Modern Portfolio Theory (MPT) and the Capital Asset Pricing Model (CAPM)

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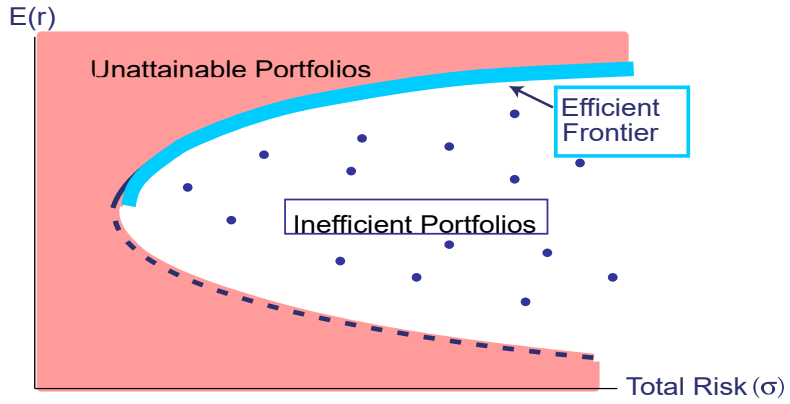
Efficient Frontier Background

Efficient frontier is the set of portfolios among all the possible portfolios of combinations of individual risky assets that offers the highest expected return for each level of risk (standard deviation).

Efficient Frontier



The efficient frontier is the upper boundary of the set of all possible portfolio risk/return combinations.

Example: Efficient Portfolios

Which of the following portfolios is *not* on the Markowitz efficient frontier?

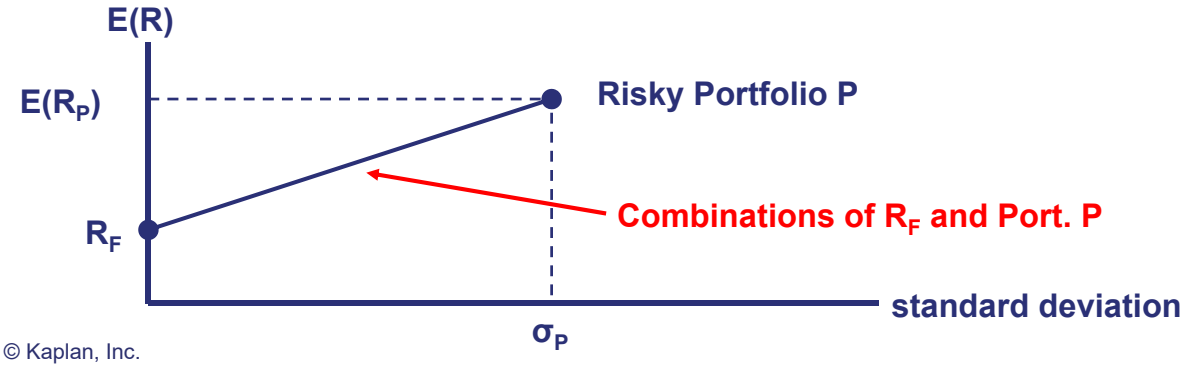
Portfolio	Expected Return	Std. Dev.
A	10%	12%
B	12%	16%
C	14%	15%

B has lower return and higher standard deviation compared to C. B can't be efficient.

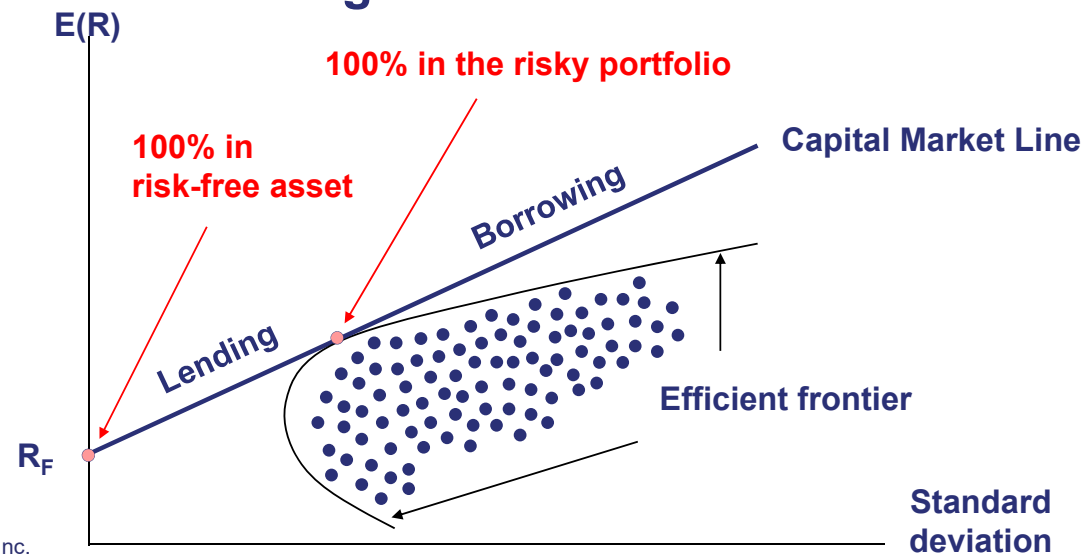
Combining Risk-Free and Risky Assets

Portfolio expected return = $W_{risky} R_p + W_{RF} R_F$

Portfolio standard deviation = $W_{risky} \sigma_p$



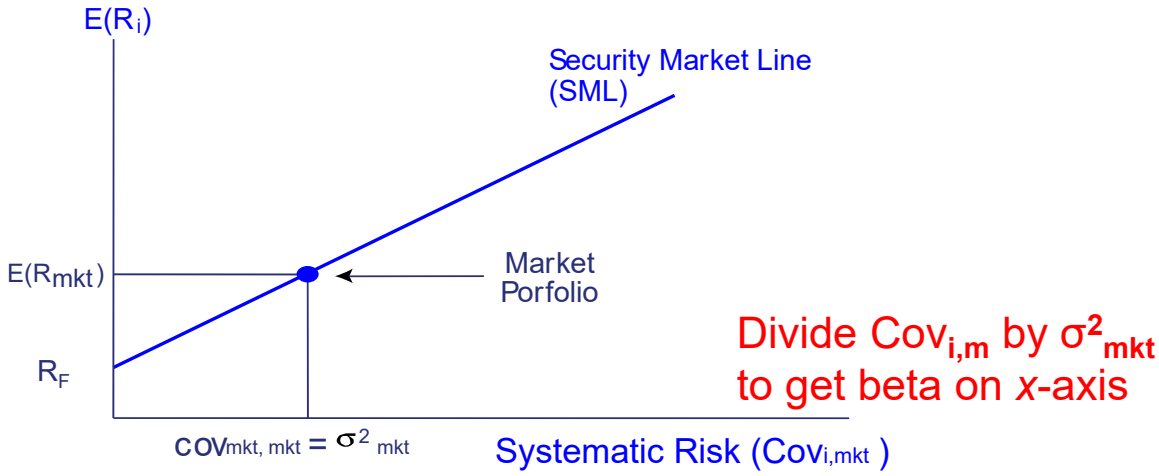
Adding a Risk-Free Asset



Assumptions of Capital Market Theory

- Information is freely available.
- There are no taxes and commissions.
- Fractional investments are possible.
- Market participants can borrow and lend at the risk-free rate.
- Individual investors cannot affect market prices.
- Investors have the same forecasts of expected returns, variances, and covariances.

Security Market Line (SML)



Divide $Cov_{i,m}$ by σ^2_{mkt} to get beta on *x*-axis

The Capital Asset Pricing Model

SML equation:

$$E(R_i) = R_F + \beta_i [E(R_{mkt}) - R_F]$$

Beta is a standardized measure of systematic risk, beta of the market portfolio is 1.

Beta measures the covariance of an asset's returns with returns on the market portfolio.

Calculating beta of asset 'i': $\beta_i = \frac{Cov_{i,mkt}}{\sigma^2_{mkt}}$

Capital Asset Pricing Model (CAPM)

- CAPM is the expected return on an asset based (only) on the asset's systematic risk or beta.
- CAPM is also used to determine the required return on an asset based on the asset's systematic risk (beta).
- Required return and expected return are the same in equilibrium.

Forecast Returns and the CAPM – Example

Stock	Price Today	E (price) in 1 year	E (dividend) in 1 year	Beta
A	$25	$27	$1.00	1.0
B	40	45	2.00	0.8
C	15	17	0.50	1.2

Are these stocks overpriced, underpriced, or at their equilibrium prices?

Show where they plot on the SML graph.

Forecast Returns and the CAPM – Example

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C	15	17	0.50	1.2

R_F

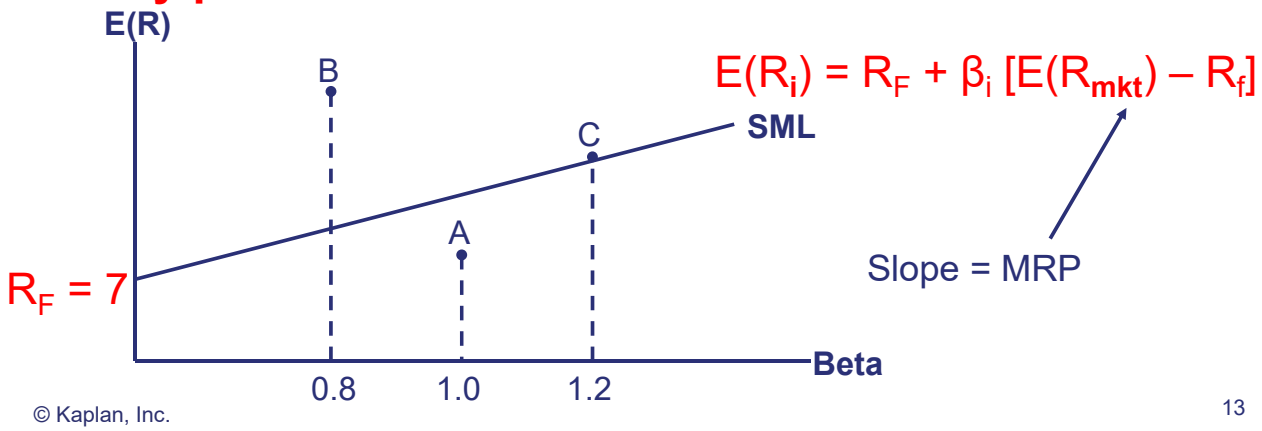
$R_F = 7\%$ $E[R_{mkt}] = 15\%$

MRP

Stock	Forecast Return	Required Return
A	(27–25+1)/25 = 12.0%	0.07+1.0(0.15–0.07) = 15.0%
B	(45–40+2)/40 = 17.5%	0.07+0.8(0.15–0.07) = 13.4%
C	(17–15+0.50)/15 = 16.6%	0.07+1.2(0.15–0.07) = 16.6%

Forecast Returns and the CAPM – Example

When securities are priced at equilibrium values, they plot on the SML.



Forecast Returns and the CAPM

Stock	Forecast Return	Required Return
A	12.0%	15.0%
B	17.5%	13.4%
C	16.6%	16.6%

Stock A is overvalued (sell it or sell it short).

Stock B is undervalued (buy it).

Stock C is properly valued (indifferent).

SML Pricing – Example

$E[R_{mkt}] = 15\%$; $E[R_{Stock\ X}] = 17\%$; $R_F = 8\%$

Stock X's beta = 1.25

Using this data and the capital asset pricing model, which of the following statements about X's stock is true? Stock X is:

A. overvalued by 1.75 percentage points.

B. properly valued.

C. undervalued by 0.25 percentage points.

$8 + 1.25(15 - 8) = 16.75$ $17 - 16.75 = 0.25$

Measures of Performance

- Modern portfolio theory and the CAPM are built upon the link between risk and return.
- Three measures exist to assess an asset's or portfolio's return with respect to its risk:
  - **Treynor measure** = risk premium divided by beta, or **systematic risk.**
  - **Sharpe measure** = risk premium divided by standard deviation, or **total risk.**
  - **Jensen's alpha** = asset's excess return over the return predicted by the CAPM.

Measures of Performance (continued)

$$\text{Treynor} = \frac{E(R_P) - R_F}{\beta_P}$$

$$\text{Sharpe} = \frac{E(R_P) - R_F}{\sigma_P}$$

$$\text{Jensen's alpha, } \alpha_p = E(R_P) - \{R_F + \beta_P [E(R_M) - R_F]\}$$

Measures of Performance (continued)

- In all three cases, the higher the better.
- Investors can apply Sharpe measure to all portfolios because it uses total risk.
- Treynor measure is more appropriate for comparing well-diversified portfolios.
- Jensen's alpha is most appropriate for comparing portfolios that have the same beta.

Tracking Error

- **Tracking error** is the standard deviation of the difference between the portfolio return and the benchmark return.
- Typically, the manager must keep the tracking error below a stated threshold.

$$\text{Tracking error} = \sigma_{R_P - R_B}$$

Information Ratio

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- **Information ratio** (IR) is the alpha of the managed portfolio relative to its benchmark divided by the tracking error.
- It is essentially a measure of how well the manager has acquired and used information compared to the average manager.

$$IR = \frac{E(R_P) - E(R_B)}{\text{tracking error}}$$

Tracking Error and Information Ratio

- Compute the tracking error and the information ratio for the following returns of a portfolio and its benchmark:

Portfolio returns (%)	Benchmark returns (%)
9.5	8.7
8.0	7.8
–2.2	–3.4
11	12.4

Tracking Error and Information Ratio (continued)

- First, compute the alpha as the difference between the returns:

Portfolio returns (%)	Benchmark returns (%)	Alpha
9.5	8.7	0.8
8.0	7.8	0.2
–2.2	–3.4	1.2
11	12.4	–1.4

- Click 2nd Data (on TI BA II+) and input the alphas as Xs. In 2nd STAT (1-V function), X-bar is average alpha = 0.2 and Sx is tracking error = 1.143.
- The information ratio is alpha/tracking error = 0.2/1.143 = 0.175

Sortino Ratio

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- The **Sortino ratio** can be interpreted as a variation of the Sharpe ratio that is more appropriate for a case where returns are not symmetric.
- We replace the risk-free rate with a minimum acceptable return, denoted R_{min}, and we replace the standard deviation with a type of downside deviation.
- A downside deviation measures the variability of only those returns that fall below the minimum acceptable return.

Sortino Ratio (continued)

- Letting R_{min} denote the minimal acceptable return and downside deviation represent the risk measure:

$$\text{Sortino ratio} = \frac{E(R_P) - R_{min}}{\text{downside deviation}}$$

Sample Exam Question

A portfolio manager received a report on his fund's performance during 2011. According to the report, the portfolio return was 2.5% with a standard deviation of 21% and a beta of 1.2. The risk-free rate over this period was 3.5%, the downside deviation of the portfolio was 16%, and the tracking error of the fund was 2%. Compute the difference between the value of the fund's Sortino ratio (assuming the risk-free rate is the minimal acceptable return) and its Sharpe ratio.

A. 0.563.

B. 0.347.

C. –0.053.

D. –0.015.

Answer

Answer: **D**

Sharpe ratio =

$[E(R_P) – R_F] / \sigma$

$(2.5 – 3.5) / 21 = –0.0476$

Sortino ratio =

$[E(R_P) – R_{min}]$ / (downside deviation)

$(2.5 – 3.5) / 16 = –0.0625$

Sample Exam Question

The information ratio of Large Hedge Fund is equal to 2 when using the S&P 500 as the benchmark index. During the same time period, standard deviation of the hedge fund returns is 4%, the hedge fund's Sharpe ratio is 3, and its tracking error against the S&P 500 is 6%. Calculate the return for the S&P 500 during the time period assuming the risk-free rate is 3%.

A. 3%.

B. 7%.

C. 11%.

D. 15%.

Answer

Answer: **A**

Sharpe ratio = 3

(Portfolio return – risk-free rate) / SD of fund = 3

(Portfolio return – 3%) / 4% = 3

Portfolio return = 15%

Information ratio = 2

(Portfolio return – S&P return) / Tracking error = 2

(15% – S&P return) / 6% = 2

S&P 500 return = 3%

Sample Exam Question

Portfolio Y has a beta of 0.8 and an expected return of 10%. The market risk premium is 6.45% and the risk-free rate is 3.7%. Jensen's Alpha measure for this portfolio is closest to:

A. 10%.

B. 8%.

C. 3%.

D. 1%.

Answer

Answer: **D**

Jensen's alpha

$= E(R_P) - [R_F + \beta_P(E(R_M) - R_F)]$

$= 10\% - [3.7\% + 0.8(6.45\%)]$

$= 1.14\%$

Example: Capital Asset Pricing Model

- The expected rate of return on a stock is 1.5 times the 16% expected rate of return from the market.
- What is the beta of the stock if the risk-free rate is 8%?

Answer

According to the CAPM for any single security or portfolio of securities *i*, the expected return in equilibrium is

$E(R_i) = R_F + \beta_i[E(R_M) - R_F]$

$24\% = 8\% + \beta(16\% - 8\%)$

$24\% = 8\% + \beta(8\%)$

Beta is then equal to = $\beta = 16 / 8 = 2$

Example: Sharpe and Treynor Measures

- Performance data for an actively managed portfolio and the S&P 500 Index is reported as:

	Portfolio	S&P 500
Return	10%	8%
Standard deviation	6%	5%
Beta	1.1	1.0

- Risk-free rate = 3%
- Determine the Sharpe and Treynor measures for the actively managed portfolio.

Answer

Sharpe measure:

= (0.10 – 0.03) / 0.06 = 1.17

Treynor measure:

= (0.10 – 0.03) / 1.1 = 0.064

Example: Sortino Ratio

- Portfolio X return = 11%
- Benchmark return = 10%
- Tracking error volatility = 6%
- Minimum acceptable return = 5%
- Risk-free rate = 4%
- Downside deviation = 20%
- What is Portfolio X's Sortino ratio?

Answer

The Sortino ratio can be interpreted as a variation of the Sharpe ratio that is more appropriate for a case where returns are not symmetric.

$$\text{Sortino ratio} = \frac{E(R_P) - R_{min}}{\text{downside devation}}$$

$$= \frac{11\% - 5\%}{20\%} = 0.3$$

The Arbitrage Pricing Theory and Multifactor Models of Risk and Return

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Multifactor Model of Risk and Return

$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$

where:

R_i = return on stock *i*

$E(R_i)$ = expected return for stock *i*

β_{ij} = *j*th factor beta for stock *i* (i.e., sensitivity of stock return to 1-unit change in the factor)

F_j = deviation of macroeconomic factor *j* from its expected value (i.e., portion of the stock's return that is unexplained by macro factors)

e_i = firm-specific return for stock *i*

Calculating Expected Returns

- A single-factor model will only consider the impact of one factor on a dependent variable (a stock's return).
- This leaves the potential for either company-specific risk or uncaptured systematic risk to influence asset returns.
- A multifactor model enables analysts to better model the impact of all systematic risk exposures to improve forecasting ability.

Hedging Exposures to Multiple Factors

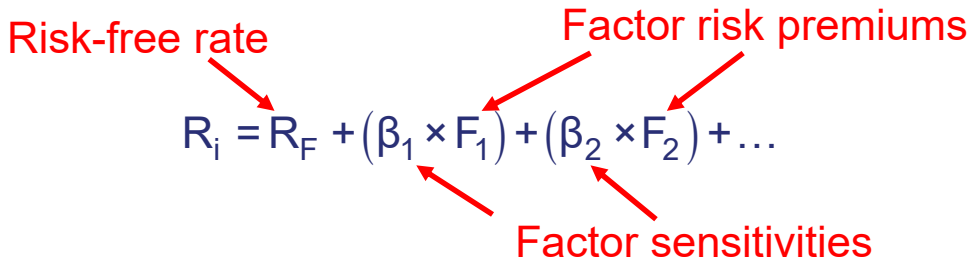
- Consider an investor who manages a portfolio with the following factor betas:
  - GDP beta = 0.50
  - Inflation risk beta = 0.25
- Assume the investor wishes to hedge away GDP factor risk, yet maintain inflation risk exposure.
- The investor should combine the original portfolio with a 50% short position in the GDP **factor portfolio** (see next slide).

Hedging Exposures to Multiple Factors (continued)

- **Factor portfolios** are well-diversified portfolios with betas equal to 1 for a single risk factor and betas equal to 0 on all remaining factors.
- In this example, the GDP factor beta on the 50% short position in the GDP factor portfolio equals –0.50, which perfectly offsets the 0.50 GDP factor beta on the original portfolio.
- The combined long and short positions hedge away GDP risk but retain inflation risk exposure.

The Arbitrage Pricing Theory Model

- APT assumes a market model of the form:



- The CAPM model is a special case of APT with only one factor—the market risk premium.

Calculate Required Return: APT

- **Problem**: Factors NOT specified by theory
- Examples of common "accepted" factors:
  1. Investor confidence risk
  2. Time horizon risk
  3. Inflation risk
  4. Business-cycle risk
  5. Market-timing risk

Do not memorize

- You will be given the factors on exam day.

Calculate Required Return: APT Example

- **Assume**:
  - Risk-free rate of 3%
  - **Three** factors and sensitivities

	Risk Premium	Sensitivity
Investor confidence risk	2%	1.1
Time horizon risk	4%	1.2
Inflation risk	3%	0.8

$R_i = 3\% + 1.1(2\%) + 1.2(4\%) + 0.8(3\%) = \mathbf{12.4\%}$

Fama-French Three-Factor Model

- APT offers no guidance as to the identification of the appropriate risk factors.
- In contrast, the Fama-French model identifies the factors.
- In addition to the market return factor, the Fama-French three-factor model specifies the following two factors:
  - SMB (small minus big): firm size factor
  - HML (high minus low): book-to-market factor

Fama-French Three-Factor Model (continued)

Fama-French three-factor model equation:

$$R_i - R_F = \alpha_i + \beta_{i,M}(R_M - R_F) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + e_i$$

- Intercept equals abnormal performance of the asset after controlling for its exposures to the market, firm size, and book-to-market factors.
- SMB and HML factors are used because historical returns are higher on small versus big firms and on high versus low book-to-market firms.

Principles for Effective Data Aggregation and Risk Reporting

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Benefits of Risk Data Aggregation

- Effective risk data aggregation and reporting benefits include:
  - An increased ability to anticipate problems
  - Enhanced ability to identify alternative routes to restore financial health in times of stress
  - Improved resolvability in the event of bank stress or failure
  - An enhanced ability to make strategic decisions, increasing the bank's efficiency, reducing the chance of loss, and ultimately increasing bank profitability

Model Development

- Financial models are used by banks for everything from analyzing risk exposures to guiding daily operations.
- Even small errors that occur in the model development process may result in serious consequences for a bank.
- Model developers must demonstrate that the data used in model development is consistent with the theory and methodologies behind the model.

Governance

- The Basel Committee on Banking Supervision has set forth principles for effective risk data aggregation and risk reporting.
- The governance principle (Principle 1) suggests that risk data aggregation should be part of the bank's overall risk management framework.
- The board and senior management should assure that adequate resources are devoted to risk data aggregation and reporting.

Data Architecture and IT Infrastructure

- The data architecture and IT infrastructure principle (Principle 2) states that a bank should design, build, and maintain data architecture and IT infrastructure which fully supports its risk data aggregation capabilities and risk reporting practices during normal times and times of stress, while still meeting the other principles.
- It stresses that banks should devote considerable financial and human resources to risk data aggregation and reporting.

Effective Risk Data Aggregation

- Banks should ensure that the data is accurate and has integrity (Principle 3), is complete (Principle 4), is timely (Principle 5), and is adaptable to the end user (Principle 6).
- In addition, the bank should not have high standards for one principle at the expense of another principle.
- Aggregated risk data should exhibit all of the features together, not in isolation.

Effective Risk Reporting Practices

- Risk reports should be accurate (Principle 7), comprehensive (Principle 8), and clear and useful (Principle 9).
- Principle 10 states that reports should be “appropriately frequent” (frequency depends on the role of the recipient).
- Principle 11 states that reports should be distributed to relevant parties in a timely fashion while maintaining confidentially.

Enterprise Risk Management and Future Trends

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Enterprise Risk Management (ERM)

- An integrated and centralized approach under ERM is significantly more effective in managing a company's risks than under the traditional silo approach of managing risks within each risk/business unit.
- ERM is a comprehensive and integrated framework for managing a firm's key risks to meet business objectives, minimize unexpected earnings volatility, and maximize firm value. It is the best way to identify and prioritize risks.

ERM Benefits and Costs

- The key motivations of an ERM initiative include integration of risk organization, integration of risk transfer, and integration of business processes, which lead to increased organizational effectiveness, better risk reporting, and improved business performance, respectively.
- ERM also allows firms to focus on the largest risks facing them.

ERM Best Practices

- A successful corporate governance framework requires that senior management and the board of directors adequately define the firm's risk appetite and risk and loss tolerance levels.
- Senior management and the board of directors should have the requisite organizational practices and processes in place to adequately control risks.

ERM Program Dimensions

- A strong ERM framework has five important dimensions: (1) targets, (2) structure, (3) identification and metrics, (4) ERM strategies, and (5) risk culture.

Risk Culture Characteristics and Challenges

- The risk culture of a firm is the goals, customs, values, and beliefs (both implicit and explicit) that influence the behaviors of employees.
- These corporate norms guide individuals in their understanding and responses to risk.
- Individuals come to firms with their own ideas and views about risk and are influenced by peers, managers, and executives.

Scenario Analysis and Stress Testing

- Scenario analysis is used by banks to better understand and plan for risks.
- Firms and regulators develop narratives to test risks, explain why variables change, and understand the effects of those changes.
- Sophisticated financial models are developed to assess the impact of various scenarios on the risks and performance of the enterprise.

Stress Testing Programs

- Since the financial crisis of 2007–2009, regulators have required banks to use scenario analysis and stress testing in capital planning.
- U.S. stress testing of banks began in 2009 with the initial Supervisory Capital Assessment Program (SCAP).
- Since 2011, the Federal Reserve has conducted annual stress tests.

Stress Testing Programs (continued)

- In addition, the Dodd-Frank Act required stress testing (Dodd-Frank Act stress tests or DFAST) and the Comprehensive Capital Analysis and Reviews (CCAR) are conducted at year-end for banks with $50 billion or more in assets.
- While the scenarios for DFAST and CCAR are the same (devised by supervisors), DFAST is more prescriptive, requires less reporting, and has limited capital action assumptions.