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MK MOBILE ROBOT WITH SIMPLIFIED CONTROLLER

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Abstract: The paper presents a simplified trajectory tracking algorithm for a two wheel MK mobile robot. The robot is driven with respect to an internal coordinate frame what complicates the control considerably. The algorithm exploits simplified model of the robot kinematics and dynamics and is based on a static linearization feedback. The behaviour of the algorithm has been illustrated with computer simulations and experiments. *Copyright © 2003 IFAC*

Keywords: Mobile robots, kinematics, dynamics, feedback linearization.

1. INTRODUCTION

Recently many researchers make considerable effort to control nonholonomic mobile robots, (d'Andréa-Novel *et al.*, 1995; Canudas de Wit *et al.*, 1996; Tchoń *et al.*, 2000). A control task, which is often analysed, is a trajectory tracking problem. Typically it is solved by applying a dynamic or static feedback linearization, (d'Andréa-Novel *et al.*, 1995; Tchoń *et al.*, 2002).

In this paper we will consider a trajectory tracking problem for the MK mobile robot, (Kabała and Wnuk, 2002), constructed at laboratories of the Department of Fundamental Cybernetics and Robotics at the Institute of Engineering Cybernetics, Wrocław University of Technology. The MK mobile robot is two wheel nonholonomic cart, whose wheels are driven with reference to its body (see figure 1), (Kabała *et al.*, 2001). The robot body is a swinging pendulum carrying batteries, wheels driving motors and a controller. Assumption that the robot moves on the plane without slipping makes it to be a nonholonomic system.

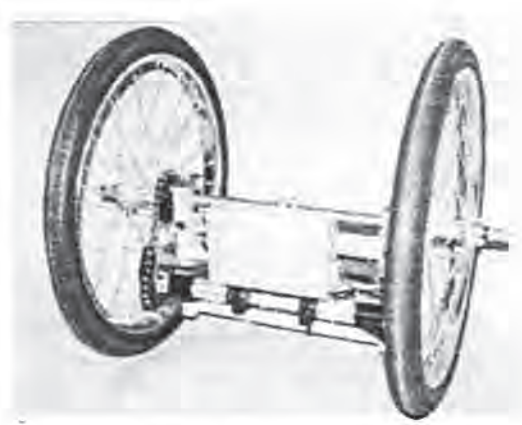

Fig. 1. MK mobile robot.

Until now several control algorithms have been proposed for the MK mobile robot, (Tchoń *et al.*, 2002; Kabała *et al.*, 2003). All of them are based on a kinematics and dynamics model of the robot, which is an affine control system with the 8-dimensional state space and two inputs. Unfortunately, complexity of this model makes all mentioned control algorithms complex as well. Computing power limitations of the MK mobile robot control unit do not allow for im-

[1] This research has been done within a statutory research project. Computer simulations have been done in MATLAB® environment provided by Wrocław Centre for Networking and Supercomputing.

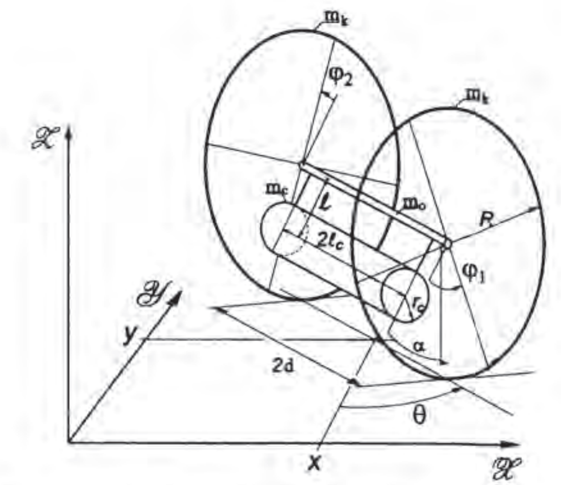

Fig. 2. MK mobile robot coordinates and parameters.

plementing any of them. Due to this fact we have proposed a new control algorithm for the cart, based on a simplified model of it, in which the state space is reduced to 6 dimensions. For such a model we choose position in Cartesian coordinates of a point from the robot body as a linearising output and design static feedback linearisation of the input-output map. This gives a system which can be controlled with a simple linear controller of the second order. Simulations and experiments show the usefulness of such a solution.

The paper is organised as follows. In next section we derive a simplified model of kinematics and dynamics of the MK mobile robot. In section 3 a control algorithm is proposed. Simulations and experimental tests of the algorithm are described in section 4. Section 5 concludes the paper.

2. ROBOT DYNAMICS AND KINEMATICS MODELS

The kinematics coordinates together with geometrical and dynamical parameters of the robot are shown in figure 2. To determine a robot model it is assumed that the robot wheels are homogeneous rings of mass m_k and radius R, and the robot body is represented by a cylinder of mass m_c, radius r_c, and length $2l_c$, moved to the distance l from the axis on which it swings, represented by a infinitely thin cylinder of mass m_o and length $2d$.

The complete kinematics and dynamics model describing the robot behaviour is derived in (Tchoń *et al.*, 2002) (it can be found in (Kabała *et al.*, 2003) as well). This model is in the form of an affine control system with 8-dimensional state space and 2 control inputs. The state coordinate vector utilised in the model consists of

$$(x, y, \varphi_1, \varphi_2, \alpha, \eta_1, \eta_2, \eta_3), \tag{1}$$

which have the following meaning:

- x, y — Cartesian coordinates of the centre of the robot axis connecting the wheels, projected on the movement plane;
- φ_1, φ_2 — rotation angles of the robot wheels with respect to the robot body;
- α — deviation angle of the robot body with respect to the vertical axis $\mathcal{Z}$;
- η_1, η_2, η_3 — auxiliary velocities (being robot centre linear velocity components resulting from the wheels and the pendulum movements, respectively).

The control inputs correspond to torques applied to propel the cart wheels.

The mentioned above model was determined with the assumptions that there is no slipping at the wheels contact points with the movement plane and that this plane is horizontal. One of the feature of this model is that its dynamical and kinematic parts cannot be decoupled, what complicates the model based control of the robot.

Assuming that the state coordinate α varies slowly, we derive the simplified version of the MK mobile robot kinematics and dynamics model obtained by taking α to be a model parameter. This model will be utilised to propose a simplified control algorithm for that cart.

With α being a model parameter the state vector describing the robot reduces to

$$(x, y, \varphi_1, \varphi_2, \eta_1, \eta_2). \tag{2}$$

The kinematics equations of the robot in these coordinates, based on the nonslipping assumption, have the form

$$\begin{aligned} \dot{x} &= (\eta_1 + \eta_2)\cos\theta, & \dot{\varphi_1} &= \frac{2}{R}\eta_1, \\ \dot{y} &= (\eta_1 + \eta_2)\sin\theta, & \dot{\varphi_2} &= \frac{2}{R}\eta_2, \end{aligned} \tag{3}$$

where $\theta = \frac{R}{2d}(\varphi_2 - \varphi_1)$.

Taking the torques propelling the cart wheels as the control inputs u_1, u_2, one obtains the dynamics model of the robot given by the equation

$$M(\alpha)\dot{\eta} + N(\alpha, \eta) = Bu, \tag{4}$$

with $M(\alpha) = M =$

$$\begin{bmatrix} \left(m_c + \frac{4}{3}m_o + 8m_k\right)d^2 + m_k R^2 + m_c\left(\frac{1}{2}r_c^2 + \frac{1}{3}l_c^2 + l^2\sin^2\alpha\right) & \left(m_c + \frac{2}{3}m_o\right)d^2 - m_k R^2 - m_c\left(\frac{1}{2}r_c^2 + \frac{1}{3}l_c^2 + l^2\sin^2\alpha\right) \\ \left(m_c + \frac{2}{3}m_o\right)d^2 - m_k R^2 - m_c\left(\frac{1}{2}r_c^2 + \frac{1}{3}l_c^2 + l^2\sin^2\alpha\right) & \left(m_c + \frac{4}{3}m_o + 8m_k\right)d^2 + m_k R^2 + m_c\left(\frac{1}{2}r_c^2 + \frac{1}{3}l_c^2 + l^2\sin^2\alpha\right) \end{bmatrix},$$

$$N(\alpha, \eta) = N = \begin{bmatrix} -2m_c l\eta_2(\eta_2 - \eta_1)\sin\alpha + \frac{4d^2k_1}{R^2}\eta_1 \\ 2m_c l\eta_1(\eta_2 - \eta_1)\sin\alpha + \frac{4d^2k_1}{R^2}\eta_2 \end{bmatrix}$$

$B = \frac{2d^2}{R}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$, $\eta = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$, and k_1, k_2 — coefficients of friction at wheels hubs and drive systems. We will call this model the simplified model of the MK mobile robot. Note that in this model it is possible to decouple the dynamics and kinematics parts. We will use it to derive a simplified controller of the MK robot.

3. CONTROL ALGORITHM

We are interested in controlling the position of a point

$$\begin{cases} \zeta_1 = x + e\cos(\theta+\delta) \\ \zeta_2 = y + e\sin(\theta+\delta) \end{cases} \tag{5}$$

(i.e. $\zeta = h(x, y, \theta)$), placed anywhere on the MK robot body, with e and δ — arbitrary chosen point parameters. We would like the robot to track a trajectory $\zeta_d(t) = (\zeta_{1d}(t), \zeta_{2d}(t))^T$ given in the Cartesian coordinates. Basically, we look for a closed loop controller to linearise the system (3)–(4).

Consider the linearization outputs (5). After differentiating these outputs we obtain

$$\begin{cases} \xi_1 = \dot{\zeta}_1 = \dot{x} - e\dot{\theta}\sin(\theta+\delta) \\ \xi_2 = \dot{\zeta}_2 = \dot{y} + e\dot{\theta}\cos(\theta+\delta). \end{cases}$$

Another differentiation gives

$$\begin{cases} \dot{\xi}_1 = (\dot{\eta}_1 + \dot{\eta}_2)\cos\theta - (\eta_1+\eta_2)\dot{\theta}\sin\theta - \\ \qquad e\ddot{\theta}\sin(\theta+\delta) - e\dot{\theta}^2\cos(\theta+\delta) \\ \dot{\xi}_2 = (\dot{\eta}_1 + \dot{\eta}_2)\sin\theta + (\eta_1+\eta_2)\dot{\theta}\cos\theta + \\ \qquad e\ddot{\theta}\cos(\theta+\delta) - e\dot{\theta}^2\sin(\theta+\delta). \end{cases} \tag{6}$$

Now using the dynamics equation (4) we can transform (6) into

$$\dot{\xi} = F + HM^{-1}Bu, \tag{7}$$

where $\xi = (\xi_1, \xi_2)^T$,

$$F = -HM^{-1}N + \begin{pmatrix} -P\dot{\theta}\eta\sin\theta - e\dot{\theta}^2\cos(\theta+\delta) \\ P\dot{\theta}\eta\cos\theta - e\dot{\theta}^2\sin(\theta+\delta) \end{pmatrix},$$

$P = (1, 1)$, and

$$H = \begin{bmatrix} \cos\theta + \frac{e}{d}\sin(\theta+\delta) & \cos\theta - \frac{e}{d}\sin(\theta+\delta) \\ \sin\theta - \frac{e}{d}\cos(\theta+\delta) & \sin\theta + \frac{e}{d}\cos(\theta+\delta) \end{bmatrix}.$$

The control that linearises the system (7) is given as

$$u = (HM^{-1}B)^{-1}(-F + w), \tag{8}$$

with new input $w = (w_1, w_2)^T$, where the decoupling matrix $HM^{-1}B$ is nonsingular if and only if H is nonsingular, since B is constant and invertible and M is nonsingular and invertible. Thus, we require that the determinant of H,

$$\det H = 2\frac{e}{d}cos\delta,$$

be nonzero, what is satisfied when

$$e \neq 0 \qquad \text{and} \qquad \delta \neq \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z},$$

i.e. the point $\zeta = (\zeta_1, \zeta_2)^T$ lies outside the axis connecting wheels centres. Substituting (8) into (7) one obtains the linearised system

$$\ddot{\zeta} = w. \tag{9}$$

For this system the regulator takes the form

$$w = \dot{\xi}_d - K_1(\xi - \xi_d) - K_0(\zeta - \zeta_d), \tag{10}$$

where K_1 and K_0 are the regulator gains (diagonal and positive defined matrices). Finally, defining $\varepsilon = \zeta - \zeta_d$, we obtain the equation for the error in the linearised coordinates

$$\ddot{\varepsilon} + K_1\dot{\varepsilon} + K_0\varepsilon = 0.$$

Choosing positive defined gains K_0, K_1 guarantees that the system consisting of the simplified model (3)–(4) and the controller (8)–(10) is stable. Figure 3 shows the block diagram of the MK mobile robot with the proposed controller.

4. SIMULATIONS AND EXPERIMENTS

To evaluate the performance of the controller based on the simplified model of the MK mobile robot we have performed both, simulations and experiments. The simulations were performed in the MATLAB® environment. For experiments the control algorithm was programmed onboard the MK mobile robot.

In simulations and experiments we have assumed robot construction parameters corresponding to the real cart parameters, (Tchoń *et al.*, 2002): $R = 0.254[m]$, $d = 0.225[m]$, $l = 0.12[m]$, $l_c = 0.165[m]$, $r_c = 0.035[m]$, $m_c = 7[kg]$, $m_o = 0.5[kg]$, $m_k = 2[kg]$, and the friction constant $k_1 = k_2 = 0.05\left[\frac{kgm^2}{s}\right]$. The control point parameters were chosen as $e = 0.1[m]$, $\delta = 0$, and the regulator gains $K_0 = 10I\left[\frac{1}{s^2}\right]$ and $K_1 = 5I\left[\frac{1}{s}\right]$. Additionally we should notice, that the real robot control torques are constrained to the interval $[-1.5, 1.5][Nm]$.

Observe, that in the control loop actual robot velocity and position in Cartesian coordinates are utilised. In experiments these values are provided by the robot odometry system according to the formulae

$$\begin{aligned} \dot{x} &= \left(\frac{R}{2}(\dot{\varphi}_1 + \dot{\varphi}_2) + R\dot{\alpha}\right)\cos\theta \\ \dot{y} &= \left(\frac{R}{2}(\dot{\varphi}_1 + \dot{\varphi}_2) + R\dot{\alpha}\right)\sin\theta. \end{aligned} \tag{11}$$

In fact, if $\dot{\alpha}$ is really small it can be omitted in the above formulae. By integrating these equations the actual Cartesian position is calculated.

We have performed simulations and experiments for the same set of trajectories. Below we show results obtained for two of them: a circle described by

$$(\zeta_{1d}(t), \zeta_{2d}(t)) = \begin{cases} \left(\cos\frac{\pi t}{4}, \sin\frac{\pi t}{4}\right) & t \in [0, 24), \\ (1, 0) & t \in [24, 32], \end{cases} \tag{12}$$

and a Lissajou figure given as

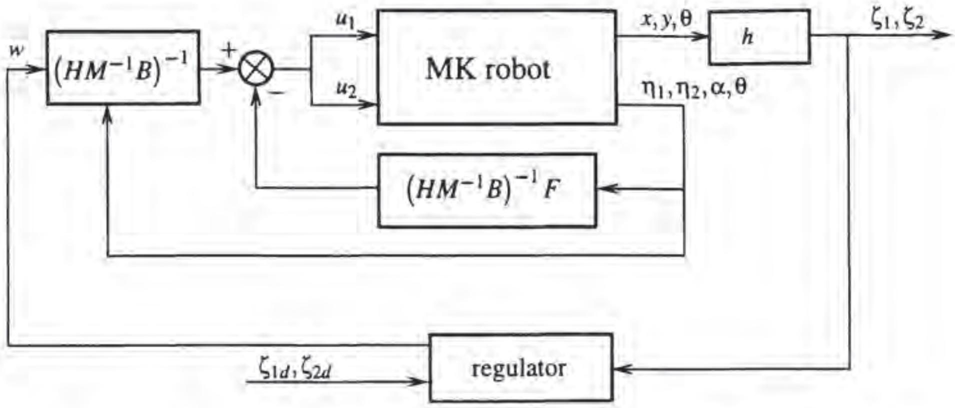

Fig. 3. Simplified model static linearization control system block diagram for the MK mobile robot.

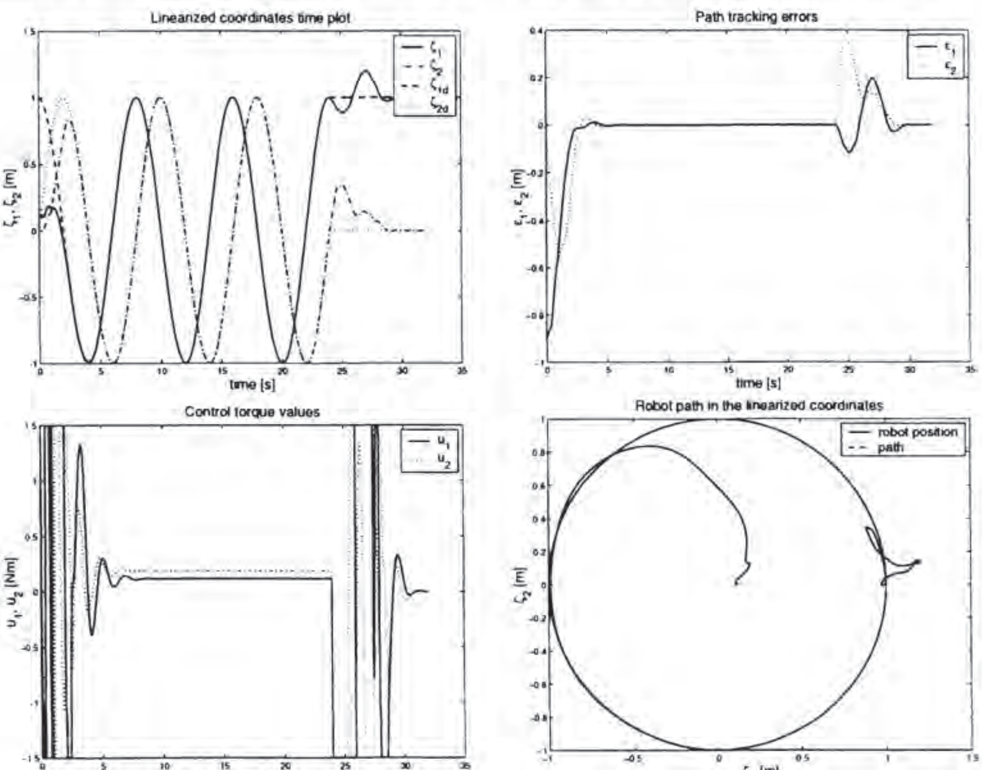

Fig. 4. Simulation results for the circular trajectory.

$$(\zeta_{1d}(t), \zeta_{2d}(t)) = \begin{cases} \left(\cos\frac{\pi t}{32} - 0.9, \sin\frac{\pi t}{16}\right) & t \in [0, 192), \\ (0.1, 0) & t \in [192, 256]. \end{cases} \quad (13)$$

Initial conditions of the cart were zero in both the cases.

Figure 4 shows the simulation results obtained for the circular trajectory. The simulation results for the Lissajou figure are shown in figure 5. The experiments results for the same trajectories are shown in figures 7 and 6, respectively.

One can notice that in all the cases the trajectories are well tracked and the errors are reasonable. The main source of them is the saturation of the control torques, which has been observed in the simulations already. However, the experiments showed other sources of errors. One of them is slipping of the robot wheels on the movement surface (due to insufficient friction between tyres and a floor). Also all floor irregularities significant influence the robot movement, what is a general problem in mobile robots control.

Additionally we want to point to the fact, that during the simulations and the experiments the body deviation angle α (unfortunately not shown in charts) does not fluctuate much and is rather held still, what justifies the assumption made to get the simplified MK robot model.

5. CONCLUSIONS

We have proposed a control algorithm for the MK mobile robot. To synthesise the algorithm a simplified model of the robot kinematics and dynamics was

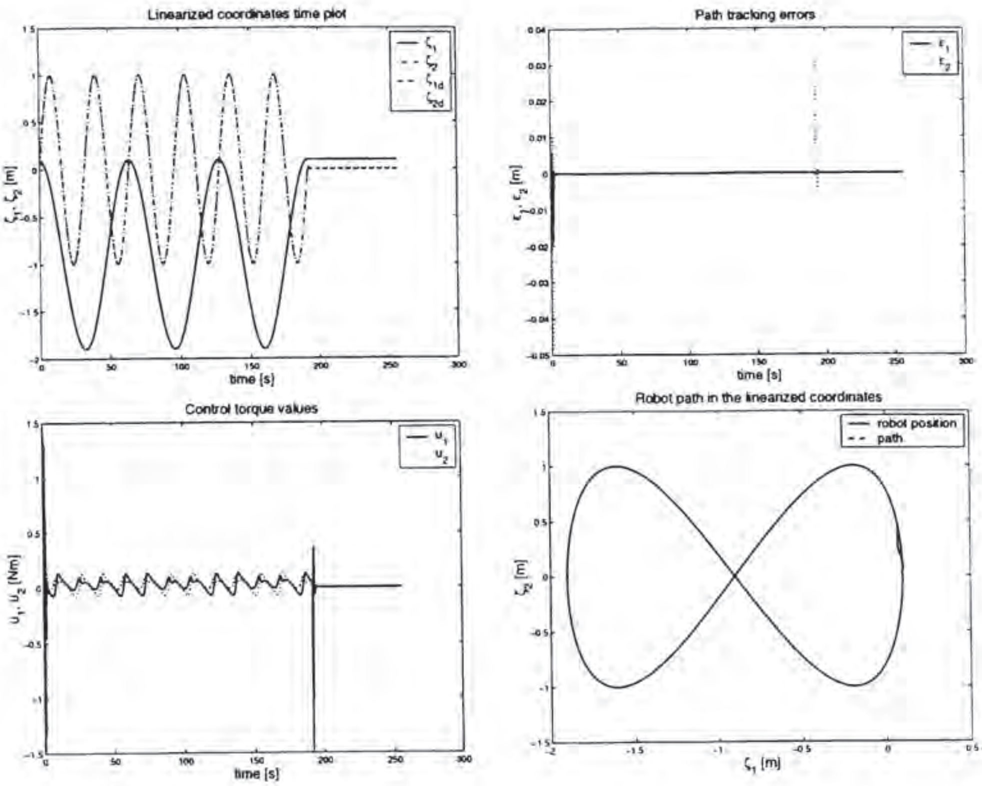

Fig. 5. Simulation results for the Lissajou trajectory.

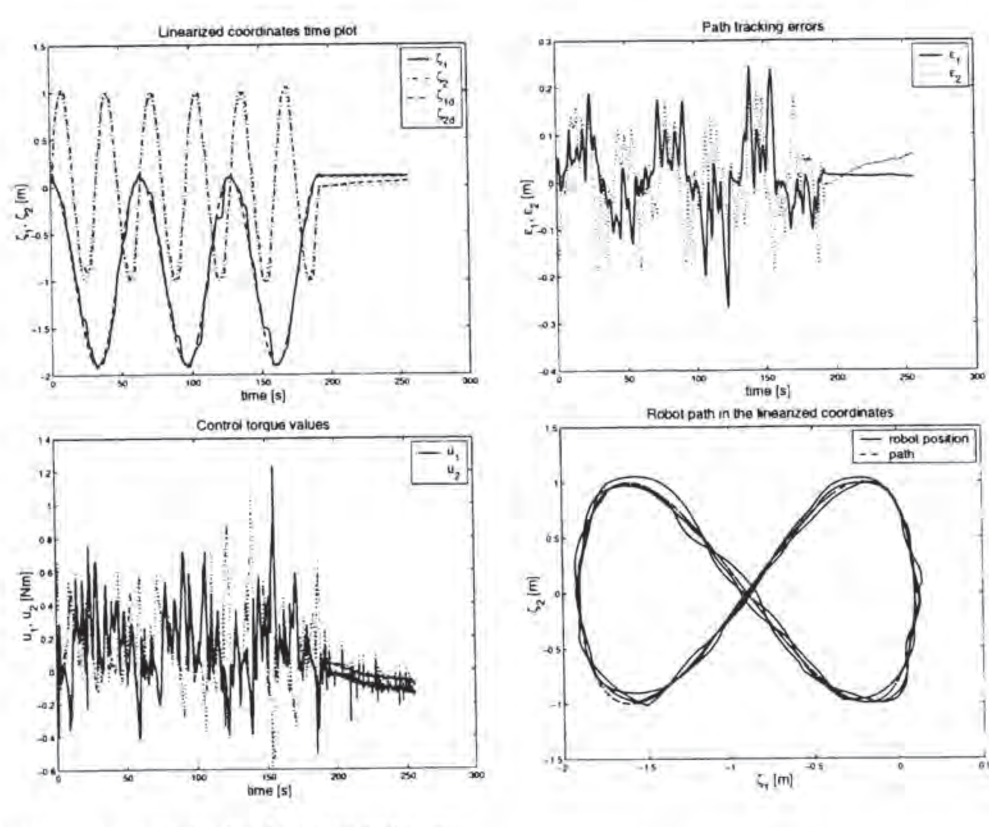

Fig. 6. Experiment results for the Lissajou trajectory.

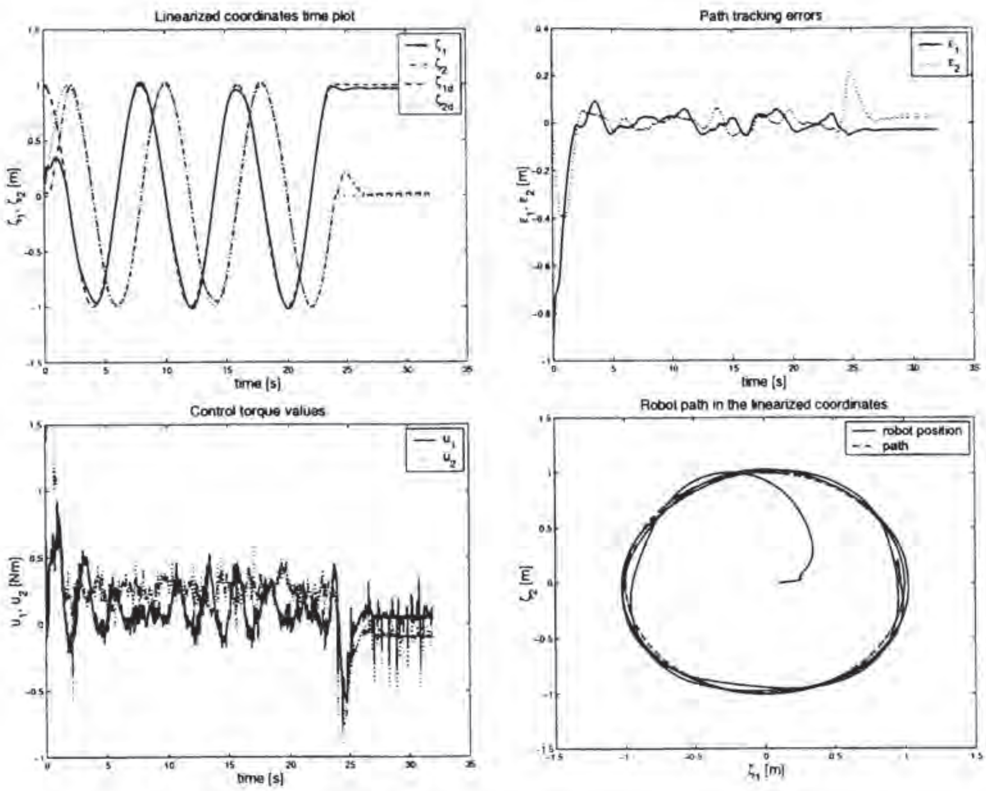

Fig. 7. Experiment results for the circular trajectory.

utilised. The algorithm allows to execute robot movements on a plane given in the Cartesian coordinates. With use of it a position of any point of the cart, situated outside the axis connecting the robot wheels centres, can be controlled.

Results of performed simulations show that nevertheless the simplification of the model the control algorithm works well and its performances are comparable with performances of other control algorithms proposed for the MK mobile robot. Moreover applied simplifications made possible utilisation of the algorithm on the real robot board. As a matter of fact, due to the robot main processor capacity [2], until now this algorithm is the only one which could be implemented in the robot controller.

Simulations and experiments results show that for different types of trajectories the tracking errors are kept close to zero even with small regulator gains. The input controls computed by the algorithm are small as well.

Planned to the near future upgrade of the robot controller will allow to implement other algorithms in the real environment and fully compare them with our algorithm. The MK mobile robot is a prototype constructed to test solutions needed in our currently developed robotic ball construction (called RoBall). Since the RoBall will have a similar type of propulsion an algorithm similar to the described one might be applied to control it.

[2] The robot main control unit is based on Motorola MC68332 32-bit microcontroller.

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