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First Phase

Simulation Implementation, Statistics and Plots

As stated in the description, we calculated and plotted ensemble averages of sojourn times and cumulative averages of number of jobs in order to determine the warmup period. We simulated the system until 500 jobs departed and again, as mentioned, we used 10 and 30 replications for each utilization value.

We first started by finding the arrival rate. Since our interarrival times come from a uniform distribution, we used the formula arrival_rate = 2/(a+b). For each different utilization value, we found the average number of busy servers in the system using the formula Ls = c * p and from there we calculated the service rate using the equation μ = λ / Ls.

After calculating the arrival rate and service rate, we coded a generic DES using simpy. In *job_generator*, *arrival*, *service* functions, besides the classical DES structure (arrival, service, departure), we also kept the required statistics in various data structures. These statistics are cumulative average of jobs and sojourn ensemble times. In each arrival event we updated the cumulative average of jobs and in each departure event we updated the sojourn ensemble times. We used this straightforward DES code both in the first phase and in the second phase where necessary.

For each different utilization value and different replication values, we ran the necessary simulations with different random seeds and calculated the necessary statistics, the ensemble averages of waiting times and the cumulative averages of the number of jobs and the confidence intervals of these statistics, and then plotted the desired graphs using these values. The x-axis of the graphs shows the number of jobs and the y-axis shows the statistic value written in the title of the graph.

The content of the graphs is quite self-explanatory. Green dots indicate the upper limit of the confidence interval, orange dots indicate the lower limit of the confidence interval and blue dots indicate the mean value. Also it should be noted that when plotting the ensemble averages, I used the moving averages of these values instead of the original ensemble averages for a smoother and easier interpretation.

After plotting the desired statistics, we determined the warmup periods for different utilization, replication and response by eyeballing and displayed them in the graphs with a red vertical line. However, we needed to convert these warmup periods from the number of departed jobs to time. For this we made an approximation and used the formula warmup_period ~= (1/departure_rate) * customer_number. This formula simply multiplies the average time each job spends in the system by the total number of departed jobs to find the approximate elapsed time. Finally, we chose the larger of the warm-up times we calculated separately for the two responses. This is because the larger one covers the bias convergence time of both responses.

Comments

In general, regardless of the response, confidence intervals become narrower and more stable as the number of repetitions increases. Also, the system reaches the warm-up period faster.

As utilization values increase, warm-up times get significantly longer and confidence intervals widen significantly.

Comparing the two responses, the ensemble averages of sojourn times converge to stationary values much faster than the cumulative averages of the number of jobs, regardless of utilization values and repetitions.

Second Phase

Average values and the confidence intervals of sojourn ensembles and average number of jobs in the system for 20 departing jobs with the following starting conditions are:

Empty System

Sojourn ensemble average: 22.918694580411206

Confidence interval of sojourn ensemble average: (15.10199099, 30.73539817)

Average number of jobs in the system: 2.43

Confidence interval of average number of jobs in the system: (1.79301539, 3.06698461)

4 Jobs in the System

Sojourn ensemble average: 22.919617867486703

Confidence interval of sojourn ensemble average: (10.54146291, 35.29777283)

Average number of jobs in the system: 3.335

Confidence interval of average number of jobs in the system: (2.46963659, 4.20036341)

Empty System with Warmup

Sojourn ensemble average: 21.761688055134357

Confidence interval of sojourn ensemble average: (13.30316632, 30.22020979)

Average number of jobs in the system: 3.4965609285050503

Confidence interval of average number of jobs in the system: (2.85114098, 4.14198088)

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Comments

Theoretically, the arrival rate of a job is 2/(a+b) = 2/(6.11 + 6.9) = 0.154. From Little's Law we know that arrival rate = L / W. In the system starting empty this ratio is 2.43/22.92 = 0.106, in the system starting with 4 jobs this ratio is 3.34/22.92 = 0.145 and in the system where we collect data after the warmup period has passed this ratio is 3.50 / 21.76 = 0.160. In fact, this result is quite expected and in line with the theory. Because the first system starts far from steady-state, but the 2nd and 3rd systems are closer to steady-state. Although this is artificially achieved in the second system, this is the result reflected in the statistics.

When we look at the average sojourn times, we find almost the same results in all three systems. This is a bit surprising. Because we would expect the third system to have a relatively higher sojourn ensemble average, which can be explained by the occurrence of low probability events or the fast convergence of the ensemble average. When we analyze the confidence intervals, the situation is as expected. As it can be deduced from the plots we drew in the first phase, the first system has a very narrow confidence interval since it is still in the initial phase and therefore far from steady-state. The other two systems have naturally wider confidence intervals.

When we look at the average number of jobs, we see quite expected results. While the last two systems are around 3.3 - 3.5, the first system is expectedly much lower. Also, we do not encounter a surprising result either when we look at the confidence intervals. The first one is much narrower and centered between a smaller mean.

Finally, we would like to mention that it is not surprising that the second and third systems give very similar results. Because the third case seems to be in a natural approximate steady state, and the second system seems to be in an artificially approximate steady state. In a way, they both reflect reality, because in real life, when we collect data from any system, it's very likely that it's not an empty system or a system that has just started. It's usually already somewhat full and functioning.