Single Photon Interference

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1 Introduction

Light has the ability to behave like a particle or like waves under different conditions. This is what is known as the wave-particle duality. Photons are essential to the understanding of the nature of light. Additionally, photons have the wave-particle duality; photons can act either as waves (with high interference) or as particles (with no interference). Only recently have we been able to detect and generate single photons[1]. When a single photons acts as wave, it interferes with itself, to much surprise. One way to observe interference is with the use of an interferometer. In an interferometer, light is divided into two paths and later recombined to produce a pattern on a screen or detector. In this experiment, we show that if a single photon is incident on a beam-splitter, it can be detected at the transmitted port, or reflected port, but not both. In order to generate single photons we use spontaneous parametric down conversion. To produce photon pairs we use a 405 nm laser diode through a barium borate crystal (BBO). These pairs go in different directions (see Figure 10); one will go straight to a detector[2] whereas the other will pass through a Mach-Zehnder interferometer.

2 Theoretical

In this section we explain the theoretical concepts.

[1]Granger et al. where the first to develop a source for single photons. For more on this see [5]

[2]This detector will act as an indicator to the presence of a single photon at the interferometer

2.1 Beam-splitter

First, let us consider why the classical interpretation of the beam-splitter is not sufficient. Consider a classical loss-less beam-splitter with electric fields of complex amplitude ε_1 incident at both its inputs (see Figure 1). If ε_2 and ε_3 are the amplitude of the reflected and

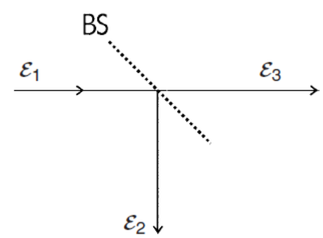

Figure 1: [4] Beam splitting in the classical sense.

transmitted beams, respectively, and r and t are the complex reflectance and transmittance of the beam-splitter[3], respectively, then

$$\varepsilon_2 = r\varepsilon_1 \text{ and } \varepsilon_3 = t\varepsilon_1$$

Since we assumed the beam-splitter to be loss-less[4],

$$|\varepsilon_1|^2 = |\varepsilon_1|^2 + |\varepsilon_3|^2.$$

More so, this condition is equivalent to,

$$|r|^2 + |t|^2 = 1.$$

If we treat the beam-splitter quantum-mechanically, we may replace ε_i by a set of annihilation operators $\hat{a}_i$,

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \begin{pmatrix} r\hat{a}_1 \\ t\hat{a}_1. \end{pmatrix} \tag{1}$$

The operations must satisfy the commutation relations:

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij},\ [\hat{a}_i, \hat{a}_j] = [\hat{a}_i^\dagger, \hat{a}_j^\dagger] = 0. \tag{2}$$

However,

$$[\hat{a}_2, \hat{a}_2^\dagger] = |r|^2[\hat{a}_1, \hat{a}_1^\dagger] = |r|^2$$
$$[\hat{a}_3, \hat{a}_3^\dagger] = |t|^2[\hat{a}_1, \hat{a}_1^\dagger] = |t|^2$$

[3] For a $50:50$ beam-splitter we would have that $|r| = |t| = \frac{1}{\sqrt{2}}$.

[4] intensity of the input beam should equal the sum of the intensities of the two output beams

and $[\hat{a}_2, \hat{a}_3^\dagger] = r\bar{t} \neq 0$. Since, the commutation relations are not satisfied, Equation (1) **cannot be the correct** quantum description of a beam-splitter. The reason is that in the classical construction of the beam-splitter, there is an unused sector, which being empty of an input field, has no effect on the output beams. However, when we consider the quantum case[5], the unused sector contains a quantized field (see Figure 2).

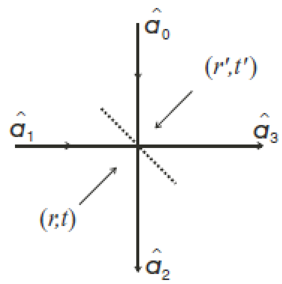

Figure 2: [4] Beam splitting quantum-mechanically

2.2 Mach-Zehnder Interferometer (MZI)

In order to observe interference with single photons we need the use of an MZI. A MZI consists of two beam-splitters and reflective mirrors (see Figure 3). In this configuration, we have interference because D_1 and D_2 cannot distinguish between with path the photons are taking. It can be shown that the probability of detecting a photon using a random beam-splitter is given by,

$$P_{10} = \left| e^{i\theta_1} tr' + e^{i\theta_2} rt' \right|^2 \tag{3}$$

where θ_1 and θ_2 are some phase shift due to being reflected[6] and r and t are the reflecting and transmitting coefficients, respectively. Then,

$$\begin{aligned} P_{10} &= (e^{i\theta_1} tr' + e^{i\theta_2} rt')\overline{(e^{i\theta_1} tr' + e^{i\theta_2} rt')} &(4)\\ &= |r'|^2|t|^2 + |t'|^2|r|^2 + 2|r||r'||t||t'|\frac{e^{i(\theta_1-\theta_2)} + e^{-i(\theta_1-\theta_2)}}{2} &(5)\\ &= |r'|^2|t|^2 + |t'|^2|r|^2 + 2|r||r'||t||t'|\cos(\theta_1-\theta_2) &(6)\end{aligned}$$

where $\theta_1 - \theta_2 \equiv \theta$ is a difference in phase between the two interferometer arms. If we had a 50:50 beam-splitter we have that $|r'| = |r| = |t| = |t'| = \frac{1}{\sqrt{2}}$ and so

$$P_{10} = \frac{1}{2}(1 + \cos\theta). \tag{7}$$

[5]See Appendix for the formalism

[6]See [6] for a more succulent discussion as to why this is.

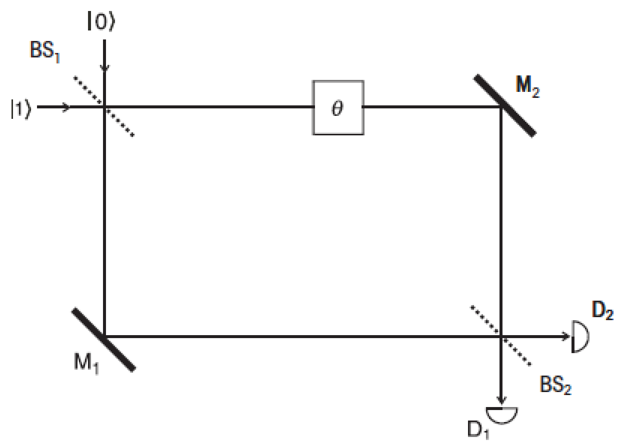

Figure 3: [4] MZI. BS_1 and BS_2 are beam-splitters, θ is the phase difference, M_1 and M_2 are mirrors, and D_1 and D_2 are detectors.

Moreover, recall that the total probability is 1 and so the probability of picking the other state is

$$P_{01} = 1 - P_{10} = \frac{1}{2}(1 - \cos\theta). \tag{8}$$

Observe that if the phase difference is nil, then

$$P_{10} = 1 \text{ and } P_{10} = 0.$$

3 Experimental Procedure

In this experiment we made use of two constitutions: a basic[7] alignment with a parametric down conversion and a Mach-Zehnder interferometer. We will explain the procedure in this section. The following equipment will be needed:

- HeNe laser
- Blue laser (405 nm)
- Beam dump
- Collimator (2)

[7] we are being extremely liberal with the definition of

- Flipper mirror (2)
- Barium borate (BBO) crystal
- Mirror (5)
- Iris diaphragms
- Fiber Optic Cables (5)
- Spectrometer
- Piezo Electric and Driver
- White Light Bulb
- Beam Splitter (2)
- Plumb-bob

One of the most essential steps to do before starting the alignment is to measure the height of the crystal that we are going to use. We have to make sure that all the mirrors, beam-splitters, iris diaphragms, and lasers are at this height.

3.1 Aligning the beam

We start by setting the HeNe laser as in Figure 4. Using the optical table, try to align the beam path along the holes of the table. It is essential that the beam path is parallel to the holes on the optical table.

3.1.1 Aligning the Beam Path to be Parallel to the Optical Table

The laser (mirror) is to be aimed by eye to be parallel to the holes on the optical table. Alternatively, we can use a plumb-bob and hover over the desired hole in a row and make the beam path hit the string on the pendulum (see Figure 5). Next, put pairs of screws along the same row of holes in two positions: a near position and a far position[8] (see Figure 6). An iris is put in the near position, close to the laser (mirror), with its mount touching the two screws in this position. The iris is then adjusted so that the beam goes through the iris. Next, the iris is placed in the far position, with its mount touching the screws. If the beam path goes through this iris, then the beam path is aligned. On the other hand, if

[8] If the beam path passes through the iris at two distinct points of same height, then the beam path is parallel to the table.

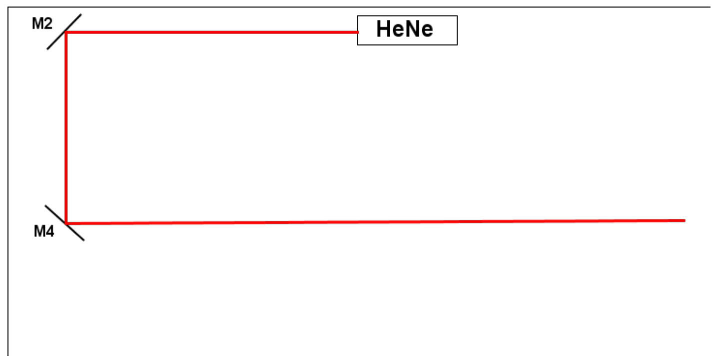

Figure 4: M2 and M4 are mirrors.

the beam path does not go through the iris in far position, then the laser (mirror) must be tilted until the beam path goes through the iris (see Figure 7 and Figure 8). Iterate this process until the alignment converges (adjust the iris when the iris is in the near position and adjust the laser when the iris is in the far position).

3.1.2 Basic Alignment

Once we have completed aligning the beam path from the HeNe laser to be parallel to the holes of the optical table, place a mirror (M2) about $10cm$ from the edge as in Figure 4 along the beam path. Make sure the beam path is hitting the center of the M2. Repeat the process outlined in the last section using mirror M2 and its reflected beam path. Next, place a second mirror (M4) along M2's reflected beam path. The location of M4 should be about $38cm$ from the top. Again, we will repeat the process outlined in the last section of beginning with a plumb bob in front of the mirror, over a whole and iterate the laser-iris alignment. Make make sure that you place M4 so that the beam path from M2 is hitting the center of M4. This last beam path should be one of the trajectories the photon can follow once it passes through our BBO crystal.

Let us begin the setup to find the second path the photon can take. We use two flipper mirrors[9], M1 and M5, as in Figure 9. Before continuing, we need to calculate where to put the mirrors. The BBO crystal should be placed about $35cm$ from the left edge of the optical table along M4's reflected beam path. Do not place the BBO crystal now, merely make a marking where it will be located. Using this information, draw a line[10] that intersects M4's reflected beam path at the BBO crystal and makes a $6°$ angle. Now, place mirror M1 in the beam path of the laser HeNe. The reflected beam path must be corrected to be parallel to the holes on the optical table. When this task is complete, place a second mirror, M5,

[9]otherwise we will lose the ability to use mirrors M2 and M4

[10]it is crucial that you do this as straight as possible

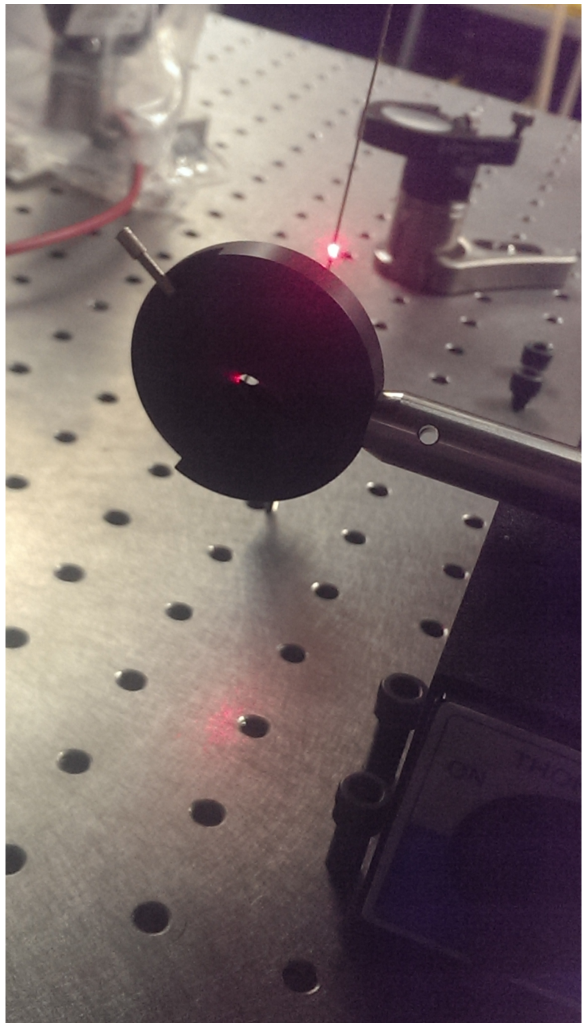

Figure 5: Laser intercepting plumb-bob

so that the reflected beam is along the line we drew[11]. Make sure M5's reflected beam path passes through the line that we drew. Repeat the process of aligning the beam parallel to the optical table. At this point we should have a crude outline of where each of the components are meant to be.

Now we add a blue laser and a mirror M3, as in Figure 10. Be careful, the blue laser can be hazardous; be sure to use protective eye-wear. Align the blue laser beam path parallel to the holes on the optical table. Place a mirror M3 so that the reflected laser path goes

[11]It should be a trivial task to find the actual location of the mirror. Construct a right triangle, it should be apparent from Figure 9, use the fact that the 6° is an interior angle and measure the distance between M2 and M1.

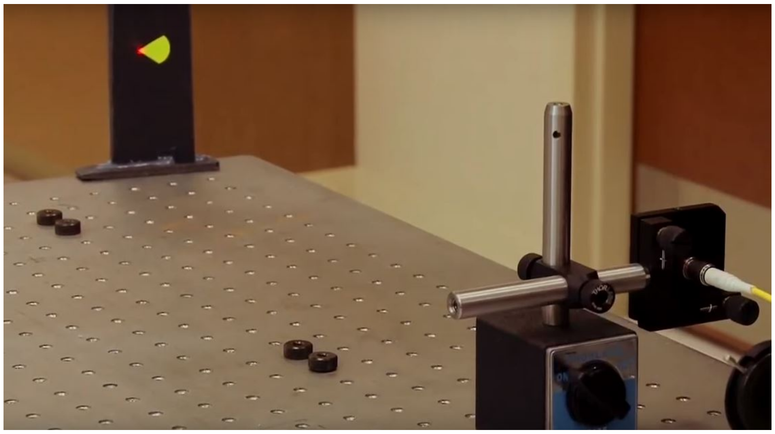

Figure 6: Screws to assure a stable iris

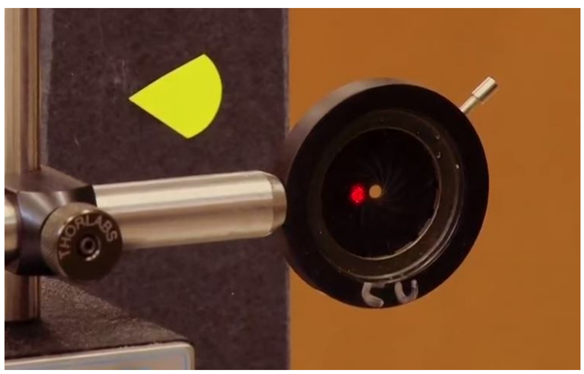

Figure 7

through the location the BBO crystal would be at. We will fine-tune this in a bit.

3.2 Detector Positioning and Alignment

With all 5 mirrors in place or roughly near their approximate positions, the detectors and beam dump need to be aligned accordingly. The beams from mirrors M3, M4, and M5 all need to converge at the location of the crystal while their end points are the beam dump, detector D1, and detector D2, respectively. Spatial optimization of the breadboard is always key. We want the detectors, ideally, 1 meter away from the crystal along the curved plane or arch at which the photons would collide with the detectors at their locations.

At this point, the crystal should be taken out, if you have not done so, and its necessary location be marked on the breadboard. To construct the arch representing the plane by

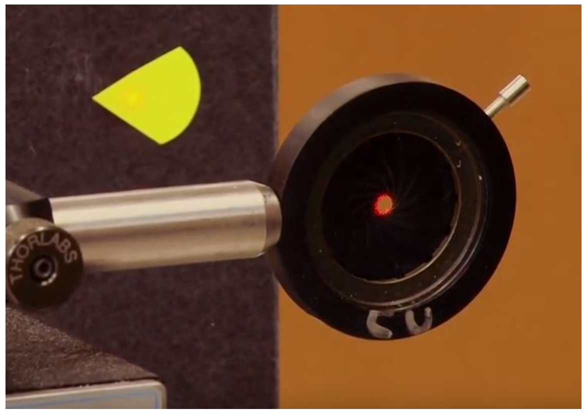

Figure 8: Beam in midst of iris

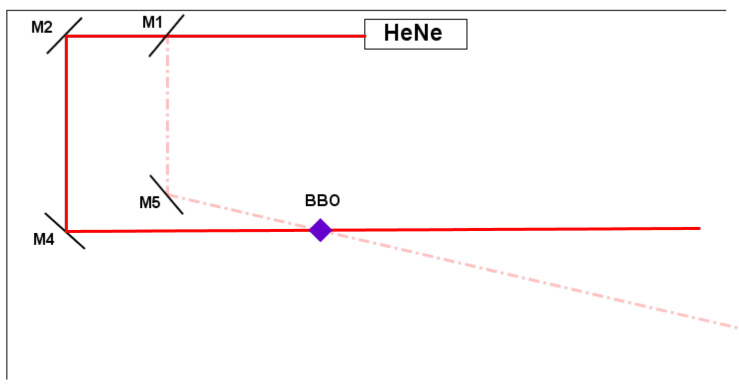

Figure 9: Adding flipper mirrors M1 and M5 and the BBO crystal

which the detectors need to be situated upon, first measure the distance away from the crystal they will be located. It will be easiest to measure the distance along the path that is parallel with that of the holes on the board - this should be from M4 to detector D1. Let us call this distance 'x'. It is suggested to construct a large compass-like device using a pivot point, string and a pencil. The distance from the point of pivot to the lead point on the pencil needs to be the same 'x' distance. The device used is shown in Figure 16.

With your arch drawn on the breadboard, Figure 11, perform the necessary calculation to determine the distance in which each incident beam upon this arch should be separated. If there are 3° between each beam along the arch, given your 'x' distance, separate the detector locations accordingly. Start with a mark for the location of detector D1 and add the separation distance for 3° twice. The first addition will be for the blue laser - where the beam dump will go - and the second addition will be for location of detector D2. With marked locations for the two detectors and beam dump on the breadboard, we may begin

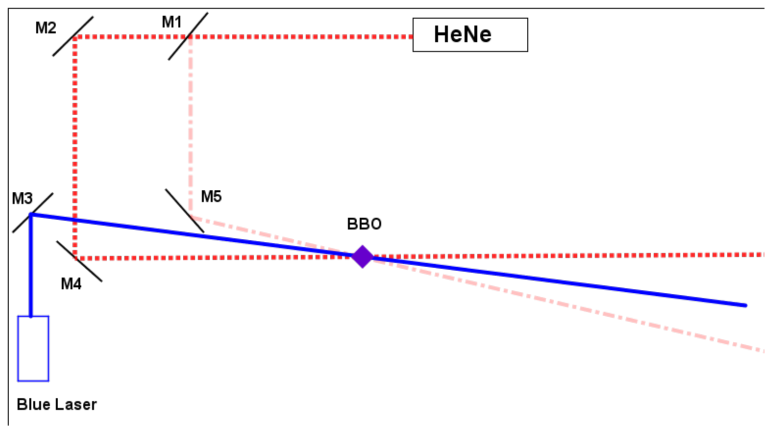

Figure 10: Adding the blue laser and mirror M3

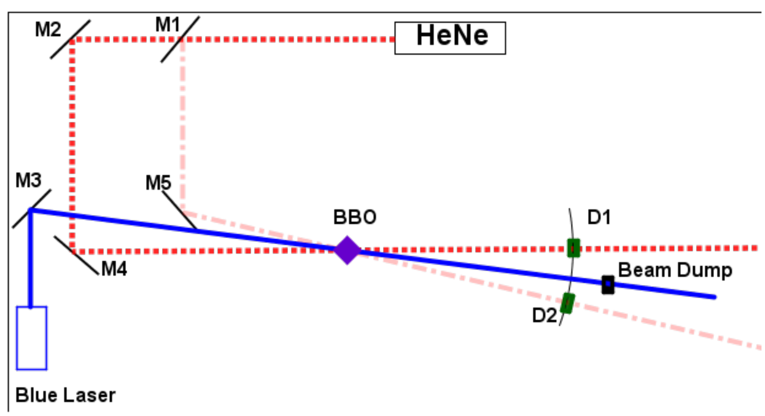

Figure 11: Using an arc to determine the locations of the detectors D1 and D2

aligning each mirror-detector system and mirror-beam dump system.

Lets begin with M3 and the beam dump. Place a plumb bob directly over the location of the crystal, the iris in front of the bob (the iris should be in between the M3 and the plumb bob), and place a second plumb bob over the marked location on the arch for the position of the beam dump. It will be also helpful to place a screen behind the second plumb bob to see the shadows cast from the strings on the plumb bobs.

Turn on the blue laser (Again, with proper eye wear on) and slightly shift M3 along its dotted path and finely adjust the sideways rotation of the mirror so that the beam simultaneously passes through the iris, is cut by the first string of the plump bob, and that shadow is cast over the second plumb bob such that the string from the second bob is in the middle of the shadow. This can be a tedious process with many fine adjustments necessary.

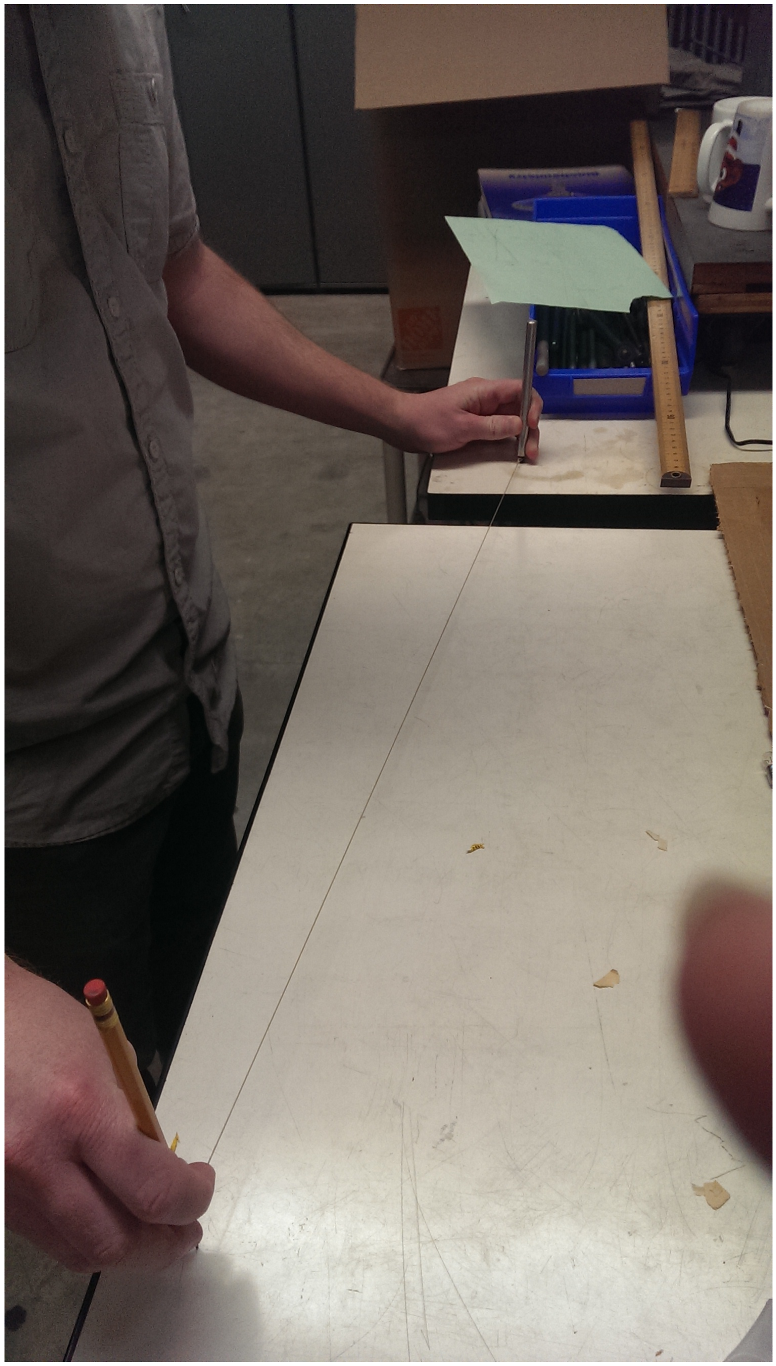

Figure 12: Compass for drawing arch

This process is to be repeated for both M4 and M5, such that the beam from M5 reaches the position of D2 and the beam from M4 reaches the position of D1. Leave the first plumb bob in place, slightly shift the iris accordingly for each mirror to have the surface of the iris be perpendicular to that of the incoming beams. Also move the plumb bob and screen accordingly for whatever system is currently being set up.

3.2.1 Inserting the BBO Crystal

Now that the 3 beams have been crudely aligned, we may insert out BBO crystal. Place the crystal directly upon the mark made for its position. The crystal must be perpendicular to the incident blue beam. It may be helpful to have the crystal mounted on a tip-tilt stage, although not necessary. To assure perpendicularity, the beam reflected from the front surface of the crystal should be going back into the blue beam. With the proper eye wear, one cannot see the beam passing through the crystal. It is recommended to use a piece of paper in the desired path of the blue beam to see its intensity upon the surface of the paper.

Figure 13: Use a screen to see the beam

3.2.2 Inserting and aligning Detectors

USE ONLY RED LASER LIGHT HERE. In order to properly place a detector into our system, we must align its corresponding collimator. Place a viewing screen roughly 5-7 cm behind the proposed location of the detector/collimator. With the aligned red laser on, place a dot on the screen where the laser hits it. Leave the screen behind the desired location of the collimator. Place the mounted collimator in the path of the beam. If the collimator is both mounted properly and in the correct location, then we will see the expanded red beam upon the screen concentric with that dot previously drawn on the screen. Adjust the collimator accordingly so that the spread beam is concentric with the dot on the screen.

Now that the collimator is in the correct location, it needs to be finely adjusted to a

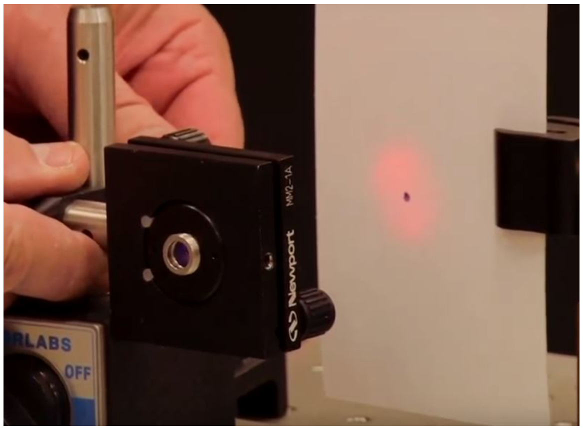

Figure 14: The dot is in the middle of the spread beam

very specific tip-tilt. Place a mirror over the front of the collimator to be flush upon its surface. The reflected beam from the mirror should go directly back into the center of the crystal. Adjust the fine adjustment knobs on the back of the collimator while holding a mirror upon its surface so that this occurs. This can be done using a small separate mirror or by mounting the filters, which has a mirror on one side, upon the front side of the collimator.

With the beam reflected upon the crystal, we may not connect a fiber optic cable to the back of the collimator. The cable **SHOULD NOT** be plugged into a detector at this time, but rather have the opposite end open so we can see the light coming out from it. Place the open end of the fiber toward a screen to view the exiting light. Now, further finely adjust the knobs on the back of the collimator while view the exiting light from the fiber upon a screen. Adjust this very finely until a nice collimated dot is shown. Try to get the brightest light exiting the fiber, like shown. This collimator-detector system is ready to go. Repeat the process for D2.

At this point, we are nearly ready to see coincidence counts. Before we do anything now, assure that the band-pass filters are mounted upon the front of each collimator, place the beam dump in its proper location in between the two collimators, and turn off the lights.

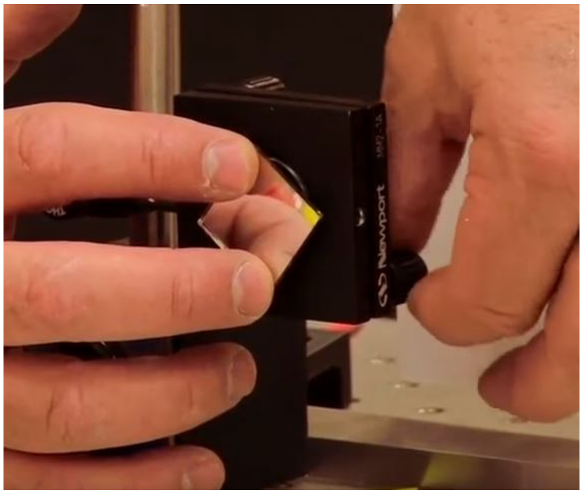

Figure 15: Reflect the beam back into the crystal

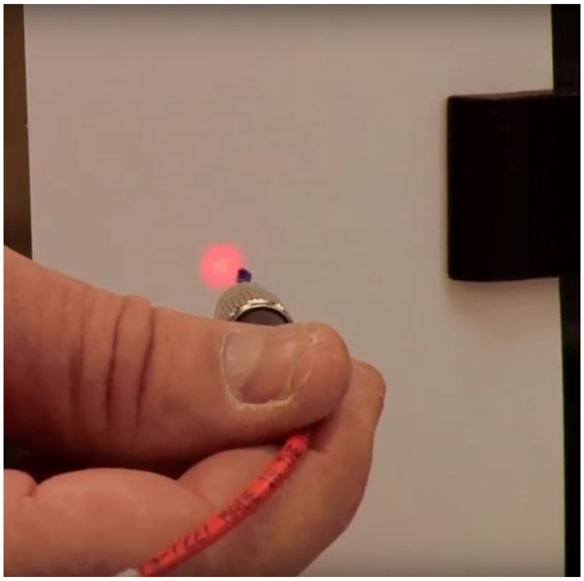

Figure 16: We want the brightest light upon the screen

3.3 Coincidence Counts

As a note, if we are not immediately seeing the magnitude of counts that we are expecting, the first thing to attempt to adjust is the crystal and THEN the collimators. If need be, we will very finely adjust the crystal and secondly finely adjust the collimators to see optimized counts from both of which. Additionally, we can adjust the mirror directing the blue beam to the BBO crystal (M3).

In Figure 17, the screen on the upper left (second from the end) is showing the coincidence counts between the two detectors. As an exaggeration, when the crystal is moved, the coincidence counts may reach a maximum or decline due to the positioning being correct, hence the peak in the graph. We want to adjust the crystal and fine tune the collimators

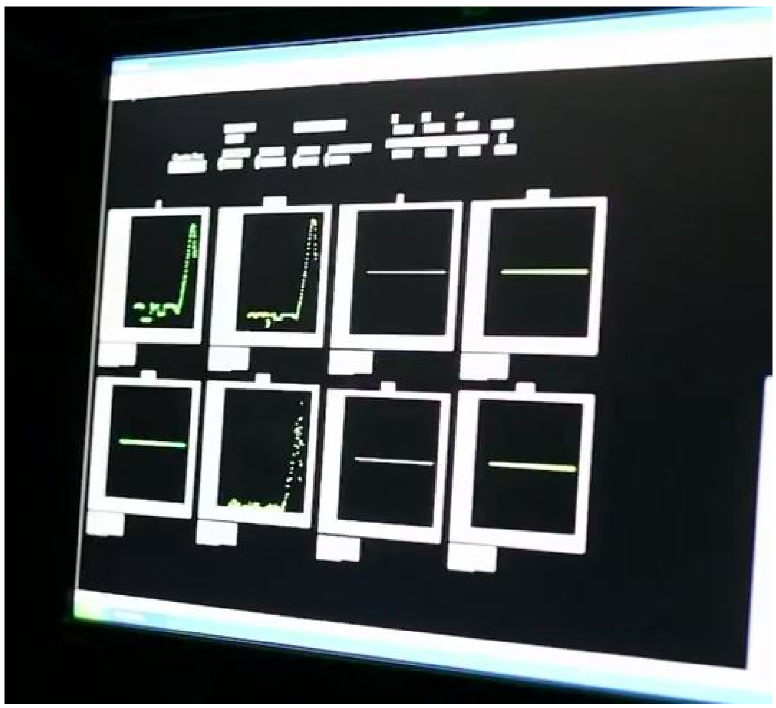

Figure 17: Move to highest coincidence

to have the maximum coincidence counts. This is shown in the figure below, however, the update period may need to be higher than .3 seconds. Open the file titled $My\_Dyn2013.vi$ (see Figure 17).

3.4 Setting up the Mach-Zehnder Interferometer

VERY IMPORTANT!!! We are now going to insert the interferometer with 1 piece at a time. Referring to Figure 11, our interferometer is going to go along the red-beam path in between the BBO crystal and detector D1, which should be directly along the holes of the table. We need to place our Piezo Electric within the translating bar of the translation stage to translate one of our mirrors which will allow our beam path to slide in and out of interference with its corresponding partner path! See Figure 18.

With the red laser on, place the first beam splitter roughly 40 cm in front of the BBO crystal with the incident beam intersecting the splitter (as best you can) right above a hole in the table such that the intended reflected beam is on the opposite side as is detector 2 (D2). Now, we will perform the same iterative process with the iris to assure that our reflected beam is perpendicular to the incident and directly along the holes in the table, or best, assured to be parallel to the holes in the table.

Next, we must place the first mirror of the interferometer to reflect the beam which as been translated from the beam splitter perpendicular to its incidence upon this mirror. NOTE: We can either make our interferometer as small as possible (with roughly 4 holes on the table in between each piece) which would be best for specifically single photon interference - Or, we can make the distance between each piece in the interferometer with roughly 7 or 8 holes of the table in between them to allow enough room to insert polarizers for the sake of performing the Quantum Eraser Experiment... With this mirror in place, again, we must perform the alignment iteration of the iris to assure our reflected beam is parallel to the holes on the optical table.

The next mirror to be placed is the mirror atop the translational stage with the piezo electric placed in the translating bar accordingly (As in figure 18). This mirror and stage is to be placed to reflect the beam which has been reflected by the initial beam splitter perpendicular to the beam hitting the mirror. Again, the beam incident upon the mirror should be (as best you can) above the holes of the table and consistently the same amount of holes away from the beam splitter as you placed the previous mirror. Once this mirror is in place, again, we perform the alignment iteration of the iris to assure our newly reflected beam is parallel to the holes (and ideally directly above them).

3.4.1 The Second Beam Splitter

We should now have the two reflected beams intersecting in space as shown in Figures 19 and 20. At this current point, our interferometer should look just like Figure 21. We are now ready to put in the last piece of the interferometer! As shown in Figure 20, we have found the spot in which our two beams intersect in space perpendicular to one another. This is exactly where our second beam splitter must go! First, place the beam splitter here roughly by hand such that we have an approximate overlap of beam pairs on both sides exiting the splitter. Lock it in once you have done so.

VERY IMPORTANT: We now need to perform a delicate iterative process similar to before. From the BBO crystal, once our incident beam hits the first beam splitter, we obtain 2 paths - we are going to block one path at a time and adjust either our 2nd beam splitter or our translational stage and mirror accordingly.

Consider our first beam splitter. If we block the beam path which is reflected, then our translated beam still bounces off of its next mirror and enters the 2nd beam splitter. We are, first, interested in the reflected beam from our 2nd beam splitter. With this current beam path, perform the alignment iteration with the iris to assure this path is parallel to

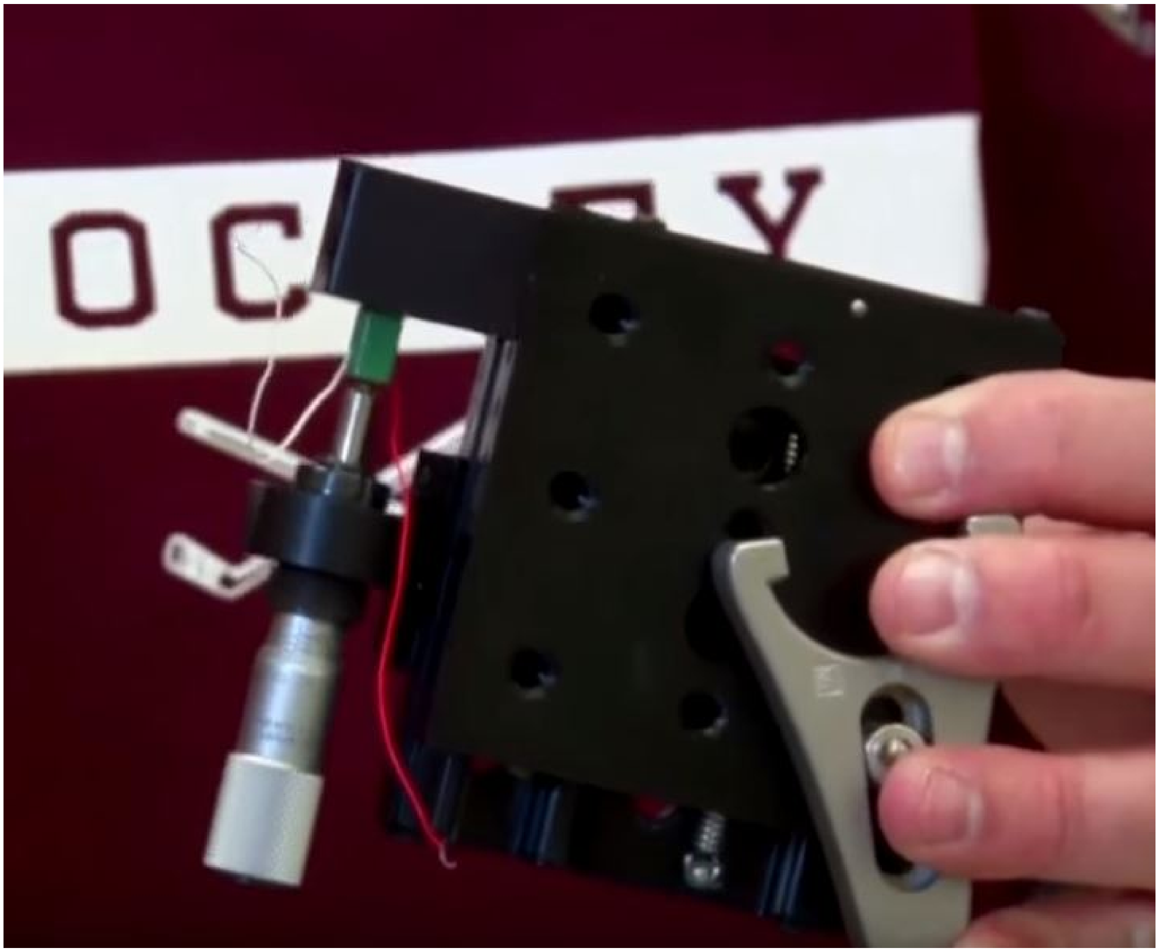

Figure 18: Piezo Driver in the Translational Stage

the holes on the table. See Figure 22.

Now, we must block the opposite beam path within the interferometer such that we can see only the beam reflected off of the mirror which is mounted upon the translational stage passing through the 2nd beam splitter. We are interested in the transmitted beam through the 2 beam splitter which should be very close to passing through where the iris just was. However, this time, we are going to adjust translational stage in performing the alignment iteration with the iris to assure this path is along the same path as the previously aligned beam. See Figure 23.

We will now iterate this iteration! Continue blocking each of the two paths and aligning each piece respectively such that when either path is no longer blocked, our two beams are lying on top of one another. You may need to repeat this several times.

We should now have 2 superimposed beams exiting our 2nd beam splitter of the inter-

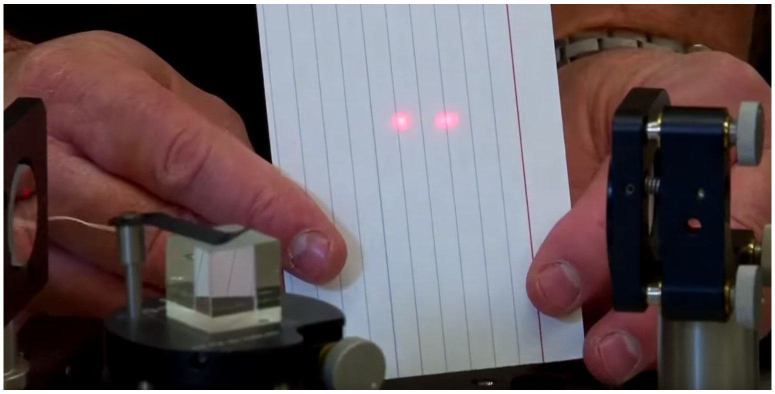

Figure 19: The two reflected beams are almost crossing in space.

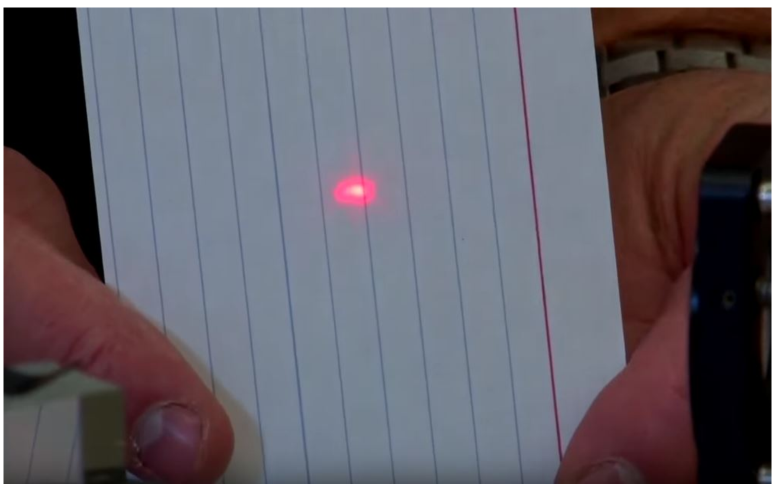

Figure 20: This is the point at which the beams over lap!

ferometer. However, they're not perfect yet! Mount and place a diverging lens in the path of our superimposed beams with a screen behind the lens to see the interference pattern of our magnified beams. Once doing so, we are most likely going to see some sort of fringes. This is good, but what does it mean and how do we fix it?

Chances are that after placing the diverging lens in front of our beam, we see some sort of fringe pattern like in Figure 24 or Figure 25, or possibly a combination of both. In Figure 24, we see several vertical lines next to one another. In this case, our two interfering beams are passing each other at an angle in the same plane parallel to the table. Another way to say this is that they are criss-crossing horizontally.

In Figure 25, we see a similar, yet opposite affect. Here, we have horizontal lines on the screen which means that our two beams are criss-crossing one another vertically in space,

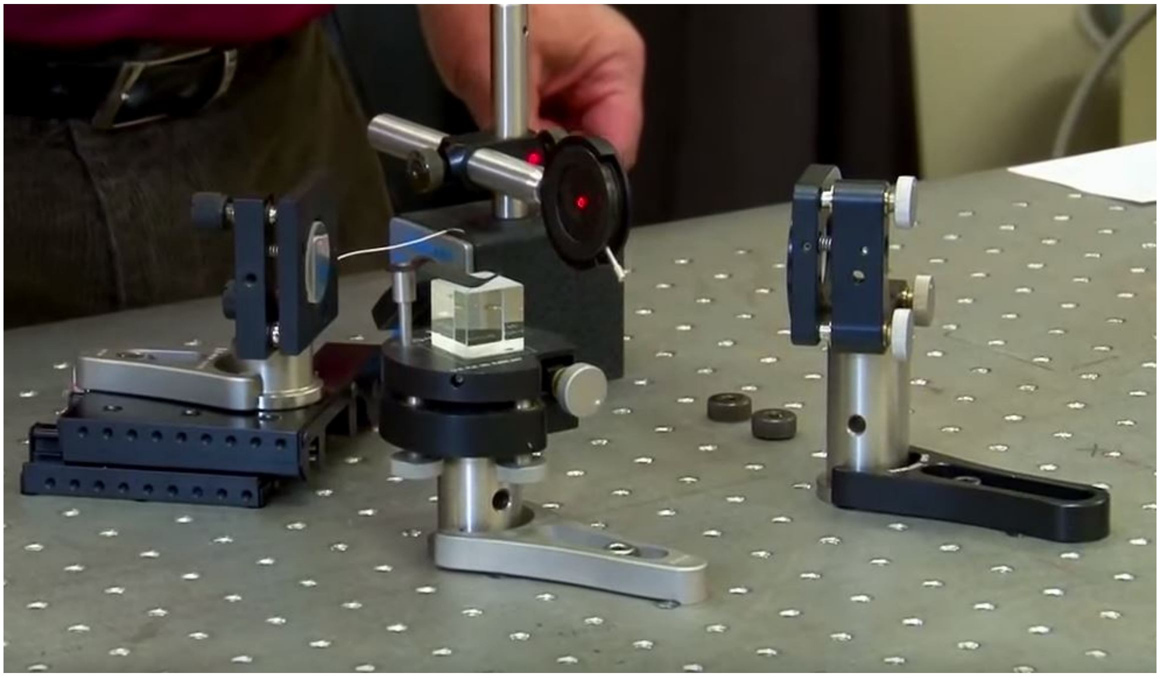

Figure 21: Current interferometer set up

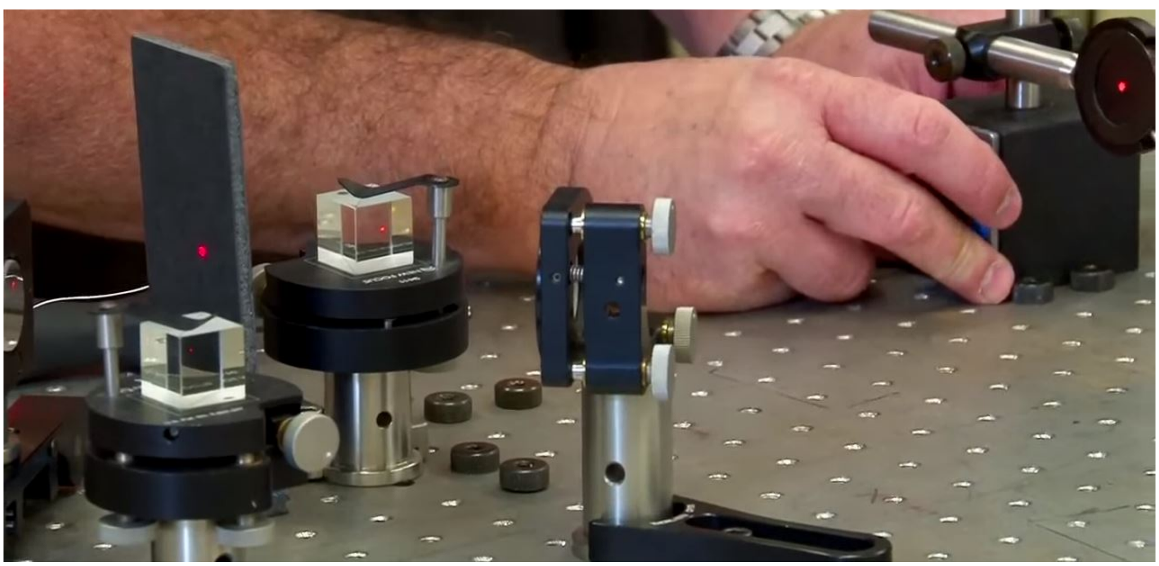

Figure 22: The adjustment of the beam splitter

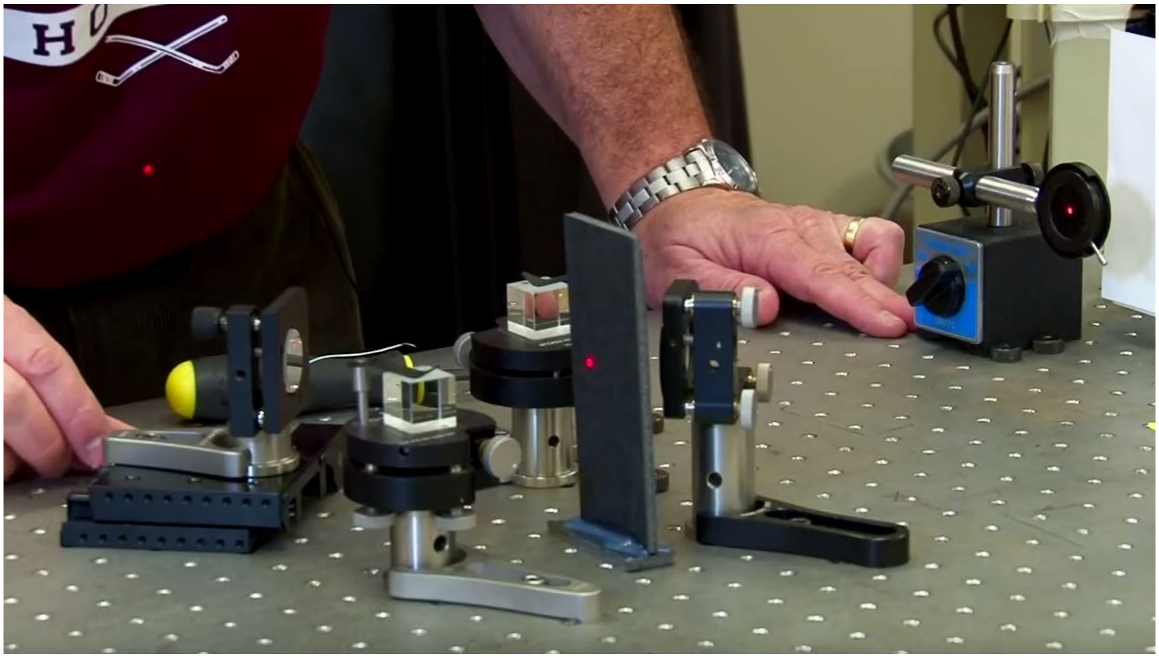

Figure 23: The adjustment of the translational stage

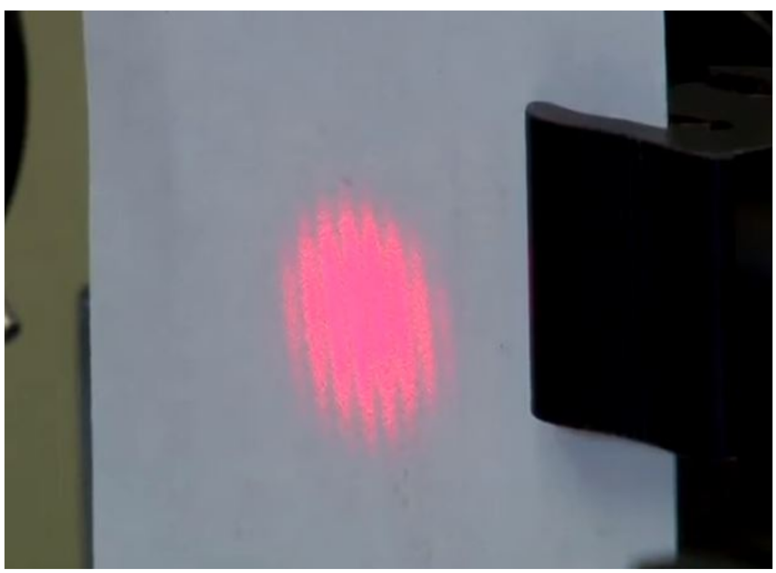

Figure 24: Our beams are criss-crossing horizontally

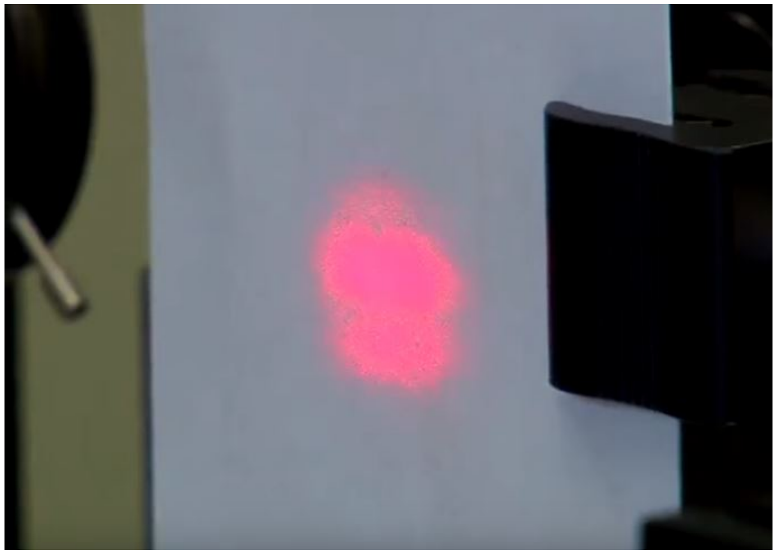

Figure 25: Our beams are criss-crossing vertically

but within the same plane perpendicular to the table.

Our goal is to have an image resemblant of Figure 26 with no fringes in either direction. In order to obtain this, we must adjust the 2nd beam-splitter either vertically or horizontally accordingly (which may seem slightly coarse). We may also, more finely, adjust the translational stage (this will only affect the vertical fringes). If we have done this well enough, then merely tapping the translational stage will result in the image jumping in an out of darkness(destructive interference).

3.4.2 White Light Interference

Our interferometer is close to being perfectly aligned, yet not quite done! In order to assure that the two arms of our interferometer are exactly the same length, we need to see wavelengths of white light interfere with themselves when passing through the Mach-Zehnder interferometer.

We are now going to mount and merely place a small light bulb just in front of our interferometer (a few centimeters before the first beam splitter). At the end of the interferometer, we must place a spectrometer to read these incoming wavelengths. We must mount and place a fiber optic cable running to a spectrometer which is then inputting to a nearby computer and a program (we simply used LoggerPro) which is compatible with the spectrometer to display the incident wavelengths of the white light exiting the spectrometer.

We can see what this should look like in Figure 27 once both are set up. Now, we should

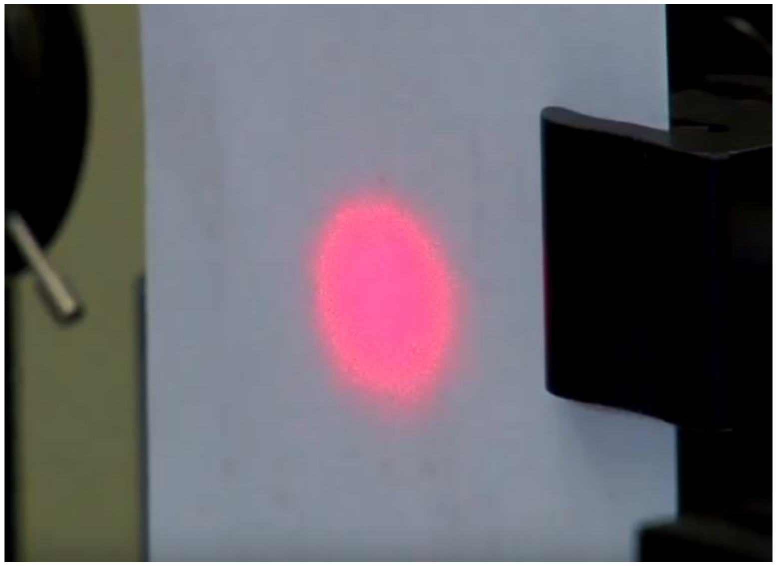

Figure 26: Our beams are constructively interfering on top of one another!

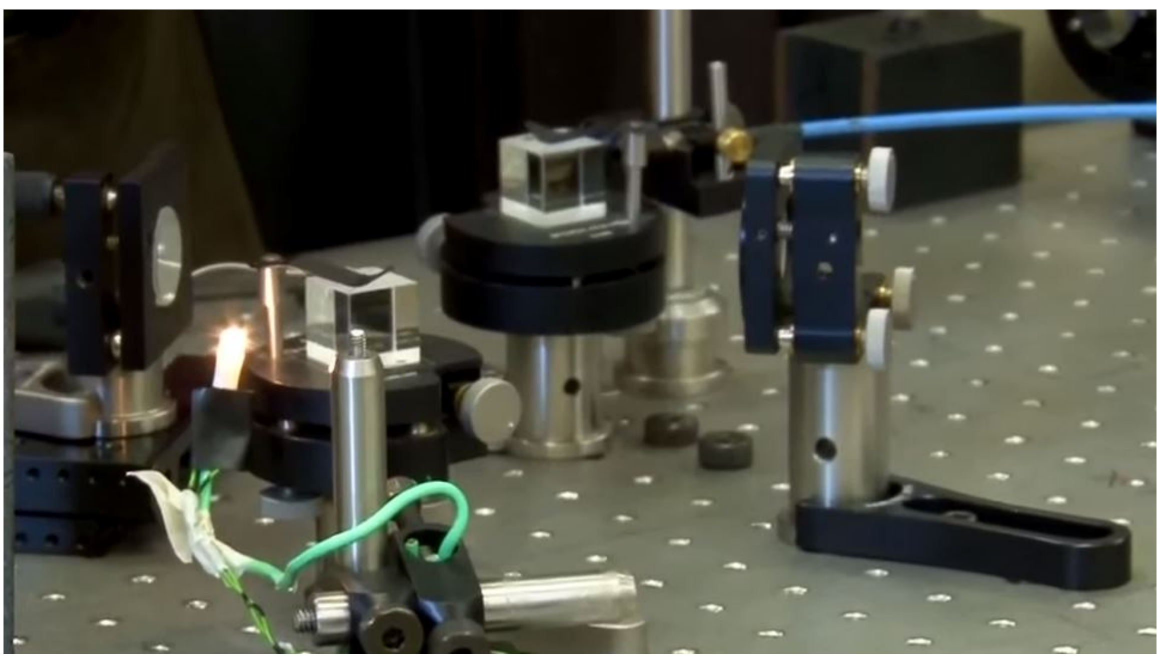

Figure 27: Both the bulb and Fiber Optic Cable are set up

also see some sort of general waveform in whatever program we may be using to view the white light spectrum. Figure 28 is an image of the LoggerPro program used. In the image, we see some wavelengths of white light are interfering constructively and some are interfering destructively. The amount of interference seen is dependent upon the distance of the two interferometer arms. By adjusting the translational stage, we are adjusting the length of one of the arms in the interferometer and effectively making them either closer or further from being equidistant.

Initially, when set up, we probably will not see distinct peaks such as Figure 28. Rather,

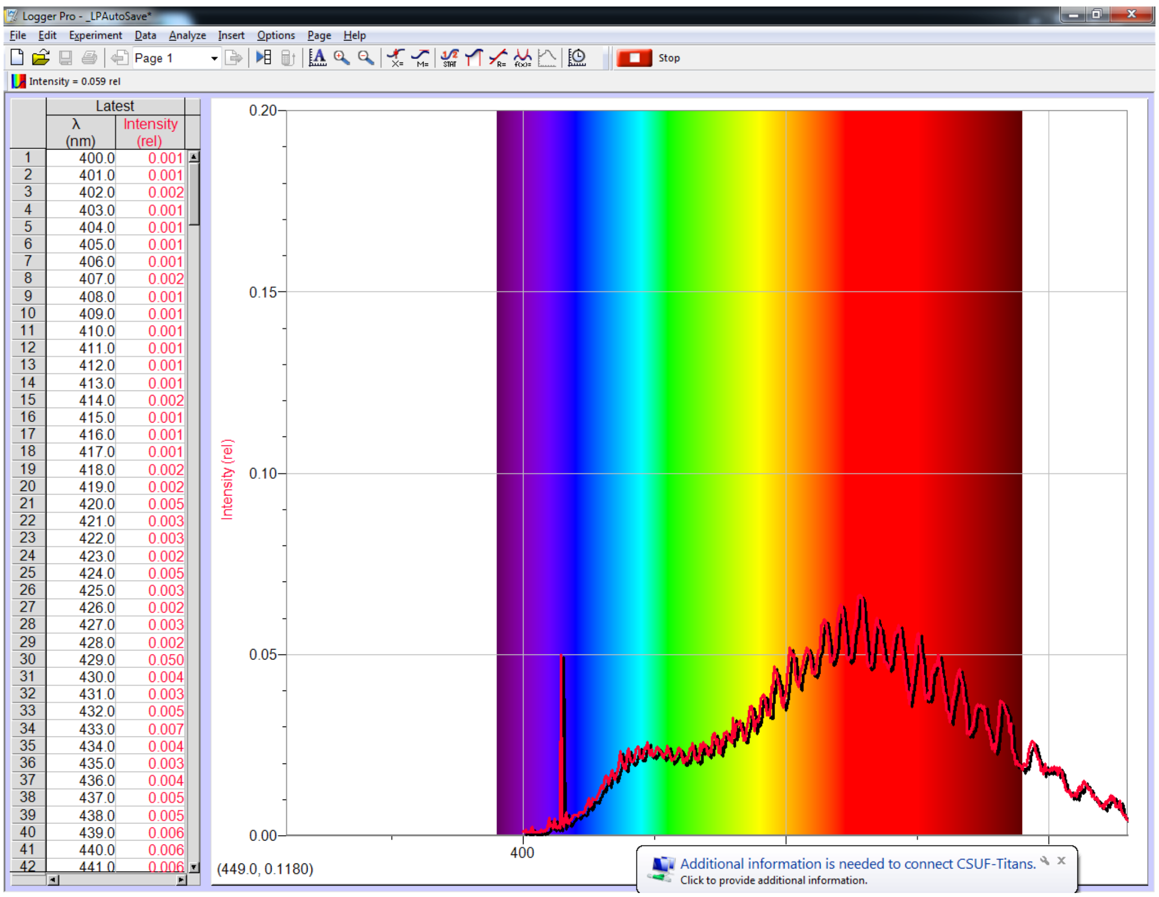

Figure 28: Some wavelengths interfering

we may, and expect to see small compact wiggles. If we do not see these wiggles or smaller waveforms after slightly adjusting the translational stage, then we need to go back and continue alignment of the interferometer (expect to do this). Once we do have the peaks, such as in Figure 28, we want to continue shifting the translational stage pushing the mirror such that our peaks separate. Eventually, if we nearest the point of the most destructive interference, in the slightest touch of the translational stage, we will see the entire spectrum jump up and down. This is what we want; our interferometer arm lengths to be exactly the same. Once we have reached this point, our interferometer is ready.

To be extra cautious (highly recommended), place the red beam back through the interferometer, double check the fringes and assure that we have a thick dot again. Once this is done, we are ready to detect interference counts!

Lastly, as before, we need to perform the same process as mentioned before in setting up a detector along this newly created path to view the destructive and constructive interference of photons with themselves.

3.4.3 Seeing Single Photon Interference

VERY IMPORTANT DO NOT TURN ON THE DETECTORS AND BLUE BEAM UNLESS THE LIGHTS ARE ALL OFF OR THE DARKROOM IS CLOSED.

Once, enclosed and the appropriate vi file is pulled up ready to be run, with the piezo driver set up, we can turn on our beam and detectors and experience Single Photon Interference. See Figure 29.

Figure 29 may not be entirely clear. Here, a trial was run in which the piezo driver increased in voltage steps by .01 volts every second until reaching a maximum of 5 volts. The upper left most box is the photon counts in time of detector 2 (without the interferometer in front of it). The next box, rightward, is the counts in time of Detector 1 (photons through interferometer). We can see that as the driver is pushing the mirror, our photons are going in and out of interference. In the next rightward box, an additional detector was set up outside of the other interferometer arm to see the phase shift of photons interfering out the other end (this is not necessary).

The most important box, is the bottom left which is showing the simultaneity of photon counts in both detector 1 and detector 2 which is where we expect to see optimum photon interference. Again, the next box over, is the interference with the addition, unnecessary detector and detector 2. Export and analyze your own data!

Figure 29: Interference, Phase Shift and Coincidence

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[1] Enrique J. Galvez. *Correlated-Photon Experiments for Undergraduate Labs*. Colgate University. 2010.

[2] Enrique J. Galvez and Mark Beck. "Quantum Optics Experiments with Single Photons for Undergraduate Laboratories". In: *Optical Society of America* (2007).

[3] Kiko Galvez. *How to set up single-photon interference experiments*. Apr. 2013. URL: https://www.youtube.com/watch?v=_SRe5W3YoQ8.

[4] Christopher Gerry and Peter Knight. *Introductory Quantum Optics*. Cambridge University Press, 2005.

[5] Philippe Grangier, Gerard Roger, and Alain Aspect. "Experimental evidence for a photon anticorrelation effect on a beam splitter: A new light on singlephoton interferences". In: *Europhysics Letters* 1 (1986), pp. 173–179.

[6] S. F. Adams K. P. Zetie and R. M. Tocknell. *How does a Mach-Zehnder interferometer work?* Jan. 2000. URL: https://www.cs.princeton.edu/courses/archive/fall06/cos576/papers/zetie_et_al_mach_zehnder00.pdf.

[7] Bo-Sture Skagerstam. *Topics in Modern Quantum Optics: Lectures presented at The 17th Symposium on Theoretical Physics - APPLIED FIELD THEORY, Seoul National University, Seoul, Korea, 1998.* 1999. URL: http://arxiv.org/abs/quant-ph/9909086.

[8] D.F. Walls and Gerard J. Milburn. *Quantum Optics*. second. Springer, 2008.

Appendix

A. Quantum Mechanics of Beam-splitters

Consider the following field operator (see Figure 2)

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \begin{pmatrix} r\hat{a}_1 + t'\hat{a}_0 \\ t\hat{a}_1 + r'\hat{a}_0 \end{pmatrix} \tag{9}$$

The operators satisfy Equation 2:

$$\begin{aligned}
[\hat{a}_2, \hat{a}_2^\dagger] = [\hat{a}_2, \bar{r}\hat{a}_1^\dagger] + [\hat{a}_2, \bar{t'}\hat{a}_0^\dagger] &= [r\hat{a}_1, \bar{r}\hat{a}_1^\dagger] + [t'\hat{a}_0, \bar{r}\hat{a}_1^\dagger] + [r\hat{a}_1, \bar{t'}\hat{a}_0^\dagger] + [t'\hat{a}_0, \bar{t'}\hat{a}_0^\dagger] \\
&= r\bar{r} + t\bar{t} \\
&= |r|^2 + |t^2| \\
[\hat{a}_2, \hat{a}_3^\dagger] = [\hat{a}_2, \bar{t}\hat{a}_1^\dagger] + [\hat{a}_2, \bar{r'}\hat{a}_0^\dagger] &= [r\hat{a}_1, \bar{t}\hat{a}_1^\dagger] + [t'\hat{a}_0, \bar{t}\hat{a}_1^\dagger] + [r\hat{a}_1, \bar{r'}\hat{a}_0^\dagger] + [t'\hat{a}_0, \bar{r'}\hat{a}_0^\dagger] \\
&= r\bar{t} + t'\bar{r'} \\
[\hat{a}_3, \hat{a}_2^\dagger] = [\hat{a}_3, \bar{r}\hat{a}_1^\dagger] + [\hat{a}_3, \bar{t'}\hat{a}_0^\dagger] &= [t\hat{a}_1, \bar{r}\hat{a}_1^\dagger] + [r'\hat{a}_0, \bar{r}\hat{a}_1^\dagger] + [t\hat{a}_1, \bar{t'}\hat{a}_0^\dagger] + [r'\hat{a}_0, \bar{t'}\hat{a}_0^\dagger] \\
&= t\bar{r} + r'\bar{t'} \\
[\hat{a}_2, \hat{a}_3] = [\hat{a}_2, t\hat{a}_1] + [\hat{a}_2, r'\hat{a}_0] &= [r\hat{a}_1, t\hat{a}_1] + [t'\hat{a}_0, t\hat{a}_1] + [r\hat{a}_1, r'\hat{a}_0] + [t'\hat{a}_0, r'\hat{a}_0] \\
&= rt + t'r' \\
[\hat{a}_2^\dagger, \hat{a}_3^\dagger] = [\hat{a}_2^\dagger, \bar{t}\hat{a}_1^\dagger] + [\hat{a}_2^\dagger, \bar{r'}\hat{a}_0^\dagger] &= [\bar{r}\hat{a}_1^\dagger, \bar{t}\hat{a}_1^\dagger] + [\bar{t'}\hat{a}_0^\dagger, \bar{t}\hat{a}_1^\dagger] + [\bar{r}\hat{a}_1^\dagger, \bar{r'}\hat{a}_0^\dagger] + [\bar{t'}\hat{a}_0^\dagger, \bar{r'}\hat{a}_0^\dagger] \\
&= \bar{r}\bar{t} + \bar{t'}\bar{r'}.
\end{aligned}$$

As long as we have $r\bar{r} + t\bar{t} = 0$, $t\bar{r} + r'\bar{t'} = 0$, $|r|^2 + |t^2| = 1$, and $rt + t'r' = \bar{r}\bar{t} + \bar{t'}\bar{r'}$. The last equality implies that $|r'| = |r|$ and $|t'| = |t|$. The phase shifts of the reflected and transmitted beam depend on the beam-splitter. For a 50:50 beam-splitter[12] (reflected beam has a $\frac{\pi}{2}$ phase shift), the input and outputs are related by,

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{a}_0 + i\hat{a}_1 \\ i\hat{a}_0 + \hat{a}_1 \end{pmatrix} \tag{10}$$

These transformations must be unitary. Let $\hat{U}$ be the unitary operator and thus

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \hat{U}^\dagger \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} \hat{U}. \tag{11}$$

[12] From [4]: If the beam-splitter is constructed as a single dielectric layer, the reflected and transmitted beams will differ in phase by a factor of $\exp(\pm i\frac{\pi}{2}) = \pm i$

We wish to consider the case of a single photon input state $|0\rangle_0|1\rangle_1 = \hat{a}_1^\dagger|0\rangle_0|0\rangle_1$. For a 50:50 beam-splitter (see Equation 10) we have,

$$\hat{a}_1^\dagger = \frac{1}{\sqrt{2}}(i\hat{a}_2^\dagger + \hat{a}_3^\dagger). \tag{12}$$

If a single photon incident at one of the input sectors of the beam splitter (see Figure 2), the other sector (the unoccupied) will be either transmitted or reflected with equal probability[13]. We show this,

$$|0\rangle_0|1\rangle_1 \to \frac{1}{\sqrt{2}}(i\hat{a}_2^\dagger + \hat{a}_3^\dagger)|0\rangle_2|0\rangle_3 = \frac{1}{\sqrt{2}}(i|1\rangle_2|0\rangle_3 + |0\rangle_2|1\rangle_3).$$

B. Mathematics of the MZI

Consider the input state $|0\rangle|1\rangle$. Assume the states propagate along the anti-clockwise path over the clockwise. If the beam-splitters satisfies Equation 10 (and thus Equation 12), then when the input state $|0\rangle|1\rangle$ passes through the first beam-splitter BS_1,

$$|0\rangle|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + i|1\rangle|0\rangle).$$

If there is a difference between the arms of the interferometer, say they are separated by a phase θ, then the first component will be changed[14] i.e.

$$\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + i|1\rangle|0\rangle) \to \frac{1}{\sqrt{2}}(\exp i\theta|0\rangle|1\rangle + i|1\rangle|0\rangle).$$

Once this reaches the second beam-splitter (BS_2),

$$|0\rangle|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + i|1\rangle|0\rangle) \tag{13}$$

$$|1\rangle|0\rangle \to \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + i|0\rangle|1\rangle) \tag{14}$$

and thus,

$$\frac{1}{\sqrt{2}}(\exp i\theta|0\rangle|1\rangle + i|1\rangle|0\rangle) \to \frac{1}{2}(\exp i\theta - 1)|0\rangle|1\rangle + \frac{i}{\sqrt{2}}(\exp i\theta + 1)|1\rangle|0\rangle).$$

From this we can find the probability that state $|0\rangle|1\rangle$ is detected

$$P_{01} = \frac{1}{2}(1 - \cos\theta)$$

[13] This is what Grangier et al. showed. Notably, no coincident counts are to be expected with photon counters placed at the outputs of the beam splitter.

[14] Actually, the mirror will contribute $e^{i\frac{\pi}{2}}$ to both components but this will irreverent

and the probability that state $|1\rangle|0\rangle$ is detected

$$P_{10} = \frac{1}{2}(1 + \cos\theta).$$

It should be obvious that if the difference between the arms of the interferometer is zero ($\theta = 0$) then we would have,

$$P_{01} = 0 \text{ and } P_{10} = 1.$$

This means that only one of the detectors will pick-up a specific state.

However, in real-life this is not as simple. The 50:50 beam-splitters are not truly 50:50 (Snell's Law). It can be shown that the probability of detecting a photon using a random beam-splitter is given by,

$$P_{10} = \left| e^{i\theta_1} tr + e^{i\theta_2} rt \right|^2 \tag{15}$$

where θ_1 and θ_2 are some phase shift due to being reflected[15]. Consider the state $|0\rangle|1\rangle$ entering the MZI and interact with the first beam splitter (BS_1),

$$|0\rangle|1\rangle \rightarrow t|0\rangle|1\rangle + r|1\rangle|0\rangle.$$

Once this reaches the second beam-splitter (BS_2) there will/might be a phase length θ_1 and θ_2

$$|0\rangle|1\rangle \rightarrow e^{i\theta_1}(r|0\rangle|1\rangle + t|1\rangle|0\rangle) \tag{16}$$

$$|1\rangle|0\rangle \rightarrow e^{i\theta_2}(t|0\rangle|1\rangle + r|1\rangle|0\rangle) \tag{17}$$

and thus the final state is given by,

$$t(e^{i\theta_1}(r|0\rangle|1\rangle + t|1\rangle|0\rangle)) + r(e^{i\theta_2}(t|0\rangle|1\rangle + r|1\rangle|0\rangle))$$

or

$$rt(e^{i\theta_1} + e^{i\theta_2})|0\rangle|1\rangle + (tte^{i\theta_1} + rre^{i\theta_2})|1\rangle|0\rangle.$$

Then,

$$P_{10} = (e^{i\theta_1} tr + e^{i\theta_2} rt)\overline{(e^{i\theta_1} tr + e^{i\theta_2} rt)} \tag{18}$$

$$= 2|t|^2|r|^2 + 2|r|^2|t|^2 \frac{e^{i(\theta_1-\theta_2)} + e^{-i(\theta_1-\theta_2)}}{2} \tag{19}$$

$$= 2|t|^2|r|^2 + 2|r|^2|t|^2 \cos(\theta_1 - \theta_2) \tag{20}$$

[15]See [6] for a more succulent discussion as to why this is.

where $\theta_1 - \theta_2 \equiv \theta$ is a difference in phase between the two interferometer arms. If we had a 50:50 beam-splitter we have that $|r| = |t| = \frac{1}{\sqrt{2}}$ and so

$$P_{10} = \frac{1}{2}(1 + \cos\theta). \tag{21}$$

Moreover, recall that the total probability is 1 and so the probability of picking the other state is

$$P_{01} = 1 - P_{10} = \frac{1}{2}(1 - \cos\theta) \tag{22}$$

as we had before.

C. Daten

Here is a closer look at the data from Figure 29.

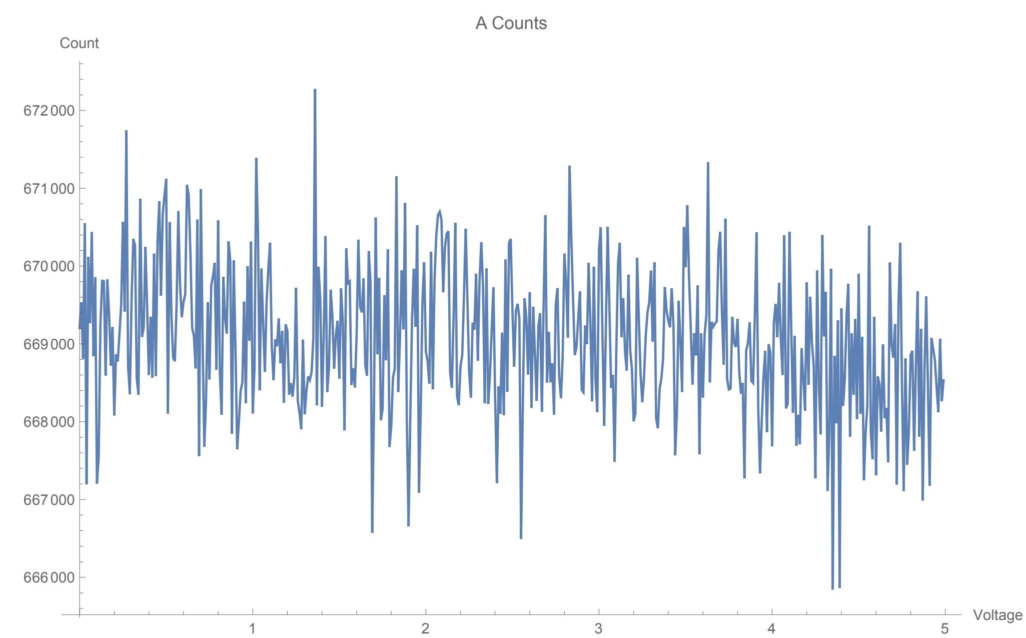

Figure 30: Counts A

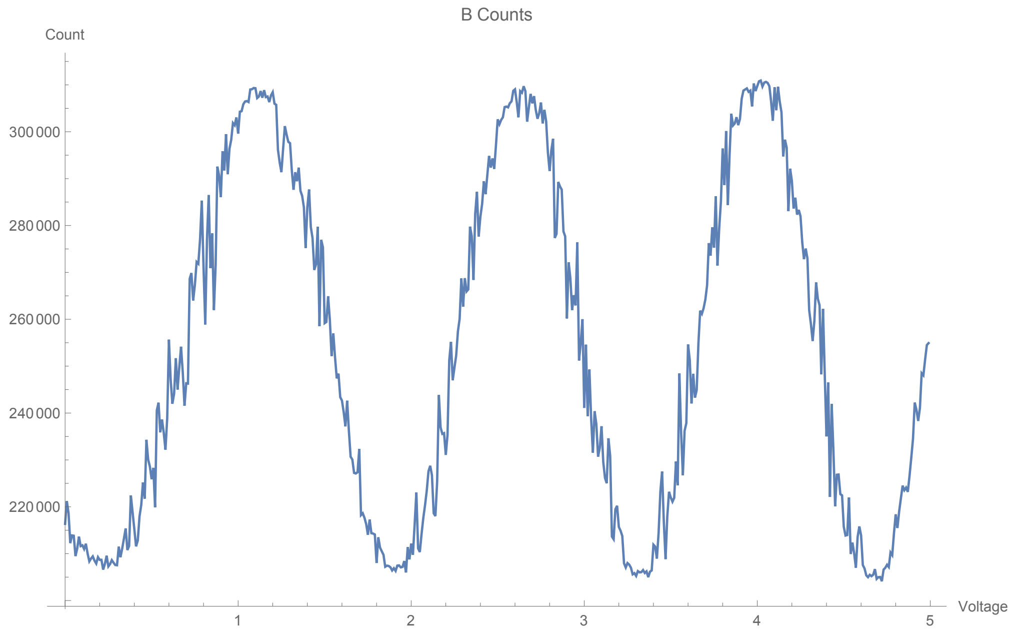

Figure 31: Counts B

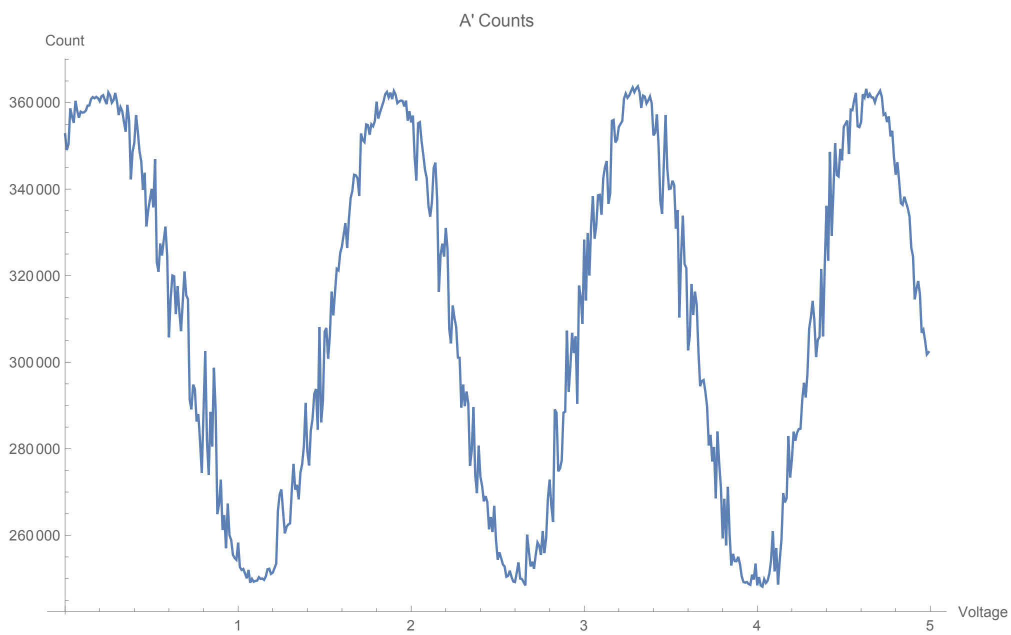

Figure 32: Counts A'

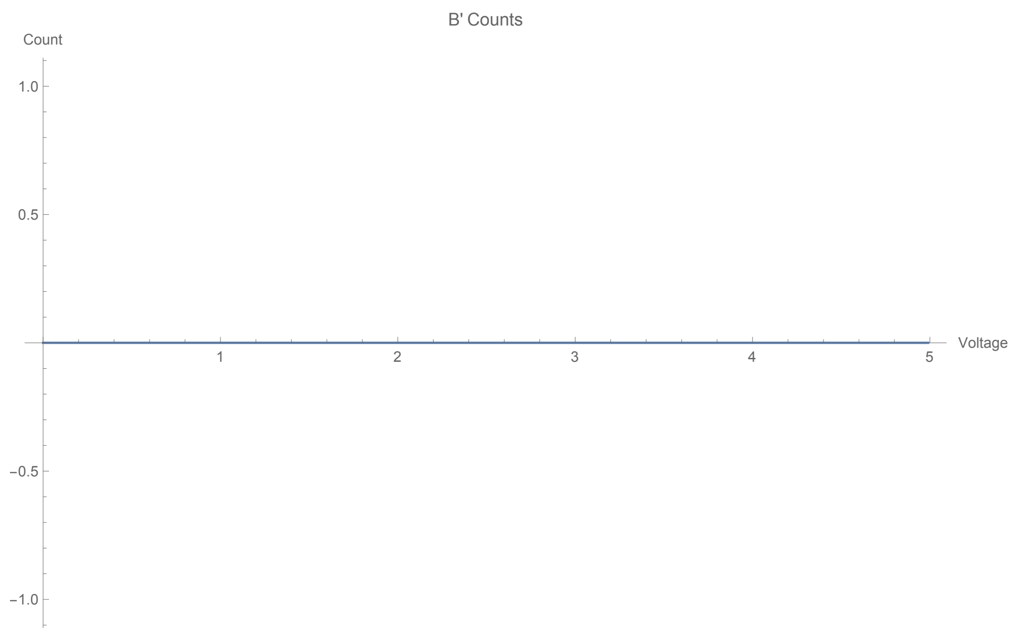

Figure 33: Counts B' (Idle)

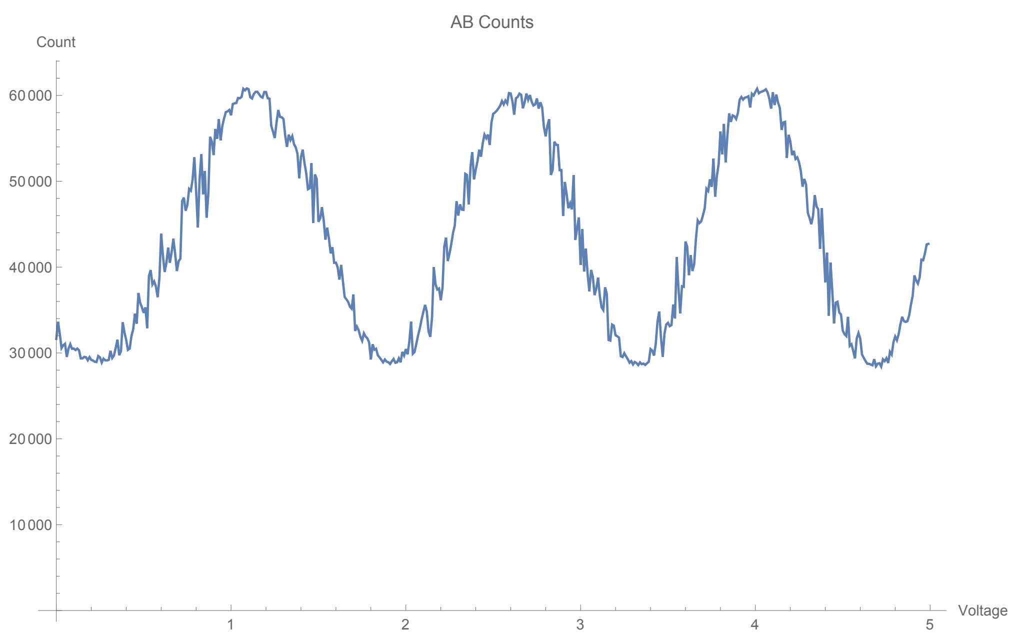

Figure 34: Counts AB

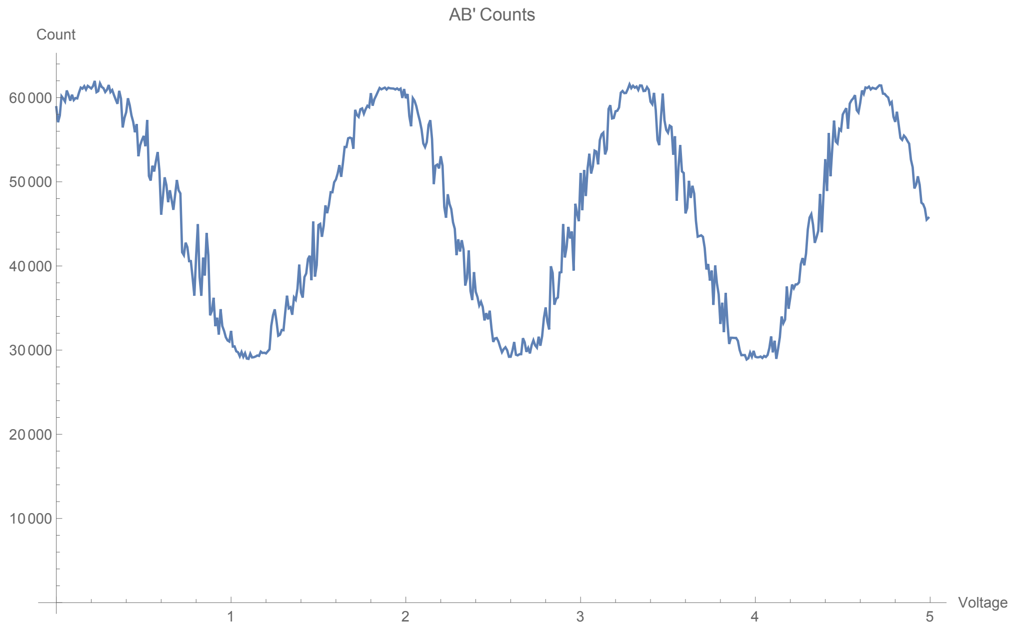

Figure 35: Counts AB'

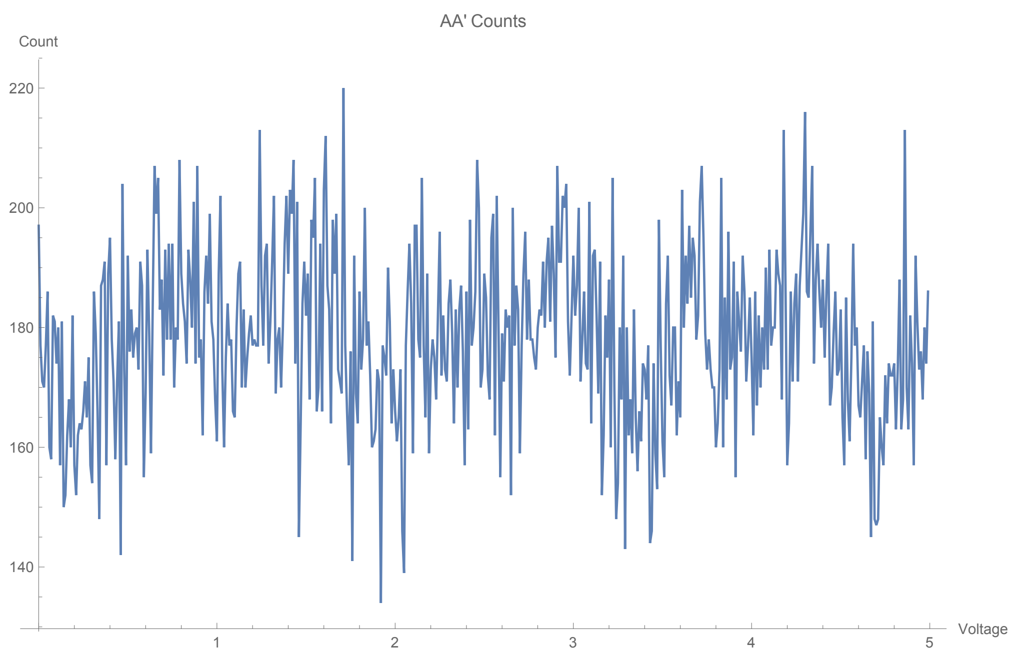

Figure 36: Counts AA'

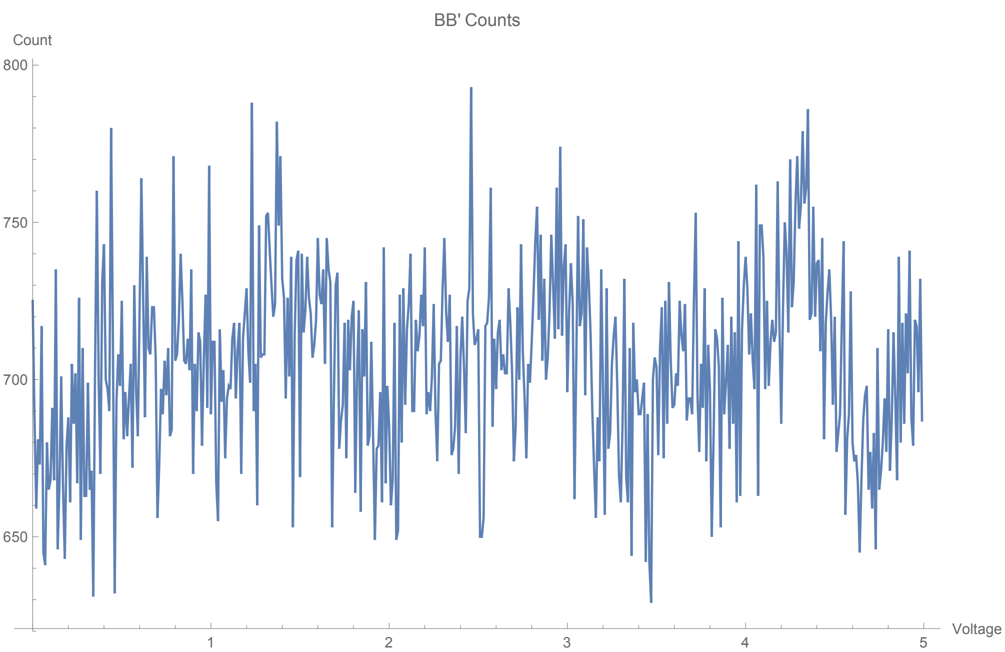

Figure 37: Counts BB’