Advanced Business Analytics - Power of Predictive Modeling 226161-0131

Adrianna Wołowiec

e-mail: *awolow1@sgh.waw.pl*

Migration model

Foundations

Introduction

Migration model is used for estimating CLV when an organization does not have to have contractual relationships with its customers.

We assume that such organizations acquire customers who might or might not generate profit during discrete, equal-length periods of time such as a month or a year.

In contrast, the retention model assumes that inactivity indicates the end of the relationship. The migration model assumes that inactivity does not necessarily signal the end of the relationship.

The migration model is appropriate for organizations in many industries, including travel, most traditional and online retailers, automotive and financial services.

Questions related to migration modelling

We would like to forecast the future value of customers and answer related questions such as:

1. What is the CLV of a customer? (The answer to this question informs a marketing decision such as whether to acquire or retain the customer.)
2. How many active customers will we have after n periods in the future and how much profit will we make?
3. If we invest more in customer retention (and could therefore increase retention rates), how would the size and profitability of our customer base change?
4. Assuming some level of retention marketing, how many customers do we need to acquire to achieve some strategic goal such as maintaining current levels of profitability or increasing the numer of active customers by 20% over the next two years?
5. The retention models do not account for customers who reactivate. How can firms correctly account for such reactivated customers?

Customer states

We assume that a customer is in some state during each time period.

An example state might be defined by having bought in the previous period.

Customers generate cash flows depending on their states and migrate between states over successive periods with certain transition probabilities.

Migration model: spreadsheet approach

The buy, no-buy model

The buy, no-buy model

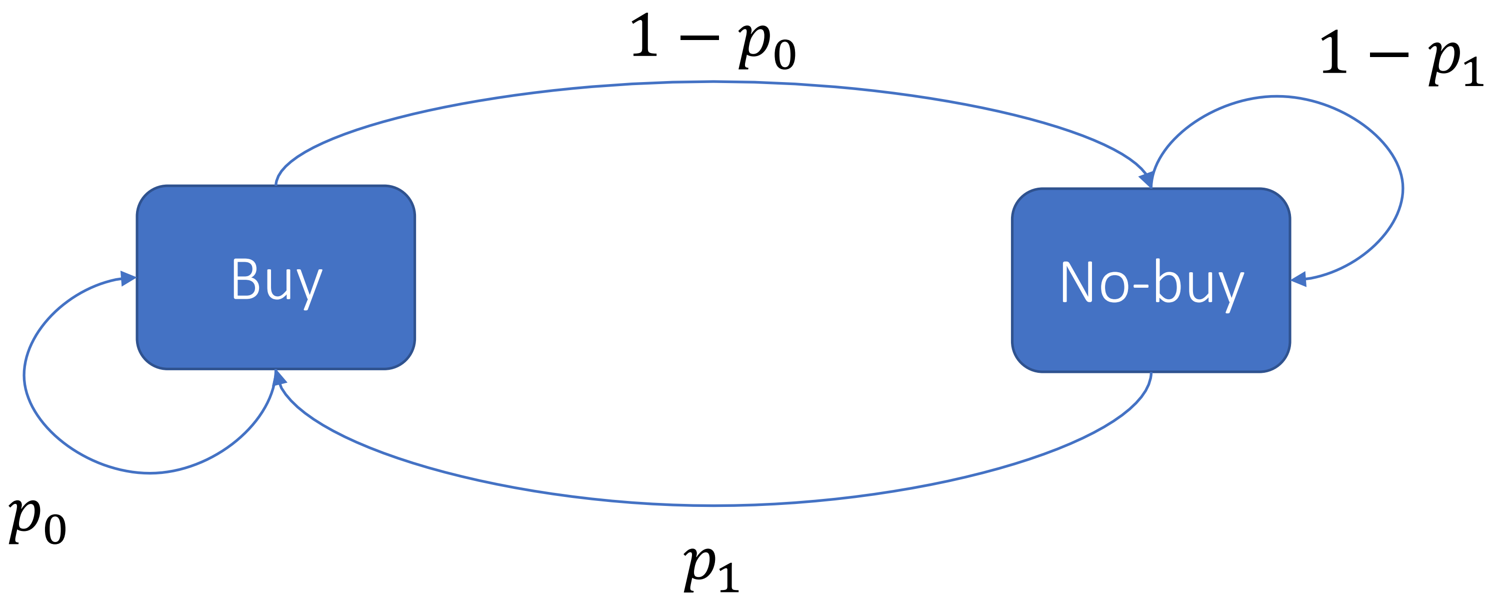
It is the simplest migration model that assumes that during each time period customers are in one of two states – „buy” or „no-buy”.

It is usually not used in practice, but can be easily generalized to the more complicated models used in practice.

A customer who is currently in the buy state can do one of two things during the next period: the customer will buy again with the transition probability p_0 or not buy with the probability $1 - p_0$. Customers who do not purchase are said to migrate to the no-buy state. Those who buy again return to the buy state.

Likewise, a customer who currently is in the no-buy state will purchase in the next period with probability p_1 (migrating back to the buy state) and not purchase with probability $1 - p_1$ (remaining in the no-buy state).

The state diagram for buy, no-buy model



Cash flows and cost

Suppose the the firm invests c on marketing costs each period and that the net contribution for a purchase is m.

Then, customers who migrateinto the buy state generate clash flows $m - c$ and those migrating to the no-buy state generate $-c$.

If we knew a customer's sequence of future states then we could compute CLV by summing the corresponding cash flows. Unfortunately, we only know the porbabilities of customers following various migration paths.

Example 1

Suppose that a firm has just acquired 1 000 customers. Because all 1000 made a purchase at the time of acquisition, they are in the buy state at time 0. Assume the probability that a customer in the buy state purchases again is $p_0 = 0{,}2$, and the probability that someone in the no-buy state purchases again is only $p_1 = 0{,}1$. The firm spends $c = \$1$ on marketing each period and buyers generate a contribution of \$20 (non-buyers generate \$0). We use a period discount rate of $d = 10\%$. Create a spreadsheet to forecast behaviour in the next two time periods.

Constraints of the model

The buy, no-buy model assumes that everyone in the buy state purchases in the next period with the same probability p_0 and everyone in the no-buy state purchases with probability p_1. This assumption is not reasonable, as the probabilities depend on factors such as the customer's recency and frequency.

Because of customer heterogeneity not all those in a state will have the same probability of purchasing in the next period. When the purchase probabilities of customers within a state are heterogeneous, the buy, no-buy model might not give accurate predictions.

The recency model

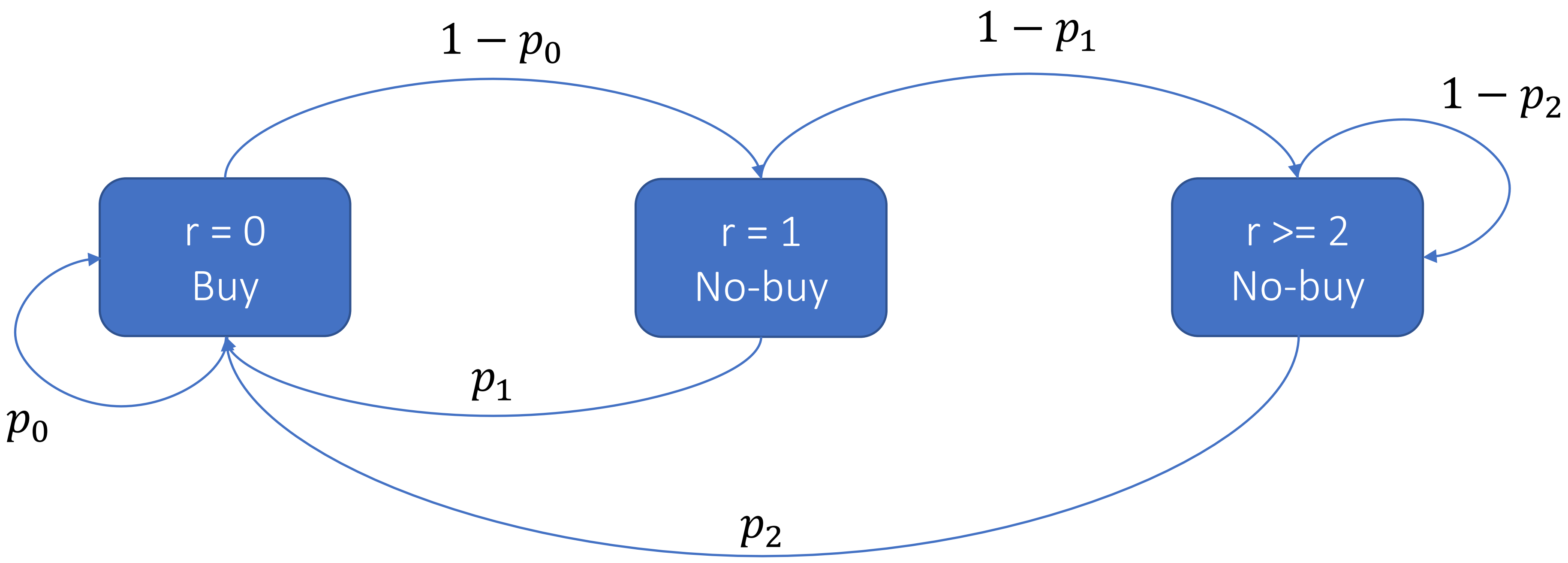
The recency model

Response probabilities often depend on customer characteristics that the firm can observe. One such charatcteristic is recency, which is the number of periods since the customer most recently purchased.

The recency model defines states based on the customer's recency. Let p_r be the probability that a customer in recency state r purchases in the current period.

For example, suppose that there are three states. Customers who have just bought in the most recent period have a recency of 0 and will buy again in the next period with probability p_0; if not, they migrate into the $r = 1$ state indicating that they have been inactive for one period. Those in the $r = 1$ state who purchase migrate back to the $r = 0$ state with probability p_1. Those who do not purchase migrate to the $r \geq 2$ state with probability $1 - p_1$.

The state diagram for the recency model



Example 2

Suppose that a firm has just acquired 1 000 customers who have just purchased and are therefore in the $r = 0$ state. Assume that the transition probabilities are as follows:

State	Probability of purchase in the next period
Recency = 0	$p_0 = 0{,}4$
Recency = 1	$p_1 = 0{,}2$
Recency >= 2	$p_2 = 0{,}1$

The firm spends $1 per customer each period on marketing and earns $20 from all buyers. The period discount rate is 10%. Create a spreadsheet to forecast buyers counts and revenues over the new periods.

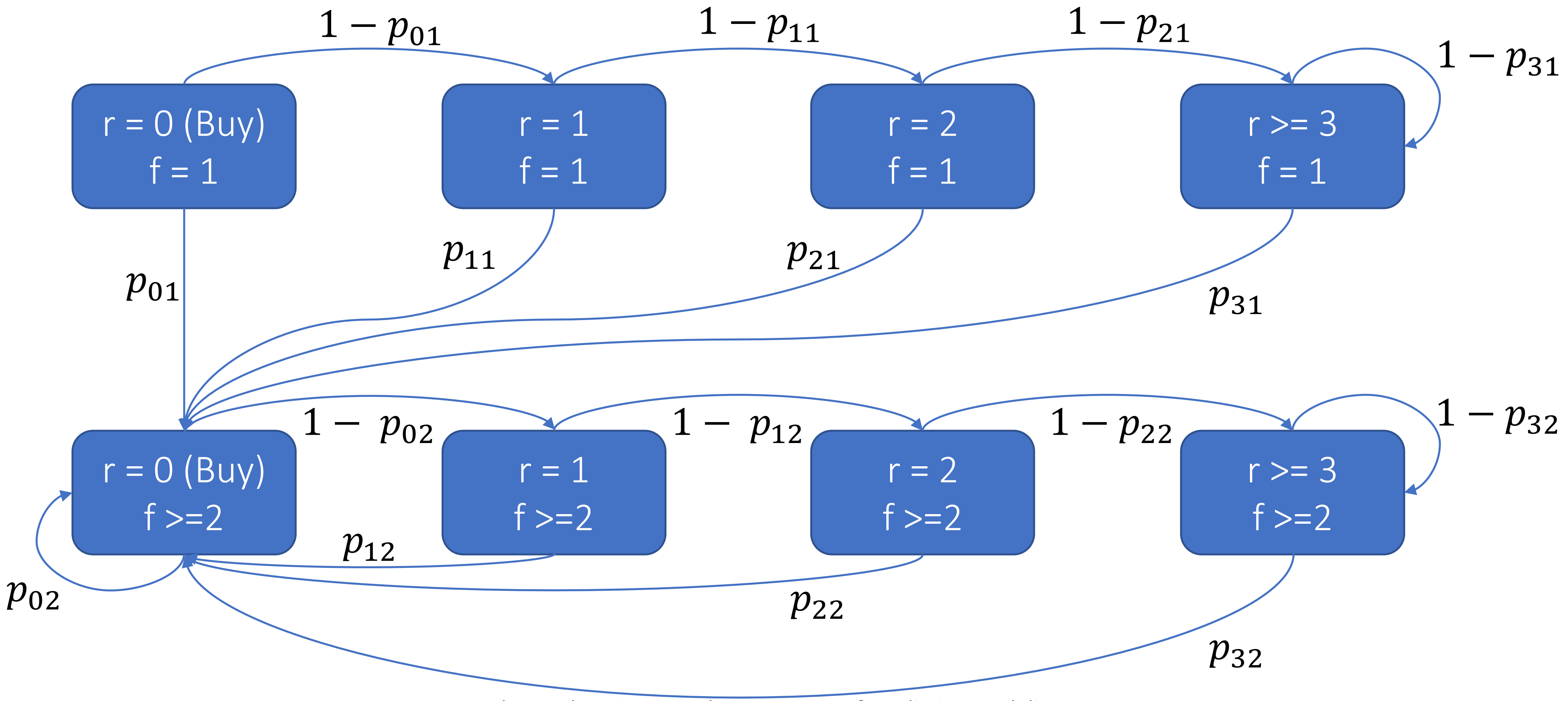
The recency-frequency model

The recency-frequency model

In addition to depending on recency, the probability of buying in the future period usually also depends on frequency, the numer of previous periods in which a customer has made a purchase.

Let p_{rf} be the probability that a customer with recency r and frequency f makes a purchase in the next period. A newly acquired customer begins in the $r = 0$, $f = 1$ state, and those who buy again in the next period transition to the $r = 0, f \geq 2$ state with the probability p_{01}. Otherwise they lapse into the $r = 1$, $f = 1$ state with the probability $1 - p_{01}$. Recent, multi-time buyers $(r = 0, f \geq 2)$ who buy again return to the same state with the probability p_{02}, otherwise they transition to the multi-time, one-period-lapsed state $(r = 0, f \geq 2)$ with the probability $1 - p_{02}$.

The state diagram for the recency-frequency model



Example 3

A firm has just acquired 1 000 customers who are now in the $r = 0$, $f = 1$ state. The transition probabilies are given in a table below.

	f = 1	**f >= 2**
r = 0	0,2	0,4
r = 1	0,1	0,2
r = 2	0,05	0,1
r >= 3	0,02	0,05

Marketing costs are \$1, the contribution is \$20 and the period discount rate is 10%. Create a spreadsheet.

Migration model: matrix approach

The matrix approach

There is an equivalent way to forecast migration models into the future that is easier and enables us to find a closed-form expression for CLV in perpetuity.

This section answers four questions:

1. What is the probability that a customer who is in the buy state, buys k periods later?
2. What is the expected profit for this customer k periods later?
3. What is the CLV for this customer?
4. What is the customer equity of a group of customers?

The answers to the first two questions are usually not of interest on their own, but we need to answer tchem in order to answer questions 3 and 4.

Initial vector

Let’s define two vector and a matrix.

The first vector, **n**, is called the initial vector and it gives the number of customers in each state at the beginning of the study (Period 0).

Let’s return to the buy, no-buy migration model discussed in Example 1. There were 1 000 customers in the buy state and 0 customers in the no-buy state. Assuming the first state is the buy one and the second is the no-buy one, the initial vector for this example is:

$$\boldsymbol{n} = \binom{1000}{0} \; or \; \boldsymbol{n}' = (1000 \quad 0)$$

Transition matrix

The transition matrix, P, consists of entries p_{ij} giving the probability that someone in state i transitions to state j at time $t+1$.

Continuing the buy, no-buy example, those in the buy state buy again with probability 0,2 and those in the no-buy state buy again with the probability 0,1. Therefore the transition matrix is:

$$\boldsymbol{P} = \begin{pmatrix} 0{,}2 & 0{,}8 \\ 0{,}1 & 0{,}9 \end{pmatrix}$$

Note that the rows of the transition matrix always sum to 1.

Value vector

The value vector, **v**, gives the value of transitioning into various states.

The buy, no-buy model assumed that the period marketing costs were c and the period contribution of buyers was m. Therefore, the value vector is

$$\boldsymbol{v} = \binom{m-c}{-c}$$

In the case of the example 1 where $c = -1$ and $m = \$20$:

$$\boldsymbol{v}' = (19 \quad -1)$$

Answer to the first question

Having defined the initial vector, transition matrix and value vector, the questions at the beginning of the section can be easily answered.

The answer to the first question, concerning the Chance that someone who boughtat time 0 buys at time t, comes from computing powers of the transition matrix. By definitione, the transition matrix gives the probability that someone in some state at time t buys during the next period. It turns out that row i and column j of the matrix $\boldsymbol{P}^k$ gives the probability that someone in state i at time t transitions to state j at time $t + k$. $\boldsymbol{P}^k$ is called the k-step transition matrix.

Answer to the first question

The two-step transition matrix for the example 1 is given by:

$$\boldsymbol{P}^2 = \begin{pmatrix} 0{,}2 & 0{,}8 \\ 0{,}1 & 0{,}9 \end{pmatrix} \begin{pmatrix} 0{,}2 & 0{,}8 \\ 0{,}1 & 0{,}9 \end{pmatrix} = \begin{pmatrix} 0{,}12 & 0{,}88 \\ 0{,}11 & 0{,}89 \end{pmatrix}$$

Someone in the buy state at time 0 has 12% chance of buying at time 2, while someone in the no-buy state at time 0 has an 11% Chance of buying two periods later.

We can estimate future customer counts by combining P and **n**. For exmaple, if we start with 1 000 customers in the buy state and none in the no-buy state, then we expect the following numer of customers in the next period:

$$\boldsymbol{n}'\boldsymbol{P} = (1000 \quad 0) \begin{pmatrix} 0{,}2 & 0{,}8 \\ 0{,}1 & 0{,}9 \end{pmatrix} = (200 \quad 800)$$

Likewise, the customer counts for period 2 are given by $\boldsymbol{n}'\boldsymbol{P}^2 = (120 \quad 880)$.

Answer to the second question

The second question concerned the expected value of a customer k periods in the future. The answer to this question is given by the vector $\boldsymbol{P}^k \boldsymbol{v}$.

$$\boldsymbol{P}^1 \boldsymbol{v} = \begin{pmatrix} 0{,}2 & 0{,}8 \\ 0{,}1 & 0{,}9 \end{pmatrix} \begin{pmatrix} 19 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Someone who bought during the most recent period has an expected profit of \$3 in the next period. Someone who did not uy during the most recent period has an expected profit of \$1 in the next period.

Multiplying these numbers by the customer counts, the profit in period 1 is $\boldsymbol{n}'\boldsymbol{P}^1\boldsymbol{v} = 3000$ and the discounted profit is $\boldsymbol{n}'\boldsymbol{P}^1\boldsymbol{v} \div 1{,}1^1 = 2727$.

Answer to the third and fourth question

The third question was about the CLV of a customer, which is easy to compute by summing the discounted expected profits going forward:

$$CLV = \sum_{t=0}^{\infty} \frac{\boldsymbol{P}^t \boldsymbol{v}}{(1+d)^t},$$

assuming a discount rate d and $\boldsymbol{P}^0 = \mathbf{I}$.

The fourth question is about the customer equity of a group of customers and is equal to:

$$CE = \sum_{t=0}^{\infty} \frac{\boldsymbol{n}'\boldsymbol{P}^t \boldsymbol{v}}{(1+d)^t} = \boldsymbol{n}' \left[\sum_{t=0}^{\infty} \left(\frac{\boldsymbol{P}}{1+d} \right)^t \right] \boldsymbol{v}$$

Answer to the third and fourth question

The expressions for CLV and CE both involve infinite sums and can be transformed into closed-form expressions:

$$CLV = \left(\boldsymbol{I} - \frac{\boldsymbol{P}}{1+d}\right)^{-1} \boldsymbol{v}$$

$$CE = \boldsymbol{n}'\left(\boldsymbol{I} - \frac{\boldsymbol{P}}{1+d}\right)^{-1} \boldsymbol{v}$$

It is easy to compute these matrix products using SAS/IML (interactive matrix language) as shown in the next example.

Example 4

Repeat example 1:

Suppose that a firm has just acquired 1 000 customers. Because all 1000 made a purchase at the time of acquisition, they are in the buy state at time 0. Assume the probability that a customer in the buy state purchases again is $p_0 = 0{,}2$, and the probability that someone in the no-buy state purchases again is only $p_1 = 0{,}1$. The firm spends $c = \$1$ on marketing each period and buyers generate a contribution of \$20 (non-buyers generate \$0). We use a period discount rate of $d = 10\%$. Forecast behaviour in the next two time periods.

Also find CLV and CE.

Estimating transition probabilities

Estimating transition probabilities

The examples discussed earlier assumed that transition probabilities are known (the transition matrix P with transition probabilities was given), but in practice they must be estimated from transaction histories. The goal of the next example is to show how to start from a tranaction history and estimate these probabilities.

Example 5

Use the data from DMEF customer lifetime value modeling competition (Malthouse, 2009) to estimate transition probabilities for a recency-only model with a period length of a year. The transactions range from January 1, 2002 through August 31, 2006. Suppose that today is September 1, 2005 and there will be four one-year recency states. Complete Task 1 of the contest, estimating customer equity (total donation amount, not discounted) of the 21 166 customers between September 1, 2006 and August 31, 2008.