**PART 1.1**

* Entities
* Patient is the person enters the system. (S)he has id.
* Nurse is the first service providing triage for the patients. Even if nurse service is full, patients must wait in the waiting queue. Also, nurse service is prerequisite for bed service.
* Bed is the second service offering healing at hospital to patients having critical condition if it is not full.
* Types of events

A : Arrival : Patient arrives enters the system

DN : Departure from triage : Nurse service is completed for the patient. According to the triage result, patient is sent to home or bed service.

DB : Treated at hospital : Bed service is completed for the patient. Patient leaves the system as healthy.

H : Healed at home : Healing time is completed at home for patients having stable condition or critical condition sent to home due to bed unavailability.

* Activities

Interarrival time : Generated an exponential rate of λ = 1

Nurse service times **:** Generated an exponential rate of mu\_t = 0,3125

Hospital healing times **:** Generated an exponential rate mu\_cb = 0,1666666667

Home healing times for stable : Generated an exponential rate of mu\_s = 0,16

Home healing times for critical : Generated an exponential rate of mu\_ch = mu\_cb / U[1.25, 1.75]

* Delays

Waiting time in triage queue : Time between the arrival of patient and entrance to the triage if nurse service is full when the patient arrives.

* System states of interest

Time : Simulation time

FEL : Future event list

Next Event : Next event that will be executed

LQ : Length of waiting queue

WQ : Waiting time of the patient leaving the queue

Healed : Total number of healed patients

Sick : Total number of sick people = total number of patients in the system

Busy Nurses : Current number of busy nurses

Occupied Beds : Current number of occupied beds

* Random variables that are used for activities
* interarrivals = [0.24107049 0.64740777 4.50411894 ... 0.25352128 0.78571413 1.11011029]
* nurse service times = [2.44192375 3.86904141 2.53139613 ... 2.33061113 1.73218938 1.01233622]
* hospital healing times = [ 9.39602558 10.31678025 0.62875763 ... 13.98098872 9.82739835 3.70913871]
* home healing times for stable = [12.22011909 1.4384001 3.35540452 ... 5.06577069 9.05110823 3.27512427]
* home healing times for critical = [ 0.10913964 31.37097383 8.64752338 ... 8.15701065 1.04104046 20.41506331]

**Flowchart**

diyagram içeren bir resim

Açıklama otomatik olarak oluşturuldu

**Pseudo Code**

**BEGIN**

**//** Event object has properties as time, type, owner id

nurses **=** 4

beds **=** 6

p\_1 **=** 0.25

termination\_limit // maximum number of healed people that stops simulation

**//** generating arrival times

arrivals[1] = 0

**FOR** i=2 **TO** arrivals.size

arrivals[i] = arrivals[i-1] + exp(1)

**END FOR**

**FOR** starting\_condition **IN** (empty, half, full)

futureEventList // sorts events according to their times in ascending order

waitingQueue // objects in waiting queue has properties entrance time and id

simulation\_time **=** 0

event\_type **=** 'A'

pid **=** 1

healed\_patients **=** 0

**IF** starting\_condition == empty

busy\_nurses = initially\_busy\_nurses **=** 0

occupied\_beds = initially\_occupied\_beds = 0

already\_in\_hospital **=** 0

**ELSEIF** starting\_condition == half

busy\_nurses = initially\_busy\_nurses **=** nurses/2

occupied\_beds = initially\_occupied\_beds = beds/2

already\_in\_hospital **=** nurses/2 + beds/2

**ELSEIF** starting\_condition == full

busy\_nurse = initially\_busy\_nurses **=** nurses

occupied\_beds = initially\_occupied\_beds = beds

already\_in\_hospital **=** nurses + beds

**FOR** i=0 **TO** busy\_nurses

futureEventList.push(Event(exp(1**/**0.3125), 'DN', pid))

pid **++**

**END FOR**

**FOR** i=0 **TO** occupied\_beds

futureEventList.push(Event(exp(1**/**0.1666666667), 'DB', pid))

pid **++**

**END FOR**

**WHILE** healed\_patients **<** termination\_limit

**IF** event\_type **==** 'A'

futureEventList**.**push(Event(arrivals[pid**-**already\_in\_hospital**+**1], 'A', pid**+**1))

**IF** busy\_nurses **<** nurses

futureEventList**.**push(Event(simulation\_time **+** exp(1**/**0.3125), 'DN', pid))

busy\_nurses **++**

**ELSE**

waitingQueue**.**put((simulation\_time, pid))

**ELSEIF** event\_type **==** 'DN'

**IF** Uniform(0, 1) <= p\_1 //stable

futureEventList**.**push(Event(simulation\_time **+** exp(1**/**0.16), 'H', pid))

**ELSE** // critical

**IF** occupied\_beds **<** beds

futureEventList**.**push(Event(simulation\_time **+** exp(1**/**0.1666666667),'DB', pid))

occupied\_beds **++**

**ELSE**

futureEventList**.**push(Event(simulation\_time **+** exp(Uniform(1.25, 1.75)**/** 0.1666666667),'H', pid))

**IF** waitingQueue**.**empty()

busy\_nurses **--**

**ELSE**

waitingPatient = waitingQueue**.**pop()

futureEventList**.**push(Event(simulation\_time **+** exp(1**/**0.3125), 'DN', waitingPatient.id))

**ELSEIF** event\_type **==** 'DB'

occupied\_beds **--**

healed\_patients **++**

**ELSEIF** event\_type **==** 'H'

healed\_patients **++**

simulation\_time, event\_type, pid **=** futureEventList**.**pop()

**ENF FOR**

**END**

**PART 2.1**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| ID | interarrival times | arrival time | nurse service time | condition  (s : stable, c: critical) | home healing time | hospital healing time |
| 1 |  | 0,000000000 | 0,000567125 | 0,964519167674982 -> c |  | 1,280412770 |
| 2 | 0,241070487 | 0,241070487 | 0,248228441 | 0,101183859095605 -> s | 7,657424040 |  |
| 3 | 0,647407769 | 0,888478256 | 0,544135251 | 0,314313178869379 -> c |  | 6,291278600 |
| 4 | 4,504118940 | 5,392597196 | 4,386731990 | 0,540291489147209 -> c |  | 8,032148110 |
| 5 | 1,074342570 | 6,466939766 | 8,558291140 |  |  |  |
| 6 | 2,533750560 | 9,000690326 | 2,222955040 | 0,580858365407692 -> c |  | 1,995603210 |
| 7 | 0,319182038 | 9,319872364 | 0,252801625 | 0,139571055815548 -> s | 0,041612336 |  |
| 8 | 1,101845720 | 10,421718084 | 0,540929031 | 0,559317570686995 -> c |  | 4,151340170 |
| 9 | 0,585289474 | 11,007007558 | 0,898865242 | 0,417490498901413 -> c |  | 6,517125070 |
| 10 | 0,228080274 | 11,235087832 | 9,008274120 |  |  |  |
| 11 | 0,082646203 | 11,317734035 | 1,265495060 | 0,51145555488414 -> c |  | 4,764165800 |
| 12 | 0,752756332 | 12,070490367 | 0,562989420 | 0,178068518205268 -> s | 0,015818572 |  |
| 13 | 0,382814961 | 12,453305328 | 1,717808650 |  |  |  |
| 14 | 0,808808754 | 13,262114082 | 4,920111180 |  |  |  |

* Condition is calculated according to the uniformly generated number between 0 and 1. If it is less than 0,25 (p1), condition is stable. If it is more than 0,25 , it is critical.
* Data are generated by 50 iterations.

|  |  |
| --- | --- |
| A(t, id) | : Arrival of patient with id at time t |
| DN(t, id) | : Departure triage of patient with id at time t |
| DB(t, id) | : Departure of patient with id from hospital at time t |
| H(t, id) | : Healing of patient with id at home at time t |

total number of busy nurses : Current number of busy nurses. It can be maximum 4 that is the total number of nurses in the system

total number of occupied beds : Current number of occupied beds. It can be maximum 6 that is the total number of beds in the system

number of healed patients : If there is healed patient at time t, it will be 1. Otherwise, empty.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| simulation time | future event list | next event | total number of busy nurses  (max4 ) | number of patients in queue | waiting time in queue | total number of occupied beds  (max 6) | home healing time | hospital healing time | number of healed patients |
| 0,000000000 | A(0,241070487,2) DN(0,000567125,1) | DN(0,000567125,1) | 1 |  |  |  |  |  |  |
| 0,000567125 | A(0,241070487,2) DB(1,280979895, 1) | A(0,241070487,2) | 0 |  |  | 1 |  | 1,280412770 |  |
| 0,241070487 | A(0,888478256,3) DN(0,489298928,2) DB(1,280979895, 1) | DN(0,489298928,2) | 1 |  |  | 1 |  |  |  |
| 0,489298928 | A(0,888478256,3) H(8,146722968,2) DB(1,280979895, 1) | A(0,888478256,3) | 0 |  |  | 1 | 7,657424040 |  |  |
| 0,888478256 | DN(1,432613507,3) H(8,146722968,2) DB(1,280979895, 1) A(5,392597196, 4) | DB(1,280979895, 1) | 1 |  |  | 1 |  |  |  |
| 1,280979895 | DN(1,432613507,3) H(8,146722968,2) A(5,392597196, 4) | DN(1,432613507,3) | 1 |  |  | 0 |  |  | 1 |
| 1,432613507 | H(8,146722968,2) A(5,392597196, 4) DB(7,723892107,3) | A(5,392597196, 4) | 0 |  |  | 1 |  | 6,291278600 |  |
| 5,392597196 | H(8,146722968,2) DB(7,723892107,3) A(6,466939766, 5) DN(9,779329186,4) | A(6,466939766, 5) | 1 |  |  | 1 |  |  |  |
| 6,466939766 | H(8,146722968,2) DB(7,723892107,3) DN(9,779329186,4) DN(15,025230906,5) A(9,000690326,6) | DB(7,723892107,3) | 2 |  |  | 1 |  |  |  |
| 7,723892107 | H(8,146722968,2) DN(9,779329186,4) DN(15,025230906,5) A(9,000690326,6) | H(8,146722968,2) | 2 |  |  | 0 |  |  | 1 |
| 8,146722968 | DN(9,779329186,4) DN(15,025230906,5) A(9,000690326,6) | A(9,000690326,6) | 2 |  |  | 0 |  |  | 1 |
| 9,000690326 | DN(9,779329186,4) DN(15,025230906,5) A(9,319872364,7) DN(11,223645366,6) | A(9,319872364,7) | 3 |  |  | 0 |  |  |  |
| 9,319872364 | DN(9,779329186,4) DN(15,025230906,5) DN(11,223645366,6) A(10,421718084,8) DN(9,572673989,7) | DN(9,572673989,7) | 4 |  |  | 0 |  |  |  |
| 9,572673989 | DN(9,779329186,4) DN(15,025230906,5) DN(11,223645366,6) A(10,421718084,8) H(9,614286325,7) | H(9,614286325,7) | 3 |  |  | 0 | 0,041612336 |  |  |
| 9,614286325 | DN(9,779329186,4) DN(15,025230906,5) DN(11,223645366,6) A(10,421718084,8) | DN(9,779329186,4) | 3 |  |  | 0 |  |  | 1 |
| 9,779329186 | DN(15,025230906,5) DN(11,223645366,6) A(10,421718084,8) DB(17,811477296,4) | A(10,421718084,8) | 2 |  |  | 1 |  | 8,032148110 |  |
| 10,421718084 | DN(15,025230906,5) DN(11,223645366,6) DB(17,811477296,4) A(11,007007558, 9) DN(10,962647115,8) | DN(10,962647115,8) | 3 |  |  | 1 |  |  |  |
| 10,962647115 | DN(15,025230906,5) DN(11,223645366,6) DB(17,811477296,4) A(11,007007558, 9) DB(15,113987285,8) | A(11,007007558, 9) | 2 |  |  | 2 |  | 4,151340170 |  |
| 11,007007558 | DN(15,025230906,5) DN(11,223645366,6) DB(17,811477296,4) DB(15,113987285,8) A(11,235087832,10) DN(11,905872800,9) | DN(11,223645366,6) | 3 |  |  | 2 |  |  |  |
| 11,223645366 | DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) A(11,235087832,10) DN(11,905872800,9) DB(13,219248576,6) | A(11,235087832,10) | 2 |  |  | 3 |  | 1,995603210 |  |
| 11,235087832 | DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DN(11,905872800,9) DB(13,219248576,6) A(11,317734035,11) DN(20,243361952,10) | A(11,317734035,11) | 3 |  |  | 3 |  |  |  |
| 11,317734035 | DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DN(11,905872800,9) DB(13,219248576,6) DN(20,243361952,10) A(12,070490367,12) DN(12,583229095,11) | DN(11,905872800,9) | 4 |  |  | 3 |  |  |  |
| 11,905872800 | DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) A(12,070490367,12) DN(12,583229095,11) DB(18,422997870,9) | A(12,070490367,12) | 3 |  |  | 4 |  | 6,517125070 |  |
| 12,070490367 | DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DN(12,583229095,11) DB(18,422997870,9) A(12,453305328,13) DN(12,633479787,12) | A(12,453305328,13) | 4 |  |  | 4 |  |  |  |
| 12,453305328 | DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DN(12,583229095,11) DB(18,422997870,9) DN(12,633479787,12) A(13,262114082,14) | DN(12,583229095,11) | 4 | 1(id=13) |  | 4 |  |  |  |
| 12,583229095 | DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DB(18,422997870,9) DN(12,633479787,12) A(13,262114082,14) DB(17,347394895,11) DN(14,301037745,13) | DN(12,633479787,12) | 4 | 0 | 0,129923767 | 5 |  | 4,764165800 |  |
| 12,633479787 | DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DB(18,422997870,9) A(13,262114082,14) DB(17,347394895,11) DN(14,301037745,13) H(12,649298359,12) | H(12,649298359,12) | 3 |  |  | 5 | 0,015818572 |  |  |
| 12,649298359 | DN(15,025230906,5) DB(17,811477296,4) DB(15,113987285,8) DB(13,219248576,6) DN(20,243361952,10) DB(18,422997870,9) A(13,262114082,14) DB(17,347394895,11) DN(14,301037745,13) | DB(13,219248576,6) | 3 |  |  | 5 |  |  | 1 |

* At simulation time 12,649298359, fifth patient is healed and next event will be departure of the patient with id 6 from the hospital.
* Until the first 5 patients are healed, 14 patients enter the system.
* There is just one patient entered the waiting queue that takes 0,129923767 because all the nurses are busy at that time.
* Decision of triage is determined according to the conditions generated from random numbers. Future event list is formed according to that decision. If the patient goes to home for healing, H event will be generated. If the patient occupies a bed for healing at the hospital, DB (Departure from bed) event will be generated at the future event list.
* Next event is determined by taking the event having the closest time to the simulation time in the future event list.
* When A event comes true, total number of busy nurses is increased while coming true of DN event decreases it. Also, coming true of DN event triggers the coming of H event or DB event according to the condition of patient. DB event affects the total number of occupied beds. The coming true of both DB and H events increases the total number of healed patients.

**PART 2.2 - Model responses**

**Number of healed patients=20**

* Empty system

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Long run marginal probability of being empty for triage or for the beds | The joint probability of both being empty | Rejection rate by bed area | Average utilization of triage nurses | Average number of occupied beds in the hospital | Patient rate that are treated at home | Average time a sick person gets better |
| 0,865408522188604 | 0,5043569730605689 | 0,16666666666666666 | 0,7597038162410228 | 4,2784810126582276 | 0,2 | 4,932192702415073 |

* Half of the nurses and half of the beds full

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Long run marginal probability of being empty for triage or for the beds | The joint probability of both being empty | Rejection rate by bed area | Average utilization of triage nurses | Average number of occupied beds in the hospital | Patient rate that are treated at home | Average time a sick person gets better |
| 0,9479639061477668 | 0,5300141367284166 | 0,20689655172413793 | 0,7228770990361053 | 4,298507462686567 | 0,20689655172413793 | 5,718208767669718 |

* All nurses and beds full

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Long run marginal probability of being empty for triage or for the beds | The joint probability of both being empty | Rejection rate by bed area | Average utilization of triage nurses | Average number of occupied beds in the hospital | Patient rate that are treated at home | Average time a sick person gets better |
| 0,9161301612671764 | 0,0809831914631936 | 0,23333333333333334 | 0,8525000325142478 | 4,967213114754099 | 0,20689655172413793 | 5,839655416845072 |

**Number of healed patients =200**

* Empty system

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Long run marginal probability of being empty for triage or for the beds | The joint probability of both being empty | Rejection rate by bed area | Average utilization of triage nurses | Average number of occupied beds in the hospital | Patient rate that are treated at home | Average time a sick person gets better |
| 0,9285912984615564 | 0,27102263195489507 | 0,08450704225352113 | 0,8232749860898935 | 3,9551282051282053 | 0,323943661971831 | 5,839000558137688 |

* Half of the nurses and half of the beds full

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Long run marginal probability of being empty for triage or for the beds | The joint probability of both being empty | Rejection rate by bed area | Average utilization of triage nurses | Average number of occupied beds in the hospital | Patient rate that are treated at home | Average time a sick person gets better |
| 0,9173162948730008 | 0,3112214240618227 | 0,09004739336492891 | 0,8182484342316042 | 3,9541734860883797 | 0,33175355450236965 | 5,828091888116171 |

* All nurses and beds full

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Long run marginal probability of being empty for triage or for the beds | The joint probability of both being empty | Rejection rate by bed area | Average utilization of triage nurses | Average number of occupied beds in the hospital | Patient rate that are treated at home | Average time a sick person gets better |
| 0,9253864103720658 | 0,25896007442566027 | 0,09905660377358491 | 0,8185699086718599 | 3,927392739273927 | 0,32547169811320753 | 5,820481613810681 |

**Number of healed patients =1000**

* Empty system

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Long run marginal probability of being empty for triage or for the beds | The joint probability of both being empty | Rejection rate by bed area | Average utilization of triage nurses | Average number of occupied beds in the hospital | Patient rate that are treated at home | Average time a sick person gets better |
| 0,8992400792143742 | 0,3119867037453868 | 0,1256133464180569 | 0,8147485409183625 | 3,956076618229855 | 0,36113837095191365 | 6,391893947468907 |

* Half of the nurses and half of the beds full

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Long run marginal probability of being empty for triage or for the beds | The joint probability of both being empty | Rejection rate by bed area | Average utilization of triage nurses | Average number of occupied beds in the hospital | Patient rate that are treated at home | Average time a sick person gets better |
| 0,8954900539901616 | 0,3118243824224693 | 0,11655239960822723 | 0,814711776985856 | 3,988418266048974 | 0,34965719882468166 | 6,302485539618888 |

* All nurses and beds full

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Long run marginal probability of being empty for triage or for the beds | The joint probability of both being empty | Rejection rate by bed area | Average utilization of triage nurses | Average number of occupied beds in the hospital | Patient rate that are treated at home | Average time a sick person gets better |
| 0,8871510925039995 | 0,2941879082680601 | 0,11655239960822723 | 0,8155448034700776 | 4,00132625994695 | 0,3476983349657199 | 6,29272946881923 |

**Comments**

First, we can observe that the initial state of the system becomes less and less important as the number of healed patients increases or as the number of events increases. From the statistics we can see that all 3 initial conditions give almost the same results in the last table.

The average healing times in all cases is about 6, which is exactly what we would expect. Because our 1/mu\_cb = 1 / 0.1666666667 and 1/mu\_s = 1/ 0.16 values correspond to approximately 6. This ratio increases only slightly as the number of iterations increases, suggesting that the relatively late recovery of those recovering at home is reflected in the system.

For utilization of each nurse, we see approximately 0.8 for all cases which is exactly equal to lambda / (nurse\_number\*mu\_T) = 1 / (4\* 0.3125) coming from the server utilization formula.

If we look at the columns “patient rate that are treated at home” and “rejection rate by bed area”, we see that the difference of these two columns is approximately equal to p\_1 = 0.25, which is exactly the expected result. Besides, if we evaluate the "rejection by bed area" statistic alone, we can see that the value of 0.11 obtained in the long run is in line with the theory. Because 1 / (alpha/mu\_cb)) = 1/ (1.5 / 0.1666666667) = 0.11111111111

The probability that the nurses are empty is 1 – lambda/ (nurses\* mu\_T) = 1 – 1 / (4\*0.3125) = 0.2, similarly probability that all the beds are empty = 1 – mu\_T/(beds \* mu\_cb) = 1- 0.3125/(6 \* 0.1666666667) = 0.69. “Long run marginal probability of being empty for triage or for the beds” is about 0.88 in the long run which is almost equal to 1 – 0.2\*0.69. Thus, this statistic also makes sense to us.

From the tables we see that the average number of occupied beds is about 4. This makes the utilization of the bed are 4/6 = 0.66. We already have calculated this above which is 1 – mu\_T/ (beds \* mu\_cb) = 1- 0.3125/ (6 \* 0.1666666667) = 0.69. It can be seen that the experimental result is quite close to the theoretical result.

All in all, it seems that our experimental statistics are very much in line with the theoretical results. We are quite satisfied with our work.