12

The R-2R Ladder DAC

Summary. The R-2R ladder DAC is another passive DAC. The R-2R ladder DAC makes use of only two resistor values—but it requires $2N$ resistors to implement an N-bit converter.

Keywords. R-2R ladder, passive DAC, buffer, superposition.

12.1 Introduction

A three-bit example of an R-2R ladder DAC is given in Figure 12.1. As we will show, the output of the DAC is

$$V_{\text{out}} = \frac{4b_2 + 2b_1 + b_0}{8}. \tag{12.1}$$

One can extend the circuit to any number of bits by continuing the pattern of R and $2R$ ohm resistors.

12.2 The Derivation

We consider the effect of any one bit being high, or, in other words, of a given digital input being equal to V_{ref}, while the rest are low—are held at 0 V. Then, we make use of the principle of superposition to arrive at (12.1). First consider the output value for the input word 100_2. In terms of voltages, we have $b_0 = b_1 = 0$ and $b_2 = V_{\text{ref}}$. Our circuit is then equivalent to the circuit of Figure 12.2. It is clear that the leftmost two resistors are equivalent to one R ohm resistor. This is in series with the next R ohm resistor. Combined, the leftmost three resistors are a $2R$ ohm resistor in parallel with the next $2R$ ohm resistor. Thus, one finds that the leftmost four resistors are equivalent to a single R ohm resistor. Combining this with the next R ohm resistor, we find

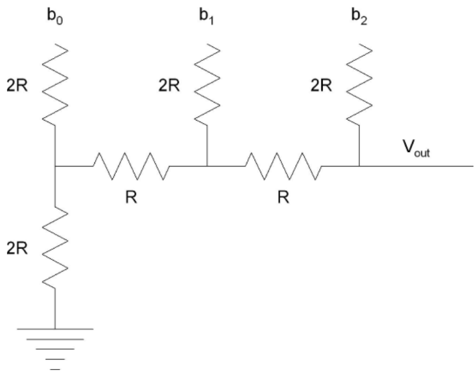

Fig. 12.1. A three-bit R-2R ladder DAC

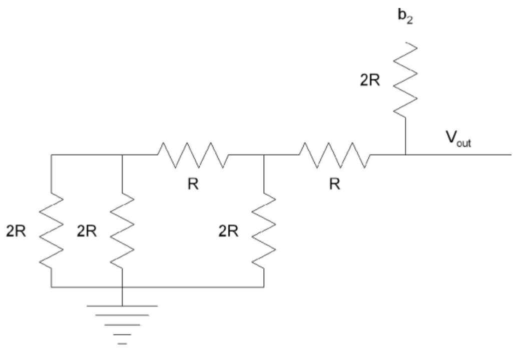

Fig. 12.2. An equivalent circuit when $b_0 = b_1 = 0$ and $b_2 = V_{\mathrm{ref}}$

that to the rest of the circuit the leftmost five resistors act as a single $2R$ ohm resistor. One finds that the circuit's output, V_{out}, is $b_2/2$.

What happens when $b_1 = V_{\mathrm{ref}}$ and $b_0 = b_2 = 0$? As we have seen, in this case the leftmost three resistors are equivalent to a single $2R$ ohm resistor. The equivalent circuit here is given by Figure 12.3. Considering the circuit in the dashed box as a two port and applying Thévenin's theorem [18], we find that the circuit can be replaced by a $b_1/2$ volt voltage source and an R ohm resistor in series; the equivalent circuit is given in Figure 12.4. It is now clear that

$$V_{\mathrm{out}} = b_1/4.$$

It is left as an exercise (see Exercise 1) to show that, when $b_0 = V_{\mathrm{ref}}$ and $b_1 = b_2 = 0$, $V_{\mathrm{out}} = b_0/8$. The principle of superposition allows us to conclude that, in general,

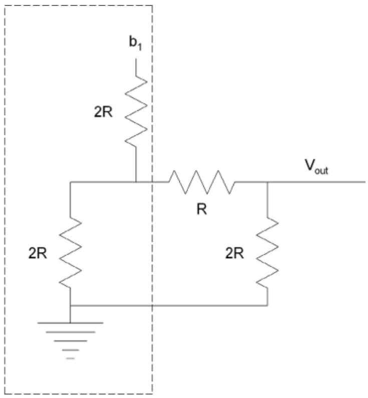

Fig. 12.3. An equivalent circuit when $b_0 = b_2 = 0$ and $b_1 = V_{\text{ref}}$

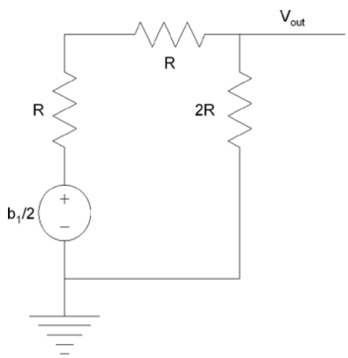

Fig. 12.4. The equivalent circuit after making use of Thévinin's theorem

$$V_{\text{out}} = \frac{4b_2 + 2b_1 + b_0}{8}.$$

The output of the DAC is proportional to the value of the binary number $b_2b_1b_0$.

12.3 Exercises

1. Show that when $b_0 = V_{\text{ref}}$ and $b_1 = b_2 = 0$, the output of the R-2R DAC is $b_0/8$.
2. Explain what should be added to the output of the circuit of Figure 12.1 in order to prevent the circuit's output from being loaded by the next stage.

3. Prove that the output of the circuit of Figure 12.5 is related to its inputs by the formula
$$V_{\mathrm{out}} = \frac{2b_1 + b_0}{4}.$$

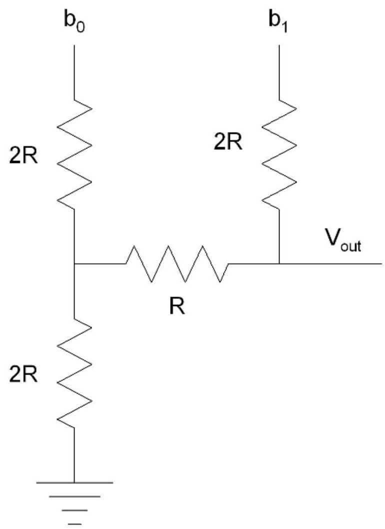

Fig. 12.5. A two-bit DAC