Department of Mechanical and Mechatronics Engineering

ME 203: Guidelines for a safe deadlift

A Report Prepared For:

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Abstract

In this report, we provide a detailed analysis of the biomechanics of the deadlift exercise. We show that the deadlift is a compound movement that can be simulated using a linked segment model, and that by applying the principles of Newton's Second Law and principles from rigid body dynamics, we are able to accurately calculate the moments, forces, and accelerations involved in the lift. We conduct our analysis using data from an elite lifter lifting a 300kg weight as a baseline. By using initial conditions, data about body composition, and time and position data, we can solve for the entire model. Our analysis reveals the effects of the lift on the shear and compression experienced by the spine, and shows that body composition factors, such as height, can significantly impact these values. We also find that the sumo deadlift technique puts less shear and compression on the spine than the traditional deadlift. Overall, our findings provide valuable insights into the biomechanics of the deadlift and can help people better understand and optimize their technique for this popular exercise.

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1 Problem Statement

The deadlift is a popular ‘compound’ exercise performed by athletes and weightlifters, alike. Compound exercises distribute loads concurrently on multiple muscle groups. For instance, the deadlift places a load on the legs, back, core, and arms, respectively. As such, many people perform the deadlift to increase their overall strength and muscle-mass. Unfortunately, people often fail to perform the deadlift in a manner that safely distributes these loads. Moreover, the discussion around technique is one that is confused; Due to a variety in body composition, there isn’t a single set of guidelines that one can follow, to minimize the risk of injury. This report offers a set of custom parameters, which if monitored, can minimize the risk of injury across all types of bodies.

2 Background

2.1 Linked Segment Model

The first step in discovering a set of guidelines to safely perform the deadlift is to quantitively model a deadlift. More specifically, establish a mathematical set of relationships that define the movement. While the body's systems become infinitely complex, many tiers of complexity offer negligible contributions to this scope. As such, it is better to abstract away these complexities and simplify the body to 5 linked segments, as shown in figure 1.

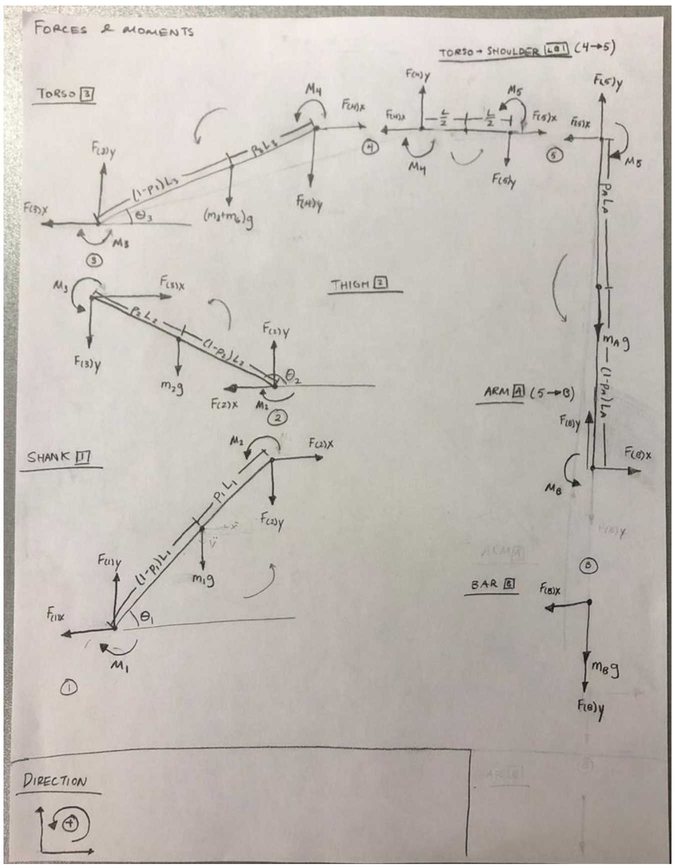

Figure 1: Simplified linked segment model

Segments	**Symbol**	**Joints**	**Symbol**
Shank	1	Ankle	1
Thigh	2	Knee	2
Torso + Head	3	Hip	3
Collar	LB1	Torso / Collar	LB1
Arm	A	Shoulder	4
		Bar	B

Table 1: Variable assignment for linked segment model

Furthermore,

- The lifter and bar are assumed to be symmetrical – enabling the linked segment analysis to remain a two-dimensional problem.
- Each 'segment' is assumed to be a uniform rod with a moment of inertia defined by formula 1.

$$\boldsymbol{I = \frac{1}{12} mL^2} \tag{1}$$

- Joints are assumed to be frictionless and massless.
- The forearm and upper arm have been added together. As they provide negligible effects to the linked segment model. The arm now represents single segment which includes the forearm and upper-arm.
- The torso is assumed to include the head, in terms of mass

To quantify the 'loads' experienced by segments, three critical metrics are calculated: force, moment, and acceleration. Using Newtons 2nd Law and principles from rigid body dynamics the linear accelerations for each joint and segment are found. In addition, force and moment analysis are conducted on each segment to find the reaction forces at each end of a segment. These analyses are shown in Figures 2, 3, 4, 5, & 6.

2.1.1 Shank

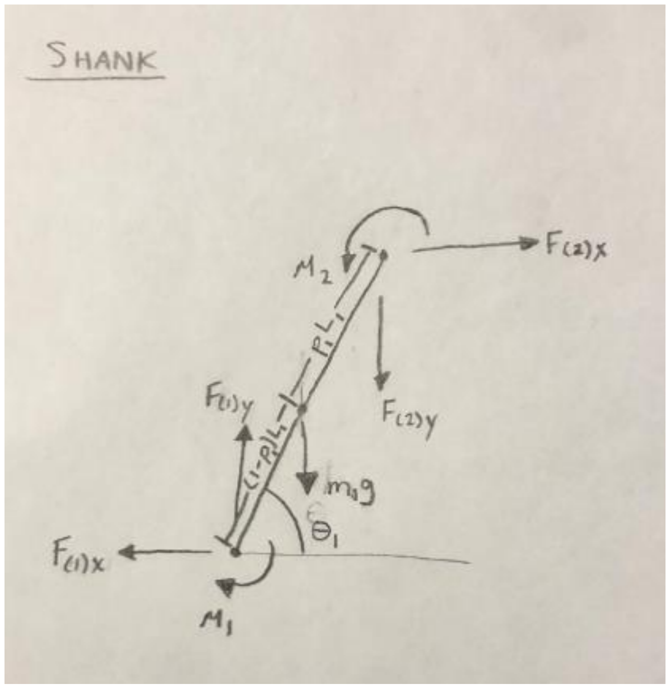

Figure 2: moments, forces, and accelerations on the shank

Point Acceleration Equations

$$\boldsymbol{\ddot{X}_1 = 0} \quad \textbf{(2)}$$

$$\ddot{Y}_1 = 0 \quad (3)$$

Center of Gravity Acceleration Equations

$$\boldsymbol{\ddot{X}_{1(cg)} = -(1-\rho_1)L_1(\ddot{\theta}_1 \sin\theta_1 + \dot{\theta}_1^2 \cos\theta_1)} \quad \textbf{(4)}$$

$$\ddot{Y}_{1(cg)} = (1-\rho_1)L_1(\ddot{\theta}_1 \cos\theta_1 - \dot{\theta}_1^2 \sin\theta_1) \quad (5)$$

Force Equations

$$\boldsymbol{F_{(1)x} = F_{(2)x} - m_1 \ddot{X}_{1(cg)}} \quad \textbf{(6)}$$

$$F_{(1)y} = F_{(2)y} + m_1(\ddot{Y}_{1(cg)} + g) \quad (7)$$

Moment Equations

$$\boldsymbol{M_1 = M_2 + I_1\ddot{\theta}_1 + F_{(1)y}(1-\rho_1)L_1 \cos\theta_1 + F_{(1)x}(1-\rho_1)L_1 \sin\theta_1 + F_{(2)y}\rho_1 L_1 \cos\theta_1 + F_{(2)x}\rho_1 L_1 \sin\theta_1} \quad \textbf{(8)}$$

2.1.2 Thigh

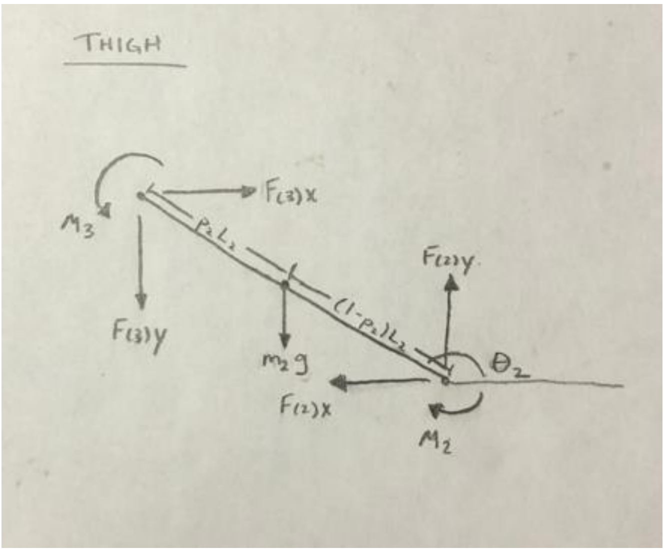

Figure 3: moments, forces, and accelerations on the thigh

Point Acceleration Equations

$$\boldsymbol{\ddot{X}_2 = -L_1(\ddot{\theta}_1 \sin\theta_1 + \dot{\theta}_1^2 \cos\theta_1)} \quad \textbf{(9)}$$

$$\ddot{Y}_2 = L_1(\ddot{\theta}_1 \cos\theta_1 - \dot{\theta}_1^2 \sin\theta_1) \quad (10)$$

Center of Gravity Acceleration Equations

$$\ddot{\boldsymbol{X}}_{2(cg)} = \ddot{\boldsymbol{X}}_2 - (\mathbf{1} - \boldsymbol{\rho}_2)\boldsymbol{L}_2(\ddot{\boldsymbol{\theta}}_2 \sin \boldsymbol{\theta}_2 + \dot{\boldsymbol{\theta}}_2^2 \cos \boldsymbol{\theta}_2) \quad \mathbf{(11)}$$

$$\ddot{Y}_{2(cg)} = \ddot{Y}_2 + (1 - \rho_2)L_2(\ddot{\theta}_2 \cos \theta_2 - \dot{\theta}_2^2 \sin \theta_2) \quad (12)$$

Force Equations

$$\boldsymbol{F}_{(2)x} = \boldsymbol{F}_{(3)x} - \boldsymbol{m}_2\ddot{\boldsymbol{X}}_{2(cg)} \quad \mathbf{(13)}$$

$$F_{(2)y} = F_{(3)y} + m_2(\ddot{Y}_{2(cg)} + g) \quad (14)$$

Moment Equations

$$\boldsymbol{M}_2 = \boldsymbol{M}_3 + \boldsymbol{I}_2\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{F}_{(2)y}(\mathbf{1} - \boldsymbol{\rho}_2)\boldsymbol{L}_2 \cos \boldsymbol{\theta}_2 + \boldsymbol{F}_{(2)x}(\mathbf{1} - \boldsymbol{\rho}_2)\boldsymbol{L}_2 \sin \boldsymbol{\theta}_2 + \boldsymbol{F}_{(3)y}\boldsymbol{\rho}_2\boldsymbol{L}_2 \cos \boldsymbol{\theta}_2 + \boldsymbol{F}_{(3)x}\boldsymbol{\rho}_2\boldsymbol{L}_2 \sin \boldsymbol{\theta}_2 \quad \mathbf{(15)}$$

2.1.3 Torso/Head

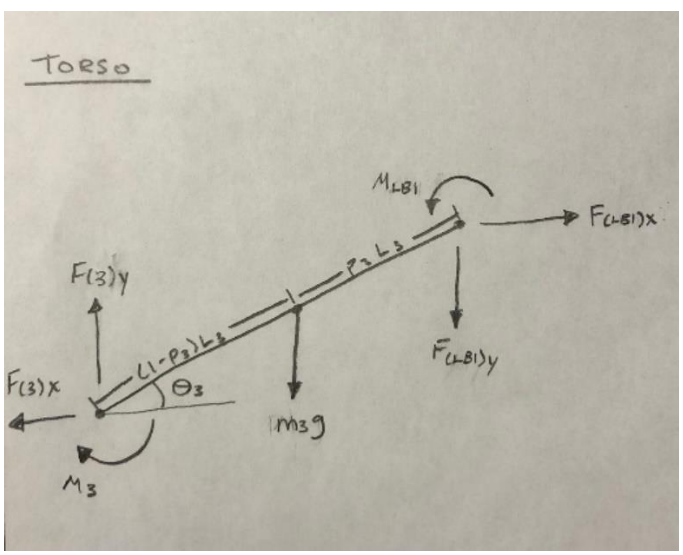

Figure 4: moments, forces, and accelerations on the trunk & head

Point Acceleration Equations

$$\boldsymbol{\ddot{X}_3 = \ddot{X}_2 - L_2(\ddot{\theta}_2 \sin\theta_2 + \dot{\theta}_2^2 \cos\theta_2)} \quad \textbf{(16)}$$

$$\ddot{Y}_3 = \ddot{Y}_2 + L_1(\ddot{\theta}_2 \cos\theta_2 - \dot{\theta}_2^2 \sin\theta_2) \quad (17)$$

Center of Gravity Acceleration Equations

$$\boldsymbol{\ddot{X}_{3(cg)} = \ddot{X}_3 - (1-\rho_3)L_3(\ddot{\theta}_3 \sin\theta_3 + \dot{\theta}_3^2 \cos\theta_3)} \quad \textbf{(18)}$$

$$\ddot{Y}_{3(cg)} = \ddot{Y}_3 + (1-\rho_3)L_3(\ddot{\theta}_3 \cos\theta_3 - \dot{\theta}_3^2 \sin\theta_3) \quad (19)$$

Force Equations

$$\boldsymbol{F_{(3)x} = F_{(4)x} - m_3\ddot{X}_{3(cg)}} \quad \textbf{(20)}$$

$$F_{(3)y} = F_{(4)y} + m_3(\ddot{Y}_{3(cg)} + g)^1 \quad (21)$$

Moment Equations

$$\boldsymbol{M_3 = M_{LB1} + I_3\ddot{\theta}_3 + F_{(3)y}(1-\rho_3)L_3 \cos\theta_3 + F_{(3)x}(1-\rho_3)L_3 \sin\theta_3 + F_{(4)y}\rho_3 L_3 \cos\theta_3 + F_{(4)x}\rho_3 L_3 \sin\theta_3} \quad \textbf{(22)}$$

[1] m_3 considers the mass of both the trunk and the head

2.1.4 Torso to Shoulder (LB1)

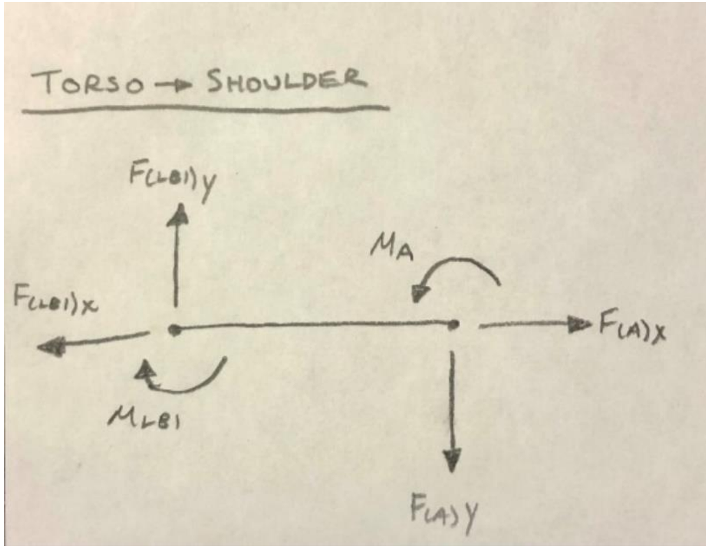

Figure 5: moments, forces, and accelerations on the torso and shoulder

Point Acceleration Equations

$$\boldsymbol{\ddot{X}_{LB1} = \ddot{X}_3 - L_3(\ddot{\theta}_3 \sin\theta_3 + \dot{\theta}_3^2 \cos\theta_3)} \tag{23}$$

$$\ddot{Y}_{LB1} = \ddot{Y}_3 + L_3(\ddot{\theta}_3 \cos\theta_3 - \dot{\theta}_3^2 \sin\theta_3) \tag{24}$$

Center of Gravity Acceleration Equations

$$\boldsymbol{\ddot{X}_{LB1(cg)} = \ddot{X}_{LB1} - \rho_{LB1} L_{LB1}(\ddot{\theta}_{LB1} \sin\theta_{LB1} + \dot{\theta}_{LB1}^2 \cos\theta_{LB1})} \tag{25}$$

$$\ddot{Y}_{LB1(cg)} = \ddot{Y}_{LB1} + \rho_{LB1} L_{LB1}(\ddot{\theta}_{LB1} \cos\theta_{LB1} - \dot{\theta}_{LB1}^2 \sin\theta_{LB1}) \tag{26}$$

Force Equations

$$\boldsymbol{F_{(LB1)x} = F_{(A)x}} \tag{27}$$

$$F_{(LB1)y} = F_{(A)y} \tag{28}$$

Moment Equations

$$\boldsymbol{M_{LB1} = L_{LB1} F_{(LB1)y} - M_A} \tag{29}$$

2.1.5 Arm (A)

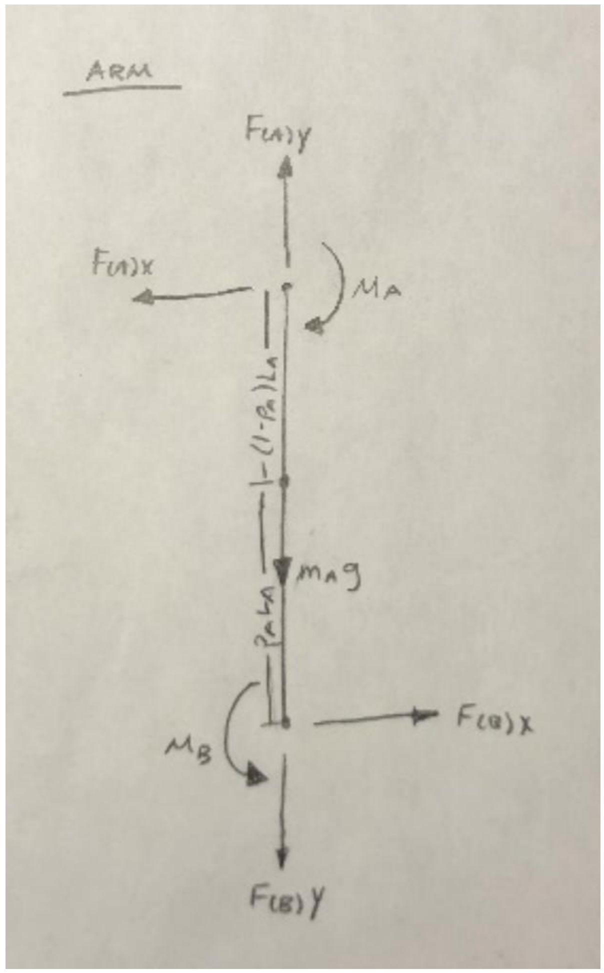

Figure 6: moments, forces, and accelerations on the arm & bar

Point Acceleration Equations

$$\boldsymbol{\ddot{X}_A = \ddot{X}_{LB1} - L_{LB1}(\ddot{\theta}_{LB1}\sin\theta_{LB1} + \dot{\theta}_{LB1}^2\cos\theta_{LB1})} \tag{30}$$

$$\ddot{Y}_A = \ddot{Y}_{LB1} + L_{LB1}(\ddot{\theta}_{LB1}\cos\theta_{LB1} - \dot{\theta}_{LB1}^2\sin\theta_{LB1}) \tag{31}$$

Center of Gravity Acceleration Equations

$$\ddot{\boldsymbol{X}}_{\boldsymbol{A}(\boldsymbol{cg})} = \ddot{\boldsymbol{X}}_{\boldsymbol{A}} - \boldsymbol{\rho}_{\boldsymbol{A}}\boldsymbol{L}_{\boldsymbol{A}}(\ddot{\boldsymbol{\theta}}_{\boldsymbol{A}} \sin \boldsymbol{\theta}_{\boldsymbol{A}} + \dot{\boldsymbol{\theta}}_{\boldsymbol{A}}^{\boldsymbol{2}} \cos \boldsymbol{\theta}_{\boldsymbol{A}}) \qquad \textbf{(32)}$$

$$\ddot{Y}_{A(cg)} = \ddot{Y}_A + \rho_A L_A(\ddot{\theta}_A \cos \theta_A - \dot{\theta}_A^2 \sin \theta_A) \qquad (33)$$

Force Equations

$$\boldsymbol{F}_{(\boldsymbol{A})\boldsymbol{x}} = \boldsymbol{F}_{(\boldsymbol{B})\boldsymbol{x}} - \boldsymbol{m}_{\boldsymbol{A}}\ddot{\boldsymbol{X}}_{\boldsymbol{A}(\boldsymbol{cg})} \qquad \textbf{(34)}$$

$$F_{(A)y} = F_{(B)y} + m_A(\ddot{Y}_{A(cg)} + g) \qquad (35)$$

Moment Equations (assuming ϴ = 90°)

$$\boldsymbol{M}_{\boldsymbol{A}} = \boldsymbol{I}_{\boldsymbol{A}}\ddot{\boldsymbol{\theta}}_{\boldsymbol{A}} + \boldsymbol{M}_{(\boldsymbol{B})} + \boldsymbol{\rho}_{\boldsymbol{A}}\boldsymbol{L}_{\boldsymbol{A}}\boldsymbol{F}_{(\boldsymbol{B})\boldsymbol{x}} + (\boldsymbol{1} - \boldsymbol{\rho}_{\boldsymbol{A}})\boldsymbol{L}_{\boldsymbol{A}}\boldsymbol{F}_{(\boldsymbol{A})\boldsymbol{x}} \qquad \textbf{(36)}$$

2.1.6 Bar (B)

Point Acceleration Equations

$$\ddot{\boldsymbol{X}}_{\boldsymbol{B}} = \ddot{\boldsymbol{X}}_{\boldsymbol{A}} - \boldsymbol{L}_{\boldsymbol{A}}(\ddot{\boldsymbol{\theta}}_{\boldsymbol{A}} \sin \boldsymbol{\theta}_{\boldsymbol{A}} + \dot{\boldsymbol{\theta}}_{\boldsymbol{A}}^{\boldsymbol{2}} \cos \boldsymbol{\theta}_{\boldsymbol{A}}) \qquad \textbf{(37)}$$

$$\ddot{Y}_B = \ddot{Y}_A + L_A(\ddot{\theta}_A \cos \theta_A - \dot{\theta}_A^2 \sin \theta_A) \qquad (38)$$

Force Equations

$$\boldsymbol{F}_{(\boldsymbol{B})\boldsymbol{x}} = -\boldsymbol{m}_{\boldsymbol{B}}\ddot{\boldsymbol{X}}_{\boldsymbol{B}} \qquad \textbf{(39)}$$

$$F_{(B)y} = m_B(\ddot{Y}_B + g) \qquad (40)$$

2.2 Data

To solve for the accelerations, forces, and moments of segment, biometric data about the lifter is required alongside time & position data of the lifter.

2.2.1 Time & Position Data

More specifically, the time and position data consist experimental values for a deadlift completed by an elite weightlifter. The concentric portion of a single repetition has been divided into 5 intervals. Position data, the segments angle with respect to horizontal, was recorded during each time interval. Position data is crucial as the placement of the external resistance has a large influence on the magnitude of joint loading [2].

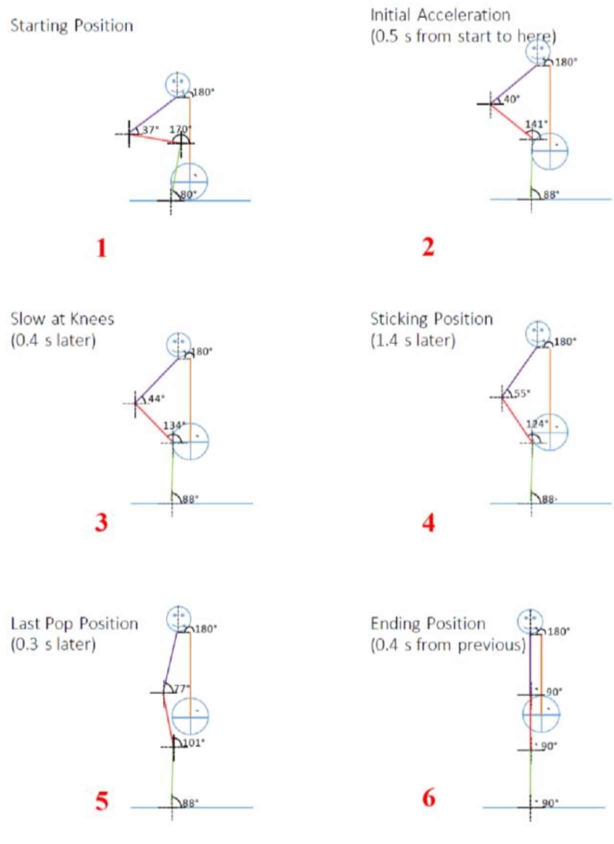

Figure 7: Elite lifter performing a deadlift

2.2.2 Biometric Data

In addition, to time and position values, biometric data for the lifter is required to solve for the force, moments, and accelerations. The length and mass of each segment are shown in table 2.

	Total Body Mass	Total Height
	180 kg	2.1 m
Limb	% of Total Body Mass	Length (m)
Ankle to Knee	0.043	0.48
Knee to Hip (Thigh)	0.11	0.44
Hip to Shoulder (Torso)	0.53	0.55
Segment B1	0	0.12
Shoulder to Hand (Arm)	0.049	0.69
Head	0.07	0

Table 2: Mass and dimensions of segments of an elite lifter

Additionally, the proximal distances to the center of mass of each segment are defined by table 3[2].

P1	*0.4459*
P2	*0.4095*
P3	*0.4486*
***P4*[3]**	*0.5000*
***P5*[4]**	*0.5280*

[2] Adolphe, M., Clerval, J., Kirchof, Z., Lacombe–Delpech, R., 2017, "Center of Mass of Human's Body Segments", Lodz University of Technology, 21(3), pp. 485–497, Table 1
[3] The shoulder (LB1) has no mass, however, we will still set a "proximal distance" based of 0.5
[4] Found using equation (50)

Table 3: Proximal distance to center of mass for a given segment

2.2.3 Additional Initial Conditions

Since the ankle is assumed to be static throughout the movement the acceleration of the joint is defined by the following equation.

$$\boldsymbol{x_1 = 0} \quad \textbf{(41)}$$

$$y_1 = 0 \quad (42)$$

Additionally, the movement is conducted from a standstill in which case:

$$\boldsymbol{\omega_1 = 0} \quad \textbf{(43)}$$

$$\omega_2 = 0 \quad (44)$$

$$\boldsymbol{\omega_3 = 0} \quad \textbf{(45)}$$

$$\omega_{lb1} = 0 \quad (46)$$

$$\boldsymbol{\omega_a = 0} \quad \textbf{(47)}$$

2.3 Additional Equations

$$\boldsymbol{\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2} \quad \textbf{(48)}$$

$$\omega_f = \omega_i + \alpha t \quad (49)$$

$$\boldsymbol{CM_{total\ arm} = \frac{m_{upper\ arm} * CM_{upper\ arm} + m_{forearm} * CM_{forearm}}{m_{upper\ arm} + m_{forearm}}} \quad \textbf{(50)}$$

$$\boldsymbol{shear = Fx_i \sin\theta + Fy_i \cos\theta - m_i g \cos\theta - M_i * \sqrt{x_i^2 + y_i^2}\cos\theta} \quad (51)$$

$$compression = Fx_i \cos\theta - Fy_i \sin\theta + m_i g \sin\theta + M_i * \sqrt{x_i^2 + y_i^2} \sin\theta \qquad (52)$$

3 Method

With the biometric data, initial conditions, and position/time values one can go about solving for the accelerations, moments, and forces for all the segments and joints. To do so, the following procedure must be followed in a sequential manner, as shown below.

3.1 Accelerations

1. Find the angular acceleration for each segment for each interval an equation from rotational kinematics. With the initial and final joint angle, time interval, and initial angular velocity one can solve for angular acceleration using the equation 48.

2. Find the angular velocity for each segment using equation 49. Use the angular acceleration calculated above, initial angular velocity, and duration of the interval.

3. Solve for the acceleration of each joint using the ***point acceleration equations***. Plug in the initial linear accelerations (initial conditions), angular velocities, accelerations, and initial and final joint angles during each interval.

   Note, the initial condition enables one to solve equations in the from i = {2 → 6}. In other words, solve for the acceleration of the knee first and end with the bar.

4. Solve for the acceleration of the center of mass of each segment using ***center of gravity acceleration equations***. Plug in the initial linear accelerations, angular velocities, and accelerations found above, the final joint angle during an interval, and the proximal distance values.

Note: Solve equations in the from i = {2→6}. In other words, solve for the acceleration of the knee first and end with the bar.

5. Repeat steps 1-4 for each interval

3.2 Forces

1. Calculate the force exerted on each joint by applying Newton's 2nd Law to all segments. Creating a free body diagram for each segment aided in deriving the individual ***force equations***. This included forces at the segment ends as well as the force of gravity acting on the segment's mass center, save for segment "LB1" which was deemed massless.

2. Since complementary forces were used at the segment ends, their direction can be taken as absolute in each iteration of segment analysis. Repeat step 1 for each segment, then each interval.
   Note: Solve equations in the from i = {6 → 2}. In other words, solve for the force of the bar first and end with the shank.

3.3 Moments

1. Calculated the moment at each joint by applying Newton's 2nd Law for moments on all segments. Use the ***moment equations*** defined *2.0 Background.*
Note: Solve equations in the from i = {6 → 2}. In other words, solve for the moment of the bar first and end with the shank.

2. Repeat step 1 for each interval.

4 Results

4.1 Interval 1→2 (0.5s)

Table 4 shows the results of the linear accelerations, forces, and moments of the first interval of the deadlift, the initial acceleration.

	x (m/s^2)	y (m/s^2)	Fx (N)	Fy (N)	Moment (Nm)
shank	0.000000	0.000000	-2650.469259	3011.358261	5615.325797
thigh	-0.537468	-0.027963	-2652.774319	2935.548788	3858.719276
torso	6.328837	-3.295656	-2583.136036	2780.069995	1191.240817
shoulder	6.179939	-3.119852	-1908.488701	2066.051494	1478.265022
arm	6.179939	-3.119852	-1908.488701	2066.051494	1404.078063
bar	6.179939	-3.119852	-1853.981640	2007.044389	0.000000

Table 4: forces, moments, and accelerations from 0-0.5s

4.2 Interval 2→3 (0.4s)

Table 5 shows the results of the linear accelerations, forces, and moments of the second interval of the deadlift in which the bar slowly crosses the knees.

	x (m/s^2)	y (m/s^2)	Fx (N)	Fy (N)	Moment (Nm)
shank	0.000000	0.000000	-411.879774	4814.426215	3188.291191
thigh	0.059854	-0.002090	-411.623077	4738.505779	3019.286302
torso	1.245159	0.776316	-396.579504	4535.208112	598.030476
shoulder	0.912810	1.109186	-281.894098	3372.063133	2412.719622
arm	0.912810	1.109186	-281.894098	3372.063133	2291.636914
bar	0.912810	1.109186	-273.843111	3275.755910	0.000000

Table 5: forces, moments, and accelerations from 0.5-0.9s

4.3 Interval 3→4 (1.4s)

Table 6 shows the results of the linear accelerations, forces, and moments of the third interval of the deadlift which is commonly known as the "sticking position".

	x (m/s^2)	y (m/s^2)	Fx (N)	Fy (N)	Moment (Nm)
shank	0.000000	0.000000	-212.823638	4465.180078	2682.791906
thigh	0.000000	0.000000	-212.823638	4389.250678	2642.592092
torso	0.507825	0.239638	-206.886201	4192.210851	868.781105
shoulder	0.494834	0.248653	-152.814722	3106.313099	2222.574807
arm	0.494834	0.248653	-152.814722	3106.313099	2111.034546
bar	0.494834	0.248653	-148.450284	3017.595783	0.000000

Table 6: forces, moments, and accelerations from 0.9-2.3s

4.4 Interval 4→5 (0.3s)

Table 7 shows the results of the linear accelerations, forces, and moments of the fourth interval of the deadlift is the most explosive as is referred to as the "last pop".

	x (m/s^2)	y (m/s^2)	Fx (N)	Fy (N)	Moment (Nm)
shank	0.000000	0.000000	63.352611	-13172.699174	-7449.510647
thigh	0.000000	0.000000	63.352611	-13248.628574	-7723.996580
torso	8.344797	-22.055078	160.919140	-13185.000809	-5909.494314
shoulder	-1.534361	-43.992209	473.841470	-10556.149892	-7552.951701
arm	-1.534361	-43.992209	473.841470	-10556.149892	-7173.905650
bar	-1.534361	-43.992209	460.308403	-10254.662805	0.000000

Table 7: forces, moments, and accelerations from 2.3-2.6s

4.5 Interval 5→6 (0.4s)

Table 8 shows the results of the linear accelerations, forces, and moments of the final stage of the deadlift.

	x (m/s^2)	y (m/s^2)	Fx (N)	Fy (N)	Moment (Nm)
shank	0.000000	0.000000	-864.481367	3574.111455	3458.454673
thigh	-0.209440	-0.000445	-865.379598	3498.183966	2861.787974
torso	2.125643	-1.827501	-842.224950	3325.316540	2271.254654
shoulder	2.006780	-1.832791	-619.733922	2463.521688	1762.655941
arm	2.006780	-1.832791	-619.733922	2463.521688	1674.196780
bar	2.006780	-1.832791	-602.034119	2393.162704	0.000000

Table 8: forces, moments, and accelerations from 2.6-3s

5 Discussion

5.1 Dynamics of an Elite Lifter

Use the set of ordinary differential equations for accelerations, forces and moments that you have developed and apply them to this lifter. Discuss what happens at the various stages during the deadlift. Use your model to discuss the following:

i. What were the periods of acceleration, constant velocity, and deceleration of the bar? Are these consistent with what you expect for someone doing a deadlift type action?

Figure 8 shows a high horizontal acceleration during the first lifting interval. These results indicate that the bar is moving away from the lifter. Following this peak, the bar begins to decelerate, while the lifter enters stages two and three. During the interval known as the "last pop" stage the bar starts to accelerate towards the lifter's body. Finally, in the final interval stage, the bar accelerates away from the lifter.

This pattern of motion would be consistent with that of a successful deadlift, especially in the popping and resting positions. In the popping stage, the lifter compresses rapidly, bringing their hips in and the bar slightly inward to explode up. This scenario is reflected in the high acceleration of the hip joint, which is 8.34m/s^2 inward as shown in Table 7. At the end of the movement the bar nears a stationary position in the lifter's hands as he begins to solely raise it, bringing his glutes inward and bringing the bar to his waist.

Since the bar travels on a relatively vertical path, analyzing the velocity in the x direction is not helpful. Conversely the velocity of the bar is worth analysis in the y direction.

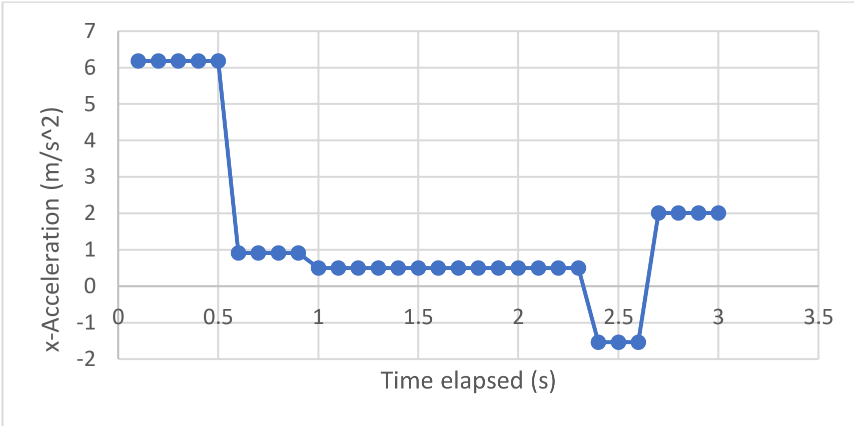

Figure 8: Horizontal Acceleration of Bar Throughout Deadlift Movement

The vertical behaviour of the bar's motion is much more relevant in the analysis of a successful deadlift simply because of the nature of the lift – the entire goal is to lift the bar straight up, with intent. The obtained vertical acceleration values are key in monitoring progression in this lift. For example, these metrics can help to determine if a lift has improved in ease by comparing vertical accelerations with past lifts of equal weight.

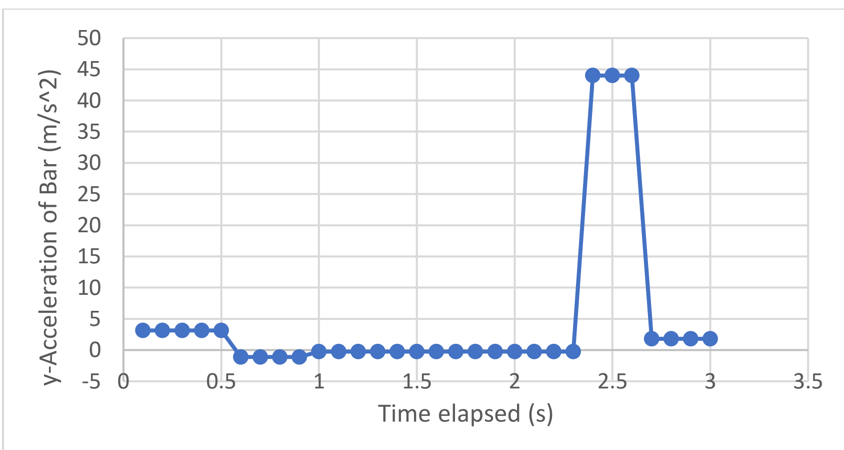

Figure 9: Vertical Acceleration of Bar Throughout Deadlift Movement

In the initial acceleration stage, it can be seen in figure 9 that the bar is lifted quickly off the ground. Acceleration quickly reduces and then remains constant throughout the two proceeding intervals until the sticking position. For the duration of the sticking position, the bar is decelerating downwards. Interestingly, this aligns with the name of the interval: “sticking position”, as this is the slowest stage in the entire movement, and lifters have the greatest tendency to fail completion of the exercise at this stage. It is in the following interval where the bar begins to experience massive vertical acceleration, the “final pop stage”.

This would align with the linked model of the elite lifter, since the hip displaces a large distance in this step, in a small amount of time. It can later be seen that the bar begins to decelerate in the closing interval. This pattern aligns with the progression of a deadlift in many ways. The quick lift off the grounds, the slow lift along the thighs, then the quick pop of the bar once the bar clears the knees, followed by the return to neutral stance at the very top.

The values obtained from the linked segment model also align with vertical velocity values obtained from a weightlifting competition as seen in Figure 10[5].

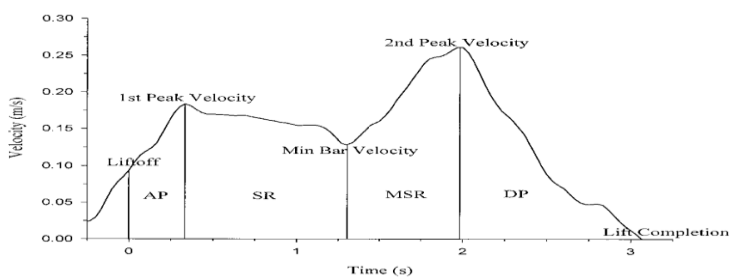

Figure 10: Vertical Velocity of the Bar Throughout Deadlift Movement

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[5] Escamilla, R. F., Francisco, A. C., Fleisig, G. S., Barrentine, S. W., Welch, C. M., Kayes, A. V., Andrews, J. R., 2000, “A three-dimensional biomechanical analysis of sumo and conventional style deadlifts,” Medicine & Science in Sports & Exercise, 32(7), pp. 1265–1275, Figure 3

ii. How much work did the lifter do in lifting the bar from the ground?

In lifting the bar from the ground, the lifter exerts **2348 joules.** The work adds gravitational potential energy to the bar and the lifter himself. The work done by the lifter can be given by the equation 55. In lifting the bar from the ground, the lifter elevates the mass center of the bar as well as numerous body segments including the shank, thighs, torso, and arm. The total work done by the lifter is determined by the summation of these changes in potential energy. The lifter does work on five total components, each of which has a center of mass denoted at the $h_{(i)}$ locations as seen in Figure x. In the starting position, the heights of the individual mass centers can be determined using equation 53 and similarly the heights of the segment themselves can be found using the same equation. Upper segments heights can simply be stacked on top of any prior segment(s) height, given each reference of theta is with respect to the horizontal. In the final stage, a simplified equation can be used to determine the mass centers' heights, given that all present joint angles are 90°. Any further assumptions and/or calculations are included in the appendix.

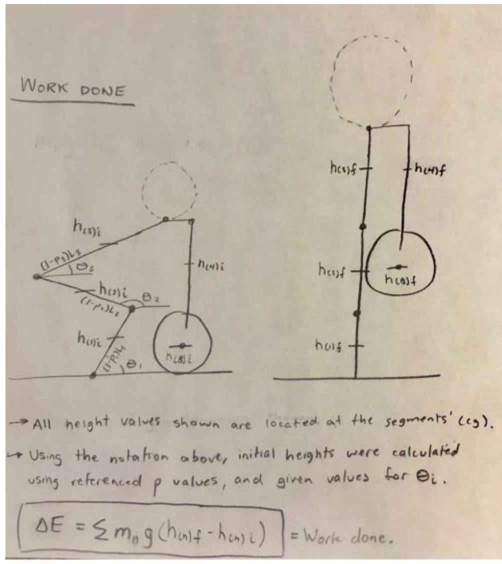

Figure 11: Work done by dead lifter through potential Energy

$$\boldsymbol{h_{n,cg} = \sum_{i=2}^{n} L_{n-1} \sin \theta_{n-1} + L_n(1-\rho_n) \sin \theta_n \;\; for\; n = 1, 2, 3} \tag{53}$$

$$\Delta h_{bar} = \Delta h_{head} = \Delta h_{arm} \tag{54}$$

$$W = \Delta E = mg\Delta h \tag{55}$$

iii. What part(s) of the lifter's body experiences the maximum forces and at what stage during the lift?

Throughout the pop stage, the lifter experiences the largest forces in the segments linked by the hip joint. Each of the thigh segment and torso segment experience some of the largest forces produced throughout the entire deadlift motion. However, the lifter has two legs so the forces in this region will be distributed evenly to each leg, reducing the force by a factor of 2. There exist similarly high forces on the shoulder and arm segments during the final pop stage as well. The trend is that the largest forces in the motion can be found in the popping stage, which includes the largest displacement of the bar in the shortest time period. Results from past studies have demonstrated that fast repetitions produce the greatest force, power, rate of force development, and muscle recruitment [4]. This is consistent with what the linked segment model results suggest.

iv. What part(s) of the lifter's body experiences the maximum moments and at what stage during the lift?

The absolute maximum moment experienced by the lifter throughout the deadlift motion occurs in the same time interval as that of the largest forces, the final pop stage. In this instance, the lifter experiences the largest moment of their shoulder joint, denoted as joint LB1. This joint is coincident with the uppermost end of the torso. It is in this location that the lifter experiences not only the absolute largest moment, but also the largest moment relative to other joints, throughout each stage of the lift, save for the final stage. This is likely the reason inexperienced lifters tend to round their upper back when nearing failure. The mechanism will relieve excessive moment off the upper back, and transfer it onto the spine, which can be cause for injury.

v. Based on your answers to iii) and iv), what kinds of injury would you expect to happen during a deadlift?

Low back structures are challenged with the largest loads during weightlifting or powerlifting competition, especially in the deadlift [5]. Based on the loading patterns of the deadlift motion, the most likely spot for injury is along the athlete's back. This is due to the extreme compression and shearing of the spine, caused by large forces on both the hip and collar joints. The extreme moments will also contribute in deforming the athlete's back, thus destroying proper form. Any deformation in the back will increase the shear along the spine, and can become very dangerous. The back injury could take place at virtually any location along the length of the torso depending on how the athlete reacts to the extreme loading.

Workplace health and safety guidelines indicate the spine should not regularly be subjected to levels of force more than a maximum of 3433N for compression of the spine and recommend an absolute maximum of 6376N. In terms of shear forces on the spine, they indicate maximum levels of 500N and absolute maxima of 1000N.

vi. Remembering that you are using a 2-D model in your analysis, what are the compression and shear forces on the spine for the case of this elite lifter?

	Interval 12	**Interval 23**	**Interval 34**	**Interval 45**	**Interval 56**
Shear	932.700289	2110.746529	1592.604847	3620.409076	842.224950
Compression	2589.416142	2589.634617	2635.170522	16397.066362	1963.087308

Table 9: Traditional Deadlift Shear and Compression Forces Per Interval

Extremely high compressive and shear forces have been reported in the spinal column with deadlifting [6]. The linked segment model indicates the max compression force on the spine is 9.36kN, max shear on the spine is 2.22kN. The maximum spinal compression and tension values were determined by accounting for the forces at the hip and collar joints, as well as the force of gravity acting on the center of mass of the spine/ torso segment. The maximum compression force was determined by summing the projection of each of the three listed forces on the torso segment. Similarly, the maximum shear force was determined by summing the projection of each of the end forces and gravitational force on the segments normal.

vii. What would be the maximum weight for this lifter to ensure staying below these levels?

After running various test trials through the linked segment model, it quickly became apparent that the limiting factor of safely conducting the lift is the shear force acting on the spine during the slow at knees stage. To ensure the shear force in the spine does not exceed the recommended 1kN at this stage, the maximum weight for this lifter should be no more than 130.5kg, less than half of the speculated weight. Figure x contains all shear and compression metrics acting on the spine throughout the deadlifting process given a bar weight of 130.5kg. At no point does the spinal compression exceed the absolute maximum limitation of 6376N, using the bar weight indicated.

	Interval 12	**Interval 23**	**Interval 34**	**Interval 45**	**Interval 56**
Shear	537.7187911	1000.865175	718.1809	624.6570991	279.5846
Compression	1553.431323	1302.746654	1240.133	3655.970419	913.6996

Table 10: Spinal shear and compression forces during the deadlift given a bar weight of 130.5kg.

viii. What would we improve going forward.

We should have verified our python scripts with more hand calculations. On the other hand, we should have used a denser set of data for position/time of the movement. 6 intervals were insufficient, and there are a lot of unknowns between these time intervals.

Another ay we could have improved our biomechanical analysis of the deadlift is by using a larger sample size of participants. This would have allowed us to gather more data and potentially identify any trends or patterns that may not have been evident with a smaller sample size. Additionally, using a wider range of participants with different body types and fitness levels would have provided a more comprehensive view of the biomechanics of the deadlift. We could also have used more advanced technology, such as 3D motion capture, to track and measure the movement of the body during the deadlift more accurately. By incorporating these improvements, our biomechanical analysis of the deadlift would have been more robust and informative.

5.2 Dynamics of Other Lifters

This part of your project is open ended for you to use your model to solve for different situations. Each member in your group should pick a different situation and write an individual discussion on what might happen in their case. Set up and analyze any additional cases that you choose (the minimum number is one additional case for each member of your group). Some examples are outlined below.

5.2.1 Relationship between the height of the lifter and safe one rep max as limited by compression and shear forces in the spine

5.2.1.1 Introduction

In a sample population, the body composition of the individuals will vary. The deadlift linked segment model can still be applied to lifters of any size. For this analysis, the metrics used for the 'Other Lifters' are simply scaled versions of the Elite Lifter. Realistically, the relative proportions would change slightly as overall segment length is scaled, but for the purpose of this analysis, this aspect will be ignored. This will result in projected data being purely theoretical. Despite this simplification, the scaling of lifter size vs conceivable one rep max will provide insight on whether extra caution should be taken by lifters from certain size demographics.

95% of males have a height in the range of 163.2cm to 193.6cm [7]. With respect to the anatomy of the elite lifter, this corresponds to scale values between 77.7% and 92.2%.

5.2.1.2 Methods and Results

To find a correlation between one metric and another, all other confounding variables should be ignored or held constant. In this case, the two metrics that are to be compared are lifter size, and one rep max, such that the compression in the spine does not exceed the recommended maximum. Thus, the shear in the spine was ignored throughout this analysis, and the compression force in the spine was forcibly held constant at 6367N. Also, the mass of the lifter was held constant.

For this analysis, the body composition of the elite lifter was used as a base model. Lifter size was determined by taking the segment lengths of the elite lifter, and scaling each of them equally, as seen in Figure x. Similarly, the lifter's height is defined as the elite lifter's height multiplied by the lifter's personal scale value. Determining height is important in deciphering a real-world range from a purely mathematical range in the data.

```
scale = 1

#segment dimensions
L_1 = scale*0.48
L_2 = scale*0.44
L_3 = scale*0.55
L_lb1 = scale*0.12
L_a = scale*0.69
```

Figure 12: Lifter size scaling method

After running the linked segment model with scale values ranging from 50% to 200%, the data showed a declining logarithmic curve, as seen in figure 12. As the lifter's size decreases, their one rep max increases non-linearly. By discarding two outliers from each end of the extreme and using curve-fitting software, the lifter's one rep max in kilograms can be determined as a power function of their height in meters, as given by equation 56.

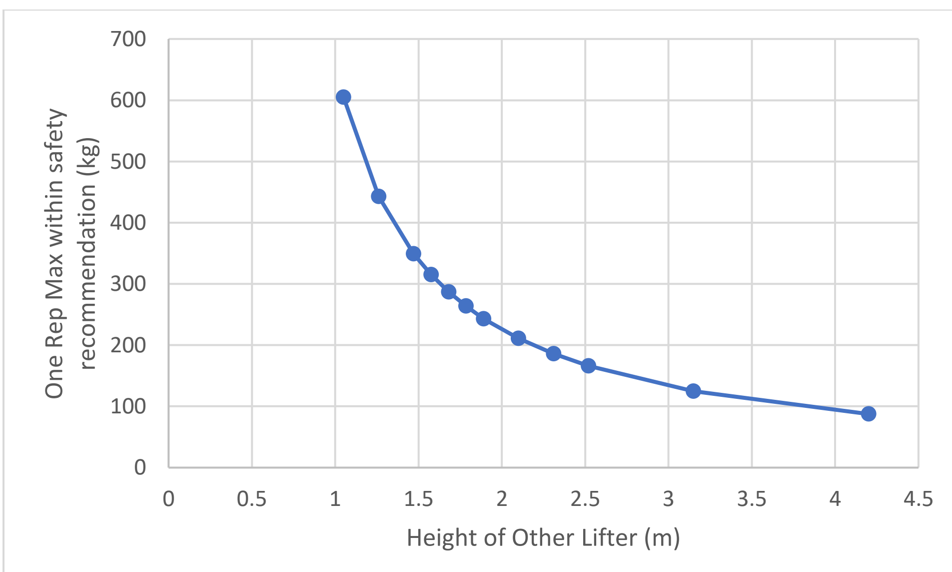

Figure 13: Safe One Rep Max vs. Height of 'Other Lifter's' dimension

$$\boldsymbol{One\ Rep\ Max = 50.6 + \frac{588}{h^{1.756}}} \qquad \textbf{(56)}$$

5.2.1.3 Discussion

In the preceding analysis various parameters were controlled to isolate the effect of the lifter's size on their one rep max within safety recommendations. At constant mass, constant spinal compression, and disregarding spinal shear, there exists a clear correlation between the assessed parameters.

The lifter's overall size increases, their capability to perform a massive one rep max diminishes. It is important that larger lifter's take note of this and are particularly cautious when performing the deadlift. They should also realize that, when lifting equivalent weights, they will be subject to increased spinal compression relative to someone smaller.

This is consistent with the results that were expected of this analysis for various reasons. Firstly, a larger lifter will experience larger moments of their joints since moment is the direct product of force and moment arm. An increased segment length will always result in a larger moment arm, which will propagate through each segment in the lifter's body. Also, the larger lifter will be required to produce larger accelerations in each segment to complete a lift in the same time frame and orientation of a smaller lifter. This is because mass center acceleration is a function of angular acceleration and radius, in this case, limb length. The angular acceleration values will remain constant across both lifters, but the segment length of larger lifters will evidently increase. This requirement to facilitate increased acceleration will in turn increase the forces acting on the joints of the larger lifters, in accordance with Newton's 2nd Law.

To extrapolate further, the larger lifter will be required to do more work in lifting bar, as each of their segment mass centers will be elevated from those of a smaller person. As the length of each segment increases and the angle joint angle remains constant, the individual center of mass heights will increase as well as the stacking heights in accordance with equation 56.

5.2.2 Iso Topic Kostubh – Sumo vs Traditional

5.2.2.1 Introduction

The technique used during a deadlift can have a significant impact on the shear and compression forces on the spine. Proper technique can help to distribute these forces evenly across the spine, reducing the risk of injury.

Sumo and traditional deadlifts are two different variations of the deadlift exercise, which distribute loads on the spine different. The main difference between the two styles is the position of the feet and hands. In a sumo deadlift, the feet are positioned wider apart, and the hands are placed inside the legs, closer to the body. These metrics are well defined in figure 13 a traditional deadlift, the feet are placed closer together and the hands are positioned outside the legs.

TABLE 1. Joint and segment angles (mean ± SD) between sumo and conventional deadlifts.

	Sumo (mean ± SD)		Conventional (mean ± SD)		
	Current	McGuigan and Wilson (17)	Current	McGuigan and Wilson (17)	Brown and Abani (2)
Liftoff (LO)					
Hip (°)	77 ± 7	77 ± 9**	72 ± 12	67 ± 5**	69 ± 5
Knee (°)	126 ± 8	127 ± 8	124 ± 9	120 ± 10	123 ± 6
Trunk (°)	33 ± 11*	25 ± 8*	24 ± 10*	17 ± 7*	23 ± 6
Thigh (°)	145 ± 9**	136 ± 6	134 ± 7**	137 ± 5	134 ± 4
Shank (°)	80 ± 4**	63 ± 11	76 ± 5**	59 ± 5	76 ± 3
Knee passing (KP)					
Hip (°)	106 ± 8*	128†	114 ± 11*	122†	134 ± 18
Knee (°)	153 ± 8*	168†	161 ± 8*	165†	165 ± 6
Trunk (°)	46 ± 7	48†	43 ± 7	43†	59 ± 14
Thigh (°)	126 ± 9**	106†	110 ± 5**	107†	105 ± 4
Shank (°)	79 ± 5**	70†	84 ± 2**	63†	90 ± 3
LO to KP range					
Hip (°)	30 ± 8**	51 ± 9	42 ± 12**	56 ± 11	65 ± 14
Knee (°)	28 ± 8*	41 ± 11	37 ± 14*	45 ± 10	42 ± 7
Trunk (°)	13 ± 10	23 ± 5	19 ± 8	26 ± 12	37 ± 13
Thigh (°)	19 ± 5	29 ± 8	24 ± 9	29 ± 5	28 ± 5
Shank (°)	−1 ± 4**	7 ± 3*	8 ± 4**	4 ± 3*	14 ± 4
Minimum bar velocity (sticking point)					
Hip (°)	111 ± 22		123 ± 17		
Knee (°)	152 ± 15*		164 ± 8*		
Trunk (°)	51 ± 13		50 ± 13		
Thigh (°)	126 ± 13**		107 ± 6**		
Shank (°)	77 ± 5**		83 ± 2**		
Foot angle (°)	42 ± 8**		14 ± 6**		

Significant differences between sumo and conventional deadlifts (** $p < 0.01$; * $p < 0.05$).
† Not statistically analyzed.

Figure 14: Sumo vs Traditional Deadlift Joint Angles (6)

5.2.2.2 Method and Results

1. Define a position and time schema for sumo squat

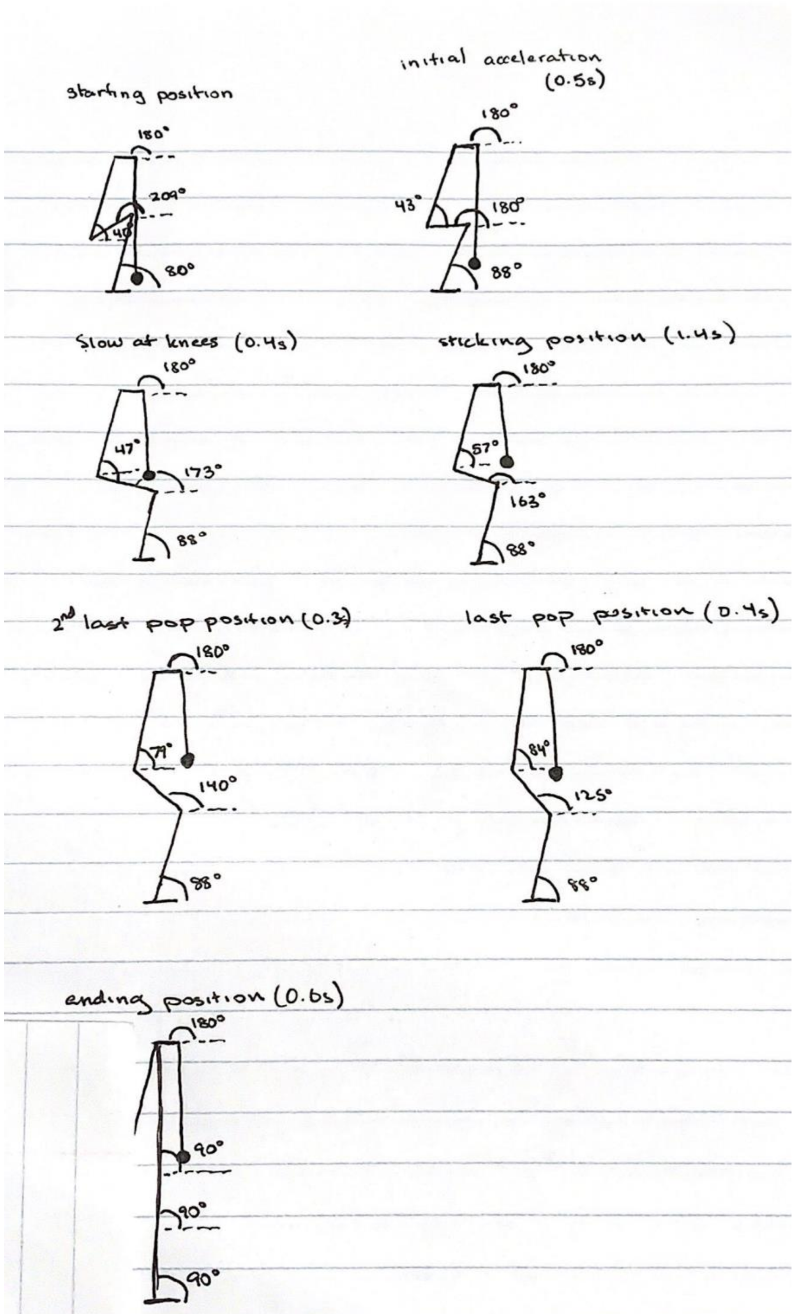

Figure 15: Sumo Deadlift estimated position and time model

2. Complete the acceleration, force, and moment analysis on the new model. Follow the procedure outlined in *3 Method*, same as that required in the analysis of the elite lifter.

Assumptions:

- Make all the same assumptions as the analysis of the elite lifter
- Use the same initial conditions
- Assume an identical body composition
- Assume both lifts are being conducted with a 300kg weight

3. Calculate shear and compression forces on the spine during the individual intervals. Compare values with the traditional deadlift shear and compression values.

	Interval 12	**Interval 23**	**Interval 34**	**Interval 45**	**Interval 56**	**Interval 67**
Shear	327.523320	2367.352015	1648.770991	1865.423049	1291.240137	742.265681
Compression	4132.802297	2650.170734	2722.086811	1336.389805	2287.145069	5419.612588

Table 11: Sumo Deadlift Shear and Compression Forces Per Interval[6]

	Interval 12	**Interval 23**	**Interval 34**	**Interval 45**	**Interval 56**
Shear	932.700289	2110.746529	1592.604847	3620.409076	842.224950
Compression	2589.416142	2589.634617	2635.170522	16397.066362	1963.087308

Table 12: Traditional Deadlift Shear and Compression Forces Per Interval

[6] Since the sumo deadlift takes more time to complete an additional interval was included in the analysis

5.2.2.3 Discussion

The sumo deadlift places less shear and compression forces on the back than the traditional deadlift because it allows for a more upright torso position.

As shown in table 11 and 12, the max compression force experienced during the sumo deadlift is 4312N whereas that of the traditional deadlift is upwards of 16000N.

Evidently the sumo deadlift is safer to conduct as a reduction in shear and compression forces reduce the tendency to deviate from a neutral spine position. This means keeping the spine in its natural alignment, without excessive rounding or arching, which ensures forces are distributed evenly across the spine.

Additionally, the sumo deadlift allows the bar to remain closer to the center of gravity of your body. This also contributes to a reduction in shear force on the spine.

Overall, proper technique is crucial for reducing the risk of injury and protecting the spine during a deadlift. By maintaining a neutral spine position, keeping the weight close to the body, and using a wide stance, you can help to distribute the shear and compression forces evenly across the spine, reducing the risk of injury.

6 Conclusion

In conclusion, our report provides a detailed analysis of the biomechanics of the deadlift exercise. We showed that the deadlift can be simulated using a linked segment model, and that by applying the principles of Newton's second law and rigid body dynamics, we were able to accurately calculate the moments, forces, and accelerations involved in the lift.

Our analysis also revealed the effects of the lift on the shear and compression experienced by the spine. We found that body composition factors, such as height, can significantly impact the amount of shear and compression experienced by the spine during a deadlift. Furthermore, we discovered that the sumo deadlift technique puts less shear and compression on the spine than the traditional deadlift, making it a potentially safer option for individuals who are at a higher risk of spinal injuries.

Overall, our findings provide valuable insights into the biomechanics of the deadlift and can help people better understand and optimize their technique for this popular exercise. By using the information presented in our report, trainers and athletes can make informed decisions about which deadlift technique is best suited for their individual needs and goals. Additionally, our findings can help researchers and medical professionals develop more effective strategies for preventing and treating injuries related to the deadlift exercise.

7 References

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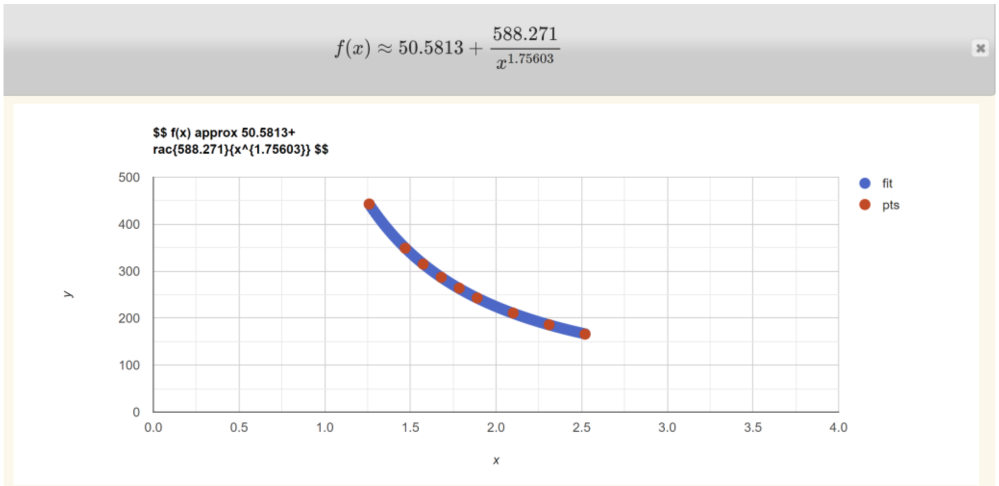
Appendices

Curve fitting of 5.2.1.3



Sumo Calculations

4.1 Interval 1-2 (0.5s)

	x_acc	y_acc	F_x	F_y	Moment
hank	0.000000	0.000000	-2872.655878	5172.094558	6880.757263
thigh	-0.537468	-0.027963	-2874.960937	5096.285085	5022.967308
torso	6.855079	1.753669	-2799.169891	4881.770094	1583.265763
shoulder	6.697184	1.921439	-2068.224235	3622.902953	2592.196142
arm	6.697184	1.921439	-2068.224235	3622.902953	2462.106377

bar	6.697184	1.921439	-2009.155075	3519.431662	0.000000

Table 13: forces, moments, and accelerations from 0-0.5s

4.2 Interval 2-3 (0.4s)

	x_acc	y_acc	F_x	F_y	Moment
shank	0.000000	0.000000	-82.524812	5053.159143	3870.793841
thigh	0.059854	-0.002090	-82.268115	4977.238707	3934.654283
torso	0.491142	1.348782	-76.040435	4767.247829	594.142361
shoulder	0.141828	1.663802	-43.799204	3543.339548	2535.268326
arm	0.141828	1.663802	-43.799204	3543.339548	2408.035492
bar	0.141828	1.663802	-42.548284	3442.140614	0.000000

Table 14: forces, moments, and accelerations from 0.5-0.9s

4.3 Interval 3-4 (1.4s)

	x_acc	y_acc	F_x	F_y	Moment

shank	0.000000	0.000000	-102.630039	4577.125430	3794.804307
thigh	0.000000	0.000000	-102.630039	4501.196030	3833.287476
torso	0.243845	0.505818	-99.779030	4301.044051	912.063204
shoulder	0.238795	0.509089	-73.744543	3186.740919	2280.121114
arm	0.238795	0.509089	-73.744543	3186.740919	2165.692882
bar	0.238795	0.509089	-71.638374	3095.726558	0.000000

Table 15: forces, moments, and accelerations from 0.9-2.3s

4.4 Interval 4-5 (0.3s)

	x_acc	y_acc	F_x	F_y	Moment
shank	0.000000	0.000000	-5370.343654	-8942.518205	126.003801
thigh	0.000000	0.000000	-5370.343654	-9018.447605	-3793.583093
torso	20.364835	-11.888464	-5132.240036	-9073.686873	-1340.561753
shoulder	11.253834	-34.168549	-3475.408903	-7522.406986	-5382.301050
arm	11.253834	-34.168549	-3475.408903	-7522.406986	-5112.189437
bar	11.253834	-34.168549	-3376.150090	-7307.564587	0.000000

Table 16: forces, moments, and accelerations from 2.3-2.6s

4.5 Interval 5-6 (0.4s)

	x_acc	y_acc	F_x	F_y	Moment
shank	0.000000	0.000000	-2557.252304	-105265.714713	-49339.912516
thigh	-0.209440	-0.000445	-2558.150534	-105341.642202	-51104.591161
torso	5.869374	-255.823522	-2491.224555	-102544.813556	-52850.167058
shoulder	5.990860	-255.827478	-1850.097498	-75975.117597	-54360.387036
arm	5.990860	-255.827478	-1850.097498	-75975.117597	-51632.302589
bar	5.990860	-255.827478	-1797.258110	-73805.243440	0.000000

Table 17: forces, moments, and accelerations from 2.6-3s

Acceleration vs time info

Time	x_accel		Time	y_accel
0			0	
0.1	6.179939		0.1	3.119852

0.2	6.179939		0.2	3.119852
0.3	6.179939		0.3	3.119852
0.4	6.179939		0.4	3.119852
0.5	6.179939		0.5	3.119852
0.6	0.91281		0.6	-1.10919
0.7	0.91281		0.7	-1.10919
0.8	0.91281		0.8	-1.10919
0.9	0.91281		0.9	-1.10919
1	0.494834		1	-0.24865
1.1	0.494834		1.1	-0.24865
1.2	0.494834		1.2	-0.24865
1.3	0.494834		1.3	-0.24865
1.4	0.494834		1.4	-0.24865
1.5	0.494834		1.5	-0.24865
1.6	0.494834		1.6	-0.24865
1.7	0.494834		1.7	-0.24865
1.8	0.494834		1.8	-0.24865
1.9	0.494834		1.9	-0.24865
2	0.494834		2	-0.24865
2.1	0.494834		2.1	-0.24865
2.2	0.494834		2.2	-0.24865
2.3	0.494834		2.3	-0.24865

2.4	-1.53436		2.4	43.99221
2.5	-1.53436		2.5	43.99221
2.6	-1.53436		2.6	43.99221
2.7	2.00678		2.7	1.832791
2.8	2.00678		2.8	1.832791
2.9	2.00678		2.9	1.832791
3	2.00678		3	1.832791

Max forces and moments found using excel max and min functions

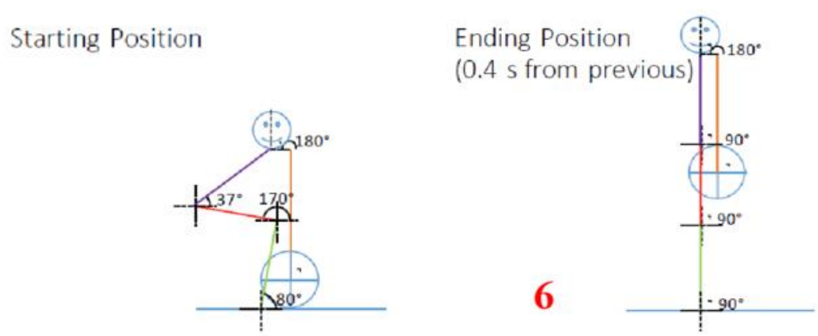
	xforce			yforce			moment
max	473.8415		max	4814.426		max	5615.326
min	-2652.77		min	-13248.6		min	-7724
	(int 1-2)			(int 4-5)			(int 4-5)
	netforce						
max	13248.78						
	(int 4-5)						

Calcs for one rep max vs height

Compression threshold					
6376		scale value	scale value	max weight lim by comp.	Actual comp

	2.1	0.4			6361
	2.1	0.5	1.05	605	6363
	2.1	0.6	1.26	443	6360
	2.1	0.7	1.47	349	6365
	2.1	0.75	1.575	315	
	2.1	0.8	1.68	287	6353
	2.1	0.85	1.785	264	
	2.1	0.9	1.89	243	6365
	2.1	1	2.1	211	6365
	2.1	1.1	2.31	186	6365
	2.1	1.2	2.52	166	6361
	2.1	1.5	3.15	125	6362
	2.1	2	4.2	87.5	6370

Calcs for determining work done by lifter



	p	1-p	
1	0.4459	0.5541	
2	0.4095	0.5905	
3	0.4486	0.5514	
4	0.528		
Initial cg			Final cg
shank	0.261927		0.265968
thigh	0.517825		0.73982
torso	0.731625		1.22327
Mass			
shank	7.74		
thigh	19.8		
torso+head	108		
arm	8.28		

*Assuming that the shoulders and arm do not move in reference to the head across the whole movement, the vertical displacement of the bar can be taken to be equal to the vertical displacement of the head. Similarly, the vertical displacement of the cg of the arm is the same as that of the head. Also, the weight of the head was transferred over to the trunk's mass to be consistent with the project simplifications.

Initial height of the head					
-Summation of y component of shank,thigh, and torso					
0.88					0.472708
					0.076405
Final Height of the head					0.330998
- Summation of shank, thigh, and torso					
1.47					0.48
					0.44
Work done is equal to change in energy					0.55
-In this case potential energy					

$$\Delta E = \sum m_i g \Delta h_i$$

$$\Delta E =$$

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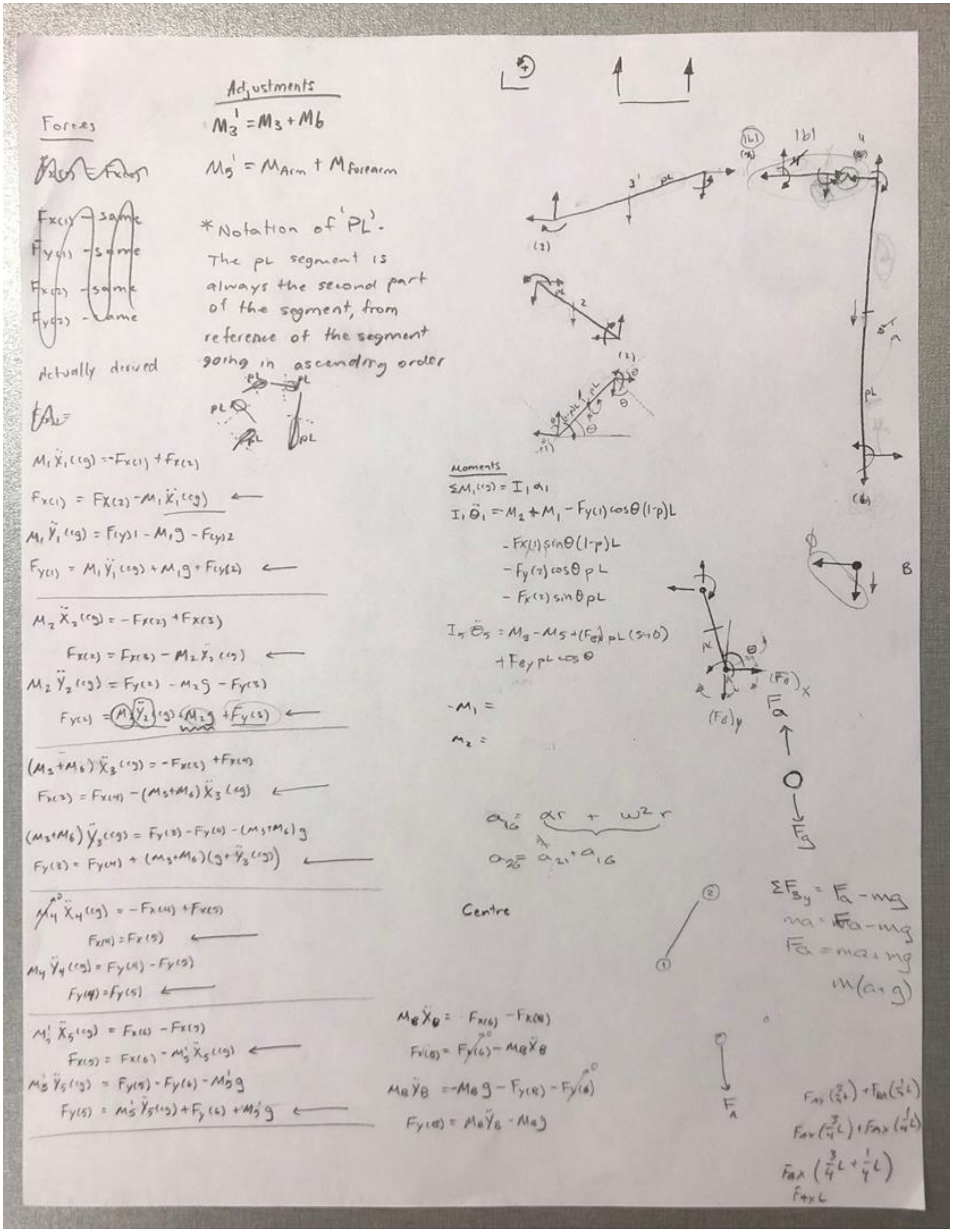
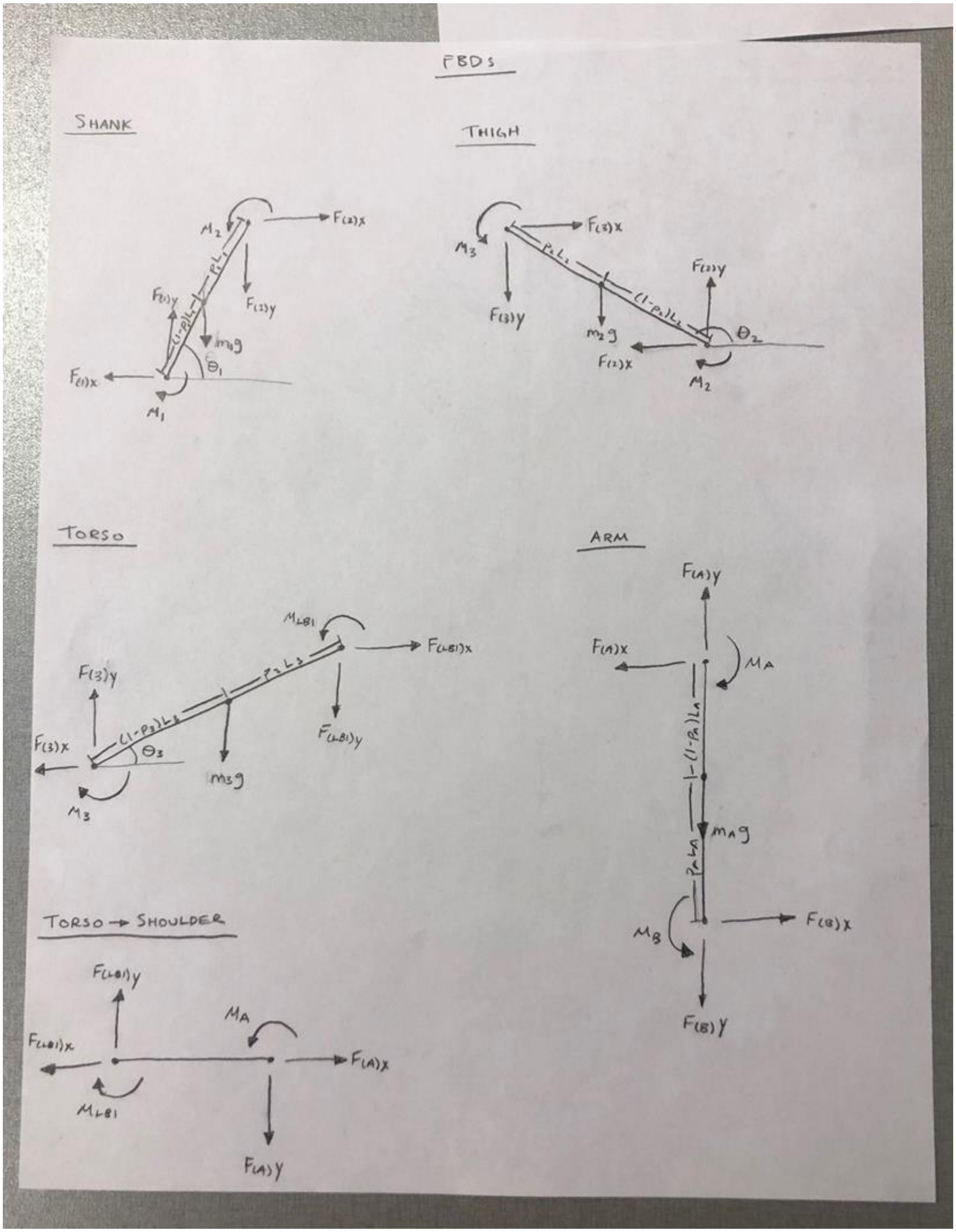
Force and Moment Calcs, FBDs, Linked Segment Model, Notations

Early force derivation and linked segment model



Individual segments



Final Force derivations, moment equation

FORCE DERIVATIONS

$\Sigma F = ma$

$\Sigma \tau = I\alpha$

$I\ddot{\Theta} = \Sigma M$

SHANK

$\Sigma F_x : m_1 \ddot{X}_1 = -F_{(1)x} + F_{(2)x}$

$\boxed{F_{(1)x} = m_1 \ddot{X}_1 + F_{(2)x}}$

$\Sigma F_y : m_1 \ddot{Y}_1 = F_{(1)y} - m_1 g - F_{(2)y}$

$F_{(1)y} = m_1 \ddot{Y}_1 + m_1 g + F_{(2)y}$

$\boxed{F_{(1)y} = F_{(2)y} + m_1(\ddot{Y}_1 + g)}$

BAR

$\boxed{m_B \ddot{X}_B = -F_{(B)x}}$

$m_B \ddot{Y}_B = -m_B g - F_{(B)y}$

$\boxed{F_{(B)y} = +m_B(\ddot{Y}_B + g)}$

changed from (-)

THIGH

$\Sigma F_x : \boxed{F_{(2)x} = m_2 \ddot{X}_2 + F_{(3)x}}$

$\Sigma F_y : \boxed{F_{(2)y} = F_{(3)y} + m_2(\ddot{Y}_2 + g)}$

TORSO

$\Sigma F_x : \boxed{F_{(3)x} = (m_3 + m_6)\ddot{X}_3 + F_{(4)x}}$

$\Sigma F_y : \boxed{F_{(3)y} = F_{(4)y} + (m_3 + m_6)(\ddot{Y}_3 + g)}$

TORSO → SHOULDER

$\Sigma F_x : \boxed{F_{(4)x} = F_{(5)x}}$

$\Sigma F_y : \boxed{F_{(4)y} = F_{(5)y}}$

ARM

$\Sigma F_x : \boxed{F_{(5)x} = m_A \ddot{X}_A + F_{(B)x}}$

$\Sigma F_y : m_A \ddot{Y}_A = F_{(5)y} + F_{(B)y} - m_A g$

$\boxed{F_{(5)y} = m_A(\ddot{Y}_A + g) + F_{(B)y}}$

changed from (-)

MOMENT DERIVATIONS $I\ddot{\Theta} = \Sigma M$

SHANK

$$I_1 \ddot{\Theta}_1 = M_2 - M_1 - F_{(1)x} \sin\Theta_1 (1-p_1)L_1 - F_{(1)y} \cos\Theta_1 (1-p_1)L_1 - F_{(2)x} \sin\Theta_1 p_1 L_1 - F_{(2)y} \cos\Theta_1 p_1 L_1$$

$$\boxed{\left.\begin{aligned} M_1 = M_2 - I_1\ddot{\Theta}_1 &- F_{(2)x} \sin\Theta_1 p_1 L_1 \\ &- F_{(2)y} \cos\Theta_1 p_1 L_1 \\ &- F_{(1)x} \sin\Theta_1 (1-p_1)L_1 \\ &- F_{(1)y} \cos\Theta_1 (1-p_1)L_1 \end{aligned}\right\} \text{for } i = 1, 2, 3, A}$$

TORSO → SHOULDER

$$I_{LB1} \ddot{\Theta}_{LB1} = M_5 - M_4 - F_{(4)y} \frac{L}{2} - F_{(5)y} \frac{L}{2}$$

$$\boxed{M_4 = M_5 - F_{(4)y} L - I_{LB1} \ddot{\Theta}_{LB1}}$$

ARM

no $\ddot{\Theta}_5$

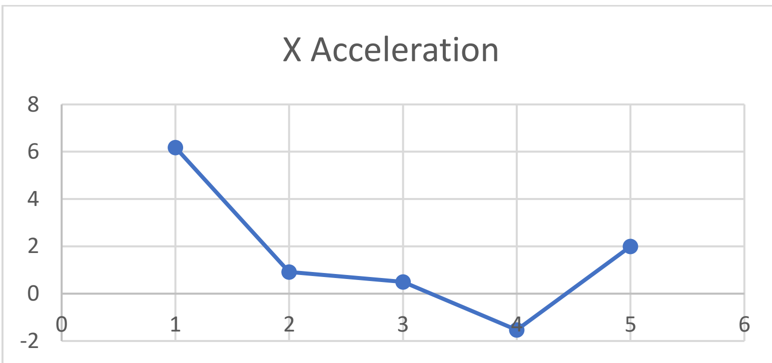
moment 0 at hands

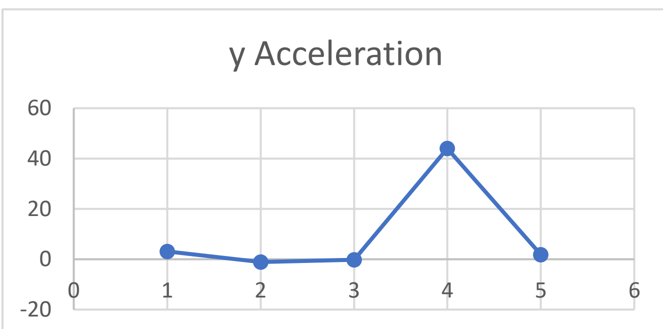
$$I_A \ddot{\Theta}_5 = \cancel{M_8} - M_5 + F_{Bx}(1-p_A)L_A + F_{(5)x}(p_A L_A)$$

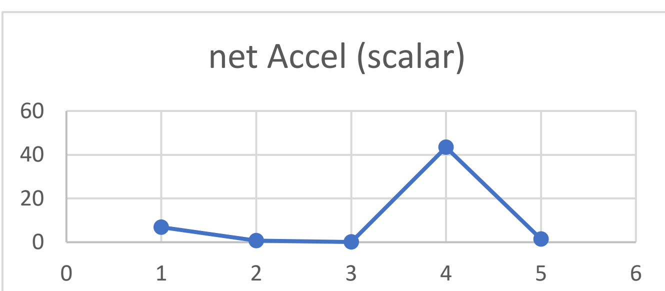
F

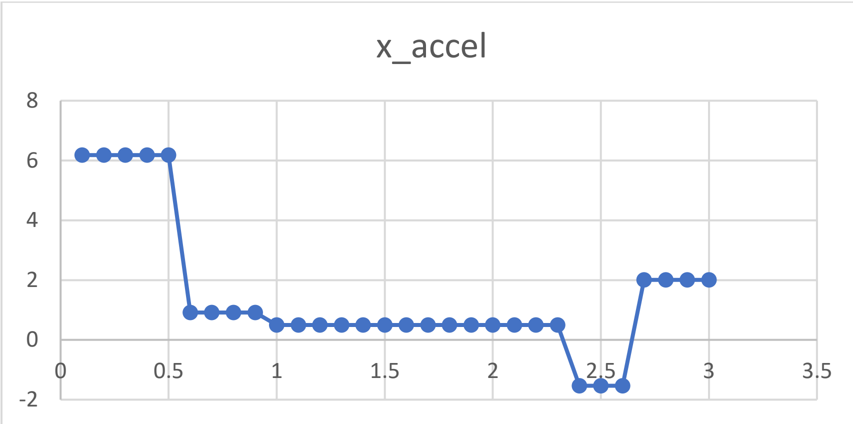
$$M_5 = F_{(B)x}(1-p_A)L_A + F_{(5)x} p_A L_A$$

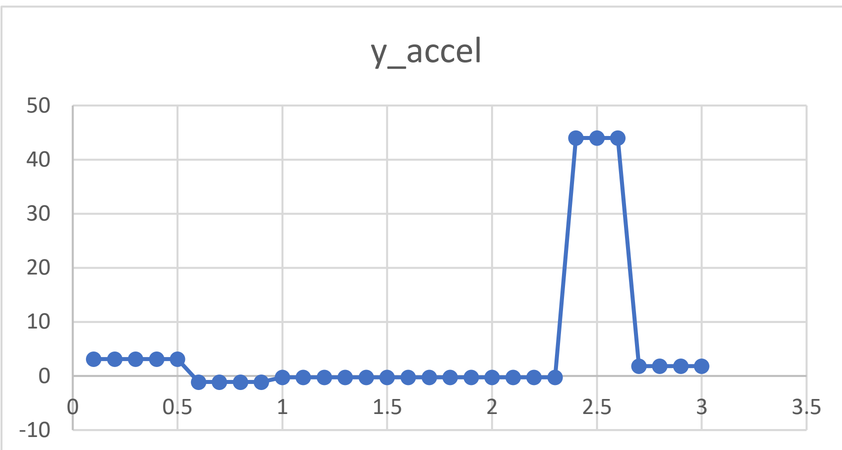
	Total Body Mass	Total Height
	180 kg	2.1 m
Limb	% of Total Body Mass	Length (m)
Ankle to Knee	0.043	0.48
Knee to Hip (Thigh)	0.11	0.44
Hip to Shoulder (Torso)	0.53	0.55
Segment B1	0	0.12
Shoulder to Hand (Arm)	0.049	0.69
Head	0.07	0

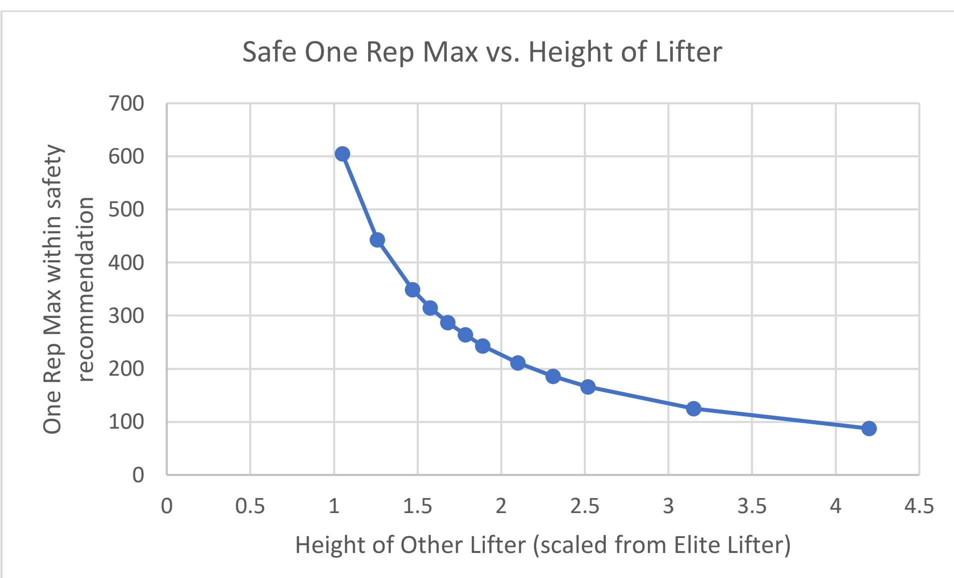


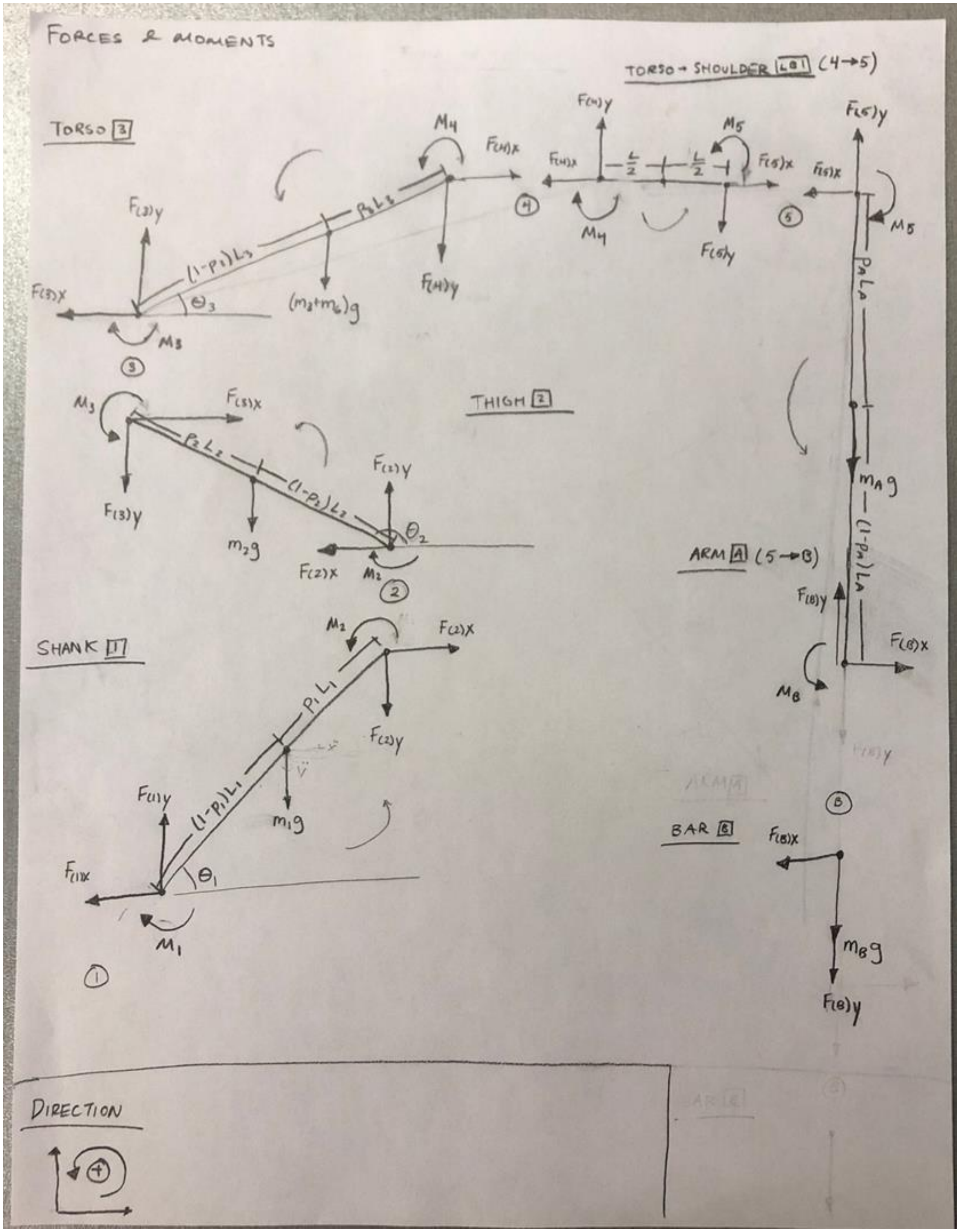




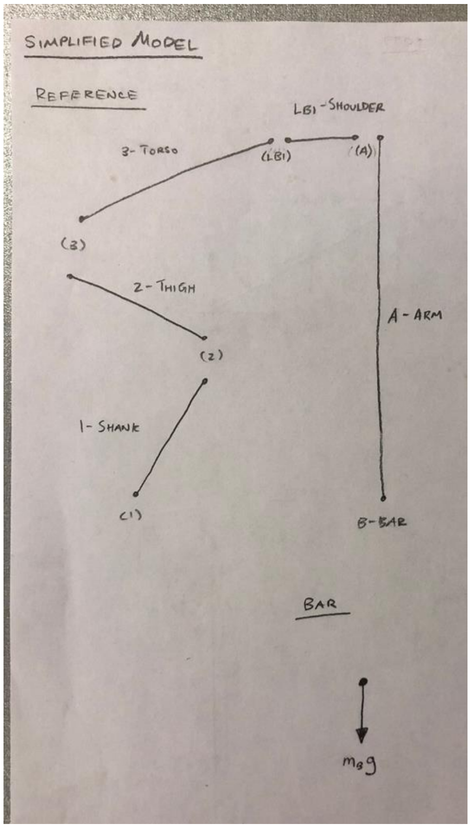

x_accel
8
6
4
2
0
-2
0
0.5
1
1.5
2
2.5
3
3.5


y_accel
50
40
30
20
10
0
-10
0
0.5
1
1.5
2
2.5
3
3.5


Safe One Rep Max vs. Height of Lifter
One Rep Max within safety recommendation
700
600
500
400
300
200
100
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Height of Other Lifter (scaled from Elite Lifter)



FORCES & MOMENTS
TORSO 3
TORSO → SHOULDER LB1 (4→5)
THIGH 2
SHANK 1
ARM A (5→B)
BAR B
DIRECTION


SIMPLIFIED MODEL
REFERENCE
LB1 - SHOULDER
3- TORSO
(LB1)
(A)
(3)
2 - THIGH
A - ARM
(2)
1- SHANK
(1)
B- BAR
BAR
$m_B g$

Log | Kostubh Agarwal & Aidan Schwartzentruber

Met in DC Library to look at textbooks and find journals related to our project. We also began to work on the initial submission.	3hrs @ Nov 1st
Met again in the basement of DC to review the first submission and submit it.	2hrs @ Nov 6th
Met in GEAR lab to begin working on the project.	4hrs @ Nov 25th
Met in GEAR lab to continue working on the project.	6hrs @ Nov 25th
Met in GEAR lab to continue working on the project.	3.5hrs @ Dec 3rd
Met in GEAR lab to continue working on the project.	5.5hrs @ Dec 4th
Met in GEAR lab to continue working on the project.	7hrs @ Dec 5th
Met on the 6th floor of E7 to complete and submit the project.	6hrs @ Dec 6th

*We communicated via text throughout this time and completed individual sections on our own

Summary – Aidan Schwartzentruber & Kostubh Agarwal

The initial submission was evenly split amongst Aidan and Kostubh. Aidan was responsible for finding two sources and Kostubh was responsible for finding 3. Afterwards they shared their findings with each other and completed the initial submission. In terms of the final report, Aidan was responsible for conducting all of the proofs and mathematical verification in part 1 of the project manual. Kostubh then double checked these calculations and revised changes with Aidan. Kostubh was then responsible for implementing the analysis via Jupyter Notebooks (rather than Excel). After the calculations and the Jupyter Notebook was complete, Aidan reviewed the code. Aidan and Kostubh went through 10+ revisions before finalizing the calculations. Some of the errors included incorrect shear and compression force algorithms, misnaming variables, and using the incorrect moment of inertia. When Aidan and Kostubh were satisfied with the calculations, they began to write the report. Kostubh was responsible for setting up the document and deciding what sections were required. After discussing and creating a point form summary for each section together, Aidan took the responsibility to write the majority of the discussion for the elite lifter. On the other hand, Kostubh took charge of the abstract, introduction, background information (although Aidan entered the equations on Word…very tedious), and method. Afterwards they each completed their respective sections in part 3 of the report: Aiden (Height vs One Rep Max) and Kostubh (Sumo vs Traditional Deadlift). The last two days before submission, they read over each others' sections, sometimes making major changes, and questioning each others logic. In the end, Aidan was responsible for making sure we correctly completed the references, and Kostubh ensured the correct formatting of the final document. Both Kostubh and Aidan enjoyed working with each other!