Hw3

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Πρόβλημα 1

A)

$$n_{11} = \|W_1^T - q_1\| \cdot b_1 = \sqrt{(-1)^2} \cdot 0,5 = 0,5$$

$$n_{12} = \| w_1^T - q_1\| b_1 = \sqrt{0^2} \cdot 0,5 = 0$$

$$a_1 = \begin{bmatrix} e^{-n_{11}^2} \\ e^{-n_{12}^2} \end{bmatrix} = \begin{bmatrix} 0,36 \\ 1 \end{bmatrix}$$

$$n_{21} = \sqrt{(-1)^2} \cdot 0,5 = 0,5 \cdot n_{22} = 0,5$$

$$a_2 = \begin{bmatrix} e^{-n_{21}^2} \\ e^{-n_{31}^2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0,36 \end{bmatrix}$$

$$U = \begin{bmatrix} z_1^T \\ z_2^T \\ z_3^T \end{bmatrix} = \begin{bmatrix} 0,36 & 1 & 1 \\ 0,77 & 0,77 & 1 \\ 1 & 0,36 & 1 \end{bmatrix}$$

$$U^T U = \begin{bmatrix} 0,36 & 0,77 & 1 \\ 1 & 0,77 & 0,36 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0,36 & 1 & 1 \\ 0,77 & 0,77 & 1 \\ 1 & 0,36 & 1 \end{bmatrix} = \begin{bmatrix} 1,72 & 1,31 & 2,13 \\ 1,31 & 1,72 & 2,13 \\ 2,13 & 2,13 & 3 \end{bmatrix}$$

$$x^T = \left[U^T U + q_1\right]^{-1} U^T t = \begin{bmatrix} 1,72 & 1,31 & 2,13 \\ 1,31 & 1,72 & 2,13 \\ 2,13 & 2,13 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 0,36 & 0,77 & 1 \\ 1 & 0,77 & 0,36 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0,26 \\ -0,26 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad W_2 = \begin{bmatrix} 0,26 \\ -0,25 \end{bmatrix}, \quad b_2 = 0$$

B) $$F(x) = t^T t - 2t^T U x + x\left[U^T U + q I\right] x = [-1\ 0\ 1]\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - 2[-1\ 0\ 1]\begin{bmatrix} 0,36 & 1 & 1 \\ 0,77 & 0,77 & 1 \\ 1 & 0,36 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} +$$

$$+ [x\ \ y\ \ 0]\begin{bmatrix} 1,72 & 1,31 & 2,13 \\ 1,31 & 1,72 & 2,13 \\ 2,13 & 2,13 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = 1,72x^2 + 1,72y^2 + 2,62xy - 1,28x + 1,28y + 2.$$

D) Για $p = 4$

$$U^T U + pI = \begin{bmatrix} 5,72 & 1,31 & 2,13 \\ 1,31 & 5,72 & 2,13 \\ 2,13 & 2,13 & 7 \end{bmatrix},$$

$$x^* = \left[U^T U + pI\right]^{-1} U^T t = \begin{bmatrix} 0,2 & -0,02 & -0,05 \\ -0,02 & 0,2 & -0,05 \\ -0,05 & -0,05 & 0,17 \end{bmatrix} \begin{bmatrix} 0,36 & 0,77 & 1 \\ 1 & 0,77 & 0,36 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0,007 & 0,008 & 0,13 \\ 0,13 & 0,08 & 0,007 \\ 0,1 & 0 & 0,1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0,14 \\ -0,14 \\ 0 \end{bmatrix}$$

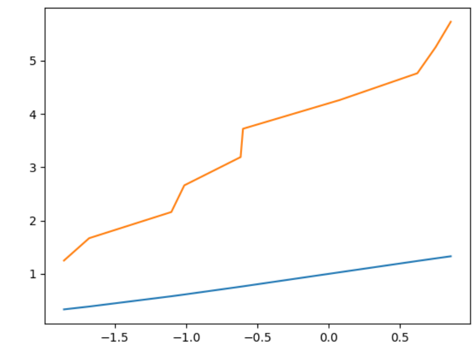
Άρα, $F(x) = 2 - 1,28x + 1,28y + \begin{bmatrix} 5,72x + 1,31y & 1,31x + 5,72y & 2,13x + 2,13y \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

$$= 5,72x^2 + 5,72y^2 + 2,62xy - 1,28x + 1,28y + 2$$

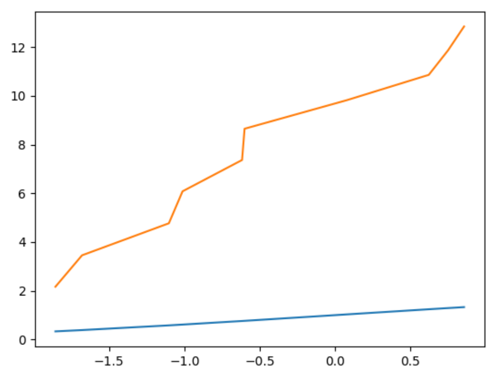
Προβλημα 2)

With a learning rate of 0.01:

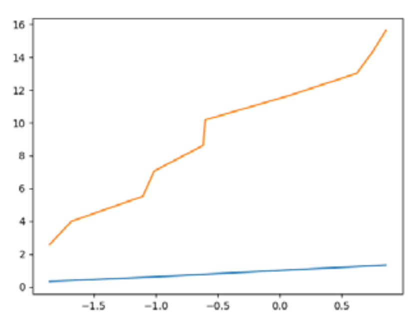
S=2



S=4



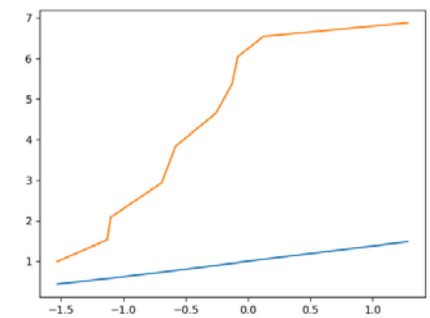
S=8



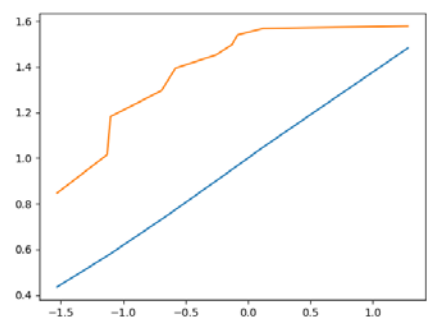
Προβλημα 3)

With a learning rate of 0.01:

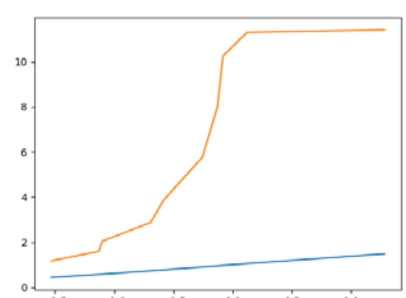
S=2



S=4



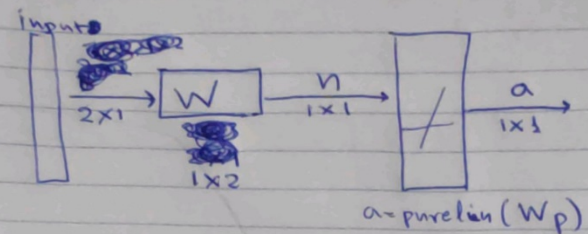
S=8



Problem-04

$\left\{ p_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}, t_1 = [75] \right\}$ $\left\{ p_2 = \begin{bmatrix} 6 \\ 3 \end{bmatrix}, t_2 = [75] \right\}$ $\left\{ p_3 = \begin{bmatrix} -6 \\ 3 \end{bmatrix}, t_3 = [-75] \right\}$

A.)



B.) Set weights and bias to zero, $W = [0\ 0]^T$ $b = 0$

$w(new) = w(old) + t_1 p_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + (75)\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 225 \\ 450 \end{bmatrix}$

$w(new) = w(old) + t_2 p_2 = \begin{bmatrix} 225 \\ 450 \end{bmatrix} + (75)\begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 675 \\ 675 \end{bmatrix}$

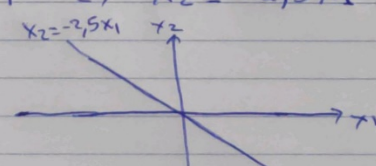
$w(new) = w(old) + t_3 p_3 = \begin{bmatrix} 675 \\ 675 \end{bmatrix} (-75)\begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.125 \\ 450 \end{bmatrix}$

So the final weight matrix is $[1.125\ \ 450]^T$

C) $1.125 x_1 + 450 x_2 = y$

Replacing y with 0 $\Rightarrow 1.125 x_1 + 450 x_2 = 0 \Rightarrow$

$x_2 = -\frac{1125}{450} x_1 \Rightarrow x_2 = -2{,}5 x_1$



Yes, it does

Problem-05:

$$P_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \qquad t_1 = 1 \qquad t_2 = -1$$

$$W = t_1 P_1^T + t_2 P_2^T = 1\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} + (-1)\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \Rightarrow$$

$$W = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Problem-06

$P_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $P_3 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $\quad a = 0.5$

$W_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $W_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

We have 2 classes, 1 subclass for each class

$$W^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

P_1:

$$a^1 = compet(n^1) = compet\begin{vmatrix} -\|W_1 - P_1\| \\ -\|W_2 - P_1\| \end{vmatrix} = compet\begin{vmatrix} -\|[1\ 0]^T - [2\ 0]^T\| \\ -\|[-1\ 0]^T - [2\ 0]^T\| \end{vmatrix} =$$

$$= compet(\begin{vmatrix} -1 \\ -3 \end{vmatrix}) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$a^2 = W^2 a^1 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$W_1(1) = W_1(0) + a \cdot (P_1 - W_1(0)) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.5\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1,5 \\ 0 \end{bmatrix}$$

P_2:

$$a^1 = compet(n^1) = compet\begin{vmatrix} -\|W_1 - P_2\| \\ -\|W_2 - P_2\| \end{vmatrix} = compet\begin{vmatrix} -\|[1.5\ 0]^T - [0\ 1]^T\| \\ -\|[-1\ 0]^T - [0\ 1]^T\| \end{vmatrix} =$$

$$= compet(\begin{vmatrix} -1,80277564 \\ -1,41421356 \end{vmatrix}) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$a^2 = W^2 a^1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

Problem-06 Continue:

$$W_1(2) = W_1(1) + a(P_2 - W_1(1)) = \begin{bmatrix} 1,5 \\ 0 \end{bmatrix} + 0{,}5\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1,5 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0,75 \\ 0,5 \end{bmatrix}$$

P_3:

$$a^1 = \text{compet}(n^1) = \text{compet}\begin{vmatrix} -\|[0,75 \quad 0,5]^T - [2 \quad 2]^T\| \\ -\|[-1 \quad 0]^T - [2 \quad 2]^T\| \end{vmatrix} =$$

$$= \text{compet}\left(\begin{vmatrix} -1,6583124 \\ -3,605\text{[illegible]} \end{vmatrix}\right) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$a^2 = W^2 a^1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$W_1(3) = W_1(2) + a(P_3 - W_1(2)) = \begin{bmatrix} 0,75 \\ 0,5 \end{bmatrix} + 0{,}5\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 0,75 \\ 0,5 \end{bmatrix}\right) = \begin{bmatrix} 1,375 \\ 1,25 \end{bmatrix}$$

P_2:

$$a^1 = \text{compet}(n^1) = \text{compet}\begin{vmatrix} -\|[1,375 \quad 1,25]^T - [0 \quad 1]^T\| \\ -\|[-1 \quad 0]^T - [0 \quad 1]^T\| \end{vmatrix} =$$

$$= \text{compet}\left(\begin{vmatrix} -1,39754249 \\ -1,41421356 \end{vmatrix}\right) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$a^2 = W^2 a^1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$W_1(4) = W_1(3) + a(P_2 - W_1(3)) = \begin{bmatrix} 1,375 \\ 1,25 \end{bmatrix} + 0.5\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1,375 \\ 1,25 \end{bmatrix}\right) = \begin{bmatrix} 0,6875 \\ 1,125 \end{bmatrix}$$

Problem-06-Continue:

P_3:

$$a^1 = \text{compet}(n^1) = \text{compet}\begin{vmatrix} -\|[0{,}6875 \quad 1{,}125]^T - [2\ 2]^T\| \\ -\|[-1\ \ 0]^T - [2\ 2]^T\| \end{vmatrix} =$$

$$= \text{compet}\left(\begin{vmatrix} -1{,}57742868 \\ -3{,}60555128 \end{vmatrix}\right) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$a^2 = W^2 a^1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$W_1(5) = W_1(4) + \alpha(P_3 - W_1(4)) = \begin{bmatrix} 0{,}6875 \\ 1{,}125 \end{bmatrix} + 0.5\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 0{,}6875 \\ 1{,}125 \end{bmatrix}\right) = \begin{bmatrix} 1{,}34375 \\ 1{,}5625 \end{bmatrix}$$

P_1:

$$a_1 = \text{compet}(n^1) = \text{compet}\begin{vmatrix} -\|[1{,}34375 \quad 1{,}5625]^T - [2\ 0]^T\| \\ -\|[-1\ \ 0]^T - [2\ 0]^T\| \end{vmatrix} =$$

$$= \text{compet}\left(\begin{vmatrix} -2{,}87207031 \\ -3 \end{vmatrix}\right) = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$

$$a^2 = W^2 a^1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$W_2(1) = W_2(0) + \alpha(P_1 - W_2(0)) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + 0.5\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

$$W_1(5) = \begin{bmatrix} 1{,}34375 \\ 1{,}5625 \end{bmatrix} \qquad W_2(1) = \begin{bmatrix} 0{,}5 \\ 0 \end{bmatrix}$$

7)

Ο κωδικας της ασκησης βρισκεται στα συννημενα.

8)

Ο κωδικας της ασκησης βρισκεται στα συννημενα.

9)

Problem-09

$M \in R^{n \times n}$ and symmetric with eigenvalues λ_i

A) If $Mx = \lambda \cdot x$ then multiplying by M from the left yields

$$MMx = M\lambda x \Leftrightarrow M^2 x = \lambda M x \Leftrightarrow M^2 x = \lambda(\lambda x) \Leftrightarrow A^2 x = \lambda^2 x$$

In fact, for every M thats multiplied to both sides, the right side "gains" a factor λ (since Mx can be substituted by λx) while the eigenvectors remain the same. It follows that multiplying both sides by A^{k-1} yields

$$M^{k-1} M x = M^{k-1} \lambda x \Leftrightarrow M^k x = \lambda(\lambda^{k-1} x) \Leftrightarrow \boxed{M^k = \lambda^k x}$$

B) Assume with no loss of generality $|\lambda_1| > |\lambda_2| > \dots > |\lambda_k|$. For any vector $x = a_1 v_1 + a_2 v_2 + \dots + a_k v_k$ that is linear combination of all eigenvectors, the normalized mapping $P = \frac{1}{\lambda_1} M$ (namely $P v_1 = \left(\frac{1}{\lambda_1} M\right) v_1 = \frac{1}{\lambda_1}(M v_1) = \frac{1}{\lambda_1} \lambda_1 v_1 = v_1$)

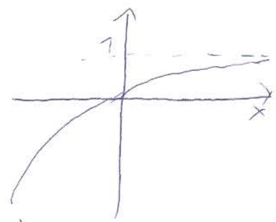
converges to the eigenvector with the largest absolute eigenvalue.

$$\lim_{k\to\infty} P^k x = \lim_{k\to\infty} \frac{1}{\lambda_1^k} M^k x = \lim_{k\to\infty} \frac{1}{\lambda_1^k} (a_1 M^k v_1 + a_2 M^k v_2 + \dots + a_k M^k v_k) =$$

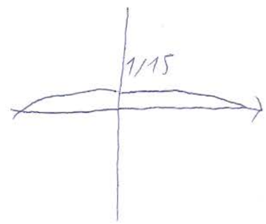
$$= \lim_{k\to\infty} \frac{1}{\lambda_1^k} (\underbrace{a_1 \lambda_1^k v_1}_{\text{steady state}} + \underbrace{a_2 \lambda_2^k v_2 + \dots + \lambda_k^k M_{v_k}^k}_{\text{transient states}}) = \boxed{a_1 v_1}$$

Πρόβλημα 10

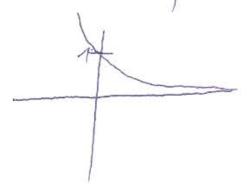
~~1) A∪B~~ $\mu_A(x)$:



, $\mu_B(x)$:



$\mu_C(x)$:



1. $A \cup B$ $\quad \mu_A(x) = \mu_B(x)$ για $x = 0{,}354$

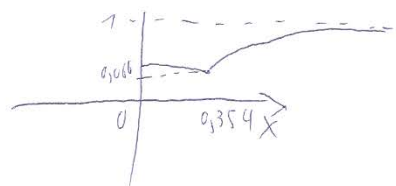
για $0 \leq x \leq 0{,}354$ $\quad \mu_B(x) > \mu_A(x)$

για $0{,}354 \leq x$ $\quad \mu_A(x) > \mu_B(x)$

$$\cancel{\text{Άρα } \mu_{A\cup B}(x) = \int_0^{0,354} \frac{1}{x^2+15}\,dx + \int_{0,354}^{\infty} \frac{x}{x+5}\,dx = \frac{\arctan\left(\frac{0,354}{\sqrt{15}}\right)}{\sqrt{15}} + \int_{0,354+5}^{\infty} \frac{u-5}{u}\,du}$$

$$\cancel{= \ldots + \Big[x - 5\ln|x+5|\Big]_{0,354}^{\infty} \to \infty}$$

$$\text{Άρα } \mu_{A\cup B}(x) = \begin{cases} \mu_B(x), & 0 \leq x \leq 0{,}354 \\ \mu_A(x), & x > 0{,}354 \\ 0, & \text{else} \end{cases}$$

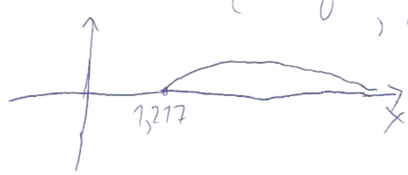


2.

$B \cap C$, Η $\mu_B(x)$ τέμνεται με την $\mu_C(x)$ στο σημείο $A(1{,}217,\ 0{,}0607)$ όπου για $x > 1{,}213 \Rightarrow$

$\mu_B(x) > \mu_C(x)$

Άρα
$$\mu_{B\cap C}(x) = \begin{cases} \frac{1}{x^2+15} - \frac{1}{10x}, & x > 1{,}217 \\ 0, & \text{else} \end{cases}$$

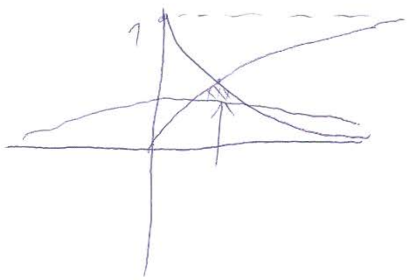


3.

$A \cup B \cup C$, Επειδή $\mu_C(x) > \mu_A(x)$ και $\mu_C(x) > \mu_B(x)$ για $0 \leq x \leq 0{,}8415$ που είναι σημείο τομής A και C , ενώ για $x > 0{,}8415$ μ_A μεγαλύτερη όλων έχουμε:

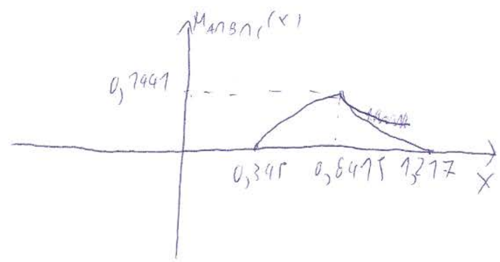
$$\mu_{A\cup B\cup C}(x) = \begin{cases} \mu_C(x), & 0 \leq x \leq 0{,}8415 \\ \mu_A(x), & x \geq 0{,}8415 \\ 0, & \text{else} \end{cases}$$

4. $A \cap B \cap C$

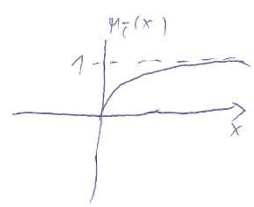


) Κάνοντας το γράφημα και για τις 3 συναρτήσεις βλέπουμε την περιοχή της τομής τους. Άρα έχουμε

$$\mu_{A\cap B\cap C}(x) = \begin{cases} \dfrac{x}{x+5} - \dfrac{1}{x^2+15} & , 0{,}354 \le x \le 0{,}6415 \\ \dfrac{1}{10^x} - \dfrac{1}{x^2+15} & , 0{,}6415 < x \le 1{,}217 \\ 0 & , \text{else} \end{cases}$$

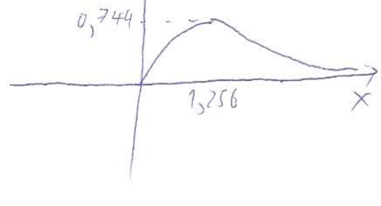


5. $\mu_{\bar{C}}(x) = 1 - \mu_C(x) = 1 - \dfrac{1}{10^x}$

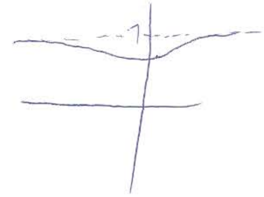


Α επειδή $\mu_{\bar{C}}(x) \ge \mu_A(x)$ $x \ge 0$

$$\mu_{A\cap \bar{C}}(x) = \begin{cases} 1 - \dfrac{1}{10^x} - \dfrac{1}{x+5} & , x > 0 \\ 0 & , \text{else} \end{cases}$$



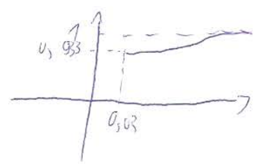
6. $\bar{B} \cup C$, $\mu_{\bar{B}}(x) = 1 - \frac{1}{x^2+15} = \frac{x^2+14}{x^2+15}$



όπου $\mu_{\bar{B}} = \mu_C$ για $x = 0,03$

και $\mu_{\bar{B}}(x) > \mu_C(x)$ για $x > 0,03$

Άρα $\mu_{\bar{B} \cup C} = \begin{cases} \frac{x^2+14}{x^2+15}, & x > 0,03 \\ 0, & \text{else} \end{cases}$



7. $\overline{A \cap B}$

$\mu_{\overline{A \cap B}}(x) = 1 - \mu_{A \cap B}(x)$, όπου $\mu_A(x) = \mu_B(x)$ για $x = 0,354$

άρα $\mu_{A \cap B}(x) = \begin{cases} \frac{1}{x^2+15} - \frac{x}{x+5}, & 0 \le x \le 0,354 \\ 0, & \text{else} \end{cases}$

Άρα $\mu_{\overline{A \cap B}}(x) = \begin{cases} 1 - \frac{1}{x^2+15} + \frac{x}{x+5}, & 0 \le x \le 0,354 \\ 0, & \text{else} \end{cases}$

8. $\bar{A} \cup \bar{B}$, επειδή $\bar{A} \cup \bar{B} = \overline{(A \wedge B)} \Rightarrow$

$\mu_{\bar{A} \cup \bar{B}}(x) = \mu_{\overline{A \wedge B}}(x)$, άρα είναι ίδια

με το προηγούμενο ερώτημα

Πρόβλημα 11)

Αφού $A \oplus B = (A \cap \bar{B}) \cup (\bar{A} \cap B)$, τότε

$$\mu_{A\oplus B} = MAX\{ MIN\{\mu_A(x), \mu_{\bar{B}}(x)\}, MIN\{\mu_{\bar{A}}(x), \mu_B(x)\}\}$$

Διότι $\mu_{A\cap B}(x) = MIN\{\mu_A(x), \mu_B(x)\}$

και $\mu_{A\cup B}(x) = MAX\{\mu_A(x), \mu_B(x)\}$

$\boxed{\text{Θέτω } K = MIN\{\mu_A(x), \mu_{\bar{B}}(x)\} \text{ και } \Lambda = MIN\{\mu_{\bar{A}}(x), \mu_B(x)\}}$

<u>Υπάρχουν 4 περιπτώσεις:</u>

1) Για $\mu_A(x) \geq 0,5 \Rightarrow \mu_{\bar{A}}(x) \leq 0,5$ και $\mu_B(x) \geq 0,5 \Rightarrow \mu_{\bar{B}}(x) \leq 0,5$ $\Rightarrow$ $K \leq 0,5$ λόγω του $\mu_{\bar{B}}(x)$, $\Lambda \leq 0,5$ λόγω του $\mu_{\bar{A}}(x)$ $\Rightarrow \mu_{A\oplus B} \leq 0,5$

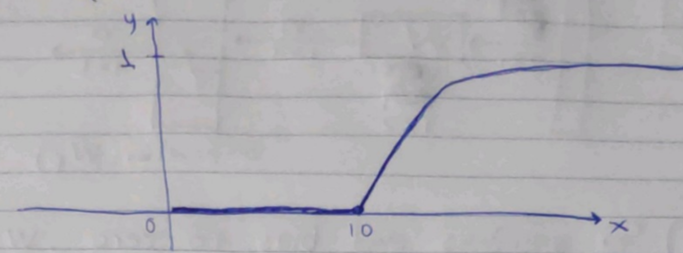
2) Για $\mu_A(x) \leq 0,5 \Rightarrow \mu_{\bar{A}}(x) \geq 0,5$ και $\mu_B(x) \geq 0,5 \Rightarrow \mu_{\bar{B}}(x) \leq 0,5$ $\Rightarrow$ $K \leq 0,5$ λόγω $\mu_{\bar{B}}(x)$, $\Lambda \leq 0,5$ λόγω $\mu_A(x)$ $\Rightarrow \mu_{A\oplus B} \leq 0,5$

3) Για $\mu_A(x) \geq 0,5 \Rightarrow \mu_{\bar{A}}(x) \leq 0,5$ και $\mu_B(x) \leq 0,5 \Rightarrow \mu_{\bar{B}}(x) \geq 0,5$ $\Rightarrow$ $K \geq 0,5$ λόγω $\mu_A, \mu_{\bar{B}} \geq 0,5$, $\Lambda \leq 0,5$ λόγω $\mu_{\bar{B}}, \mu_A \leq 0,5$ $\Rightarrow \mu_{A\oplus B} \geq 0,5$

Άρα δεν ισχύει η εξίσωση που αναφέρεται στο ερώτημα για τη 3η περίπτωση.

Problem-12:

$$\mu_A(x) = \begin{cases} 0 & , x \le 10 \\ \dfrac{1}{1+\dfrac{1}{(x-10)^2}} & , x > 10 \end{cases}$$



$$\mu(\lambda x + (1-\lambda)y) \geq \lambda\mu(x) + (1-\lambda)\mu(y), \ \lambda \in [0,1]$$

$x \neq y \in (10, \infty)$