Furkan Keskin – 2018400150
Sinem Kocoglu – 2020400339

PART 1.1

- Entities
  - Patient is the person enters the system. (S)he has id.
  - Nurse is the first service providing triage for the patients. Even if nurse service is full, patients must wait in the waiting queue. Also, nurse service is prerequisite for bed service.
  - Bed is the second service offering healing at hospital to patients having critical condition if it is not full.

- Types of events
  A : Arrival : Patient arrives enters the system
  DN : Departure from triage : Nurse service is completed for the patient. According to the triage result, patient is sent to home or bed service.
  DB : Treated at hospital : Bed service is completed for the patient. Patient leaves the system as healthy.
  H : Healed at home : Healing time is completed at home for patients having stable condition or critical condition sent to home due to bed unavailability.

- Activities
  Interarrival time : Generated an exponential rate of $\lambda = 1$
  Nurse service times **:** Generated an exponential rate of mu_t = 0,3125
  Hospital healing times **:** Generated an exponential rate mu_cb = 0,1666666667
  Home healing times for stable : Generated an exponential rate of mu_s = 0,16
  Home healing times for critical : Generated an exponential rate of mu_ch = mu_cb / U[1.25, 1.75]

- Delays
  Waiting time in triage queue : Time between the arrival of patient and entrance to the triage if nurse service is full when the patient arrives.

- System states of interest

  Time : Simulation time
  FEL : Future event list
  Next Event : Next event that will be executed
  LQ : Length of waiting queue
  WQ : Waiting time of the patient leaving the queue

Healed : Total number of healed patients
Sick : Total number of sick people = total number of patients in the system
Busy Nurses : Current number of busy nurses
Occupied Beds : Current number of occupied beds

- Random variables that are used for activities
  - interarrivals = [0.24107049 0.64740777 4.50411894 ... 0.25352128 0.78571413 1.11011029]
  - nurse service times = [2.44192375 3.86904141 2.53139613 ... 2.33061113 1.73218938 1.01233622]
  - hospital healing times = [9.39602558 10.31678025 0.62875763 ... 13.98098872 9.82739835 3.70913871]
  - home healing times for stable = [12.22011909 1.4384001 3.35540452 ... 5.06577069 9.05110823 3.27512427]
  - home healing times for critical = [0.10913964 31.37097383 8.64752338 ... 8.15701065 1.04104046 20.41506331]

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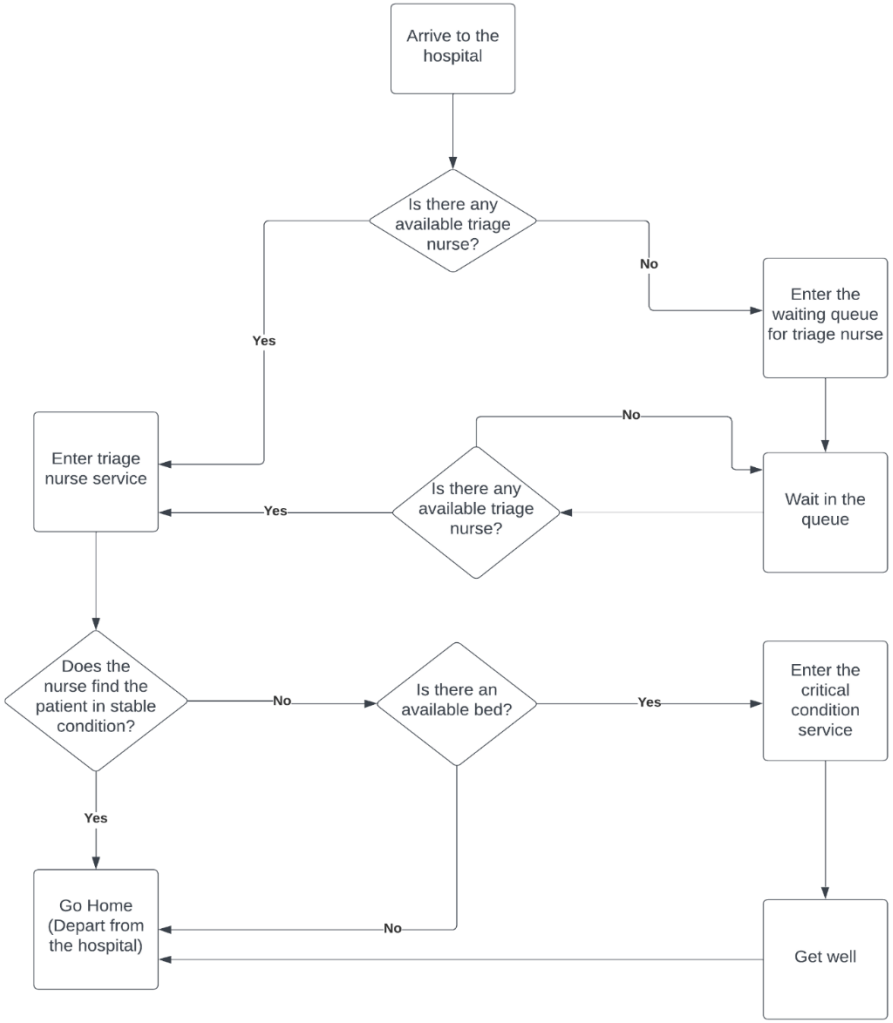
Flowchart



Furkan Keskin – 2018400150
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Pseudo Code

```
BEGIN

    // Event object has properties as time, type, owner id

    nurses = 4
    beds = 6
    p_1 = 0.25
    termination_limit  // maximum number of healed people that stops simulation

    // generating arrival times
    arrivals[1] = 0
    FOR i=2 TO arrivals.size
         arrivals[i] = arrivals[i-1] + exp(1)
    END FOR

    FOR starting_condition IN (empty, half, full)
        futureEventList // sorts events according to their times in ascending order
        waitingQueue    // objects in waiting queue has properties entrance time and id

        simulation_time = 0
        event_type = 'A'
        pid = 1
        healed_patients = 0

        IF starting_condition == empty
              busy_nurses = initially_busy_nurses = 0
              occupied_beds = initially_occupied_beds = 0
              already_in_hospital = 0

        ELSEIF starting_condition == half
              busy_nurses = initially_busy_nurses = nurses/2
              occupied_beds = initially_occupied_beds = beds/2
              already_in_hospital = nurses/2 + beds/2

        ELSEIF starting_condition == full
```

```
      busy_nurse = initially_busy_nurses = nurses
      occupied_beds = initially_occupied_beds = beds
      already_in_hospital = nurses + beds

  FOR i=0 TO busy_nurses
      futureEventList.push(Event(exp(1/0.3125), 'DN', pid))
      pid ++
  END FOR


  FOR i=0 TO occupied_beds
      futureEventList.push(Event(exp(1/0.1666666667), 'DB', pid))
      pid ++
  END FOR


  WHILE healed_patients < termination_limit
      IF event_type == 'A'
         futureEventList.push(Event(arrivals[pid-already_in_hospital+1], 'A', pid+1))

          IF busy_nurses < nurses
              futureEventList.push(Event(simulation_time + exp(1/0.3125), 'DN', pid))
              busy_nurses ++
          ELSE
              waitingQueue.put((simulation_time, pid))

      ELSEIF event_type == 'DN'
         IF Uniform(0, 1) <= p_1  //stable
              futureEventList.push(Event(simulation_time + exp(1/0.16), 'H', pid))
         ELSE    // critical
              IF occupied_beds < beds
                  futureEventList.push(Event(simulation_time + exp(1/0.1666666667),'DB', pid))
              occupied_beds ++
              ELSE
                  futureEventList.push(Event(simulation_time + exp(Uniform(1.25, 1.75)/ 0.1666666667),'H',
pid))
```

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```
                IF waitingQueue.empty()
                    busy_nurses --

                ELSE
                    waitingPatient = waitingQueue.pop()
                    futureEventList.push(Event(simulation_time + exp(1/0.3125), 'DN', waitingPatient.id))


            ELSEIF event_type == 'DB'
               occupied_beds --
               healed_patients ++
            ELSEIF event_type == 'H'
               healed_patients ++

            simulation_time, event_type, pid = futureEventList.pop()

    ENF FOR
END
```

Furkan Keskin – 2018400150
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PART 2.1

ID	interarrival times	arrival time	nurse service time	condition (s : stable, c: critical)	home healing time	hospital healing time
1		0,000000000	0,000567125	0,964519167674982 -> c		1,280412770
2	0,241070487	0,241070487	0,248228441	0,101183859095605 -> s	7,657424040	
3	0,647407769	0,888478256	0,544135251	0,314313178869379 -> c		6,291278600
4	4,504118940	5,392597196	4,386731990	0,540291489147209 -> c		8,032148110
5	1,074342570	6,466939766	8,558291140			
6	2,533750560	9,000690326	2,222955040	0,580858365407692 -> c		1,995603210
7	0,319182038	9,319872364	0,252801625	0,139571055815548 -> s	0,041612336	
8	1,101845720	10,421718084	0,540929031	0,559317570686995 -> c		4,151340170
9	0,585289474	11,007007558	0,898865242	0,417490498901413 -> c		6,517125070
10	0,228080274	11,235087832	9,008274120			
11	0,082646203	11,317734035	1,265495060	0,51145555488414 -> c		4,764165800
12	0,752756332	12,070490367	0,562989420	0,178068518205268 -> s	0,015818572	
13	0,382814961	12,453305328	1,717808650			
14	0,808808754	13,262114082	4,920111180			

- Condition is calculated according to the uniformly generated number between 0 and 1. If it is less than 0,25 (p1), condition is stable. If it is more than 0,25 , it is critical.
- Data are generated by 50 iterations.

Furkan Keskin – 2018400150
Sinem Kocoglu – 2020400339

A(t, id) : Arrival of patient with id at time t
DN(t, id) : Departure triage of patient with id at time t
DB(t, id) : Departure of patient with id from hospital at time t
H(t, id) : Healing of patient with id at home at time t

total number of busy nurses : Current number of busy nurses. It can be maximum 4 that is the total number of nurses in the system

total number of occupied beds : Current number of occupied beds. It can be maximum 6 that is the total number of beds in the system

number of healed patients : If there is healed patient at time t, it will be 1. Otherwise, empty.

simulation time	future event list	next event	total number of busy nurses (max4)	number of patients in queue	waiting time in queue	total number of occupied beds (max 6)	home healing time	hospital healing time	number of healed patients
0,000000000	A(0,241070487,2) DN(0,000567125,1)	DN(0,000567125,1)	1						
0,000567125	A(0,241070487,2) DB(1,280979895, 1)	A(0,241070487,2)	0			1		1,280412770	
0,241070487	A(0,888478256,3) DN(0,489298928,2) DB(1,280979895, 1)	DN(0,489298928,2)	1			1			
0,489298928	A(0,888478256,3) H(8,146722968,2) DB(1,280979895, 1)	A(0,888478256,3)	0			1	7,657424040		
0,888478256	DN(1,432613507,3) H(8,146722968,2)	DB(1,280979895, 1)	1			1			

	DB(1,280979895, 1) A(5,392597196, 4)								
1,280979895	DN(1,432613507,3) H(8,146722968,2) A(5,392597196, 4)	DN(1,432613507,3)	1			0			1
1,432613507	H(8,146722968,2) A(5,392597196, 4) DB(7,723892107,3)	A(5,392597196, 4)	0			1		6,291278600	
5,392597196	H(8,146722968,2) DB(7,723892107,3) A(6,466939766, 5) DN(9,779329186,4)	A(6,466939766, 5)	1			1			
6,466939766	H(8,146722968,2) DB(7,723892107,3) DN(9,779329186,4) DN(15,025230906,5) A(9,000690326,6)	DB(7,723892107,3)	2			1			
7,723892107	H(8,146722968,2) DN(9,779329186,4) DN(15,025230906,5) A(9,000690326,6)	H(8,146722968,2)	2			0			1
8,146722968	DN(9,779329186,4) DN(15,025230906,5) A(9,000690326,6)	A(9,000690326,6)	2			0			1
9,000690326	DN(9,779329186,4) DN(15,025230906,5) A(9,319872364,7) DN(11,223645366,6)	A(9,319872364,7)	3			0			
9,319872364	DN(9,779329186,4) DN(15,025230906,5) DN(11,223645366,6	DN(9,572673989,7)	4			0			

	) A(10,421718084,8) DN(9,572673989,7)								
9,572673989	DN(9,779329186,4) DN(15,025230906,5) DN(11,223645366,6) A(10,421718084,8) H(9,614286325,7)	H(9,614286325,7)	3			0	0,041612336		
9,614286325	DN(9,779329186,4) DN(15,025230906,5) DN(11,223645366,6) A(10,421718084,8)	DN(9,779329186,4)	3			0			1
9,779329186	DN(15,025230906,5) DN(11,223645366,6) A(10,421718084,8) DB(17,811477296,4)	A(10,421718084,8)	2			1		8,032148110	
10,421718084	DN(15,025230906,5) DN(11,223645366,6) DB(17,811477296,4) A(11,007007558, 9) DN(10,962647115,8)	DN(10,962647115,8)	3			1			
10,962647115	DN(15,025230906,5) DN(11,223645366,6) DB(17,811477296,4) A(11,007007558, 9) DB(15,113987285,8)	A(11,007007558, 9)	2			2		4,151340170	

11,007007558	DN(15,025230906,5) DN(11,223645366,6) DB(17,811477296,4) DB(15,113987285,8) A(11,235087832,10) DN(11,905872800,9)	DN(11,223645366,6)	3			2			
11,223645366	DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) A(11,235087832,10) DN(11,905872800,9) DB(13,219248576,6)	A(11,235087832,10)	2			3		1,995603210	
11,235087832	DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DN(11,905872800,9) DB(13,219248576,6) A(11,317734035,11) DN(20,243361952,10)	A(11,317734035,11)	3			3			
11,317734035	DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DN(11,905872800,9)	DN(11,905872800,9)	4			3			

	DB(13,219248576,6) DN(20,243361952,10) A(12,070490367,12) DN(12,583229095,11)								
11,905872800	DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) A(12,070490367,12) DN(12,583229095,11) DB(18,422997870,9)	A(12,070490367,12)	3			4		6,517125070	
12,070490367	DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DN(12,583229095,11) DB(18,422997870,9) A(12,453305328,13) DN(12,633479787,12)	A(12,453305328,13)	4			4			
12,453305328	DN(15,025230906,5) DB(17,811477296,4)	DN(12,583229095,11)	4	1(id=13)		4			

	DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DN(12,583229095,11) DB(18,422997870,9) DN(12,633479787,12) A(13,262114082,14)								
12,583229095	DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DB(18,422997870,9) DN(12,633479787,12) A(13,262114082,14) DB(17,347394895,11) DN(14,301037745,13)	DN(12,633479787,12)	4	0	0,129923767	5		4,764165800	
12,633479787	DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DB(18,422997870,9)	H(12,649298359,12)	3			5	0,015818572		

	A(13,262114082,14) DB(17,347394895,11) DN(14,301037745,13) H(12,649298359,12)								
12,649298359	DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DB(18,422997870,9) A(13,262114082,14) DB(17,347394895,11) DN(14,301037745,13)	DB(13,219248576,6)	3			5			1

- At simulation time 12,649298359, fifth patient is healed and next event will be departure of the patient with id 6 from the hospital.
- Until the first 5 patients are healed, 14 patients enter the system.
- There is just one patient entered the waiting queue that takes 0,129923767 because all the nurses are busy at that time.
- Decision of triage is determined according to the conditions generated from random numbers. Future event list is formed according to that decision. If the patient goes to home for healing, H event will be generated. If the patient occupies a bed for healing at the hospital, DB (Departure from bed) event will be generated at the future event list.
- Next event is determined by taking the event having the closest time to the simulation time in the future event list.
- When A event comes true, total number of busy nurses is increased while coming true of DN event decreases it. Also, coming true of DN event triggers the coming of H event or DB event according to the condition of patient. DB event affects the total number of occupied beds. The coming true of both DB and H events increases the total number of healed patients.

Furkan Keskin – 2018400150
Sinem Kocoglu – 2020400339

PART 2.2 - Model responses

Number of healed patients=20

- Empty system

Long run marginal probability of being empty for triage or for the beds	The joint probability of both being empty	Rejection rate by bed area	Average utilization of triage nurses	Average number of occupied beds in the hospital	Patient rate that are treated at home	Average time a sick person gets better
0,865408522188604	0,5043569730605689	0,16666666666666666	0,7597038162410228	4,2784810126582276	0,2	4,932192702415073

- Half of the nurses and half of the beds full

Long run marginal probability of being empty for triage or for the beds	The joint probability of both being empty	Rejection rate by bed area	Average utilization of triage nurses	Average number of occupied beds in the hospital	Patient rate that are treated at home	Average time a sick person gets better
0,9479639061477668	0,5300141367284166	0,20689655172413793	0,7228770990361053	4,298507462686567	0,20689655172413793	5,718208767669718

- All nurses and beds full

Long run marginal probability of being empty for triage or for the beds	The joint probability of both being empty	Rejection rate by bed area	Average utilization of triage nurses	Average number of occupied beds in the hospital	Patient rate that are treated at home	Average time a sick person gets better
0,9161301612671764	0,0809831914631936	0,23333333333333334	0,8525000325142478	4,967213114754099	0,20689655172413793	5,839655416845072

Furkan Keskin – 2018400150
Sinem Kocoglu – 2020400339

Number of healed patients =200

- Empty system

Long run marginal probability of being empty for triage or for the beds	The joint probability of both being empty	Rejection rate by bed area	Average utilization of triage nurses	Average number of occupied beds in the hospital	Patient rate that are treated at home	Average time a sick person gets better
0,9285912984615564	0,27102263195489507	0,08450704225352113	0,8232749860898935	3,9551282051282053	0,32394366197183168	5,8390005581376848

- Half of the nurses and half of the beds full

Long run marginal probability of being empty for triage or for the beds	The joint probability of both being empty	Rejection rate by bed area	Average utilization of triage nurses	Average number of occupied beds in the hospital	Patient rate that are treated at home	Average time a sick person gets better
0,9173162948730008	0,3112214240618227	0,09004739336492891	0,8182484342316042	3,9541734860883797	0,33175355450236965	5,8280918881161711

- All nurses and beds full

Long run marginal probability of being empty for triage or for the beds	The joint probability of both being empty	Rejection rate by bed area	Average utilization of triage nurses	Average number of occupied beds in the hospital	Patient rate that are treated at home	Average time a sick person gets better
0,9253864103720658	0,25896007442566027	0,09905660377358491	0,8185699086718599	3,927392739273927	0,32547169811320753	5,8204816138106811

Furkan Keskin – 2018400150
Sinem Kocoglu – 2020400339

Number of healed patients =1000

- Empty system

Long run marginal probability of being empty for triage or for the beds	The joint probability of both being empty	Rejection rate by bed area	Average utilization of triage nurses	Average number of occupied beds in the hospital	Patient rate that are treated at home	Average time a sick person gets better
0,8992400792143742	0,3119867037453868	0,1256133464180569	0,8147485409183625	3,956076618229855	0,36113837095191365	6,391893947468907

- Half of the nurses and half of the beds full

Long run marginal probability of being empty for triage or for the beds	The joint probability of both being empty	Rejection rate by bed area	Average utilization of triage nurses	Average number of occupied beds in the hospital	Patient rate that are treated at home	Average time a sick person gets better
0,8954900539901616	0,3118243824224693	0,11655239960822723	0,814711776985856	3,988418266048974	0,34965719882468166	6,302485539618888

- All nurses and beds full

Long run marginal probability of being empty for triage or for the beds	The joint probability of both being empty	Rejection rate by bed area	Average utilization of triage nurses	Average number of occupied beds in the hospital	Patient rate that are treated at home	Average time a sick person gets better
0,8871510925039995	0,2941879082680601	0,11655239960822723	0,8155448034700776	4,00132625994695	0,34769833496571949	6,292729468819236

Furkan Keskin – 2018400150
Sinem Kocoglu – 2020400339

Comments

First, we can observe that the initial state of the system becomes less and less important as the number of healed patients increases or as the number of events increases. From the statistics we can see that all 3 initial conditions give almost the same results in the last table.

The average healing times in all cases is about 6, which is exactly what we would expect. Because our 1/mu_cb = 1 / 0.1666666667 and 1/mu_s = 1/ 0.16 values correspond to approximately 6. This ratio increases only slightly as the number of iterations increases, suggesting that the relatively late recovery of those recovering at home is reflected in the system.

For utilization of each nurse, we see approximately 0.8 for all cases which is exactly equal to lambda / (nurse_number*mu_T) = 1 / (4* 0.3125) coming from the server utilization formula.

If we look at the columns “patient rate that are treated at home” and “rejection rate by bed area”, we see that the difference of these two columns is approximately equal to p_1 = 0.25, which is exactly the expected result. Besides, if we evaluate the "rejection by bed area" statistic alone, we can see that the value of 0.11 obtained in the long run is in line with the theory. Because 1 / (alpha/mu_cb)) = 1/ (1.5 / 0.1666666667) = 0.11111111111

The probability that the nurses are empty is 1 – lambda/ (nurses* mu_T) = 1 – 1 / (4*0.3125) = 0.2, similarly probability that all the beds are empty = 1 – mu_T/(beds * mu_cb) = 1- 0.3125/(6 * 0.1666666667) = 0.69. “Long run marginal probability of being empty for triage or for the beds” is about 0.88 in the long run which is almost equal to 1 – 0.2*0.69. Thus, this statistic also makes sense to us.

From the tables we see that the average number of occupied beds is about 4. This makes the utilization of the bed are 4/6 = 0.66. We already have calculated this above which is 1 – mu_T/ (beds * mu_cb) = 1- 0.3125/ (6 * 0.1666666667) = 0.69. It can be seen that the experimental result is quite close to the theoretical result.

All in all, it seems that our experimental statistics are very much in line with the theoretical results. We are quite satisfied with our work.