

Optimal design for universal multiport interferometers

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Universal multiport interferometers, which can be programmed to implement any linear transformation between multiple channels, are emerging as a powerful tool for both classical and quantum photonics. These interferometers are typically composed of a regular mesh of beam splitters and phase shifters, allowing for straightforward fabrication using integrated photonic architectures and ready scalability. The current, standard design for universal multiport interferometers is based on work by Reck *et al.* [Phys. Rev. Lett. 73, 58 (1994)]. We demonstrate a new design for universal multiport interferometers based on an alternative arrangement of beam splitters and phase shifters, which outperforms that by Reck *et al.* Our design requires half the optical depth of the Reck design and is significantly more robust to optical losses.

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1. INTRODUCTION

Reconfigurable universal multiport interferometers, which can implement any linear transformation between several optical channels, are emerging as a powerful tool for fields such as microwave photonics [1,2], optical networking [3,4], and quantum photonics [5,6]. Such devices are typically built using planar meshes of beam splitters, which are easy to fabricate and to individually control, as recent demonstrations of large, yet non-universal, interferometers have shown [7,8]. While it had been known for some time that useful operations could be performed by such meshes [9], the seminal work by Reck *et al.* [5] demonstrated that a specific triangular mesh of 2×2 beam splitters and phase shifters could be programmed, using a simple analytical method, to implement any unitary transformation between a set of optical channels. Continued interest in universal multiport interferometers for classical and quantum applications has led to new applications and programming procedures for the same interferometer design [10,11]. Recent demonstrations of universal multiport interferometers are based on this design and have been used to interfere up to six channels [6].

In this article, we propose a new design (Fig. 1), which outperforms the Reck design in two key respects. First, our new design achieves the minimal optical depth, requiring roughly half the depth of the Reck design, which is important for minimizing

optical losses and reducing fabrication resources. Second, the natural symmetry of this new design makes it significantly more robust to fabrication errors caused by mismatched optical losses. Our finding is based on a new mathematical decomposition of a unitary matrix. We use this decomposition both to prove the universality of the design and to construct an efficient algorithm to program interferometers based on it. In the following, we first provide an overview of both the Reck design and of our new design and discuss some advantages of the latter. We then explain the general principles of our decomposition procedure using a 5×5 universal transformation as an example. Finally, we quantitatively compare the loss tolerance of our design to that of the Reck design and discuss their tolerance to error.

2. BACKGROUND

An ideal, lossless multiport interferometer between N channels performs an optical transformation which can be described by an $N \times N$ unitary scattering matrix U acting on electric fields as $\mathbf{E}_{\text{out}} = U\mathbf{E}_{\text{in}}$. Equivalently, in quantum optics, U describes the transformation of the creation or annihilation operators of the input modes to those of the output modes.

Within this framework, the following transformation between channels m and n ($m = n - 1$),

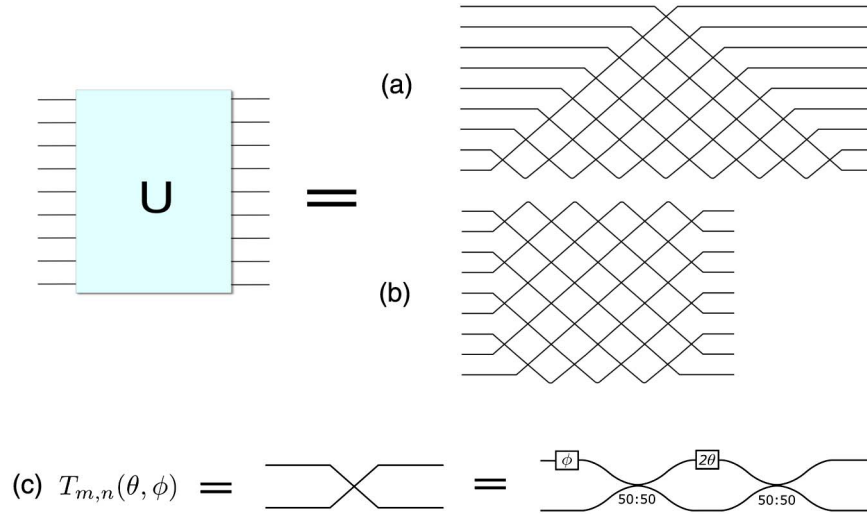


Fig. 1. Universal N -mode multipoint interferometer (shown here for $N = 9$) can be implemented using a mesh of $N(N - 1)/2$ beam splitters such as (a) the one proposed by Reck *et al.* or (b) the one we demonstrate in this paper. As shown in (c), a line corresponds to an optical mode, and crossings between two modes correspond to a variable beam splitter described by a $T_{m,n}(\theta, \phi)$ matrix, which can be implemented by a Mach-Zehnder interferometer consisting of two 50:50 directional couplers, preceded by a phase shift at one input port. Although the total number of beam splitters in both interferometers is identical, our scheme clearly has a much shorter optical depth and therefore suffers less propagation loss. This reduction in optical depth stems from the fact that each mode crosses its nearest neighbor at the first possible occasion in contrast to the Reck scheme, where the top modes must propagate for some distance before interacting with other modes. Furthermore, the high symmetry inherent to our design improves the loss tolerance of the interferometer, as we show in the main text.

$$T_{m,n}(\theta, \phi) = \begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 1 & & & & & \vdots \\ \vdots & & \ddots & & & & \vdots \\ \vdots & & & e^{i\phi} \cos \theta & -\sin \theta & & \vdots \\ \vdots & & & e^{i\phi} \sin \theta & \cos \theta & & \vdots \\ \vdots & & & & & \ddots & \vdots \\ \vdots & & & & & & 1 & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 1 \end{bmatrix}, \quad (1)$$

corresponds to a lossless beam splitter between channels m and n with reflectivity $\cos \theta$ ($\theta \in [0, \pi/2]$) and a phase shift ϕ ($\phi \in [0, 2\pi]$) at input m . In the following, we will generally omit the explicit dependence of these $T_{m,n}(\theta, \phi)$ matrices on θ and ϕ for notational simplicity.

Both our scheme and the scheme by Reck *et al.* [5] are based on analytical methods of decomposing the U matrix into a product of $T_{m,n}$ matrices. Specifically, these schemes provide an explicit algorithm for writing any unitary matrix U as

$$U = D \left(\prod_{(m,n) \in S} T_{m,n} \right), \quad (2)$$

where S defines a specific ordered sequence of two-mode transformations and where D is a diagonal matrix with complex elements with a modulus equal to one on the diagonal. A physical interferometer composed of beam splitters and phase shifters in the configuration defined by S , with values defined by the θ and ϕ in the $T_{m,n}$ matrices, will therefore implement transformation U . We note that D is physically irrelevant for most applications but can be implemented in an interferometer nonetheless by phase shifts on all individual channels at the output of an interferometer.

The formalism developed here for unitary transformations describing lossless $N \times N$ interferometers can be extended to include any $M \times N$ linear (non-unitary) transformation. Indeed, it has been noted that any $M \times N$ linear transformation, with, for example, $M \leq N$ (resp. $M \geq N$), can be either directly embedded within a $2N \times 2N$ (resp. $2M \times 2M$) unitary transformation [12] to within a scaling factor, or, in a more compact way, implemented by 2 separate $N \times N$ (resp. $M \times M$) interferometers connected to each other via phase and amplitude modulators [10]. Furthermore, realistic, lossy interferometers can also be included in our formalism simply by rescaling U by a loss factor, as we explain later. Therefore, our design for universal multipoint interferometers, as well as that by Reck *et al.* [5], can be used to implement any linear transformation to within a scaling factor on any number of input and output channels. While our design is very general and can be used for any interferometric transformation, more efficient architectures may exist for specific values of N or M when $N \neq M$ (see [13] for N or $M = 1$, for example) or for specific transformations (see [14] for implementing Fourier transforms, for example).

3. OVERVIEW OF THE TWO DESIGNS

Schematic views of the Reck design and of our design are presented in Fig. 1. Figure 1(a) presents the Reck design, in which the matrix decomposition method determines a sequence S that corresponds to a triangular mesh of beam splitters. Figure 1(b) presents our design, in which every mode crosses its nearest neighbor at the first possible occasion. Our design has a shorter optical depth and is more symmetrical than the Reck design. We note that both interferometers use the same minimal number $N(N - 1)/2$ of beam splitters to implement an $N \times N$ interferometer [5].

We define the depth of an interferometer as the longest path through the interferometer, enumerated by counting the number of beam splitters traversed by that path. It is important to minimize the optical depth of an interferometer because the resulting circuits can then be more compact, which is an important factor for the fabrication of planar waveguide circuits. Furthermore, propagation losses are reduced for an interferometer with a smaller depth. It is easy to see that our design has the minimal possible optical depth, since every channel crosses its nearest neighbor at the first possible occasion. Specifically, for an $N \times N$ interferometer, the Reck design has an optical depth of $2N - 3$, whereas our design has an optical depth of N . To illustrate this, the longest path through the interferometer shown in Fig. 1(a) follows the edges of the triangle and crosses $2N - 3 = 15$ beam splitters, whereas the longest paths through the interferometer in Fig. 1(b) cross $N = 9$ beam splitters. The increased symmetry of our design also leads to significantly better loss tolerance, as discussed in a subsequent paragraph.

4. DECOMPOSITION METHOD

In the following, we present our decomposition method, which allows us to analytically calculate the values of the beam splitter elements $T_{m,n}$ in our design. Beyond its use in proving that our design is capable of implementing universal interferometric transformations, this method directly provides a recipe for programming such interferometers. Our decomposition method relies on two important properties of the $T_{m,n}$ matrices. Firstly, for any given unitary matrix U , there are specific values of θ and ϕ that make any target element in row m or n of matrix $T_{m,n}U$ zero, as per Reck *et al.* [5]. We will refer to this process as nulling that element of U and will still refer to the modified matrix after this operation as U . Secondly, we note that any target element in column n or m of U can also be nulled by multiplying U from the right by a $T_{m,n}^{-1}$ matrix.

Our algorithm, shown in Fig. 2 for the 5×5 case, consists of nulling elements of U one by one in such a way that every $T_{m,n}$ and $T_{m,n}^{-1}$ matrix used in the process completely determines both the reflectivity and phase shift of one beam splitter and phase shifter. The sequence of $T_{m,n}$ and $T_{m,n}^{-1}$ matrices must both correspond to the desired order of beam splitters in the interferometer and guarantee that the nulled elements of U are not affected by subsequent operations. In the Reck decomposition, the entire matrix can be nulled using either only $T_{m,n}$ matrices or only $T_{m,n}^{-1}$ matrices while still making sure nulled elements of U are not affected by subsequent operations. Both matrices are necessary to verify this condition in our decomposition. As illustrated in Fig. 2, we null successive diagonals of U by alternating between $T_{m,n}$ and $T_{m,n}^{-1}$ matrices in such a way that every nulled diagonal in the matrix corresponds to one diagonal line of beam splitters through the interferometer.

At the end of the decomposition process, we obtain the following expression for a 5×5 matrix:

$$T_{4,5}T_{3,4}T_{2,3}T_{1,2}T_{4,5}^{-1}T_{3,4}^{-1}T_{2,3}^{-1}T_{1,2}^{-1} = D, \quad (3)$$

where D is a diagonal matrix, as in Eq. (2). This can be rewritten as

$$U = T_{3,4}^{-1}T_{4,5}^{-1}T_{1,2}^{-1}T_{2,3}^{-1}T_{3,4}^{-1}DT_{1,2}T_{2,3}T_{3,4}T_{1,2}. \quad (4)$$

It is easy to demonstrate that if D consists of single-mode phase shifts, then for any $T_{m,n}^{-1}$ matrix, one can find a matrix

D' of single-mode phases and a matrix $T_{m,n}$ such that $T_{m,n}^{-1}D = D'T_{m,n}$. The previous equation can therefore be rewritten as

$$U = D'T_{3,4}T_{4,5}T_{1,2}T_{2,3}T_{3,4}T_{4,5}T_{1,2}T_{2,3}T_{3,4}T_{1,2}, \quad (5)$$

which, mirroring Eq. (2), completes our decomposition.

By construction, Eq. (5) physically corresponds to the multiport interferometer shown in Fig. 2, and the values of the θ and ϕ of the $T_{m,n}$ matrices in this equation determine the values of the beam splitters and phase shifts that must be programmed to implement U . This decomposition principle can be generalized to any N , and an explicit general algorithm is given in Supplement 1. We also note that this algorithm can be used to inform the design of fixed interferometric circuits, such as those demonstrated in [15,16], in which the same arrangement of beam splitters was used to provide specific instances of random interference. Our algorithm can also be used to implement any desired optical interference between any type of optical mode and in any optical platform beyond integrated photonics.

We note that whereas the decomposition method presented here can straightforwardly be applied to program a universal multiport interferometer built according to our design, other programming schemes may exist. For instance, for the Reck design, self-configuring methods have recently been proposed [10,11] and demonstrated [17,18]. These methods can be advantageous for some applications in that they do not require full characterization of the circuit elements corresponding to the $T_{m,n}$ matrices. The existence of such schemes for our design is beyond the scope of this article.

5. LOSS TOLERANCE

Optical loss is unavoidable in realistic interferometers, and finding methods to mitigate its effects is an integral part of any photonic scheme. In the following, we study the tolerance of multiport interferometers built according to our decomposition to loss and compare their performance to interferometers built and programmed according to the Reck design.

We first distinguish between two types of loss. Balanced loss in a multiport interferometer, in which every path through the interferometer experiences the same loss, preserves the target interference to within an overall scaling factor. This is generally acceptable for applications in the classical domain, such as optical switching or microwave photonics. In the quantum domain, although loss severely affects the scalability of quantum experiments, post-selection can, in some situations, be used to recover the desired interference pattern. We note that propagation loss in an interferometer is expected to contribute to balanced loss, since every physical path length in an interferometer must be matched to within the coherence length of the input light to maintain high-fidelity interference. However, propagation loss must therefore be proportional to the longest path through the interferometer (i.e., the optical depth), so interferometers built according to our design will suffer from only about half the propagation loss of an interferometer built according to the Reck design.

Unbalanced loss, where different paths through the interferometer experience different losses, can be difficult to characterize and, critically, can result in poor fidelity to the intended operation [19–22]. Unequal losses between paths in the interferometer are typically caused by beam splitters, which are unavoidably lossy

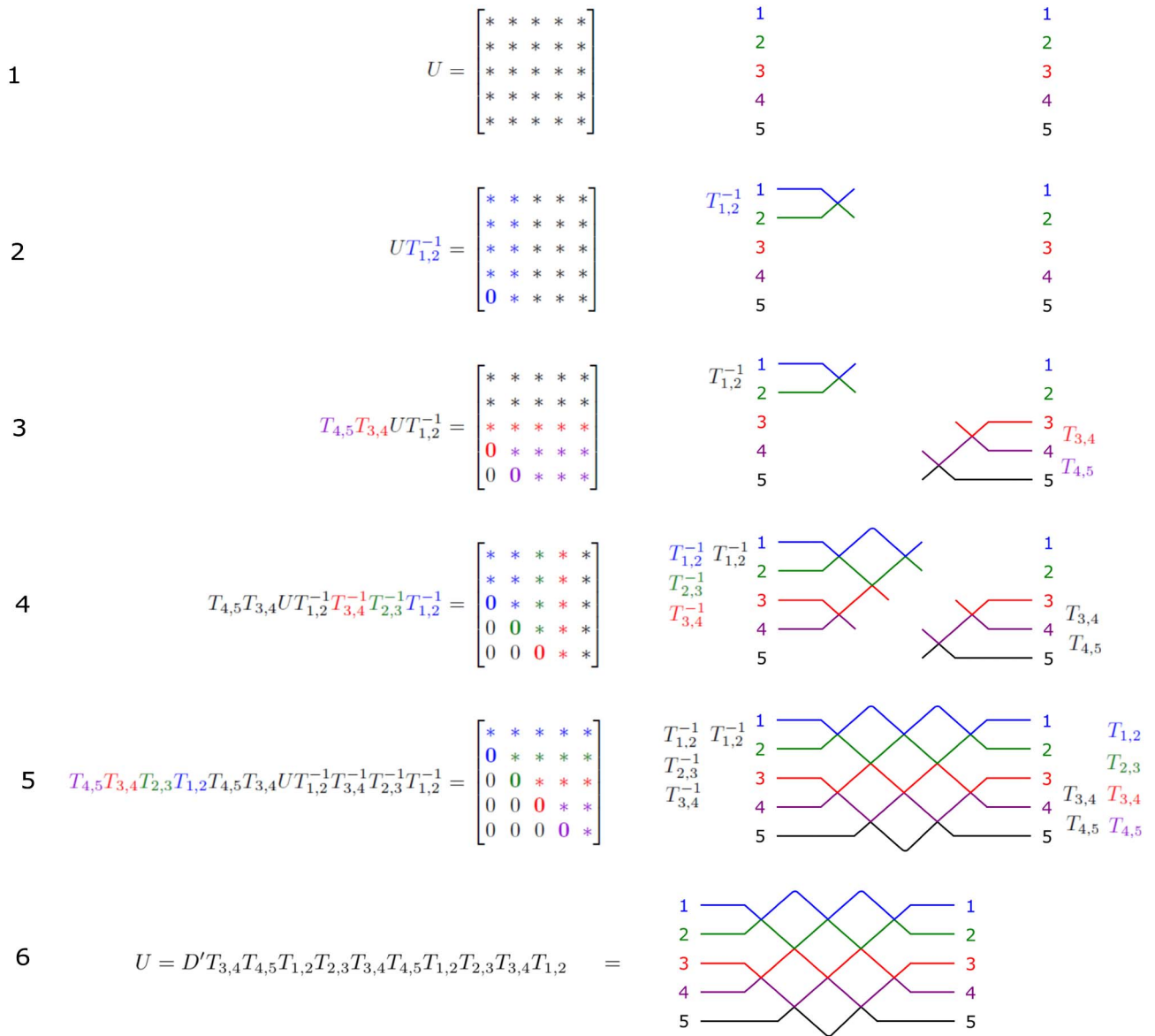


Fig. 2. Illustration of the algorithm for programming a universal multiport interferometer for a 5×5 interferometer. The left-hand side presents our decomposition procedure, and the right-hand side shows how our decomposition corresponds to building up the corresponding interferometer. (1) We start with any random unitary matrix U , and a blank interferometer. (2) We first null the bottom-left element of U with a $T_{1,2}^{-1}$ matrix, which causes the first two columns of U to mix. This corresponds to adding the top-left beam splitter in the interferometer. (3) We then null the next two elements of U using a $T_{3,4}$ matrix followed by a $T_{4,5}$ matrix, which correspond to the two bottom-right beam splitters in the interferometer. $T_{3,4}$ mixes rows 3 and 4, and $T_{4,5}$ mixes rows 4 and 5. Since both the (4,1) and (5,1) elements of U had been nulled, they are not affected by $T_{4,5}$. (4) and (5) At every step in the algorithm, we null a successive diagonal of the updated U matrix by alternating between $T_{m,n}$ and $T_{m,n}^{-1}$ matrices, which corresponds to adding diagonal lines of beam splitters to the interferometer. $T_{m,n}$ (resp. $T_{m,n}^{-1}$) matrices of a given color cause the rows (resp. columns) m and n , which are shown in the same color, to mix and null the corresponding element of that color in U . It is clear from this process that once a matrix element has been nulled, no subsequent operation can modify it. (6) After step 5, U is a lower triangular matrix, which by virtue of its unitarity must be diagonal. As explained in the main text, we can then write U in the way shown here, which by construction exactly corresponds to the desired interferometer.

due to additional bending losses and scattering. To compare the tolerance of multiport interferometers to the unbalanced losses caused by beam splitters, we adopt the following procedure. For a given N , we generate 500 random unitary matrices [23], implement our decomposition, add loss to both outputs of all the resulting beam splitters, and compare the fidelities in the overall transformations. We use a simple loss model that assumes equal insertion loss for every beam splitter, and we quantify the fidelity of the transformation implemented by a lossy $N \times N$

experimental interferometer, U_{exp} , to the intended transformation U using the following metric,

$$F(U_{\text{exp}}, U) = \left| \frac{\text{tr}(U^\dagger U_{\text{exp}})}{\sqrt{N \text{tr}(U_{\text{exp}}^\dagger U_{\text{exp}})}} \right|^2, \quad (6)$$

which corresponds to a standard fidelity measure, normalized so we do not distinguish between matrices that differ by only

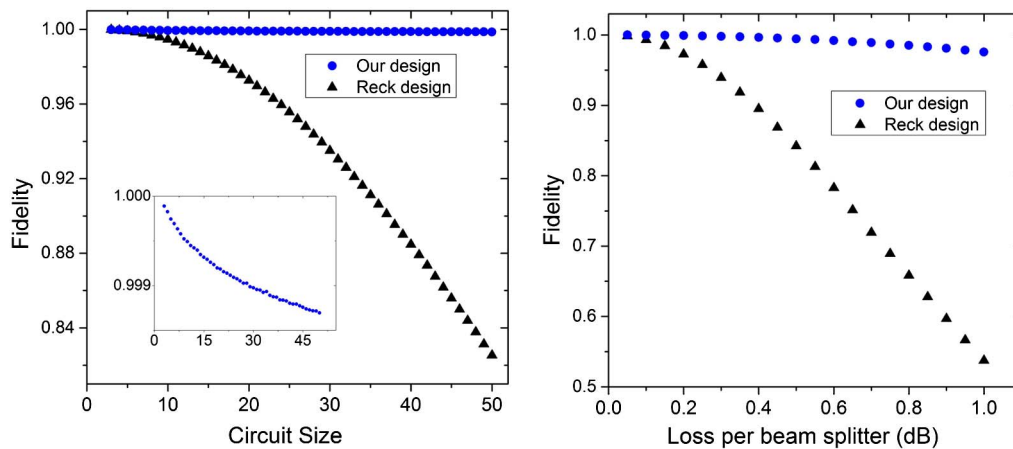


Fig. 3. Left: average fidelity for an interferometer with a constant loss of 0.2 dB per beam splitter (as in the universal multiport interferometer in [6]) for interferometers built according to the Reck design (black) and our design (blue), for different interferometer sizes. Inset: close-up of the fidelity in our design. Right: fidelity as a function of loss for interferometers implementing 20×20 transformations. We see from our results that our design is much more loss-tolerant than the Reck design and maintains high fidelity with the target unitary even in the case of high loss. This is because the mismatched path lengths in the Reck design causes the loss to severely affect the resulting interference. See [24] for underlying data.

a constant multiplicative factor. This allows us to focus on unbalanced loss instead of balanced loss in our simulations.

Figure 3 shows our simulation results for both a fixed loss and varying interferometer sizes and for a fixed interferometer size and varying loss. We conclude the interferometers that are implemented in our design are significantly more tolerant to unbalanced loss than those implemented in the Reck design. This is because in the Reck design, different paths through the interferometer go through different numbers of beam splitters, so they all experience different losses and the resulting interference is degraded. In our design, the path lengths are better matched, so equally distributed loss within the interferometer does not strongly affect the resulting interference. We note that whereas unbalanced loss can be compensated for in the Reck design by adding loss to shorter paths, for example, by adding dummy beam splitters to the shorter paths in the interferometer, as proposed by Miller [11], this is inefficient and it is better to start with a fundamentally loss-resistant interferometer.

6. ERROR TOLERANCE

In a realistic interferometer, there will always be some error when setting the values for the phases and beam splitter reflectivities. Furthermore, if the 2×2 elements are composed of single Mach-Zehnder interferometers, imperfections in the beam splitters will make it difficult to reach high values of transmission or reflection. These errors will affect the circuit fidelity in both our design and in the Reck design. Since the overall error in the interferometer caused by these individual errors depends on the total number of beam splitters, the layout of the interferometer mainly affects how that error is distributed among the output ports. Therefore, the average error caused by these imperfections is roughly equal in the Reck design and in our design.

However, we note that the problem of error can, if necessary, be mitigated by concatenating imperfect beam splitters to create one ideal reconfigurable beam splitter [11,25,26], at the expense of additional loss. Our design is particularly well suited for implementing such a solution, since a uniform beam splitter loss does not strongly affect the circuit fidelity.

7. CONCLUSION

In conclusion, we have demonstrated a design for universal multiport interferometers which outperforms the design proposed by Reck *et al.* [5] in several respects. Our design is programmed using a new method for decomposing unitary matrices into a sequence of beam splitters, requires half the optical depth, and, significantly, not only suffers less propagation loss but is more loss-tolerant than the previous design.

We expect that our compact and loss-tolerant design for fully programmable universal multiport interferometers will play an important role in the development of optical processors for both classical and quantum applications. Furthermore, we anticipate that our matrix decomposition method will be of use in its own right for other systems that use mathematical structures analogous to beam splitters and phase shifters, such as ion traps [27] and some architectures for superconducting circuits [28,29].

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COMPETING FINANCIAL INTERESTS

The authors have jointly applied for a patent for the work presented in this paper.

See [Supplement 1](#) for supporting content.

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