Introduction to Structural Equation Modeling

Mplus Demonstration Notes

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Chapter 1
Introduction, Background, and Multiple Regression

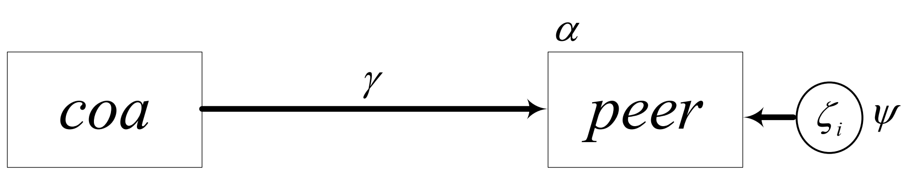
Multiple Regression Analysis of Deviant Peer Affiliations

The data for this demonstration were provided by Dr. Chassin from the Adolescent and Family Development Project, housed at Arizona State University. ***Note that these data were generously provided for strictly pedagogical purposes and should not be used for any other purposes beyond this workshop.*** The sample includes 316 adolescents, between 10-16 years of age. The study was designed to assess the association between parental alcoholism and adolescent substance use and psychopathology. The data are in the text file **`afdp.dat`**. The variables in the data set that we will use are

`peer`	adolescent report on peer substance use and peer tolerance of use
`coa`	parent report of alcoholism diagnosis where 0=non-alcoholic and 1=alcoholic
`gen`	gender where 0=girl and 1=boy
`age`	age measured in years at assessment
`stress`	self report measure of uncontrollable negative life stressful events
`emotion`	self report measure of temperamental emotional expressiveness
`negaff`	self report measure of depression and anxiety

Single Predictor Regression[1]

The single-predictor regression model of COA status predicting deviant peer affiliations is shown in the diagram below:



The model is of the form $peer_i = \alpha + \gamma coa_i + \zeta_i$ where $peer_i$ is an individual's report of their level of association with deviant peers, coa_i is an adolescent's parent report of parent alcoholism status, α represents mean association with deviant peers for children of non-alcoholics (i.e., where $coa_i = 0$), and γ is the expected increase in deviant peer associations for children of alcoholics (i.e. a one unit increase in coa_i).

We assume that both **`coa`** and **`peer`** contain no measurement error, that $\zeta \sim N(0,\psi)$ for all individuals i, and that ζ and coa_i are uncorrelated.

The Mplus input file that fits this model is provided in **`ch01_1.inp`** and is shown next:

[1] The initial model building procedure presented in Chapter 1 of the lecture notes used OLS estimation in traditional regression whereas here we use ML as a way of introducing the SEM procedure in Mplus. The results are virtually identical.

```
Title:
 AFDP Example 1
Data:
 file = afdp.dat;
Variable:
 names = id coa age gen stress emotion negaff peer;
 usevariables =  peer coa;
Analysis:
 estimator=ml;
Model:
 peer on coa;
Output:
 stand;
```

Mplus requires that every statement ends in a semicolon. The equal sign is interchangeable with the phrase 'are'. The `TITLE` command is optional and it provides a title for the analysis. The `DATA` command specifies the name and location of the data source. If no directory path is specified (as above), the default location is in the same folder in which the Mplus input file is saved. If the data are saved in a different location from the input file, a path may be specified as follows "`file=D:\cba\data\ch1\afdp.dat;`"

The `VARIABLE` command names all of the variables included in the data set (`names` = …) and the `usevariables` statement specifies which variables are to be used for this analysis.

In the `ANALYSIS` section, we specify that we are using maximum likelihood (ML) estimation. In the context of multiple regression, ML and OLS estimates of regression coefficients are equivalent. The residual variance estimate provided by ML is different, however, and biased at low sample size. It is asymptotically unbiased (at large N).

The `MODEL` command specifies the single variable regression model. In Mplus, the dependent variable is placed on the left and it is regressed '`on`' the predictor(s).

Finally, the `OUTPUT` section requests standardized estimates in addition to the unstandardized estimates that are provided by default. Let us now turn to the output, first examining the analysis summary:

```
SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         316

Number of dependent variables                                    1
Number of independent variables                                  1
Number of continuous latent variables                            0

Observed dependent variables

  Continuous
   PEER
```

```
Observed independent variables
   COA
Estimator                                                        ML
Information matrix                                         OBSERVED
Maximum number of iterations                                   1000
Convergence criterion                                     0.500D-04
Maximum number of steepest descent iterations                    20
```

We have not specified more than one group in our sample, so the number of groups is one. We confirm that there are 316 observations in the data set. Our model has a single dependent variable (**peer**) and a single independent variable (**COA**) with no latent variables. We have requested ML estimation (the observed information matrix is used to calculate standard errors) We have not altered the default convergence values.

Because the model information appears to be correct, we turn to model fit.

```
MODEL FIT INFORMATION

Number of Free Parameters                        3

Loglikelihood

          H0 Value                        -247.533
          H1 Value                        -247.533

Information Criteria

          Akaike (AIC)                     501.065
          Bayesian (BIC)                   512.333
          Sample-Size Adjusted BIC         502.817
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              0.000
          Degrees of Freedom                     0
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.000
          90 Percent C.I.                    0.000  0.000
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                1.000
          TLI                                1.000
```

```
Chi-Square Test of Model Fit for the Baseline Model

          Value                              8.620
          Degrees of Freedom                     1
          P-Value                           0.0033

SRMR (Standardized Root Mean Square Residual)

          Value                              0.000
```

Multiple regression models are just identified (i.e., every piece of information provided by the sample is 'used up' to estimate model parameters so that no degrees of freedom remain). Therefore, the model fits the data perfectly and it is not worthwhile to interpret the model chi-square, the CFI, TLI, or RMSEA (these fit indices will be discussed in later sections). Information criteria (AIC and BIC) will also be discussed later.

It is, however, worth pausing here to note that Mplus counts parameters differently than most SEM programs. For this model we would normally count 5 parameters, the mean and variance of **COA**, the intercept and slope of the regression of **Peer** on **COA**, and the residual variance of **Peer**. But Mplus does not count the mean or variance of the predictor, **COA**, as parameters of the model. Nor does it count covariances among predictors (though there are no such covariances in the current model) as free parameters. Fortunately, Mplus also does not count these parameters when determining the number of observed moments, so the degrees of freedom for the chi-square work out regardless (e.g., here it neither counts the mean and variance of COA as observed moments nor does it count them as estimated moments, leaving a net difference of zero when calculating the degrees of freedom).

The estimates for the model are shown next.

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.176      0.060      2.955      0.003

 Intercepts
    PEER               0.298      0.043      6.885      0.000

 Residual Variances
    PEER               0.280      0.022     12.570      0.000
```

Recall that our model is

$$peer_i = \alpha + \gamma coa_i + \zeta_i$$

`PEER on COA` is the slope parameter estimate $\hat{\gamma}$ and the `PEER` intercept is $\hat{\alpha}$. The `PEER Residual Variance` represents $\hat{\psi}$, the estimated variance of ζ. `Estimate` is the ML point estimate of each parameter, `S.E.` is the standard error of the estimate, `Est./S.E.` is the zvalue for the Wald test of the null hypothesis test that the parameter is significantly different from zero in the population, and the `Two-Tailed P-Value` is the p-value associated with the z-statistic. We see that the average non-COA has a score of .298 on **peer** and the average COA has a score that is .176 units higher than non-COAs on **peer** (.298 + .176 = .474). Both the intercept and slope are significantly different from zero. Finally, the variance in deviant peer association that is not explained by COA status is approximately .280, and this residual variance is significantly different from zero. This indicates that, although COA status is a significant predictor of **peer**, COA status does not fully account for affiliation with deviant peers.

Because **peer** is not on an intrinsically meaningful scale, we cannot easily interpret the differences among the regression parameters or residual variances. To better interpret these results, we requested the standardized solution, shown here:

```
STDYX Standardization

                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.164      0.055      2.995      0.003

 Intercepts
    PEER               0.555      0.087      6.376      0.000

 Residual Variances
    PEER               0.973      0.018     54.204      0.000


STDY Standardization

                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.328      0.109      3.016      0.003

 Intercepts
    PEER               0.555      0.087      6.376      0.000

 Residual Variances
    PEER               0.973      0.018     54.204      0.000

STD Standardization

                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.176      0.060      2.955      0.003
```

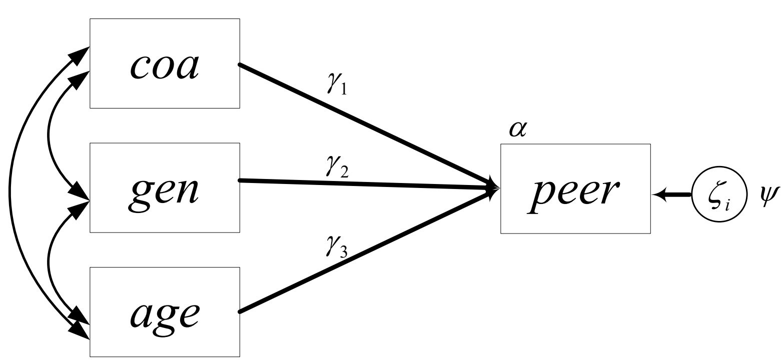
```
 Intercepts
    PEER                0.298      0.043      6.885      0.000

 Residual Variances
    PEER                0.280      0.022     12.570      0.000

R-SQUARE

    Observed                                         Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value
    PEER               0.027      0.018      1.498      0.134
```

Mplus provides three types of standardized solutions. The first type, labeled `STDYX`, is the typical type of standardized solution. Here, both the independent and dependent variables have been standardized to have a variance of 1 (Note that Mplus does not, however, also make the means 0, as one might expect). Thus, we say that a one standard deviation increase in COA status is associated with a .164 standard deviation increase in deviant peer associations. It may be more useful to retain the original scaling of **`COA`** while standardizing **`peer`**. This type of standardization is represented in the `STDY` output. We can interpret these results by saying that, the average COA affiliates with deviant peers about .328 standard deviations more than the average non-COA. The `STD` standardization is the least useful for our purposes; it standardizes the coefficients using the variances of the latent variables, and these are not relevant in the current model.

Finally, `R-SQUARE` is the estimated proportion of variance in **`peer`** that is accounted for by the models (i.e., **`COA`**, the only predictor included in this model). We have explained less than 3% of the total variance in **`peer`** with the **`COA`** predictor. Note, however, that measured variable regression models assume that no measurement error is present. If measurement error is present, the estimated relationship among the variables in the model may be attenuated and the R^2 value will also be underestimated.

Multiple Regression Model with COA Status, Gender, and Age as Predictors

We now expand the model to include gender and age as additional predictors of deviant peer relations as shown in the diagram below. All of the predictors are implicitly allowed to covary with one another, but these covariances are not estimated model parameters.



The model is of the form $peer_i = \alpha + \gamma_1 coa_i + \gamma_2 gen_i + \gamma_3 age_i + \zeta_i$. Each γ is interpreted as the effect of the associated predictor on **`peer`**, holding the other predictors constant. Unless all predictors are uncorrelated with one another, these estimates will change depending on which other predictors are included in the model. α represents the expected value of **`peer`** when all predictors are equal to zero.

We assume that none of the variables in our model contain measurement error, that $\zeta \sim N(0,\psi)$ for all individuals *i,* that the error variance is constant for all predictors, and that ζ is uncorrelated with all of the predictors in the model.

The Mplus input file that fits this model is provided in **`ch01_2.inp`** and is shown below:

```
Title:
 AFDP Example 2
Data:
 file= afdp.dat;
Variable:
 names = id coa age gen stress emotion negaff peer;
 usevariables =  peer coa age gen;
Analysis:
 estimator=ml;
Model:
 peer on coa age gen;
Output:
 stand;
```

Input for this example differs from the first example in two ways. First, **`gen`** and **`age`** have been included on the `usevariables` line. Second, we have indicated that we would like to add these variables as additional predictors of **`peer`** by placing them after the `on` statement in the `MODEL` command.

After confirming that the analysis summary reflects the model that we wanted to estimate, we turn to model fit.

```
MODEL FIT INFORMATION

Number of Free Parameters                        5

Loglikelihood

          H0 Value                        -217.301
          H1 Value                        -217.301

Information Criteria

          Akaike (AIC)                     444.601
          Bayesian (BIC)                   463.380
          Sample-Size Adjusted BIC         447.521
            (n* = (n + 2) / 24)
```

```
Chi-Square Test of Model Fit

          Value                              0.000
          Degrees of Freedom                     0
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.000
          90 Percent C.I.                    0.000  0.000
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                1.000
          TLI                                1.000

Chi-Square Test of Model Fit for the Baseline Model

          Value                             69.084
          Degrees of Freedom                     3
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.000
```

As before, our regression model is just identified. We turn to the model estimates.

```
MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.210      0.054      3.858      0.000
    GEN               -0.048      0.055     -0.877      0.381
    AGE                0.151      0.019      7.993      0.000

 Intercepts
    PEER              -1.610      0.250     -6.454      0.000

 Residual Variances
    PEER               0.232      0.018     12.570      0.000
```

From the above, we see that **`gen`** is not a significant predictor of **`peer`** when **`COA`** and **`age`** are also included in the model (p = .381). However, **`age`** is significantly related with **`peer`** such that older adolescents are more likely to associate with deviant peers. We estimate that a one year increase in **`age`** is associated with an increase in a .151 rating on the deviant peer association variable. **`COA`** remains a significant predictor of **`peer`**, and the regression parameter estimate has not changed much from the single predictor model to the multiple predictor model relative to its standard error (i.e., it has increased from .18 to .21). This indicates that **`COA`** is not highly multicollinear with **`gen`** or **`age`**.

The regression coefficients can be interpreted as follows. The slope of **`peer`** on **`COA`** is the average effect of being a child of an alcoholic, holding age and gender constant. The slope of **`age`** is the average effect of **`age`** on **`peer`**, holding COA status and gender constant. The intercept is slightly less informative in this model because **`age`** has not been centered. It now represents the average deviant peer association score for female, non-COAs who are zero years old. This is obviously outside of the range of our data.

To better interpret these results, and to get a sense for the relative contribution of each predictor on a standardized scale, we requested the standardized solution, shown here:

```
STDYX Standardization
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 PEER     ON
    COA                0.195      0.050      3.917      0.000
    GEN               -0.044      0.051     -0.877      0.380
    AGE                0.406      0.047      8.693      0.000

 Intercepts
    PEER              -2.999      0.416     -7.203      0.000

 Residual Variances
    PEER               0.804      0.040     20.057      0.000

STDY Standardization
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 PEER     ON
    COA                0.391      0.099      3.963      0.000
    GEN               -0.089      0.101     -0.878      0.380
    AGE                0.281      0.031      9.073      0.000

 Intercepts
    PEER              -2.999      0.416     -7.203      0.000

 Residual Variances
    PEER               0.804      0.040     20.057      0.000

STD Standardization
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.210      0.054      3.858      0.000
    GEN               -0.048      0.055     -0.877      0.381
    AGE                0.151      0.019      7.993      0.000
```

```
 Intercepts
    PEER               -1.610      0.250     -6.454      0.000

 Residual Variances
    PEER                0.232      0.018     12.570      0.000

R-SQUARE
    Observed                                        Two-Tailed
    Variable         Estimate       S.E.  Est./S.E.    P-Value

    PEER                0.196      0.040      4.901      0.000
```

Because **age** and **gen** are in meaningful units, it may again be most useful to interpret the STDY estimates. After controlling for age and gender, COAs affiliate with deviant peers about .391 standard deviation units more than non-COAs. Each additional year of age is associated with a .281 standard deviation increase in self-reported affiliation with deviant peers. It is also useful to examine the STDYX output to get a sense for the relative influence of each predictor because predictor units are not on comparable scales. From this output, we see that **age** is a relatively stronger predictor of **peer** than **COA**, and that both are much stronger relative predictors of **peer** than **gen**.

The multiple predictor regression model explains more of the variance in peer than the single predictor model. Age and gender account for an additional 17% of the variance in **peer**, over and above COA status. Still, approximately 80% of the variance in **peer** remains unexplained by the variables in our model.

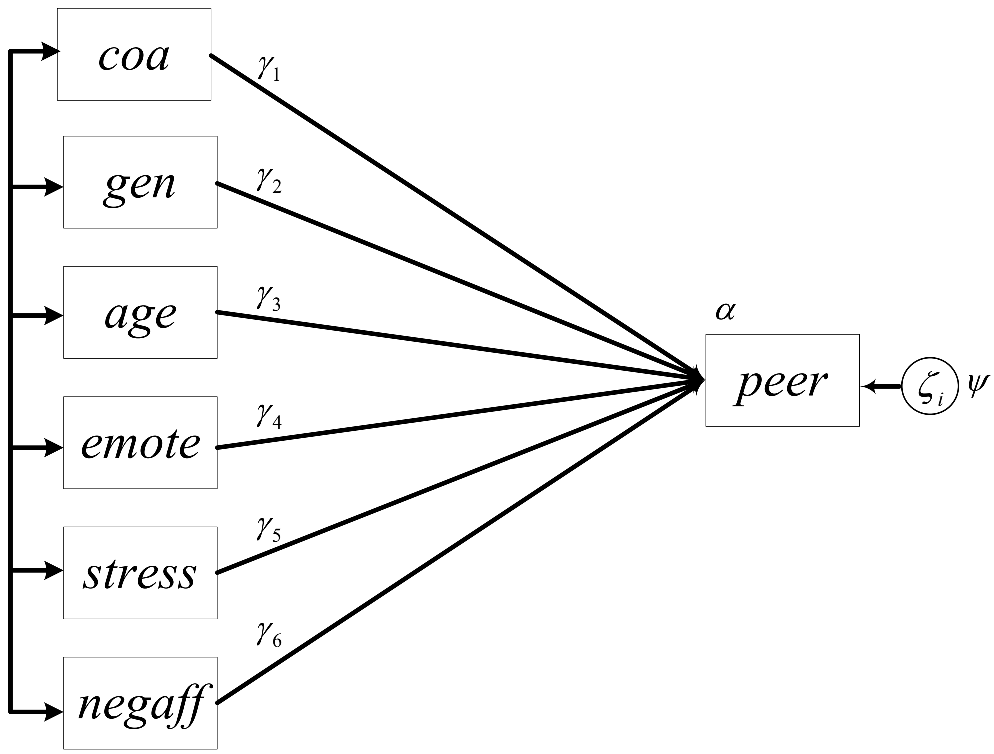
Multiple Regression Model with COA Status, Gender, Age, Stress, Emotion, and Negative Affect as Predictors

To see if we can explain more of the variance in deviant peer associations, we expand the model to include **stress**, **emotion**, and **negaff,** while retaining **COA**, **gen**, and **age**.

The model is now

$$peer_i = \alpha + \gamma_1 coa_i + \gamma_2 gen_i + \gamma_3 age_i + \gamma_4 emote_i + \gamma_5 stress_i + \gamma_6 negaff_i + \zeta_i$$

with the same assumptions as before. Although **gen** was not a significant predictor of deviant peer relations in the last model, we do not exclude it from this model because it may covary with the new predictors in such a way that it is important to include it as a control variable in the model. The full model is shown in the diagram below.



The Mplus input file that fits this model is provided in **`ch01_3.inp`** and is shown below:

```
Title:
 AFDP Example 3
Data:
 file= afdp.dat;
Variable:
 names = id coa age gen stress emotion negaff peer;
 usevariables =  peer coa age gen stress emotion negaff;
Analysis:
 estimator=ml;
Model:
 peer on coa age gen stress emotion negaff;
Output:
 stand;
```

As before, we simply include the new predictors in the `usevariables` statement and include them as predictors of **`peer`** in the `MODEL` command.

We now turn to model fit.

```
MODEL FIT INFORMATION

Number of Free Parameters                        8

Loglikelihood

          H0 Value                        -201.791
          H1 Value                        -201.791

Information Criteria

          Akaike (AIC)                     419.582
          Bayesian (BIC)                   449.628
          Sample-Size Adjusted BIC         424.254
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              0.000
          Degrees of Freedom                     0
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.000
          90 Percent C.I.                    0.000  0.000
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                1.000
          TLI                                1.000

Chi-Square Test of Model Fit for the Baseline Model

          Value                            100.103
          Degrees of Freedom                     6
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.000
```

As before, the model is saturated so this information is of little use here. We therefore turn to the model estimates.

```
MODEL RESULTS

                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.137      0.055      2.492      0.013
    GEN               -0.027      0.052     -0.509      0.611
    AGE                0.140      0.018      7.660      0.000
    EMOTION            0.030      0.058      0.520      0.603
    STRESS             0.111      0.044      2.546      0.011
    NEGAFF             0.109      0.030      3.660      0.000

 Intercepts
    PEER              -1.934      0.267     -7.233      0.000

 Residual Variances
    PEER               0.210      0.017     12.570      0.000
```

We see that **gen** is still not a significant predictor of **peer** when other predictors are included in the model (*p* = .611), but that **COA** and **age** remain statistically significant even though the value of their regression coefficients have changed. Importantly, the effect of **COA** on **peer** is not as strong after controlling for **emotion**, **stress**, and **negaff**, indicating that these variables are somewhat related to one another. These new variables could potentially mediate the relationship between **COA** and **peer**; we will later explore this possibility with a path analysis model. Of the new variables, it appears that stressful life events and negative affect are significant predictors of association with deviant peers, but self-reported emotional expressiveness is not significantly related to deviant peer association, after controlling for **age**, **gen**, **stress**, **negaff**, and **COA**.

Let us examine the standardized model results to get a better sense of our findings.

```
STDYX Standardization

                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.127      0.051      2.504      0.012
    GEN               -0.025      0.048     -0.509      0.611
    AGE                0.378      0.047      8.137      0.000
    EMOTION            0.028      0.054      0.520      0.603
    STRESS             0.140      0.055      2.559      0.010
    NEGAFF             0.195      0.053      3.704      0.000

 Intercepts
    PEER              -3.602      0.443     -8.123      0.000

 Residual Variances
    PEER               0.728      0.043     17.058      0.000
```

```
STDY Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.255      0.101      2.516      0.012
    GEN               -0.049      0.097     -0.509      0.610
    AGE                0.262      0.031      8.447      0.000
    EMOTION            0.056      0.108      0.520      0.603
    STRESS             0.206      0.080      2.571      0.010
    NEGAFF             0.204      0.055      3.737      0.000

 Intercepts
    PEER              -3.602      0.443     -8.123      0.000

 Residual Variances
    PEER               0.728      0.043     17.058      0.000


STD Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    COA                0.137      0.055      2.492      0.013
    GEN               -0.027      0.052     -0.509      0.611
    AGE                0.140      0.018      7.660      0.000
    EMOTION            0.030      0.058      0.520      0.603
    STRESS             0.111      0.044      2.546      0.011
    NEGAFF             0.109      0.030      3.660      0.000

 Intercepts
    PEER              -1.934      0.267     -7.233      0.000

 Residual Variances
    PEER               0.210      0.017     12.570      0.000


R-SQUARE

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    PEER               0.272      0.043      6.357      0.000
```

From the `STDYX` output, we see that **`age`** is the strongest relative predictor of **`peer`**, followed by **`negaff`**, **`stress`**, **`COA`**, **`emotion`**, and then **`gen`**. The final model explains about 27% of the total variance in peer.

Chapter 2
Path Analysis: Part I

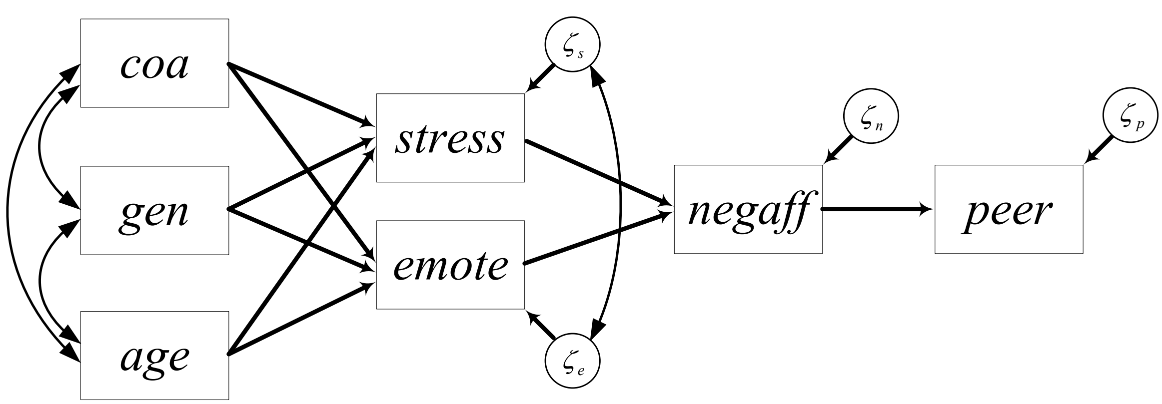
Path Analysis with Deviant Peer Affiliation Data

The data for this demonstration were provided by Dr. Laurie Chassin from the Adolescent and Family Development Project, housed at Arizona State University. ***Note that these data were generously provided for strictly pedagogical purposes and should not be used for any other purposes beyond this workshop.*** The sample includes 316 adolescents, between 10-16 years of age. The study was designed to assess the association between parental alcoholism and adolescent substance use and psychopathology. The data are in the text file **`afdp.dat`**. The variables in the data set that we will use are

`peer`	adolescent report on peer substance use and peer tolerance of use
`coa`	parent report of alcoholism diagnosis where 0=non-alcoholic and 1=alcoholic
`gen`	gender where 0=girl and 1=boy
`age`	age measured in years at assessment
`stress`	self report measure of uncontrollable negative life stressful events
`emotion`	self report measure of temperamental emotional expressiveness
`negaff`	self report measure of depression and anxiety

Path Analysis of Theoretical Peer Affiliation Model

Theory dictates that alcoholic parents increase the number of stressful life events that their children experience, leading to an increase in child negative affect. Further, children of alcoholics are hypothesized to have higher levels of emotionality, leading to more negative affect. Negative affect is thought to be related to higher rates of affiliation with deviant peers. Stressful life events and emotionality should covary, but we hypothesize no directional relation among these variables. We allow **`age`** and gender to predict **`stress`** and **`emote`**, and we allow all exogenous characteristics (**`coa`**, **`gen`**, and **`age`**) to covary.



The model is of the form

$$\begin{pmatrix} y_{stress_i} \\ y_{emote_i} \\ y_{negaff_i} \\ y_{peer_i} \end{pmatrix} = \begin{pmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \\ \alpha_{4i} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} coa_i \\ gen_i \\ age_i \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta_{31} & \beta_{32} & 0 & 0 \\ 0 & 0 & \beta_{43} & 0 \end{pmatrix} \begin{pmatrix} stress_i \\ emote_i \\ negaff_i \\ peer_i \end{pmatrix} + \begin{pmatrix} \zeta_{1i} \\ \zeta_{2i} \\ \zeta_{3i} \\ \zeta_{4i} \end{pmatrix} \text{where}$$

$$COV(\zeta_i) = \begin{pmatrix} \psi_{11} & & & \\ \psi_{21} & \psi_{22} & & \\ 0 & 0 & \psi_{33} & \\ 0 & 0 & 0 & \psi_{44} \end{pmatrix}$$

The Mplus input file that fits this model is provided in **`ch02_1.inp`** and is shown below:

```
title:
 Initial hypothesized mediational model with no direct effects;
data:
 file=afdp.dat;
variable:
 names =  id coa age gen stress emotion negaff peer;
 usevariables =  coa age gen stress emotion negaff peer;
analysis:
 estimator=ml;
model:
 peer on negaff;
 negaff on stress emotion;
 stress emotion on gen age coa;

 stress with emotion;

output:
 stand residual;
```

We have seen most of this code in Chapter 1, so here we will focus only on the `MODEL` command.

In Chapter 1, **`peer`** was regressed on all of the other variables in the model. Here, we specify only the *direct* regression paths using the `on` statement. For instance, because **`peer`** is only directly influenced by **`negaff`**, we write: `peer on negaff`. The program will "know" that **`stress`** and **`emotion`** are indirectly related to **`peer`** because of the statement: `negaff on stress emote`. This statement tells Mplus that both **`stress`** and **`emote`** are direct predictors of **`negaff`**. The program uses the information provided in the input statements to populate the matrices that form the model equations given above. Both **`stress`** and **`emote`** have an identical set of predictors; thus, it is convenient to include both of these variables on the left side of an `on` statement with the shared predictors on the right side of the `on` statement.

As a default, Mplus allows all exogenous variables to covary but it fixes the residual covariances among endogenous variables to zero. The residual terms of **`stress`** and **`emote`** are freed to covary by using the `with` statement.

We request standardized parameter estimates using the `stand` statement in the `OUTPUT` command to aid interpretation of the results. Finally, the `residual` statement computes the difference between the value of the observed sample statistics and the model-implied counterpart. Standardized residuals are obtained by dividing the residual by the standard deviation of the difference between the observed sample statistic and the model-implied counterpart and are approximately distributed as z-scores.

Typically, we would evaluate model fit prior to interpreting parameter estimates. For pedagogical purposes, however, we will put aside a discussion of model fit until Chapter 3 and move directly to parameter estimates for our model.

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    NEGAFF             0.176      0.030      5.892      0.000

 NEGAFF   ON
    STRESS             0.246      0.078      3.134      0.002
    EMOTE              0.553      0.106      5.206      0.000

 STRESS   ON
    GEN               -0.016      0.073     -0.215      0.830
    AGE                0.002      0.025      0.078      0.938
    COA                0.451      0.072      6.223      0.000

 EMOTE    ON
    GEN               -0.048      0.056     -0.843      0.399
    AGE               -0.027      0.019     -1.374      0.170
    COA                0.110      0.056      1.963      0.050

 STRESS   WITH
    EMOTE              0.112      0.019      5.896      0.000

 Intercepts
    STRESS             0.687      0.333      2.066      0.039
    EMOTE              2.341      0.258      9.088      0.000
    NEGAFF             1.527      0.207      7.373      0.000
    PEER              -0.118      0.091     -1.298      0.194

 Residual Variances
    STRESS             0.412      0.033     12.570      0.000
    EMOTE              0.247      0.020     12.570      0.000
    NEGAFF             0.778      0.062     12.570      0.000
    PEER               0.260      0.021     12.570      0.000
```

```
STANDARDIZED MODEL RESULTS

STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    NEGAFF             0.315      0.051      6.207      0.000

 NEGAFF   ON
    STRESS             0.175      0.055      3.173      0.002
    EMOTE              0.290      0.054      5.400      0.000

 STRESS   ON
    GEN               -0.011      0.053     -0.215      0.830
    AGE                0.004      0.053      0.078      0.938
    COA                0.331      0.050      6.593      0.000

 EMOTE  ON
    GEN               -0.047      0.056     -0.844      0.398
    AGE               -0.077      0.056     -1.378      0.168
    COA                0.110      0.055      1.974      0.048

 STRESS   WITH
    EMOTE              0.352      0.049      7.131      0.000

 Intercepts
    STRESS             1.010      0.493      2.050      0.040
    EMOTE              4.663      0.529      8.813      0.000
    NEGAFF             1.594      0.251      6.357      0.000
    PEER              -0.220      0.166     -1.324      0.185

 Residual Variances
    STRESS             0.890      0.033     26.846      0.000
    EMOTE              0.979      0.016     61.855      0.000
    NEGAFF             0.848      0.037     22.828      0.000
    PEER               0.901      0.032     28.253      0.000

STDY Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    NEGAFF             0.315      0.051      6.207      0.000

 NEGAFF   ON
    STRESS             0.175      0.055      3.173      0.002
    EMOTE              0.290      0.054      5.400      0.000
```

```
 STRESS   ON
    GEN               -0.023      0.107     -0.215      0.830
    AGE                0.003      0.037      0.078      0.938
    COA                0.663      0.098      6.777      0.000

 EMOTE    ON
    GEN               -0.095      0.112     -0.845      0.398
    AGE               -0.053      0.039     -1.380      0.168
    COA                0.219      0.111      1.980      0.048

 STRESS   WITH
    EMOTE              0.352      0.049      7.131      0.000

 Intercepts
    STRESS             1.010      0.493      2.050      0.040
    EMOTE              4.663      0.529      8.813      0.000
    NEGAFF             1.594      0.251      6.357      0.000
    PEER              -0.220      0.166     -1.324      0.185

 Residual Variances
    STRESS             0.890      0.033     26.846      0.000
    EMOTE              0.979      0.016     61.855      0.000
    NEGAFF             0.848      0.037     22.828      0.000
    PEER               0.901      0.032     28.253      0.000


STD Standardization

                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    NEGAFF             0.176      0.030      5.892      0.000

 NEGAFF   ON
    STRESS             0.246      0.078      3.134      0.002
    EMOTE              0.553      0.106      5.206      0.000

 STRESS   ON
    GEN               -0.016      0.073     -0.215      0.830
    AGE                0.002      0.025      0.078      0.938
    COA                0.451      0.072      6.223      0.000

 EMOTE    ON
    GEN               -0.048      0.056     -0.843      0.399
    AGE               -0.027      0.019     -1.374      0.170
    COA                0.110      0.056      1.963      0.050

 STRESS   WITH
    EMOTE              0.112      0.019      5.896      0.000
```

```
 Intercepts
    STRESS             0.687      0.333      2.066      0.039
    EMOTE              2.341      0.258      9.088      0.000
    NEGAFF             1.527      0.207      7.373      0.000
    PEER              -0.118      0.091     -1.298      0.194

 Residual Variances
    STRESS             0.412      0.033     12.570      0.000
    EMOTE              0.247      0.020     12.570      0.000
    NEGAFF             0.778      0.062     12.570      0.000
    PEER               0.260      0.021     12.570      0.000


R-SQUARE

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    STRESS             0.110      0.033      3.305      0.001
    EMOTE              0.021      0.016      1.304      0.192
    NEGAFF             0.152      0.037      4.079      0.000
    PEER               0.099      0.032      3.103      0.002
```

We see that the hypothesized pathways to deviant peer affiliation do contain statistically significant components. To aid in interpretation, standardized values are included in the model path diagram, shown below. We report partially standardized effects for coa, gen, and age (from `stdy` output), and fully standardized effects (from `stdyx` output) elsewhere.

Recall that `stdy` provides partially standardized effects in which only the outcome variables are standardized. These are most useful when examining the effects of coding variables (e.g., coa and gen) or predictors with natural metrics (e.g., age). These partially standardized parameter estimates represent the expected change in standard deviation units in y given a one unit increase in x.

By comparison, `stdyx` provides fully standardized estimates. This standardization method standardizes both the predictors and the outcome variables so that parameter estimates represent the expected change in standard deviation units in y given a one standard deviation increase in x. Only a modest amount of the total variance in any of these variables has been explained by the model, as shown by the `RSQUARE` output.

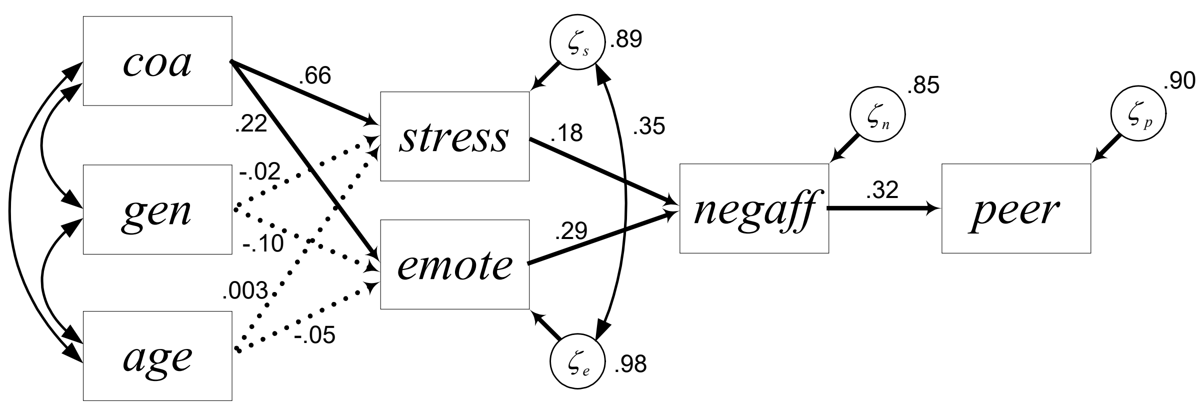
We can also examine the raw and normalized estimated residuals between the observed covariance matrix and the model-implied covariance matrix. (M*plus* provides both *standardized* and *normalized* estimates; these are quite similar and we focus on normalized here). Note there are no residuals for the variable means given that mean structure is fully saturated and are thus reproduced perfectly.

Partial output for the model-implied covariance matrix and raw and normalized residuals is:

```
Model Estimated Covariances/Correlations/Residual Correlations
              STRESS        EMOTION       NEGAFF        PEER          COA
              ________      ________      ________      ________      ________
 STRESS         0.463
 EMOTION        0.125         0.252
 NEGAFF         0.183         0.170         0.917
 PEER           0.032         0.030         0.162         0.288
 COA            0.112         0.029         0.044         0.008         0.249
 AGE           -0.019        -0.059        -0.037        -0.007        -0.055
 GEN           -0.003        -0.010        -0.006        -0.001         0.002

Model Estimated Covariances/Correlations/Residual Correlations
              AGE           GEN
              ________      ________
 AGE            2.095
 GEN           -0.070         0.249

Residuals for Covariances/Correlations/Residual Correlations
              STRESS        EMOTION       NEGAFF        PEER          COA
              ________      ________      ________      ________      ________
 STRESS         0.000
 EMOTION        0.000         0.000
 NEGAFF         0.000         0.000         0.000
 PEER           0.055         0.006         0.000         0.000
 COA            0.000         0.000        -0.004         0.036         0.000
 AGE            0.000         0.000         0.247         0.314         0.000
 GEN            0.000         0.000        -0.042        -0.021         0.000

Residuals for Covariances/Correlations/Residual Correlations
              AGE           GEN
              ________      ________
 AGE            0.000
 GEN            0.000         0.000


Normalized Residuals for Covariances/Correlations/Residual Correlations
              STRESS        EMOTION       NEGAFF        PEER          COA
              ________      ________      ________      ________      ________
 STRESS         0.000
 EMOTION        0.000         0.000
 NEGAFF         0.000         0.000         0.000
 PEER           2.626         0.394         0.000         0.000
 COA            0.000         0.000        -0.159         2.375         0.000
 AGE            0.000         0.000         3.130         6.681         0.000
 GEN            0.000         0.000        -1.541        -1.383         0.000

Normalized Residuals for Covariances/Correlations/Residual Correlations
              AGE           GEN
              ________      ________
 AGE            0.000
 GEN            0.000         0.000
```

The first matrix labeled "`Model Estimated Covariances`" is the model-implied covariance matrix; the second matrix labeled "`Residuals for Covariances`" is the raw difference between the observed and model-implied covariance matrices; the final matrix labeled "`Normalized Residuals for Covariances`" is the matrix of normalized residuals that are scaled to follow a standard normal distribution. These residuals suggest that the model is doing a poor job in reproducing the observed covariances between several variables, most notably between *age* and *peer*, between *age* and *negaff*, and between *stress* and *peer*.

Finally, we can compactly summarize the parameter estimates in path diagram form. The effects from *coa, gen,* and *age* are partially standardized and all others are fully standardized.



Chapter 3
Path Analysis: Part II

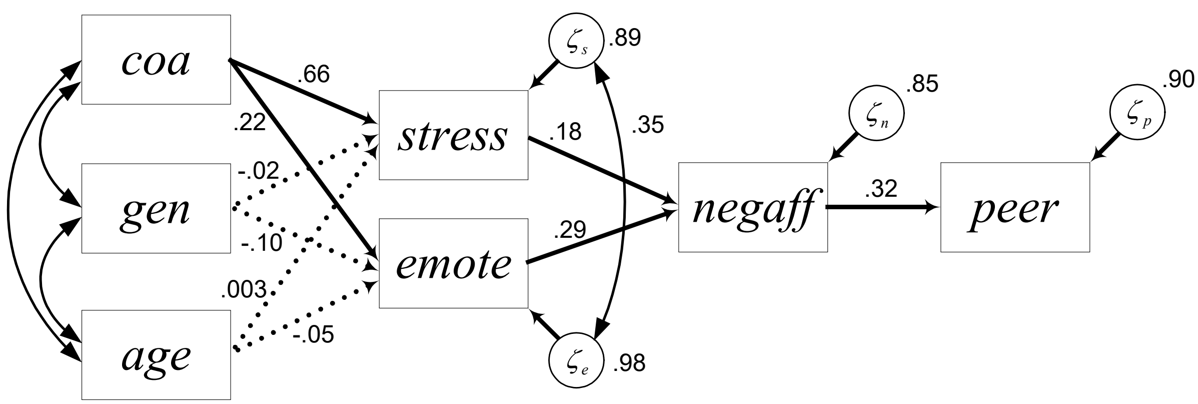
Path Analysis with Deviant Peer Affiliation Data

The data for this demonstration were provided by Dr. Laurie Chassin from the Adolescent and Family Development Project, housed at Arizona State University. ***Note that these data were generously provided for strictly pedagogical purposes and should not be used for any other purposes beyond this workshop.*** The sample includes 316 adolescents, between 10-16 years of age. The study was designed to assess the association between parental alcoholism and adolescent substance use and psychopathology. The data are in the text file **`afdp.dat`**. The variables in the data set that we will use are

`peer`	adolescent report on peer substance use and peer tolerance of use
`coa`	parent report of alcoholism diagnosis where 0=non-alcoholic and 1=alcoholic
`gen`	gender where 0=girl and 1=boy
`age`	age measured in years at assessment
`stress`	self report measure of uncontrollable negative life stressful events
`emotion`	self report measure of temperamental emotional expressiveness
`negaff`	self report measure of depression and anxiety

Theoretical Model

We tested our hypothesized theoretical model of pathways to deviant peer affiliation for COAs and non-COAs. The original model is illustrated below with standardized parameter estimates overlaid on the path diagram:



The input file for this model was shown in Chapter 2 (and is included again here as **`ch03_0.inp`**). We are now ready to discuss model fit.

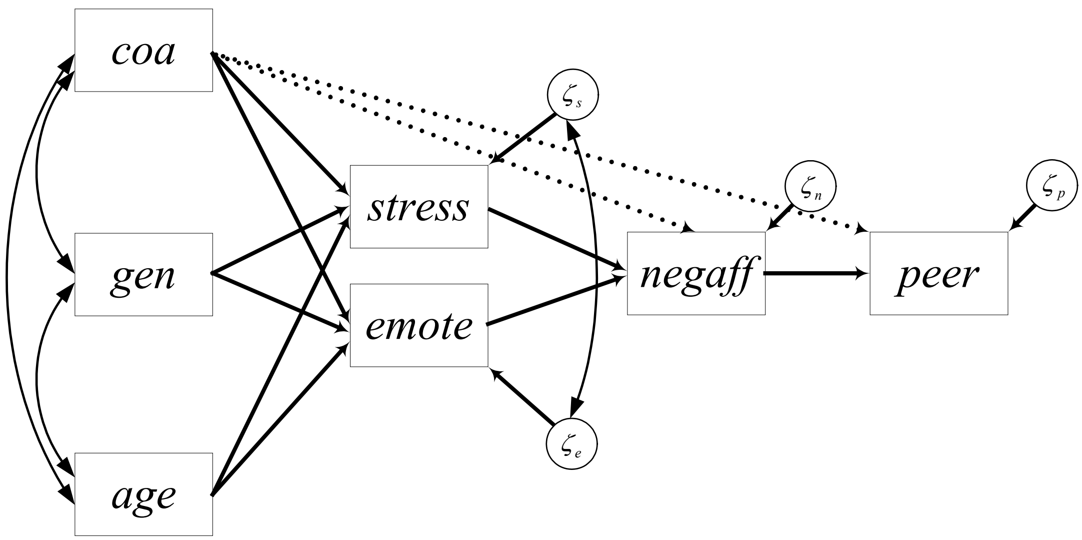
```
Loglikelihood

          H0 Value                        -1158.938
          H1 Value                        -1118.351

Information Criteria

          Akaike (AIC)                     2353.875
          Bayesian (BIC)                   2421.479
          Sample-Size Adjusted BIC         2364.387
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              81.173
          Degrees of Freedom                      8
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.170
          90 Percent C.I.                     0.138  0.205
          Probability RMSEA <= .05            0.000

CFI/TLI

          CFI                                 0.686
          TLI                                 0.293

Chi-Square Test of Model Fit for the Baseline Model

          Value                             251.027
          Degrees of Freedom                     18
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.085
```

Mplus lists the number of free parameters as 18, but recall that it does not count the means, variances, or covariances of the exogenous predictors (**coa**, **gen**, and **age**) as estimated parameters. The number of free parameters reported by Mplus for this model thus does not equal what we computed by the *t*-rule (27 free parameters, including 3 means, 3 variances, and 3 covariances for **coa**, **gen**, and **age**; 16 + 9 = 27). The *t*-rule still works out to 8 degrees of freedom for the chi-square, however, because the observed means, variances, and covariances of the covariates are also not counted in the number of sample means and variances *k*.

Fit statistics indicate that the model does not fit the data well. The chi square-distributed likelihood ratio test of model fit rejects the null hypothesis that the model fits the data. Additionally, the CFI and the TLI are far lower than .9, the standard lower bound for a good-fitting model. Finally, the 90% confidence interval for the RMSEA does not even include .10 at the lower bound, indicating terrible fit. As such, we cannot interpret the obtained parameter estimates with any confidence given the severe misfit of the model.

Modification to Peer Affiliation Model: Likelihood Ratio Test

Since the theoretical model of deviant peer affiliation that was presented in Chapter 2 did not fit the data well, we will consider model modifications to improve our representation of the data. First, theory might suggest that it is an excessively severe restriction to require that the influence of parental alcoholism be conveyed entirely by the mediators. Thus we can allow **`coa`** to directly predict **`negaff`** and **`peer`**, over and above its indirect relationship with these variables via **`stress`** and **`emote`**. The new paths are illustrated with dotted lines in the figure below.



The model is now of the form:

$$\begin{pmatrix} y_{stress_i} \\ y_{emote_i} \\ y_{negaff_i} \\ y_{peer_i} \end{pmatrix} = \begin{pmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \\ \alpha_{4i} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & 0 & 0 \\ \gamma_{41} & 0 & 0 \end{pmatrix} \begin{pmatrix} coa_i \\ gen_i \\ age_i \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta_{31} & \beta_{32} & 0 & 0 \\ 0 & 0 & \beta_{43} & 0 \end{pmatrix} \begin{pmatrix} stress_i \\ emote_i \\ negaff_i \\ peer_i \end{pmatrix} + \begin{pmatrix} \zeta_{1i} \\ \zeta_{2i} \\ \zeta_{3i} \\ \zeta_{4i} \end{pmatrix}$$

where

$$COV(\boldsymbol{\zeta}_i) = \begin{pmatrix} \psi_{11} & & & \\ \psi_{21} & \psi_{22} & & \\ 0 & 0 & \psi_{33} & \\ 0 & 0 & 0 & \psi_{44} \end{pmatrix}$$

The Mplus input file that fits this model is provided in **ch03_1.inp** and is shown below:

```
Title:
 Adding direct effects of COA
Data:
 file=afdp.dat;
Variable:
 names =  id coa age gen stress emote negaff peer;
 usevariables =  coa age gen stress emote negaff peer;
Analysis:
 estimator=ml;
Model:
 peer on negaff coa;
 negaff on stress emote coa;
 stress emote on gen age coa;
 stress with emote;
Output:
 stand;
```

The only difference between this code and the input file in Chapter 2 is that **coa** is included on the right hand side of the `on` statement for **peer** and **negaff**.

Let us now turn to the model fit to determine whether freeing these two regression parameters has led to a significant improvement in model fit.

```
MODEL FIT INFORMATION

Number of Free Parameters                        20

Loglikelihood

          H0 Value                        -1155.511
          H1 Value                        -1118.351

Information Criteria

          Akaike (AIC)                     2351.023
          Bayesian (BIC)                   2426.138
          Sample-Size Adjusted BIC         2362.703
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              74.321
          Degrees of Freedom                      6
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.190
          90 Percent C.I.                     0.153  0.230
          Probability RMSEA <= .05            0.000
```

```
CFI/TLI

          CFI                                0.707
          TLI                                0.120

Chi-Square Test of Model Fit for the Baseline Model

          Value                            251.027
          Degrees of Freedom                    18
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.081
```

The likelihood for the original model (Model A) was $T_A = 81.17$ on 8 degrees of freedom. We have used two addition model parameters for the modified model (Model B), and the likelihood test statistic for this model is 74.32. These models are nested, so we can conduct a Likelihood Ratio Test (LRT) to determine whether Model B fits significantly better than Model A:

$$T_\Delta = T_A - T_B = 81.17 - 74.32 = 6.85$$
$$df_\Delta = df_A - df_B = 8 - 6 = 2$$
$$T_\Delta \sim \chi^2(df_\Delta) \rightarrow p = .033$$

Given that $p < .05$, we can reject the null hypothesis that there is no difference in model fit between Model A and Model B. There is thus support for including the two additional paths. However, given that the sample size is rather large and the effect size (gain in model improvement) is rather small, the LRT suggests only a trivial gain in model fit associated with this modification.

Side notes:

- This computation of the LRT would be invalid if the default estimator in Mplus, MLR (ML with robust standard errors and test statistics), was used to fit the model. With MLR, likelihood ratio tests must be conducted in a different way.
- The command MODEL TEST can also be used to perform tests of multiple parameters. This command produces a multivariate Wald chi-square test that is asymptotically equivalent to the LRT but will differ somewhat in small samples.

Turning to relative tests of model fit, however, we see that the model fit is still terrible. Our attempt to modify the model based upon a priori hypotheses has failed to produce an acceptable model.

Modification to Peer Affiliation Model: Modification Indices

Returning to the original hypothesized model, we will take a different approach to model modification. We can request that Mplus suggest changes to our model based on the expected change in model chi-square if a fixed parameter were freed; these are the modification indices. The corresponding Mplus input file is provided in **ch03_2.inp** and is shown below:

```
Title:
 Initial hypothesized mediational model with MIs;
Data:
 file=afdp.dat;
Variable:
 names =  id coa age gen stress emote negaff peer;
 usevariables =  coa age gen stress emote negaff peer;
Analysis:
 estimator=ml;
Model:
 peer on negaff;
 negaff on stress emote;
 stress emote on gen age coa;
 stress with emote;
Output:
 stand;
 mod;
```

This input file has been only slightly changed from the input filed used in Chapter 2. Now, a `mod` statement is included under the `OUTPUT` command to ask for Mplus to print modification indices. By default, Mplus only shows modification indices that result in an expected change of 10 units or greater in the model chi square value. To over-ride this default, use `mod(x)` where 'x' is the minimum modification index desired. Writing `mod(0)` will show all possible modification indices.

Model modification indices are shown below:

```
MODEL MODIFICATION INDICES

NOTE:  Modification indices for direct effects of observed dependent
variables regressed on covariates may not be included.  To include
these, request MODINDICES (ALL).

Minimum M.I. value for printing the modification index    10.000

                            M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

ON Statements

NEGAFF   ON AGE            11.890     0.119      0.119        0.179
PEER     ON AGE            42.540     0.129      0.129        0.348

WITH Statements

AGE      WITH NEGAFF       10.877     0.236      0.236        0.185
AGE      WITH PEER         44.535     0.275      0.275        0.373
```

`M.I.` denotes the expected improvement in the model chi square value if the modification is accepted. `E.P.C.` denotes the expected parameter value if the modification is implemented.

`Std E.P.C.` re-scales the EPC. according to the variance of latent variables in the model and `StdYX E.P.C.` re-scales the EPC. according to the variance of the latent variables and the exogenous measured variables in the model. Here, `Std E.P.C.` is equivalent to the EPC. because there are no latent variables in the model. It may be useful to rely on standardized EPC. values in order to get a sense of the relative magnitude of each modification.

Note the overlap in suggested modification indices. This tells us that our original model does not allow for a significant relation between **age** and **negaff** or between **age** and **peer**. These suggestions are not independent from one another; allowing **negaff** to relate directly with **age** would imply an increased correlation between **negaff** and **peer**.

It is more theoretically justifiable to regress **peer** on **age** than to regress **negaff** on **age**; thus, we will proceed with this model modification.

After adding **age** as a predictor of **peer** in the Mplus code by including **age** to the right of the `on` statement for **peer** (not shown -- see **ch03_2a.inp**), we obtain the following model fit:

```
MODEL FIT INFORMATION

Number of Free Parameters                        19

Loglikelihood

          H0 Value                        -1135.535
          H1 Value                        -1118.351

Information Criteria

          Akaike (AIC)                     2309.070
          Bayesian (BIC)                   2380.429
          Sample-Size Adjusted BIC         2320.166
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              34.368
          Degrees of Freedom                      7
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.111
          90 Percent C.I.                     0.076  0.150
          Probability RMSEA <= .05            0.003
```

```
CFI/TLI

          CFI                                0.883
          TLI                                0.698

Chi-Square Test of Model Fit for the Baseline Model

          Value                            251.027
          Degrees of Freedom                    18
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.056
```

We can statistically compare the fit of this model with that of the original model using a LRT:

$$T_\Delta = T_A - T_B = 81.17 - 34.37 = 46.80$$
$$df_\Delta = df_A - df_B = 8 - 7 = 1$$
$$T_\Delta \sim \chi^2(df_\Delta) \rightarrow p < .001$$

Thus, including age as a predictor of deviant peer affiliation has significantly improved the fit of the model. However, the relative model fit is still poor. We can again examine the modification indices to determine whether a justifiable model modification would lead to further improvements in model fit.

```
Minimum M.I. value for printing the modification index    10.000

                            M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

ON Statements

NEGAFF   ON AGE            11.886     0.119      0.119         0.179
PEER     ON STRESS         13.381     0.149      0.149         0.192
PEER     ON COA            11.853     0.185      0.185         0.175

WITH Statements

COA      WITH PEER         11.857     0.046      0.046         0.194
AGE      WITH NEGAFF       10.877     0.236      0.236         0.185
```

The MIs continue to suggest that **`negaff`** be directly regressed on **`age`**, even after we have regressed **`peer`** on **`age`**. However, the MIs also suggest regressing **`peer`** on **`coa`**, a modification that we consider to be the most theoretically defensible of the suggested modifications.

Proceeding with this modification (see **`ch03_2b.inp`**), we obtain the following model fit:

```
MODEL FIT INFORMATION

Number of Free Parameters                        20

Loglikelihood

          H0 Value                        -1129.488
          H1 Value                        -1118.351

Information Criteria

          Akaike (AIC)                     2298.977
          Bayesian (BIC)                   2374.092
          Sample-Size Adjusted BIC         2310.657
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              22.274
          Degrees of Freedom                      6
          P-Value                            0.0011

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.093
          90 Percent C.I.                     0.054  0.135
          Probability RMSEA <= .05            0.038

CFI/TLI

          CFI                                 0.930
          TLI                                 0.790

Chi-Square Test of Model Fit for the Baseline Model

          Value                             251.027
          Degrees of Freedom                     18
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.043
```

A LRT would show that the inclusion of a direct path from **`coa`** to **`peer`** results in a significant improvement to the model fit; however, we are still not satisfied with the relative model fit. Once again, we turn to the modification indices.

```
MODEL MODIFICATION INDICES

Minimum M.I. value for printing the modification index    10.000

                            M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

ON Statements

NEGAFF   ON AGE           11.885     0.119      0.119        0.179

WITH Statements

AGE      WITH NEGAFF      10.877     0.236      0.236        0.185
```

Modification indices persist in suggesting that **`age`** is directly related with **`negaff`**. This is the only remaining modification that has an expected change in model fit that is greater than 10. Because this is not an unreasonable modification, we directly regress **`negaff`** on age and re-run the model (see **`ch03_2c.inp`**) and examine the resulting model fit, shown below.

```
MODEL FIT INFORMATION

Number of Free Parameters                       21

Loglikelihood

          H0 Value                       -1123.429
          H1 Value                       -1118.351

Information Criteria

          Akaike (AIC)                    2288.858
          Bayesian (BIC)                  2367.728
          Sample-Size Adjusted BIC        2301.122
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             10.156
          Degrees of Freedom                     5
          P-Value                           0.0709

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.057
          90 Percent C.I.                    0.000  0.108
          Probability RMSEA <= .05           0.345

CFI/TLI

          CFI                                0.978
          TLI                                0.920
```

```
Chi-Square Test of Model Fit for the Baseline Model

          Value                              251.027
          Degrees of Freedom                      18
          P-Value                             0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                0.027
```

By including three data-driven but theoretically acceptable modifications to our original model, we have obtained good model fit. The CFI and the TLI are both above .9 and the 90% confidence interval for the RMSEA includes 0. Note, however, that the confidence interval also includes values greater than .10, so the model fit is not outstanding.

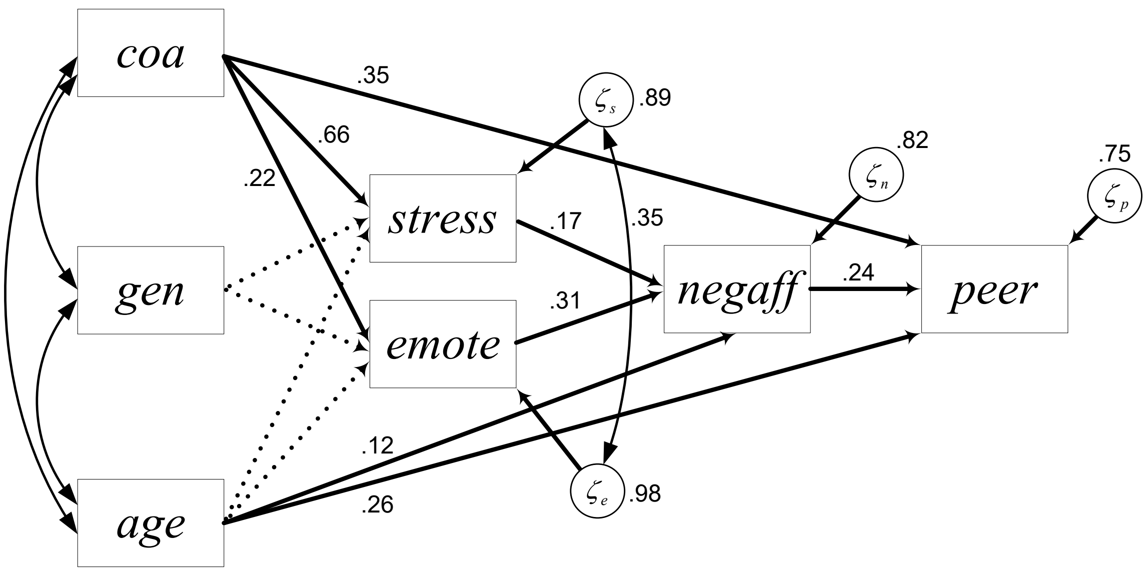
We turn now to the raw and standardized parameter estimates associated with our final model.

```
MODEL RESULTS

                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    NEGAFF             0.137      0.028      4.941      0.000
    AGE                0.138      0.018      7.524      0.000
    COA                0.185      0.053      3.511      0.000

 NEGAFF   ON
    STRESS             0.243      0.077      3.157      0.002
    EMOTION            0.582      0.105      5.568      0.000
    AGE                0.119      0.034      3.515      0.000

 STRESS   ON
    GEN               -0.016      0.073     -0.215      0.830
    AGE                0.002      0.025      0.078      0.938
    COA                0.451      0.072      6.223      0.000

 EMOTION  ON
    GEN               -0.048      0.056     -0.843      0.399
    AGE               -0.027      0.019     -1.374      0.170
    COA                0.110      0.056      1.963      0.050

 STRESS   WITH
    EMOTION            0.112      0.019      5.896      0.000

 Intercepts
    STRESS             0.687      0.333      2.066      0.039
    EMOTION            2.341      0.258      9.088      0.000
    NEGAFF            -0.038      0.489     -0.078      0.938
    PEER              -1.856      0.239     -7.772      0.000
```

```
 Residual Variances
    STRESS              0.412       0.033     12.570      0.000
    EMOTION             0.247       0.020     12.570      0.000
    NEGAFF              0.749       0.060     12.570      0.000
    PEER                0.216       0.017     12.570      0.000

STANDARDIZED MODEL RESULTS


STDYX Standardization

                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    NEGAFF              0.244       0.048      5.058      0.000
    AGE                 0.372       0.046      8.008      0.000
    COA                 0.172       0.049      3.547      0.000

 NEGAFF   ON
    STRESS              0.173       0.054      3.193      0.001
    EMOTION             0.305       0.053      5.787      0.000
    AGE                 0.179       0.050      3.561      0.000

 STRESS   ON
    GEN                -0.011       0.053     -0.215      0.830
    AGE                 0.004       0.053      0.078      0.938
    COA                 0.331       0.050      6.593      0.000

 EMOTION  ON
    GEN                -0.047       0.056     -0.844      0.398
    AGE                -0.077       0.056     -1.378      0.168
    COA                 0.110       0.055      1.974      0.048

 STRESS   WITH
    EMOTION             0.352       0.049      7.131      0.000

 Intercepts
    STRESS              1.010       0.493      2.050      0.040
    EMOTION             4.663       0.529      8.813      0.000
    NEGAFF             -0.040       0.510     -0.078      0.938
    PEER               -3.457       0.385     -8.978      0.000

 Residual Variances
    STRESS              0.890       0.033     26.846      0.000
    EMOTION             0.979       0.016     61.855      0.000
    NEGAFF              0.816       0.039     20.748      0.000
    PEER                0.748       0.042     17.760      0.000
```

```
STDY Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PEER     ON
    NEGAFF             0.244      0.048      5.058      0.000
    AGE                0.257      0.031      8.303      0.000
    COA                0.345      0.096      3.581      0.000

 NEGAFF   ON
    STRESS             0.173      0.054      3.193      0.001
    EMOTE              0.305      0.053      5.787      0.000
    AGE                0.124      0.034      3.595      0.000

 STRESS   ON
    GEN               -0.023      0.107     -0.215      0.830
    AGE                0.003      0.037      0.078      0.938
    COA                0.663      0.098      6.777      0.000

 EMOTE    ON
    GEN               -0.095      0.112     -0.845      0.398
    AGE               -0.053      0.039     -1.380      0.168
    COA                0.219      0.111      1.980      0.048

 STRESS   WITH
    EMOTE              0.352      0.049      7.131      0.000

 Intercepts
    STRESS             1.010      0.493      2.050      0.040
    EMOTE              4.663      0.529      8.813      0.000
    NEGAFF            -0.040      0.510     -0.078      0.938
    PEER              -3.457      0.385     -8.978      0.000

 Residual Variances
    STRESS             0.890      0.033     26.846      0.000
    EMOTE              0.979      0.016     61.855      0.000
    NEGAFF             0.816      0.039     20.748      0.000
    PEER               0.748      0.042     17.760      0.000
```

The final path diagram with standardized estimates overlaid is shown below. Note that, as before, partially standardized estimates (from the `STDY Standardization` output) are reported for regression paths emanating from the exogenous variables, **COA**, **gen**, and **age**, as all of these predictors have meaningful metrics.



We originally hypothesized that **coa** would lead to an increase in uncontrolled stressful life events, and this was supported by the model. COA status leads to a moderately high increase in **stress**. We hypothesized that being a COA would lead to an increase in emotionality, and this was supported: **coa** leads to a small-to-moderate increase in **emote**. We hypothesized that stress and **emote** would increase **negaff**, and model results suggest a small, positive effect of **stress** on **negaff** and a moderate effect of **emote** on **negaff**. Finally, we hypothesized that negative affect would increase affiliation with deviant peers, and we estimated a moderately positive direct relation between **negaff** and **peer**.

Additionally, we found support for a direct effect of **coa** on **peer**, suggesting that **stress**, **emote**, and **negaff** do not fully account for the total relation between COA status and affiliation with deviant peers. Furthermore, age appears to account for a significant amount of the variation in negative affect and affiliation with deviant peers, but age does not appear to be related to uncontrolled stressful life events or emotionality.

Tests of Direct, Indirect, and Specific Effects

Significant links in a mediational pathway are not sufficient to infer mediation. In order to formally test the full mediation effect, we need an inferential test of the entire specific indirect effect in question. Here, we want to know whether the specific mediational pathway of COA status on deviant peer affiliation via stressful events and negative affect is statistically significant, whether the specific mediational effect of **coa** on **peer** via emotionality and negative affect is significant, and whether the overall indirect effect of **coa** on **peer** is significant. Finally, we would like to have an estimate of the total effect of **coa** on **peer** considering both direct and indirect pathways.

The Mplus input for testing the mediational pathways is included in the file **ch03_3.inp** and is shown below:

```
Title:
 Initial hypothesized mediational model with added
 params from MIs and tests of indirect effects;
Data:
 file=afdp.dat;
Variable:
 names =  id coa age gen stress emotion negaff peer;
 usevariables =  coa age gen stress emotion negaff peer;
Analysis:
 estimator=ml;
 bootstrap=1000;
Model:
 peer on negaff age coa; ! age coa added on mi's;
 negaff on stress emotion age; !age added on mi's;
 stress emotion on gen age coa;
 stress with emotion;
Model Indirect:
 peer ind coa;
Output:
 stand cinterval(bootstrap);
```

Within Mplus, it is possible to make comments that do not affect the program by writing an exclamation mark before comments and including a semicolon after the comment is complete. There are three changes with respect to the prior code.

First, we have included `bootstrap=1000;` under the `Analysis` statement; this invokes the bootstrap estimation procedure for all parameters in the model and requests that the data be resampled with replacement 1000 times. Second, we include `cinterval(bootstrap)` under the `Output` statement; this provides bootstrapped confidence intervals of various widths for all parameter estimates.

Finally, the `MODEL INDIRECT` command allows tests of mediational effects. Above, we have specified that we want tests of mediational effects of **coa** on **peer** using the statement `peer ind coa`. As a result, Mplus will provide tests of all direct, indirect, and specific pathways for this mediational effect. If we had included a particular mediation pathway (e.g., `peer ind negaff stress coa`) on the right hand side of the equation, Mplus would only provide information regarding that specific pathway. However, we requested output related to all mediational pathways from **coa** to **peer**.

As we noted in lecture, the current best practices is to conduct inferential tests of indirect effects using bootstrapped confidence intervals. We’ll thus examine the point estimates of the indirect effect and conduct the inferential tests using the bootstrapped CIs.

Model output and parameter estimates have been shown previously. The output corresponding to the indirect and bootstrapped commands is shown below:

```
CONFIDENCE INTERVALS OF TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

                Lower .5%  Lower 2.5%    Lower 5%    Estimate    Upper 5%  Upper 2.5%   Upper .5%

 Effects from COA to PEER

 Total              0.068       0.107       0.125       0.209       0.288       0.303       0.336
 Total indirect     0.005       0.008       0.010       0.024       0.042       0.047       0.055

 Specific indirect

   PEER
   NEGAFF
   STRESS
   COA              0.001       0.003       0.005       0.015       0.029       0.033       0.040

   PEER
   NEGAFF
   EMOTION
   COA             -0.003       0.000       0.001       0.009       0.018       0.021       0.025

 Direct
   PEER
   COA              0.040       0.090       0.104       0.185       0.263       0.281       0.308
```

The total effect of **`coa`** on **`peer`** is equal to .209 and represents a combination of the direct effect (.185) and the indirect effect (.024). The 95% CI is equal to .107 and .303; because this does not contain zero, the total effect is deemed to be significant (p<.05).

Examining the specific mediational pathways, we see that the pathway from **`coa`→`stress`→`negaff`→`peer`** is equal to .015 (95% CI=.003, .033) and is significant. The biological pathway from **`coa`→`emote`→`negaff`→`peer`** is equal to .009 (95% CI=0, .021) and thus does not reach statistical significance (because the lower CI is equal to zero).

In sum, examining the specific indirect mediational pathways has provided a more nuanced understanding of how parent alcoholism is related to their children’s affiliation with deviant peers.

Chapter 4
Confirmatory Factor Analysis

Confirmatory Factor analysis of Holzinger-Swineford Data

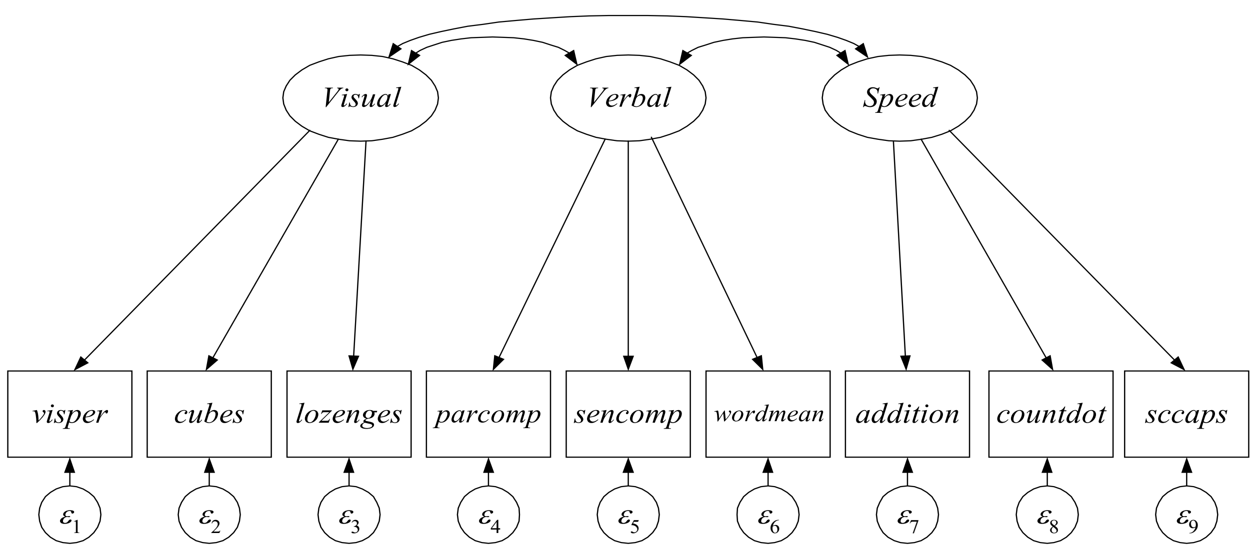
The data for this demonstration were provided by Holzinger & Swineford in their 1939 monograph *A Study in Factor Analysis: The Stability of a Bi-Factor Solution*. The sample includes 301 7th and 8th grade students, between 11-16 years of age, drawn from two schools. The data is in the text file **`hs.dat`**. The variables in the data set that we will use are

`visperc`	visual perception test in which participants select the next image in a series
`cubes`	visual perception test in which participants must mentally rotate a cube
`lozenges`	visual perception test involving mental “flipping” of a parallelogram (“lozenge”)
`parcomp`	paragraph comprehension test
`sencomp`	sentence completion task in which participants select most appropriate word to put at the end of a sentence
`wordmean`	verbal ability test in which participants must select a word most similar in meaning to a word used in a sentence.
`addition`	participants have 2 minutes to complete as many 2-number addition problems as they can
`countdot`	participants have 4 minutes to count the number of dots in each of a series of dot pictures
`sccaps`	participants have 3 minutes to indicate whether capital letters are composed entirely of straight lines or include curved lines.

Other variables in the data not included in the models fit here are **`school`**, **`female`**, **`age`**, and **`month`**.

Initial Model with Standardized Factors

The hypothesized model for the data includes three factors and is shown in the diagram below:



The model is of the form

$$\begin{pmatrix} visper_i \\ cubes_i \\ lozenges_i \\ parcomp_i \\ sencomp_i \\ wordmean_i \\ addition_i \\ countdot_i \\ sccaps_i \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \\ \nu_7 \\ \nu_8 \\ \nu_9 \end{pmatrix} + \begin{pmatrix} \lambda_{11} & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & 0 & 0 \\ 0 & \lambda_{42} & 0 \\ 0 & \lambda_{52} & 0 \\ 0 & \lambda_{62} & 0 \\ 0 & 0 & \lambda_{73} \\ 0 & 0 & \lambda_{83} \\ 0 & 0 & \lambda_{93} \end{pmatrix} \begin{pmatrix} visual_i \\ verbal_i \\ speed_i \end{pmatrix} + \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \\ \varepsilon_{5i} \\ \varepsilon_{6i} \\ \varepsilon_{7i} \\ \varepsilon_{8i} \\ \varepsilon_{9i} \end{pmatrix}$$

$$COV(\boldsymbol{\varepsilon}_i) = \boldsymbol{\Theta} = DIAG(\theta_{11}, \theta_{22}, \theta_{33}, \theta_{44}, \theta_{55}, \theta_{66}, \theta_{77}, \theta_{88}, \theta_{99})$$

$$E(\boldsymbol{\eta}_i) = \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}; \quad COV(\boldsymbol{\eta}_i) = \boldsymbol{\Psi} = \begin{pmatrix} \psi_{11} & & \\ \psi_{21} & \psi_{22} & \\ \psi_{31} & \psi_{32} & \psi_{33} \end{pmatrix}$$

Recall that to identify the model we must set the scale of the latent variables. Two options for doing so are to (1) standardize the latent variables or (2) set the intercept and factor loading of one item per factor to zero and one, respectively. We shall begin with the standardized scaling by setting these parameters as fixed:

$$E(\boldsymbol{\eta}_i) = \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \quad COV(\boldsymbol{\eta}_i) = \boldsymbol{\Psi} = \begin{pmatrix} 1 & & \\ \psi_{21} & 1 & \\ \psi_{31} & \psi_{32} & 1 \end{pmatrix}$$

The Mplus input file that fits this model is provided in **`ch04_1.inp`** and is shown below:

```
TITLE:
  Holzinger-Swineford 3-Factor Model;

DATA:
  FILE IS hs.dat;

VARIABLE:
  NAMES ARE school female age month visperc cubes lozenges
            parcomp sencomp wordmean addition countdot sccaps;
  USEVARIABLES ARE visperc cubes lozenges
            parcomp sencomp wordmean addition countdot sccaps;

DEFINE:
  addition=addition/4;
  countdot=countdot/4;
  sccaps=sccaps/4;
```

```
ANALYSIS:
  ESTIMATOR=ML;

MODEL:
  [visual@0 verbal@0 speed@0];
  visual@1 verbal@1 speed@1;
  visual by visperc* cubes lozenges;
  verbal by parcomp* sencomp wordmean;
  speed by addition* countdot sccaps;

OUTPUT:
  sampstat stdyx mod;
```

We have seen most of these commands before, so here we will highlight only portions of the code, especially new code that involves the use of latent variables.

In the `VARIABLE` section, we have used the `USEVARIABLES` statement to indicate the subset of variables in the data set to be included in our models.

In thee `DEFINE` section, we rescaled the variables **`addition`**, **`countdot`**, and **`sccaps`** by dividing by four. This was done to facilitate model estimation, as numerical instability problems can arise when using variables on widely differing scales. Dividing these variables by four brings their standard deviations closer to the standard deviations of the other observed variables. Note that this linear transformation does **not** affect model fit.

The first two statements in `MODEL` section set the means and variances of the latent variables to zero and one. Note that the `@` symbol indicates a fixed value; that is, it sets a parameter to a specified value and no estimate is produced. Means are referenced by variable names within square brackets, whereas variances are referenced simply by variable names (no brackets).

The following three statements in `MODEL` section indicate which observed variables load on the factors. Factor loadings are indicated via the `BY` statement. The asterisks indicate parameters to be estimated. Note that we use asterisks on the first variable listed for each factor. This is done to override the Mplus default in which the first item on each factor is used to set the scale of the latent factor and thus fixes the loading to a value of one. Because we are manually setting the scale of the factors by fixing the means and variances to 0 and 1, respectively, we do **not** want the first item to be used as a scaling indicator. The other items do not require an asterisk because Mplus assumes the loadings are to be freely estimated (although we could also include asterisks on those variables as well just for completeness; the resulting models would be exactly the same).

In the `OUTPUT` section, we have requested the sample means and covariance matrix, standardized estimates (standardizing both observed, “y”, and latent, “x”, variables), and modification indices.

Let us now turn to the output, considering first the fit indices for the model:

```
MODEL FIT INFORMATION

Number of Free Parameters                           30

Loglikelihood

          H0 Value                           -8326.241
          H1 Value                           -8283.589

Information Criteria

          Akaike (AIC)                       16712.483
          Bayesian (BIC)                     16823.696
          Sample-Size Adjusted BIC           16728.553
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                                 85.306
          Degrees of Freedom                        24
          P-Value                               0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                               0.092
          90 Percent C.I.                        0.071  0.114
          Probability RMSEA <= .05               0.001

CFI/TLI

          CFI                                    0.931
          TLI                                    0.896

Chi-Square Test of Model Fit for the Baseline Model

          Value                                918.852
          Degrees of Freedom                        36
          P-Value                               0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                  0.060
```

We see here that the fit indices indicate that the model does not fit the data particularly well.

The estimates for the model are shown next. These values must be interpreted cautiously, given the lack of fit of the model.

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 VISUAL   BY
    VISPERC            5.398      0.499     10.808      0.000
    CUBES              1.992      0.323      6.164      0.000
    LOZENGES           5.249      0.621      8.458      0.000

 VERBAL   BY
    PARCOMP            2.969      0.170     17.457      0.000
    SENCOMP            4.406      0.250     17.601      0.000
    WORDMEAN           6.416      0.376     17.051      0.000

 SPEED    BY
    ADDITION           3.562      0.427      8.339      0.000
    COUNTDOT           3.655      0.377      9.685      0.000
    SCCAPS             6.029      0.698      8.643      0.000

 VERBAL   WITH
    VISUAL             0.458      0.063      7.224      0.000

 SPEED    WITH
    VISUAL             0.470      0.086      5.456      0.000
    VERBAL             0.283      0.071      3.959      0.000

 Means
    VISUAL             0.000      0.000    999.000    999.000
    VERBAL             0.000      0.000    999.000    999.000
    SPEED              0.000      0.000    999.000    999.000

 Intercepts
    VISPERC           29.615      0.403     73.473      0.000
    CUBES             24.352      0.271     89.855      0.000
    LOZENGES          18.003      0.521     34.579      0.000
    PARCOMP            9.183      0.201     45.694      0.000
    SENCOMP           17.362      0.297     58.452      0.000
    WORDMEAN          15.299      0.441     34.667      0.000
    ADDITION          24.069      0.360     66.767      0.000
    COUNTDOT          27.635      0.291     94.854      0.000
    SCCAPS            48.367      0.523     92.546      0.000

 Variances
    VISUAL             1.000      0.000    999.000    999.000
    VERBAL             1.000      0.000    999.000    999.000
    SPEED              1.000      0.000    999.000    999.000

 Residual Variances
    VISPERC           19.766      4.286      4.612      0.000
    CUBES             18.141      1.668     10.875      0.000
```

```
    LOZENGES          54.034      6.085      8.880      0.000
    PARCOMP            3.341      0.432      7.739      0.000
    SENCOMP            7.140      0.927      7.702      0.000
    WORDMEAN          17.454      2.129      8.199      0.000
    ADDITION          26.428      2.894      9.131      0.000
    COUNTDOT          12.189      2.291      5.320      0.000
    SCCAPS            45.868      7.335      6.253      0.000
```

Note that the z-statistic and p-value for the factor means and variances are all listed as `999.000`. This simply indicates that these parameters were not estimated, and hence no inferential tests were conducted on their values.

Because the items are on different scales, we cannot easily interpret the differences among the factor loadings or residual variances. To better interpret these results, we requested the standardized solution, shown here:

```
STANDARDIZED MODEL RESULTS


STDYX Standardization

                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 VISUAL   BY
    VISPERC            0.772      0.058     13.415      0.000
    CUBES              0.424      0.063      6.752      0.000
    LOZENGES           0.581      0.058      9.942      0.000

 VERBAL   BY
    PARCOMP            0.852      0.023     37.605      0.000
    SENCOMP            0.855      0.022     38.526      0.000
    WORDMEAN           0.838      0.024     35.596      0.000

 SPEED    BY
    ADDITION           0.570      0.058      9.771      0.000
    COUNTDOT           0.723      0.062     11.614      0.000
    SCCAPS             0.665      0.066     10.064      0.000

 VERBAL   WITH
    VISUAL             0.458      0.063      7.224      0.000

 SPEED    WITH
    VISUAL             0.470      0.086      5.456      0.000
    VERBAL             0.283      0.071      3.959      0.000

 Means
    VISUAL             0.000      0.000    999.000    999.000
    VERBAL             0.000      0.000    999.000    999.000
    SPEED              0.000      0.000    999.000    999.000
```

```
 Intercepts
    VISPERC            4.235      0.182     23.272      0.000
    CUBES              5.179      0.219     23.669      0.000
    LOZENGES           1.993      0.100     20.010      0.000
    PARCOMP            2.634      0.122     21.617      0.000
    SENCOMP            3.369      0.149     22.623      0.000
    WORDMEAN           1.998      0.100     20.027      0.000
    ADDITION           3.848      0.167     23.030      0.000
    COUNTDOT           5.467      0.230     23.754      0.000
    SCCAPS             5.334      0.225     23.716      0.000

 Variances
    VISUAL             1.000      0.000    999.000    999.000
    VERBAL             1.000      0.000    999.000    999.000
    SPEED              1.000      0.000    999.000    999.000

 Residual Variances
    VISPERC            0.404      0.089      4.551      0.000
    CUBES              0.821      0.053     15.437      0.000
    LOZENGES           0.662      0.068      9.747      0.000
    PARCOMP            0.275      0.039      7.127      0.000
    SENCOMP            0.269      0.038      7.084      0.000
    WORDMEAN           0.298      0.039      7.546      0.000
    ADDITION           0.676      0.066     10.175      0.000
    COUNTDOT           0.477      0.090      5.298      0.000
    SCCAPS             0.558      0.088      6.350      0.000

R-SQUARE

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    VISPERC            0.596      0.089      6.708      0.000
    CUBES              0.179      0.053      3.376      0.001
    LOZENGES           0.338      0.068      4.971      0.000
    PARCOMP            0.725      0.039     18.803      0.000
    SENCOMP            0.731      0.038     19.263      0.000
    WORDMEAN           0.702      0.039     17.798      0.000
    ADDITION           0.324      0.066      4.885      0.000
    COUNTDOT           0.523      0.090      5.807      0.000
    SCCAPS             0.442      0.088      5.032      0.000
```

The standardized factor loadings can be compared directly (e.g., **`visperc`** is a better indicator than **`cubes`** for the **`visual`** factor). Furthermore, the residual variances are now interpretable as the proportion of variance unexplained by the latent factors (or 1 – communality). The communalities themselves are reported in the section labeled `R-SQUARE`. Items with high communality are generally regarded as better items.

One odd thing to note is that the indicator intercepts are not zero, as you would expect if the observed variables had been standardized by the usual way of deviating the mean and dividing by the standard deviation. In Mplus, the standardized solution merely rescales the observed variables to have standard deviations of one, and does not center the variables to have means of zero. This is of no consequence here, as the mean structure is saturated and of little interest.

Given the poor model fit, we may wish to examine modification indices to get an idea of where the model may be misspecified. The modification indices for the model are shown here:

```
MODEL MODIFICATION INDICES

Minimum M.I. value for printing the modification index    10.000

                            M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

BY Statements

VISUAL   BY ADDITION       18.625    -2.182     -2.182        -0.349
VISUAL   BY SCCAPS         36.417     4.671      4.671         0.515

WITH Statements

COUNTDOT WITH ADDITION     34.124    15.423     15.423         0.859
SCCAPS   WITH COUNTDOT     14.942   -19.038    -19.038        -0.805
```

Here we see the largest modification indices are associated with a cross-loading for **`sccaps`** on **`visual`**, and a correlated uniqueness for **`countdot`** with **`addition`**. It is important to keep in mind that both modification indices may be related to the same misspecification.

Before proceeding to respecify the model, let us also consider how the model would be input into Mplus if we chose to scale the latent variables using scaling items rather than standardizing.

Initial Model with Scaling Items

We shall choose the first indicator for each factor to be the scaling item. The intercept and factor loading for each scaling item is set to zero and one, respectively. This scaling option permits the means and variances of the latent factors to be estimated. The model is thus now specified as

$$\begin{pmatrix} visper_i \\ cubes_i \\ lozenges_i \\ parcomp_i \\ sencomp_i \\ wordmean_i \\ addition_i \\ countdot_i \\ sccaps_i \end{pmatrix} = \begin{pmatrix} 0 \\ \nu_2 \\ \nu_3 \\ 0 \\ \nu_5 \\ \nu_6 \\ 0 \\ \nu_8 \\ \nu_9 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \lambda_{52} & 0 \\ 0 & \lambda_{62} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \lambda_{83} \\ 0 & 0 & \lambda_{93} \end{pmatrix} \begin{pmatrix} visual_i \\ verbal_i \\ speed_i \end{pmatrix} + \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \\ \varepsilon_{5i} \\ \varepsilon_{6i} \\ \varepsilon_{7i} \\ \varepsilon_{8i} \\ \varepsilon_{9i} \end{pmatrix}$$

where

$$COV(\boldsymbol{\varepsilon}_i) = \boldsymbol{\Theta} = DIAG(\theta_{11}, \theta_{22}, \theta_{33}, \theta_{44}, \theta_{55}, \theta_{66}, \theta_{77}, \theta_{88}, \theta_{99})$$

$$E(\boldsymbol{\eta}_i) = \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}; \quad COV(\boldsymbol{\eta}_i) = \boldsymbol{\Psi} = \begin{pmatrix} \psi_{11} & & \\ \psi_{21} & \psi_{22} & \\ \psi_{31} & \psi_{32} & \psi_{33} \end{pmatrix}$$

and all elements in $\boldsymbol{\alpha}$ and $\boldsymbol{\Psi}$ are now estimated. The corresponding Mplus input file is **`ch04_2.inp`** . The only difference in the code from the previous specification is in the `MODEL` section, shown here:

```
MODEL:
  [visual verbal speed];
  visual verbal speed;
  visual by visperc@1 cubes lozenges;
  verbal by parcomp@1 sencomp wordmean;
  speed by addition@1 countdot sccaps;
  [visperc@0 parcomp@0 addition@0];
```

In this specification, the factor means and variances are freely estimated (note absence of `@` symbol in the first two lines). The factor loadings for **`visperc`**, **`parcomp`**, and **`addition`** are set to one (note `@1` in `BY` statements). Finally, in the last line, the intercepts for **`visperc`**, **`parcomp`**, and **`addition`** are set to zero (note `@0` within square brackets). The remaining item intercepts are not referenced because these are freely estimated by default.

[As a side note, the defaults used in Mplus define a hybrid method of scaling to set the metric of the latent factors. For example, we could use the following simplified code:

```
MODEL:
  visual by visperc  cubes lozenges;
  verbal by parcomp  sencomp wordmean;
  speed  by addition countdot sccaps;
```

and this would define a model in which the factor loading for the first item on each factor is fixed to 1.0, the variance of the latent factor is freely estimated, the mean of the latent factor is set to zero, and all item intercepts are freely estimated. The model fit and standardized loadings all remain identical to the models considered here.]

The resulting output for the model code presented in the first box above is:

```
MODEL FIT INFORMATION

Number of Free Parameters                              30

Loglikelihood

          H0 Value                              -8326.241
          H1 Value                              -8283.589
```

```
Information Criteria

          Akaike (AIC)                    16712.483
          Bayesian (BIC)                  16823.696
          Sample-Size Adjusted BIC        16728.553
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              85.306
          Degrees of Freedom                     24
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.092
          90 Percent C.I.                     0.071  0.114
          Probability RMSEA <= .05            0.001

CFI/TLI

          CFI                                 0.931
          TLI                                 0.896

Chi-Square Test of Model Fit for the Baseline Model

          Value                             918.852
          Degrees of Freedom                     36
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.060
```

Note that the tests of model fit are identical to the standardized factor model presented previously. The two models are equivalent, and merely scaled differently.

The difference in scales is apparent when considering the model estimates:

```
MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 VISUAL   BY
    VISPERC            1.000      0.000    999.000    999.000
    CUBES              0.369      0.073      5.067      0.000
    LOZENGES           0.972      0.156      6.220      0.000

 VERBAL   BY
    PARCOMP            1.000      0.000    999.000    999.000
    SENCOMP            1.484      0.087     17.128      0.000
    WORDMEAN           2.161      0.131     16.481      0.000
```

```
 SPEED    BY
    ADDITION           1.000       0.000    999.000    999.000
    COUNTDOT           1.026       0.131      7.851      0.000
    SCCAPS             1.693       0.305      5.543      0.000

 VERBAL   WITH
    VISUAL             7.348       1.434      5.124      0.000

 SPEED    WITH
    VISUAL             9.047       1.911      4.735      0.000
    VERBAL             2.993       0.851      3.518      0.000

 Means
    VISUAL            29.615       0.403     73.473      0.000
    VERBAL             9.183       0.201     45.694      0.000
    SPEED             24.069       0.360     66.766      0.000

 Intercepts
    VISPERC            0.000       0.000    999.000    999.000
    CUBES             13.424       2.173      6.178      0.000
    LOZENGES         -10.797       4.656     -2.319      0.020
    PARCOMP            0.000       0.000    999.000    999.000
    SENCOMP            3.734       0.825      4.524      0.000
    WORDMEAN          -4.545       1.249     -3.639      0.000
    ADDITION           0.000       0.000    999.000    999.000
    COUNTDOT           2.939       3.167      0.928      0.353
    SCCAPS             7.622       7.379      1.033      0.302

 Variances
    VISUAL            29.135       5.391      5.404      0.000
    VERBAL             8.816       1.010      8.729      0.000
    SPEED             12.688       3.044      4.168      0.000

 Residual Variances
    VISPERC           19.766       4.286      4.612      0.000
    CUBES             18.141       1.668     10.875      0.000
    LOZENGES          54.036       6.085      8.881      0.000
    PARCOMP            3.341       0.432      7.739      0.000
    SENCOMP            7.140       0.927      7.703      0.000
    WORDMEAN          17.454       2.129      8.200      0.000
    ADDITION          26.430       2.895      9.130      0.000
    COUNTDOT          12.192       2.292      5.321      0.000
    SCCAPS            45.856       7.337      6.250      0.000
```

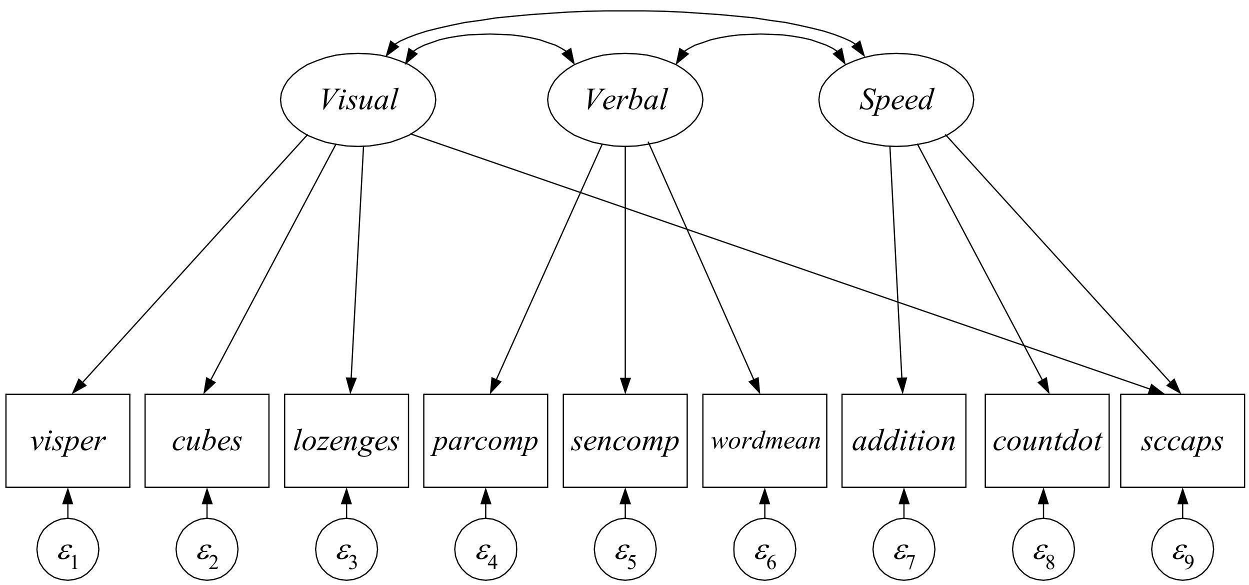
Factor loadings and intercepts are now interpreted in a relative scale, with reference to the scaling item. Further, because the factor variances are no longer set to one, the `WITH` estimates are factor covariances rather than factor correlations.

Again, though, the difference in metric across the tests makes interpretation of differences in intercepts and loadings somewhat difficult. The standardized solution can again be considered to aid in interpretation. We do not present it here as it is identical to the standardized solution shown previously. Similarly, modification indices are no different with this scaling option and are not repeated here.

We now consider respecification of the model, returning to the standardized scaling option to set the metric of the latent variables.

Model Modification

We first introduce a cross loading of **`sccaps`** on **`visual`**, as shown in the diagram below:



The model is now of the form

$$\begin{pmatrix} visper_i \\ cubes_i \\ lozenges_i \\ parcomp_i \\ sencomp_i \\ wordmean_i \\ addition_i \\ countdot_i \\ sccaps_i \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \\ \nu_7 \\ \nu_8 \\ \nu_9 \end{pmatrix} + \begin{pmatrix} \lambda_{11} & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & 0 & 0 \\ 0 & \lambda_{42} & 0 \\ 0 & \lambda_{52} & 0 \\ 0 & \lambda_{62} & 0 \\ 0 & 0 & \lambda_{73} \\ 0 & 0 & \lambda_{83} \\ \lambda_{91} & 0 & \lambda_{93} \end{pmatrix} \begin{pmatrix} visual_i \\ verbal_i \\ speed_i \end{pmatrix} + \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \\ \varepsilon_{5i} \\ \varepsilon_{6i} \\ \varepsilon_{7i} \\ \varepsilon_{8i} \\ \varepsilon_{9i} \end{pmatrix}$$

where

$$COV(\boldsymbol{\varepsilon}_i) = \boldsymbol{\Theta} = DIAG(\theta_{11}, \theta_{22}, \theta_{33}, \theta_{44}, \theta_{55}, \theta_{66}, \theta_{77}, \theta_{88}, \theta_{99})$$

$$E\left(\boldsymbol{\eta}_i\right)=\boldsymbol{\alpha}=\begin{pmatrix}0\\0\\0\end{pmatrix};\quad COV\left(\boldsymbol{\eta}_i\right)=\boldsymbol{\Psi}=\begin{pmatrix}1 & & \\ \psi_{21} & 1 & \\ \psi_{31} & \psi_{32} & 1\end{pmatrix}$$

The difference between the modified model and the original CFA is that the element in 9th row (corresponding to **sccaps**) and 1st column (corresponding to **visual**) of the factor loading matrix has been freed from zero to λ_{91}.

The new cross-loading can be included by modifying the MODEL syntax in Mplus. The Mplus input file that fits this model is provided in **ch04_3.inp**.

```
MODEL:
  [visual@0 verbal@0 speed@0];
  visual@1 verbal@1 speed@1;
  visual by visperc* cubes lozenges sccaps;
  verbal by parcomp* sencomp wordmean;
  speed by addition* countdot sccaps;
```

Notice that **sccaps** now appears twice in the MODEL syntax: once on **speed** and once on **visual**. Factor loadings for **sccaps** are freely estimated for both factors.

The resulting output is shown here:

```
MODEL FIT INFORMATION

Number of Free Parameters                          31

Loglikelihood

          H0 Value                          -8309.780
          H1 Value                          -8283.589

Information Criteria

          Akaike (AIC)                      16681.560
          Bayesian (BIC)                    16796.480
          Sample-Size Adjusted BIC          16698.166
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                                52.382
          Degrees of Freedom                       23
          P-Value                              0.0004

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                              0.065
          90 Percent C.I.                       0.042  0.089
          Probability RMSEA <= .05              0.133
```

```
CFI/TLI

          CFI                                 0.967
          TLI                                 0.948

Chi-Square Test of Model Fit for the Baseline Model

          Value                             918.852
          Degrees of Freedom                     36
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.041
```

Freeing the cross-loading significantly improved the model fit according to a likelihood ratio test of the original CFA and the model estimated above:

$$\chi^2\left(df_{Original} - df_{CrossLoad}\right) = \chi^2{}_{Original} - \chi^2{}_{CrossLoad}$$

$$\chi^2\left(24-23\right) = 85.306 - 52.382$$

$$\chi^2\left(1\right) = 32.924, p < .001$$

Other fit indices suggest that the modified model may have satisfactory fit to the data.

The estimates for the model are shown next. To the extent that we are justified in including the new cross loading, we can have more faith in these estimates due to improved model fit.

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 VISUAL   BY
    VISPERC            5.308      0.464     11.433      0.000
    CUBES              2.045      0.314      6.518      0.000
    LOZENGES           5.333      0.592      9.004      0.000
    SCCAPS             3.480      0.573      6.078      0.000

 VERBAL   BY
    PARCOMP            2.967      0.170     17.443      0.000
    SENCOMP            4.411      0.250     17.627      0.000
    WORDMEAN           6.414      0.376     17.046      0.000

 SPEED    BY
    ADDITION           3.830      0.429      8.923      0.000
    COUNTDOT           4.021      0.378     10.629      0.000
    SCCAPS             4.049      0.610      6.641      0.000
```

```
VERBAL   WITH
   VISUAL            0.453      0.063      7.178      0.000

 SPEED    WITH
   VISUAL            0.301      0.080      3.754      0.000
   VERBAL            0.206      0.071      2.925      0.003

 Means
   VISUAL            0.000      0.000    999.000    999.000
   VERBAL            0.000      0.000    999.000    999.000
   SPEED             0.000      0.000    999.000    999.000

 Intercepts
   VISPERC          29.615      0.403     73.474      0.000
   CUBES            24.352      0.271     89.855      0.000
   LOZENGES         18.003      0.521     34.579      0.000
   PARCOMP           9.183      0.201     45.694      0.000
   SENCOMP          17.362      0.297     58.452      0.000
   WORDMEAN         15.299      0.441     34.667      0.000
   ADDITION         24.069      0.360     66.766      0.000
   COUNTDOT         27.635      0.291     94.854      0.000
   SCCAPS           48.367      0.523     92.546      0.000

 Variances
   VISUAL            1.000      0.000    999.000    999.000
   VERBAL            1.000      0.000    999.000    999.000
   SPEED             1.000      0.000    999.000    999.000

 Residual Variances
   VISPERC          20.724      3.757      5.516      0.000
   CUBES            17.925      1.634     10.969      0.000
   LOZENGES         53.150      5.764      9.221      0.000
   PARCOMP           3.353      0.432      7.770      0.000
   SENCOMP           7.098      0.926      7.662      0.000
   WORDMEAN         17.482      2.128      8.217      0.000
   ADDITION         24.450      2.930      8.344      0.000
   COUNTDOT          9.383      2.467      3.803      0.000
   SCCAPS           45.234      5.000      9.047      0.000

STANDARDIZED MODEL RESULTS

STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 VISUAL   BY
   VISPERC           0.759      0.051     14.774      0.000
   CUBES             0.435      0.060      7.219      0.000
   LOZENGES          0.590      0.054     10.840      0.000
   SCCAPS            0.384      0.060      6.431      0.000
```

```
 VERBAL   BY
    PARCOMP            0.851      0.023     37.556      0.000
    SENCOMP            0.856      0.022     38.651      0.000
    WORDMEAN           0.838      0.024     35.584      0.000

 SPEED    BY
    ADDITION           0.612      0.058     10.646      0.000
    COUNTDOT           0.795      0.061     12.941      0.000
    SCCAPS             0.447      0.063      7.137      0.000

 VERBAL   WITH
    VISUAL             0.453      0.063      7.178      0.000

 SPEED    WITH
    VISUAL             0.301      0.080      3.754      0.000
    VERBAL             0.206      0.071      2.925      0.003

 Means
    VISUAL             0.000      0.000    999.000    999.000
    VERBAL             0.000      0.000    999.000    999.000
    SPEED              0.000      0.000    999.000    999.000

 Intercepts
    VISPERC            4.235      0.182     23.273      0.000
    CUBES              5.179      0.219     23.669      0.000
    LOZENGES           1.993      0.100     20.010      0.000
    PARCOMP            2.634      0.122     21.617      0.000
    SENCOMP            3.369      0.149     22.623      0.000
    WORDMEAN           1.998      0.100     20.027      0.000
    ADDITION           3.848      0.167     23.030      0.000
    COUNTDOT           5.467      0.230     23.754      0.000
    SCCAPS             5.334      0.225     23.716      0.000

 Variances
    VISUAL             1.000      0.000    999.000    999.000
    VERBAL             1.000      0.000    999.000    999.000
    SPEED              1.000      0.000    999.000    999.000

 Residual Variances
    VISPERC            0.424      0.078      5.433      0.000
    CUBES              0.811      0.052     15.464      0.000
    LOZENGES           0.651      0.064     10.128      0.000
    PARCOMP            0.276      0.039      7.154      0.000
    SENCOMP            0.267      0.038      7.049      0.000
    WORDMEAN           0.298      0.039      7.561      0.000
    ADDITION           0.625      0.070      8.873      0.000
    COUNTDOT           0.367      0.098      3.756      0.000
    SCCAPS             0.550      0.059      9.304      0.000
```

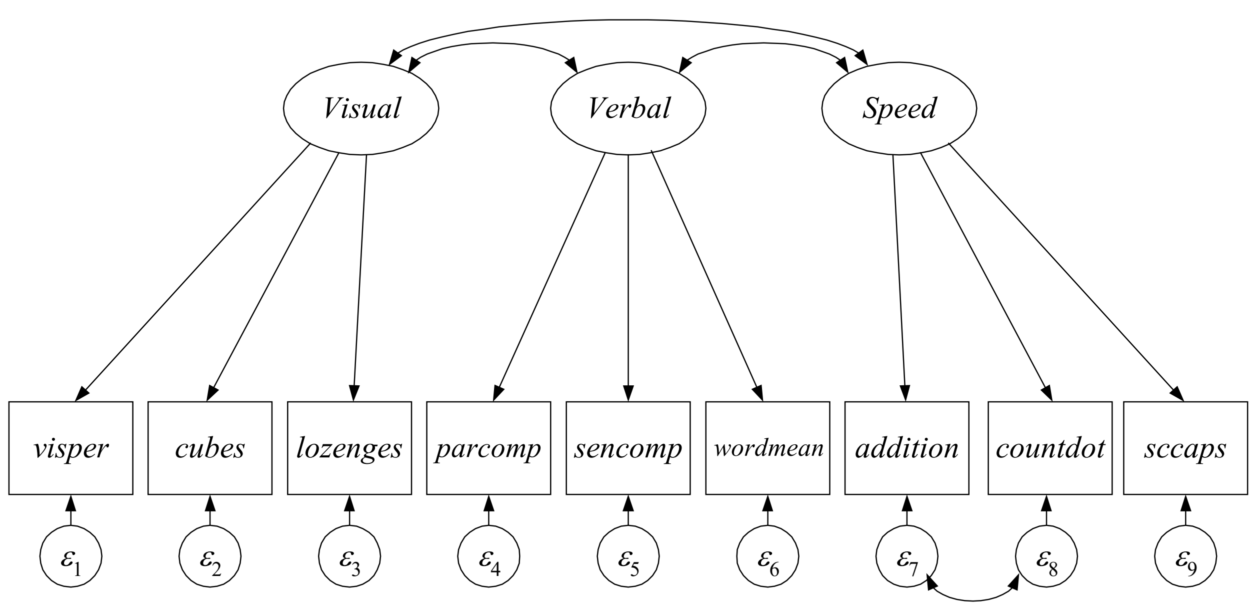
```
R-SQUARE

    Observed                                                Two-Tailed
    Variable         Estimate       S.E.  Est./S.E.    P-Value

    VISPERC             0.576      0.078      7.387      0.000
    CUBES               0.189      0.052      3.610      0.000
    LOZENGES            0.349      0.064      5.420      0.000
    PARCOMP             0.724      0.039     18.778      0.000
    SENCOMP             0.733      0.038     19.325      0.000
    WORDMEAN            0.702      0.039     17.792      0.000
    ADDITION            0.375      0.070      5.323      0.000
    COUNTDOT            0.633      0.098      6.471      0.000
    SCCAPS              0.450      0.059      7.607      0.000
```

In the above, note that **sccaps** loads significantly on **visual**. The residual variance of **sccaps** remains fairly high and the correlation between **visual** and **speed** is still statistically significant. The new modification indices are reported below:

```
MODEL MODIFICATION INDICES

Minimum M.I. value for printing the modification index    10.000

                            M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

No modification indices above the minimum value.
```

Note that no further changes are suggested for the model.

Next, we illustrate how accepting a different model modification might affect conclusions drawn from the model. We allow the residual errors for **addition** and **countdot** to correlate, as shown in the diagram below:



The model is now of the form

$$\begin{pmatrix} visper_i \\ cubes_i \\ lozenges_i \\ parcomp_i \\ sencomp_i \\ wordmean_i \\ addition_i \\ countdot_i \\ sccaps_i \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \\ \nu_7 \\ \nu_8 \\ \nu_9 \end{pmatrix} + \begin{pmatrix} \lambda_{11} & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & 0 & 0 \\ 0 & \lambda_{42} & 0 \\ 0 & \lambda_{52} & 0 \\ 0 & \lambda_{62} & 0 \\ 0 & 0 & \lambda_{73} \\ 0 & 0 & \lambda_{83} \\ 0 & 0 & \lambda_{93} \end{pmatrix} \begin{pmatrix} visual_i \\ verbal_i \\ speed_i \end{pmatrix} + \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \\ \varepsilon_{5i} \\ \varepsilon_{6i} \\ \varepsilon_{7i} \\ \varepsilon_{8i} \\ \varepsilon_{9i} \end{pmatrix}$$

where

$$COV(\boldsymbol{\varepsilon}_i) = \boldsymbol{\Theta} = \begin{pmatrix} \theta_{11} & & & & & & & & \\ 0 & \theta_{22} & & & & & & & \\ 0 & 0 & \theta_{33} & & & & & & \\ 0 & 0 & 0 & \theta_{44} & & & & & \\ 0 & 0 & 0 & 0 & \theta_{55} & & & & \\ 0 & 0 & 0 & 0 & 0 & \theta_{66} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{77} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{87} & \theta_{88} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{99} \end{pmatrix}$$

$$E(\boldsymbol{\eta}_i) = \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \quad COV(\boldsymbol{\eta}_i) = \boldsymbol{\Psi} = \begin{pmatrix} 1 & & \\ \psi_{21} & 1 & \\ \psi_{31} & \psi_{32} & 1 \end{pmatrix}$$

The difference between the modified model and the original CFA is that $\boldsymbol{\Theta}$ is no longer diagonal; it contains a covariance between the residual terms for **`addition`** and **`countdot`**. This change can be included in the model by modifying the MODEL syntax in Mplus. The Mplus input file that fits this model is provided in **`ch04_4.inp`**.

```
MODEL:
  [visual@0 verbal@0 speed@0];
  visual@1 verbal@1 speed@1;
  visual by visperc* cubes lozenges;
  verbal by parcomp* sencomp wordmean;
  speed by addition* countdot sccaps;
  addition with countdot;
```

The difference between this input and the original input is the final statement: "`addition with countdot;`" In Mplus, when observed variable names are joined by a `with` statement, it indicates that the residuals associated with those variables should be correlated. The output associated with the above input is shown below.

```
MODEL FIT INFORMATION

Number of Free Parameters                        31

Loglikelihood

          H0 Value                        -8310.225
          H1 Value                        -8283.589

Information Criteria

          Akaike (AIC)                    16682.450
          Bayesian (BIC)                  16797.370
          Sample-Size Adjusted BIC        16699.056
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              53.272
          Degrees of Freedom                     23
          P-Value                            0.0003

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.066
          90 Percent C.I.                     0.043  0.090
          Probability RMSEA <= .05            0.118

CFI/TLI

          CFI                                 0.966
          TLI                                 0.946

Chi-Square Test of Model Fit for the Baseline Model

          Value                             918.852
          Degrees of Freedom                     36
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.043
```

Allowing a single residual correlation among items resulted in a statistically significantly improvement in model fit according to the chi-square difference test:

$$\chi^2\left(df_{Original} - df_{ResCorr}\right) = \chi^2{}_{Original} - \chi^2{}_{ResCorr}$$
$$\chi^2\left(24 - 23\right) = 85.306 - 53.272$$
$$\chi^2\left(1\right) = 32.034, p < .001$$

Other fit indices suggest that the modified model may have satisfactory fit to the data.

The estimates for the model are shown next. To the extent that we are justified in including the added cross loading, we can have more faith in these estimates due to improved model fit.

This model is not nested with the alternative modified model; however, the two models provide nearly equivalent fit (RMSEA=.065 versus .066; TLI=.948 versus .946). Thus, model selection should be based on which modification is most plausible, and upon the interpretability of parameter estimates.

```
MODEL RESULTS

                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 VISUAL   BY
    VISPERC            5.308      0.462     11.488      0.000
    CUBES              2.037      0.313      6.510      0.000
    LOZENGES           5.323      0.591      9.003      0.000

 VERBAL   BY
    PARCOMP            2.967      0.170     17.445      0.000
    SENCOMP            4.411      0.250     17.620      0.000
    WORDMEAN           6.413      0.376     17.040      0.000

 SPEED    BY
    ADDITION           2.202      0.397      5.552      0.000
    COUNTDOT           2.383      0.387      6.150      0.000
    SCCAPS             8.667      0.996      8.702      0.000

 VERBAL   WITH
    VISUAL             0.457      0.064      7.109      0.000

 SPEED    WITH
    VISUAL             0.544      0.078      6.935      0.000
    VERBAL             0.270      0.068      3.942      0.000

 ADDITION WITH
    COUNTDOT          10.140      1.906      5.320      0.000

 Means
    VISUAL             0.000      0.000    999.000    999.000
    VERBAL             0.000      0.000    999.000    999.000
    SPEED              0.000      0.000    999.000    999.000
```

```
 Intercepts
    VISPERC           29.615      0.403     73.473      0.000
    CUBES             24.352      0.271     89.855      0.000
    LOZENGES          18.003      0.521     34.579      0.000
    PARCOMP            9.183      0.201     45.694      0.000
    SENCOMP           17.362      0.297     58.452      0.000
    WORDMEAN          15.299      0.441     34.667      0.000
    ADDITION          24.069      0.360     66.766      0.000
    COUNTDOT          27.635      0.291     94.855      0.000
    SCCAPS            48.367      0.523     92.547      0.000

 Variances
    VISUAL             1.000      0.000    999.000    999.000
    VERBAL             1.000      0.000    999.000    999.000
    SPEED              1.000      0.000    999.000    999.000

 Residual Variances
    VISPERC           20.730      3.725      5.565      0.000
    CUBES             17.960      1.632     11.005      0.000
    LOZENGES          53.255      5.752      9.259      0.000
    PARCOMP            3.350      0.432      7.757      0.000
    SENCOMP            7.099      0.928      7.648      0.000
    WORDMEAN          17.496      2.129      8.217      0.000
    ADDITION          34.268      2.914     11.761      0.000
    COUNTDOT          19.870      2.079      9.560      0.000
    SCCAPS             7.087     15.934      0.445      0.656
```

Note that the newly freed parameter, "`addition with countdots`," is statistically significantly different from zero. Note also the substantial reduction in residual variance estimates for items loading on **speed**. Specifically, even though **sccaps** is not directly involved in the residual correlation between **addition** and **countdot**, its residual variance was substantially reduced.

By accounting for the local dependence between **addition** and **countdot**, **speed** was able to account for more of the variance in **sccaps**. This is further evidenced by the fact that **sccaps** is now the highest loading on the factor.

```
STANDARDIZED MODEL RESULTS


STDYX Standardization

                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 VISUAL   BY
    VISPERC            0.759      0.051     14.894      0.000
    CUBES              0.433      0.060      7.210      0.000
    LOZENGES           0.589      0.054     10.841      0.000
```

```
 VERBAL   BY
    PARCOMP            0.851      0.023     37.548      0.000
    SENCOMP            0.856      0.022     38.582      0.000
    WORDMEAN           0.838      0.024     35.549      0.000

 SPEED    BY
    ADDITION           0.352      0.059      5.991      0.000
    COUNTDOT           0.471      0.070      6.712      0.000
    SCCAPS             0.956      0.101      9.424      0.000

 VERBAL   WITH
    VISUAL             0.457      0.064      7.109      0.000

 SPEED    WITH
    VISUAL             0.544      0.078      6.935      0.000
    VERBAL             0.270      0.068      3.942      0.000

 ADDITION WITH
    COUNTDOT           0.389      0.054      7.241      0.000

 Means
    VISUAL             0.000      0.000    999.000    999.000
    VERBAL             0.000      0.000    999.000    999.000
    SPEED              0.000      0.000    999.000    999.000

 Intercepts
    VISPERC            4.235      0.182     23.272      0.000
    CUBES              5.179      0.219     23.669      0.000
    LOZENGES           1.993      0.100     20.010      0.000
    PARCOMP            2.634      0.122     21.617      0.000
    SENCOMP            3.369      0.149     22.624      0.000
    WORDMEAN           1.998      0.100     20.027      0.000
    ADDITION           3.848      0.167     23.030      0.000
    COUNTDOT           5.467      0.230     23.754      0.000
    SCCAPS             5.334      0.225     23.717      0.000

 Variances
    VISUAL             1.000      0.000    999.000    999.000
    VERBAL             1.000      0.000    999.000    999.000
    SPEED              1.000      0.000    999.000    999.000

 Residual Variances
    VISPERC            0.424      0.077      5.480      0.000
    CUBES              0.812      0.052     15.606      0.000
    LOZENGES           0.653      0.064     10.187      0.000
    PARCOMP            0.276      0.039      7.143      0.000
    SENCOMP            0.267      0.038      7.038      0.000
    WORDMEAN           0.298      0.039      7.562      0.000
    ADDITION           0.876      0.041     21.166      0.000
    COUNTDOT           0.778      0.066     11.743      0.000
    SCCAPS             0.086      0.194      0.445      0.657
```

```
R-SQUARE

    Observed                                              Two-Tailed
    Variable         Estimate       S.E.  Est./S.E.    P-Value

    VISPERC             0.576      0.077      7.447      0.000
    CUBES               0.188      0.052      3.605      0.000
    LOZENGES            0.347      0.064      5.420      0.000
    PARCOMP             0.724      0.039     18.774      0.000
    SENCOMP             0.733      0.038     19.291      0.000
    WORDMEAN            0.702      0.039     17.774      0.000
    ADDITION            0.124      0.041      2.995      0.003
    COUNTDOT            0.222      0.066      3.356      0.001
    SCCAPS              0.914      0.194      4.712      0.000
```

Standardized results indicate that `addition` and `countdots` are moderately correlated above and beyond the correlation implied by the common `speed` factor (r = .389).

As with the previous example, introducing this single new parameter resulted in no further sizeable modification indices.

```
MODEL MODIFICATION INDICES

Minimum M.I. value for printing the modification index    10.000

                            M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

No modification indices above the minimum value.
```

Chapter 5
Structural Equation Models with Latent Variables

Structural Equation Modeling of Şenol-Durak and Ayvaşik's Posttraumatic Growth Data

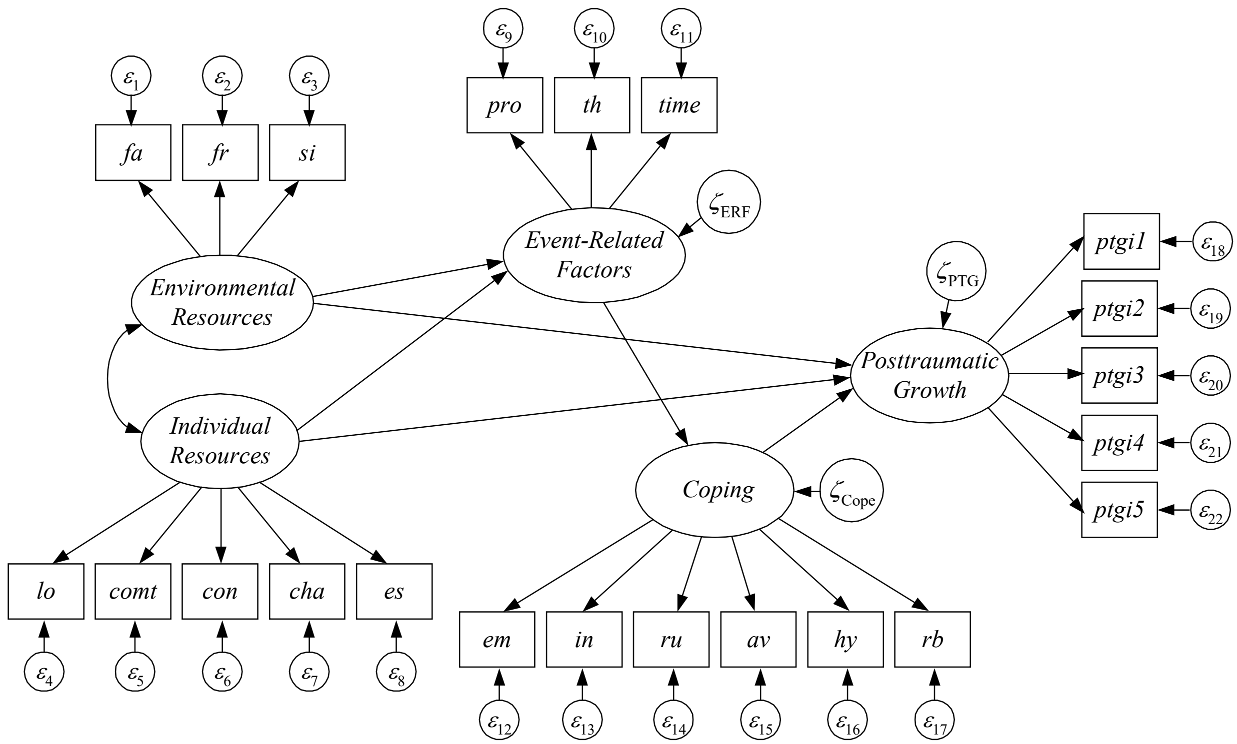
The data for this demonstration were provided by Şenol-Durak & Ayvaşik in their 2010 *Journal of Health Psychology* manuscript, "Factors associated with posttraumatic growth among the spouses of myocardial infraction patients." The sample includes 132 spouses of myocardial infarction patients. The correlation matrix as well as the means and standard deviations for the variables were provided by the authors. This information is in the text file **`mip.dat`**. The variables in the data set that we will use are

`fa`	social support from family	
`fr`	social support from friends	*Environmental Resources*
`si`	social support from significant others	
`lo`	Locus of Control Scale score	
`comt`	Commitment score	
`con`	Control score	*Individual Resources*
`cha`	Challenge score	
`es`	Rosenberg Self Esteem Scale score	
`pro`	subjective evaluation of prognosis	
`th`	threat to future health	*Event Related Factors*
`time`	time since diagnosis	
`em`	emotion focused coping	
`in`	indirect coping	
`ru`	rumination	*Cognitive Process Coping*
`av`	avoidance	
`hy`	hypervigilance	
`rb`	religious beliefs	
`ptgi1`	improved relationships	
`ptgi2`	new possibilities for one's life	
`ptgi3`	greater appreciation of life	*Posttraumatic Growth*
`ptgi4`	greater sense of personal strength	
`ptgi5`	spiritual development	

Refer to the article for definitions of variables not included in the model.

Initial Hypothesized Model

The hypothesized model for the data predicts that both individual and environmental resources directly lead to increased posttraumatic growth, but also indirectly lead to somewhat decreased posttraumatic growth by reducing event-related hardship, thus decreasing the need for coping and reducing opportunities for posttraumatic growth. The hypothesized model also predicts that neither environmental nor individual resources have a direct impact on cognitive coping. Further, the effect of event-related factors on posttraumatic growth is hypothesized to be purely mediated by coping.



We can also express the model using matrix algebra, as shown on the next page.

The measurement model is:

$$\begin{bmatrix} fa_i \\ fr_i \\ si_i \\ lo_i \\ comt_i \\ con_i \\ cha_i \\ es_i \\ pro_i \\ th_i \\ time_i \\ em_i \\ in_i \\ ru_i \\ av_i \\ hy_i \\ rb_i \\ ptgi_{1i} \\ ptgi_{2i} \\ ptgi_{3i} \\ ptgi_{4i} \\ ptgi_{5i} \end{bmatrix} = \begin{bmatrix} \nu_{1i} \\ \nu_{2i} \\ \nu_{3i} \\ \nu_{4i} \\ \nu_{5i} \\ \nu_{6i} \\ \nu_{7i} \\ \nu_{8i} \\ \nu_{9i} \\ \nu_{10i} \\ \nu_{11i} \\ \nu_{12i} \\ \nu_{13i} \\ \nu_{14i} \\ \nu_{15i} \\ \nu_{16i} \\ \nu_{17i} \\ \nu_{18i} \\ \nu_{19i} \\ \nu_{20i} \\ \nu_{21i} \\ \nu_{22i} \end{bmatrix} + \begin{bmatrix} \lambda_{11} & 0 & 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 & 0 & 0 \\ \lambda_{31} & 0 & 0 & 0 & 0 \\ 0 & \lambda_{42} & 0 & 0 & 0 \\ 0 & \lambda_{52} & 0 & 0 & 0 \\ 0 & \lambda_{62} & 0 & 0 & 0 \\ 0 & \lambda_{72} & 0 & 0 & 0 \\ 0 & \lambda_{82} & 0 & 0 & 0 \\ 0 & 0 & \lambda_{93} & 0 & 0 \\ 0 & 0 & \lambda_{10,3} & 0 & 0 \\ 0 & 0 & \lambda_{11,3} & 0 & 0 \\ 0 & 0 & 0 & \lambda_{12,4} & 0 \\ 0 & 0 & 0 & \lambda_{13,4} & 0 \\ 0 & 0 & 0 & \lambda_{14,4} & 0 \\ 0 & 0 & 0 & \lambda_{15,4} & 0 \\ 0 & 0 & 0 & \lambda_{16,4} & 0 \\ 0 & 0 & 0 & \lambda_{17,4} & 0 \\ 0 & 0 & 0 & 0 & \lambda_{18,5} \\ 0 & 0 & 0 & 0 & \lambda_{19,5} \\ 0 & 0 & 0 & 0 & \lambda_{20,5} \\ 0 & 0 & 0 & 0 & \lambda_{21,5} \\ 0 & 0 & 0 & 0 & \lambda_{22,5} \end{bmatrix} \begin{bmatrix} \eta_{ERi} \\ \eta_{IRi} \\ \eta_{ERFi} \\ \eta_{COPE_i} \\ \eta_{PTGi} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \\ \varepsilon_{5i} \\ \varepsilon_{6i} \\ \varepsilon_{7i} \\ \varepsilon_{8i} \\ \varepsilon_{9i} \\ \varepsilon_{10i} \\ \varepsilon_{11i} \\ \varepsilon_{12i} \\ \varepsilon_{13i} \\ \varepsilon_{14i} \\ \varepsilon_{15i} \\ \varepsilon_{16i} \\ \varepsilon_{17i} \\ \varepsilon_{18i} \\ \varepsilon_{19i} \\ \varepsilon_{20i} \\ \varepsilon_{21i} \\ \varepsilon_{22i} \end{bmatrix}$$

where $\mathbf{\Theta} = DIAG\left(\theta_{11}, \theta_{22}, \ldots, \theta_{22,22}\right)$

The latent variable model is:

$$\begin{bmatrix} \eta_{ERi} \\ \eta_{IRi} \\ \eta_{ERFi} \\ \eta_{COPE_i} \\ \eta_{PTGi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \beta_{31} & \beta_{32} & 0 & 0 & 0 \\ 0 & 0 & \beta_{43} & 0 & 0 \\ \beta_{51} & \beta_{52} & 0 & \beta_{54} & 0 \end{bmatrix} \begin{bmatrix} \eta_{ERi} \\ \eta_{IRi} \\ \eta_{ERFi} \\ \eta_{COPE_i} \\ \eta_{PTGi} \end{bmatrix} + \begin{bmatrix} \zeta_{ERi} \\ \zeta_{IRi} \\ \zeta_{ERFi} \\ \zeta_{COPE_i} \\ \zeta_{PTGi} \end{bmatrix}$$

where $\mathbf{\Psi} = \begin{bmatrix} 1 & & & & \\ \psi_{21} & 1 & & & \\ 0 & 0 & 1 & & \\ 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Note that the means/intercepts and (residual) variances of the factors have been fixed to 0 and 1, respectively to scale the latent variables. In class, we used the *t*-rule and the two-step rule to verify that the model is identified. We can thus go on to specify the model in Mplus.

The Mplus input file that fits this model is provided in **`ch05_1.inp`** and is shown below:

```
TITLE:
  Senol-Durak & Ayvasik SEM;

DATA:
  FILE IS mip.dat;
  TYPE IS FULLCORR MEANS STDEVIATIONS;
  NOBSERVATIONS=132;

VARIABLE:
  NAMES ARE ptgi ptgi1 ptgi2 ptgi3 ptgi4 ptgi5 marital fa fr si child
   child18 age gender depres comt con cha es lo pro th diord time
   problem em in ru av hy relipart rb;

  USEVARIABLES ARE fa fr si lo comt con cha es
            pro th time em in ru av hy rb
            ptgi1 ptgi2 ptgi3 ptgi4 ptgi5;

ANALYSIS:
  ESTIMATOR=ML;

MODEL:
  ER by fa* fr si;
  IR by lo*-1 comt con cha es;
  ERF by pro*1 th*-1 time*1;
  CPP by em*1 in*-1 ru av hy rb;
  PTG by ptgi1* ptgi2 ptgi3 ptgi4 ptgi5;
  [fa fr si];
  [lo comt con cha es];
  [pro th time];
  [em in ru av hy rb];
  [ptgi1 ptgi2 ptgi3 ptgi4 ptgi5];
  [ER@0 IR@0 ERF@0 CPP@0 PTG@0];
  ER@1 IR@1 ERF@1 CPP@1 PTG@1;
  ER with IR;
  ERF on ER IR;
  CPP on ERF;
  PTG on ER IR CPP;

OUTPUT:
  sampstat stdyx mod;
```

We have seen most of these commands before, so here we will highlight only portions of the code.

Mplus accepts either raw data or summary data (i.e., means, standard deviations, and correlations among variables). As a default, Mplus assumes that data are in raw format. Since the data from this example are in summary form, we used the `DATA` command to tell the program that we are inputting a correlation matrix by writing: `TYPE IS FULLCORR MEANS STDEVIATIONS`. We then included the sample size with `NOBSERVATIONS`.

As before, the measurement models are specified under `MODEL` using the `by` statements. Here, we have used asterisks to allow all factor loadings to be freely estimated. Mplus defaults to fixing the first factor loading to 1, so it is only necessary to place an asterisk next to the first loading on each factor in order to ensure that all factor loadings are freely estimated.

In the journal article, some of the estimated factor loadings were negative. Due to rotational indeterminacy, a measurement model will fit equally well if all of its factor loadings are directionally flipped such that positive loadings are negative and negative loadings are positive. We provided starting values to ensure the model would converge to the most interpretable solution. Thus, for example, event related factors (**`ERF`**) was given positive starting values for **`pro`** and **`time`** and a negative starting value for **`th`** so that this factor has a positive valence (higher values are better, e.g., indicating perception of better prognosis and less life threat). This was achieved by placing a starting value after an asterisk (e.g., 1 or -1).

We have standardized the factors in order to identify the measurement models. Factor means (and intercepts for endogenous factors) were fixed to zero by placing `@0` next to each factor name inside of square brackets. In Mplus, square brackets denote means and intercepts and `@` is used to constrain parameters to a fixed value. Factor variances (and residual variances for endogenous factors) were constrained to 1 by placing an `@1` next to each factor name on a line without brackets.

Structural covariances are specified in Mplus using the `with` statement, and regression parameters are specified using the `on` statement (outcome variable `on` predictor variable).

We have requested sample statistics (`sampstat`), the typical standardized solution (with both the predictor and outcome standardizes; `stdyx`), and modification indices (`mod`) under the `OUTPUT` command.

Let us now turn to the fit indices for the model:

```
Number of Free Parameters                              73

Loglikelihood

          H0 Value                              -8012.004
          H1 Value                              -7837.004

Information Criteria

          Akaike (AIC)                          16170.009
          Bayesian (BIC)                        16380.453
          Sample-Size Adjusted BIC              16149.553
            (n* = (n + 2) / 24)
```

```
Chi-Square Test of Model Fit

          Value                              350.000
          Degrees of Freedom                     202
          P-Value                             0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                             0.075
          90 Percent C.I.                      0.061  0.087
          Probability RMSEA <= .05             0.002

CFI/TLI

          CFI                                  0.848
          TLI                                  0.826

Chi-Square Test of Model Fit for the Baseline Model

          Value                             1205.231
          Degrees of Freedom                     231
          P-Value                             0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                0.101
```

The fit indices indicate that the model does not fit the data well. Rather than interpreting the parameter estimates, we will examine modification indices to get a sense for what might be causing the model to fit poorly, keeping in mind that any model modification must be theoretically justifiable.

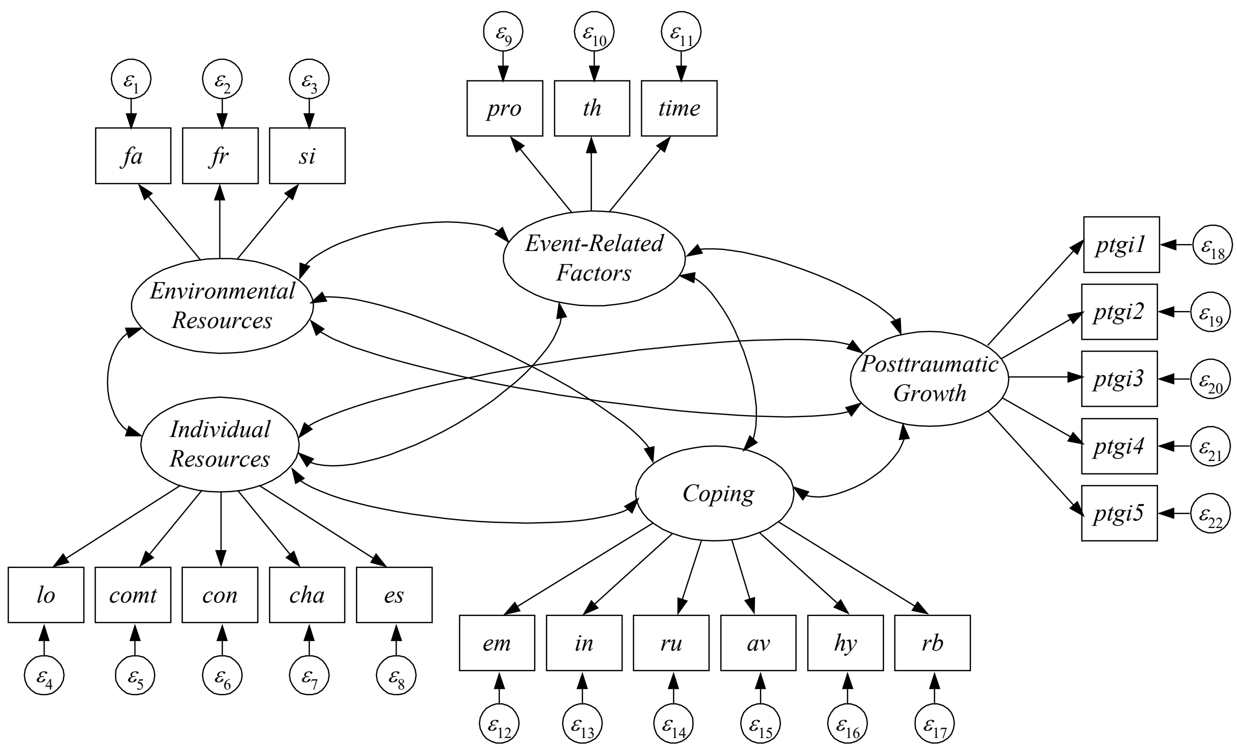
```
Minimum M.I. value for printing the modification index    10.000

                            M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

BY Statements

IR       BY PTGI5          11.877    -0.730     -0.730        -0.241
PTG      BY CHA            10.321     0.590      0.688         0.282

WITH Statements

CHA      WITH COMT         27.844     2.684      2.684         0.546
CHA      WITH CON          12.343    -2.215     -2.215        -0.381
EM       WITH CON          10.695    -8.717     -8.717        -0.304
IN       WITH FA           14.250    -7.574     -7.574        -0.333
IN       WITH EM           21.340   -26.655    -26.655        -0.410
HY       WITH RU           32.248    34.384     34.384         4.461
```

Modification indices suggest that the largest improvement to the model chi-square could be achieved by allowing hypervigilance to correlate with rumination, over and above the correlation implied by the coping factor, allowing challenge to correlate with commitment over and above the individual resources factor, and allowing indirect and emotional coping to correlate above and beyond the correlation implied by the coping factor. These modifications reflect misspecification in the measurement model.

Confirmatory Factor Analysis

When building a structural equation model, a useful strategy to avoid complex misspecification is to begin by ensuring that the simplest foundation of the overall model, the measurement model, is correctly specified. Once the measurement model has been properly specified, the next step is to incorporate structural parameters. Thus, we turn next to a CFA with saturated covariances among factors. This strategy will allow us to get measurement right so that measurement misspecification is not confounded with structural misfit.

The CFA model is provided in **`ch05_2.inp`**, shown below.



We omit discussion of the input file because CFA estimation was discussed in the previous chapter. The resulting model fit is shown below.

```
MODEL FIT INFORMATION

Number of Free Parameters                        76

Loglikelihood

          H0 Value                        -8011.322
          H1 Value                        -7837.004

Information Criteria

          Akaike (AIC)                    16174.645
          Bayesian (BIC)                  16393.738
          Sample-Size Adjusted BIC        16153.348
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             348.636
          Degrees of Freedom                    199
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.075
          90 Percent C.I.                     0.062  0.088
          Probability RMSEA <= .05            0.001

CFI/TLI

          CFI                                 0.846
          TLI                                 0.822

Chi-Square Test of Model Fit for the Baseline Model

          Value                            1205.231
          Degrees of Freedom                    231
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.099
```

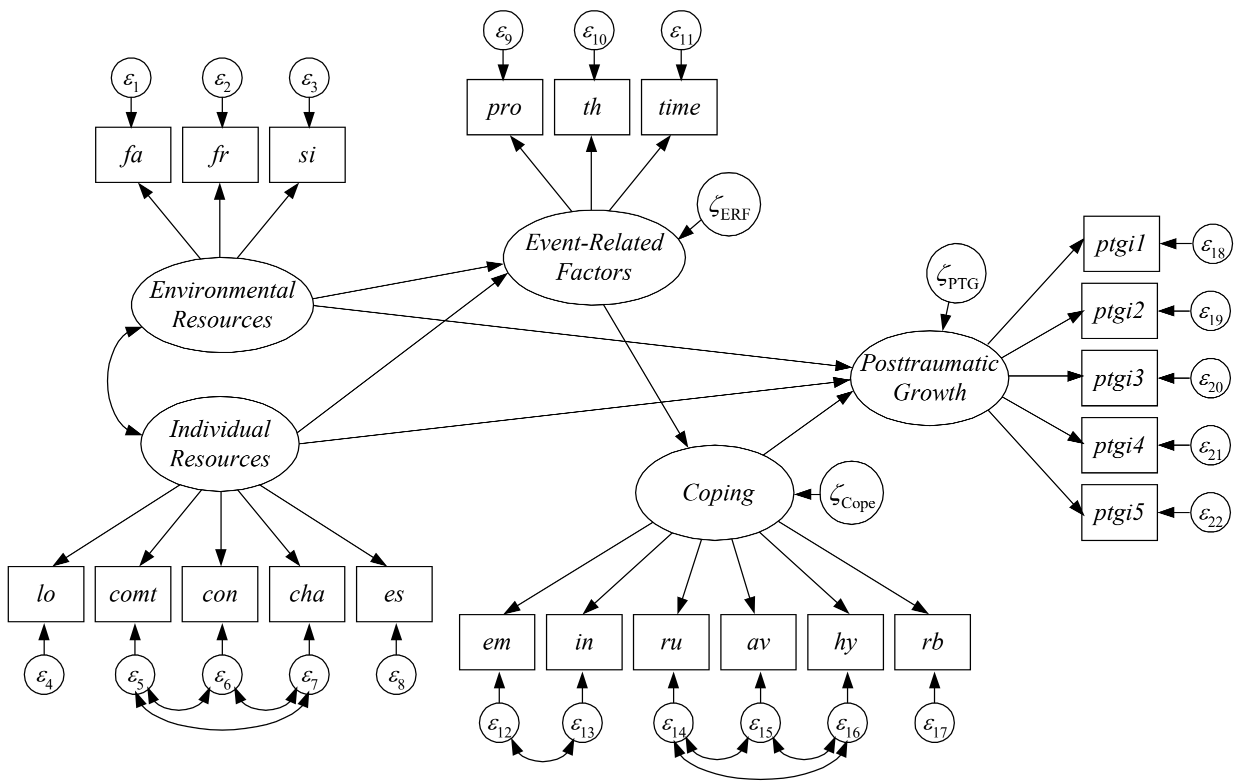
The model still does not fit the data well, confirming our hypothesis that the measurement model, and not the structural model, is misspecified. Indeed, we can conduct a likelihood ratio test comparing the CFA model with the hypothesized model because the hypothesized model is a constrained version of the CFA with three structural parameters fixed to zero:

$$\Delta\chi^2(3) = 350.00 - 348.64 = 1.36, p = .71$$

Combining this information with the information provided earlier from the modification indices, we can conclude that the measurement model requires respecification. Theoretically, allowing some residuals to correlate (as suggested by the MIs) makes sense because some factors include multiple subscale scores as indicators. When combined with other items from different scales, we would expect some degree of local dependence. Specifically, the **`IR`** factor includes three Psychological Hardiness subscale scores as indicators (**`comt`**, **`con`**, and **`cha`**), but also two indicators from independent scales (**`lo`** and **`es`**). The coping factor includes two indicators from the Ways of Coping Inventory (**`em`** and **`in`**), three indicators from the Impact of Event Scale (**`ru`**, **`av`**, and **`hy`**), and a religious beliefs score from another scale (**`rb`**).

Revised Model

We now introduce correlated uniquenesses for **`comt`**, **`con`**, and **`cha`** on the individual resources factor, between **`em`** and **`in`** on the coping factor, and among **`ru`**, **`av`**, and **`hy`** on the coping factor.



The new correlated uniquenesses can be included by modifying the MODEL syntax in Mplus. The Mplus input file that fits this model is provided in **`ch05_3.inp`**.

```
MODEL:
  ER by fa* fr si;
  IR by lo*-1 comt con cha es;
  ERF by pro*1 th*-1 time*;
  CPP by em*1 in*-1 ru* av* hy* rb*;
  PTG by ptgi1* ptgi2 ptgi3 ptgi4 ptgi5;
  [fa fr si];
  [lo comt con cha es];
  [pro th time];
  [em in ru av hy rb];
  [ptgi1 ptgi2 ptgi3 ptgi4 ptgi5];
  [ER@0 IR@0 ERF@0 CPP@0 PTG@0];
  ER@1 IR@1 ERF@1 CPP@1 PTG@1;
  ER with IR;
  ERF on ER IR;
  CPP on ERF;
  PTG on ER IR CPP;
  comt with con cha;
  con with cha;
  em with in;
  ru with av hy;
  av with hy;
```

Mplus does not use separate names for uniquenesses/residuals/disturbances. Instead, uniquenesses or disturbances are referred to by the referent variable. Thus, covariances between uniquenesses are specified via the `with` statement just as covariances among variables are. The line `con with cha` thus includes a covariance between the uniquenesses of **con** and **cha**.

We use the `MODEL INDIRECT` command to request calculation of the total indirect effects of one variable on another. The `IND` statement is used to request an estimate of the indirect effect of a predictor on an outcome by listing the outcome first, followed by `IND`, and then the predictor. Here, we have requested indirect effect estimates of **ERF**, **IR**, and **ER** on **PTG**. (Note we are not using the bootstrap procedures described in Chapter 3 because that procedure requires raw data and here we are fitting the model to the summary statistics.)

```
MODEL INDIRECT:
  PTG IND ERF;
  PTG IND IR;
  PTG IND ER;
```

The resulting output is shown here:

```
MODEL FIT INFORMATION

Number of Free Parameters                       80

Loglikelihood

          H0 Value                       -7972.143
          H1 Value                       -7837.004
```

```
Information Criteria

          Akaike (AIC)                    16104.287
          Bayesian (BIC)                  16334.911
          Sample-Size Adjusted BIC        16081.869
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             270.278
          Degrees of Freedom                    195
          P-Value                            0.0003

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.054
          90 Percent C.I.                     0.037  0.069
          Probability RMSEA <= .05            0.325

CFI/TLI

          CFI                                 0.923
          TLI                                 0.908

Chi-Square Test of Model Fit for the Baseline Model

          Value                            1205.231
          Degrees of Freedom                    231
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.090
```

Adding 7 free parameters to the hypothesized model resulted in a significant improvement in model fit:

$\Delta\chi^2(7) = 350.00 - 270.28 = 79.72, p < .001$.

Other fit indices suggest that the modified model has a satisfactory fit to the data.

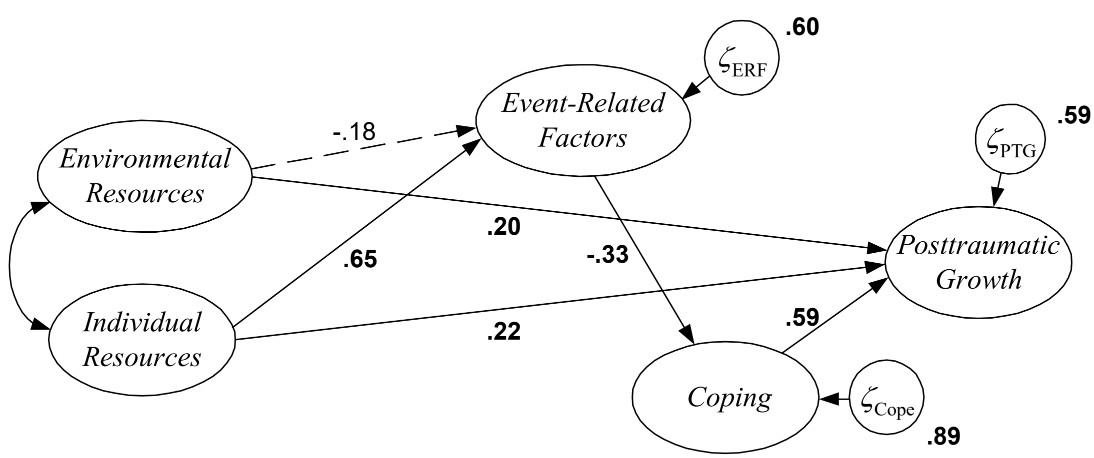
Parameter estimates are presented below.

```
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 ER       BY
    FA                 1.317      0.362      3.637      0.000
    FR                 5.855      0.767      7.633      0.000
    SI                 4.457      0.776      5.740      0.000

 IR       BY
    LO               -11.555      1.675     -6.897      0.000
    COMT               1.028      0.290      3.538      0.000
    CON                2.312      0.325      7.105      0.000
    CHA                1.141      0.264      4.324      0.000
    ES                 3.007      0.551      5.453      0.000
 ERF      BY
    PRO                0.347      0.103      3.365      0.001
    TH                -0.342      0.114     -3.012      0.003
    TIME               1.316      0.743      1.771      0.076

 CPP      BY
    EM                 6.060      1.103      5.495      0.000
    IN                -3.100      0.668     -4.644      0.000
    RU                 4.173      0.798      5.229      0.000
    AV                 2.958      0.587      5.039      0.000
    HY                 3.725      0.625      5.963      0.000
    RB                 0.221      0.088      2.515      0.012

 PTG      BY
    PTGI1              6.036      0.633      9.538      0.000
    PTGI2              4.578      0.484      9.457      0.000
    PTGI3              2.974      0.365      8.146      0.000
    PTGI4              2.259      0.261      8.649      0.000
    PTGI5              1.879      0.214      8.794      0.000

 ERF      ON
    ER                -0.236      0.181     -1.305      0.192
    IR                 0.834      0.293      2.840      0.005

 CPP      ON
    ERF               -0.276      0.130     -2.120      0.034

 PTG      ON
    ER                 0.259      0.118      2.201      0.028
    IR                 0.284      0.139      2.039      0.041
    CPP                0.722      0.168      4.299      0.000
```

IR has a significant direct effect on **ERF** ($\gamma = .834$; S.E. = .293; *p* = .005)and **PTG** ($\gamma = .284$; S.E. = .139; *p* = .041). **ER** has a significant direct effect on **PTG** ($\gamma = .259$; S.E. = .118; *p* = .28) and **coping** ($\gamma = -.276$; S.E. = .130; *p* = .034). **Coping** is significantly related to **PTG** ($\gamma = .722$; S.E. = .168 *p* < .001). Next we can view the standardized parameter estimates.

```
STDYX Standardization

ERF      ON
    ER                 -0.183      0.131     -1.404      0.160
    IR                  0.648      0.139      4.671      0.000

 CPP      ON
    ERF                -0.334      0.135     -2.478      0.013

 PTG      ON
    ER                  0.200      0.087      2.286      0.022
    IR                  0.219      0.100      2.192      0.028
    CPP                 0.590      0.091      6.497      0.000
Variances
    ER                  1.000      0.000    999.000    999.000
    IR                  1.000      0.000    999.000    999.000

 Residual Variances
    ERF                 0.604      0.168      3.602      0.000
    CPP                 0.888      0.090      9.859      0.000
    PTG                 0.593      0.102      5.795      0.000
```

We focus on the structural parameter estimates in this chapter because interpretation of measurement models has been discussed previously. The standardized structural parameter estimates have been drawn on the path diagram below to more easily comprehend the model results. The non-significant path from environmental resources to event-related factors is dashed. All other paths are statistically significant and shown with solid lines.



Standardized results suggest that environmental and individual resources have a moderate, direct, positive influence on posttraumatic growth, cognitive coping has a strong, direct, positive influence on posttraumatic growth, individual resources strongly predict more event-related factors (shorter time since prognosis, poorer prognosis, and greater threat), and more positive event-related factors predicts moderately less cognitive coping. Individual resources and event related factors have a complex relationship with posttraumatic growth. To better understand these relationships, we must consider direct, indirect, and total effects.

```
TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

                                                       Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Effects from ERF to PTG

  Total               -0.199      0.100     -1.982      0.047
  Total indirect      -0.199      0.100     -1.982      0.047

  Specific indirect

    PTG
    CPP
    ERF               -0.199      0.100     -1.982      0.047


Effects from IR to PTG

  Total                0.118      0.132      0.893      0.372
  Total indirect      -0.166      0.090     -1.833      0.067

  Specific indirect

    PTG
    CPP
    ERF
    IR                -0.166      0.090     -1.833      0.067

  Direct
    PTG
    IR                 0.284      0.139      2.039      0.041


Effects from ER to PTG

  Total                0.306      0.123      2.494      0.013
  Total indirect       0.047      0.041      1.152      0.249

  Specific indirect

    PTG
    CPP
    ERF
    ER                 0.047      0.041      1.152      0.249

  Direct
    PTG
    ER                 0.259      0.118      2.201      0.028
```

For each predictor-to-outcome effect, the Mplus output first presents the total effect (along with standard errors and significance tests). Then it breaks down the total effect by presenting the total indirect effect and the direct effect. In some cases, the entire effect is indirect (e.g., `ERF to PTG`). If the indirect effect consists of multiple pathways, the indirect effect is further divided to show the specific indirect effect for each pathway. In this example, each predictor only had one indirect pathways affecting **PTG**.

We will closely examine the effect of **IR** on **PTG**. We start by noting that the total effect of **IR** on **PTG** is non-significant. However, upon closer examination, it is apparent that **IR** is related to **PTG** both directly and indirectly, but that these effects are in opposite directions such that the net, total effect is nearly zero. The direct effect of **IR** on **PTG** is significant and positive, but the indirect effect is marginally significant and negative.

Appendix A: How to Use the Mplus Diagrammer

This appendix presents a worked example drawn from class demonstrating the creation of Mplus code using the Mplus diagrammer that is newly available in Version 7. Prior versions of Mplus do not contain this option.

Holzinger-Swineford CFA (Chapter 4)

The data for this demonstration were provided by Holzinger & Swineford in their 1939 monograph *A Study in Factor Analysis: The Stability of a Bi-Factor Solution*. The sample includes 301 7th and 8th grade students, between 11-16 years of age, drawn from two schools. The data is in the text file **`hs.dat`**. The variables in the data set that we will use are

`visperc` visual perception test in which participants select the next image in a series

`cubes` visual perception test in which participants must mentally rotate a cube

`lozenges` visual perception test involving mental "flipping" of a parallelogram ("lozenge")

`parcomp` paragraph comprehension test

`sencomp` sentence completion task in which participants select most appropriate word to put at the end of a sentence

`wordmean` verbal ability test in which participants must select a word most similar in meaning to a word used in a sentence.

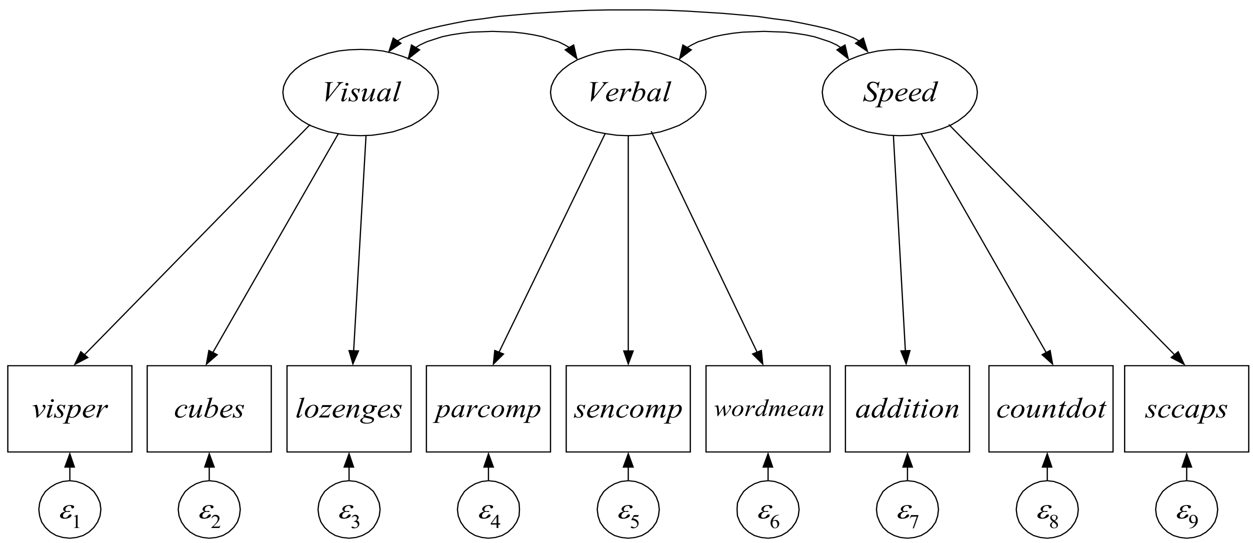
`addition` participants have 2 minutes to complete as many 2-number addition problems as they can

`countdot` participants have 4 minutes to count the number of dots in each of a series of dot pictures

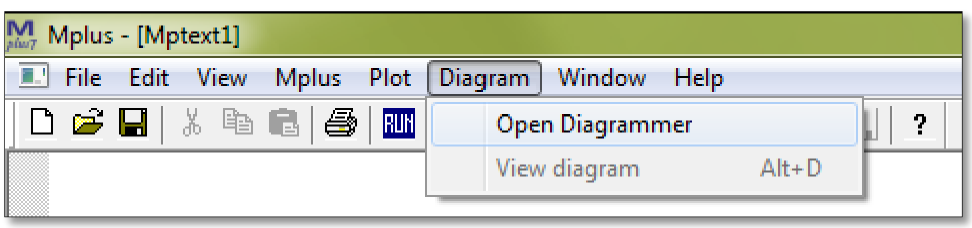
`sccaps` participants have 3 minutes to indicate whether capital letters are composed entirely of straight lines or include curved lines.

Other variables in the data not included in the models fit here are **`school`**, **`female`**, **`age`**, and **`month`**.

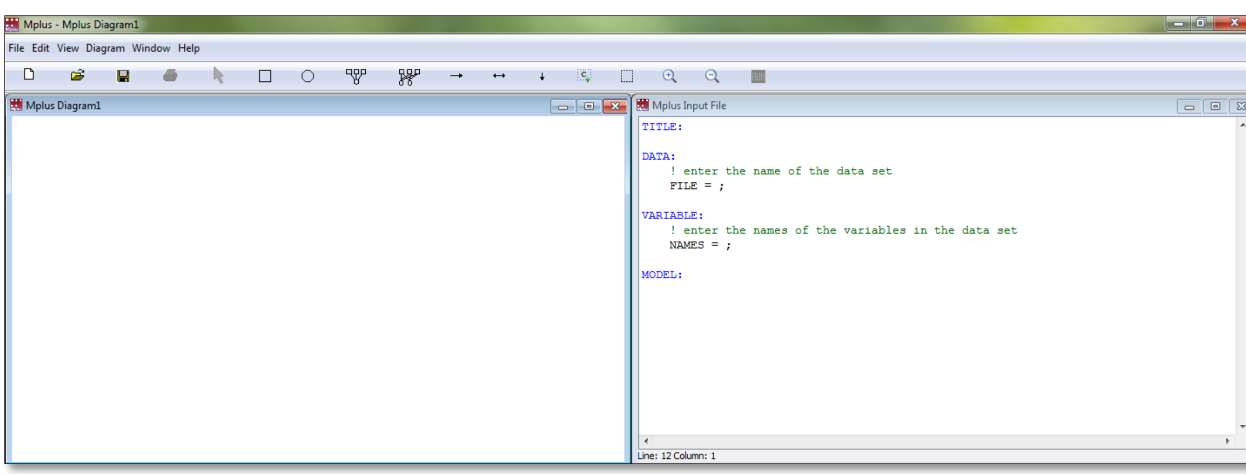
The following instructions will detail how to create a diagram using the diagrammer to replicate example **`ch04_1.inp`**. Our original path diagram of the model is shown below. We want to reproduce this same structure using the diagrammer utility.



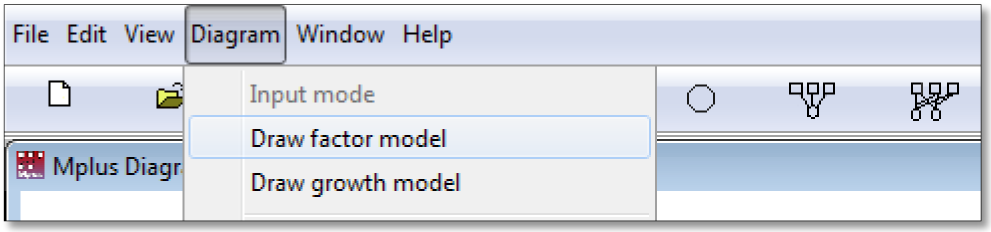
Upon opening Mplus, a blank input file appears in the editor window. From this input file, select “Diagram” then “Open Diagrammer.”



The following window will appear. The left side is the workspace for creating a diagram. The right side shows an input file that will update as you create your diagram in the workspace.

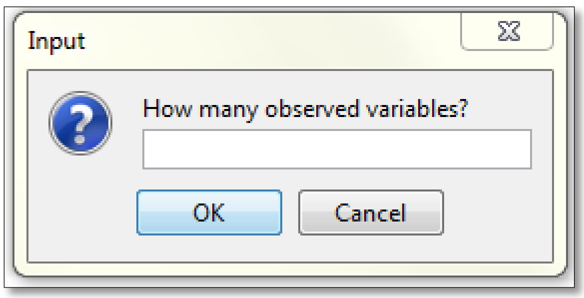


We begin by creating a factor model. This can be done in one of two ways. From the "Diagram" menu, select "Draw factor model."

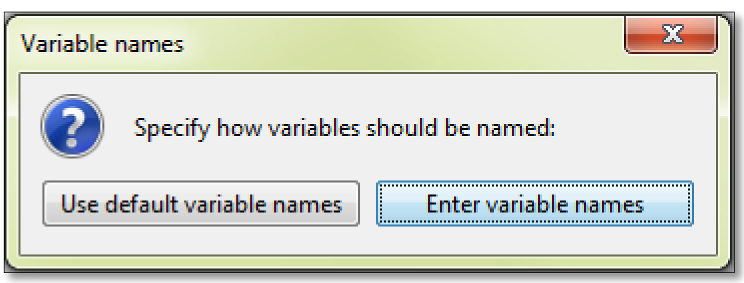


This action could also be accomplished by selecting the  symbol.

After selecting this option, clicking in the open diagram space will prompt a window that asks:

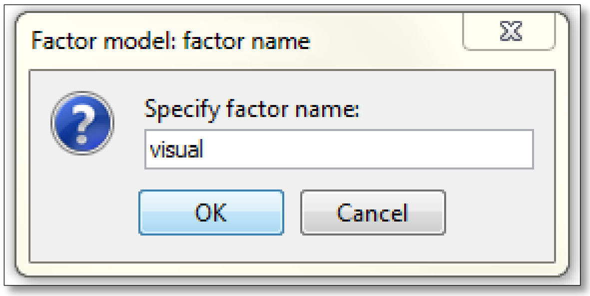


This is the number of observed variables for one of your factors. We will enter the number 3, to indicate that our first factor (visual) has three indicators. Mplus then asks how the variables should be named.

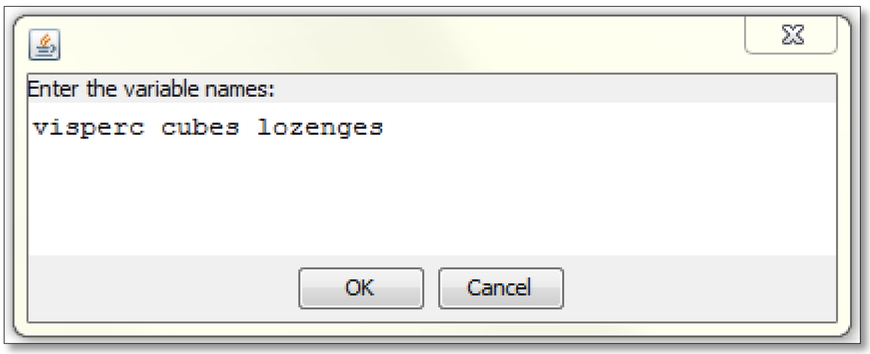


If "Use default variable names" is selected, then the variables will be labeled y1-y3 and the factor will be labeled as f. We will select "Enter variable names" in order to provide Mplus with the actual names of the factor and the variables. Note that these variable names must correspond to those used in a later step in which the data file is imported and variables are defined. In contrast, because the latent factors are not in the data file and are inferred by the model, these can be named anything you please.

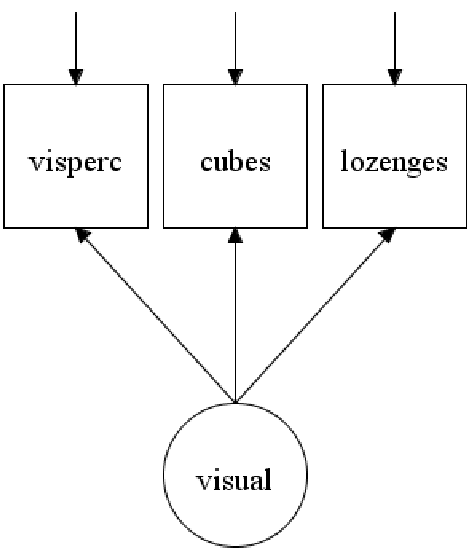
We first provide the factor name in the prompt.

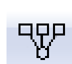


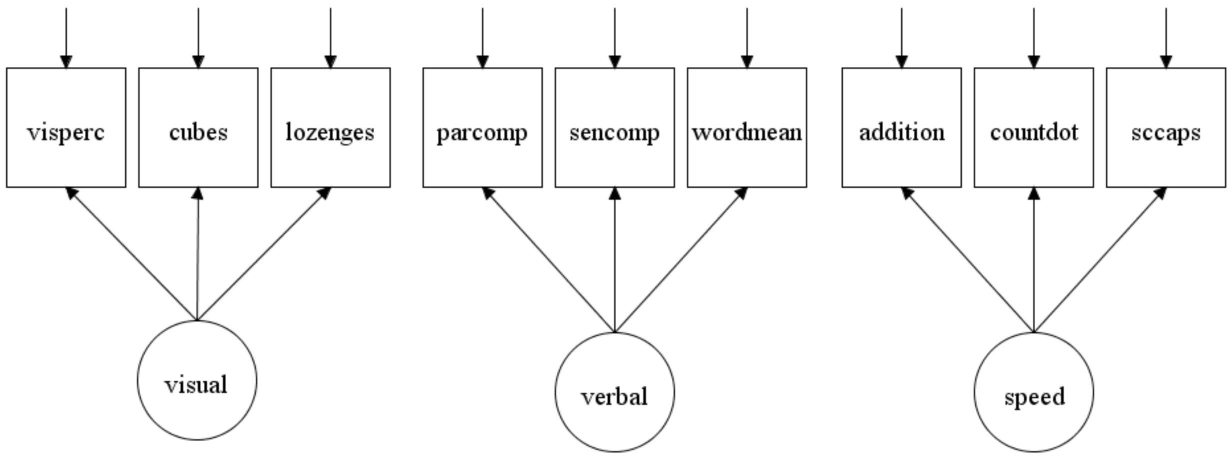
After entering the factor name, a window appears for us to enter the variable names. We enter the variable names below, separated with a single space.



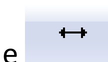
Below is the result of our work thus far:

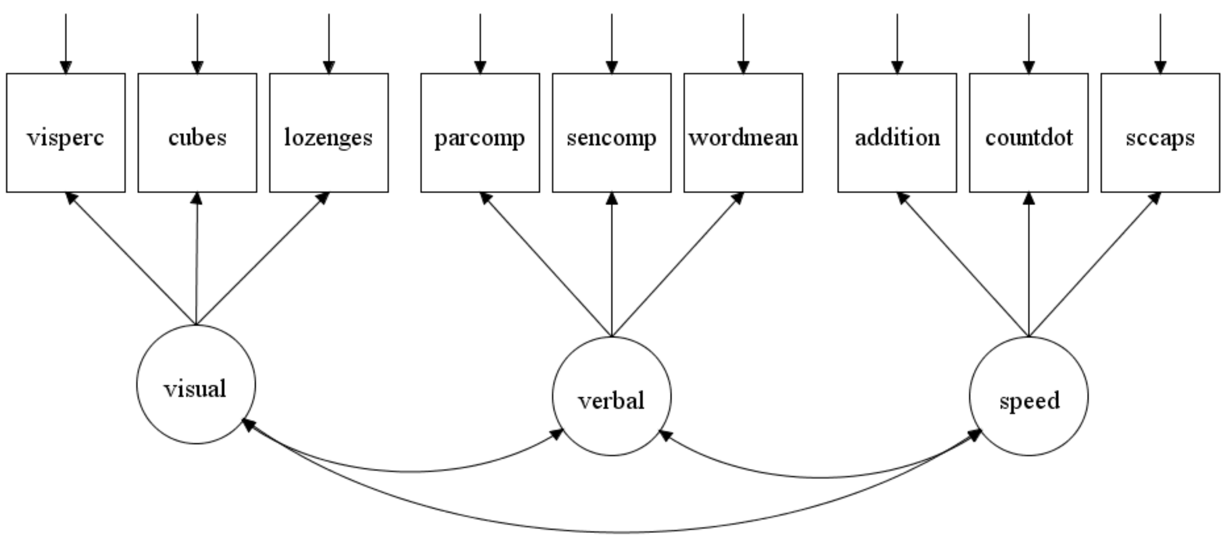


Because the "Draw factor model"  option is still selected on the top ribbon, clicking on the workspace again will prompt the creation of another factor. We repeat the above steps to create our other two factors. Below is the result.



We are finished creating factors. We now want to draw covariance arrows between the factors.

Covariance arrows can be added by selecting the  option from the top ribbon. After selecting this double headed arrow, click on the circle for the first factor, then the second factor. This will draw a covariance arrow between the two factors you selected. Repeat this step to add more covariance arrows. Our final diagram is shown below.



As we have created the diagram, the input file on the right side of the screen has been updated with our model. At this point, our diagram has generated the following model code:

```
TITLE:

DATA:
      ! enter the name of the data set
      FILE = ;

VARIABLE:
      ! enter the names of the variables in the data set
      NAMES = ;

MODEL:
      visual BY visperc cubes lozenges;
      verbal BY parcomp sencomp wordmean;
      speed BY addition countdot sccaps;
      verbal WITH visual;
      speed WITH verbal;
      speed WITH visual;
```

We can modify the above code to include the name of the data file, the names of the variables, and the means and variances for the latent factors. You will type the path of the data file as it is located on your computer in the blank space of "`FILE = ;`". Alternatively, if your input file is saved in the same folder as your data set, then you only need to identify the data file by its name. For example, we would enter "`FILE = hs.dat;`" if our data file is in the same location as our input file. In the "`NAMES = `" statement, we list the variable names in the order that they appear in the data set. Because the variables in our analysis are only a subset of all of the variables, we separately list the variables included in our analysis in the "`USEVARIABLES = `" statement. Thus, we would include **visperc**, **cubes**, **lozenges**, **parcomp**, **sencomp**, **wordmean**, **addition**, **countdot**, and **sccaps** in the "`USEVARIABLES = `" statement. The manually modified code is now:

```
DATA:
      ! enter the name of the data set
      FILE = hs.dat;

VARIABLE:
      ! enter the names of the variables in the data set
      NAMES = school female age month visperc cubes lozenges
              parcomp sencomp wordmean addition countdot sccaps;
```

We need to make a few final augmentations to the input code before running it.

By default, to set the metric of the latent factor Mplus fixes the first factor loading of each item to 1.0 and freely estimates the variance of the latent factor. This strategy is just fine, although in this example we would like to set the metric of the factor by fixing the factor variance to 1.0

and freely estimating all of the factor loadings. To do this we need to put asterisks (*) next to the first indicator in each `BY` line so that Mplus will freely estimate the first loading of each factor (all other loadings are freely estimated by default -- asterisks could be included with each indicator, but this would be redundant with the defaults); next, we need to set the mean and variance of each factor to 0 and 1, respectively. Now we have defined the metric of the latent factors as intended. The `MODEL` section now reads:

```
MODEL:
      visual BY visperc* cubes lozenges;
      verbal BY parcomp* sencomp wordmean;
      speed BY addition* countdot sccaps;
      verbal WITH visual;
      speed WITH verbal;
      speed WITH visual;
      [visual@0 verbal@0 speed@0];
      visual@1 verbal@1 speed@1;
```

Finally, for reasons we describe in the lecture notes, we would like to linearly rescale three variables so that the corresponding metric is more similar to the remaining variables. To do this we included a "`DEFINE:`" command to rescale the variables **`addition`**, **`countdot`**, and **`sccaps`** to facilitate model estimation. Dividing these variables by 4 brings their standard deviations closer to the standard deviations of the other variables. Under the `VARIABLE` section, we thus add:

```
DEFINE:
      addition = addition/4;
      countdot = countdot/4;
      sccaps = sccaps/4;
```

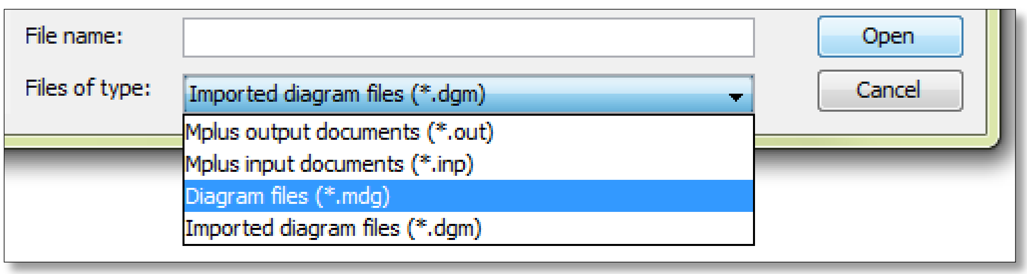
The results of our modifications to the code results in the following complete script:

```
DATA:
      ! enter the name of the data set
      FILE = hs.dat;

VARIABLE:
      ! enter the names of the variables in the data set
      NAMES = school female age month visperc cubes lozenges
              parcomp sencomp wordmean addition countdot sccaps;
      USEVARIABLES = visperc cubes lozenges
              parcomp sencomp wordmean addition countdot sccaps;

DEFINE:
      addition = addition/4;
      countdot = countdot/4;
      sccaps = sccaps/4;

ANALYSIS:
      estimator=ML;
```

```
MODEL:
     visual BY visperc* cubes lozenges;
     verbal BY parcomp* sencomp wordmean;
     speed BY addition* countdot sccaps;
     verbal WITH visual;
     speed WITH verbal;
     speed WITH visual;
     [visual@0 verbal@0 speed@0];
     visual@1 verbal@1 speed@1;
```

In the process of creating the diagram, you may want to save your progress before you are finished. In order to do so, make sure you have clicked on some portion of the diagram (that is, not the input file). Click “File” then “Save as” and save the diagram file at a desired location on your computer. The file will be saved with a .mdg extension.

In order to reopen this file, do not double-click it. Instead, reopen Mplus and navigate to the diagrammer. From the diagrammer, select “File” then “Open.” For the “Files of type:” option, select Diagram files (*.mdg) and then select your diagram file.



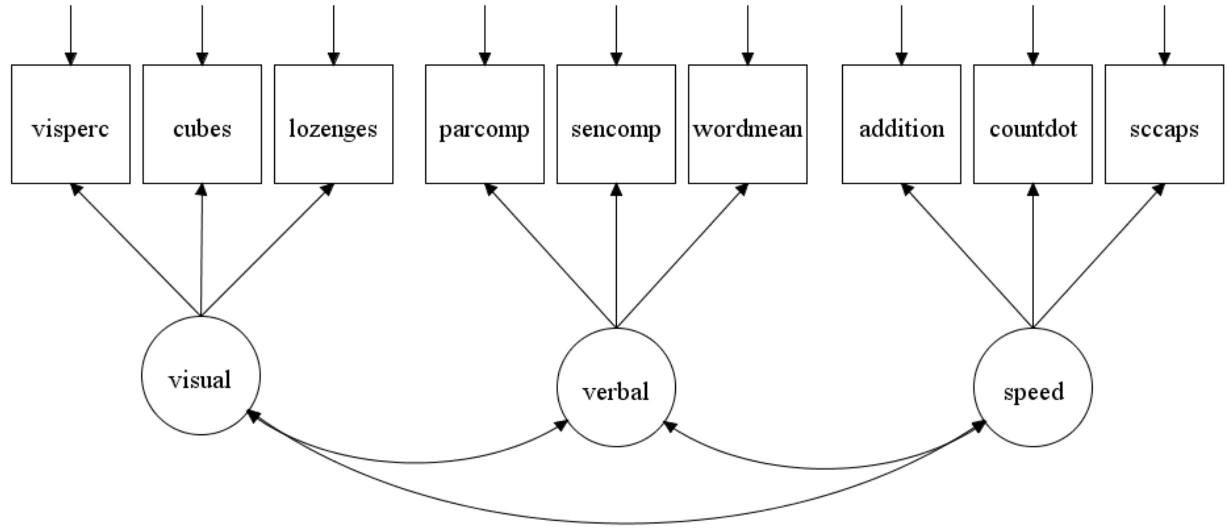
The diagram file that you previously saved will now reopen, and the input window will again be populated with code.

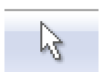
We are now ready to run the input file. In the top ribbon of the diagrammer, select the RUN button. The resulting output is:

```
MODEL FIT INFORMATION

Number of Free Parameters                        30

Loglikelihood

          H0 Value                        -8326.241
          H1 Value                        -8283.589

Information Criteria

          Akaike (AIC)                    16712.483
          Bayesian (BIC)                  16823.696
          Sample-Size Adjusted BIC        16728.553
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              85.306
          Degrees of Freedom                     24
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.092
          90 Percent C.I.                     0.071  0.114
          Probability RMSEA <= .05            0.001

CFI/TLI

          CFI                                 0.931
          TLI                                 0.896

Chi-Square Test of Model Fit for the Baseline Model

          Value                             918.852
          Degrees of Freedom                     36
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.060
```

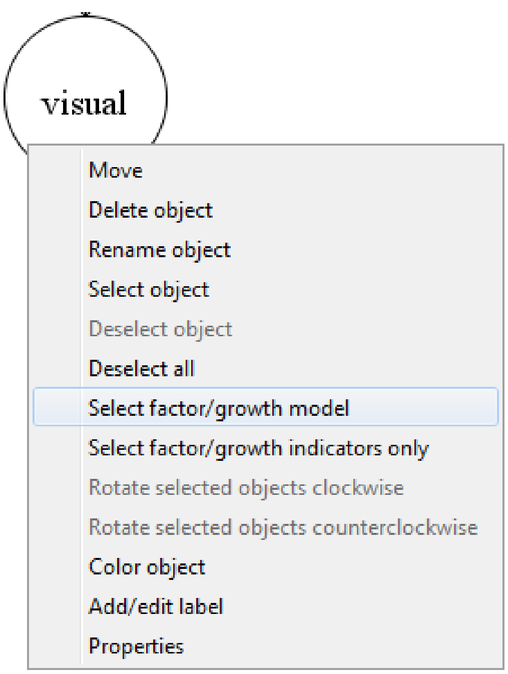
We see that these indices of fit exactly match the notes for the output of **`ch04_1.inp`**.

In the process of creating the diagram, you may want to move elements around to your liking. We now detail a few basic editing steps for the diagrammer.

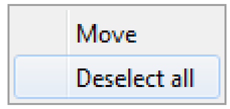


We will select  from the top ribbon to begin editing the figure.

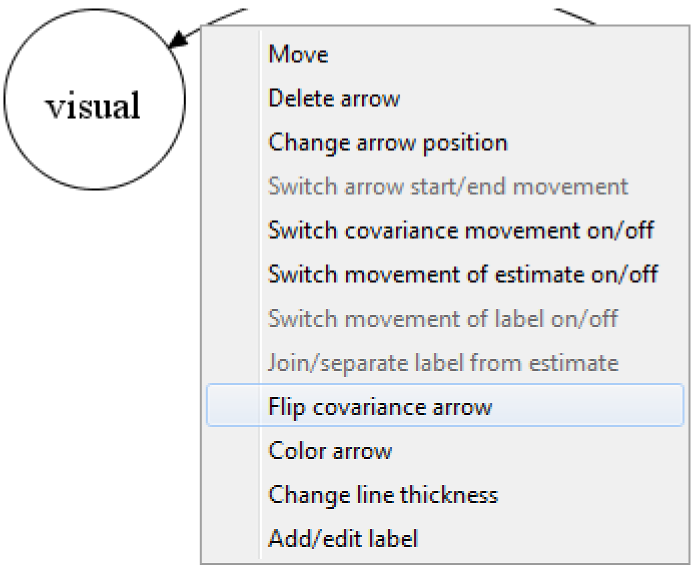
Selecting any individual square or circle will move that individual piece of the figure, not the figure in its entirety. In order to move the factor and its indicators, right click the circle and select “Select factor/growth model.”



Doing so will select the entire factor and its indicators so that it can be moved around the workspace. This can be done by dragging the figure with your cursor, or by simply clicking where you would like for it to go. When you are done moving the factor, simply right click any portion of the workspace and select "Deselect all."

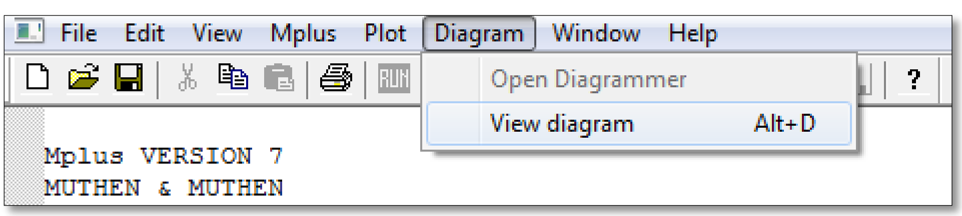


After doing this, the covariance arrows between the factors may not be positioned as you like. In order to change the location of the covariance arrow, simply right click the arrow and select "Flip covariance arrow." Alternatively, you may more specifically change its location by selecting "Change arrow position" and selecting a location from the drop down menu.

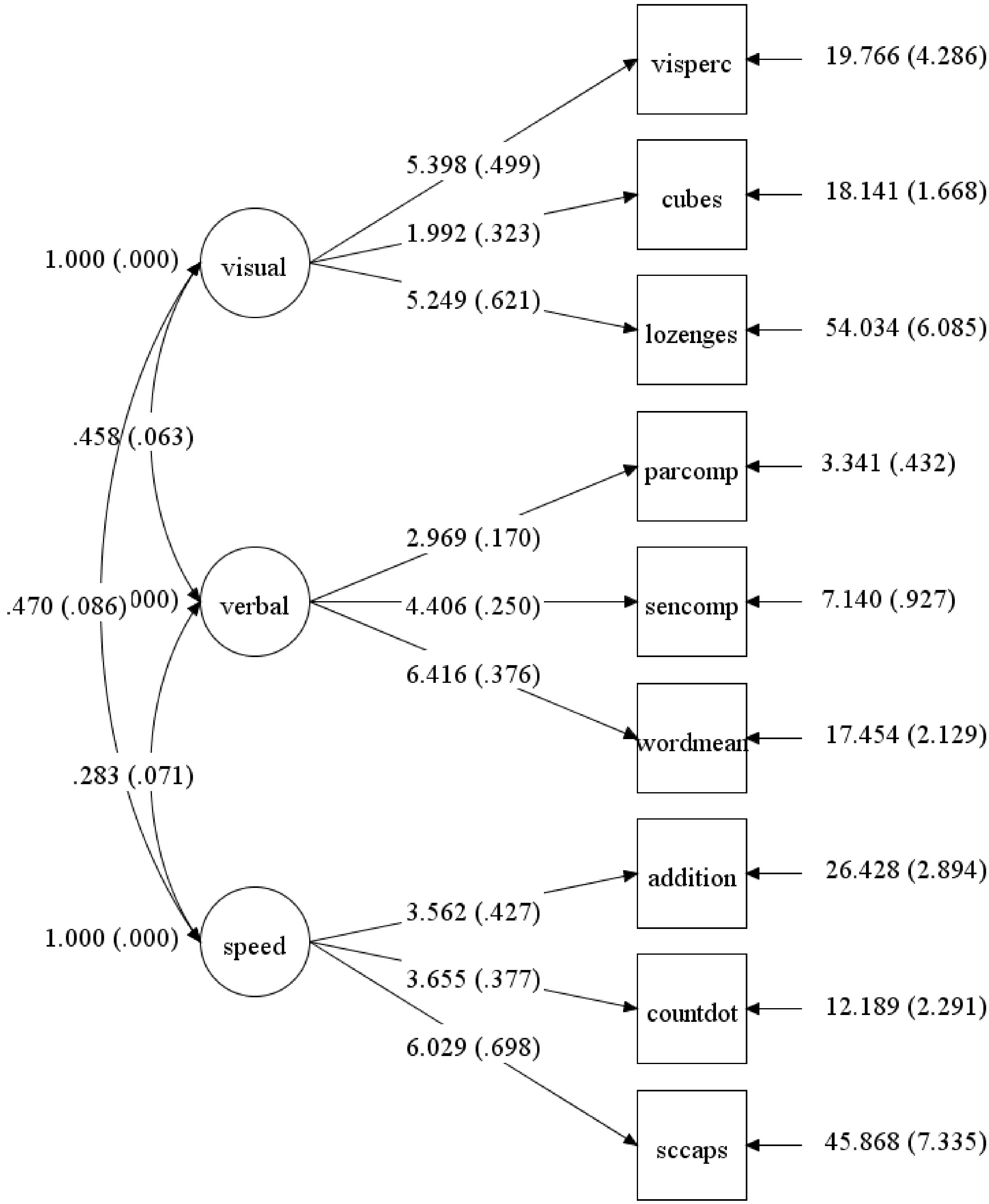


Creating a Diagram from Mplus Code

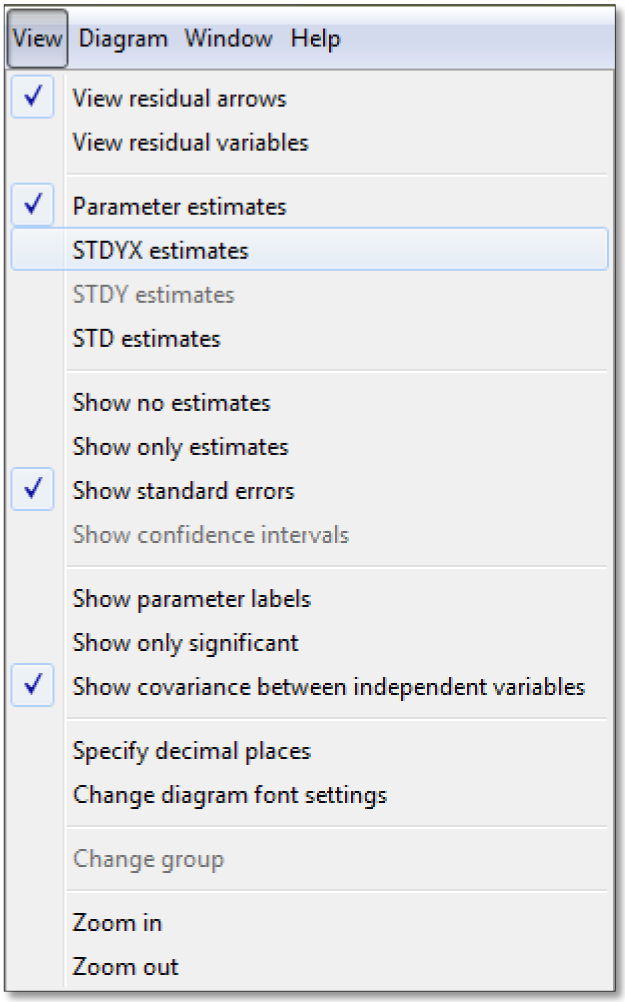
The example above uses the diagrammer to create the Mplus code that corresponds to a particular diagram. However, we can also use manually written code to create the corresponding diagram. For example, we can open the code corresponding to our Chapter 4 example named **`ch04_4.inp`** and run the program in the usual way. After running a full input file, the diagram can be viewed from the output window by selecting "Diagram" then "View Diagram."

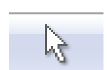


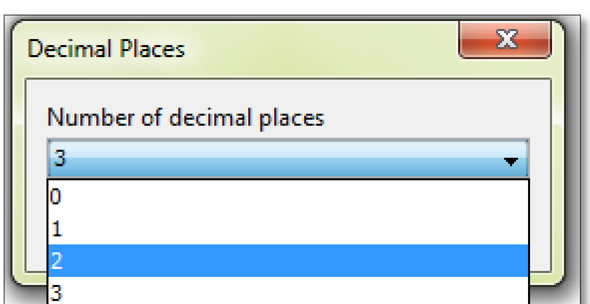
The diagram that corresponds to the code written in **`ch04_1.inp`** is:



As you can tell, this default is sometimes not optimal. We see that some values are occluding each other, and we may not need results to three decimal places. Additionally, we may prefer to view standardized estimates. In the "View" menu of the diagrammer, there are options to edit the figure. In the view menu, we select "STDYX estimates" in order to show the standardized parameter estimates.



In the final diagram, we moved the verbal factor using the  so that all parameter estimates were visible. We also changed the number of decimal places to 2 decimals by selecting "Specify decimal places" and selecting 2.



The final diagram is shown below.