Determining Damping With Lasers and Photo Diodes

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Table of contents

Vibration

- Vibration is the periodic motion of a body or system of connected bodies displaced from a position of equilibrium.
    1. Free vibration occurs when the motion is maintained by gravitational or elastic restoring forces, such as the swinging motion of a pendulum.
    2. Forced vibration is caused by an external periodic or intermittent force applied to the system.
- Both of these types of vibration can either be damped or undamped.
- Undamped vibrations can continue indefinitely because frictional effects are neglected in the analysis.

Introduction to Damping

- In reality both internal and external frictional forces are present, the motion of all vibrating bodies is actually damped.
- Damping is a phenomenon by which mechanical energy is dissipated, usually converted as thermal energy.
- We can represent this motion mathematically!

Introduction to Damping

- There are three types of damping:
  1. Structural Damping
  2. Coulomb Damping
  3. Viscous Damping
- Structural damping is caused by the inherent intermolecular friction.
- Coulomb damping is caused by rubbing between surfaces.

Viscous damping

- This type of damping is caused by fluid friction, viscosity.
- For example when the system vibrates in a viscous medium such as air or water.
- In vicious damping, the damping force $\mathbf{F}_d$ is is proportional to the negative velocity, since it is against the motion, that is

$$\mathbf{F}_d = -c\mathbf{v} = -c\frac{d}{dt}(\mathbf{x}(t)) = -c\dot{\mathbf{x}}$$

where $\mathbf{x}$(t) the motion of the object dependent on time, c is the proportionality constant.

Equation of Motion

- Recall the elastic restoring force and Newton's 2^{nd} Law

$$\mathbf{F} = -k\mathbf{x}(t)$$

and

$$\sum \mathbf{F} = m\mathbf{a} = m\ddot{\mathbf{x}}(t)$$

- Together with the damping force, we can model the motion by the following equation

$$m\ddot{\mathbf{x}}(t) + c\dot{\mathbf{x}}(t) + k\mathbf{x}(t) = 0 \tag{1}$$

where m is the mass, k is the stiffness of the material.

Equation of Motion

- This equation is a second order linear ODE.
- As usual guess a solution of the form

$$\mathbf{x}(t) = e^{\lambda t}$$

which will yield

$$e^{\lambda t}(m\lambda^2 + c\lambda + k) = 0,$$

or simply that

$$m\lambda^2 + c\lambda + k = 0$$

- Solving this by utilizing the quadratic equation we get

$$\lambda = -\frac{c}{2m} \pm \frac{(c^2 - 4km)^{1/2}}{2m}$$

Equation of Motion

- Notice that the behaviors of the system will depend on

$$c^2 - 4km.$$

We will call the critical damping coefficient c_c the c that is equivalent to

$$c_c^2 - 4km = 0 \rightarrow c_c = 2Sqrt(km).$$

Notice also that $Sqrt(\frac{k}{m}) = \omega_n$, where ω_n is the natural frequency.

- We define the damping ratio, ζ, to be

$$\zeta = \frac{c}{c_c} = \frac{c}{2Sqrt(km)}. \tag{2}$$

Equation of Motion

- We can rewrite λ as: $\lambda = -\zeta\omega_n \pm \omega_n(\zeta^2 - 1)^{\frac{1}{2}}$
- Our general solution, $\zeta \neq 1$ with A, B as constants of integration:

$$\mathbf{x}(t) = e^{-\zeta\omega_n t}(Ae^{\omega_n tSqrt(\zeta^2-1)} + Be^{-\omega_n Sqrt(\zeta^2-1)t}) \tag{3}$$

Type of Damping

- We have three cases to consider $\zeta < 1$, $\zeta = 1$, and $\zeta > 1$
- $\zeta < 1$ is considered to be underdamped motion: if that happens our solution will be complex.
- $\zeta = 1$ is considered to be critically damp: we get one real solution, for λ.
- $\zeta > 1$ is considered to be overdamped motion: we get to real solutions for λ

Type of Damping

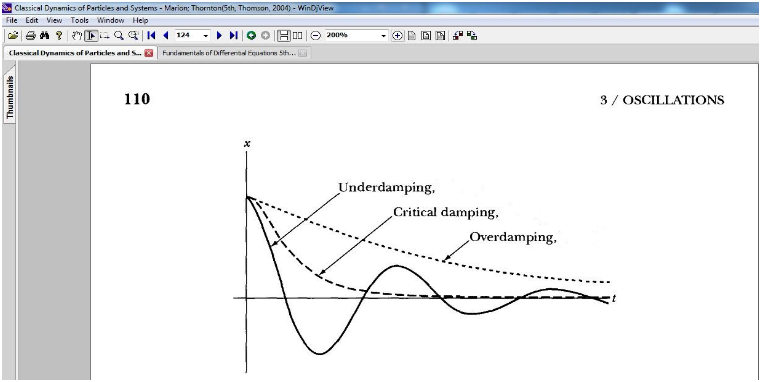

Figure: Different Damping, from [3]

Underdamp Motion

- $\lambda = -\zeta\omega_n \pm \omega_n \mathrm{i}(1-\zeta^2)^{\frac{1}{2}}$
- We define the damped natural frequency to be

$$\omega_d = \omega_n Sqrt(1-\zeta^2)$$

- Our general solution in this case is:

$$\mathbf{x}(t) = De^{-\zeta\omega_n t} sin(\omega_d t + \phi) \tag{4}$$

Were D is the initial displacement, or amplitude

Underdamp Motion

- An alternative way of witting (4), with $\beta = \zeta\omega_n$ is

$$\mathbf{x}(t) = Ee^{-\beta t}cos(\omega_d t - \delta). \tag{5}$$

With this the ratio of the amplitude of two successive maxima is

$$\frac{Ee^{-\beta t}}{Ee^{-\beta(t-\tau)}} = e^{\beta\tau}, \tag{6}$$

where $\tau = \frac{2Pi}{\omega_n}$, the period of motion.

the Quality Factor, Q

- It measures resonance.
- It is the number of free oscillations an oscillator completes before decaying to zero
- If the Q factor is large we have light damping
- If the Q factor is small we have heavy damping
- for small ζ we have that Q and ζ are related by the following equation

$$\zeta = \frac{1}{2Q} \tag{7}$$

The Experiment

- In the experiment consisted of shinning diffuse laser light at two diodes
- Then we placed a material between the laser and the diodes in such a way that it casts a shadow on the diodes
- Then we vibrate the material and analyze the signals coming of the diodes

The Diodes

- The two diodes are set up using a zero bias or photovoltaic mode
- This mean that the light hitting the diode causes the driving voltage using the photovoltaic effect
- When light hits the diode, electrons are moved from the valence band to the conduction band
- The electrons then cause a driving voltage as they move through the Electric field built into the diodes via electron deficient regions

The Diodes

- Here is the diagram for the photovoltaic mode used.

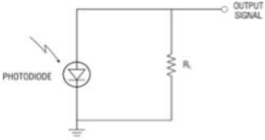

Figure: Schematic with Diode

- We placed two such circuits on a single bread board with the diodes place adjacent to one another with some space in between.

The Op-Amp

- Using an LM301 operational amplifier, we analyzed the signals from the two diodes.
- The Op-amp takes in the signal from the diodes and subtracts them using negative feedback

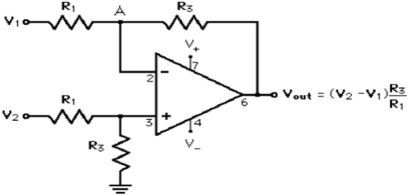

Figure: LM301 with Schematic

The Op-Amp

- Here V_1 and V_2 are the signals from the diode
- The Op-amp supplies negative feedback in order to make the signal at pin 2 and 3 equal
- So if we assume $V_A = V_B$ and we observe the rules for a voltage dividers ,

$$V_{out} = (V_2 - V_1)\frac{R_3}{R_1} \tag{8}$$

- Thus by making $R_3 = 100R_1$ we increase our signal to noise ratio by a factor of 100

The Laser

- In order to apply enough light to the diodes we used a curved lens to refract the light of the laser to cover an are slightly larger than the area of diodes

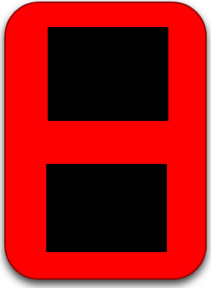

Figure: Light shinning on Diodes

The Material

- We placed the material in a clamping devise between the laser and the diodes in such a way that the shadow of the material barley overlapped the diodes

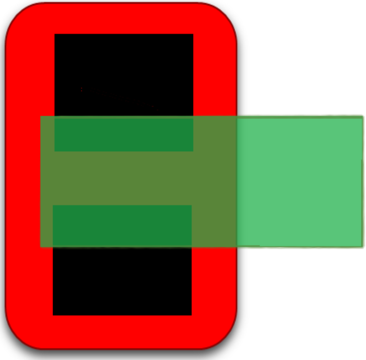

Figure: Material covering diodes

The Material

- When the material vibrates it blocks one of the diodes and thus decreases one of the signals and so the output of the op-amp changes
- When the shadow blocks the bottom diode $V_1 = 0$, the output signal of the op-amp achieves a maximum as $V_2 - V_1 = V_2$
- When the shadow blocks the top diode $V_2 = 0$ so the output signal of the op-amp achieves a maximum as $V_2 - V_1 = -V_1$

The Signal and Obtaining the Data

- As the amplitude of vibrations decreases more light falls on both diodes. So the difference in the signals at the maximums and minimums becomes smaller over time.
- After obtaining the signal we measured the original peak to peak then found at what time the peak to peak signal had dropped to $\frac{1}{e}$ the size, that is when time,t, was equal to τ.
- Next are some of the resulting signals.

Results

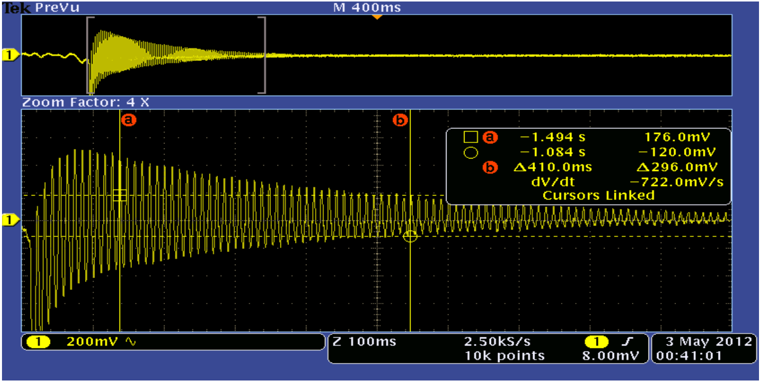

Figure: Metallic Ruler

Results

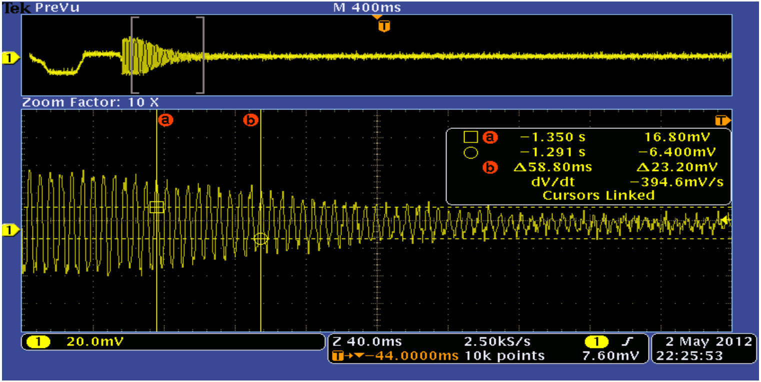

Figure: Wood Stick

Results

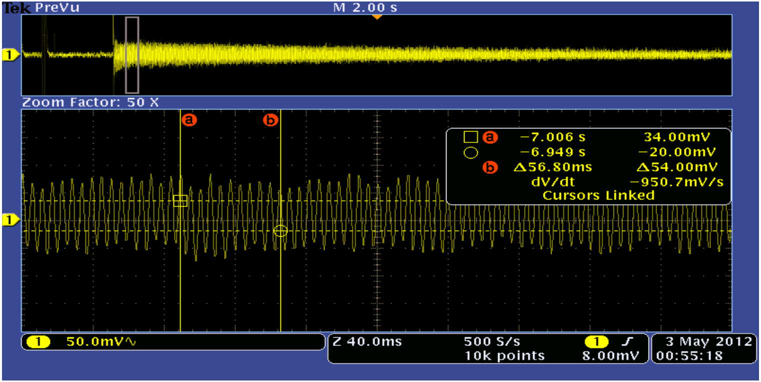

Figure: Tuning Fork

Results

Material	Frequency (Hz)	τ(s)	Q=τ*f*π	B=1/τ
Plastic Ruler	47.6	.344	51.44	2.91
Wood Stick	172.4	.136	73.7	7.35
Plastic Binder Clip	20.7	1.356	88.2	.737
Metal Ruler	84.7	.720	191.6	1.39
Thick Metal	96.2	.447	135.1	2.24
Tuning Fork	181.2	10.712±.5	6098±280	.0936±.0046

[1] R. K. Nagle; E. B. Staff; A. D. Snider, *Fundamentals of Differential Equations 5th Edition*, Addison Wesley, New York, 2010.

[2] S. G. Kelly, *Fundamentals of Mechanical Vibrations 2nd Edition*, McGraw Hill, 2000.

[3] S. T. Thornton; J. B. Marion, *Classical Dynamics of Particles and Systems*. Thomson Brooke/Cole, Belmont, 2004.

W. Rudin. *Principles of Mathematical Analysis*, McGraw-Hill, New York, 1953.