Modern Portfolio Theory (MPT) and the Capital Asset Pricing Model (CAPM)

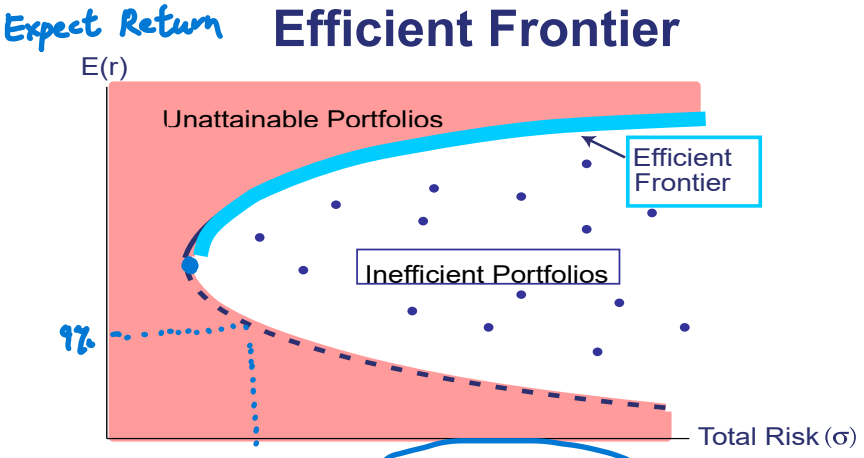
KAPLAN SCHWESER

Efficient Frontier Background

有效前沿

Efficient frontier is the set of portfolios among all the possible portfolios of combinations of individual risky assets that offers the highest expected return for each level of risk (standard deviation).

Efficient Frontier

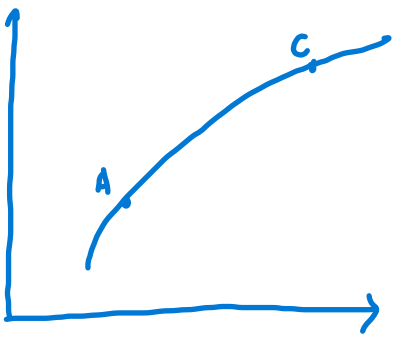


The efficient frontier is the upper boundary of the set of all possible portfolio risk/return combinations.

Example: Efficient Portfolios

Which of the following portfolios is *not* on the Markowitz efficient frontier?

Portfolio	Expected Return	Std. Dev.
→ A	10%	12%
→ B	12%	16%
C	14%	15%

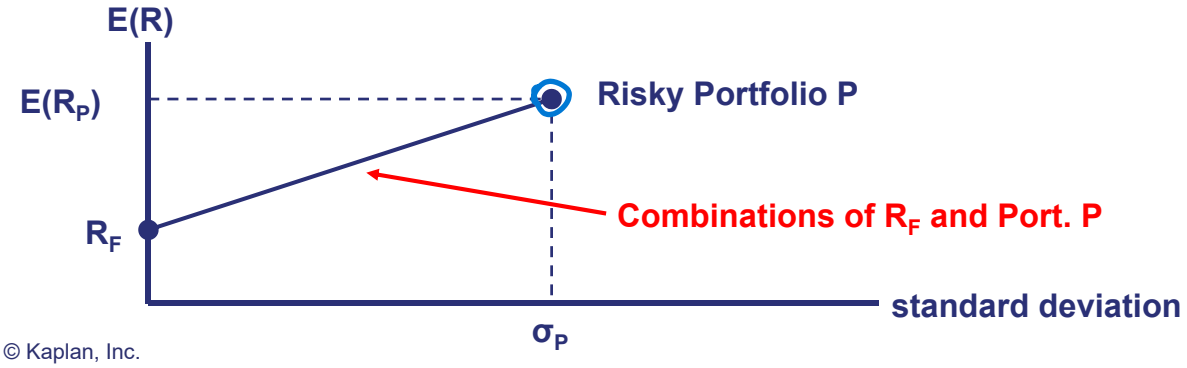


B has lower return and higher standard deviation compared to C. B can't be efficient.

Combining Risk-Free and Risky Assets

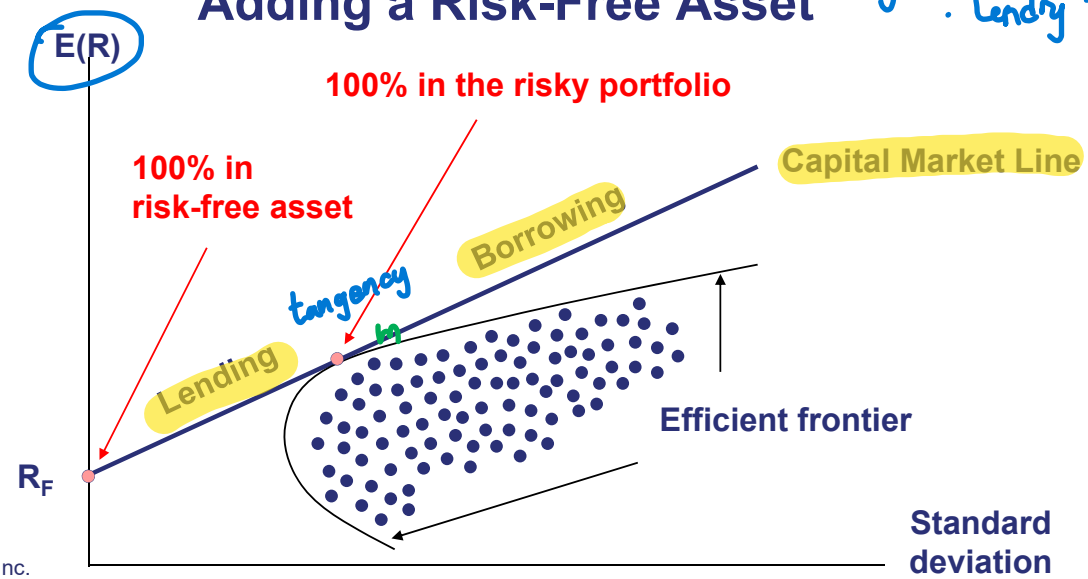
Portfolio expected return = $W_{risky} R_p + W_{RF} R_F$

Portfolio standard deviation = $W_{risky} \sigma_p$



Adding a Risk-Free Asset

e.g. Gov bond · lendry to gov

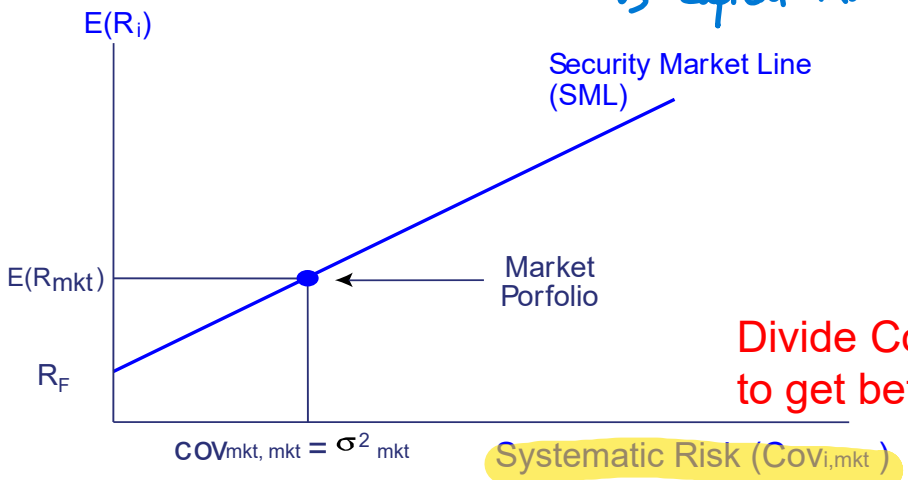


Assumptions of Capital Market Theory

- Information is freely available. ✓
- There are no taxes and commissions. ✓ (佣金)
- Fractional investments are possible. (零碎投資)
- Market participants can borrow and lend at the risk-free rate.
- Individual investors cannot affect market prices.
- Investors have the same forecasts of expected returns, variances, and covariances.

Security Market Line (SML)

vs Capital Market Line



SIGMA

Divide $Cov_{i,m}$ by σ^2_{mkt} to get beta on *x*-axis

The Capital Asset Pricing Model

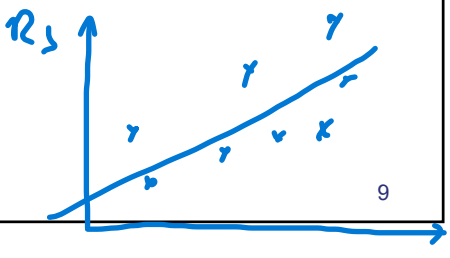
SML equation:

$$E(R_i) = R_F + \beta_i[E(R_{mkt}) - R_F]$$

Beta is a standardized measure of systematic risk, beta of the market portfolio is 1.

Beta measures the covariance of an asset's returns with returns on the market portfolio.

Calculating beta of asset 'i': $\beta_i = \frac{Cov_{i,mkt}}{\sigma^2_{mkt}}$



Capital Asset Pricing Model (CAPM)

- CAPM is the expected return on an asset based (only) on the asset's systematic risk or beta.
- CAPM is also used to determine the required return on an asset based on the asset's systematic risk (beta).
- Required return and expected return are the same in equilibrium.

Forecast Returns and the CAPM – Example

Stock	Price Today	E (price) in 1 year	E (dividend) in 1 year	Beta
A	$25	$27	$1.00	1.0
B	40	45	2.00	0.8
C	15	17	0.50	1.2

Are these stocks overpriced, underpriced, or at their equilibrium prices?

Show where they plot on the SML graph.

Forecast Returns and the CAPM – Example

Stock	Price Today	E (price) in 1 year	E (dividend) in 1 year	Beta
A	$25	$27	$1.00	1.0
B	40	45	2.00	0.8
C	15	17	0.50	1.2

R_F

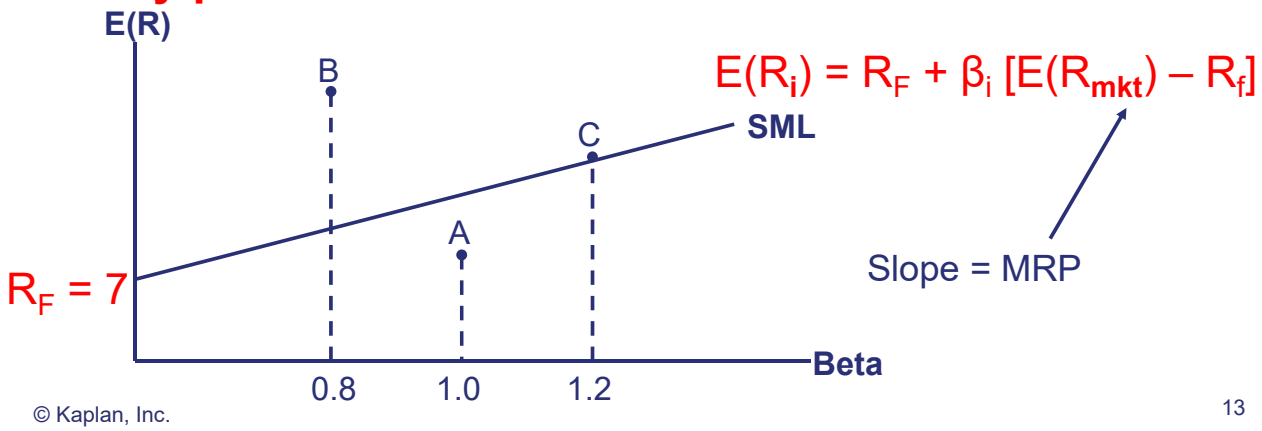
$R_F = 7\%$ $E[R_{mkt}] = 15\%$

MRP

Stock	Forecast Return	Required Return
A	(27–25+1)/25 = 12.0%	0.07+1.0(0.15–0.07) = 15.0%
B	(45–40+2)/40 = 17.5%	0.07+0.8(0.15–0.07) = 13.4%
C	(17–15+0.50)/15 = 16.6%	0.07+1.2(0.15–0.07) = 16.6%

Forecast Returns and the CAPM – Example

When securities are priced at equilibrium values, they plot on the SML.



Forecast Returns and the CAPM

Stock	Forecast Return	Required Return
A	12.0%	15.0%
B	17.5%	13.4%
C	16.6%	16.6%

Stock A is overvalued (sell it or sell it short).

Stock B is undervalued (buy it).

Stock C is properly valued (indifferent).

SML Pricing – Example

$E[R_{mkt}] = 15\%$; $E[R_{Stock\ X}] = 17\%$; $R_F = 8\%$

Stock X's beta = 1.25

Using this data and the capital asset pricing model, which of the following statements about X's stock is true? Stock X is:

A. overvalued by 1.75 percentage points.

B. properly valued.

C. undervalued by 0.25 percentage points.

8 + 1.25(15 – 8) = 16.75 17 – 16.75 = 0.25

Measures of Performance

- Modern portfolio theory and the CAPM are built upon the link between risk and return.
- Three measures exist to assess an asset's or portfolio's return with respect to its risk:
  - **Treynor measure** = risk premium divided by beta, or **systematic risk.**
  - **Sharpe measure** = risk premium divided by standard deviation, or **total risk.**
  - **Jensen's alpha** = asset's excess return over the return predicted by the CAPM.

Measures of Performance (continued)

$$\text{Treynor} = \frac{E(R_P) - R_F}{\beta_P}$$

$$\text{Sharpe} = \frac{E(R_P) - R_F}{\sigma_P}$$

$$\text{Jensen's alpha, } \alpha_p = E(R_P) - \{R_F + \beta_P[E(R_M) - R_F]\}$$

Measures of Performance (continued)

- In all three cases, the higher the better.
- Investors can apply Sharpe measure to all portfolios because it uses total risk.
- Treynor measure is more appropriate for comparing well-diversified portfolios.
- Jensen's alpha is most appropriate for comparing portfolios that have the same beta.

Tracking Error

大 -> 較大嘅變動範圍

- **Tracking error** is the standard deviation of the difference between the portfolio return and the benchmark return.
  指數
- Typically, the manager must keep the tracking error below a stated threshold.

$$\text{Tracking error} = \sigma_{R_P - R_B}$$

Information Ratio

正數->超過市場預期嘅回報

- **Information ratio** (IR) is the alpha of the managed portfolio relative to its benchmark divided by the tracking error.
- It is essentially a measure of how well the manager has acquired and used information compared to the average manager.

$$IR = \frac{E(R_P) - E(R_B)}{\text{tracking error}}$$

Tracking Error and Information Ratio

- Compute the tracking error and the information ratio for the following returns of a portfolio and its benchmark:

Portfolio returns (%)	Benchmark returns (%)
9.5	8.7
8.0	7.8
–2.2	–3.4
11	12.4

Tracking Error and Information Ratio (continued)

- First, compute the alpha as the difference between the returns:

Portfolio returns (%)	Benchmark returns (%)	Alpha
9.5	8.7	0.8
8.0	7.8	0.2
–2.2	–3.4	1.2
11	12.4	–1.4

- Click 2nd Data (on TI BA II+) and input the alphas as Xs. In 2nd STAT (1-V function), X-bar is average alpha = 0.2 and Sx is tracking error = 1.143.
- The information ratio is alpha/tracking error = 0.2/1.143 = 0.175

索蒂諾比率
Sortino Ratio

- The **Sortino ratio** can be interpreted as a variation of the Sharpe ratio that is more appropriate for a case where returns are not symmetric.
- We replace the risk-free rate with a minimum acceptable return, denoted R_{min}, and we replace the standard deviation with a type of downside deviation.
- A downside deviation measures the variability of only those returns that fall below the minimum acceptable return.

夏普比率
回報不對稱
無風險利率

Sortino Ratio (continued)

- Letting R_{min} denote the minimal acceptable return and downside deviation represent the risk measure:

$$\text{Sortino ratio} = \frac{E(R_P) - R_{min}}{\text{downside deviation}}$$

Sample Exam Question

A portfolio manager received a report on his fund's performance during 2011. According to the report, the portfolio return was 2.5% with a standard deviation of 21% and a beta of 1.2. The risk-free rate over this period was 3.5%, the downside deviation of the portfolio was 16%, and the tracking error of the fund was 2%. Compute the difference between the value of the fund's Sortino ratio (assuming the risk-free rate is the minimal acceptable return) and its Sharpe ratio.

A. 0.563.

B. 0.347.

C. –0.053.

D. –0.015.

Answer

Answer: **D**

Sharpe ratio =

$[E(R_P) – R_F] / \sigma$

$(2.5 – 3.5) / 21 = –0.0476$

Sortino ratio =

$[E(R_P) – R_{min}]$ / (downside deviation)

$(2.5 – 3.5) / 16 = –0.0625$

Sample Exam Question

The information ratio of Large Hedge Fund is equal to 2 when using the S&P 500 as the benchmark index. During the same time period, standard deviation of the hedge fund returns is 4%, the hedge fund's Sharpe ratio is 3, and its tracking error against the S&P 500 is 6%. Calculate the return for the S&P 500 during the time period assuming the risk-free rate is 3%.

A. 3%.

B. 7%.

C. 11%.

D. 15%.

Answer

Answer: **A**

Sharpe ratio = 3

(Portfolio return – risk-free rate) / SD of fund = 3

(Portfolio return – 3%) / 4% = 3

Portfolio return = 15%

Information ratio = 2

(Portfolio return – S&P return) / Tracking error = 2

(15% – S&P return) / 6% = 2

S&P 500 return = 3%

Sample Exam Question

Portfolio Y has a beta of 0.8 and an expected return of 10%. The market risk premium is 6.45% and the risk-free rate is 3.7%. Jensen's Alpha measure for this portfolio is closest to:

A. 10%.

B. 8%.

C. 3%.

D. 1%.

Answer

Answer: **D**

Jensen's alpha

$= E(R_P) - [R_F + \beta_P(E(R_M) - R_F)]$

$= 10\% - [3.7\% + 0.8(6.45\%)]$

$= 1.14\%$

Example: Capital Asset Pricing Model

- The expected rate of return on a stock is 1.5 times the 16% expected rate of return from the market.
- What is the beta of the stock if the risk-free rate is 8%?

Answer

According to the CAPM for any single security or portfolio of securities *i*, the expected return in equilibrium is

$E(R_i) = R_F + \beta_i[E(R_M) - R_F]$

$24\% = 8\% + \beta(16\% - 8\%)$

$24\% = 8\% + \beta(8\%)$

Beta is then equal to = $\beta = 16 / 8 = 2$

Example: Sharpe and Treynor Measures

- Performance data for an actively managed portfolio and the S&P 500 Index is reported as:

	Portfolio	S&P 500
Return	10%	8%
Standard deviation	6%	5%
Beta	1.1	1.0

- Risk-free rate = 3%
- Determine the Sharpe and Treynor measures for the actively managed portfolio.

Answer

Sharpe measure:

= (0.10 – 0.03) / 0.06 = 1.17

Treynor measure:

= (0.10 – 0.03) / 1.1 = 0.064

Example: Sortino Ratio

- Portfolio X return = 11%
- Benchmark return = 10%
- Tracking error volatility = 6%
- Minimum acceptable return = 5%
- Risk-free rate = 4%
- Downside deviation = 20%
- What is Portfolio X's Sortino ratio?

Answer

The Sortino ratio can be interpreted as a variation of the Sharpe ratio that is more appropriate for a case where returns are not symmetric.

$$\text{Sortino ratio} = \frac{E(R_P) - R_{min}}{\text{downside devation}}$$

下行偏差

$$= \frac{11\% - 5\%}{20\%} = 0.3$$