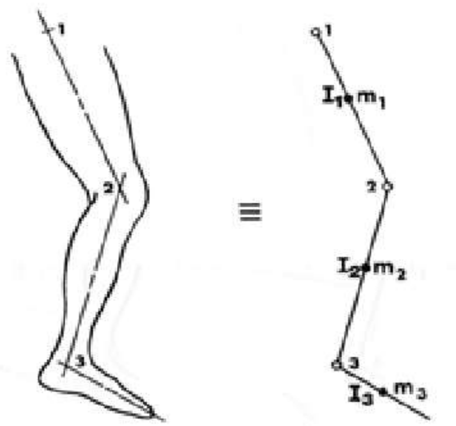
Department of Mechanical
Engineering ME 203
CHALLENGE 2

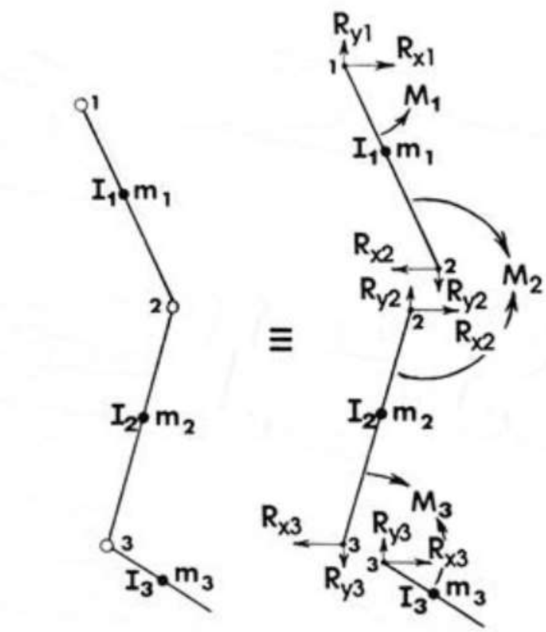
Linked Segment Model for Deadlift

Some information on Linked Segment Models

The joints are replaced by pin (hinge) joints and segments are replaced by masses and moments of inertia which are assigned to the center of mass of each segment. The segments are broken at the joints and moment/reaction forces designated appropriately at the connections.



Anatomical Model   Link Segment Model



Link-Segment Model   Free-Body Diagram

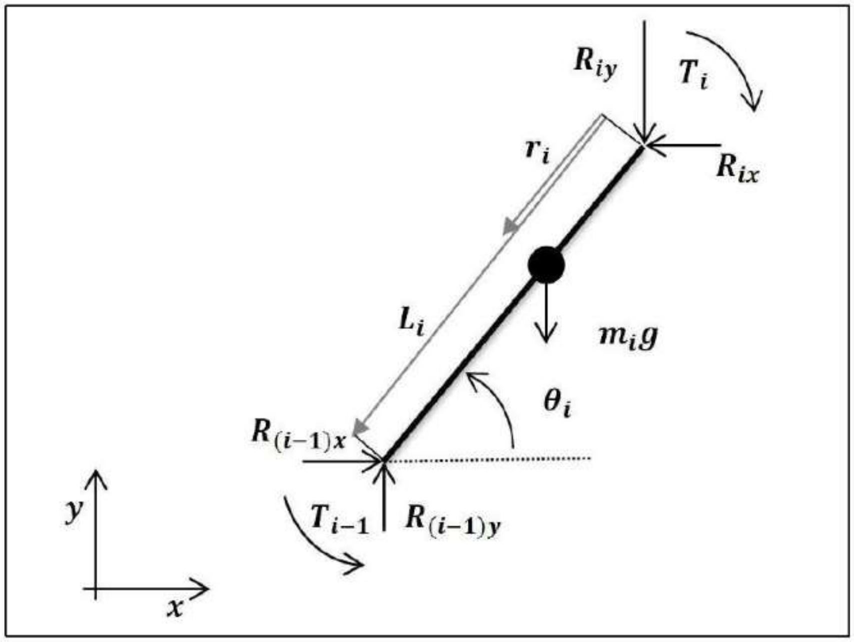


Figure 2: 2 dimensional free body diagram of a generalized segment.

$$0 = -m_i a_{ix} + R_{(i-1)x} - R_{ix}$$
$$0 = -m_i a_{iy} + R_{(i-1)y} - R_{iy} - m_i g$$
$$0 = -I_i \alpha_i + T_{i-1} - T_i + R_{(i-1)x}(L_i - r_i)\sin\theta_i - R_{(i-1)y}(L_i - r_i)\cos\theta_i + R_{ix} r_i \sin\theta_i - R_{iy} r_i \cos\theta_i$$

$R_{(i-1)x}, R_{(i-1)y}$	Intersegmental force at the inferior joint in x and y directions, respectively
R_{ix}, R_{iy}	Intersegmental force at the superior joint in x and y directions, respectively (equal to zero for segment 7)
T_{i-1}	Net joint torque at inferior joint
T_i	Net joint torque at superior joint (equal to zero for segment 7)
L_i	Segment length
r_i	Distance from superior joint to segment mass center
θ_i	Segment angle relative to x (horizontal) axis
m_i	Segmental mass
g	Gravitational constant
α_i	Segment angular acceleration
a_{ix}, a_{iy}	Segment mass center acceleration in x and y directions, respectively
I_i	Segment mass moment of inertia with respect to its mass center

The Model

Nomenclature

1	Shank
2	Thigh
3	Trunk
4	Arm
5	Forearm
6	Head and neck
B2	Bar and hand
(B1)	Level of 7th cervical vertebrae at center of neck
(B2)	Center of bar and hand
(1)	Center of ankle joint
(2)	Center of knee joint
(3)	Center of hip joint
(4)	Segment connecting *(B1)* and center of shoulder joint
(5)	Center of elbow joint
g	Acceleration due to gravity ($-9.8\ m \cdot s^{2-1}$)
M	Mass (kg)
W	Weight (N); W = Mg
M	Torque (Nm)
F	Force (N)
I	Moment of inertia (kg m^2)
L	Segment length (m)
L_{B1}	Length from 7th cervical vertebrae to center of shoulder joint
ρ	Percent of segment length to center of gravity
o	Location of center of gravity (cg)
X	Horizontal position (m) or direction
$\dot{X}$	Horizontal velocity ($m \cdot s^{-1}$)
$\ddot{X}$	Horizontal acceleration ($m \cdot s^{2-1}$)
Y	Vertical position (m) or direction
$\dot{Y}$	Vertical velocity ($m \cdot s^{2-1}$)
$\ddot{Y}$	Vertical acceleration ($m \cdot s^{2-1}$)
Θ	Angular position (rad)
$\dot{\Theta}$	Angular velocity ($rad \cdot s^{-1}$)
$\ddot{\Theta}$	Angular acceleration ($rad \cdot s^{2-1}$)
p	Proximal end of segment
d	Distal end of segment

Figure 3: Nomenclature for the Linked Segment Model of the Deadlift

$$\ddot{X}_{(1)} = 0$$

$$\ddot{Y}_{(1)} = 0$$

$$\left.\begin{aligned} \ddot{X}_{(i+1)} &= \ddot{X}_{(i)} - L_i(\ddot{\Theta}_i \sin \Theta_i + \dot{\Theta}_i^2 \cos \Theta_i) \\ \ddot{Y}_{(i+1)} &= \ddot{Y}_{(i)} + L_i(\ddot{\Theta}_i \cos \Theta_i - \dot{\Theta}_i^2 \sin \Theta_i) \end{aligned}\right\} \text{ where } i = 1, 2, 4$$

$$\ddot{X}_{(B1)} = \ddot{X}_{(3)} - L_3(\ddot{\Theta}_3 \sin \Theta_3 + \dot{\Theta}_3^2 \cos \Theta_3)$$

$$\ddot{Y}_{(B1)} = \ddot{Y}_{(3)} + L_3(\ddot{\Theta}_3 \cos \Theta_3 - \dot{\Theta}_3^2 \sin \Theta_3)$$

$$\ddot{X}_{(4)} = \ddot{X}_{(B1)} - L_{B1}(\ddot{\Theta}_{B1} \sin \Theta_{B1} + \dot{\Theta}_{B1}^2 \cos \Theta_{B1})$$

$$\ddot{Y}_{(4)} = \ddot{Y}_{(B1)} + L_{B1}(\ddot{\Theta}_{B1} \cos \Theta_{B1} - \dot{\Theta}_{B1}^2 \sin \Theta_{B1})$$

$$\ddot{X}_{(B2)} = \ddot{X}_{(5)} - L_5(\ddot{\Theta}_5 \sin \Theta_5 + \dot{\Theta}_5^2 \cos \Theta_5)$$

$$\ddot{Y}_{(B2)} = \ddot{Y}_{(5)} + L_5(\ddot{\Theta}_5 \cos \Theta_5 - \dot{\Theta}_5^2 \sin \Theta_5)$$

$$\left.\begin{aligned} \ddot{X}_i(cg) &= \ddot{X}_{(i)} - (1 - \rho_i)L_i(\ddot{\Theta}_i \sin \Theta_i + \dot{\Theta}_i^2 \cos \Theta_i) \\ \ddot{Y}_i(cg) &= \ddot{Y}_{(i)} + (1 - \rho_i)L_i(\ddot{\Theta}_i \cos \Theta_i - \dot{\Theta}_i^2 \sin \Theta_i) \end{aligned}\right\} \text{ where } i = 1, 2, 3$$

$$\left.\begin{aligned} \ddot{X}_i(cg) &= \ddot{X}_{(i)} - \rho_i L_i(\ddot{\Theta}_i \sin \Theta_i + \dot{\Theta}_i^2 \cos \Theta_i) \\ \ddot{Y}_i(cg) &= \ddot{Y}_{(i)} + \rho_i L_i(\ddot{\Theta}_i \cos \Theta_i - \dot{\Theta}_i^2 \sin \Theta_i) \end{aligned}\right\} \text{ where } i = 4, 5$$

$$\ddot{X}_6(cg) = \ddot{X}_{(B1)} - \rho_6 L_6(\ddot{\Theta}_6 \sin \Theta_6 + \dot{\Theta}_6^2 \cos \Theta_6)$$

$$\ddot{Y}_6(cg) = \ddot{Y}_{(B1)} + \rho_6 L_6(\ddot{\Theta}_6 \cos \Theta_6 - \dot{\Theta}_6^2 \sin \Theta_6)$$

Figure 4: Starting Equations for Accelerations for the Linked Segment Model of the Deadlift

$$F_{X(B2)} = -M_{B2}\ddot{X}_{(B2)}$$

$$F_{Y(B2)} = M_{B2}g + M_{B2}\ddot{Y}_{(B2)}$$

$$F_{X(5)} = F_{X(B2)} - M_5\ddot{X}_5(cg)$$

$$F_{Y(5)} = F_{Y(B2)} + M_5g + M_5\ddot{Y}_5(cg)$$

$$F_{X(4)} = F_{X(5)} - M_4\ddot{X}_4(cg)$$

$$F_{Y(4)} = F_{Y(5)} + M_4g + M_4\ddot{Y}_4(cg)$$

$$F_{X(6)} = -M_6\ddot{X}_6(cg)$$

$$F_{Y(6)} = M_6\ddot{X}_6(cg) + M_6g$$

$$F_{X(B1)} = F_{X(4)} + F_{X(6)}$$

$$F_{Y(B1)} = F_{Y(4)} + F_{Y(6)}$$

$$F_{X(3)} = F_{X(B1)} - M_3\ddot{X}_3(cg)$$

$$F_{Y(3)} = F_{Y(B1)} + M_3g + M_3\ddot{Y}_3(cg)$$

$$F_{X(2)} = F_{X(3)} - M_2\ddot{X}_2(cg)$$

$$F_{Y(2)} = F_{Y(3)} + M_2g + M_2\ddot{Y}_2(cg)$$

$$F_{X(1)} = F_{X(2)} - M_1\ddot{X}_1(cg)$$

$$F_{Y(1)} = F_{Y(2)} + M_1g + M_1\ddot{Y}_1(cg)$$

Figure 5: Starting Equations for Forces for the Linked Segment Model of the Deadlift

(Note: M that is not in italics is Mass)

$$M_6 = I_6\ddot{\Theta}_6 + F_{Y(6)}\rho_6 L_6 \cos\Theta_6 + F_{X(6)}\rho_6 L_6 \sin\Theta_6$$

$$\begin{aligned} M_5 = I_5\ddot{\Theta}_5 &- F_{Y(B2)}(1 - \rho_5)L_5\cos\Theta_5 \\ &- F_{X(B2)}(1 - \rho_5)L_5\sin\Theta_5 - F_{Y(5)}\rho_5 L_5\cos\Theta_5 \\ &- F_{X(5)}\rho_5 L_5 \sin\Theta_5 \end{aligned}$$

$$\begin{aligned} M_4 = I_4\ddot{\Theta}_4 &- F_{Y(4)}\rho_4 L_4\cos\Theta_4 \\ &- F_{X(4)}\rho_4 L_4 \sin\Theta_4 \\ &- F_{Y(5)}(1 - \rho_4)L_4\cos\Theta_4 \\ &- F_{X(5)}(1 - \rho_4)L_4\sin\Theta_4 + M_5 \end{aligned}$$

$$\begin{aligned} M_{L_{B1}} = M_4 + M_6 &- F_{Y(4)}L_{B1}\cos\Theta_{L_{B1}} \\ &- F_{X(4)}L_{B1}\sin\Theta_{L_{B1}} \end{aligned}$$

$$\begin{aligned} M_3 = I_3\ddot{\Theta}_3 &+ F_{Y(B1)}\rho_3 L_3\cos\Theta_3 + F_{X(B1)}\rho_3 L_3\sin\Theta_3 \\ &+ F_{Y(3)}(1 + \rho_3)L_3\cos\Theta_3 \\ &+ F_{X(3)}(1 + \rho_3)L_3\sin\Theta_3 + M_{L_{B1}} \end{aligned}$$

$$\left.\begin{aligned} M_i = I_i\ddot{\Theta}_i &+ F_{Y(i)}(1 - \rho_i)L_i\cos\Theta_i \\ &+ F_{X(i)}(1 - \rho_i)L_i\sin\Theta_i \\ &+ F_{Y(i+1)}\rho_i L_i\cos\Theta_i \\ &+ F_{X(i+1)}\rho_i L_i\sin\Theta_i + M_{i+1} \end{aligned}\right\}\text{where } i = 1,2$$

Figure 6: Starting Equations for Moments for the Linked Segment Model of the Deadlift

(Note: M that is in italics is Moment)

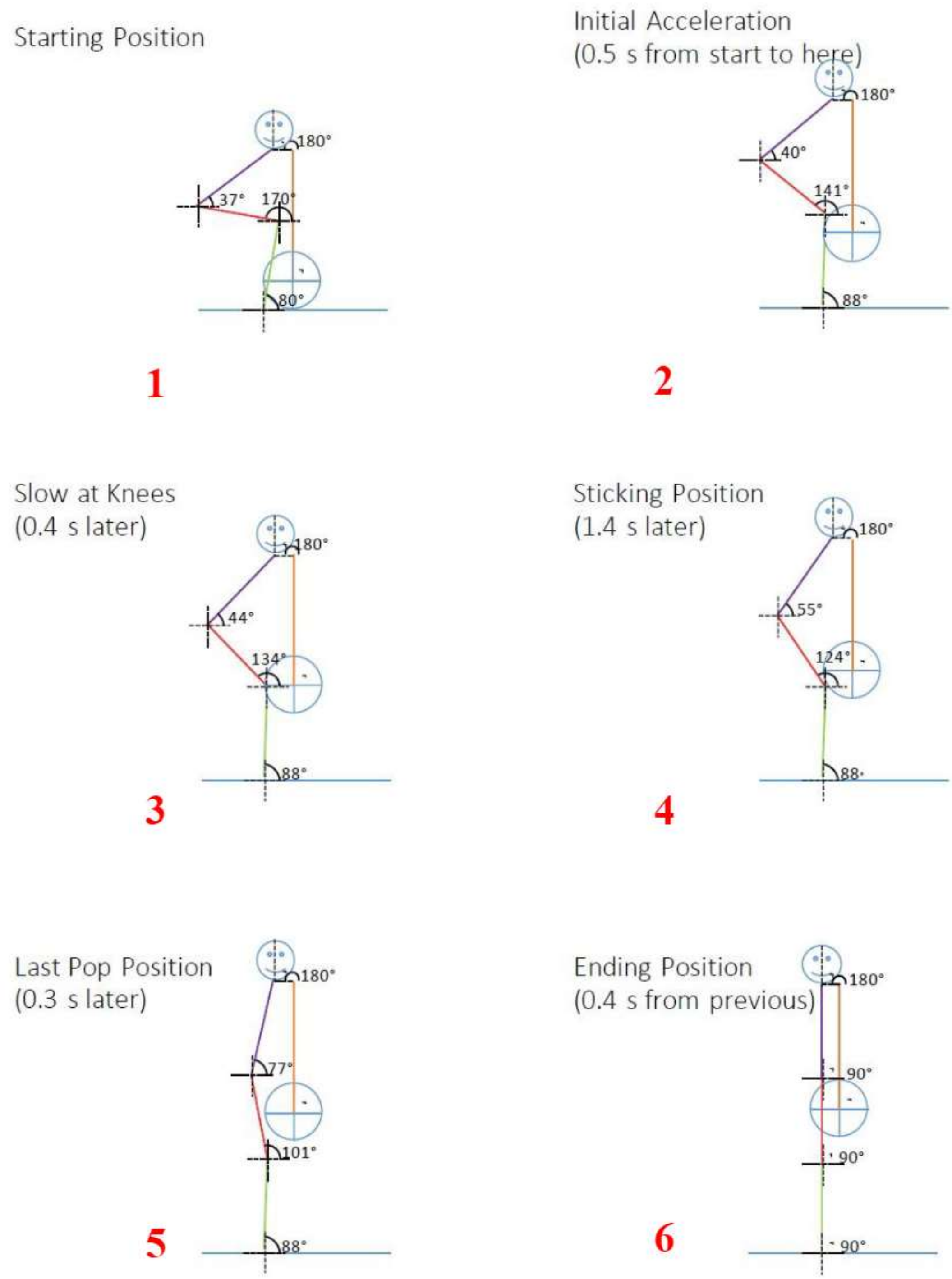


Figure 7: Six stages in the deadlift that should be included in your initial model for the elite lifter

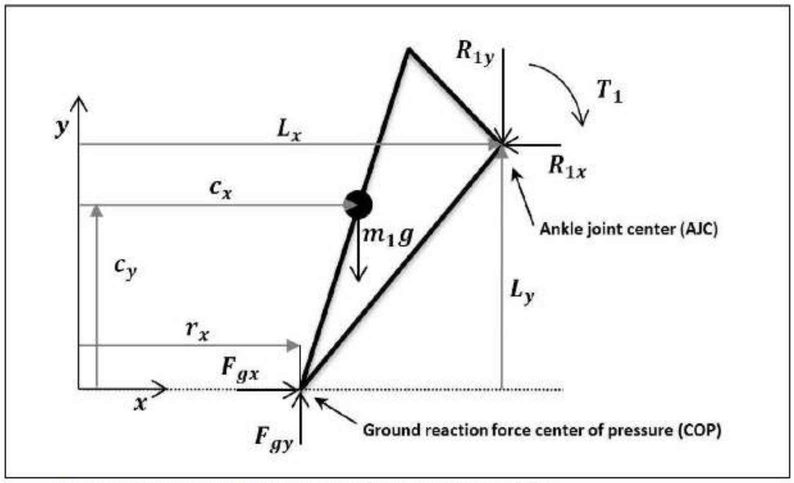


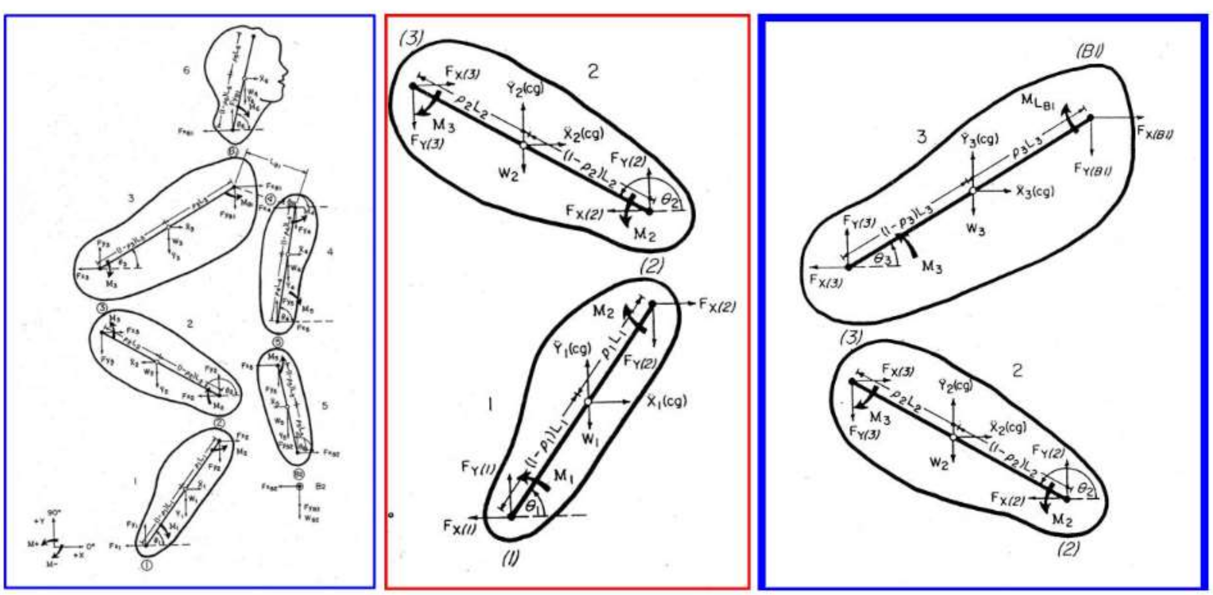
Figure 3: Free body diagram of foot segment at takeoff.

$$0 = -m_1 a_{1x} + F_{gx} - R_{1x} \tag{2.7}$$

$$0 = -m_1 a_{1y} + F_{gy} - R_{1y} - m_1 g \tag{2.8}$$

$$0 = -I_1 \alpha_1 - T_1 + F_{gx} c_y - F_{gy}(c_x - r_x) + R_{1x}(L_y - c_y) - R_{1y}(L_x - c_x) \tag{2.9}$$

R_{1x}, R_{1y}	Intersegmental force at the ankle joint center in x and y directions, respectively.
F_{gx}, F_{gy}	Ground reaction force in x and y directions, respectively
T_1	Net joint torque at ankle
L_x, L_y	Distance from origin to ankle joint center in the x and y directions, respectively
r_x	Distance from origin to the COP in the x
c_x, c_y	Distance from origin to foot mass center in the x and y directions, respectively
m_1	Foot segment mass
g	Gravitational constant
α_1	Foot angular acceleration
a_{1x}, a_{1y}	Foot mass center acceleration in x and y directions, respectively
I_1	Foot mass moment of inertia with respect to its mass center



Additional Figures