2} = \boxed{307\ \mu\text{W/m}^2}$$

Goal Solution

What is the average magnitude of the Poynting vector 5.00 miles from a radio transmitter broadcasting isotropically with an average power of 250 kW?

G: As the distance from the source is increased, the power per unit area will decrease, so at a distance of 5 miles from the source, the power per unit area will be a small fraction of the Poynting vector near the source.

O: The Poynting vector is the power per unit area, where A is the surface area of a sphere with a 5-mile radius.

A: The Poynting vector is $$S_{av} = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{4\pi r^2}$$

In meters, $$r = (5.00 \text{ mi})(1609 \text{ m / mi}) = 8045 \text{ m}$$

and the magnitude is $$S = \frac{250 \times 10^3 \text{ W}}{(4\pi)(8045)^2} = 3.07 \times 10^{-4} \text{ W / m}^2$$

L: The magnitude of the Poynting vector ten meters from the source is 199 W/m^2, on the order of a million times larger than it is 5 miles away! It is surprising to realize how little power is actually received by a radio (at the 5-mile distance, the signal would only be about 30 nW, assuming a receiving area of about 1 cm^2).

34.14 $$I = \frac{100 \text{ W}}{4\pi(1.00 \text{ m})^2} = 7.96 \text{ W/m}^2$$

$$u = \frac{I}{c} = 2.65 \times 10^{-8} \text{ J/m}^3 = 26.5 \text{ nJ/m}^3$$

(a) $u_E = \frac{1}{2} u = \boxed{13.3 \text{ nJ/m}^3}$

(b) $u_B = \frac{1}{2} u = \boxed{13.3 \text{ nJ/m}^3}$

(c) $I = \boxed{7.96 \text{ W/m}^2}$

34.15 Power output = (power input)(efficiency)

Thus, $$\text{Power input} = \frac{\text{power out}}{\text{eff}} = \frac{1.00 \times 10^6 \text{ W}}{0.300} = 3.33 \times 10^6 \text{ W}$$

and $$A = \frac{\mathsf{P}}{I} = \frac{3.33 \times 10^6 \text{ W}}{1.00 \times 10^3 \text{ W/m}^2} = \boxed{3.33 \times 10^3 \text{ m}^2}$$

***34.16** $I = \dfrac{B_{\max}^2 c}{2\mu_0} = \dfrac{\mathcal{P}}{4\pi r^2}$

$$B_{\max} = \sqrt{\left(\frac{\mathcal{P}}{4\pi r^2}\right)\left(\frac{2\mu_0}{c}\right)} = \sqrt{\frac{\left(10.0\times10^3\right)(2)\left(4\pi\times10^{-7}\right)}{4\pi\left(5.00\times10^3\right)^2\left(3.00\times10^8\right)}} = \boxed{5.16\times10^{-10}\ \text{T}}$$

Since the magnetic field of the Earth is approximately 5×10^{-5} T, the Earth's field is some 100,000 times stronger.

34.17 (a) $\mathcal{P} = I^2R = 150$ W; $A = 2\pi rL = 2\pi(0.900 \times 10^{-3}\ \text{m})(0.0800\ \text{m}) = 4.52 \times 10^{-4}\ \text{m}^2$

$S = \dfrac{\mathcal{P}}{A} = \boxed{332\ \text{kW/m}^2}$ (points radially inward)

(b) $B = \mu_0 \dfrac{I}{2\pi r} = \dfrac{\mu_0(1.00)}{2\pi(0.900\times10^{-3})} = \boxed{222\ \mu\text{T}}$

$E = \dfrac{\Delta V}{\Delta x} = \dfrac{IR}{L} = \dfrac{150\ \text{V}}{0.0800\ \text{m}} = \boxed{1.88\ \text{kV/m}}$

Note: $S = \dfrac{EB}{\mu_0} = 332\ \text{kW/m}^2$

34.18 (a) $\mathbf{E}\cdot\mathbf{B} = (80.0\mathbf{i} + 32.0\mathbf{j} - 64.0\mathbf{k})(\text{N/C})\cdot(0.200\mathbf{i} + 0.0800\mathbf{j} + 0.290\mathbf{k})\mu\text{T}$

$$\mathbf{E}\cdot\mathbf{B} = (16.0 + 2.56 - 18.56)\text{N}^2\cdot\text{s/C}^2\cdot\text{m} = \boxed{0}$$

(b) $$\mathbf{S} = \frac{1}{\mu_0}\,\mathbf{E}\times\mathbf{B} = \frac{(80.0\mathbf{i} + 32.0\mathbf{j} - 64.0\mathbf{k})(\text{N/C})\times(0.200\mathbf{i} + 0.0800\mathbf{j} + 0.290\mathbf{k})\mu\text{T}}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}$$

$$\mathbf{S} = \frac{(6.40\mathbf{k} - 23.2\mathbf{j} - 6.40\mathbf{k} + 9.28\mathbf{i} - 12.8\mathbf{j} + 5.12\mathbf{i})10^{-6}\ \text{W/m}^2}{4\pi\times10^{-7}}$$

$$\mathbf{S} = \boxed{(11.5\mathbf{i} - 28.6\mathbf{j})\ \text{W/m}^2} = 30.9\ \text{W/m}^2 \text{ at } -68.2° \text{ from the } +x \text{ axis}$$

34.19 We call the current I_{rms} and the intensity I. The power radiated at this frequency is

$$\mathcal{P} = (0.0100)(\Delta V_{\text{rms}})I_{\text{rms}} = \frac{0.0100(\Delta V_{\text{rms}})^2}{R} = 1.31\ \text{W}$$

If it is isotropic, the intensity one meter away is

$$I = \frac{\mathcal{P}}{A} = \frac{1.31\ \text{W}}{4\pi(1.00\ \text{m})^2} = 0.104\ \text{W/m}^2 = S_{\text{av}} = \frac{c}{2\mu_0}\,B_{\text{max}}^2$$

$$B_{\text{max}} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(0.104\ \text{W/m}^2\right)}{3.00\times10^8\ \text{m/s}}} = \boxed{29.5\ \text{nT}}$$

***34.20** (a) $$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}}\times100\% = \left(\frac{700\ \text{W}}{1400\ \text{W}}\right)\times100\% = \boxed{50.0\%}$$

(b) $$S_{\text{av}} = \frac{\mathcal{P}}{A} = \frac{700\ \text{W}}{(0.0683\ \text{m})(0.0381\ \text{m})} = 2.69\times10^5\ \text{W/m}^2$$

$$\mathbf{S}_{\text{av}} = \boxed{269\ \text{kW/m}^2 \text{ toward the oven chamber}}$$

(c) $$S_{\text{av}} = \frac{E_{\text{max}}^2}{2\mu_0 c}$$

$$E_{\text{max}} = \sqrt{2\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)\left(2.69\times10^5\ \frac{\text{W}}{\text{m}^2}\right)} = 1.42\times10^4\ \frac{\text{V}}{\text{m}} = \boxed{14.2\ \text{kV/m}}$$

34.21 (a) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{7.00 \times 10^5 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.33 \text{ mT}}$

(b) $I = \dfrac{E_{\max}^2}{2\mu_0 c} = \dfrac{(7.00 \times 10^5)^2}{2(4\pi \times 10^{-7})(3.00 \times 10^8)} = \boxed{650 \text{ MW/m}^2}$

(c) $I = \dfrac{\mathcal{P}}{A}$: $\mathcal{P} = IA = (6.50 \times 10^8 \text{ W/m}^2)\dfrac{\pi}{4}(1.00 \times 10^{-3} \text{ m})^2 = \boxed{510 \text{ W}}$

34.22 Power $= SA = \dfrac{E_{\max}^2}{2\mu_0 c}(4\pi r^2)$; solving for r, $r = \sqrt{\dfrac{\mathcal{P}\mu_0 c}{E_{\max}^2 2\pi}} = \sqrt{\dfrac{(100 \text{ W})\mu_0 c}{2\pi(15.0 \text{ V/m})^2}} = \boxed{5.16 \text{ m}}$

34.23 (a) $I = \dfrac{(10.0 \times 10^{-3})\text{W}}{\pi(0.800 \times 10^{-3} \text{ m})^2} = \boxed{4.97 \text{ kW/m}^2}$

(b) $u_{\text{av}} = \dfrac{I}{c} = \dfrac{4.97 \times 10^3 \text{ J/m}^2 \cdot \text{s}}{3.00 \times 10^8 \text{ m/s}} = \boxed{16.6\ \mu\text{J/m}^3}$

34.24 (a) $E = cB = (3.00 \times 10^8 \text{ m/s})(1.80 \times 10^{-6} \text{ T}) = \boxed{540 \text{ V/m}}$

(b) $u_{\text{av}} = \dfrac{B^2}{\mu_0} = \dfrac{(1.80 \times 10^{-6})^2}{4\pi \times 10^{-7}} = \boxed{2.58\ \mu\text{J/m}^3}$

(c) $S_{\text{av}} = cu_{\text{av}} = (3.00 \times 10^8)(2.58 \times 10^{-6}) = \boxed{773 \text{ W/m}^2}$

(d) This is $\boxed{\text{77.3\% of the flux in Example 34.5}}$. It may be cloudy, or the Sun may be setting.

34.25 For complete absorption, $P = \dfrac{S}{c} = \dfrac{25.0}{3.00 \times 10^8} = \boxed{83.3 \text{ nPa}}$

***34.26** (a) $\mathcal{P} = (S_{av})(A) = (6.00 \text{ W/m}^2)(40.0 \times 10^{-4} \text{ m}^2) = 2.40 \times 10^{-2} \text{ J/s}$

In one second, the total energy U impinging on the mirror is 2.40×10^{-2} J. The momentum p transferred each second for total reflection is

$p = \dfrac{2U}{c} = \dfrac{2(2.40 \times 10^{-2} \text{ J})}{3.00 \times 10^8 \text{ m/s}} = \boxed{1.60 \times 10^{-10} \dfrac{\text{kg} \cdot \text{m}}{\text{s}}}$ (each second)

(b) $F = \dfrac{dp}{dt} = \dfrac{1.60 \times 10^{-10} \text{ kg} \cdot \text{m/s}}{1 \text{ s}} = \boxed{1.60 \times 10^{-10} \text{ N}}$

34.27 (a) The radiation pressure is $\frac{(2)(1340 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}^2} = 8.93 \times 10^{-6} \text{ N/m}^2$

Multiplying by the total area, $A = 6.00 \times 10^5 \text{ m}^2$ gives: $F = \boxed{5.36 \text{ N}}$

(b) The acceleration is: $a = \frac{F}{m} = \frac{5.36 \text{ N}}{6000 \text{ kg}} = \boxed{8.93 \times 10^{-4} \text{ m/s}^2}$

(c) It will take a time t where: $d = \frac{1}{2}at^2$

or $t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2\left(3.84 \times 10^8 \text{ m}\right)}{\left(8.93 \times 10^{-4} \text{ m/s}^2\right)}} = 9.27 \times 10^5 \text{ s} = \boxed{10.7 \text{ days}}$

34.28 The pressure P upon the mirror is $P = \frac{2S_{\text{av}}}{c}$

where A is the cross-sectional area of the beam and $S_{\text{av}} = \frac{\mathcal{P}}{A}$

The force on the mirror is then $F = PA = \frac{2}{c}\left(\frac{\mathcal{P}}{A}\right)A = \frac{2\mathcal{P}}{c}$

Therefore, $F = \frac{2(100 \times 10^{-3})}{(3 \times 10^8)} = \boxed{6.67 \times 10^{-10} \text{ N}}$

34.29 $I = \frac{\mathcal{P}}{\pi r^2} = \frac{E_{\max}^2}{2\mu_0 c}$

(a) $E_{\max} = \sqrt{\frac{\mathcal{P}\left(2\mu_0 c\right)}{\pi r^2}} = \boxed{1.90 \text{ kN/C}}$

(b) $\frac{15 \times 10^{-3} \text{ J/s}}{3.00 \times 10^8 \text{ m/s}}(1.00 \text{ m}) = \boxed{50.0 \text{ pJ}}$

(c) $p = \frac{U}{c} = \frac{5 \times 10^{-11}}{3.00 \times 10^8} = \boxed{1.67 \times 10^{-19} \text{ kg} \cdot \text{m/s}}$

34.30 (a) If P_S is the total power radiated by the Sun, and r_E and r_M are the radii of the orbits of the planets Earth and Mars, then the intensities of the solar radiation at these planets are:

$$I_E = \frac{\mathsf{P}_S}{4\pi r_E^2} \qquad \text{and} \qquad I_M = \frac{\mathsf{P}_S}{4\pi r_M^2}$$

Thus, $$I_M = I_E\left(\frac{r_E}{r_M}\right)^2 = \left(1340\ \text{W/m}^2\right)\left(\frac{1.496\times 10^{11}\ \text{m}}{2.28\times 10^{11}\ \text{m}}\right)^2 = \boxed{577\ \text{W/m}^2}$$

(b) Mars intercepts the power falling on its circular face:

$$\mathsf{P}_M = I_M\left(\pi R_M^2\right) = \left(577\ \text{W/m}^2\right)\pi\left(3.37\times 10^6\ \text{m}\right)^2 = \boxed{2.06\times 10^{16}\ \text{W}}$$

(c) If Mars behaves as a perfect absorber, it feels pressure $P = \dfrac{S_M}{c} = \dfrac{I_M}{c}$

and force $F = PA = \dfrac{I_M}{c}\left(\pi R_M^2\right) = \dfrac{\mathsf{P}_M}{c} = \dfrac{2.06\times 10^{16}\ \text{W}}{3.00\times 10^8\ \text{m/s}} = \boxed{6.87\times 10^7\ \text{N}}$

(d) The attractive gravitational force exerted on Mars by the Sun is

$$F_g = \frac{GM_SM_M}{r_M^2} = \frac{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times 10^{30}\ \text{kg}\right)\left(6.42\times 10^{23}\ \text{kg}\right)}{\left(2.28\times 10^{11}\ \text{m}\right)^2} = 1.64\times 10^{21}\ \text{N}$$

which is $\boxed{\sim 10^{13}\ \text{times stronger}}$ than the repulsive force of (c).

34.31 (a) The total energy absorbed by the surface is

$$U = \left(\tfrac{1}{2}I\right)At = \left[\tfrac{1}{2}\left(750\ \frac{\text{W}}{\text{m}^2}\right)\right]\left(0.500\times 1.00\ \text{m}^2\right)\left(60.0\ \text{s}\right) = \boxed{11.3\ \text{kJ}}$$

(b) The total energy incident on the surface in this time is $2U = 22.5$ kJ, with $U = 11.3$ kJ being absorbed and $U = 11.3$ kJ being reflected. The total momentum transferred to the surface is

$$p = \left(\text{momentum from absorption}\right) + \left(\text{momentum from reflection}\right)$$

$$p = \left(\frac{U}{c}\right) + \left(\frac{2U}{c}\right) = \frac{3U}{c} = \frac{3\left(11.3\times 10^3\ \text{J}\right)}{3.00\times 10^8\ \text{m/s}} = \boxed{1.13\times 10^{-4}\ \text{kg}\cdot\text{m/s}}$$

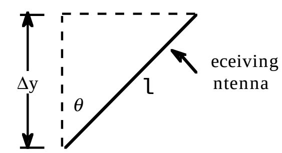
34.32 $S_{\text{av}} = \dfrac{\mu_0 J_{\max}^2 c}{8}$ or $570 = \dfrac{(4\pi\times 10^{-7})J_{\max}^2(3.00\times 10^8)}{8}$ so $\boxed{J_{\max} = 3.48\ \text{A/m}^2}$

34.33 (a) $\mathsf{P} = S_{\text{av}} A = \left(\dfrac{\mu_0 J_{\max}^2 c}{8}\right) A$

$$\mathsf{P} = \left(\frac{4\pi \times 10^{-7}(10.0)^2(3.00\times 10^8)}{8}\right)(1.20 \times 0.400) = \boxed{2.26\text{ kW}}$$

(b) $S_{\text{av}} = \dfrac{\mu_0 J_{\max}^2 c}{8} = \dfrac{(4\pi \times 10^{-7}(10.0)^2(3.00\times 10^8)}{8} = \boxed{4.71\text{ kW/m}^2}$

***34.34** $\mathsf{P} = \dfrac{(\Delta V)^2}{R}$ or $\mathsf{P} \propto (\Delta V)^2$



$\Delta V = (-)E_y \cdot \Delta y = E_y \cdot l \cos\theta$

$\Delta V \propto \cos\theta$ so $\mathsf{P} \propto \cos^2\theta$

(a) $\theta = 15.0°$: $\mathsf{P} = \mathsf{P}_{\max}\cos^2(15.0°) = 0.933\mathsf{P}_{\max} = \boxed{93.3\%}$

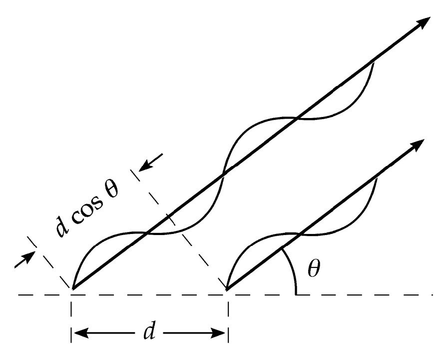
(b) $\theta = 45.0°$: $\mathsf{P} = \mathsf{P}_{\max}\cos^2(45.0°) = 0.500\mathsf{P}_{\max} = \boxed{50.0\%}$

(c) $\theta = 90.0°$: $\mathsf{P} = \mathsf{P}_{\max}\cos^2(90.0°) = \boxed{0}$

34.35 (a) Constructive interference occurs when $d\cos\theta = n\lambda$ for some integer n.

$\cos\theta = n\dfrac{\lambda}{d} = n\left(\dfrac{\lambda}{\lambda/2}\right) = 2n \quad n = 0, \pm 1, \pm 2, \ldots$

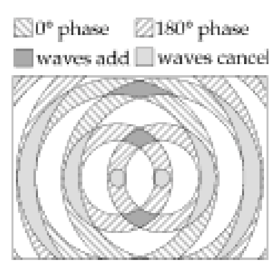
$\therefore$ $\boxed{\text{strong signal @ } \theta = \cos^{-1} 0 = 90°, 270°}$



(b) Destructive interference occurs when

$d\cos\theta = \left(\dfrac{2n+1}{2}\right)\lambda$: $\qquad \cos\theta = 2n + 1$

$\therefore$ $\boxed{\text{weak signal @ } \theta = \cos^{-1}(\pm 1) = 0°, 180°}$



Goal Solution

Two radio-transmitting antennas are separated by half the broadcast wavelength and are driven in phase with each other. In which directions are (a) the strongest and (b) the weakest signals radiated?

G: The strength of the radiated signal will be a function of the location around the two antennas and will depend on the interference of the waves.

O: A diagram helps to visualize this situation. The two antennas are driven in phase, which means that they both create maximum electric field strength at the same time, as shown in the diagram. The radio EM waves travel radially outwards from the antennas, and the received signal will be the vector sum of the two waves.

A: (a) Along the perpendicular bisector of the line joining the antennas, the distance is the same to both transmitting antennas. The transmitters oscillate in phase, so along this line the two signals will be received in phase, constructively interfering to produce a maximum signal strength that is twice the amplitude of one transmitter.

(b) Along the extended line joining the sources, the wave from the more distant antenna must travel one-half wavelength farther, so the waves are received 180° out of phase. They interfere destructively to produce the weakest signal with zero amplitude.

L: Radio stations may use an antenna array to direct the radiated signal toward a highly-populated region and reduce the signal strength delivered to a sparsely-populated area.

34.36 $\lambda = \dfrac{c}{f} = 536\text{ m}$ so $h = \dfrac{\lambda}{4} = \boxed{134\text{ m}}$

$\lambda = \dfrac{c}{f} = 188\text{ m}$ so $h = \dfrac{\lambda}{4} = \boxed{46.9\text{ m}}$

34.37 For the proton: $\Sigma F = ma \Rightarrow qvB\sin 90.0° = mv^2/R$

The period and frequency of the proton’s circular motion are therefore:

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB} = \frac{2\pi\left(1.67\times10^{-27}\text{ kg}\right)}{\left(1.60\times10^{-19}\text{ C}\right)(0.350\text{ T})} = 1.87\times10^{-7}\text{ s} \qquad f = 5.34\times10^{6}\text{ Hz}.$$

The charge will radiate at this same frequency, with $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{5.34\times10^{6}\text{ Hz}} = \boxed{56.2\text{ m}}$

34.38 For the proton, $\Sigma F = ma$ yields $qvB\sin 90.0° = \dfrac{mv^2}{R}$

The period of the proton’s circular motion is therefore: $T = \dfrac{2\pi R}{v} = \dfrac{2\pi m}{qB}$

The frequency of the proton's motion is $f = 1/T$

The charge will radiate electromagnetic waves at this frequency, with $\lambda = \dfrac{c}{f} = cT = \boxed{\dfrac{2\pi mc}{qB}}$

***34.39** From the electromagnetic spectrum chart and accompanying text discussion, the following identifications are made:

Frequency f	Wavelength, $\lambda = c/f$	Classification
$2\text{ Hz} = 2\times 10^0\text{ Hz}$	150 Mm	Radio
$2\text{ kHz} = 2\times 10^3\text{ Hz}$	150 km	Radio
$2\text{ MHz} = 2\times 10^6\text{ Hz}$	150 m	Radio
$2\text{ GHz} = 2\times 10^9\text{ Hz}$	15 cm	Microwave
$2\text{ THz} = 2\times 10^{12}\text{ Hz}$	150 μm	Infrared
$2\text{ PHz} = 2\times 10^{15}\text{ Hz}$	150 nm	Ultraviolet
$2\text{ EHz} = 2\times 10^{18}\text{ Hz}$	150 pm	x-ray
$2\text{ ZHz} = 2\times 10^{21}\text{ Hz}$	150 fm	Gamma ray
$2\text{ YHz} = 2\times 10^{24}\text{ Hz}$	150 am	Gamma Ray

Wavelength, λ	Frequency $f = c/\lambda$	Classification
$2\text{ km} = 2\times 10^3\text{ m}$	$1.5\times 10^5\text{ Hz}$	Radio
$2\text{ m} = 2\times 10^0\text{ m}$	$1.5\times 10^8\text{ Hz}$	Radio
$2\text{ mm} = 2\times 10^{-3}\text{ m}$	$1.5\times 10^{11}\text{ Hz}$	Microwave
$2\ \mu\text{m} = 2\times 10^{-6}\text{ m}$	$1.5\times 10^{14}\text{ Hz}$	Infrared
$2\text{ nm} = 2\times 10^{-9}\text{ m}$	$1.5\times 10^{17}\text{ Hz}$	Ultraviolet/x-ray
$2\text{ pm} = 2\times 10^{-12}\text{ m}$	$1.5\times 10^{20}\text{ Hz}$	x-ray/Gamma ray
$2\text{ fm} = 2\times 10^{-15}\text{ m}$	$1.5\times 10^{23}\text{ Hz}$	Gamma ray
$2\text{ am} = 2\times 10^{-18}\text{ m}$	$1.5\times 10^{26}\text{ Hz}$	Gamma ray

***34.40** (a) $f = \dfrac{c}{\lambda} = \dfrac{3\times 10^8\text{ m/s}}{1.7\text{ m}}\ \boxed{\sim 10^8\text{ Hz}}\ \boxed{\text{radio wave}}$

(b) 1000 pages, 500 sheets, is about 3 cm thick so one sheet is about 6×10^{-5} m thick

$$f = \frac{3\times 10^8\text{ m/s}}{6\times 10^{-5}\text{ m}}\ \boxed{\sim 10^{13}\text{ Hz}}\ \boxed{\text{infrared}}$$

***34.41** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times 10^8\text{ m/s}}{5.50\times 10^{-7}\text{ m}} = \boxed{5.45\times 10^{14}\text{ Hz}}$

34.42 (a) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^8\text{ m/s}}{1150\times 10^3/\text{s}} = 261\text{ m}$ so $\dfrac{180\text{ m}}{261\text{ m}} = \boxed{0.690\text{ wavelengths}}$

(b) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^8\text{ m/s}}{98.1\times 10^6/\text{s}} = 3.06\text{ m}$ so $\dfrac{180\text{ m}}{3.06\text{ m}} = \boxed{58.9\text{ wavelengths}}$

34.43 (a) $f\lambda = c$ gives $(5.00 \times 10^{19}\text{ Hz})\lambda = 3.00 \times 10^{8}\text{ m/s}$: $\boxed{\lambda = 6.00 \times 10^{-12}\text{ m} = 6.00\text{ pm}}$

(b) $f\lambda = c$ gives $(4.00 \times 10^{9}\text{ Hz})\lambda = 3.00 \times 10^{8}\text{ m/s}$: $\boxed{\lambda = 0.075\text{ m} = 7.50\text{ cm}}$

***34.44** Time to reach object $= \frac{1}{2}$(total time of flight) $= \frac{1}{2}(4.00 \times 10^{-4}\text{ s}) = 2.00 \times 10^{-4}\text{ s}$

Thus, $d = vt = (3.00 \times 10^{8}\text{ m/s})(2.00 \times 10^{-4}\text{ s}) = 6.00 \times 10^{4}\text{ m} = \boxed{60.0\text{ km}}$

34.45 The time for the radio signal to travel 100 km is: $t_r = \dfrac{100 \times 10^{3}\text{ m}}{3.00 \times 10^{8}\text{ m/s}} = 3.33 \times 10^{-4}\text{ s}$

The sound wave to travel 3.00 m across the room in: $t_s = \dfrac{3.00\text{ m}}{343\text{ m/s}} = 8.75 \times 10^{-3}\text{ s}$

Therefore, $\boxed{\text{listeners 100 km away}}$ will receive the news before the people in the newsroom by a total time difference of

$\Delta t = 8.75 \times 10^{-3}\text{ s} - 3.33 \times 10^{-4}\text{ s} = 8.41 \times 10^{-3}\text{ s}$

***34.46** The wavelength of an ELF wave of frequency 75.0 Hz is $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\text{ m/s}}{75.0\text{ Hz}} = 4.00 \times 10^{6}\text{ m}$

The length of a quarter-wavelength antenna would be $L = 1.00 \times 10^{6}\text{ m} = \boxed{1.00 \times 10^{3}\text{ km}}$

or $L = (1000\text{ km})\left(\dfrac{0.621\text{ mi}}{1.00\text{ km}}\right) = \boxed{621\text{ mi}}$

Thus, while the project may be theoretically possible, it is not very practical.

34.47 (a) For the AM band, $\lambda_{\max} = \dfrac{c}{f_{\min}} = \dfrac{3.00 \times 10^{8}\text{ m/s}}{540 \times 10^{3}\text{ Hz}} = \boxed{556\text{ m}}$

$$\lambda_{\min} = \dfrac{c}{f_{\max}} = \dfrac{3.00 \times 10^{8}\text{ m/s}}{1600 \times 10^{3}\text{ Hz}} = \boxed{187\text{ m}}$$

(b) For the FM band, $\lambda_{\max} = \dfrac{c}{f_{\min}} = \dfrac{3.00 \times 10^{8}\text{ m/s}}{88.0 \times 10^{6}\text{ Hz}} = \boxed{3.41\text{ m}}$

$$\lambda_{\min} = \dfrac{c}{f_{\max}} = \dfrac{3.00 \times 10^{8}\text{ m/s}}{108 \times 10^{6}\text{ Hz}} = \boxed{2.78\text{ m}}$$

34.48 CH_4: $f_{\min} = 66$ MHz $\quad \lambda_{\max} = \boxed{4.55\text{ m}}$

$f_{\max} = 72$ MHz $\quad \lambda_{\min} = \boxed{4.17\text{ m}}$

CH_6: $f_{\min} = 82$ MHz $\quad \lambda_{\max} = \boxed{3.66\text{ m}}$

$f_{\max} = 88$ MHz $\quad \lambda_{\min} = \boxed{3.41\text{ m}}$

CH_8: $f_{\min} = 180$ MHz $\quad \lambda_{\max} = \boxed{1.67\text{ m}}$

$f_{\max} = 186$ MHz $\quad \lambda_{\min} = \boxed{1.61\text{ m}}$

34.49 (a) $\mathscr{P} = SA = (1340\text{ W/m}^2)4\pi(1.496 \times 10^{11}\text{ m})^2 = \boxed{3.77 \times 10^{26}\text{ W}}$

(b) $S = \dfrac{cB_{\max}^2}{2\mu_0}$ so $\quad B_{\max} = \sqrt{\dfrac{2\mu_0 S}{c}} = \sqrt{\dfrac{2(4\pi \times 10^{-7}\text{ N/A}^2)(1340\text{ W/m}^2)}{3.00 \times 10^8\text{ m/s}}} = \boxed{3.35\ \mu\text{T}}$

$S = \dfrac{E_{\max}^2}{2\mu_0 c}$ so $\quad E_{\max} = \sqrt{2\mu_0 cS} = \sqrt{2\left(4\pi \times 10^{-7}\right)\left(3.00 \times 10^8\right)(1340)} = \boxed{1.01\text{ kV/m}}$

***34.50** Suppose you cover a 1.7 m-by-0.3 m section of beach blanket. Suppose the elevation angle of the Sun is 60°. Then the target area you fill in the Sun's field of view is

$(1.7\text{ m})(0.3\text{ m})(\cos 30°) = 0.4\text{ m}^2$

Now $I = \dfrac{\mathscr{P}}{A} = \dfrac{E}{At}$: $\quad E = IAt = 1340\ \dfrac{\text{W}}{\text{m}^2}(0.6)(0.5)(0.4\text{ m}^2)\ \ 3600\text{ s}\ \boxed{\sim 10^6\text{ J}}$

34.51 (a) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}(BA\cos\theta) = -A\dfrac{d}{dt}(B_{\max}\cos\omega t\cos\theta) = AB_{\max}\,\omega(\sin\omega t\cos\theta)$

$\mathcal{E}(t) = 2\pi f B_{\max} A\sin 2\pi f t\cos\theta = 2\pi^2 r^2 f B_{\max}\cos\theta\sin 2\pi f t$

Thus, $\boxed{\mathcal{E}_{\max} = 2\pi^2 r^2 f B_{\max}\cos\theta}$, where θ is the angle between the magnetic field and the normal to the loop.

(b) If **E** is vertical, then **B** is horizontal, so the $\boxed{\text{plane of the loop should be vertical}}$ and the $\boxed{\text{plane should contain the line of sight to the transmitter}}$.

34.52 (a) $F_{grav} = \dfrac{GM_s m}{R^2} = \left(\dfrac{GM_s}{R^2}\right)\rho(4/3)\pi r^3$

where M_s = mass of Sun, r = radius of particle and R = distance from Sun to particle.

Since $F_{\text{rad}} = \dfrac{S\pi r^2}{c}$, $\qquad \dfrac{F_{rad}}{F_{grav}} = \left(\dfrac{1}{r}\right)\left(\dfrac{3SR^2}{4cGM_s\rho}\right) \propto \dfrac{1}{r}$

(b) From the result found in part (a), when $F_{\text{grav}} = F_{\text{rad}}$, we have $\qquad r = \dfrac{3SR^2}{4cGM_s\,\rho}$

$$r = \frac{3\left(214\ \text{W/m}^2\right)\left(3.75\times10^{11}\ \text{m}\right)^2}{4\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)\left(1500\ \text{kg/m}^3\right)\left(3.00\times10^8\ \text{m/s}\right)} = \boxed{3.78\times10^{-7}\ \text{m}}$$

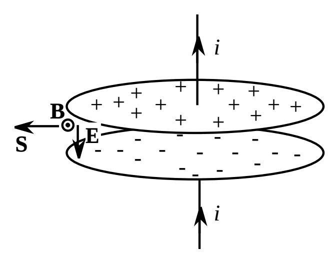
34.53 (a) $B_{\text{max}} = \dfrac{E_{\text{max}}}{c} = \boxed{6.67\times10^{-16}\ \text{T}}$

(b) $S_{\text{av}} = \dfrac{E_{\text{max}}^2}{2\mu_0 c} = \boxed{5.31\times10^{-17}\ \text{W/m}^2}$

(c) $\mathcal{P} = S_{\text{av}}A = \boxed{1.67\times10^{-14}\ \text{W}}$

(d) $F = PA = \left(\dfrac{S_{av}}{c}\right)A = \boxed{5.56\times10^{-23}\ \text{N}}$ (≈ weight of 3000 H atoms!)

***34.54** (a) The electric field between the plates is $E = \Delta V/\ell$, directed downward in the figure. The magnetic field between the plate's edges is $B = \mu_0 i/2\pi r$ counterclockwise.



The Poynting vector is: $\quad \mathbf{S} = \dfrac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = \boxed{\dfrac{(\Delta V)i}{2\pi r\ell}\ \text{(radially outward)}}$

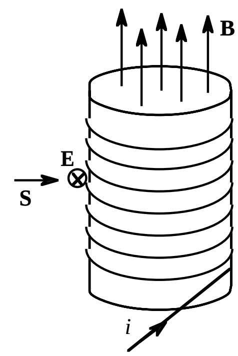
(b) The lateral surface area surrounding the electric field volume is

$A = 2\pi r\ell$, so the power output is $\mathcal{P} = SA = \left(\dfrac{(\Delta V)i}{2\pi r\ell}\right)(2\pi r\ell) = \boxed{(\Delta V)i}$

(c) As the capacitor charges, the polarity of the plates and hence the direction of the electric field is unchanged. Reversing the current reverses the direction of the magnetic field, and therefore the Poynting vector.

$\boxed{\text{The Poynting vector is now directed radially inward.}}$

***34.55** (a) The magnetic field in the enclosed volume is directed upward, with magnitude $B = \mu_0 ni$ and increasing at the rate $\frac{dB}{dt} = \mu_0 n \frac{di}{dt}$. The changing magnetic field induces an electric field around any circle of radius r, according to Faraday's Law:



$$E(2\pi r) = -\mu_0 n\left(\frac{di}{dt}\right)\left(\pi r^2\right) \qquad E = -\frac{\mu_0 nr}{2}\left(\frac{di}{dt}\right)$$

or $$\mathbf{E} = \frac{\mu_0 nr}{2}\left(\frac{di}{dt}\right) \text{ (clockwise)}$$

Then, $$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = \frac{1}{\mu_0}\left[\frac{\mu_0 nr}{2}\left(\frac{di}{dt}\right)\right](\mu_0 ni) \text{ inward,}$$

or the Poynting vector is $$\mathbf{S} = \boxed{\frac{\mu_0 n^2 ri}{2}\left(\frac{di}{dt}\right) \text{ (radially inward)}}$$

(b) The power flowing into the volume is $\mathcal{P} = SA_{\text{lat}}$ where A_{lat} is the lateral area perpendicular to $\mathbf{S}$. Therefore,

$$\mathcal{P} = \left[\frac{\mu_0 n^2 ri}{2}\left(\frac{di}{dt}\right)\right](2\pi r\ell) = \boxed{\mu_0 \pi n^2 r^2 \ell i\left(\frac{di}{dt}\right)}$$

(c) Taking A_{cross} to be the cross-sectional area perpendicular to $\mathbf{B}$, the induced voltage between the ends of the inductor, which has $N = n\ell$ turns, is

$$\Delta V = |\mathcal{E}| = N\left(\frac{dB}{dt}\right)A_{\text{cross}} = n\ell\left(\mu_0 n\frac{di}{dt}\right)\left(\pi r^2\right) = \mu_0 \pi n^2 r^2 \ell\left(\frac{di}{dt}\right)$$

and it is observed that $$\boxed{\mathcal{P} = (\Delta V)i}$$

***34.56** (a) The power incident on the mirror is: $$\mathcal{P}_I = IA = \left(1340\ \frac{\text{W}}{\text{m}^2}\right)\left[\pi(100\text{ m})^2\right] = 4.21\times10^7\text{ W}$$

The power reflected through the atmosphere is $$\mathcal{P}_R = 0.746\left(4.21\times10^7\text{ W}\right) = \boxed{3.14\times10^7\text{ W}}$$

(b) $$S = \frac{\mathcal{P}_R}{A} = \frac{3.14\times10^7\text{ W}}{\pi\left(4.00\times10^3\text{ m}\right)^2} = \boxed{0.625\text{ W/m}^2}$$

(c) Noon sunshine in Saint Petersburg produces this power-per-area on a horizontal surface:

$$\mathcal{P}_N = 0.746\left(1340\text{ W/m}^2\right)\sin 7.00° = 122\text{ W/m}^2$$

The radiation intensity received from the mirror is

$$\left(\frac{0.625\text{ W/m}^2}{122\text{ W/m}^2}\right)100\% = \boxed{0.513\%}$$ of that from the noon Sun in January.

34.57 $u = \frac{1}{2}\epsilon_0 E_{\max}^2$ (Equation 34.21)

$$E_{\max} = \sqrt{\frac{2u}{\epsilon_0}} = \boxed{95.1\ \mathrm{mV/m}}$$

***34.58** The area over which we model the antenna as radiating is the lateral surface of a cylinder,

$$A = 2\pi r \ell = 2\pi\left(4.00\times10^{-2}\ \mathrm{m}\right)(0.100\ \mathrm{m}) = 2.51\times10^{-2}\ \mathrm{m}^2$$

(a) The intensity is then: $S = \frac{\mathcal{P}}{A} = \frac{0.600\ \mathrm{W}}{2.51\times10^{-2}\ \mathrm{m}^2} = \boxed{23.9\ \mathrm{W/m^2}}$

(b) The standard is: $0.570\ \frac{\mathrm{mW}}{\mathrm{cm}^2} = 0.570\left(\frac{\mathrm{mW}}{\mathrm{cm}^2}\right)\left(\frac{1.00\times10^{-3}\ \mathrm{W}}{1.00\ \mathrm{mW}}\right)\left(\frac{1.00\times10^{4}\ \mathrm{cm}^2}{1.00\ \mathrm{m}^2}\right) = 5.70\ \frac{\mathrm{W}}{\mathrm{m}^2}$

While it is on, the telephone is over the standard by $\frac{23.9\ \mathrm{W/m^2}}{5.70\ \mathrm{W/m^2}} = \boxed{4.19\ \text{times}}$

34.59 (a) $B_{\max} = \frac{E_{\max}}{c} = \frac{175\ \mathrm{V/m}}{3.00\times10^{8}\ \mathrm{m/s}} = \boxed{5.83\times10^{-7}\ \mathrm{T}}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.0150\ \mathrm{m})} = \boxed{419\ \mathrm{rad/m}}$$

$$\omega = kc = \boxed{1.26\times10^{11}\ \mathrm{rad/s}}$$

Since **S** is along x, and **E** is along y, **B** must be in $\boxed{\text{the } z \text{ direction}}$. (That is $\mathbf{S} \propto \mathbf{E}\times\mathbf{B}$.)

(b) $S_{\text{av}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \boxed{40.6\ \mathrm{W/m^2}}$

(c) $P_r = \frac{2S}{c} = \boxed{2.71\times10^{-7}\ \mathrm{N/m^2}}$

(d) $a = \frac{\Sigma F}{m} = \frac{PA}{m} = \frac{\left(2.71\times10^{-7}\ \mathrm{N/m^2}\right)\left(0.750\ \mathrm{m^2}\right)}{0.500\ \mathrm{kg}} = \boxed{4.06\times10^{-7}\ \mathrm{m/s^2}}$

***34.60** (a) At steady-state, $\mathcal{P}_{in} = \mathcal{P}_{out}$ and the power radiated out is $\mathcal{P}_{out} = e\sigma AT^4$.

Thus, $$0.900\left(1000\ \frac{\text{W}}{\text{m}^2}\right)A = (0.700)\left(5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)AT^4$$

or $$T = \left[\frac{900\ \text{W/m}^2}{0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{388\ \text{K}} = 115°\text{C}$$

(b) The box of horizontal area A, presents projected area $A\sin 50.0°$ perpendicular to the sunlight. Then by the same reasoning,

$$0.900\left(1000\ \frac{\text{W}}{\text{m}^2}\right)A\sin 50.0° = (0.700)\left(5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)AT^4$$

or $$T = \left[\frac{\left(900\ \text{W/m}^2\right)\sin 50.0°}{0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{363\ \text{K}} = 90.0\ °\text{C}$$

34.61 (a) $P = \dfrac{F}{A} = \dfrac{I}{c}$

$$F = \frac{IA}{c} = \frac{\mathcal{P}}{c} = \frac{100\ \text{J/s}}{3.00\times10^8\ \text{m/s}} = 3.33\times10^{-7}\ \text{N} = (110\ \text{kg})a$$

$a = 3.03\times10^{-9}\ \text{m/s}^2$ and $x = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2x}{a}} = 8.12\times10^4\ \text{s} = \boxed{22.6\ \text{h}}$$

(b) $0 = (107\ \text{kg})v - (3.00\ \text{kg})(12.0\ \text{m/s} - v) = (107\ \text{kg})v - 36.0\ \text{kg}\cdot\text{m/s} + (3.00\ \text{kg})v$

$$v = \frac{36.0}{110} = 0.327\ \text{m/s}$$

$t = \boxed{30.6\ \text{s}}$

Goal Solution

An astronaut, stranded in space 10.0 m from his spacecraft and at rest relative to it, has a mass (including equipment) of 110 kg. Since he has a 100-W light source that forms a directed beam, he decides to use the beam as a photon rocket to propel himself continuously toward the spacecraft. (a) Calculate how long it takes him to reach the spacecraft by this method. (b) Suppose, instead, he decides to throw the light source away in a direction opposite the spacecraft. If the mass of the light source has a mass of 3.00 kg and, after being thrown, moves at 12.0 m/s **relative to the recoiling astronaut**, how long does it take for the astronaut to reach the spacecraft?

G: Based on our everyday experience, the force exerted by photons is too small to feel, so it may take a very long time (maybe days!) for the astronaut to travel 10 m with his "photon rocket." Using the momentum of the thrown light seems like a better solution, but it will still take a while (maybe a few minutes) for the astronaut to reach the spacecraft because his mass is so much larger than the mass of the light source.

O: In part (a), the radiation pressure can be used to find the force that accelerates the astronaut toward the spacecraft. In part (b), the principle of conservation of momentum can be applied to find the time required to travel the 10 m.

A: (a) Light exerts on the astronaut a pressure $P = F/A = S/c$, and a force of

$$F = \frac{SA}{c} = \frac{\mathcal{P}}{c} = \frac{100 \text{ J / s}}{3.00 \times 10^{8} \text{ m / s}} = 3.33 \times 10^{-7} \text{ N}$$

By Newton's 2nd law, $$a = \frac{F}{m} = \frac{3.33 \times 10^{-7} \text{ N}}{110 \text{ kg}} = 3.03 \times 10^{-9} \text{ m / s}^2$$

This acceleration is constant, so the distance traveled is $x = \frac{1}{2}at^2$, and the amount of time it travels is

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(10.0 \text{ m})}{3.03 \times 10^{-9} \text{ m / s}^2}} = 8.12 \times 10^{4} \text{ s} = 22.6 \text{ h}$$

(b) Because there are no external forces, the momentum of the astronaut before throwing the light is the same as afterwards when the now 107-kg astronaut is moving at speed v towards the spacecraft and the light is moving away from the spacecraft at $(12.0 \text{ m / s} - v)$. Thus, $\mathbf{p}_i = \mathbf{p}_f$ gives

$$0 = (107 \text{ kg})v - (3.00 \text{ kg})(12.0 \text{ m / s} - v)$$

$$0 = (107 \text{ kg})v - (36.0 \text{ kg} \cdot \text{m / s}) + (3.00 \text{ kg})v$$

$$v = \frac{36.0}{110} = 0.327 \text{ m / s}$$

$$t = \frac{x}{v} = \frac{10.0 \text{ m}}{0.327 \text{ m / s}} = 30.6 \text{ s}$$

L: Throwing the light away is certainly a more expedient way to reach the spacecraft, but there is not much chance of retrieving the lamp unless it has a very long cord. How long would the cord need to be, and does its length depend on how hard the astronaut throws the lamp? (You should verify that the minimum cord length is 367 m, independent of the speed that the lamp is thrown.)

34.62 The 38.0% of the intensity $S = 1340\ \dfrac{\text{W}}{\text{m}^2}$ that is reflected exerts a pressure $P_1 = \dfrac{2S_r}{c} = \dfrac{2(0.380)S}{c}$

The absorbed light exerts pressure $P_2 = \dfrac{S_a}{c} = \dfrac{(0.620)S}{c}$

Altogether the pressure at the subsolar point on Earth is

(a) $P_{tot} = P_1 + P_2 = \dfrac{1.38S}{c} = \dfrac{1.38(1340\ \text{W/m}^2)}{3.00 \times 10^8\ \text{m/s}} = \boxed{6.16 \times 10^{-6}\ \text{Pa}}$

(b) $\dfrac{P_a}{P_{tot}} = \dfrac{1.01 \times 10^5\ \text{N/m}^2}{6.16 \times 10^{-6}\ \text{N/m}^2} = \boxed{1.64 \times 10^{10}\ \text{times smaller than atmospheric pressure}}$

34.63 Think of light going up and being absorbed by the bead which presents a face area πr_b^2 .

The light pressure is $P = \dfrac{S}{c} = \dfrac{I}{c}$.

(a) $F_l = \dfrac{I\pi r_b^2}{c} = mg = \rho \dfrac{4}{3} \pi r_b^3\ g$ and $I = \dfrac{4\rho gc}{3}\left(\dfrac{3m}{4\pi\rho}\right)^{1/3} = \boxed{8.32 \times 10^7\ \text{W/m}^2}$

(b) $\mathscr{P} = IA = (8.32 \times 10^7\ \text{W/m}^2)\pi(2.00 \times 10^{-3}\ \text{m})^2 = \boxed{1.05\ \text{kW}}$

34.64 Think of light going up and being absorbed by the bead which presents face area πr_b^2 .

If we take the bead to be perfectly absorbing, the light pressure is $P = \dfrac{S_{av}}{c} = \dfrac{I}{c} = \dfrac{F_l}{A}$

(a) $F_l = F_g$ so $I = \dfrac{F_l c}{A} = \dfrac{F_g c}{A} = \dfrac{mgc}{\pi r_b{}^2}$

From the definition of density, $\rho = \dfrac{m}{V} = \dfrac{m}{\frac{4}{3}\pi r_b{}^3}$ so $\dfrac{1}{r_b} = \left(\frac{4}{3}\pi\rho \big/ m\right)^{1/3}$

Substituting for r_b, $I = \dfrac{mgc}{\pi}\left(\dfrac{4\pi\rho}{3m}\right)^{2/3} = gc\left(\dfrac{4\rho}{3}\right)^{2/3}\left(\dfrac{m}{\pi}\right)^{1/3} = \boxed{\dfrac{4\rho gc}{3}\left(\dfrac{3m}{4\pi\rho}\right)^{1/3}}$

(b) $\mathscr{P} = IA = \boxed{\dfrac{\pi r^2 4\rho gc}{3}\left(\dfrac{3m}{4\pi\rho}\right)^{1/3}}$

34.65 The mirror intercepts power

$$P = I_1 A_1 = (1.00\times 10^3\ \text{W/m}^2)\pi(0.500\ \text{m})^2 = 785\ \text{W}$$

In the image,

(a) $I_2 = \dfrac{P}{A_2} = \dfrac{785\ \text{W}}{\pi(0.0200\ \text{m})^2} = \boxed{625\ \text{kW/m}^2}$

(b) $I_2 = \dfrac{E_{\max}^2}{2\mu_0 c}$ so $E_{\max} = (2\mu_0 c\, I_2)^{1/2} = [2(4\pi\times 10^{-7})(3.00\times 10^8)(6.25\times 10^5)]^{1/2} = \boxed{21.7\ \text{kN/C}}$

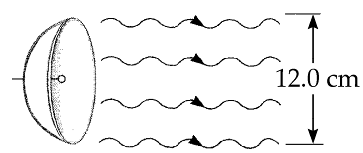
$B_{\max} = \dfrac{E_{\max}}{c} = \boxed{72.4\ \mu\text{T}}$

(c) $0.400\,P\,t = mc\,\Delta T$

$$0.400(785\ \text{W})t = (1.00\ \text{kg})\left(4186\ \frac{\text{J}}{\text{kg}\cdot\text{C}^\circ}\right)(100^\circ\text{C} - 20.0^\circ\text{C})$$

$$t = \frac{3.35\times 10^5\ \text{J}}{314\ \text{W}} = 1.07\times 10^3\ \text{s} = \boxed{17.8\ \text{min}}$$

34.66 (a) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^8\ \text{m/s}}{20.0\times 10^9\ \text{s}^{-1}} = \boxed{1.50\ \text{cm}}$



(b) $U = P(\Delta t) = \left(25.0\times 10^3\ \dfrac{\text{J}}{\text{s}}\right)\left(1.00\times 10^{-9}\ \text{s}\right) = 25.0\times 10^{-6}\ \text{J} = \boxed{25.0\ \mu\text{J}}$

(c) $u_{\text{av}} = \dfrac{U}{V} = \dfrac{U}{(\pi r^2)\ell} = \dfrac{U}{(\pi r^2)c(\Delta t)} = \dfrac{25.0\times 10^{-6}\ \text{J}}{\pi(0.0600\ \text{m})^2\left(3.00\times 10^8\ \text{m/s}\right)\left(1.00\times 10^{-9}\ \text{s}\right)}$

$u_{\text{av}} = 7.37\times 10^{-3}\ \text{J/m}^3 = \boxed{7.37\ \text{mJ/m}^3}$

(d) $E_{\max} = \sqrt{\dfrac{2u_{\text{av}}}{\epsilon_0}} = \sqrt{\dfrac{2\left(7.37\times 10^{-3}\ \text{J/m}^3\right)}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}} = 4.08\times 10^4\ \text{V m} = \boxed{40.8\ \text{kV/m}}$

$B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{4.08\times 10^4\ \text{V/m}}{3.00\times 10^8\ \text{m/s}} = 1.36\times 10^{-4}\ \text{T} = \boxed{136\ \mu\text{T}}$

(e) $F = PA = \left(\dfrac{S}{c}\right)A = \left(\dfrac{c\,u_{\text{av}}}{c}\right)A = u_{\text{av}}A = \left(7.37\times 10^{-3}\ \dfrac{\text{J}}{\text{m}^3}\right)\pi(0.0600\ \text{m})^2 = 8.33\times 10^{-5}\ \text{N} = \boxed{83.3\ \mu\text{N}}$

34.67 (a) On the right side of the equation, $\dfrac{C^2\left(m/s^2\right)^2}{\left(C^2/N\cdot m^2\right)\left(m/s\right)^3} = \dfrac{N\cdot m^2\cdot C^2\cdot m^2\cdot s^3}{C^2\cdot s^4\cdot m^3} = \dfrac{N\cdot m}{s} = \dfrac{J}{s} = W$

(b) $F = ma = qE$ or $a = \dfrac{qE}{m} = \dfrac{\left(1.60\times10^{-19}\text{ C}\right)\left(100\text{ N/C}\right)}{9.11\times10^{-31}\text{ kg}} = \boxed{1.76\times10^{13}\text{ m/s}^2}$

The radiated power is then: $\mathcal{P} = \dfrac{q^2a^2}{6\pi\epsilon_0c^3} = \dfrac{\left(1.60\times10^{-19}\right)^2\left(1.76\times10^{13}\right)^2}{6\pi\left(8.85\times10^{-12}\right)\left(3.00\times10^{8}\right)^3} = \boxed{1.75\times10^{-27}\text{ W}}$

(c) $F = ma_r = m\left(\dfrac{v^2}{r}\right) = qvB$ so $v = \dfrac{qBr}{m}$

The proton accelerates at $a = \dfrac{v^2}{r} = \dfrac{q^2B^2r}{m^2} = \dfrac{\left(1.60\times10^{-19}\right)^2(0.350)^2(0.500)}{\left(1.67\times10^{-27}\right)^2} = 5.62\times10^{14}\text{ m/s}^2$

The proton then radiates $\mathcal{P} = \dfrac{q^2a^2}{6\pi\epsilon_0c^3} = \dfrac{\left(1.60\times10^{-19}\right)^2\left(5.62\times10^{14}\right)^2}{6\pi\left(8.85\times10^{-12}\right)\left(3.00\times10^{8}\right)^3} = \boxed{1.80\times10^{-24}\text{ W}}$

34.68 $P = \dfrac{S}{c} = \dfrac{\text{Power}}{Ac} = \dfrac{\mathcal{P}}{2\pi r\ell c} = \dfrac{60.0\text{ W}}{2\pi(0.0500\text{ m})(1.00\text{ m})(3.00\times10^{8}\text{ m \textbackslash\ s})} = \boxed{6.37\times10^{-7}\text{ Pa}}$

34.69 $F = PA = \dfrac{SA}{c} = \dfrac{(\mathcal{P}/A)A}{c} = \dfrac{\mathcal{P}}{c}$, $\tau = F\left(\dfrac{\ell}{2}\right) = \dfrac{\mathcal{P}\ell}{2c}$, and $\tau = \kappa\theta$

Therefore, $\theta = \dfrac{\mathcal{P}\ell}{2c\kappa} = \dfrac{\left(3.00\times10^{-3}\right)(0.0600)}{2\left(3.00\times10^{8}\right)\left(1.00\times10^{-11}\right)} = \boxed{3.00\times10^{-2}\text{ deg}}$

***34.70** We take R to be the planet's distance from its star. The planet, of radius r, presents a $\boxed{\text{projected area } \pi r^2}$ perpendicular to the starlight. $\boxed{\text{It radiates over area } 4\pi r^2.}$

At steady-state, $\mathcal{P}_{in} = \mathcal{P}_{out}$: $eI_{\text{in}}\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4$

$e\left(\dfrac{6.00\times10^{23}\text{ W}}{4\pi R^2}\right)\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4$ so that $6.00\times10^{23}\text{ W} = 16\pi\sigma R^2T^4$

$$R = \sqrt{\frac{6.00\times10^{23}\text{ W}}{16\pi\sigma T^4}} = \sqrt{\frac{6.00\times10^{23}\text{ W}}{16\pi\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(310\text{ K})^4}} = \boxed{4.77\times10^{9}\text{ m} = 4.77\text{ Gm}}$$

34.71 The light intensity is $I = S_{av} = \dfrac{E^2}{2\mu_0 c}$

The light pressure is $P = \dfrac{S}{c} = \dfrac{E^2}{2\mu_0 c^2} = \frac{1}{2}\epsilon_0 E^2$

For the asteroid, $PA = ma$ and $a = \boxed{\dfrac{\epsilon_0 E^2 A}{2m}}$

34.72 $f = 90.0\text{ MHz}$, $E_{max} = 2.00 \times 10^{-3}\text{ V/m} = 200\text{ mV/m}$

(a) $\lambda = \dfrac{c}{f} = \boxed{3.33\text{ m}}$

$T = \dfrac{1}{f} = 1.11 \times 10^{-8}\text{ s} = \boxed{11.1\text{ ns}}$

$B_{max} = \dfrac{E_{max}}{c} = 6.67 \times 10^{-12}\text{ T} = \boxed{6.67\text{ pT}}$

(b) $\boxed{\mathbf{E} = (2.00\text{ mV/m})\cos 2\pi\left(\dfrac{x}{3.33\text{ m}} - \dfrac{t}{11.1\text{ ns}}\right)\mathbf{j}}$ $\boxed{\mathbf{B} = (6.67\text{ pT})\mathbf{k}\cos 2\pi\left(\dfrac{x}{3.33\text{ m}} - \dfrac{t}{11.1\text{ ns}}\right)}$

(c) $I = \dfrac{E_{max}^2}{2\mu_0 c} = \dfrac{(2.00 \times 10^{-3})^2}{2(4\pi \times 10^{-7})(3.00 \times 10^8)} = \boxed{5.31 \times 10^{-9}\text{ W/m}^2}$

(d) $I = cu_{av}$ so $u_{av} = \boxed{1.77 \times 10^{-17}\text{ J/m}^3}$

(e) $P = \dfrac{2I}{c} = \dfrac{(2)(5.31 \times 10^{-9})}{3.00 \times 10^8} = \boxed{3.54 \times 10^{-17}\text{ Pa}}$

Chapter 35 Solutons

35.1 The Moon's radius is 1.74×10^6 m and the Earth's radius is 6.37×10^6 m. The total distance traveled by the light is:

$$d = 2(3.84 \times 10^8 \text{ m} - 1.74 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m}) = 7.52 \times 10^8 \text{ m}$$

This takes 2.51 s, so $v = \dfrac{7.52 \times 10^8 \text{ m}}{2.51 \text{ s}} = 2.995 \times 10^8 \text{ m/s} = \boxed{299.5 \text{ Mm/s}}$

35.2 $\Delta x = ct$; $\quad c = \dfrac{\Delta x}{t} = \dfrac{2(1.50 \times 10^8 \text{ km})(1000 \text{ m/km})}{(22.0 \text{ min})(60.0 \text{ s/min})} = 2.27 \times 10^8 \text{ m/s} = \boxed{227 \text{ Mm/s}}$

35.3 The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next: $t = 2\ell/c$

$\theta = \omega t = \omega\left(\dfrac{2\ell}{c}\right)$ so $\omega = \dfrac{c\theta}{2\ell} = \dfrac{(2.998 \times 10^8)[2\pi / (720)]}{2(11.45 \times 10^3)} = \boxed{114 \text{ rad/s}}$

The returning light would be blocked by a tooth at one-half the angular speed, giving another data point.

35.4 (a) For the light beam to make it through both slots, the time for the light to travel the distance d must equal the time for the disk to rotate through the angle θ, if c is the speed of light,

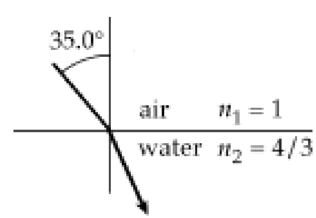
$\dfrac{d}{c} = \dfrac{\theta}{\omega}$, so $\boxed{c = \dfrac{d\omega}{\theta}}$

(b) We are given that

$d = 2.50 \text{ m}$, $\quad \theta = \dfrac{1.00°}{60.0}\left(\dfrac{\pi \text{ rad}}{180°}\right) = 2.91 \times 10^{-4} \text{ rad}$, $\quad \omega = 5555 \dfrac{\text{rev}}{\text{s}}\left(\dfrac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = 3.49 \times 10^4 \text{ rad/s}$

$$c = \frac{d\omega}{\theta} = \frac{(2.50 \text{ m})(3.49 \times 10^4 \text{ rad/s})}{2.91 \times 10^{-4} \text{ rad}} = 3.00 \times 10^8 \text{ m/s} = \boxed{300 \text{ Mm/s}}$$

35.5 Using Snell's law, $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_1$



$\theta_2 = \boxed{25.5°}$ $\qquad \lambda_2 = \dfrac{\lambda_1}{n_1} = \boxed{442 \text{ nm}}$

35.6 (a) $f = \dfrac{c}{\lambda} = \dfrac{3.00 \times 10^8 \text{ m/s}}{6.328 \times 10^{-7} \text{ m}} = \boxed{4.74 \times 10^{14} \text{ Hz}}$

(b) $\lambda_{\text{glass}} = \dfrac{\lambda_{\text{air}}}{n} = \dfrac{632.8 \text{ nm}}{1.50} = \boxed{422 \text{ nm}}$

(c) $v_{\text{glass}} = \dfrac{c_{\text{air}}}{n} = \dfrac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s} = \boxed{200 \text{ Mm/s}}$

35.7 $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$\sin \theta_1 = 1.333 \sin 45.0°$

$\sin \theta_1 = (1.33)(0.707) = 0.943$

$\theta_1 = 70.5° \rightarrow \boxed{19.5° \text{ above the horizon}}$

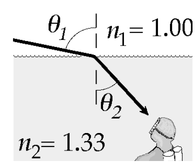


Figure for Goal Solution

Goal Solution

An underwater scuba diver sees the Sun at an apparent angle of 45.0° from the vertical. What is the actual direction of the Sun?

G: The sunlight refracts as it enters the water from the air. Because the water has a higher index of refraction, the light slows down and bends toward the vertical line that is normal to the interface. Therefore, the elevation angle of the Sun above the water will be less than 45° as shown in the diagram to the right, even though it appears to the diver that the sun is 45° above the horizon.

O: We can use Snell's law of refraction to find the precise angle of incidence.

A: Snell's law is: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

which gives $\sin \theta_1 = 1.333 \sin 45.0°$

$\sin \theta_1 = (1.333)(0.707) = 0.943$

The sunlight is at $\theta_1 = 70.5°$ to the vertical, so the Sun is $19.5°$ above the horizon.

L: The calculated result agrees with our prediction. When applying Snell's law, it is easy to mix up the index values and to confuse angles-with-the-normal and angles-with-the-surface. Making a sketch and a prediction as we did here helps avoid careless mistakes.

***35.8** (a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00 \sin 30.0° = n \sin 19.24°$

$n = \boxed{1.52}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{6.328\times10^{-7} \text{ m}} = \boxed{4.74\times10^{14} \text{ Hz}}$ in air and in syrup.

(d) $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.52} = 1.98\times10^8 \text{ m/s} = \boxed{198 \text{ Mm/s}}$

(b) $\lambda = \dfrac{v}{f} = \dfrac{1.98\times10^8 \text{ m/s}}{4.74\times10^{14} \text{ /s}} = \boxed{417 \text{ nm}}$

35.9 (a) Flint Glass: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.66} = 1.81\times10^8 \text{ m/s} = \boxed{181 \text{ Mm/s}}$

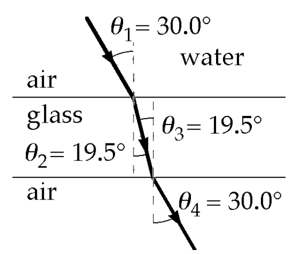
(b) Water: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.333} = 2.25\times10^8 \text{ m/s} = \boxed{225 \text{ Mm/s}}$

(c) Cubic Zirconia: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{2.20} = 1.36\times10^8 \text{ m/s} = \boxed{136 \text{ Mm/s}}$

35.10 $n_1 \sin\theta_1 = n_2 \sin\theta_2$; $1.333 \sin 37.0° = n_2 \sin 25.0°$

$n_2 = 1.90 = \dfrac{c}{v}$; $v = \dfrac{c}{1.90} = 1.58\times10^8 \text{ m/s} = \boxed{158 \text{ Mm/s}}$

35.11 $n_1 \sin\theta_1 = n_2 \sin\theta_2$; $\theta_2 = \sin^{-1}\left(\dfrac{n_1 \sin\theta_1}{n_2}\right)$



$\theta_2 = \sin^{-1}\left\{\dfrac{(1.00)(\sin 30°)}{1.50}\right\} = \boxed{19.5°}$

θ_2 and θ_3 are alternate interior angles formed by the ray cutting parallel normals. So, $\theta_3 = \theta_2 = \boxed{19.5°}$.

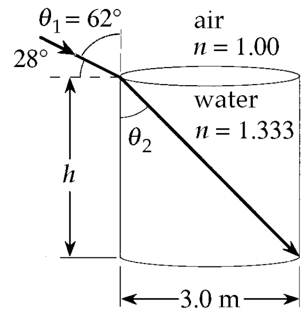
$1.50 \sin\theta_3 = (1.00)\sin\theta_4$

$\theta_4 = \boxed{30.0°}$

35.12 (a) Water $\lambda = \frac{\lambda_0}{n} = \frac{436 \text{ nm}}{1.333} = \boxed{327 \text{ nm}}$

(b) Glass $\lambda = \frac{\lambda_0}{n} = \frac{436 \text{ nm}}{1.52} = \boxed{287 \text{ nm}}$

***35.13** $\sin\theta_1 = n_w \sin\theta_2$



$$\sin\theta_2 = \frac{1}{1.333}\sin\theta_1 = \frac{1}{1.333}\sin(90.0° - 28.0°) = 0.662$$

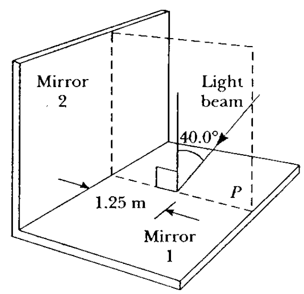
$$\theta_2 = \sin^{-1} 0.662 = 41.5°$$

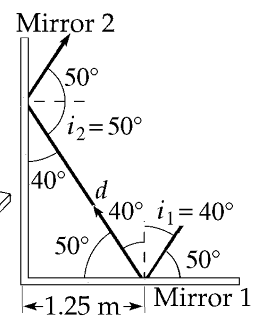
$$h = \frac{d}{\tan\theta_2} = \frac{3.00 \text{ m}}{\tan 41.5°} = \boxed{3.39 \text{ m}}$$

35.14 (a) From geometry, 1.25 m = $d \sin 40.0°$

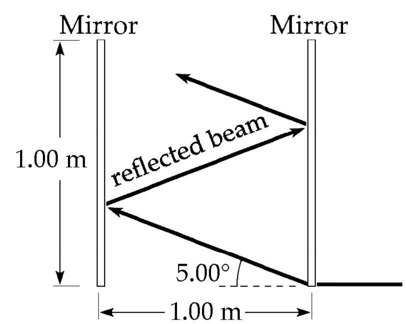
so $d = \boxed{1.94 \text{ m}}$

(b) $\boxed{50.0° \text{ above horizontal}}$, or parallel to the incident ray





***35.15** The incident light reaches the left-hand mirror at distance



(1.00 m) tan 5.00° = 0.0875 m

above its bottom edge. The reflected light first reaches the right-hand mirror at height

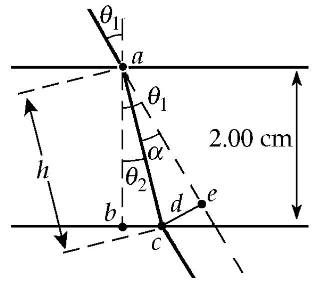
2(0.0875 m) = 0.175 m

It bounces between the mirrors with this distance between points of contact with either.

Since $\frac{1.00 \text{ m}}{0.175 \text{ m}} = 5.72$, the light reflects

$\boxed{\text{five times from the right-hand mirror and six times from the left.}}$

***35.16** At entry, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ or $1.00 \sin 30.0° = 1.50 \sin\theta_2$



$\theta_2 = 19.5°$

The distance h the light travels in the medium is given by

$$\cos\theta_2 = \frac{(2.00\text{ cm})}{h} \quad \text{or} \quad h = \frac{(2.00\text{ cm})}{\cos 19.5°} = 2.12\text{ cm}$$

The angle of deviation upon entry is $\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$

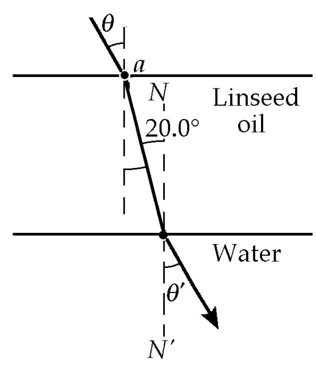
The offset distance comes from $\sin\alpha = \frac{d}{h}$: $d = (2.21\text{ cm})\sin 10.5° = \boxed{0.388\text{ cm}}$

***35.17** The distance, h, traveled by the light is $h = \frac{2.00\text{ cm}}{\cos 19.5°} = 2.12\text{ cm}$

The speed of light in the material is $v = \frac{c}{n} = \frac{3.00\times10^8\text{ m/s}}{1.50} = 2.00\times10^8\text{ m/s}$

Therefore, $t = \frac{h}{v} = \frac{2.12\times10^{-2}\text{ m}}{2.00\times10^8\text{ m/s}} = 1.06\times10^{-10}\text{ s} = \boxed{106\text{ ps}}$

***35.18** Applying Snell's law at the air-oil interface,



$n_{air}\sin\theta = n_{oil}\sin 20.0°$ yields $\boxed{\theta = 30.4°}$

Applying Snell's law at the oil-water interface

$n_w \sin\theta' = n_{oil}\sin 20.0°$ yields $\boxed{\theta' = 22.3°}$

***35.19** time difference = (time for light to travel 6.20 m in ice) – (time to travel 6.20 m in air)

$$\Delta t = \frac{6.20\text{ m}}{v_{ice}} - \frac{6.20\text{ m}}{c} \quad \text{but} \quad v = \frac{c}{n}$$

$$\Delta t = (6.20\text{ m})\left(\frac{1.309}{c} - \frac{1}{c}\right) = \frac{(6.20\text{ m})}{c}(0.309) = 6.39\times10^{-9}\text{ s} = \boxed{6.39\text{ ns}}$$

***35.20** Consider glass with an index of refraction of 1.5, which is 3 mm thick The speed of light in the glass is

$$\frac{3\times10^{8}\text{ m/s}}{1.5}=2\times10^{8}\text{ m/s}$$

The extra travel time is $$\frac{3\times10^{-3}\text{ m}}{2\times10^{8}\text{ m/s}}-\frac{3\times10^{-3}\text{ m}}{3\times10^{8}\text{ m/s}}\ \boxed{\sim 10^{-11}\text{ s}}$$

For light of wavelength 600 nm in vacuum and wavelength $\frac{600\text{ nm}}{1.5}=400\text{ nm}$ in glass,

the extra optical path, in wavelengths, is $$\frac{3\times10^{-3}\text{ m}}{4\times10^{-7}\text{ m}}-\frac{3\times10^{-3}\text{ m}}{6\times10^{-7}\text{ m}}\ \boxed{\sim 10^{3}\text{ wavelengths}}$$

***35.21** Refraction proceeds according to $$(1.00)\sin\theta_1=(1.66)\sin\theta_2 \qquad (1)$$

(a) For the normal component of velocity to be constant, $$v_1\cos\theta_1=v_2\cos\theta_2$$

or $$(c)\cos\theta_1=(c/1.66)\cos\theta_2 \qquad (2)$$

We multiply Equations (1) and (2), obtaining: $$\sin\theta_1\cos\theta_1=\sin\theta_2\cos\theta_2$$

or $$\sin 2\theta_1=\sin 2\theta_2$$

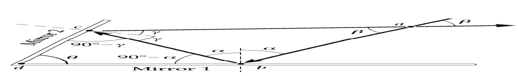
The solution $\theta_1=\theta_2=0$ does not satisfy Equation (2) and must be rejected. The physical solution is $2\theta_1=180°-2\theta_2$ or $\theta_2=90.0°-\theta_1$. Then Equation (1) becomes:

$$\sin\theta_1=1.66\cos\theta_1,\ \text{ or }\ \tan\theta_1=1.66$$

which yields $$\theta_1=\boxed{58.9°}$$

(b) Light entering the glass slows down and makes a smaller angle with the normal. Both effects reduce the velocity component parallel to the surface of the glass, so that component cannot remain constant, or will remain constant only in the trivial case $\boxed{\theta_1=\theta_2=0}$

35.22 See the sketch showing the path of the light ray. α and γ are angles of incidence at mirrors 1 and 2.



For triangle abca, $2\alpha+2\gamma+\beta=180°$

or $$\beta=180°-2(\alpha+\gamma) \qquad (1)$$

Now for triangle bcdb,

$$(90.0°-\alpha)+(90.0°-\gamma)+\theta=180°$$

or $$\theta=\alpha+\gamma \qquad (2)$$

Substituting Equation (2) into Equation (1) gives $\boxed{\beta = 180° - 2\theta}$

Note: From Equation (2), $\gamma = \theta - \alpha$. Thus, the ray will follow a path like that shown only if $\alpha < \theta$. For $\alpha > \theta$, γ is negative and multiple reflections from each mirror will occur before the incident and reflected rays intersect.

35.23 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface. Then,

$$n(x) = 1.000 + \left(\frac{n - 1.000}{h}\right)x$$

(a) The total time required to traverse the atmosphere is

$$t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n-1.000}{h}\right)x\right]dx = \frac{h}{c} + \frac{(n-1.000)}{ch}\left(\frac{h^2}{2}\right) = \frac{h}{c}\left(\frac{n+1.000}{2}\right)$$

$$t = \frac{20.0 \times 10^3 \text{ m}}{3.00 \times 10^8 \text{ m/s}}\left(\frac{1.005 + 1.000}{2}\right) = \boxed{66.8\ \mu\text{s}}$$

(b) The travel time in the absence of an atmosphere would be h/c. Thus, the time in the presence of an atmosphere is

$\left(\frac{n+1.000}{2}\right) = 1.0025$ times larger or $\boxed{0.250\% \text{ longer}}$.

35.24 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface. Then,

$$n(x) = 1.000 + \left(\frac{n - 1.000}{h}\right)x$$

(a) The total time required to traverse the atmosphere is

$$t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n-1.000}{h}\right)x\right]dx = \frac{h}{c} + \frac{(n-1.000)}{ch}\left(\frac{h^2}{2}\right) = \boxed{\frac{h}{c}\left(\frac{n+1.000}{2}\right)}$$

(b) The travel time in the absence of an atmosphere would be h/c. Thus, the time in the presence of an atmosphere is

$\boxed{\left(\frac{n+1.000}{2}\right) \text{ times larger}}$

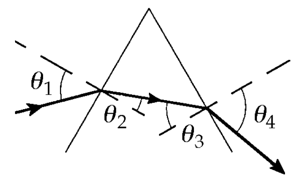
35.25 From Fig. 35.20 $n_v = 1.470$ at 400 nm and $n_r = 1.458$ at 700 nm

Then $(1.00)\sin\theta = 1.470 \sin\theta_v$ and $(1.00)\sin\theta = 1.458\sin\theta_r$

$$\delta_r - \delta_v = \theta_r - \theta_v = \sin^{-1}\left(\frac{\sin\theta}{1.458}\right) - \sin^{-1}\left(\frac{\sin\theta}{1.470}\right)$$

$$\Delta\delta = \sin^{-1}\left(\frac{\sin 30.0°}{1.458}\right) - \sin^{-1}\left(\frac{\sin 30.0°}{1.470}\right) = \boxed{0.171°}$$

35.26 $n_1 \sin\theta_1 = n_2 \sin\theta_2$ so $\theta_2 = \sin^{-1}\left(\dfrac{n_1 \sin\theta_1}{n_2}\right)$



$$\theta_2 = \sin^{-1}\left(\frac{(1.00)(\sin 30.0°)}{1.50}\right) = \boxed{19.5°}$$

$$\theta_3 = \left(\left[(90.0° - 19.5°) + 60.0°\right] - 180°\right) + 90.0° = \boxed{40.5°}$$

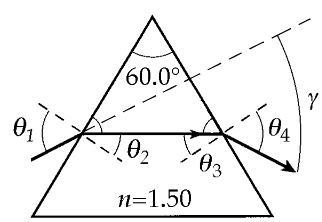
$n_3 \sin\theta_3 = n_4 \sin\theta_4$ so $\theta_4 = \sin^{-1}\left(\dfrac{n_3 \sin\theta_3}{n_4}\right) = \sin^{-1}\left(\dfrac{(1.50)(\sin 40.5°)}{1.00}\right) = \boxed{77.1°}$

35.27 Taking Φ to be the apex angle and $\delta_{\min}$ to be the angle of minimum deviation, from Equation 35.9, the index of refraction of the prism material is

$$n = \frac{\sin\left(\dfrac{\Phi + \delta_{\min}}{2}\right)}{\sin(\Phi/2)}$$

Solving for $\delta_{\min}$, $\delta_{\min} = 2\sin^{-1}\left(n \sin\dfrac{\Phi}{2}\right) - \Phi = 2\sin^{-1}\left[(2.20)\sin(25.0°)\right] - 50.0° = \boxed{86.8°}$

35.28 $n(700 \text{ nm}) = 1.458$



(a) $(1.00)\sin 75.0° = 1.458 \sin\theta_2$; $\theta_2 = \boxed{41.5°}$

(b) Let $\theta_3 + \beta = 90.0°$, $\theta_2 + \alpha = 90.0°$; then $\alpha + \beta + 60.0° = 180°$

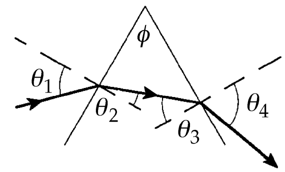
So $60.0° - \theta_2 - \theta_3 = 0 \Rightarrow 60.0° - 41.5° = \theta_3 = \boxed{18.5°}$

(c) $1.458 \sin 18.5° = 1.00 \sin\theta_4$ $\theta_4 = \boxed{27.6°}$

(d) $\gamma = (\theta_1 - \theta_2) + [\beta - (90.0° - \theta_4)]$

$$\gamma = 75.0° - 41.5° + (90.0° - 18.5°) - (90.0° - 27.6°) = \boxed{42.6°}$$

35.29 For the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$



Using the figure to the right, $(\theta_2)_{\text{violet}} = \sin^{-1}\left(\dfrac{\sin\ 50.0°}{1.66}\right) = 27.48°$

$$(\theta_2)_{\text{red}} = \sin^{-1}\left(\frac{\sin\ 50.0°}{1.62}\right) = 28.22°$$

For the outgoing ray, $\theta_3' = 60.0° - \theta_2$ and $\sin\theta_4 = n\sin\theta_3$

$$(\theta_4)_{\text{violet}} = \sin^{-1}[1.66\ \sin\ 32.52°] = 63.17°$$

$$(\theta_4)_{\text{red}} = \sin^{-1}[1.62\ \sin\ 31.78°] = 58.56°$$

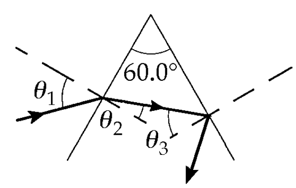
The dispersion is the difference $\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}} = 63.17° - 58.56° = \boxed{4.61°}$

35.30 $$n = \frac{\sin\left(\dfrac{\Phi+\delta_{\min}}{2}\right)}{\sin(\Phi/2)}$$

For small Φ, $\delta_{\min} \approx \Phi$ so $\dfrac{\Phi+\delta_{\min}}{2}$ is also a small angle. Then, using the small angle approximation ($\sin\theta \approx \theta$ when $\theta << 1$ rad), we have:

$$n \approx \frac{(\Phi+\delta_{\min})/2}{\Phi/2} = \frac{\Phi+\delta_{\min}}{\Phi}$$ or $\boxed{\delta_{\min} \approx (n-1)\Phi}$ where Φ is in radians.

35.31 At the first refraction, $(1.00)\sin\theta_1 = n\sin\theta_2$



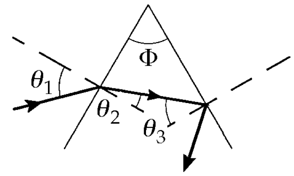
The critical angle at the second surface is given by

$$n\sin\theta_3 = 1.00, \quad \text{or} \quad \theta_3 = \sin^{-1}\left(\frac{1.00}{1.50}\right) = 41.8°.$$

But, $\theta_2 = 60.0° - \theta_3$. Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3 < 41.8°$), it is necessary that $\theta_2 > 18.2°$. Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes

$$\sin\theta_1 > (1.50)\sin(18.2°) = 0.468, \quad \text{or} \quad \theta_1 > \boxed{27.9°}$$

35.32 At the first refraction, $(1.00)\sin\theta_1 = n\sin\theta_2$. The critical angle at the second surface is given by



$n\sin\theta_3 = 1.00$, or $\theta_3 = \sin^{-1}(1.00/n)$

But $(90.0° - \theta_2) + (90.0° - \theta_3) + \Phi = 180°$, which gives $\theta_2 = \Phi - \theta_3$.

Thus, to have $\theta_3 < \sin^{-1}(1.00/n)$ and avoid total internal reflection at the second surface, it is necessary that $\theta_2 > \Phi - \sin^{-1}(1.00/n)$. Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes

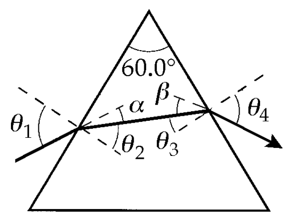
$$\sin\theta_1 > n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right] \quad \text{or} \quad \theta_1 > \boxed{\sin^{-1}\left(n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]\right)}$$

Through the application of trigonometric identities, $\theta_1 > \boxed{\sin^{-1}\left(\sqrt{n^2-1}\,\sin\Phi - \cos\Phi\right)}$

35.33 $n = \dfrac{\sin(\delta+\phi)}{\sin(\phi/2)}$ so $1.544\sin\left(\frac{1}{2}\phi\right) = \sin\left(5° + \frac{1}{2}\phi\right) = \cos\left(\frac{1}{2}\phi\right)\sin 5° + \sin\left(\frac{1}{2}\phi\right)\cos 5°$

$\tan\left(\frac{1}{2}\phi\right) = \dfrac{\sin 5°}{1.544 - \cos 5°}$ and $\phi = \boxed{18.1°}$

***35.34** Note for use in every part: $\Phi + (90.0° - \theta_2) + (90.0° - \theta_3) = 180°$



so $\theta_3 = \Phi - \theta_2$

At the first surface is $\alpha = \theta_1 - \theta_2$

At exit, the deviation is $\beta = \theta_4 - \theta_3$

The total deviation is therefore $\delta = \alpha + \beta = \theta_1 + \theta_4 - \theta_2 - \theta_3 = \theta_1 + \theta_4 - \Phi$

(a) At entry: $n_1\sin\theta_1 = n_2\sin\theta_2$ or $\theta_2 = \sin^{-1}\left(\dfrac{\sin 48.6°}{1.50}\right) = 30.0°$

Thus, $\theta_3 = 60.0° - 30.0° = 30.0°$

At exit: $1.50\sin 30.0° = 1.00\sin\theta_4$ or $\theta_4 = \sin^{-1}[1.50\sin(30.0°)] = 48.6°$

so the path through the prism is symmetric when $\theta_1 = 48.6°$.

(b) $\delta = 48.6° + 48.6° - 60.0° = \boxed{37.2°}$

(c) At entry: $\sin\theta_2 = \dfrac{\sin 45.6°}{1.50} \Rightarrow \theta_2 = 28.4°$ $\theta_3 = 60.0° - 28.4° = 31.6°$

At exit: $\sin\theta_4 = 1.50\sin(31.6°) \Rightarrow \theta_4 = 51.7°$ $\delta = 45.6° + 51.7° - 60.0° = \boxed{37.3°}$

(d) At entry: $\sin\theta_2 = \dfrac{\sin 51.6°}{1.50} \Rightarrow \theta_2 = 31.5°$ $\theta_3 = 60.0° - 31.5° = 28.5°$

At exit: $\sin\theta_4 = 1.50\sin(28.5°) \Rightarrow \theta_4 = 45.7°$ $\delta = 51.6° + 45.7° - 60.0° = \boxed{37.3°}$

35.35 $n \sin\theta = 1$. From Table 35.1,

(a) $\theta = \sin^{-1}\left(\frac{1}{2.419}\right) = \boxed{24.4°}$

(b) $\theta = \sin^{-1}\left(\frac{1}{1.66}\right) = \boxed{37.0°}$

(c) $\theta = \sin^{-1}\left(\frac{1}{1.309}\right) = \boxed{49.8°}$

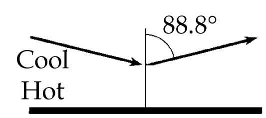
35.36 $\sin\theta_c = \frac{n_2}{n_1}$; $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

(a) Diamond: $\theta_c = \sin^{-1}\left(\frac{1.333}{2.419}\right) = \boxed{33.4°}$

(b) Flint glass: $\theta_c = \sin^{-1}\left(\frac{1.333}{1.66}\right) = \boxed{53.4°}$

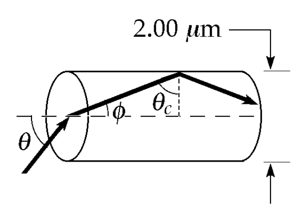
(c) Ice: Since $n_2 > n_1$, $\boxed{\text{there is no critical angle}}$.

35.37 $\sin\theta_c = \frac{n_2}{n_1}$ (Equation 35.10)



$n_2 = n_1 \sin 88.8° = (1.0003)(0.9998) = \boxed{1.000\ 08}$

***35.38** $\sin\theta_c = \frac{n_{air}}{n_{pipe}} = \frac{1.00}{1.36} = 0.735$ $\theta_c = 47.3°$



Geometry shows that the angle of refraction at the end is

$$\theta_r = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$$

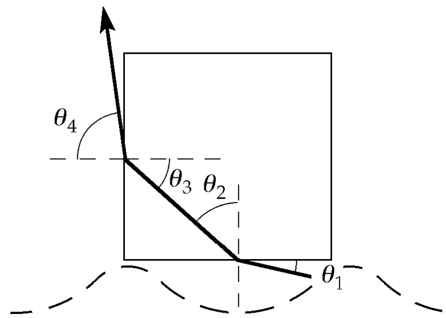
Then, Snell's law at the end, $1.00 \sin\theta = 1.36 \sin 42.7°$

gives $\boxed{\theta = 67.2°}$

35.39 For total internal reflection, $n_1 \sin\theta_1 = n_2 \sin 90.0°$

$(1.50)\sin\theta_1 = (1.33)(1.00)$ or $\theta_1 = \boxed{62.4°}$

35.40 To avoid internal reflection and come out through the vertical face, light inside the cube must have



$\theta_3 < \sin^{-1}(1/n)$

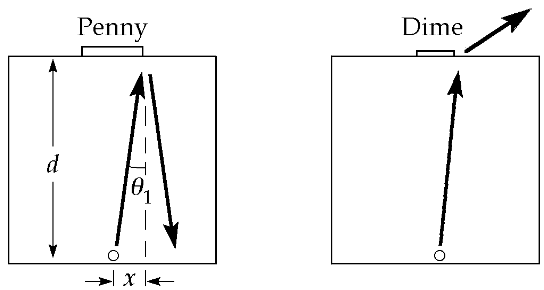
So $\quad \theta_2 > 90.0° - \sin^{-1}(1/n)$

But $\quad \theta_1 < 90.0° \quad$ and $\quad n \sin\theta_2 < 1$

In the critical case, $\sin^{-1}(1/n) = 90.0° - \sin^{-1}(1/n)$

$1/n = \sin 45.0° \qquad \boxed{n = 1.41}$

35.41 From Snell's law, $\quad n_1 \sin\theta_1 = n_2 \sin\theta_2$



At the extreme angle of viewing, $\theta_2 = 90.0°$

$(1.59)(\sin\theta_1) = (1.00)\cdot\sin 90.0°$

So $\quad \theta_1 = 39.0°$

Therefore, the depth of the air bubble is

$$\frac{r_d}{\tan\theta_1} < d < \frac{r_p}{\tan\theta_1}$$

or $\quad \boxed{1.08\text{ cm} < d < 1.17\text{ cm}}$

***35.42** (a) $\dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{v_2}{v_1}$ and $\theta_2 = 90.0°$ at the critical angle

$$\frac{\sin 90.0°}{\sin\theta_c} = \frac{1850\text{ m/s}}{343\text{ m/s}} \quad \text{so} \quad \theta_c = \sin^{-1} 0.185 = \boxed{10.7°}$$

(b) Sound can be totally reflected if it is traveling in the medium where it travels slower: $\boxed{\text{air}}$

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

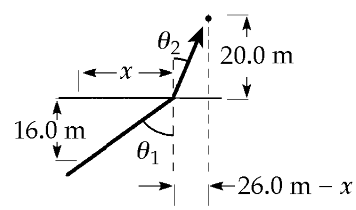
***35.43** For plastic with index of refraction $\boxed{n \geq 1.42}$ surrounded by air, the critical angle for total internal reflection is given by

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) \leq \sin^{-1}\left(\frac{1}{1.42}\right) = 44.8°$$

In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from both the sides of the slab and from facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark. To frustrate total internal reflection in the gasoline, the index of refraction of the plastic should be $\boxed{n < 2.12}$, since

$$\theta_c = \sin^{-1}\left(\tfrac{1.50}{2.12}\right) = 45.0°.$$

***35.44** Assume the lifeguard's path makes angle θ_1 with the north-south normal to the shoreline, and angle θ_2 with this normal in the water. By Fermat's principle, his path should follow the law of refraction:



$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{7.00 \text{ m/s}}{1.40 \text{ m/s}} = 5.00 \quad \text{or} \quad \theta_2 = \sin^{-1}\left(\frac{\sin \theta_1}{5}\right)$$

The lifeguard on land travels eastward a distance $x = (16.0 \text{ m})\tan \theta_1$. Then in the water, he travels $26.0 \text{ m} - x = (20.0 \text{ m})\tan \theta_2$ further east. Thus, $26.0 \text{ m} = (16.0 \text{ m})\tan \theta_1 + (20.0 \text{ m})\tan \theta_2$

or $$26.0 \text{ m} = (16.0 \text{ m})\tan \theta_1 + (20.0 \text{ m})\tan\left[\sin^{-1}\left(\frac{\sin \theta_1}{5}\right)\right]$$

We home in on the solution as follows:

θ_1 (deg)	50.0	60.0	54.0	54.8	54.81
right-hand side	22.2 m	31.2 m	25.3 m	25.99 m	26.003 m

The lifeguard should start running at $\boxed{54.8° \text{ east of north}}$.

***35.45** Let the air and glass be medium 1 and 2, respectively. By Snell's law, $n_2 \sin \theta_2 = n_1 \sin \theta_1$

or $1.56 \sin \theta_2 = \sin \theta_1$

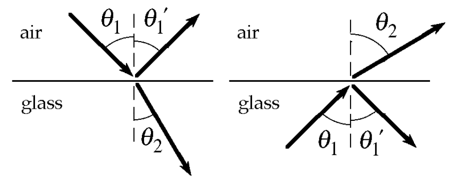
But the conditions of the problem are such that $\theta_1 = 2\theta_2$. $1.56 \sin \theta_2 = \sin 2\theta_2$

We now use the double-angle trig identity suggested. $1.56 \sin \theta_2 = 2 \sin \theta_2 \cos \theta_2$

or $\cos \theta_2 = \dfrac{1.56}{2} = 0.780$

Thus, $\theta_2 = 38.7°$ and $\theta_1 = 2\theta_2 = \boxed{77.5°}$

***35.46** (a) $\theta_1' = \theta_1 = \boxed{30.0°}$



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$(1.00)\sin 30.0° = 1.55 \sin\theta_2$

$\theta_2 = \boxed{18.8°}$

(b) $\theta_1' = \theta_1 = \boxed{30.0°}$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{1.55 \sin 30.0°}{1}\right) = \boxed{50.8°}$$

(c) and (d) The other entries are computed similarly, and are shown in the table below.

(c) air into glass, angles in degrees			(d) glass into air, angles in degrees		
incidence	reflection	refraction	incidence	reflection	refraction
0	0	0	0	0	0
10.0	10.0	6.43	10.0	10.0	15.6
20.0	20.0	12.7	20.0	20.0	32.0
30.0	30.0	18.8	30.0	30.0	50.8
40.0	40.0	24.5	40.0	40.0	85.1
50.0	50.0	29.6	50.0	50.0	none*
60.0	60.0	34.0	60.0	60.0	none*
70.0	70.0	37.3	70.0	70.0	none*
80.0	80.0	39.4	80.0	80.0	none*
90.0	90.0	40.2	90.0	90.0	none*

*total internal reflection

35.47 For water, $\sin\theta_c = \dfrac{1}{4/3} = \dfrac{3}{4}$

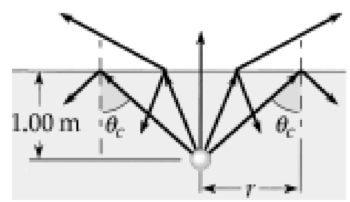


Figure for Goal Solution

Thus $\theta_c = \sin^{-1}(0.750) = 48.6°$

and $d = 2\left[(1.00 \text{ m})\tan\theta_c\right]$

$d = (2.00 \text{ m})\tan 48.6° = \boxed{2.27 \text{ m}}$

Goal Solution

A small underwater pool light is 1.00 m below the surface. The light emerging from the water forms a circle on the water's surface. What is the diameter of this circle?

G: Only the light that is directed upwards and hits the water's surface at less than the critical angle will be transmitted to the air so that someone outside can see it. The light that hits the surface farther from the center at an angle greater than θ_c will be totally reflected within the water, unable to be seen from the outside. From the diagram above, the diameter of this circle of light appears to be about 2 m.

O: We can apply Snell's law to find the critical angle, and the diameter can then be found from the geometry.

A: The critical angle is found when the refracted ray just grazes the surface (θ_2 = 90°). The index of refraction of water is $n_2 = 1.33$, and $n_1 = 1.00$ for air, so

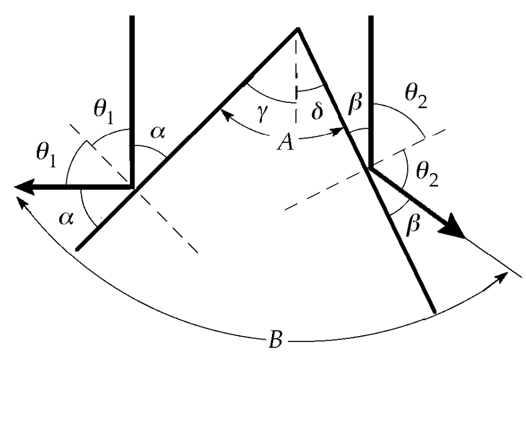
$n_1 \sin\theta_c = n_2 \sin 90°$ gives $\theta_c = \sin^{-1}\left(\frac{1}{1.333}\right) = \sin^{-1}(0.750) = 48.6°$

The radius then satisfies $\tan\theta_c = \frac{r}{(1.00\text{ m})}$

So the diameter is $d = 2r = 2(1.00\text{ m})\tan 48.6° = 2.27\text{ m}$

L: Only the light rays within a 97.2° cone above the lamp escape the water and can be seen by an outside observer (Note: this angle does not depend on the depth of the light source). The path of a light ray is always reversible, so if a person were located beneath the water, they could see the whole hemisphere above the water surface within this cone; this is a good experiment to try the next time you go swimming!

***35.48** Call θ_1 the angle of incidence and of reflection on the left face and θ_2 those angles on the right face. Let α represent the complement of θ_1 and β be the complement of θ_2. Now $\alpha = \gamma$ and $\beta = \delta$ because they are pairs of alternate interior angles. We have



$$A = \gamma + \delta = \alpha + \beta$$

and $B = \alpha + A + \beta = \alpha + \beta + A = \boxed{2A}$

***35.49** (a) We see the Sun swinging around a circle in the extended plane of our parallel of latitude. Its angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{86\ 400 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s}$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

$$v = r\omega = (2.37 \text{ m})(7.27 \times 10^{-5} \text{ rad/s}) = 1.72 \times 10^{-4} \text{ m/s} = \boxed{0.172 \text{ mm/s}}$$

(b) The mirror folds into the cell the motion that would occur in a room twice as wide: $v = r\omega = 2(0.174 \text{ mm/s}) = \boxed{0.345 \text{ mm/s}}$

(c) and (d)

As the Sun moves southward and upward at 50.0°, we may regard the corner of the window as fixed, and both patches of light move $\boxed{\text{northward and downward at } 50.0°}$.

***35.50** By Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

With $v = \dfrac{c}{n}$, $\dfrac{c}{v_1}\sin\theta_1 = \dfrac{c}{v_2}\sin\theta_2$ or $\dfrac{\sin\theta_1}{v_1} = \dfrac{\sin\theta_2}{v_2}$

This is also true for sound. Here, $\dfrac{\sin 12.0°}{340 \text{ m/s}} = \dfrac{\sin\theta_2}{1510 \text{ m/s}}$

$$\theta_2 = \arcsin(4.44 \sin 12.0°) = \boxed{67.4°}$$

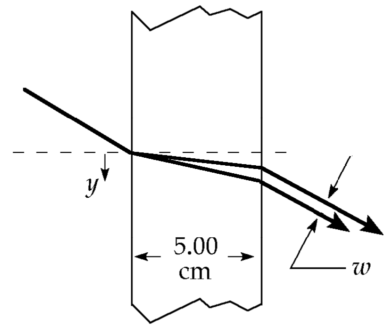
***35.51** (a) $$n = \frac{c}{v} = \frac{2.998 \times 10^8 \text{ m/s}}{\left(61.15 \dfrac{\text{km}}{\text{hr}}\right)\left(\dfrac{1.00 \text{ h}}{3600 \text{ s}}\right)\left(\dfrac{1.00 \times 10^3 \text{ m}}{1.00 \text{ km}}\right)} = \boxed{1.76 \times 10^7}$$

(b) $n_1 \sin\theta_1 = n_2 \sin\theta_2$ so $(1.76 \times 10^7)\sin\theta_1 = (1.00)\sin 90.0°$

$$\theta_1 = \boxed{3.25 \times 10^{-6} \text{ degree}}$$

This problem is misleading. The speed of energy transport is slow, but the speed of the wavefront advance is normally fast. The condensate's index of refraction is not far from unity.

***35.52** Violet light:



$$(1.00)\sin 25.0° = 1.689 \sin \theta_2 \Rightarrow \theta_2 = 14.490°$$

$$y_v = (5.00 \text{ cm})\tan \theta_2 = (5.00 \text{ cm})\tan 14.490° = 1.2622 \text{ cm}$$

Red Light:

$$(1.00)\sin 25.0° = 1.642 \sin \theta_2 \Rightarrow \theta_2 = 14.915°$$

$$y_R = (5.00 \text{ cm})\tan 14.915° = 1.3318 \text{ cm}$$

The emergent beams are both at 25.0° from the normal. Thus,

$w = \Delta y \cos 25.0°$ where $\Delta y = 1.3318 \text{ cm} - 1.2622 \text{ cm} = 0.0396 \text{ cm}$

$$w = (0.396 \text{ mm})\cos 25.0° = \boxed{0.359 \text{ mm}}$$

35.53 Horizontal light rays from the setting Sun pass above the hiker. The light rays are twice refracted and once reflected, as in Figure (b) below, by just the certain special raindrops at 40.0° to 42.0° from the hiker's shadow, and reach the hiker as the rainbow.

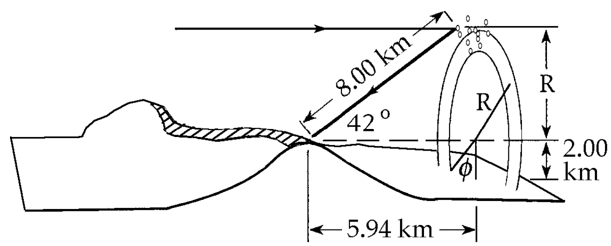


Figure (a)

The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius R of the circle of droplets is

$$R = (8.00 \text{ km})(\sin 42.0°) = 5.35 \text{ km}$$

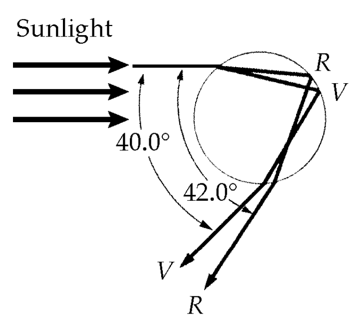


Figure (b)

Then the angle ϕ, between the vertical and the radius where the bow touches the ground, is given by

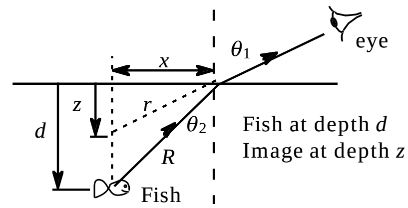
$$\cos \phi = \frac{2.00 \text{ km}}{R} = \frac{2.00 \text{ km}}{5.35 \text{ km}} = 0.374 \quad \text{or} \quad \phi = 68.1°$$

The angle filled by the visible bow is $360° - (2 \times 68.1°) = 224°$, so the visible bow is

$$\frac{224°}{360°} = \boxed{62.2\% \text{ of a circle}}$$

35.54 From Snell's law, $(1.00)\sin\theta_1 = \frac{4}{3}\sin\theta_2$

$x = R\sin\theta_2 = r\sin\theta_1$



so $$\frac{r}{R} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{3}{4}$$

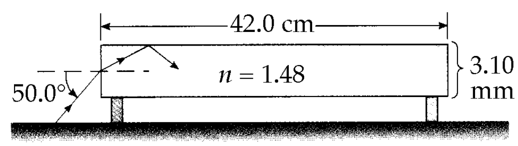
$$\frac{\text{apparent depth}}{\text{actual depth}} = \frac{z}{d} = \frac{r\cos\theta_1}{R\cos\theta_2} = \frac{3}{4}\frac{\cos\theta_1}{\sqrt{1-\sin^2\theta_2}}$$

But $$\sin^2\theta_2 = \left(\frac{3}{4}\sin\theta_1\right)^2 = \frac{9}{16}\left(1-\cos^2\theta_1\right)$$

So $$\frac{z}{d} = \frac{3}{4}\frac{\cos\theta_1}{\sqrt{1-\frac{9}{16}+\frac{9}{16}\cos^2\theta_1}} = \frac{3}{4}\frac{\cos\theta_1}{\sqrt{\frac{7+9\cos^2\theta_1}{16}}} \quad \text{or} \quad \boxed{z = \frac{3d\cos\theta_1}{\sqrt{7+9\cos^2\theta_1}}}$$

35.55 As the beam enters the slab, $(1.00)\sin 50.0° = (1.48)\sin\theta_2$ giving $\theta_2 = 31.2°$. The beam then strikes the top of the slab at $x_1 = 1.55\text{ mm}/\tan(31.2°)$ from the left end. Thereafter, the beam strikes a face each time it has traveled a distance of $2x_1$ along the length of the slab. Since the slab is 420 mm long, the beam has an additional $420\text{ mm} - x_1$ to travel after the first reflection. The number of additional reflections is

$$\frac{420\text{ mm} - x_1}{2x_1} = \frac{420\text{ mm} - 1.55\text{ mm}/\tan(31.2°)}{3.10\text{ mm}/\tan(31.2°)} = 81.5$$



or 81 reflections since the answer must be an integer. The total number of reflections made in the slab is then $\boxed{82}$.

***35.56** (a) $$\frac{S_1'}{S_1} = \left[\frac{n_2-n_1}{n_2+n_1}\right]^2 = \left[\frac{1.52-1.00}{1.52+1.00}\right]^2 = \boxed{0.0426}$$

(b) If medium 1 is glass and medium 2 is air, $$\frac{S_1'}{S_1} = \left[\frac{n_2-n_1}{n_2+n_1}\right]^2 = \left[\frac{1.00-1.52}{1.00+1.52}\right]^2 = 0.0426;$$

There is $\boxed{\text{no difference}}$

(c) $$\frac{S_1'}{S_1} = \left[\frac{1.76\times10^7-1.00}{1.76\times10^7+1.00}\right]^2 = \left[\frac{1.76\times10^7+1.00-2.00}{1.76\times10^7+1.00}\right]^2$$

$$\frac{S_1'}{S_1} = \left[1.00-\frac{2.00}{1.76\times10^7+1.00}\right]^2 \approx 1.00 - 2\left(\frac{2.00}{1.76\times10^7+1.00}\right) = 1.00-2.27\times10^{-7} \quad \text{or} \quad \boxed{100\%}$$

This suggests he appearance would be $\boxed{\text{very shiny, reflecting practically all incident light}}$. See, however, the note concluding the solution to problem 35.51.

***35.57** (a) With $n_1 = 1$ and $n_2 = n$, the reflected fractional intensity is $\dfrac{S_1'}{S_1} = \left(\dfrac{n-1}{n+1}\right)^2$.

The remaining intensity must be transmitted:

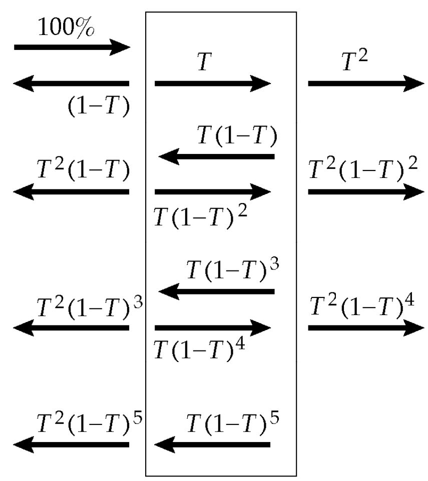
$$\frac{S_2}{S_1} = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{(n+1)^2 - (n-1)^2}{(n+1)^2} = \frac{n^2 + 2n + 1 - n^2 + 2n - 1}{(n+1)^2} = \boxed{\frac{4n}{(n+1)^2}}$$

(b) At entry, $\dfrac{S_2}{S_1} = 1 - \left(\dfrac{n-1}{n+1}\right)^2 = \dfrac{4(2.419)}{(2.419+1)^2} = 0.828$

At exit, $\dfrac{S_3}{S_2} = 0.828$

Overall, $\dfrac{S_3}{S_1} = \left(\dfrac{S_3}{S_2}\right)\left(\dfrac{S_2}{S_1}\right) = (0.828)^2 = 0.685$ or $\boxed{68.5\%}$

***35.58** Define $T = \dfrac{4n}{(n+1)^2}$ as the transmission coefficient for one encounter with an interface. For diamond and air, it is 0.828, as in problem 57.



As shown in the figure, the total amount transmitted is

$$T^2 + T^2(1-T)^2 + T^2(1-T)^4 + T^2(1-T)^6 + \ldots + T^2(1-T)^{2n} + \ldots$$

We have $1 - T = 1 - 0.828 = 0.172$ so the total transmission is

$$(0.828)^2\left[1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots\right]$$

To sum this series, define $F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots$.
Note that $(0.172)^2 F = (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots$, and
$1 + (0.172)^2 F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots = F$.

Then, $1 = F - (0.172)^2 F$ or $F = \dfrac{1}{1-(0.172)^2}$

The overall transmission is then $\dfrac{(0.828)^2}{1-(0.172)^2} = 0.706$ or $\boxed{70.6\%}$

35.59 $n \sin 42.0° = \sin 90.0°$ so $n = \dfrac{1}{\sin 42.0°} = 1.49$

$\sin \theta_1 = n \sin 18.0°$ and $\sin \theta_1 = \dfrac{\sin 18.0°}{\sin 42.0°}$

$\theta_1 = \boxed{27.5°}$

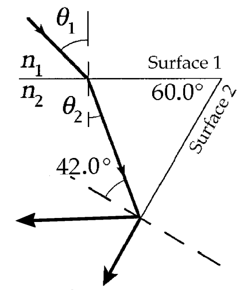


Figure for Goal Solution

Goal Solution

The light beam shown in Figure P35.59 strikes surface 2 at the critical angle. Determine the angle of incidence θ_1.

G: From the diagram it appears that the angle of incidence is about 40°.

O: We can find θ_1 by applying Snell's law at the first interface where the light is refracted. At surface 2, knowing that the 42.0° angle of reflection is the critical angle, we can work backwards to find θ_1.

A: Define n_1 to be the index of refraction of the surrounding medium and n_2 to be that for the prism material. We can use the critical angle of 42.0° to find the ratio n_2/n_1:

$$n_2 \sin 42.0° = n_1 \sin 90.0°$$

So, $$\frac{n_2}{n_1} = \frac{1}{\sin 42.0°} = 1.49$$

Call the angle of refraction θ_2 at the surface 1. The ray inside the prism forms a triangle with surfaces 1 and 2, so the sum of the interior angles of this triangle must be 180°. Thus,

$$(90.0° - \theta_2) + 60.0° + (90.0° - 42.0°) = 180°$$

Therefore, $\theta_2 = 18.0°$

Applying Snell's law at surface 1, $n_1 \sin \theta_1 = n_2 \sin 18.0°$

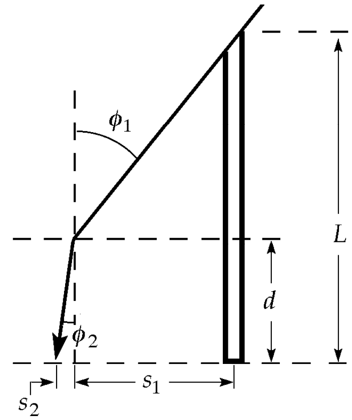
$$\sin \theta_1 = (n_2/n_1) \sin \theta_2 = (1.49) \sin 18.0°$$

$$\theta_1 = 27.5°$$

L: The result is a bit less than the 40.0° we expected, but this is probably because the figure is not drawn to scale. This problem was a bit tricky because it required four key concepts (refraction, reflection, critical angle, and geometry) in order to find the solution. One practical extension of this problem is to consider what would happen to the exiting light if the angle of incidence were varied slightly. Would all the light still be reflected off surface 2, or would some light be refracted and pass through this second surface?

35.60 Light passing the top of the pole makes an angle of incidence $\phi_1 = 90.0° - \theta$. It falls on the water surface at distance

$$s_1 = \frac{(L-d)}{\tan\theta} \text{ from the pole,}$$

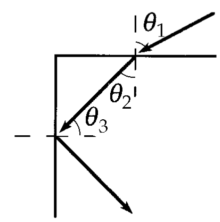


and has an angle of refraction ϕ_2 from $(1.00)\sin\phi_1 = n\sin\phi_2$. Then $s_2 = d\tan\phi_2$ and the whole shadow length is

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\sin\phi_1}{n}\right)\right)$$

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\cos\theta}{n}\right)\right) = \frac{2.00\text{ m}}{\tan 40.0°} + (2.00\text{ m})\tan\left(\sin^{-1}\left(\frac{\cos 40.0°}{1.33}\right)\right) = \boxed{3.79\text{ m}}$$

35.61 (a) For polystyrene *surrounded by air*, internal reflection requires



$$\theta_3 = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

Then from the geometry, $\theta_2 = 90.0° - \theta_3 = 47.8°$

From Snell's law, $\sin\theta_1 = (1.49)\sin 47.8° = 1.10$

This has no solution. Therefore, total internal reflection $\boxed{\text{always happens}}$.

(b) For polystyrene *surrounded by water*, $\theta_3 = \sin^{-1}\left(\frac{1.33}{1.49}\right) = 63.2°$

and $\theta_2 = 26.8°$

From Snell's law, $\theta_1 = \boxed{30.3°}$

(c) $\boxed{\text{No internal refraction is possible}}$ since the beam is initially traveling in a medium of lower index of refraction.

***35.62** $\delta = \theta_1 - \theta_2 = 10.0°$ and $n_1\sin\theta_1 = n_2\sin\theta_2$ with $n_1 = 1,\ n_2 = \frac{4}{3}$

Thus, $\theta_1 = \sin^{-1}(n_2\sin\theta_2) = \sin^{-1}[n_2\sin(\theta_1 - 10.0°)]$

(You can use a calculator to home in on an approximate solution to this equation, testing different values of θ_1 until you find that $\theta_1 = \boxed{36.5°}$. Alternatively, you can solve for θ_1 exactly, as shown below.)

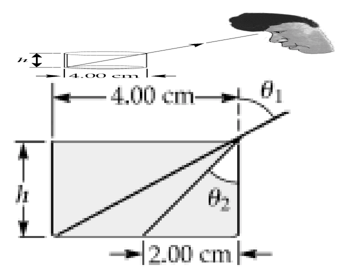
We are given that $\sin\theta_1 = \frac{4}{3}\sin(\theta_1 - 10.0°)$

This is the sine of a difference, so $\frac{3}{4}\sin\theta_1 = \sin\theta_1 \cos 10.0° - \cos\theta_1 \sin 10.0°$

Rearranging, $\sin 10.0° \cos\theta_1 = \left(\cos 10.0° - \frac{3}{4}\right)\sin\theta_1$

$$\frac{\sin 10.0°}{\cos 10.0° - 0.750} = \tan\theta_1 \quad \text{and} \quad \theta_1 = \tan^{-1} 0.740 = \boxed{36.5°}$$

35.63 $\tan\theta_1 = \frac{4.00\text{ cm}}{h}$ and $\tan\theta_2 = \frac{2.00\text{ cm}}{h}$



$$\tan^2\theta_1 = (2.00\tan\theta_2)^2 = 4.00\tan^2\theta_2$$

$$\frac{\sin^2\theta_1}{(1-\sin^2\theta_1)} = 4.00\frac{\sin^2\theta_2}{(1-\sin^2\theta_2)} \qquad (1)$$

Snell's law in this case is: $n_1\sin\theta_1 = n_2\sin\theta_2$

$$\sin\theta_1 = 1.333\ \sin\theta_2$$

Squaring both sides, $\sin^2\theta_1 = 1.777\ \sin^2\theta_2 \qquad (2)$

Substituting (2) into (1), $\frac{1.777\sin^2\theta_2}{1-1.777\sin^2\theta_2} = 4.00\frac{\sin^2\theta_2}{1-\sin^2\theta_2}$

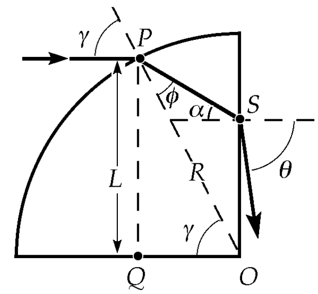
Defining $x = \sin^2\theta$, $\frac{0.444}{(1-1.777x)} = \frac{1}{(1-x)}$

Solving for x, $0.444 - 0.444x = 1 - 1.777x$ and $x = 0.417$

From x we can solve for θ_2: $\theta_2 = \sin^{-1}\sqrt{0.417} = 40.2°$

Thus, the height is $h = \frac{(2.00\text{ cm})}{\tan\theta_2} = \frac{(2.00\text{ cm})}{\tan(40.2°)} = \boxed{2.37\text{ cm}}$

35.64 Observe in the sketch that the angle of incidence at point P is γ, and using triangle OPQ: $\sin\gamma = L/R$.



Also, $$\cos\gamma = \sqrt{1-\sin^2\gamma} = \frac{\sqrt{R^2-L^2}}{R}$$

Applying Snell's law at point P, $(1.00)\sin\gamma = n\sin\phi$

Thus, $$\sin\phi = \frac{\sin\gamma}{n} = \frac{L}{nR}$$

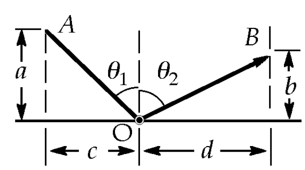
and $$\cos\phi = \sqrt{1-\sin^2\phi} = \frac{\sqrt{n^2R^2-L^2}}{nR}$$

From triangle OPS, $\phi + (\alpha + 90.0°) + (90.0° - \gamma) = 180°$ or the angle of incidence at point S is $\alpha = \gamma - \phi$. Then, applying Snell's law at point S gives $(1.00)\sin\theta = n\sin\alpha = n\sin(\gamma-\phi)$, or

$$\sin\theta = n[\sin\gamma\cos\phi - \cos\gamma\sin\phi] = n\left[\left(\frac{L}{R}\right)\frac{\sqrt{n^2R^2-L^2}}{nR} - \frac{\sqrt{R^2-L^2}}{R}\left(\frac{L}{nR}\right)\right]$$

$$\sin\theta = \frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right) \quad \text{and} \quad \theta = \boxed{\sin^{-1}\left[\frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)\right]}$$

35.65 To derive the law of *reflection*, locate point O so that the time of travel from point A to point B will be minimum.



The *total* light path is $L = a\sec\theta_1 + b\sec\theta_2$

The time of travel is $t = \left(\frac{1}{v}\right)(a\sec\theta_1 + b\sec\theta_2)$

If point O is displaced by dx, then

$$dt = \left(\frac{1}{v}\right)(a\sec\theta_1\tan\theta_1\,d\theta_1 + b\sec\theta_2\tan\theta_2\,d\theta_2) = 0 \qquad (1)$$

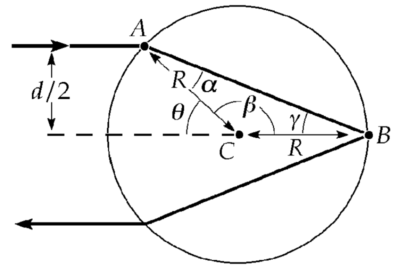
(since for minimum time $dt = 0$).

Also, $c + d = a\tan\theta_1 + b\tan\theta_2 = \text{constant}$

so, $$a\sec^2\theta_1\,d\theta_1 + b\sec^2\theta_2\,d\theta_2 = 0 \qquad (2)$$

Divide equations (1) and (2) to find $\boxed{\theta_1 = \theta_2}$

35.66 As shown in the sketch, the angle of incidence at point A is:



$$\theta = \sin^{-1}\left[\frac{(d/2)}{R}\right] = \sin^{-1}\left[\frac{1.00\text{ m}}{2.00\text{ m}}\right] = 30.0°$$

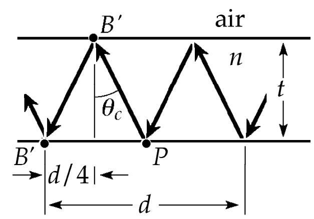
If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the center line CB of the cylinder . In the isosceles triangle ABC, $\gamma = \alpha$ and $\beta = 180° - \theta$. Therefore, $\alpha + \beta + \gamma = 180°$ becomes

$$2\alpha + 180° - \theta = 180° \quad \text{or} \quad \alpha = \frac{\theta}{2} = 15.0°$$

Then, applying Snell's law at point A, $n \sin\alpha = (1.00)\sin\theta$

or $$n = \frac{\sin\theta}{\sin\alpha} = \frac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$$

35.67 (a) At the boundary of the air and glass, the critical angle is given by



$$\sin\theta_c = \frac{1}{n}$$

Consider the critical ray PBB': $\tan\theta_c = \frac{d/4}{t}$ or $\frac{\sin\theta_c}{\cos\theta_c} = \frac{d}{4t}$

Squaring the last equation gives: $\frac{\sin^2\theta_c}{\cos^2\theta_c} = \frac{\sin^2\theta_c}{1-\sin^2\theta_c} = \left(\frac{d}{4t}\right)^2$

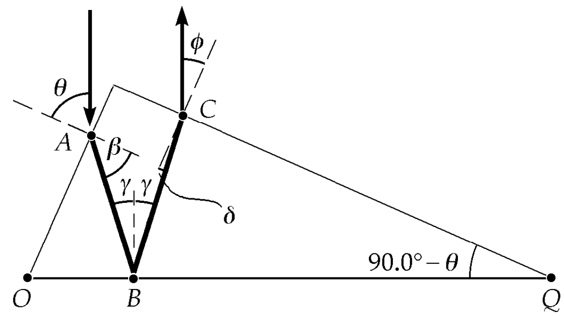
Since $\sin\theta_c = \frac{1}{n}$, this becomes $\frac{1}{n^2-1} = \left(\frac{d}{4t}\right)^2$ or $\boxed{n = \sqrt{1+(4t/d)^2}}$

(b) Solving for d, $d = \frac{4t}{\sqrt{n^2-1}}$

Thus, if $n = 1.52$ and $t = 0.600$ cm, $d = \frac{4(0.600\text{ cm})}{\sqrt{(1.52)^2-1}} = \boxed{2.10\text{ cm}}$

(c) Since violet light has a larger index of refraction, it will lead to a smaller critical angle and the inner edge of the white halo will be tinged with $\boxed{\text{violet}}$ light.

35.68 From the sketch, observe that the angle of incidence at point A is the same as the prism angle θ at point O. Given that $\theta = 60.0°$, application of Snell's law at point A gives



$1.50 \sin\beta = 1.00 \sin 60.0°$ or $\beta = 35.3°$

From triangle AOB, we calculate the angle of incidence (and reflection) at point B.

$\theta + (90.0° - \beta) + (90.0° - \gamma) = 180°$ so $\gamma = \theta - \beta = 60.0° - 35.3° = 24.7°$

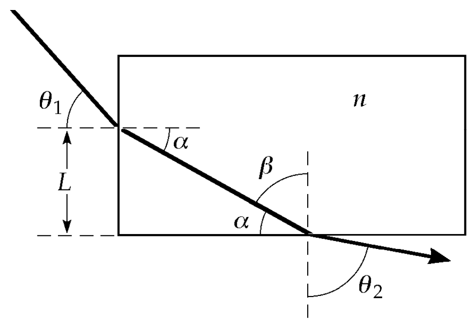
Now, using triangle BCQ: $(90.0° - \gamma) + (90.0° - \delta) + (90.0° - \theta) = 180°$

Thus the angle of incidence at point C is $\delta = (90.0° - \theta) - \gamma = 30.0° - 24.7° = 5.30°$

Finally, Snell's law applied at point C gives $1.00 \sin\phi = 1.50 \sin 5.30°$

or $\phi = \sin^{-1}(1.50 \sin 5.30°) = \boxed{7.96°}$

35.69 (a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$, Snell's law at the first surface gives



$n \sin\alpha = (1.00)\sin 45.0°$ (1)

Observe that the angle of incidence at the second surface is $\beta = 90.0° - \alpha$. Thus, Snell's law at the second surface yields

$n \sin\beta = n \sin(90.0° - \alpha) = (1.00)\sin 76.0°$, or

$n \cos\alpha = \sin 76.0°$ (2)

Dividing Equation (1) by Equation (2), $\tan\alpha = \dfrac{\sin 45.0°}{\sin 76.0°} = 0.729$ or $\alpha = 36.1°$

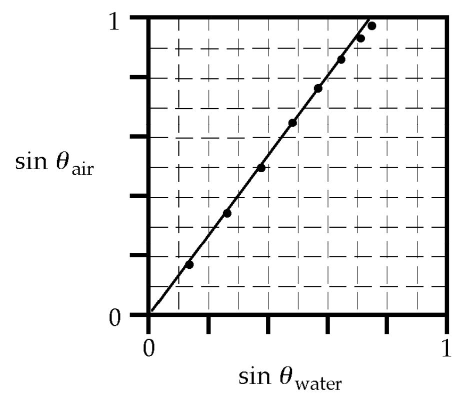
Then, from Equation (1), $n = \dfrac{\sin 45.0°}{\sin\alpha} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$

(b) From the sketch, observe that the distance the light travels in the plastic is $d = L/\sin\alpha$. Also, the speed of light in the plastic is $v = c/n$, so the time required to travel through the plastic is

$$t = \frac{d}{v} = \frac{nL}{c \sin\alpha} = \frac{(1.20)(0.500\text{ m})}{(3.00\times10^{8}\text{ m/s})\sin 36.1°} = 3.40\times10^{-9}\text{ s} = \boxed{3.40\text{ ns}}$$

35.70

$\sin\theta_1$	$\sin\theta_2$	$\sin\theta_1 / \sin\theta_2$
0.174	0.131	1.3304
0.342	0.261	1.3129
0.500	0.379	1.3177
0.643	0.480	1.3385
0.766	0.576	1.3289
0.866	0.647	1.3390
0.940	0.711	1.3220
0.985	0.740	1.3315



The straightness of the graph line demonstrates Snell's proportionality. The slope of the line is $\overline{n} = 1.3276 \pm 0.01$

and $n = \boxed{1.328 \pm 0.8\%}$

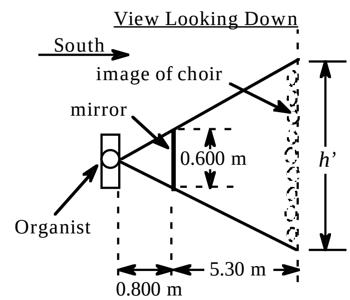
Chapter 36 Solutions

***36.1** I stand 40 cm from my bathroom mirror. I scatter light which travels to the mirror and back to me in time

$$\frac{0.8\text{ m}}{3\times 10^{8}\text{ m/s}}\ \boxed{\sim 10^{-9}\text{ s}}$$

showing me a view of myself as I was at that look-back time. I'm no Dorian Gray!

***36.2** The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror.



The image of the choir is $0.800\text{ m} + 5.30\text{ m} = 6.10\text{ m}$ from the organist. Using similar triangles:

$$\frac{h'}{0.600\text{ m}} = \frac{6.10\text{ m}}{0.800\text{ m}} \quad \text{or} \quad h' = (0.600\text{ m})\left(\frac{6.10\text{ m}}{0.800\text{ m}}\right) = \boxed{4.58\text{ m}}$$

36.3 The flatness of the mirror is described by $R = \infty$, $f = \infty$, and $1/f = 0$. By our general mirror equation,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}, \quad \text{or} \quad q = -p$$

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

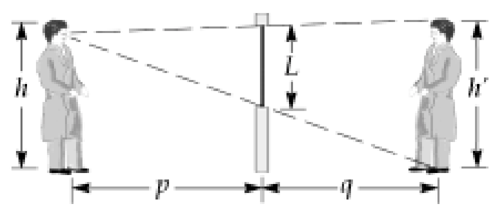


Figure for Goal Solution

$$M = \frac{-q}{p} = 1 = \frac{h'}{h} \quad \text{so} \quad h' = h = 70.0''$$

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

$$h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2}$$

Thus, the mirror must be $\boxed{\text{at least 35.0'' high}}$.

Goal Solution

Determine the minimum height of a vertical flat mirror in which a person 5'10" in height can see his or her full image. (A ray diagram would be helpful.)

G: A diagram with the optical rays that create the image of the person is shown above. From this diagram, it appears that the mirror only needs to be about half the height of the person.

O: The required height of the mirror can be found from the mirror equation, where this flat mirror is described by

$$R = \infty,\ f = \infty, \text{ and } 1/f = 0.$$

A: The general mirror equation is $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, so with $f = \infty$, $q = -p$

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M = \frac{-q}{p} = 1 = \frac{h'}{h}$$

so $h' = h = 70.0$ in.

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of the image, as shown. From the geometry of the similar triangles, we see that the length of the mirror must be:

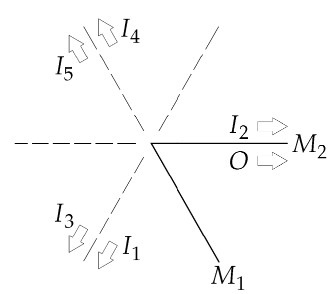
$$L = h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2} = \frac{70.0 \text{ in}}{2} = 35.0 \text{ in}.$$

Thus, the mirror must be at least 35.0 in high.

L: Our result agrees with our prediction from the ray diagram. Evidently, a full-length mirror only needs to be a half-height mirror! On a practical note, the vertical positioning of such a mirror is also important for the person to be able to view his or her full image. To allow for some variation in positioning and viewing by persons of different heights, most full-length mirrors are about 5' in length.

36.4 A graphical construction produces 5 images, with images I_1 and I_2 directly into the mirrors from the object O,

and (O, I_3, I_4) and (I_1, I_2, I_5) forming the vertices of equilateral triangles.



***36.5** (1) The first image in the left mirror is 5.00 ft behind the mirror, or $\boxed{10.0\text{ ft}}$ from the position of the person.

(2) The first image in the right mirror is located 10.0 ft behind the right mirror, but this location is 25.0 ft from the left mirror. Thus, the second image in the left mirror is 25.0 ft behind the mirror, or $\boxed{30.0\text{ ft}}$ from the person.

(3) The first image in the left mirror forms an image in the right mirror. This first image is 20.0 ft from the right mirror, and, thus, an image 20.0 ft behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 35.0 ft. The third image in the left mirror is, thus, 35.0 ft behind the mirror, or $\boxed{40.0\text{ ft}}$ from the person.

***36.6** For a concave mirror, both R and f are positive. We also know that $f = \frac{R}{2} = 10.0\text{ cm}$

(a) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{40.0\text{ cm}} = \frac{3}{40.0\text{ cm}}$, and $\boxed{q = 13.3\text{ cm}}$

$M = \frac{q}{p} = -\frac{13.3\text{ cm}}{40.0\text{ cm}} = \boxed{-0.333}$

The image is 13.3 cm in front of the mirror, is $\boxed{\text{real, and inverted}}$.

(b) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{20.0\text{ cm}} = \frac{1}{20.0\text{ cm}}$, and $\boxed{q = 20.0\text{ cm}}$

$M = \frac{q}{p} = -\frac{20.0\text{ cm}}{20.0\text{ cm}} = \boxed{-1.00}$

The image is 20.0 cm in front of the mirror, is $\boxed{\text{real, and inverted}}$.

(c) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{10.0\text{ cm}} = 0$ Thus, q = infinity.

$\boxed{\text{No image is formed.}}$ The rays are reflected parallel to each other.

***36.7** $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = -\frac{1}{0.275\text{ m}} - \frac{1}{10.0\text{ m}}$ gives $\boxed{q = -0.267\text{ m}}$

Thus, the image is $\boxed{\text{virtual}}$.

$M = \frac{-q}{p} = -\frac{-0.267}{10.0\text{ m}} = \boxed{0.0267}$

Thus, the image is $\boxed{\text{upright}}$ $(+M)$ and $\boxed{\text{diminished}}$ $(|M| < 1)$

***36.8** With radius 2.50 m, the cylindrical wall is a highly efficient mirror for sound, with focal length

$$f = \frac{R}{2} = 1.25 \text{ m}$$

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance q from the back of the niche, where

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \text{so} \qquad \frac{1}{2.00 \text{ m}} + \frac{1}{q} = \frac{1}{1.25 \text{ m}} \qquad q = \boxed{3.33 \text{ m}}$$

36.9 (a) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{(30.0 \text{ cm})} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$

$$\frac{1}{q} = -\frac{2}{(40.0 \text{ cm})} - \frac{1}{(30.0 \text{ cm})} = -0.0833 \text{ cm}^{-1} \qquad \text{so} \qquad q = \boxed{-12.0 \text{ cm}}$$

$$M = \frac{-q}{p} = -\frac{(-12.0 \text{ cm})}{(30.0 \text{ cm})} = \boxed{0.400}$$

(b) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{(60.0 \text{ cm})} + \frac{1}{q} = -\frac{2}{(40.0 \text{ cm})}$

$$\frac{1}{q} = -\frac{2}{(40.0 \text{ cm})} - \frac{1}{(60.0 \text{ cm})} = -0.0666 \text{ cm}^{-1} \qquad \text{so} \qquad q = \boxed{-15.0 \text{ cm}}$$

$$M = -\frac{q}{p} = -\frac{(-15.0 \text{ cm})}{(60.0 \text{ cm})} = \boxed{0.250}$$

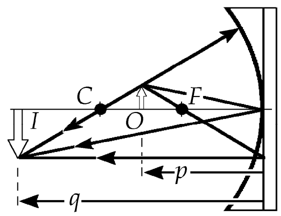
(c) Since $M > 0$, the images are $\boxed{\text{upright}}$.

36.10 (a) $M = -\frac{q}{p}$. For a real image, $q > 0$ so in this case $M = -4.00$

$q = -pM = 120$ cm and from $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$

$$R = \frac{2pq}{(p+q)} = \frac{2(30.0 \text{ cm})(120 \text{ cm})}{(150 \text{ cm})} = \boxed{48.0 \text{ cm}}$$

(b)



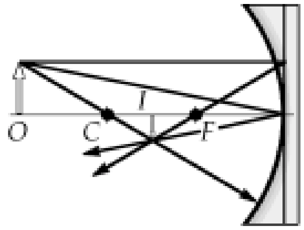
36.11 (a) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{(60.0\text{ cm})}-\frac{1}{(90.0\text{ cm})}$

$q=\boxed{45.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{(45.0\text{ cm})}{(90.0\text{ cm})}=\boxed{-0.500}$

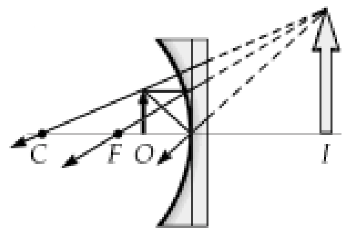
(b) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{(60.0\text{ cm})}-\frac{1}{(20.0\text{ cm})}$,

$q=\boxed{-60.0\text{ cm}}$ and $M=-\frac{q}{p}=-\frac{(-60.0\text{ cm})}{(20.0\text{ cm})}=\boxed{3.00}$

(c) The image in (a) is real, inverted and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are similar to Figures 36.15(a) and 36.15(b), respectively.



(a)



(b)

Figures for Goal Solution

Goal Solution

A concave mirror has a radius of curvature of 60.0 cm. Calculate the image position and magnification of an object placed in front of the mirror (a) at a distance of 90.0 cm and (b) at a distance of 20.0 cm. (c) In each case, draw ray diagrams to obtain the image characteristics.

G: It is always a good idea to first draw a ray diagram for any optics problem. This gives a qualitative sense of how the image appears relative to the object. From the ray diagrams above, we see that when the object is 90 cm from the mirror, the image will be real, inverted, diminished, and located about 45 cm in front of the mirror, midway between the center of curvature and the focal point. When the object is 20 cm from the mirror, the image is be virtual, upright, magnified, and located about 50 cm behind the mirror.

O: The mirror equation can be used to find precise quantitative values.

A: (a) The mirror equation is applied using the sign conventions listed in the text.

$$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text{or} \quad \frac{1}{90.0\text{ cm}}+\frac{1}{q}=\frac{2}{60.0\text{ cm}} \quad \text{so} \quad q = 45.0\text{ cm (real, in front of the mirror)}$$

$$M=\frac{-q}{p}=-\frac{45.0\text{ cm}}{90.0\text{ cm}}=-0.500 \text{ (inverted)}$$

(b) $$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text{or} \quad \frac{1}{20.0\text{ cm}}+\frac{1}{q}=\frac{2}{60.0\text{ cm}} \quad \text{so} \quad q = -60.0\text{ cm (virtual, behind the mirror)}$$

$$M=-\frac{q}{p}=-\frac{-60.0\text{ cm}}{20.0\text{ cm}}=3.00 \text{ (upright)}$$

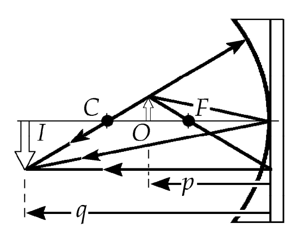
L: The calculated image characteristics agree well with our predictions. It is easy to miss a minus sign or to make a computational mistake when using the mirror-lens equation, so the qualitative values obtained from the ray diagrams are useful for a check on the reasonableness of the calculated values.

36.12 For a concave mirror, R and f are positive. Also, for an erect image, M is positive. Therefore, $M=-\frac{q}{p}=4$ and $q=-4p$.

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q} \quad \text{becomes} \quad \frac{1}{40.0\text{ cm}}=\frac{1}{p}-\frac{1}{4p}=\frac{3}{4p}; \quad \text{from which, } p=\boxed{30.0\text{ cm}}$$

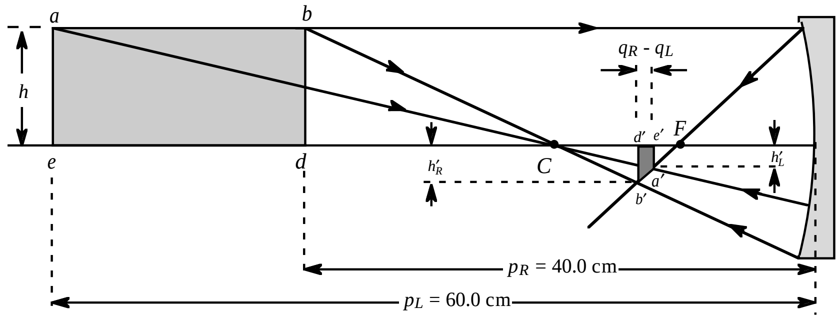
36.13 (a) $q=(p+5.00\text{ m})$ and, since the image must be real, $M=-\frac{q}{p}=-5$ or $q=5p$. Therefore, $p+5.00=5p$ or $p=1.25$ m and $q=6.25$ m.

From $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$, $R=\frac{2pq}{(q+p)}=\frac{2(1.25)(6.25)}{(6.25+1.25)}=\boxed{2.08\text{ m (concave)}}$



(b) From part (a), $p=1.25$ m; the mirror should be $\boxed{1.25\text{ m}}$ in front of the object.

36.14 (a) The image is the trapezoid $a'b'd'e'$ as shown in the ray diagram.



(b) To find the area of the trapezoid, the image distances, q_R and q_L, along with the heights h'_R and h'_L, must be determined. The mirror equation,

$$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text{becomes} \quad \frac{1}{40.0\text{ cm}}+\frac{1}{q_R}=\frac{2}{20.0\text{ cm}} \quad \text{or} \quad q_R=13.3\text{ cm}$$

$$h_R=hM_R=h\left(\frac{-q_R}{p_R}\right)=(10.0\text{ cm})\left(\frac{-13.3\text{ cm}}{40.0\text{ cm}}\right)=-3.33\text{ cm}$$

Also $$\frac{1}{60.0\text{ cm}}+\frac{1}{q_L}=\frac{2}{20.0\text{ cm}} \quad \text{or} \quad q_L=12.0\text{ cm}$$

$$h_L=hM_L=(10.0\text{ cm})\left(\frac{-12.0\text{ cm}}{60.0\text{ cm}}\right)=-2.00\text{ cm}$$

The area of the trapezoid is the sum of the area of a square plus the area of a triangle:

$$A_t=A_1+A_2=(q_R-q_L)h_L+\frac{1}{2}(q_R-q_L)(h_R-h_L)=\boxed{3.56\text{ cm}^2}$$

36.15 Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image (q = – 10.0 cm) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

(concave side: $R=|R|$, $q=-30.0$ cm) $$\frac{1}{p}-\frac{1}{30.0}=\frac{2}{|R|}, \quad \text{or} \quad \frac{2}{|R|}=\frac{30.0\text{ cm}-p}{(30.0\text{ cm})p} \qquad [1]$$

(convex side: $R=-|R|$, $q=-10.0$ cm) $$\frac{1}{p}-\frac{1}{10.0}=-\frac{2}{|R|}, \quad \text{or} \quad \frac{2}{|R|}=\frac{p-10.0\text{ cm}}{(10.0\text{ cm})p} \qquad [2]$$

(a) Equating Equations (1) and (2) gives: $\frac{30.0\text{ cm}-p}{3.00}=p-10.0\text{ cm}$ or $p=15.0$ cm Thus, her face is $\boxed{15.0\text{ cm}}$ from the hubcap.

(b) Using the above result ($p=15.0$ cm) in Equation [1] gives:

$$\frac{2}{|R|}=\frac{30.0\text{ cm}-15.0\text{ cm}}{(30.0\text{ cm})(15.0\text{ cm})} \quad \text{or} \quad \frac{2}{|R|}=\frac{1}{30.0\text{ cm}}, \quad \text{and} \quad |R|=60.0\text{ cm}$$

The radius of the hubcap is $\boxed{60.0\text{ cm}}$.

36.16 $\quad \dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{f} \qquad f=\dfrac{R}{2} = -1.50\text{ cm}$

$$\boxed{q=-\frac{15.0}{11.0}\text{ cm (behind mirror)}} \qquad M=\frac{-q}{p} = \boxed{\frac{1}{11.0}}$$

36.17 (a) The image starts from a point whose height above the mirror vertex is given by

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R} \qquad \frac{1}{3.00\text{ m}}+\frac{1}{q}=\frac{1}{0.500\text{ m}} \text{ Therefore,} \qquad q=0.600\text{ m}$$

As the ball falls, p decreases and q increases. Ball and image pass when $q_1=p_1$. When this is true,

$$\frac{1}{p_1}+\frac{1}{p_1}=\frac{1}{0.500\text{ m}}=\frac{2}{p_1} \qquad \text{or} \qquad p_1=1.00\text{ m}.$$

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when $p_2=q_2=0$.

(b) The falling ball passes its real image when it has fallen

$$3.00\text{ m}-1.00\text{ m}=2.00\text{ m}=\tfrac{1}{2}gt^2,\text{ or when } t=\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}} = \boxed{0.639\text{ s}}.$$

The ball reaches its virtual image when it has traversed

$$3.00\text{ m}-0=3.00\text{ m}=\tfrac{1}{2}gt^2,\text{ or at } t=\sqrt{\frac{2(3.00\text{ m})}{9.80\text{ m/s}^2}} = \boxed{0.782\text{ s}}.$$

36.18 When $R\to\infty$, Equation 36.8 for a spherical surface becomes $q=-p(n_2/n_1)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate:

$$q_{B1}=-\left(\frac{1.33}{1.66}\right)(8.00\text{ cm})=-6.41\text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass or 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$$q_{B2} = -\left(\frac{1.00}{1.33}\right)(18.41\text{ cm}) = -13.84\text{ cm} \qquad \text{or} \qquad 13.84\text{ cm below the water surface.}$$

Now find image the water forms of the *top* surface of the glass.

$$q_3 = -\left(\frac{1}{1.33}\right)(12.0\text{ cm}) = -9.02\text{ cm, or } 9.02\text{ cm below the water surface.}$$

Therefore, the apparent thickness of the glass is $\Delta t = 13.84\text{ cm} - 9.02\text{ cm} = \boxed{4.82\text{ cm}}$

36.19 $\dfrac{n_1}{p} + \dfrac{n_2}{q} = \dfrac{n_2 - n_1}{R} = 0$ and $R \to \infty$

$$q = -\frac{n_2}{n_1}\,p = -\frac{1}{1.309}(50.0\text{ cm}) = -38.2\text{ cm}$$

Thus, the virtual image of the dust speck is $\boxed{\text{38.2 cm below the top surface}}$ of the ice.

***36.20** $\dfrac{n_1}{p} + \dfrac{n_2}{q} = \dfrac{n_2 - n_1}{R}$ so $\dfrac{1.00}{\infty} + \dfrac{1.40}{21.0\text{ mm}} = \dfrac{1.40 - 1.00}{6.00\text{ mm}}$ and $0.0667 = 0.0667$

They agree. $\boxed{\text{The image is inverted, real and diminished.}}$

36.21 From Equation 36.8, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$

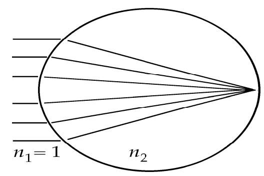
Solve for q to find $q = \frac{n_2 R p}{p(n_2 - n_1) - n_1 R}$

In this case, $n_1 = 1.50$, $n_2 = 1.00$, $R = -15.0$ cm, and $p = 10.0$ cm,

So $q = \frac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00 - 1.50) - (1.50)(-15.0\text{ cm})} = -8.57$ cm

Therefore, the $\boxed{\text{apparent depth is 8.57 cm}}$.

36.22 $p = \infty$ and $q = +2R$



$$\frac{1.00}{p} + \frac{n_2}{q} = \frac{n_2 - 1.00}{R}$$

$0 + \frac{n_2}{2R} = \frac{n_2 - 1.00}{R}$ so $\boxed{n_2 = 2.00}$

36.23 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$ because $\frac{1.00}{p} + \frac{1.50}{q} = \frac{1.50 - 1.00}{6.00\text{ cm}} = \frac{1.00}{12.0\text{ cm}}$

(a) $\frac{1}{20.0\text{ cm}} + \frac{1.50}{q} = \frac{1.00}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}} - \frac{1.00}{20.0\text{ cm}}\right]} = \boxed{45.0\text{ cm}}$

(b) $\frac{1.00}{10.0\text{ cm}} + \frac{1.50}{q} = \frac{1.00}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}} - \frac{1.00}{10.0\text{ cm}}\right]} = \boxed{-90.0\text{ cm}}$

(c) $\frac{1.00}{3.00\text{ cm}} + \frac{1.50}{q} = \frac{1.00}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}} - \frac{1.00}{3.00\text{ cm}}\right]} = \boxed{-6.00\text{ cm}}$

36.24 For a plane surface, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $q = -\frac{n_2 p}{n_1}$.

Thus, the magnitudes of the rate of change in the image and object positions are related by

$$\left|\frac{dq}{dt}\right| = \frac{n_2}{n_1}\left|\frac{dp}{dt}\right|$$

If the fish swims toward the wall with a speed of 2.00 cm/s, the speed of the image is given by

$$v_{\text{image}} = \left|\frac{dq}{dt}\right| = \frac{1.00}{1.33}(2.00\text{ cm/s}) = \boxed{1.50\text{ cm/s}}$$

36.25 $\dfrac{n_1}{p}+\dfrac{n_2}{q}=\dfrac{n_2-n_1}{R}$ $\qquad n_1=1.33 \qquad n_2=1.00 \qquad p=+10.0\text{ cm} \qquad R=-15.0\text{ cm}$

$q=-9.01\text{ cm}$, or the fish appears to be $\boxed{9.01\text{ cm inside the bowl}}$

***36.26** Let R_1 = outer radius and R_2 = inner radius

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.50-1)\left(\frac{1}{2.00\text{ m}}-\frac{1}{2.50\text{ cm}}\right)=\frac{0.0500}{\text{cm}} \qquad \text{so} \qquad f=\boxed{20.0\text{ cm}}$$

36.27 (a) $\dfrac{1}{f}=(n-1)\left[\dfrac{1}{R_1}-\dfrac{1}{R_2}\right]=(0.440)\left[\dfrac{1}{(12.0\text{ cm})}-\dfrac{1}{(-18.0\text{ cm})}\right]$: $f=\boxed{16.4\text{ cm}}$

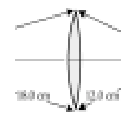


Figure for Goal Solution

(b) $\dfrac{1}{f}=(0.440)\left[\dfrac{1}{(18.0\text{ cm})}-\dfrac{1}{(-12.0\text{ cm})}\right]$: $\qquad f=\boxed{16.4\text{ cm}}$

<u>Goal Solution</u>

The left face of a biconvex lens has a radius of curvature of magnitude 12.0 cm, and the right face has a radius of curvature of magnitude 18.0 cm. The index of refraction of the glass is 1.44. (a) Calculate the focal length of the lens. (b) Calculate the focal length if the radii of curvature of the two faces are interchanged.

G: Since this is a biconvex lens, the center is thicker than the edges, and the lens will tend to converge incident light rays. Therefore it has a positive focal length. Exchanging the radii of curvature amounts to turning the lens around so the light enters the opposite side first. However, this does not change the fact that the center of the lens is still thicker than the edges, so we should not expect the focal length of the lens to be different (assuming the thin-lens approximation is valid).

O: The lens makers' equation can be used to find the focal length of this lens.

A: The centers of curvature of the lens surfaces are on opposite sides, so the second surface has a negative radius:

(a) $\dfrac{1}{f}=(n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)=(1.44-1.00)\left(\dfrac{1}{12.0\text{ cm}}-\dfrac{1}{-18.0\text{ cm}}\right)$ $\qquad$ so $\qquad f=16.4\text{ cm}$

(b) $\dfrac{1}{f}=(0.440)\left(\dfrac{1}{18.0\text{ cm}}-\dfrac{1}{-12.0\text{ cm}}\right)$ $\qquad$ so $\qquad f=16.4\text{ cm}$

L: As expected, reversing the orientation of the lens does not change what it does to the light, as long as the lens is relatively thin (variations may be noticed with a thick lens). The fact that light rays can be traced forward or backward through an optical system is sometimes referred to as the **principle of reversibility**. We can see that the focal length of this biconvex lens is about the same magnitude as the average radius of curvature. A few approximations, useful as checks, are that a symmetric biconvex lens with radii of magnitude R will have focal length $f\approx R$; a plano-convex lens with radius R will have $f\approx R/2$; and a symmetric biconcave lens has $f\approx -R$. These approximations apply when the lens has $n\approx 1.5$, which is typical of many types of clear glass and plastic.

***36.28** For a converging lens, f is positive. We use $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{f}$.

(a) $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = \dfrac{1}{20.0\text{ cm}} - \dfrac{1}{40.0\text{ cm}} = \dfrac{1}{40.0\text{ cm}}$ $\boxed{q = 40.0\text{ cm}}$

$M = -\dfrac{q}{p} = -\dfrac{40.0}{40.0} = \boxed{-1.00}$

The image is $\boxed{\text{real, inverted}}$, and located 40.0 cm past the lens.

(b) $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = \dfrac{1}{20.0\text{ cm}} - \dfrac{1}{20.0\text{ cm}} = 0$ $\boxed{q = \text{infinity}}$

$\boxed{\text{No image}}$ is formed. The rays emerging from the lens are parallel to each other.

(c) $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = \dfrac{1}{20.0\text{ cm}} - \dfrac{1}{10.0\text{ cm}} = -\dfrac{1}{20.0\text{ cm}}$ $\boxed{q = -20.0\text{ cm}}$

$M = \dfrac{-q}{p} = -\dfrac{-20.0}{10.0} = \boxed{2.00}$

The image is $\boxed{\text{upright, virtual}}$, and 20.0 cm in front of the lens.

***36.29** (a) $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = \dfrac{1}{25.0\text{ cm}} - \dfrac{1}{26.0\text{ cm}}$ $q = \boxed{650\text{ cm}}$

The image is $\boxed{\text{real, inverted, and enlarged}}$.

(b) $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = \dfrac{1}{25.0\text{ cm}} - \dfrac{1}{24.0\text{ cm}}$ $q = \boxed{-600\text{ cm}}$

The image is $\boxed{\text{virtual, upright, and enlarged}}$.

36.30 (a) $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} \Rightarrow \dfrac{1}{(32.0\text{ cm})} + \dfrac{1}{(8.00\text{ cm})} = \dfrac{1}{f}$ so $\boxed{f = 6.40\text{ cm}}$

(b) $M = \dfrac{-q}{p} = \dfrac{-(8.00\text{ cm})}{(32.00\text{ cm})} = \boxed{-0.250}$

(c) Since $f > 0$, the lens is $\boxed{\text{converging}}$.

36.31 We are looking at an enlarged, upright, virtual image:

$$M = \frac{h'}{h} = 2 = -\frac{q}{p} \qquad \text{so} \qquad p = -\frac{q}{2} = -\frac{-2.84 \text{ cm}}{2} = +1.42 \text{ cm}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \text{gives} \qquad \frac{1}{1.42 \text{ cm}} + \frac{1}{-2.84 \text{ cm}} = \frac{1}{f}$$

$$f = \boxed{2.84 \text{ cm}}$$

***36.32** To use the lens as a magnifying glass, we form an upright, virtual image:

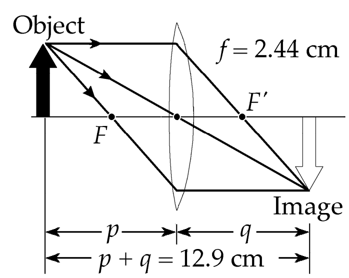
$$M = +2.00 = \frac{-q}{p} \qquad \text{or} \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

We eliminate $q = -2.00p$::

$$\frac{1}{p} + \frac{1}{-2.00p} = \frac{1}{15.0 \text{ cm}} \qquad \text{or} \qquad \frac{-2.00 + 1.00}{-2.00p} = \frac{1}{15.0 \text{ cm}}$$

Solving, $p = \boxed{7.50 \text{ cm}}$

36.33 (a) Note that $q = 12.9 \text{ cm} - p$



so

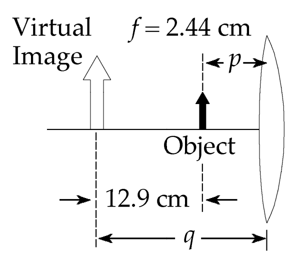
$$\frac{1}{p} + \frac{1}{12.9 - p} = \frac{1}{2.44}$$

which yields a quadratic in p: $-p^2 + 12.9p = 31.5$

which has solutions $\boxed{p = 9.63 \text{ cm or } p = 3.27 \text{ cm}}$

Both solutions are valid.

(b) For a virtual image, $-q = p + 12.9$ cm



$$\frac{1}{p} - \frac{1}{12.9 + p} = \frac{1}{2.44}$$

or $p^2 + 12.9p = 31.8$

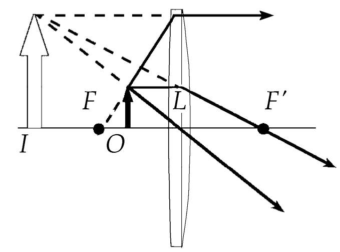
from which $\boxed{p = 2.10 \text{ cm}}$ or $p = -15.0$ cm.

We must have a real object so the -15.0 cm solution must be rejected.

36.34 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$: $\frac{1}{p} + \frac{1}{-30.0\text{ cm}} = \frac{1}{12.5\text{ cm}}$

$p = 8.82$ cm $M = -\frac{q}{p} = -\frac{(-30.0)}{8.82} = \boxed{3.40\text{, upright}}$

(b) See the figure to the right.



***36.35** $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$: $p^{-1} + q^{-1} = \text{constant}$

We may differentiate through with respect to p: $-1p^{-2} - 1q^{-2}\frac{dq}{dp} = 0$

$$\frac{dq}{dp} = -\frac{q^2}{p^2} = -M^2$$

36.36 The image is inverted: $M = \frac{h'}{h} = \frac{-1.80\text{ m}}{0.0240\text{ m}} = -75.0 = \frac{-q}{p}$ $q = 75.0p$

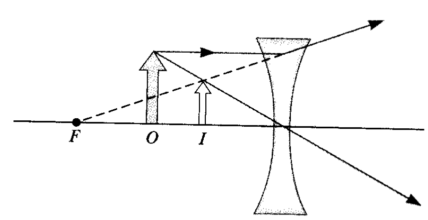
(b) $q + p = 3.00\text{ m} = 75.0p + p$ $p = \boxed{39.5\text{ mm}}$

(a) $q = 2.96$ m $\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.0395\text{ m}} + \frac{1}{2.96\text{ m}}$

$f = \boxed{39.0\text{ mm}}$

36.37 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\frac{1}{(20.0\text{ cm})} + \frac{1}{q} = \frac{1}{(-32.0\text{ cm})}$

so $q = -\left(\frac{1}{20.0} + \frac{1}{32.0}\right)^{-1} = \boxed{-12.3\text{ cm}}$



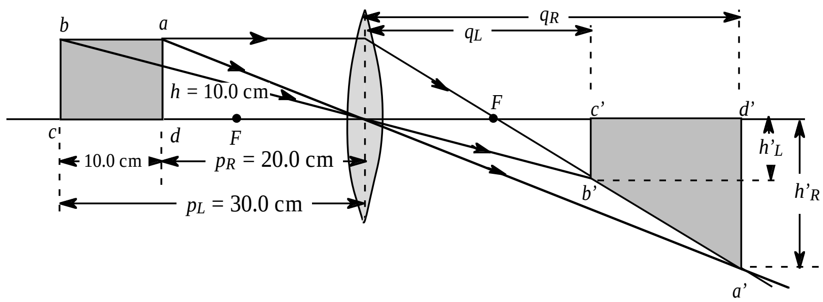
The image is 12.3 cm to the left of the lens.

(b) $M = -\frac{q}{p} = -\frac{(-12.3\text{ cm})}{(20.0\text{ cm})} = \boxed{0.615}$

(c) See the ray diagram above.

36.38 (a) $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.50-1)\left[\frac{1}{15.0\text{ cm}}-\frac{1}{(-12.0\text{ cm})}\right]$, or $\boxed{f=13.3\text{ cm}}$

(b) Ray Diagram:



(c) To find the area, first find q_R and q_L, along with the heights h'_R and h'_L, using the thin lens equation.

$\frac{1}{p_R}+\frac{1}{q_R}=\frac{1}{f}$ becomes: $\frac{1}{20.0\text{ cm}}+\frac{1}{q_R}=\frac{1}{13.3\text{ cm}}$ or $q_R=40.0\text{ cm}$

$$h'_R=hM_R=h\left(\frac{-q_R}{p_R}\right)=(10.0\text{ cm})(-2.00)=-20.0\text{ cm}$$

$\frac{1}{30.0\text{ cm}}+\frac{1}{q_L}=\frac{1}{13.3\text{ cm}}$: $q_L=24.0\text{ cm}$

$$h'_L=hM_L=(10.0\text{ cm})(-0.800)=-8.00\text{ cm}$$

Thus, the area of the image is: Area $=\left|q_R-q_L\right|\left|h'_L\right|+\frac{1}{2}\left|q_R-q_L\right|\left|h'_R-h'_L\right|=\boxed{224\text{ cm}^2}$

36.39 (a) The distance from the object to the lens is p, so the image distance is $q=5.00\text{ m}-p$.

Thus, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes: $\frac{1}{p}+\frac{1}{5.00\text{ m}-p}=\frac{1}{0.800\text{ m}}$

This reduces to a quadratic equation: $p^2-(5.00\text{ m})p+(4.00\text{ m})=0$

which yields $\boxed{p=4.00\text{ m, or } p=1.00\text{ m}}$.

Thus, there are two possible object distances, both corresponding to real objects.

(b) For $p=4.00$ m: $q=5.00\text{ m}-4.00\text{ m}=1.00\text{ m}$: $M=-\frac{1.00\text{ m}}{4.00\text{ m}}=\boxed{-0.250}$.

For p = 1.00 m: $q=5.00\text{ m}-1.00\text{ m}=4.00\text{ m}$: $M=-\frac{4.00\text{ m}}{1.00\text{ m}}=\boxed{-4.00}$.

Both images are $\boxed{\text{real and inverted}}$, but the magnifications are different, with one being larger than the object and the other smaller.

36.40 (a) The image distance is: $q = d - p$. Thus, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes $\frac{1}{p} + \frac{1}{d-p} = \frac{1}{f}$

This reduces to a quadratic equation: $p^2 + (-d)p + (fd) = 0$

which yields: $\boxed{p = \frac{d \pm \sqrt{d^2 - 4fd}}{2} = \left(\frac{d}{2}\right) \pm \sqrt{\frac{d^2}{4} - fd}}$

Since $f < d/4$, both solutions are meaningful and the two solutions are not equal to each other. Thus, there are two distinct lens positions that form an image on the screen.

(b) The smaller solution for p gives a larger value for q, with a $\boxed{\text{real, enlarged, inverted image}}$.

The larger solution for p describes a $\boxed{\text{real, diminished, inverted image}}$.

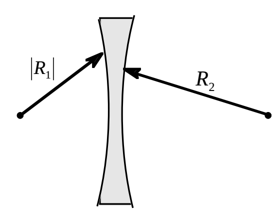
***36.41** To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0$ mm). For the closer object:

$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f}$ becomes $\frac{1}{2000 \text{ mm}} + \frac{1}{q_2} = \frac{1}{65.0 \text{ mm}}$ and $q_2 = (65.0 \text{ mm})\left(\frac{2000}{2000 - 65.0}\right)$

The lens must be moved $\boxed{\text{away from the film}}$ by a distance

$$D = q_2 - q_1 = (65.0 \text{ mm})\left(\frac{2000}{2000 - 65.0}\right) - 65.0 \text{ mm} = \boxed{2.18 \text{ mm}}$$

***36.42** (a) The focal length of the lens is given by

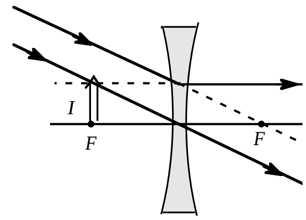


$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.53 - 1.00)\left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}}\right)$$

$f = -34.7$ cm

Note that R_1 is negative because the center of curvature of the first surface is on the virtual image side.

When $p = \infty$, the thin lens equation gives $q = f$. Thus, the violet image of a very distant object is formed at $\boxed{q = -34.7\text{ cm}}$. The image is $\boxed{\text{virtual, upright, and diminished}}$.



(b) The same ray diagram and image characteristics apply for red light. Again, $q = f$, and now

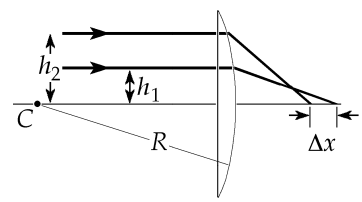
$$\frac{1}{f} = (1.51 - 1.00)\left(\frac{1}{-32.5\text{ cm}} - \frac{1}{42.5\text{ cm}}\right) \text{ giving } f = \boxed{-36.1\text{ cm}}.$$

36.43 Ray h_1 is undeviated at the plane surface and strikes the second surface at angle of incidence given by

$$\theta_1 = \sin^{-1}\left(\frac{h_1}{R}\right) = \sin^{-1}\left(\frac{0.500\text{ cm}}{20.0\text{ cm}}\right) = 1.43°$$

Then, $(1.00)\sin\theta_2 = (1.60)\sin\theta_1 = (1.60)\left(\frac{0.500}{20.0\text{ cm}}\right)$ so $\theta_2 = 2.29°$

The angle this emerging ray makes with the horizontal is



$$\theta_2 - \theta_1 = 0.860°$$

It crosses the axis at a point farther out by f_1 where

$$f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)} = \frac{0.500\text{ cm}}{\tan(0.860°)} = 33.3\text{ cm}$$

The point of exit for this ray is distant axially from the lens vertex by

$$20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (0.500\text{ cm})^2} = 0.00625\text{ cm}$$

so ray h_1 crosses the axis at this distance from the vertex:

$$x_1 = 33.3\text{ cm} - 0.00625\text{ cm} = 33.3\text{ cm}$$

Now we repeat this calculation for ray h_2: $\theta_1 = \sin^{-1}\left(\frac{12.0\text{ cm}}{20.0\text{ cm}}\right) = 36.9°$

$$(1.00)\sin\theta_2 = (1.60)\sin\theta_1 = (1.60)\left(\frac{12.00}{20.0}\right) \qquad \theta_2 = 73.7°$$

$$f_2 = \frac{h_2}{\tan(\theta_2 - \theta_1)} = \frac{12.0\text{ cm}}{\tan(36.8°)} = 16.0\text{ cm}$$

$$x_2 = (16.0\text{ cm})\left(20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (12.0\text{ cm})^2}\right) = 12.0\text{ cm}$$

Now $\Delta x = 33.3\text{ cm} - 12.0\text{ cm} = \boxed{21.3\text{ cm}}$

36.44 For starlight going through Nick's glasses, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

$$\frac{1}{\infty}+\frac{1}{(-0.800\text{ m})}=\frac{1}{f}=-1.25\text{ diopters}$$

For a nearby object, $\frac{1}{p}+\frac{1}{(-0.180\text{ m})}=-1.25\text{ m}^{-1}$, so $p=\boxed{23.2\text{ cm}}$

36.45 $P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{\infty}-\frac{1}{0.250\text{ m}}=-4.00\text{ diopters}=\boxed{-4.00\text{ diopters, a diverging lens}}$

36.46 Consider an object at infinity, imaged at the person's far point:

$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ $\quad\frac{1}{\infty}+\frac{1}{q}=-4.00\text{ m}^{-1}$ $\quad q=-25.0\text{ cm}$

The person's far point is $25.0\text{ cm}+2.00\text{ cm}=27.0\text{ cm}$ from his eyes. For the contact lenses we want

$$\frac{1}{\infty}+\frac{1}{(-0.270\text{ m})}=\frac{1}{f}=\boxed{-3.70\text{ diopters}}$$

36. 47 First, we use the thin lens equation to find the object distance: $\frac{1}{p}+\frac{1}{(-25.0\text{ cm})}=\frac{1}{10.0\text{ cm}}$

Then, $p=7.14\text{ cm}$ and Then, $M=-\frac{q}{p}=-\frac{(-25.0\text{ cm})}{7.14\text{ cm}}=\boxed{3.50}$

36.48 (a) From the thin lens equation: $\frac{1}{p}+\frac{1}{(-25.0\text{ cm})}=\frac{1}{5.00\text{ cm}}$ or $p=\boxed{4.17\text{ cm}}$

(b) $M=-\frac{q}{p}=1+\frac{25.0\text{ cm}}{f}=1+\frac{25.0\text{ cm}}{5.00\text{ cm}}=\boxed{6.00}$

36.49 Using Equation 36.20, $M=-\left(\frac{L}{f_o}\right)\left(\frac{25.0\text{ cm}}{f_e}\right)=-\left(\frac{23.0\text{ cm}}{0.400\text{ cm}}\right)\left(\frac{25.0\text{ cm}}{2.50\text{ cm}}\right)=\boxed{-575}$

36.50 $$M = M_1 m_e = M_1\left(\frac{25.0\text{ cm}}{f_e}\right) \Rightarrow f_e = \left(\frac{M_1}{M}\right)(25.0\text{ cm}) = \left(\frac{-12.0}{-140}\right)(25.0\text{ cm}) = \boxed{2.14\text{ cm}}$$

36.51 $f_o = 20.0\text{ m}$ $\quad f_e = 0.0250\text{ m}$

(a) The angular magnification produced by this telescope is: $m = -\frac{f_o}{f_e} = \boxed{-800}$

(b) Since $m < 0$, the image is $\boxed{\text{inverted}}$.

***36.52** (a) The lensmaker's equation $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

gives $$q=\frac{1}{1/f-1/p}=\frac{1}{\left(\frac{p-f}{fp}\right)}=\frac{fp}{p-f}$$

Then, $$M=\frac{h'}{h}=-\frac{q}{p}=-\frac{f}{p-f}$$

gives $$\boxed{h'=\frac{hf}{f-p}}$$

(b) For $p>>f$, $f-p\approx -p$. Then, $h'=\boxed{-\frac{hf}{p}}$.

(c) Suppose the telescope observes the space station at the zenith:

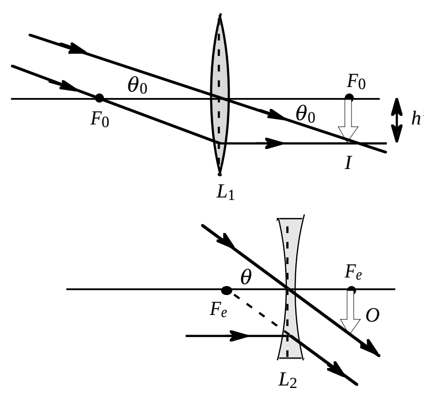
$$h'=-\frac{hf}{p}=-\frac{(108.6\text{ m})(4.00\text{ m})}{407\times 10^{3}\text{ m}}=\boxed{-1.07\text{ mm}}$$

***36.53** (b) Call the focal length of the objective f_o and that of the eyepiece $-|f_e|$. The distance between the lenses is $f_o-|f_e|$. The objective forms a real diminished inverted image of a very distant object at $q_1=f_o$. This image is a virtual object for the eyepiece at $p_2=-|f_e|$.

For it $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{-|f_e|}+\frac{1}{q}=\frac{1}{-|f_e|}$, $\frac{1}{q_2}=0$

and $\boxed{q_2=\infty}$

(a) The user views the image as $\boxed{\text{virtual}}$. Letting h' represent the height of the first image, $\theta_o=h'/f_o$ and $\theta=h'/|f_e|$. The angular magnification is



$$m=\frac{\theta}{\theta_o}=\frac{h'/|f_e|}{h'/f_o}=\frac{f_o}{|f_e|}$$

(c) Here, $f_o-|f_e|=10.0\text{ cm}$ and $\frac{f_o}{|f_e|}=3.00$.

Thus, $|f_e|=\frac{f_o}{3.00}$ and $\frac{2}{3}f_o=10.0\text{ cm}$.

$$f_o=\boxed{15.0\text{ cm}}$$

$|f_e|=5.00\text{ cm}$ and $f_e=\boxed{-5.00\text{ cm}}$

***36.54** Let I_0 represent the intensity of the light from the nebula and θ_o its angular diameter. With the first telescope, the image diameter h' on the film is given by $\theta_o = -h'/f_o$ as $h' = -\theta_o(2000 \text{ mm})$.

The light power captured by the telescope aperture is $\mathsf{P}_1 = I_0 A_1 = I_0\left[\pi(200 \text{ mm})^2/4\right]$, and the light energy focused on the film during the exposure is $E_1 = \mathsf{P}_1 t_1 = I_0\left[\pi(200 \text{ mm})^2/4\right](1.50 \text{ min})$.

Likewise, the light power captured by the aperture of the second telescope is $\mathsf{P}_2 = I_0 A_2 = I_0\left[\pi(60.0 \text{ mm})^2/4\right]$ and the light energy is $E_2 = I_0\left[\pi(60.0 \text{ mm})^2/4\right]t_2$. Therefore, to have the same light energy per unit area, it is necessary that

$$\frac{I_0\left[\pi(60.0 \text{ mm})^2/4\right]t_2}{\pi\left[\theta_o(900 \text{ mm})^2/4\right]} = \frac{I_0\left[\pi(200 \text{ mm})^2/4\right](1.50 \text{ min})}{\pi\left[\theta_o(2000 \text{ mm})^2/4\right]}$$

The required exposure time with the second telescope is

$$t_2 = \frac{(200 \text{ mm})^2(900 \text{ mm})^2}{(60.0 \text{ mm})^2(2000 \text{ mm})^2}(1.50 \text{ min}) = \boxed{3.38 \text{ min}}$$

36.55 Only a diverging lens gives an upright diminished image. The image is virtual and

$$d = p - |q| = p + q: \qquad M = -\frac{q}{p} \qquad \text{so} \qquad q = -Mp \qquad \text{and} \qquad d = p - Mp$$

$$p = \frac{d}{1-M}: \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{-Mp} = \frac{-M+1}{-Mp} = \frac{(1-M)^2}{-Md}$$

$$f = \frac{-Md}{(1-M)^2} = \frac{-(0.500)(20.0 \text{ cm})}{(1-0.500)^2} = \boxed{-40.0 \text{ cm}}$$

36.56 If $M < 1$, the lens is diverging and the image is virtual. $d = p - |q| = p + q$

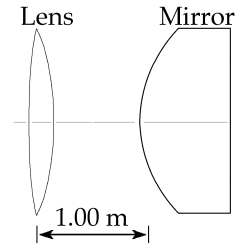
$$M = -\frac{q}{p} \qquad \text{so} \qquad q = -Mp \qquad \text{and} \qquad d = p - Mp$$

$$p = \frac{d}{1-M}: \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{-Mp} = \frac{-M+1}{-Mp} = \frac{(1-M)^2}{-Md} \qquad \boxed{f = \frac{-Md}{(1-M)^2}}$$

If $M > 1$, the lens is converging and the image is still virtual.

Now $d = -q - p$. We obtain in this case $\boxed{f = \frac{Md}{(M-1)^2}}$

***36.57** Start with the first pass through the lens.



$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{80.0\text{ cm}} - \frac{1}{100\text{ cm}} \qquad q_1 = 400\text{ cm to right of lens}$$

For the mirror, $p_2 = -300$ cm

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{-50.0\text{ cm}} - \frac{1}{-300\text{ cm}} \qquad q_2 = -60.0\text{ cm}$$

For the second pass through the lens, $p_3 = 160$ cm

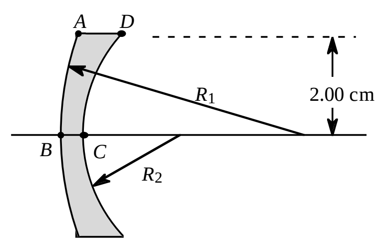
$$\frac{1}{q_3} = \frac{1}{f_1} - \frac{1}{p_3} = \frac{1}{80.0\text{ cm}} - \frac{1}{160\text{ cm}} \qquad q_3 = \boxed{160\text{ cm to left of lens}}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{400\text{ cm}}{100\text{ cm}} = -4.00 \qquad M_2 = -\frac{q_2}{p_2} = -\frac{-60.0\text{ cm}}{-300\text{ cm}} = -\frac{1}{5}$$

$$M_3 = -\frac{q_3}{p_3} = -\frac{160\text{ cm}}{160\text{ cm}} = -1 \qquad M = M_1 M_2 M_3 = \boxed{-0.800}$$

Since $M < 0$ the final image is $\boxed{\text{inverted}}$.

***36.58** (a) $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$



$$\frac{1}{-65.0\text{ cm}} = (1.66-1)\left(\frac{1}{50.0\text{ cm}} - \frac{1}{R_2}\right)$$

$$\frac{1}{R_2} = \frac{1}{50.0\text{ cm}} + \frac{1}{42.9\text{ cm}} \qquad \text{so} \qquad R_2 = \boxed{23.1\text{ cm}}$$

(b) The distance along the axis from B to A is

$$R_1 - \sqrt{R_1^2 - (2.00\text{ cm})^2} = 50.0\text{ cm} - \sqrt{(50.0\text{ cm})^2 - (2.00\text{ cm})^2} = 0.0400\text{ cm}$$

Similarly, the axial distance from C to D is

$$23.1\text{ cm} - \sqrt{(23.1\text{ cm})^2 - (2.00\text{ cm})^2} = 0.0868\text{ cm}$$

Then, $AD = 0.100\text{ cm} - 0.0400\text{ cm} + 0.0868\text{ cm} = \boxed{0.147\text{ cm}}$.

***36.59** $\dfrac{1}{q_1} = \dfrac{1}{f_1} - \dfrac{1}{p_1} = \dfrac{1}{10.0\text{ cm}} - \dfrac{1}{12.5\text{ cm}}$ so $q_1 = 50.0$ cm (to left of mirror)

This serves as an object for the lens (a virtual object), so

$\dfrac{1}{q_2} = \dfrac{1}{f_2} - \dfrac{1}{p_2} = \dfrac{1}{-16.7\text{ cm}} - \dfrac{1}{-25.0\text{ cm}}$ $q_2 = -50.3$ cm (to right of lens)

Thus, the final image is located $\boxed{\text{25.3 cm to right of mirror}}$.

$$M_1 = -\frac{q_1}{p_1} = -\frac{50.0\text{ cm}}{12.5\text{ cm}} = -4.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{-50.3\text{ cm}}{-25.0\text{ cm}} = -2.01$$

$$M = M_1M_2 = \boxed{8.05}$$

Thus, the final image is $\boxed{\text{virtual, upright}}$, 8.05 times the size of object, and 25.3 cm to right of the mirror.

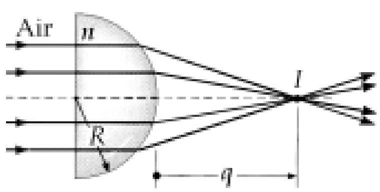
36.60 We first find the focal length of the mirror.

$\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{10.0\text{ cm}} + \dfrac{1}{8.00\text{ cm}} = \dfrac{9}{40.0\text{ cm}}$ and $f = 4.44$ cm

Hence, if $p = 20.0$ cm, $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = \dfrac{1}{4.44\text{ cm}} - \dfrac{1}{20.0\text{ cm}} = \dfrac{15.56}{88.8\text{ cm}}$

Thus, $q = \boxed{5.71\text{ cm}}$, real

36.61 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R = -6.00$ cm



The incident rays are parallel, so $p = \infty$.

Then, $\dfrac{n_1}{p} + \dfrac{n_2}{q} = \dfrac{n_2 - n_1}{R}$ becomes $0 + \dfrac{1}{q} = \dfrac{(1.00 - 1.56)}{-6.00\text{ cm}}$ and $\boxed{q = 10.7\text{ cm}}$

***36.62** (a) $I = \dfrac{\mathcal{P}}{4\pi r^2} = \dfrac{4.50\text{ W}}{4\pi\left(1.60\times10^{-2}\text{ m}\right)^2} = \boxed{1.40\text{ kW/m}^2}$

(b) $I = \dfrac{\mathcal{P}}{4\pi r^2} = \dfrac{4.50\text{ W}}{4\pi(7.20\text{ m})^2} = \boxed{6.91\text{ mW/m}^2}$

(c) $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{f} \Rightarrow \dfrac{1}{7.20\text{ m}}+\dfrac{1}{q}=\dfrac{1}{0.350\text{ m}}$ so $q = 0.368\text{ m}$ and

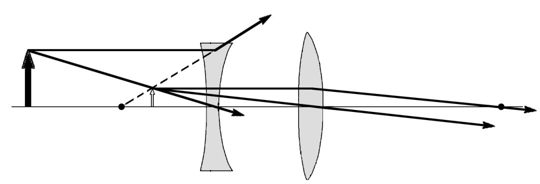
$M = \dfrac{h'}{3.20\text{ cm}} = -\dfrac{q}{p} = -\dfrac{0.368\text{ m}}{7.20\text{ m}}$ $\qquad h' = \boxed{0.164\text{ cm}}$

(d) The lens intercepts power given by $\mathcal{P} = IA = \left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\frac{1}{4}\pi(0.150\text{ m})^2\right]$

and puts it all onto the image where $I = \dfrac{\mathcal{P}}{A} = \dfrac{\left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\pi(15.0\text{ cm})^2/4\right]}{\pi(0.164\text{ cm})^2/4} = \boxed{58.1\text{ W/m}^2}$

36.63 From the thin lens equation, $q_1 = \dfrac{f_1 p_1}{p_1 - f_1} = \dfrac{(-6.00\text{ cm})(12.0\text{ cm})}{12.0\text{ cm} - (-6.00\text{ cm})} = -4.00\text{ cm}$

When we require that $q_2 \to \infty$, the thin lens equation becomes $p_2 = f_2$;



In this case, $p_2 = d - (-4.00\text{ cm})$

Therefore, $d + 4.00\text{ cm} = f_2 = 12.0\text{ cm}$

and $d = \boxed{8.00\text{ cm}}$

***36.64** (a) For the light the mirror intercepts, $\mathcal{P} = I_0 A = I_0 \pi R_a^2$

$350\text{ W} = \left(1000\text{ W/m}^2\right)\pi R_a^2$ and $R_a = \boxed{0.334\text{ m or larger}}$

(b) In $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{f}=\dfrac{2}{R}$ we have $p \to \infty$ so $q = \dfrac{R}{2}$.

$M = \dfrac{h'}{h} = -\dfrac{q}{p}$, so $h' = -q(h/p) = -\left(\dfrac{R}{2}\right)\left[0.533°\left(\dfrac{\pi\text{ rad}}{180°}\right)\right] = -\left(\dfrac{R}{2}\right)(9.30\text{ m rad})$

where h/p is the angle the Sun subtends. The intensity at the image is then

$$I = \frac{\mathcal{P}}{\pi h'^2/4} = \frac{4I_0\pi R_a^2}{\pi h'^2} = \frac{4I_0 R_a^2}{(R/2)^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$$

$120\times10^3\text{ W/m}^2 = \dfrac{16\left(1000\text{ W/m}^2\right)R_a^2}{R^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$ so $\boxed{\dfrac{R_a}{R} = 0.0255\text{ or larger}}$

36.65 For the mirror, $f = R/2 = +1.50$ m. In addition, because the distance to the Sun is so much larger than any other figures, we can take $p = \infty$. The mirror equation,

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, then gives $q = f = \boxed{1.50\text{ m}}$.

Now, in $M = -\frac{q}{p} = \frac{h'}{h}$, the magnification is nearly zero, but we can be more precise: $\frac{h}{p}$ is the angular diameter of the object. Thus, the image diameter is

$$h' = -\frac{hq}{p} = (-0.533°)\left(\frac{\pi}{180}\text{ rad/deg}\right)(1.50\text{ m}) = -0.140\text{ m} = \boxed{-1.40\text{ cm}}$$

36.66 (a) The lens makers' equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, becomes:

$$\frac{1}{5.00\text{ cm}} = (n-1)\left[\frac{1}{9.00\text{ cm}} - \frac{1}{(-11.0\text{ cm})}\right] \qquad \text{giving} \qquad n = \boxed{1.99}.$$

(b) As the light passes through the lens for the first time, the thin lens equation

$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$ becomes: $\frac{1}{8.00\text{ cm}} + \frac{1}{q_1} = \frac{1}{5.00\text{ cm}}$

or $q_1 = 13.3\text{ cm}$, and $M_1 = -\frac{q_1}{p_1} = -\frac{13.3\text{ cm}}{8.00\text{ cm}} = -1.67$

This image becomes the object for the concave mirror with:

$p_m = 20.0\text{ cm} - q_1 = 20.0\text{ cm} - 13.3\text{ cm} = 6.67\text{ cm}$, and $f = \frac{R}{2} = +4.00\text{ cm}$.

The mirror equation becomes: $\frac{1}{6.67\text{ cm}} + \frac{1}{q_m} = \frac{1}{4.00\text{ cm}}$

giving $q_m = 10.0\text{ cm}$ and $M_2 = -\frac{q_m}{p_m} = -\frac{10.0\text{ cm}}{6.67\text{ cm}} = -1.50$

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:

$p_3 = 20.0\text{ cm} - q_m = +10.0\text{ cm}$

The thin lens equation yields: $\frac{1}{10.0\text{ cm}} + \frac{1}{q_3} = \frac{1}{5.00\text{ cm}}$

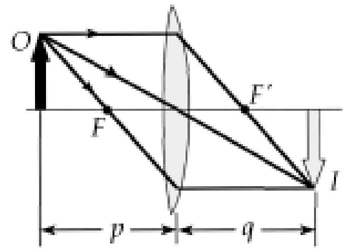
or $q_3 = 10.0\text{ cm}$, and $M_3 = -\frac{q_3}{p_3} = -\frac{10.0\text{ cm}}{10.0\text{ cm}} = -1.00$.

The final image is a real image located $\boxed{\text{10.0 cm to the left of the lens}}$.

The overall magnification is $M_{\text{total}} = M_1 M_2 M_3 = \boxed{-2.50}$.

(c) Since the total magnification is negative, this final image is $\boxed{\text{inverted}}$.

36.67 In the original situation, $p_1 + q_1 = 1.50$ m



In the final situation, $p_2 = p_1 + 0.900$ m

and $q_2 = q_1 - 0.900$ m.

Our lens equation is $\dfrac{1}{p_1} + \dfrac{1}{q_1} = \dfrac{1}{f} = \dfrac{1}{p_2} + \dfrac{1}{q_2}$

Substituting, we have $\dfrac{1}{p_1} + \dfrac{1}{1.50\text{ m} - p_1} = \dfrac{1}{p_1 + 0.900} + \dfrac{1}{0.600 - p_1}$

Adding the fractions, $\dfrac{1.50\text{ m} - p_1 + p_1}{p_1(1.50\text{ m} - p_1)} = \dfrac{0.600 - p_1 + p_1 + 0.900}{(p_1 + 0.900)(0.600 - p_1)}$

Simplified, this becomes $p_1(1.50\text{ m} - p_1) = (p_1 + 0.900)(0.600 - p_1)$

(a) Thus, $p_1 = \dfrac{0.540}{1.80}\text{ m} = \boxed{0.300\text{ m}}$

$p_2 = p_1 + 0.900 = \boxed{1.20\text{ m}}$

(b) $\dfrac{1}{f} = \dfrac{1}{0.300\text{ m}} + \dfrac{1}{1.50\text{ m} - 0.300\text{ m}}$ and $f = \boxed{0.240\text{ m}}$

(c) The second image is $\boxed{\text{real, inverted, and diminished}}$, with $M = -\dfrac{q_2}{p_2} = \boxed{-0.250}$

36.68 As the light passes through, the lens attempts to form an image at distance q_1 where

$$\frac{1}{q_1} = \frac{1}{f} - \frac{1}{p_1} \qquad \text{or} \qquad q_1 = \frac{fp_1}{p_1 - f}$$

This image serves as a virtual object for the mirror with $p_2 = -q_1$. The plane mirror then forms an image located at $q_2 = -p_2 = +q_1$ above the mirror and lens.

This second image serves as a virtual object ($p_3 = -q_2 = -q_1$) for the lens as the light makes a return passage through the lens. The final image formed by the lens is located at distance q_3 above the lens where

$$\frac{1}{q_3} = \frac{1}{f} - \frac{1}{p_3} = \frac{1}{f} + \frac{1}{q_1} = \frac{1}{f} + \frac{p_1 - f}{fp_1} = \frac{2p_1 - f}{fp_1} \qquad \text{or} \qquad q_3 = \frac{fp_1}{2p_1 - f}$$

If the final image coincides with the object, it is necessary to require $q_3 = p_1$, or $\dfrac{fp_1}{2p_1 - f} = p_1$.

This yields the solution $\boxed{p_1 = f}$ or $\boxed{\text{the object must be located at the focal point of the lens}}$.

36.69 For the objective: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{3.40\text{ mm}}+\frac{1}{q}=\frac{1}{3.00\text{ mm}}$ so $q=25.5\text{ mm}$

The objective produces magnification $M_1=-q/p=-\frac{25.5\text{ mm}}{3.40\text{ mm}}=-7.50$

For the eyepiece as a simple magnifier, $m_e=\frac{25.0\text{ cm}}{f}=\frac{25.0\text{ cm}}{2.50\text{ cm}}=10.0$

and overall $M=M_1m_e=\boxed{-75.0}$

36.70 (a) Start with the second lens: This lens must form a virtual image located 19.0 cm to the left of it (i.e., $q_2=-19.0\text{ cm}$). The required object distance for this lens is then

$$p_2=\frac{q_2 f_2}{q_2-f_2}=\frac{(-19.0\text{ cm})(20.0\text{ cm})}{-19.0\text{ cm}-20.0\text{ cm}}=\frac{380\text{ cm}}{39.0}$$

The image formed by the first lens serves as the object for the second lens. Therefore, the image distance for the first lens is

$$q_1=50.0\text{ cm}-p_2=50.0\text{ cm}-\frac{380\text{ cm}}{39.0}=\frac{1570\text{ cm}}{39.0}$$

The distance the original object must be located to the left of the first lens is then given by

$$\frac{1}{p_1}=\frac{1}{f_1}-\frac{1}{q_1}=\frac{1}{10.0\text{ cm}}-\frac{39.0}{1570\text{ cm}}=\frac{157-39.0}{1570\text{ cm}}=\frac{118}{1570\text{ cm}} \quad\text{or}\quad p_1=\frac{1570\text{ cm}}{118}=\boxed{13.3\text{ cm}}$$

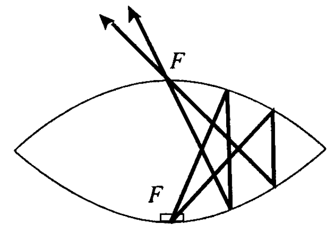
(b) $$M=M_1M_2=\left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right)=\left[\left(\frac{1570\text{ cm}}{39.0}\right)\left(\frac{118}{1570\text{ cm}}\right)\right]\left[\frac{(-19.0\text{ cm})(39.0)}{380\text{ cm}}\right]=\boxed{-5.90}$$

(c) Since $M<0$, the final image is $\boxed{\text{inverted}}$.

36.71 (a) $$P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{(0.0224\text{ m})}+\frac{1}{\infty}=\boxed{44.6\text{ diopters}}$$

(b) $$P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{(0.330\text{ m})}+\frac{1}{\infty}=\boxed{3.03\text{ diopters}}$$

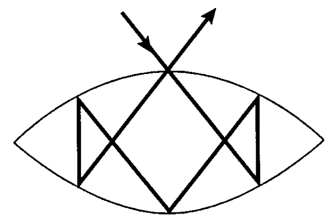
36.72 The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).



The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror. Thus, the $\boxed{\text{image is real, inverted, and actual size}}$.

For the upper mirror:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \Rightarrow \frac{1}{7.50\text{ cm}}+\frac{1}{q_1}=\frac{1}{7.50\text{ cm}}: \; q_1=\infty$$



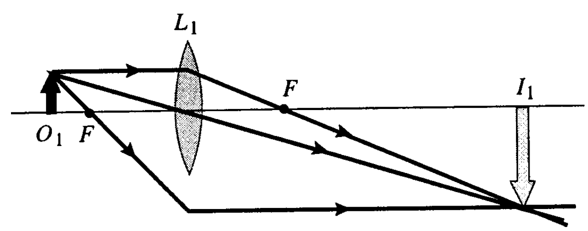
For the lower mirror:

$$\frac{1}{\infty}+\frac{1}{q_2}=\frac{1}{7.50\text{ cm}}: \qquad q_2=7.50\text{ cm}$$

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

36.73 (a) Lens one:

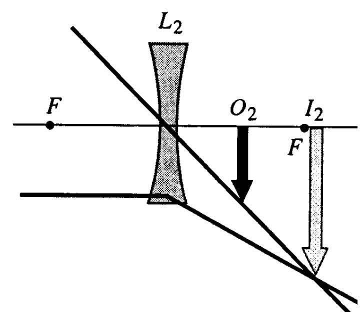
$$\frac{1}{40.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{30.0\text{ cm}}: \qquad q_1=120\text{ cm}$$



$$M_1=-\frac{q_1}{p_1}=-\frac{120\text{ cm}}{40.0\text{ cm}}=-3.00$$

This real image is a virtual object for the second lens, at

$$p_2=110\text{ cm}-120\text{ cm}=-10.0\text{ cm}$$



$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{-20.0\text{ cm}}: \qquad q_2=\boxed{20.0\text{ cm}}$$

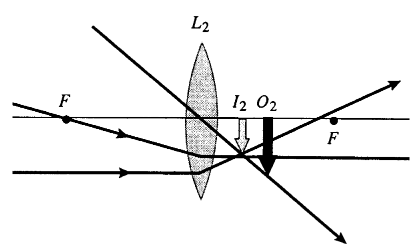
$$M_2=-\frac{q_2}{p_2}=-\frac{20.0\text{ cm}}{(-10.0\text{ cm})}=+2.00$$

$$M_{\text{overall}}=M_1M_2=\boxed{-6.00}$$

(b) $M_{\text{overall}}<0$, so final image is $\boxed{\text{inverted}}$.

(c) Lens two converging: $$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{20.0\text{ cm}}$$

$$q_2=\boxed{6.67\text{ cm}}$$



$$M_2=-\frac{6.67\text{ cm}}{(-10.0\text{ cm})}=+0.667$$

$$M_{\text{overall}}=M_1M_2=\boxed{-2.00}$$

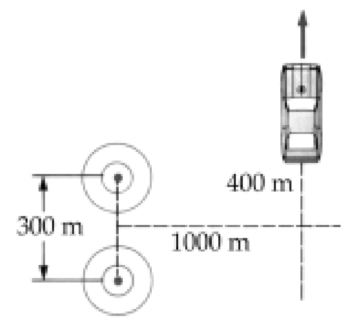
Again, $M_{\text{overall}}<0$ and the final image is $\boxed{\text{inverted}}$.

Chapter 37 Solutions

37.1 $\Delta y_{\text{bright}} = \dfrac{\lambda L}{d} = \dfrac{(632.8 \times 10^{-9})(5.00)}{2.00 \times 10^{-4}}$ m = $\boxed{1.58 \text{ cm}}$

37.2 $y_{\text{bright}} = \dfrac{\lambda L}{d} m$ For $m = 1$, $\lambda = \dfrac{yd}{L} = \dfrac{(3.40 \times 10^{-3} \text{ m})(5.00 \times 10^{-4} \text{ m})}{3.30 \text{ m}} = \boxed{515 \text{ nm}}$

37.3 Note, with the conditions given, the small angle approximation **does not work well**. That is, sin θ, tan θ, and θ are significantly different. The approach to be used is outlined below.



(a) At the $m = 2$ maximum, $\tan\theta = \dfrac{400 \text{ m}}{1000 \text{ m}} = 0.400$

$\theta = 21.8°$ so $\lambda = \dfrac{d \sin\theta}{m} = \dfrac{(300 \text{ m}) \sin 21.8°}{2} = \boxed{55.7 \text{ m}}$

(b) The next minimum encountered is the $m = 2$ minimum; and at that point,

$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$ which becomes $d \sin\theta = \frac{5}{2}\lambda$

or $\sin\theta = \dfrac{5\lambda}{2d} = \dfrac{5(55.7 \text{ m})}{2(300 \text{ m})} = 0.464$ and $\theta = 27.7°$

so $y = (1000 \text{ m}) \tan 27.7° = 524$ m

Therefore, the car must travel an additional $\boxed{124 \text{ m}}$.

37.4 $\lambda = \dfrac{v}{f} = \dfrac{354 \text{ m/s}}{2000/\text{s}} = 0.177$ m

(a) $d \sin\theta = m\lambda$ so $(0.300 \text{ m}) \sin\theta = 1(0.177 \text{ m})$ and $\theta = \boxed{36.2°}$

(b) $d \sin\theta = m\lambda$ so $d \sin 36.2° = 1(0.0300 \text{ m})$ and $d = \boxed{5.08 \text{ cm}}$

(c) $(1.00 \times 10^{-6} \text{ m}) \sin 36.2° = 1\lambda$ so $\lambda = 590$ nm

$f = \dfrac{c}{\lambda} = \dfrac{3.00 \times 10^{8} \text{ m/s}}{5.90 \times 10^{-7} \text{ m}} = \boxed{508 \text{ THz}}$

37.5 For the tenth minimum, $m = 9$. Using Equation 37.3, $\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$

Also, $\tan\theta = \frac{y}{L}$. For small θ, $\sin\theta \approx \tan\theta$. Thus,

$$d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y} = \frac{9.5(589\times10^{-9}\text{ m})(2.00\text{ m})}{7.26\times10^{-3}\text{ m}} = 1.54\times10^{-3}\text{ m} = \boxed{1.54\text{ mm}}$$

<u>Goal Solution</u>

Young's double-slit experiment is performed with 589-nm light and a slit-to-screen distance of 2.00 m. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

G: For the situation described, the observed interference pattern is very narrow, (the minima are less than 1 mm apart when the screen is 2 m away). In fact, the minima and maxima are so close together that it would probably be difficult to resolve adjacent maxima, so the pattern might look like a solid blur to the naked eye. Since the angular spacing of the pattern is inversely proportional to the slit width, we should expect that for this narrow pattern, the space between the slits will be larger than the typical fraction of a millimeter, and certainly much greater than the wavelength of the light ($d >> \lambda = 589$ nm).

O: Since we are given the location of the tenth minimum for this interference pattern, we should use the equation for **destructive interference** from a double slit. The figure for Problem 7 shows the critical variables for this problem.

A: In the equation $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$,

The first minimum is described by $m = 0$ and the tenth by $m = 9$: $\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$

Also, $\tan\theta = y/L$, but for small θ, $\sin\theta \approx \tan\theta$. Thus, $d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y}$

$$d = \frac{9.5(5890\cdot10^{-10}\text{ m})(2.00\text{ m})}{7.26\cdot10^{-3}\text{ m}} = 1.54\cdot10^{-3}\text{ m} = 1.54\text{ mm} = 1.54\text{ mm}$$

L: The spacing between the slits is relatively large, as we expected (about 3 000 times greater than the wavelength of the light). In order to more clearly distinguish between maxima and minima, the pattern could be expanded by increasing the distance to the screen. However, as L is increased, the overall pattern would be less bright as the light expands over a larger area, so that beyond some distance, the light would be too dim to see.

***37.6** $\lambda = \dfrac{340\text{ m/s}}{2000\text{ Hz}} = 0.170\text{ m}$

Maxima are at $d\sin\theta = m\lambda$:

$m = 0$ gives $\theta = 0°$

$m = 1$ gives $\sin\theta = \dfrac{\lambda}{d} = \dfrac{0.170\text{ m}}{0.350\text{ m}}$ $\theta = 29.1°$

$m = 2$ gives $\sin\theta = \dfrac{2\lambda}{d} = 0.971$ $\theta = 76.3°$

$m = 3$ gives $\sin\theta = 1.46$ No solution.

Minima at $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$:

$m = 0$ gives $\sin\theta = \dfrac{\lambda}{2d} = 0.243$ $\theta = 14.1°$

$m = 1$ gives $\sin\theta = \dfrac{3\lambda}{2d} = 0.729$ $\theta = 46.8°$

$m = 2$ gives No solution.

$\boxed{\text{So we have maxima at } 0°, 29.1°, \text{ and } 76.3° \text{ and minima at } 14.1° \text{ and } 46.8°.}$

37.7 (a) For the bright fringe,

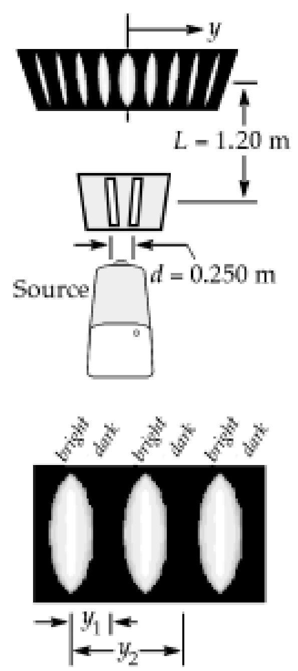
$y_{\text{bright}} = \dfrac{m\lambda L}{d}$ where $m = 1$

$$y = \frac{(546.1\times10^{-9}\text{ m})(1.20\text{ m})}{0.250\times10^{-3}\text{ m}} = 2.62\times10^{-3}\text{ m} = \boxed{2.62\text{ mm}}$$

(b) For the dark bands, $y_{\text{dark}} = \dfrac{\lambda L}{d}\left(m + \dfrac{1}{2}\right)$; $m = 0, 1, 2, 3, \ldots$

$$y_2 - y_1 = \frac{\lambda L}{d}\left[\left(1+\frac{1}{2}\right) - \left(0+\frac{1}{2}\right)\right] = \frac{\lambda L}{d}(1) = \frac{(546.1\times10^{-9}\text{ m})(1.20\text{ m})}{0.250\times10^{-3}\text{ m}}$$

$\Delta y = \boxed{2.62\text{ mm}}$



Figures for Goal Solution

Goal Solution

A pair of narrow, parallel slits separated by 0.250 mm is illuminated by green light (λ = 546.1 nm). The interference pattern is observed on a screen 1.20 m away from the plane of the slits. Calculate the distance (a) from the central maximum to the first bright region on either side of the central maximum and (b) between the first and second dark bands.

G: The spacing between adjacent maxima and minima should be fairly uniform across the pattern as long as the width of the pattern is much less than the distance to the screen (so that the small angle approximation is valid). The separation between fringes should be at least a millimeter if the pattern can be easily observed with a naked eye.

O: The bright regions are areas of constructive interference and the dark bands are destructive interference, so the corresponding double-slit equations will be used to find the y distances.

It can be confusing to keep track of four different symbols for distances. Three are shown in the drawing to the right. Note that:

y is the unknown distance from the bright central maximum (m = 0) to another maximum or minimum on either side of the center of the interference pattern.

λ is the wavelength of the light, determined by the source.

A: (a) For **very small** θ $\quad \sin\theta \approx \tan\theta$ and $\tan\theta = y/L$

and the equation for constructive interference $\quad \sin\theta = m\lambda/d$ (Eq. 37.2)

becomes $\quad y_{\text{bright}} \approx (\lambda L/d)m$ (Eq. 37.5)

Substituting values, $\quad y_{\text{bright}} = \dfrac{(546\times10^{-9}\text{ m})(1.20\text{ m})}{0.250\times10^{-3}\text{ m}}(1) = 2.62\text{ mm}$

(b) If you have trouble remembering whether Equation 37.5 or Eq. 37.6 applies to a given situation, you can instead remember that the first bright band is in the center, and dark bands are halfway between bright bands. Thus, Eq. 37.5 describes them all, with $m = 0,\ 1,\ 2\ .\ .\ .$ for bright bands, and with $m = 0.5,\ 1.5,\ 2.5\ .\ .\ .$ for dark bands. The dark band version of Eq. 37.5 is simply Eq. 37.6:

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(m + \tfrac{1}{2}\right)$$

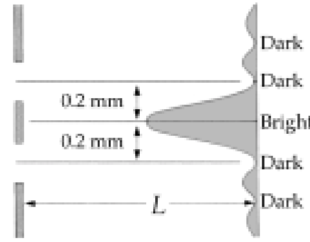
$$\Delta y_{\text{dark}} = \left(1 + \tfrac{1}{2}\right)\frac{\lambda L}{d} - \left(0 + \tfrac{1}{2}\right)\frac{\lambda L}{d} = \frac{\lambda L}{d} = 2.62\text{ mm}$$

L: This spacing is large enough for easy resolution of adjacent fringes. The distance between minima is the same as the distance between maxima. We expected this equality since the angles are small:

$$\theta = (2.62\text{ mm})/(1.20\text{ m}) = 0.00218\text{ rad} = 0.125°$$

When the angular spacing exceeds about 3°, then $\sin\theta$ differs from $\tan\theta$ when written to three significant figures.

37.8 Taking $m = 0$ and $y = 0.200$ mm in Equation 37.6 gives



$$L \approx \frac{2dy}{\lambda} = \frac{2(0.400\times10^{-3}\text{ m})(0.200\times10^{-3}\text{ m})}{442\times10^{-9}\text{ m}} = 0.362\text{ m}$$

$$L \approx \boxed{36.2\text{ cm}}$$

Geometric optics incorrectly predicts bright regions opposite the slits and darkness in between. But, as this example shows, interference can produce just the opposite.

37.9 Location of A = central maximum,

Location of B = first minimum.

So, $$\Delta y = [y_{\min} - y_{\max}] = \frac{\lambda L}{d}\left(0+\frac{1}{2}\right) - 0 = \frac{1}{2}\frac{\lambda L}{d} = 20.0\text{ m}$$

Thus, $$d = \frac{\lambda L}{2(20.0\text{ m})} = \frac{(3.00\text{ m})(150\text{ m})}{40.0\text{ m}} = \boxed{11.3\text{ m}}$$

37.10 At 30.0°, $d\sin\theta = m\lambda$

$(3.20\times10^{-4}\text{ m})\sin 30.0° = m(500\times10^{-9}\text{ m})$ so $m = 320$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead.

There are $\boxed{641\text{ maxima}}$.

37.11 $$\phi = \frac{2\pi}{\lambda} d\sin\theta \approx \frac{2\pi}{\lambda} d\left(\frac{y}{L}\right)$$

(a) $$\phi = \frac{2\pi}{(5.00\times10^{-7}\text{ m})}(1.20\times10^{-4}\text{ m})\sin\left(0.500^\circ\right) = \boxed{13.2\text{ rad}}$$

(b) $$\phi = \frac{2\pi}{(5.00\times10^{-7}\text{ m})}(1.20\times10^{-4}\text{ m})\left(\frac{5.00\times10^{-3}\text{ m}}{1.20\text{ m}}\right) = \boxed{6.28\text{ rad}}$$

(c) If $\phi = 0.333 \text{ rad} = \dfrac{2\pi d \sin\theta}{\lambda}$, $\qquad \theta = \sin^{-1}\left(\dfrac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\dfrac{(5.00\times10^{-7}\text{ m})(0.333\text{ rad})}{2\pi(1.20\times10^{-4}\text{ m})}\right]$

$$\theta = \boxed{1.27\times10^{-2}\text{ deg}}$$

(d) If $d\sin\theta = \dfrac{\lambda}{4}$, $\qquad \theta = \sin^{-1}\left(\dfrac{\lambda}{4d}\right) = \sin^{-1}\left[\dfrac{5\times10^{-7}\text{ m}}{4(1.20\times10^{-4}\text{ m})}\right]$

$$\theta = \boxed{5.97\times10^{-2}\text{ deg}}$$

37.12 The path difference between rays 1 and 2 is: $\delta = d\sin\theta_1 - d\sin\theta_2$

For constructive interference, this path difference must be equal to an integral number of wavelengths: $d\sin\theta_1 - d\sin\theta_2 = m\lambda$, or

$$\boxed{d(\sin\theta_1 - \sin\theta_2) = m\lambda}$$

37.13 (a) The path difference $\delta = d\sin\theta$ and when $L >> y$

$$\delta \approx \frac{yd}{L} = \frac{(1.80\times10^{-2}\text{ m})(1.50\times10^{-4}\text{ m})}{1.40\text{ m}} = 1.93\times10^{-6}\text{ m} = \boxed{1.93\ \mu\text{m}}$$

(b) $\dfrac{\delta}{\lambda} = \dfrac{1.93\times10^{-6}\text{ m}}{6.43\times10^{-7}\text{ m}} = 3.00$, or $\boxed{\delta = 3.00\ \lambda}$

(c) Point P will be a $\boxed{\text{maximum}}$ since the path difference is an integer multiple of the wavelength.

37.14 (a) $\dfrac{I}{I_{\max}} = \cos^2\left(\dfrac{\phi}{2}\right)$ (Equation 37.11)

Therefore, $\phi = 2\cos^{-1}\left(\dfrac{I}{I_{\max}}\right)^{1/2} = 2\cos^{-1}(0.640)^{1/2} = \boxed{1.29\text{ rad}}$

(b) $\delta = \dfrac{\lambda\phi}{2\pi} = \dfrac{(486\text{ nm})(1.29\text{ rad})}{2\pi} = \boxed{99.8\text{ nm}}$

37.15 $I_{av} = I_{max} \cos^2\left(\dfrac{\pi d \sin\theta}{\lambda}\right)$

For small θ, $\sin\theta = \dfrac{y}{L}$ and $I_{av} = 0.750 I_{max}$

$$y = \frac{\lambda L}{\pi d} \cos^{-1}\left(\frac{I_{av}}{I_{max}}\right)^{1/2}$$

$$y = \frac{(6.00 \times 10^{-7})(1.20\text{ m})}{\pi(2.50 \times 10^{-3}\text{ m})} \cos^{-1}\left(\frac{0.750 I_{max}}{I_{max}}\right)^{1/2} = \boxed{48.0\ \mu\text{m}}$$

37.16 $I = I_{max} \cos^2\left(\dfrac{\pi y d}{\lambda L}\right)$

$$\frac{I}{I_{max}} = \cos^2\left[\frac{\pi\left(6.00 \times 10^{-3}\text{ m}\right)\left(1.80 \times 10^{-4}\text{ m}\right)}{\left(656.3 \times 10^{-9}\text{ m}\right)(0.800\text{ m})}\right] = \boxed{0.987}$$

37.17 (a) From Equation 37.8,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda} \cdot \frac{y}{\sqrt{y^2 + D^2}}$$

$$\phi \approx \frac{2\pi y d}{\lambda D} = \frac{2\pi\left(0.850 \times 10^{-3}\text{ m}\right)\left(2.50 \times 10^{-3}\text{ m}\right)}{\left(600 \times 10^{-9}\text{ m}\right)(2.80\text{ m})} = \boxed{7.95\text{ rad}}$$

(b) $$\frac{I}{I_{max}} = \frac{\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta\right)}{\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta_{max}\right)} = \frac{\cos^2\dfrac{\phi}{2}}{\cos^2 m\pi}$$

$$\frac{I}{I_{max}} = \cos^2\frac{\phi}{2} = \cos^2\left(\frac{7.95\text{ rad}}{2}\right) = \boxed{0.453}$$

Goal Solution

Two narrow parallel slits separated by 0.850 mm are illuminated by 600-nm light, and the viewing screen is 2.80 m away from the slits. (a) What is the phase difference between the two interfering waves on a screen at a point 2.50 mm from the central bright fringe? (b) What is the ratio of the intensity at this point to the intensity at the center of a bright fringe?

G: It is difficult to accurately predict the relative intensity at the point of interest without actually doing the calculation. The waves from each slit could meet in phase ($\phi = 0$) to produce a bright spot of **constructive interference**, out of phase ($\phi = 180°$) to produce a dark region of **destructive interference**, or most likely the phase difference will be somewhere between these extremes, $0 < \phi < 180°$, so that the relative intensity will be $0 < I/I_{\max} < 1$.

O: The phase angle depends on the path difference of the waves according to Equation 37.8. This phase difference is used to find the average intensity at the point of interest. Then the relative intensity is simply this intensity divided by the maximum intensity.

A: (a) Using the variables shown in the diagram for problem 7 we have,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\left(\frac{y}{\sqrt{y^2+L^2}}\right) \cong \frac{2\pi yd}{\lambda L} = \frac{2\pi\left(0.850 \cdot 10^{-3}\text{ m}\right)\left(0.00250\text{ m}\right)}{\left(600 \cdot 10^{-9}\text{ m}\right)\left(2.80\text{ m}\right)} = 7.95\text{ rad} = 2\pi + 1.66\text{ rad} = 95.5$$

(b) $$\frac{I}{I_{\max}} = \frac{\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)}{\cos^2\left(\frac{\pi d}{\lambda}\sin\theta_{\max}\right)} = \frac{\cos^2\left(\frac{\phi}{2}\right)}{\cos^2(m\pi)} = \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{95.5}{2}\right) = 0.452$$

L: It appears that at this point, the waves show **partial interference** so that the combination is about half the brightness found at the central maximum. We should remember that the equations used in this solution do not account for the diffraction caused by the finite width of each slit. This diffraction effect creates an "envelope" that diminishes in intensity away from the central maximum as shown by the dotted line in Figures 37.13 and P37.60. Therefore, the relative intensity at y = 2.50 mm will actually be slightly less than 0.452.

37.18 (a) The resultant amplitude is

$$E_r = E_0 \sin \omega t + E_0 \sin(\omega t + \phi) + E_0 \sin(\omega t + 2\phi), \qquad \text{where} \qquad \phi = \frac{2\pi}{\lambda} d\sin\theta .$$

$$E_r = E_0\left(\sin \omega t + \sin \omega t \cos \phi + \cos \omega t \sin \phi + \sin \omega t \cos 2\phi + \cos \omega t \sin 2\phi\right)$$

$$E_r = E_0(\sin \omega t)\left(1 + \cos\phi + 2\cos^2\phi - 1\right) + E_0(\cos \omega t)(\sin\phi + 2\sin\phi\cos\phi)$$

$$E_r = E_0(1 + 2\cos\phi)(\sin \omega t \cos\phi + \cos \omega t \sin\phi) = E_0(1 + 2\cos\phi)\sin(\omega t + \phi)$$

Then the intensity is $I \propto E_r^2 = E_0^2 (1 + 2\cos\phi)^2 \left(\frac{1}{2}\right)$

where the time average of $\sin^2(\omega t + \phi)$ is 1/2.

From one slit alone we would get intensity $I_{\max} \propto E_0^2 \left(\frac{1}{2}\right)$ so $\boxed{I = I_{\max}\left[1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right]^2}$

(b) Look at the $N = 3$ graph in Figure 37.13. Minimum intensity is zero, attained where $\cos\phi = -1/2$. One relative maximum occurs at $\cos\phi = -1.00$, where $I = I_{\max}$.

The larger local maximum happens where $\cos\phi = +1.00$, giving $I = 9.00\, I_0$.

The ratio of intensities at primary versus secondary maxima is $\boxed{9.00}$.

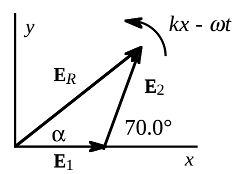
***37.19** (a) We can use $\sin A + \sin B = 2\sin(A/2 + B/2)\cos(A/2 - B/2)$ to find the sum of the two sine functions to be

$$E_1 + E_2 = (24.0\ \text{kN/C})\sin(15x - 4.5t + 35.0°)\cos 35.0°$$

$$E_1 + E_2 = (19.7\ \text{kN/C})\sin(15x - 4.5t + 35.0°)$$

Thus, the total wave has amplitude $\boxed{19.7\ \text{kN/C}}$ and has a constant phase difference of $\boxed{35.0°}$ from the first wave.

(b) In units of kN/C, the resultant phasor is



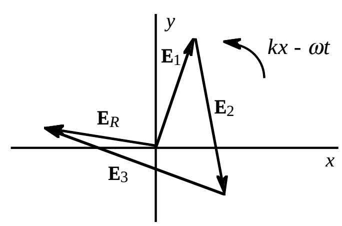
$$\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 = (12.0\,\mathbf{i}) + (12.0\cos 70.0°\,\mathbf{i} + 12.0\sin 70.0°\,\mathbf{j}) = 16.1\,\mathbf{i} + 11.3\,\mathbf{j}$$

$$\mathbf{E}_R = \sqrt{(16.1)^2 + (11.3)^2} \text{ at } \tan^{-1}(11.3/16.1) = \boxed{19.7\ \text{kN/C at } 35.0°}$$

(c) $\mathbf{E}_R = 12.0\cos 70.0°\,\mathbf{i} + 12.0\sin 70.0°\,\mathbf{j}$

$+15.5\cos 80.0°\,\mathbf{i} - 15.5\sin 80.0°\,\mathbf{j}$

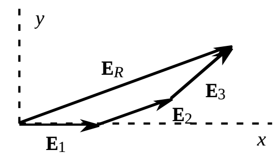
$+17.0\cos 160°\,\mathbf{i} + 17.0\sin 160°\,\mathbf{j}$



$$\mathbf{E}_R = -9.18\,\mathbf{i} + 1.83\,\mathbf{j} = \boxed{9.36\ \text{kN/C at } 169°}$$

The wave function of the total wave is $E_P = (9.36\ \text{kN/C})\sin(15x - 4.5t + 169°)$

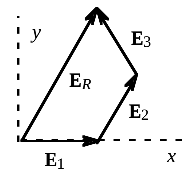
37.20 (a) $\mathbf{E}_R = E_0\left[\mathbf{i} + \left(\mathbf{i}\cos 20.0° + \mathbf{j}\sin 20.0°\right) + \left(\mathbf{i}\cos 40.0° + \mathbf{j}\sin 40.0°\right)\right]$



$\mathbf{E}_R = E_0\left[2.71\mathbf{i} + 0.985\mathbf{j}\right] = 2.88E_0$ at $20.0° = \boxed{2.88E_0 \text{ at } 0.349 \text{ rad}}$

$E_P = 2.88E_0 \sin\left(\omega t + 0.349\right)$

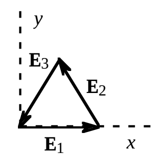
(b) $\mathbf{E}_R = E_0\left[\mathbf{i} + \left(\mathbf{i}\cos 60.0° + \mathbf{j}\sin 60.0°\right) + \left(\mathbf{i}\cos 120° + \mathbf{j}\sin 120°\right)\right]$



$\mathbf{E}_R = E_0\left[1.00\mathbf{i} + 1.73\mathbf{j}\right] = 2.00E_0$ at $60.0° = \boxed{2.00E_0 \text{ at } \pi/3 \text{ rad}}$

$E_P = 2.00E_0 \sin\left(\omega t + \pi/3\right)$

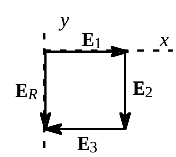
(c) $\mathbf{E}_R = E_0\left[\mathbf{i} + \left(\mathbf{i}\cos 120° + \mathbf{j}\sin 120°\right) + \left(\mathbf{i}\cos 240° + \mathbf{j}\sin 240°\right)\right]$



$\mathbf{E}_R = E_0\left[0\mathbf{i} + 0\mathbf{j}\right] = \boxed{0}$

$E_P = 0$

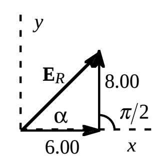
(d) $\mathbf{E}_R = E_0\left[\mathbf{i} + \left(\mathbf{i}\cos 3\pi/2 + \mathbf{j}\sin 3\pi/2\right) + \left(\mathbf{i}\cos 3\pi + \mathbf{j}\sin 3\pi\right)\right]$



$\mathbf{E}_R = E_0\left[0\mathbf{i} - 1.00\mathbf{j}\right] = E_0$ at $270° = \boxed{E_0 \text{ at } 3\pi/2 \text{ rad}}$

$E_P = E_0 \sin\left(\omega t + 3\pi/2\right)$

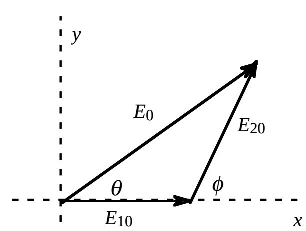
37.21 $\mathbf{E}_R = 6.00\mathbf{i} + 8.00\mathbf{j} = \sqrt{(6.00)^2 + (8.00)^2}$ at $\tan^{-1}\left(8.00/6.00\right)$



$\mathbf{E}_R = 10.0$ at $53.1° = 10.0$ at 0.927 rad

$E_P = \boxed{10.0 \sin\left(100\pi t + 0.927\right)}$

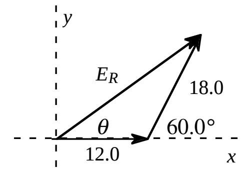
37.22 If $E_1 = E_{10}\sin\omega t$ and $E_2 = E_{20}\sin\left(\omega t + \phi\right)$, then by phasor addition, the amplitude of $\mathbf{E}$ is



$$E_0 = \sqrt{\left(E_{10} + E_{20}\cos\phi\right)^2 + \left(E_{20}\sin\phi\right)^2} = \boxed{\sqrt{E_{10}^{\,2} + 2E_{10}E_{20}\cos\phi + E_{20}^{\,2}}}$$

and the phase angle is found from $\boxed{\sin\theta = \dfrac{E_{20}\sin\phi}{E_0}}$

37.23 $\mathbf{E}_R = 12.0\mathbf{i} + (18.0\cos 60.0°\mathbf{i} + 18.0\sin 60.0°\mathbf{j})$



$\mathbf{E}_R = 21.0\mathbf{i} + 15.6\mathbf{j} = 26.2$ at 36.6°

$E_P = \boxed{26.2\sin(\omega t + 36.6°)}$

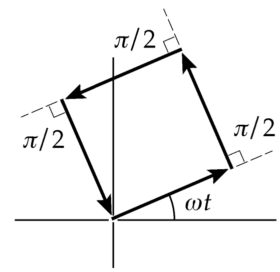
37.24 Constructive interference occurs where $m = 0, 1, 2, 3, \ldots,$ for

$$\left(\frac{2\pi x_1}{\lambda} - 2\pi f t + \frac{\pi}{6}\right) - \left(\frac{2\pi x_2}{\lambda} - 2\pi f t + \frac{\pi}{8}\right) = 2\pi m \quad \frac{2\pi(x_1 - x_2)}{\lambda} + \left(\frac{\pi}{6} - \frac{\pi}{8}\right) = 2\pi m$$

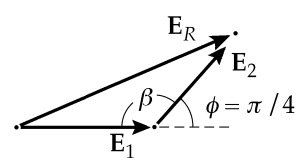
$$\frac{(x_1 - x_2)}{\lambda} + \frac{1}{12} - \frac{1}{16} = m \qquad \boxed{x_1 - x_2 = \left(m - \frac{1}{48}\right)\lambda \quad m = 0, 1, 2, 3, \ldots}$$

37.25 See the figure to the right:

$\boxed{\phi = \pi / 2}$



37.26 $E_R^2 = E_1^2 + E_2^2 - 2E_1E_2\cos\beta$, where $\beta = 180 - \phi$.



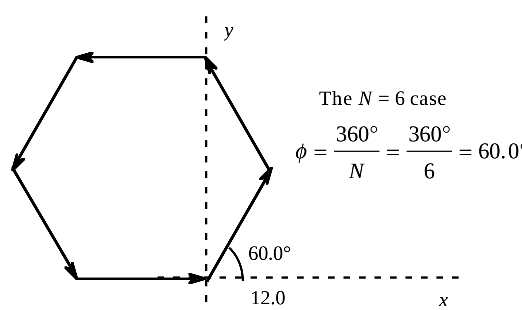
Since $I \propto E^2$,

$$I_R = \boxed{I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$

37.27 Take $\boxed{\phi = 360°/N}$ where N defines the number of coherent sources. Then,

$$E_R = \sum_{m=1}^{N} E_0 \sin(\omega t + m\phi) = 0$$

In essence, the set of N electric field components complete a full circle and return to zero.



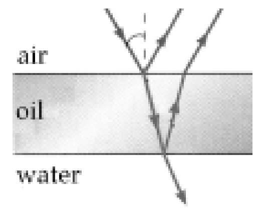
***37.28** Light reflecting from the first surface suffers phase reversal. Light reflecting from the second surface does not, but passes twice through the thickness t of the film. So, for constructive interference, we require

$$\frac{\lambda_n}{2} + 2t = \lambda_n$$

where $\lambda_n = \frac{\lambda}{n}$ is the wavelength in the material. Then $$2t = \frac{\lambda_n}{2} = \frac{\lambda}{2n}$$

$\lambda = 4nt = 4 \times 1.33 \times 115\text{ nm} = \boxed{612\text{ nm}}$

37.29 (a) The light reflected from the top of the oil film undergoes phase reversal. Since 1.45 > 1.33, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have



$2nt = \left(m + \frac{1}{2}\right)\lambda$ or $$\lambda_m = \frac{2nt}{\left(m + \frac{1}{2}\right)} = \frac{2(1.45)(280\text{ nm})}{\left(m + \frac{1}{2}\right)}$$

Substituting for m, we have $m = 0$: $\lambda_0 = 1620$ nm (infrared)

$m = 1$: $\lambda_1 = 541$ nm (green)

$m = 2$: $\lambda_2 = 325$ nm (ultraviolet)

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in the reflected light is $\boxed{\text{green}}$.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$2nt = m\lambda$ or $$\lambda_m = \frac{2nt}{m} = \frac{812\text{ nm}}{m}$$

Substituting for m gives: $m = 1$, $\lambda_1 = 812$ nm (near infrared)

$m = 2$, $\lambda_2 = 406$ nm (violet)

$m = 3$, $\lambda_3 = 271$ nm (ultraviolet)

Of these. the only wavelength visible to the human eye (and hence the dominate wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is $\boxed{\text{violet}}$.

37.30 Since $1 < 1.25 < 1.33$, light reflected both from the top and from the bottom surface of the oil suffers phase reversal.

For constructive interference we require $2t = \dfrac{m\lambda_{\text{cons}}}{n}$

and for destructive interference, $2t = \dfrac{\left(m + \frac{1}{2}\right)\lambda_{\text{des}}}{n}$

Then $\dfrac{\lambda_{\text{cons}}}{\lambda_{\text{dest}}} = 1 + \dfrac{1}{2m} = \dfrac{640\text{ nm}}{512\text{ nm}} = 1.25$ and $m = 2$

Therefore, $t = \dfrac{2(640\text{ nm})}{2(1.25)} = \boxed{512\text{ nm}}$

37.31 Treating the anti-reflectance coating like a camera-lens coating, $2t = \left(m + \dfrac{1}{2}\right)\dfrac{\lambda}{n}$

Let $m = 0$: $t = \dfrac{\lambda}{4n} = \dfrac{3.00\text{ cm}}{4(1.50)} = \boxed{0.500\text{ cm}}$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar—to 1.50 cm—now creating maximum reflection!

37.32 $2nt = \left(m + \dfrac{1}{2}\right)\lambda$ so $t = \left(m + \dfrac{1}{2}\right)\dfrac{\lambda}{2n}$

Minimum $t = \left(\dfrac{1}{2}\right)\dfrac{(500\text{ nm})}{2(1.30)} = \boxed{96.2\text{ nm}}$

37.33 Since the light undergoes a 180° phase change at each surface of the film, the condition for *constructive* interference is $2nt = m\lambda$, or $\lambda = 2nt/m$. The film thickness is $t = 1.00 \times 10^{-5}\text{ cm} = 1.00 \times 10^{-7}\text{ m} = 100\text{ nm}$. Therefore, the wavelengths intensified in the reflected light are

$$\lambda = \frac{2(1.38)(100\text{ nm})}{m} = \frac{276\text{ nm}}{m} \quad \text{where} \quad m = 1, 2, 3, \ldots$$

or $\lambda_1 = 276\text{ nm}$, $\lambda_2 = 138\text{ nm}$, ... All reflection maxima are in the ultraviolet and beyond. $\boxed{\text{No visible wavelengths are intensified.}}$

***37.34** (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase reversal and light reflected from the back does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film: $2t = \lambda/n$.

$$t = \frac{\lambda}{2n} = \frac{656.3 \text{ nm}}{2(1.378)} = \boxed{238 \text{ nm}}$$

(b) The filter will expand. As t increases in $2nt = \lambda$, so does $\boxed{\lambda \text{ increase}}$.

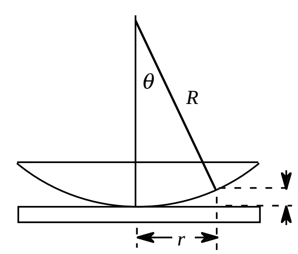
(c) Destructive interference for reflected light happens also for λ in $2nt = 2\lambda$,

or $\lambda = 1.378(238 \text{ nm}) = \boxed{328 \text{ nm}}$ (near ultraviolet) .

37.35 If the path length $\Delta = \lambda$, the transmitted light will be bright. Since $\Delta = 2d = \lambda$,

$$d_{\min} = \frac{\lambda}{2} = \frac{580 \text{ nm}}{2} = \boxed{290 \text{ nm}}$$

37.36 The condition for bright fringes is



$$2t + \frac{\lambda}{2n} = m\frac{\lambda}{n} \qquad m = 1,\ 2,\ 3,\ \ldots$$

From the sketch, observe that

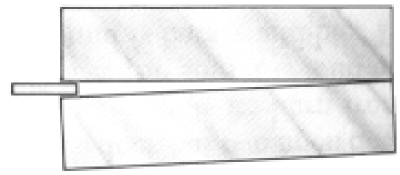
$$t = R(1 - \cos\theta) \approx R\left(1 - 1 + \frac{\theta^2}{2}\right) = \frac{R}{2}\left(\frac{r}{R}\right)^2 = \frac{r^2}{2R}$$

The condition for a bright fringe becomes $\dfrac{r^2}{R} = \left(m - \dfrac{1}{2}\right)\dfrac{\lambda}{n}$.

Thus, for fixed m and λ, $nr^2 =$ constant.

Therefore, $n_{\text{liquid}} r_f^2 = n_{\text{air}} r_i^2$ and $n_{\text{liquid}} = (1.00)\dfrac{(1.50 \text{ cm})^2}{(1.31 \text{ cm})^2} = \boxed{1.31}$

37.37 For destructive interference in the air, $2t = m\lambda$.

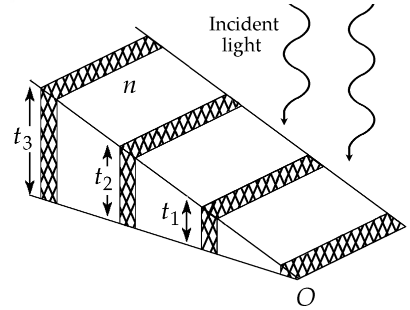


For 30 dark fringes, including the one where the plates meet,

$$t = \frac{29(600\text{ nm})}{2} = 8.70 \times 10^{-6}\text{ m}$$

Therefore, the <u>radius</u> of the wire is

$$r = \frac{d}{2} = \frac{8.70\ \mu\text{m}}{2} = \boxed{4.35\ \mu\text{m}}$$



<u>Goal Solution</u>
An air wedge is formed between two glass plates separated at one edge by a very fine wire as shown in Figure P37.37. When the wedge is illuminated from above by 600-nm light, 30 dark fringes are observed. Calculate the radius of the wire.

G: The radius of the wire is probably less than 0.1 mm since it is described as a "very fine wire."

O: Light reflecting from the bottom surface of the top plate undergoes no phase shift, while light reflecting from the top surface of the bottom plate is shifted by π, and also has to travel an extra distance $2t$, where t is the thickness of the air wedge.

For destructive interference, $2t = m\lambda \quad (m = 0, 1, 2, 3, \ldots)$

The first dark fringe appears where $m = 0$ at the line of contact between the plates. The 30th dark fringe gives for the diameter of the wire $2t = 29\lambda$, and $t = 14.5\lambda$.

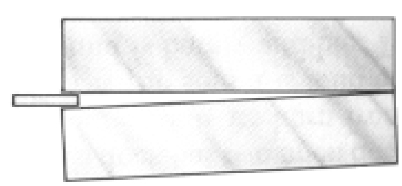
A: $r = \frac{t}{2} = 7.25\lambda = 7.25\left(600 \times 10^{-9}\text{ m}\right) = 4.35\ \mu\text{m}$

L: This wire is not only less than 0.1 mm; it is even thinner than a typical human hair (~ 50 μm).

37.38 For destructive interference, $2t = \frac{\lambda m}{n}$.

At the position of the maximum thickness of the air film,

$$m = \frac{2tn}{\lambda} = \frac{2(4.00 \times 10^{-5}\text{ m})(1.00)}{5.461 \times 10^{-7}\text{ m}} = 146.5$$



The greatest integer value is m = 146.

Therefore, including the dark band at zero thickness, there are $\boxed{\text{147 dark fringes}}$.

***37.39** For total darkness, we want destructive interference for reflected light for both 400 nm and 600 nm. With phase reversal at just one reflecting surface, the condition for destructive interference is

$$2n_{\text{air}}t = m\lambda \qquad m = 0,\ 1,\ 2,\ \ldots$$

The least common multiple of these two wavelengths is 1200 nm, so we get no reflected light at $2(1.00)t = 3(400\text{ nm}) = 2(600\text{ nm}) = 1200\text{ nm}$, so t = 600 nm at this second dark fringe.

By similar triangles, $$\frac{600\text{ nm}}{x} = \frac{0.0500\text{ mm}}{10.0\text{ cm}},$$

or the distance from the contact point is $$x = \left(600\times10^{-9}\text{ m}\right)\left(\frac{0.100\text{ m}}{5.00\times10^{-5}\text{ m}}\right) = \boxed{1.20\text{ mm}}$$

37.40 $$2t = m\lambda \quad \Rightarrow \quad m = \frac{2t}{\lambda} = \frac{2(1.80\times10^{-4}\text{ m})}{550.5\times10^{-9}\text{ m}} = \boxed{654\text{ dark fringes}}$$

37.41 When the mirror on one arm is displaced by Δl, the path difference increases by $2\Delta l$. A shift resulting in the formation of successive dark (or bright) fringes requires a path length change of one-half wavelength. Therefore, $2\Delta l = m\lambda/2$, where in this case, m = 250.

$$\Delta l = m\frac{\lambda}{4} = \frac{(250)\left(6.328\times10^{-7}\text{ m}\right)}{4} = \boxed{39.6\ \mu\text{m}}$$

37.42 Distance = $2(3.82\times10^{-4}\text{ m}) = 1700\lambda$ $\qquad \lambda = 4.49\times10^{-7}\text{ m} = \boxed{449\text{ nm}}$

The light is $\boxed{\text{blue}}$

37.43 Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \dfrac{2L}{\lambda}$. It changes to $m_2 = \dfrac{2L}{\lambda/n} = \dfrac{2nL}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \dfrac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{35\left(633\times10^{-9}\text{ m}\right)}{2(0.0300\text{ m})} = \boxed{1.000\ 369}$$

37.44 Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \frac{2L}{\lambda}$. It changes to $m_2 = \frac{2L}{\lambda/n} = \frac{2nL}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \frac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = \boxed{1 + \frac{N\lambda}{2L}}$$

37.45 The wavelength is $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{60.0 \times 10^6 \text{ s}^{-1}} = 5.00 \text{ m}$.

Along the line AB the two traveling waves going in opposite directions add to give a standing wave. The two transmitters are exactly 2.00 wavelengths apart and the signal from B, when it arrives at A, will always be in phase with transmitter B. Since B is 180° out of phase with A, the two signals always interfere destructively at the position of A.

The first antinode (point of constructive interference) is located at distance

$\frac{\lambda}{4} = \frac{5.00 \text{ m}}{4} = \boxed{1.25 \text{ m}}$ from the node at A.

***37.46** My middle finger has width d = 2 cm.

(a) Two adjacent directions of constructive interference for 600-nm light are described by

$d \sin\theta = m\lambda$ $\quad\theta_0 = 0$ $\quad(2 \times 10^{-2} \text{ m}) \sin\theta_1 = 1\,(6 \times 10^{-7} \text{ m})$

Thus, $\quad\theta_1 = 2 \times 10^{-3}$ degree

and $\quad\theta_1 - \theta_0 \boxed{\sim 10^{-3} \text{ degree}}$

(b) Choose $\theta_1 = 20°$ $\quad 2 \times 10^{-2} \text{ m} \sin 20° = 1\lambda$ $\quad\lambda = 7$ mm

Millimeter waves are $\boxed{\text{microwaves}}$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{7 \times 10^{-3} \text{ m}} \boxed{\sim 10^{11} \text{ Hz}}$$

37.47 If the center point on the screen is to be a dark spot rather than bright, passage through the plastic must delay the light by one-half wavelength. Calling the thickness of the plastic t.

$\frac{t}{\lambda}+\frac{1}{2}=\frac{t}{(\lambda/n)}=\frac{nt}{\lambda}$ or $t=\boxed{\frac{\lambda}{2(n-1)}}$ where n is the index of refraction for the plastic.

***37.48** No phase shift upon reflection from the upper surface (glass to air) of the film, but there will be a shift of $\lambda/2$ due to the reflection at the lower surface of the film (air to metal). The total phase difference in the two reflected beams is then $\delta = 2nt + \lambda/2$. For constructive interference, $\delta = m\lambda$, or $2(1.00)t + \lambda/2 = m\lambda$. Thus, the film thickness for the mth order bright fringe is:

$$t_m = \left(m - \frac{1}{2}\right)\frac{\lambda}{2} = m\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4},$$

and the thickness for the $m - 1$ bright fringe is: $$t_{m-1} = (m-1)\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}.$$

Therefore, the change in thickness required to go from one bright fringe to the next is $\Delta t = t_m - t_{m-1} = \lambda/2$. To go through 200 bright fringes, the change in thickness of the air film must be: $200(\lambda/2) = 100\lambda$. Thus, the increase in the length of the rod is

$$\Delta L = 100\lambda = 100\left(5.00 \times 10^{-7}\text{ m}\right) = 5.00 \times 10^{-5}\text{ m},$$

From $\Delta L = L_i\alpha(\Delta T)$, we have: $$\alpha = \frac{\Delta L}{L_i(\Delta T)} = \frac{5.00 \times 10^{-5}\text{ m}}{(0.100\text{ m})(25.0°\text{C})} = \boxed{20.0 \times 10^{-6}\ °\text{C}^{-1}}$$

***37.49** Since 1 < 1.25 < 1.34, light reflected from top and bottom surfaces of the oil undergoes phase reversal. The path difference is then $2t$, which must be equal to

$$m\lambda_n = \frac{m\lambda}{n}$$

for maximum reflection, with $m = 1$ for the given first-order condition and $n = 1.25$. So

$$t = \frac{m\lambda}{2n} = \frac{1(500\text{ nm})}{2(1.25)} = 200\text{ nm}$$

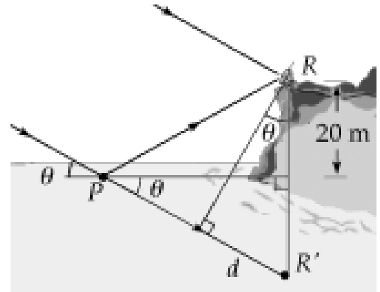
The volume we assume to be constant: $1.00\text{ m}^3 = (200\text{ nm})A$

$$A = \frac{1.00\text{ m}^3}{200(10^{-9}\text{ m})} = 5.00 \times 10^6\text{ m}^2 = \boxed{5.00\text{ km}^2}$$

37.50 For destructive interference, the path length must differ by $m\lambda$. We may treat this problem as a double slit experiment if we remember the light undergoes a $\pi/2$-phase shift at the mirror. The second slit is the mirror image of the source, 1.00 cm below the mirror plane. Using Equation 37.5,

$$y_{\text{dark}} = \frac{m\lambda L}{d} = \frac{1(5.00 \times 10^{-7}\text{ m})(100\text{ m})}{(2.00 \times 10^{-2}\text{ m})} = \boxed{2.50\text{ mm}}$$

37.51 One radio wave reaches the receiver R directly from the distant source at an angle θ above the horizontal. The other wave undergoes phase reversal as it reflects from the water at P.



Constructive interference first occurs for a path difference of

$$d = \frac{\lambda}{2} \qquad (1)$$

The angles θ in the figure are equal because they each form part of a right triangle with a shared angle at R'.

It is equally far from P to R as from P to R', the mirror image of the telescope.

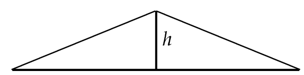
So the path difference is $d = 2(20.0 \text{ m}) \sin\theta = (40.0 \text{ m}) \sin\theta$

The wavelength is $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{60.0 \times 10^6 \text{ Hz}} = 5.00 \text{ m}$

Substituting for d and λ in Equation (1), $(40.0 \text{ m}) \sin\theta = \frac{5.00 \text{ m}}{2}$

Solving for the angle θ, $\sin\theta = \frac{5.00 \text{ m}}{80.0 \text{ m}}$ and $\boxed{\theta = 3.58°}$

37.52 $2\sqrt{(15.0 \text{ km})^2 + h^2} = 30.175 \text{ km}$



$(15.0 \text{ km})^2 + h^2 = 227.63$ $h = \boxed{1.62 \text{ km}}$

37.53 From Equation 37.13, $\frac{I}{I_{\max}} = \cos^2\left(\frac{\pi y d}{\lambda L}\right)$

Let λ_2 equal the wavelength for which $\frac{I}{I_{\max}} \to \frac{I_2}{I_{\max}} = 0.640$

Then $\lambda_2 = \frac{\pi y d / L}{\cos^{-1}(I_2/I_{\max})^{1/2}}$

But $\frac{\pi y d}{L} = \lambda_1 \cos^{-1}\left(\frac{I_1}{I_{\max}}\right)^{1/2} = (600 \text{ nm}) \cos^{-1}(0.900) = 271 \text{ nm}$

Substituting this value into the expression for λ_2, $\lambda_2 = \frac{271 \text{ nm}}{\cos^{-1}(0.640^{1/2})} = \boxed{421 \text{ nm}}$

Note that in this problem, $\cos^{-1}\left(\frac{I}{I_{\max}}\right)^{1/2}$ must be expressed in radians.

37.54 For Young's experiment, use $\delta = d\sin\theta = m\lambda$. Then, at the point where the two bright lines coincide,

$d\sin\theta = m_1\lambda_1 = m_2\lambda_2$ so $\dfrac{\lambda_1}{\lambda_2} = \dfrac{540}{450} = \dfrac{m_2}{m_1} = \dfrac{6}{5}$

$$\sin\theta = \frac{6\lambda_2}{d} = \frac{6(450\text{ nm})}{0.150\text{ mm}} = 0.0180$$

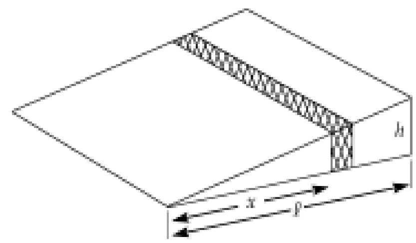
Since $\sin\theta \approx \theta$ and $L = 1.40$ m, $x = \theta L = (0.0180)(1.40\text{ m}) = \boxed{2.52\text{ cm}}$

37.55 For dark fringes, $2nt = m\lambda$

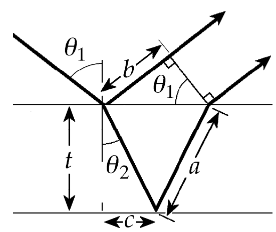
and at the edge of the wedge, $t = \dfrac{84(500\text{ nm})}{2}$.

When submerged in water, $2nt = m\lambda$

$m = \dfrac{2(1.33)(42)(500\text{ nm})}{500\text{ nm}}$ so $m + 1 = \boxed{113\text{ dark fringes}}$



***37.56** At entrance, $1.00\sin 30.0° = 1.38\sin\theta_2$ $\theta_2 = 21.2°$



Call t the unknown thickness. Then

$\cos 21.2° = \dfrac{t}{a}$ $\qquad a = \dfrac{t}{\cos 21.2°}$

$\tan 21.2° = \dfrac{c}{t}$ $\qquad c = t\tan 21.2°$

$\sin\theta_1 = \dfrac{b}{2c}$ $\qquad b = 2t\tan 21.2°\sin 30.0°$

The net shift for the second ray, including the phase reversal on reflection of the first, is

$$2an - b - \frac{\lambda}{2}$$

where the factor n accounts for the shorter wavelength in the film. For constructive interference, we require

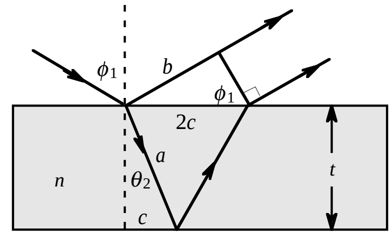
$$2an - b - \frac{\lambda}{2} = m\lambda$$

The minimum thickness will be given by $2an - b - \dfrac{\lambda}{2} = 0.$

$$\frac{\lambda}{2} = 2an - b = 2\frac{nt}{\cos 21.2°} - 2t(\tan 21.2°)\sin 30.0°$$

$$\frac{590\text{ nm}}{2} = \left(\frac{2\times 1.38}{\cos 21.2°} - 2\tan 21.2°\sin 30.0°\right)t = 2.57t \qquad t = \boxed{115\text{ nm}}$$

37.57 The shift between the two reflected waves is $\delta = 2na - b - \lambda/2$ where a and b are as shown in the ray diagram, n is the index of refraction, and the factor of $\lambda/2$ is due to phase reversal at the top surface. For constructive interference, $\delta = m\lambda$ where m has integer values. This condition becomes



$$2na - b = \left(m + \frac{1}{2}\right)\lambda \qquad (1)$$

From the figure's geometry, $a = \dfrac{t}{\cos\theta_2}$, $c = a\sin\theta_2 = \dfrac{t\sin\theta_2}{\cos\theta_2}$, $b = 2c\sin\phi_1 = \dfrac{2t\sin\theta_2}{\cos\theta_2}\sin\phi_1$

Also, from Snell's law, $\sin\phi_1 = n\sin\theta_2$. Thus, $b = \dfrac{2nt\sin^2\theta_2}{\cos\theta_2}$

With these results, the condition for constructive interference given in Equation (1) becomes:

$$2n\left(\frac{t}{\cos\theta_2}\right) - \frac{2nt\sin^2\theta_2}{\cos\theta_2} = \frac{2nt}{\cos\theta_2}\left(1 - \sin^2\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda \qquad \text{or} \qquad \boxed{2nt\cos\theta_2 = \left(m + \frac{1}{2}\right)\lambda}$$

37.58 (a) Minimum: $2nt = m\lambda_2$ $\qquad m = 0,\ 1,\ 2, \ldots$

Maximum: $2nt = \left(m' + \frac{1}{2}\right)\lambda_1$ $\qquad m' = 0,\ 1,\ 2, \ldots$

for $\lambda_1 > \lambda_2$, $\left(m' + \frac{1}{2}\right) < m$ $\qquad$ so $\qquad m' = m - 1$

Then $\qquad 2nt = m\lambda_2 = \left(m - \frac{1}{2}\right)\lambda_1$

$2m\lambda_2 = 2m\lambda_1 - \lambda_1$ $\qquad$ so $\qquad \boxed{m = \dfrac{\lambda_1}{2(\lambda_1 - \lambda_2)}}$

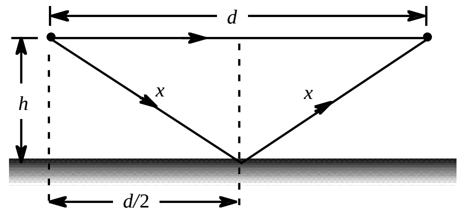
(b) $m = \dfrac{500}{2(500 - 370)} = 1.92 \to 2$ (wavelengths measured to $\pm$ 5 nm)

[Minimum]: $\qquad 2nt = m\lambda_2 \qquad 2(1.40)t = 2(370\text{ nm}) \qquad t = 264\text{ nm}$

[Maximum]: $2nt = \left(m - 1 + \frac{1}{2}\right)\lambda = 1.5\lambda$ $\quad 2(1.40)t = (1.5)500\text{ nm} \qquad t = 268\text{ nm}$

Film thickness = $\boxed{266\text{ nm}}$

37.59 From the sketch, observe that



$$x = \sqrt{h^2 + (d/2)^2} = \frac{\sqrt{4h^2 + d^2}}{2}$$

Including the phase reversal due to reflection from the ground, the total shift between the two waves is $\delta = 2x - d - \lambda/2$.

(a) For constructive interference, the total shift must be an integral number of wavelengths, or $\delta = m\lambda$ where m = 0, 1, 2, 3, ...

Thus, $2x - d = \left(m + \frac{1}{2}\right)\lambda$ or $\lambda = \frac{4x - 2d}{2m + 1}$

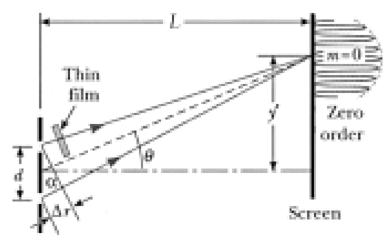
For the longest wavelength, m = 0, giving $\lambda = 4x - 2d = \boxed{2\sqrt{4h^2 + d^2} - 2d}$

(b) For destructive interference, $\delta = \left(m - \frac{1}{2}\right)\lambda$ where m = 1, 2, 3, ...

Thus, $2x - d = m\lambda$ or $\lambda = \frac{2x - d}{m}$.

For the longest wavelength, m = 1 giving $\lambda = 2x - d = \boxed{\sqrt{4h^2 + d^2} - d}$

37.60 Call t the thickness of the sheet. The central maximum corresponds to zero phase difference. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. As light advances through distance t in air, the number of cycles it goes through is t / λ_a.



The number of cycles in the sheet is $\frac{t}{(\lambda_a / n)} = \frac{nt}{\lambda_a}$

Thus, the sheet introduces phase difference $\phi = 2\pi\left(\frac{nt}{\lambda_a} - \frac{t}{\lambda_a}\right)$

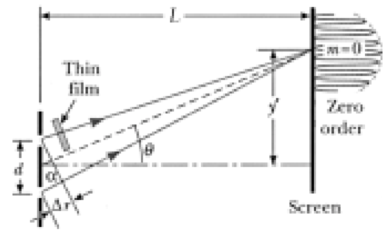
The corresponding difference in **path length** is $\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = \frac{2\pi}{\lambda_a}(nt - t)\frac{\lambda_a}{2\pi} = (n-1)t$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel, so the angle θ may be expressed as $\tan\theta = \Delta r/d = y'/L$.

Substituting for Δr and solving for y' gives

$$y' = \Delta r\left(\frac{L}{d}\right) = \frac{t(n-1)L}{d} = \frac{(5.00 \times 10^{-5}\text{ m})(1.50 - 1)(1.00\text{ m})}{(3.00 \times 10^{-4}\text{ m})} = 0.0833\text{ m} = \boxed{8.33\text{ cm}}$$

37.61 Call t the thickness of the film. The central maximum corresponds to zero phase difference. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. The phase difference ϕ is



$$\phi = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)$$

The corresponding difference in **path length** Δr is $\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)\left(\frac{\lambda_a}{2\pi}\right) = t(n-1)$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel.

Thus the angle θ may be expressed as $\tan\theta = \frac{\Delta r}{d} = \frac{y'}{L}$

Eliminating Δr by substitution, $\frac{y'}{L} = \frac{t(n-1)}{d}$ gives $\boxed{y' = \frac{t(n-1)L}{d}}$

Goal Solution

Consider the double-slit arrangement shown in Figure P37.60, where the slit separation is d and the slit to screen distance is L. A sheet of transparent plastic having an index of refraction n and thickness t is placed over the upper slit. As a result, the central maximum of the interference pattern moves upward a distance y'. Find y'.

G: Since the film shifts the pattern upward, we should expect y' to be proportional to n, t, and L.

O: The film increases the optical path length of the light passing through the upper slit, so the physical distance of this path must be shorter for the waves to meet in phase ($\phi = 0$) to produce the central maximum. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film.

A: First calculate the additional phase difference due to the plastic. Recall that the relation between phase difference and path difference is $\phi = 2\pi\delta/\lambda$. The presence of plastic affects this by changing the wavelength of the light, so that the phase change of the light in air and plastic, as it travels over the thickness t is

$$\phi_{air} = \frac{2\pi t}{\lambda_{air}} \quad \text{and} \quad \phi_{plastic} = \frac{2\pi t}{\lambda_{air}/n}$$

Thus, plastic causes an additional phase change of $\Delta\phi = \frac{2\pi t}{\lambda_{air}}(n-1)$

Next, in order to interfere constructively, we must calculate the additional distance that the light from the bottom slit must travel.

$$\Delta r = \frac{\Delta\phi\,\lambda_{air}}{2\pi} = t(n-1)$$

In the small angle approximation we can write $\Delta r = y'd/L$, so $y' = \frac{t(n-1)L}{dL}$

L: As expected, y' is proportional to t and L. It increases with increasing n, being proportional to $(n-1)$. It is also inversely proportional to the slit separation d, which makes sense since slits that are closer together make a wider interference pattern.

37.62 $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{1.50 \times 10^{6}\ \text{Hz}} = 200\ \text{m}$

For destructive interference, the path difference is one-half wavelength.

Thus, $\dfrac{\lambda}{2} = 100\ \text{m} = x + \sqrt{x^2 + \left(2.00 \times 10^4\ \text{m}\right)^2} - 2.00 \times 10^4\ \text{m}\,,$

or $2.01 \times 10^4\ \text{m} - x = \sqrt{x^2 + \left(2.00 \times 10^4\ \text{m}\right)^2}$

Squaring and solving, $x = \boxed{99.8\ \text{m}}$

37.63 (a) Constructive interference in the reflected light requires $2t = \left(m + \frac{1}{2}\right)\lambda$. The first bright ring has $m = 0$ and the 55th has $m = 54$, so at the edge of the lens

$$t = \frac{54.5(650 \times 10^{-9}\ \text{m})}{2} = 17.7\ \mu\text{m}$$

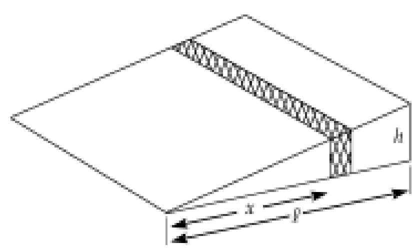
Now from the geometry in Figure 37.18, the distance from the center of curvature down to the flat side of the lens is

$\sqrt{R^2 - r^2} = R - t$ or $R^2 - r^2 = R^2 - 2Rt + t^2$

$$R = \frac{r^2 + t^2}{2t} = \frac{(5.00 \times 10^{-2}\ \text{m})^2 + (1.77 \times 10^{-5}\ \text{m})^2}{2(1.77 \times 10^{-5}\ \text{m})} = \boxed{70.6\ \text{m}}$$

(b) $\dfrac{1}{f} = (\text{n} - 1)\left(\dfrac{1}{R_2} - \dfrac{1}{R_2}\right) = 0.520\left(\dfrac{1}{\infty} - \dfrac{1}{-70.6\ \text{m}}\right)$ so $f = \boxed{136\ \text{m}}$

37.64 Bright fringes occur when $2t = \dfrac{\lambda}{n}\left(m + \dfrac{1}{2}\right)$



and dark fringes occur when $2t = \left(\dfrac{\lambda}{n}\right)m$

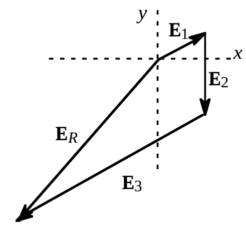
The thickness of the film at x is $t = \left(\dfrac{h}{\ell}\right)x$.

Therefore, $\boxed{x_{\text{bright}} = \dfrac{\lambda \ell}{2hn}\left(m + \dfrac{1}{2}\right)}$ and $\boxed{x_{\text{dark}} = \dfrac{\lambda \ell m}{2hn}}$

37.65 $\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = \left[\cos\frac{\pi}{6} + 3.00\cos\frac{7\pi}{2} + 6.00\cos\frac{4\pi}{3}\right]\mathbf{i} + \left[\sin\frac{\pi}{6} + 3.00\sin\frac{7\pi}{2} + 6.00\sin\frac{4\pi}{3}\right]\mathbf{j}$

$\mathbf{E}_R = -2.13\,\mathbf{i} - 7.70\,\mathbf{j}$

$\mathbf{E}_R = \sqrt{(-2.13)^2 + (-7.70)^2}$ at $\tan^{-1}\left(\frac{-7.70}{-2.13}\right) = 7.99$ at 4.44 rad

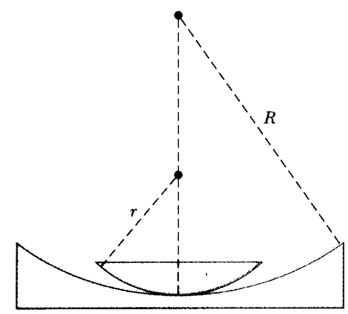


Thus, $E_P = \boxed{7.99\sin(\omega t + 4.44\text{ rad})}$

37.66 For bright rings the gap t between surfaces is given by $2t = \left(m + \frac{1}{2}\right)\lambda$ The first bright ring has $m = 0$ and the hundredth has $m = 99$.

So, $t = \frac{1}{2}(99.5)\left(500\times10^{-9}\text{ m}\right) = 24.9\ \mu\text{m}$.

Call r_b the ring radius. From the geometry of the figure at the right,

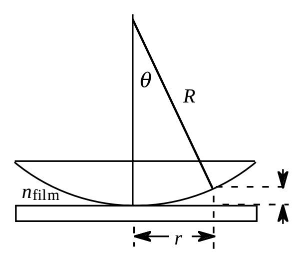


$t = r - \sqrt{r^2 - r_b^2} - \left(R - \sqrt{R^2 - r_b^2}\right)$

Since $r_b << r$, we can expand in series: $t = r - r\left(1 - \frac{1}{2}\frac{r_b^2}{r^2}\right) - R + R\left(1 - \frac{1}{2}\frac{r_b^2}{R^2}\right) = \frac{1}{2}\frac{r_b^2}{r} - \frac{1}{2}\frac{r_b^2}{R}$

$$r_b = \left[\frac{2t}{1/r - 1/R}\right]^{1/2} = \left[\frac{2\left(24.9\times10^{-6}\text{ m}\right)}{1/4.00\text{ m} - 1/12.0\text{ m}}\right]^{1/2} = \boxed{1.73\text{ cm}}$$

***37.67** The shift between the waves reflecting from the top and bottom surfaces of the film at the point where the film has thickness t is $\delta = 2tn_{\text{film}} + (\lambda/2)$, with the factor of $\lambda/2$ being due to a phase reversal at *one* of the surfaces.



For the dark rings (destructive interference), the total shift should be $\delta = \left(m + \frac{1}{2}\right)\lambda$ with $m = 0, 1, 2, 3, \ldots$. This requires that $t = m\lambda/2n_{\text{film}}$.

To find t in terms of r and R, $R^2 = r^2 + (R-t)^2$ so $r^2 = 2Rt + t^2$

Since t is much smaller than R, $t^2 << 2Rt$ and $r^2 \approx 2Rt = 2R\left(\frac{m\lambda}{2n_{\text{film}}}\right)$.

Thus, where m is an integer, $\boxed{r \approx \sqrt{\frac{m\lambda R}{n_{\text{film}}}}}$

37.68 (a) Bright bands are observed when $2nt = \left(m + \frac{1}{2}\right)\lambda$

Hence, the first bright band ($m = 0$) corresponds to $nt = \lambda/4$.

Since $\frac{x_1}{x_2} = \frac{t_1}{t_2}$, we have $x_2 = x_1\left(\frac{t_2}{t_1}\right) = x_1\left(\frac{\lambda_2}{\lambda_1}\right) = (3.00\text{ cm})\left(\frac{680\text{ nm}}{420\text{ nm}}\right) = \boxed{4.86\text{ cm}}$

(b) $t_1 = \frac{\lambda_1}{4n} = \frac{420\text{ nm}}{4(1.33)} = \boxed{78.9\text{ nm}}$ $t_2 = \frac{\lambda_2}{4n} = \frac{680\text{ nm}}{4(1.33)} = \boxed{128\text{ nm}}$

(c) $\theta \approx \tan\theta = \frac{t_1}{x_1} = \frac{78.9\text{ nm}}{3.00\text{ cm}} = \boxed{2.63\times 10^{-6}\text{ rad}}$

37.69 $2h\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ bright

$2h\left(\frac{\Delta y}{2L}\right) = \frac{1}{2}\lambda$ so $h = \frac{L\lambda}{2\Delta y} = \frac{(2.00\text{ m})\left(606\times 10^{-9}\text{ m}\right)}{2\left(1.2\times 10^{-3}\text{ m}\right)} = \boxed{0.505\text{ mm}}$

37.70 Superposing the two vectors, $E_R = \left|\mathbf{E}_1 + \mathbf{E}_2\right|$

$$E_R = \left|\mathbf{E}_1 + \mathbf{E}_2\right| = \sqrt{\left(E_0 + \frac{E_0}{3}\cos\phi\right)^2 + \left(\frac{E_0}{3}\sin\phi\right)^2} = \sqrt{E_0^2 + \frac{2}{3}E_0^2\cos\phi + \frac{E_0^2}{9}\cos^2\phi + \frac{E_0^2}{9}\sin^2\phi}$$

$$E_R = \sqrt{\frac{10}{9}E_0^2 + \frac{2}{3}E_0^2\cos\phi}$$

Since intensity is proportional to the square of the amplitude,

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\cos\phi$$

Using the trigonometric identity $\cos\phi = 2\cos^2\frac{\phi}{2} - 1$, this becomes

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\left(2\cos^2\frac{\phi}{2} - 1\right) = \frac{4}{9}I_{\max} + \frac{4}{3}I_{\max}\cos^2\frac{\phi}{2},$$

or $\boxed{I = \frac{4}{9}I_{\max}\left(1 + 3\cos^2\frac{\phi}{2}\right)}$

CHAPTER 38

38.1 $\sin\theta = \dfrac{\lambda}{a} = \dfrac{6.328\times10^{-7}}{3.00\times10^{-4}} = 2.11\times10^{-3}$

$\dfrac{y}{1.00\text{ m}} = \tan\theta \approx \sin\theta = \theta$ (for small θ)

$2y = \boxed{4.22\text{ mm}}$

38.2 The positions of the first-order minima are $y/L \approx \sin\theta = \pm\lambda/a$. Thus, the spacing between these two minima is $\Delta y = 2(\lambda/a)L$ and the wavelength is

$$\lambda = \left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right) = \left(\frac{4.10\times10^{-3}\text{ m}}{2}\right)\left(\frac{0.550\times10^{-3}\text{ m}}{2.06\text{ m}}\right) = \boxed{547\text{ nm}}$$

38.3 $\dfrac{y}{L} = \sin\theta = \dfrac{m\lambda}{a}$ $\quad \Delta y = 3.00\times10^{-3}\text{ m}$ $\quad \Delta m = 3 - 1 = 2$ $\quad$ and $\quad a = \dfrac{\Delta m\lambda L}{\Delta y}$

$$a = \frac{(2)(690\times10^{-9}\text{ m})(0.500\text{ m})}{3.00\times10^{-3}\text{ m}} = \boxed{2.30\times10^{-4}\text{ m}}$$

***38.4** For destructive interference,

$\sin\theta = m\dfrac{\lambda}{a} = \dfrac{\lambda}{a} = \dfrac{5.00\text{ cm}}{36.0\text{ cm}} = 0.139$ $\quad$ and $\quad \theta = 7.98°$

$\dfrac{d}{L} = \tan\theta$ gives $d = L\tan\theta = (6.50\text{ m})\tan 7.98° = 0.912\text{ m}$

$d = \boxed{91.2\text{ cm}}$

***38.5** If the speed of sound is 340 m/s,

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{650 \text{ /s}} = 0.523 \text{ m}$$

Diffraction minima occur at angles described by $a \sin \theta = m\lambda$

$1.10 \text{ m} \sin \theta_1 = 1(0.523 \text{ m})$ $\theta_1 = 28.4°$

$1.10 \text{ m} \sin \theta_2 = 2(0.523 \text{ m})$ $\theta_2 = 72.0°$

$1.10 \text{ m} \sin \theta_3 = 3(0.523 \text{ m})$ θ_3 nonexistent

Maxima appear straight ahead at $\boxed{0°}$ and left and right at an angle given approximately by

$(1.10 \text{ m}) \sin \theta_x = 1.5(0.523 \text{ m})$ $\theta_x \boxed{\approx 46°}$

There is no solution to $a \sin \theta = 2.5\lambda$, so our answer is already complete, with $\boxed{\text{three}}$ sound maxima.

38.6 (a) $\sin \theta = \dfrac{y}{L} = \dfrac{m\lambda}{a}$

Therefore, for first minimum, $m = 1$ and

$$L = \frac{ay}{m\lambda} = \frac{\left(7.50 \times 10^{-4} \text{ m}\right)\left(8.50 \times 10^{-4} \text{ m}\right)}{(1)\left(587.5 \times 10^{-9} \text{ m}\right)} = \boxed{1.09 \text{ m}}$$

(b) $w = 2y_1$ yields $y_1 = 0.850$ mm

$$w = 2\left(0.850 \times 10^{-3} \text{ m}\right) = \boxed{1.70 \text{ mm}}$$

38.7 $\sin \theta \approx \dfrac{y}{L} = \dfrac{4.10 \times 10^{-3} \text{ m}}{1.20 \text{ m}}$

$$\frac{\beta}{2} = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi\left(4.00 \times 10^{-4} \text{ m}\right)}{546.1 \times 10^{-9} \text{ m}}\left(\frac{4.10 \times 10^{-3} \text{ m}}{1.20 \text{ m}}\right) = 7.86 \text{ rad}$$

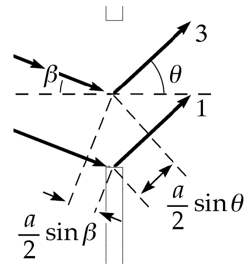
$$\frac{I}{I_{\max}} = \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = \left[\frac{\sin(7.86)}{7.86}\right]^2 = \boxed{1.62 \times 10^{-2}}$$

38.8 Bright fringes will be located approximately midway between adjacent dark fringes. Therefore, for the second bright fringe, let m = 2.5 and use

$$\sin\theta = m\lambda/a \approx y/L \;.$$

The wavelength will be $\lambda \approx \dfrac{ay}{mL} = \dfrac{(0.800\times10^{-3}\text{ m})(1.40\times10^{-3}\text{ m})}{2.5(0.800\text{ m})} = 5.60\times10^{-7}\text{ m} = \boxed{560\text{ nm}}$

38.9 Equation 38.1 states that $\sin\theta = m\lambda/a$, where $m = \pm1, \pm2, \pm3, \ldots$. The requirement for m = 1 is from an analysis of the extra path distance traveled by ray 1 compared to ray 3 in Figure 38.5. This extra distance must be equal to $\lambda/2$ for destructive interference. When the source rays approach the slit at an angle β, there is a distance added to the path difference (of ray 1 compared to ray 3) of $(a/2)\sin\beta$ Then, for destructive interference,



$$\frac{a}{2}\sin\beta + \frac{a}{2}\sin\theta = \frac{\lambda}{2} \qquad \text{so} \qquad \sin\theta = \frac{\lambda}{a} - \sin\beta.$$

Dividing the slit into 4 parts leads to the 2nd order minimum: $\sin\theta = \dfrac{2\lambda}{a} - \sin\beta$

Dividing the slit into 6 parts gives the third order minimum: $\sin\theta = \dfrac{3\lambda}{a} - \sin\beta$

Generalizing, we obtain the condition for the mth order minimum: $\sin\theta = \dfrac{m\lambda}{a} - \sin\beta$

***38.10** (a) Double-slit interference maxima are at angles given by $d\sin\theta = m\lambda$.

For $m = 0$, $\theta_0 = \boxed{0°}$

For $m = 1$, $(2.80\ \mu\text{m})\sin\theta = 1(0.5015\ \mu\text{m})$: $\theta_1 = \sin^{-1}(0.179) = \boxed{10.3°}$

Similarly, for $m = 2$, 3, 4, 5 and 6, $\theta_2 = \boxed{21.0°}$, $\theta_3 = \boxed{32.5°}$, $\theta_4 = \boxed{45.8°}$, $\theta_5 = \boxed{63.6°}$, and $\theta_6 = \sin^{-1}(1.07) = \text{nonexistent}$.

Thus, there are $5+5+1 = \boxed{\text{11 directions for interference maxima}}$.

(b) We check for missing orders by looking for single-slit diffraction minima, at $a\sin\theta = m\lambda$.

For m = 1, $(0.700\ \mu\text{m})\sin\theta = 1(0.5015\ \mu\text{m})$ and $\theta_1 = 45.8°$.

Thus, there is no bright fringe at this angle. There are only $\boxed{\text{nine bright fringes}}$, at $\boxed{\theta = 0°, \pm10.3°, \pm21.0°, \pm32.5°, \text{ and } \pm63.6°}$.

(c) $I = I_{\max}\left[\dfrac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$

At $\theta = 0°$, $\dfrac{\sin\theta}{\theta} \to 1$ and $\dfrac{I}{I_{\max}} \to \boxed{1.00}$

At $\theta = 10.3°$, $\dfrac{\pi a \sin\theta}{\lambda} = \dfrac{\pi(0.700\ \mu\text{m})\sin 10.3°}{0.5015\ \mu\text{m}} = 0.785\ \text{rad} = 45.0°$

$$\frac{I}{I_{\max}} = \left[\frac{\sin 45.0°}{0.785}\right]^2 = \boxed{0.811}$$

Similarly, at $\theta = 21.0°$, $\dfrac{\pi a \sin\theta}{\lambda} = 1.57\ \text{rad} = 90.0°$ and $\dfrac{I}{I_{\max}} = \boxed{0.405}$

At $\theta = 32.5°$, $\dfrac{\pi a \sin\theta}{\lambda} = 2.36\ \text{rad} = 135°$ and $\dfrac{I}{I_{\max}} = \boxed{0.0901}$

At $\theta = 63.6°$, $\dfrac{\pi a \sin\theta}{\lambda} = 3.93\ \text{rad} = 225°$ and $\dfrac{I}{I_{\max}} = \boxed{0.0324}$

38.11 $\sin\theta = \dfrac{\lambda}{a} = \dfrac{5.00\times 10^{-7}\ \text{m}}{5.00\times 10^{-4}} = \boxed{1.00\times 10^{-3}\ \text{rad}}$

38.12 $\theta_{\min} = \dfrac{y}{L} = 1.22\dfrac{\lambda}{D}$

$y = \dfrac{(1.22)(5.00\times 10^{-7})(0.0300)}{7.00\times 10^{-3}} = \boxed{2.61\ \mu\text{m}}$

y = radius of star-image
L = length of eye
λ = 500 nm
D = pupil diameter
θ = half angle

38.13 Following Equation 38.9 for diffraction from a circular opening, the beam spreads into a cone of half-angle

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{(632.8\times 10^{-9}\ \text{m})}{(0.00500\ \text{m})} = 1.54\times 10^{-4}\ \text{rad}$$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$$r_{\text{beam}} = \theta_{\min}\,(1.00\times 10^4\ \text{m}) = 1.544\ \text{m}$$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = \boxed{3.09\ \text{m}}$

Goal Solution

A helium-neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser.

G: A typical laser pointer makes a spot about 5 cm in diameter at 100 m, so the spot size at 10 km would be about 100 times bigger, or about 5 m across. Assuming that this HeNe laser is similar, we could expect a comparable beam diameter.

O: We assume that the light is parallel and not diverging as it passes through and fills the circular aperture. However, as the light passes through the circular aperture, it will spread from diffraction according to Equation 38.9.

A: The beam spreads into a cone of half-angle $\theta_{\min} = 1.22\dfrac{\lambda}{D} = 1.22\dfrac{\left(632.8 \times 10^{-9}\ \text{m}\right)}{(0.00500\ \text{m})} = 1.54 \times 10^{-4}\ \text{rad}$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$$r_{\text{beam}} = \theta_{\min}\left(1.00 \times 10^{4}\ \text{m}\right) = 1.54\ \text{m}$$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = 3.09\ \text{m}$

L: The beam is several meters across as expected, and is about 600 times larger than the laser aperture. Since most HeNe lasers are low power units in the mW range, the beam at this range would be so spread out that it would be too dim to see on a screen.

38.14 $\theta_{\min} = 1.22\dfrac{\lambda}{D} = \dfrac{d}{L}$ $\quad 1.22\left(\dfrac{5.80 \times 10^{-7}\ \text{m}}{4.00 \times 10^{-3}\ \text{m}}\right) = \dfrac{d}{1.80\ \text{mi}}\left(\dfrac{1\ \text{mi}}{1609\ \text{m}}\right)$ $\quad d = \boxed{0.512\ \text{m}}$

The shortening of the wavelength inside the patriot's eye does not change the answer.

38.15 By Rayleigh's criterion, two dots separated center-to-center by 2.00 mm would overlap when

$$\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$$

Thus, $L = \dfrac{dD}{1.22\lambda} = \dfrac{\left(2.00 \times 10^{-3}\ \text{m}\right)\left(4.00 \times 10^{-3}\ \text{m}\right)}{1.22\left(500 \times 10^{-9}\ \text{nm}\right)} = \boxed{13.1\ \text{m}}$

38.16 $D = 1.22\dfrac{\lambda}{\theta_{\min}} = \dfrac{1.22(5.00 \times 10^{-7})}{1.00 \times 10^{-5}}\ \text{m} = \boxed{6.10\ \text{cm}}$

38.17 $\theta_{\min} = 1.22\left(\dfrac{\text{wavelength}}{\text{pupil diameter}}\right) = \dfrac{(\text{distance between sources})}{L}$

Thus, $\dfrac{1.22\lambda}{d} = \dfrac{w}{vt}$, or $w = \dfrac{1.22\lambda(vt)}{d}$

Taillights are red. Take $\lambda \approx 650$ nm: $w \approx \dfrac{1.22\left(650\times10^{-9}\text{ m}\right)(20.0\text{ m/s})(600\text{ s})}{5.00\times10^{-3}\text{ m}} = \boxed{1.90\text{ m}}$

38.18 $\theta_{\min} = 1.22\left(\dfrac{\text{wavelength}}{\text{pupil diameter}}\right) = \dfrac{(\text{distance between sources})}{L}$ so $\dfrac{1.22\lambda}{d} = \dfrac{w}{vt}$

$w = \boxed{\dfrac{1.22\lambda(vt)}{d}}$ where $\lambda \approx 650$ nm is the average wavelength radiated by the red taillights.

38.19 $\dfrac{1.22\lambda}{D} = \dfrac{d}{L}$ $\lambda = \dfrac{c}{f} = 0.0200$ m $D = 2.10$ m $L = 9000$ m

$d = 1.22\,\dfrac{(0.0200\text{ m})(9000\text{ m})}{2.10\text{ m}} = \boxed{105\text{ m}}$

38.20 Apply Rayleigh's criterion, $\theta_{\min} = \dfrac{x}{D} = 1.22\dfrac{\lambda}{d}$

where $\theta_{\min} =$ half-angle of light cone, $x =$ radius of spot, $\lambda =$ wavelength of light, $d =$ diameter of telescope, $D =$ distance to Moon.

Then, the diameter of the spot on the Moon is

$$2x = 2\left(1.22\frac{\lambda D}{d}\right) = \frac{2(1.22)\left(694.3\times10^{-9}\text{ m}\right)\left(3.84\times10^{8}\text{ m}\right)}{2.70\text{ m}} = \boxed{241\text{ m}}$$

38.21 For 0.100° angular resolution, $1.22\dfrac{\left(3.00\times10^{-3}\text{ m}\right)}{D} = (0.100°)\left(\dfrac{\pi}{180°}\right)$ $D = \boxed{2.10\text{ m}}$

38.22 $L = 88.6\times10^{9}$ m, $D = 0.300$ m, $\lambda = 590\times10^{-9}$ m

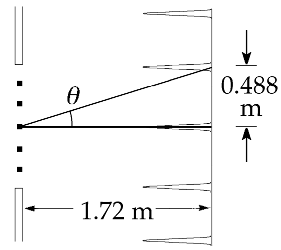
(a) $1.22\dfrac{\lambda}{D} = \theta_{\min} = \boxed{2.40\times10^{-6}\text{ rad}}$

(b) $d = \theta_{\min}L = \boxed{213\text{ km}}$

38.23 $d = \dfrac{1.00\text{ cm}}{2000} = \dfrac{1.00 \times 10^{-2}\text{ m}}{2000} = 5.00\ \mu\text{m}$

$\sin\theta = \dfrac{m\lambda}{d} = \dfrac{1\left(640 \times 10^{-9}\text{ m}\right)}{5.00 \times 10^{-6}\text{ m}} = 0.128 \qquad \theta = \boxed{7.35°}$

38.24 The principal maxima are defined by



$$d\sin\theta = m\lambda \quad m = 0, 1, 2, \ldots$$

For $m = 1$, $\qquad \lambda = d\sin\theta$

where θ is the angle between the central ($m = 0$) and the first order ($m = 1$) maxima. The value of θ can be determined from the information given about the distance between maxima and the grating-to-screen distance. From the figure,

$\tan\theta = \dfrac{0.488\text{ m}}{1.72\text{ m}} = 0.284 \qquad$ so $\qquad \theta = 15.8° \qquad$ and $\qquad \sin\theta = 0.273$

The distance between grating "slits" equals the reciprocal of the number of grating lines per centimeter

$d = \dfrac{1}{5310\text{ cm}^{-1}} = 1.88 \times 10^{-4}\text{ cm} = 1.88 \times 10^{3}\text{ nm}$

The wavelength is $\lambda = d\sin\theta = (1.88 \times 10^{3}\text{ nm})(0.273) = \boxed{514\text{ nm}}$

38.25 The grating spacing is $\qquad d = \dfrac{(1.00 \times 10^{-2}\text{ m})}{4500} = 2.22 \times 10^{-6}\text{ m}$

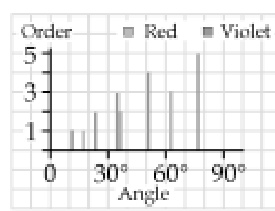


Figure for Goal Solution

In the 1st-order spectrum, diffraction angles are given by

$\sin\theta = \dfrac{\lambda}{d}$: $\qquad \sin\theta_1 = \dfrac{656 \times 10^{-9}\text{ m}}{2.22 \times 10^{-6}\text{ m}} = 0.295$

so that for red $\qquad \theta_1 = 17.17°$

and for violet $\qquad \sin\theta_2 = \dfrac{434 \times 10^{-9}\text{ m}}{2.22 \times 10^{-6}\text{ m}} = 0.195$

so that $\qquad \theta_2 = 11.26°$

The angular separation is in first-order, $\Delta\theta = 17.17° - 11.26° = \boxed{5.91°}$

In the second-order spectrum, $\Delta\theta = \sin^{-1}\left(\frac{2\lambda_1}{d}\right) - \sin^{-1}\left(\frac{2\lambda_2}{d}\right) = \boxed{13.2°}$

Again, in the third order, $\Delta\theta = \sin^{-1}\left(\frac{3\lambda_1}{d}\right) - \sin^{-1}\left(\frac{3\lambda_2}{d}\right) = \boxed{26.5°}$

Since the red line does not appear in the fourth-order spectrum, the answer is complete.

Goal Solution

The hydrogen spectrum has a red line at 656 nm and a violet line at 434 nm. What is the angular separation between two spectral lines obtained with a diffraction grating that has 4500 lines/cm?

G: Most diffraction gratings yield several spectral orders within the 180° viewing range, which means that the angle between red and violet lines is probably 10° to 30°.

O: The angular separation is the difference between the angles corresponding to the red and violet wavelengths for each visible spectral order according to the diffraction grating equation, $d\sin\theta = m\lambda$.

A: The grating spacing is $d = \left(1.00 \times 10^{-2}\text{ m}\right)/4500\text{ lines} = 2.22 \times 10^{-6}\text{ m}$

In the first-order spectrum ($m = 1$), the angles of diffraction are given by $\sin\theta = \lambda/d$:

$$\sin\theta_{1r} = \frac{656 \times 10^{-9}\text{ m}}{2.22 \times 10^{-6}\text{ m}} = 0.295 \quad \text{so} \quad \theta_{1r} = 17.17°$$

$$\sin\theta_{1v} = \frac{434 \times 10^{-9}\text{ m}}{2.22 \times 10^{-6}\text{ m}} = 0.195 \quad \text{so} \quad \theta_{1v} = 11.26°$$

The angular separation is $\Delta\theta_1 = \theta_{1r} - \theta_{1v} = 17.17 - 11.26 = 5.91$

In the 2nd-order ($m = 2$) $\Delta\theta_2 = \sin^{-1}\left(\frac{2\lambda_r}{d}\right) - \sin^{-1}\left(\frac{2\lambda_v}{d}\right) = 13.2°$

In the third order ($m = 3$), $\Delta\theta_3 = \sin^{-1}\left(\frac{3\lambda_r}{d}\right) - \sin^{-1}\left(\frac{3\lambda_v}{d}\right) = 26.5°$

In the fourth order, the red line is not visible: $\theta_{4r} = \sin^{-1}(4\lambda_r / d) = \sin^{-1}(1.18)$ does not exist

L: The full spectrum is visible in the first 3 orders with this diffraction grating, and the fourth is partially visible. We can also see that the pattern is dispersed more for higher spectral orders so that the angular separation between the red and blue lines increases as m increases. It is also worth noting that the spectral orders can overlap (as is the case for the second and third order spectra above), which makes the pattern look confusing if you do not know what you are looking for.

38.26 $\sin\theta = 0.350$: $\quad d = \dfrac{\lambda}{\sin\theta} = \dfrac{632.8\text{ nm}}{0.350} = 1.81\times10^{3}\text{ nm}$

Line spacing = $\boxed{1.81\ \mu\text{m}}$

***38.27** (a) $d = \dfrac{1}{3660\text{ lines/cm}} = 2.732\times10^{-4}\text{ cm} = 2.732\times10^{-6}\text{ m} = 2732\text{ nm}$

$\lambda = \dfrac{d\sin\theta}{m}$: $\quad$ At $\theta = 10.09°$ $\quad \lambda = \boxed{478.7\text{ nm}}$

At $\theta = 13.71°$, $\quad \lambda = \boxed{647.6\text{ nm}}$

At $\theta = 14.77°$, $\quad \lambda = \boxed{696.6\text{ nm}}$

(b) $d = \dfrac{\lambda}{\sin\theta_1}$ $\quad$ and $\quad \lambda = d\sin\theta_2$ $\quad$ so $\quad \sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2\lambda}{\left(\dfrac{\lambda}{\sin\theta_1}\right)} = 2\sin\theta_1$

Therefore, if $\theta_1 = 10.09°$ then $\sin\theta_2 = 2\sin(10.09°)$ gives $\theta_2 = \boxed{20.51°}$

Similarly, for $\theta_1 = 13.71°$, $\theta_2 = \boxed{28.30°}$ and for $\theta_1 = 14.77°$, $\theta_2 = \boxed{30.66°}$

38.28 $d = \dfrac{1}{800/\text{mm}} = 1.25\times10^{-6}\text{ m}$

The blue light goes off at angles $\sin\theta_m = \dfrac{m\lambda}{d}$: $\quad \theta_1 = \sin^{-1}\left(\dfrac{1\times5.00\times10^{-7}\text{ m}}{1.25\times10^{-6}\text{ m}}\right) = 23.6°$

$\theta_2 = \sin^{-1}(2\times0.400) = 53.1°$

$\theta_3 = \sin^{-1}(3\times0.400) = \text{nonexistent}$

The red end of the spectrum is at $\quad \theta_1 = \sin^{-1}\left(\dfrac{1\times7.00\times10^{-7}\text{ m}}{1.25\times10^{-6}\text{ m}}\right) = 34.1°$

$\theta_2 = \sin^{-1}(2\times0.560) = \text{nonexistent}$

So only the first-order spectrum is complete, and $\boxed{\text{it does not overlap}}$ the second-order spectrum.

38.29 (a) From Equation 38.12, $R = Nm$ where $N = (3000 \text{ lines/cm})(4.00 \text{ cm}) = 1.20 \times 10^4$ lines.

In the 1st order, $R = (1)(1.20 \times 10^4 \text{ lines}) = \boxed{1.20 \times 10^4}$

In the 2nd order, $R = (2)(1.20 \times 10^4 \text{ lines}) = \boxed{2.40 \times 10^4}$

In the 3rd order, $R = (3)(1.20 \times 10^4 \text{ lines}) = \boxed{3.60 \times 10^4}$

(b) From Equation 38.11, $R = \dfrac{\lambda}{\Delta\lambda}$:

In the 3rd order, $\Delta\lambda = \dfrac{\lambda}{R} = \dfrac{400 \text{ nm}}{3.60 \times 10^4} = 0.0111 \text{ nm} = \boxed{11.1 \text{ pm}}$

38.30 $\sin\theta = \dfrac{m\lambda}{d}$

Therefore, taking the ends of the visible spectrum to be $\lambda_v = 400$ nm and $\lambda_r = 750$ nm, the ends the different order spectra are:

End of second order: $\sin\theta_{2r} = \dfrac{2\lambda_r}{d} = \dfrac{1500 \text{ nm}}{d}$

Start of third order: $\sin\theta_{3v} = \dfrac{2\lambda_v}{d} = \dfrac{1200 \text{ nm}}{d}$

Thus, it is seen that $\boxed{\theta_{2r} > \theta_{3v} \text{ and these orders must overlap}}$ regardless of the value of the grating spacing *d*.

38.31 (a) $Nm = \dfrac{\lambda}{\Delta\lambda}$ $\quad N(1) = \dfrac{531.7 \text{ nm}}{0.19 \text{ nm}} = \boxed{2800}$

(b) $\dfrac{1.32 \times 10^{-2} \text{ m}}{2800} = \boxed{4.72\ \mu\text{m}}$

38.32 $d\sin\theta = m\lambda$ and, differentiating, $d(\cos\theta)d\theta = md\lambda$ or $d\sqrt{1-\sin^2\theta}\,\Delta\theta \approx m\Delta\lambda$

$d\sqrt{1 - m^2\lambda^2 / d^2}\,\Delta\theta \approx m\Delta\lambda$ so $\boxed{\Delta\theta \approx \dfrac{\Delta\lambda}{\sqrt{d^2 / m^2 - \lambda^2}}}$

38.33 $$d=\frac{1.00\times10^{-3}\ \text{m/mm}}{250\ \text{lines/mm}}=4.00\times10^{-6}\ \text{m}=4000\ \text{nm}$$

$$d\sin\theta=m\lambda\quad\Rightarrow\quad m=\frac{d\sin\theta}{\lambda}$$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$$m_{\text{max}}=\frac{d\sin\theta_{\text{max}}}{\lambda}=\frac{(4000\ \text{nm})\sin 90.0°}{700\ \text{nm}}=5.71\quad\text{or}\quad\boxed{\text{5 orders is the maximum}}\,.$$

(b) The highest order in which the violet end of the spectrum can be seen is:

$$m_{\text{max}}=\frac{d\sin\theta_{\text{max}}}{\lambda}=\frac{(4000\ \text{nm})\sin 90.0°}{400\ \text{nm}}=10.0\quad\text{or}\quad\boxed{\text{10 orders in the short - wavelength region}}$$

38.34 $$d=\frac{1}{4200/\text{cm}}=2.38\times10^{-6}\ \text{m}=2380\ \text{nm}$$

$$d\sin\theta=m\lambda\qquad\text{or}\qquad\theta=\sin^{-1}\left(\frac{m\lambda}{d}\right)\qquad\text{and}\qquad y=L\tan\theta=L\tan\left[\sin^{-1}\left(\frac{m\lambda}{d}\right)\right]$$

Thus, $$\Delta y=L\left\{\tan\left[\sin^{-1}\left(\frac{m\lambda_2}{d}\right)\right]-\tan\left[\sin^{-1}\left(\frac{m\lambda_1}{d}\right)\right]\right\}$$

For $m=1$, $$\Delta y=(2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{589.6}{2380}\right)\right]-\tan\left[\sin^{-1}\left(\frac{589}{2380}\right)\right]\right\}=0.554\ \text{mm}$$

For $m=2$, $$\Delta y=(2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{2(589.6)}{2380}\right)\right]-\tan\left[\sin^{-1}\left(\frac{2(589)}{2380}\right)\right]\right\}=1.54\ \text{mm}$$

For $m=3$, $$\Delta y=(2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{3(589.6)}{2380}\right)\right]-\tan\left[\sin^{-1}\left(\frac{3(589)}{2380}\right)\right]\right\}=5.04\ \text{mm}$$

Thus, the observed order must be $\boxed{m=2}$.

38.35 $2d\sin\theta=m\lambda$: $$\lambda=\frac{2d\sin\theta}{m}=\frac{2(0.353\times10^{-9}\ \text{m})\sin(7.60°)}{(1)}=9.34\times10^{-11}\ \text{m}=\boxed{0.0934\ \text{nm}}$$

38.36 $$2d\sin\theta=m\lambda\Rightarrow d=\frac{m\lambda}{2\sin\theta}=\frac{(1)(0.129\ \text{nm})}{2\sin(8.15°)}=\boxed{0.455\ \text{nm}}$$

38.37 $2d\sin\theta = m\lambda$ so $\sin\theta = \dfrac{m\lambda}{2d} = \dfrac{1(0.140\times10^{-9}\text{ m})}{2(0.281\times10^{-9}\text{ m})} = 0.249$ and $\boxed{\theta = 14.4°}$

38.38 $\sin\theta_m = \dfrac{m\lambda}{2d}$: $\sin 12.6° = \dfrac{1\lambda}{2d} = 0.218$

$\sin\theta_2 = \dfrac{2\lambda}{2d} = 2(0.218)$ so $\theta_2 = 25.9°$

$\boxed{\text{Three}}$ other orders appear: $\theta_3 = \sin^{-1}(3\times0.218) = 40.9°$

$\theta_4 = \sin^{-1}(4\times0.218) = 60.8°$

$\theta_5 = \sin^{-1}(5\times0.218) = \text{nonexistent}$

38.39 $2d\sin\theta = m\lambda$ $\theta = \sin^{-1}\left[\dfrac{m\lambda}{2d}\right] = \sin^{-1}\left[\dfrac{2\times0.166}{2\times0.314}\right] = \boxed{31.9°}$

***38.40** Figure 38.25 of the text shows the situation. $2d\sin\theta = m\lambda$ or $\lambda = \dfrac{2d\sin\theta}{m}$

$$m = 1 \quad\Rightarrow\quad \lambda_1 = \frac{2(2.80\text{ m})\sin 80.0°}{1} = \boxed{5.51\text{ m}}$$

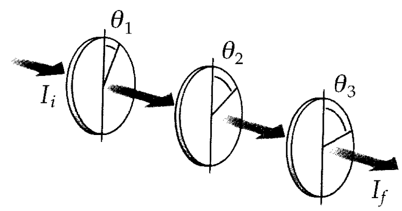
$$m = 2 \quad\Rightarrow\quad \lambda_2 = \frac{2(2.80\text{ m})\sin 80.0°}{2} = \boxed{2.76\text{ m}}$$

$$m = 3 \quad\Rightarrow\quad \lambda_3 = \frac{2(2.80\text{ m})\sin 80.0°}{3} = \boxed{1.84\text{ m}}$$

***38.41** The average value of the cosine-squared function is one-half, so the first polarizer transmits $\dfrac{1}{2}$ the light. The second transmits $\cos^2 30.0° = \dfrac{3}{4}$.

$$I_f = \frac{1}{2}\times\frac{3}{4}\,I_i = \boxed{\frac{3}{8}I_i}$$

38.42 (a) $\theta_1 = 20.0°, \quad \theta_2 = 40.0°, \quad \theta_3 = 60.0°$



$$I_f = I_i \cos^2(\theta_1 - 0°)\cos^2(\theta_2 - \theta_1)\cos^2(\theta_3 - \theta_2)$$

$$I_f = (10.0 \text{ units})\cos^2(20.0°)\cos^2(20.0°)\cos^2(20.0°) = \boxed{6.89 \text{ units}}$$

(b) $\theta_1 = 0°, \quad \theta_2 = 30.0°, \quad \theta_3 = 60.0°$

$$I_f = (10.0 \text{ units})\cos^2(0°)\cos^2(30.0°)\cos^2(30.0°) = \boxed{5.63 \text{ units}}$$

38.43 $I = I_{max}\cos^2\theta \qquad \Rightarrow \qquad \theta = \cos^{-1}\left(\dfrac{I}{I_{max}}\right)^{1/2}$

(a) $\dfrac{I}{I_{max}} = \dfrac{1}{3.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{1}{3.00}\right)^{1/2} = \boxed{54.7°}$

(b) $\dfrac{I}{I_{max}} = \dfrac{1}{5.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{1}{5.00}\right)^{1/2} = \boxed{63.4°}$

(c) $\dfrac{I}{I_{max}} = \dfrac{1}{10.0} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{1}{10.0}\right)^{1/2} = \boxed{71.6°}$

38.44 By Brewster's law, $\qquad n = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$

38.45 $\sin\theta_c = \dfrac{1}{n} \qquad$ or $\qquad n = \dfrac{1}{\sin\theta_c} = \dfrac{1}{\sin 34.4°} = 1.77$

Also, $\tan\theta_p = n$. Thus, $\theta_p = \tan^{-1}(n) = \tan^{-1}(1.77) = \boxed{60.5°}$

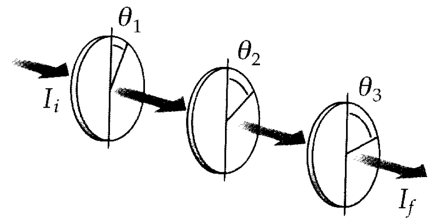
38.46 $\sin\theta_c = \dfrac{1}{n} \qquad$ and $\qquad \tan\theta_p = n$

Thus, $\sin\theta_c = \dfrac{1}{\tan\theta_p} \qquad$ or $\qquad \boxed{\cot\theta_p = \sin\theta_c}$

38.47 Complete polarization occurs at Brewster's angle $\tan\theta_p = 1.33$ $\theta_p = 53.1°$

Thus, the Moon is $\boxed{36.9°}$ above the horizon.

38.48 For incident unpolarized light of intensity $I_{\max}$:



After transmitting 1st disk: $I = \left(\frac{1}{2}\right)I_{\max}$

After transmitting 2nd disk: $I = \left(\frac{1}{2}\right)I_{\max}\cos^2\theta$

After transmitting 3rd disk: $I = \left(\frac{1}{2}\right)I_{\max}\left(\cos^2\theta\right)\cos^2(90°-\theta)$

where the angle between the first and second disk is $\theta = \omega t$.

Using trigonometric identities $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ and $\cos^2(90-\theta) = \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

we have $I = \frac{1}{2}I_{\max}\left[\frac{(1+\cos 2\theta)}{2}\right]\left[\frac{(1-\cos 2\theta)}{2}\right] = \frac{1}{8}I_{\max}(1-\cos^2 2\theta) = \frac{1}{8}I_{\max}\left(\frac{1}{2}\right)(1-\cos 4\theta)$

Since $\theta = \omega t$, the intensity of the emerging beam is given by $\boxed{I = \frac{1}{16}I_{\max}(1-\cos 4\omega t)}$

38.49 Let the first sheet have its axis at angle θ to the original plane of polarization, and let each further sheet have its axis turned by the same angle.

The first sheet passes intensity $I_{\max}\cos^2\theta$.

The second sheet passes $I_{\max}\cos^4\theta$,

nd the nth sheet lets through $I_{\max}\cos^{2n}\theta \ge 0.90 I_{\max}$ where $\theta = 45°/n$

Try different integers to find $\cos^{2\times 5}\left(\frac{45°}{5}\right) = 0.885$, $\cos^{2\times 6}\left(\frac{45°}{6}\right) = 0.902$,

(a) So $n = \boxed{6}$

(b) $\theta = \boxed{7.50°}$

***38.50** Consider vocal sound moving at 340 m/s and of frequency 3000 Hz. Its wavelength is

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{3000 \text{ Hz}} = 0.113 \text{ m}$$

If your mouth, for horizontal dispersion, behaves similarly to a slit 6.00 cm wide, then $a\sin\theta = m\lambda$ predicts no diffraction minima. You are a nearly isotropic source of this sound. It spreads out from you nearly equally in all directions. On the other hand, if you use a megaphone with width 60.0 cm at its wide end, then $a\sin\theta = m\lambda$ predicts the first diffraction minimum at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{0.113 \text{ m}}{0.600 \text{ m}}\right) = 10.9°$$

This suggests that the sound is radiated mostly toward the front into a diverging beam of angular diameter only about 20°. With less sound energy wasted in other directions, more is available for your intended auditors. We could check that a distant observer to the side or behind you receives less sound when a megaphone is used.

38.51 The first minimum is at $a \sin\theta = 1\lambda$.

This has no solution if $\dfrac{\lambda}{a} > 1$

or if $a < \lambda = \boxed{632.8 \text{ nm}}$

38.52 $x = 1.22\dfrac{\lambda}{d}D = 1.22\left(\dfrac{5.00\times 10^{-7} \text{ m}}{5.00\times 10^{-3} \text{ m}}\right)\left(250\times 10^{3} \text{ m}\right) = \boxed{30.5 \text{ m}}$

$D = 250 \times 10^3$ m
$\lambda = 5.00 \times 10^{-7}$ m
$d = 5.00 \times 10^{-3}$ m

38.53 $d = \dfrac{1}{400/\text{mm}} = 2.50 \times 10^{-6}$ m

(a) $d \sin\theta = m\lambda$ $\theta_a = \sin^{-1}\left(\dfrac{2 \times 541 \times 10^{-9} \text{ m}}{2.50 \times 10^{-6} \text{ m}}\right) = \boxed{25.6°}$

(b) $\lambda = \dfrac{541 \times 10^{-9} \text{ m}}{1.33} = 4.07 \times 10^{-7}$ m

$\theta_b = \sin^{-1}\left(\dfrac{2 \times 4.07 \times 10^{-7} \text{ m}}{2.50 \times 10^{-6} \text{ m}}\right) = \boxed{19.0°}$

(c) $d \sin\theta_a = 2\lambda$ $d \sin\theta_b = \dfrac{2\lambda}{n}$ $n \sin\theta_b = 1 \sin\theta_a$

***38.54** (a) $\lambda = \frac{v}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{1.40\times10^{9}\ \text{s}^{-1}} = 0.214\ \text{m}$

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{0.214\ \text{m}}{3.60\times10^{4}\ \text{m}}\right) = \boxed{7.26\ \mu\text{rad}} = 7.26\ \mu\text{rad}\left(\frac{180\times60\times60\ \text{s}}{\pi}\right) = \boxed{1.50\ \text{arc seconds}}$$

(b) $\theta_{\min} = \frac{d}{L}$: $\quad d = \theta_{\min}L = \left(7.26\times10^{-6}\ \text{rad}\right)\left(26\ 000\ \text{ly}\right) = \boxed{0.189\ \text{ly}}$

(c) $\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{500\times10^{-9}\ \text{m}}{12.0\times10^{-3}\ \text{m}}\right) = \boxed{50.8\ \mu\text{rad}}$ (10.5 seconds of arc)

(d) $d = \theta_{\min}L = \left(50.8\times10^{-6}\ \text{rad}\right)\left(30.0\ \text{m}\right) = 1.52\times10^{-3}\ \text{m} = \boxed{1.52\ \text{mm}}$

38.55 $\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{(2.00\ \text{m})}{(10.0\ \text{m})} = \boxed{0.244\ \text{rad} = 14.0°}$

38.56 With a grazing angle of 36.0°, the angle of incidence is 54.0°

$\tan\theta_p = n = \tan 54.0° = 1.38$

In the liquid, $\lambda_n = \lambda / n = 750\ \text{nm} / 1.38 = \boxed{545\ \text{nm}}$

38.57 (a) $d\sin\theta = m\lambda$, or $d = \frac{m\lambda}{\sin\theta} = \frac{3\left(500\times10^{-9}\ \text{m}\right)}{\sin 32.0°} = 2.83\ \mu\text{m}$

Therefore, lines per unit length $= \frac{1}{d} = \frac{1}{2.83\times10^{-6}\ \text{m}}$

or lines per unit length $= 3.53\times10^{5}/\text{m} = \boxed{3.53\times10^{3}/\text{cm}}$.

(b) $\sin\theta = \frac{m\lambda}{d} = \frac{m\left(500\times10^{-9}\ \text{m}\right)}{2.83\times10^{-6}\ \text{m}} = m(0.177)$

For $\sin\theta \le 1.00$, we must have $\quad m(0.177) \le 1.00$ or $m \le 5.65$

Therefore, the highest order observed is $\quad m = 5$

Total number primary maxima observed is $\quad 2m+1 = \boxed{11}$

Goal Solution

Light of wavelength 500 nm is incident normally on a diffraction grating. If the third-order maximum of the diffraction pattern is observed at 32.0°, (a) what is the number of rulings per centimeter for the grating? (b) Determine the total number of primary maxima that can be observed in this situation.

G: The diffraction pattern described in this problem seems to be similar to previous problems that have diffraction gratings with 2 000 to 5 000 lines/mm. With the third-order maximum at 32°, there are probably 5 or 6 maxima on each side of the central bright fringe, for a total of 11 or 13 primary maxima.

O: The diffraction grating equation can be used to find the grating spacing and the angles of the other maxima that should be visible within the 180° viewing range.

A: (a) Use Equation 38.10, $d\sin\theta = m\lambda$

$$d = \frac{m\lambda}{\sin\theta} = \frac{(3)(5.00\times10^{-7}\text{ m})}{\sin(32.0°)} = 2.83\times10^{-6}\text{ m}$$

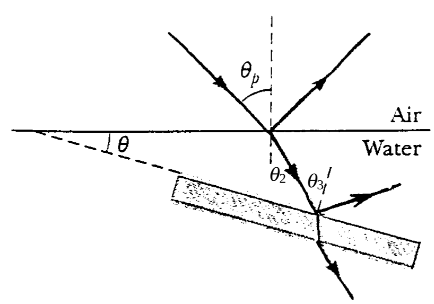
Thus, the grating gauge is $\frac{1}{d} = 3.534\times10^{5}\text{ lines / m} = 3530\text{ lines / cm}$ ◊

(b) $$\sin\theta = m\left(\frac{\lambda}{d}\right) = \frac{m(5.00\times10^{-7}\text{ m})}{2.83\times10^{-6}\text{ m}} = m(0.177)$$

For $\sin\theta \le 1$, we require that $m(1.77) \le 1$ or $m \le 5.65$. Since m must be an integer, its maximum value is really 5. Therefore, the total number of maxima is $2m+1 = 11$

L: The results agree with our predictions, and apparently there are 5 maxima on either side of the central maximum. If more maxima were desired, a grating with **fewer** lines/cm would be required; however, this would reduce the ability to resolve the difference between lines that appear close together.

38.58 For the air-to-water interface,



$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} \qquad \theta_p = 53.1°$$

and $(1.00)\sin\theta_p = (1.33)\sin\theta_2$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1°}{1.33}\right) = 36.9°$$

For the water-to-glass interface, $\tan\theta_p = \tan\theta_3 = \frac{n_{\text{glass}}}{n_{\text{water}}} = \frac{1.50}{1.33}$ so $\theta_3 = 48.4°$

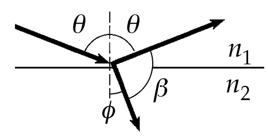
The angle between surfaces is $\theta = \theta_3 - \theta_2 = \boxed{11.5°}$

38.59 The limiting resolution between lines $\theta_{\min} = 1.22\dfrac{\lambda}{D} = 1.22\dfrac{\left(550\times10^{-9}\text{ m}\right)}{\left(5.00\times10^{-3}\text{ m}\right)} = 1.34\times10^{-4}$ rad

Assuming a picture screen with vertical dimension l, the minimum viewing distance for no visible lines is found from $\theta_{\min} = (l/485)/L$. The desired ratio is then

$$\frac{L}{l} = \frac{1}{485\,\theta_{\min}} = \frac{1}{485\left(1.34\times10^{-4}\text{ rad}\right)} = \boxed{15.4}$$

38.60 (a) Applying Snell's law gives $n_2 \sin\phi = n_1 \sin\theta$. From the sketch, we also see that:



$$\theta + \phi + \beta = \pi, \quad \text{or} \quad \phi = \pi - (\theta + \beta)$$

Using the given identity: $\sin\phi = \sin\pi\cos(\theta+\beta) - \cos\pi\sin(\theta+\beta)$,

which reduces to: $\sin\phi = \sin(\theta+\beta)$.

Applying the identity again: $\sin\phi = \sin\theta\cos\beta + \cos\theta\sin\beta$

Snell's law then becomes: $n_2\left(\sin\theta\cos\beta + \cos\theta\sin\beta\right) = n_1\sin\theta$

or (after dividing by $\cos\theta$): $n_2(\tan\theta\cos\beta + \sin\beta) = n_1\tan\theta$.

Solving for $\tan\theta$ gives: $\boxed{\tan\theta = \dfrac{n_2\sin\beta}{n_1 - n_2\cos\beta}}$

(b) If $\beta = 90.0°$, $n_1 = 1.00$, and $n_2 = n$, the above result becomes:

$\tan\theta = \dfrac{n(1.00)}{1.00 - 0}$, or $n = \tan\theta$, which is Brewster's law.

38.61 (a) From Equation 38.1, $\theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right)$

In this case $m = 1$ and $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{7.50\times10^{9}\text{ Hz}} = 4.00\times10^{-2}$ m

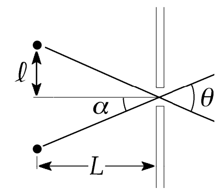
Thus, $\theta = \sin^{-1}\left(\dfrac{4.00\times10^{-2}\text{ m}}{6.00\times10^{-2}\text{ m}}\right) = \boxed{41.8°}$

(b) From Equation 38.4, $\dfrac{I}{I_{\max}} = \left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$ where $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$

When $\theta = 15.0°$, $\beta = \dfrac{2\pi(0.0600\text{ m})\sin 15.0°}{0.0400\text{ m}} = 2.44\text{ rad}$

and $\dfrac{I}{I_{\max}} = \left[\dfrac{\sin(1.22\text{ rad})}{1.22\text{ rad}}\right]^2 = \boxed{0.593}$

(c) $\sin\theta = \dfrac{\lambda}{a}$ so $\theta = 41.8°$:



This is the minimum angle subtended by the two sources at the slit. Let α be the half angle between the sources, each a distance $\ell = 0.100$ m from the center line and a distance L from the slit plane. Then,

$L = \ell\cot\alpha = (0.100\text{ m})\cot(41.8/2) = \boxed{0.262\text{ m}}$

38.62 $\dfrac{I}{I_{\max}} = \dfrac{1}{2}(\cos^2 45.0°)(\cos^2 45.0°) = \boxed{\dfrac{1}{8}}$

38.63 (a) The E and O rays, in phase at the surface of the plate, will have a phase difference

$\theta = (2\pi/\lambda)\delta$

after traveling distance d through the plate. Here δ is the difference in the *optical path* lengths of these rays. The optical path length between two points is the product of the actual path length d and the index of refraction. Therefore,

$\delta = |dn_O - dn_E|$

The absolute value is used since n_O/n_E may be more or less than unity. Therefore,

$$\theta = \left(\frac{2\pi}{\lambda}\right)|dn_O - dn_E| = \boxed{\left(\frac{2\pi}{\lambda}\right)d|n_O - n_E|}$$

(b) $d = \dfrac{\lambda\theta}{2\pi|n_O - n_E|} = \dfrac{(550\times10^{-9}\text{ m})(\pi/2)}{2\pi|1.544 - 1.553|} = 1.53\times10^{-5}\text{ m} = \boxed{15.3\ \mu\text{m}}$

***38.64** For a diffraction grating, the locations of the principal maxima for wavelength λ are given by $\sin\theta = m\lambda/d \approx y/L$. The grating spacing may be expressed as $d = a/N$ where a is the width of the grating and N is the number of slits. Thus, the screen locations of the maxima become

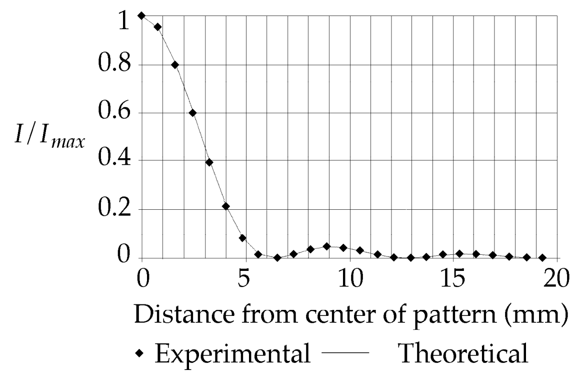
$y = NLm\lambda / a$. If two nearly equal wavelengths are present, the difference in the screen locations of corresponding maxima is

$$\Delta y = \frac{NLm(\Delta\lambda)}{a}$$

For a single slit of width a, the location of the first diffraction minimum is $\sin\theta = \lambda/a \approx y/L$, or $y = (L/a)\lambda$. If the two wavelengths are to be just resolved by Rayleigh's criterion, $y = \Delta y$ from above. Therefore,

$$\left(\frac{L}{a}\right)\lambda = \frac{NLm(\Delta\lambda)}{a}$$ or the resolving power of the grating is $\boxed{R'' \frac{\lambda}{\Delta\lambda} = Nm}$.

38.65 The first minimum in the single-slit diffraction pattern occurs at



$$\sin\theta = \frac{\lambda}{a} » \frac{y_{\min}}{L}$$

Thus, the slit width is given by

$$a = \frac{\lambda L}{y_{\min}}$$

For a minimum located at $y_{\min} = 6.36 \text{ mm} \pm 0.08 \text{ mm}$,

the width is $a = \dfrac{(632.8 \cdot 10^{-9} \text{ m})(1.00 \text{ m})}{6.36 \cdot 10^{-3} \text{ m}} = \boxed{99.5\ \mu\text{m} \pm 1\%}$

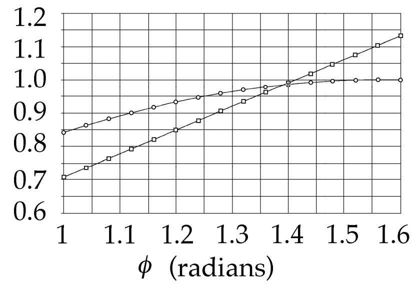
38.66 (a) From Equation 38.4, $$\frac{I}{I_{\max}} = \left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2$$

If we define $\phi \equiv \beta/2$ this becomes $\dfrac{I}{I_{\max}} = \left[\dfrac{\sin\phi}{\phi}\right]^2$

Therefore, when $\dfrac{I}{I_{\max}} = \dfrac{1}{2}$ we must have $\dfrac{\sin\phi}{\phi} = \dfrac{1}{\sqrt{2}}$, or $\boxed{\sin\phi = \dfrac{\phi}{\sqrt{2}}}$

(b) Let $y_1 = \sin\phi$ and $y_2 = \dfrac{\phi}{\sqrt{2}}$.

A plot of y_1 and y_2 in the range $1.00 \le \phi \le \pi/2$ is shown to the right.



The solution to the transcendental equation is found to be $\boxed{\phi = 1.39 \text{ rad}}$.

(c) $\beta = \dfrac{2\pi a \sin\theta}{\lambda} = 2\phi$

gives $\sin\theta = \left(\dfrac{\phi}{\pi}\right)\left(\dfrac{\lambda}{a}\right) = 0.443\dfrac{\lambda}{a}$.

If $\dfrac{\lambda}{a}$ is small, then $\theta \approx 0.443\dfrac{\lambda}{a}$.

This gives the half-width, measured away from the maximum at $\theta = 0$. The pattern is symmetric, so the full width is given by

$$\Delta\theta = 0.443\frac{\lambda}{a} - \left(-0.443\frac{\lambda}{a}\right) = \boxed{\frac{0.886\lambda}{a}}$$

38.67

ϕ	$\sqrt{2}\sin\phi$	
1	1.19	bigger than ϕ
2	1.29	smaller than ϕ
1.5	1.41	smaller
1.4	1.394	
1.39	1.391	bigger
1.395	1.392	
1.392	1.3917	smaller
1.3915	1.39154	bigger
1.39152	1.39155	bigger
1.3916	1.391568	smaller
1.39158	1.391563	
1.39157	1.391560	
1.39156	1.391558	
1.391559	1.3915578	
1.391558	1.3915575	
1.391557	1.3915573	
1.3915574	1.3915574	

We get the answer to seven digits after 17 steps. Clever guessing, like using the value of $\sqrt{2}\sin\phi$ as the next guess for ϕ, could reduce this to around 13 steps.

***38.68** In $I = I_{max}\left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2$ find $\frac{dI}{d\beta} = I_{max}\left(\frac{2\sin(\beta/2)}{(\beta/2)}\right)\left[\frac{(\beta/2)\cos(\beta/2)(1/2) - \sin(\beta/2)(1/2)}{(\beta/2)^2}\right]$

and require that it be zero. The possibility $\sin(\beta/2) = 0$ locates all of the minima and the central maximum, according to

$\beta/2 = 0,\ \pi,\ 2\pi,\ \ldots;$ $\beta = \frac{2\pi a \sin\theta}{\lambda} = 0,\ 2\pi,\ 4\pi,\ \ldots;$ $a\sin\theta = 0,\ \lambda,\ 2\lambda,\ \ldots.$

The side maxima are found from $\frac{\beta}{2}\cos\left(\frac{\beta}{2}\right) - \sin\left(\frac{\beta}{2}\right) = 0$, or $\tan\left(\frac{\beta}{2}\right) = \frac{\beta}{2}$.

This has solutions $\boxed{\frac{\beta}{2} = 4.4934}$, $\boxed{\frac{\beta}{2} = 7.7253}$, and others, giving

(a) $\pi a \sin\theta = 4.4934\lambda$ $\boxed{a\sin\theta = 1.4303\lambda}$

(b) $\pi a \sin\theta = 7.7253\lambda$ $\boxed{a\sin\theta = 2.4590\lambda}$

***38.69** (a) We require $\theta_{min} = 1.22\frac{\lambda}{D} = \frac{\text{radius of diffraction disk}}{L} = \frac{D}{2L}$.

Then $\boxed{D^2 = 2.44\lambda L}$

(b) $D = \sqrt{2.44(500\times10^{-9}\text{ m})(0.150\text{ m})} = \boxed{428\ \mu\text{m}}$

Chapter 39 Solutions

39.1 In the rest frame,

$$p_i = m_1 v_{1i} + m_2 v_{2i} = (2000 \text{ kg})(20.0 \text{ m/s}) + (1500 \text{ kg})(0 \text{ m/s}) = 4.00 \times 10^4 \text{ kg} \cdot \text{m/s}$$

$$p_f = (m_1 + m_2)v_f = (2000 \text{ kg} + 1500 \text{ kg})v_f$$

Since $p_i = p_f$, $\quad v_f = \dfrac{4.00 \times 10^4 \text{ kg} \cdot \text{m/s}}{2000 \text{ kg} + 1500 \text{ kg}} = 11.429 \text{ m/s}$

In the moving frame, these velocities are all reduced by +10.0 m/s.

$$v'_{1i} = v_{1i} - v' = 20.0 \text{ m/s} - (+10.0 \text{ m/s}) = 10.0 \text{ m/s}$$

$$v'_{2i} = v_{2i} - v' = 0 \text{ m/s} - (+10.0 \text{ m/s}) = -10.0 \text{ m/s}$$

$$v'_f = 11.429 \text{ m/s} - (+10.0 \text{ m/s}) = 1.429 \text{ m/s}$$

Our initial momentum is then

$$p'_i = m_1 v'_{1i} + m_2 v'_{2i} = (2000 \text{ kg})(10.0 \text{ m/s}) + (1500 \text{ kg})(-10.0 \text{ m/s}) = 5000 \text{ kg} \cdot \text{m/s}$$

and our final momentum is

$$p'_f = (2000 \text{ kg} + 1500 \text{ kg})\, v'_f = (3500 \text{ kg})(1.429 \text{ m/s}) = 5000 \text{ kg} \cdot \text{m/s}$$

39.2 (a) $v = v_T + v_B = \boxed{60.0 \text{ m/s}}$

(b) $v = v_T - v_B = \boxed{20.0 \text{ m/s}}$

(c) $v = \sqrt{v_T^2 + v_B^2} = \sqrt{20^2 + 40^2} = \boxed{44.7 \text{ m/s}}$

39.3 The first observer watches some object accelerate under applied forces. Call the instantaneous velocity of the object v_1. The second observer has constant velocity v_{21} relative to the first, and measures the object to have velocity $v_2 = v_1 - v_{21}$.

The second observer measures an acceleration of $\quad a_2 = \dfrac{dv_2}{dt} = \dfrac{dv_1}{dt}$

This is the same as that measured by the first observer. In this nonrelativistic case, they measure the same forces as well. Thus, the second observer also confirms that $\Sigma F = ma$.

39.4 The laboratory observer notes Newton's second law to hold: $F_1 = ma_1$

(where the subscript 1 implies the measurement was made in the laboratory frame of reference). The observer in the accelerating frame measures the acceleration of the mass as $a_2 = a_1 - a'$

(where the subscript 2 implies the measurement was made in the accelerating frame of reference, and the primed acceleration term is the acceleration of the accelerated frame with respect to the laboratory frame of reference). If Newton's second law held for the accelerating frame, that observer would then find valid the relation

$F_2 = ma_2$ or $F_1 = ma_2$

(since $F_1 = F_2$ and the mass is unchanged in each). But, instead, the accelerating frame observer will find that $F_2 = ma_2 - ma'$ which is *not* Newton's second law.

***39.5** $L = L_p\sqrt{1 - v^2/c^2} \Rightarrow v = c\sqrt{1 - \left(L/L_p\right)^2}$

Taking $L = L_p/2$ where $L_p = 1.00$ m gives $v = c\sqrt{1 - \left(\dfrac{L_p/2}{L_p}\right)^2} = c\sqrt{1 - \dfrac{1}{4}} = \boxed{0.866c}$

39.6 $\Delta t = \dfrac{\Delta t_p}{\left[1 - (v/c)^2\right]^{1/2}}$ so $v = c\left[1 - \left(\dfrac{\Delta t_p}{\Delta t}\right)^2\right]^{1/2}$

For $\Delta t = 2\Delta t_p \Rightarrow v = c\left[1 - \left(\dfrac{\Delta t_p}{2\Delta t_p}\right)^2\right]^{1/2} = c\left[1 - \dfrac{1}{4}\right]^{1/2} = \boxed{0.866c}$

***39.7** (a) $\gamma = \dfrac{1}{\sqrt{1 - (v/c)^2}} = \dfrac{1}{\sqrt{1 - (0.500)^2}} = \dfrac{2}{\sqrt{3}}$

The time interval between pulses as measured by the Earth observer is

$$\Delta t = \gamma \Delta t_p = \frac{2}{\sqrt{3}}\left(\frac{60.0\text{ s}}{75.0}\right) = 0.924\text{ s}$$

Thus, the Earth observer records a pulse rate of $\dfrac{60.0\text{ s/min}}{0.924\text{ s}} = \boxed{64.9/\text{min}}$

(b) At a relative speed $v = 0.990c$, the relativistic factor γ increases to 7.09 and the pulse rate recorded by the Earth observer decreases to $\boxed{10.6/\text{min}}$. That is, the life span of the astronaut (reckoned by the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

39.8 The observed length of an object moving at speed v is $L = L_p\sqrt{1 - v^2 / c^2}$ with L_p as the proper length. For the two ships, we know $L_2 = L_1$, $L_{2p} = 3L_{1p}$, and $v_1 = 0.350c$

Thus, $L_2^2 = L_1^2$ and $9L_{1p}{}^2\left(1 - \frac{v_2{}^2}{c^2}\right) = L_{1p}{}^2\left[1 - (0.350)^2\right]$

giving $9 - 9\frac{v_2^2}{c^2} = 0.878,$ or $v_2 = \boxed{0.950c}$

***39.9** $\Delta t = \gamma \Delta t_p = \frac{\Delta t_p}{\sqrt{1 - v^2 / c^2}}$ so $\Delta t_p = \left(\sqrt{1 - v^2 / c^2}\right)\Delta t \approx \left(1 - \frac{v^2}{2c^2}\right)\Delta t$ and $\Delta t - \Delta t_p = \left(\frac{v^2}{2c^2}\right)\Delta t$

If $v = 1000\ \text{km/h} = \frac{1.00 \times 10^6\ \text{m}}{3600\ \text{s}} = 277.8\ \text{m/s},$ then $\frac{v}{c} = 9.26 \times 10^{-7}$

and $\left(\Delta t - \Delta t_p\right) = (4.28 \times 10^{-13})(3600\ \text{s}) = 1.54 \times 10^{-9}\ \text{s} = \boxed{1.54\ \text{ns}}$

39.10 $\gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - (0.950)^2} = 0.312$

(a) astronauts' time: $\Delta t_p = \gamma^{-1}\Delta t = (0.312)(4.42\ \text{yr}) = \boxed{1.38\ \text{yr}}$

(b) astronauts' distance: $L = \gamma^{-1}\Delta L_p = (0.312)(4.20\ \text{ly}) = \boxed{1.31\ \text{ly}}$

39.11 The spaceship appears length-contracted to the Earth observer as given by

$L = L_p\sqrt{1 - v^2/c^2}$ or $L^2 = L_p{}^2\left(1 - v^2/c^2\right)$

Also, the contracted length is related to the time required to pass overhead by:

$L = vt$ or $L^2 = v^2t^2 = \frac{v^2}{c^2}(ct)^2$

Equating these two expressions gives $L_p{}^2 - L_p{}^2\frac{v^2}{c^2} = (ct)^2\frac{v^2}{c^2}$ or $\left[L_p{}^2 + (ct)^2\right]\frac{v^2}{c^2} = L_p{}^2$

Using the given values: $L_p = 300\ \text{m}$ and $t = 7.50 \times 10^{-7}\ \text{s}$

this becomes $(1.41 \times 10^5\ \text{m}^2)\frac{v^2}{c^2} = 9.00 \times 10^4\ \text{m}^2$

giving $v = \boxed{0.800c}$

Goal Solution

A spaceship with a proper length of 300 m takes 0.750 μs seconds to pass an Earth observer. Determine its speed as measured by the Earth observer.

G: We should first determine if the spaceship is traveling at a relativistic speed: classically, $v = (300\text{m})/(0.750\ \mu\text{s}) = 4.00 \times 10^8$ m/s, which is faster than the speed of light (impossible)! Quite clearly, the relativistic correction must be used to find the correct speed of the spaceship, which we can guess will be close to the speed of light.

O: We can use the contracted length equation to find the speed of the spaceship in terms of the proper length and the time. The time of 0.750 μs is the **proper time** measured by the Earth observer, because it is the time interval between two events that she sees as happening at the same point in space. The two events are the passage of the front end of the spaceship over her stopwatch, and the passage of the back end of the ship.

A: $L = L_p / \gamma$, with $L = v\Delta t$: $$v\Delta t = L_p\left(1 - v^2 / c^2\right)^{1/2}$$

Squaring both sides, $$v^2\Delta t^2 = L_p{}^2\left(1 - v^2 / c^2\right)$$

$$v^2 c^2 = L_p{}^2 c^2 / \Delta t^2 - v^2 L_p{}^2 / \Delta t^2$$

Solving for the velocity, $$v = \frac{c L_p / \Delta t}{\sqrt{c^2 + L_p{}^2 / \Delta t^2}}$$

So $$v = \frac{\left(3.00 \times 10^8\right)(300\ \text{m})\Big/\left(0.750 \times 10^{-6}\ \text{s}\right)}{\sqrt{\left(3.00 \times 10^8\right)^2 + (300\ \text{m})^2\Big/\left(0.750 \times 10^{-6}\ \text{s}\right)^2}} = 2.40 \times 10^8\ \text{m / s}$$

L: The spaceship is traveling at 0.8c. We can also verify that the general equation for the speed reduces to the classical relation $v = L_p / \Delta t$ when the time is relatively large.

39.12 The spaceship appears to be of length L to Earth observers,

where $$L = L_p\left(1 - \frac{v^2}{c^2}\right)^{1/2} \quad \text{and} \quad L = vt$$

$$vt = L_p\left(1 - \frac{v^2}{c^2}\right)^{1/2} \quad \text{so} \quad v^2 t^2 = L_p{}^2\left(1 - \frac{v^2}{c^2}\right)$$

Solving for v, $$v^2\left(t^2 + \frac{L_p{}^2}{c^2}\right) = L_p{}^2 \qquad \boxed{\frac{v}{c} = L_p\left(c^2 t^2 + L_p{}^2\right)^{-1/2}}$$

***39.13** For $\frac{v}{c} = 0.990$, $\gamma = 7.09$

(a) The muon's lifetime as measured in the Earth's rest frame is $\Delta t = \frac{4.60 \text{ km}}{0.990c}$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}\left[\frac{4.60 \times 10^3 \text{ m}}{0.990\left(3.00 \times 10^8 \text{ m/s}\right)}\right] = \boxed{2.18\ \mu\text{s}}$$

(b) $L = L_p\sqrt{1-(v/c)^2} = \frac{L_p}{\gamma} = \frac{4.60 \times 10^3 \text{ m}}{7.09} = \boxed{649 \text{ m}}$

39.14 We find Carpenter's speed: $\frac{GMm}{r^2} = \frac{mv^2}{r}$

$$v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.37 \times 10^6 + 0.160 \times 10^6)}\right]^{1/2} = 7.82 \text{ km / s}$$

Then the time period of one orbit, $T = \frac{2\pi(R+h)}{v} = \frac{2\pi(6.53 \times 10^6)}{7.82 \times 10^3} = 5.25 \times 10^3 \text{ s}$

(a) The time difference for 22 orbits is $\Delta t - \Delta t_p = (\gamma - 1)\Delta t_p = \left[\left(1 - v^2/c^2\right)^{-1/2} - 1\right](22T)$

$$\Delta t - \Delta t_p \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)(22T) = \frac{1}{2}\left(\frac{7.82 \times 10^3 \text{ m / s}}{3 \times 10^8 \text{ m / s}}\right)^2 22\left(5.25 \times 10^3 \text{ s}\right) = \boxed{39.2\ \mu\text{s}}$$

(b) For one orbit, $\Delta t - \Delta t_p = \frac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$. The press report is $\boxed{\text{accurate to one digit}}$.

39.15 For pion to travel 10.0 m in Δt in our frame, $10.0 \text{ m} = v\Delta t = v(\gamma\Delta t_p) = \frac{v(26.0 \times 10^{-9} \text{ s})}{\sqrt{1 - v^2/c^2}}$

Solving for the velocity, $(3.85 \times 10^8 \text{ m / s})^2(1 - v^2/c^2) = v^2$

$$1.48 \times 10^{17} \text{ m}^2/\text{s}^2 = v^2(1 + 1.64)$$

$$v = 2.37 \times 10^8 \text{ m / s} = \boxed{0.789c}$$

***39.16** $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.01$ so $\boxed{v = 0.140c}$

***39.17** (a) Since your ship is identical to his, and you are at rest with respect to your own ship, its length is $\boxed{20.0\text{ m}}$.

(b) His ship is in motion relative to you, so you see its length contracted to $\boxed{19.0\text{ m}}$.

(c) We have $L = L_p\sqrt{1 - v^2/c^2}$

from which $\dfrac{L}{L_p} = \dfrac{19.0\text{ m}}{20.0\text{ m}} = 0.950 = \sqrt{1 - \dfrac{v^2}{c^2}}$ and $\boxed{v = 0.312c}$

***39.18** (a) $\Delta t = \gamma \Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \dfrac{15.0\text{ yr}}{\sqrt{1-(0.700)^2}} = \boxed{21.0\text{ yr}}$

(b) $d = v(\Delta t) = [0.700c](21.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](21.0\text{ yr}) = \boxed{14.7\text{ ly}}$

(c) The astronauts see Earth flying out the back window at $0.700c$:

$d = v(\Delta t_p) = [0.700c](15.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](15.0\text{ yr}) = \boxed{10.5\text{ ly}}$

(d) Mission control gets signals for 21.0 yr while the battery is operating, and then for 14.7 years after the battery stops powering the transmitter, 14.7 ly away: $21.0\text{ yr} + 14.7\text{ yr} = \boxed{35.7\text{ yr}}$

***39.19** The orbital speed of the Earth is as described by $\Sigma F = ma$: $\dfrac{Gm_S m_E}{r^2} = \dfrac{m_E v^2}{r}$

$$v = \sqrt{\frac{Gm_S}{r}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}{1.496\times10^{11}\text{ m}}} = 2.98\times10^4\text{ m s}$$

The maximum frequency received by the extraterrestrials is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1+v/c}{1-v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}} = 57.005\,66\times10^6\text{ Hz}$$

The minimum frequency received is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1-v/c}{1+v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}} = 56.994\,34\times10^6\text{ Hz}$$

The difference, which lets them figure out the speed of our planet, is

$$(57.005\,66 - 56.994\,34)\times10^6\text{ Hz} = \boxed{1.13\times10^4\text{ Hz}}$$

39.20 (a) Let f_c be the frequency as seen by the car. Thus, $f_c = f_{\text{source}}\sqrt{\dfrac{c+v}{c-v}}$

and, if f is the frequency of the reflected wave, $f = f_c\sqrt{\dfrac{c+v}{c-v}}$

Combining gives $\boxed{f = f_{\text{source}}\dfrac{(c+v)}{(c-v)}}$

(b) Using the above result, $f(c-v) = f_{\text{source}}(c+v)$

which gives $(f - f_{\text{source}})c = (f + f_{\text{source}})v \approx 2f_{\text{source}}v$

The beat frequency is then $f_b = f - f_{\text{source}} = \dfrac{2f_{\text{source}}v}{c} = \boxed{\dfrac{2v}{\lambda}}$

(c) $$f_b = \frac{(2)(30.0\ \text{m/s})(10.0\times10^9\ \text{Hz})}{3.00\times10^8\ \text{m/s}} = \frac{(2)(30.0\ \text{m/s})}{(0.0300\ \text{m})} = 2000\ \text{Hz} = \boxed{2.00\ \text{kHz}}$$

$$\lambda = \frac{c}{f_{\text{source}}} = \frac{3.00\times10^8\ \text{m/s}}{10.0\times10^9\ \text{Hz}} = 3.00\ \text{cm}$$

(d) $v = \dfrac{f_b\lambda}{2}$ so $\Delta v = \dfrac{\Delta f_b\lambda}{2} = \dfrac{(5\ \text{Hz})(0.0300\ \text{m})}{2} = \boxed{0.0750\ \text{m/s} \approx 0.2\ \text{mi / h}}$

39.21 (a) When the source moves away from an observer, the observed frequency is

$$f_{\text{obs}} = f_{\text{source}}\left(\frac{c-v_s}{c+v_s}\right)^{1/2} \qquad \text{where} \qquad v_s = v_{\text{source}}$$

When $v_s << c$, the binomial expansion gives

$$\left(\frac{c-v_s}{c+v_s}\right)^{1/2} = \left[1-\left(\frac{v_s}{c}\right)\right]^{1/2}\left[1+\left(\frac{v_s}{c}\right)\right]^{-1/2} \approx \left(1-\frac{v_s}{2c}\right)\left(1-\frac{v_s}{2c}\right) \approx \left(1-\frac{v_s}{c}\right)$$

So, $$f_{\text{obs}} \approx f_{\text{source}}\left(1-\frac{v_s}{c}\right)$$

The observed wavelength is found from $c = \lambda_{\text{obs}}f_{\text{obs}} = \lambda f_{\text{source}}$:

$$\lambda_{\text{obs}} = \frac{\lambda f_{\text{source}}}{f_{\text{obs}}} \approx \frac{\lambda f_{\text{source}}}{f_{\text{source}}(1-v_s/c)} = \frac{\lambda}{1-v_s/c}$$

$$\Delta\lambda = \lambda_{\text{obs}} - \lambda = \lambda\left(\frac{1}{1-v_s/c}-1\right) = \lambda\left(\frac{1}{1-v_s/c}-1\right) = \lambda\left(\frac{v_s/c}{1-v_s/c}\right)$$

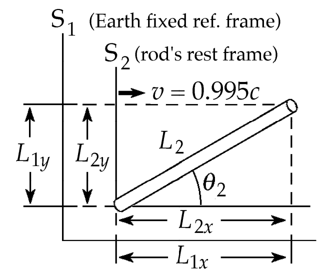
Since $1 - v_s/c \approx 1$, $\boxed{\dfrac{\Delta\lambda}{\lambda} \approx \dfrac{v_{\text{source}}}{c}}$

(b) $$v_{\text{source}} = c\left(\frac{\Delta\lambda}{\lambda}\right) = c\left(\frac{20.0\ \text{nm}}{397\ \text{nm}}\right) = \boxed{0.050\,4c}$$

39.22 $$u'_x = \frac{u_x - v}{1 - u_x v / c^2} = \frac{0.950c - 0.750c}{1 - 0.950 \times 0.750} = \boxed{0.696c}$$

39.23 $$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{-0.750c - 0.750c}{1 - (-0.750)(0.750)} = \boxed{-0.960c}$$

***39.24** $\gamma = 10.0$ We are also given: $L_1 = 2.00$ m, and $\theta_1 = 30.0°$ (both measured in a reference frame moving relative to the rod).



Thus, $L_{1x} = L_1 \cos\theta_1 = (2.00 \text{ m})(0.867) = 1.73$ m

and $L_{1y} = L_1 \sin\theta_1 = (2.00 \text{ m})(0.500) = 1.00$ m

L_{2x} = a "proper length" is related to L_{1x}

by $L_{1x} = L_{2x}/\gamma$

Therefore, $L_{2x} = 10.0 L_{1x} = 17.3$ m and $L_{2y} = L_{1y} = 1.00$ m

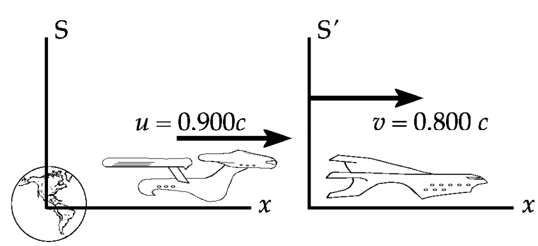
(Lengths perpendicular to the motion are unchanged).

(a) $L_2 = \sqrt{(L_{2x})^2 + (L_{2y})^2}$ gives $\boxed{L_2 = 17.4 \text{ m}}$

(b) $\theta_2 = \tan^{-1}\dfrac{L_{2y}}{L_{2x}}$ gives $\boxed{\theta_2 = 3.30°}$

39.25 u_x = Enterprise velocity

v = Klingon velocity



From Equation 39.16,

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{0.900c - 0.800c}{1 - (0.900)(0.800)} = \boxed{0.357c}$$

***39.26** (a) From Equation 39.13, $\Delta x' = \gamma(\Delta x - v\,\Delta t)$,

$$0 = \gamma\left[2.00\text{ m} - v\left(8.00\times10^{-9}\text{ s}\right)\right]$$

$$v = \frac{2.00\text{ m}}{8.00\times10^{-9}\text{ s}} = \boxed{2.50\times10^{8}\text{ m/s}}$$

$$\gamma = \frac{1}{\sqrt{1-\left(2.50\times10^{8}\text{ m/s}\right)^2 / \left(3.00\times10^{8}\text{ m/s}\right)^2}} = 1.81$$

(b) From Equation 39.11, $x' = \gamma(x - vt) = 1.81\left[3.00\text{ m} - \left(2.50\times10^{8}\text{ m/s}\right)\left(1.00\times10^{-9}\text{ s}\right)\right] = \boxed{4.97\text{ m}}$

(c)
$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = 1.81\left[1.00\times10^{-9}\text{ s} - \frac{\left(2.50\times10^{8}\text{ m/s}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)^2}(3.00\text{ m})\right]$$

$$t' = \boxed{-1.33\times10^{-8}\text{ s}}$$

39.27 $p = \gamma m u$

(a) For an electron moving at $0.0100c$, $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = \dfrac{1}{\sqrt{1-(0.0100)^2}} = 1.00005 \approx 1.00$

Thus, $p = 1.00\left(9.11\times10^{-31}\text{ kg}\right)(0.0100)\left(3.00\times10^{8}\text{ m / s}\right) = \boxed{2.73\times10^{-24}\text{ kg}\cdot\text{m/s}}$

(b) Following the same steps as used in part (a), we find at $0.500c$

$\gamma = 1.15$ and $p = \boxed{1.58\times10^{-22}\text{ kg}\cdot\text{m/s}}$

(c) At $0.900c$, $\gamma = 2.29$ and $p = \boxed{5.64\times10^{-22}\text{ kg}\cdot\text{m/s}}$

***39.28** Using the relativistic form, $p = \dfrac{mu}{\sqrt{1-(u/c)^2}} = \gamma m u$,

we find the difference Δp from the classical momentum, mu: $\Delta p = \gamma m u - m u = (\gamma - 1)mu$

(a) The difference is 1.00% when $(\gamma-1)mu = 0.0100\,\gamma m u$:

$$\gamma = \frac{1}{0.990} = \frac{1}{\sqrt{1-(u/c)^2}} \;\Rightarrow\; 1-(u/c)^2 = (0.990)^2 \qquad \text{or} \qquad u = \boxed{0.141c}$$

(b) The difference is 10.0% when $(\gamma-1)mu = 0.100\,\gamma m u$:

$$\gamma = \frac{1}{0.900} = \frac{1}{\sqrt{1-(u/c)^2}} \;\Rightarrow\; 1-(u/c)^2 = (0.900)^2 \qquad \text{or} \qquad u = \boxed{0.436c}$$

***39.29** $\dfrac{p - mu}{mu} = \dfrac{\gamma mu - mu}{mu} = \gamma - 1$

$$\gamma - 1 = \frac{1}{\sqrt{1 - (u/c)^2}} - 1 \approx 1 + \frac{1}{2}\left(\frac{u}{c}\right)^2 - 1 = \frac{1}{2}\left(\frac{u}{c}\right)^2$$

$$\frac{p - mu}{mu} = \frac{1}{2}\left(\frac{90.0 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2 = \boxed{4.50 \times 10^{-14}}$$

39.30 $p = \dfrac{mu}{\sqrt{1 - (u/c)^2}}$ becomes $1 - \dfrac{u^2}{c^2} = \dfrac{m^2u^2}{p^2}$

which gives: $1 = u^2\left(\dfrac{m^2}{p^2} + \dfrac{1}{c^2}\right)$

or $c^2 = u^2\left(\dfrac{m^2c^2}{p^2} + 1\right)$ and $\boxed{u = \dfrac{c}{\sqrt{\dfrac{m^2c^2}{p^2} + 1}}}$

***39.31** Relativistic momentum must be conserved:

For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2 = p_1$

or $\gamma_2 m_2 u_2 = \gamma_1 m_1 u_1 = \dfrac{2.50 \times 10^{-28} \text{ kg}}{\sqrt{1 - (0.893)^2}} \times (0.893c)$

or $\dfrac{(1.67 \times 10^{-27} \text{ kg})u_2}{\sqrt{1 - (u_2/c)^2}} = (4.960 \times 10^{-28} \text{ kg})c$

and $u_2 = \boxed{0.285c}$

Goal Solution

An unstable particle at rest breaks into two fragments of **unequal** mass. The rest mass of the lighter fragment is 2.50×10^{-28} kg, and that of the heavier fragment is 1.67×10^{-27} kg. If the lighter fragment has a speed of $0.893c$ after the breakup, what is the speed of the heavier fragment?

G: The heavier fragment should have a speed less than that of the lighter piece since the momentum of the system must be conserved. However, due to the relativistic factor, the ratio of the speeds will not equal the simple ratio of the particle masses, which would give a speed of $0.134c$ for the heavier particle.

O: Relativistic momentum of the system must be conserved. For the total momentum to be zero after the fission, as it was before, $\mathbf{p}_1 + \mathbf{p}_2 = 0$, where we will refer to the lighter particle with the subscript '1', and to the heavier particle with the subscript '2.'

A: $\gamma_2 m_2 v_2 + \gamma_1 m_1 v_1 = 0$ so $\gamma_2 m_2 v_2 + \left(\dfrac{2.50 \times 10^{-28}\text{ kg}}{\sqrt{1 - 0.893^2}}\right)(0.893c) = 0$

Rearranging, $\left(\dfrac{1.67 \times 10^{-27}\text{ kg}}{\sqrt{1 - v_2^{\,2}/c^2}}\right)\dfrac{v_2}{c} = -4.96 \times 10^{-28}\text{ kg}$

Squaring both sides, $\left(2.79 \times 10^{-54}\right)\left(\dfrac{v_2}{c}\right)^2 = \left(2.46 \times 10^{-55}\right)\left(1 - \dfrac{v_2^{\,2}}{c^2}\right)$ and $v_2 = -0.285c$

We choose the negative sign only to mean that the two particles must move in opposite directions. The speed, then, is $|v_2| = 0.285c$

L: The speed of the heavier particle is less than the lighter particle, as expected. We can also see that for this situation, the relativistic speed of the heavier particle is about twice as great as was predicted by a simple non-relativistic calculation.

39.32 $\Delta E = (\gamma_1 - \gamma_2)mc^2$. For an electron, $mc^2 = 0.511$ MeV.

(a) $\Delta E = \left(\sqrt{\dfrac{1}{(1 - 0.810)}} - \sqrt{\dfrac{1}{(1 - 0.250)}}\right)mc^2 = \boxed{0.582\text{ MeV}}$

(b) $\Delta E = \left(\sqrt{\dfrac{1}{1 - (0.990)^2}} - \sqrt{\dfrac{1}{1 - 0.810}}\right)mc^2 = \boxed{2.45\text{ MeV}}$

39.33 $E = \gamma mc^2 = 2mc^2$, or $\gamma = 2$

Thus, $\dfrac{u}{c} = \sqrt{1 - (1/\gamma)^2} = \dfrac{\sqrt{3}}{2}$, or $u = \dfrac{c\sqrt{3}}{2}$.

The momentum is then $p = \gamma mu = 2m\left(\dfrac{c\sqrt{3}}{2}\right) = \left(\dfrac{mc^2}{c}\right)\sqrt{3} = \left(\dfrac{938.3\text{ MeV}}{c}\right)\sqrt{3} = \boxed{1.63 \times 10^3\ \dfrac{\text{MeV}}{c}}$

***39.34** The relativistic kinetic energy of an object of mass m and speed u is $K_r = \left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$

For $u = 0.100c$, $K_r = \left(\frac{1}{\sqrt{1-0.0100}} - 1\right)mc^2 = 0.005038\,mc^2$

The classical equation $K_c = \frac{1}{2}mu^2$ gives $K_c = \frac{1}{2}m(0.100c)^2 = 0.005000\,mc^2$

different by $\frac{0.005038 - 0.005000}{0.005038} = 0.751\%$

For still smaller speeds the agreement will be still better.

39.35 (a) $E_R = mc^2 = (1.67\times10^{-27}\text{ kg})(2.998\times10^8\text{ m/s})^2 = 1.50\times10^{-10}\text{ J} = \boxed{938\text{ MeV}}$

(b) $E = \gamma mc^2 = \frac{1.50\times10^{-10}\text{ J}}{[1-(0.95c/c)^2]^{1/2}} = 4.81\times10^{-10}\text{ J} = \boxed{3.00\times10^3\text{ MeV}}$

(c) $K = E - mc^2 = 4.81\times10^{-10}\text{ J} - 1.50\times10^{-10}\text{ J} = 3.31\times10^{-10}\text{ J} = \boxed{2.07\times10^3\text{ MeV}}$

***39.36** (a) $KE = E - E_R = 5E_R$

$E = 6E_R = 6(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})^2 = 4.92\times10^{-13}\text{ J} = \boxed{3.07\text{ MeV}}$

(b) $E = \gamma mc^2 = \gamma E_R$

Thus, $\gamma = \frac{E}{E_r} = 6 = \frac{1}{\sqrt{1-u^2/c^2}}$ which yields $\boxed{u = 0.986c}$

39.37 The relativistic density is

$$\frac{E_R}{c^2 V} = \frac{mc^2}{c^2 V} = \frac{m}{V} = \frac{m}{(L_p)(L_p)\left[L_p\sqrt{1-(u/c)^2}\right]} = \frac{8.00\text{ g}}{(1.00\text{ cm})^3\sqrt{1-(0.900)^2}} = \boxed{18.4\text{ g/cm}^3}$$

***39.38** We must conserve both mass-energy and relativistic momentum. With subscript 1 referring to the $0.868c$ particle and subscript 2 to the $0.987c$ particle,

$$\gamma_1 = \frac{1}{\sqrt{1-(0.868)^2}} = 2.01 \qquad \text{and} \qquad \gamma_2 = \frac{1}{\sqrt{1-(0.987)^2}} = 6.22$$

Conservation of mass-energy gives $E_1 + E_2 = E_{\text{total}}$ which is $\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_{\text{total}} c^2$

or $2.01 m_1 + 6.22 m_2 = 3.34\times10^{-27}\ \text{kg}$

This reduces to: $m_1 + 3.09 m_2 = 1.66\times10^{-27}\ \text{kg}$ [1]

Since the momentum after must equal zero, $p_1 = p_2$ gives $\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2$

or $(2.01)(0.868c)m_1 = (6.22)(0.987c)m_2$

which becomes $m_1 = 3.52 m_2$ [2]

Solving [1] and [2] simultaneously, $m_1 = \boxed{8.84\times10^{-28}\ \text{kg}}$ and $m_2 = \boxed{2.51\times10^{-28}\ \text{kg}}$

39.39 $E = \gamma mc^2$, $p = \gamma mu$; $E^2 = (\gamma mc^2)^2$; $p^2 = (\gamma mu)^2$;

$$E^2 - p^2c^2 = (\gamma mc^2)^2 - (\gamma mu)^2c^2 = \gamma^2\left((mc^2)^2 - (mc)^2u^2\right) = (mc^2)^2\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-1} = (mc^2)^2 \quad \text{Q.E.D.}$$

39.40 (a) $K = 50.0\ \text{GeV}$

$$mc^2 = \left(1.67\times10^{-27}\ \text{kg}\right)\left(2.998\times10^8\ \text{m/s}\right)^2\left(\frac{1}{1.60\times10^{-10}\ \text{J/GeV}}\right) = 0.938\ \text{GeV}$$

$$E = K + mc^2 = 50.0\ \text{GeV} + 0.938\ \text{GeV} = 50.938\ \text{GeV}$$

$$E^2 = p^2c^2 + \left(mc^2\right)^2 \quad \Rightarrow \quad p = \sqrt{\frac{E^2 - \left(mc^2\right)^2}{c^2}} = \sqrt{\frac{(50.938\ \text{GeV})^2 - (0.938\ \text{GeV})^2}{c^2}}$$

$$p = 50.9\ \frac{\text{GeV}}{c} = \left(\frac{50.9\ \text{GeV}}{3.00\times10^8\ \text{m/s}}\right)\left(\frac{1.60\times10^{-10}\ \text{J}}{1\ \text{GeV}}\right) = \boxed{2.72\times10^{-17}\ \text{kg}\cdot\text{m/s}}$$

(b) $$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-(u/c)^2}} \quad \Rightarrow \quad u = c\sqrt{1-\left(mc^2/E\right)^2}$$

$$v = \left(3.00\times10^8\ \text{m/s}\right)\sqrt{1-\left(\frac{0.938\ \text{GeV}}{50.938\ \text{GeV}}\right)^2} = \boxed{2.9995\times10^8\ \text{m/s}}$$

39.41 (a) $q(\Delta V) = K = (\gamma - 1)m_e c^2$

Thus, $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = 1 + \dfrac{q(\Delta V)}{m_e c^2}$ from which $\boxed{u = 0.302c}$

(b) $K = (\gamma - 1)m_e c^2 = q(\Delta V) = \left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.50 \times 10^4\ \text{J/C}\right) = \boxed{4.00 \times 10^{-15}\ \text{J}}$

39.42 (a) $E = \gamma mc^2 = 20.0\ \text{GeV}$ with $mc^2 = 0.511\ \text{MeV}$ for electrons. Thus, $\gamma = \dfrac{20.0 \times 10^9\ \text{eV}}{0.511 \times 10^6\ \text{eV}} = \boxed{3.91 \times 10^4}$

(b) $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = 3.91 \times 10^4$ from which $\boxed{u = 0.999\,999\,999\,7c}$

(c) $L = L_p\sqrt{1-(u/c)^2} = \dfrac{L_p}{\gamma} = \dfrac{3.00 \times 10^3\ \text{m}}{3.91 \times 10^4} = 7.67 \times 10^{-2}\ \text{m} = \boxed{7.67\ \text{cm}}$

39.43 Conserving total momentum, $p_{\text{Before decay}} = p_{\text{after decay}} = 0$: $\quad p_\nu = p_\mu = \gamma m_\mu u = \gamma(206\, m_e)u$

Conservation of mass-energy gives: $\quad E_\mu + E_\nu = E_\pi$

$$\gamma m_\mu c^2 + p_\nu c = m_\pi c^2$$

$$\gamma(206\, m_e) + \frac{p_\nu}{c} = 270\, m_e$$

Substituting from the momentum equation above, $\quad \gamma(206\, m_e) + \gamma(206\, m_e)\dfrac{u}{c} = 270\, m_e$

or $\quad \gamma\left(1 + \dfrac{u}{c}\right) = \dfrac{270}{206} = 1.31 \quad \Rightarrow \quad \dfrac{u}{c} = 0.264$

Then, $K_\mu = (\gamma - 1)m_\mu c^2 = (\gamma - 1)206\left(m_e c^2\right) = \left(\dfrac{1}{\sqrt{1-(0.264)^2}} - 1\right)206(0.511\ \text{MeV}) = \boxed{3.88\ \text{MeV}}$

Also, $E_\nu = E_\pi - E_\mu = m_\pi c^2 - \gamma m_\mu c^2 = (270 - 206\gamma)m_e c^2$

$$E_\nu = \left(270 - \frac{206}{\sqrt{1-(0.264)^2}}\right)(0.511\ \text{MeV}) = \boxed{28.8\ \text{MeV}}$$

***39.44** Let a 0.3-kg flag be run up a flagpole 7 m high.

We put into it energy $\quad mgh = 0.3\ \text{kg}(9.8\ \text{m/s}^2)\ 7\ \text{m} \approx 20\ \text{J}$

So we put into it extra mass $\Delta m = \frac{E}{c^2} = \frac{20 \text{ J}}{(3\times10^8 \text{ m/s})^2} = 2\times10^{-16}\text{ kg}$

for a fractional increase of $\frac{2\times10^{16}\text{ kg}}{0.3\text{ kg}}\ \boxed{\sim10^{-15}}$

***39.45** $E = 2.86\times10^5$ J. Also, the mass-energy relation says that $E = mc^2$.

Therefore, $m = \frac{E}{c^2} = \frac{2.86\times10^5\text{ J}}{(3.00\times10^8\text{ m / s})^2} = \boxed{3.18\times10^{-12}\text{ kg}}$

No, a mass loss of this magnitude (out of a total of 9.00 g) $\boxed{\text{could not be detected}}$.

39.46 (a) $K = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-u^2/c^2}}-1\right)mc^2 = 0.25mc^2 = \boxed{2.25\times10^{22}\text{ J}}$

(b) $E = m_{\text{fuel}}c^2$ so $m_{\text{fuel}} = \frac{2.25\times10^{22}}{9.00\times10^{16}} = \boxed{2.50\times10^5\text{ kg}}$

39.47 $\Delta m = \frac{E}{c^2} = \frac{\mathcal{P}t}{c^2} = \frac{0.800(1.00\times10^9\text{ J/s})(3.00\text{ yr})(3.16\times10^7\text{ s/yr})}{(3.00\times10^8\text{ m/s})^2} = \boxed{0.842\text{ kg}}$

39.48 Since the total momentum is zero before decay, it is necessary that after the decay

$$p_{\text{nucleus}} = p_{\text{photon}} = \frac{E_\gamma}{c} = \frac{14.0\text{ keV}}{c}$$

Also, for the recoiling nucleus, $E^2 = p^2c^2 + (mc^2)^2$ with $mc^2 = 8.60\times10^{-9}\text{ J} = 53.8\text{ GeV}$

Thus, $(mc^2+K)^2 = (14.0\text{ keV})^2 + (mc^2)^2$ or $\left(1+\frac{K}{mc^2}\right)^2 = \left(\frac{14.0\text{ keV}}{mc^2}\right)^2 + 1$

So $1+\frac{K}{mc^2} = \sqrt{1+\left(\frac{14.0\text{ keV}}{mc^2}\right)^2} \approx 1+\frac{1}{2}\left(\frac{14.0\text{ keV}}{mc^2}\right)^2$ (Binomial Theorem)

and $K \approx \frac{(14.0\text{ keV})^2}{2mc^2} = \frac{(14.0\times10^3\text{ eV})^2}{2(53.8\times10^9\text{ eV})} = \boxed{1.82\times10^{-3}\text{ eV}}$

39.49 $\mathcal{P} = \frac{dE}{dt} = \frac{d(mc^2)}{dt} = c^2\frac{dm}{dt} = 3.77\times10^{26}\text{ W}$

Thus, $\dfrac{dm}{dt} = \dfrac{3.77 \times 10^{26}\ \text{J/s}}{\left(3.00 \times 10^{8}\ \text{m/s}\right)^2} = \boxed{4.19 \times 10^{9}\ \text{kg/s}}$

39.50 $2m_e c^2 = 1.02$ MeV: $E_\gamma \geq \boxed{1.02\ \text{MeV}}$

39.51 The moving observer sees the charge as stationary, so she says it feels no magnetic force.

$q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = q(\mathbf{E}' + 0)$ and $\boxed{\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}}$

***39.52** (a) When $K_e = K_p$, $m_e c^2 \left(\gamma_e - 1\right) = m_p c^2 \left(\gamma_p - 1\right)$

In this case, $m_e c^2 = 0.511$ MeV, $m_p c^2 = 938$ MeV and $\gamma_e = \left[1 - (0.750)^2\right]^{-1/2} = 1.5119$

Substituting, $\gamma_p = 1 + \dfrac{m_e c^2 (\gamma_e - 1)}{m_p c^2} = 1 + \dfrac{(0.511\ \text{MeV})(1.5119 - 1)}{938\ \text{MeV}} = 1.000279$

but $\gamma_p = \dfrac{1}{\left[1 - \left(u_p/c\right)^2\right]^{1/2}}$. Therefore, $u_p = c\sqrt{1 - \gamma_p^{-2}} = \boxed{0.0236c}$

(b) When $p_e = p_p$, $\gamma_p m_p u_p = \gamma_e m_e u_e$ or $\gamma_p u_p = \dfrac{\gamma_e m_e u_e}{m_p}$.

Thus, $\gamma_p u_p = \dfrac{(1.5119)\left(0.511\ \text{MeV}/c^2\right)(0.750c)}{938\ \text{MeV}/c^2} = 6.1772 \times 10^{-4}\ c$

and $\dfrac{u_p}{c} = 6.1772 \times 10^{-4} \sqrt{1 - \left(\dfrac{u_p}{c}\right)^2}$ which yields $u_p = \boxed{6.18 \times 10^{-4}\ c} = 185\ \text{km/s}$

39.53 (a) $10^{13}\ \text{MeV} = (\gamma - 1) m_p c^2$ so $\gamma \approx 10^{10}$ $v_p \approx c$

$t' = \dfrac{t}{\gamma} = \dfrac{10^5\ \text{yr}}{10^{10}} = 10^{-5}\ \text{yr} \sim \boxed{10^2\ \text{s}}$

(b) $d' = ct'$ $\boxed{\sim 10^{11}\ \text{m}}$

Goal Solution

The cosmic rays of highest energy are protons, which have kinetic energy on the order of 10^{13} MeV. (a) How long would it take a proton of this energy to travel across the Milky Way galaxy, having a diameter on the order of ~10^5 light-years, as measured in the proton's frame? (b) From the point of view of the proton, how many kilometers across is the galaxy?

G: We can guess that the energetic cosmic rays will be traveling close to the speed of light, so the time it takes a proton to traverse the Milky Way will be much less in the proton's frame than 10^5 years. The galaxy will also appear smaller to the high-speed protons than the galaxy's proper diameter of 10^5 light-years.

O: The kinetic energy of the protons can be used to determine the relativistic γ-factor, which can then be applied to the time dilation and length contraction equations to find the time and distance in the proton's frame of reference.

A: The relativistic kinetic energy of a proton is $K = (\gamma - 1)mc^2 = 10^{13}$ MeV

Its rest energy is $$mc^2 = \left(1.67\times10^{-27}\text{ kg}\right)\left(2.998\times10^8\ \frac{\text{m}}{\text{s}}\right)^2\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ kg}\cdot\text{m}^2/\text{s}^2}\right) = 938\text{ MeV}$$

So 10^{13} MeV $= (\gamma - 1)(938\text{ MeV})$, and therefore $\gamma = 1.07\times10^{10}$

The proton's speed in the galaxy's reference frame can be found from $\gamma = 1\big/\sqrt{1 - v^2/c^2}$:

$$1 - v^2/c^2 = 8.80\times10^{-21} \quad\text{and}\quad v = c\sqrt{1 - 8.80\times10^{-21}} = \left(1 - 4.40\times10^{-21}\right)c \approx 3.00\times10^8\text{ m/s}$$

The proton's speed is nearly as large as the speed of light. In the galaxy frame, the traversal time is $\Delta t = x/v = 10^5\text{ light-years}/c = 10^5\text{ years}$

(a) This is dilated from the proper time measured in the proton's frame. The proper time is found from $\Delta t = \gamma\Delta t_p$:

$$\Delta t_p = \Delta t/\gamma = 10^5\text{ yr}\big/1.07\times10^{10} = 9.38\times10^{-6}\text{ years} = 296\text{ s} \sim \text{a few hundred seconds}$$

(b) The proton sees the galaxy moving by at a speed nearly equal to c, passing in 296 s:

$$\Delta L_p = v\Delta t_p = \left(3.00\times10^8\right)(296\text{ s}) = 8.88\times10^7\text{ km} \sim 10^8\text{ km}$$

$$\Delta L_p = \left(8.88\times10^{10}\text{ m}\right)\left(9.46\times10^{15}\text{ m/ly}\right) = 9.39\times10^{-6}\text{ ly} \sim 10^{-5}\text{ ly}$$

L: The results agree with our predictions, although we may not have guessed that the protons would be traveling so close to the speed of light! The calculated results should be rounded to zero significant figures since we were given order of magnitude data. We should also note that the relative speed of motion v and the value of γ are the same in both the proton and galaxy reference frames.

39.54 Take the primed frame as:

(a) The mother ship: $u_x = \dfrac{u'_{x'} + v}{1 + u'_{x'}v/c^2} = \dfrac{v+v}{1+v^2/c^2} = \dfrac{2v}{1+v^2/c^2} = \dfrac{2(0.500c)}{1+(0.500)^2} = \boxed{0.800c}$

(b) The shuttle: $u_x = \dfrac{v + \dfrac{2v}{1+v^2/c^2}}{1 + \dfrac{v}{c^2}\left(\dfrac{2v}{1+v^2/c^2}\right)} = \dfrac{3v + v^3/c^2}{1+3v^2/c^2} = \dfrac{3(0.500c)+(0.500c)^3/c^2}{1+3(0.500)^2} = \boxed{0.929c}$

39.55 $\dfrac{\Delta mc^2}{mc^2} = \dfrac{4(938.78\text{ MeV}) - 3728.4\text{ MeV}}{4(938.78\text{ MeV})} \times 100\% = \boxed{0.712\%}$

39.56 $d_{\text{earth}} = vt_{\text{earth}} = v\gamma t_{\text{astro}}$ so $2.00\times10^6\text{ yr}\cdot c = v\dfrac{1}{\sqrt{1-v^2/c^2}}30.0\text{ yr}$

$\sqrt{1-v^2/c^2} = (v/c)(1.50\times10^{-5})$ $\qquad 1-\dfrac{v^2}{c^2} = \dfrac{v^2}{c^2}(2.25\times10^{-10})$

$1 = \dfrac{v^2}{c^2}(1+2.25\times10^{-10})$ so $\dfrac{v}{c} = \left(1+2.25\times10^{-10}\right)^{-1/2} = 1-\tfrac{1}{2}(2.25\times10^{-10})$

$$\boxed{\frac{v}{c} = 1 - 1.12\times10^{-10}}$$

***39.57** (a) Take the spaceship as the primed frame, moving toward the right at $v = +0.600c$. Then $u'_x = +0.800c$, and

$$u_x = \frac{u'_x + v}{1+(u'_x v)/c^2} = \frac{0.800c+0.600c}{1+(0.800)(0.600)} = \boxed{0.946c}$$

(b) $L = \dfrac{L_p}{\gamma} = (0.200\text{ ly})\sqrt{1-(0.600)^2} = \boxed{0.160\text{ ly}}$

(c) The aliens observe the 0.160-ly distance closing because the probe nibbles into it from one end at $0.800c$ and the Earth reduces it at the other end at $0.600c$. Thus,

$$\text{time} = \frac{0.160\text{ ly}}{0.800c+0.600c} = \boxed{0.114\text{ yr}}$$

(d) $K = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2 = \left(\dfrac{1}{\sqrt{1-(0.946)^2}} - 1\right)(4.00\times10^5\text{ kg})(3.00\times10^8\text{ m/s})^2 = \boxed{7.50\times10^{22}\text{ J}}$

39.58 In this case, the proper time is T_0 (the time measured by the students on a clock at rest relative to them). The dilated time measured by the professor is: $\Delta t = \gamma T_0$

where $\Delta t = T + t$. Here T is the time she waits before sending a signal and t is the time required for the signal to reach the students.

Thus, we have: $T + t = \gamma T_0$ (1)

To determine the travel time t, realize that the distance the students will have moved beyond the professor before the signal reaches them is: $d = v(T + t)$

The time required for the signal to travel this distance is: $t = \dfrac{d}{c} = \left(\dfrac{v}{c}\right)(T + t)$

Solving for t gives: $t = \dfrac{(v/c)T}{1-(v/c)}$

Substituting this into equation (1) yields: $T + \dfrac{(v/c)T}{1-(v/c)} = \gamma T_0$

or $T = (1 - v/c)^{-1} = \gamma T_0$

Then $T = T_0 \dfrac{1-(v/c)}{\sqrt{1-(v^2/c^2)}} = T_0 \dfrac{1-(v/c)}{\sqrt{[1+(v/c)][1-(v/c)]}} = \boxed{T_0\sqrt{\dfrac{1-(v/c)}{1+(v/c)}}}$

39.59 Look at the situation from the instructor's viewpoint since they are at rest relative to the clock, and hence measure the proper time. The Earth moves with velocity $v = -0.280c$ relative to the instructors while the students move with a velocity $u' = -0.600c$ relative to Earth. Using the velocity addition equation, the velocity of the students relative to the instructors (and hence the clock) is:

$$u = \frac{v + u'}{1 + vu'/c^2} = \frac{(-0.280c) - (0.600c)}{1 + (-0.280c)(-0.600c)/c^2} = -0.753c \text{ (students relative to clock)}$$

(a) With a proper time interval of $\Delta t_p = 50.0$ min, the time interval measured by the students is:

$\Delta t = \gamma \Delta t_p$ with $\gamma = \dfrac{1}{\sqrt{1-(0.753c)^2/c^2}} = 1.52$

Thus, the students measure the exam to last $T = 1.52(50.0\text{ min}) = \boxed{76.0\text{ minutes}}$

(b) The duration of the exam as measured by observers on Earth is:

$\Delta t = \gamma \Delta t_p$ with $\gamma = \dfrac{1}{\sqrt{1-(0.280c)^2/c^2}}$ so $T = 1.04(50.0\text{ min}) = \boxed{52.1\text{ minutes}}$

***39.60** The energy which arrives in one year is $E = \mathcal{P}t = \left(1.79\times10^{17}\ \text{J / s}\right)\left(3.16\times10^{7}\ \text{s}\right) = 5.66\times10^{24}\ \text{J}$

Thus, $m = \dfrac{E}{c^2} = \dfrac{5.66\times10^{24}\ \text{J}}{\left(3.00\times10^{8}\ \text{m / s}\right)^2} = \boxed{6.28\times10^{7}\ \text{kg}}$

***39.61** The observer sees the proper length of the tunnel, 50.0 m, but sees the train contracted to length

$L = L_p\sqrt{1 - v^2/c^2} = 100\ \text{m}\sqrt{1-(0.950)^2} = 31.2\ \text{m}$

shorter than the tunnel by $50.0 - 31.2 = \boxed{18.8\ \text{m}}$ so it is completely within the tunnel.

***39.62** If the energy required to remove a mass m from the surface is equal to its mass energy mc^2, then

$$\frac{GM_s m}{R_g} = mc^2$$

and $$R_g = \frac{GM_s}{c^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2\ /\ \text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(3.00\times10^{8}\ \text{m / s})^2} = 1.47\times10^{3}\ \text{m} = \boxed{1.47\ \text{km}}$$

39.63 (a) At any speed, the momentum of the particle is given by $p = \gamma m u = \dfrac{mu}{\sqrt{1-(u/c)^2}}$

Since $F = qE = \dfrac{dp}{dt}$

$$qE = \frac{d}{dt}\left[mu\left(1 - u^2/c^2\right)^{-1/2}\right]$$

$$qE = m\left(1-u^2/c^2\right)^{-1/2}\frac{du}{dt} + \frac{1}{2}mu\left(1-u^2/c^2\right)^{-3/2}\left(2u/c^2\right)\frac{du}{dt}$$

So $\dfrac{qE}{m} = \dfrac{du}{dt}\left[\dfrac{1-u^2/c^2+u^2/c^2}{\left(1-u^2/c^2\right)^{3/2}}\right]$ and $\boxed{a = \dfrac{du}{dt} = \dfrac{qE}{m}\left(1-\dfrac{u^2}{c^2}\right)^{3/2}}$

(b) As $u \to c$, $\boxed{a \to 0}$

(c) $\displaystyle\int_0^v \frac{du}{\left(1-u^2\ /\ c^2\right)^{3/2}} = \int_{t=0}^{t}\frac{qE}{m}dt$ so $\boxed{u = \dfrac{qEct}{\sqrt{m^2c^2+q^2E^2t^2}}}$

$$x = \int_0^t u\,dt = qEc\int_0^t \frac{t\,dt}{\sqrt{m^2c^2+q^2E^2t^2}} = \boxed{\frac{c}{qE}\left(\sqrt{m^2c^2+q^2E^2t^2} - mc\right)}$$

***39.64** (a) $f_{\text{observed}} = f_{\text{source}}\sqrt{\dfrac{1+v/c}{1-v/c}}$ implies $\dfrac{c}{\lambda+\Delta\lambda} = \dfrac{c}{\lambda}\sqrt{\dfrac{1+v/c}{1-v/c}}$,

or $$\sqrt{\frac{1-v/c}{1+v/c}} = \frac{\lambda+\Delta\lambda}{\lambda}$$

and $$\boxed{1+\frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1-v/c}{1+v/c}}}$$

(b) $$1+\frac{550\text{ nm}-650\text{ nm}}{650\text{ nm}} = \sqrt{\frac{1-v/c}{1+v/c}} = 0.846$$

$$1-\frac{v}{c} = (0.846)^2\left(1+\frac{v}{c}\right) = 0.716+0.716\left(\frac{v}{c}\right)$$

$$v = 0.166c = \boxed{4.97\times10^7\ \text{m/s}}$$

39.65 (a) An observer at rest relative to the mirror sees the light travel a distance

$$D = 2d - x = 2\left(1.80\times10^{12}\text{ m}\right) - (0.800c)t$$

where $x = (0.800c)t$ is the distance the ship moves toward the mirror in time t. Since this observer agrees that the speed of light is c, the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2(1.80\times10^{12}\text{ m})}{3.00\times10^8\text{ m / s}} - 0.800t = \boxed{6.67\times10^3\text{ s}}$$

(b) The observer in the rocket sees a length-contracted initial distance to the mirror of:

$$L = d\sqrt{1-\frac{v^2}{c^2}} = \left(1.80\times10^{12}\text{ m}\right)\sqrt{1-\frac{(0.800c)^2}{c^2}} = 1.08\times10^{12}\text{ m},$$

and the mirror moving toward the ship at speed $v = 0.800c$. Thus, he measures the distance the light travels as:

$$D = 2\left(1.08\times10^{12}\text{ m} - y\right)$$

where $y = (0.800c)(t/2)$ is the distance the mirror moves toward the ship before the light reflects off it. This observer also measures the speed of light to be c, so the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2}{c}\left[1.08\times10^{12}\text{ m} - (0.800c)\frac{t}{2}\right],\ \text{which gives } t = \boxed{4.00\times10^3\text{ s}}$$

39.66 (a) An observer at rest relative to the mirror sees the light travel a distance $D = 2d - x$, where $x = vt$ is the distance the ship moves toward the mirror in time t. Since this observer agrees that the speed of light is c, the time for it to travel distance D is

$$t = \frac{D}{c} = \frac{2d - vt}{c} = \boxed{\frac{2d}{c+v}}$$

(b) The observer in the rocket sees a length-contracted initial distance to the mirror of

$$L = d\sqrt{1 - \frac{v^2}{c^2}}$$

and the mirror moving toward the ship at speed v. Thus, he measures the distance the light travels as

$$D = 2(L - y)$$

where $y = vt/2$ is the distance the mirror moves toward the ship before the light reflects off it. This observer also measures the speed of light to be c, so the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2}{c}\left(d\sqrt{1 - \frac{v^2}{c^2}} - \frac{vt}{2}\right) \quad \text{so} \quad (c+v)t = \frac{2d}{c}\sqrt{(c+v)(c-v)} \quad \text{or} \quad \boxed{t = \frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}$$

39.67 (a) Since Mary is in the same reference frame, S′, as Ted, she observes the ball to have the same speed Ted observes, namely $|u_x'| = \boxed{0.800c}$.

(b) $$\Delta t' = \frac{L_p}{|u_x'|} = \frac{1.80 \times 10^{12}\ \text{m}}{0.800(3.00 \times 10^8\ \text{m/s})} = \boxed{7.50 \times 10^3\ \text{s}}$$

(c) $$L = L_p\sqrt{1 - \frac{v^2}{c^2}} = (1.80 \times 10^{12}\ \text{m})\sqrt{1 - \frac{(0.600c)^2}{c^2}} = \boxed{1.44 \times 10^{12}\ \text{m}}$$

Since $v = 0.600c$ and $u_x' = -0.800c$, the velocity Jim measures for the ball is

$$u_x = \frac{u_x' + v}{1 + u_x' v/c^2} = \frac{(-0.800c) + (0.600c)}{1 + (-0.800)(0.600)} = \boxed{-0.385c}$$

(d) Jim observes the ball and Mary to be initially separated by 1.44×10^{12} m. Mary's motion at $0.600c$ and the ball's motion at $0.385c$ nibble into thi distance from both ends. The gap closes at the rate $0.600c + 0.385c = 0.985c$, so the ball and catcher meet after a time

$$\Delta t = \frac{1.44 \times 10^{12}\ \text{m}}{0.985(3.00 \times 10^{18}\ \text{m / s})} = \boxed{4.88 \times 10^3\ \text{s}}$$

39.68 (a) $L_0^2 = L_{0x}^2 + L_{0y}^2$ and $L^2 = L_x^2 + L_y^2$

The motion is in the x direction: $L_y = L_{0y} = L_0 \sin\theta_0$

$$L_x = L_{0x}\sqrt{1-(v/c)^2} = (L_0 \cos\theta_0)\sqrt{1-(v/c)^2}$$

Thus,
$$L^2 = L_0{}^2\cos^2\theta_0\left[1-\left(\frac{v}{c}\right)^2\right] + L_0{}^2\sin^2\theta_0 = L_0{}^2\left[1-\left(\frac{v}{c}\right)^2\cos^2\theta_0\right]$$

or
$$\boxed{L = L_0\left[1-(v/c)^2\cos^2\theta_0\right]^{1/2}}$$

(b)
$$\tan\theta = \frac{L_y}{L_x} = \frac{L_{0y}}{L_{0x}\sqrt{1-(v/c)^2}} = \boxed{\gamma\tan\theta_0}$$

39.69 (a) First, we find the velocity of the stick relative to S′ using $L = L_p\sqrt{1-(u_x')^2/c^2}$

Thus
$$u_x' = \pm\, c\sqrt{1-(L/L_p)^2}$$

Selecting the negative sign because the stick moves in the negative x direction in S′ gives:

$$u_x' = -c\sqrt{1-\left(\frac{0.500\text{ m}}{1.00\text{ m}}\right)^2} = -0.866c$$ so the speed is $|u_x'| = \boxed{0.866c}$

Now determine the velocity of the stick relative to S, using the measured velocity of the stick relative to S′ and the velocity of S′ relative to S. From the velocity addition equation, we have:

$$u_x = \frac{u_x' + v}{1+vu_x'/c^2} = \frac{(-0.866c)+(0.600c)}{1+(0.600c)(-0.866c)} = -0.554c$$ and the speed is $|u_x| = \boxed{0.554c}$

(b) Therefore, the contracted length of the stick as measured in S is:

$$L = L_p\sqrt{1-(u_x/c)^2} = (1.00\text{ m})\sqrt{1-(0.554)^2} = \boxed{0.833\text{ m}}$$

39.70 (b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that $\boxed{\text{the two stars blew up simultaneously}}$.

(a) We in the spaceship moving past the hermit do not calculate the explosions to be simultaneous. We see the distance we have traveled from the Sun as

$$L = L_p\sqrt{1-(v/c)^2} = (6.00\text{ ly})\sqrt{1-(0.800)^2} = 3.60\text{ ly}$$

We see the Sun flying away from us at $0.800c$ while the light from the Sun approaches at $1.00c$. Thus, the gap between the Sun and its blast wave has opened at $1.80c$, and the time we calculate to have elapsed since the Sun exploded is

$$3.60\text{ ly}/1.80c = 2.00\text{ yr}.$$

We see Tau Ceti as moving toward us at $0.800c$, while its light approaches at $1.00c$, only $0.200c$ faster. We see the gap between that star and its blast wave as 3.60 ly and growing at $0.200c$. We calculate that it must have been opening for

$$3.60\text{ ly}/0.200c = 18.0\text{ yr}$$

and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

***39.71** The unshifted frequency is $f_{\text{source}} = \dfrac{c}{\lambda} = \dfrac{3.00\times 10^8\text{ m/s}}{394\times 10^{-9}\text{ m}} = 7.61\times 10^{14}\text{ Hz}$

We observe frequency $f = \dfrac{3.00\times 10^8\text{ m/s}}{475\times 10^{-9}\text{ m}} = 6.32\times 10^{14}\text{ Hz}$

Then $f = f_{\text{source}}\sqrt{\dfrac{1+v/c}{1-v/c}}$

gives: $6.32 = 7.61\sqrt{\dfrac{1+v/c}{1-v/c}}$

or $\dfrac{1+v/c}{1-v/c} = (0.829)^2$

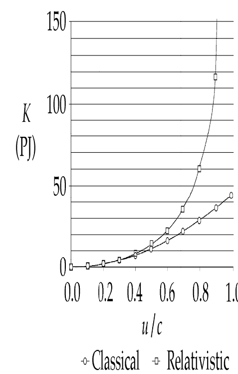
Solving for v yields: $v = -0.185c = \boxed{0.185c\ (\text{away})}$

39.72 Take $m = 1.00$ kg.

The classical kinetic energy is $K_c = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\left(\frac{u}{c}\right)^2 = \left(4.50\times10^{16}\text{ J}\right)\left(\frac{u}{c}\right)^2$

and the actual kinetic energy is $K_r = \left(\frac{1}{\sqrt{1-(u/c)^2}}-1\right)mc^2 = \left(9.00\times10^{16}\text{ J}\right)\left(\frac{1}{\sqrt{1-(u/c)^2}}-1\right)$

u/c	K_c (J)	K_r (J)
0.000	0.000	0.000
0.100	0.045×10^{16}	0.0453×10^{16}
0.200	0.180×10^{16}	0.186×10^{16}
0.300	0.405×10^{16}	0.435×10^{16}
0.400	0.720×10^{16}	0.820×10^{16}
0.500	1.13×10^{16}	1.39×10^{16}
0.600	1.62×10^{16}	2.25×10^{16}
0.700	2.21×10^{16}	3.60×10^{16}
0.800	2.88×10^{16}	6.00×10^{16}
0.900	3.65×10^{16}	11.6×10^{16}
0.990	4.41×10^{16}	54.8×10^{16}



$K_c = 0.990K_r$ when $\frac{1}{2}(u/c)^2 = 0.990\left[\frac{1}{\sqrt{1-(u/c)^2}}-1\right]$, yielding $u = \boxed{0.115c}$

Similarly, $K_c = 0.950K_r$ when $u = \boxed{0.257c}$

and $K_c = 0.500K_r$ when $u = \boxed{0.786c}$

39.73 $$\Delta m = \frac{E}{c^2} = \frac{mc(\Delta T)}{c^2} = \frac{\rho Vc(\Delta T)}{c^2} = \frac{\left(1030\text{ kg / m}^3\right)\left(1.40\times10^9\right)\left(10^3\text{ m}\right)^3\left(4186\text{ J / kg}\cdot{}^\circ\text{C}\right)\left(10.0\ {}^\circ\text{C}\right)}{\left(3.00\times10^8\text{ m / s}\right)^2}$$

$$\Delta m = \boxed{6.71\times10^8\text{ kg}}$$

Chapter 40 Solutions

40.1 $T = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{560\times10^{-9}\ \text{m}} = \boxed{5.18\times10^{3}\ \text{K}}$

***40.2** (a) $\lambda_{\max} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} \sim \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{10^{4}\ \text{K}}\ \boxed{\sim10^{-7}\ \text{m}}$ $\boxed{\text{ultraviolet}}$.

(b) $\lambda_{\max} \sim \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{10^{7}\ \text{K}}\ \boxed{\sim10^{-10}\ \text{m}}$. $\boxed{\gamma-\text{ray}}$

40.3 (a) Using $\lambda_{\max}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$

we get $\lambda_{\max} = \dfrac{2.898\times10^{-3}\ \text{m}}{2900\ \text{K}} = 9.99\times10^{-7}\ \text{m} = \boxed{999\ \text{nm}}$

(b) The $\boxed{\text{peak wavelength is in the infrared}}$ region of the electromagnetic spectrum, which is much wider than the visible region of the spectrum.

40.4 Planck's radiation law gives intensity-per-wavelength. Taking E to be the photon energy and n to be the number of photons emitted each second, we multiply by area and wavelength range to have energy-per-time leaving the hole:

$$\mathsf{P} = \frac{2\pi hc^2(\lambda_2-\lambda_1)\pi(d/2)^2}{\left(\frac{\lambda_1+\lambda_2}{2}\right)^5\left(e^{\frac{2hc}{(\lambda_1+\lambda_2)k_BT}}-1\right)} = En = nhf \qquad \text{where} \qquad E = hf = \frac{2hc}{\lambda_1+\lambda_2}$$

$$n = \frac{\mathsf{P}}{E} = \frac{8\pi^2cd^2(\lambda_2-\lambda_1)}{(\lambda_1+\lambda_2)^4\left(e^{2hc/(\lambda_1+\lambda_2)k_BT}-1\right)} = \frac{8\pi^2\left(3.00\times10^{8}\ \text{m/s}\right)\left(5.00\times10^{-5}\ \text{m}\right)^2\left(1.00\times10^{-9}\ \text{m}\right)}{\left(1001\times10^{-9}\ \text{m}\right)^4\left(e^{\frac{2\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1001\times10^{-9}\ \text{m}\right)\left(1.38\times10^{-23}\ \text{J/K}\right)\left(7.50\times10^{3}\ \text{K}\right)}}-1\right)}$$

$$n = \frac{5.90\times10^{16}\ /\ \text{s}}{\left(e^{3.84}-1\right)} = \boxed{1.30\times10^{15}\ /\ \text{s}}$$

***40.5** (a) $P = eA\sigma T^4 = 1\left(20.0\times10^{-4}\ \text{m}^2\right)\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(5000\ \text{K})^4 = \boxed{7.09\times10^4\ \text{W}}$

(b) $\lambda_{\text{max}}T = \lambda_{\text{max}}(5000\ \text{K}) = 2.898\times10^{-3}\ \text{m}\cdot\text{K} \Rightarrow \lambda_{\text{max}} = \boxed{580\ \text{nm}}$

(c) We compute: $\dfrac{hc}{k_BT} = \dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{\left(1.38\times10^{-23}\ \text{J/K}\right)(5000\ \text{K})} = 2.88\times10^{-6}\ \text{m}$

The power per wavelength interval is $P(\lambda) = AI(\lambda) = \dfrac{2\pi hc^2A}{\lambda^5\left[\exp\left(hc/\lambda k_BT\right)-1\right]}$, and

$$2\pi hc^2A = 2\pi\left(6.626\times10^{-34}\right)\left(3.00\times10^8\right)^2\left(20.0\times10^{-4}\right) = 7.50\times10^{-19}\ \frac{\text{J}\cdot\text{m}^4}{\text{s}}$$

$$P(580\ \text{nm}) = \frac{7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}}{\left(580\times10^{-9}\ \text{m}\right)^5\left[\exp\left(2.88\ \mu\text{m}/0.580\ \mu\text{m}\right)-1\right]} = \frac{1.15\times10^{13}\ \text{J/m}\cdot\text{s}}{e^{4.973}-1} = \boxed{7.99\times10^{10}\ \text{W/m}}$$

(d) - (i) The other values are computed similarly:

	λ	hc/k_BT	$e^{hc/\lambda k_BT}-1$	$2\pi hc^2A/\lambda^5$	$P(\lambda)$, W/m
(d)	1.00 nm	2882.6	7.96×10^{1251}	7.50×10^{26}	9.42×10^{-1226}
(e)	5.00 nm	576.5	2.40×10^{250}	2.40×10^{23}	1.00×10^{-227}
(f)	400 nm	7.21	1347	7.32×10^{13}	5.44×10^{10}
(c)	580 nm	4.97	143.5	1.15×10^{13}	7.99×10^{10}
(g)	700 nm	4.12	60.4	4.46×10^{12}	7.38×10^{10}
(h)	1.00 mm	0.00288	0.00289	7.50×10^{-4}	0.260
(i)	10.0 cm	2.88×10^{-5}	2.88×10^{-5}	7.50×10^{-14}	2.60×10^{-9}

(j) We approximate the area under the $P(\lambda)$ versus λ curve, between 400 nm and 700 nm, as two trapezoids:

$$P \approx \frac{\left[(5.44+7.99)\times10^{10}\ \frac{\text{W}}{\text{m}}\right]\left[(580-400)\times10^{-9}\ \text{m}\right]}{2} + \frac{\left[(7.99+7.38)\times10^{10}\ \frac{\text{W}}{\text{m}}\right]\left[(700-580)\times10^{-9}\ \text{m}\right]}{2}$$

$P = 2.13\times10^4\ \text{W}$ so the power radiated as visible light is $\boxed{\text{approximately 20 kW}}$.

40.6 (a) $\mathsf{P} = eA\sigma T^4$, so

$$T = \left(\frac{\mathsf{P}}{eA\sigma}\right)^{1/4} = \left[\frac{3.77\times10^{26}\ \text{W}}{1\left[4\pi\left(6.96\times10^{8}\ \text{m}\right)^2\right]\left(5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)}\right]^{1/4} = \boxed{5.75\times10^{3}\ \text{K}}$$

(b) $$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{5.75\times10^{3}\ \text{K}} = 5.04\times10^{-7}\ \text{m} = \boxed{504\ \text{nm}}$$

40.7 (a) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(620\times10^{12}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$$

(b) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.10\times10^{9}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.28\times10^{-5}\ \text{eV}}$$

(c) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(46.0\times10^{6}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.91\times10^{-7}\ \text{eV}}$$

(d) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{620\times10^{12}\ \text{Hz}} = 4.84\times10^{-7}\ \text{m} = \boxed{484\ \text{nm, visible light (blue)}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{3.10\times10^{9}\ \text{Hz}} = 9.68\times10^{-2}\ \text{m} = \boxed{9.68\ \text{cm, radio wave}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{46.0\times10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m, radio wave}}$$

40.8 $$E = hf = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{589.3\times10^{-9}\ \text{m}} = 3.37\times10^{-19}\ \text{J/photon}$$

$$n = \frac{\mathsf{P}}{E} = \frac{10.0\ \text{J/s}}{3.37\times10^{-19}\ \text{J/photon}} = \boxed{2.96\times10^{19}\ \text{photons/s}}$$

40.9 Each photon has an energy $E = hf = (6.626\times10^{-34})(99.7\times10^{6}) = 6.61\times10^{-26}\ \text{J}$

This implies that there are $$\frac{150\times10^{3}\ \text{J/s}}{6.61\times10^{-26}\ \text{J/photons}} = \boxed{2.27\times10^{30}\ \text{photons/s}}$$

***40.10** Energy of a single 500-nm photon:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})(3.00\times10^{8}\ \mathrm{m/s})}{500\times10^{-9}\ \mathrm{m}} = 3.98\times10^{-19}\ \mathrm{J}$$

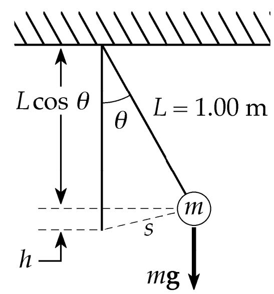
The energy entering the eye each second

$$E = \mathcal{P}t = (IA)t = (4.00\times10^{-11}\ \mathrm{W/m^2})\frac{\pi}{4}(8.50\times10^{-3}\ \mathrm{m})^2(1.00\ \mathrm{s}) = 2.27\times10^{-15}\ \mathrm{J}$$

The number of photons required to yield this energy

$$n = \frac{E}{E_\gamma} = \frac{2.27\times10^{-15}\ \mathrm{J}}{3.98\times10^{-19}\ \mathrm{J/photon}} = \boxed{5.71\times10^{3}\ \text{photons}}$$

40.11 We take $\theta = 0.0300$ radians. Then the pendulum's total energy is



$$E = mgh = mg(L - L\cos\theta)$$

$$E = (1.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(1.00 - 0.9995) = 4.41\times10^{-3}\ \mathrm{J}$$

The frequency of oscillation is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{g/L} = 0.498\ \mathrm{Hz}$

The energy is quantized, $E = nhf$

Therefore, $n = \frac{E}{hf} = \frac{4.41\times10^{-3}\ \mathrm{J}}{(6.626\times10^{-34}\ \mathrm{J\cdot s})(0.498\ \mathrm{s^{-1}})} = \boxed{1.34\times10^{31}}$

40.12 The radiation wavelength of $\lambda' = 500$ nm that is observed by observers on Earth is not the true wavelength, λ, emitted by the star because of the Doppler effect. The true wavelength is related to the observed wavelength using:

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-(v/c)}{1+(v/c)}}$$

$$\lambda = \lambda'\sqrt{\frac{1-(v/c)}{1+(v/c)}} = (500\ \mathrm{nm})\sqrt{\frac{1-(0.280)}{1+(0.280)}} = 375\ \mathrm{nm}$$

The temperature of the star is given by $\lambda_{\max}T = 2.898\times10^{-3}\ \mathrm{m\cdot K}$:

$$T = \frac{2.898\times10^{-3}\ \mathrm{m\cdot K}}{\lambda_{\max}} = \frac{2.898\times10^{-3}\ \mathrm{m\cdot K}}{375\times10^{-9}} = \boxed{7.73\times10^{3}\ \mathrm{K}}$$

40.13 This follows from the fact that at low T or long λ, the exponential factor in the denominator of Planck's radiation law is large compared to 1, so the factor of 1 in the denominator can be neglected. In this approximation, one arrives at *Wien's radiation law*.

***40.14** Planck's radiation law is $$I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$$

Using the series expansion $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

Planck's law reduces to $$I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left[\left(1 + hc/\lambda k_B T + \ \ldots\right) - 1\right]} \approx \frac{2\pi hc^2}{\lambda^5\left(hc/\lambda k_B T\right)} = \frac{2\pi c k_B T}{\lambda^4}$$

which is the Rayleigh-Jeans law, for very long wavelengths.

40.15 (a) $$\lambda_c = \frac{hc}{\phi} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(4.20\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = \boxed{296\ \text{nm}}$$

$$f_c = \frac{c}{\lambda_c} = \frac{3.00\times10^{8}\ \text{m/s}}{296\times10^{-9}\ \text{m}} = \boxed{1.01\times10^{15}\ \text{Hz}}$$

(b) $$\frac{hc}{\lambda} = \phi + e(\Delta V_S): \quad \frac{(6.626\times10^{-34})(3.00\times10^{8})}{180\times10^{-9}} = (4.20\ \text{eV})(1.60\times10^{-19}\ \text{J/eV}) + (1.60\times10^{-19})(\Delta V_S)$$

Therefore, $\boxed{\Delta V_S = 2.71\ \text{V}}$

40.16 $$K_{\max} = \tfrac{1}{2}mv_{\max}^2 = \frac{1}{2}(9.11\times10^{-31})(4.60\times10^{5})^2 = 9.64\times10^{-20}\ \text{J} = 0.602\ \text{eV}$$

(a) $$\phi = E - K_{\max} = \frac{1240\ \text{eV}\cdot\text{nm}}{625\ \text{nm}} - 0.602\ \text{eV} = \boxed{1.38\ \text{eV}}$$

(b) $$f_c = \frac{\phi}{h} = \frac{1.38\ \text{eV}}{6.626\times10^{-34}\ \text{J}\cdot\text{s}}\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right) = \boxed{3.34\times10^{14}\ \text{Hz}}$$

40.17 (a) $\lambda_c = \dfrac{hc}{\phi}$ Li: $\lambda_c = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{(2.30\text{ eV})(1.60\times10^{-19}\text{ J / eV})} = 540\text{ nm}$

Be: $\lambda_c = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{(3.90\text{ eV})(1.60\times10^{-19}\text{ J / eV})} = 318\text{ nm}$

Hg: $\lambda_c = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{(4.50\text{ eV})(1.60\times10^{-19}\text{ J / eV})} = 276\text{ nm}$

$\lambda < \lambda_c$ for photo current. $\boxed{\text{Thus, only lithium will exhibit the photoelectric effect.}}$

(b) For lithium, $\dfrac{hc}{\lambda} = \phi + K_{\max}$

$$\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{400\times10^{-9}\text{ m}} = (2.30\text{ eV})(1.60\times10^{-19}) + K_{\max}$$

$$K_{\max} = 1.29\times10^{-19}\text{ J} = \boxed{0.808\text{ eV}}$$

40.18 From condition (i), $hf = e(\Delta V_{S1}) + \phi_1$ and $hf = e(\Delta V_{S2}) + \phi_2$

$$(\Delta V_{S1}) = (\Delta V_{S2}) + 1.48\text{ V}$$

Then $\phi_2 - \phi_1 = 1.48\text{ eV}$

From condition (ii), $h f_{c1} = \phi_1 = 0.600hf_{c2} = 0.600\phi_2$

$$\phi_2 - 0.600\phi_2 = 1.48\text{ eV}$$

$$\boxed{\phi_2 = 3.70\text{ eV}} \quad \boxed{\phi_1 = 2.22\text{ eV}}$$

40.19 (a) $e(\Delta V_S) = \dfrac{hc}{\lambda} - \phi \quad \rightarrow \quad \phi = \dfrac{1240\text{ nm}\cdot\text{eV}}{546.1\text{ nm}} - 0.376\text{ eV} = \boxed{1.90\text{ eV}}$

(b) $e(\Delta V_S) = \dfrac{hc}{\lambda} - \phi = \dfrac{1240\text{ nm}\cdot\text{eV}}{587.5\text{ nm}} - 1.90\text{ eV} \quad \rightarrow \quad \Delta V_S = \boxed{0.216\text{ V}}$

Goal Solution

Two light sources are used in a photoelectric experiment to determine the work function for a particular metal surface. When green light from a mercury lamp (λ = 546.1 nm) is used, a retarding potential of 0.376 V reduces the photocurrent to zero. (a) Based on this measurement, what is the work function for this metal? (b) What stopping potential would be observed when using the yellow light from a helium discharge tube (λ = 587.5 nm)?

G: According to Table 40.1, the work function for most metals is on the order of a few eV, so this metal is probably similar. We can expect the stopping potential for the yellow light to be slightly lower than 0.376 V since the yellow light has a longer wavelength (lower frequency) and therefore less energy than the green light.

O: In this photoelectric experiment, the green light has sufficient energy hf to overcome the work function of the metal ϕ so that the ejected electrons have a maximum kinetic energy of 0.376 eV. With this information, we can use the photoelectric effect equation to find the work function, which can then be used to find the stopping potential for the less energetic yellow light.

A: (a) Einstein's photoelectric effect equation is $K_{\max} = hf - \phi$, and the energy required to raise an electron through a 1 V potential is 1 eV, so that $K_{\max} = eV_s = 0.376$ eV.

A photon from the mercury lamp has energy: $$hf = \frac{hc}{\lambda} = \frac{\left(4.14 \times 10^{-15}\ \text{eV} \cdot \text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{546.1 \times 10^{-9}\ \text{m}}$$

$$E = hf = 2.27\ \text{eV}$$

Therefore, the work function for this metal is: $$\phi = hf - K_{\max} = 2.27\ \text{eV} - (0.376\ \text{eV}) = 1.90\ \text{eV}$$

(b) For the yellow light, $\lambda = 587.5$ nm, and $$hf = \frac{hc}{\lambda} = \frac{\left(4.14 \times 10^{-15}\ \text{eV} \cdot \text{s}\right)\left(3.00 \times 10^{8}\ \text{m / s}\right)}{587.5 \times 10^{-9}\ \text{m}}$$

$$E = 2.11\ \text{eV}$$

Therefore, $K_{\max} = hf - \phi = 2.11\ \text{eV} - 1.90\ \text{eV} = 0.216\ \text{eV}$, so $V_s = 0.216$ V

L: The work function for this metal is lower than we expected, and does not correspond with any of the values in Table 40.1. Further examination in the **CRC Handbook of Chemistry and Physics** reveals that all of the metal elements have work functions between 2 and 6 eV. However, a single metal's work function may vary by about 1 eV depending on impurities in the metal, so it is just barely possible that a metal might have a work function of 1.90 eV.

The stopping potential for the yellow light is indeed lower than for the green light as we expected. An interesting calculation is to find the wavelength for the lowest energy light that will eject electrons from this metal. That threshold wavelength for $K_{\max}$ = 0 is 658 nm, which is red light in the visible portion of the electromagnetic spectrum.)

40.20 From the photoelectric equation, we have: $e(\Delta V_{S1}) = E_{\gamma 1} - \phi$ and $e(\Delta V_{S2}) = E_{\gamma 2} - \phi$

Since $\Delta V_{S2} = 0.700(\Delta V_{S1})$, then $e(\Delta V_{S2}) = 0.700(E_{\gamma 1} - \phi) = E_{\gamma 2} - \phi$

or $(1 - 0.700)\phi = E_{\gamma 2} - 0.700 E_{\gamma 1}$

and the work function is: $\phi = \dfrac{E_{\gamma 2} - 0.700 E_{\gamma 1}}{0.300}$

The photon energies are: $E_{\gamma 1} = \dfrac{hc}{\lambda_1} = \dfrac{1240 \text{ nm} \cdot \text{eV}}{410 \text{ eV}} = 3.03 \text{ eV}$

and $E_{\gamma 2} = \dfrac{hc}{\lambda_2} = \dfrac{1240 \text{ nm} \cdot \text{eV}}{445 \text{ eV}} = 2.79 \text{ eV}$

Thus, the work function is $\phi = \dfrac{2.79 \text{ eV} - 0.700(3.03 \text{ eV})}{0.300} = 2.23 \text{ eV}$

and we recognize this as characteristic of $\boxed{\text{potassium}}$.

***40.21** The energy needed is $E = 1.00 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$

The energy absorbed in time t is $E = Pt = (IA)t$

so $t = \dfrac{E}{IA} = \dfrac{1.60 \times 10^{-19} \text{ J}}{(500 \text{ J/s} \cdot \text{m}^2)[\pi(2.82 \times 10^{-15} \text{ m})^2]} = 1.28 \times 10^7 \text{ s} = \boxed{148 \text{ days}}$

The gross failure of the classical theory of the photoelectric effect contrasts with the success of quantum mechanics.

***40.22** Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{\max} = hf - \phi$, or

$$K_{\max} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{200 \times 10^{-9} \text{ m}} \left(\frac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) - 4.70 \text{ eV} = 1.51 \text{ eV}$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$$V = \frac{k_e Q}{r} \quad \text{or} \quad Q = \frac{rV}{k_e} = \frac{(5.00 \times 10^{-2} \text{ m})(1.51 \text{ N} \cdot \text{m/C})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = \boxed{8.41 \times 10^{-12} \text{ C}}$$

40.23 (a) By having the photon source move toward the metal, the incident photons are Doppler shifted to higher frequencies, and hence, higher energy.

(b) If $v = 0.280c$, $f' = f\sqrt{\dfrac{1+v/c}{1-v/c}} = \left(7.00\times10^{14}\right)\sqrt{\dfrac{1.28}{0.720}} = 9.33\times10^{14}\text{ Hz}$

Therefore, $\phi = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(9.33\times10^{14}\text{ Hz}\right) = 6.18\times10^{-19}\text{ J} = \boxed{3.87\text{ eV}}$

(c) At $v = 0.900c$, $f = 3.05\times10^{15}\text{ Hz}$

and $K_{\max} = hf - \phi = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.05\times10^{15}\text{ Hz}\right)\left(\dfrac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) - 3.87\text{ eV} = \boxed{8.78\text{ eV}}$

***40.24** $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{700\times10^{-9}\text{ m}} = 2.84\times10^{-19}\text{ J} = \boxed{1.78\text{ eV}}$

$p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{700\times10^{-9}\text{ m}} = \boxed{9.47\times10^{-28}\text{ kg}\cdot\text{m/s}}$

40.25 (a) $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta) = \dfrac{6.626\times10^{-34}}{(9.11\times10^{-31})(3.00\times10^{8})}(1-\cos 37.0°) = \boxed{4.88\times10^{-13}\text{ m}}$

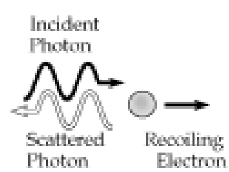
(b) $E_0 = hc/\lambda_0$: $\left(300\times10^{3}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right) = \left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\text{ m/s}\right)/\lambda_0$

$\lambda_0 = 4.14\times10^{-12}\text{ m}$ and $\lambda' = \lambda_0 + \Delta\lambda = 4.63\times10^{-12}\text{ m}$

$E' = \dfrac{hc}{\lambda'} = \dfrac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{4.63\times10^{-12}\text{ m}} = 4.30\times10^{14}\text{ J} = \boxed{268\text{ keV}}$

(c) $K_e = E_0 - E' = 300\text{ keV} - 268.5\text{ keV} = \boxed{31.5\text{ keV}}$

40.26 This is Compton scattering through 180°:



$E_0 = \dfrac{hc}{\lambda_0} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(0.110\times10^{-9}\text{ m})(1.60\times10^{-19}\text{ J/eV})} = 11.3\text{ keV}$

$\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta) = (2.43\times10^{-12}\text{ m})(1-\cos 180°) = 4.86\times10^{-12}\text{ m}$

$\lambda' = \lambda_0 + \Delta\lambda = 0.115\text{ nm}$ so $E' = \dfrac{hc}{\lambda'} = 10.8\text{ keV}$

Momentum conservation: $\dfrac{h}{\lambda_0}\,\mathbf{i} = \dfrac{h}{\lambda'}(-\mathbf{i}) + p_e(\mathbf{i})$ and $p_e = h\left(\dfrac{1}{\lambda_0} - \dfrac{1}{\lambda'}\right)$

$p_e = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(\dfrac{\left(3.00\times10^{8}\text{ m/s}\right)/c}{1.60\times10^{-19}\text{ J/eV}}\right)\left(\dfrac{1}{0.110\times10^{-9}\text{ m}} + \dfrac{1}{0.115\times10^{-9}\text{ m}}\right) = \boxed{22.1\text{ keV}/c}$

Energy conservation: $11.3\text{ keV} = 10.8\text{ keV} + K_e$ so that $\boxed{K_e = 478\text{ eV}}$

Check: $E^2 = p^2c^2 + m_e^2c^4$ or $(m_ec^2 + K_e)^2 = (pc)^2 + (m_ec^2)^2$

$(511\text{ keV} + 0.478\text{ keV})^2 = (22.1\text{ keV})^2 + (511\text{ keV})^2$

$2.62 \times 10^{11} = 2.62 \times 10^{11}$

40.27 $K_e = E_0 - E'$

With $K_e = E'$, $E' = E_0 - E'$: $E' = \dfrac{E_0}{2}$

$\lambda' = \dfrac{hc}{E'} = \dfrac{hc}{\frac{1}{2}E_0} = 2\dfrac{hc}{E_0} = 2\lambda_0$ $\lambda' = \lambda_0 + \lambda_C(1 - \cos\theta)$

$2\lambda_0 = \lambda_0 + \lambda_C(1 - \cos\theta)$ $1 - \cos\theta = \dfrac{\lambda_0}{\lambda_C} = \dfrac{0.00160}{0.00243} \to \theta = \boxed{70.0°}$

40.28 We may write down four equations, not independent, in the three unknowns λ_0, λ', and v using the conservation laws:

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + \gamma m_ec^2 - m_ec^2 \qquad \text{(Energy conservation)}$$

$$\frac{h}{\lambda_0} = \gamma m_e v\cos 20.0° \quad \text{(momentum in } x\text{-direction)}$$

$$0 = \frac{h}{\lambda'} - \gamma m_e v\sin 20.0° \qquad \text{(momentum in } y\text{-direction)}$$

and Compton's equation $\lambda' - \lambda_0 = \dfrac{h}{m_ec}(1 - \cos 90.0°)$.

It is easiest to ignore the energy equation and, using the two momentum equations, write

$$\frac{h/\lambda_0}{h/\lambda'} = \frac{\gamma m_e v\cos 20.0°}{\gamma m_e v\sin 20.0°} \quad \text{or} \quad \lambda_0 = \lambda'\tan 20.0°$$

Then, the Compton equation becomes $\lambda' - \lambda'\tan 20.0° = 0.00243\text{ nm}$,

or $\lambda' = \dfrac{0.00243\text{ nm}}{1 - \tan 20.0°} = 0.00382\text{ nm} = \boxed{3.82\text{ pm}}$

40.29 (a) Conservation of momentum in the x direction gives: $p_\gamma = p'_\gamma \cos\theta + p_e \cos\phi$

or since $\theta = \phi$, $$\frac{h}{\lambda_0} = \left(p_e + \frac{h}{\lambda'}\right)\cos\theta \qquad [1]$$

Conservation of momentum in the y direction gives: $0 = p'_\gamma \sin\theta - p_e \sin\theta$,

which (neglecting the trivial solution $\theta = 0$) gives: $$p_e = p'_\gamma = \frac{h}{\lambda'} \qquad [2]$$

Substituting [2] into [1] gives: $\frac{h}{\lambda_0} = \frac{2h}{\lambda'}\cos\theta$, or $$\lambda' = 2\lambda_0 \cos\theta \qquad [3]$$

Then the Compton equation is $$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

giving $$2\lambda_0 \cos\theta - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

or $$2\cos\theta - 1 = \frac{hc}{\lambda_0}\frac{1}{m_e c^2}(1 - \cos\theta)$$

Since $E_\gamma = \frac{hc}{\lambda_0}$, this may be written as: $$2\cos\theta - 1 = \left(\frac{E_\gamma}{m_e c^2}\right)(1 - \cos\theta)$$

which reduces to: $$\left(2 + \frac{E_\gamma}{m_e c^2}\right)\cos\theta = 1 + \frac{E_\gamma}{m_e c^2}$$

or $\cos\theta = \frac{m_e c^2 + E_\gamma}{2m_e c^2 + E_\gamma} = \frac{0.511\text{ MeV} + 0.880\text{ MeV}}{1.02\text{ MeV} + 0.880\text{ MeV}} = 0.732$ so that $\boxed{\theta = \phi = 43.0°}$

(b) Using Equation (3): $E'_\gamma = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0(2\cos\theta)} = \frac{E_\gamma}{2\cos\theta} = \frac{0.880\text{ MeV}}{2\cos 43.0°} = 0.602\text{ MeV} = \boxed{602\text{ keV}}$

Then, $$p'_\gamma = \frac{E'_\gamma}{c} = 0.602\ \text{MeV}/c = \boxed{3.21 \times 10^{-22}\ \text{kg}\cdot\text{m/s}}$$

(c) From Equation (2), $$p_e = p'_\gamma = 0.602\ \text{MeV}/c = \boxed{3.21 \times 10^{-22}\ \text{kg}\cdot\text{m/s}}$$

From energy conservation: $$K_e = E_\gamma - E'_\gamma = 0.880\text{ MeV} - 0.602\text{ MeV} = 0.278\text{ MeV} = \boxed{278\text{ keV}}$$

40.30 The energy of the incident photon is $E_0 = p_\gamma c = hc/\lambda_0$.

(a) Conserving momentum in the x direction gives

$$p_\gamma = p_e \cos\phi + p'_\gamma \cos\theta, \text{ or since } \phi = \theta, \quad \frac{E_0}{c} = \left(p_e + p'_\gamma\right)\cos\theta \qquad [1]$$

Conserving momentum in the y direction (with $\phi = \theta$) yields

$$0 = p'_\gamma \sin\theta - p_e \sin\theta, \quad \text{or} \quad p_e = p'_\gamma = \frac{h}{\lambda'} \qquad [2]$$

Substituting Equation [2] into Equation [1] gives

$$\frac{E_0}{c} = \left(\frac{h}{\lambda'} + \frac{h}{\lambda'}\right)\cos\theta, \quad \text{or} \quad \lambda' = \frac{2hc}{E_0}\cos\theta \qquad [3]$$

By the Compton equation, $\lambda' - \lambda_0 = \dfrac{h}{m_e c}(1 - \cos\theta)$, $\qquad \dfrac{2hc}{E_0}\cos\theta - \dfrac{2hc}{E_0} = \dfrac{h}{m_e c}(1 - \cos\theta)$

which reduces to $\qquad \left(2m_e c^2 + E_0\right)\cos\theta = m_e c^2 + E_0$

Thus, $\qquad \boxed{\phi = \theta = \cos^{-1}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)}$

(b) From Equation [3], $\qquad \lambda' = \dfrac{2hc}{E_0}\cos\theta = \dfrac{2hc}{E_0}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$

Therefore,

$$E'_\gamma = \frac{hc}{\lambda'} = \frac{hc}{(2hc/E_0)\left(m_e c^2 + E_0\right)/\left(2m_e c^2 + E_0\right)} = \boxed{\frac{E_0}{2}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)},$$

and

$$p'_\gamma = \frac{E'_\gamma}{c} = \boxed{\frac{E_0}{2c}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$$

(c) From conservation of energy, $\qquad K_e = E_0 - E'_\gamma = E_0 - \dfrac{E_0}{2}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$

or

$$K_e = \frac{E_0}{2}\left(\frac{2m_e c^2 + 2E_0 - 2m_e c^2 - E_0}{m_e c^2 + E_0}\right) = \boxed{\frac{E_0^2}{2\left(m_e c^2 + E_0\right)}}$$

Finally, from Equation (2), $\qquad p_e = p'_\gamma = \boxed{\dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$

40.31 (a) Thanks to Compton we have four equations in the unknowns ϕ, v, and λ':

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + \gamma m_e c^2 - m_e c^2 \qquad \text{(energy conservation)} \qquad [1]$$

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'} \cos 2\phi + \gamma m_e v \cos \phi \qquad \text{(momentum in } x \text{ direction)} \qquad [2]$$

$$0 = \frac{h}{\lambda'} \sin 2\phi - \gamma m_e v \sin \phi \qquad \text{(momentum in } y \text{ direction)} \qquad [3]$$

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos 2\phi) \qquad \text{(Compton equation)} \qquad [4]$$

Using $\sin 2\phi = 2 \sin \phi \cos \phi$ in Equation [3] gives $\gamma m_e v = \frac{2h}{\lambda'} \cos \phi$.

Substituting this into Equation [2] and using $\cos 2\phi = 2\cos^2 \phi - 1$ yields

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'}(2\cos^2\phi - 1) + \frac{2h}{\lambda'} \cos^2\phi = \frac{h}{\lambda'}(4\cos^2\phi - 1),$$

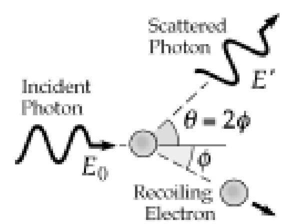
or $\lambda' = 4\lambda_0 \cos^2\phi - \lambda_0$ [5]

Substituting the last result into the Compton equation gives

$$4\lambda_0 \cos^2\phi - 2\lambda_0 = \frac{h}{m_e c}\left[1 - \left(2\cos^2\phi - 1\right)\right] = 2\frac{hc}{m_e c^2}\left(1 - \cos^2 \phi\right).$$

With the substitution $\lambda_0 = hc/E_0$, this reduces to

$$\cos^2 \phi = \frac{m_e c^2 + E_0}{2m_e c^2 + E_0} = \frac{1+x}{2+x} \quad \text{where} \quad x \equiv \frac{E_0}{m_e c^2}.$$



For $x = \frac{0.700 \text{ MeV}}{0.511 \text{ MeV}} = 1.37$, this gives $\phi = \cos^{-1}\sqrt{\frac{1+x}{2+x}} = \boxed{33.0°}$

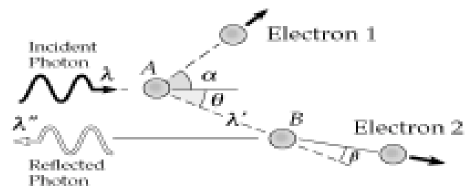
(b) From Equation [5], $\lambda' = \lambda_0\left(4\cos^2\phi - 1\right) = \lambda_0\left[4\left(\frac{1+x}{2+x}\right) - 1\right] = \lambda_0\left(\frac{2+3x}{2+x}\right)$.

Then, Equation [1] becomes

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda_0}\left(\frac{2+x}{2+3x}\right) + \gamma m_e c^2 - m_e c^2 \quad \text{or} \quad \frac{E_0}{m_e c^2} - \frac{E_0}{m_e c^2}\left(\frac{2+x}{2+3x}\right) + 1 = \gamma.$$

Thus, $\gamma = 1 + x - x\left(\frac{2+x}{2+3x}\right)$, and with $x = 1.37$ we get $\gamma = 1.614$.

Therefore, $\frac{v}{c} = \sqrt{1 - \gamma^{-2}} = \sqrt{1 - 0.384} = 0.785$ or $v = \boxed{0.785c}$.



40.32 $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$

$$\lambda'' - \lambda' = \frac{h}{m_e c}\left[1 - \cos(\pi - \theta)\right]$$

$$\lambda'' - \lambda = \frac{h}{m_e c} - \frac{h}{m_e c}\cos(\pi - \theta) + \frac{h}{m_e c} - \frac{h}{m_e c}\cos\theta$$

Now $\cos(\pi - \theta) = -\cos\theta$, so $\quad \lambda'' - \lambda = 2\dfrac{h}{m_e c} = \boxed{0.00486\text{ nm}}$

40.33 (a) $K = \frac{1}{2}m_e v^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(1.40\times10^6\text{ m/s}\right)^2 = 8.93\times10^{-19}\text{ J} = 5.58\text{ eV}$

$$E_0 = \frac{hc}{\lambda_0} = \frac{1240\text{ eV}\cdot\text{nm}}{0.800\text{ nm}} = 1550\text{ eV}$$

$E' = E_0 - K$, and $\lambda' = \dfrac{hc}{E'} = \dfrac{1240\text{ eV}\cdot\text{nm}}{1550\text{ eV} - 5.58\text{ eV}} = 0.803\text{ nm}$

$$\Delta\lambda = \lambda' - \lambda_0 = 0.00288\text{ nm} = \boxed{2.88\text{ pm}}$$

(b) $\Delta\lambda = \lambda_C(1 - \cos\theta) \Rightarrow \cos\theta = 1 - \dfrac{\Delta\lambda}{\lambda_C} = 1 - \dfrac{0.00288\text{ nm}}{0.00243\text{ nm}} = -0.189$, so $\boxed{\theta = 101°}$

***40.34** Maximum energy loss appears as maximum increase in wavelength, which occurs for scattering angle 180°. Then $\Delta\lambda = (1 - \cos 180°)(h/mc) = 2h/mc$ where m is the mass of the target particle. The fractional energy loss is

$$\frac{E_0 - E'}{E_0} = \frac{hc/\lambda_0 - hc/\lambda'}{hc/\lambda_0} = \frac{\lambda' - \lambda_0}{\lambda'} = \frac{\Delta\lambda}{\lambda_0 + \Delta\lambda} = \frac{2h/mc}{\lambda_0 + 2h/mc}$$

Further, $\lambda_0 = hc/E_0$, so $\dfrac{E_0 - E'}{E_0} = \dfrac{2h/mc}{hc/E_0 + 2h/mc} = \dfrac{2E_0}{mc^2 + 2E_0}$.

(a) For scattering from a free electron, $mc^2 = 0.511$ MeV, so

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511\text{ MeV})}{0.511\text{ MeV} + 2(0.511\text{ MeV})} = \boxed{0.667}$$

(b) For scattering from a free proton, $mc^2 = 938$ MeV, and

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511\text{ MeV})}{938\text{ MeV} + 2(0.511\text{ MeV})} = \boxed{0.00109}$$

40.35 Start with Balmer's equation, $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, or $\lambda = \frac{(4n^2 / R_H)}{(n^2 - 4)}$.

Substituting $R_H = 1.0973732 \times 10^7 \text{ m}^{-1}$, we obtain

$$\lambda = \frac{(3.645 \times 10^{-7} \text{ m})n^2}{n^2 - 4} = \frac{364.5n^2}{n^2 - 4} \text{ nm}, \quad \text{where } n = 3, 4, 5, \ldots$$

40.36 (a) Using $\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, for $n_f = 2$, and $n_i \geq 3$, we get:

$$\lambda = \frac{4n^2}{R_H\left(n^2 - 4\right)} = \frac{4n^2}{\left(2.00 \times 10^7 \text{ m}^{-1}\right)\left(n^2 - 4\right)} = \frac{(200.0)n^2}{n^2 - 4} \text{ nm}$$

This says that $200 \text{ nm} \leq \lambda \leq 360 \text{ nm}$, which is $\boxed{\text{ultraviolet}}$.

(b) Using $n \geq 3$, $\lambda = \frac{4n^2}{R_H\left(n^2 - 4\right)} = \frac{4n^2}{\left(0.500 \times 10^7 \text{ m}^{-1}\right)\left(n^2 - 4\right)} = \frac{(800.0)n^2}{n^2 - 4} \text{ nm}$

This says that $800 \text{ nm} \leq \lambda \leq 1440 \text{ nm}$, which is in the $\boxed{\text{infrared}}$.

40.37 (a) Lyman series: $\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right)$ $n = 2, 3, 4, \ldots$

$\frac{1}{\lambda} = \frac{1}{94.96 \times 10^{-9}} = (1.097 \times 10^7)\left(1 - \frac{1}{n^2}\right)$ $\boxed{n = 5}$

(b) Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$ $n = 4, 5, 6, \ldots$

The shortest wavelength for this series corresponds to $n = \infty$ for ionization

$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{9} - \frac{1}{n^2}\right)$ For $n = \infty$, this gives $\lambda = 820$ nm

This is larger than 94.96 nm, so this wave length $\boxed{\text{cannot be associated with the Paschen series}}$

Brackett series: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$ $n = 5, 6, 7, \ldots$

$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{16} - \frac{1}{n^2}\right)$ $n = \infty$ for ionization $\lambda_{min} = 1458$ nm

Once again this wavelength $\boxed{\text{cannot be associated with the Brackett series}}$

40.38 (a) $\lambda_{\min} = \dfrac{hc}{E_{\max}}$

Lyman ($n_f = 1$): $\lambda_{\min} = \dfrac{hc}{|E_1|} = \dfrac{1240\text{ eV}\cdot\text{nm}}{13.6\text{ eV}} = \boxed{91.2\text{ nm}}$ (Ultraviolet)

Balmer ($n_f = 2$): $\lambda_{\min} = \dfrac{hc}{|E_2|} = \dfrac{1240\text{ eV}\cdot\text{nm}}{\left(\frac{1}{4}\right)13.6\text{ eV}} = \boxed{365\text{ nm}}$ (UV)

Paschen ($n_f = 3$): $\lambda_{\min} = \ . \ . \ . \ = 3^2(91.2\text{ nm}) = \boxed{821\text{ nm}}$ (Infrared)

Bracket ($n_f = 4$): $\lambda_{\min} = \ . \ . \ . \ = 4^2(91.2\text{ nm}) = \boxed{1460\text{ nm}}$ (IR)

(b) $E_{\max} = \dfrac{hc}{\lambda_{\min}}$

Lyman: $E_{\max} = \boxed{13.6\text{ eV}}$ $\left(=|E_1|\right)$

Balmer: $E_{\max} = \boxed{3.40\text{ eV}}$ $\left(=|E_2|\right)$

Paschen: $E_{\max} = \boxed{1.51\text{ eV}}$ $\left(=|E_3|\right)$

Brackett: $E_{\max} = \boxed{0.850\text{ eV}}$ $\left(=|E_4|\right)$

40.39 Liquid O_2 $\lambda_{\text{abs}} = 1269\text{ nm}$

$E = \dfrac{hc}{\lambda} = \dfrac{1.2398\times10^{-6}}{1.269\times10^{-6}} = 0.977\text{ eV}$ for each molecule.

For two molecules, $\lambda = \dfrac{hc}{2E} = \boxed{634\text{ nm, red}}$

By absorbing the red photons, the liquid O_2 appears to be blue.

***40.40** (a) $v_1 = \sqrt{\dfrac{k_e e^2}{m_e r_1}}$ where $r_1 = (1)^2 a_0 = 0.00529\text{ nm} = 5.29\times10^{-11}\text{ m}$

$$v_1 = \sqrt{\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)\left(5.29\times10^{-11}\text{ m}\right)}} = \boxed{2.19\times10^6\ \text{m/s}}$$

(b) $K_1 = \frac{1}{2}m_e v_1^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(2.19\times10^6\ \text{m/s}\right)^2 = 2.18\times10^{-18}\text{ J} = \boxed{13.6\text{ eV}}$

(c) $U_1 = -\dfrac{k_e e^2}{r_1} = -\dfrac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{5.29\times10^{-11}\text{ m}} = -4.35\times10^{-18}\text{ J} = \boxed{-27.2\text{ eV}}$

40.41 (a) $r_2^2 = (0.0529\ \text{nm})(2)^2 = \boxed{0.212\ \text{nm}}$

(b) $m_e v_2 = \sqrt{\dfrac{m_e k_e e^2}{r_2}} = \sqrt{\dfrac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{0.212\times10^{-9}\ \text{m}}} = \boxed{9.95\times10^{-25}\ \text{kg}\cdot\text{m/s}}$

(c) $L_2 = m_e v_2 r_2 = \left(9.95\times10^{-25}\ \text{kg}\cdot\text{m/s}\right)\left(0.212\times10^{-9}\ \text{m}\right) = \boxed{2.11\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}}$

(d) $K_2 = \frac{1}{2} m_e v_2^2 = \dfrac{(m_e v_2)^2}{2m_e} = \dfrac{\left(9.95\times10^{-25}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)} = 5.43\times10^{-19}\ \text{J} = \boxed{3.40\ \text{eV}}$

(e) $U_2 = -\dfrac{k_e e^2}{r_2} = -\dfrac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{0.212\times10^{-9}\ \text{m}} = -1.09\times10^{-18}\ \text{J} = \boxed{-6.80\ \text{eV}}$

(f) $E_2 = K_2 + U_2 = 3.40\ \text{eV} - 6.80\ \text{eV} = \boxed{-3.40\ \text{eV}}$

40.42 $\Delta E = (13.6\ \text{eV})\left(\dfrac{1}{n_i^2} - \dfrac{1}{n_f^2}\right)$

Where for $\Delta E > 0$ we have absorption and for $\Delta E < 0$ we have emission.

(A) for $n_i = 2$ and $n_f = 5$ $\Delta E = 2.86$ eV (absorption)

(B) for $n_i = 5$ and $n_f = 3$ $\Delta E = -0.967$ eV (emission)

(C) for $n_i = 7$ and $n_f = 4$ $\Delta E = -0.572$ eV (emission)

(D) for $n_i = 4$ and $n_f = 7$ $\Delta E = 0.572$ eV (absorption)

(a) $E = \dfrac{hc}{\lambda}$ so the shortest wavelength is emitted in transition $\boxed{\text{B}}$.

(b) The atom gains most energy in transition $\boxed{\text{A}}$.

(c) The atom loses energy in transitions $\boxed{\text{B and C}}$.

40.43 (b) $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right) = (1.097\times10^{7}\ \text{m}^{-1})\left(\dfrac{1}{2^2} - \dfrac{1}{6^2}\right)$ so $\boxed{\lambda = 410\ \text{nm}}$

(a) $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m / s})}{410\times10^{-9}\ \text{m}} = 4.85\times10^{-19}\ \text{J} = \boxed{3.03\ \text{eV}}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}}{410\times10^{-9}} = \boxed{7.32\times10^{14}\ \text{Hz}}$

***40.44** We use $E_n = \dfrac{-13.6\text{ eV}}{n^2}$

To ionize the atom when the electron is in the n^{th} level, it is necessary to add an amount of energy given by

$$E = -E_n = \frac{13.6\text{ eV}}{n^2}$$

(a) Thus, in the ground state where $n = 1$, we have $\boxed{E = 13.6\text{ eV}}$

(b) In the $n = 3$ level, $E = \dfrac{13.6\text{ eV}}{9} = \boxed{1.51\text{ eV}}$

***40.45** Starting with $\frac{1}{2} m_e v^2 = \dfrac{k_e e^2}{2r}$, we have $v^2 = \dfrac{k_e e^2}{m_e r}$

and using $r_n = \dfrac{n^2\hbar^2}{m_e k_e e^2}$

gives $v_n^{\;2} = \dfrac{k_e e^2}{m_e \dfrac{n^2\hbar^2}{m_e k_e e^2}}$ or $v_n = \dfrac{k_e e^2}{n\hbar}$

***40.46** (a) The velocity of the moon in its orbit is $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(3.84 \times 10^8\text{ m})}{2.36 \times 10^6\text{ s}} = 1.02 \times 10^3\text{ m/s}$

So, $L = mvr = (7.36 \times 10^{22}\text{ kg})(1.02 \times 10^3\text{ m/s})(3.84 \times 10^8\text{ m}) = \boxed{2.89 \times 10^{34}\text{ kg} \cdot \text{m}^2/\text{s}}$

(b) We have $L = n\hbar$

or $n = \dfrac{L}{\hbar} = \dfrac{2.89 \times 10^{34}\text{ kg} \cdot \text{m}^2/\text{s}}{1.055 \times 10^{-34}\text{ J} \cdot \text{s}} = \boxed{2.74 \times 10^{68}}$

(c) We have $n\hbar = L = mvr = m(GM_e/r)^{1/2} r$,

so $r = \dfrac{\hbar^2}{m^2 GM_e} n^2 = Rn^2$ and $\dfrac{\Delta r}{r} = \dfrac{(n+1)^2 R - n^2 R}{n^2 R} = \dfrac{2n+1}{n^2}$

which is approximately equal to $\dfrac{2}{n} = \boxed{7.30 \times 10^{-69}}$

40.47 The batch of excited atoms must make these six transitions to get back to state one: $2 \to 1$, and also $3 \to 2$ and $3 \to 1$, and also $4 \to 3$ and $4 \to 2$ and $4 \to 1$. Thus, the incoming light must have just enough energy to produce the $1 \to 4$ transition. It must be the third line of the Lyman series in the absorption spectrum of hydrogen. The absorbing atom changes from energy

$$E_i = -\frac{13.6 \text{ eV}}{1^2} = -13.6 \text{ eV} \quad \text{to} \quad E_f = -\frac{13.6 \text{ eV}}{4^2} = -0.850 \text{ eV},$$

so the incoming photons have wavelength

$$\lambda = \frac{hc}{E_f - E_i} = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{-0.850 \text{ eV} - (-13.6 \text{ eV})}\left(\frac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 9.75 \times 10^{-8} \text{ m} = \boxed{97.5 \text{ nm}}$$

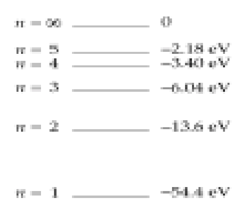
40.48 Each atom gives up its kinetic energy in emitting a photon, so

$$\frac{1}{2} m v^2 = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(1.216 \times 10^{-7} \text{ m})} = 1.63 \times 10^{-18} \text{ J}$$

$$v = \boxed{4.42 \times 10^4 \text{ m/s}}$$

40.49 (a) The energy levels of a hydrogen-like ion whose charge number is Z are given by

$$E_n = (-13.6 \text{ eV})\frac{Z^2}{n^2}$$



Thus for He lium $(Z = 2)$, the energy levels are

$$\boxed{E_n = -\frac{54.4 \text{ eV}}{n^2} \quad n = 1, 2, 3, \ldots}$$

(b) For He^+, $Z = 2$, so we see that the ionization energy (the energy required to take the electron from the $n = 1$ to the $n = \infty$ state is

$$E = E_\infty - E_1 = 0 - \frac{(-13.6 \text{ eV})(2)^2}{(1)^2} = \boxed{54.4 \text{ eV}}$$

40.50 $r = \dfrac{n^2\hbar^2}{Zm_e k_e e^2} = \dfrac{n^2}{Z}\left(\dfrac{\hbar^2}{m_e k_e e^2}\right)$; $n = 1$

$$r = \frac{1}{Z}\left[\frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{(9.11\times10^{-31}\ \text{kg})(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}\right] = \frac{5.29\times10^{-11}\ \text{m}}{Z}$$

(a) For He^{+}, $Z = 2$ $\quad r = \dfrac{5.29\times10^{-11}\ \text{m}}{2} = 2.65\times10^{-11}\ \text{m} = \boxed{0.0265\ \text{nm}}$

(b) For Li^{2+}, $Z = 3$ $\quad r = \dfrac{5.29\times10^{-11}\ \text{m}}{3} = 1.77\times10^{-11}\ \text{m} = \boxed{0.0177\ \text{nm}}$

(c) For Be^{3+}, $Z = 4$ $\quad r = \dfrac{5.29\times10^{-11}\ \text{m}}{4} = 1.32\times10^{-11}\ \text{m} = \boxed{0.0132\ \text{nm}}$

40.51 Since $F = qvB = \dfrac{mv^2}{r}$ we have $qrB = mv$,

or $qr^2B = mvr = n\hbar$ so $\boxed{r_n = \sqrt{\dfrac{n\hbar}{qB}}}$

40.52 (a) The time for one complete orbit is: $T = \dfrac{2\pi r}{v}$

From Bohr's quantization postulate, $L = m_e vr = n\hbar$, we see that $v = \dfrac{n\hbar}{m_e r}$

Thus, the orbital period becomes:

$$T = \frac{2\pi m_e r^2}{n\hbar} = \frac{2\pi m_e (a_0 n^2)^2}{n\hbar} = \frac{2\pi m_e a_0^2}{\hbar}n^3 \quad \text{or} \quad T = t_0 n^3 \quad \text{where}$$

$$t_0 = \frac{2\pi m_e a_0^2}{\hbar} = \frac{2\pi(9.11\times10^{-31}\ \text{kg})(0.0529\times10^{-9}\ \text{m})^2}{(1.055\times10^{-34}\ \text{J}\cdot\text{s})} = \boxed{1.52\times10^{-16}\ \text{s}}$$

(b) With $n = 2$, we have $T = 8t_0 = 8(1.52\times10^{-16}\ \text{s}) = 1.21\times10^{-15}\ \text{s}$

Thus, if the electrons stay in the $n = 2$ state for 10 μs, it will make

$$\frac{10.0\times10^{-6}\ \text{s}}{1.21\times10^{-15}\ \text{s / rev}} = \boxed{8.23\times10^{9}\ \text{revolutions}} \text{ of the nucleus}$$

(c) $\boxed{\text{Yes, for } 8.23\times10^{9} \text{ "electron years"}}$

***40.53** $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(1.67\times10^{-27}\text{ kg})(1.00\times10^{6}\text{ m/s})} = \boxed{3.97\times10^{-13}\text{ m}}$

40.54 (a) $\dfrac{p^2}{2m} = (50.0)(1.60\times10^{-19}\text{ J})$

$p = 3.81\times10^{-24}\text{ kg}\cdot\text{m/s}$

$\lambda = \dfrac{h}{p} = \boxed{0.174\text{ nm}}$

(b) $\dfrac{p^2}{2m} = (50.0\times10^{3})(1.60\times10^{-19}\text{ J})$

$p = 1.20\times10^{-22}\text{ kg}\cdot\text{m/s}$

$\lambda = \dfrac{h}{p} = \boxed{5.49\times10^{-12}\text{ m}}$

The relativistic answer is slightly more precise:

$$\lambda = \frac{h}{p} = \frac{hc}{\left[(mc^2+K)^2 - m^2c^4\right]^{1/2}} = 5.37\times10^{-12}\text{ m}$$

***40.55** (a) Electron: $\lambda = \dfrac{h}{p}$ and $K = \frac{1}{2}m_e v^2 = \dfrac{m_e^2v^2}{2m_e} = \dfrac{p^2}{2m_e}$

so $p = \sqrt{2m_eK}$

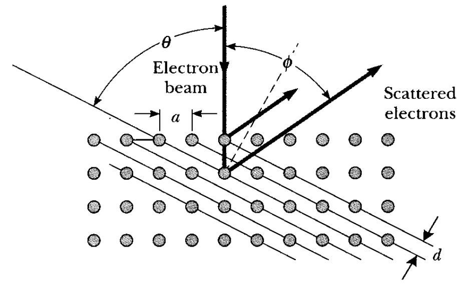
and $\lambda = \dfrac{h}{\sqrt{2m_eK}} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)(3.00)\left(1.60\times10^{-19}\text{ J}\right)}}$

$\lambda = 7.09\times10^{-10}\text{ m} = \boxed{0.709\text{ nm}}$

(b) Photon: $\lambda = c/f$ and $E = hf$ so $f = E/h$ and

$\lambda = \dfrac{hc}{E} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(3.00)(1.60\times10^{-19}\text{ J})} = 4.14\times10^{-7}\text{ m} = \boxed{414\text{ nm}}$

40.56 From the Bragg condition (Eq. 38.13),



$$m\lambda = 2d\sin\theta = 2d\cos(\phi/2)$$

But, $d = a\sin(\phi/2)$ where a is the lattice spacing. Thus, with $m = 1$,

$$\lambda = 2a\sin(\phi/2)\cos(\phi/2) = a\sin\phi$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\text{ kg})(54.0\times1.60\times10^{-19}\text{ J})}} = 1.67\times10^{-10}\text{ m}$$

Therefore, the lattice spacing is

$$a = \frac{\lambda}{\sin\phi} = \frac{1.67\times10^{-10}\text{ m}}{\sin 50.0°} = 2.18\times10^{-10}\text{ m} = \boxed{0.218\text{ nm}}$$

***40.57** (a) $\lambda \sim 10^{-14}$ m or less.

$$p = \frac{h}{\lambda} \sim \frac{6.6\times10^{-34}\text{ J}\cdot\text{s}}{10^{-14}\text{ m}} = 10^{-19}\text{ kg}\cdot\text{m/s or more.}$$

The energy of the electron is

$$E = \sqrt{p^2c^2 + m_e^2c^4} \sim \left[(10^{-19})^2(3\times10^8)^2 + (9\times10^{-31})^2(3\times10^8)^4\right]^{1/2} \sim 10^{-11}\text{ J} \sim 10^8\text{ eV or more,}$$

so that $K = E - m_ec^2 \sim 10^8\text{ eV} - (0.5\times10^6\text{ eV})$ $\boxed{\sim 10^8\text{ eV}}$ or more.

(b) The electric potential energy of the electron would be

$$U_e = \frac{k_eq_1q_2}{r} \sim \frac{(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10^{-19}\text{ C})(-e)}{10^{-14}\text{ m}} \sim -10^5\text{ eV}$$

With its kinetic energy much larger than its negative potential energy, $\boxed{\text{the electron would immediately escape the nucleus}}$.

Goal Solution

The nucleus of an atom is on the order of 10^{-14} m in diameter. For an electron to be confined to a nucleus, its de Broglie wavelength would have to be of this order of magnitude or smaller. (a) What would be the kinetic energy of an electron confined to this region? (b) On the basis of this result, would you expect to find an electron in a nucleus? Explain.

G: The de Broglie wavelength of a normal ground-state orbiting electron is on the order 10^{-10} m (the diameter of a hydrogen atom), so with a shorter wavelength, the electron would have more kinetic energy if confined inside the nucleus. If the kinetic energy is much greater than the potential energy from its attraction with the positive nucleus, then the electron will escape from its electrostatic potential well.

O: If we try to calculate the velocity of the electron from the de Broglie wavelength, we find that

$$v = \frac{h}{m_e \lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(10^{-14}\ \text{m}\right)} = 7.27\times10^{10}\ \text{m / s}$$

which is not possible since it exceeds the speed of light. Therefore, we must use the relativistic energy expression to find the kinetic energy of this fast-moving electron.

A: (a) The relativistic kinetic energy of a particle is $K = E - mc^2$, where $E^2 = (pc)^2 + \left(mc^2\right)^2$, and the momentum is $p = h/\lambda$:

$$p = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} = 6.63\times10^{-20}\ \text{N}\cdot\text{s}$$

$$E = \sqrt{\left(1.99\times10^{-11}\ \text{J}\right)^2 + \left(8.19\times10^{-14}\ \text{J}\right)^2} = 1.99\times10^{-11}\ \text{J}$$

$$K = E - mc^2 = \frac{1.99\times10^{-11}\ \text{J} - 8.19\times10^{-14}\ \text{J}}{1.60\times10^{-19}\ \text{J / eV}} = 124\ \text{MeV} \sim 100\ \text{MeV}$$

(b) The electrostatic potential energy of the electron 10^{-14} m away from a positive proton is :

$$U = -k_e e^2/r = -\frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{10^{-14}\ \text{m}} = -2.30\times10^{-14}\ \text{J} \sim -0.1\ \text{MeV}$$

L: Since the kinetic energy is nearly 1000 times greater than the potential energy, the electron would immediately escape the proton's attraction and would not be confined to the nucleus.

It is also interesting to notice in the above calculations that the rest energy of the electron is negligible compared to the momentum contribution to the total energy.

40.58 (a) From $E = \gamma m_e c^2$ $\qquad \gamma = \dfrac{20.0 \times 10^3 \text{ MeV}}{0.511 \text{ MeV}} = \boxed{3.91 \times 10^4}$

(b) $p \approx \dfrac{E}{c}$ $\qquad$ (for $m_e c^2 << pc$)

$$p = \frac{(2.00 \times 10^4 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{3.00 \times 10^8 \text{ m/s}} = \boxed{1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}}$$

(c) $$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}} = \boxed{6.22 \times 10^{-17} \text{ m}}$$

Since the size of a nucleus is on the order of 10^{-14} m, the 20-GeV electrons would be small enough to go through the nucleus.

40.59 (a) $E^2 = p^2c^2 + m^2c^4$

with $E = hf$, $\qquad p = \dfrac{h}{\lambda}$, $\qquad$ and $\qquad mc = \dfrac{h}{\lambda_C}$

so $\qquad h^2 f^2 = \dfrac{h^2c^2}{\lambda^2} + \dfrac{h^2c^2}{\lambda_C^2}$ $\qquad$ and $\qquad \left(\dfrac{f}{c}\right)^2 = \dfrac{1}{\lambda^2} + \dfrac{1}{\lambda_C^2}$ $\qquad$ (Eq. 1)

(b) For a photon $\qquad f/c = 1/\lambda$.

The third term $1/\lambda_C$ in Equation 1 for electrons and other massive particles shows that $\boxed{\text{they will always have a different frequency from photons of the same wavelength}}$

40.60 (a) The wavelength of the student is $\lambda = h/p = h/mv$. If w is the width of the diffraction aperture, then we need $w \le 10.0\lambda = 10.0(h/mv)$, so that

$$v \le 10.0\frac{h}{mw} = 10.0\left(\frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(80.0 \text{ kg})(0.750 \text{ m})}\right) = \boxed{1.10 \times 10^{-34} \text{ m/s}}$$

(b) Using $t = \dfrac{d}{v}$ we get: $\qquad t \ge \dfrac{0.150 \text{ m}}{1.10 \times 10^{-34} \text{ m/s}} = \boxed{1.36 \times 10^{33} \text{ s}}$

(c) $\boxed{\text{No}}$. The minimum time to pass through the door is over 10^{15} times the age of the Universe.

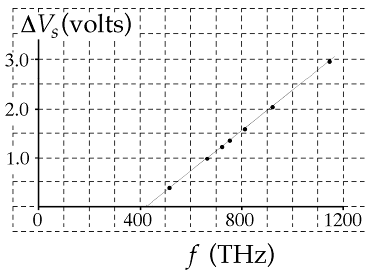
40.61 The de Broglie wavelength is: $\lambda = \dfrac{h}{\gamma m_e v}$

The Compton wavelength is: $\lambda_C = \dfrac{h}{m_e c}$

Therefore, we see that to have $\lambda = \lambda_C$, it is necessary that $\gamma v = c$.

This give: $\dfrac{v}{\sqrt{1 - v^2 / c^2}} = c$, or $\left(\dfrac{v}{c}\right)^2 = 1 - \left(\dfrac{v}{c}\right)^2$, yielding $v = \boxed{\dfrac{c}{\sqrt{2}}}$.

40.62 $\Delta V_S = \left(\dfrac{h}{e}\right) f - \dfrac{\phi}{e}$



From two points on the graph $0 = \left(\dfrac{h}{e}\right)\left(4.1 \times 10^{14} \text{ Hz}\right) - \dfrac{\phi}{e}$

and $3.3 \text{ V} = \left(\dfrac{h}{e}\right)\left(12 \times 10^{14} \text{ Hz}\right) - \dfrac{\phi}{e}$

Combining these two expressions we find:

(a) $\phi = \boxed{1.7 \text{ eV}}$

(b) $\dfrac{h}{e} = \boxed{4.2 \times 10^{-15} \text{ V} \cdot \text{s}}$

(c) At the cutoff wavelength $\dfrac{hc}{\lambda_c} = \phi = \left(\dfrac{h}{e}\right)\dfrac{ec}{\lambda_c}$

$$\lambda_c = (4.2 \times 10^{-15} \text{ V} \cdot \text{s})(1.6 \times 10^{-19} \text{ C}) \frac{(3.0 \times 10^8 \text{ m/s})}{(1.7 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})} = \boxed{730 \text{ nm}}$$

40.63 $K_{\max} = \dfrac{q^2 B^2 R^2}{2 m_e} = \dfrac{(1.60 \times 10^{-19} \text{ C})^2 (2.00 \times 10^{-5} \text{ T})^2 (0.200 \text{ m})^2}{2(9.11 \times 10^{-31} \text{ kg})} = 2.25 \times 10^{-19} \text{ J} = 1.40 \text{ eV} = hf - \phi$

$$\phi = hf - K_{\max} = \frac{hc}{\lambda} - K_{\max} = \frac{(4.14 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{450 \times 10^{-9} \text{ m}} - 1.40 \text{ eV} = \boxed{1.36 \text{ eV}}$$

40.64 From the path the electrons follow in the magnetic field, the maximum kinetic energy is seen to be:

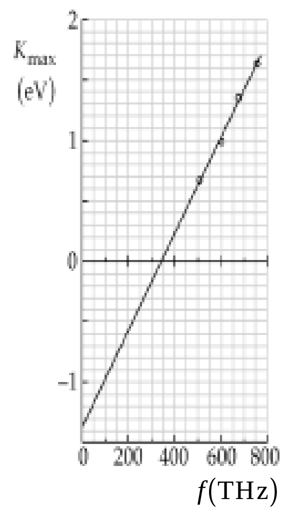
$$K_{\max} = \frac{e^2B^2R^2}{2m_e}$$

From the photoelectric equation, $K_{\max} = hf - \phi = \dfrac{hc}{\lambda} - \phi$

Thus, the work function is $\phi = \dfrac{hc}{\lambda} - K_{\max} = \boxed{\dfrac{hc}{\lambda} - \dfrac{e^2B^2R^2}{2m_e}}$

40.65 We want an Einstein plot of $K_{\max}$ versus f

λ, nm	f, 10^{14} Hz	$K_{\max}$, eV
588	5.10	0.67
505	5.94	0.98
445	6.74	1.35
399	7.52	1.63



(a) slope $= \dfrac{0.402 \text{ eV}}{10^{14} \text{ Hz}} \pm 8\%$

(b) $e(\Delta V_S) = hf - \phi$

$$h = (0.402)\frac{1.60 \times 10^{-19} \text{ J} \cdot \text{s}}{10^{14}} = \boxed{6.4 \times 10^{-34} \text{ J} \cdot \text{s} \pm 8\%}$$

(c) $K_{\max} = 0$ at $f \approx 344 \times 10^{12}$ Hz

$$\phi = hf = 2.32 \times 10^{-19} \text{ J} = \boxed{1.4 \text{ eV}}$$

40.66 $$\Delta\lambda = \frac{h}{m_p c}(1 - \cos\theta) = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})}{(1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m / s})}(0.234) = 3.09 \times 10^{-16} \text{ m}$$

$$\lambda_0 = \frac{hc}{E_0} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m / s})}{(200 \text{ MeV})(1.60 \times 10^{-13} \text{ J / MeV})} = 6.20 \times 10^{-15} \text{ m}$$

$$\lambda' = \lambda_0 + \Delta\lambda = 6.51 \times 10^{-15} \text{ m}$$

(a) $E_\gamma = \dfrac{hc}{\lambda'} = \boxed{191 \text{ MeV}}$

(b) $K_p = \boxed{9.20 \text{ MeV}}$

40.67 M is the mass of the positron which equals m_e, the mass of the electron.

So $$\mu \equiv \text{reduced mass} = \frac{m_e M}{m_e + M} = \frac{m_e}{2}$$

$$r_{\text{pos}} = \frac{n^2\hbar^2}{Z\mu k_e e^2} = \frac{n^2\hbar^2}{Z(m_e/2)k_e e^2} = \frac{2n^2\hbar^2}{Zm_e k_e e^2} \quad \text{or} \quad r_{\text{pos}} = 2r_{\text{Hyd}} = \boxed{\left(1.06\times10^{-10}\ \text{m}\right)n^2}$$

This is the separation of the two particles.

$$E_{\text{pos}} = -\frac{\mu k_e{}^2 e^4}{2\hbar^2}\frac{1}{n^2} = -\frac{m_e k_e{}^2 e^4}{4\hbar^2}\left(\frac{1}{n^2}\right); \quad n = 1, 2, 3, \ldots \quad \text{or} \quad E_{\text{pos}} = \frac{E_{\text{Hyd}}}{2} = \boxed{\frac{-6.80\ \text{eV}}{n^2}}$$

Goal Solution

Positronium is a hydrogen-like atom consisting of a positron (a positively charged electron) and an electron revolving around each other. Using the Bohr model, find the allowed radii (relative to the center of mass of the two particles) and the allowed energies of the system.

G: Since we are told that positronium is like hydrogen, we might expect the allowed radii and energy levels to be about the same as for hydrogen: $r = a_0 n^2 = \left(5.29\times10^{-11}\ \text{m}\right)n^2$ and $E_n = (-13.6\ \text{eV})/n^2$.

O: Similar to the textbook calculations for hydrogen, we can use the quantization of angular momentum of positronium to find the allowed radii and energy levels.

A: Let r represent the distance between the electron and the positron. The two move in a circle of radius $r/2$ around their center of mass with opposite velocities. The total angular momentum is quantized according to

$$L_n = \frac{mvr}{2} + \frac{mvr}{2} = n\hbar, \quad \text{where} \quad n = 1, 2, 3, \ldots$$

For each particle, $\Sigma F = ma$ expands to $$\frac{k_e e^2}{r^2} = \frac{mv^2}{r/2}$$

We can eliminate $v = \dfrac{n\hbar}{mr}$ to find $$\frac{k_e e^2}{r} = \frac{2mn^2\hbar}{m^2 r^2}$$

So the separation distances are $$r = \frac{2n^2\hbar^2}{mk_e e^2} = 2a_0 n^2 = \left(1.06\times10^{-10}\ \text{m}\right)n^2$$

The orbital radii are $r/2 = a_0 n^2$, the same as for the electron in hydrogen.

The energy can be calculated from $$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 - \frac{k_e e^2}{r}$$

Since $mv^2 = \dfrac{k_e e^2}{2r}$, $$E = \frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = -\frac{k_e e^2}{2r} = \frac{-k_e e^2}{4a_0 n^2} = -\frac{6.80\ \text{eV}}{n^2}$$

L: It appears that the allowed radii for positronium are twice as large as for hydrogen, while the energy levels are half as big. One way to explain this is that in a hydrogen atom, the proton is much more massive than the electron, so the proton remains nearly stationary with essentially no kinetic energy. However, in positronium, the positron and electron have the same mass and therefore both have kinetic energy that separates them from each other and reduces their total energy compared with hydrogen.

40.68 Isolate the terms involving ϕ in Equations 40.12 and 40.13. Square and add to eliminate ϕ.

$$h^2\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]=\gamma^2 m_e^2 v^2$$

Solve for $\frac{v^2}{c^2}=\frac{b}{\left(b+c^2\right)}$:

$$b=\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

Substitute into Eq. 40.11:

$$1+\left(\frac{h}{m_e c}\right)\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]=\gamma=\sqrt{1-\frac{b}{b+c^2}}$$

Square each side:

$$c^2+\frac{2hc}{m_e}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]+\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]^2=c^2+\left(\frac{h^2}{m_e^2}\right)\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

From this we get Eq. 40.10:

$$\lambda'-\lambda_0=\left(h/m_e c\right)\left[1-\cos\theta\right]$$

40.69 $hf=\Delta E=\frac{4\pi^2 m_e k_e^2 e^4}{2h^2}\left(\frac{1}{(n-1)^2}-\frac{1}{n^2}\right)$ so $f=\frac{2\pi^2 m_e k_e^2 e^4}{h^3}\left(\frac{2n-1}{(n-1)^2 n^2}\right)$

As n approaches infinity, we have f approaching $\frac{2\pi^2 m_e k_e^2 e^4}{h^3}\frac{2}{n^3}$

The classical frequency is $f=\frac{v}{2\pi r}=\frac{1}{2\pi}\sqrt{\frac{k_e e^2}{m_e}}\frac{1}{r^{3/2}}$ where $r=\frac{n^2h^2}{4\pi m_e k_e e^2}$

Using this equation to eliminate r from the expression for f, $f=\frac{2\pi^2 m_e k_e^2 e^4}{h^3}\frac{2}{n^3}$

40.70 Show that if all of the energy of a photon is transmitted to an electron, momentum will not be conserved.

Energy: $\frac{hc}{\lambda_0}=\frac{hc}{\lambda'}+K_e=m_e c^2(\gamma-1)$ if $\frac{hc}{\lambda'}=0$ (1)

Momentum: $\frac{h}{\lambda_0}=\frac{h}{\lambda'}+\gamma m_e v=\gamma m_e v$ if $\lambda'=\infty$ (2)

From (1), $\gamma=\frac{h}{\lambda_0 m_e c}+1$ (3)

$$v=c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2} \quad (4)$$

Substitute (3) and (4) into (2) and show the inconsistency:

$$\frac{h}{\lambda_0}=\left(1+\frac{h}{\lambda_0 m_e c}\right)m_e c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2}=\frac{\lambda_0 m_e c+h}{\lambda_0}\sqrt{\frac{h(h+2\lambda_0 m_e c)}{(h+\lambda_0 m_e c)^2}}=\frac{h}{\lambda_0}\sqrt{\frac{h+2\lambda_0 m_e c}{h}}$$

This is impossible, so all of the energy of a photon cannot be transmitted to an electron.

40.71 Begin with momentum expressions: $p=\frac{h}{\lambda}$, and $p=\gamma mv=\gamma mc\left(\frac{v}{c}\right)$.

Equating these expressions, $\gamma\left(\dfrac{v}{c}\right)=\left(\dfrac{h}{mc}\right)\dfrac{1}{\lambda}=\dfrac{\lambda_C}{\lambda}$

Thus, $\dfrac{(v/c)^2}{1-(v/c)^2}=\left(\dfrac{\lambda_C}{\lambda}\right)^2$

or $$\left(\frac{v}{c}\right)^2=\left(\frac{\lambda_C}{\lambda}\right)^2-\left(\frac{\lambda_C}{\lambda}\right)^2\left(\frac{v}{c}\right)^2=\frac{(\lambda_C/\lambda)^2}{1+(\lambda_C/\lambda)^2}=\frac{1}{(\lambda/\lambda_C)^2+1}$$

giving $\boxed{v=\dfrac{c}{\sqrt{1+(\lambda\ /\ \lambda_C)^2}}}$

40.72 (a) The energy of the ground state is: $E_1=-\dfrac{hc}{\lambda_{\text{series limit}}}=-\dfrac{1240\text{ eV}\cdot\text{nm}}{152.0\text{ nm}}=\boxed{-8.16\text{ eV}}$

From the wavelength of the L_α line, we see: $E_2-E_1=\dfrac{hc}{\lambda}=\dfrac{1240\text{ nm}\cdot\text{eV}}{202.6\text{ nm}}=6.12\text{ eV}$

$E_2=E_1+6.12\text{ eV}=\boxed{-2.04\text{ eV}}$

Using the wavelength of the L_β line gives: $E_3-E_1=\dfrac{1240\text{ nm}\cdot\text{eV}}{170.9\text{ nm}}=7.26\text{ eV}$

so $E_3=\boxed{-0.902\text{ eV}}$

Next, using the L_γ line gives: $E_4-E_1=\dfrac{1240\text{ nm}\cdot\text{eV}}{162.1\text{ nm}}=7.65\text{ eV}$

and $E_4=\boxed{-0.508\text{ eV}}$

From the L_δ line, $E_5-E_1=\dfrac{1240\text{ nm}\cdot\text{eV}}{158.3\text{ nm}}=7.83\text{ eV}$

so $E_5=\boxed{-0.325\text{ eV}}$

(b) For the Balmer series, $\dfrac{hc}{\lambda}=E_i-E_2$, or $\lambda=\dfrac{1240\text{ nm}\cdot\text{eV}}{E_i-E_2}$

For the α line, $E_i=E_3$ and so $\lambda_\alpha=\dfrac{1240\text{ nm}\cdot\text{eV}}{(-0.902\text{ eV})-(-2.04\text{ eV})}=\boxed{1090\text{ nm}}$

Similarly, the wavelengths of the β line, γ line, and the short wavelength limit are found to be: $\boxed{811\text{ nm}}$, $\boxed{724\text{ nm}}$, and $\boxed{609\text{ nm}}$.

(c) Computing 60.0% of the wavelengths of the spectral lines shown on the energy-level diagram gives:

0.600(202.6 nm) = $\boxed{122\text{ nm}}$, 0.600(170.9 nm) = $\boxed{103\text{ nm}}$, 0.600(162.1 nm) = $\boxed{97.3\text{ nm}}$, 0.600(158.3 nm) = $\boxed{95.0\text{ nm}}$, and 0.600(152.0 nm) = $\boxed{91.2\text{ nm}}$.

These are seen to be the wavelengths of the α, β, γ, and δ lines as well as the short wavelength limit for the Lyman series in Hydrogen.

(d) The observed wavelengths could be the result of Doppler shift when the source moves away from the Earth. The required speed of the source is found from

$$\frac{f'}{f}=\frac{\lambda}{\lambda'}=\sqrt{\frac{1-(v/c)}{1+(v/c)}}=0.600 \qquad \text{yielding} \qquad \boxed{v = 0.471c}$$

40.73 (a) Starting with Planck's law, $$I(\lambda,T)=\frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T}-1\right]}$$

the total power radiated per unit area $$\int_0^\infty I(\lambda,T)\,d\lambda=\int_0^\infty \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T}-1\right]}\,d\lambda .$$

Change variables by letting $$x=\frac{hc}{\lambda k_B T}$$

and $$dx=-\frac{hc\,d\lambda}{k_B T\lambda^2}$$

Note that as λ varies from $0\to\infty$, x varies from $\infty\to 0$.

Then $$\int_0^\infty I(\lambda,T)\,d\lambda=-\frac{2\pi k_B^4T^4}{h^3c^2}\int_\infty^0\frac{x^3}{\left(e^x-1\right)}dx=\frac{2\pi k_B^4T^4}{h^3c^2}\left(\frac{\pi^4}{15}\right)$$

Therefore, $$\boxed{\int_0^\infty I(\lambda,T)\,d\lambda=\left(\frac{2\pi^5k_B^4}{15h^3c^2}\right)T^4=\sigma T^4}$$

(b) From part (a), $$\sigma=\frac{2\pi^5k_B^4}{15h^3c^2}=\frac{2\pi^5\left(1.38\times10^{-23}\text{ J/K}\right)^4}{15\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^3\left(3.00\times10^8\text{ m/s}\right)^2}$$

$$\sigma=\boxed{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}$$

***40.74** Planck's law states $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]} = 2\pi hc^2 \lambda^{-5}\left[e^{hc/\lambda k_B T} - 1\right]^{-1}$

To find the wavelength at which this distribution has a maximum, compute

$$\frac{dI}{d\lambda} = 2\pi hc^2\left\{-5\lambda^{-6}\left[e^{hc/\lambda k_B T} - 1\right]^{-1} - \lambda^{-5}\left[e^{hc/\lambda k_B T} - 1\right]^{-2} e^{hc/\lambda k_B T}\left(-\frac{hc}{\lambda^2 k_B T}\right)\right\} = 0$$

$$\frac{dI}{d\lambda} = \frac{2\pi hc^2}{\lambda^6\left[e^{hc/\lambda k_B T} - 1\right]}\left\{-5 + \frac{hc}{\lambda k_B T}\frac{e^{hc/\lambda k_B T}}{\left[e^{hc/\lambda k_B T} - 1\right]}\right\} = 0$$

Letting $x = \dfrac{hc}{\lambda k_B T}$, the condition for a maximum becomes $\dfrac{xe^x}{e^x - 1} = 5$.

We zero in on the solution to this transcendental equation by iterations as shown in the table below. The solution is found to be

x	$xe^x/(e^x-1)$
4.00000	4.0746294
4.50000	4.5505521
5.00000	5.0339183
4.90000	4.9367620
4.95000	4.9853130
4.97500	5.0096090
4.96300	4.9979452
4.96900	5.0037767
4.96600	5.0008609
4.96450	4.9994030
4.96550	5.0003749
4.96500	4.9998890
4.96525	5.0001320
4.96513	5.0000153
4.96507	4.9999570
4.96510	4.9999862
4.965115	5.0000008

$$x = \frac{hc}{\lambda_{\max} k_B T} = 4.965115 \qquad \text{and} \qquad \lambda_{\max} T = \frac{hc}{4.965115\, k_B}$$

Thus, $\lambda_{\max} T = \dfrac{\left(6.626075 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.997925 \times 10^8\ \text{m/s}\right)}{4.965115\left(1.380658 \times 10^{-23}\ \text{J/K}\right)} = \boxed{2.897755 \times 10^{-3}\ \text{m}\cdot\text{K}}$

This result is very close to Wien's experimental value of $\lambda_{\max} T = 2.898 \times 10^{-3}\ \text{m}\cdot\text{K}$

for this constant.

40.75

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda' - \lambda_0$$

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0 + \Delta\lambda} = hc\left[\lambda_0 + \frac{h}{m_e c}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1}$$

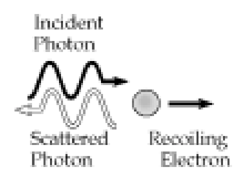
$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1} = E_0\left[1 + \frac{E_0}{m_e c^2}(1-\cos\theta)\right]^{-1}$$

40.76

$$r_1 = \frac{(1)^2\hbar^2}{Z\mu k_e e^2} = \frac{\hbar^2}{(82)(207 m_e)k_e e^2} = \frac{a_0}{(82)(207)} = \frac{0.0529\text{ nm}}{(82)(207)} = \boxed{3.12\text{ fm}}$$

$$E_1 = \frac{-13.6\text{ eV}}{(1)^2}\left(\frac{207}{1}\right)\left(\frac{82}{1}\right)^2 = \boxed{-18.9\text{ MeV}}$$

40.77 This is a case of Compton scattering with a scattering angle of 180°.



$$\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c}\left(1-\cos 180°\right) = \frac{2h}{m_e c}$$

$$E_0 = \frac{hc}{\lambda_0}, \quad \text{so} \quad \lambda_0 = \frac{hc}{E_0} \quad \text{and} \quad \lambda' = \lambda_0 + \Delta\lambda = \frac{hc}{E_0} + \frac{2h}{m_e c} = \frac{hc}{E_0}\left(1 + \frac{2E_0}{m_e c^2}\right)$$

The kinetic energy of the recoiling electron is then

$$K = E_0 - \frac{hc}{\lambda'} = E_0 - \frac{E_0}{\left(1 + 2E_0/m_e c^2\right)} = E_0\left(\frac{1 + 2E_0/m_e c^2 - 1}{1 + 2E_0/m_e c^2}\right) = \frac{2E_0^2/m_e c^2}{1 + 2E_0/m_e c^2}$$

Defining $a \equiv E_0/m_e c^2$, the kinetic energy can be written as

$$K = \frac{2E_0 a}{1+2a} = \frac{2(hf)a}{1+2a} = \boxed{2hfa(1+2a)^{-1}}$$

where f is the frequency of the incident photon.

40.78 (a) Planck's radiation law predicts maximum intensity at a wavelength $\lambda_{\max}$ we find from

$$\frac{dI}{d\lambda}=0=\frac{d}{d\lambda}\left\{2\pi hc^2\lambda^{-5}\left[e^{(hc/\lambda k_B T)}-1\right]^{-1}\right\}$$

$$0=2\pi hc^2\lambda^{-5}(-1)\left[e^{(hc/\lambda k_B T)}-1\right]^{-2}e^{(hc/\lambda k_B T)}\left(-hc/\lambda^2 k_B T\right)+2\pi hc^2(-5)\lambda^{-6}\left[e^{(hc/\lambda k_B T)}-1\right]^{-1}$$

or $$\frac{-hc\,e^{(hc/\lambda k_B T)}}{\lambda^7 k_B T\left[e^{(hc/\lambda k_B T)}-1\right]^2}+\frac{5}{\lambda^6\left[e^{(hc/\lambda k_B T)}-1\right]}=0$$

which reduces to $$5\left(\lambda k_B T/hc\right)\left[e^{(hc/\lambda k_B T)}-1\right]=e^{(hc/\lambda k_B T)}$$

Define $x=hc/\lambda k_B T$. Then we require $5e^x-5=xe^x$.

Numerical solution of this transcendental equation gives $x=4.965$ to four digits. So $\lambda_{\max}=hc/4.965k_B T$, in agreement with Wien's law.

The intensity radiated over all wavelengths is $$\int_0^\infty I(\lambda,T)d\lambda=A+B=\int_0^\infty\frac{2\pi hc^2\,d\lambda}{\lambda^5\left[e^{(hc/\lambda k_B T)}-1\right]}$$

Again, define $x=hc/\lambda k_B T$ so $\lambda=hc/xk_B T$ and $d\lambda=-\left(hc/x^2 k_B T\right)dx$

Then, $$A+B=\int_{x=\infty}^0\frac{-2\pi hc^2x^5k_B^5T^5\;hc\,dx}{h^5c^5x^2\,k_B T\left(e^x-1\right)}=\frac{2\pi k_B^4T^4}{h^3c^2}\int_0^\infty\frac{x^3dx}{\left(e^x-1\right)}$$

The integral is tabulated as $\pi^4/15$, so (in agreement with Stefan's law) $$A+B=\frac{2\pi^5k_B^4T^4}{15h^3c^2}$$

The intensity radiated over wavelengths shorter than $\lambda_{\max}$ is

$$\int_0^{\lambda_{\max}}I(\lambda,T)d\lambda=A=\int_0^{\lambda_{\max}}\frac{2\pi hc^2\,d\lambda}{\lambda^5\left[e^{(hc/\lambda k_B T)}-1\right]}$$

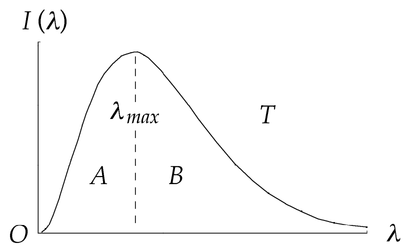
With $x=hc/\lambda k_B T$, this similarly becomes $$A=\frac{2\pi k_B^4T^4}{h^3c^2}\int_{4.965}^\infty\frac{x^3\,dx}{e^x-1}$$

So the fraction of power or of intensity radiated at wavelengths shorter than $\lambda_{\max}$ is

$$\frac{A}{A+B}=\frac{\dfrac{2\pi k_B^4T^4}{h^3c^2}\left(\dfrac{\pi^4}{15}-\displaystyle\int_0^{4.965}\frac{x^3\,dx}{e^x-1}\right)}{\dfrac{2\pi^5k_B^4T^4}{15h^3c^2}}=\boxed{1-\frac{15}{\pi^4}\int_0^{4.965}\frac{x^3\,dx}{e^x-1}}$$

(b) Here are some sample values of the integrand, along with a sketch of the curve:

x	$x^3\left(e^x-1\right)^{-1}$
0.000	0.00
0.100	9.51×10^{-3}
0.200	3.61×10^{-2}
1.00	0.582
2.00	1.25
3.00	1.42
4.00	1.19
4.90	0.883
4.965	0.860



Approximating the integral by trapezoids gives $\frac{A}{A+B}\approx 1-\frac{15}{\pi^4}(4.870)=\boxed{0.2501}$

40.79 $\lambda_C=\frac{h}{m_e c}$ and $\lambda=\frac{h}{p}$: $\frac{\lambda_C}{\lambda}=\frac{h/m_e c}{h/p}=\frac{p}{m_e c}$;

$E^2=c^2p^2+(m_ec^2)^2$: $p=\sqrt{\frac{E^2}{c^2}-(m_ec)^2}$

$$\frac{\lambda_C}{\lambda}=\frac{1}{m_ec}\sqrt{\frac{E^2}{c^2}-(m_ec)^2}=\sqrt{\frac{1}{(m_ec)^2}\left[\frac{E^2}{c^2}-(m_ec)^2\right]}=\sqrt{\left(\frac{E}{m_ec^2}\right)^2-1}$$

40.80 $p=mv=\sqrt{2mE}=\sqrt{2\left(1.67\times10^{-27}\text{ kg}\right)(0.0400\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}$

$\lambda=\frac{h}{mv}=1.43\times10^{-10}\text{ m}=\boxed{0.143\text{ nm}}$

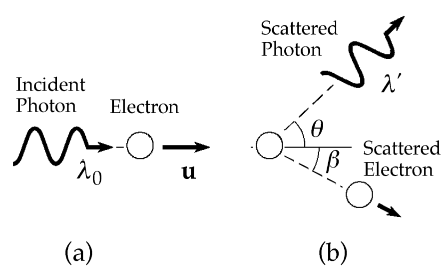
This is of the same order of magnitude as the spacing between atoms in a crystal so diffraction should appear.

40.81 Let u' represent the final speed of the electron and let $\gamma'=\left(1-u'^2/c^2\right)^{-1/2}$. We must eliminate β and u' from the three conservation equations:

$\frac{hc}{\lambda_0}+\gamma m_ec^2=\frac{hc}{\lambda'}+\gamma' m_ec^2$ [1]

$\frac{h}{\lambda_0}+\gamma m_eu-\frac{h}{\lambda'}\cos\theta=\gamma' m_eu'\cos\beta$ [2]

$\frac{h}{\lambda'}\sin\theta=\gamma' m_eu'\sin\beta$ [3]



Square Equations [2] and [3] and add:

$$\frac{h^2}{\lambda_0^2}+\gamma^2 m_e^2u^2+\frac{h^2}{\lambda'^2}+\frac{2h\gamma m_e u}{\lambda_0}-\frac{2h^2\cos\theta}{\lambda_0\lambda'}-\frac{2h\gamma m_e u\cos\theta}{\lambda'}=\gamma'^2 m_e^2u'^2$$

$$\frac{h^2}{\lambda_0^2}+\frac{h^2}{\lambda'^2}+\gamma^2 m_e^2u^2+\frac{2h\gamma m_e u}{\lambda_0}-\frac{2h\gamma m_e u\cos\theta}{\lambda'}-\frac{2h^2\cos\theta}{\lambda_0\lambda'}=\frac{m_e^2u'^2}{1-u'^2/c^2}$$

Call the left-hand side b. Then $b-\frac{bu'^2}{c^2}=m_e^2u'^2$ and $u'^2=\frac{b}{m_e^2+b/c^2}=\frac{c^2b}{m_e^2c^2+b}$

Now square Equation [1] and substitute to eliminate γ':

$$\frac{h^2}{\lambda^2}+\gamma^2m_e^2c^2+\frac{h^2}{\lambda'^2}+\frac{2h\gamma m_e c}{\lambda_0}-\frac{2h^2}{\lambda_0\lambda'}-\frac{2h\gamma m_e c}{\lambda'}=\frac{m_e^2c^2}{1-u'^2/c^2}=m_e^2c^2+b$$

So we have

$$\frac{h^2}{\lambda_0^2}+\frac{h^2}{\lambda'^2}+\gamma^2m_e^2c^2+\frac{2h\gamma m_e c}{\lambda_0}-\frac{2h\gamma m_e c}{\lambda'}-\frac{2h^2}{\lambda_0\lambda'}$$
$$=m_ec^2+\frac{h^2}{\lambda_0^2}+\frac{h^2}{\lambda'^2}+\gamma^2m_e^2u^2+\frac{2h\gamma m_e u}{\lambda_0}-\frac{2h\gamma m_e u\cos\theta}{\lambda'}-\frac{2h^2\cos\theta}{\lambda_0\lambda'}$$

Multiply through by $\lambda_0\lambda'/m_e^2c^2$

$$\lambda_0\lambda'\gamma^2+\frac{2h\lambda'\gamma}{m_ec}-\frac{2h\lambda_0\gamma}{m_ec}-\frac{2h^2}{m_e^2c^2}=\lambda_0\lambda'+\frac{\lambda_0\lambda'\gamma^2u^2}{c^2}+\frac{2h\lambda' u\gamma}{m_ec^2}-\frac{2h\gamma\lambda_0u\cos\theta}{m_ec^2}-\frac{2h^2\cos\theta}{m_e^2c^2}$$

$$\lambda_0\lambda'\left(\gamma^2-1-\frac{\gamma^2u^2}{c^2}\right)+\frac{2h\gamma\lambda'}{m_ec}\left(1-\frac{u}{c}\right)=\frac{2h\gamma\lambda_0}{m_ec}\left(1-\frac{u\cos\theta}{c}\right)+\frac{2h^2}{m_e^2c^2}(1-\cos\theta)$$

The first term is zero. Then $\lambda'=\lambda_0\left(\frac{1-(u\cos\theta)/c}{1-u/c}\right)+\frac{h\gamma^{-1}}{m_ec}\left(\frac{1}{1-u/c}\right)(1-\cos\theta)$

Since $\gamma^{-1}=\sqrt{1-(u/c)^2}=\sqrt{(1-u/c)(1+u/c)}$

this result may be written as $\boxed{\lambda'=\lambda_0\left(\frac{1-(u\cos\theta)/c}{1-u/c}\right)+\frac{h}{m_ec}\sqrt{\frac{1+u/c}{1-u/c}}(1-\cos\theta)}$

Chapter 41 Solutions

41.1 (a) $\lambda = \dfrac{h}{mv} = \dfrac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{\left(1.67 \times 10^{-27} \text{ kg}\right)(0.400 \text{ m/s})} = \boxed{9.92 \times 10^{-7} \text{ m}}$

(b) For destructive interference in a multiple-slit experiment,

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

with $m = 0$ for the first minimum. Then,

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = 0.0284°$$

$\dfrac{y}{L} = \tan\theta$ so $y = L\tan\theta = (10.0 \text{ m})(\tan\ 0.0284°) = \boxed{4.96 \text{ mm}}$

(c) We cannot say the neutron passed through one slit. We can only say it passed through the slits.

41.2 Consider the first bright band away from the center: $d \sin\theta = m\lambda$

$$\left(6.00 \times 10^{-8} \text{ m}\right)\sin\left(\tan^{-1}\left[\frac{0.400}{200}\right]\right) = 1\lambda = 1.20 \times 10^{-10} \text{ m}$$

$\lambda = \dfrac{h}{m_e v}$ so $m_e v = \dfrac{h}{\lambda}$ and

$$K = \tfrac{1}{2} m_e v^2 = \frac{m_e{}^2 v^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} = e(\Delta V)$$

$$\Delta V = \frac{h^2}{2em_e\lambda^2} = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{2\left(1.60 \times 10^{-19} \text{ C}\right)\left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.20 \times 10^{-10} \text{ m}\right)^2} = \boxed{105 \text{ V}}$$

41.3 (a) The wavelength of a non-relativistic particle of mass m is given by $\lambda = h/p = h/\sqrt{2mK}$ where the kinetic energy K is in joules. If the neutron kinetic energy K_n is given in electron volts, its kinetic energy in joules is $K = \left(1.60 \times 10^{-19} \text{ J/eV}\right)K_n$ and the equation for the wavelength becomes

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2\left(1.67 \times 10^{-27} \text{ kg}\right)\left(1.60 \times 10^{-19} \text{ J/eV}\right)K_n}} = \boxed{\frac{2.87 \times 10^{-11}}{\sqrt{K_n}} \text{ m}}$$

where K_n is expressed in electron volts.

(b) If $K_n = 1.00 \text{ keV} = 1000 \text{ eV}$, then

41.4 $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}}$, so $K = \dfrac{h^2}{2m\lambda^2}$

If the particles are electrons and $\lambda \sim 0.1 \text{ nm} = 10^{-10} \text{ m}$, the kinetic energy in electron volts is

$$K = \frac{\left(6.626\times10^{-34} \text{ J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31} \text{ kg}\right)\left(10^{-10} \text{ m}\right)^2}\left(\frac{1 \text{ eV}}{1.602\times10^{-19} \text{ J}}\right) = \boxed{\sim 10^2 \text{ eV}}$$

41.5 $\lambda = \dfrac{h}{p}$ $\qquad p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{1.00\times10^{-11} \text{ m}} = 6.63\times10^{-23} \text{ kg}\cdot\text{m/s}$

(a) electrons: $K_e = \dfrac{p^2}{2m_e} = \dfrac{\left(6.63\times10^{-34} \text{ J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\right)} \text{ J} = \boxed{15.1 \text{ keV}}$

The relativistic answer is more precisely correct:

$$K_e = \left(p^2c^2 + m_e{}^2c^4\right)^{1/2} - m_ec^2 = 14/.9 \text{ keV}$$

(b) photons: $E_\gamma = pc = \left(6.63\times10^{-23}\right)\left(3.00\times10^8\right) = \boxed{124 \text{ keV}}$

41.6 The theoretical limit of the electron microscope is the wavelength of the electrons. If $K_e = 40.0 \text{ keV}$, then $E = K_e + m_ec^2 = 551 \text{ keV}$ and

$$p = \frac{1}{c}\sqrt{E^2 - m_e{}^2c^4} = \frac{\sqrt{(551 \text{ keV})^2 - (511 \text{ keV})^2}}{3.00\times10^8 \text{ m/s}}\left(\frac{1.60\times10^{-16} \text{ J}}{1.00 \text{ keV}}\right) = 1.10\times10^{-22} \text{ kg}\cdot\text{m/s}$$

The electron wavelength, and hence the theoretical limit of the microscope, is then

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{1.10\times10^{-22} \text{ kg}\cdot\text{m/s}} = 6.03\times10^{-12} \text{ m} = \boxed{6.03 \text{ pm}}$$

41.7 $E = K + m_ec^2 = 1.00 \text{ MeV} + 0.511 \text{ MeV} = 1.51 \text{ MeV}$

$p^2c^2 = \sqrt{E^2 - m_e{}^2c^4} = \sqrt{(1.51 \text{ MeV})^2 - (0.511 \text{ MeV})^2}$ so
$p = 1.42 \text{ MeV}/c$

$$\lambda = \frac{h}{p} = \frac{hc}{1.42 \text{ MeV}} = \frac{\left(6.626\times10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00\times10^8 \text{ m/s}\right)}{\left(1.42\times10^6\right)\left(1.60\times10^{-19} \text{ J}\right)} = 8.74\times10^{-13} \text{ m}$$

Suppose the array is like a flat diffraction grating with openings 0.250 nm apart:
$d\sin\theta = m\lambda$

41.8 (a) $\Delta p \Delta x = m\Delta v \Delta x \geq \hbar/2$ so

$$\Delta v \geq \frac{h}{4\pi m \Delta x} = \frac{2\pi \text{ J}\cdot\text{s}}{4\pi(2.00 \text{ kg})(1.00 \text{ m})} = \boxed{0.250 \text{ m/s}}$$

(b) The duck might move by $(0.25 \text{ m/s})(5 \text{ s}) = 1.25 \text{ m}$. With original position uncertainty of 1.00 m, we can think of Δx growing to $1.00 \text{ m} + 1.25 \text{ m} = \boxed{2.25 \text{ m}}$

41.9 For the electron,

$$\Delta p = m_e \Delta v = \left(9.11\times10^{-31} \text{ kg}\right)(500 \text{ m/s})\left(1.00\times10^{-4}\right) = 4.56\times10^{-32} \text{ kg}\cdot\text{m/s}$$

$$\Delta x = \frac{h}{4\pi\Delta p} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{4\pi\left(4.56\times10^{-32} \text{ kg}\cdot\text{m/s}\right)} = \boxed{1.16 \text{ mm}}$$

For the bullet, $\Delta p = m\Delta v = (0.0200 \text{ kg})(500 \text{ m/s})\left(1.00\times10^{-4}\right) = 1.00\times10^{-3} \text{ kg}\cdot\text{m/s}$

$$\Delta x = \frac{h}{4\pi\Delta p} = \boxed{5.28\times10^{-32} \text{ m}}$$

Goal Solution

An electron ($m_e = 9.11\times10^{-31}$ kg) and a bullet $(m = 0.0200 \text{ kg})$ each have a speed of 500 m/s, accurate to within 0.0100%. Within what limits could we determine the position of the objects?

G: It seems reasonable that a tiny particle like an electron could be located within a more narrow region than a bigger object like a bullet, but we often find that the realm of the very small does not obey common sense.

O: Heisenberg's uncertainty principle can be used to find the uncertainty in position from the uncertainty in the momentum.

A: The uncertainty principle states: $\Delta x \Delta p_x \geq \hbar/2$ where $\Delta p_x = m\Delta v$ and $h = \hbar/2\pi$.

Both the electron and bullet have a velocity uncertainty,
$\Delta v = (0.000100)(500 \text{ m/s}) = 0.0500 \text{ m/s}$

For the electron, the minimum uncertainty in position is

$$\Delta x = \frac{h}{4\pi m\Delta v} = \frac{6.63\times10^{-34} \text{ J}\cdot\text{s}}{4\pi\left(9.11\times10^{-31} \text{ kg}\right)(0.0500 \text{ m/s})} = 1.16 \text{ mm}$$

For the bullet,

$$\Delta x = \frac{h}{4\pi m\Delta v} = \frac{6.63\times10^{-34} \text{ J}\cdot\text{s}}{4\pi(0.0200 \text{ kg})(0.0500 \text{ m/s})} = 5.28\times10^{-32} \text{ m}$$

41.10 $\frac{\Delta y}{x} = \frac{\Delta p_y}{p_x}$ and $d\Delta p_y \ge h/4\pi$ Eliminate Δp_y and solve for x.

$$x = 4\pi p_x(\Delta y)\frac{d}{h} = 4\pi\left(1.00\times10^{-3}\text{ kg}\right)(100\text{ m/s})\left(1.00\times10^{-2}\text{ m}\right)\frac{\left(2.00\times10^{-3}\text{ m}\right)}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)} =$$

$\boxed{3.79\times10^{28}\text{ m}}$

This is 190 times greater than the diameter of the Universe!

41.11 $\Delta p\Delta x \ge \frac{\hbar}{2}$ so $\Delta p = m_e\Delta v \ge \frac{h}{4\pi\,\Delta x}$

$$\Delta v \ge \frac{h}{4\pi\,m_e\Delta x} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(9.11\times10^{-31}\text{ kg}\right)\left(5.00\times10^{-11}\text{ m}\right)} = \boxed{1.16\times10^{6}\text{ m/s}}$$

41.12 With $\Delta x = 2\times10^{-15}$ m, the uncertainty principle requires $\Delta p_x \ge \frac{\hbar}{2\Delta x} = 2.6\times10^{-20}\text{ kg}\cdot\text{m/s}$
The average momentum of the particle bound in a stationary nucleus is zero. The uncertainty in momentum measures the root-mean-square momentum, so we take $p_{rms} = 3\times10^{-20}\text{ kg}\cdot\text{m/s}$.
For an electron, the non-relativistic approximation $p = m_e v$ would predict $v = 3\times10^{10}$ m/s, while v cannot be greater than c.

Thus, a better solution would be $$E = \left[\left(m_e c^2\right)^2 + (pc)^2\right]^{1/2} = 56\text{ MeV} = \gamma\,m_e c^2$$

$$\gamma \approx 110 = \frac{1}{\sqrt{1 - v^2/c^2}}$$ so

$$v \approx 0.99996c$$

For a proton, $v = p/m$ gives $v = 1.8\times10^{7}$ m / s, less than one-tenth the speed of light.

41.13 (a) At the top of the ladder, the woman holds a pellet inside a small region Δx_i. Thus, the uncertainty principle requires her to release it with typical horizontal momentum $\Delta p_x = m\Delta v_x = \hbar/2\Delta x_i$. It falls to the floor in time given by $H = 0 + \frac{1}{2}gt^2$ as $t = \sqrt{2H/g}$, so the total width of the impact points is

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}} = \Delta x_i + \frac{A}{\Delta x_i},$$ where

$A = \frac{\hbar}{\ldots}\sqrt{\frac{2H}{\ldots}}$

so $\Delta x_i = \sqrt{A}$, and the minimum width of the impact points is

$$\left(\Delta x_f\right)_{\min} = \left(\Delta x_i + \frac{A}{\Delta x_i}\right)\Bigg|_{\Delta x_i = \sqrt{A}} = 2\sqrt{A} = \boxed{\left(\frac{2\hbar}{m}\right)^{1/2} = \left(\frac{2H}{g}\right)^{1/4}}$$

(b) $$\left(\Delta x_f\right)_{\min} = \left[\frac{2\left(1.0546\times10^{-34}\ \mathrm{J\cdot s}\right)}{5.00\times10^{-4}\ \mathrm{kg}}\right]^{1/2}\left[\frac{2(2.00\ \mathrm{m})}{9.80\ \mathrm{m/s^2}}\right]^{1/4} = \boxed{5.19\times10^{-16}\ \mathrm{m}}$$

41.14 Probability $P = \int_{-a}^{a} |\psi(x)|^2 = \int_{-a}^{a} \frac{a}{\pi(x^2 + a^2)} dx = \left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right) \tan^{-1} \frac{x}{a}\Big|_{-a}^{a}$

$$P = \frac{1}{\pi}\left[\tan^{-1} 1 - \tan^{-1}(-1)\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \boxed{1/2}$$

41.15 (a) $\psi(x) = A \sin\left(\frac{2\pi x}{\lambda}\right) = A \sin\left(5.00 \times 10^{10} x\right)$

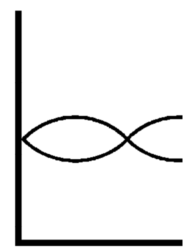
so $\frac{2\pi}{\lambda} = 5.00 \times 10^{10} \text{ m}^{-1}$ $\qquad \lambda = \frac{2\pi}{\left(5.00 \times 10^{10}\right)} = \boxed{1.26 \times 10^{-10} \text{ m}}$

(b) $p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.26 \times 10^{-10} \text{ m}} = \boxed{5.27 \times 10^{-24} \text{ kg} \cdot \text{m/s}}$

(c) $m = 9.11 \times 10^{-31}$ kg

$$K = \frac{p^2}{2m} = \frac{\left(5.27 \times 10^{-24} \text{ kg} \cdot \text{m/s}\right)^2}{\left(2 \times 9.11 \times 10^{-31} \text{ kg}\right)} = 1.52 \times 10^{-17} \text{ J} = \frac{1.52 \times 10^{-17} \text{ J}}{1.602 \times 10^{-19} \text{ J/eV}} = \boxed{95.5 \text{ eV}}$$

41.16 For an electron to "fit" into an infinitely deep potential well, an integral number of half-wavelengths must equal the width of the well.



$\frac{n\lambda}{2} = 1.00 \times 10^{-9} \text{ m}$ $\qquad$ so

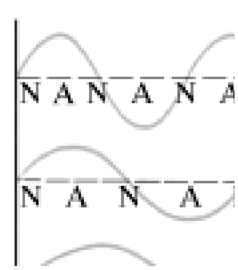
$$\lambda = \frac{2.00 \times 10^{-9}}{n} = \frac{h}{p}$$

(a) Since $K = \frac{p^2}{2m_e} = \frac{\left(h^2/\lambda^2\right)}{2m_e} = \frac{h^2}{2m_e} \frac{n^2}{\left(2 \times 10^{-9}\right)^2} = \left(0.377 n^2\right) \text{ eV}$

For $K \approx 6 \text{ eV}$, $\boxed{n = 4}$

(b) With $n = 4$, $\boxed{K = 6.03 \text{ eV}}$

41.17 (a) We can draw a diagram that parallels our treatment of standing mechanical waves. In each state, we measure the distance d from one node to another (N to N), and base our solution upon that:



Since $\qquad d_{N \text{ to } N} = \frac{\lambda}{2} \qquad$ and

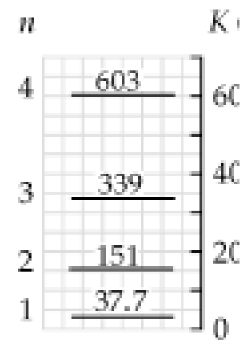
h

Next,

$$K = \frac{p^2}{2m_e} = \frac{h^2}{8m_e d} = \frac{1}{d^2}\left[\frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)}\right]$$

Evaluating, $K = \dfrac{6.02\times10^{-38}\ \text{J}\cdot\text{m}^2}{d^2}$

$$K = \frac{3.77\times10^{-19}\ \text{eV}\cdot\text{m}^2}{d^2}$$



In state 1, $d = 1.00\times10^{-10}\ \text{m}$
$K_1 = 37.7\ \text{eV}$

In state 2, $d = 5.00\times10^{-11}\ \text{m}$
$K_2 = 151\ \text{eV}$

In state 3, $d = 3.33\times10^{-11}\ \text{m}$
$K_3 = 339\ \text{eV}$

In state 4, $d = 2.50\times10^{-11}\ \text{m}$
$K_4 = 603\ \text{eV}$

(b) When the electron falls from state 2 to state 1, it puts out energy

$$E = 151\ \text{eV} - 37.7\ \text{eV} = 113\ \text{eV} = hf = \frac{hc}{\lambda}$$

into emitting a photon of wavelength

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(113\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 11.0\ \text{nm}$$

The wavelengths of the other spectral lines we find similarly:

Transition						
$E(\text{eV})$						
$\lambda(\text{nm})$						

41.18 $E_1 = 2.00 \text{ eV} = 3.20 \times 10^{-19} \text{ J}$

For the ground-state, $$E_1 = \frac{h^2}{8m_e L^2}$$

(a) $$L = \frac{h}{\sqrt{8m_e E_1}} = 4.34 \times 10^{-10} \text{ m} = \boxed{0.434 \text{ nm}}$$

(b) $$\Delta E = E_2 - E_1 = 4\left(\frac{h^2}{8m_e L^2}\right) - \left(\frac{h^2}{8m_e L^2}\right) = \boxed{6.00 \text{ eV}}$$

41.19 $$\Delta E = \frac{hc}{\lambda} = \left(\frac{h^2}{8m_e L^2}\right)\left[2^2 - 1^2\right] = \frac{3h^2}{8m_e L^2}$$

$$L = \sqrt{\frac{3h\lambda}{8m_e c}} = 7.93 \times 10^{-10} \text{ m} = \boxed{0.793 \text{ nm}}$$

41.20 $$\Delta E = \frac{hc}{\lambda} = \left(\frac{h^2}{8m_e L^2}\right)\left[2^2 - 1^2\right] = \frac{3h^2}{8m_e L^2}$$

so $$L = \boxed{\sqrt{\frac{3h\lambda}{8m_e c}}}$$

41.21 $$E_n = \frac{n^2 h^2}{8mL^2}$$

so $$\Delta E = E_2 - E_1 = \frac{3h^2}{8mL^2} = \frac{3(hc)^2}{8mc^2 L^2}$$

and $$\Delta E = hf = \frac{hc}{\lambda}$$

Hence, $$\lambda = \frac{8mc^2 L^2}{3hc} = \frac{8\left(938 \times 10^6 \text{ eV}\right)\left(1.00 \times 10^{-5} \text{ nm}\right)^2}{3(1240 \text{ eV} \cdot \text{nm})}$$

$$\lambda = \boxed{2.02 \times 10^{-4} \text{ nm (gamma ray)}}$$

$$E = \frac{hc}{\lambda} = \boxed{6.15 \text{ MeV}}$$

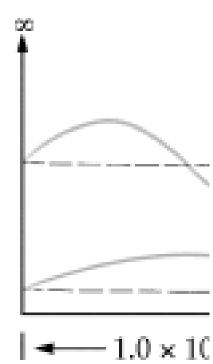


Figure for Goal Soluti

Goal Solution
The nuclear potential energy that binds protons and neutrons in a nucleus is often approximated by a square well. Imagine a proton confined in an infinitely high square well of width 10.0 fm, a typical nuclear diameter. Calculate the wavelength and energy associated with the photon emitted when the proton moves from the $n = 2$ state to the ground state. In what region of the electromagnetic spectrum does this wavelength belong?

G: Nuclear radiation from nucleon transitions is usually in the form of high energy gamma rays with short wavelengths.

O: The energy of the particle can be obtained from the wavelengths of the standing waves corresponding to each level. The transition between energy levels will result in the emission of a photon with this energy difference.

A: At level 1, the node-to-node distance of the standing wave is $1.00\times10^{-14}\text{ m}$, so the wavelength is twice this distance: $h/p = 2.00\times10^{-14}\text{ m}$. The proton's kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{-14}\text{ m}\right)^2} = \frac{3.29\times10^{-13}\text{ J}}{1.60\times10^{-19}\text{ J/eV}} = 2.06\text{ MeV}$$

In the first excited state, level 2, the node-to-node distance is two times smaller than in state 1. The momentum is two times larger and the energy is four times larger: $K = 8.23\text{ MeV}$.

The proton has mass, has charge, moves slowly compared to light in a standing-wave state, and stays inside the nucleus. When it falls from level 2 to level 1, its energy change is

$$2.06\text{ MeV} - 8.23\text{ MeV} = -6.17\text{ MeV}$$

Therefore, we know that a photon (a traveling wave with no mass and no charge) is emitted at the speed of light, and that it has an energy of $+6.17\text{ MeV}$.

Its frequency is
$$f = \frac{E}{h} = \frac{\left(6.17\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{6.63\times10^{-34}\text{ J}\cdot\text{s}} = 1.49\times10^{21}\text{ Hz}$$

and its wavelength is
$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{1.49\times10^{21}\text{ s}^{-1}} = 2.02\times10^{-13}\text{ m}$$

This is a gamma ray, according to Figure 34.17.

L: The radiated photons are energetic gamma rays as we expected for a nuclear transition. In the above calculations, we assumed that the proton was not relativistic ($v < 0.1c$), but we should check this assumption for the highest energy state we examined ($n = 2$):

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(8.23\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{1.67\times10^{-27}\text{ kg}}} = 3.97\times10^{7}\text{ m/s} = 0.133c$$

This appears to be a borderline case where we should probably use relativistic equations, but our classical treatment should give reasonable results, within $(0.133)^2 = 1\%$ accuracy.

41.22 $\lambda = 2D$ for the lowest energy state

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2}{8mD} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{8\left[4\left(1.66\times10^{-27}\text{ kg}\right)\right]\left(1.00\times10^{-14}\text{ m}\right)^2} = 8.27\times10^{-14}\text{ J} =$$

$$\boxed{0.517\text{ MeV}}$$

$$p = \frac{h}{\lambda} = \frac{h}{2D} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\left(1.00\times10^{-14}\text{ m}\right)} = \boxed{3.31\times10^{-20}\text{ kg}\cdot\text{m/s}}$$

41.23 $E_n = \left(\dfrac{h^2}{8mL^2}\right)n^2$

$$E_1 = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{-14}\text{ m}\right)^2} = 8.21\times10^{-14}\text{ J}$$

$E_1 = \boxed{0.513\text{ MeV}}$ $E_2 = 4E_1 = \boxed{2.05\text{ MeV}}$ $E_3 = 9E_1 = \boxed{4.62\text{ MeV}}$

41.24 (a) $$\langle x\rangle = \int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_0^L x\left(\frac{1}{2} - \frac{1}{2}\cos\frac{4\pi x}{L}\right)dx$$

$$\langle x\rangle = \frac{1}{L}\frac{x^2}{2}\bigg|_0^L - \frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L} + \cos\frac{4\pi x}{L}\right]_0^L = \boxed{L/2}$$

(b) $$\text{Probability} = \int_{0.490L}^{0.510L}\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \left[\frac{1}{L}x - \frac{1}{L}\frac{L}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.490L}^{0.510L}$$

$$\text{Probability} = 0.20 - \frac{1}{4\pi}\left(\sin 2.04\pi - \sin 1.96\pi\right) = \boxed{5.26\times10^{-5}}$$

(c) $$\text{Probability} = \left[\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.240L}^{0.260L} = \boxed{3.99\times10^{-2}}$$

(d) In the $n = 2$ graph in Figure 41.11 (b), it is more probable to find the particle either near

$x = \dfrac{L}{4}$ or $x = \dfrac{3L}{4}$ than at the center, where the probability density is zero.

$$\int_{x=0}^{L} A^2 \sin^2\left(\frac{n\pi x}{L}\right)dx = A^2\left(\frac{L}{2}\right) = 1 \qquad \text{or} \qquad \boxed{A = \sqrt{\frac{2}{L}}}$$

41.26 The desired probability is $P = \int_{x=0}^{x=L/4} |\psi|^2 dx = \frac{2}{L}\int_0^{L/4} \sin^2\left(\frac{2\pi x}{L}\right)dx$

where $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Thus,

$$P = \left(\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right)\Bigg|_0^{L/4} = \left(\frac{1}{4} - 0 - 0 + 0\right) = \boxed{0.250}$$

41.27 In $0 \le x \le L$, the argument $2\pi x/L$ of the sine function ranges from 0 to 2π. The probability density $(2/L)\sin^2(2\pi x/L)$ reaches maxima at $\sin\theta = 1$ and $\sin\theta = -1$ at

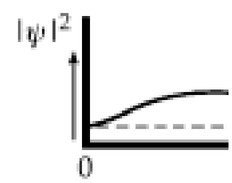
$$\frac{2\pi x}{L} = \frac{\pi}{2} \qquad \text{and} \qquad \frac{2\pi x}{L} = \frac{3\pi}{2}$$

$\therefore$ The most probable positions of the particle are at $\boxed{x = \frac{L}{4} \quad \text{and} \quad x = \frac{3L}{4}}$

41.28 (a) The probability is $P = \int_0^{L/3} |\psi|^2 dx = \int_0^{L/3} \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_0^{L/3}\left(\frac{1}{2} - \frac{1}{2}\cos\frac{2\pi x}{L}\right)dx$

$$P = \left(\frac{x}{L} - \frac{1}{2\pi}\sin\frac{2\pi x}{L}\right)\Bigg|_0^{L/3} = \left(\frac{1}{3} - \frac{1}{2\pi}\sin\frac{2\pi}{3}\right) = \left(\frac{1}{3} - \frac{\sqrt{3}}{4\pi}\right) = \boxed{0.196}$$

(b) The probability density is symmetric about $x = L/2$. Thus, the probability of finding the particle between $x = 2L/3$ and $x = L$ is the same 0.196. Therefore, the probability of finding it in the range $L/3 \le x \le 2L/3$ is $P = 1.00 - 2(0.196) = 0.609$.



(c) Classically, the electron moves back and forth with constant speed between the walls, and the probability of finding the electron is the same for all points between the walls. Thus, the classical probability of finding the electron in any range equal to one-third of the available space is $P_{\text{classical}} = \boxed{1/3}$.

41.29 The ground state energy of a particle (mass m) in a 1-dimensional box of width L is $E_1 = \dfrac{h^2}{8mL^2}$.

(a) For a proton $\left(m = 1.67 \times 10^{-27}\ \text{kg}\right)$ in a 0.200-nm wide box:

$$E_1 = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(2.00 \times 10^{-10}\ \text{m}\right)^2} = 8.22 \times 10^{-22}\ \text{J} = \boxed{5.13 \times 10^{-3}\ \text{eV}}$$

(b) For an electron $\left(m = 9.11 \times 10^{-31}\ \text{kg}\right)$ in the same size box:

$$E_1 = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(2.00 \times 10^{-10}\ \text{m}\right)^2} = 1.51 \times 10^{-18}\ \text{J} = \boxed{9.41\ \text{eV}}$$

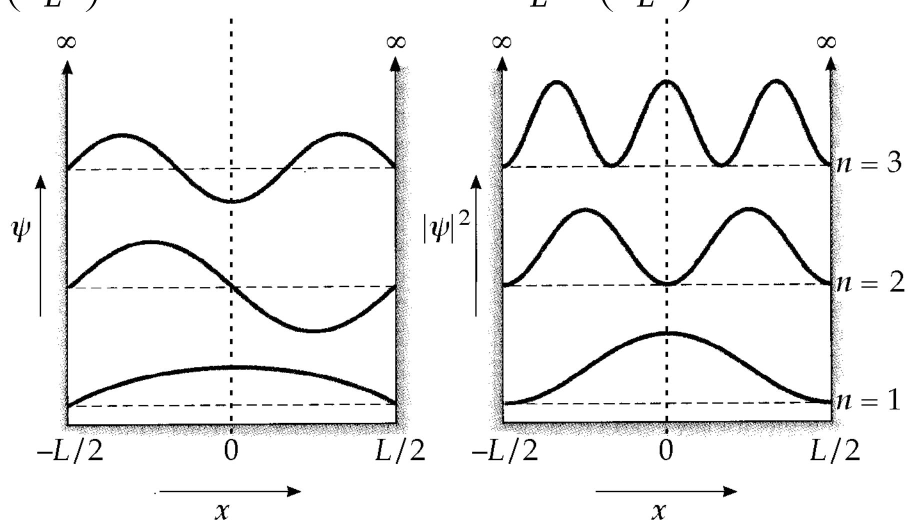
(c) The electron has a much higher energy because it is much less massive.

41.30 (a)

$$\psi_1(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right) \qquad P_1(x) = \left|\psi_1(x)\right|^2 = \frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$$

$$\psi_2(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right) \qquad P_2(x) = \left|\psi_2(x)\right|^2 = \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$$

$$\psi_3(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right) \qquad P_3(x) = \left|\psi_3(x)\right|^2 = \frac{2}{L}\cos\left(\frac{3\pi x}{L}\right)$$



41.31 We have $\psi = Ae^{i(kx-\omega t)}$ and $\dfrac{\partial^2\psi}{\partial x^2} = -k^2\psi$

Schrödinger's equation: $$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi = \frac{2m}{\hbar^2}(E-U)\psi$$

Since $k^2 = \dfrac{(2\pi)^2}{\lambda^2} = \dfrac{(2\pi p)^2}{h^2} = \dfrac{p^2}{\hbar^2}$ and $(E-U) = p^2/2m$

41.32 $\psi(x)=A\cos kx+B\sin kx$ $\qquad \dfrac{\partial\psi}{\partial x}=-kA\sin kx+kB\cos kx$

$$\frac{\partial^2\psi}{\partial x^2}=-k^2A\cos kx-k^2B\sin kx \qquad -\frac{2m}{h^2}(E-U)\psi=-\frac{2mE}{h^2}(A\cos kx+B\sin kx)$$

Therefore the Schrödinger equation is satisfied if

$$\frac{\partial^2\psi}{\partial x^2}=\left(-\frac{2m}{h^2}\right)(E-U)\psi \qquad \text{or}$$

$$-k^2(A\cos kx+B\sin kx)=\left(-\frac{2mE}{h^2}\right)(A\cos kx+B\sin kx)$$

This is true as an identity (functional equality) for all x if $\boxed{E=\dfrac{h^2k^2}{2m}}$

41.33 Problem 45 in Ch. 16 helps students to understand how to draw conclusions from an identity.

(a) $\psi(x)=A\left(1-\dfrac{x^2}{L^2}\right)$ $\qquad \dfrac{d\psi}{dx}=-\dfrac{2Ax}{L^2}$ $\qquad \dfrac{\partial^2\psi}{\partial x^2}=-\dfrac{2A}{L^2}$

Schrödinger's equation $\qquad \dfrac{\partial^2\psi}{\partial x^2}=-\dfrac{2m}{h^2}(E-U)\psi$

becomes

$$-\frac{2A}{L^2}=\frac{2m}{h^2}EA\left(1-\frac{x^2}{L^2}\right)+\frac{2m}{h^2}\frac{\left(-h^2x^2\right)A\left(1-\frac{x^2}{L^2}\right)}{mL^2\left(L^2-x^2\right)}$$

$$-\frac{1}{L^2}=-\frac{mE}{h^2}+\frac{mEx^2}{h^2L^2}-\frac{x^2}{L^4}$$

This will be true for all x if both $\qquad \dfrac{1}{L^2}=\dfrac{mE}{h^2}$ $\qquad$ and $\qquad \dfrac{mE}{h^2L^2}-\dfrac{1}{L^4}=0$

Both of these conditions are satisfied for a particle of energy $\boxed{E=\dfrac{h^2}{L^2m}}$.

(b) For normalization,

$$1=\int_{-L}^{L}A^2\left(1-\frac{x^2}{L^2}\right)^2dx=A^2\int_{-L}^{L}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx$$

$$1=A^2\left[x-\frac{2x^3}{3L^2}+\frac{x^5}{5L^4}\right]_{-L}^{L}=A^2\left[L-\frac{2}{3}L+\frac{L}{5}+L-\frac{2}{3}L+\frac{L}{5}\right]=A^2\frac{16L}{15} \qquad \boxed{A=\sqrt{\frac{15}{16L}}}$$

(c)

$$P = \frac{47}{81} = \boxed{0.580}$$

41.34 (a) Setting the total energy E equal to zero and rearranging the Schrödinger equation to isolate the potential energy function gives

$$U(x)=\left(\frac{\hbar^2}{2m}\right)\frac{1}{\psi}\frac{d^2\psi}{dx^2}$$

If $\psi(x)=Axe^{-x^2/L^2}$ Then $\dfrac{d^2\psi}{dx^2}=\left(4Ax^3-6AxL^2\right)\dfrac{e^{-x^2/L^2}}{L^4}$

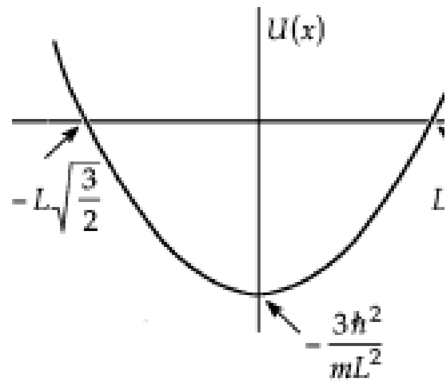
or

$$\frac{d^2\psi}{dx^2}=\frac{\left(4x^2-6L^2\right)}{L^4}\psi(x)$$

and

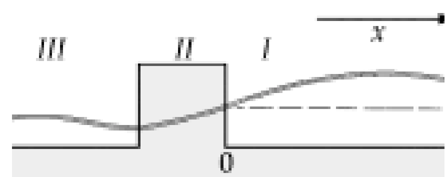
$$U(x)=\frac{\hbar^2}{2mL^2}\left(\frac{4x^2}{L^2}-6\right)$$

$\boxed{\text{See figure to the right}}$.



41.35 (a) $\boxed{\text{See figure to the right}}$.

(b) The wavelength of the transmitted wave traveling to the left is the same as the original wavelength, which equals $\boxed{2L}$.



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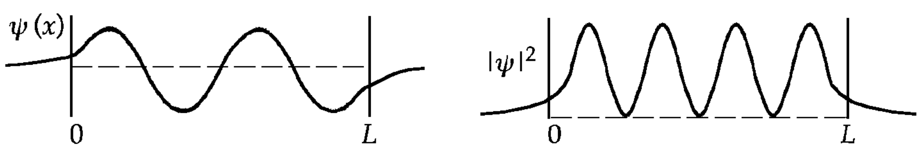
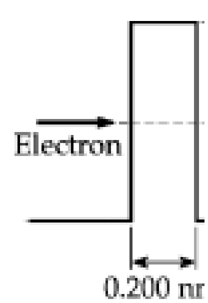


41.37 $T=e^{-2CL}$ (Use Equation 41.17)



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$$2CL=\frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(8.00\times10^{-19}\right)}}{1.055\times10^{-34}}\left(2.00\times10^{-10}\right)=4.58$$

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Goal Solution

An electron with kinetic energy $E = 5.00\ \text{eV}$ is incident on a barrier with thickness $L = 0.100\ \text{nm}$ and height $U = 10.0\ \text{eV}$ (Fig. P41.37). What is the probability that the electron (a) will tunnel through the barrier and (b) will be reflected?

G: Since the barrier energy is higher than the kinetic energy of the electron, transmission is not likely, but should be possible since the barrier is not infinitely high or thick.

O: The probability of transmission is found from the transmission coefficient equation 41.18.

A: The transmission coefficient is

$$C = \frac{\sqrt{2m(U-E)}}{h} = \frac{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)(10.0\ \text{eV} - 5.00\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}/2\pi} = 1.14\times10^{10}\ \text{m}^{-1}$$

(a) The probability of transmission is

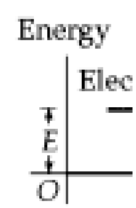
$$T = e^{-2CL} = e^{-2\left(1.14\times10^{10}\ \text{m}^{-1}\right)\left(2.00\times10^{-10}\ \text{m}\right)} = e^{-4.58} = 0.0103$$

(b) If the electron does not tunnel, it is reflected, with probability $1 - 0.0103 = 0.990$

L: Our expectation was correct: there is only a 1% chance that the electron will penetrate the barrier. This tunneling probability would be greater if the barrier were thinner, shorter, or if the kinetic energy of the electron were greater.

41.38

$$C = \frac{\sqrt{2\left(9.11\times10^{-31}\right)(5.00-4.50)\left(1.60\times10^{-19}\right)\ \text{kg}\cdot\text{m/s}}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}$$



$$T = e^{-2CL} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)\left(950\times10^{-12}\ \text{m}\right)\right] = \exp(-6.88)$$

$$T = \boxed{1.03\times10^{-3}}$$

41.39 From problem 38, $C = 3.62\times10^{9}\ \text{m}^{-1}$

$$10^{-6} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L\right]$$

Taking logarithms, $-13.816 = -2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L$

New $L = 1.91\ \text{nm}$

41.40 With the wave function proportional to e^{-CL}, the transmission coefficient and the tunneling current are proportional to $|\psi|^2$, to e^{-CL}.

Then, $$\frac{I(0.500\text{ nm})}{I(0.515\text{ nm})}=\frac{e^{-2(10.0\ /\text{nm})(0.500\text{ nm})}}{e^{-2(10.0\ /\text{nm})(0.515\text{ nm})}}=e^{20.0(0.015)}=\boxed{1.35}$$

41.41 With transmission coefficient e^{-CL}, the fractional change in transmission is

$$\frac{e^{-2(10.0\ /\text{nm})L}-e^{-2(10.0\ /\text{nm})(L+0.00200\text{ nm})}}{e^{-2(10.0\ /\text{nm})L}}=1-e^{29.0(0.00200)}=0.0392=\boxed{3.92\%}$$

41.42 $\psi=Be^{-(m\omega/2\hbar)x^2}$ so $\dfrac{d\psi}{dx}=-\left(\dfrac{m\omega}{\hbar}\right)x\psi$ and $\dfrac{d^2\psi}{dx^2}=\left(\dfrac{m\omega}{\hbar}\right)^2x^2\psi+\left(-\dfrac{m\omega}{\hbar}\right)\psi$

Substituting into Equation 41.19 gives

$$\left(\frac{m\omega}{\hbar}\right)^2x^2\psi+\left(-\frac{m\omega}{\hbar}\right)\psi=\left(\frac{2mE}{\hbar^2}\right)\psi+\left(\frac{m\omega}{\hbar}\right)^2x^2\psi$$

which is satisfied provided that $E=\dfrac{\hbar\omega}{2}$.

41.43 Problem 45 in Chapter 16 helps students to understand how to draw conclusions from an identity.

$\psi=Axe^{-bx^2}$ so $\dfrac{d\psi}{dx}=Ae^{-bx^2}-2bx^2Ae^{-bx^2}$

and

$$\frac{d^2\psi}{dx^2}=-2bxAe^{-bx^2}-4bxAe^{-bx^2}+4b^2x^3Ae^{-bx^2}=-6b\psi+4b^2x^2\psi$$

Substituting into Equation 41.19, $$-6b\psi+4b^2x^2\psi=-\left(\frac{2mE}{\hbar}\right)\psi+\left(\frac{m\omega}{\hbar}\right)^2x^2\psi$$

For this to be true as an identity, it must be true for all values of x.

So we must have both $-6b=-\dfrac{2mE}{\hbar^2}$ and $4b^2=\left(\dfrac{m\omega}{\hbar}\right)^2$

(a) Therefore $$\boxed{b=\frac{m\omega}{2\hbar}}$$

(b) and $$E=\frac{3b\hbar^2}{m}=\boxed{\frac{3}{2}\hbar\omega}$$

(c) The wave function is that of the $\boxed{\text{first excited state}}$.

41.44 The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator:

$$\frac{hc}{\lambda} = \hbar\omega = \hbar\sqrt{\frac{k}{m}} \qquad \text{so}$$

$$\lambda = 2\pi c\sqrt{\frac{m}{k}} = 2\pi\left(3.00\times10^{8}\text{ m/s}\right)\left(\frac{9.11\times10^{-31}\text{ kg}}{8.99\text{ N/m}}\right)^{1/2} = \boxed{600\text{ nm}}$$

41.45 (a) With $\psi = Be^{-(m\omega/2\hbar)x^2}$, the normalization condition $\int_{\text{all}}|\psi|^2 dx = 1$

becomes $$1 = \int_{-\infty}^{\infty} B^2 e^{-2(m\omega/2\hbar)x^2}dx = 2B^2\int_0^{\infty} e^{-2(m\omega/2\hbar)x^2}dx = 2B^2\frac{1}{2}\sqrt{\frac{\pi}{m\omega/\hbar}}$$

where Table B.6 in Appendix B was used to evaluate the integral.

Thus, $1 = B^2\sqrt{\dfrac{\pi\hbar}{m\omega}}$ and $\boxed{B = \left(\dfrac{m\omega}{\pi\hbar}\right)^{1/4}}$

(b) For small δ, the probability of finding the particle in the range $-\delta/2 \le x \le \delta/2$ is

$$\int_{-\delta/2}^{\delta/2}|\psi|^2 dx = \delta|\psi(0)|^2 = \delta B^2 e^{-0} = \boxed{\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}}$$

41.46 (a) With $<x>=0$ and $<p_x>=0$, the average value of x^2 is $(\Delta x)^2$ and the average value of $p_x{}^2$ is $(\Delta p_x)^2$. Then $\Delta x \ge \hbar/2\Delta p_x$ requires

$$E \ge \frac{p_x{}^2}{2m} + \frac{k}{2}\frac{\hbar^2}{4p_x{}^2} = \boxed{\frac{p_x{}^2}{2m} + \frac{k\hbar^2}{8p_x{}^2}}$$

(b) To minimize this as a function of $p_x{}^2$, we require $$\frac{dE}{dp_x{}^2} = 0 = \frac{1}{2m} + \frac{k\hbar^2}{8}(-1)\frac{1}{p_x{}^4}$$

Then $$\frac{k\hbar^2}{8p_x{}^4} = \frac{1}{2m}$$

$$p_x{}^2 = \left(\frac{2mk\hbar^2}{8}\right)^{1/2} = \frac{\hbar\sqrt{mk}}{2}$$

and $$E \ge \frac{\hbar\sqrt{mk}}{2(2m)} + \frac{k\hbar^2 3}{8\hbar\sqrt{mk}} = \frac{\hbar}{4}\sqrt{\frac{k}{m}} + \frac{\hbar}{4}\sqrt{\frac{k}{m}}.$$

41.47 Suppose the marble has mass 20 g. Suppose the wall of the box is 12 cm high and 2 mm thick. While it is inside the wall,

$$U = mgy = (0.02\text{ kg})(9.8\text{ m/s}^2)(0.12\text{ m}) = 0.0235\text{ J}$$

and $$E = K = \frac{1}{2}mv^2 = \frac{1}{2}(0.02\text{ kg})(0.8\text{ m/s})^2 = 0.0064\text{ J}$$

Then $$C = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2(0.02\text{ kg})(0.0171\text{ J})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = 2.5\times10^{32}\text{ m}^{-1}$$

and the transmission coefficient is

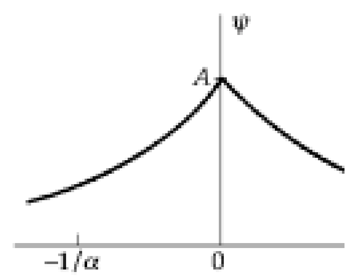
$$e^{-2CL} = e^{-2\left(2.5\times10^{32}\right)\left(2\times10^{-3}\right)} = e^{-10\times10^{29}} = e^{-2.30\left(4.3\times10^{29}\right)} = 10^{-4.3\times10^{29}} = \boxed{\sim 10^{-10^{30}}}$$

41.48 (a) $\lambda = 2L = \boxed{2.00\times10^{-10}\text{ m}}$

(b) $p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2.00\times10^{-10}\text{ m}} = \boxed{3.31\times10^{-24}\text{ kg}\cdot\text{m/s}}$

(c) $E = \dfrac{p^2}{2m} = \boxed{0.172\text{ eV}}$

41.49 (a) $\boxed{\text{See the first figure to the right}}$.



(b) $\boxed{\text{See the second figure to the right}}$.

(c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$

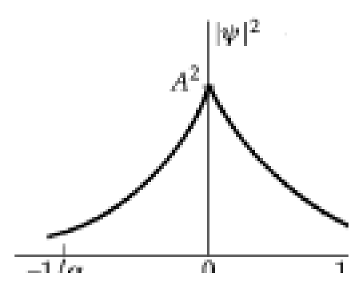
(d) Since ψ is symmetric,

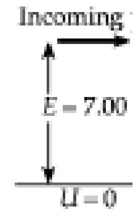
$$\int_{-\infty}^{\infty}|\psi|^2 dx = 2\int_0^{\infty}|\psi|^2 dx = 1$$

or

$$2A^2\int_0^{\infty} e^{-2\alpha x}dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty} - e^0\right) = 1$$

This gives $A = \sqrt{\alpha}$





(a) Use Schrödinger's equation

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-U)\psi$$

with solutions $\psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$ [region *I*]

$$\psi_2 = Ce^{ik_2x}$$ [region *II*]

Where $k_1 = \dfrac{\sqrt{2mE}}{\hbar}$

and

$$k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$$

Then, matching functions and derivatives at $x = 0$: $(\psi_1)_0 = (\psi_2)_0 \Rightarrow A + B = C$

and

$$\left(\frac{d\psi_1}{dx}\right)_0 = \left(\frac{d\psi_2}{dx}\right)_0 \Rightarrow k_1(A-B) = k_2C$$

Then $B = \dfrac{1-k_2/k_1}{1+k_2/k_1}A$

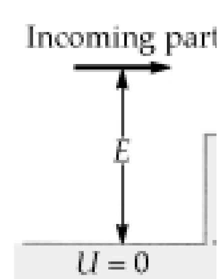
$$C = \frac{2}{1+k_2/k_1}A$$

Incident wave Ae^{ikx} reflects Be^{-ikx}, with probability $R = \dfrac{B^2}{A^2} = \dfrac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2} = \boxed{\dfrac{(k_1-k_2)^2}{(k_1+k_2)^2}}$

(b) With $E = 7.00$ eV and $U = 5.00$ eV, $\dfrac{k_2}{k_1} = \sqrt{\dfrac{E-U}{E}} = \sqrt{\dfrac{2.00}{7.00}} = 0.535$

The reflection probability is $R = \dfrac{(1-0.535)^2}{(1+0.535)^2} = \boxed{0.0920}$

The probability of transmission is $T = 1 - R = \boxed{0.908}$



41.51 $R = \dfrac{(k_1-k_2)^2}{(k_1+k_2)^2} = \dfrac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2}$

$\dfrac{\hbar^2k^2}{2m} = E - U$ for constant U

$\dfrac{h^2k_1^{\ 2}}{2m} = E$ since $U = 0$ (1)

Dividing (2) by (1), $$\frac{k_2{}^2}{k_1{}^2} = 1 - \frac{U}{E} = 1 - \frac{1}{2} = \frac{1}{2}$$ so

$$\frac{k_2}{k_1} = \frac{1}{\sqrt{2}}$$

and therefore,

$$R = \frac{\left(1 - 1/\sqrt{2}\right)^2}{\left(1 + 1/\sqrt{2}\right)^2} = \frac{\left(\sqrt{2} - 1\right)^2}{\left(\sqrt{2} + 1\right)^2} = \boxed{0.0294}$$

41.52 (a) The wave functions and probability densities are the same as those shown in the two lower curves in Figure 41.11 of the textbook.

(b)

$$P_1 = \int_{0.150\text{ nm}}^{0.350\text{ nm}} |\psi_1|^2 dx = \left(\frac{2}{1.00\text{ nm}}\right)\int_{0.150\text{ nm}}^{0.350\text{ nm}} \sin^2\left(\frac{\pi x}{1.00\text{ nm}}\right)dx = \frac{2.00}{\text{nm}}\left[\frac{x}{2} - \frac{1.00\text{ nm}}{4\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right]_{0.150\text{ nm}}^{0.350\text{ nm}}$$

In the above result we used $\int \sin^2 ax dx = (x/2) - (1/4a)\sin(2ax)$

$$P_1 = \frac{1.00}{\text{nm}}\left(x - \frac{1.00\text{ nm}}{2\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right)_{0.150\text{ nm}}^{0.350\text{ nm}}$$

$$P_1 = \frac{1.00}{\text{nm}}\left\{0.350\text{ nm} - 0.150\text{ nm} - \frac{1.00\text{ nm}}{2\pi}\left[\sin(0.700\pi) - \sin(0.300\pi)\right]\right\} = \boxed{0.200}$$

(c) $$P_2 = \frac{2}{1.00}\int_{0.150}^{0.350}\sin^2\left(\frac{2\pi x}{1.00}\right)dx = 2.00\left[\frac{x}{2} - \frac{1.00}{8\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$$

$$P_2 = 1.00\left[x - \frac{1.00}{4\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350} = 1.00\left\{(0.350 - 0.150) - \frac{1.00}{4\pi}\left[\sin(1.40\pi) - \sin(0.600\pi)\right]\right\} =$$
$\boxed{0.351}$

(d) Using $E_n = \dfrac{n^2h^2}{8mL^2}$, we find that $E_1 = \boxed{0.377\text{ eV}}$ and $E_2 = \boxed{1.51\text{ eV}}$

41.53 (a) $mgy_i = \frac{1}{2}mv_f^2$ $\qquad v_f = \sqrt{2gy_i} = \sqrt{2(9.80\text{ m/s}^2)(50.0\text{ m})} = 31.3\text{ m/s}$

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(75.0\text{ kg})(31.3\text{ m/s})} = \boxed{2.82\times10^{-37}\text{ m}}$$ (not observable)

(b) $\Delta E\Delta t \geq \hbar/2$ so $\Delta E \geq \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(5.00\times10^{-3}\text{ s})} = \boxed{1.06\times10^{-32}\text{ J}}$

(c) $$\frac{\Delta E}{E} = \frac{1.06\times10^{-32}\text{ J}}{(75.0\text{ kg})(9.80\text{ m/s}^2)(50.0\text{ m})} = \boxed{2.87\times10^{-35}\ \%}$$

41.54 From the uncertainty principle $\Delta E\Delta t - h/2$ or $\Delta\left(mc^2\right)\Delta t = h/2$. Therefore,

$$\frac{\Delta m}{m} = \frac{h}{4\pi c^2(\Delta t)m} = \frac{h}{4\pi(\Delta t)E_R} = \frac{6.626\times 10^{-34}\ \mathrm{J\cdot s}}{4\pi\left(8.70\times 10^{-17}\ \mathrm{s}\right)(135\ \mathrm{MeV})}\left(\frac{1\,\mathrm{MeV}}{1.60\times 10^{-13}\ \mathrm{J}}\right) =$$

$\boxed{2.81\times 10^{-8}}$

41.55 (a) $f = \dfrac{E}{h} = \dfrac{180\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-19}\text{ J}}{1.00\text{ eV}}\right) = \boxed{4.34\times10^{14}\text{ Hz}}$

(b) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{4.34\times10^{14}\text{ Hz}} = 6.91\times10^{-7}\text{ m} = \boxed{691\text{ nm}}$

(c) $\Delta E\Delta t \geq \dfrac{\hbar}{2}$ so

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{h}{4\pi\,\Delta t} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(2.00\times10^{-8}\text{ s}\right)} = 2.64\times10^{-29}\text{ J} = \boxed{1.65\times10^{-10}\text{ eV}}$$

41.56 (a) $f = \boxed{\dfrac{E}{h}}$

(b) $\lambda = \dfrac{c}{f} = \boxed{\dfrac{hc}{E}}$

(c) $\Delta E\Delta t \geq \dfrac{\hbar}{2}$ so $\Delta E \geq \dfrac{\hbar}{2\Delta t} = \boxed{\dfrac{h}{4\pi T}}$

41.57 $\left\langle x^2\right\rangle = \int_{-\infty}^{\infty} x^2\left|\psi\right|^2 dx$

For a one-dimensional box of width L, $\psi_n = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi x}{L}\right)$

Thus, $\left\langle x^2\right\rangle = \dfrac{2}{L}\int_0^L x^2\sin^2\left(\dfrac{n\pi x}{L}\right)dx = \boxed{\dfrac{L^2}{3} - \dfrac{L^2}{2n^2\pi^2}}$ (from integral tables)

41.58 (a) $\int_{-\infty}^{\infty}\left|\psi\right|^2 dx = 1$ becomes

$$A^2\int_{-L/4}^{L/4}\cos^2\left(\frac{2\pi x}{L}\right)dx = A^2\left(\frac{L}{2\pi}\right)\left[\frac{\pi x}{L} + \frac{1}{4}\sin\left(\frac{4\pi x}{L}\right)\right]_{-L/4}^{L/4} = A^2\left(\frac{L}{2\pi}\right)\left(\frac{\pi}{2}\right) = 1$$

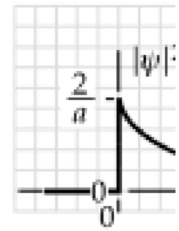
or $A^2 = \dfrac{4}{L}$ and $\boxed{A = \dfrac{2}{\sqrt{L}}}$

(b) The probability of finding the particle between 0 and $L/8$ is

$$\int_0^{L/8}\left|\psi\right|^2 dx = A^2\int_0^{L/8}\cos^2\left(\frac{2\pi x}{L}\right)dx = \frac{1}{4} + \frac{1}{2\pi} = \boxed{0.409}$$

41.59 For a particle with wave function $\psi(x) = \sqrt{\dfrac{2}{a}}e^{-x/a}$ for $x > 0$ and 0 for $x < 0$

(a) $\left|\psi(x)\right|^2 = 0,\ x < 0$ and $\left|\psi^2(x)\right| = \dfrac{2}{a}e^{-2x/a},\ \ x > 0$



(b) $\text{Prob}(x<0) = \int_{-\infty}^{0}\left|\psi(x)\right|^2 dx = \int_{-\infty}^{0}(0)\,dx = \boxed{0}$

(c) Normalization

$$\int_{-\infty}^{\infty}\left|\psi(x)\right|^2 dx = \int_{-\infty}^{0}|\psi|^2 dx + \int_{0}^{\infty}|\psi|^2 dx = 1$$

$$\int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty}(2/a)e^{-2x/a}dx = 0 - e^{-2x/a}\Big|_{0}^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

$$\text{Prob}(0<x<a) = \int_{0}^{a}|\psi|^2 dx = \int_{0}^{a}(2/a)\,e^{-2x/a}dx = e^{-2x/a}\Big|_{0}^{a} = 1 - e^{-2} =$$
$\boxed{0.865}$

41.60 (a) $\lambda = \dfrac{h}{p} = \dfrac{hc}{\sqrt{E^2 - m_e{}^2c^4}} = \dfrac{hc}{\sqrt{\left(m_ec^2 + K\right)^2 - \left(m_ec^2\right)^2}}$

$$\lambda = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\sqrt{(576\ \text{keV})^2 - (511\ \text{keV})^2}}\left(\frac{1\ \text{keV}}{1.60\times10^{-16}\ \text{J}}\right) = \boxed{4.68\times10^{-12}\ \text{m}}$$

(b) $50.0\lambda = \boxed{2.34\times10^{-10}\ \text{m}}$

41.61 (a) $\Delta x\Delta p \ge \hbar/2$ so if $\Delta x = r,\ \Delta p \ge \boxed{\hbar/2r}$

(b) Choosing $\Delta p = \dfrac{\hbar}{r},\ K = \dfrac{p^2}{2m_e} = \dfrac{(\Delta p)^2}{2m_e} = \boxed{\dfrac{\hbar^2}{2m_e r^2}}$

$U = -\dfrac{k_e e^2}{r}$, so $E = K + U = \boxed{\dfrac{\hbar^2}{2m_e r^2} - \dfrac{k_e e^2}{r}}$

(c) To minimize E,

$dE \quad \hbar \quad k_e e^2 \quad \hbar^2$

Then, $E = \dfrac{\hbar^2}{2m_e}\left(\dfrac{m_e k_e e^2}{\hbar^2}\right)^2 - k_e e^2\left(\dfrac{m_e k_e e^2}{\hbar^2}\right) = -\left(\dfrac{m_e k_e^2 e^4}{2\hbar^2}\right) = \boxed{-13.6 \text{ eV}}$

41.62 (a) The requirement that $\dfrac{n\lambda}{2} = L$ so $p = \dfrac{h}{\lambda} = \dfrac{nh}{2L}$ is still valid.

$$E = \sqrt{(pc)^2 + \left(mc^2\right)^2} \quad \Rightarrow \quad E_n = \sqrt{\left(\frac{nhc}{2L}\right)^2 + \left(mc^2\right)^2}$$

$$K_n = E_n - mc^2 = \boxed{\sqrt{\left(\frac{nhc}{2L}\right)^2 + \left(mc^2\right)^2} - mc^2}$$

(b) Taking $L = 1.00 \times 10^{-12}$ m, $m = 9.11 \times 10^{-31}$ kg, and $n = 1$, we find $K_1 = \boxed{4.69 \times 10^{-14} \text{ J}}$

Nonrelativistic, $E_1 = \dfrac{h^2}{8mL^2} = \dfrac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)^2}{8\left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.00 \times 10^{-12} \text{ m}\right)^2} = 6.02 \times 10^{-14} \text{ J}$

Comparing this to K_1, we see that this value is too large by $\boxed{28.6\%}$.

41.63 (a) $U = \dfrac{e^2}{4\pi\epsilon_0 d}\left[-1 + \dfrac{1}{2} - \dfrac{1}{3} + \left(-1 + \dfrac{1}{2}\right) + (-1)\right] = \dfrac{(-7/3)e^2}{4\pi\epsilon_0 d} = \boxed{-\dfrac{7k_e e^2}{3d}}$

(b) From Equation 41.9, $K = 2E_1 = \dfrac{2h^2}{8m_e\left(9d^2\right)} = \boxed{\dfrac{h^2}{36m_e d^2}}$

(c) $E = U + K$ and $\dfrac{dE}{dd} = 0$ for a minimum: $\dfrac{7k_e e^2}{3d^2} - \dfrac{h^2}{18m_e d^3} = 0$

$$d = \frac{3h^2}{(7)\left(18k_e e^2 m_e\right)} = \frac{h^2}{42m_e k_e e^2} = \frac{\left(6.626 \times 10^{-34}\right)^2}{(42)\left(9.11 \times 10^{-31}\right)\left(8.99 \times 10^9\right)\left(1.602 \times 10^{-19} \text{ C}\right)^2} =$$
$\boxed{0.0499 \text{ nm}}$

(d) Since the lithium spacing is a, where $Na^3 = V$, and the density is Nm/V, where m is the mass of one atom, we get:

$$a = \left(\frac{Vm}{Nm}\right)^{1/3} = \left(\frac{m}{\text{density}}\right)^{1/3} = \left(\frac{1.66 \times 10^{-27} \text{ kg} \times 7}{530 \text{ kg}}\right)^{1/3} \text{ m} = 2.80 \times 10^{-10} \text{ m} = \boxed{0.280 \text{ nm}}$$

(5.62 times larger than c).

41.64 (a) $\psi = Bxe^{-(m\omega/2h)x^2}$

$$\frac{d\psi}{dx} = Be^{-(m\omega/2h)x^2} + Bx\left(\frac{-m\omega}{2h}\right)2xe^{-(m\omega/2h)x^2} = Be^{-(m\omega/2h)x^2} - B\left(\frac{m\omega}{h}\right)x^2e^{-(m\omega/2h)x^2}$$

$$\frac{d^2\psi}{dx^2} = Bx\left(\frac{-m\omega}{h}\right)xe^{-(m\omega/2h)x^2} - B\left(\frac{m\omega}{h}\right)2xe^{-(m\omega/2h)x^2} - B\left(\frac{m\omega}{h}\right)x^2\left(\frac{-m\omega}{h}\right)xe^{-(m\omega/2h)x^2}$$

$$\frac{d^2\psi}{dx^2} = -3B\left(\frac{m\omega}{h}\right)xe^{-(m\omega/2h)x^2} + B\left(\frac{m\omega}{h}\right)x^3e^{-(m\omega/2h)x^2}$$

Substituting into the Schrödinger Equation (41.19), we have

$$-3B\left(\frac{m\omega}{h}\right)xe^{-(m\omega/2h)x^2} + B\left(\frac{m\omega}{h}\right)x^3e^{-(m\omega/2h)x^2} = -\frac{2mE}{h^2}Bxe^{-(m\omega/2h)x^2} + \left(\frac{m\omega}{h}\right)^2 x^2Bxe^{-(m\omega/2h)x^2}$$

This is true if $-3\omega = -\frac{2E}{h}$; it is true if $\boxed{E = \frac{3}{2}h\omega}$

(b) We never find the particle at $\boxed{x = 0}$ because $\psi = 0$ there.

(c) ψ is maximized if $\frac{d\psi}{dx} = 0 = 1 - x^2\left(\frac{m\omega}{h}\right)$, which is true at $\boxed{x = \pm\sqrt{\frac{h}{m\omega}}}$

(d) We require $\int_{-\infty}^{\infty}|\psi|^2 dx = 1$:

$$1 = \int_{-\infty}^{\infty} B^2x^2e^{-(m\omega/h)x^2}dx = 2B^2\int_0^{\infty} x^2e^{-(m\omega/h)x^2}dx = 2B^2\frac{1}{4}\sqrt{\frac{\pi}{(m\omega/h)^3}} = \frac{B^2}{2}\frac{\pi^{1/2}h^{3/2}}{(m\omega)^{3/2}}$$

Then $\quad B = \frac{2^{1/2}}{\pi^{1/4}}\left(\frac{m\omega}{h}\right)^{3/4} = \boxed{\left(\frac{4m^3\omega^3}{\pi h^3}\right)^{1/4}}$

(e) At $x = 2\sqrt{h/m\omega}$, the potential energy is $\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(4h/m\omega) = 2h\omega$. This is larger than the total energy $3h\omega/2$, so there is $\boxed{\text{zero}}$ classical probability of finding the particle here.

(f) Probability $= |\psi|^2dx = \left(Bxe^{-(m\omega/2h)x^2}\right)^2\delta = \delta B^2x^2e^{-(m\omega/h)x^2}$

Probability $= \delta\frac{2}{\pi^{1/2}}\left(\frac{m\omega}{h}\right)^{3/2}\left(\frac{4h}{m\omega}\right)e^{-(m\omega/h)4(h/m\omega)} = \boxed{8\delta\left(\frac{m\omega}{h\pi}\right)^{1/2}e^{-4}}$

41.65 (a) $\int_0^L |\psi|^2 dx = 1$: $\quad A^2 \int_0^L \left[\sin^2\left(\frac{\pi x}{L}\right) + 16\sin^2\left(\frac{2\pi x}{L}\right) + 8\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\right] dx = 1$

$$A^2\left[\left(\frac{L}{2}\right) + 16\left(\frac{L}{2}\right) + 8\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx\right] = 1$$

$$A^2\left[\frac{17L}{2} + 16\int_0^L \sin^2\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)dx\right] = A^2\left[\frac{17L}{2} + \frac{16L}{3\pi}\sin^3\left(\frac{\pi x}{L}\right)\Big|_{x=0}^{x=L}\right] = 1$$

$A^2 = \frac{2}{17L}$, so the normalization constant is $\boxed{A = \sqrt{2/17L}}$

(b) $\int_{-a}^{a} |\psi|^2 dx = 1$:

$$\int_{-a}^{a}\left[|A|^2\cos^2\left(\frac{\pi x}{2a}\right) + |B|^2\sin^2\left(\frac{\pi x}{a}\right) + 2|A||B|\cos\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{a}\right)\right]dx = 1$$

The first two terms are $|A|^2 a$ and $|B|^2 a$. The third term is:

$$2|A||B|\int_{-a}^{a}\cos\left(\frac{\pi x}{2a}\right)\left[2\sin\left(\frac{\pi x}{2a}\right)\cos\left(\frac{\pi x}{2a}\right)\right]dx = 4|A||B|\int_{-a}^{a}\cos^2\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{2a}\right)dx = \frac{8a|A||B|}{3\pi}\cos^3\left(\frac{\pi x}{2a}\right)\Big|_{-a}^{a} = 0$$

so that $a\left(|A|^2 + |B|^2\right) = 1$, giving $\boxed{|A|^2 + |B|^2 = 1/a}$.

41.66 With one slit open $\quad P_1 = |\psi_1|^2$ or $P_2 = |\psi_2|^2$

With both slits open, $\quad P = |\psi_1 + \psi_2|^2$

At a maximum, the wave functions are in phase $\quad P_{max} = \left(|\psi_1| + |\psi_2|\right)^2$

At a minimum, the wave functions are out of phase $\quad P_{min} = \left(|\psi_1| - |\psi_2|\right)^2$

Now $\frac{P_1}{P_2} = \frac{|\psi_1|^2}{|\psi_2|^2} = 25.0$, so $\quad \frac{|\psi_1|}{|\psi_2|} = 5.00$

‘41.67 (a) The light is unpolarized. It contains both horizontal and vertical field oscillations.

(b) The interference pattern appears, but with diminished overall intensity.

(c) The results are the same in each case.

(d) The interference pattern appears and disappears as the polarizer turns, with alternately increasing and decreasing contrast between the bright and dark fringes. The intensity on the screen is precisely zero at the center of a dark fringe four times in each revolution, when the filter axis has turned by 45°, 135°, 225°, and 315° from the vertical.

(e) Looking at the overall light energy arriving at the screen, we see a low-contrast interference pattern. After we sort out the individual photon runs into those for trial 1, those for trial 2, and those for trial 3, we have the original results replicated: The runs for trials 1 and 2 form the two blue graphs in Figure 41.3, and the runs for trial 3 build up the red graph.

Chapter 42 Solutions

42.1 (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{\text{Au}}}{r} \qquad \text{or} \qquad r_{\text{min}} = \frac{k_e(2e)(79e)}{E}$$

$$r_{\text{min}} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)158\left(1.60\times10^{-19}\ \text{C}\right)^2}{(4.00\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)} = \boxed{5.68\times10^{-14}\ \text{m}}$$

(b) The maximum force exerted on the alpha particle is

$$F_{\text{max}} = \frac{k_e q_\alpha q_{\text{Au}}}{r_{\text{min}}^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)158\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(5.68\times10^{-14}\ \text{m}\right)^2} = \boxed{11.3\ \text{N}}\ \text{away from the nucleus}$$

42.2 (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{\text{T}}}{r} \quad \text{or} \quad r_{\text{min}} = \frac{k_e(2e)(Ze)}{E} = \boxed{\frac{2Zk_e e^2}{E}}$$

(b) The maximum force exerted on the alpha particle is

$$F_{\text{max}} = \frac{k_e q_\alpha q_{\text{T}}}{r_{\text{min}}^2} = 2Zk_e e^2\left(\frac{E}{2Zk_e e^2}\right)^2 = \boxed{\frac{E^2}{2Zk_e e^2}}\ \text{away from the target nucleus}$$

42.3 (a) The photon has energy 2.28 eV.

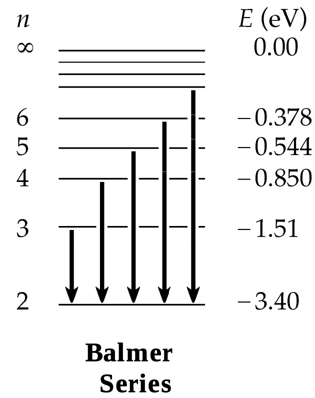
And $(13.6\ \text{eV})/2^2 = 3.40\ \text{eV}$ is required to ionize a hydrogen atom from state $n = 2$. So while the photon cannot ionize a hydrogen atom pre-excited to $n = 2$, it can ionize a hydrogen atom in the $n = \boxed{3}$ state, with energy

$$-\frac{13.6\ \text{eV}}{3^2} = -1.51\ \text{eV}$$

(b) The electron thus freed can have kinetic energy $K_e = 2.28\ \text{eV} - 1.51\ \text{eV} = 0.769\ \text{eV} = \frac{1}{2}m_e v^2$

$$v = \sqrt{\frac{2(0.769)(1.60\times10^{-19})\text{J}}{(9.11\times10^{-31})\text{kg}}} = \boxed{520\ \text{km/s}}$$

***42.4** (a) Longest wavelength implies lowest frequency and smallest energy: the electron falls from $n = 3$ to $n = 2$, losing energy



$$-\frac{13.6\text{ eV}}{3^2} + \frac{13.6\text{ eV}}{2^2} = \boxed{1.89\text{ eV}}$$

The photon frequency is $f = \Delta E/h$ and its wavelength is

$$\lambda = \frac{c}{f} = \frac{ch}{\Delta E} = \frac{(3.00\times10^8\text{ m / s})(6.626\times10^{-34}\text{ J}\cdot\text{s})}{1.89\text{ eV}}\left(\frac{\text{eV}}{1.60\times10^{-19}\text{ J}}\right)$$

$$\lambda = \boxed{656\text{ nm}}$$

(b) The biggest energy loss is for an electron to fall from ionization, $n = \infty$, to the $n = 2$ state.

It loses energy $$-\frac{13.6\text{ eV}}{\infty} + \frac{13.6\text{ eV}}{2^2} = \boxed{3.40\text{ eV}}$$

to emit light of wavelength $$\lambda = \frac{hc}{\Delta E} = \frac{(3.00\times10^8\text{ m / s})(6.626\times10^{-34}\text{ J}\cdot\text{s})}{3.40\text{ eV}(1.60\times10^{-19}\text{ J / eV})} = \boxed{365\text{ nm}}$$

42.5 (a) For positronium, $\mu = \frac{m_e}{2}$, so λ_{32} = (656 nm)2 = 1312 nm = $\boxed{1.31\ \mu\text{m}}$ (infrared region) .

(b) For He$^+$, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$, so λ_{32} = (656/4) nm = $\boxed{164\text{ nm}}$ (ultraviolet region) .

Goal Solution

A general expression for the energy levels of one-electron atoms and ions is

$$E_n = -\left(\frac{\mu k_e{}^2 q_1{}^2 q_2{}^2}{2\hbar^2 n^2}\right)$$

where k_e is the Coulomb constant, q_1 and q_2 are the charges of the two particles, and μ is the reduced mass, given by $\mu = m_1 m_2 / (m_1 + m_2)$. In Problem 4 we found that the wavelength for the $n = 3$ to $n = 2$ transition of the hydrogen atom is 656.3 nm (visible red light). What are the wavelengths for this same transition in (a) positronium, which consists of an electron and a positron, and (b) singly ionized helium? (**Note**: A positron is a positively charged electron.)

G: The reduced mass of positronium is **less** than hydrogen, so the photon energy will be **less** for positronium than for hydrogen. This means that the wavelength of the emitted photon will be **longer** than 656.3 nm. On the other hand, helium has about the same reduced mass but more charge than hydrogen, so its transition energy will be **larger**, corresponding to a wavelength **shorter** than 656.3 nm.

O: All the factors in the above equation are constant for this problem except for the reduced mass and the nuclear charge. Therefore, the wavelength corresponding to the energy difference for the transition can be found simply from the ratio of mass and charge variables.

A: For hydrogen, $\mu = \dfrac{m_p m_e}{m_p + m_e} \approx m_e$

The photon energy is $\Delta E = E_3 - E_2$

Its wavelength is $\lambda = 656.3\text{ nm}$, where $\lambda = \dfrac{c}{f} = \dfrac{hc}{\Delta E}$

(a) For positronium, $\mu = \dfrac{m_e m_e}{m_e + m_e} = \dfrac{m_e}{2}$

so the energy of each level is one half as large as in hydrogen, which we could call "protonium." The photon energy is inversely proportional to its wavelength, so for positronium,

$\lambda_{32} = 2(656.3\text{ nm}) = 1313\text{ nm}$ (in the infrared region)

(b) For He^+, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$,

so the transition energy is $2^2 = 4$ times larger than hydrogen. Then,

$\lambda_{32} = \left(\dfrac{656}{4}\right)\text{ nm} = 164\text{ nm}$ (in the ultraviolet region)

L: As expected, the wavelengths for positronium and helium are respectively larger and smaller than for hydrogen. Other energy transitions should have wavelength shifts consistent with this pattern. It is important to remember that the reduced mass is not the total mass, but is generally close in magnitude to the smaller mass of the system (hence the name **reduced** mass).

***42.6** (a) For a particular transition from n_i to n_f, $\Delta E_{\text{H}} = -\dfrac{\mu_{\text{H}} k_e^2 e^4}{2\hbar^2}\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right) = \dfrac{hc}{\lambda_{\text{H}}}$

and $\Delta E_{\text{D}} = -\dfrac{\mu_{\text{D}} k_e^2 e^4}{2\hbar^2}\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right) = \dfrac{hc}{\lambda_{\text{D}}}$

where $\mu_{\text{H}} = \dfrac{m_e m_p}{m_e + m_p}$ and $\mu_{\text{D}} = \dfrac{m_e m_{\text{D}}}{m_e + m_{\text{D}}}$

By division, $\dfrac{\Delta E_{\text{H}}}{\Delta E_{\text{D}}} = \dfrac{\mu_{\text{H}}}{\mu_{\text{D}}} = \dfrac{\lambda_{\text{D}}}{\lambda_{\text{H}}}$ or $\lambda_{\text{D}} = \left(\dfrac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}$

Then, $\boxed{\lambda_{\text{H}} - \lambda_{\text{D}} = \left(1 - \dfrac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}}$

(b) $$\frac{\mu_{\text{H}}}{\mu_{\text{D}}} = \left(\frac{m_e m_p}{m_e + m_p}\right)\left(\frac{m_e + m_{\text{D}}}{m_e m_{\text{D}}}\right) = \frac{(1.007\ 276\text{ u})(0.000\ 549\text{ u} + 2.013\ 553\text{ u})}{(0.000\ 549\text{ u} + 1.007\ 276\text{ u})(2.013\ 553\text{ u})} = 0.999728$$

$$\lambda_{\text{H}} - \lambda_{\text{D}} = (1 - 0.999\ 728)(656.3\text{ nm}) = \boxed{0.179\text{ nm}}$$

42.7 (a) In the 3d subshell, $n = 3$ and $\ell = 2$, we have

n	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+1	+1	0	0	-1	-1	-2	-2
m_s	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2

(A total of 10 states)

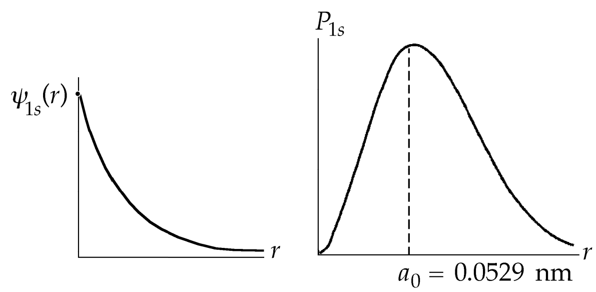
(b) In the 3p subshell, $n = 3$ and $\ell = 1$, we have

n	3	3	3	3	3	3
ℓ	1	1	1	1	1	1
m_ℓ	+1	+1	+0	+0	-1	-1
m_s	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2

(A total of 6 states)

42.8 $$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \qquad \text{(Eq. 42.3)}$$

$$P_{1s}(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0} \qquad \text{(Eq. 42.7)}$$



42.9 (a) $$\int |\psi|^2 dV = 4\pi \int_0^\infty |\psi|^2 r^2\, dr = 4\pi\left(\frac{1}{\pi a_0^3}\right)\int_0^\infty r^2 e^{-2r/a_0}\, dr$$

Using integral tables, $$\int |\psi|^2\, dV = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{r=0}^{r=\infty} = \left(-\frac{2}{a_0^2}\right)\left(-\frac{a_0^2}{2}\right) = \boxed{1}$$

so the wave function as given is normalized.

(b) $$P_{a_0/2 \to 3a_0/2} = 4\pi \int_{a_0/2}^{3a_0/2} |\psi|^2 r^2\, dr = 4\pi\left(\frac{1}{\pi a_0^3}\right)\int_{a_0/2}^{3a_0/2} r^2 e^{-2r/a_0}\, dr$$

Again, using integral tables,

$$P_{a_0/2 \to 3a_0/2} = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{a_0/2}^{3a_0/2} = -\frac{2}{a_0^2}\left[e^{-3}\left(\frac{17a_0^2}{4}\right) - e^{-1}\left(\frac{5a_0^2}{4}\right)\right] = \boxed{0.497}$$

42.10 $\psi = \frac{1}{\sqrt{3}}\frac{1}{(2a_0)^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$ so $P_r = 4\pi r^2\left|\psi^2\right| = 4\pi r^2\frac{r^2}{24a_0^5}e^{-r/a_0}$

Set $\frac{dP}{dr} = \frac{4\pi}{24a_0^5}\left[4r^3 e^{-r/a_0} + r^4\left(-\frac{1}{a_0}\right)e^{-r/a_0}\right] = 0$

Solving for r, this is a maximum at $\boxed{r = 4a_0}$

42.11 $\psi = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$ $\quad \frac{2}{r}\frac{d\psi}{dr} = \frac{-2}{r\sqrt{\pi a_0^5}}e^{-r/a_0} = \frac{2}{ra_0}\psi$ $\quad \frac{d^2\psi}{dr^2} = \frac{1}{\sqrt{\pi a_0^7}}e^{-r/a_0} = \frac{1}{a_0^2}\psi$

$$-\frac{\hbar^2}{2m_e}\left(\frac{1}{a_0^2} - \frac{2}{ra_0}\right)\psi - \frac{e^2}{4\pi\epsilon_0 r}\psi = E\psi$$

But $a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2}$, so $-\frac{e^2}{8\pi\epsilon_0 a_0} = E$ or $\boxed{E = -\frac{k_e e^2}{2a_0}}$

This is true, so the Schrödinger equation is satisfied.

42.12 The hydrogen ground-state radial probability density is

$$P(r) = 4\pi r^2\left|\psi_{1s}\right|^2 = \frac{4r^2}{a_0^3}\exp\left(-\frac{2r}{a_0}\right)$$

The number of observations at $2a_0$ is, by proportion

$$N = 1000\frac{P(2a_0)}{P(a_0/2)} = 1000\frac{(2a_0)^2}{(a_0/2)^2}\frac{e^{-4a_0/a_0}}{e^{-a_0/a_0}} = 1000(16)e^{-3} = \boxed{797 \text{ times}}$$

42.13 (a) For the d state, $\ell = 2$, $\quad L = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}\ \text{J}\cdot\text{s}$

(b) For the f state, $\ell = 3$, $\quad L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{12}\hbar} = 3.65 \times 10^{-34}\ \text{J}\cdot\text{s}$

***42.14** $L = \sqrt{\ell(\ell+1)}\hbar$ so $4.714 \times 10^{-34} = \sqrt{\ell(\ell+1)}\dfrac{6.626 \times 10^{-34}}{2\pi}$

$$\ell(\ell+1) = \frac{\left(4.714 \times 10^{-34}\right)^2 (2\pi)^2}{\left(6.626 \times 10^{-34}\right)^2} = 1.998 \times 10^1 \approx 20 = 4(4+1) \qquad \text{so} \qquad \boxed{\ell = 4}$$

42.15 The 5th excited state has $n = 6$, energy $\dfrac{-13.6 \text{ eV}}{36} = -0.378 \text{ eV}$

The atom loses this much energy: $\dfrac{hc}{\lambda} = \dfrac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(1090 \times 10^{-9} \text{ m})(1.60 \times 10^{-19} \text{ J/eV})} = 1.14 \text{ eV}$

to end up with energy $-0.378 \text{ eV} - 1.14 \text{ eV} = -1.52 \text{ eV}$

which is the energy in state 3: $-\dfrac{13.6 \text{ eV}}{3^3} = -1.51 \text{ eV}$

While $n = 3$, ℓ can be as large as 2, giving angular momentum $\sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\,h}$

42.16 For a 3d state, $n = 3$ and $\ell = 2$. Therefore, $L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{2(2+1)}\hbar = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34} \text{ J} \cdot \text{s}$

m_ℓ can have the values $-2, -1, 0, 1$, and 2, so $\boxed{L_z \text{ can have the values} -2\hbar, -\hbar, 0, \text{ and } 2\hbar}$

Using the relation $\cos\theta = L_z / L$, we find that the possible values of θ are equal to

$\boxed{145°, 114°, 90.0°, 65.9°, \text{ and } 35.3°}$.

42.17 (a) $n = 1$: For $n = 1$, $\ell = 0$, $m_\ell = 0$, $m_s = \pm\dfrac{1}{2}$, $\rightarrow$ 2 sets

n	ℓ	m_ℓ	m_s
1	0	0	$-1/2$
1	0	0	$+1/2$

$2n^2 = 1(1)^2 = \boxed{2}$

(b) For $n = 2$, we have

n	ℓ	m_ℓ	m_s
2	0	0	$\pm 1/2$
2	1	-1	$\pm 1/2$
2	1	0	$\pm 1/2$
2	1	1	$\pm 1/2$

yields 8 sets; $2n^2 = 2(2)^2 = \boxed{8}$

Note that the number is twice the number of m_l values. Also, for each l there are $(2l+1)$ different m_l values. Finally, l can take on values ranging from 0 to $n-1$. So the general expression is

$$s = \sum_{0}^{n-1} 2(2l+1)$$

The series is an arithmetic progression: 2 + 6 + 10 + 14, the sum of which is

$s = \frac{n}{2}\left[2a + (n-1)d\right]$ where $a = 2,\ d = 4$: $\quad s = \frac{n}{2}\left[4 + (n-1)4\right] = 2n^2$

(c) $n = 3$: $2(1) + 2(3) + 2(5) = 2 + 6 + 10 = 18$ $\quad 2n^2 = 2(3)^2 = \boxed{18}$

(d) $n = 4$: $2(1) + 2(3) + 2(5) + 2(7) = 32$ $\quad 2n^2 = 2(4)^2 = \boxed{32}$

(e) $n = 5$: $32 + 2(9) = 32 + 18 = 50$ $\quad 2n^2 = 2(5)^2 = \boxed{50}$

42.18 $\mu_B = \frac{e\hbar}{2m_e}$ $\quad e = 1.60 \times 10^{-19}$ C $\quad \hbar = 1.055 \times 10^{-34}$ J · s $\quad m_e = 9.11 \times 10^{-31}$ kg

$\mu_B = \boxed{9.27 \times 10^{-24} \text{ J/T} = 5.79 \times 10^{-5} \text{ eV/T}}$

42.19 (a) Density of a proton: $\rho = \frac{m}{V} = \frac{1.67 \times 10^{-27}\text{ kg}}{(4/3)\pi(1.00 \times 10^{-15}\text{ m})^3} = \boxed{3.99 \times 10^{17}\text{ kg/m}^3}$

(b) Size of model electron: $r = \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left(\frac{3 \times 9.11 \times 10^{-31}\text{ kg} \cdot \text{m}^3}{4\pi \times 3.99 \times 10^{17}\text{ kg}}\right)^{1/3} = \boxed{8.17 \times 10^{-17}\text{ m}}$

(c) Moment of inertia: $I = \frac{2}{5}mr^2 = \frac{2}{5}(9.11 \times 10^{-31}\text{ kg})(8.17 \times 10^{-17}\text{ m})^2 = 2.43 \times 10^{-63}\text{ kg} \cdot \text{m}^2$

$$L_z = I\omega = \frac{\hbar}{2} = \frac{Iv}{r}$$

Therefore, $v = \frac{\hbar r}{2I} = \frac{\left(6.626 \times 10^{-34}\text{ J} \cdot \text{s}\right)(8.17 \times 10^{-17}\text{ m})}{2\pi(2)\left(2.43 \times 10^{-63}\text{ kg} \cdot \text{m}^2\right)} = \boxed{1.77 \times 10^{12}\text{ m/s}}$

(d) This is $\boxed{5.91 \times 10^3 \text{ times larger}}$ than the speed of light.

42.20 (a) $L = mvr = m\frac{2\pi r}{T}r = \sqrt{\ell(\ell+1)}\hbar = \sqrt{(\ell^2+\ell)}\hbar \approx \ell\hbar$

$$(5.98\times10^{24}\text{ kg})\frac{2\pi(1.496\times10^{11}\text{ m})^2}{3.156\times10^7\text{ s}} = \ell\hbar \quad \text{so} \quad \frac{2.66\times10^{40}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = \ell = \boxed{2.52\times10^{74}}$$

(b) $|E| = |-U+K| = |-K| = \frac{1}{2}mv^2 = \frac{1}{2}\frac{mr^2}{mr^2}mv^2 = \frac{1}{2}\frac{L^2}{mr^2} = \frac{1}{2}\frac{\ell(\ell+1)\hbar^2}{mr^2} \approx \frac{1}{2}\frac{\ell^2\hbar^2}{mr^2}$

$$\frac{dE}{d\ell} = \frac{1}{2}\frac{2\ell\hbar^2}{mr^2}\frac{\ell}{\ell} = 2\frac{E}{\ell} \text{ so} \qquad dE = 2\frac{E}{\ell}d\ell = 2\frac{\frac{1}{2}(5.98\times10^{24}\text{ kg})\left(\frac{2\pi\times1.496\times10^{11}\text{ m}}{3.156\times10^7\text{ s}}\right)^2 1}{2.52\times10^{74}}$$

$$\Delta E = \frac{5.30\times10^{33}\text{ J}}{2.52\times10^{74}} = \boxed{2.10\times10^{-41}\text{ J}}$$

***42.21** $\mu_n = \frac{e\hbar}{2m_p}$ $\quad e = 1.60\times10^{-19}$ C $\quad \hbar = 1.055\times10^{-34}$ J · s $\quad m_p = 1.67\times10^{-27}$ kg

(a) $\mu_n = \boxed{5.05\times10^{-27}\text{ J/T}} = \boxed{31.6\text{ neV/T}}$

(b) $\frac{\mu_n}{\mu_B} = \frac{1}{1836} = \frac{m_e}{m_p}$

Apparently it is harder to "spin up" a nucleus than a electron, because of its greater mass.

42.22 In the N shell, $n = 4$. For $n = 4$, ℓ can take on values of 0, 1, 2, and 3. For each value of ℓ, m_ℓ can be $-\ell$ to ℓ in integral steps. Thus, the maximum value for m_ℓ is 3. Since $L_z = m_\ell\hbar$, the maximum value for L_z is $L_z = \boxed{3\hbar}$.

42.23 The 3d subshell has $\ell = 2$, and $n = 3$. Also, we have $s = 1$.
Therefore, we can have $\boxed{n = 3;\ \ell = 2;\ m_\ell = -2, -1, 0, 1, 2;\ s = 1;\text{ and } m_s = -1, 0, 1}$, leading to the following table:

n	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
m_ℓ	–2	–2	–2	–1	–1	–1	0	0	0	1	1	1	2	2	2
s	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
m_s	–1	0	1	–1	0	1	–1	0	1	–1	0	1	–1	0	1

42.24 (a) $\boxed{1s^2 2s^2 2p^4}$

(b) For the 1s electrons, $n = 1$, $\ell = 0$, $m_\ell = 0$, $m_s = +1/2$ and $-1/2$
For the two 2s electrons, $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = +1/2$ and $-1/2$
For the four 2p electrons, $n = 2$; $\ell = 1$; $m_\ell = -1, 0$, or 1; and $m_s = +1/2$ or $-1/2$

42.25 The $\boxed{\text{4}s\text{ subshell fills first}}$, for potassium and calcium, before the 3d subshell starts to fill for scandium through zinc. Thus, we would first suppose that $[\text{Ar}]3d^4 4s^2$ would have lower energy than $[\text{Ar}]3d^5 4s^1$. But the latter has more unpaired spins, six instead of four, and Hund's rule suggests that this could give the latter configuration lower energy. In fact it must, for $[\text{Ar}]3d^5 4s^1$ is the ground state for chromium.

***42.26** (a) For electron one and also for electron two, $n = 3$ and $\ell = 1$. The possible states are listed here in columns giving the other quantum numbers:

electron one	m_ℓ	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	m_s	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
electron two	m_ℓ	1	0	0	–1	–1	1	0	0	–1	–1	1	1	0	–1	–1
	m_s	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

electron one	m_ℓ	0	0	0	0	0	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
	m_s	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
electron two	m_ℓ	1	1	0	–1	–1	1	1	0	0	–1	1	1	0	0	–1
	m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

There are thirty allowed states, since electron one can have any of three possible values for m_ℓ for both spin up and spin down, amounting to six states, and the second electron can have any of the other five states.

(b) Were it not for the exclusion principle, there would be $\boxed{36}$ possible states, six for each electron independently.

42.27

Shell	K	L				M									N
n	1	2				3									4
ℓ	0	0	1			0	1			2					0
m_ℓ	0	0	1	0	–1	0	1	0	–1	2	1	0	–1	–2	0
m_s	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓
count	1 2	3 4			10	12			18	21				30	20
	He	Be			Ne	Mg			Ar					Zn	Ca

(a) $\boxed{\text{zinc or copper}}$

(b) $\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10}}$ or $\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^1 3d^{10}}$

42.28 Listing subshells in the order of filling, we have for element 110,

$1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^{10}\,4p^6\,5s^2\,4d^{10}\,5p^6\,6s^2\,4f^{14}\,5d^{10}\,6p^6\,7s^2\,5f^{14}\,6d^8$

In order of increasing principal quantum number, this is

$$\boxed{1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^{10}\,4s^2\,4p^6\,4d^{10}\,4f^{14}\,5s^2\,5p^6\,5d^{10}\,5f^{14}\,6s^2\,6p^6\,6d^8\,7s^2}$$

42.29 (a)

$n + l$	1	2	3	4	5	6	7
subshell	$1s$	$2s$	$2p$, $3s$	$3p$, $4s$	$3d$, $4p$, $5s$	$4d$, $5p$, $6s$	$4f$, $5d$, $6p$, $7s$

(b)

$Z = 15$:	Filled subshells:	$1s$, $2s$, $2p$, $3s$ (12 electrons)
	Valence subshell:	3 electrons in $3p$ subshell
	Prediction:	Valance = +3 or −5
	Element is phosphorus	Valence +3 or −5 (Prediction correct)
$Z = 47$:	Filled subshells:	$1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$ (38 electrons)
	Outer subshell:	9 electrons in $4d$ subshell
	Prediction:	Valence = −1
	Element is silver,	(Prediction fails) Valence is +1
$Z = 86$:	Filled subshells:	$1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$, $4d$, $5p$, $6s$, $4f$, $5d$, $6p$ (86 electrons)
	Prediction	Outer subshell is full: inert gas
	Element is radon, inert	(Prediction correct)

42.30 Electronic configuration: Sodium to Argon

$[1s^2\,2s^2\,2p^6]\ +3s^1 \rightarrow Na^{11}$
$+3s^2 \rightarrow Mg^{12}$
$+3s^2\,3p^1 \rightarrow Al^{13}$
$+3s^2\,3p^2 \rightarrow Si^{14}$
$+3s^2\,3p^3 \rightarrow P^{15}$
$+3s^2\,3p^4 \rightarrow S^{16}$
$+3s^2\,3p^5 \rightarrow Cl^{17}$
$+3s^2\,3p^6 \rightarrow Ar^{18}$
$[1s^2\,2s^2\,2p^6\,3s^2\,3p^6]4s^1 \rightarrow K^{19}$

***42.31** $\boxed{n = 3,\ \ell = 0,\ m_\ell = 0}$

ψ_{300} corresponds to $E_{300} = -\dfrac{Z^2 E_0}{n^2} = -\dfrac{2^2(13.6)}{(3)^2} = \boxed{-6.05 \text{ eV}}$

$\boxed{n = 3,\ \ell = 1,\ m_\ell = -1, 0, 1}$

ψ_{31-1}, ψ_{310}, ψ_{311} have the same energy since n is the same.

For $\boxed{n = 3,\ \ell = 2,\ m_\ell = -2, -1, 0, 1, 2}$

ψ_{32-2}, ψ_{32-1}, ψ_{320}, ψ_{321}, ψ_{322} have the same energy since n is the same.

All states are degenerate.

42.32 $E = \dfrac{hc}{\lambda} = e(\Delta V) \Rightarrow \dfrac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{10.0 \times 10^{-9} \text{ m}} = (1.60 \times 10^{-19})(\Delta V)$

$\Delta V = \boxed{124 \text{ V}}$

***42.33** $E_{\text{photon max}} = \dfrac{hc}{\lambda_{\min}} = e(\Delta V) = 40.0 \text{ keV}$

$$\lambda_{\min} = \frac{hc}{E_{\max}} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{40.0 \times 10^3 \text{ eV}}\left(\frac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = \boxed{0.031\,0 \text{ nm}}$$

42.34 Some electrons can give all their kinetic energy $K_e = e(\Delta V)$ to the creation of a single photon of x-radiation, with

$hf = \dfrac{hc}{\lambda} = e(\Delta V)$

$$\lambda = \frac{hc}{e(\Delta V)} = \frac{(6.6261 \times 10^{-34} \text{ J} \cdot \text{s})(2.9979 \times 10^8 \text{ m/s})}{(1.6022 \times 10^{-19} \text{ C})(\Delta V)} = \boxed{\frac{1240 \text{ nm} \cdot \text{V}}{\Delta V}}$$

42.35 Following Example 42.7, $E_\gamma = \dfrac{3}{4}(42 - 1)^2(13.6 \text{ eV}) = 1.71 \times 10^4 \text{ eV} = 2.74 \times 10^{-15} \text{ J}$

$f = 4.14 \times 10^{18}$ Hz and $\lambda = \boxed{0.072\,5 \text{ nm}}$

42.36 The K_β x-rays are emitted when there is a vacancy in the (n = 1) K shell and an electron from the (n = 3) M shell falls down to fill it. Then this electron is shielded by nine electrons originally and by one in its final state.

$$\frac{hc}{\lambda} = -\frac{13.6(Z-9)^2}{3^2}\ \text{eV} + \frac{13.6(Z-1)^2}{1^2}\ \text{eV}$$

$$\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m / s})}{(0.152\times10^{-9}\ \text{m})(1.60\times10^{-19}\ \text{J / eV})} = 13.6\ \text{eV}\left(-\frac{Z^2}{9}+\frac{18Z}{9}-\frac{81}{9}+Z^2-2Z+1\right)$$

$8.17\times10^3\ \text{eV} = 13.6\ \text{eV}\left(\frac{8Z^2}{9}-8\right)$ so $601 = \frac{8Z^2}{9} - 8$ and $Z = 26$ $\boxed{\text{Iron}}$

42.37 (a) Suppose the electron in the M shell is shielded from the nucleus by two K plus seven L electrons. Then its energy is

$$-\frac{13.6\ \text{eV}(83-9)^2}{3^2} = -8.27\ \text{keV}$$

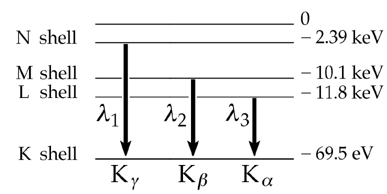
Suppose, after it has fallen into the vacancy in the L shell, it is shielded by just two K-shell electrons. Then its energy is

$$\frac{-13.6\ \text{eV}(83-2)^2}{2^2} = -22.3\ \text{keV}$$

Thus the electron's energy loss is the photon energy: $(22.3-8.27)\ \text{keV} = \boxed{14.0\ \text{keV}}$

(b) $\Delta E = \frac{hc}{\lambda}$ so $\lambda = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}\ (3.00\times10^8\ \text{m / s})}{14.0\times10^3\times1.60\times10^{-19}\ \text{J}} = \boxed{8.85\times10^{-11}\ \text{m}}$

***42.38** $E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{\lambda} = \frac{1.240\ \text{keV}\cdot\text{nm}}{\lambda}$



for λ_1 = 0.0185 nm, E = 67.11 keV

λ_2 = 0.0209 nm, E = 59.4 keV

λ_3 = 0.0215 nm, E = 57.7 keV

The ionization energy for K shell = 69.5 keV, so, the ionization energies for the other shells are: $\boxed{\text{L shell} = 11.8\ \text{keV}}$: $\boxed{\text{M shell} = 10.1\ \text{keV}}$: $\boxed{\text{N shell} = 2.39\ \text{keV}}$

***42.39** (a) The outermost electron in sodium has a 3*s* state for its ground state. The longest wavelength means minimum photon energy and smallest step on the energy level diagram. Since $n = 3$, n' must be 4. With $\ell = 0$, ℓ' must be $\boxed{1}$, since ℓ must change by 1 in a photon absorption process.

(b) $$\frac{1}{330\times 10^{-9}\text{ m}} = \left(1.097\times 10^{7}\ \frac{1}{\text{m}}\right)\left[\frac{1}{(3-1.35)^2} - \frac{1}{(4-\delta_1)^2}\right]$$

$$0.276 = \frac{1}{(1.65)^2} - \frac{1}{(4-\delta_1)^2} = 0.367 - \frac{1}{(4-\delta_1)^2} \quad \text{so} \quad (4-\delta_1)^2 = 10.98 \quad \text{and} \quad \boxed{\delta_1 = 0.686}$$

42.40 $$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(3.00\times 10^{8}\text{ m / s})}{(2.10\text{ eV})(1.60\times 10^{19}\text{ J / eV})} = \boxed{590\text{ nm}}$$

***42.41** We require $A = u_f B = \dfrac{16\pi^2 \hbar}{\lambda^3}B$ or $$u_f = \frac{16\pi^2\hbar}{\lambda^3} = \frac{16\pi^2\left(1.055\times 10^{-34}\text{ J}\cdot\text{s}\right)}{\left(645\times 10^{-9}\text{ m}\right)^3} = \boxed{6.21\times 10^{-14}\ \frac{\text{J}\cdot\text{s}}{\text{m}^3}}$$

42.42 $$f = \frac{E}{h} = \boxed{2.82\times 10^{13}\text{ s}^{-1}}$$

$$\lambda = \frac{c}{f} = \boxed{10.6\ \mu\text{m}}, \quad \boxed{\text{infrared}}$$

42.43 $$E = \mathcal{P}t = (1.00\times 10^{6}\text{ W})(1.00\times 10^{-8}\text{ s}) = 0.0100\text{ J}$$

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626\times 10^{-34})(3.00\times 10^{8})}{694.3\times 10^{-9}}\text{ J} = 2.86\times 10^{-19}\text{ J}$$

$$N = \frac{E}{E_\gamma} = \frac{0.0100}{2.86\times 10^{-19}} = \boxed{3.49\times 10^{16}\text{ photons}}$$

Goal Solution
A ruby laser delivers a 10.0-ns pulse of 1.00 MW average power. If the photons have a wavelength of 694.3 nm, how many are contained in the pulse?

G: Lasers generally produce concentrated beams that are bright (except for IR or UV lasers that produce invisible beams). Since our eyes can detect light levels as low as a few photons, there are probably at least 1000 photons in each pulse.

O: From the pulse width and average power, we can find the energy delivered by each pulse. The number of photons can then be found by dividing the pulse energy by the energy of each photon, which is determined from the photon wavelength.

A: The energy in each pulse is $E = \mathcal{P}t = (1.00\times10^6 \text{ W})(1.00\times10^{-8} \text{ s}) = 1.00\times10^{-2} \text{ J}$

The energy of each photon is $E_\gamma = hf = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34})(3.00\times10^8)}{694.3\times10^{-9}} \text{ J} = 2.86\times10^{-19} \text{ J}$

So $N = \dfrac{E}{E_\gamma} = \dfrac{1.00\times10^{-2} \text{ J}}{2.86\times10^{-19} \text{ J / photon}} = 3.49\times10^{16} \text{ photons}$

L: With 10^{16} photons/pulse, this laser beam should produce a bright red spot when the light reflects from a surface, even though the time between pulses is generally much longer than the width of each pulse. For comparison, this laser produces more photons in a single ten-nanosecond pulse than a typical 5 mW helium-neon laser produces over a full second (about 1.6×10^{16} photons/second).

***42.44** In $G = e^{\sigma(n_u - n_l)L}$ we require $1.05 = e^{(1.00\times10^{-18} \text{ m}^2)(n_u - n_l)(0.500 \text{ m})}$

Thus, $\ln(1.05) = (5.00\times10^{-19} \text{ m}^3)(n_u - n_l)$ so $n_u - n_l = \dfrac{\ln(1.05)}{5.00\times10^{-19} \text{ m}^3} = \boxed{9.76\times10^{16} \text{ m}^{-3}}$

42.45 (a) $\dfrac{N_3}{N_2} = \dfrac{N_g e^{-E_3/(k_B\cdot300 \text{ K})}}{N_g e^{-E_2/(k_B\cdot300 \text{ K})}} = e^{-(E_3 - E_2)/(k_B\cdot300 \text{ K})} = e^{-hc/\lambda(k_B\cdot300 \text{ K})}$

where λ is the wavelength of light radiated in the $3\to2$ transition:

$$\frac{N_3}{N_2} = e^{-(6.626\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})/(632.8\times10^{-9} \text{ m})(1.38\times10^{-23} \text{ J/K})(300 \text{ K})} = e^{-75.8} = \boxed{1.22\times10^{-33}}$$

(b) $$\frac{N_3}{N_2} = e^{hc/\lambda k_B T} = e^{-(6.626\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})/(694.3\times10^{-9} \text{ m})(1.38\times10^{-23} \text{ J/K})(4.00 \text{ K})} = e^{-5187}$$

To avoid overflowing your calculator, note that $10 = e^{\ln 10}$. Take

$$\frac{N_3}{N_2} = e^{\ln 10\times(-5187/\ln 10)} = \boxed{10^{-2253}}$$

***42.46** $N_u/N_l = e^{-(E_u - E_l)/k_B T}$ where the subscript u refers to an upper energy state and the subscript l to a lower energy state.

(a) Since $E_u - E_l = E_{\text{photon}} = hc/\lambda$, $N_u/N_l = e^{-hc/\lambda k_B T}$

Thus, we require $1.02 = e^{-hc/\lambda k_B T}$ or $\ln(1.02) = -\dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(694.3\times10^{-9}\text{ m})(1.38\times10^{-23}\text{ J/K})T}$

$$T = -\frac{2.07\times10^4\text{ K}}{\ln(1.02)} = \boxed{-1.05\times10^6\text{ K}}$$

A negative-temperature state is not achieved by cooling the system below 0 K, but by heating it above $T = \infty$, for as $T \to \infty$ the populations of upper and lower states approach equality.

(b) Because $E_u - E_l > 0$, and in any real equilibrium state $T > 0$, $e^{-(E_u - E_l)/k_B T} < 1$ and $N_u < N_l$.

Thus, a population inversion cannot happen in thermal equilibrium.

42.47 (a) $I = \dfrac{(3.00\times10^{-3}\text{ J})}{(1.00\times10^{-9}\text{ s})\pi(15.0\times10^{-6}\text{ m})^2} = \boxed{4.24\times10^{15}\text{ W/m}^2}$

(b) $(3.00\times10^{-3}\text{ J})\dfrac{(0.600\times10^{-9}\text{ m})^2}{(30.0\times10^{-6}\text{ m})^2} = \boxed{1.20\times10^{-12}\text{ J}} = 7.50\text{ MeV}$

***42.48** (a) The energy difference between these two states is equal to the energy that is absorbed.

Thus, $E = E_2 - E_1 = \dfrac{(-13.6\text{ eV})}{4} - \dfrac{(-13.6\text{ eV})}{1} = 10.2\text{ eV} = \boxed{1.63\times10^{-18}\text{ J}}$

(b) We have $E = \dfrac{3}{2}k_B T$, or $T = \dfrac{2}{3k_B}E = \dfrac{2(1.63\times10^{-18}\text{ J})}{3(1.38\times10^{-23}\text{ J/K})} = \boxed{7.88\times10^4\text{ K}}$

42.49 $r_{\text{av}} = \int_0^\infty rP(r)dr = \int_0^\infty \left(\dfrac{4r^3}{a_0^3}\right)(e^{-2r/a_0})dr$

Make a change of variables with $\dfrac{2r}{a_0} = x$ and $dr = \dfrac{a_0}{2}dx$.

Then $r_{\text{av}} = \dfrac{a_0}{4}\int_0^\infty x^3 e^{-x}dx = \dfrac{a_0}{4}\left[-x^3e^{-x} + 3\left(-x^2e^{-x} + 2e^{-x}(-x-1)\right)\right]\Big|_0^\infty = \boxed{\dfrac{3}{2}a_0}$

***42.50** $$\left\langle \frac{1}{r} \right\rangle = \int_0^\infty \frac{4r^2}{a_0^3} e^{-2r/a_0} \frac{1}{r} dr = \frac{4}{a_0^3} \int_0^\infty r e^{-(2/a_0)r} \, dr = \frac{4}{a_0^3} \frac{1}{(2/a_0)^2} = \boxed{\frac{1}{a_0}}$$

We compare this to $\frac{1}{\langle r \rangle} = \frac{1}{3a_0/2} = \frac{2}{3a_0}$, and find that the average reciprocal value is $\boxed{\text{NOT}}$ the reciprocal of the average value.

42.51 The wave equation for the $2s$ state is given by Eq. 42.7: $\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}} \left(\frac{1}{a_0} \right)^{3/2} \left[2 - \frac{r}{a_0} \right] e^{-r/2a_0}$

(a) Taking $r = a_0 = 0.529 \times 10^{-10}$ m, we find

$$\psi_{2s}(a_0) = \frac{1}{4\sqrt{2\pi}} \left(\frac{1}{0.529 \times 10^{-10} \text{ m}} \right)^{3/2} [2-1] e^{-1/2} = \boxed{1.57 \times 10^{14} \text{ m}^{-3/2}}$$

(b) $$\left| \psi_{2s}(a_0) \right|^2 = \left(1.57 \times 10^{14} \text{ m}^{-3/2} \right)^2 = \boxed{2.47 \times 10^{28} \text{ m}^{-3}}$$

(c) Using Equation 42.5 and the results to (b) gives $P_{2s}(a_0) = 4\pi a_0^2 \left| \psi_{2s}(a_0) \right|^2 = \boxed{8.69 \times 10^8 \text{ m}^{-1}}$

***42.52** We define the reduced mass to be μ, and the ground state energy to be E_1:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{207 m_e m_p}{207 m_e + m_p} = \frac{207 \left(9.11 \times 10^{-31} \text{ kg} \right) \left(1.67 \times 10^{-27} \text{ kg} \right)}{207 \left(9.11 \times 10^{-31} \text{ kg} \right) + \left(1.67 \times 10^{-27} \text{ kg} \right)} = 1.69 \times 10^{-28} \text{ kg}$$

$$E_1 = -\frac{\mu k_e^2 q_1^2 q_2^2}{2\text{h}^2 (1)^2} = -\frac{\left(1.69 \times 10^{-28} \text{ kg} \right) \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right)^2 \left(1.60 \times 10^{-19} \text{ C} \right)^3 e}{2 \left(1.055 \times 10^{-34} \text{ J} \cdot \text{s} \right)^2} = -2.52 \times 10^3 \text{ eV}$$

To ionize the muonium "atom" one must supply energy $\boxed{+2.52 \text{ keV}}$.

42.53 (a) $(3.00 \times 10^8 \text{ m/s})(14.0 \times 10^{-12} \text{ s}) = \boxed{4.20 \text{ mm}}$

(b) $E = \frac{hc}{\lambda} = 2.86 \times 10^{-19}$ J $\qquad N = \frac{3.00 \text{ J}}{2.86 \times 10^{-19} \text{ J}} = \boxed{1.05 \times 10^{19} \text{ photons}}$

(c) $V = (4.20 \text{ mm})\pi(3.00 \text{ mm})^2 = 119 \text{ mm}^3$ $\qquad n = \frac{1.05 \times 10^{19}}{119} = \boxed{8.82 \times 10^{16} \text{ mm}^{-3}}$

42.54 (a) The length of the pulse is $\Delta L = \boxed{ct}$

(b) The energy of each photon is $E_\gamma = \dfrac{hc}{\lambda}$ so $N = \dfrac{E}{E_\gamma} = \boxed{\dfrac{E\lambda}{hc}}$

(c) $V = \Delta L \pi \dfrac{d^2}{4}$ $n = \dfrac{N}{V} = \boxed{\left(\dfrac{4}{ct\pi d^2}\right)\left(\dfrac{E\lambda}{hc}\right)}$

42.55 We use $\psi_{2s}(r) = \dfrac{1}{4}(2\pi a_0^3)^{-1/2}\left(2 - \dfrac{r}{a_0}\right)e^{-r/2a_0}$

By Equation 42.5, $P(r) = 4\pi r^2\psi^2 = \dfrac{1}{8}\left(\dfrac{r^2}{a_0^3}\right)\left(2 - \dfrac{r}{a_0}\right)^2 e^{-r/a_0}$

(a)
$$\frac{dP(r)}{dr} = \frac{1}{8}\left[\frac{2r}{a_0^3}\left(2 - \frac{r}{a_0}\right)^2 - \frac{2r^2}{a_0^3}\left(\frac{1}{a_0}\right)\left(2 - \frac{r}{a_0}\right) - \frac{r^2}{a_0^3}\left(2 - \frac{r}{a_0}\right)^2\left(\frac{1}{a_0}\right)\right]e^{-r/a_0} = 0$$

or
$$\frac{1}{8}\left(\frac{r}{a_0^3}\right)\left(2 - \frac{r}{a_0}\right)\left[2\left(2 - \frac{r}{a_0}\right) - \frac{2r}{a_0} - \frac{r}{a_0}\left(2 - \frac{r}{a_0}\right)\right]e^{-r/a_0} = 0$$

Therefore we require the roots of $\dfrac{dP}{dr} = 0$ at $r = 0$, $r = 2a_0$, and $r = \infty$ to be minima with $P(r) = 0$.

$[\,.\,.\,.\,.\,.\,] = 4 - (6r/a_0) + (r/a_0)^2 = 0$ with solutions $r = \left(3 \pm \sqrt{5}\right)a_0$.

We substitute the last two roots into $P(r)$ to determine the most probable value:

When $r = \left(3 - \sqrt{5}\right)a_0 = 0.7639a_0$, then $P(r) = 0.0519/a_0$
When $r = \left(3 + \sqrt{5}\right)a_0 = 5.236a_0$, then $P(r) = 0.191/a_0$

Therefore, the most probable value of r is $\left(3 + \sqrt{5}\right)a_0 = \boxed{5.236a_0}$

(b) $\displaystyle\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}\left(\frac{r^2}{a_0{}^3}\right)\left(2 - \frac{r}{a_0}\right)^2 e^{-r/a_0}\,dr$ Let $u = \dfrac{r}{a_0}$, $dr = a_0\,du$,

$$\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}u^2(4 - 4u + u^2)e^{-u}\,dr = \int_0^\infty \frac{1}{8}(u^4 - 4u^3 + 4u^2)e^{-u}\,du = -\frac{1}{8}(u^4 + 4u^2 + 8u + 8)e^{-u}\Big|_0^\infty = 1$$

$\boxed{\text{This is as desired}}$.

***42.56** $\Delta z = \dfrac{at^2}{2} = \dfrac{1}{2}\left(\dfrac{F_z}{m_0}\right)t^2 = \dfrac{\mu_z(dB_z/dz)}{2m_0}\left(\dfrac{\Delta x}{v}\right)^2$ and $\mu_z = \dfrac{e\hbar}{2m_e}$

$$\frac{dB_z}{dz} = \frac{2m_0(\Delta z)v^2 2m_e}{\Delta x^2 e\hbar} = \frac{2(108)(1.66\times10^{-27}\text{ kg})(10^4\text{ m}^2/\text{s}^2)2(9.11\times10^{-31}\text{ kg})(10^{-3}\text{ m})}{(1.00\text{ m}^2)(1.60\times10^{-19}\text{ C})(1.05\times10^{-34}\text{ J}\cdot\text{s})} = \boxed{0.389\text{ T/m}}$$

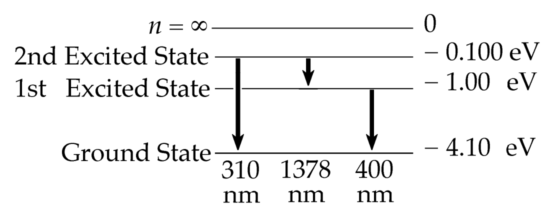
42.57 With one vacancy in the K shell, excess energy $\Delta E \approx -(Z-1)^2(13.6\text{ eV})\left(\frac{1}{2^2}-\frac{1}{1^2}\right) = 5.40\text{ keV}$

We suppose the outermost $4s$ electron is shielded by 20 electrons inside its orbit:

$$E_{\text{ionization}} \approx \frac{2^2(13.6\text{ eV})}{4^2} = 3.40\text{ eV}$$

Note the experimental ionization energy is 6.76 eV. $K = \Delta E - E_{\text{ionization}} \approx \boxed{5.39\text{ keV}}$

***42.58** $E = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{\lambda} = \Delta E$



$\lambda_1 = 310$ nm, so $\Delta E_1 = 4.00$ eV

$\lambda_2 = 400$ nm, $\Delta E_2 = 3.10$ eV

$\lambda_3 = 1378$ nm, $\Delta E_3 = 0.900$ eV

and the ionization energy = 4.10 eV

The energy level diagram having the fewest levels and consistent with these energies is shown at the right.

42.59 (a) One molecule's share of volume

Al: $V = \frac{\text{mass per molecule}}{\text{density}} = \left(\frac{27.0\text{ g/mol}}{6.02\times10^{23}\text{ molecules/mole}}\right)\left(\frac{1.00\times10^{-6}\text{ m}^3}{2.70\text{ g}}\right) = 1.66\times10^{-29}\text{ m}^3$

$\sqrt[3]{V} = \boxed{2.55\times10^{-10}\text{ m} \sim 10^{-1}\text{ nm}}$

U: $V = \left(\frac{238\text{ g}}{6.02\times10^{23}\text{ molecules}}\right)\left(\frac{1.00\times10^{-6}\text{ m}^3}{18.9\text{ g}}\right) = 2.09\times10^{-29}\text{ m}^3$

$\sqrt[3]{V} = \boxed{2.76\times10^{-10}\text{ m} \sim 10^{-1}\text{ nm}}$

(b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge, $+Ze-(Z-1)e = +e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is a_0/Z.

42.60 (a) No orbital magnetic moment to consider: higher energy for $\boxed{\begin{matrix}N\\S\end{matrix}}\ \boxed{\begin{matrix}N\\S\end{matrix}}$ parallel magnetic moments, for $\boxed{\text{antiparallel spins}}$ of the electron and proton.

(b) $E = \dfrac{hc}{\lambda} = 9.42 \times 10^{-25}\text{ J} = \boxed{5.89\ \mu\text{eV}}$

(c) $\Delta E \Delta t \approx \dfrac{\hbar}{2}$ so $\Delta E \approx \dfrac{1.055 \times 10^{-34}\text{ J}\cdot\text{s}}{2\left(10^{7}\text{ yr}\right)\left(3.16 \times 10^{7}\text{ s/yr}\right)}\left(\dfrac{1.00\text{ eV}}{1.60 \times 10^{-19}\text{ J}}\right) = \boxed{1.04 \times 10^{-30}\text{ eV}}$

42.61 $P = \displaystyle\int_{2.50 a_0}^{\infty} \frac{4r^2}{a_0^{\,3}} e^{-2r/a_0}\,dr = \tfrac{1}{2}\int_{5.00}^{\infty} z^2 e^{-z}\,dz$ where $z \equiv \dfrac{2r}{a_0}$

$$P = -\frac{1}{2}(z^2 + 2z + 2)e^{-z}\Big|_{5.00}^{\infty} = -\frac{1}{2}[0] + \frac{1}{2}(25.0 + 10.0 + 2.00)e^{-5} = \left(\frac{37}{2}\right)(0.00674) = \boxed{0.125}$$

Goal Solution

For hydrogen in the 1*s* state, what is the probability of finding the electron farther than $2.50 a_o$ from the nucleus?

G: From the graph shown in Figure 42.8, it appears that the probability of finding the electron beyond $2.5\ a_0$ is about 20%.

O: The precise probability can be found by integrating the 1*s* radial probability distribution function from $r = 2.50 a_o$ to ∞.

A: The general radial probability distribution function is $P(r) = 4\pi r^2 |\psi|^2$

With $\psi_{1s} = \left(\pi a_0^{\,3}\right)^{-1/2} e^{-r/a_0}$ it is $P(r) = 4r^2 a_0^{\,-3} e^{-2r/a_0}$

The required probability is then $P = \displaystyle\int_{2.50a_0}^{\infty} P(r)dr = \int_{2.50a_0}^{\infty} \frac{4r^2}{a_0^{\,3}} e^{-2r/a_0} dr$

Let $z = 2r/a_0$ and $dz = 2dr/a_0$: $P = \tfrac{1}{2}\displaystyle\int_{5.00}^{\infty} z^2 e^{-z} dz$

Performing this integration by parts, $P = -\tfrac{1}{2}\left(z^2 + 2z + 2\right)e^{-z}\Big]_{5.00}^{\infty}$

$$P = -\tfrac{1}{2}(0) + \tfrac{1}{2}(25.0 + 10.0 + 2.00)e^{-5.00} = \left(\tfrac{37}{2}\right)(0.00674) = 0.125$$

L: The probability of 12.5% is less than the 20% we estimated, but close enough to be a reasonable result. In comparing the 1*s* probability density function with the others in Figure 42.8, it appears that the ground state is the most narrow, indicating that a 1*s* electron will probably be found in the narrow range of 0 to 4 Bohr radii, and most likely at $r = a_0$.

42.62 The probability, P, of finding the electron within the Bohr radius is

$$P = \int_{r=0}^{a_0} P_{1s}(r)\,dr = \int_{r=0}^{a_0} \left(\frac{4r^2}{a_0^{\,3}}\right) e^{-2r/a_0}\,dr$$

Defining $z \equiv 2r/a_0$, this becomes

$$P = -\frac{1}{2}(z^2 + 2z + 2)e^{-z}\Big|_0^2 = -\frac{1}{2}\left[(4+4+2)e^{-2} - (0+0+2)e^0\right] = \frac{1}{2}\left(2 - \frac{10}{e^2}\right) = \boxed{0.323}$$

The electron is likely to be within the Bohr radius about one-third of the time. The Bohr model indicates *none* of the time.

42.63 (a) For a classical atom, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{1}{4\pi e_0}\frac{e^2}{r^2 m_e}$$

$$E = -\frac{e^2}{4\pi e_0 r} + \frac{m_e v^2}{2} = -\frac{e^2}{8\pi e_0 r} \quad \text{so} \quad \frac{dE}{dt} = \frac{e^2}{8\pi e_0 r^2}\frac{dr}{dt} = \frac{-1}{6\pi e_0}\frac{e^2 a^2}{c^3} = \frac{-e^2}{6\pi e_0 c^3}\left(\frac{e^2}{4\pi e_0 r^2 m_e}\right)^2$$

Therefore, $$\frac{dr}{dt} = -\frac{e^4}{12\pi^2 e_0^{\,2} r^2 m_e^{\,2} c^3}$$

(b) $$-\int_{r=2.00\times10^{-10}\ \text{m}}^{r=0} 12\pi^2 e_0^{\,2} r^2 m_e^{\,2} c^3\,dr = e^4 \int_{t=0}^{T} dt$$

$$\frac{12\pi^2 e_0^{\,2} m_e^{\,2} c^3}{e^4}\frac{r^3}{3}\Bigg|_0^{2.00\times10^{-10}} = T = \boxed{8.46\times10^{-10}\ \text{s}}$$

Since atoms last a lot longer than 0.8 ns, the classical laws (fortunately!) do not hold for systems of atomic size.

42.64 (a) $+3e - 0.85e - 0.85e = \boxed{1.30e}$

(b) The valence electron is in an $n = 2$ state, with energy

$$\frac{-13.6\ \text{eV}\ Z_{\text{eff}}^2}{n^2} = \frac{-13.6\ \text{eV}(1.30)^2}{2^2} = -5.75\ \text{eV}$$

To ionize the atom you must put in $\boxed{+5.75\ \text{eV}}$

This differs from the experimental value by 6%, so we could say the effective value of Z is accurate within 3%.

42.65 $\Delta E = 2\mu_B B = hf$ so $2\left(9.27\times10^{-24}\text{ J / T}\right)\left(0.350\text{ T}\right) = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)f$

and $f = \boxed{9.79\times10^{9}\text{ Hz}}$

42.66 The photon energy is $E_4 - E_3 = 20.66 - 18.70\text{ eV} = 1.96\text{ eV} = \dfrac{hc}{\lambda}$

$$\lambda = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{1.96\times1.60\times10^{-19}\text{ J}} = \boxed{633\text{ nm}}$$

42.67 (a) $\dfrac{1}{\alpha} = \dfrac{\hbar c}{k_e e^2} = \dfrac{\left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\right)}{2\pi\left(8.99\times10^{9}\right)\left(1.60\times10^{-19}\right)^2} = \boxed{137}$

(b) $\dfrac{\lambda_C}{r_e} = \dfrac{h}{mc}\dfrac{mc^2}{k_e e^2} = \dfrac{hc}{ke^2} = \boxed{\dfrac{2\pi}{\alpha}}$

(c) $\dfrac{a_0}{\lambda_C} = \dfrac{\hbar^2}{mk_e e^2}\dfrac{mc}{h} = \dfrac{1}{2\pi}\dfrac{\hbar c}{k_e e^2} = \dfrac{137}{2\pi} = \boxed{\dfrac{1}{2\pi\alpha}}$

(d) $\dfrac{1/R_H}{a_0} = \dfrac{1}{R_H a_0} = \dfrac{4\pi c\hbar^3}{mk_e^2 e^4}\dfrac{mk_e e^2}{\hbar^2} = 4\pi\dfrac{\hbar c}{k_e e^2} = \boxed{\dfrac{4\pi}{\alpha}}$

42.68 $\psi = \frac{1}{4}(2\pi)^{-1/2}\left(\dfrac{1}{a_0}\right)^{3/2}\left(2 - \dfrac{r}{a_0}\right)e^{-r/2a_0} = A\left(2 - \dfrac{r}{a_0}\right)e^{-r/2a_0}$ $\qquad \dfrac{\partial^2\psi}{\partial r^2} = \left(\dfrac{Ae^{-r/2a_0}}{a_0^2}\right)\left(\dfrac{3}{2} - \dfrac{r}{4a_0}\right)$

Substituting into Schrödinger's equation and dividing by ψ,

$$\frac{1}{a_0^2}\left(\frac{1}{2} - \frac{r}{4a_0}\right) = -\frac{2m}{\hbar^2}[E-U]\left(2 - \frac{r}{a_0}\right)$$

Now $$E - U = \frac{\hbar^2}{2m\,a_0^2}\left(\frac{1}{4}\right) - \frac{\left(ke^2/4a_0\right)\left(m/\hbar^2\right)}{\left(m/\hbar^2\right)} = -\frac{1}{4}\left(\frac{\hbar^2}{2m\,a_0^2}\right)$$

and $$\left(\frac{1}{a_0^2}\right)\left(\frac{1}{2} - \frac{r}{4a_0}\right) = \frac{1}{4a_0^2}\left(2 - \frac{r}{a_0}\right)$$ $\therefore\ \psi$ is a solution.

***42.69** The beam intensity is reduced by absorption of photons into atoms in the lower state. The number of transitions per time and per area is $-BN_l I(x) n\,dx/c$. The beam intensity is increased by stimulating emission in atoms in the upper state, with transition rate $+BN_u I(x) n\,dx/c$. The net rate of change in photon numbers per area is then $-B(N_l - N_u) I(x) n\,dx/c$.

Each photon has energy hf, so the net change in intensity is

$$dI(x) = -hfB(N_l - N_u) I(x) n\,dx/c = -hfB\Delta N\, I(x) n\,dx/c$$

Then, $\dfrac{dI(x)}{I(x)} = -\dfrac{hfB\Delta N\, n}{c}dx$ so $\displaystyle\int_{I_0}^{I(L)} \frac{dI(x)}{I(x)} = \int_{x=0}^{L}\left(-\frac{hfB\Delta N\, n}{c}\right)dx$

$$\ln[I(L)] - \ln[I_0] = \ln\left[\frac{I(L)}{I_0}\right] = -\frac{hfB\Delta N\, n}{c}(L-0)$$

$$I(L) = I_0\, e^{-hfB\Delta N\, nL/c} = I_0\, e^{-\alpha L}$$

This result is also expressed in problem 42.44 as $\dfrac{I(L)}{I_0} = G = e^{-\sigma(n_l - n_u)L} = e^{+\sigma(n_u - n_l)L}$

***42.70** (a) Suppose the atoms move in the $+x$ direction. The absorption of a photon by an atom is a completely inelastic collision, described by

$$mv_i\,\mathbf{i} + \frac{h}{\lambda}(-\mathbf{i}) = mv_f\,\mathbf{i} \quad \text{so} \quad v_f - v_i = -\frac{h}{m\lambda}$$

This happens promptly every time an atom has fallen back into the ground state, so it happens every $10^{-8}\text{ s} = \Delta t$. Then,

$$a = \frac{v_f - v_i}{\Delta t} = -\frac{h}{m\lambda\,\Delta t} \sim -\frac{6.626\times 10^{-34}\text{ J}\cdot\text{s}}{(10^{-25}\text{ kg})(500\times 10^{-9}\text{ m})(10^{-8}\text{ s})} \quad \boxed{-10^{6}\text{ m/s}^2}$$

(b) With constant average acceleration, $v_f^2 = v_i^2 + 2a(\Delta x)$

$$0 \sim (10^3\text{ m/s})^2 + 2(-10^6\text{ m/s}^2)\Delta x \quad \text{so} \quad \Delta x \sim \frac{(10^3\text{ m/s})^2}{10^6\text{ m/s}^2} \quad \boxed{\sim 1\text{ m}}$$

Chapter 43 Solutions

43.1 (a) $F = \dfrac{q^2}{4\pi e_0 r^2} = \dfrac{(1.60\times10^{-19})^2(8.99\times10^9)}{(5.00\times10^{-10})^2}\ \text{N} = \boxed{0.921\times10^{-9}\ \text{N}}$ toward the other ion.

(b) $U = \dfrac{-q^2}{4\pi e_0 r} = -\dfrac{(1.60\times10^{-19})^2(8.99\times10^9)}{5.00\times10^{-10}}\ \text{J} \approx \boxed{-2.88\ \text{eV}}$

***43.2** We are told $\quad \text{K} + \text{Cl} + 0.7\ \text{eV} \rightarrow \text{K}^+ + \text{Cl}^-$

and $\quad \text{Cl} + \text{e}^- \rightarrow \text{Cl}^- + 3.6\ \text{eV}$

or $\quad \text{Cl}^- \rightarrow \text{Cl} + \text{e}^- - 3.6\ \text{eV}$

By substitution, $\quad \text{K} + \text{Cl} + 0.7\ \text{eV} \rightarrow \text{K}^+ + \text{Cl} + \text{e}^- - 3.6\ \text{eV}$

$$\text{K} + 4.3\ \text{eV} \rightarrow \text{K}^+ + \text{e}^-$$

or the ionization energy of potassium is $\boxed{4.3\ \text{eV}}$

43.3 (a) Minimum energy of the molecule is found from

$$\frac{dU}{dr} = -12Ar^{-13} + 6Br^{-7} = 0, \quad \text{yielding} \quad \boxed{r_0 = \left[\frac{2A}{B}\right]^{1/6}}$$

(b) $E = U|_{r=\infty} - U|_{r=r_0} = 0 - \left[\dfrac{A}{4A^2/B^2} - \dfrac{B}{2A/B}\right] = -\left[\dfrac{1}{4} - \dfrac{1}{2}\right]\dfrac{B^2}{A} = \boxed{\dfrac{B^2}{4A}}$

This is also the equal to the binding energy, the amount of energy given up by the two atoms as they come together to form a molecule.

(c) $r_0 = \left[\dfrac{2\left(0.124\times10^{-120}\ \text{eV}\cdot\text{m}^{12}\right)}{1.488\times10^{-60}\ \text{eV}\cdot\text{m}^6}\right]^{1/6} = 7.42\times10^{-11}\ \text{m} = \boxed{74.2\ \text{pm}}$

$$E = \frac{\left(1.488\times10^{-60}\ \text{eV}\cdot\text{m}^6\right)^2}{4\left(0.124\times10^{-120}\ \text{eV}\cdot\text{m}^{12}\right)} = \boxed{4.46\ \text{eV}}$$

***43.4** At the boiling or condensation temperature, $\quad k_B T \approx 10^{-3}\ \text{eV} = 10^{-3}\left(1.6\times10^{-19}\ \text{J}\right)$

$$T \approx \frac{1.6\times10^{-22}\ \text{J}}{1.38\times10^{-23}\ \text{J/K}} \quad \boxed{\sim 10\ \text{K}}$$

43.5 $\mu = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{132.9(126.9)}{132.9 + 126.9}\left(1.66 \times 10^{-27}\ \text{kg}\right) = 1.08 \times 10^{-25}\ \text{kg}$

$$I = \mu r^2 = (1.08 \times 10^{-25}\ \text{kg})(0.127 \times 10^{-9}\ \text{m})^2 = 1.74 \times 10^{-45}\ \text{kg} \cdot \text{m}^2$$

(a) $E = \frac{1}{2} I \omega^2 = \dfrac{(I\omega)^2}{2I} = \dfrac{J(J+1)\hbar^2}{2I}$

$J = 0$ gives $E = 0$

$J = 1$ gives $E = \dfrac{\hbar^2}{I} = \dfrac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})^2}{4\pi^2(1.74 \times 10^{-45}\ \text{kg} \cdot \text{m}^2)} = 6.41 \times 10^{-24}\ \text{J} = \boxed{40.0\ \mu\text{eV}}$

$hf = 6.41 \times 10^{-24}\ \text{J} - 0$ to $f = \boxed{9.66 \times 10^9\ \text{Hz}}$

(b) $f = \dfrac{E_1}{h} = \dfrac{\hbar^2}{hI} = \dfrac{h}{4\pi^2 \mu r^2} \propto r^{-2}$ $\boxed{\text{If } r \text{ is 10\% too small, } f \text{ is 20\% too large.}}$

43.6 $hf = \Delta E = \dfrac{\hbar^2}{2I}\left[2(2+1)\right] - \dfrac{\hbar^2}{2I}\left[1(1+1)\right] = \dfrac{\hbar^2}{2I}(4)$

$$I = \frac{4(h/2\pi)^2}{2hf} = \frac{h}{2\pi^2 f} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{2\pi^2\left(2.30 \times 10^{11}\ \text{Hz}\right)} = \boxed{1.46 \times 10^{-46}\ \text{kg} \cdot \text{m}^2}$$

43.7 For the HCl molecule in the $J = 1$ rotational energy level, we are given $r_0 = 0.1275\ \text{nm}$.

$$E_{\text{rot}} = \frac{\hbar^2}{2I} J(J+1)$$

Taking $J = 1$, we have $E_{\text{rot}} = \dfrac{\hbar^2}{I} = \frac{1}{2} I \omega^2$ or $\omega = \sqrt{\dfrac{2\hbar^2}{I^2}} = \sqrt{2}\dfrac{\hbar}{I}$

The moment of inertia of the molecule is given by Equation 43.3. $I = \mu r_0^2 = \left(\dfrac{m_1 m_2}{m_1 + m_2}\right) r_0^2$

$$I = \left[\frac{(1\ \text{u})(35\ \text{u})}{1\ \text{u} + 35\ \text{u}}\right] r_0^2 = (0.972\ \text{u})(1.66 \times 10^{-27}\ \text{kg / u})(1.275 \times 10^{-10}\ \text{m})^2 = 2.62 \times 10^{-47}\ \text{kg} \cdot \text{m}^2$$

Therefore, $\omega = \sqrt{2}\dfrac{\hbar}{I} = \sqrt{2}\dfrac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{2.62 \times 10^{-47}\ \text{kg} \cdot \text{m}^2} = \boxed{5.69 \times 10^{12}\ \text{rad / s}}$

Goal Solution

An HCl molecule is excited to its first rotational-energy level, corresponding to $J=1$. If the distance between its nuclei is 0.1275 nm, what is the angular speed of the molecule about its center of mass?

G: For a system as small as a molecule, we can expect the angular speed to be much faster than the few rad/s typical of everyday objects we encounter.

O: The rotational energy is given by the angular momentum quantum number, J. The angular speed can be calculated from this kinetic rotational energy and the moment of inertia of this one-dimensional molecule.

A: For the HCl molecule in the $J=1$ rotational energy level, we are given $r_0 = 0.1275$ nm.

$$E_{rot} = \frac{\hbar}{2I} J(J+1) \qquad \text{so with } J=1, \qquad E_{rot} = \frac{\hbar^2}{I} = \frac{1}{2} I\omega^2 \qquad \text{and} \qquad \omega = \sqrt{\frac{2\hbar^2}{I^2}} = \frac{\hbar\sqrt{2}}{I}$$

The moment of inertia of the molecule is given by: $I = \mu r_0^2 = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r_0^2 = \left[\frac{(1\text{ u})(35\text{ u})}{1\text{ u} + 35\text{ u}}\right] r_0^2$

$$I = (0.972\text{ u})(1.66\times10^{-27}\text{ kg / u})(1.275\times10^{-10}\text{ m})^2 = 2.62\times10^{-47}\text{ kg}\cdot\text{m}^2$$

Therefore, $\omega = \sqrt{2}\frac{\hbar}{I} = \sqrt{2}\left(\frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2.62\times10^{-47}\text{ kg}\cdot\text{m}^2}\right) = 5.69\times10^{12}\text{ rad / s}$

L: This angular speed is more than a billion times faster than the spin rate of a music CD, which rotates at 200 to 500 revolutions per minute, or $\omega = 20$ rad / s to 50 rad / s.

43.8 $I = m_1 r_1^2 + m_2 r_2^2$ where $m_1 r_1 = m_2 r_2$ and $r_1 + r_2 = r$

Then $r_1 = \frac{m_2 r_2}{m_1}$ so $\frac{m_2 r_2}{m_1} + r_2 = r$ and $r_2 = \frac{m_1 r}{m_1 + m_2}$

Also, $r_2 = \frac{m_1 r_1}{m_2}$ Thus, $r_1 + \frac{m_1 r_1}{m_2} = r$ and $r_1 = \frac{m_2 r}{m_1 + m_2}$

$$I = m_1 \frac{m_2^2 r^2}{(m_1+m_2)^2} + \frac{m_2 m_1^2 r^2}{(m_1+m_2)^2} = \frac{m_1 m_2 r^2 (m_2 + m_1)}{(m_1+m_2)^2} = \frac{m_1 m_2 r^2}{m_1 + m_2} = \boxed{\mu r^2}$$

43.9 (a) $\mu = \frac{22.99(35.45)}{(22.99 + 35.45)}\left(1.66\times10^{-27}\text{ kg}\right) = 2.32\times10^{-26}\text{ kg}$

$$I = \mu r^2 = \left(2.32\times10^{-26}\text{ kg}\right)\left(0.280\times10^{-9}\text{ m}\right)^2 = \boxed{1.81\times10^{-45}\text{ kg}\cdot\text{m}^2}$$

(b) $\frac{hc}{\lambda} = \frac{\hbar^2}{2I} 2(2+1) - \frac{\hbar^2}{2I} 1(1+1) = \frac{3\hbar^2}{I} - \frac{\hbar^2}{I} = \frac{2\hbar^2}{I} = \frac{2h^2}{4\pi^2 I}$

$$\lambda = \frac{c 4\pi^2 I}{2h} = \frac{\left(3.00\times10^8\text{ m / s}\right)4\pi^2\left(1.81\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}{2\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)} = \boxed{1.62\text{ cm}}$$

43.10 The energy of a rotational transition is $\Delta E = \left(\hbar^2/I\right)J$ where J is the rotational quantum number of the higher energy state (see Equation 43.7). We do not know J from the data. However,

$$\Delta E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \mathrm{J\cdot s}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)}{\lambda}\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right)$$

For each observed wavelength,

λ (mm)	ΔE (eV)
0.1204	0.01032
0.0964	0.01288
0.0804	0.01544
0.0690	0.01800
0.0604	0.02056

The $\Delta E's$ consistently increase by 0.00256 eV. $E_1 = \hbar^2/I = 0.00256$ eV

and $$I = \frac{\hbar^2}{E_1} = \frac{\left(1.055\times10^{-34}\ \mathrm{J\cdot s}\right)^2}{(0.00256\ \mathrm{eV})}\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right) = \boxed{2.72\times10^{-47}\ \mathrm{kg\cdot m^2}}$$

For the HCl molecule, the internuclear radius is $r = \sqrt{\dfrac{I}{\mu}} = \sqrt{\dfrac{2.72\times10^{-47}}{1.62\times10^{-27}}}\ \mathrm{m} = 0.130\ \mathrm{nm}$

43.11 $$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{35}{36}\times1.66\times10^{-27}\ \mathrm{kg} = 1.61\times10^{-27}\ \mathrm{kg}$$

$$\Delta E_{\mathrm{vib}} = \hbar\sqrt{\frac{k}{\mu}} = \left(1.055\times10^{-34}\right)\sqrt{\frac{480}{1.61\times10^{-27}}} = 5.74\times10^{-20}\ \mathrm{J} = \boxed{0.358\ \mathrm{eV}}$$

43.12 (a) Minimum amplitude of vibration of HI is

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}hf:\ A = \sqrt{\frac{hf}{k}} = \sqrt{\frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})(6.69\times10^{13}\ /\ \mathrm{s})}{320\ \mathrm{N\ /\ m}}} = 1.18\times10^{-11}\ \mathrm{m} = \boxed{0.0118\ \mathrm{nm}}$$

(b) For HF, $$A = \sqrt{\frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})(8.72\times10^{13}\ /\ \mathrm{s})}{970\ \mathrm{N\ /\ m}}} = 7.72\times10^{-12}\ \mathrm{m} = \boxed{0.00772\ \mathrm{nm}}$$

Since HI has the smaller k, it is more weakly bound.

43.13 (a) The reduced mass of the O_2 is $\mu = \dfrac{(16\text{ u})(16\text{ u})}{(16\text{ u})+(16\text{ u})} = 8\text{ u} = 8\left(1.66\times10^{-27}\text{ kg}\right) = 1.33\times10^{-26}\text{ kg}$

The moment of inertia is then $I = \mu r^2 = \left(1.33\times10^{-26}\text{ kg}\right)\left(1.20\times10^{-10}\text{ m}\right)^2 = 1.91\times10^{-46}\text{ kg}\cdot\text{m}^2$

The rotational energies are $E_{\text{rot}} = \dfrac{\hbar^2}{2I}J(J+1) = \dfrac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.91\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}J(J+1)$

Thus $E_{\text{rot}} = \left(2.91\times10^{-23}\text{ J}\right)J(J+1)$

And for $J = 0, 1, 2,$ $E_{\text{rot}} = \boxed{0,\ 3.64\times10^{-4}\text{ eV},\ 1.09\times10^{-3}\text{ eV}}$

(b) $E_{\text{vib}} = \left(v+\dfrac{1}{2}\right)\hbar\sqrt{\dfrac{k}{\mu}} = \left(v+\dfrac{1}{2}\right)\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\sqrt{\dfrac{1177\text{ N / m}}{8(1.66\times10^{-27}\text{ kg})}}$

$E_{\text{vib}} = \left(v+\dfrac{1}{2}\right)\left(3.14\times10^{-20}\text{ J}\right)\left(\dfrac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \left(v+\dfrac{1}{2}\right)(0.196\text{ eV})$

For $v = 0, 1, 2,$ $\boxed{E_{\text{vib}} = 0.0982\text{ eV},\ 0.295\text{ eV},\ 0.491\text{ eV}}$

43.14 In Benzene, the carbon atoms are each 0.110 nm from the axis and each hydrogen atom is $(0.110+0.100\text{ nm}) = 0.210\text{ nm}$ from the axis. Thus, $I = \Sigma mr^2$:

$$I = 6\left(1.99\times10^{-26}\text{ kg}\right)\left(0.110\times10^{-9}\text{ m}\right)^2 + 6\left(1.67\times10^{-27}\text{ kg}\right)\left(0.210\times10^{-9}\text{ m}\right)^2 = 1.89\times10^{-45}\text{ kg}\cdot\text{m}^2$$

The allowed rotational energies are then

$$E_{\text{rot}} = \frac{\hbar^2}{2I}J(J+1) = \frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.89\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}J(J+1) = \left(2.95\times10^{-24}\text{ J}\right)J(J+1) = \left(18.4\times10^{-6}\text{ eV}\right)J(J+1)$$

$$E_{\text{rot}} = \boxed{(18.4\ \mu\text{eV})J(J+1)\quad\text{where } J = 0, 1, 2, 3, \ldots}$$

The first five of these allowed energies are: $E_{\text{rot}} = 0,\ 36.9\ \mu\text{eV},\ 111\ \mu\text{eV},\ 221\ \mu\text{eV},\text{ and }369\ \mu\text{eV}$

43.15 $hf = \dfrac{h^2}{4\pi^2 I}J$ where the rotational transition is from $J-1$ to J,

where $f = 6.42\times10^{13}\text{ Hz}$ and $I = 1.46\times10^{-46}\text{ kg}\cdot\text{m}^2$ from Example 43.1.

$$J = \frac{4\pi^2 If}{h} = \frac{4\pi^2(1.46\times10^{-46}\text{ kg}\cdot\text{m}^2)(6.42\times10^{13}\text{ / s})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{558}$$

***43.16** The emission energies are the same as the absorption energies, but the final state must be below $(v=1,\ J=0)$. The transition must satisfy $\Delta J = \pm 1$, so it must end with $J=1$. To be lower in energy, it must be $(v=0,\ J=1)$. The emitted photon energy is therefore

$$hf_{\text{photon}} = \left(E_{\text{vib}}\big|_{v=1} + E_{\text{rot}}\big|_{J=0}\right) - \left(E_{\text{vib}}\big|_{v=0} + E_{\text{rot}}\big|_{J=1}\right) = \left(E_{\text{vib}}\big|_{v=1} - E_{\text{vib}}\big|_{v=0}\right) - \left(E_{\text{rot}}\big|_{J=1} - E_{\text{rot}}\big|_{J=0}\right)$$

$$hf_{\text{photon}} = hf_{\text{vib}} - hf_{\text{rot}}$$

Thus, $f_{\text{photon}} = f_{\text{vib}} - f_{\text{rot}} = 6.42\times10^{13}\text{ Hz} - 1.15\times10^{11}\text{ Hz} = \boxed{6.41\times10^{13}\text{ Hz}}$

***43.17** The moment of inertia about the molecular axis is $I_x = \frac{2}{5}mr^2 + \frac{2}{5}mr^2 = \frac{4}{5}m\left(2.00\times10^{-15}\text{ m}\right)^2$

The moment of inertia about a perpendicular axis is $I_y = m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2 = \frac{m}{2}\left(2.00\times10^{-10}\text{ m}\right)^2$

The allowed rotational energies are $E_{\text{rot}} = \left(\hbar^2/2I\right)J(J+1)$, so the energy of the first excited state is $E_1 = \hbar^2/I$. The ratio is therefore

$$\frac{E_{1,x}}{E_{1,y}} = \frac{\left(\hbar^2/I_x\right)}{\left(\hbar^2/I_y\right)} = \frac{I_y}{I_x} = \frac{\frac{1}{2}m\left(2.00\times10^{-10}\text{ m}\right)^2}{\frac{4}{5}m\left(2.00\times10^{-15}\text{ m}\right)} = \frac{5}{8}\left(10^5\right)^2 = \boxed{6.25\times10^9}$$

***43.18** Consider a cubical salt crystal of edge length 0.1 mm.

The number of atoms is $\left(\dfrac{10^{-4}\text{ m}}{0.261\times10^{-9}\text{ m}}\right)^3$ $\boxed{\sim 10^{17}}$

This number of salt crystals would have volume $\left(10^{-4}\text{ m}\right)^3\left(\dfrac{10^{-4}\text{ m}}{0.261\times10^{-9}\text{ m}}\right)^3$ $\boxed{\sim 10^5\text{ m}^3}$

If it is cubic, it has edge length 40 m.

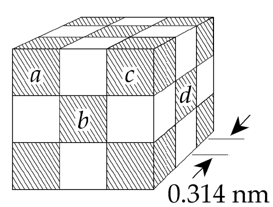
43.19 $$U = -\frac{\alpha k_e e^2}{r_0}\left(1-\frac{1}{m}\right) = -(1.7476)(8.99\times10^9)\frac{(1.60\times10^{-19})^2}{(0.281\times10^{-9})}\left(1-\frac{1}{8}\right) = -1.25\times10^{-18}\text{ J} = \boxed{-7.84\text{ eV}}$$

43.20 Visualize a K^+ ion at the center of each shaded cube, a Cl^- ion at the center of each white one.

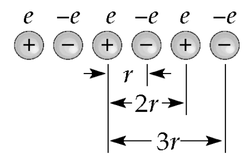
The distance ab is $\sqrt{2}\,(0.314\text{ nm}) = \boxed{0.444\text{ nm}}$

Distance ac is $2(0.314\text{ nm}) = \boxed{0.628\text{ nm}}$

Distance ad is $\sqrt{2^2 + (\sqrt{2})^2}\,(0.314\text{ nm}) = \boxed{0.769\text{ nm}}$



43.21 $U = -\dfrac{k_e e^2}{r} - \dfrac{k_e e^2}{r} + \dfrac{k_e e^2}{2r} + \dfrac{k_e e^2}{2r} - \dfrac{k_e e^2}{3r} - \dfrac{k_e e^2}{3r} + \dfrac{k_e e^2}{4r} + \dfrac{k_e e^2}{4r} - \;\ldots\; = -\dfrac{2k_e e^2}{r}\left(1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \;\ldots\right)$



But, $\ln(1+x) = 1 - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \;\ldots$

so, $U = -\dfrac{2k_e e^2}{r}\ln 2$, or $\boxed{U = -k_e \alpha \dfrac{e^2}{r} \quad \text{where} \quad \alpha = 2\ln 2}$

43.22 $E_F = \dfrac{h^2}{2m}\left(\dfrac{3n_e}{8\pi}\right)^{2/3} = \left[\dfrac{(6.625\times 10^{-34}\text{ J}\cdot\text{s})^2}{2(9.11\times 10^{-31}\text{ kg})(1.60\times 10^{-19}\text{ J / eV})}\right](3/8\pi)^{2/3} n^{2/3}$

$E_F = (3.65\times 10^{-19})n^{2/3}$ eV with n measured in electrons / m^3

43.23 The density of conduction electrons n is given by $E_F = \dfrac{h^2}{2m}\left(\dfrac{3n_e}{8\pi}\right)^{2/3}$

or $n_e = \dfrac{8\pi}{3}\left(\dfrac{2mE_F}{h^2}\right)^{3/2} = \dfrac{8\pi}{3}\dfrac{\left[2(9.11\times 10^{-31}\text{ kg})(5.48)(1.60\times 10^{-19}\text{ J})\right]^{3/2}}{(6.626\times 10^{-34}\text{ J}\cdot\text{s})^3} = 5.80\times 10^{28}\text{ m}^{-3}$

The number-density of silver atoms is

$$n_{Ag} = \left(10.6\times 10^3\text{ kg / m}^3\right)\left(\frac{1\text{ atom}}{108\text{ u}}\right)\left(\frac{1\text{ u}}{1.66\times 10^{-27}\text{ kg}}\right) = 5.91\times 10^{28}\text{ m}^{-3}$$

So an average atom contributes $\dfrac{5.80}{5.91} = \boxed{0.981\text{ electron to the conduction band}}$

43.24 (a) $\frac{1}{2}mv^2 = 7.05$ eV

$$v = \sqrt{\frac{2(7.05\text{ eV})(1.60\times 10^{-19}\text{ J / eV})}{9.11\times 10^{-31}\text{ kg}}} = \boxed{1.57\times 10^6\text{ m / s}}$$

(b) $\boxed{\text{Larger than } 10^{-4}\text{ m / s by ten orders of magnitude.}}$ However, the energy of an electron at room temperature is typically $k_B T = \frac{1}{40}$ eV.

43.25 For sodium, $M = 23.0\ \text{g/mol}$ and $\rho = 0.971\ \text{g/cm}^3$.

(a) $$n_e = \frac{N_A \rho}{M} = \frac{\left(6.02\times10^{23}\ \text{electrons/mol}\right)\left(0.971\ \text{g/cm}^3\right)}{23.0\ \text{g/mol}}$$

$$n_e = 2.54\times10^{22}\ \text{electrons/cm}^3 = \boxed{2.54\times10^{28}\ \text{electrons/m}^3}$$

(b) $$E_F = \left(\frac{h^2}{2m}\right)\left(\frac{3n_e}{8\pi}\right)^{2/3} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)}\left[\frac{3\left(2.54\times10^{28}\ \text{m}^{-3}\right)}{8\pi}\right]^{2/3} = 5.05\times10^{-19}\ \text{J} = \boxed{3.15\ \text{eV}}$$

(c) $$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2\left(5.05\times10^{-19}\ \text{J}\right)}{9.11\times10^{-31}\ \text{kg}}} = \boxed{1.05\times10^6\ \text{m/s}}$$

***43.26** The melting point of silver is 1234 K. Its Fermi energy at 300 K is 5.48 eV. The approximate fraction of electrons excited is

$$\frac{k_B T}{E_F} = \frac{\left(1.38\times10^{-23}\ \text{J/K}\right)(1234\ \text{K})}{(5.48\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)} \approx \boxed{2\%}$$

43.27 Taking $E_F = 5.48\ \text{eV}$ for sodium at 800 K,

$$f = \left[e^{(E-E_F)/k_B T} + 1\right]^{-1} = 0.950$$

$$e^{(E-E_F)/k_B T} = (1/0.950) - 1 = 0.0526$$

$$\frac{E-E_F}{k_B T} = \ln(0.0526) = -2.94$$

$$E - E_F = -2.94\frac{(1.38\times10^{-23})(800)\text{J}}{1.60\times10^{-19}\ \text{J/eV}} = -0.203\ \text{eV} \qquad \text{or} \qquad \boxed{E = 5.28\ \text{eV}}$$

Goal Solution

Calculate the energy of a conduction electron in silver at 800 K if the probability of finding an electron in that state is 0.950. The Fermi energy is 5.48 eV at this temperature.

G: Since there is a 95% probability of finding the electron in this state, its energy should be slightly less than the Fermi energy, as indicated by the graph in Figure 43.21.

O: The electron energy can be found from the Fermi-Dirac distribution function.

A: Taking $E_F = 5.48$ eV for silver at 800 K, and given $f(E) = 0.950$, we find

$$f(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} = 0.950 \qquad \text{or} \qquad e^{(E-E_F)/k_BT} = \frac{1}{0.950} - 1 = 0.05263$$

$$\frac{E - E_F}{k_BT} = \ln(0.05263) = -2.944 \qquad \text{so} \qquad E - E_F = -2.944 k_BT = -2.944\left(1.38\times10^{-23}\ \text{J / K}\right)(800\ \text{K})$$

$$E = E_F - 3.25\times10^{-20}\ \text{J} = 5.48\ \text{eV} - 0.203\ \text{eV} = 5.28\ \text{eV}$$

L: As expected, the energy of the electron is slightly less than the Fermi energy, which is about 5 eV for most metals. There is very little probability of finding an electron significantly above the Fermi energy in a metal.

43.28 $d = 1.00$ mm, so $V = \left(1.00\times10^{-3}\ \text{m}\right)^3 = 1.00\times10^{-9}\ \text{m}^3$

The **density of states** is $g(E) = CE^{1/2} = \dfrac{8\sqrt{2}\,\pi m^{3/2}}{h^3}E^{1/2}$

or
$$g(E) = \frac{8\sqrt{2}\,\pi\left(9.11\times10^{-31}\ \text{kg}\right)^{3/2}}{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^3}\sqrt{(4.00\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}$$

$$g(E) = 8.50\times10^{46}\ \text{m}^{-3}\cdot\text{J}^{-1} = 1.36\times10^{28}\ \text{m}^{-3}\cdot\ \text{eV}^{-1}$$

So, the total number of electrons is

$$N = [g(E)](\Delta E)V = \left(1.36\times10^{28}\ \text{m}^{-3}\cdot\ \text{eV}^{-1}\right)(0.0250\ \text{eV})\left(1.00\times10^{-9}\ \text{m}^3\right) = \boxed{3.40\times10^{17}\ \text{electrons}}$$

43.29 $E_{\text{av}} = \dfrac{1}{n_e}\displaystyle\int_0^\infty EN(E)\,dE$

At $T = 0$, $N(E) = 0$ for $E > E_\text{F}$;

Since $f(E) = 1$ for $E < E_\text{F}$ and $f(E) = 0$ for $E > E_\text{F}$, we can take $N(E) = CE^{1/2}$

$$E_{\text{av}} = \frac{1}{n_e}\int_0^{E_\text{F}} CE^{3/2}\,dE = \frac{C}{n_e}\int_0^{E_\text{F}} E^{3/2}\,dE = \frac{2C}{5n_e}E^{5/2}$$

But from Equation 43.24, $\dfrac{C}{n_e} = \dfrac{3}{2}E_\text{F}{}^{-3/2}$, so that $E_{\text{av}} = \left(\dfrac{2}{5}\right)\left(\dfrac{3}{2}E_\text{F}{}^{-3/2}\right)E_\text{F}{}^{5/2} = \boxed{\dfrac{3}{5}E_\text{F}}$

43.30 Consider first the wave function in x. At $x = 0$ and $x = L$, $\psi = 0$.

Therefore, $\sin k_x L = 0$ and $k_x L = \pi, 2\pi, 3\pi, \ldots$

Similarly, $\sin k_y L = 0$ and $k_y L = \pi, 2\pi, 3\pi, \ldots$

$\sin k_z L = 0$ and $k_z L = \pi, 2\pi, 3\pi, \ldots$

$$\psi = A \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right) \sin\left(\frac{n_z \pi z}{L}\right)$$

From $\dfrac{d^2\psi}{dx^2} + \dfrac{d^2\psi}{dy^2} + \dfrac{d^2\psi}{dz^2} = \dfrac{2m_e}{\hbar^2}(U - E)\psi$, we have inside the box, where $U = 0$,

$$\left(-\frac{n_x^2\pi^2}{L^2} - \frac{n_y^2\pi^2}{L^2} - \frac{n_z^2\pi^2}{L^2}\right)\psi = \frac{2m_e}{\hbar^2}(-E)\psi \qquad \boxed{E = \frac{\hbar^2\pi^2}{2m_e L^2}\left(n_x^2 + n_y^2 + n_z^2\right) \qquad n_x, n_y, n_z = 1, 2, 3, \ldots}$$

Outside the box we require $\psi = 0$.

The minimum energy state inside the box is $n_x = n_y = n_z = 1$, with $E = \dfrac{3\hbar^2\pi^2}{2m_e L^2}$

43.31 (a) The density of states at energy E is $g(E) = CE^{1/2}$

Hence, the required ratio is $\dfrac{g(8.50\text{ eV})}{g(7.00\text{ eV})} = \dfrac{C(8.50)^{1/2}}{C(7.00)^{1/2}} = \boxed{1.10}$

(b) From Eq. 43.22, the number of occupied states having energy E is $N(E) = \dfrac{CE^{1/2}}{e^{(E-E_F)/k_B T} + 1}$

Hence, the required ratio is $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \dfrac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\dfrac{e^{(7.00-7.00)/k_B T} + 1}{e^{(8.50-7.00)/k_B T} + 1}\right]$

At $T = 300\text{ K}$, $k_B T = 4.14\times10^{-21}\text{ J} = 0.0259\text{ eV}$, $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \dfrac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\dfrac{2.00}{e^{(1.50)/0.0259} + 1}\right]$

And $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \boxed{1.55\times10^{-25}}$

Comparing this result with that from part (a), we conclude that very few states with $E > E_F$ are occupied.

43.32 (a) $E_g = 1.14$ eV for Si

$hf = 1.14\text{ eV} = (1.14\text{ eV})(1.60\times10^{-19}\text{ J/eV}) = 1.82\times10^{-19}\text{ J}$ so $f \geq \boxed{2.75\times10^{14}\text{ Hz}}$

(b) $c = \lambda f$; $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{2.75\times10^{14}\text{ Hz}} = 1.09\times10^{-6}\text{ m} = \boxed{1.09\ \mu\text{m}}$ (in the infrared region)

43.33 Photons of energy greater than 2.42 eV will be absorbed. This means wavelength shorter than

$$\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{2.42\times1.60\times10^{-19}\text{ J}} = 514\text{ nm}$$

All the hydrogen Balmer lines except for the red line at 656 nm will be absorbed.

43.34 $$E_g = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{650\times10^{-9}\text{ m}}\text{ J} \approx \boxed{1.91\text{ eV}}$$

43.35 If $\lambda \le 1.00\times10^{-6}$ m, then photons of sunlight have energy

$$E \ge \frac{hc}{\lambda_{\max}} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{1.00\times10^{-6}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 1.24\text{ eV}$$

Thus, the energy gap for the collector material should be $\boxed{E_g \le 1.24\text{ eV}}$. Since Si has an energy gap $E_g \approx 1.14$ eV, it will absorb radiation of this energy and greater. Therefore, $\boxed{\text{Si is acceptable}}$ as a material for a solar collector.

Goal Solution

Most solar radiation has a wavelength of 1 μm or less. What energy gap should the material in a solar cell have in order to absorb this radiation? Is silicon appropriate (see Table 43.5)?

G: Since most photovoltaic solar cells are made of silicon, this semiconductor seems to be an appropriate material for these devices.

O: To absorb the longest-wavelength photons, the energy gap should be no larger than the photon energy.

A: The minimum photon energy is

$$hf = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m / s}\right)}{10^{-6}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 1.24\text{ eV}$$

Therefore, the energy gap in the absorbing material should be smaller than 1.24 eV.

L: So silicon, with gap of 1.14 eV < 1.24 eV, is an appropriate material for absorbing solar radiation.

***43.36** If the photon energy is 5.5 eV or higher, the diamond window will absorb. Here,

$$(hf)_{\max} = \frac{hc}{\lambda_{\min}} = 5.50\text{ eV}: \qquad \lambda_{\min} = \frac{hc}{5.5\text{ eV}} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{(5.5\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}$$

$$\lambda_{\min} = 2.26\times10^{-7}\text{ m} = \boxed{226\text{ nm}}$$

43.37 $I = I_0\left(e^{e(\Delta V)/k_B T} - 1\right)$ Thus, $e^{e(\Delta V)/k_B T} = 1 + I/I_0$

and $$\Delta V = \frac{k_B T}{e}\ln\left(1 + I/I_0\right)$$

At $T = 300\text{ K}$, $$\Delta V = \frac{\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})}{1.60\times10^{-19}\text{ C}}\ln\left(1+\frac{I}{I_0}\right) = (25.9\text{ mV})\ln\left(1+\frac{I}{I_0}\right)$$

(a) If $I = 9.00I_0$, $\Delta V = (25.9\text{ mV})\ln(10.0) = \boxed{59.5\text{ mV}}$

(b) If $I = -0.900I_0$, $\Delta V = (25.9\text{ mV})\ln(0.100) = \boxed{-59.5\text{ mV}}$

The basic idea behind a semiconductor device is that a large current or charge can be controlled by a small control voltage.

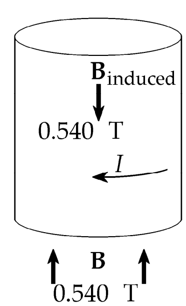
43.38 The voltage across the diode is about 0.6 V. The voltage drop across the resistor is $(0.025\text{ A})(150\ \Omega) = 3.75\text{ V}$. Thus, $\mathcal{E} - 0.6\text{ V} - 3.8\text{ V} = 0$ and $\mathcal{E} = \boxed{4.4\text{ V}}$

***43.39** First, we evaluate I_0 in $I = I_0\left(e^{e(\Delta V)/k_B T} - 1\right)$, given that $I = 200\text{ mA}$ when $\Delta V = 100\text{ mV}$ and $T = 300\text{ K}$.

$$\frac{e(\Delta V)}{k_B T} = \frac{\left(1.60\times10^{-19}\text{ C}\right)(0.100\text{ V})}{\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})} = 3.86 \text{ so} \qquad I_0 = \frac{I}{e^{e(\Delta V)/k_B T} - 1} = \frac{200\text{ mA}}{e^{3.86} - 1} = 4.28\text{ mA}$$

If $\Delta V = -100\text{ mV}$, $\dfrac{e(\Delta V)}{k_B T} = -3.86$; and the current will be

$$I = I_0\left(e^{e(\Delta V)/k_B T} - 1\right) = (4.28\text{ mA})\left(e^{-3.86} - 1\right) = \boxed{-4.19\text{ mA}}$$



43.40 (a) $\boxed{\text{See the figure at right.}}$

(b) For a surface current around the outside of the cylinder as shown,

$$B = \frac{N\mu_0 I}{\ell} \quad \text{or} \quad NI = \frac{B\ell}{\mu_0} = \frac{(0.540\text{ T})(2.50\times10^{-2}\text{ m})}{(4\pi\times10^{-7})\text{ T}\cdot\text{m / A}} = \boxed{10.7\text{ kA}}$$

43.41 By Faraday's law (Equation 32.1), $\frac{\Delta\Phi_B}{\Delta t} = L\frac{\Delta I}{\Delta t} = A\frac{\Delta B}{\Delta t}$.

Thus, $$\Delta I = \frac{A(\Delta B)}{L} = \frac{\pi(0.0100\text{ m})^2(0.0200\text{ T})}{3.10\times10^{-8}\text{ H}} = \boxed{203\text{ A}}$$

The direction of the induced current is such as to maintain the B-field through the ring.

Goal Solution

Determine the current generated in a superconducting ring of niobium metal 2.00 cm in diameter if a 0.0200-T magnetic field in a direction perpendicular to the ring is suddenly decreased to zero. The inductance of the ring is 3.10×10^{-8} H.

G: The resistance of a superconductor is zero, so the current is limited only by the change in magnetic flux and self-inductance. Therefore, unusually large currents (greater than 100 A) are possible.

O: The change in magnetic field through the ring will induce an emf according to Faraday's law of induction. Since we do not know how fast the magnetic field is changing, we must use the ring's inductance and the geometry of the ring to calculate the magnetic flux, which can then be used to find the current.

A: From Faraday's law (Eq. 31.1), we have

$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = A\frac{\Delta B}{\Delta t} = L\frac{\Delta I}{\Delta t} \quad \text{or} \quad \Delta I = \frac{A\Delta B}{L} = \frac{\pi(0.0100\text{ m})^2(0.0200\text{ T})}{3.10\times10^{-8}\text{ H}} = 203\text{ A}$$

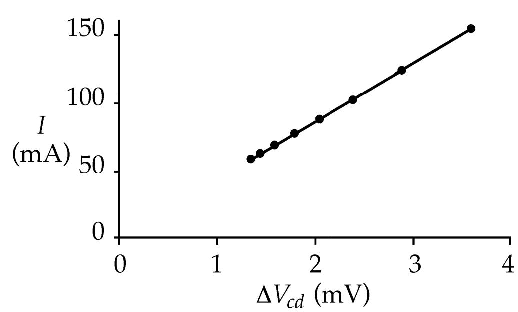
The current is directed so as to produce its own magnetic field in the direction of the original field.

L: This induced current should remain constant as long as the ring is superconducting. If the ring failed to be a superconductor (e.g. if it warmed above the critical temperature), the metal would have a non-zero resistance, and the current would quickly drop to zero. It is interesting to note that we were able to calculate the current in the ring without knowing the emf. In order to calculate the emf, we would need to know how quickly the magnetic field goes to zero.

43.42 (a) $\Delta V = IR$

If $R = 0$, then $\Delta V = 0$, even when $I \neq 0$.

(b) The graph shows a direct proportionality.



$$\text{Slope} = \frac{1}{R} = \frac{\Delta I}{\Delta V} = \frac{(155-57.8)\text{ mA}}{(3.61-1.356)\text{ mV}} = 43.1\ \Omega^{-1}$$

$$R = \boxed{0.0232\ \Omega}$$

(c) Expulsion of magnetic flux and therefore fewer current-carrying paths could explain the decrease in current.

***43.43** (a) Since the interatomic potential is the same for both molecules, the spring constant is the same.

Then $f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$ where $\mu_{12} = \frac{(12\text{ u})(16\text{ u})}{12\text{ u} + 16\text{ u}} = 6.86\text{ u}$ and $\mu_{14} = \frac{(14\text{ u})(16\text{ u})}{14\text{ u} + 16\text{ u}} = 7.47\text{ u}$

Therefore,

$$f_{14} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{14}}} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{12}}\left(\frac{\mu_{12}}{\mu_{14}}\right)} = f_{12}\sqrt{\frac{\mu_{12}}{\mu_{14}}} = \left(6.42\times10^{13}\text{ Hz}\right)\sqrt{\frac{6.86\text{ u}}{7.47\text{ u}}} = \boxed{6.15\times10^{13}\text{ Hz}}$$

(b) The equilibrium distance is the same for both molecules.

$$I_{14} = \mu_{14}r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right)\mu_{12}r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right)I_{12}$$

$$I_{14} = \left(\frac{7.47\text{ u}}{6.86\text{ u}}\right)\left(1.46\times10^{-46}\text{ kg}\cdot\text{m}^2\right) = \boxed{1.59\times10^{-46}\text{ kg}\cdot\text{m}^2}$$

(c) The molecule can move to the $(v=1,\ J=9)$ state or to the $(v=1,\ J=11)$ state. The energy it can absorb is either

$$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\tfrac{1}{2}\right)hf_{14} + 9(9+1)\frac{\hbar^2}{2I_{14}}\right] - \left[\left(0+\tfrac{1}{2}\right)hf_{14} + 10(10+1)\frac{\hbar^2}{2I_{14}}\right],$$

or $$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\tfrac{1}{2}\right)hf_{14} + 11(11+1)\frac{\hbar^2}{2I_{14}}\right] - \left[\left(0+\tfrac{1}{2}\right)hf_{14} + 10(10+1)\frac{\hbar^2}{2I_{14}}\right].$$

The wavelengths it can absorb are then

$$\lambda = \frac{c}{f_{14} - 10\hbar/(2\pi I_{14})} \quad\text{or}\quad \lambda = \frac{c}{f_{14} + 11\hbar/(2\pi I_{14})}$$

These are: $$\lambda = \frac{3.00\times10^8\text{ m/s}}{6.15\times10^{13}\text{ Hz} - \frac{10\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)}{2\pi\left(1.59\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}} = \boxed{4.96\ \mu\text{m}}$$

and $$\lambda = \frac{3.00\times10^8\text{ m/s}}{6.15\times10^{13}\text{ Hz} + \frac{11\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)}{2\pi\left(1.59\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}} = \boxed{4.79\ \mu\text{m}}$$

43.44 For the N_2 molecule, $k = 2297 \text{ N/m}$, $m = 2.32 \times 10^{-26} \text{ kg}$, $r = 1.20 \times 10^{-10} \text{ m}$, $\mu = m/2$

$\omega = \sqrt{k/\mu} = 4.45 \times 10^{14} \text{ rad/s}$, $I = \mu r^2 = (1.16 \times 10^{-26} \text{ kg})(1.20 \times 10^{-10} \text{ m})^2 = 1.67 \times 10^{-46} \text{ kg} \cdot \text{m}^2$

For a rotational state sufficient to allow a transition to the first exited vibrational state,

$$\frac{\hbar^2}{2I} J(J+1) = \hbar\omega \quad \text{so} \quad J(J+1) = \frac{2I\omega}{\hbar} = \frac{2(1.67 \times 10^{-46})(4.45 \times 10^{14})}{1.055 \times 10^{-34}} = 1410$$

Thus $\boxed{J = 37}$

43.45 $\Delta E_{\max} = 4.5 \text{ eV} = \left(v + \frac{1}{2}\right)\hbar\omega$ so $\dfrac{(4.5 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})(8.28 \times 10^{14} \text{ s}^{-1})} \ge \left(v + \frac{1}{2}\right)$

$8.25 > 7.5$ $\boxed{v = 7}$

43.46 With 4 van der Waal bonds per atom pair or 2 electrons per atom, the total energy of the solid is

$$E = 2(1.74 \times 10^{-23} \text{ J/atom})\left(\frac{6.02 \times 10^{23} \text{ atoms}}{4.00 \text{ g}}\right) = \boxed{5.23 \text{ J/g}}$$

43.47 The total potential energy is given by Equation 43.16: $U_{\text{total}} = -\alpha \dfrac{k_e e^2}{r} + \dfrac{B}{r^m}$

The total potential energy has its minimum value U_0 at the equilibrium spacing, $r = r_0$. At this point, $\left. dU/dr \right|_{r=r_0} = 0$,

or $$\left.\frac{dU}{dr}\right|_{r=r_0} = \frac{d}{dr}\left.\left(-\alpha \frac{k_e e^2}{r} + \frac{B}{r^m}\right)\right|_{r=r_0} = \alpha \frac{k_e e^2}{r_0^2} - \frac{mB}{r_0^{m+1}} = 0$$

Thus, $$B = \alpha \frac{k_e e^2}{m} r_0^{m-1}$$

Substituting this value of B into U_{total}, $$U_0 = -\alpha \frac{k_e e^2}{r_0} + \alpha \frac{k_e e^2}{m} r_0^{m-1}\left(\frac{1}{r_0^m}\right) = \boxed{-\alpha \frac{k_e e^2}{r_0}\left(1 - \frac{1}{m}\right)}$$

***43.48** Suppose it is a harmonic-oscillator potential well. Then, $\frac{1}{2}hf + 4.48 \text{ eV} = \frac{3}{2}hf + 3.96 \text{ eV}$ is the depth of the well below the dissociation point. We see $hf = 0.520 \text{ eV}$, so the depth of the well is

$$\tfrac{1}{2}hf + 4.48 \text{ eV} = \tfrac{1}{2}(0.520 \text{ eV}) + 4.48 \text{ eV} = \boxed{4.74 \text{ eV}}$$

***43.49** (a) For equilibrium, $\frac{dU}{dx}=0$: $\qquad \frac{d}{dx}\left(Ax^{-3}-Bx^{-1}\right)=-3Ax^{-4}+Bx^{-2}=0$

$x\to\infty$ describes one equilibrium position, but the stable equilibrium position is at $3Ax_0^{-2}=B$.

$$x_0=\sqrt{\frac{3A}{B}}=\sqrt{\frac{3\left(0.150\text{ eV}\cdot\text{nm}^3\right)}{3.68\text{ eV}\cdot\text{nm}}}=\boxed{0.350\text{ nm}}$$

(b) The depth of the well is given by $\qquad U_0=U\big|_{x=x_0}=\frac{A}{x_0^3}-\frac{B}{x_0}=\frac{AB^{3/2}}{3^{3/2}A^{3/2}}-\frac{BB^{1/2}}{3^{1/2}A^{1/2}}$

$$U_0=U\big|_{x=x_0}=-\frac{2B^{3/2}}{3^{3/2}A^{1/2}}=-\frac{2(3.68\text{ eV}\cdot\text{nm})^{3/2}}{3^{3/2}\left(0.150\text{ eV}\cdot\text{nm}^3\right)^{1/2}}=\boxed{-7.02\text{ eV}}$$

(c) $F_x=-\frac{dU}{dx}=3Ax^{-4}-Bx^{-2}$

To find the maximum force, we determine finite x_m such that $\qquad \left.\frac{dF_x}{dx}\right|_{x=x_m}=0$

Thus, $\left[-12Ax^{-5}+2Bx^{-3}\right]_{x=x_0}=0$ $\qquad$ so that $\qquad x_m=\left(\frac{6A}{B}\right)^{1/2}$

Then $\qquad F_{\max}=3A\left(\frac{B}{6A}\right)^2-B\left(\frac{B}{6A}\right)=-\frac{B^2}{12A}=-\frac{(3.68\text{ eV}\cdot\text{nm})^2}{12\left(0.150\text{ eV}\cdot\text{nm}^3\right)}$

or $\qquad F_{\max}=-7.52\ \frac{\text{eV}}{\text{nm}}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)=-1.20\times10^{-9}\text{ N}=\boxed{-1.20\text{ nN}}$

43.50 (a) For equilibrium, $\frac{dU}{dx}=0$: $\qquad \frac{d}{dx}\left(Ax^{-3}-Bx^{-1}\right)=-3Ax^{-4}+Bx^{-2}=0$

$x\to\infty$ describes one equilibrium position, but the stable equilibrium position is at

$3Ax_0^{-2}=B$ $\quad$ or $\quad \boxed{x_0=\sqrt{3A/B}}$

(b) The depth of the well is given by $\qquad U_0=U\big|_{x=x_0}=\frac{A}{x_0^3}-\frac{B}{x_0}=\frac{AB^{3/2}}{3^{3/2}A^{3/2}}-\frac{BB^{1/2}}{3^{1/2}A^{1/2}}=\boxed{-2\sqrt{\frac{B^3}{27A}}}$

(c) $F_x=-\frac{dU}{dx}=3Ax^{-4}-Bx^{-2}$

To find the maximum force, we determine finite x_m such that

$\left.\frac{dF_x}{dx}\right|_{x=x_m}=\left[-12Ax^{-5}+2Bx^{-3}\right]_{x=x_0}=0$ $\qquad$ then $\qquad F_{\max}=3A\left(\frac{B}{6A}\right)^2-B\left(\frac{B}{6A}\right)=\boxed{-\frac{B^2}{12A}}$

***43.51** (a) At equilibrium separation, $r = r_e$, $\left.\frac{dU}{dr}\right|_{r=r_e} = -2aB\left[e^{-a(r_e - r_0)} - 1\right]e^{-a(r_e - r_0)} = 0$

We have neutral equilibrium as $r_e \to \infty$ and stable equilibrium at $e^{-a(r_e - r_0)} = 1$,

or $r_e = \boxed{r_0}$

(b) At $r = r_0$, $U = 0$. As $r \to \infty$, $U \to B$. The depth of the well is $\boxed{B}$.

(c) We expand the potential in a Taylor series about the equilibrium point:

$$U(r) \approx U(r_0) + \left.\frac{dU}{dr}\right|_{r=r_0}(r - r_0) + \frac{1}{2}\left.\frac{d^2U}{dr^2}\right|_{r=r_0}(r - r_0)^2$$

$$U(r) \approx 0 + 0 + \tfrac{1}{2}(-2Ba)\left[-ae^{-2(r-r_0)} - ae^{-(r-r_0)}\left(e^{-2(r-r_0)} - 1\right)\right]_{r=r_0}(r - r_0)^2 \approx Ba^2(r - r_0)^2$$

This is of the form $\tfrac{1}{2}kx^2 = \tfrac{1}{2}k(r - r_0)^2$

for a simple harmonic oscillator with $k = 2Ba^2$

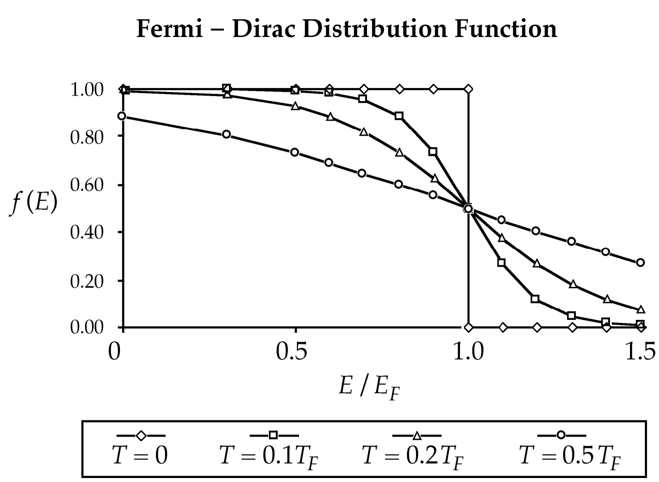
Then the molecule vibrates with frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{a}{2\pi}\sqrt{\frac{2B}{\mu}} = \boxed{\frac{a}{\pi}\sqrt{\frac{B}{2\mu}}}$

(d) The zero-point energy is $\tfrac{1}{2}\hbar\omega = \tfrac{1}{2}hf = \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}$

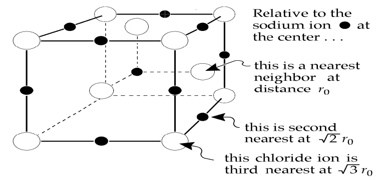
Therefore, to dissociate the molecule in its ground state requires energy $\boxed{B - \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}}$

43.52

	$T=0$		$T=0.1T_F$		$T=0.2T_F$		$T=0.5T_F$	
E/E_F	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$
0	$e^{-\infty}$	1.00	$e^{-10.0}$	1.000	$e^{-5.00}$	0.993	$e^{-2.00}$	0.881
0.500	$e^{-\infty}$	1.00	$e^{-5.00}$	0.993	$e^{-2.50}$	0.924	$e^{-1.00}$	0.731
0.600	$e^{-\infty}$	1.00	$e^{-4.00}$	0.982	$e^{-2.00}$	0.881	$e^{-0.800}$	0.690
0.700	$e^{-\infty}$	1.00	$e^{-3.00}$	0.953	$e^{-1.50}$	0.818	$e^{-0.600}$	0.646
0.800	$e^{-\infty}$	1.00	$e^{-2.00}$	0.881	$e^{-1.00}$	0.731	$e^{-0.400}$	0.599
0.900	$e^{-\infty}$	1.00	$e^{-1.00}$	0.731	$e^{-0.500}$	0.622	$e^{-0.200}$	0.550
1.00	e^{0}	0.500	e^{0}	0.500	e^{0}	0.500	e^{0}	0.500
1.10	$e^{+\infty}$	0.00	$e^{1.00}$	0.269	$e^{0.500}$	0.378	$e^{0.200}$	0.450
1.20	$e^{+\infty}$	0.00	$e^{2.00}$	0.119	$e^{1.00}$	0.269	$e^{0.400}$	0.401
1.30	$e^{+\infty}$	0.00	$e^{3.00}$	0.0474	$e^{1.50}$	0.182	$e^{0.600}$	0.354
1.40	$e^{+\infty}$	0.00	$e^{4.00}$	0.0180	$e^{2.00}$	0.119	$e^{0.800}$	0.310
1.50	$e^{+\infty}$	0.00	$e^{5.00}$	0.00669	$e^{2.50}$	0.0759	$e^{1.00}$	0.269



43.53 (a) There are 6 Cl^- ions at distance $r = r_0$. The contribution of these ions to the electrostatic potential energy is $-6k_e e^2 / r_0$.



There are 12 Na^+ ions at distance $r = \sqrt{2}\, r_0$. Their contribution to the electrostatic potential energy is $+12 k_e e^2 / \sqrt{2}\, r_0$. Next, there are 8 Cl^- ions at distance $r = \sqrt{3}\, r_0$. These contribute a term of $-8k_e e^2 / \sqrt{3}\, r_0$ to the electrostatic potential energy.

To three terms, the electrostatic potential energy is:

$$U = \left(-6 + \frac{12}{\sqrt{2}} - \frac{8}{\sqrt{3}}\right)\frac{k_e e^2}{r_0} = -2.13\,\frac{k_e e^2}{r_0} \quad \text{or} \quad \boxed{U = -\alpha \frac{k_e e^2}{r_0} \text{ with } \alpha = 2.13}$$

(b) The fourth term consists of 6 Na^+ at distance $r = 2r_0$. Thus, to four terms,

$$U = (-2.13 + 3)\frac{k_e e^2}{r_0} = 0.866\,\frac{k_e e^2}{r_0}$$

So we see that the electrostatic potential energy is not even attractive to 4 terms, and that the infinite series $\boxed{\text{does not converge rapidly}}$ when groups of atoms corresponding to nearest neighbors, next-nearest neighbors, etc. are added together.

Chapter 44 Solutions

***44.1** An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons.

So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

$$35\text{ kg}\left(\frac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim 10^{28}\text{ protons}}\quad\text{and}\quad\boxed{\sim 10^{28}\text{ neutrons}}$$

The electron number is precisely equal to the proton number, $\boxed{\sim 10^{28}\text{ electrons}}$

44.2 $\frac{1}{2}mv^2 = q(\Delta V)$ and $\frac{mv^2}{r} = qvB \quad\Rightarrow\quad 2m(\Delta V) = qr^2B^2$

$$r = \sqrt{\frac{2m(\Delta V)}{qB^2}} = \left[\frac{2(1000\text{ V})}{(1.60\times10^{-19}\text{ C})(0.200\text{ T})^2}\right]^{1/2}\sqrt{m}$$

$$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{m}$$

(a) For ^{12}C: $m = 12$ u and

$$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{12(1.66\times10^{-27}\text{ kg})} = 0.0789\text{ m} = \boxed{7.89\text{ cm}}$$

For ^{13}C: $$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{13(1.66\times10^{-27}\text{ kg})} = 0.0821\text{ m} = \boxed{8.21\text{ cm}}$$

(b) With $r_1 = \sqrt{\frac{2m_1(\Delta V)}{qB^2}}$ and $r_2 = \sqrt{\frac{2m_2(\Delta V)}{qB^2}}$,

the ratio gives $\boxed{\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}}$.

$\frac{r_1}{r_2} = \frac{7.89\text{ cm}}{8.21\text{ cm}} = 0.961$ and $\sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{12\text{ u}}{13\text{ u}}} = 0.961$ so they do agree.

***44.3** (a) $F = k_e \dfrac{Q_1Q_2}{r^2} = (8.99\times 10^9\ \mathrm{N\cdot m^2/C^2})\dfrac{(2)(6)(1.60\times 10^{-19}\ \mathrm{C})^2}{(1.00\times 10^{-14}\ \mathrm{m})^2} = \boxed{27.6\ \mathrm{N}}$

(b) $a = \dfrac{F}{m} = \dfrac{27.6\ \mathrm{N}}{6.64\times 10^{-27}\ \mathrm{kg}} = \boxed{4.17\times 10^{27}\ \mathrm{m/s^2}}$ away from the nucleus..

(c) $U = k_e \dfrac{Q_1Q_2}{r} = (8.99\times 10^9\ \mathrm{N\cdot m^2/C^2})\dfrac{(2)(6)(1.60\times 10^{-19}\ \mathrm{C})^2}{(1.00\times 10^{-14}\ \mathrm{m})} = 2.76\times 10^{-13}\ \mathrm{J} = \boxed{1.73\ \mathrm{MeV}}$

44.4 $E_\alpha = 7.70$ MeV

(a) $d_{\min} = \dfrac{4k_eZe^2}{mv^2} = \dfrac{2k_eZe^2}{E_\alpha} = \dfrac{2(8.99\times 10^9)(79)(1.60\times 10^{-19})^2}{7.70(1.60\times 10^{-13})} = 29.5\times 10^{-15}\ \mathrm{m} = \boxed{29.5\ \mathrm{fm}}$

(b) The de Broglie wavelength of the α is

$$\lambda = \frac{h}{m_\alpha v_\alpha} = \frac{h}{\sqrt{2m_\alpha E_\alpha}} = \frac{6.626\times 10^{-34}}{\sqrt{2(6.64\times 10^{-27})7.70(1.60\times 10^{-13})}} = 5.18\times 10^{-15}\ \mathrm{m} = \boxed{5.18\ \mathrm{fm}}$$

(c) Since $\boxed{\lambda \text{ is much less than the distance of closest approach}}$, the α may be considered a particle.

44.5 (a) The initial kinetic energy of the alpha particle must equal the electrostatic potential energy at the distance of closest approach.

$$K_i = U_f = \frac{k_eqQ}{r_{\min}}$$

$$r_{\min} = \frac{k_eqQ}{K_i} = \frac{\left(8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}\right)(2)(79)\left(1.60\times 10^{-19}\ \mathrm{C}\right)^2}{(0.500\ \mathrm{MeV})\left(1.60\times 10^{-13}\ \mathrm{J/MeV}\right)} = \boxed{4.55\times 10^{-13}\ \mathrm{m}}$$

(b) Since $K_i = \frac{1}{2}m_\alpha v_i^2 = \dfrac{k_eqQ}{r_{\min}}$,

$$v_i = \sqrt{\frac{2k_eqQ}{m_\alpha r_{\min}}} = \sqrt{\frac{2\left(8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}\right)(2)(79)\left(1.60\times 10^{-19}\ \mathrm{C}\right)^2}{(4.00\ \mathrm{u})\left(1.66\times 10^{-27}\ \mathrm{kg/u}\right)\left(3.00\times 10^{-13}\ \mathrm{m}\right)}} = \boxed{6.04\times 10^6\ \mathrm{m/s}}$$

Goal Solution

(a) Use energy methods to calculate the distance of closest approach for a head-on collision between an alpha particle having an initial energy of 0.500 MeV and a gold nucleus (^{197}Au) at rest. (Assume the gold nucleus remains at rest during the collision.) (b) What minimum initial speed must the alpha particle have in order to get as close as 300 fm?

G: The positively charged alpha particle ($q = +2e$) will be repelled by the positive gold nucleus ($Q = +79e$), so that the particles probably will not touch each other in this electrostatic "collision." Therefore, the closest the alpha particle can get to the gold nucleus would be if the two nuclei did touch, in which case the distance between their centers would be about 6 fm (using $r = r_0 A^{1/3}$ for the radius of each nucleus). To get this close, or even within 300 fm, the alpha particle must be traveling very fast, probably close to the speed of light (but of course v must be less than c).

O: At the distance of closest approach, $r_{\min}$, the initial kinetic energy will equal the electrostatic potential energy between the alpha particle and gold nucleus.

A: (a) $K_\alpha = U = k_e \dfrac{qQ}{r_{\min}}$ and $r_{\min} = k_e \dfrac{qQ}{K_\alpha} = \dfrac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(2)(79)(1.60 \times 10^{-19} \text{ C})^2}{(0.500 \text{ MeV})(1.60 \times 10^{-13} \text{ J / MeV})} = 455 \text{ fm}$

(b) Since $K_\alpha = \frac{1}{2} m v^2 = k_e \dfrac{qQ}{r_{\min}}$

$$v = \sqrt{\frac{2k_e qQ}{m r_{\min}}} = \sqrt{\frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2)(79)(1.60 \times 10^{-19} \text{ C})^2}{4(1.66 \times 10^{-27} \text{ kg})(3.00 \times 10^{-13} \text{ m})}} = 6.04 \times 10^6 \text{ m / s}$$

L: The minimum distance in part (a) is about 100 times greater than the combined radii of the particles. For part (b), the alpha particle must have more than 0.5 MeV of energy since it gets closer to the nucleus than the 455 fm found in part (a). Even so, the speed of the alpha particle in part (b) is only about 2% of the speed of light, so we are justified in not using a relativistic approach. In solving this problem, we ignored the effect of the electrons around the gold nucleus that tend to "screen" the nucleus so that the alpha particle sees a reduced positive charge. If this screening effect were considered, the potential energy would be slightly reduced and the alpha particle could get closer to the gold nucleus for the same initial energy.

***44.6** It must start with kinetic energy equal to $K_i = U_f = k_e qQ/r_f$. Here r_f stands for the sum of the radii of the $^{4}_{2}$He and $^{197}_{79}$Au nuclei, computed as

$$r_f = r_0 A_1^{1/3} + r_0 A_2^{1/3} = \left(1.20 \times 10^{-15} \text{ m}\right)\left(4^{1/3} + 197^{1/3}\right) = 8.89 \times 10^{-15} \text{ m}$$

Thus,
$$K_i = U_f = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(2)(79)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{8.89 \times 10^{-15} \text{ m}} = 4.09 \times 10^{-12} \text{ J} = \boxed{25.6 \text{ MeV}}$$

44.7 (a) $r = r_0 A^{1/3} = (1.20 \times 10^{-15}\text{ m})(4)^{1/3} = \boxed{1.90 \times 10^{-15}\text{ m}}$

(b) $r = r_0 A^{1/3} = (1.20 \times 10^{-15}\text{ m})(238)^{1/3} = \boxed{7.44 \times 10^{-15}\text{ m}}$

***44.8** From $r = r_0 A^{1/3}$, the radius of uranium is $r_U = r_0(238)^{1/3}$.

Thus, if $r = \frac{1}{2} r_U$ then $r_0 A^{1/3} = \frac{1}{2} r_0 (238)^{1/3}$

from which $\boxed{A = 30}$

44.9 The number of nucleons in a star of two solar masses is

$$A = \frac{2\left(1.99 \times 10^{30}\text{ kg}\right)}{1.67 \times 10^{-27}\text{ kg/nucleon}} = 2.38 \times 10^{57}\text{ nucleons}$$

Therefore $r = r_0 A^{1/3} = \left(1.20 \times 10^{-15}\text{ m}\right)\left(2.38 \times 10^{57}\right)^{1/3} = \boxed{16.0\text{ km}}$

***44.10** $V = \frac{4}{3}\pi r^3 = 4.16 \times 10^{-5}\text{ m}^3$

$m = \rho V = (2.31 \times 10^{17}\text{ kg / m}^3)(4.16 \times 10^{-5}\text{ m}^3) = 9.61 \times 10^{12}\text{ kg}$ and

$F = G\frac{m_1 m_2}{r^2} = (6.67 \times 10^{-11}\text{ Nm}^2\text{ / kg}^2)\frac{(9.61 \times 10^{12}\text{ kg})^2}{(1.00\text{ m})^2} = \boxed{6.16 \times 10^{15}\text{ N}}$ toward the other ball.

44.11 The stable nuclei that correspond to magic numbers are:

Z magic: ^{2}He ^{8}O ^{20}Ca ^{28}Ni ^{50}Sn ^{82}Pb 126

N magic: T_1^3, He_2^4, N_7^{15}, O_8^{16}, Cl_{17}^{37}, K_{19}^{39}, Ca_{20}^{40}, V_{23}^{51}, Cr_{24}^{52}, Sr_{38}^{88}, Y_{39}^{89},

Zr_{40}^{90}, Xe_{54}^{136}, Ba_{56}^{138}, La_{57}^{139}, Ce_{58}^{140}, Pr_{59}^{141}, Nd_{60}^{142}, Pb_{82}^{208}, Bi_{83}^{209}, Po_{84}^{210}

44.12 Of the 102 stable nuclei listed in Table A.3,

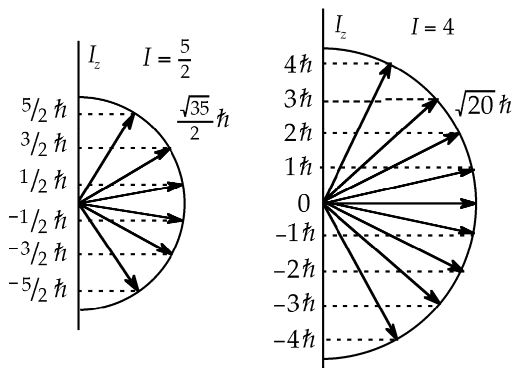
(a) Even Z, Even N $\boxed{48}$

(b) Even Z, Odd N $\boxed{6}$

(c) Odd Z, Even N $\boxed{44}$

(d) Odd Z, Odd N $\boxed{4}$

44.13



(a) (b)

44.14 (a) $f_n = \dfrac{|2\mu B|}{h} = \dfrac{2(1.9135)(5.05\times10^{-27}\text{ J / T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{29.2\text{ MHz}}$

(b) $f_p = \dfrac{2(2.7928)(5.05\times10^{-27}\text{ J / T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{42.6\text{ MHz}}$

(c) In the Earth's magnetic field, $f_p = \dfrac{2(2.7928)(5.05\times10^{-27})(50.0\times10^{-6})}{6.626\times10^{-34}} = \boxed{2.13\text{ kHz}}$

44.15 Using atomic masses as given in Table A.3,

(a) For $^2H_1$, $\dfrac{-2.014\,102 + 1(1.008\,665) + 1(1.007\,825)}{2}$

$$E_b = (0.00119\,4 \text{ u})\left(\frac{931.5 \text{ MeV}}{\text{u}}\right) = \boxed{1.11 \text{ MeV/nucleon}}$$

(b) For ^{4}He, $\dfrac{2(1.008\,665) + 2(1.007\,825) - 4.002\,602}{4}$

$E_b = 0.00759\text{ u} = \boxed{7.07\text{ MeV/nucleon}}$

(c) For $^{56}\text{Fe}_{26}$, $30(1.008\,665) + 26(1.007\,825) - 55.934\,940 = 0.528\text{ u}$

$E_b = \dfrac{0.528}{56} = 0.00944\text{ u} = \boxed{8.79\text{ MeV/nucleon}}$

(d) For $^{238}\text{U}_{92}$, $146(1.008\,665) + 92(1.007\,825) - 238.050\,784 = 1.934\,2\text{ u}$

$E_b = \dfrac{1.934\,2}{238} = 0.00813\text{ u} = \boxed{7.57\text{ MeV/nucleon}}$

44.16 $\Delta M = Zm_{\text{H}} + Nm_{\text{n}} - M$ $\dfrac{\text{BE}}{A} = \dfrac{\Delta M(931.5)}{A}$

Nuclei	Z	N	M in u	ΔM in u	BE/A in MeV
^{55}Mn	25	30	54.938048	0.517527	8.765
^{56}Fe	26	30	55.934940	0.528460	8.786
^{59}Co	27	32	58.933198	0.555357	8.768

$\therefore$ ^{56}Fe has a greater BE/A than its neighbors. This tells us finer detail than is shown in Figure 44.8.

44.17 (a) The neutron-to-proton ratio, $(A - Z)/Z$ is greatest for $\boxed{^{139}_{55}\text{Cs}}$ and is equal to 1.53.

(b) $\boxed{^{139}\text{La}}$ has the largest binding energy per nucleon of 8.378 MeV.

(c) ^{139}Cs with a mass of 138.913 u. We locate the nuclei carefully on Figure 44.3, the neutron-proton plot of stable nuclei. $\boxed{\text{Cesium}}$ appears to be farther from the center of the zone of stability. Its instability means extra energy and extra mass.

44.18 Use Equation 44.4.

The $^{23}_{11}$Na, $\dfrac{E_b}{A} = 8.11\text{ MeV/nucleon}$

and for $^{23}_{12}$Mg, $\dfrac{E_b}{A} = 7.90\text{ MeV/nucleon}$

The binding energy per nucleon is greater for $^{23}_{11}$Na by $\boxed{0.210\text{ MeV}}$. (There is less proton repulsion in Na^{23}.)

44.19 The binding energy of a nucleus is $E_b(\text{MeV}) = \left[Z\,M(\text{H}) + Nm_n - M\left({}^{A}_{Z}\text{X}\right)\right](931.494\ \text{MeV/u})$

For ${}^{15}_{8}\text{O}$: $E_b = \left[8(1.007\ 825\ \text{u}) + 7(1.008\ 665\ \text{u}) - 15.003\ 065\ \text{u}\right](931.494\ \text{MeV/u}) = 111.96\ \text{MeV}$

For ${}^{15}_{7}\text{N}$: $E_b = \left[7(1.007\ 825\ \text{u}) + 8(1.008\ 665\ \text{u}) - 15.000\ 108\ \text{u}\right](931.494\ \text{MeV/u}) = 115.49\ \text{MeV}$

Therefore, $\boxed{\text{the binding energy of } {}^{15}_{7}\text{N is larger by 3.54 MeV}}$.

44.20 (a) The radius of the ${}^{40}\text{Ca}$ nucleus is: $R = r_0 A^{1/3} = \left(1.20 \times 10^{-15}\ \text{m}\right)(40)^{1/3} = 4.10 \times 10^{-15}\ \text{m}$

The energy required to overcome electrostatic repulsion is

$$U = \frac{3k_eQ^2}{5R} = \frac{3\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[20\left(1.60 \times 10^{-19}\ \text{C}\right)\right]^2}{5\left(4.10 \times 10^{-15}\ \text{m}\right)} = 1.35 \times 10^{-11}\ \text{J} = \boxed{84.1\ \text{MeV}}$$

(b) The binding energy of ${}^{40}_{20}\text{Ca}$ is

$$E_b = \left[20(1.007\ 825\ \text{u}) + 20(1.008\ 665\ \text{u}) - 39.962\ 591\ \text{u}\right](931.5\ \text{MeV/u}) = \boxed{342\ \text{MeV}}$$

(c) The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.

44.21 Removal of a neutron from ${}^{43}_{20}\text{Ca}$ would result in the residual nucleus, ${}^{42}_{20}\text{Ca}$. If the required separation energy is S_n, the overall process can be described by

$$\text{mass}\left({}^{43}_{20}\text{Ca}\right) + S_n = \text{mass}\left({}^{42}_{20}\text{Ca}\right) + \text{mass(n)}$$

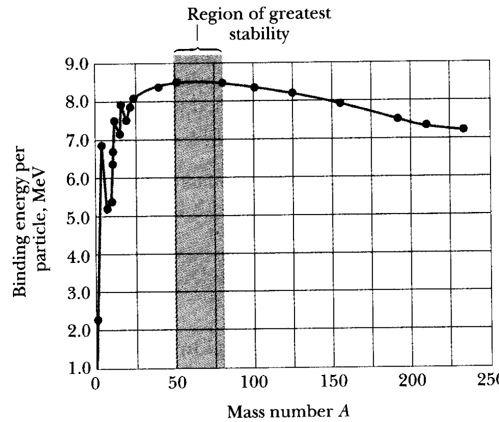
$$S_n = (41.958\ 618 + 1.008\ 665 - 42.958\ 767)\ \text{u} = (0.008\ 516\ \text{u})(931.5\ \text{MeV/u}) = \boxed{7.93\ \text{MeV}}$$

44.22 (a) The first term overstates the importance of volume and the second term *subtracts* this overstatement.

(b) For spherical volume $\dfrac{\frac{4}{3}\pi R^3}{4\pi R^2} = \boxed{\dfrac{R}{3}}$ For cubical volume $\dfrac{R^3}{6R^2} = \boxed{\dfrac{R}{6}}$

The maximum binding energy or lowest state of energy is achieved by building "nearly" spherical nuclei.

44.23 $\Delta E_b = E_{bf} - E_{bi}$



For $A = 200$, $\frac{E_b}{A} = 7.4$ MeV

so $E_{bi} = 200(7.4\text{ MeV}) = 1480\text{ MeV}$

For $A \approx 100$, $E_b/A \approx 8.4$ MeV

so $E_{bf} = 2(100)(8.4\text{ MeV}) = 1680\text{ MeV}$

$\Delta E_b = E_{bf} - E_{bi}$

$E_b = 1680\text{ MeV} - 1480\text{ MeV} = \boxed{200\text{ MeV}}$

44.24 (a) "Volume" term: $E_1 = C_1 A = (15.7\text{ MeV})(56) = 879\text{ MeV}$

"Surface" term: $E_2 = -C_2 A^{2/3} = -(17.8\text{ MeV})(56)^{2/3} = -260\text{ MeV}$

"Coulomb" term: $E_3 = -C_3 \frac{Z(Z-1)}{A^{1/3}} = -(0.71\text{ MeV})\frac{(26)(25)}{(56)^{1/3}} = -121\text{ MeV}$

"Asymmetry" term: $E_4 = C_4 \frac{(A-2Z)^2}{A} = -(23.6\text{ MeV})\frac{(56-52)^2}{56} = -6.74\text{ MeV}$

$\boxed{E_b = 491\text{ MeV}}$

(b) $\frac{E_1}{E_b} = 179\%$; $\frac{E_2}{E_b} = -53.0\%$, $\frac{E_3}{E_b} = -24.6\%$; $\frac{E_4}{E_b} = -1.37\%$

44.25 $\frac{dN}{dt} = -\lambda N$ so $\lambda = \frac{1}{N}\left(-\frac{dN}{dt}\right) = (1.00\times 10^{-15})(6.00\times 10^{11}) = 6.00\times 10^{-4}\text{ s}^{-1}$

$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{1.16\times 10^{3}\text{ s}}$ (= 19.3 min)

***44.26** $R = R_0 e^{-\lambda t} = (6.40\text{ mCi})e^{-\left(\frac{\ln 2}{8.04\text{ d}}\right)(40.2\text{ d})} = (6.40\text{ mCi})\left(e^{-\ln 2}\right)^5 = (6.40\text{ mCi})\left(\frac{1}{2^5}\right) = \boxed{0.200\text{ mCi}}$

44.27 (a) From $R = R_0 e^{-\lambda t}$,

$$\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \left(\frac{1}{4.00\text{ h}}\right)\ln\left(\frac{10.0}{8.00}\right) = 5.58\times 10^{-2}\text{ h}^{-1} = \boxed{1.55\times 10^{-5}\text{ s}^{-1}} \qquad T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4\text{ h}}$$

(b) $$N_0 = \frac{R_0}{\lambda} = \frac{10.0\times 10^{-3}\text{ Ci}}{1.55\times 10^{-5}/\text{s}}\left(\frac{3.70\times 10^{10}/\text{s}}{1\text{ Ci}}\right) = \boxed{2.39\times 10^{13}\text{ atoms}}$$

(c) $$R = R_0 e^{-\lambda t} = (10.0\text{ mCi})\exp(-5.58\times 10^{-2}\times 30.0) = \boxed{1.87\text{ mCi}}$$

<u>Goal Solution</u>

A freshly prepared sample of a certain radioactive isotope has an activity of 10.0 mCi. After 4.00 h, its activity is 8.00 mCi. (a) Find the decay constant and half-life. (b) How many atoms of the isotope were contained in the freshly prepared sample? (c) What is the sample's activity 30.0 h after it is prepared?

G: Over the course of 4 hours, this isotope lost 20% of its activity, so its half-life appears to be around 10 hours, which means that its activity after 30 hours (~3 half-lives) will be about 1 mCi. The decay constant and number of atoms are not so easy to estimate.

O: From the rate equation, $R = R_0 e^{-\lambda t}$, we can find the decay constant λ, which can then be used to find the half life, the original number of atoms, and the activity at any other time, t.

A: (a) $$\lambda = \frac{1}{t}\ln\left(\frac{R_o}{R}\right) = \left(\frac{1}{(4.00\text{ h})(60.0\text{ s / h})}\right)\ln\left(\frac{10.0\text{ mCi}}{8.00\text{ mCi}}\right) = 1.55\times 10^{-5}\text{ s}^{-1}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.0558\text{ h}^{-1}} = 12.4\text{ h}$$

(b) The number of original atoms can be found if we convert the initial activity from curies into becquerels (decays per second): $1\text{ Ci} \equiv 3.7\times 10^{10}\text{ Bq}$

$$R_0 = 10.0\text{ mCi} = \left(10.0\times 10^{-3}\text{ Ci}\right)\left(3.70\times 10^{10}\text{ Bq / Ci}\right) = 3.70\times 10^{8}\text{ Bq}$$

Since $R_0 = \lambda N_0$, $$N_0 = \frac{R_0}{\lambda} = \frac{3.70\times 10^{8}\text{ decays / s}}{1.55\times 10^{-5}\text{ s}} = 2.39\times 10^{13}\text{ atoms}$$

(c) $$R = R_o e^{-\lambda t} = (10.0\text{ mCi})e^{-(5.58\times 10^{-2}\text{ h}^{-1})(30.0\text{ h})} = 1.87\text{ mCi}$$

L: Our estimate of the half life was about 20% short because we did not account for the non-linearity of the decay rate. Consequently, our estimate of the final activity also fell short, but both of these calculated results are close enough to be reasonable.

The number of atoms is much less than one mole, so this appears to be a very small sample. To get a sense of how small, we can assume that the molar mass is about 100 g/mol, so the sample has a mass of only $m \approx \left(2.4\times 10^{13}\text{ atoms}\right)\left(100\text{ g / mol}\right)\Big/\left(6.02\times 10^{23}\text{ atoms / mol}\right) \approx 0.004\ \mu\text{g}$

This sample is so small it cannot be measured by a commercial mass balance! The problem states that this sample was “freshly prepared,” from which we assumed that **all** the atoms within the sample are initially radioactive. Generally this is not true, so that N_0 only accounts for the formerly radioactive atoms, and does not include additional atoms in the sample that were not radioactive. Realistically then, the sample mass should be significantly greater than our above estimate.

44.28 $R = R_0 e^{-\lambda t}$ where $\lambda = \dfrac{\ln 2}{26.0 \text{ h}} = 0.0266/\text{h}$

$\dfrac{R}{R_0} = 0.100 = e^{-\lambda t}$ so $\ln(0.100) = -\lambda t$

$2.30 = \left(\dfrac{0.0266}{\text{h}}\right) t$ $t = \boxed{86.4 \text{ h}}$

44.29 The number of nuclei which decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$

First we find λ: $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{0.693}{64.8 \text{ h}} = 0.0107 \text{ h}^{-1} = 2.97 \times 10^{-6} \text{ s}^{-1}$

and $N_0 = \dfrac{R_0}{\lambda} = \dfrac{(40.0\ \mu\text{Ci})(3.70 \times 10^4 \text{ cps}/\mu\text{Ci})}{2.97 \times 10^{-6} \text{ s}^{-1}} = 4.98 \times 10^{11}$ nuclei

Substituting these values, $N_1 - N_2 = \left(4.98 \times 10^{11}\right)\left[e^{-(0.0107 \text{ h}^{-1})(10.0 \text{ h})} - e^{-(0.0107 \text{ h}^{-1})(12.0 \text{ h})}\right]$

Hence, the number of nuclei decaying during the interval is $N_1 - N_2 = \boxed{9.47 \times 10^9 \text{ nuclei}}$

44.30 The number of nuclei which decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$

First we find λ: $\lambda = \dfrac{\ln 2}{T_{1/2}}$

so $e^{-\lambda t} = e^{\ln 2(-t/T_{1/2})} = 2^{-t/T_{1/2}}$ and $N_0 = \dfrac{R_0}{\lambda} = \dfrac{R_0 T_{1/2}}{\ln 2}$

Substituting in these values $N_1 - N_2 = \dfrac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\dfrac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}$

44.31 $R = \lambda N = \left(\dfrac{\ln 2}{5.27 \text{ yr}}\right)\left(\dfrac{1.00 \text{ g}}{59.93 \text{ g/mol}}\right)\left(6.02 \times 10^{23}\right)$

$R = \left(1.32 \times 10^{21} \dfrac{\text{decays}}{\text{yr}}\right)\left(\dfrac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}}\right) = 4.18 \times 10^{13} \text{ Bq}$

44.32 (a) $^{65}_{28}\mathrm{Ni}^*$

(b) $^{211}_{82}\mathrm{Pb}$

(c) $^{55}_{27}\mathrm{Co}$

(d) $^{0}_{-1}\mathrm{e}$

(e) $^{1}_{1}\mathrm{H}$ (or p)

44.33 $$Q = \left(M_{^{238}\mathrm{U}} - M_{^{234}\mathrm{Th}} - M_{^{4}\mathrm{He}}\right)(931.5 \text{ MeV/u})$$

$$Q = (238.050\,784 - 234.043\,593 - 4.002\,602)\,\mathrm{u}\,(931.5 \text{ MeV/u}) = \boxed{4.27 \text{ MeV}}$$

44.34 $$N_C = \left(\frac{0.0210 \text{ g}}{12.0 \text{ g/mol}}\right)\left(6.02\times10^{23} \text{ molecules/mol}\right)$$

$\left(N_C = 1.05\times10^{21} \text{ carbon atoms}\right)$ of which 1 in 7.70×10^{11} is a $^{14}\mathrm{C}$ atom

$(N_0)_{^{14}\mathrm{C}} = 1.37\times10^9$, $\quad \lambda_{^{14}\mathrm{C}} = \dfrac{\ln 2}{5730 \text{ yr}} = 1.21\times10^{-4} \text{ yr}^{-1} = 3.83\times10^{-12} \text{ s}^{-1}$

$$R = \lambda N = \lambda N_0 e^{-\lambda t}$$

At $t = 0$, $\quad R_0 = \lambda N_0 = \left(3.83\times10^{-12} \text{ s}^{-1}\right)\left(1.37\times10^9\right)\left[\dfrac{7(86400 \text{ s})}{1 \text{ week}}\right] = 3.17\times10^3 \ \dfrac{\text{decays}}{\text{week}}$

At time t, $\quad R = \dfrac{837}{0.88} = 951$ decays/week

Taking logarithms, $\quad \ln\dfrac{R}{R_0} = -\lambda t \quad$ so $\quad t = \dfrac{-1}{\lambda}\ln\left(\dfrac{R}{R_0}\right)$

$$t = \frac{-1}{1.21\times10^{-4} \text{ yr}^{-1}}\ln\left(\frac{951}{3.17\times10^3}\right) = \boxed{9.96\times10^3 \text{ yr}}$$

44.35 In the decay $^{3}_{1}\mathrm{H} \to {}^{3}_{2}\mathrm{He} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}$, the energy released is $E = (\Delta m)c^2 = \left[M_{^{3}_{1}\mathrm{H}} - M_{^{3}_{2}\mathrm{He}}\right]c^2$ since the antineutrino is massless and the mass of the electron is accounted for in the masses of $^{3}_{1}\mathrm{H}$ and $^{3}_{2}\mathrm{He}$.

Thus, $\quad E = [3.016\,049\,\mathrm{u} - 3.016\,029\,\mathrm{u}]\,(931.5 \text{ MeV/u}) = 0.0186 \text{ MeV} = \boxed{18.6 \text{ keV}}$

44.36 (a) For e^+ decay,

$$Q = \left(M_X - M_Y - 2m_e\right)c^2 = \left[39.962\ 591\ \text{u} - 39.964\ 000\ \text{u} - 2\left(0.0000\ 549\ \text{u}\right)\right]\left(931.5\ \text{MeV/u}\right)$$

$$Q = -2.34\ \text{MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(b) For alpha decay,

$$Q = \left(M_X - M_\alpha - M_Y\right)c^2 = \left[97.905\ 287\ \text{u} - 4.002\ 602\ \text{u} - 93.905\ 085\ \text{u}\right]\left(931.5\ \text{MeV/u}\right)$$

$$Q = -2.24\ \text{MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(c) For alpha decay,

$$Q = \left(M_X - M_\alpha - M_Y\right)c^2 = \left[143.910\ 082\ \text{u} - 4.002\ 602\ \text{u} - 139.905\ 434\ \text{u}\right]\left(931.5\ \text{MeV/u}\right)$$

$$Q = 1.91\ \text{MeV}$$

Since $Q > 0$, the decay $\boxed{\text{can occur}}$ spontaneously.

44.37 (a) $\boxed{e^- + p \rightarrow n + \nu}$

(b) For nuclei, $\quad {}^{15}\text{O} + e^- \rightarrow {}^{15}\text{N} + \nu$.

Add seven electrons to both sides to obtain $\boxed{{}^{15}_{8}\text{O atom} \rightarrow {}^{15}_{7}\text{N atom} + \nu}$.

(c) From Table A.3, $\quad m\left({}^{15}\text{O}\right) = m\left({}^{15}\text{N}\right) + \dfrac{Q}{c^2}$

Δm = 15.003 065 u – 15.000 108 u = 0.002 957 u

Q = (931.5 MeV / u)(0.002 957 u) = $\boxed{2.75\ \text{MeV}}$

44.38 (a) Let N be the number of ^{238}U nuclei and N' be ^{206}Pb nuclei.

Then $N = N_0 e^{-\lambda t}$ and $N_0 = N + N'$ so $N = (N + N')e^{-\lambda t}$ or $e^{\lambda t} = 1 + \frac{N'}{N}$

Taking logarithms, $\lambda t = \ln\left(1 + \frac{N'}{N}\right)$ where $\lambda = (\ln 2)/T_{1/2}$.

Thus, $t = \left(\frac{T_{1/2}}{\ln 2}\right)\ln\left(1 + \frac{N'}{N}\right)$

If $\frac{N}{N'} = 1.164$ for the $^{238}\text{U} \rightarrow {}^{206}\text{Pb}$ chain with $T_{1/2} = 4.47 \times 10^9$ yr, the age is:

$$t = \left(\frac{4.47 \times 10^9 \text{ yr}}{\ln 2}\right)\ln\left(1 + \frac{1}{1.164}\right) = \boxed{4.00 \times 10^9 \text{ yr}}$$

(b) From above, $e^{\lambda t} = 1 + \frac{N'}{N}$. Solving for $\frac{N}{N'}$ gives $\frac{N}{N'} = \frac{e^{-\lambda t}}{1 - e^{-\lambda t}}$

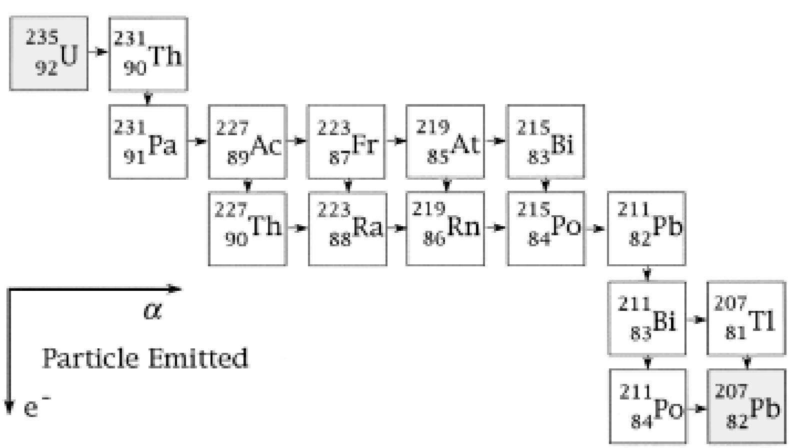
With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 7.04 \times 10^8$ yr for the $^{235}\text{U} \rightarrow {}^{207}\text{Pb}$ chain,

$$\lambda t = \left(\frac{\ln 2}{T_{1/2}}\right)t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{7.04 \times 10^8 \text{ yr}} = 3.938 \quad \text{and} \quad \boxed{\frac{N}{N'} = 0.0199}$$

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 1.41 \times 10^{10}$ yr for the $^{232}\text{Th} \rightarrow {}^{208}\text{Pb}$ chain,

$$\lambda t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{1.41 \times 10^{10} \text{ yr}} = 0.1966 \quad \text{and} \quad \boxed{\frac{N}{N'} = 4.60}$$

44.39



***44.40** (a) $4.00\ \text{pCi/L} = \left(\dfrac{4.00\times10^{-12}\ \text{Ci}}{1\ \text{L}}\right)\left(\dfrac{3.70\times10^{10}\ \text{Bq}}{1\ \text{Ci}}\right)\left(\dfrac{1.00\times10^{3}\ \text{L}}{1\ \text{m}^3}\right) = \boxed{148\ \text{Bq/m}^3}$

(b) $N = \dfrac{R}{\lambda} = R\left(\dfrac{T_{1/2}}{\ln 2}\right) = \left(148\ \dfrac{\text{Bq}}{\text{m}^3}\right)\left(\dfrac{3.82\ \text{d}}{\ln 2}\right)\left(\dfrac{86\ 400\ \text{s}}{1\ \text{d}}\right) = \boxed{7.05\times10^{7}\ \text{atoms/m}^3}$

(c) $\text{mass} = \left(7.05\times10^{7}\ \dfrac{\text{atoms}}{\text{m}^3}\right)\left(\dfrac{1\ \text{mol}}{6.02\times10^{23}\ \text{atoms}}\right)\left(\dfrac{222\ \text{g}}{1\ \text{mol}}\right) = 2.60\times10^{-14}\ \dfrac{\text{g}}{\text{m}^3}$

Since air has a density of $1.20\ \text{kg/m}^3$, the fraction consisting of radon is

$\text{fraction} = \dfrac{2.60\times10^{-14}\ \text{g/m}^3}{1.20\ \text{kg/m}^3} = \boxed{2.17\times10^{-17}}$

***44.41** Number remaining: $N = N_0 e^{-(\ln 2)t/T_{1/2}}$

Fraction remaining: $\dfrac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$

(a) With $T_{1/2} = 3.82\ \text{d}$ and $t = 7.00\ \text{d}$, $\dfrac{N}{N_0} = e^{-(\ln 2)(7.00)/(3.82)} = \boxed{0.281}$

(b) When $t = 1.00\ \text{yr} = 365.25\ \text{d}$, $\dfrac{N}{N_0} = e^{-(\ln 2)(365.25)/(3.82)} = \boxed{1.65\times10^{-29}}$

(c) $\boxed{\text{Radon is continuously created}}$ as one daughter in the series of decays starting from the long-lived isotope ^{238}U.

44.42 $Q = \left[M_{^{27}\text{Al}} + M_{\alpha} - M_{^{30}\text{P}} - m_n\right]c^2$

$Q = [26.981\ 538 + 4.002\ 602 - 29.978\ 307 - 1.008\ 665]\,\text{u}\ (931.5\ \text{MeV/u}) = \boxed{-2.64\ \text{MeV}}$

44.43 (a) ${}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \rightarrow {}^{197}_{79}\text{Au}^* \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}$

(b) Consider adding 79 electrons: ${}^{197}_{79}\text{Au atom} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{80}\text{Hg atom} + \bar{\nu} + Q$

$Q = \left[M_{^{197}\text{Au}} + m_{\text{n}} - M_{^{198}\text{Hg}}\right]c^2$

$Q = [196.966\ 543 + 1.008\ 665 - 197.966\ 743]\,\text{u}\ (931.5\ \text{MeV/u}) = \boxed{7.89\ \text{MeV}}$

***44.44** (a) For X, $A = 24 + 1 - 4 = 21$ and $Z = 12 + 0 - 2 = 10$, so X is $\boxed{{}^{21}_{10}\text{Ne}}$

(b) $A = 235 + 1 - 90 - 2 = 144$ and $Z = 92 + 0 - 38 - 0 = 54$, so X is $\boxed{{}^{144}_{54}\text{Xe}}$

(c) $A = 2 - 2 = 0$ and $Z = 2 - 1 = +1$, so X must be a positron.

As it is ejected, so is a neutrino: $\boxed{X = {}^{0}_{1}\text{e}^{+}}$ and $\boxed{X' = {}^{0}_{0}\nu}$

***44.45** Neglect recoil of product nucleus, (i.e., do not require momentum conservation). The energy balance gives $K_{\text{emerging}} = K_{\text{incident}} + Q$. To find Q:

$$Q = \left[\left(M_{\text{H}} + M_{\text{Al}}\right) - \left(M_{\text{Si}} + m_n\right)\right]c^2$$

$$Q = \left[(1.007\ 825 + 26.981\ 528) - (26.986\ 721 + 1.008\ 665)\right]\text{u}\ (931.5\ \text{MeV/u}) = -5.61\ \text{MeV}$$

Thus, $K_{\text{emerging}} = 6.61\ \text{MeV} - 5.61\ \text{MeV} = \boxed{1.00\ \text{MeV}}$

***44.46** (a) ${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \to {}^{13}_{6}\text{C} + {}^{1}_{1}\text{H}$

The product nucleus is $\boxed{{}^{13}_{6}\text{C}}$

(b) ${}^{13}_{6}\text{C} + {}^{1}_{1}\text{H} \to {}^{10}_{5}\text{B} + {}^{4}_{2}\text{He}$

The product nucleus is $\boxed{{}^{10}_{5}\text{B}}$

44.47 ${}^{9}_{4}\text{Be} + 1.666\ \text{MeV} \to {}^{8}_{4}\text{Be} + {}^{1}_{0}\text{n}$, so $M_{{}^{8}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} - \dfrac{Q}{c^2} - m_n$

$$M_{{}^{8}_{4}\text{Be}} = 9.012\ 174\ \text{u} - \frac{(-1.666\ \text{MeV})}{931.5\ \text{MeV/u}} - 1.008\ 665\ \text{u} = \boxed{8.005\ 3\ \text{u}}$$

${}^{9}_{4}\text{Be} + {}^{1}_{0}\text{n} \to {}^{10}_{4}\text{Be} + 6.810\ \text{MeV}$, so $M_{{}^{10}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} + m_n - \dfrac{Q}{c^2}$

$$M_{{}^{10}_{4}\text{Be}} = 9.012\ 174\ \text{u} + 1.008\ 665\ \text{u} - \frac{6.810\ \text{MeV}}{931.5\ \text{MeV/u}} = \boxed{10.013\ 5\ \text{u}}$$

Goal Solution

Using the Q values of appropriate reactions and from Table 44.5, calculate the masses of ^{8}Be and ^{10}Be in atomic mass units to four decimal places.

G: The mass of each isotope in atomic mass units will be approximately the number of nucleons (8 or 10), also called the mass number. The electrons are much less massive and contribute only about 0.03% to the total mass.

O: In addition to summing the mass of the subatomic particles, the net mass of the isotopes must account for the binding energy that holds the atom together. Table 44.5 includes the energy released for each nuclear reaction. Precise atomic masses values are found in Table A.3.

A: The notation $^{9}\text{Be}\,(\gamma,\ \text{n})\ ^{8}\text{Be}$ with $Q = -1.666\ \text{MeV}$

means $^{9}\text{Be} + \gamma \rightarrow\ ^{8}\text{Be} + \text{n} - 1.666\ \text{MeV}$

Therefore $$m\left(^{8}\text{Be}\right) = m\left(^{9}\text{Be}\right) - m_n + \frac{1.666\ \text{MeV}}{931.5\ \text{MeV / u}}$$

$$m\left(^{8}\text{Be}\right) = 9.012174 - 1.008665 + 0.001789 = 8.0053\ \text{u}$$

The notation $^{9}\text{Be}\,(\text{n},\ \gamma)\ ^{10}\text{Be}$ with $Q = 6.810\ \text{MeV}$

means $^{9}\text{Be} + \text{n} \rightarrow\ ^{10}\text{Be} + \gamma + 6.810\ \text{MeV}$

$$m\left(^{10}\text{Be}\right) = m\left(^{9}\text{Be}\right) + m_n + \frac{6.810\ \text{MeV}}{931.5\ \text{MeV / u}}$$

$$m\left(^{10}\text{Be}\right) = 9.012174 + 1.008665 - 0.001789 = 10.0135\ \text{u}$$

L: As expected, both isotopes have masses slightly greater than their mass numbers. We were asked to calculate the masses to four decimal places, but with the available data, the results could be reported accurately to as many as six decimal places.

44.48 $$^{236}_{92}\text{U} \rightarrow\ ^{90}_{37}\text{Rb} +\ ^{143}_{55}\text{Cs} + 3\,^{1}_{0}\text{n},$$

so $$Q = \left[M_{^{236}_{92}\text{U}} - M_{^{90}_{37}\text{Rb}} - M_{^{143}_{55}\text{Cs}} - 3m_n\right]c^2$$

From Table A.3,

$$Q = \left[236.045\,562 - 89.914\,811 - 142.927\,220 - 3(1.008\,665)\right]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{165\ \text{MeV}}$$

44.49 $$\frac{N_1}{N_2} = \frac{N_0 - N_0 e^{-\lambda T_h/2}}{N_0 e^{-\lambda T_h/2} - N_0 e^{-\lambda T_h}} = \frac{1 - e^{-\ln 2/2}}{e^{-\ln 2/2} - e^{-\ln 2}} = \frac{1 - 2^{-1/2}}{2^{-1/2} - 2^{-1}} = \boxed{\sqrt{2}}$$

***44.50** $^1_1\text{H} + {}^7_3\text{Li} \to {}^7_4\text{Be} + {}^1_0\text{n}$

$$Q = \left[(M_{\text{H}} + M_{\text{Li}}) - (M_{\text{Be}} + M_{\text{n}})\right](931.5 \text{ MeV / u})$$

$$Q = \left[(1.007\ 825 \text{ u} + 7.016\ 003 \text{ u}) - (7.016\ 928 \text{ u} + 1.008\ 665 \text{ u})\right](931.5 \text{ MeV / u})$$

$$Q = (-1.765 \times 10^{-3} \text{ u})(931.5 \text{ MeV / u}) = -1.644 \text{ MeV}$$

Thus, $$KE_{\text{min}} = \left(1 + \frac{m_{\text{incident projectile}}}{m_{\text{target nucleus}}}\right)|Q| = \left(1 + \frac{1.007\ 825}{7.016\ 003}\right)(1.644 \text{ MeV}) = \boxed{1.88 \text{ MeV}}$$

44.51 (a) $$N_0 = \frac{\text{mass}}{\text{mass per atom}} = \frac{1.00 \text{ kg}}{(239.05 \text{ u})\left(1.66 \times 10^{-27} \text{ kg/u}\right)} = \boxed{2.52 \times 10^{24}}$$

(b) $$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{\left(2.412 \times 10^4 \text{ yr}\right)\left(3.156 \times 10^7 \text{ s/yr}\right)} = 9.106 \times 10^{-13} \text{ s}^{-1}$$

$$R_0 = \lambda N_0 = \left(9.106 \times 10^{-13} \text{ s}^{-1}\right)\left(2.52 \times 10^{24}\right) = \boxed{2.29 \times 10^{12} \text{ Bq}}$$

(c) $$R = R_0 e^{-\lambda t}, \quad \text{so} \quad t = \frac{-1}{\lambda} \ln\left(\frac{R}{R_0}\right) = \frac{1}{\lambda} \ln\left(\frac{R_0}{R}\right)$$

$$t = \frac{1}{9.106 \times 10^{-13} \text{ s}^{-1}} \ln\left(\frac{2.29 \times 10^{12} \text{ Bq}}{0.100 \text{ Bq}}\right) = 3.38 \times 10^{13} \text{ s}\left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = \boxed{1.07 \times 10^6 \text{ yr}}$$

44.52 (a) $^{57}_{27}\text{Co} \to {}^{57}_{26}\text{Fe} + {}^{0}_{+1}\text{e} + {}^{0}_{0}\nu$

The Q – value for this positron emission is $Q = \left[M_{^{57}\text{Co}} - M_{^{57}\text{Fe}} - 2m_e\right]c^2$

$$Q = \left[56.936\ 294 - 56.935\ 396 - 2(0.000\ 549)\right]\text{u}\ (931.5 \text{ MeV/u}) = -0.186 \text{ MeV}$$

Since $Q < 0$, this reaction $\boxed{\text{cannot spontaneously occur}}$.

(b) $^{14}_{6}\text{C} \to {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\bar{\nu}$

The Q – value for this e^- decay is $Q = \left[M_{^{14}\text{C}} - M_{^{14}\text{N}}\right]c^2$.

$$Q = \left[14.003\ 242 - 14.003\ 074\right]\text{u}\ (931.5 \text{ MeV/u}) = 0.156 \text{ MeV} = 156 \text{ keV}$$

Since $Q > 0$, the decay $\boxed{\text{can spontaneously occur}}$.

(c) The energy released in the reaction of (b) is shared by the electron and neutrino. Thus, $\boxed{K_e \text{ can range from zero to 156 keV}}$.

44.53 (a) $r = r_0 A^{1/3} = 1.20 \times 10^{-15} A^{1/3}$ m. When $A = 12$, $r = \boxed{2.75 \times 10^{-15}\ \text{m}}$

(b) $F = \dfrac{k_e(Z-1)e^2}{r^2} = \dfrac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2 / \text{C}^2)(Z-1)(1.60 \times 10^{-19}\ \text{C})^2}{r^2}$

When $Z = 6$ and $r = 2.75 \times 10^{-15}$ m, $F = \boxed{152\ \text{N}}$

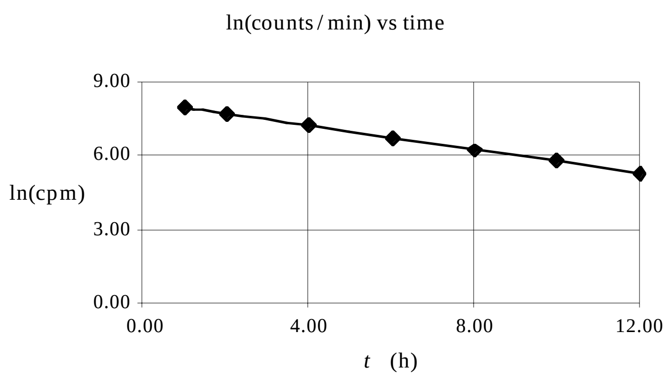
(c) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{k_e(Z-1)e^2}{r} = \dfrac{(8.99 \times 10^9)(1.6 \times 10^{-19})^2(Z-1)}{r}$

When $Z = 6$ and $r = 2.75 \times 10^{-15}$ m, $U = 4.19 \times 10^{-13}\ \text{J} = \boxed{2.62\ \text{MeV}}$

(d) $A = 238$; $Z = 92$, $r = \boxed{7.44 \times 10^{-15}\ \text{m}}$ $F = \boxed{379\ \text{N}}$

and $U = 2.82 \times 10^{-12}\ \text{J} = \boxed{17.6\ \text{MeV}}$

44.54 (a)



A least-square fit to the graph yields: $\lambda = -\text{slope} = -\left(-0.250\ \text{h}^{-1}\right) = 0.250\ \text{h}^{-1}$,

and $\ln\left(\text{cpm}\right)\Big|_{t=0} = \text{intercept} = 8.30$

(b) $\lambda = 0.250\ \text{h}^{-1}\left(\dfrac{1\ \text{h}}{60.0\ \text{min}}\right) = \boxed{4.17 \times 10^{-3}\ \text{min}^{-1}}$

$T_{1/2} = \dfrac{\ln 2}{\lambda} = \dfrac{\ln 2}{4.17 \times 10^{-3}\ \text{min}^{-1}} = 166\ \text{min} = \boxed{2.77\ \text{h}}$

(c) From (a), intercept = $\ln(\text{cpm})_0 = 8.30$.

Thus, $\left(\text{cpm}\right)_0 = e^{8.30}\ \text{counts/min} = \boxed{4.02 \times 10^3\ \text{counts/min}}$

(d) $N_0 = \dfrac{R_0}{\lambda} = \dfrac{1}{\lambda}\dfrac{\left(\text{cpm}\right)_0}{\text{Eff}} = \dfrac{4.02 \times 10^3\ \text{counts/min}}{\left(4.17 \times 10^{-3}\ \text{min}^{-1}\right)(0.100)} = \boxed{9.65 \times 10^6\ \text{atoms}}$

44.55 (a) Because the reaction $p \to n + e^+ + \nu$ would violate the law of $\boxed{\text{conservation of energy}}$,

$m_p = 1.007\ 276$ u $\quad m_n = 1.008\ 665$ u $\quad m_{e^+} = 5.49 \times 10^{-4}$ u $\qquad$ Note that $\quad m_n + m_{e^+} > m_p$

(b) The $\boxed{\text{required energy can come from the electrostatic repulsion}}$ of protons in the nucleus.

(c) Add seven electrons to both sides of the reaction for nuclei $\quad {}^{13}_{7}\text{N} \to {}^{13}_{6}\text{C} + e^+ + \nu$

to obtain the reaction for neutral atoms $\quad {}^{13}_{7}\text{N atom} \to {}^{13}_{6}\text{C atom} + e^+ + e^- + \nu$

$$Q = c^2\left[m\left({}^{13}\text{N}\right) - m\left({}^{13}\text{C}\right) - m_{e^+} - m_{e^-} - m_\nu\right]$$

$$Q = (931.5 \text{ MeV / u})\left[13.005\ 738 - 13.003\ 355 - 2\left(5.49 \times 10^{-4}\right) - 0\right]\text{u}$$

$$Q = (931.5 \text{ MeV/u})(1.285 \times 10^{-3} \text{ u}) = \boxed{1.20 \text{ MeV}}$$

44.56 (a) If we assume all the ^{87}Sr came from ^{87}Rb, then $N = N_0 e^{-\lambda t}$ yields

$$t = \frac{-1}{\lambda}\ln\left(\frac{N}{N_0}\right) = \frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_0}{N}\right), \quad \text{where} \quad N = N_{{}^{87}\text{Rr}} \text{ and } N_0 = N_{{}^{87}\text{Sr}} + N_{{}^{87}\text{Rb}}.$$

$$t = \frac{\left(4.75 \times 10^{10} \text{ yr}\right)}{\ln 2}\ln\left(\frac{1.82 \times 10^{10} + 1.07 \times 10^{9}}{1.82 \times 10^{10}}\right) = \boxed{3.91 \times 10^{9} \text{ yr}}$$

(b) It could be $\boxed{\text{no longer}}$. The rock could be younger if some ^{87}Sr were originally present.

44.57 (a) Let us assume that the parent nucleus (mass M_p) is initially at rest, and let us denote the masses of the daughter nucleus and alpha particle by M_d and M_α, respectively. Applying the equations of conservation of momentum and energy for the alpha decay process gives

$$M_d v_d = M_\alpha v_\alpha \qquad (1)$$

$$M_p c^2 = M_d c^2 + M_\alpha c^2 + \tfrac{1}{2} M_\alpha v_\alpha^2 + \tfrac{1}{2} M_d v_d^2 \qquad (2)$$

The disintegration energy Q is given by $\quad Q = (M_p - M_d - M_\alpha)c^2 = \tfrac{1}{2} M_\alpha v_\alpha^2 + \tfrac{1}{2} M_d v_d^2 \qquad (3)$

Eliminating v_d from Equations (1) and (3) gives

$$Q = \tfrac{1}{2} M_d\left(\frac{M_\alpha}{M_d} v_\alpha\right)^2 + \tfrac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2}\frac{M_\alpha^2}{M_d} v_\alpha^2 + \tfrac{1}{2} M_\alpha v_\alpha^2 = \tfrac{1}{2} M_\alpha v_\alpha^2\left(1 + \frac{M_\alpha}{M_d}\right) = \boxed{K_\alpha\left(1 + \frac{M_\alpha}{M_d}\right)}$$

(b) $$K_\alpha = \frac{Q}{1 + \left(M_\alpha / M_d\right)} = \frac{4.87 \text{ MeV}}{1 + (4 / 222)} = \boxed{4.78 \text{ MeV}}$$

44.58 (a) The reaction is ${}^{145}_{61}\text{Pm} \rightarrow {}^{141}_{59}\text{Pr} + \alpha$

(b) $Q = (M_{\text{Pm}} - M_\alpha - M_{\text{Pr}})931.5 = (144.912\,745 - 4.002\,602 - 140.907\,647)931.5 = \boxed{2.32\text{ MeV}}$

(c) The alpha and daughter have equal and opposite momenta $\quad p_\alpha = p_d$

$$E_\alpha = \frac{p_\alpha^2}{2m_\alpha} \qquad E_d = \frac{p_d^2}{2m_d}$$

$$\frac{E_\alpha}{E_{\text{tot}}} = \frac{E_\alpha}{E_\alpha + E_d} = \frac{\dfrac{p_\alpha^2}{2m_\alpha}}{\dfrac{p_\alpha^2}{2m_a} + \dfrac{p_\alpha^2}{2m_d}} = \frac{\dfrac{1}{2m_\alpha}}{\dfrac{1}{2m_\alpha} + \dfrac{1}{2m_d}} = \frac{m_d}{m_d + m_\alpha} = \frac{141}{141+4} = \boxed{97.2\%} \text{ or } 2.26\text{ MeV}$$

This is carried away by the alpha

44.59 (a) If ΔE is the energy difference between the excited and ground states of the nucleus of mass M, and hf is the energy of the emitted photon, conservation of energy gives

$$\Delta E = hf + E_r \tag{1}$$

Where E_r is the recoil energy of the nucleus, which can be expressed as

$$E_r = \frac{Mv^2}{2} = \frac{(Mv)^2}{2M} \tag{2}$$

Since momentum must also be conserved, we have

$$Mv = \frac{hf}{c} \tag{3}$$

Hence, E_r can be expressed as $\quad E_r = \dfrac{(hf)^2}{2Mc^2}$.

When $hf << Mc^2$, we can make the approximation that $hf \approx \Delta E$, so $\quad E_r \approx \boxed{\dfrac{(\Delta E)^2}{2Mc^2}}$

(b) $E_r = \dfrac{(\Delta E)^2}{2Mc^2}$ where $\Delta E = 0.0144$ MeV and $Mc^2 = (57\text{ u})(931.5\text{ MeV/u}) = 5.31 \times 10^4$ MeV

Therefore, $$E_r = \frac{(1.44 \times 10^{-2}\text{ MeV})^2}{(2)(5.31 \times 10^4\text{ MeV})} = \boxed{1.94 \times 10^{-3}\text{ eV}}$$

***44.60** (a) One liter of milk contains this many ^{40}K nuclei:

$$N = (2.00\text{ g})\left(\frac{6.02\times10^{23}\text{ nuclei/mol}}{39.1\text{ g/mol}}\right)\left(\frac{0.0117}{100}\right) = 3.60\times10^{18}\text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.28\times10^{9}\text{ yr}}\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right) = 1.72\times10^{-17}\text{ s}^{-1}$$

$$R = \lambda N = \left(1.72\times10^{-17}\text{ s}^{-1}\right)\left(3.60\times10^{18}\right) = \boxed{61.8\text{ Bq}}$$

(b) For the iodine, $R = R_0 e^{-\lambda t}$ with $\lambda = \dfrac{\ln 2}{8.04\text{ d}}$.

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{8.04\text{ d}}{\ln 2}\ln\left(\frac{2000}{61.8}\right) = \boxed{40.3\text{ d}}$$

***44.61** (a) For cobalt-56, $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{77.1\text{ d}}\left(\dfrac{365.25\text{ d}}{1\text{ yr}}\right) = 3.28\text{ yr}^{-1}$.

The elapsed time from July 1054 to July 2000 is 946 yr.

$R = R_0 e^{-\lambda t}$ implies $\dfrac{R}{R_0} = e^{-\lambda t} = e^{-\left(3.28\text{ yr}^{-1}\right)(946\text{ yr})} = e^{-3106} = e^{-(\ln 10)1349} = \boxed{\sim 10^{-1349}}$

(b) For carbon-14, $\lambda = \dfrac{\ln 2}{5730\text{ yr}} = 1.21\times10^{-4}\text{ yr}^{-1}$

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(1.21\times10^{-4}\text{ yr}^{-1}\right)(946\text{ yr})} = e^{-0.114} = \boxed{0.892}$$

***44.62** We have $N_{235} = N_{0,\,235}e^{-\lambda_{235}t}$ and $N_{238} = N_{0,\,238}e^{-\lambda_{238}t}$

$$\frac{N_{235}}{N_{238}} = 0.00725 = e^{\left(-(\ln 2)t/T_{h,\,235} + (\ln 2)t/T_{h,\,238}\right)}$$

Taking logarithms, $-4.93 = \left(-\dfrac{\ln 2}{0.704\times10^{9}\text{ yr}} + \dfrac{\ln 2}{4.47\times10^{9}\text{ yr}}\right)t$

or $-4.93 = \left(-\dfrac{1}{0.704\times10^{9}\text{ yr}} + \dfrac{1}{4.47\times10^{9}\text{ yr}}\right)(\ln 2)t$

$$t = \frac{-4.93}{\left(-1.20\times10^{-9}\text{ yr}^{-1}\right)\ln 2} = \boxed{5.94\times10^{9}\text{ yr}}$$

44.63 (a) Add two electrons to both sides of the reaction to have it in energy terms: $4\,{}^1_1\text{H atom} \to {}^4_2\text{He atom} + Q$

$$Q = \Delta mc^2 = \left[4\,M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2$$

$$Q = \left[4(1.007\ 825\ \text{u}) - 4.002\ 602\ \text{u}\right](931.5\ \text{MeV/u})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = \boxed{4.28\times10^{-12}\ \text{J}}$$

(b) $$N = \frac{1.99\times10^{30}\ \text{kg}}{1.67\times10^{-27}\ \text{kg/atom}} = \boxed{1.19\times10^{57}\ \text{atoms}} = 1.19\times10^{57}\ \text{protons}$$

(c) The energy that could be created by this many protons in this reaction is:

$$(1.19\times10^{57}\ \text{protons})\left(\frac{4.28\times10^{-12}\ \text{J}}{4\ \text{protons}}\right) = 1.27\times10^{45}\ \text{J}$$

$$\mathsf{P} = \frac{E}{t} \qquad \text{so} \qquad t = \frac{E}{\mathsf{P}} = \frac{1.27\times10^{45}\ \text{J}}{3.77\times10^{26}\ \text{W}} = 3.38\times10^{18}\ \text{s} = \boxed{107\ \text{billion years}}$$

44.64 (a) $$Q = \left[M_{{}^9\text{Be}} + M_{{}^4\text{He}} - M_{{}^{12}\text{C}} - m_n\right]c^2$$

$$Q = \left[9.012\ 174\ \text{u} + 4.002\ 602\ \text{u} - 12.000\ 000\ \text{u} - 1.008\ 665\ \text{u}\right](931.5\ \text{MeV/u}) = \boxed{5.69\ \text{MeV}}$$

(b) $$Q = \left[2\,M_{{}^2\text{H}} - M_{{}^3\text{He}} - m_n\right]$$

$$Q = \left[2(2.014\ 102) - 3.016\ 029 - 1.008\ 665\right]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{3.27\ \text{MeV (exothermic)}}$$

44.65 $E = -\boldsymbol{\mu}\cdot\mathbf{B}$ so the energies are $E_1 = +\mu B$ and $E_2 = -\mu B$

$\mu = 2.7928\mu_\text{n}$ and $\mu_\text{n} = 5.05\times10^{-27}\ \text{J/T}$

$$\Delta E = 2\mu B = 2(2.7928)(5.05\times10^{-27}\ \text{J/T})(12.5\ \text{T}) = 3.53\times10^{-25}\ \text{J} = \boxed{2.20\times10^{-6}\ \text{eV}}$$

44.66 (a) $$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5.27\ \text{yr}}\left(\frac{1\ \text{yr}}{3.156\times 10^{7}\ \text{s}}\right) = 4.17\times 10^{-9}\ \text{s}^{-1}$$

$$t = 30.0\ \text{months} = (2.50\ \text{yr})\left(\frac{3.156\times 10^{7}\ \text{s}}{1\ \text{yr}}\right) = 7.89\times 10^{7}\ \text{s}$$

$$R = R_0 e^{-\lambda t} = (\lambda N_0)e^{-\lambda t}$$

so $$N_0 = \left(\frac{R}{\lambda}\right)e^{\lambda t} = \left[\frac{(10.0\ \text{Ci})(3.70\times 10^{10}\ \text{Bq/Ci})}{4.17\times 10^{-9}\ \text{s}^{-1}}\right]e^{(4.17\times 10^{-9}\ \text{s}^{-1})(7.89\times 10^{7}\ \text{s})}$$

$$N_0 = 1.23\times 10^{20}\ \text{nuclei}$$

$$\text{Mass} = (1.23\times 10^{20}\ \text{atoms})\left(\frac{59.93\ \text{g/mol}}{6.02\times 10^{23}\ \text{atoms/mol}}\right) = 1.23\times 10^{-2}\ \text{g} = \boxed{12.3\ \text{mg}}$$

(b) We suppose that each decaying nucleus promptly puts out both a beta particle and two gamma rays, for

$$Q = (0.310 + 1.17 + 1.33)\ \text{MeV} = 2.81\ \text{Mev}$$

$$\mathcal{P} = QR = (2.81\ \text{Mev})(1.6\times 10^{-13}\ \text{J/MeV})(3.70\times 10^{11}\ \text{s}^{-1}) = \boxed{0.166\ \text{W}}$$

44.67 For an electric charge density $\rho = \dfrac{Ze}{\frac{4}{3}\pi R^{3}}$

Using Gauss's Law inside the sphere,

$$E\cdot 4\pi r^{2} = \frac{\frac{4}{3}\pi r^{3}}{\epsilon_0}\frac{Ze}{\frac{4}{3}\pi R^{3}}: \qquad E = \frac{1}{4\pi\epsilon_0}\frac{Zer}{R^{3}} \quad (r \le R)$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Ze}{r^{2}} \quad (r \ge R)$$

We now find the electrostatic energy: $U = \int_{r=0}^{\infty}\frac{1}{2}\epsilon_0 E^{2}4\pi r^{2}\,dr$

$$U = \frac{1}{2}\epsilon_0\int_0^R\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{Z^2e^2r^2}{R^6}4\pi r^2\,dr + \frac{1}{2}\epsilon_0\int_R^\infty\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{Z^2e^2}{r^4}4\pi r^2\,dr = \frac{Z^2e^2}{8\pi\epsilon_0}\left[\frac{R^5}{5R^6} + \frac{1}{R}\right] = \boxed{\frac{3}{20}\frac{Z^2e^2}{\pi\epsilon_0 R}}$$

44.68 (a) For the electron capture, ${}^{93}_{43}\mathrm{Tc} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{93}_{42}\mathrm{Mo} + \gamma$

The disintegration energy is $Q = \left[M_{^{93}\mathrm{Tc}} - M_{^{93}\mathrm{Mo}}\right]c^2$.

$$Q = [92.910\ 2 - 92.906\ 8]\,\mathrm{u}\ (931.5\ \mathrm{MeV/u}) = 3.17\ \mathrm{MeV}\ >\ 2.44\ \mathrm{MeV}$$

Electron capture is allowed $\boxed{\text{to all specified excited states}}$ in ${}^{93}_{42}\mathrm{Mo}$.

For positron emission, ${}^{93}_{43}\mathrm{Tc} \rightarrow {}^{93}_{42}\mathrm{Mo} + {}^{0}_{+1}\mathrm{e} + \gamma$

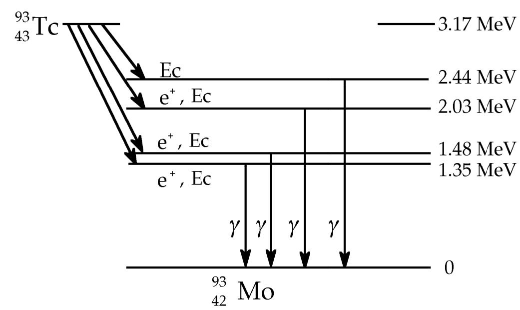
The disintegration energy is $Q' = \left[M_{^{93}\mathrm{Tc}} - M_{^{93}\mathrm{Mo}} - 2m_e\right]c^2$.

$$Q' = [92.910\ 2 - 92.906\ 8 - 2(0.000\ 549)]\,\mathrm{u}\ (931.5\ \mathrm{MeV/u}) = 2.14\ \mathrm{MeV}$$

Positron emission can reach

$\boxed{\text{the 1.35, 1.48, and 2.03 MeV states}}$

but there is insufficient energy to reach the 2.44 MeV state.



(b) The daughter nucleus in both forms of decay is $\boxed{{}^{93}_{42}\mathrm{Mo}}$.

44.69 $K = \frac{1}{2}mv^2$,

so $$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.0400\ \mathrm{eV})(1.60\times10^{-19}\ \mathrm{J/eV})}{1.67\times10^{-27}\ \mathrm{kg}}} = 2.77\times10^{3}\ \mathrm{m\ s}$$

The time for the trip is $t = \dfrac{x}{v} = \dfrac{1.00\times10^{4}\ \mathrm{m}}{2.77\times10^{3}\ \mathrm{m/s}} = 3.61\ \mathrm{s}$

The number of neutrons finishing the trip is given by $N = N_0 e^{-\lambda t}$.

The fraction decaying is $1 - \dfrac{N}{N_0} = 1 - e^{-(\ln 2)t/T_{1/2}} = 1 - e^{-(\ln 2)(3.61\ \mathrm{s}/624\ \mathrm{s})} = 0.004\ 00 = \boxed{0.400\%}$

44.70 (a) At threshold, the particles have no kinetic energy relative to each other. That is, they move like two particles which have suffered a perfectly inelastic collision. Therefore, in order to calculate the reaction threshold energy, we can use the results of a perfectly inelastic collision. Initially, the projectile M_a moves with velocity v_a while the target M_X is at rest. We have from momentum conservation: $M_a v_a = (M_a + M_X)v_c$

The initial energy is: $E_i = \frac{1}{2} M_a v_a^2$

The final kinetic energy is: $E_f = \frac{1}{2}(M_a + M_X)v_c^2 = \frac{1}{2}(M_a + M_X)\left[\frac{M_a v_a}{M_a + M_X}\right]^2 = \left[\frac{M_a}{(M_a + M_X)}\right]E_i$

From this, we see that E_f is always less than E_i and the *loss* in energy, $E_i - E_f$, is given by

$$E_i - E_f = \left[1 - \frac{M_a}{M_a + M_X}\right]E_i = \left[\frac{M_X}{M_a + M_X}\right]E_i$$

In this problem, the energy loss is the disintegration energy $-Q$ and the initial energy is the threshold energy E_{th}. Therefore,

$$-Q = \left[\frac{M_X}{M_a + M_X}\right]E_{th} \quad \text{or} \quad E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = \boxed{-Q\left[1 + \frac{M_a}{M_X}\right]}$$

(b) First, calculate the Q – value for the reaction: $Q = \left[M_{^{14}\text{N}} + M_{^{4}\text{He}} - M_{^{17}\text{O}} - M_{^{1}\text{H}}\right]c^2$

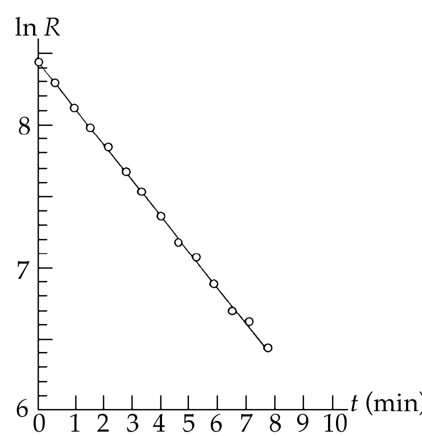
$$Q = [14.003\ 074 + 4.002\ 602 - 16.999\ 132 - 1.007\ 825]\text{u}\ (931.5\ \text{MeV/u}) = -1.19\ \text{MeV}$$

Then, $E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -(-1.19\ \text{MeV})\left[1 + \frac{4.002\ 602}{14.003\ 074}\right] = \boxed{1.53\ \text{MeV}}$

44.71 $R = R_0 \exp(-\lambda t)$

$\ln R = \ln R_0 - \lambda t$ (the equation of a straight line)

$|\text{slope}| = \lambda$



The logarithmic plot shown in Figure P44.71 is fitted by

$\ln R = 8.44 - 0.262t$.

If t is measured in minutes, then the decay constant λ is 0.262 per minute. The half-life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.262\ /\ \text{min}} = \boxed{2.64\ \text{min}}$$

The reported half-life of ^{137}Ba is 2.55 min. The difference reflects experimental uncertainties.

Chapter 45 Solutions

***45.1** $\Delta m = (m_n + M_{\text{U}}) - (M_{\text{Zr}} + M_{\text{Te}} + 3m_n)$

$\Delta m = (1.008\ 665 \text{ u} + 235.043\ 924 \text{ u}) - (97.912\ 0 \text{ u} + 134.908\ 7 \text{ u} + 3(1.008\ 665 \text{ u}))$

$\Delta m = 0.205\ 89 \text{ u} = 3.418 \times 10^{-28} \text{ kg}$ so $Q = \Delta mc^2 = 3.076 \times 10^{-11} \text{ J} = \boxed{192 \text{ MeV}}$

45.2 Three different fission reactions are possible: ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{144}_{54}\text{Xe} + 2\,{}^{1}_{0}\text{n}$ $\boxed{{}^{144}_{54}\text{Xe}}$

${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{143}_{54}\text{Xe} + 3\,{}^{1}_{0}\text{n}$ $\boxed{{}^{143}_{54}\text{Xe}}$ ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{142}_{54}\text{Xe} + 4\,{}^{1}_{0}\text{n}$ $\boxed{{}^{142}_{54}\text{Xe}}$

45.3 ${}^{1}_{0}\text{n} + {}^{232}_{90}\text{Th} \rightarrow {}^{233}_{90}\text{Th} \rightarrow {}^{233}_{91}\text{Pa} + e^- + \bar{\nu}$ ${}^{233}_{91}\text{Pa} \rightarrow {}^{233}_{92}\text{U} + e^- + \bar{\nu}$

45.4 ${}^{1}_{0}\text{n} + {}^{238}_{92}\text{U} \rightarrow {}^{239}_{92}\text{U} \rightarrow {}^{239}_{93}\text{Np} + e^- + \bar{\nu}$ ${}^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + e^- + \bar{\nu}$

45.5 (a) $Q = (\Delta m)c^2 = \left[m_n + M_{\text{U235}} - M_{\text{Ba141}} - M_{\text{Kr92}} - 3m_n\right]c^2$

$\Delta m = \left[(1.008\ 665 + 235.043\ 924) - (140.913\ 9 + 91.897\ 3 + 3 \times 1.008\ 665)\right]\text{u} = 0.215\ 39 \text{ u}$

$Q = (0.215\ 39 \text{ u})(931.5 \text{ MeV/u}) = \boxed{201 \text{ MeV}}$

(b) $f = \dfrac{\Delta m}{m_i} = \dfrac{0.215\ 39 \text{ u}}{236.052\ 59 \text{ u}} = 9.13 \times 10^{-4} = \boxed{0.0913\%}$

45.6 If the electrical power output of 1000 MW is 40.0% of the power derived from fission reactions, the power output of the fission process is

$$\frac{1000 \text{ MW}}{0.400} = \left(2.50 \times 10^9 \ \frac{\text{J}}{\text{s}}\right)\left(\frac{8.64 \times 10^4}{\text{d}}\right) = 2.16 \times 10^{14} \text{ J/d}$$

The number of fissions per day is $\left(2.16 \times 10^{14} \ \dfrac{\text{J}}{\text{d}}\right)\left(\dfrac{1 \text{ fission}}{200 \times 10^6 \text{ eV}}\right)\left(\dfrac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 6.74 \times 10^{24} \text{ d}^{-1}$

This also is the number of ${}^{235}\text{U}$ nuclei used, so the mass of ${}^{235}\text{U}$ used per day is

$$\left(6.74 \times 10^{24} \ \frac{\text{nuclei}}{\text{d}}\right)\left(\frac{235 \text{ g/mol}}{6.02 \times 10^{23} \text{ nuclei/mol}}\right) = 2.63 \times 10^3 \text{ g/d} = \boxed{2.63 \text{ kg/d}}$$

In contrast, a coal-burning steam plant producing the same electrical power uses more than 6×10^6 kg/d of coal.

45.7 The available energy to do work is 0.200 times the energy content of the fuel.

$$(1.00\text{ kg fuel})\left(\frac{0.0340\ ^{235}\text{U}}{\text{fuel}}\right)\left(\frac{1000\text{ g}}{1\text{ kg}}\right)\left(\frac{1\text{ mol}}{235\text{ g}}\right)\left(\frac{6.02\times10^{23}}{\text{mol}}\right)\left(\frac{(208)(1.60\times10^{-13}\text{ J})}{\text{fission}}\right)$$

$$\left(2.90\times10^{12}\text{ J}\right)(0.200) = 5.80\times10^{11}\text{ J} = \left(1.00\times10^{5}\text{ N}\right)\cdot d$$

$d = 5.80 \times 10^6$ m = $\boxed{5.80\text{ Mm}}$

Goal Solution

Suppose enriched uranium containing 3.40% of the fissionable isotope $^{235}_{92}\text{U}$ is used as fuel for a ship. The water exerts an average frictional drag of 1.00×10^{5} N on the ship. How far can the ship travel per kilogram of fuel? Assume that the energy released per fission event is 208 MeV and that the ship's engine has an efficiency of 20.0%.

G: Nuclear fission is much more efficient for converting mass to energy than burning fossil fuels. However, without knowing the rate of diesel fuel consumption for a comparable ship, it is difficult to estimate the nuclear fuel rate. It seems plausible that a ship could cross the Atlantic ocean with only a few kilograms of nuclear fuel, so a reasonable range of uranium fuel consumption might be 10 km / kg to 10 000 km / kg.

O: The fuel consumption rate can be found from the energy released by the nuclear fuel and the work required to push the ship through the water.

A: One kg of enriched uranium contains 3.40% $^{235}_{92}\text{U}$ so $m_{235} = (1000\text{ g})(0.0340) = 34.0\text{ g}$

In terms of number of nuclei, this is equivalent to

$$N_{235} = (34.0\text{ g})\left(\frac{1}{235\text{ g / mol}}\right)\left(6.02\times10^{23}\text{ atoms / mol}\right) = 8.71\times10^{22}\text{ nuclei}$$

If all these nuclei fission, the thermal energy released is equal to

$$\left(8.71\times10^{22}\text{ nuclei}\right)\left(208\ \frac{\text{MeV}}{\text{nucleus}}\right)\left(1.602\times10^{-19}\text{ J/eV}\right) = 2.90\times10^{12}\text{ J}$$

Now, for the engine, $\text{efficiency} = \dfrac{\text{work output}}{\text{heat input}}$ or $e = \dfrac{fd\cos\theta}{Q_h}$

So the distance the ship can travel per kilogram of uranium fuel is

$$d = \frac{eQ_h}{f\cos(0)} = \frac{0.200\left(2.90\times10^{12}\text{ J}\right)}{1.00\times10^{5}\text{ N}} = 5.80\times10^{6}\text{ m}$$

L: The ship can travel 5 800 km / kg of uranium fuel, which is on the high end of our prediction range. The distance between New York and Paris is 5 851 km, so this ship could cross the Atlantic ocean on just one kilogram of uranium fuel.

45.8 (a) For a sphere: $V = \frac{4}{3}\pi r^3$ and $r = \left(\frac{3V}{4\pi}\right)^{1/3}$ so $\frac{A}{V} = \frac{4\pi r^2}{(4/3)\pi r^3} = \boxed{4.84V^{-1/3}}$

(b) For a cube: $V = \ell^3$ and $\ell = V^{1/3}$ so $\frac{A}{V} = \frac{6\ell^2}{\ell^3} = \boxed{6V^{-1/3}}$

(c) For a parallelepiped: $V = 2a^3$ and $a = \left(\frac{V}{2}\right)^{1/3}$ so $\frac{A}{V} = \frac{\left(2a^2 + 8a^2\right)}{2a^3} = \boxed{6.30V^{-1/3}}$

(d) Therefore, the $\boxed{\text{sphere has the least leakage}}$ and the $\boxed{\text{parallelepiped has the greatest leakage}}$ for a given volume.

45.9 mass of ^{235}U available $\approx (0.007)\left(10^9 \text{ metric tons}\right)\left(\frac{10^6 \text{ g}}{1 \text{ metric ton}}\right) = 7 \times 10^{12}$ g

number of nuclei $\sim \left(\frac{7 \times 10^{12} \text{ g}}{235 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \frac{\text{nuclei}}{\text{mol}}\right) = 1.8 \times 10^{34}$ nuclei

The energy available from fission (at 208 MeV/event) is

$$E \sim \left(1.8 \times 10^{34} \text{ events}\right)(208 \text{ MeV / event})\left(1.60 \times 10^{-13} \text{ J / MeV}\right) = 6.0 \times 10^{23} \text{ J}$$

This would last for a time of $t = \frac{E}{\mathcal{P}} \sim \frac{6.0 \times 10^{23} \text{ J}}{7.0 \times 10^{12} \text{ J/s}} = \left(8.6 \times 10^{10} \text{ s}\right)\left(\frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}}\right) \sim \boxed{3000 \text{ yr}}$

45.10 In one minute there are $\frac{60.0 \text{ s}}{1.20 \text{ ms}} = 5.00 \times 10^4$ fissions.

So the rate increases by a factor of $(1.000\,25)^{50000} = \boxed{2.68 \times 10^5}$

45.11 $\mathcal{P} = 10.0 \text{ MW} = 1.00 \times 10^7$ J/s

If each decay delivers 1.00 MeV = 1.60×10^{-13} J, then the number of decays/s = $\boxed{6.25 \times 10^{19} \text{ Bq}}$

45.12 (a) The Q value for the D-T reaction is 17.59 MeV.

Heat content in fuel for D-T reaction: $\dfrac{(17.59\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(5\text{ u})(1.66\times10^{-27}\text{ kg/u})}=3.39\times10^{14}\text{ J/kg}$

$$r_{\text{DT}}=\frac{(3.00\times10^{9}\text{ J / s})(3600\text{ s / hr})}{(3.39\times10^{14}\text{ J / kg})(10^{-3}\text{ kg / g})}=\boxed{31.9\text{ g/h burning of D and T}}$$

(b) Heat content in fuel for D-D reaction: $Q=\frac{1}{2}(3.27+4.03)=3.65$ MeV average of two Q values

$$\frac{(3.65\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(4\text{ u})(1.66\times10^{-27}\text{ kg/u})}=8.80\times10^{13}\text{ J/kg}$$

$$r_{\text{DD}}=\frac{(3.00\times10^{9}\text{ J / s})(3600\text{ s / hr})}{(8.80\times10^{12}\text{ J / kg})(10^{-3}\text{ kg / g})}=\boxed{122\text{ g/h burning of D}}$$

45.13 (a) At closest approach, the electrostatic potential energy equals the total energy E.

$$U_f=\frac{k_e(Z_1e)(Z_2e)}{r_{\min}}=E: \qquad E=\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\text{ C}\right)^2 Z_1Z_2}{1.00\times10^{-14}\text{ m}}=\boxed{\left(2.30\times10^{-14}\text{ J}\right)Z_1Z_2}$$

(b) For both the D-D and the D-T reactions, $Z_1=Z_2=1$. Thus, the minimum energy required in both cases is

$$E=\left(2.30\times10^{-14}\text{ J}\right)\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)=\boxed{0.144\text{ MeV}}$$

45.14 (a) $r_f=r_{\text{D}}+r_{\text{T}}=\left(1.20\times10^{-15}\text{ m}\right)\left[(2)^{1/3}+(3)^{1/3}\right]=\boxed{3.24\times10^{-15}\text{ m}}$

(b) $$U_f=\frac{k_ee^2}{r_f}=\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{3.24\times10^{-15}\text{ m}}=7.10\times10^{-14}\text{ J}=\boxed{444\text{ keV}}$$

(c) Conserving momentum, $m_{\text{D}}v_i=(m_{\text{D}}+m_{\text{T}})v_f$, or $v_f=\left(\dfrac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)v_i=\boxed{\frac{2}{5}v_i}$

(d) $K_i+U_i=K_f+U_f$: $\qquad K_i+0=\frac{1}{2}(m_{\text{D}}+m_{\text{T}})v_f^2+U_f=\frac{1}{2}(m_{\text{D}}+m_{\text{T}})\left(\dfrac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)^2v_i^2+U_f$

$$K_i+0=\left(\frac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)\left(\tfrac{1}{2}m_{\text{D}}v_i^2\right)+U_f=\left(\frac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)K_i+U_f$$

$\left(1-\dfrac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)K_i=U_f$: $\qquad K_i=U_f\left(\dfrac{m_{\text{D}}+m_{\text{T}}}{m_{\text{T}}}\right)=\frac{5}{3}(444\text{ keV})=\boxed{740\text{ keV}}$

(e) $\boxed{\text{Possibly by tunneling.}}$

45.15 (a) Average KE per particle is $\frac{3}{2}k_BT = \frac{1}{2}mv^2$.

Therefore, $v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\text{ J/K})(4.00\times10^{8}\text{ K})}{2(1.67\times10^{-27}\text{ kg})}} = \boxed{2.23\times10^{6}\text{ m/s}}$

(b) $t = \frac{x}{v} \sim \frac{0.1\text{ m}}{10^{6}\text{ m/s}}\ \boxed{\sim 10^{-7}\text{ s}}$

45.16 (a) $V = (317\times10^{6}\text{ mi}^3)\left(\frac{1609\text{ m}}{1\text{ mi}}\right)^3 = 1.32\times10^{18}\text{ m}^3$

$$m_{\text{water}} = \rho V = (10^{3}\text{ kg/m}^3)(1.32\times10^{18}\text{ m}^3) = 1.32\times10^{21}\text{ kg}$$

$$m_{H_2} = \left(\frac{M_{H_2}}{M_{H_2O}}\right)m_{H_2O} = \left(\frac{2.016}{18.015}\right)(1.32\times10^{21}\text{ kg}) = 1.48\times10^{20}\text{ kg}$$

$$m_{\text{Deuterium}} = (0.0300\%)m_{H_2} = (0.0300\times10^{-2})(1.48\times10^{20}\text{ kg}) = 4.43\times10^{16}\text{ kg}$$

The number of deuterium nuclei in this mass is

$$N = \frac{m_{\text{Deuterium}}}{m_{\text{Deuteron}}} = \frac{4.43\times10^{16}\text{ kg}}{(2.014\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 1.33\times10^{43}$$

Since two deuterium nuclei are used per fusion, ${}^2_1\text{H} + {}^2_1\text{H} \to {}^4_2\text{He} + Q$, the number of events is $N/2 = 6.63\times10^{42}$.

The energy released per event is

$$Q = \left[M_{^2\text{H}} + M_{^2\text{H}} - M_{^4\text{He}}\right]c^2 = [2(2.014\,102) - 4.002\,602]\text{u}\,(931.5\text{ MeV/u}) = 23.8\text{ MeV}$$

The total energy available is then

$$E = \left(\frac{N}{2}\right)Q = (6.63\times10^{42})(23.8\text{ MeV})\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) = \boxed{2.52\times10^{31}\text{ J}}$$

(b) The time this energy could possibly meet world requirements is

$$t = \frac{E}{\mathsf{P}} = \frac{2.52\times10^{31}\text{ J}}{100(7.00\times10^{12}\text{ J/s})} = (3.61\times10^{16}\text{ s})\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right) = \boxed{1.14\times10^{9}\text{ yr}}\ \sim 1\text{ billion years.}$$

45.17 (a) Including both ions and electrons, the number of particles in the plasma is $N = 2nV$ where n is the ion density and V is the volume of the container. Application of Equation 21.6 gives the total energy as

$$E = \frac{3}{2}Nk_BT = 3nVk_BT = 3\left(2.0\times10^{13}\ \text{cm}^{-3}\right)\left[\left(50\ \text{m}^3\right)\left(\frac{10^6\ \text{cm}^3}{1\ \text{m}^3}\right)\right]\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.0\times10^8\ \text{K}\right)$$

$$E = \boxed{1.7\times10^7\ \text{J}}$$

(b) From Table 20.2, the heat of vaporization of water is $L_v = 2.26\times10^6$ J/kg. The mass of water that could be boiled away is

$$m = \frac{E}{L_v} = \frac{1.7\times10^7\ \text{J}}{2.26\times10^6\ \text{J/kg}} = \boxed{7.3\ \text{kg}}$$

45.18 (a) Lawson's criterion for the D-T reaction is $n\tau \ge 10^{14}\ \text{s/cm}^3$. For a confinement time of $\tau = 1.00$ s, this requires a minimum ion density of $n = \boxed{10^{14}\ \text{cm}^{-3}}$

(b) At the ignition temperature of $T = 4.5\times10^7$ K and the ion density found above, the plasma pressure is

$$P = 2nk_BT = 2\left[\left(10^{14}\ \text{cm}^{-3}\right)\left(\frac{10^6\text{cm}^3}{1\ \text{m}^3}\right)\right]\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.5\times10^7\ \text{K}\right) = \boxed{1.24\times10^5\ \text{J/m}^3}$$

(c) The required magnetic energy density is then

$$u_B = \frac{B^2}{2\mu_0} \ge 10P = 10\left(1.24\times10^5\ \text{J/m}^3\right) = 1.24\times10^6\ \text{J/m}^3,$$

$$B \ge \sqrt{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(1.24\times10^6\ \text{J/m}^3\right)} = \boxed{1.77\ \text{T}}$$

45.19 Let the number of ^{6}Li atoms, each having mass 6.015 u, be N_6 while the number of ^{7}Li atoms, each with mass 7.016 u, is N_7.

Then, $N_6 = 7.50\%$ of $N_{\text{total}} = 0.0750\left(N_6 + N_7\right)$, or $N_7 = \left(\frac{0.925}{0.0750}\right)N_6$

Also, total mass $= \left[N_6(6.015\ \text{u}) + N_7(7.016\ \text{u})\right]\left(1.66\times10^{-27}\ \text{kg/u}\right) = 2.00$ kg,

or $$N_6\left[(6.015\ \text{u}) + \left(\frac{0.925}{0.0750}\right)(7.016\ \text{u})\right]\left(1.66\times10^{-27}\ \text{kg/u}\right) = 2.00\ \text{kg}.$$

This yields $N_6 = \boxed{1.30\times10^{25}}$ as the number of ^{6}Li atoms and

$$N_7 = \left(\frac{0.925}{0.0750}\right)\left(1.30\times10^{25}\right) = \boxed{1.61\times10^{26}}$$ as the number of ^{7}Li atoms.

45.20 The number of nuclei in 1.00 metric ton of trash is

$$N = 1000\text{ kg}(1000\text{ g / kg})\left(6.02\times10^{23}\text{ nuclei / mol}\right)\Big/(56.0\text{ g / mol}) = 1.08\times10^{28}\text{ nuclei}$$

At an average charge of 26.0 e / nucleus, $q = \left(1.08\times10^{28}\right)(26.0)\left(1.60\times10^{-19}\right) = 4.47\times10^{10}\text{ C}$

Therefore $t = \dfrac{q}{I} = \dfrac{4.47\times10^{10}}{1.00\times10^{6}} = 4.47\times10^{4}\text{ s} = \boxed{12.4\text{ h}}$

45.21 $$N_0 = \frac{\text{mass present}}{\text{mass of nucleus}} = \frac{5.00\text{ kg}}{(89.9077\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right)} = 3.35\times10^{25}\text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{29.1\text{ yr}} = 2.38\times10^{-2}\text{ yr}^{-1} = 4.52\times10^{-8}\text{ min}^{-1}$$

$$R_0 = \lambda N_0 = \left(4.52\times10^{-8}\text{ min}^{-1}\right)\left(3.35\times10^{25}\right) = 1.52\times10^{18}\text{ counts/min}$$

$$\frac{R}{R_0} = e^{-\lambda t} = \frac{10.0\text{ counts/min}}{1.52\times10^{18}\text{ counts/min}} = 6.60\times10^{-18} \quad \text{and} \quad \lambda t = -\ln\left(6.60\times10^{-18}\right) = 39.6$$

giving $t = \dfrac{39.6}{\lambda} = \dfrac{39.6}{2.38\times10^{-2}\text{ yr}^{-1}} = \boxed{1.66\times10^{3}\text{ yr}}$

45.22 Source: 100 mrad of 2-MeV γ-rays / h at a 1.00-m distance.

(a) For γ-rays, dose in rem = dose in rad.

Thus a person would have to stand $\boxed{\text{10.0 hours}}$ to receive 1.00 rem from a 100-mrad / h source.

(b) If the γ-radiation is emitted isotropically, the dosage rate falls off as $1/r^2$.

Thus a dosage 10.0 mrad / h would be received at a distance $r = \sqrt{10.0}\text{ m} = \boxed{3.16\text{ m}}$.

45.23 (a) The number of x-rays taken per year is

$$n = (8\text{ x-ray/d})(5\text{ d/wk})(50\text{ wk/yr}) = 2.0\times10^{3}\text{ x-ray/yr}$$

The average dose per photograph is $\dfrac{5.0\text{ rem/yr}}{2.0\times10^{3}\text{ x-ray/yr}} = \boxed{2.5\times10^{-3}\text{ rem/x-ray}}$

(b) The technician receives low-level background radiation at a rate of 0.13 rem/yr. The dose of 5.0 rem/yr received as a result of the job is

$$\frac{5.0 \text{ rem/yr}}{0.13 \text{ rem/yr}} = \boxed{38 \text{ times background levels}}$$

45.24 (a) $I = I_0 e^{-\mu x}$, so $$x = \frac{1}{\mu}\ln\left(\frac{I_0}{I}\right)$$

With $\mu = 1.59\ \text{cm}^{-1}$, the thickness when $I = I_0/2$ is $$x = \frac{1}{1.59\ \text{cm}^{-1}}\ln(2) = \boxed{0.436\ \text{cm}}$$

(b) When $\frac{I_0}{I} = 1.00\times 10^4$, $$x = \frac{1}{1.59\ \text{cm}^{-1}}\ln\left(1.00\times 10^4\right) = \boxed{5.79\ \text{cm}}$$

45.25 $1\ \text{rad} = 10^{-2}\ \text{J / kg}$ $\quad Q = mc\Delta T\quad \mathcal{P}t = mc\Delta T$

$$t = \frac{mc\Delta T}{\mathcal{P}} = \frac{m(4186\ \text{J / kg}\cdot{}^\circ\text{C})(50.0\ {}^\circ\text{C})}{(10)\left(10^{-2}\ \text{J / kg}\cdot\text{s}\right)(m)} = \boxed{2.09\times 10^6\ \text{s}}\ \approx 24\ \text{days!}$$

Note that power is the product of dose rate and mass.

45.26 $$\frac{Q}{m} = \frac{\text{absorbed energy}}{\text{unit mass}} = (1000\ \text{rad})\frac{10^{-2}\ \text{J / kg}}{1\ \text{rad}} = 10.0\ \text{J / kg}$$

The rise in body temperature is calculated from $Q = mc\,\Delta T$ where $c = 4186\ \text{J/kg}\cdot{}^\circ\text{C}$ for water and the human body

$$\Delta T = \frac{Q}{mc} = (10.0\ \text{J / kg})\frac{1}{4186\ \text{J / kg}\cdot{}^\circ\text{C}} = \boxed{2.39\times 10^{-3}\ {}^\circ\text{C}}\quad \text{(Negligible)}$$

45.27 If half of the 0.140-MeV gamma rays are absorbed by the patient, the total energy absorbed is

$$E = \frac{(0.140\ \text{MeV})}{2}\left[\left(\frac{1.00\times 10^{-8}\ \text{g}}{98.9\ \text{g/mol}}\right)\left(\frac{6.02\times 10^{23}\ \text{nuclei}}{1\ \text{mol}}\right)\right] = 4.26\times 10^{12}\ \text{MeV}$$

$$E = \left(4.26\times 10^{12}\ \text{MeV}\right)\left(1.60\times 10^{-13}\ \text{J/MeV}\right) = 0.682\ \text{J}$$

Thus, the dose received is $$\text{Dose} = \frac{0.682\ \text{J}}{60.0\ \text{kg}}\left(\frac{1\ \text{rad}}{10^{-2}\ \text{J/kg}}\right) = \boxed{1.14\ \text{rad}}$$

45.28 The nuclei initially absorbed are $N_0 = \left(1.00\times10^{-9}\text{ g}\right)\left(\dfrac{6.02\times10^{23}\text{ nuclei/mol}}{89.9\text{ g/mol}}\right) = 6.70\times10^{12}$

The number of decays in time t is $\Delta N = N_0 - N = N_0\left(1-e^{-\lambda t}\right) = N_0\left(1-e^{-(\ln 2)t/T_{1/2}}\right)$

At the end of 1 year, $\dfrac{t}{T_{1/2}} = \dfrac{1.00\text{ yr}}{29.1\text{ yr}} = 0.0344$

and $\Delta N = N_0 - N = \left(6.70\times10^{12}\right)\left(1-e^{-0.0238}\right) = 1.58\times10^{11}$

The energy deposited is $E = \left(1.58\times10^{11}\right)(1.10\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right) = 0.0277\text{ J}$

Thus, the dose received is $\text{Dose} = \left(\dfrac{0.0277\text{ J}}{70.0\text{ kg}}\right) = \boxed{3.96\times10^{-4}\text{ J/kg}} = 0.0396\text{ rad}$

45.29 (a) $\dfrac{E}{E_\beta} = \dfrac{\frac{1}{2}C(\Delta V)^2}{0.500\text{ MeV}} = \dfrac{\frac{1}{2}\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)^2}{(0.500\text{ MeV})\left(1.60\times10^{-13}\text{ J / MeV}\right)} = \boxed{3.12\times10^{7}}$

(b) $N = \dfrac{Q}{e} = \dfrac{C(\Delta V)}{e} = \dfrac{\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)}{1.60\times10^{-19}\text{ C}} = \boxed{3.12\times10^{10}\text{ electrons}}$

45.30 (a) $\text{amplification} = \dfrac{\text{energy discharged}}{E} = \dfrac{\frac{1}{2}C(\Delta V)^2}{E} = \boxed{\dfrac{C(\Delta V)^2}{2E}}$

(b) $N = \dfrac{\text{charge released}}{\text{charge of electron}} = \boxed{\dfrac{C(\Delta V)}{e}}$

45.31 (a) $E_I = 10.0\text{ eV}$ is the energy required to liberate an electron from a dynode. Let n_i be the number of electrons incident upon a dynode, each having gained energy $e(\Delta V)$ as it was accelerated to this dynode. The number of electrons that will be freed from this dynode is $N_i = n_i e(\Delta V)/E_I$:

At the first dynode, $n_i = 1$ and $N_1 = \dfrac{(1)e(100\text{ V})}{10.0\text{ eV}} = \boxed{10^{1}\text{ electrons}}$

(b) For the second dynode, $n_i = N_1 = 10^1$, so $N_2 = \dfrac{(10^1)e(100\text{ V})}{10.0\text{ eV}} = 10^2$.

At the third dynode, $n_i = N_2 = 10^2$ and $N_3 = \dfrac{(10^2)e(100\text{ V})}{10.0\text{ eV}} = 10^3$.

Observing the developing pattern, we see that the number of electrons incident on the seventh and last dynode is $n_7 = N_6 = \boxed{10^6}$.

(c) The number of electrons incident on the last dynode is $n_7 = 10^6$. The total energy these electrons deliver to that dynode is given by

$$E = n_i e(\Delta V) = 10^6\, e(700\text{ V} - 600\text{ V}) = \boxed{10^8\text{ eV}}$$

***45.32** (a) The average time between slams is $60\text{ min}/38 = 1.6\text{ min}$. Sometimes, the actual interval is nearly zero. Perhaps about equally as often, it is $2\times 1.6\text{ min}$. Perhaps about half as often, it is $4\times 1.6\text{ min}$. Somewhere around $5\times 1.6\text{ min} = \boxed{8.0\text{ min}}$, the chances of randomness producing so long a wait get slim, so such a long wait might likely be due to mischief.

(b) The midpoints of the time intervals are separated by 5.00 minutes. We use $R = R_0 e^{-\lambda t}$. Subtracting the background counts,

$$337 - 5(15) = [372 - 5(15)]e^{-(\ln 2/T_{1/2})(5.00\text{ min})}$$

or $$\ln\left(\frac{262}{297}\right) = \ln(0.882) = -3.47\text{ min}/T_{1/2}$$ which yields $T_{1/2} = \boxed{27.6\text{ min}}$.

(c) As in the random events in part (a), we imagine a ± 5 count counting uncertainty. The smallest likely value for the half-life is then given by

$$\ln\left(\frac{262-5}{297+5}\right) = -3.47\text{ min}/T_{1/2},\ \text{or}\ \left(T_{1/2}\right)_{\min} = 21.1\text{ min}$$

The largest credible value is found from

$$\ln\left(\frac{262+5}{297-5}\right) = -3.47\text{ min}/T_{1/2},\ \text{yielding}\ \left(T_{1/2}\right)_{\max} = 38.8\text{ min}$$

Thus, $T_{1/2} = \left(\dfrac{38.8+21.1}{2}\right) \pm \left(\dfrac{38.8-21.1}{2}\right)\text{ min} = (30\pm 9)\text{ min} = \boxed{30\text{ min} \pm 30\%}$

45.33 The initial specific activity of ^{59}Fe in the steel,

$$(R/m)_0 = \frac{20.0\ \mu\text{Ci}}{0.200\ \text{kg}} = \frac{100\ \mu\text{Ci}}{\text{kg}}\left(\frac{3.70\times 10^4\ \text{Bq}}{1\ \mu\text{Ci}}\right) = 3.70\times 10^6\ \text{Bq/kg}$$

After 1000 h, $\frac{R}{m} = (R/m)_0 e^{-\lambda t} = (3.70\times 10^6\ \text{Bq/kg})e^{-(6.40\times 10^{-4}\ \text{h}^{-1})(1000\ \text{h})} = 1.95\times 10^6\ \text{Bq/kg}$

The activity of the oil, $R_{\text{oil}} = \left(\frac{800}{60.0}\ \text{Bq/liter}\right)(6.50\ \text{liters}) = 86.7\ \text{Bq}$

Therefore, $m_{\text{in oil}} = \frac{R_{\text{oil}}}{(R/m)} = \frac{86.7\ \text{Bq}}{1.95\times 10^6\ \text{Bq/kg}} = 4.45\times 10^{-5}\ \text{kg}$

So that wear rate is $\frac{4.45\times 10^{-5}\ \text{kg}}{1000\ \text{h}} = \boxed{4.45\times 10^{-8}\ \text{kg/h}}$

***45.34** The half-life of ^{14}O is 70.6 s, so the decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{70.6\ \text{s}} = 0.009\ 82\ \text{s}^{-1}$

The ^{14}O nuclei remaining after five min is $N = N_0 e^{-\lambda t} = \left(10^{10}\right)e^{-\left(0.009\ 82\ \text{s}^{-1}\right)(300\ \text{s})} = 5.26\times 10^8$

The number of these in one cubic centimeter of blood is

$$N' = N\left(\frac{1.00\ \text{cm}^3}{\text{total vol. of blood}}\right) = \left(5.26\times 10^8\right)\left(\frac{1.00\ \text{cm}^3}{2000\ \text{cm}^3}\right) = 2.63\times 10^5$$

and their activity is $R = \lambda N' = \left(0.009\ 82\ \text{s}^{-1}\right)\left(2.63\times 10^5\right) = 2.58\times 10^3\ \text{Bq}$ $\boxed{\sim 10^3\ \text{Bq}}$

***45.35** (a) The number of photons is $10^4\ \text{MeV}/1.04\ \text{MeV} = 9.62\times 10^3$. Since only 50% of the photons are detected, the number of ^{65}Cu nuclei decaying is twice this value, or 1.92×10^4. In two half-lives, three-fourths of the original nuclei decay, so $\frac{3}{4}N_0 = 1.92\times 10^4$ and $N_0 = 2.56\times 10^4$. This is 1% of the ^{65}Cu, so the number of ^{65}Cu is 2.56×10^6 $\boxed{\sim 10^6}$.

(b) Natural copper is 69.17% ^{63}Cu and 30.83% ^{65}Cu. Thus, if the sample contains N_{Cu} copper atoms, the number of atoms of each isotope is
$N_{63} = 0.6917 N_{\text{Cu}}$ and $N_{65} = 0.3083 N_{\text{Cu}}$.

Therefore, $\frac{N_{63}}{N_{65}} = \frac{0.6917}{0.3083}$ or $N_{63} = \left(\frac{0.6917}{0.3083}\right)N_{65} = \left(\frac{0.6917}{0.3083}\right)\left(2.56\times 10^6\right) = 5.75\times 10^6$

The total mass of copper present is then $m_{\text{Cu}} = (62.93\ \text{u})N_{63} + (64.93\ \text{u})N_{65}$:

$$m_{\text{Cu}} = \left[(62.93)\left(5.75\times 10^6\right) + (64.93)\left(2.56\times 10^6\right)\right]\text{u}\left(1.66\times 10^{-24}\ \text{g/u}\right) = 8.77\times 10^{-16}\ \text{g}\ \boxed{\sim 10^{-15}\ \text{g}}$$

45.36 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production – decay)

$$\frac{dN}{dt} = R - \lambda N \qquad \text{so} \qquad dN = (R - \lambda N)dt$$

The variables are separable. $\displaystyle\int_{N=0}^{N} \frac{dN}{R - \lambda N} = \int_{t=0}^{t} dt$

$$-\frac{1}{\lambda}\ln\left(\frac{R - \lambda N}{R}\right) = t \qquad \text{so} \qquad \ln\left(\frac{R - \lambda N}{R}\right) = -\lambda t$$

$$\left(\frac{R - \lambda N}{R}\right) = e^{-\lambda t} \qquad \text{and} \qquad 1 - \frac{\lambda}{R}N = e^{-\lambda t}$$

Therefore, $N = \boxed{\dfrac{R}{\lambda}\left(1 - e^{-\lambda t}\right)}$

(b) The maximum number of radioactive nuclei would be $\boxed{R / \lambda}$

45.37 (a) At 6×10^8 K, each carbon nucleus has thermal energy of

$$\tfrac{3}{2}k_B T = (1.5)(8.62 \times 10^{-5}\text{ eV / K})(6 \times 10^8\text{ K}) = \boxed{8 \times 10^4\text{ eV}}$$

(b) The energy released is $E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Ne}^{20}\right) - m\left(\text{He}^4\right)\right]c^2$

$$E = (24.000\,000 - 19.992\,435 - 4.002\,602)(931.5)\text{ MeV} = \boxed{4.62\text{ MeV}}$$

In the second reaction, $E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Mg}^{24}\right)\right](931.5)\text{MeV / u}$

$$E = (24.000\,000 - 23.985\,042)(931.5)\text{MeV} = \boxed{13.9\text{ MeV}}$$

(c) The energy released is the energy of reaction of the # of carbon nuclei in a 2.00-kg sample, which corresponds to

$$\Delta E = \left(2.00 \times 10^3\text{ g}\right)\left(\frac{6.02 \times 10^{23}\text{ atoms / mol}}{12.0\text{ g / mol}}\right)\left(\frac{4.62\text{ MeV / fusion event}}{2\text{ nuclei / fusion event}}\right)\left(\frac{1\text{ kWh}}{2.25 \times 10^{19}\text{ MeV}}\right)$$

$$\Delta E = \frac{\left(1.00 \times 10^{26}\right)(4.62)}{2\left(2.25 \times 10^{19}\right)}\text{ kWh} = \boxed{1.03 \times 10^7\text{ kWh}}$$

45.38 (a) Suppose each ^{235}U fission releases 208 MeV of energy. Then, the number of nuclei that must have undergone fission is

$$N = \frac{\text{total release}}{\text{energy per nuclei}} = \frac{5\times 10^{13}\ \text{J}}{(208\ \text{MeV})\left(1.60\times 10^{-13}\ \text{J/MeV}\right)} = \boxed{1.5\times 10^{24}\ \text{nuclei}}$$

(b) $$\text{mass} = \left(\frac{1.5\times 10^{24}\ \text{nuclei}}{6.02\times 10^{23}\ \text{nuclei/mol}}\right)(235\ \text{g/mol}) \approx \boxed{0.6\ \text{kg}}$$

45.39 For a typical ^{235}U, $Q = 208$ MeV; and the initial mass is 235 u. Thus, the fractional energy loss is

$$\frac{Q}{mc^2} = \frac{208\ \text{MeV}}{(235\ \text{u})(931.5\ \text{MeV/u})} = 9.50\times 10^{-4} = \boxed{0.0950\%}$$

For the D-T fusion reaction, $Q = 17.6$ MeV

The initial mass is $m = (2.014\ \text{u}) + (3.016\ \text{u}) = 5.03\ \text{u}$

The fractional loss in this reaction is $\dfrac{Q}{mc^2} = \dfrac{17.6\ \text{MeV}}{(5.03\ \text{u})(931.5\ \text{MeV/u})} = 3.75\times 10^{-3} = \boxed{0.375\%}$

$$\frac{0.375\%}{0.0950\%} = 3.95 \quad \text{or} \quad \boxed{\text{the fractional loss in D} - \text{T fusion is about 4 times that in } ^{235}\text{U fission}}$$

.

45.40 To conserve momentum, the two fragments must move in opposite directions with speeds v_1 and v_2 such that

$$m_1 v_1 = m_2 v_2 \quad \text{or} \quad v_2 = \left(\frac{m_1}{m_2}\right) v_1$$

The kinetic energies after the break-up are then

$$K_1 = \tfrac{1}{2} m_1 v_1^2 \quad \text{and} \quad K_2 = \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_2 \left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right) K_1$$

The fraction of the total kinetic energy carried off by m_1 is $\dfrac{K_1}{K_1 + K_2} = \dfrac{K_1}{K_1 + (m_1/m_2)K_1} = \boxed{\dfrac{m_2}{m_1 + m_2}}$

and the fraction carried off by m_2 is $1 - \dfrac{m_2}{m_1 + m_2} = \boxed{\dfrac{m_1}{m_1 + m_2}}$

45.41 The decay constant is $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{(12.3\text{ yr})(3.16\times10^{7}\text{ s/yr})} = 1.78\times10^{-9}\text{ s}^{-1}$

The tritium in the plasma decays at a rate of

$$R = \lambda N = \left(1.78\times10^{-9}\text{ s}^{-1}\right)\left[\left(\frac{2.00\times10^{14}}{\text{cm}^3}\right)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right)\left(50.0\text{ m}^3\right)\right]$$

$$R = 1.78\times10^{13}\text{ Bq} = \left(1.78\times10^{13}\text{ Bq}\right)\left(\frac{1\text{ Ci}}{3.70\times10^{10}\text{ Bq}}\right) = \boxed{482\text{ Ci}}$$

$\boxed{\text{The fission inventory is } \dfrac{4\times10^{10}\text{ Ci}}{482\text{ Ci}} \sim 10^8 \text{ times greater}}$ than this amount.

Goal Solution

The half-life of tritium is 12.3 yr. If the TFTR fusion reactor contained 50.0 m^3 of tritium at a density equal to 2.00×10^{14} ions / cm^3, how many curies of tritium were in the plasma? Compare this value with a fission inventory (the estimated supply of fissionable material) of 4×10^{10} Ci.

G: It is difficult to estimate the activity of the tritium in the fusion reactor without actually calculating it; however, we might expect it to be a small fraction of the fission (not fusion) inventory.

O: The decay rate (activity) can be found by multiplying the decay constant λ by the number of ${}^3_1\text{H}$ particles. The decay constant can be found from the half-life of tritium, and the number of particles from the density and volume of the plasma.

A: The number of Hydrogen-3 nuclei is

$$N = \left(50.0\text{ m}^3\right)\left(2.00\times10^{14}\ \frac{\text{particles}}{\text{m}^3}\right)\left(100\ \frac{\text{cm}}{\text{m}}\right)^3 = 1.00\times10^{22}\text{ particles}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{12.3\text{ yr}}\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right) = 1.78\times10^{-9}\text{ s}^{-1}$$

The activity is then

$$R = \lambda N = \left(1.78\times10^{-9}\text{ s}^{-1}\right)\left(1.00\times10^{22}\text{ nuclei}\right) = 1.78\times10^{13}\text{ Bq} = \left(1.78\times10^{13}\text{ Bq}\right)\left(\frac{1\text{ Ci}}{3.70\times10^{10}\text{ Bq}}\right) = 482\text{ Ci}$$

L: Even though 482 Ci is a large amount of radioactivity, it is smaller than 4.00×10^{10} Ci by about a hundred million. Therefore, loss of containment is a smaller hazard for a fusion power reactor than for a fission reactor.

45.42 Momentum conservation: $0 = m_{\text{Li}}\mathbf{v}_{\text{Li}} + m_\alpha \mathbf{v}_\alpha$, or, $m_{\text{Li}} v_{\text{Li}} = m_\alpha v_\alpha$

Thus, $$K_{\text{Li}} = \frac{1}{2} m_{\text{Li}} v_{\text{Li}}^2 = \frac{1}{2}\frac{\left(m_{\text{Li}} v_{\text{Li}}\right)^2}{m_{\text{Li}}} = \frac{\left(m_\alpha v_\alpha\right)^2}{2m_{\text{Li}}} = \left(\frac{m_\alpha^2}{2m_{\text{Li}}}\right) v_\alpha^2$$

$$K_{\text{Li}} = \left(\frac{(4.002\,6 \text{ u})^2}{2(7.016\,9 \text{ u})}\right)\left(9.30\times 10^6 \text{ m/s}\right)^2 = (1.14 \text{ u})\left(9.30\times 10^6 \text{ m/s}\right)^2$$

$$K_{\text{Li}} = 1.14\left(1.66\times 10^{-27} \text{ kg}\right)\left(9.30\times 10^6 \text{ m/s}\right)^2 = 1.64\times 10^{-13} \text{ J} = \boxed{1.02 \text{ MeV}}$$

45.43 The complete fissioning of 1.00 gram of U^{235} releases

$$\Delta Q = \frac{(1.00 \text{ g})}{235 \text{ grams / mol}}\left(6.02\times 10^{23} \ \frac{\text{atoms}}{\text{mol}}\right)\left(200 \ \frac{\text{MeV}}{\text{fission}}\right)\left(1.60\times 10^{-13} \ \frac{\text{J}}{\text{MeV}}\right) = 8.20\times 10^{10} \text{ J}$$

If all this energy could be utilized to convert m kilograms of 20.0°C water to 400°C steam (see Chapter 20 of text for values), then

$$\Delta Q = mc_w\,\Delta T + mL_v + mc_s\,\Delta T$$

$$\Delta Q = m\left[(4186 \text{ J / kg}\cdot{}^\circ\text{C})(80.0\ {}^\circ\text{C}) + 2.26\times 10^6 \text{ J / kg} + (2010 \text{ J / kg}\cdot{}^\circ\text{C})(300\ {}^\circ\text{C})\right]$$

Therefore $$m = \frac{8.20\times 10^{10} \text{ J}}{3.20\times 10^6 \text{ J / kg}} = \boxed{2.56\times 10^4 \text{ kg}}$$

45.44 When mass m of ^{235}U undergoes complete fission, releasing 200 MeV per fission event, the total energy released is:

$$Q = \left(\frac{m}{235 \text{ g/mol}}\right) N_A (200 \text{ MeV})$$ where N_A is Avogadro's number.

If all this energy could be utilized to convert a mass m_w of liquid water at T_c into steam at T_h, then,

$$Q = m_w\left[c_w\left(100^\circ\text{C} - T_c\right) + L_v + c_s\left(T_h - 100^\circ\text{C}\right)\right]$$

where c_w is the specific heat of liquid water, L_v is the latent heat of vaporization, and c_s is the specific heat of steam. Solving for the mass of water converted gives

$$m_w = \frac{Q}{\left[c_w\left(100^\circ\text{C} - T_c\right) + L_v + c_s\left(T_h - 100^\circ\text{C}\right)\right]} = \boxed{\frac{m N_A (200 \text{ MeV})}{(235 \text{ g/mol})\left[c_w\left(100^\circ\text{C} - T_c\right) + L_v + c_s\left(T_h - 100^\circ\text{C}\right)\right]}}$$

45.45 (a) The number of molecules in 1.00 liter of water (mass = 1000 g) is

$$N = \left(\frac{1.00 \times 10^3 \text{ g}}{18.0 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \text{ molecules/mol}\right) = 3.34 \times 10^{25} \text{ molecules}$$

The number of deuterium nuclei contained in these molecules is

$$N' = \left(3.34 \times 10^{25} \text{ molecules}\right)\left(\frac{1 \text{ deuteron}}{3300 \text{ molecules}}\right) = 1.01 \times 10^{22} \text{ deuterons}$$

Since 2 deuterons are consumed per fusion event, the number of events possible is $N'/2 = 5.07 \times 10^{21}$ reactions, and the energy released is

$$E_{\text{fusion}} = \left(5.07 \times 10^{21} \text{ reactions}\right)\left(3.27 \text{ MeV/reaction}\right) = 1.66 \times 10^{22} \text{ MeV}$$

$$E_{\text{fusion}} = \left(1.66 \times 10^{22} \text{ MeV}\right)\left(1.60 \times 10^{-13} \text{ J/MeV}\right) = \boxed{2.65 \times 10^{9} \text{ J}}$$

(b) In comparison to burning 1.00 liter of gasoline, the energy from the fusion of deuterium is

$$\frac{E_{\text{fusion}}}{E_{\text{gasoline}}} = \frac{2.65 \times 10^9 \text{ J}}{3.40 \times 10^7 \text{ J}} = \boxed{78.0 \text{ times larger}}.$$

45.46 The number of nuclei in 0.155 kg of ^{210}Po is

$$N_0 = \left(\frac{155 \text{ g}}{209.98 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \text{ nuclei/g}\right) = 4.44 \times 10^{23} \text{ nuclei}$$

The half-life of ^{210}Po is 138.38 days, so the decay constant is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(138.38 \text{ d})\left(8.64 \times 10^4 \text{ s/d}\right)} = 5.80 \times 10^{-8} \text{ s}^{-1}$$

The initial activity is $R_0 = \lambda N_0 = \left(5.80 \times 10^{-8} \text{ s}^{-1}\right)\left(4.44 \times 10^{23} \text{ nuclei}\right) = 2.58 \times 10^{16} \text{ Bq}$

The energy released in each $^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$ reaction is $Q = \left[M_{^{210}_{84}\text{Po}} - M_{^{206}_{82}\text{Pb}} - M_{^{4}_{2}\text{He}}\right]c^2$:

$$Q = [209.982\,848 - 205.974\,440 - 4.002\,602]\text{u}\left(931.5 \frac{\text{MeV}}{\text{u}}\right) = 5.41 \text{ MeV}$$

Thus, assuming a conversion efficiency of 1.00%, the initial power output of the battery is

$$\mathsf{P} = (0.0100)R_0 Q = (0.0100)\left(2.58 \times 10^{16} \frac{\text{decays}}{\text{s}}\right)\left(5.41 \frac{\text{MeV}}{\text{decay}}\right)\left(1.60 \times 10^{-13} \frac{\text{J}}{\text{MeV}}\right) = \boxed{223 \text{ W}}$$

45.47 (a) The thermal power transferred to the water is $\mathcal{P}_w = 0.970(\text{waste heat})$

$$\mathcal{P}_w = 0.970(3065 - 1000)\,\text{MW} = 2.00 \times 10^9\ \text{J/s}$$

r_w is the mass of heated per hour: $r_w = \frac{\mathcal{P}_w}{c(\Delta T)} = \frac{(2.00 \times 10^9\ \text{J/s})(3600\ \text{s/h})}{(4186\ \text{J/kg}\cdot{}^\circ\text{C})(3.50\ {}^\circ\text{C})} = \boxed{4.91 \times 10^8\ \text{kg/h}}$

The volume used per hour is $\frac{4.91 \times 10^8\ \text{kg/h}}{1.00 \times 10^3\ \text{kg/m}^3} = \boxed{4.91 \times 10^5\ \text{m}^3/\text{h}}$

(b) The ^{235}U fuel is consumed at a rate $r_f = \left(\frac{3065 \times 10^6\ \text{J/s}}{7.80 \times 10^{10}\ \text{J/g}}\right)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = \boxed{0.141\ \text{kg/h}}$

***45.48** (a) $\Delta V = 4\pi r^2(\Delta r) = 4\pi(14.0 \times 10^3\ \text{m})^2(0.05\ \text{m}) = 1.23 \times 10^8\ \text{m}^3$ $\boxed{\sim 10^8\ \text{m}^3}$

(b) The force on the next layer is determined by atmospheric pressure.

$$W = P(\Delta V) = \left(1.013 \times 10^5\ \frac{\text{N}}{\text{m}^2}\right)(1.23 \times 10^8\ \text{m}^3) = 1.25 \times 10^{13}\ \text{J} \quad \boxed{\sim 10^{13}\ \text{J}}$$

(c) $1.25 \times 10^{13}\ \text{J} = \frac{1}{10}(\text{yield})$, so $\text{yield} = 1.25 \times 10^{14}\ \text{J}$ $\boxed{\sim 10^{14}\ \text{J}}$

(d) $\frac{1.25 \times 10^{14}\ \text{J}}{4.2 \times 10^9\ \text{J/ton TNT}} = 2.97 \times 10^4\ \text{ton TNT} \sim 10^4\ \text{ton TNT}$ or $\boxed{\sim 10\ \text{kilotons}}$

***45.49** (a) $V = \ell^3 = \frac{m}{\rho}$, so $\ell = \left(\frac{m}{\rho}\right)^{1/3} = \left(\frac{70.0\ \text{kg}}{18.7 \times 10^3\ \text{kg/m}^3}\right)^{1/3} = \boxed{0.155\ \text{m}}$

(b) Add 92 electrons to both sides of the given nuclear reaction. Then it becomes ${}^{238}_{92}\text{U atom} \to 8\,{}^{4}_{2}\text{He atom} + {}^{206}_{82}\text{Pb atom} + Q_{\text{net}}$.

$$Q_{\text{net}} = \left[M_{{}^{238}_{92}\text{U}} - 8M_{{}^{4}_{2}\text{He}} - M_{{}^{206}_{82}\text{Pb}}\right]c^2 = [238.050\,784 - 8(4.002\,602) - 205.974\,440]\,\text{u}\ (931.5\ \text{MeV/u})$$

$$Q_{\text{net}} = \boxed{51.7\ \text{MeV}}$$

(c) If there is a single step of decay, the number of decays per time is the decay rate R and the energy released in each decay is Q. Then the energy released per time is $\boxed{\mathcal{P} = QR}$. If there is a series of decays in steady state, the equation is still true, with Q representing the net decay energy.

(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N = \left(\frac{7.00 \times 10^4 \text{ g}}{238 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \text{ nuclei/mol}\right) = 1.77 \times 10^{26} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47 \times 10^9 \text{ yr}} = 1.55 \times 10^{-10} \ \frac{1}{\text{yr}}$$

$$R = \lambda N = \left(1.55 \times 10^{-10} \ \frac{1}{\text{yr}}\right)\left(1.77 \times 10^{26} \text{ nuclei}\right) = 2.75 \times 10^{16} \text{ decays/yr},$$

so $$\mathcal{P} = QR = (51.7 \text{ MeV})\left(2.75 \times 10^{16} \ \frac{1}{\text{yr}}\right)\left(1.60 \times 10^{-13} \text{ J/MeV}\right) = \boxed{2.27 \times 10^5 \text{ J/yr}}$$

(e) dose in rem = dose in rad x RBE

$$5.00 \ \frac{\text{rem}}{\text{yr}} = \left(\text{dose in } \frac{\text{rad}}{\text{yr}}\right) 1.10, \text{ giving } \left(\text{dose in } \frac{\text{rad}}{\text{yr}}\right) = 4.55 \ \frac{\text{rad}}{\text{yr}}$$

The allowed whole-body dose is then $$(70.0 \text{ kg})\left(4.55 \ \frac{\text{rad}}{\text{yr}}\right)\left(\frac{10^{-2} \text{ J/kg}}{1 \text{ rad}}\right) = \boxed{3.18 \text{ J/yr}}$$

45.50 $$E_T \equiv E(\text{thermal}) = \tfrac{3}{2} k_B T = 0.039 \text{ eV}$$

$$E_T = \left(\tfrac{1}{2}\right)^n E$$ where $n \equiv$ number of collisions, and $$0.039 = \left(\tfrac{1}{2}\right)^n \left(2.0 \times 10^6\right)$$

Therefore, $n = 25.6 = \boxed{26 \text{ collisions}}$

45.51 From conservation of energy: $K_\alpha + K_n = Q$ or $\frac{1}{2} m_\alpha v_\alpha{}^2 + \frac{1}{2} m_n v_n{}^2 = 17.6 \text{ MeV}$

Conservation of momentum: $m_\alpha v_\alpha = m_n v_n$ or $v_\alpha = \left(\dfrac{m_n}{m_\alpha}\right) v_n$.

The energy equation becomes: $$\tfrac{1}{2} m_\alpha \left(\frac{m_n}{m_\alpha}\right)^2 v_n^2 + \tfrac{1}{2} m_n v_n^2 = \left(\frac{m_n + m_\alpha}{m_\alpha}\right)\left(\tfrac{1}{2} m_n v_n^2\right) = 17.6 \text{ MeV}$$

Thus, $$K_n = \left(\frac{m_\alpha}{m_n + m_\alpha}\right)(17.6 \text{ MeV}) = \left(\frac{4.002\,602}{1.008\,665 + 4.002\,602}\right) = \boxed{14.1 \text{ MeV}}$$

Goal Solution

Assuming that a deuteron and a triton are at rest when they fuse according to

$$^{2}\text{H} + {}^{3}\text{H} \rightarrow {}^{4}\text{He} + \text{n} + 17.6\text{ MeV},$$

determine the kinetic energy acquired by the neutron.

G: The products of this nuclear reaction are an alpha particle and a neutron, with total kinetic energy of 17.6 MeV. In order to conserve momentum, the lighter neutron will have a larger velocity than the more massive alpha particle (which consists of two protons and two neutrons). Since the kinetic energy of the particles is proportional to the square of their velocities but only linearly proportional to their mass, the neutron should have the larger kinetic energy, somewhere between 8.8 and 17.6 MeV.

O: Conservation of linear momentum and energy can be applied to find the kinetic energy of the neutron. We first suppose the particles are moving nonrelativistically.

A: The momentum of the alpha particle and that of the neutron must add to zero, so their velocities must be in opposite directions with magnitudes related by

$$m_n\mathbf{v}_n + m_\alpha \mathbf{v}_\alpha = 0 \qquad \text{or} \qquad (1.0087\text{ u})v_n = (4.0026\text{ u})v_\alpha$$

At the same time, their kinetic energies must add to 17.6 MeV

$$E = \tfrac{1}{2}m_n v_n^2 + \tfrac{1}{2}m_\alpha v_\alpha^2 = \tfrac{1}{2}(1.0087\text{ u})v_n^2 + \tfrac{1}{2}(4.0026\text{ u})v_\alpha^2 = 17.6\text{ MeV}$$

Substitute $v_\alpha = 0.2520 v_n$: $\quad E = (0.50435\text{ u})v_n^2 + (0.12710\text{ u})v_n^2 = 17.6\text{ MeV}\left(\dfrac{1\text{ u}}{931.494\text{ MeV}/c^2}\right)$

$$v_n = \sqrt{\frac{0.0189c^2}{0.63145}} = 0.173c = 5.19\times10^7\text{ m/s}$$

Since this speed is not too much greater than $0.1c$, we can get a reasonable estimate of the kinetic energy of the neutron from the classical equation,

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.0087\text{ u})(0.173c)\left(\frac{931.494\text{ MeV}/c^2}{\text{u}}\right) = 14.1\text{ MeV}$$

L: The kinetic energy of the neutron is within the range we predicted. For a more accurate calculation of the kinetic energy, we should use relativistic expressions. Conservation of momentum gives

$$\gamma_n m_n \mathbf{v}_n + \gamma_\alpha m_\alpha \mathbf{v}_\alpha = 0 \qquad 1.0087\frac{v_n}{\sqrt{1 - v_n^2/c^2}} = 4.0026\frac{v_\alpha}{\sqrt{1 - v_\alpha^2/c^2}}$$

yielding $\quad \dfrac{v_\alpha^2}{c^2} = \dfrac{v_n^2}{15.746c^2 - 14.746v_n^2}$

Then $\quad (\gamma_n - 1)m_n c^2 + (\gamma_\alpha - 1)m_\alpha c^2 = 17.6\text{ MeV}$

and $v_n = 0.171c$, implying that $(\gamma_n - 1)m_n c^2 = 14.0\text{ MeV}$

45.52 From Table A.3, the half-life of ^{32}P is 14.26 d. Thus, the decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{14.26\text{ d}} = 0.0486\text{ d}^{-1} = 5.63 \times 10^{-7}\text{ s}^{-1}.$$

$$N_0 = \frac{R_0}{\lambda} = \frac{5.22 \times 10^{6}\text{ decay/s}}{5.63 \times 10^{-7}\text{ s}^{-1}} = 9.28 \times 10^{12}\text{ nuclei}$$

At $t = 10.0$ days, the number remaining is

$$N = N_0 e^{-\lambda t} = \left(9.28 \times 10^{12}\text{ nuclei}\right) e^{-\left(0.0486\text{ d}^{-1}\right)(10.0\text{ d})} = 5.71 \times 10^{12}\text{ nuclei}$$

so the number of decays has been $N_0 - N = 3.57 \times 10^{12}$ and the energy released is

$$E = \left(3.57 \times 10^{12}\right)(700\text{ keV})\left(1.60 \times 10^{-16}\ \frac{\text{J}}{\text{keV}}\right) = 0.400\text{ J}$$

If this energy is absorbed by 100 g of tissue, the absorbed dose is

$$\text{Dose} = \left(\frac{0.400\text{ J}}{0.100\text{ kg}}\right)\left(\frac{1\text{ rad}}{10^{-2}\text{ J/kg}}\right) = \boxed{400\text{ rad}}$$

45.53 (a) The number of Pu nuclei in 1.00 kg = $\dfrac{6.02 \times 10^{23}\text{ nuclei/mol}}{239.05\text{ g/mol}}(1000\text{ g})$

The total energy = $(25.2 \times 10^{23}\text{ nuclei})(200\text{ MeV}) = 5.04 \times 10^{26}$ MeV

$E = (5.04 \times 10^{26}\text{ MeV})(4.44 \times 10^{-20}\text{ kWh/MeV}) = \boxed{2.24 \times 10^{7}\text{ kWh}}$ or 22 million kWh

(b) $E = \Delta mc^2 = (3.016\,049\text{ u} + 2.014\,102\text{ u} - 4.002\,602\text{ u} - 1.008\,665\text{ u})(931.5\text{ MeV/u})$

$E = \boxed{17.6\text{ MeV for each D-T fusion}}$

(c) E_n = (Total number of D nuclei)$(17.6)(4.44 \times 10^{-20})$

$E_n = (6.02 \times 10^{23})(1000/2.014)(17.6)(4.44 \times 10^{-20}) = \boxed{2.34 \times 10^{8}\text{ kWh}}$

(d) E_n = the number of C atoms in 1.00 kg $\times$ 4.20 eV

$E_n = (6.02 \times 10^{26}/12.0)(4.20 \times 10^{-6}\text{ MeV})(4.44 \times 10^{-20}) = \boxed{9.36\text{ kWh}}$

(e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels.

***45.54** Add two electrons to both sides of the given reaction. Then $4\,{}^1_1\text{H atom} \to {}^4_2\text{He atom} + Q$

where $Q = (\Delta m)c^2 = \left[4(1.007\ 825) - 4.002\ 602\right]\text{u}\ (931.5\ \text{MeV/u}) = 26.7\ \text{MeV}$

or $Q = (26.7\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right) = 4.28\times10^{-12}\ \text{J}$

The proton fusion rate is then

$$\text{rate} = \frac{\text{power output}}{\text{energy per proton}} = \frac{3.77\times10^{26}\ \text{J/s}}{\left(4.28\times10^{-12}\ \text{J}\right)/(4\ \text{protons})} = \boxed{3.53\times10^{38}\ \text{protons/s}}$$

***45.55** (a) $Q_{\text{I}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}}\right]c^2$, and $Q_{\text{II}} = \left[M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}} + M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} + M_{\text{D}} - M_{\text{E}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives ${}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} + {}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} \to {}^4_2\text{He} + 2\nu + Q_{\text{net}}$

or $4\,{}^1_1\text{H} + 2\,{}^{\ 0}_{-1}\text{e} \to {}^4_2\text{He} + 2\nu + Q_{\text{net}}$

Adding two electrons to each side $4\,{}^1_1\text{H atom} \to {}^4_2\text{He atom} + Q_{\text{net}}$

Thus, $Q_{\text{net}} = \left[4M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2 = \left[4(1.007\ 825) - 4.002\ 602\right]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{26.7\ \text{MeV}}$

45.56 (a) The mass of the pellet is $m = \rho V = \left(0.200\ \frac{\text{g}}{\text{cm}^3}\right)\left[\frac{4\pi}{3}\left(\frac{1.50\times10^{-2}\ \text{cm}}{2}\right)^3\right] = 3.53\times10^{-7}\ \text{g}$

The pellet consists of equal numbers of ${}^2\text{H}$ and ${}^3\text{H}$ atoms, so the average atomic weight is 2.50 and the total number of atoms is

$$N = \left(\frac{3.53\times10^{-7}\ \text{g}}{2.50\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right) = 8.51\times10^{16}\ \text{atoms}$$

When the pellet is vaporized, the plasma will consist of $2N$ particles (N nuclei and N electrons). The total energy delivered to the plasma is 1.00% of 200 kJ or 2.00 kJ. The temperature of the plasma is found from $E = (2N)\left(\frac{3}{2}k_{\text{B}}T\right)$ as

$$T = \frac{E}{3Nk_{\text{B}}} = \frac{2.00\times10^3\ \text{J}}{3\left(8.51\times10^{16}\right)\left(1.38\times10^{-23}\ \text{J/K}\right)} = \boxed{5.68\times10^8\ \text{K}}$$

(b) Each fusion event uses 2 nuclei, so $N/2$ events will occur. The energy released will be

$$E = \left(\frac{N}{2}\right)Q = \left(\frac{8.51\times10^{16}}{2}\right)(17.59\ \text{MeV})\left(1.60\times10^{-13}\ \frac{\text{J}}{\text{MeV}}\right) = 1.20\times10^5\ \text{J} = \boxed{120\ \text{kJ}}$$

***45.57** (a) The solar-core temperature of 15 MK gives particles enough kinetic energy to overcome the Coulomb-repulsion barrier to ${}^1_1\text{H} + {}^3_2\text{He} \to {}^4_2\text{He} + \text{e}^+ + \nu$, estimated as $k_e(e)(2e)/r$. The Coulomb barrier to Bethe's fifth and eight reactions is like $k_e(e)(7e)/r$, larger by $\frac{7}{2}$ times, so the temperature should be like $\frac{7}{2}(15 \times 10^6\text{ K}) \approx \boxed{5 \times 10^7\text{ K}}$.

(b) For ${}^{12}\text{C} + {}^1\text{H} \to {}^{13}\text{N} + Q$,

$$Q_1 = (12.000\,000 + 1.007\,825 - 13.005\,738)(931.5\text{ MeV}) = \boxed{1.94\text{ MeV}}$$

For the second step, add seven electrons to both sides to have: ${}^{13}\text{N atom} \to {}^{13}\text{C atom} + \text{e}^- + \text{e}^+ + Q$.

$$Q_2 = [13.005\,738 - 13.003\,355 - 2(0.000\,549)](931.5\text{ MeV}) = \boxed{1.20\text{ MeV}}$$

$$Q_3 = Q_7 = 2(0.000\,549)(931.5\text{ MeV}) = \boxed{1.02\text{ MeV}}$$

$$Q_4 = [13.003\,355 + 1.007\,825 - 14.003\,074](931.5\text{ MeV}) = \boxed{7.55\text{ MeV}}$$

$$Q_5 = [14.003\,074 + 1.007\,825 - 15.003\,065](931.5\text{ MeV}) = \boxed{7.30\text{ MeV}}$$

$$Q_6 = [15.003\,065 - 15.000\,108 - 2(0.000\,549)](931.5\text{ MeV}) = \boxed{1.73\text{ MeV}}$$

$$Q_8 = [15.000\,108 + 1.007\,825 - 12 - 4.002\,602](931.5\text{ MeV}) = \boxed{4.97\text{ MeV}}$$

The sum is $\boxed{26.7\text{ MeV}}$, the same as for the proton-proton cycle.

(c) Not all of the energy released heats the star. When a neutrino is created, it will likely fly directly out of the star without interacting with any other particle.

45.58 (a) $\dfrac{I_2}{I_1} = \dfrac{I_0 e^{-\mu_2 x}}{I_0 e^{-\mu_1 x}} = \boxed{e^{-(\mu_2 - \mu_1)x}}$

(b) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40 - 41.0)(0.100)} = e^{3.56} = \boxed{35.2}$

(c) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40 - 41.0)(1.00)} = e^{35.6} = \boxed{2.89 \times 10^{15}}$

Thus, a 1.00-cm aluminum plate has essentially removed the long-wavelength x-rays from the beam.

Chapter 46 Solutions

46.1 Assuming that the proton and antiproton are left nearly at rest after they are produced, the energy of the photon E, must be

$$E = 2E_0 = 2(938.3 \text{ MeV}) = 1876.6 \text{ MeV} = 3.00 \times 10^{-10} \text{ J}$$

Thus, $E = hf = 3.00 \times 10^{-10}$ J

$$f = \frac{3.00 \times 10^{-10} \text{ J}}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{4.53 \times 10^{23} \text{ Hz}}$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8} \text{ m/s}}{4.53 \times 10^{23} \text{ Hz}} = \boxed{6.62 \times 10^{-16} \text{ m}}$$

46.2 The minimum energy is released, and hence the minimum frequency photons are produced, when the proton and antiproton are at rest when they annihilate.

That is, $E = E_0$ and $K = 0$. To conserve momentum, each photon must carry away one-half the energy. Thus,

$$E_{\min} = hf_{\min} = \frac{(2E_0)}{2} = E_0 = 938.3 \text{ MeV}$$

Thus, $f_{\min} = \dfrac{(938.3 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{2.27 \times 10^{23} \text{ Hz}}$

$$\lambda = \frac{c}{f_{\min}} = \frac{3.00 \times 10^{8} \text{ m/s}}{2.27 \times 10^{23} \text{ Hz}} = \boxed{1.32 \times 10^{-15} \text{ m}}$$

***46.3** In $\gamma \rightarrow p^{+} + p^{-}$, we start with energy 2.09 GeV

we end with energy $938.3 \text{ MeV} + 938.3 \text{ MeV} + 95.0 \text{ MeV} + K_2$

where K_2 is the kinetic energy of the second proton.

Conservation of energy gives $\boxed{K_2 = 118 \text{ MeV}}$

Goal Solution

A photon with an energy $E_\gamma = 2.09$ GeV creates a proton-antiproton pair in which the proton has a kinetic energy of 95.0 MeV. What is the kinetic energy of the antiproton? ($m_p c^2 = 938.3$ MeV).

G: An antiproton has the same mass as a proton, so it seems reasonable to expect that both particles will have similar kinetic energies.

O: The total energy of each particle is the sum of its rest energy and its kinetic energy. Conservation of energy requires that the total energy before this pair production event equal the total energy after.

A: $E_\gamma = \left(E_{Rp} + K_p\right) + \left(E_{R\bar{p}} + K_{\bar{p}}\right)$

The energy of the photon is given as $E_\gamma = 2.09 \text{ GeV} = 2.09 \times 10^3$ MeV. From Table 46.2, we see that the rest energy of both the proton and the antiproton is

$$E_{Rp} = E_{R\bar{p}} = m_p c^2 = 938.3 \text{ MeV}$$

If the kinetic energy of the proton is observed to be 95.0 MeV, the kinetic energy of the antiproton is

$$K_{\bar{p}} = E_\gamma - E_{R\bar{p}} - E_{Rp} - K_p = 2.09 \times 10^3 \text{ MeV} - 2(938.5 \text{ MeV}) - 95.0 \text{ MeV} = 118 \text{ MeV}$$

L: The kinetic energy of the antiproton is slightly (~20%) greater than the proton. The two particles most likely have different shares in momentum of the gamma ray, and therefore will not have equal energies, either.

***46.4** The reaction is $\mu^+ + e^- \to \nu + \nu$

muon-lepton number before reaction: (–1) + (0) = –1

electron-lepon number before reaction: (0) + (1) = 1

Therefore, after the reaction, the muon-lepton number must be –1. Thus, one of the neutrinos must be the anti-neutrino associated with muons, and one of the neutrinos must be the neutrino associated with electrons:

$$\boxed{\bar{\nu}_\mu \quad \text{and} \quad \nu_e}$$

Then $\mu^+ + e^- \to \bar{\nu}_\mu + \nu_e$

46.5 The creation of a virtual Z^0 boson is an energy fluctuation $\Delta E = 93 \times 10^9$ eV. It can last no longer than $\Delta t = \hbar/2\Delta E$ and move no farther than

$$c(\Delta t) = \frac{hc}{4\pi\,\Delta E} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{4\pi\left(93 \times 10^9 \text{ eV}\right)}\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 1.06 \times 10^{-18} \text{ m} = \boxed{\sim 10^{-18} \text{ m}}$$

46.6 (a) $$\Delta E = (m_n - m_p - m_e)c^2$$

From Table A-3, $\Delta E = (1.008\,665 - 1.007\,825)931.5 = \boxed{0.782\text{ MeV}}$

(b) Assuming the neutron at rest, momentum is conserved, $p_p = p_e$

relativistic energy is conserved, $\sqrt{(m_pc^2)^2 + p_p^2c^2} + \sqrt{(m_ec^2)^2 + p_e^2c^2} = m_nc^2$

Since $p_p = p_e$, $\sqrt{(938.3)^2 + (pc)^2} + \sqrt{(0.511)^2 + (pc)^2} = 939.6\text{ MeV}$

Solving the algebra $pc = 1.19\text{ MeV}$

If $p_ec = \gamma m_e v_e c = 1.19$ MeV, then $\dfrac{\gamma v_e}{c} = \dfrac{1.19\text{ MeV}}{0.511\text{ MeV}} = \dfrac{x}{\sqrt{1-x^2}} = 2.33$ where $x = \dfrac{v_e}{c}$

Solving, $x^2 = (1 - x^2)5.43$ and $x = v_e/c = 0.919$

$$\boxed{v_e = 0.919c}$$

Then $m_pv_p = \gamma_e m_e v_e$:

$$v_p = \frac{\gamma_e m_e v_e c}{m_p c} = \frac{(1.19\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(1.67\times10^{-27})(3.00\times10^8)} = 3.80\times10^5\text{ m/s} = \boxed{380\text{ km/s}}$$

(c) $\boxed{\text{The electron is relativistic, the proton is not.}}$

***46.7** The time for a particle traveling with the speed of light to travel a distance of 3×10^{-15} m is

$$\Delta t = \frac{d}{v} = \frac{3\times10^{-15}\text{ m}}{3\times10^8\text{ m/s}} = \boxed{\sim 10^{-23}\text{ s}}$$

***46.8** With energy 938.3 MeV, the time that a virtual proton could last is at most Δt in $\Delta E \Delta t \sim \hbar$.

The distance it could move is at most

$$c\,\Delta t \sim \frac{\hbar c}{\Delta E} = \frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})(3\times10^8\text{ m/s})}{(938.3)(1.6\times10^{-13}\text{ J})} = \boxed{\sim 10^{-16}\text{ m}}$$

46.9 By Table 46.2, $M_{\pi^0} = 135\ \text{MeV}/c^2$

Therefore, $E_\gamma = \boxed{67.5\ \text{MeV}}$ for each photon

$$p = \frac{E_\gamma}{c} = \boxed{67.5\ \frac{\text{MeV}}{c}} \quad \text{and} \quad f = \frac{E_\gamma}{h} = \boxed{1.63 \times 10^{22}\ \text{Hz}}$$

***46.10** In $? + p^+ \to n + \mu^+$, charge conservation requires the unknown particle to be neutral. Baryon number conservation requires baryon number = 0. The muon-lepton number of ? must be –1.

So the unknown particle must be $\boxed{\bar{\nu}_\mu}$.

46.11 $\Omega^+ \to \overline{\Lambda}^0 + K^+$

$\overline{K}^0_S \to \pi^+ + \pi^-$ (or $\pi^0 + \pi^0$)

$\overline{\Lambda}^0 \to \overline{p} + \pi^+$

$\overline{n} \to \overline{p} + e^+ + \nu_e$

46.12 (a) $p + \overline{p} \to \mu^+ + e^-$ $\boxed{L_e}$ $0 + 0 \to 0 + 1$ and $\boxed{L_\mu}$ $0 + 0 \to -1 + 0$

(b) $\pi^- + p \to p + \pi^+$ $\boxed{\text{charge}}$ $-1 + 1 \to +1 + 1$

(c) $p + p \to p + \pi^+$ $\boxed{\text{baryon number}}$ $1 + 1 \to 1 + 0$

(d) $p + p \to p + p + n$ $\boxed{\text{baryon number}}$ $1 + 1 \to 1 + 1 + 1$

(e) $\gamma + p \to n + \pi^0$ $\boxed{\text{charge}}$ $0 + 1 \to 0 + 0$

***46.13** (a) Baryon number and charge are conserved, with values of 0 + 1 = 0 + 1 and 1 + 1 = 1 + 1 in both reactions.

(b) $\boxed{\text{Strangeness is } \textit{not} \text{ conserved}}$ in the second reaction.

46.14 Baryon number conservation $\boxed{\text{allows the first and forbids the second}}$.

46.15 (a) $\pi^- \to \mu^- + \boxed{\bar{\nu}_\mu}$ $\quad L_\mu$: $0 \to 1-1$

(b) $K^+ \to \mu^+ + \boxed{\nu_\mu}$ $\quad L_\mu$: $0 \to -1+1$

(c) $\boxed{\bar{\nu}_e} + p^+ \to n + e^+$ $\quad L_e$: $-1+0 \to 0-1$

(d) $\boxed{\nu_e} + n \to p^+ + e^-$ $\quad L_e$: $1+0 \to 0+1$

(e) $\boxed{\nu_\mu} + n \to p^+ + \mu^-$ $\quad L_\mu$: $1+0 \to 0+1$

(f) $\mu^- \to e^- + \boxed{\bar{\nu}_e} + \boxed{\nu_\mu}$ $\quad L_\mu$: $1 \to 0+0+1$ and L_e: $0 \to 1-1+0$

***46.16** Momentum conservation requires the pions to have equal speeds.

The total energy of each is $\quad 497.7\text{ MeV}/2$

so $E^2 = p^2c^2 + (mc^2)^2$ gives $\quad (248.8\text{ MeV})^2 = (pc)^2 + (139.6\text{ MeV})^2$

Solving, $\quad pc = 206\text{ MeV} = \gamma mvc = \dfrac{mc^2}{\sqrt{1-(v/c)^2}}\left(\dfrac{v}{c}\right)$

$$\frac{pc}{mc^2} = \frac{206\text{ MeV}}{139.6\text{ MeV}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right) = 1.48$$

$(v/c) = 1.48\sqrt{1-(v/c)^2}$ and $\quad (v/c)^2 = 2.18\left[1-(v/c)^2\right] = 2.18 - 2.18(v/c)^2$

$3.18(v/c)^2 = 2.18$ so $\quad \dfrac{v}{c} = \sqrt{\dfrac{2.18}{3.18}} = 0.828$ and $\quad \boxed{v = 0.828c}$

46.17 (a) $p^+ \to \pi^+ + \pi^0$ $\quad \boxed{\text{Baryon number}}$ is violated: $1 \to 0 + 0$

(b) $p^+ + p^+ \to p^+ + p^+ + \pi^0$ $\quad$ This reaction $\boxed{\text{can occur}}$.

(c) $p^+ + p^+ \to p^+ + \pi^+$ $\quad \boxed{\text{Baryon number}}$ is violated: $1 + 1 \to 1 + 0$

(d) $\pi^+ \to \mu^+ + \nu_\mu$ $\quad$ This reaction $\boxed{\text{can occur}}$.

(e) $n^0 \to p^+ + e^- + \bar{\nu}_e$ $\quad$ This reaction $\boxed{\text{can occur}}$.

(f) $\pi^+ \to \mu^+ + n$ $\quad$ Violates $\boxed{\text{baryon number}}$: $0 \to 0 + 1$

Violates $\boxed{\text{muon-lepton number}}$: $0 \to -1 + 0$

46.18 (a) $p \to e^+ + \gamma$ Baryon number: $+1 \to 0 + 0$ $\Delta B \neq 0$, so baryon number is violated.

(b) From conservation of momentum: $p_e = p_\gamma$

Then, for the positron, $E_e^2 = (p_e c)^2 + E_{0,e}^2$ becomes $E_e^2 = (p_\gamma c)^2 + E_{0,e}^2 = E_\gamma^2 + E_{0,e}^2$

From conservation of energy: $E_{0,p} = E_e + E_\gamma$ or $E_e = E_{0,p} - E_\gamma$

so $E_e^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$.

Equating this to the result from above gives $E_\gamma^2 + E_{0,e}^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$,

or $$E_\gamma = \frac{E_{0,p}^2 - E_{0,e}^2}{2E_{0,p}} = \frac{(938.3\text{ MeV})^2 - (0.511\text{ MeV})^2}{2(938.3\text{ MeV})} = \boxed{469\text{ MeV}}$$

Thus, $E_e = E_{0,p} - E_\gamma = 938.3\text{ MeV} - 469\text{ MeV} = \boxed{469\text{ MeV}}$

Also, $p_\gamma = \dfrac{E_\gamma}{c} = \boxed{469\text{ MeV/c}}$ and $p_e = p_\gamma = \boxed{469\text{ MeV}/c}$

(c) The total energy of the positron is $E_e = 469\text{ MeV}$.

But, $E_e = \gamma E_{0,e} = \dfrac{E_{0,e}}{\sqrt{1-(v/c)^2}}$ so $\sqrt{1-(v/c)^2} = \dfrac{E_{0,e}}{E_e} = \dfrac{0.511\text{ MeV}}{469\text{ MeV}} = 1.09 \times 10^{-3}$

which yields: $\boxed{v = 0.999\,999\,4c}$

***46.19** The relevant conservation laws are: $\Delta L_e = 0$, $\Delta L_\mu = 0$, and $\Delta L_\tau = 0$.

(a) $\pi^+ \to \pi^0 + e^+ + ?$ $L_e:\ 0 \to 0 - 1 + L_e \Rightarrow L_e = 1$ and we have a $\boxed{\nu_e}$

(b) $? + p \to \mu^- + p + \pi^+$ $L_\mu:\ L_\mu + 0 \to +1 + 0 + 0 \Rightarrow L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

(c) $\Lambda^0 \to p + \mu^- + ?$ $L_\mu:\ 0 \to 0 + 1 + L_\mu \Rightarrow L_\mu = -1$ and we have a $\boxed{\bar{\nu}_\mu}$

(d) $\tau^+ \to \mu^+ + ? + ?$ $L_\mu:\ 0 \to -1 + L_\mu \Rightarrow L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

$L_\tau:\ +1 \to 0 + L_\tau \Rightarrow L_\tau = 1$ and we have a $\boxed{\bar{\nu}_\tau}$

Conclusion for (d): $L_\mu = 1$ for one particle, and $L_\tau = 1$ for the other particle.

We have $\boxed{\nu_\mu}$ and $\boxed{\bar{\nu}_\tau}$.

46.20 The $\rho^0 \to \pi^+ + \pi^-$ decay must occur via the strong interaction.

The $K_S^0 \to \pi^+ + \pi^-$ decay must occur via the weak interaction.

46.21 (a) $\Lambda^0 \to p + \pi^-$ Strangeness: $-1 \to 0 + 0$ (strangeness is $\boxed{\text{not conserved}}$)

(b) $\pi^- + p \to \Lambda^0 + K^0$ Strangeness: $0 + 0 \to -1 + 1$ (0 = 0 and strangeness is $\boxed{\text{conserved}}$)

(c) $\bar{p} + p \to \bar{\Lambda}^0 + \Lambda^0$ Strangeness: $0 + 0 \to +1 - 1$ (0 = 0 and strangeness is $\boxed{\text{conserved}}$)

(d) $\pi^- + p \to \pi^- + \Sigma^+$ Strangeness: $0 + 0 \to 0 - 1$ ($0 \neq -1$: strangeness is $\boxed{\text{not conserved}}$)

(e) $\Xi^- \to \Lambda^0 + \pi^-$ Strangeness: $-2 \to -1 + 0$ ($-2 \neq -1$ so strangeness is $\boxed{\text{not conserved}}$)

(f) $\Xi^0 \to p + \pi^-$ Strangeness: $-2 \to 0 + 0$ ($-2 \neq 0$ so strangeness is $\boxed{\text{not conserved}}$)

46.22 (a) $\mu^- \to e^- + \gamma$ L_e: $0 \to 1 + 0$, and L_μ: $1 \to 0$

(b) $n \to p + e^- + \nu_e$ L_e: $0 \to 0 + 1 + 1$

(c) $\Lambda^0 \to p + \pi^0$ Strangeness: $-1 \to 0 + 0$, and charge: $0 \to +1 + 0$

(d) $p \to e^+ + \pi^0$ Baryon number: $+1 \to 0 + 0$

(e) $\Xi^0 \to n + \pi^0$ Strangeness: $-2 \to 0 + 0$

***46.23** (a) $\pi^- + p \to 2\eta$ violates conservation of baryon number as $0 + 1 \to 0$. $\boxed{\text{not allowed}}$

(b) $K^- + n \to \Lambda^0 + \pi^-$
Baryon number = $0 + 1 \to 1 + 0$ Charge = $-1 + 0 \to 0 - 1$
Strangeness, $-1 + 0 \to -1 + 0$ Lepton number, $0 \to 0$
The interaction may occur via the $\boxed{\text{strong interaction}}$ since all are conserved.

(c) $K^- \to \pi^- + \pi^0$
Strangeness, $-1 \to 0 + 0$ Baryon number, $0 \to 0$
Lepton number, $0 \to 0$ Charge, $-1 \to -1 + 0$
Strangeness is violated by one unit, but everything else is conserved. Thus, the reaction can occur via the $\boxed{\text{weak interaction}}$, but not the strong or electromagnetic interaction.

(d) $\Omega^- \to \Xi^- + \pi^0$
Baryon number, $1 \to 1 + 0$ Lepton number, $0 \to 0$
Charge, $-1 \to -1 + 0$ Strangeness, $-3 \to -2 + 0$
May occur by $\boxed{\text{weak interaction}}$, but not by strong or electromagnetic.

(e) $\eta \to 2\gamma$
Baryon number, $0 \to 0$ Lepton number, $0 \to 0$
Charge, $0 \to 0$ Strangeness, $0 \to 0$

No conservation laws are violated, but photons are the mediators of the electromagnetic interaction. Also, the lifetime of the η is consistent with the $\boxed{\text{electromagnetic interaction}}$.

***46.24** (a) $\Xi^- \to \Lambda^0 + \mu^- + \nu_\mu$

Baryon number: $+1 \to +1 + 0 + 0$ Charge: $-1 \to 0 - 1 + 0$

L_e: $0 \to 0 + 0 + 0$ L_μ: $0 \to 0 + 1 + 1$

L_τ: $0 \to 0 + 0 + 0$ Strangeness: $-2 \to -1 + 0 + 0$

Conserved quantities are: $\boxed{B\text{, charge, } L_e\text{, and } L_\tau}$

(b) $K_S^0 \to 2\pi^0$

Baryon number: $0 \to 0$ Charge: $0 \to 0$

L_e: $0 \to 0$ L_μ: $0 \to 0$

L_τ: $0 \to 0$ Strangeness: $+1 \to 0$

Conserved quantities are: $\boxed{B\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(c) $K^- + p \to \Sigma^0 + n$

Baryon number: $0 + 1 \to 1 + 1$ Charge: $-1 + 1 \to 0 + 0$

L_e: $0 + 0 \to 0 + 0$ L_μ: $0 + 0 \to 0 + 0$

L_τ: $0 + 0 \to 0 + 0$ Strangeness: $-1 + 0 \to -1 + 0$

Conserved quantities are: $\boxed{S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(d) $\Sigma^0 \to \Lambda^0 + \gamma$

Baryon number: $+1 \to 1 + 0$ Charge: $0 \to 0$

L_e: $0 \to 0 + 0$ L_μ: $0 \to 0 + 0$

L_τ: $0 \to 0 + 0$ Strangeness: $-1 \to -1 + 0$

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(e) $e^+ + e^- \to \mu^+ + \mu^-$

Baryon number: $0 + 0 \to 0 + 0$ Charge: $+1 - 1 \to +1 - 1$

L_e: $-1 + 1 \to 0 + 0$ L_μ: $0 + 0 \to +1 - 1$

L_τ: $0 + 0 \to 0 + 0$ Strangeness: $0 + 0 \to 0 + 0$

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(f) $\overline{p} + n \to \overline{\Lambda}^0 + \Sigma^-$

Baryon number: $-1 + 1 \to -1 + 1$ Charge: $-1 + 0 \to 0 - 1$

L_e: $0 + 0 \to 0 + 0$ L_μ: $0 + 0 \to 0 + 0$

L_τ: $0 + 0 \to 0 + 0$ Strangeness: $0 + 0 \to +1 - 1$

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

***46.25** (a) $K^+ + p \to \underline{\ ?\ } + p$

The strong interaction conserves everything.

Baryon number,	$0+1 \to B+1$	so	$B = 0$
Charge,	$+1+1 \to Q+1$	so	$Q = +1$
Lepton numbers,	$0+0 \to L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$+1+0 \to S+0$	so	$S = 1$

The conclusion is that the particle must be positively charged, a non-baryon, with strangeness of +1. Of particles in Table 46.2, it can only be the $\boxed{K^+}$. Thus, this is an elastic scattering process.

The weak interaction conserves all but strangeness, and $\Delta S = \pm 1$.

(b) $\Omega^- \to \underline{\ ?\ } + \pi^-$

Baryon number,	$+1 \to B+0$	so	$B = 1$
Charge,	$-1 \to Q-1$	so	$Q = 0$
Lepton numbers,	$0 \to L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$-3 \to S+0$	so $\Delta S = 1$:	$S = -2$

The particle must be a neutral baryon with strangeness of –2. Thus, it is the $\boxed{\Xi^0}$.

(c) $K^+ \to \underline{\ ?\ } + \mu^+ + \nu_\mu$:

Baryon number,	$0 \to B+0+0$	so	$B = 0$
Charge,	$+1 \to Q+1+0$	so	$Q = 0$
Lepton Numbers	L_e, $0 \to L_e + 0 + 0$	so	$L_e = 0$
	L_μ, $0 \to L_\mu - 1 + 1$	so	$L_\mu = 0$
	L_τ, $0 \to L_\tau + 0 + 0$	so	$L_\tau = 0$
Strangeness:	$1 \to S+0+0$	so	$\Delta S = \pm 1$ (for weak interaction): $S = 0$

The particle must be a neutral meson with strangeness = 0 $\Rightarrow$ $\boxed{\pi^0}$.

***46.26** (a)

	proton	u	u	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	e	$2e/3$	$2e/3$	$-e/3$	e

(b)

	neutron	u	d	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

***46.27** (a) The number of protons $N_p = 1000\text{ g}\left(\dfrac{6.02\times 10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\dfrac{10\text{ protons}}{\text{molecule}}\right) = 3.34\times 10^{26}\text{ protons}$

and there are $$N_n = (1000\text{ g})\left(\frac{6.02\times 10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\frac{8\text{ neutrons}}{\text{molecule}}\right) = 2.68\times 10^{26}\text{ neutrons}$$

So there are for electric neutrality $\boxed{3.34\times 10^{26}\text{ electrons}}$

The up quarks have number $2\times 3.34\times 10^{26} + 2.68\times 10^{26} = \boxed{9.36\times 10^{26}\text{ up quarks}}$

and there are $2\times 2.68\times 10^{26} + 3.34\times 10^{26} = \boxed{8.70\times 10^{26}\text{ down quarks}}$

(b) Model yourself as 65 kg of water. Then you contain $65\times 3.34\times 10^{26}$ $\boxed{\sim 10^{28}\text{ electrons}}$

$65\times 9.36\times 10^{26}$ $\boxed{\sim 10^{29}\text{ up quarks}}$

$65\times 8.70\times 10^{26}$ $\boxed{\sim 10^{29}\text{ down quarks}}$

Only these fundamental particles form your body. You have no strangeness, charm, topness or bottomness.

46.28 (a)

	K^0	d	$\bar{s}$	total
strangeness	1	0	1	1
baryon number	0	1/3	–1/3	0
charge	0	$-e/3$	$e/3$	0

(b)

	Λ^0	u	d	s	total
strangeness	–1	0	0	–1	–1
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

46.29 Quark composition of proton = uud and of neutron = udd.

Thus, if we neglect binding energies, we may write $$m_p = 2m_u + m_d \qquad (1)$$

and $$m_n = m_u + 2m_d \qquad (2)$$

Solving simultaneously, we find

$$m_u = \frac{1}{3}(2m_p - m_n) = \frac{1}{3}\left[2(938.3\text{ MeV}/c^2) - 939.6\text{ MeV}/c^2\right] = \boxed{312\text{ MeV}/c^2}$$

and from either (1) or (2), $m_d = \boxed{314\text{ MeV}/c^2}$

***46.30** In the first reaction, $\pi^- + p \to K^0 + \Lambda^0$, the quarks in the particles are: $\bar{u}d + uud \to d\bar{s} + uds$. There is a net of 1 up quark both before and after the reaction, a net of 2 down quarks both before and after, and a net of zero strange quarks both before and after. Thus, the reaction conserves the net number of each type of quark.

In the second reaction, $\pi^- + \mathrm{p} \to \mathrm{K}^0 + \mathrm{n}$, the quarks in the particles are: $\overline{\mathrm{u}}\mathrm{d} + \mathrm{uud} \to \mathrm{d}\overline{\mathrm{s}} + \mathrm{udd}$. In this case, there is a net of 1 up and 2 down quarks before the reaction but a net of 1 up, 3 down, and 1 anti-strange quark after the reaction. Thus, the reaction does not conserve the net number of each type of quark.

46.31 (a) $\pi^- + \mathrm{p} \to \mathrm{K}^0 + \Lambda^0$

In terms of constituent quarks: $\boxed{\overline{\mathrm{u}}\mathrm{d} + \mathrm{uud} \to \mathrm{d}\overline{\mathrm{s}} + \mathrm{uds}}$.

up quarks: $-1 + 2 \to 0 + 1$, or $1 \to 1$

down quarks: $1 + 1 \to 1 + 1$, or $2 \to 2$

strange quarks: $0 + 0 \to -1 + 1$, or $0 \to 0$

(b) $\pi^+ + \mathrm{p} \to \mathrm{K}^+ + \Sigma^+ \Rightarrow$ $\boxed{\mathrm{u}\overline{\mathrm{d}} + \mathrm{uud} \to \mathrm{u}\overline{\mathrm{s}} + \mathrm{uus}}$

up quarks: $1 + 2 \to 1 + 2$, or $3 \to 3$

down quarks: $-1 + 1 \to 0 + 0$, or $0 \to 0$

strange quarks: $0 + 0 \to -1 + 1$, or $0 \to 0$

(c) $\mathrm{K}^- + \mathrm{p} \to \mathrm{K}^+ + \mathrm{K}^0 + \Omega^- \Rightarrow$ $\boxed{\overline{\mathrm{u}}\mathrm{s} + \mathrm{uud} \to \mathrm{u}\overline{\mathrm{s}} + \mathrm{d}\overline{\mathrm{s}} + \mathrm{sss}}$

up quarks: $-1 + 2 \to 1 + 0 + 0$, or $1 \to 1$

down quarks: $0 + 1 \to 0 + 1 + 0$, or $1 \to 1$

strange quarks: $1 + 0 \to -1 - 1 + 3$, or $1 \to 1$

(d) $\mathrm{p} + \mathrm{p} \to \mathrm{K}^0 + \mathrm{p} + \pi^+ + \underline{\ ?\ } \Rightarrow$ $\mathrm{uud} + \mathrm{uud} \to \mathrm{d}\overline{\mathrm{s}} + \mathrm{uud} + \mathrm{u}\overline{\mathrm{d}} + \underline{\ ?\ }$

The quark combination of ? must be such as to balance the last equation for up, down, and strange quarks.

up quarks: $2 + 2 = 0 + 2 + 1 + ?$ (has 1 u quark)

down quarks: $1 + 1 = 1 + 1 - 1 + ?$ (has 1 d quark)

strange quarks: $0 + 0 = -1 + 0 + 0 + ?$ (has 1 s quark)

quark composite = uds = $\boxed{\Lambda^0 \text{ or } \Sigma^0}$

46.32 $\Sigma^0 + \mathrm{p} \to \Sigma^+ + \gamma + X$ dds + uud → uds + 0 + ?

The left side has a net 3d, 2u and 1s. The right-hand side has 1d, 1u, and 1s leaving 2d and 1u missing.

$\boxed{\text{The unknown particle is a neutron, udd.}}$

Baryon and strangeness numbers are conserved.

***46.33** Compare the given quark states to the entries in Tables 46.4 and 46.5.

(a) $\mathrm{suu} = \boxed{\Sigma^+}$

(b) $\overline{\mathrm{u}}\mathrm{d} = \boxed{\pi^-}$

(c) $\overline{\mathrm{s}}\mathrm{d} = \boxed{\mathrm{K}^0}$

(d) $\mathrm{ssd} = \boxed{\Xi^-}$

***46.34** (a) $\overline{\mathrm{u}}\,\overline{\mathrm{u}}\,\overline{\mathrm{d}}$: charge = $\left(-\frac{2}{3}e\right)+\left(-\frac{2}{3}e\right)+\left(\frac{1}{3}e\right) = \boxed{-e}$. This is the $\boxed{\text{antiproton}}$.

(b) $\overline{\mathrm{u}}\,\overline{\mathrm{d}}\,\overline{\mathrm{d}}$: charge = $\left(-\frac{2}{3}e\right)+\left(\frac{1}{3}e\right)+\left(\frac{1}{3}e\right) = \boxed{0}$. This is the $\boxed{\text{antineutron}}$.

***46.35** Section 39.4 says $f_{\text{observer}} = f_{\text{source}}\sqrt{\dfrac{1+v_a/c}{1-v_a/c}}$

The velocity of approach, v_a, is the negative of the velocity of mutual recession: $v_a = -v$.

Then, $\dfrac{c}{\lambda'} = \dfrac{c}{\lambda}\sqrt{\dfrac{1-v/c}{1+v/c}}$ and $\boxed{\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}}}$

46.36 $v = HR$ (Equation 46.7) $H = \dfrac{(1.7\times 10^{-2}\text{ m/s})}{\text{ly}}$

(a) $v\,(2.00\times 10^6\text{ ly}) = 3.4\times 10^4\text{ m/s}$ $\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}} = 590(1.0001133) = \boxed{590.07\text{ nm}}$

(b) $v\,(2.00\times 10^8\text{ ly}) = 3.4\times 10^6\text{ m/s}$ $\lambda' = 590\sqrt{\dfrac{1+0.01133}{1-0.01133}} = \boxed{597\text{ nm}}$

(c) $v\,(2.00\times 10^9\text{ ly}) = 3.4\times 10^7\text{ m/s}$ $\lambda' = 590\sqrt{\dfrac{1+0.1133}{1-0.1133}} = \boxed{661\text{ nm}}$

46.37 (a) $\dfrac{\lambda'}{\lambda} = \dfrac{650\text{ nm}}{434\text{ nm}} = 1.50 = \sqrt{\dfrac{1+v/c}{1-v/c}}$ $\dfrac{1+v/c}{1-v/c} = 2.24$

$v = 0.383c$, $\boxed{\text{38.3\% the speed of light}}$

(b) Equation 46.7, $v = HR$ $R = \dfrac{v}{H} = \dfrac{(0.383)(3.00\times 10^8)}{(1.7\times 10^{-2})} = \boxed{6.76\times 10^9\text{ light years}}$

Goal Solution

A distant quasar is moving away from Earth at such high speed that the blue 434-nm hydrogen line is observed at 650 nm, in the red portion of the spectrum. (a) How fast is the quasar receding? You may use the result of Problem 35. (b) Using Hubble's law, determine the distance from Earth to this quasar.

G: The problem states that the quasar is moving very fast, and since there is a significant red shift of the light, the quasar must be moving away from Earth at a relativistic speed ($v > 0.1c$). Quasars are very distant astronomical objects, and since our universe is estimated to be about 15 billion years old, we should expect this quasar to be ~10^9 light-years away.

O: As suggested, we can use the equation in Problem 35 to find the speed of the quasar from the Doppler red shift, and this speed can then be used to find the distance using Hubble's law.

A: (a) $\frac{\lambda'}{\lambda} = \frac{650 \text{ nm}}{434 \text{ nm}} = 1.498 = \sqrt{\frac{1+v/c}{1-v/c}}$ or squared, $\frac{1+v/c}{1-v/c} = 2.243$

Therefore, $v = 0.383c$ or 38.3% the speed of light

(b) Hubble's law asserts that the universe is expanding at a constant rate so that the speeds of galaxies are proportional to their distance R from Earth, $v = HR$

so, $R = \frac{v}{H} = \frac{(0.383)\left(3.00 \times 10^8 \text{ m / s}\right)}{\left(1.70 \times 10^{-2} \text{ m / s} \cdot \text{ly}\right)} = 6.76 \times 10^9 \text{ ly}$

L: The speed and distance of this quasar are consistent with our predictions. It appears that this quasar is quite far from Earth but not the most distant object in the visible universe.

***46.38** (a) $\lambda'_n = \lambda_n \sqrt{\frac{1+v/c}{1-v/c}} = (Z+1)\lambda_n$ $\qquad \frac{1+v/c}{1-v/c} = (Z+1)^2$

$$1 + \frac{v}{c} = (Z+1)^2 - \left(\frac{v}{c}\right)(Z+1)^2$$

$$\left(\frac{v}{c}\right)(Z^2 + 2Z + 2) = Z^2 + 2Z$$

$$v = \boxed{c\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)}$$

(b) $R = \frac{v}{H} = \boxed{\frac{c}{H}\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)}$

***46.39** The density of the Universe is $\rho = 1.20\rho_c = 1.20\left(3H^2/8\pi G\right)$.

Consider a remote galaxy at distance r. The mass interior to the sphere below it is

$$M = \rho\left(\frac{4\pi r^3}{3}\right) = 1.20\left(\frac{3H^2}{8\pi G}\right)\left(\frac{4\pi r^3}{3}\right) = \frac{0.600 H^2 r^3}{G}$$

both now and in the future when it has slowed to rest from its current speed $v = Hr$. The energy of this galaxy is constant as it moves to apogee distance R:

$$\frac{1}{2}mv^2 - \frac{GmM}{r} = 0 - \frac{GmM}{R} \qquad \text{so} \qquad \frac{1}{2}mH^2r^2 - \frac{Gm}{r}\left(\frac{0.600H^2r^3}{G}\right) = 0 - \frac{Gm}{R}\left(\frac{0.600H^2r^3}{G}\right)$$

$$-0.100 = -0.600\frac{r}{R} \qquad \text{so} \qquad R = 6.00r$$

The Universe will expand by a factor of $\boxed{6.00}$ from its current dimensions.

***46.40** (a) $k_B T \approx 2m_p c^2$ so $T \approx \dfrac{2m_pc^2}{k_B} = \dfrac{2(938.3\text{ MeV})}{\left(1.38\times10^{-23}\text{ J/K}\right)}\left(\dfrac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right)$ $\boxed{\sim 10^{13}\text{ K}}$

(b) $k_B T \approx 2m_e c^2$ $T \approx \dfrac{2m_ec^2}{k_B} = \dfrac{2(0.511\text{ MeV})}{\left(1.38\times10^{-23}\text{ J/K}\right)}\left(\dfrac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right)$ $\boxed{\sim 10^{10}\text{ K}}$

***46.41** (a) Wien's law: $\lambda_{\max}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$

Thus, $\lambda_{\max} = \dfrac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{T} = \dfrac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{2.73\text{ K}} = 1.06\times10^{-3}\text{ m} = \boxed{1.06\text{ mm}}$

(b) This is a $\boxed{\text{microwave}}$.

***46.42** (a) $L = \sqrt{\dfrac{\hbar G}{c^3}} = \sqrt{\dfrac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)}{\left(3.00\times10^8\text{ m/s}\right)^3}} = \boxed{1.61\times10^{-35}\text{ m}}$

(b) This time is given as $T = \dfrac{L}{c} = \dfrac{1.61\times10^{-35}\text{ m}}{3.00\times10^8\text{ m/s}} = \boxed{5.38\times10^{-44}\text{ s}}$,

which is approximately equal to the ultra-hot epoch.

46.43 (a) $\Delta E \Delta t \approx \hbar$, and $\Delta t \approx \frac{r}{c} = \frac{1.4 \times 10^{-15}\ \text{m}}{3.0 \times 10^{8}\ \text{m/s}} = 4.7 \times 10^{-24}\ \text{s}$

$$\Delta E \approx \frac{\hbar}{\Delta t} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{4.7 \times 10^{-24}\ \text{s}} = 2.3 \times 10^{-11}\ \text{J}\left(\frac{1\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) = 1.4 \times 10^{2}\ \text{MeV}$$

$$m = \frac{\Delta E}{c^2} \approx 1.4 \times 10^{2}\ \text{MeV}/c^2 \quad \boxed{\sim 10^{2}\ \text{MeV}/c^2}$$

(b) From Table 46.2, $m_\pi c^2 = 139.6$ MeV, $\boxed{\text{a pi-meson}}$

***46.44** (a) $\pi^- + \text{p} \rightarrow \Sigma^+ + \pi^0$ is forbidden by $\boxed{\text{charge conservation}}$

(b) $\mu^- \rightarrow \pi^- + \nu_e$ is forbidden by $\boxed{\text{energy conservation}}$

(c) $\text{p} \rightarrow \pi^+ + \pi^+ + \pi^-$ is forbidden by $\boxed{\text{baryon number conservation}}$

46.45 The total energy in neutrinos emitted per second by the Sun is:

$(0.4)(4\pi)(1.5 \times 10^{11})^2 = 1.1 \times 10^{23}$ W

Over 10^9 years, the Sun emits 3.6×10^{39} J in neutrinos. This represents an annihilated mass

$mc^2 = 3.6 \times 10^{39}$ J

$m = 4.0 \times 10^{22}$ kg

About $\boxed{\text{1 part in 50,000,000}}$ of the Sun's mass, over 10^9 years, has been lost to neutrinos.

Goal Solution

The energy flux carried by neutrinos from the Sun is estimated to be on the order of 0.4 W/m^2 at Earth's surface. Estimate the fractional mass loss of the Sun over 10^9 years due to the radiation of neutrinos. (The mass of the Sun is 2×10^{30} kg. The Earth-Sun distance is 1.5×10^{11} m.)

G: Our Sun is estimated to have a life span of about 10 billion years, so in this problem, we are examining the radiation of neutrinos over a considerable fraction of the Sun's life. However, the mass carried away by the neutrinos is a very small fraction of the total mass involved in the Sun's nuclear fusion process, so even over this long time, the mass of the Sun may not change significantly (probably less than 1%).

O: The change in mass of the Sun can be found from the energy flux received by the Earth and Einstein's famous equation, $E = mc^2$.

A: Since the neutrino flux from the Sun reaching the Earth is 0.4 W/m^2, the total energy emitted per second by the Sun in neutrinos in all directions is

$$\left(0.4 \text{ W / m}^2\right)\left(4\pi r^2\right) = \left(0.4 \text{ W / m}^2\right)(4\pi)\left(1.5 \times 10^{11} \text{ m}\right)^2 = 1.13 \times 10^{23} \text{ W}$$

In a period of 10^9 yr, the Sun emits a total energy of

$$\left(1.13 \times 10^{23} \text{ J / s}\right)\left(10^9 \text{ yr}\right)\left(3.156 \times 10^7 \text{ s / yr}\right) = 3.57 \times 10^{39} \text{ J}$$

in the form of neutrinos. This energy corresponds to an annihilated mass of

$$E = m_\nu c^2 = 3.57 \times 10^{39} \text{ J} \qquad \text{so} \qquad m_\nu = 3.97 \times 10^{22} \text{ kg}$$

Since the Sun has a mass of about 2×10^3 kg, this corresponds to a loss of only about 1 part in 50 000 000 of the Sun's mass over 10^9 yr in the form of neutrinos.

L: It appears that the neutrino flux changes the mass of the Sun by so little that it would be difficult to measure the difference in mass, even over its lifetime!

46.46 $\text{p} + \text{p} \to \text{p} + \pi^+ + X$

We suppose the protons each have 70.4 MeV of kinetic energy. From conservation of momentum, particle X has zero momentum and thus zero kinetic energy. Conservation of energy then requires

$$M_\text{p}c^2 + M_\pi c^2 + M_X c^2 = \left(M_\text{p}c^2 + K_\text{p}\right) + \left(M_\text{p}c^2 + K_\text{p}\right)$$

$$M_X c^2 = M_\text{p}c^2 + 2K_\text{p} - M_\pi c^2 = 938.3 \text{ MeV} + 2(70.4 \text{ MeV}) - 139.6 \text{ MeV} = 939.5 \text{ MeV}$$

X must be a neutral baryon of rest energy 939.5 MeV. Thus X is a $\boxed{\text{neutron}}$.

***46.47** We find the number N of neutrinos: $10^{46}\text{ J} = N(6\text{ MeV}) = N(6 \times 1.6 \times 10^{-13}\text{ J})$

$$N = 1.0 \times 10^{58}\text{ neutrinos}$$

The intensity at our location is

$$\frac{N}{A} = \frac{N}{4\pi r^2} = \frac{1.0\times 10^{58}}{4\pi(1.7\times 10^5\text{ ly})^2}\left(\frac{1\text{ ly}}{(3.0\times 10^8\text{ m/s})(3.16\times 10^7\text{ s})}\right)^2 = 3.1\times 10^{14}\text{ / m}^2$$

The number passing through a body presenting $5000\text{ cm}^2 = 0.50\text{ m}^2$ is then

$$\left(3.1\times 10^{14}\,\frac{1}{\text{m}^2}\right)\left(0.50\text{ m}^2\right) = 1.5\times 10^{14} \quad\text{or}\quad \boxed{\sim 10^{14}}$$

***46.48** By relativistic energy conservation,

$$E_\gamma + m_e c^2 = \frac{3m_e c^2}{\sqrt{1 - v^2/c^2}} \qquad (1)$$

By relativistic momentum conservation,

$$\frac{E_\gamma}{c} = \frac{3m_e v}{\sqrt{1 - v^2/c^2}} \qquad (2)$$

Dividing (2) by (1),

$$X = \frac{E_\gamma}{E_\gamma + m_e c^2} = \frac{v}{c}$$

Subtracting (2) from (1),

$$m_e c^2 = \frac{3m_e c^2}{\sqrt{1 - X^2}} - \frac{3m_e c^2 X}{\sqrt{1 - X^2}}$$

Solving, $1 = \dfrac{3 - 3X}{\sqrt{1 - X^2}}$ and $X = \dfrac{4}{5}$ so $E_\gamma = 4m_e c^2 = \boxed{2.04\text{ MeV}}$

46.49 $m_\Lambda c^2 = 1115.6\text{ MeV}$ $\Lambda^0 \to \text{p} + \pi^-$

$m_p c^2 = 938.3\text{ MeV}$ $\qquad m_\pi c^2 = 139.6\text{ MeV}$

The difference between starting mass-energy and final mass-energy is the kinetic energy of the products.

$$K_p + K_\pi = 37.7\text{ MeV} \quad\text{and}\quad p_p = p_\pi = p$$

Applying conservation of relativistic energy,

$$\left[\sqrt{(938.3)^2 + p^2c^2} - 938.3\right] + \left[\sqrt{(139.6)^2 + p^2c^2} - 139.6\right] = 37.7\text{ MeV}$$

Solving the algebra yields $p_\pi c = p_p c = 100.4\text{ MeV}$

Then,

$$K_p = \sqrt{(m_p c^2)^2 + (100.4)^2} - m_p c^2 = \boxed{5.35\text{ MeV}}$$

$$K_\pi = \sqrt{(139.6)^2 + (100.4)^2} - 139.6 = \boxed{32.3\text{ MeV}}$$

46.50 Momentum of proton is $qBr = \left(1.60 \times 10^{-19}\ \text{C}\right)\left(0.250\ \text{kg / C} \cdot \text{s}\right)\left(1.33\ \text{m}\right)$

$p_p = 5.32 \times 10^{-20}\ \dfrac{\text{kg} \cdot \text{m}}{\text{s}}$ $\quad cp_p = 1.60 \times 10^{-11}\ \dfrac{\text{kg m}^2}{\text{s}^2} = 1.60 \times 10^{-11}\ \text{J} = 99.8\ \text{MeV}$

Therefore, $p_p = 99.8\ \dfrac{\text{MeV}}{c}$

The total energy of the proton is $E_p = \sqrt{E_0^2 + (cp)^2} = \sqrt{(938.3)^2 + (99.8)^2} = 944\ \text{MeV}$

For pion, the momentum qBr is the same (as it must be from conservation of momentum in a 2-particle decay).

$p_\pi = 99.8\ \dfrac{\text{MeV}}{c}$ $\quad E_{0\pi} = 139.6\ \text{MeV}$

$$E_\pi = \sqrt{E_0^2 + (cp)^2} = \sqrt{(139.6)^2 + (99.8)^2} = 172\ \text{MeV}$$

Thus, $E_{\text{Total after}} = E_{\text{Total before}} = \text{Rest Energy}$

Rest Energy of unknown particle = 944 MeV + 172 MeV = 1116 MeV (This is a Λ^0 particle!)

Mass = $\boxed{1116\ \text{MeV}/c^2}$

46.51 $\Sigma^0 \to \Lambda^0 + \gamma$

From Table 46.2, $m_\Sigma = 1192.5\ \text{MeV}/c^2$ and $m_\Lambda = 1115.6\ \text{MeV}/c^2$

Conservation of energy requires $E_{0,\Sigma} = \left(E_{0,\Lambda} + K_\Lambda\right) + E_\gamma$,

or $$1192.5\ \text{MeV} = \left(1115.6\ \text{MeV} + \frac{p_\Lambda^2}{2m_\Lambda}\right) + E_\gamma$$

Momentum conservation gives $|p_\Lambda| = |p_\gamma|$, so the last result may be written as

$$1192.5\ \text{MeV} = \left(1115.6\ \text{MeV} + \frac{p_\gamma^2}{2m_\Lambda}\right) + E_\gamma$$

or $$1192.5\ \text{MeV} = \left(1115.6\ \text{MeV} + \frac{p_\gamma^2 c^2}{2m_\Lambda c^2}\right) + E_\gamma$$

Recognizing that $m_\Lambda c^2 = 1115.6\ \text{MeV}$ and $p_\gamma c = E_\gamma$,

we now have $$1192.5\ \text{MeV} = 1115.6\ \text{MeV} + \frac{E_\gamma^2}{2(1115.6\ \text{MeV})} + E_\gamma$$

Solving this quadratic equation, $E_\gamma \approx \boxed{74.4\ \text{MeV}}$

46.52 $p + p \to p + n + \pi^+$

The total momentum is zero before the reaction. Thus, all three particles present after the reaction may be at rest and still conserve momentum. This will be the case when the incident protons have minimum kinetic energy. Under these conditions, conservation of energy gives

$$2\left(m_p c^2 + K_p\right) = m_p c^2 + m_n c^2 + m_\pi c^2$$

so the kinetic energy of each of the incident protons is

$$K_p = \frac{m_n c^2 + m_\pi c^2 - m_p c^2}{2} = \frac{(939.6 + 139.6 - 938.3)\text{ MeV}}{2} = \boxed{70.4\text{ MeV}}$$

46.53 Time-dilated lifetime: $T = \gamma T_0 = \dfrac{0.900 \times 10^{-10}\text{ s}}{\sqrt{1 - v^2/c^2}} = \dfrac{0.900 \times 10^{-10}\text{ s}}{\sqrt{1 - (0.960)^2}} = 3.214 \times 10^{-10}\text{ s}$

distance $= (0.960)(3.00 \times 10^8\text{ m/s})(3.214 \times 10^{-10}\text{ s}) = \boxed{9.26\text{ cm}}$

46.54 $\pi^- \to \mu^- + \bar{\nu}_\mu$

From the conservation laws, $m_\pi c^2 = 139.5\text{ MeV} = E_\mu + E_\nu$ [1]

and $p_\mu = p_\nu, \quad E_\nu = p_\nu c$

Thus, $E_\mu^2 = \left(p_\mu c\right)^2 + (105.7\text{ MeV})^2 = \left(p_\nu c\right)^2 + (105.7\text{ MeV})^2$

or $E_\mu^2 - E_\nu^2 = (105.7\text{ MeV})^2$ [2]

Since $E_\mu + E_\nu = 139.5\text{ MeV}$ [1]

and $\left(E_\mu + E_\nu\right)\left(E_\mu - E_\nu\right) = (105.7\text{ MeV})^2$ [2]

then $E_\mu - E_\nu = \dfrac{(105.7\text{ MeV})^2}{139.5\text{ MeV}} = 80.1$ [3]

Subtracting [3] from [1], $2E_\nu = 59.4\text{ MeV}$ and $\boxed{E_\nu = 29.7\text{ MeV}}$

***46.55** The expression $e^{-E/k_BT}\,dE$ gives the fraction of the photons that have energy between E and $E+dE$. The fraction that have energy between E and infinity is

$$\frac{\int_E^\infty e^{-E/k_BT}\,dE}{\int_0^\infty e^{-E/k_BT}\,dE}=\frac{\int_E^\infty e^{-E/k_BT}\left(-dE/k_BT\right)}{\int_0^\infty e^{-E/k_BT}\left(-dE/k_BT\right)}=\frac{e^{-E/k_BT}\Big|_E^\infty}{e^{-E/k_BT}\Big|_0^\infty}=e^{-E/k_BT}$$

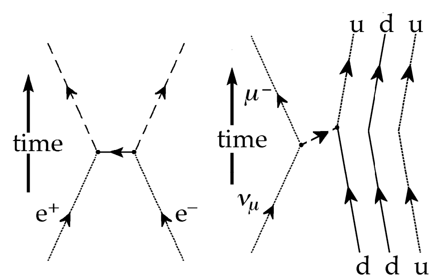
We require T when this fraction has a value of 0.0100 (i.e., 1.00%)

and $E=1.00\text{ eV}=1.60\times10^{-19}\text{ J}$

Thus, $0.0100=e^{-\left(1.60\times10^{-19}\text{ J}\right)/\left(1.38\times10^{-23}\text{ J/K}\right)T}$

or $\ln\left(0.0100\right)=-\dfrac{1.60\times10^{-19}\text{ J}}{\left(1.38\times10^{-23}\text{ J/K}\right)T}=-\dfrac{1.16\times10^4\text{ K}}{T}$ giving $T=\boxed{2.52\times10^3\text{ K}}$

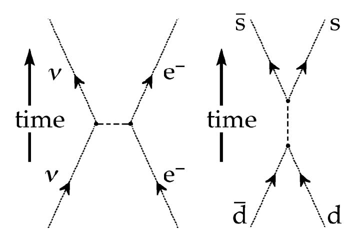
46.56 (a) This diagram represents the annihilation of an electron and an antielectron. From charge and lepton-number conservation at either vertex, the exchanged particle must be an electron, $\boxed{e^-}$.



(a) (b)

(b) This is the tough one. A neutrino collides with a neutron, changing it into a proton with release of a muon. This is a weak interaction. The exchanged particle has charge +1*e* and is a $\boxed{W^+}$.

46.57 (a) The mediator of this weak interaction is a $\boxed{Z^0\text{ boson}}$.



(a) **(b)**

(b) The Feynman diagram shows a down quark and its antiparticle annihilating each other. They can produce a particle carrying energy, momentum, and angular momentum, but zero charge, zero baryon number, and, it may be, no color charge. In this case the product particle is a $\boxed{\text{photon}}$. For conservation of both energy and momentum, we would expect two photons; but momentum need not be strictly conserved, according to the uncertainty principle, if the photon travels a sufficiently short distance before producing another matter-antimatter pair of particles, as shown in Figure P46.57. Depending on the color charges of the d and $\overline{\text{d}}$ quarks, the ephemeral particle could also be a $\boxed{\text{gluon}}$, as suggested in the discussion of Figure 46.13(b).