

Highway Discretionary Lane-change Decision and Control Using Model Predictive Control

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Abstract—To enable vehicles to perform automatic lane change amidst the random traffic flow on highways, this paper introduces a decision-making and control method for vehicle lane-change based on Model Predictive Control (MPC). This approach divides the driving control of vehicles on highways into two parts: lane-change decision and lane-change control, both of which are solved using the MPC method. In the lane-change decision module, the minimum driving costs for each lane are computed and compared by solving the MPC problem to make lane-change decisions. In the lane-change control module, a dynamic bicycle model is incorporated, and a multi-objective cost function is designed to obtain the optimal control inputs for the lane-change process. The proposed lane-change decision and control methods are simulated and validated within the SUMO platform under random highway traffic conditions.

Index Terms—Autonomous Vehicle, Lane-change, Model Predictive Control

I. INTRODUCTION

The quest for autonomous driving systems that seamlessly blend with the complex tapestry of human-driven traffic has become a focal point in the field of intelligent transportation. Central to this pursuit is the development of advanced control strategies that enable automated vehicles to execute critical maneuvers with precision and safety.

Lane change, which necessitates the integration of both longitudinal and lateral control, is considered one of the most complex driving maneuvers. Despite the inherent risks and the fact that improper lane-changes are often implicated in accidents, it has been observed that lane-changes executed by autonomous vehicles can effectively increase the average speed of the vehicle [1]. The decision-making process for lane-change is thus an indispensable aspect of autonomous driving. It involves evaluating the feasibility of the lane-change and ensuring that the maneuver is conducted with safety and efficiency in mind [2]. Autonomous vehicles, by perceiving road conditions and the information of surrounding vehicles, can plan the optimal driving path. This not only saves travel time for the vehicle itself but also contributes to optimizing traffic flow, enhancing road capacity, and reducing the incidence of traffic accidents.

In recent years, MPC has emerged as a potent paradigm in the orchestration of such maneuvers due to its forward-looking capabilities and constraint-handling properties. By leveraging a model of the vehicle's dynamics and the surrounding environment, MPC anticipates future states and derives optimal control inputs that adhere to the constraints of the road and vehicle capabilities.

The execution of lane-change maneuvers by drivers involves not only the consideration of their own vehicle but also the attention to nearby traffic participants. The MPC method has been proven to be an attractive approach in the domain of autonomous vehicle control [3], [4]. MPC uses a dynamic vehicle model to predict future states and determines an optimal control sequence for each moment, aiming to minimize a predefined performance index while satisfying constraints on control and state variables [5]. Mukai and Kawabe developed a lane-change assistance system based on MPC that formulates the problem of generating optimal lane-change paths as a mixed-integer programming problem, solving it with multi-parametric programming [6]. With existing infrastructure, lane measurement and control can be performed, thereby facilitating lane-change decisions [7]. Utilizing this technology, Kamal et al. designed an Economical Adaptive Cruise Control (EACC) lane-change model capable of real-time selection of the optimal lane for fuel economy [8], [9]. Tejeddin proposed a Multi-Lane Adaptive Cruise Controller (MLACC), designed to determine the optimal speed and driving lane in real-time [10]. Compared to Kamal's model, the MLACC model accounts for road grade and lane-change benefit thresholds. However, it only updates vehicles ahead when generating traffic flow and does not consider the relationship between the ego vehicle and following vehicles. Karlsson outlines a control system for self-driving cars to navigate highway exits, integrating a new trajectory planner and decision manager to enhance safety and comfort [11]. Bae introduces a real-time control framework for autonomous lane changes in heavy traffic using cooperative behavior prediction and MPC enhanced by Recurrent Neural Networks in 2020 [12]. And in 2022 he developed an online framework for smooth-path lane changes in dense traffic, integrating MPC with GANs for maneuver generation, and enhancing performance with adaptive safety

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measures and noise reduction, validated by simulations for efficiency and safety [13]. Ammour addresses autonomous driving safety by presenting an MPC-based collision avoidance algorithm accounting for dynamic traffic, with decision making and trajectory planning simplified for efficiency, and validated through simulations with mixed integer formulations and Sigmoid-based safety constraints [14]. Bhattacharyya and Vahidi introduces aiMPC, an adaptive motion planning framework for automated highway merging that optimizes vehicle interactions through mixed integer quadratic programming and inverse optimal control, demonstrating effective merging in simulations [15].

A controlling method combining lane-change decision and control, inspired by the previous work [10], is presented here. We propose a method based on MPC that integrates decision-making and vehicle control, capable of manipulating the vehicle's intended actions and specific controls in a phased manner. This approach accounts not only for the motion control of the vehicle itself but also incorporates interactions with surrounding traffic into its calculations. Furthermore, this method conveniently facilitates subsequent improvements to individual modules. Through this holistic control strategy, an autonomous vehicle can dynamically adapt to the rapidly changing conditions of the road environment, ensuring both the safety and efficiency of lane-change maneuvers.

The paper is organized in the following manner. Section II describes the lane-change decision method based on MPC. Section III details the establishment of a dynamic bicycle model and the control of the vehicle based on MPC. Section IV presents the simulation results and analysis.

II. LANE-CHANGE DECISION

When the ego vehicle maintains a straight course on the road and the distance to the lead vehicle in the current lane is less than Δs_{ref} , a lane-change can be considered. By comparing the driving cost of the current lane with that of the adjacent lanes, the optimal lane for travel is determined, and a decision is made to either change lane or stay in the current lane.

A. Modelling

$$x = [\Delta s \quad \Delta s_f \quad v \quad v_l \quad v_f \quad a \quad j]^T \quad (1)$$

$$u = a \quad (2)$$

$$x(k+1) = Ax(k) + Bu(k), \quad (3)$$

where

$$A = \begin{bmatrix} 1 & 0 & -T_s & T_s & 0 & 0 & 0 \\ 0 & 1 & T_s & 0 & -T_s & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & T_s & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_s} & 0 \end{bmatrix} \quad (4)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{T_s} \end{bmatrix}^T \quad (5)$$

To address the lane-change decision problem, it is essential to establish a mathematical model for the single-lane car-following problem. In this model, we consider the ego vehicle and the preceding and following vehicles on the current lane as point masses. For the Adaptive Cruise Control (ACC) problem, take the distance Δs between the ego vehicle and the leader, the distance Δs_f between the ego vehicle and the follower, the speed v of the ego vehicle, the speed v_l of the leading vehicle, the speed v_f of the following vehicle, the acceleration a of the ego vehicle, and the jerk j of the ego vehicle as state variables, and take the acceleration a of the ego vehicle as the control variable. The state space variables and control variables are shown as formula (1)(2).

To represent the kinematic equations of a vehicle in state-space form, formula (3) can be obtained. The length of time step T_s is set as 0.1s.

B. Cost Function

$$\left\{ \begin{array}{l} J = \sum_{l=t}^{t+T} \left| \frac{v(l|t) - v_{ref}}{v_{ref}} \right| + \sum_{l=t}^{t+T} \lambda_1 m_1 \quad (6a) \\ \quad + \sum_{l=t}^{t+T} \lambda_2 m_2 + \sum_{l=t}^{t+T} \lambda_3 m_3 \\ \text{s.t. } m_1 = j(l|t) \quad (6b) \\ m_2 = \begin{cases} 0, \Delta s(l|t) \geq \Delta s_d(l|t) \\ \frac{\Delta s_d(l|t) - \Delta s(l|t)}{\Delta s_{ref}}, \\ \Delta s(l|t) < \Delta s_d(l|t) \end{cases} \quad (6c) \\ m_3 = \begin{cases} 0, \Delta s_f(l|t) \geq \Delta s_{d,f}(l|t) \\ \text{or } l_{sw} = 0 \\ \frac{\Delta s_{d,f}(l|t) - \Delta s_f(l|t)}{\Delta s_{ref}}, \\ \Delta s_f(l|t) < \Delta s_{d,f}(l|t) \end{cases} \quad (6d) \\ \Delta s_d(l|t) = d_0 + t_h v(l|t) \quad (6e) \\ \Delta s_{d,f}(l|t) = d_0 + t_h v_f(l|t) \quad (6f) \\ \Delta s_{ref} = 50 \text{ m} \quad (6g) \end{array} \right.$$

When optimizing for multiple objectives simultaneously, a composite cost function that is the sum of individual terms representing different objectives is commonly used. This approach allows one to balance and prioritize the various aspects of the vehicle's performance that are important for the driving task. The cost function used for lane-change decisions in this paper is presented in formula (6a), and the description of each variable in cost function can be referred to in TABLE I.

The initial term of the cost function pertains to the velocity component. Within the scope of this study's scenario, which necessitates the vehicle's continuous gap-seeking and overtaking maneuvers, the maintenance of a comparatively high traveling speed is of paramount importance. The reference speed v_{ref} is slightly higher than the speeds of surrounding vehicles.

m_1 represents the jerk of the ego vehicle. However, comfort is not considered the most critical criterion; hence, the weight assigned to this term is relatively low.

TABLE I
PARAMETERS OF COST FUNCTION IN LANE-CHANGE DECISION

Parameters	Description	Value
v_{ref}	A desired speed higher than that of the surrounding vehicles	27 m/s
t_h	Time headway during car-following	1.5 s
d_0	The reference following distance when the ego vehicle is stationary	5 m
λ_1	Jerk weight term	0.2
λ_2	Leading vehicle distance weight term	1
λ_3	Following vehicle distance weight term	0.2
l_{sw}	Lane-change decision. 0 refers to stay, 1 refers to switch	0 or 1
u_d	Desired control effort	
J_{th}	Lane-change threshold of driving cost	0.3
k_p	Penalty term on lane-change	0.1
t	Current time step	
l	Predictive time step	

m_2 represents cost related to the leading vehicle. When ego vehicle is too close to the leader, the driving cost begins to rise. However, when Δs is far enough, meaning the actual distance is further than the desired distance Δs_d , it can be assumed that the leading vehicle does not interfere with the movement of ego vehicle, allowing the vehicle to accelerate freely within its lane. Therefore, the driving cost at this moment is considered to be 0.

When ego vehicle stays in its current lane, the following vehicle should be ignored. Consequently, within the computation of the driving cost for ego vehicle in its lane, the following distance term, denoted as m_3 , is assigned a value of 0. In contrast, when assessing the driving cost for potential maneuvers into adjacent lanes, the influence of the following vehicle is similar to the m_2 scenario.

C. Constraints

Undoubtedly, both lane-change and car-following behaviors necessitate conditions where collision avoidance is paramount. Therefore, imposing a minimum longitudinal distance constraint between vehicles is necessary. To define the boundaries of following distance and avoid collision, hard constraints are added:

$$\begin{cases} \Delta s(l | t) > d_{safe} = 2b \\ \Delta s_f(l | t) > d_{safe} = 2b \end{cases} \quad (7a)$$

$$(7b)$$

where $b = 5\text{m}$ represents the body length of the ego vehicle.

Additionally, considering the actual acceleration and deceleration performance of the vehicles, it's necessary to impose hard constraints on the vehicle's acceleration.

$$a \in [a_{min}, a_{max}] \quad (8)$$

where $a_{min} = -4.5 \text{ m/s}^2$, $a_{max} = 2.6 \text{ m/s}^2$.

D. Future State Prediction

On highways, sudden stops or spurts in speed by leading vehicles are uncommon, with surrounding vehicles predominantly maintaining a steady pace. Consequently, it is a reasonable assumption that v_l will remain constant over the predictive horizon. Therefore, this paper posits that the controller measures the leading vehicle's velocity v_l at the

present time step k and continuously applies this value in subsequent calculations within the predictive horizon, with a prediction horizon of $N = 50$ selected. This approach to resolving the MPC issue is known as the "Frozen Time" implementation.

E. Optimal Lane Selection

At each moment k , the ego vehicle retrieves information of surrounding vehicles, including the relative distance $\Delta s_{l,i}$ and velocity $v_{l,i}$ of the leading and following vehicles in the current lane and adjacent lanes, where $i \in \{r, c, l\}$ denotes the lane. Subsequently, it first determines whether the left and right lanes exist and whether there are conditions for lane-change.

For all available lanes, the model optimizes the minimum future driving cost and the corresponding control inputs using a MPC approach. It then determines the optimal lane at the current moment and executes the decision to either remain in the current lane or change lanes, as detailed in Algorithm 1.

Regarding the safety conditions during lane-change, considering that the speeds of vehicles in the traffic flow other than the ego vehicle are generally stable, a constant value is used as the safety distance criterion, set as $d_{safe} = 3b = 15\text{m}$. Thus, as long as the longitudinal distances with the leading and following vehicles in the adjacent lanes are both greater than d_{safe} , it is considered to meet the safety conditions for lane change.

III. LANE-CHANGE CONTROL

The previous chapter elaborates the methodology for discretionary lane-change decisions at each time step. This chapter will introduce the control mechanisms engaged to maneuver the vehicle during the lane-change process subsequent to the decision, also using the MPC method.

At each discrete time step, the control of the vehicle can be construed as a composite function of acceleration a and the steering angle δ of the front wheels. The objective of the proposed controller is to ascertain the optimal control combination and apply it to the ego vehicle.

A. Dynamic Bicycle Model

In a highway scenario, using the dynamic bicycle model can make the calculation of its driving trajectory more accurate.

Algorithm 1: Determination of l_{sw} and u_d

Input: $l_i, \Delta s_i, \Delta s_{f,i}, v, v_{l,i}, v_{f,i}, a, j$
Output: l_{sw}, u_d

```

1 while MLACC engaged do
2    $(J_c, u_{d,c}) \leftarrow$ 
      $\text{NMPC}(l_c, \Delta s_c, \Delta s_{f,c}, v, v_{l,c}, v_{f,c}, a, j)$ 
3   if  $J_c \leq J_{th}$  then
4     return  $l_{sw} \leftarrow 0, u_d \leftarrow u_{d,c}$ 
5   else
6      $(J_r, u_{d,r}) \leftarrow$ 
        $\text{NMPC}(l_r, \Delta s_r, \Delta s_{f,r}, v, v_{l,r}, v_{f,r}, a, j)$ 
7      $(J_l, u_{d,l}) \leftarrow$ 
        $\text{NMPC}(l_l, \Delta s_l, \Delta s_{f,l}, v, v_{l,l}, v_{f,l}, a, j)$ 
8     if  $(1 + k_p)J_r < J_c$  and  $J_r \leq J_l$  then
9       return  $l_{sw} \leftarrow -1, u_d \leftarrow u_{d,r}$ 
10    else if  $(1 + k_p)J_l < J_c$  and  $J_l < J_r$  then
11      return  $l_{sw} \leftarrow 1, u_d \leftarrow u_{d,l}$ 
12    else
13      return  $l_{sw} \leftarrow 0, u_d \leftarrow u_{d,c}$ 
14    end
15  end
16 end

```

The vehicle dynamics model used in this article is described as formula (9)(10)(11). The state variables respectively denotes longitudinal position, lateral position, yaw angle, lateral velocity, longitudinal velocity, yaw rate.

$$X = [x \ y \ \varphi \ u \ v \ \omega]^\top \quad (9)$$

$$U = [a \ \delta]^\top \quad (10)$$

where

$$\dot{X} = f(X, U) = \begin{bmatrix} u \cos \varphi - v \sin \varphi \\ v \cos \varphi + u \sin \varphi \\ \omega \\ a + v\omega - \frac{1}{m}F_{Y_1} \sin \delta \\ -u\omega + \frac{1}{m}(F_{Y_1} \cos \delta + F_{Y_2}) \\ \frac{1}{I_z}(l_f F_{Y_1} \cos \delta - l_r F_{Y_2}) \end{bmatrix} \quad (11)$$

When the front wheel steering angle of the vehicle is very small, it can be approximately obtained:

$$\cos \delta \approx 1 \quad (12)$$

$$\sin \delta \approx \delta \quad (13)$$

$$F_{Y_1} = k_f \alpha_1 \approx k_f \left(\frac{v + l_f \omega}{u} - \delta \right) \quad (14)$$

$$F_{Y_2} = k_r \alpha_2 \approx k_r \left(\frac{v - l_r \omega}{u} \right) \quad (15)$$

The parameters of the vehicle model are as shown in TABLE II.

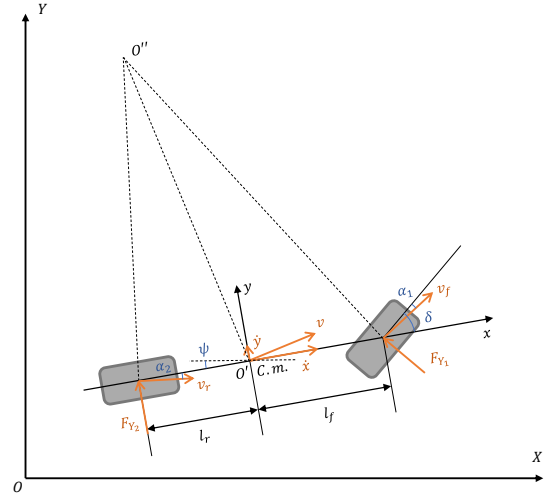


Fig. 1. Dynamic bicycle model

TABLE II
BASIC PARAMETERS OF SIMULATION VEHICLE

Parameter	Description	Value
m	mass of the vehicle	1470 kg
k_f	front axle equivalent sideslip stiffness	-100000 N/rad
k_r	rear axle equivalent sideslip stiffness	-100000 N/rad
l_f	distance between C.m. and front axle	1.085 m
l_r	distance between C.m. and rear axle	2.503 m
I_z	yaw inertia of vehicle body	2400 kg·m ²

B. Cost Function

When the vehicle is driving in its lane without making a lane-change decision, the lateral control objective of the ego vehicle is to maintain the center of the lane, and the longitudinal control objective is to follow the preceding vehicle.

When the vehicle makes a lane-change decision, the reference lateral position y_{ref} is updated to correspond to the centerline coordinates of the targeted adjacent lane. The lateral control aim for the ego vehicle is to converge toward y_{ref} , while the longitudinal control objective shifts to maintaining a safe following distance behind the leading vehicle in the target lane.

Similar to the cost function in lane-change decision, the ego vehicle needs to maintain a high reference speed v_{ref} , to ensure it can overtake other vehicles while driving. Another important factor is the lateral deviation of the vehicle's center of mass from the centerline of the lane. The designed controller should minimize this deviation to keep the vehicle driving straight. Additionally, for a smooth driving experience, it's necessary to control the acceleration a , steering angle δ , rate of change in acceleration Δa , and rate of change in

steering angle $\Delta\delta$.

$$\begin{aligned}
J = & \sum_{l=t}^{t+T} \lambda_v \|v(l|t) - v_{ref}\|^2 + \sum_{l=t}^{t+T} \lambda_y \|y(l|t) - y_{ref}\|^2 \\
& + \sum_{l=t}^{t+T} \lambda_l \max(0, d_l(l|t) - d_{safe}) \\
& + \sum_{l=t}^{t+T} \lambda_\delta \|\delta(l|t)\|^2 + \sum_{l=t+1}^{t+T-1} \lambda_{\Delta\delta} \|\delta(l|t) - \delta(l-1|t)\|^2 \\
& + \sum_{l=t}^{t+T} \lambda_a \|a(l|t)\|^2 + \sum_{l=t+1}^{t+T-1} \lambda_{\Delta a} \|a(l|t) - a(l-1|t)\|^2 \\
\text{s.t. } & d_l(l|t) = d_l(l-1|t) + [v(l-1|t) - v_l] T_s
\end{aligned} \tag{16}$$

TABLE III
WEIGHT PARAMETERS OF COST FUNCTION

Parameters	Value	Parameters	Value
λ_v	1	λ_δ	100000
λ_y	100	$\lambda_{\Delta\delta}$	10000
λ_l	-100	λ_a	1
		$\lambda_{\Delta a}$	50

To maintain a safe distance d_{safe} from the leading vehicle while also maximizing the use of gaps in traffic for overtaking, it is advisable to employ either a cost or constraint to limit this term. Utilizing a constraint may render the MPC problem infeasible when another vehicle suddenly cuts in front or when control deviation causes the distance to the leading vehicle to fall below d_{safe} . Therefore, this study employs a cost function to regulate the following distance. When the distance to the leading vehicle exceed d_{safe} , the cost remains at 0. If the distance falls below d_{safe} , the cost rises sharply in a linear fashion. This approach not only sustain the distance from the leading vehicle above d_{safe} as much as possible, but also guarantee that the problem remains solvable even when the distance contract to less than d_{safe} .

C. Constraints

- The velocity of the ego vehicle should comply with the speed limit regulations of the highway.
- The ego vehicle is situated on a three-lane road, and constraints should be applied to the road boundaries.
- The aforementioned dynamic bicycle model holds true only when the steering angle of the front wheels is small; hence, it is necessary to impose constraints on the values of the front wheel steering angle.
- Considering the vehicle's actual acceleration and braking capabilities, constraints on acceleration must be applied.

$$\begin{aligned}
\begin{cases} v(l|t) \in [v_{min}, v_{max}] \end{cases} & \tag{17a} \\
\begin{cases} y(l|t) \in [y_{min}, y_{max}] \end{cases} & \tag{17b} \\
\begin{cases} \delta(l|t) \in [\delta_{min}, \delta_{max}] \end{cases} & \tag{17c} \\
\begin{cases} a(l|t) \in [a_{min}, a_{max}] \end{cases} & \tag{17d}
\end{aligned}$$

where

$$\begin{aligned}
v_{min} &= 20 \text{ m/s}, v_{max} = 30 \text{ m/s}; \\
y_{min} &= -9.6 \text{ m}, y_{max} = 0 \text{ m}; \\
\delta_{min} &= -5^\circ, \delta_{max} = 5^\circ; \\
a_{min} &= -4.5 \text{ m/s}^2, a_{max} = 2.6 \text{ m/s}^2.
\end{aligned}$$

IV. SIMULATION

A. Platform & Rules

This article uses SUMO as the simulation platform. The road vehicle distribution is randomly generated, placing all vehicles at time zero, and then the simulation begins. For surrounding vehicles, the Intelligent Driver Model(IDM) car-following model and LC2013 lane-change model in SUMO are used, canceling the tendency to drive to the right, and overtaking is allowed in all three lanes.

B. Result & Analysis

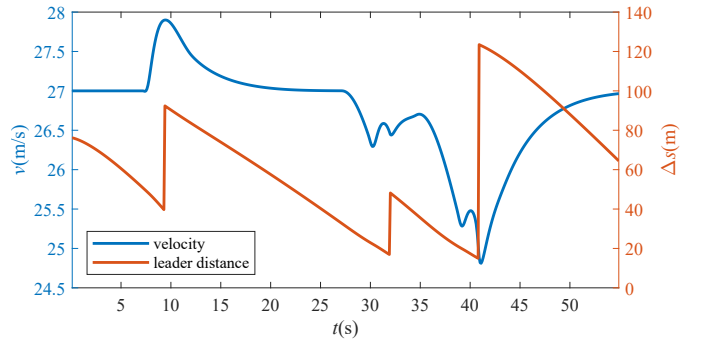


Fig. 2. Velocity(blue) and leader distance(red) of ego vehicle.

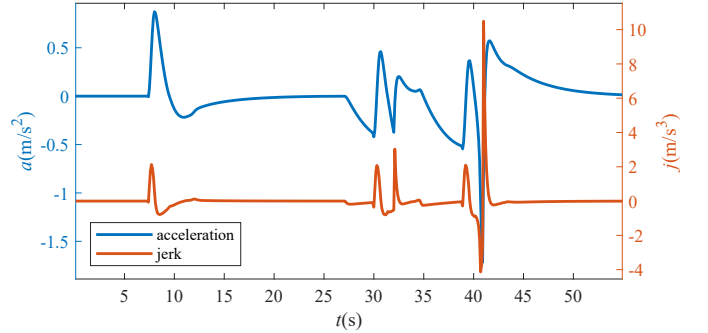


Fig. 3. Acceleration(blue) and jerk(red) of ego vehicle.

Initially, when the distance to the vehicle ahead in the current lane is substantial, the ego vehicle maintains a steady cruising speed. As it closes in on the leading vehicle, reducing the gap to less than Δs_{ref} , the ego vehicle decides to execute a lane change to the right. During the maneuver, the vehicle slightly increases its speed to align more swiftly with the center of the adjacent lane and then returns to the reference speed v_{ref} once it stabilizes in the new lane. If the overtaking space is not sufficiently clear, or the relative positioning of the vehicles does not meet the safe criteria for lane-change, the ego vehicle opts to decelerate and follow the leading vehicle,

adjusting its relative position with surrounding traffic. It waits and seeks an ideal gap within the traffic flow to safely complete the lane-change.

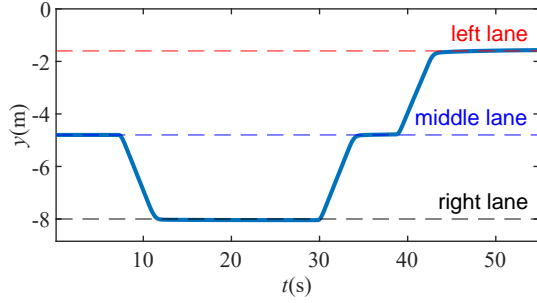


Fig. 4. Lateral distance of ego vehicle.

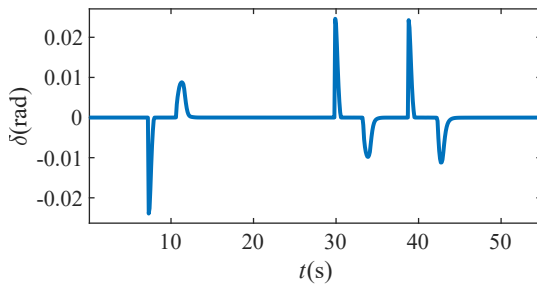


Fig. 5. Steering angle of ego vehicle.

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