Shadow Sensoring: Determining Damping with Lasers and Photodiodes

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Abstract. With this experiment, we will become acquainted with the notion of damping of various materials.

I. INTRODUCTION

In this experiment, we investigated the damping of various materials using a shadow technique. We had a laser on one end and using a convex lens, converging so that we could have the laser hit both of the sensors. Once the laser hits the sensor, we clamp a flexible material in between the lens and the sensor. Then when we flick the material, the sensors will pick up when the laser is shining on it and when it is not. Thus if we connect it to an oscilloscope, we will get a signal that is on when the laser hits the diode and off when the shadow of the material cover the diode. From this, we will get an underdamp oscillation and we will do analysis on this.

In this lab, we are dealing with underdamp vibrational motion. There are two types of vibration: only the gravitational or elastic restoring forces maintain free vibration, in which motion, for example a pendulum hanging from a string; the second is a forced vibration in which an external periodic force is applied to the system. Both of these can be considered damped or undamped motion, but realistically speaking all motion is damped motion. The difference lies that in undamped motion we analyze the system neglecting the frictional effects generation, such as two surfaces coming into contact to each other.

As far as damping goes, there are three types: structural, coulomb, and viscous damping. Structural damping is caused by intermolecular friction and coulomb damping is caused by two surfaces meeting each other. The last case is viscous damping, in which the damping is caused by viscosity. In reality all these damping are present but for sake of argument suppose that the material, as it is being flick , only experiences viscous damping, the other, are, in most cases negligible, in our experiment I will talk about this. Thus if the motion is damped by a viscous medium, such as air or water. Then a mathematical model, i.e. a second order linear differential equation, can analyze this type of motion. We have that the damping force is $\boldsymbol{F}_d = -c\,\boldsymbol{v}(t)$, where $\boldsymbol{v}$ is the velocity at time t and c is just proportionality constant. If we combine this with Newton's 2nd Law, along with the elastic restoring force, $\boldsymbol{F}_R = -k\boldsymbol{x}(t)$, where k is proportionality constant referred to as rate, or the spring constant and $\boldsymbol{x} = \boldsymbol{x}(t)$ is the position at time t. Together with the second law we get that the sum of forces is equal to $\boldsymbol{F} = \mathrm{m}\,\mathbf{a}(t)$, where m is the mass of the object and $\mathbf{a}(t)$, is the acceleration of the object at time t. If we write acceleration and velocity as functions of position, we get the following equation:

$$m\,\boldsymbol{x}''(t) + c\,\boldsymbol{x}'(t) + k\,\boldsymbol{x}(t) = 0,$$

a second order linear ordinary equation, which has solution of the form: $\boldsymbol{x}(t) = \mathrm{Exp}[\lambda t]$, which will yield that $\mathrm{Exp}[\lambda t](m\lambda^2 + c\lambda + k) = 0$, since the exponential cannot be zero it will lead that the polynomial in the parenthesis will be zero, thus

$$\lambda = \frac{-c}{2m} \pm \frac{\sqrt{c^2 - 4\mathrm{km}}}{2m}$$

and the state of the system depends on the discriminant of the solution. When the discriminant equals zero we get a state known as critical damping that is when $c_c = c = 2\sqrt{\mathrm{km}}$. One important aspect of this motion is the damping ratio, that is comparing the damping ratio, ζ, of a material with that of when there is critical damping, that is $\zeta = \frac{c}{c_c} = \frac{2}{2\sqrt{\mathrm{km}}}$. After some math, we can get that if we let $\sqrt{\frac{k}{m}} = \omega_n$, we can get:

$$\lambda = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

which will lead to our general solution to be:

$$\boldsymbol{x}(t) = e^{-\zeta\omega_n t}\left(Ae^{\omega_n t\sqrt{\zeta^2-1}} + Be^{-\omega_n t\sqrt{\zeta^2-1}}\right)$$

We have three cases to consider now when the damping ratio, ζ is equal to, less than, or greater than one. In viscous damping we get that the damping ratio is less than one; with this we will complex solutions, since the discriminant will be less than zero and complex solutions in motion imply that we have oscillation. So we get solutions of the form:

$$x(t) = e^{-\zeta\omega_n t}(D\ \mathrm{Sin}[\omega_d t + \phi])$$

where the D is the initial amplitude and the damped natural frequency: $\omega_d = \omega_n\sqrt{1-\zeta^2}$. If we write $\beta = \zeta\omega_n$ we get that the amplitude of the two successive maxima is:

$$\frac{De^{-\beta t}}{De^{-\beta(t-\tau)}} = e^{\beta\tau}$$

where we have that $\tau = \frac{2\mathrm{Pi}}{\omega_n}$, is the period of motion.

In the experiment we calculated graphically what $\frac{1}{e}$ is and we calculated a quantity called the quality factor Q. This factor measures resonance and it measures the number of oscillation it completes before it reaches $\frac{1}{e}$ it original size. For a small Q factor we have high damping and when it is very large we have that is light damping. For viscous damping we have the following relation to the damping ratio: $\zeta = \frac{1}{2Q}$.

II. EXPERIMENT

THE SETUP

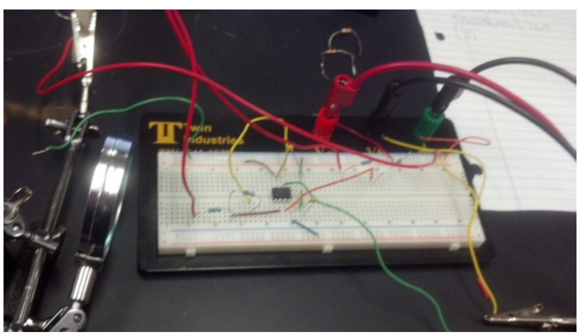


Figure 2.A Op-amp

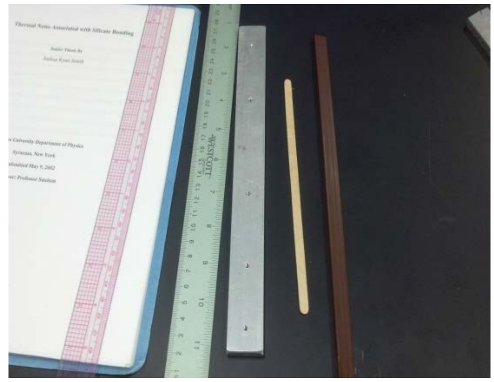


Figure 2.B The Materials used

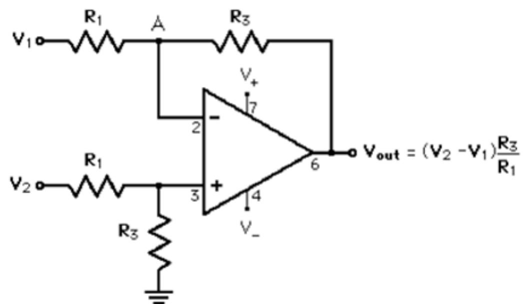


Figure 2.C Op-amp Schematic LM 301

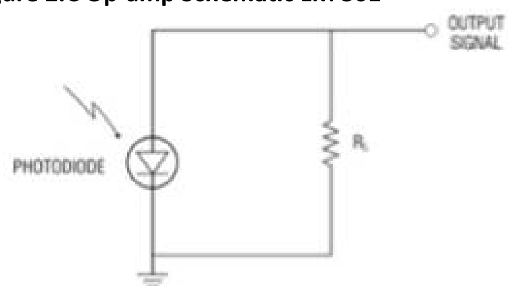


Figure 2.D Diode Schematic

For this section we utilized the following equipment:

- Convex, converging, lens
- Various materials
  - Pink Plastic Ruler, very thin
  - Metalic Ruler (greenish)
  - Wood stick, from a popsicle
  - Chunk of metal
  - Something that goes on report cover, the brown thing in figure 2.B
  - Tuning Fork, not in figure 2.B
- LM 301 Op Amp
- Function generator
- Tektronix Oscilloscope
- Photodiodes (two)

- Clamping mechanism
- Wires
- Breadboard (2 for our set up)
- Laser

THE EXPERIMENT

In this experiment we shined a diffused laser light at two diodes that we strategically put into position, see figure 3.A. After that we clamped a material between the converging point of the lens, as the laser is being shined through, in such a way that the shadow of the material is on both diodes, Figure 3.B. The schematic of the diode is in figure 2.D, this is for one diode, in this schematic we have that it is connected to an output signal, this output signal is either V_2 or V_1, we used the top diode as the positive reference, voltage was positive and in the bottom diode, signal has negative voltage.

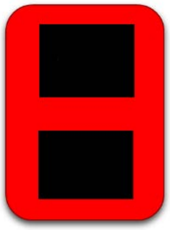


Figure 3.A The laser shinning on the diodes

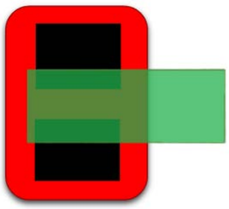


Figure 3.B The shadow of the material on the diodes

The op-amp used was a LM301 operational amplifier, schematic figure 2.C. What it did was to take the signal from the diodes and subtracted using negative feedback. The signal generated was equal to $V_{\text{out}} = (V_2 - V_1)\frac{\mathrm{R}_3}{\mathrm{R}_1}$.

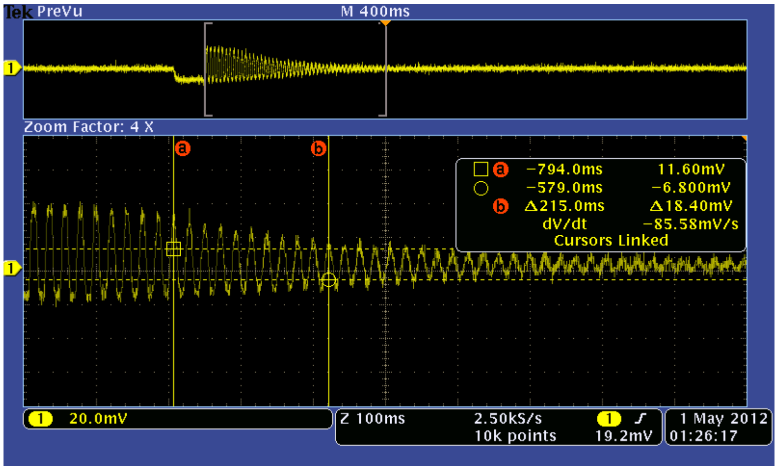
After this was set off we hit the material so that is start to vibrate and we analyzed the signal generated by the diodes. These two diodes are set up using a zero bias photovoltaic mode, see figure 2.D for the schematic. By the photovoltaic effect there is a creation of voltage in the photodiodes upon exposure to light. This meaning that when the laser shines through the diode the signal outputted will be, high and when the shadow covers it we get the signal that is low, or zero voltage. As the material is vibrating it block one of the diodes and then part of both and thus decreases one of the signals and so the output of the op-amp changes. Say for example that the shadow blocks the bottom diode, that is it has voltage, $V_1 = 0$ then the output signal of the maximal op-amp is equal to $V_2 - V_1 = V_2$ if it block the top one, then that one has a voltage, $V_2 - V_2 = 0$ and thus has a maximal op-amp output signal of, $V_2 - V_1 = -V_1$. Now as the amplitude of the vibration decreases more light if shinned through on both diodes, meaning that the difference in signals at the maximums and minimums is smaller.

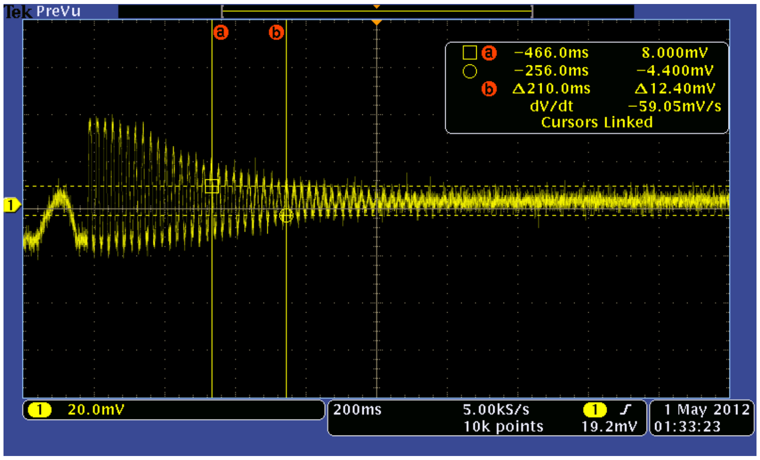
III. RESULTS AND DISCUSSION

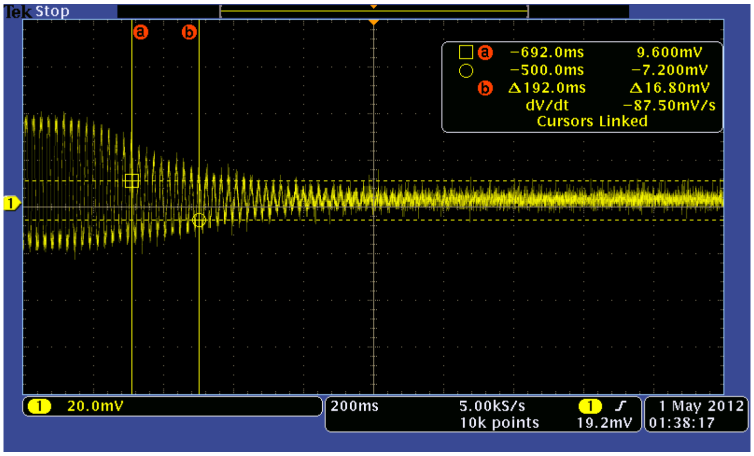
RESULTS

The following as screen captures of the signal we obtain for each trial, presented in order, i.e. first trial and so on, with the exception of the first and sixth material each had three trials, the first had four and the last one had one:

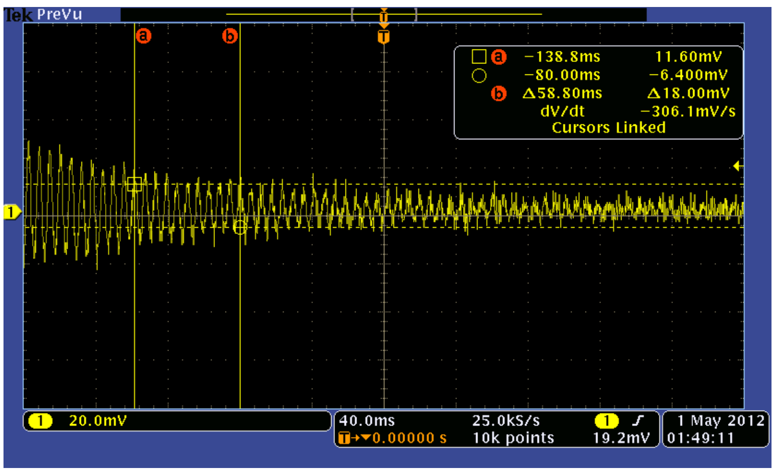
The pink plastic ruler:

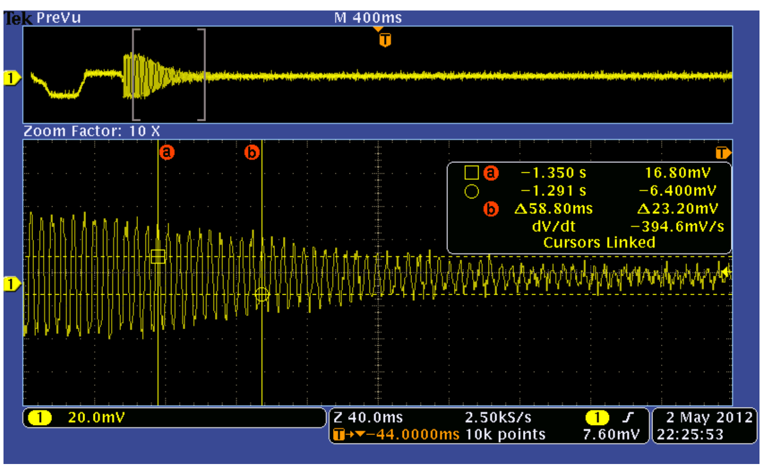


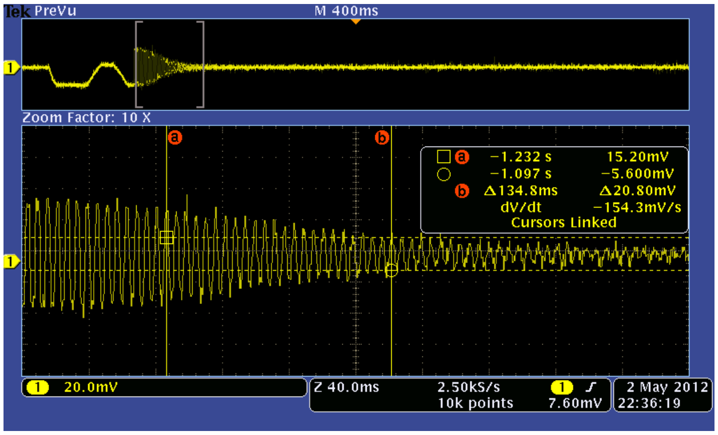




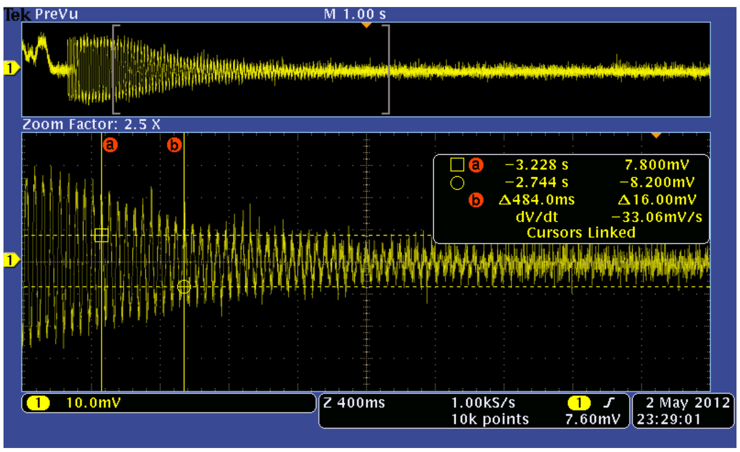
For the wood stick (popsicle):

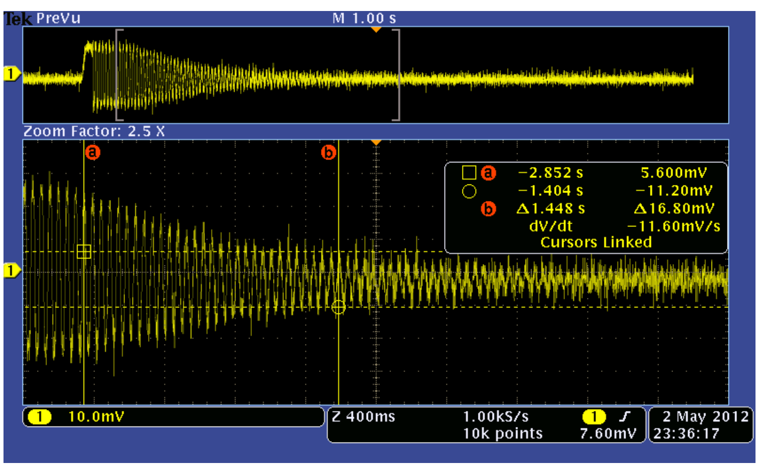


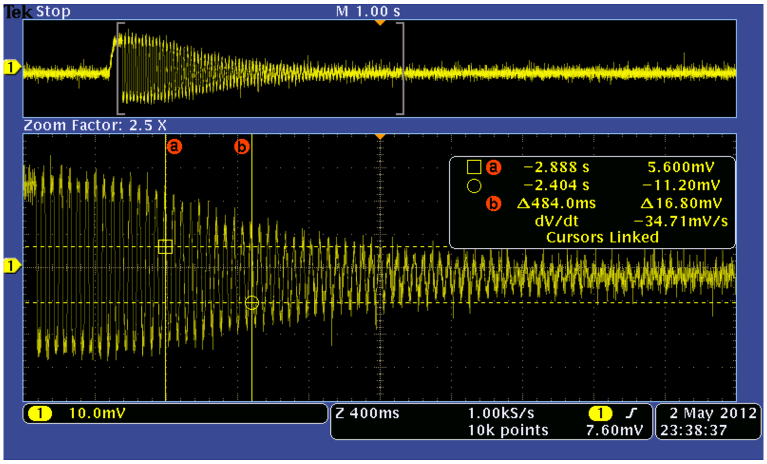




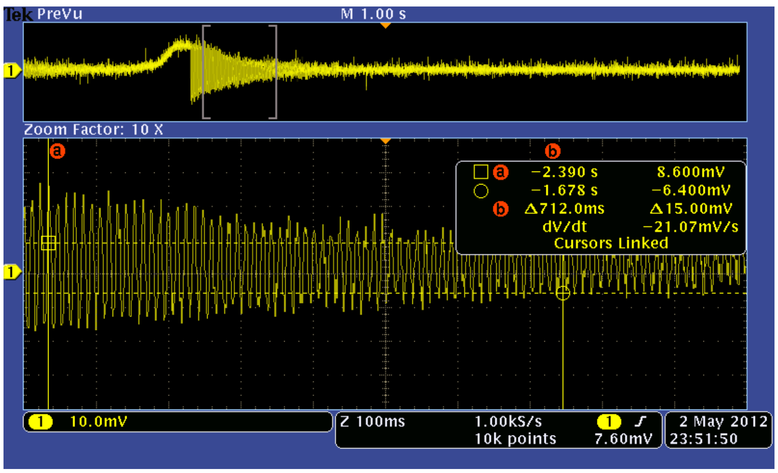
Brown Plastic:

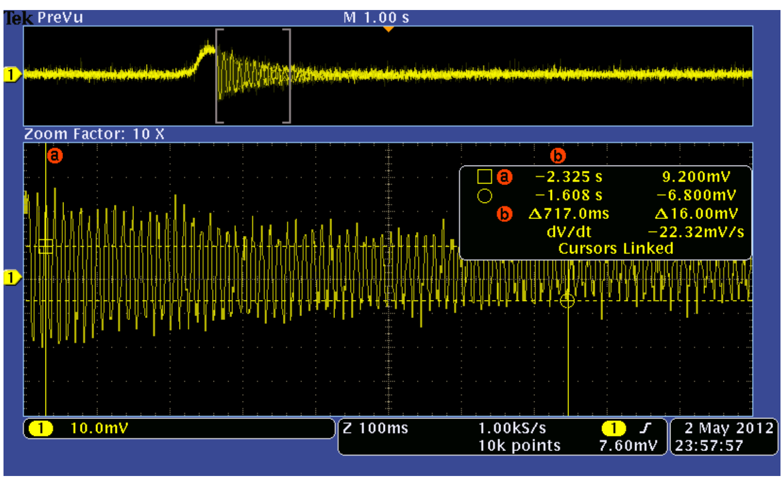


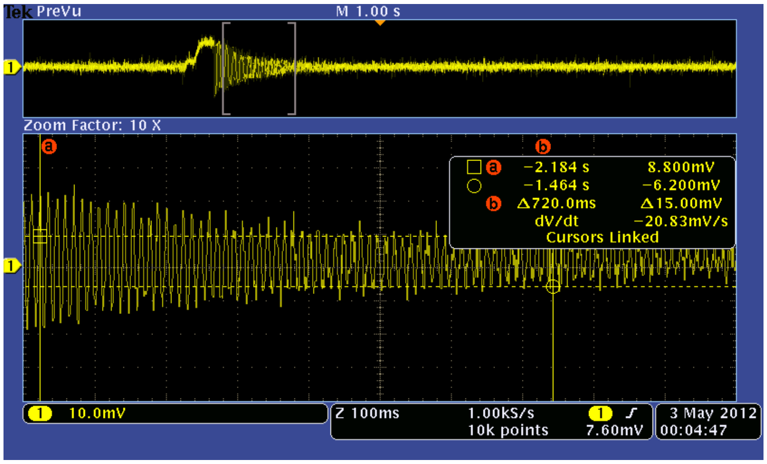




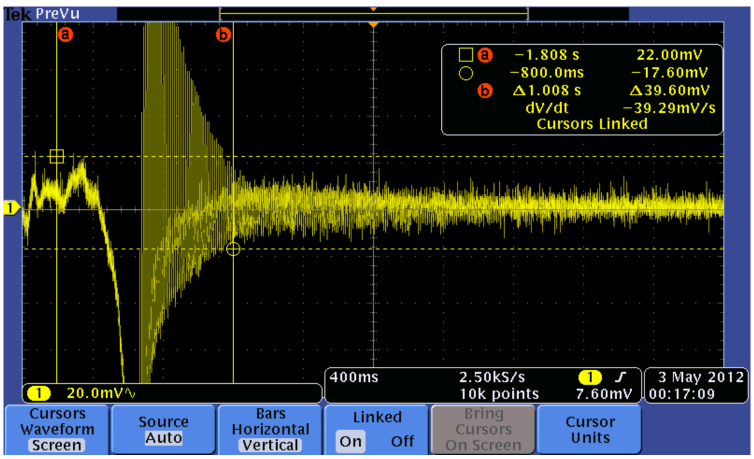
Green Metal Ruler:

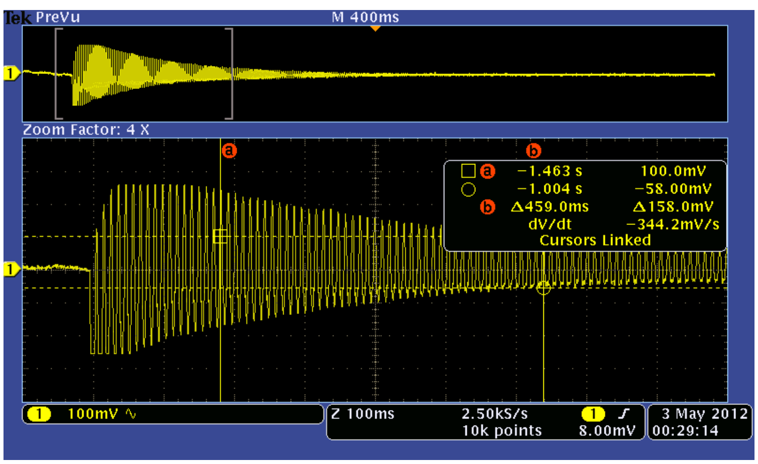


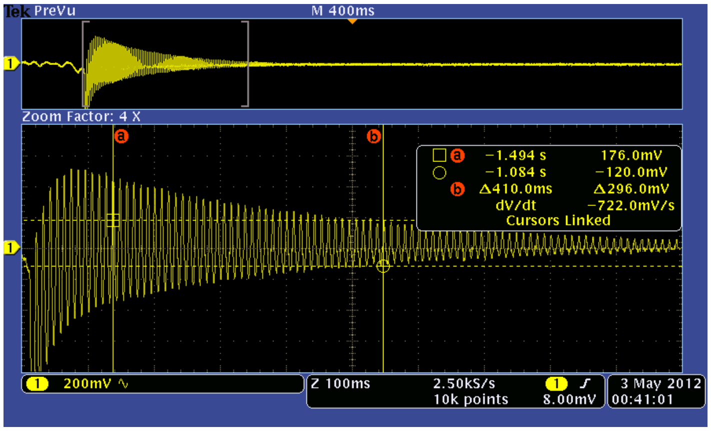




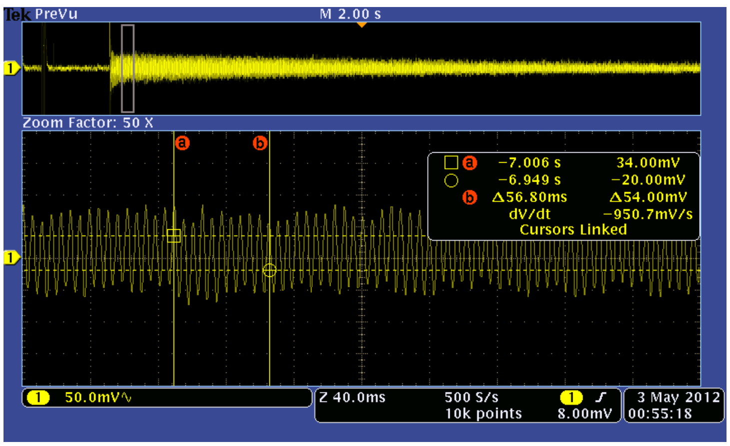
Chunk of Metal:







Tuning Fork:



Here is the data captured in tubular form:

Material	f [Hz]	τ [sec]	$Q = f \pi \tau$	$\beta = \frac{1}{\tau}$
Ruler	46.5	0.322	47	3.10559
Ruler	47.6	0.344	51.44	2.906977
Ruler	52.1	0.354	55.9	2.824859
wood stick	172.4	0.136	73.7	7.352941
wood stick	172.4	0.141	76.4	7.092199
wood stick	172.4	0.134	72.6	7.462687
Binder Plastic Stick (Brown)	20.7	1.356	88.2	0.737463
Binder Plastic Stick (Brown)	21	1.448	95.5	0.690608
Binder Plastic Stick (Brown)	20.7	1.328	86.4	0.753012
Metal Ruler	84.7	0.712	191.9	1.404494
Metal Ruler	84	0.717	189.2	1.3947
Metal Ruler	84.7	0.72	191.6	1.388889

{Chunk of Metal}	*{96.2}*	*{0.252}*	*{76.2}*	*{3.968254}*
Chunk of Metal	96.2	0.447	135.1	2.237136
Chunk of Metal	96.2	0.459	138.7	2.178649
Chunk of Metal	94.3	0.41	121.5	2.439024
Tuning Fork (320Hz)	181.2Hz	$10.712 \pm .5$	6098 ± 280	1/10.712

{Italic} is outlier point, see discussion as to why.

DISCUSSION AND REMARKS

Our data was as expected; the way we capture τ was as follows: after we obtained the signal we measured the original peak to peak then found at what time the peak to peak signal had dropped to $\frac{1}{e}$ the size, that was when the time was equal to τ. Initially we started with the resistors equal, $R_1 = R_3$, we thought this would make our experiment easier, but we were wrong. When it came to do the chunk of metal, our signal to noise ratio was rather small, and since the material had very small vibration we were getting interference with the noise. To overcome this we decided to increase the signal to noise ratio, we initially increased both to 1kOhm and then we increased $R_3 = 100R_1$, thus our signal to noise ratio increased by a factor of 100. This yielded better results, unfortunately for us this idea only came to us in the latter stage of the experiment. Recall the damping coefficient I derived in the beginning, $\zeta = \frac{1}{2Q}$, if we have a big quality factor our damping ration would be rather small very close to zero, notice that for any quality factor the damping coefficient will always be less than one. From our results we see that the higher the quality factor is the lower the damping ratio it is, thus it has a smaller damping coefficient, meaning the system is damped less. Another problem that I would like to call to attention was in the same trial with the chunk of metal. It was hard to clamp and make it vibrate, Danny was putting additional force to the base of the clamp this might have had affected some of our result, additionally the way it was clamped some of the energy was lost due to the clamp mechanism we had; this energy was lost to the table. Since the material is vibrating, the section that is clamped is being exerted with coulomb damping; the material and the clamp are sliding on each other thus friction was occurring, since the metal was bending up and down but it was forced to stay in position by the clamp. The energy is lost in the form of heat. A exception, to some extent, was the tuning fork the way it was design is such that the handle of the fork does not vibrate just the top U section, realistically we cannot say no energy was lost to the clamp mechanism but relatively less than the other material and much less than the chunk of metal. We saw that the tuning fork had the greatest quality factor, thus we can assume that is has a small damping coefficient, and it should come to no surprise, it is made so that it is not damped by grabbing the handle, but rather it dies out due to the viscous material, the air. Our data captured was very comforting with very small deviation from the mean of each of the trials. Again with the metal ruler I have posted one of our data trials (the first one it is the only one in curly brackets and it is italicized) before we switch the signal to noise ratio, consider it as an outlier. I included it for the sake of argument, the noise affected our results in this case but once we had increased the ratio we had obtained better results, the noise was reduced and we could focus on the signal.

IX. CONCLUSION

In conclusion we were able to investigate damping of various materials using a sensor to find the quality factor and from this we could rate the relative damping coefficients of various materials, the higher the quality factor, the smaller the damping coefficient is and thus we would get more oscillations before the signal, or the motion, ended. I have mentioned some improvements to this experiment such as increasing the signal to noise ratio by 100, or even higher, using large resistors, one much larger than the other. The main problem we had was the clamping mechanism. Some of the energy was lost due to this clamping device, using a device that can reduce this energy lost to the clamp can help yield better results. For example if we found the point in the material which vibrate the least we could try to clamp that part and make the material vibrate and the energy lost should be less and thus our results might be better. Our initial hypothesis was that the higher the quality factor was the smaller the damping coefficient. This was shown to be correct as the discussion and data had showed, refer to the table and screen capture of the oscilloscope.

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