Kostyantyn Hrytsyuk & Khrystyna Kubatska. Finances Project

```r
require(dplyr)
```

```
## Loading required package: dplyr

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
require(ggplot2)
```

```
## Loading required package: ggplot2
```

```r
require(xts)
```

```
## Loading required package: xts

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

##
## Attaching package: 'xts'

## The following objects are masked from 'package:dplyr':
##
##     first, last
```

```r
require(rugarch)
```

```
## Loading required package: rugarch

## Loading required package: parallel

##
## Attaching package: 'rugarch'

## The following object is masked from 'package:stats':
##
##     sigma
```

```r
require(PerformanceAnalytics)
```

```
## Loading required package: PerformanceAnalytics
```

```
##
## Attaching package: 'PerformanceAnalytics'

## The following object is masked from 'package:graphics':
##
##      legend
```

```r
require(quantmod)
```

```
## Loading required package: quantmod

## Loading required package: TTR

## Registered S3 method overwritten by 'quantmod':
##    method            from
##    as.zoo.data.frame zoo

## Version 0.4-0 included new data defaults. See ?getSymbols.
```

```r
library(skewt)
```

```r
# Configure propet path to the Funds.csv file here
# setwd('./')
```

```r
read_funds <- function(lf) {
  dfs <- list()
 browser()
  for (f in 1:length(lf)) {
      cond <- substr(lf[f], nchar(lf[f])-3, nchar(lf[f])) == '.csv'
      if (cond) {
      temp <- data.frame(read.csv(lf[f], stringsAsFactors = FALSE))
      temp$Date <- as.Date(temp$Date)
      temp$Close <- as.numeric(temp$Close)
      dfs[[f]] <- temp[,c(1,5)]
    }
  }

  return(dfs)
}

funds_names <- c("Vanguard", "Blackrock", "Statestreet",
                 "JPmorgan", "Bankmellon", "Allianz")
```

```r
# Forming set of parameters for different GARCH model
get_garch_specs <- function() {

  # Standard GARCH with normal distribution of errors
  norm_garch_spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                        variance.model = list(model = 'sGARCH'),
                        distribution.model = 'norm')

  # GJR GARCH with normal distribution of errors
  norm_gjr_spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                        variance.model = list(model = 'gjrGARCH'),
                        distribution.model = 'norm')

  # Standard GARCH with skewed Student t distribution of errors
  sstd_garch_spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
```

```r
                            variance.model = list(model = 'sGARCH'),
                            distribution.model = 'sstd')

  # GJR GARCH with skewed Student t distribution of errors
  sstd_gjr_spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                            variance.model = list(model = 'gjrGARCH'),
                            distribution.model = 'sstd')

  garch_specs <- list(norm_garch_spec, norm_gjr_spec,
                      sstd_garch_spec, sstd_gjr_spec)

  return(garch_specs)
}

# Apply GARCH model to our data
get_garch_fits <- function(fund) {
  garch_specs <- get_garch_specs()

  garch_fits <- list()

  for (s in 1:length(garch_specs)) {
    suppressWarnings(garch_fits[[s]] <- ugarchfit(data = fund,
                                                spec = garch_specs[[s]]))
  }

  return(garch_fits)
}

# Visualizing standardized residuals for models
visualize_residuals <- function(garch_fits) {
  for (f in 1:length(garch_fits)) {
    chart.Histogram(residuals(garch_fits[[f]], standardize = T),
                methods = c('add.normal', 'add.density'),
                main = paste('Standardized residuals of',names(garch_fits)[f]))
  }
}

# Models validation
test_garch_models <- function(garch_fits) {
  for (f in 1:length(garch_fits)) {
      standard_residuals <- residuals(garch_fits[[f]], standardize = T)
      p <- acf(abs(standard_residuals),22, plot = F)
      plot(p, main = names(garch_fits)[f])
      cat('\n', names(garch_fits)[f],'\n')
      print(Box.test(abs(standard_residuals), 22, type = 'Ljung-Box'))
  }
}

#Coefficients
print_garch_coefficients <- function(garch_fits) {
  for (f in 1:length(garch_fits)) {
    cat('\nCoefficients of', names(garch_fits)[f], '\n')
    print(round(garch_fits[[f]]@fit$matcoef,10))
```

```r
      cat('\nRobust coefficients of', names(garch_fits)[f], '\n')
      print(round(garch_fits[[f]]@fit$robust.matcoef,10))
  }
}


# Models comparing
compare_garch_models <- function(garch_fits, short_model_names) {
  model_comparison <- data.frame()

  for (f in 1:length(garch_fits)) {
      temp <- data.frame()
      temp[1,1] <- likelihood(garch_fits[[f]])
      inf_criterion <- infocriteria(garch_fits[[f]])

      temp <- rbind(temp, inf_criterion)

      model_comparison <- c(model_comparison, temp)
  }
  model_comparison <- as.data.frame(model_comparison)

  rownames(model_comparison) <- c('Likelihood',rownames(inf_criterion))
  colnames(model_comparison) <- short_model_names
  print(model_comparison)
}

# Visualizing impact of negative previous return on variance
visualize_dependecy_ret_var <- function(garch_fits, short_model_names) {
  p <- ggplot()

  for (f in 1:length(garch_fits)) {
    garch_news <- as.data.frame(newsimpact(garch_fits[[f]])[1:2])

    model_name <- short_model_names[f]
    model_name <- enquo(model_name)

    p <- p + geom_line(data = garch_news,
            aes(x = zx, y = zy, color = !!model_name))
  }

  p <- p + labs(x = 'Error', y = 'Variance',
        title = 'Dependence of variance on errors in different models') +
    theme(plot.title = element_text(hjust = 0.5))

  print(p)
}


# Visualizing volatility
visualizing_volatility <- function(garch_fits, short_model_names, fund_vol) {
  p <- ggplot()
  garch_vol <- list()
  for (f in 1:length(garch_fits)) {
    garch_vol[[f]] <- sigma(garch_fits[[f]])
```

```r
    model_name <- short_model_names[f]
    model_name <- enquo(model_name)

    p <- p + geom_line(data = garch_vol[[f]], aes(x = index(garch_vol[[f]][,1]),
                                        y = garch_vol[[f]][,1],
                                        color = !!model_name), alpha = 0.2)
  }

  names(garch_vol) <- short_model_names

  p <- p + geom_line(data = fund_vol, aes(y = fund_vol[,1], x = index(fund_vol[,1]),
                                    color = 'Actual volatility')) +
      labs(x = 'Date', y = 'Volatility',
            title = 'Volatility constructed by different models') +
      theme(plot.title = element_text(hjust = 0.5))

  suppressMessages(suppressWarnings(print(p)))

  return(garch_vol)
}

# Predicting volatility for n.ahead periods
predict_volatility <- function(garch_fits, garch_vol,
                              fund_vol, short_model_names) {
  fund_tail_volatility <- tail(fund_vol,10)
  predict_results <- data.frame(fund_tail_volatility)
  garch_sst <- c()
  for (f in 1:length(garch_fits)) {
    garch_forecast <- ugarchforecast(fitORspec = garch_fits[[f]],
                                      data = garch_vol[[f]], n.ahead = 10)

    predict_results <- cbind(predict_results, sigma(garch_forecast))

    garch_sst[[f]] <- sum(fund_tail_volatility - sigma(garch_forecast))
    names(garch_sst)[f] <- paste('TES for',short_model_names[f])

  }
  names(predict_results)[2:ncol(predict_results)] <- short_model_names

  # Total error sum for models
  print(garch_sst)

  #Comparing predicted volatility for models with actual one
  print(predict_results)
}

# Data loading
df <- read.csv('Funds.csv', stringsAsFactors = F)
df <- df[,-1]
df$Date <- as.Date(df$Date)

# Transforming data from df to xts
funds <- xts(df[,2:ncol(df)], order.by = df$Date)
```

```r
funds <- na.omit(funds)
rm(df)

# Subseting original data for April of 2020 year
funds_red <- funds['/202004']
```

```r
get_volatiles <- function(funds, width = 22, time_scale = 1, funds_names) {
    vol_df <- data.frame()

    for (i in 1:ncol(funds)) {
      fund <- funds[,i]
      temp <- rollapply(data = fund, width = 22, FUN = 'sd.annualized', scale = time_scale)

      if (nrow(vol_df) == 0) {
        vol_df <- temp
      } else {
        vol_df <- cbind(vol_df, temp)
      }

    }
    names(vol_df) <- funds_names
    return(vol_df)
}

visualize_funds_lines <- function(funds, y_axis_label = 'Volatility') {

  for (i in 1:ncol(funds)) {
    temp <- funds[,i]
    temp_mean <- mean(temp, na.rm = TRUE)

    p <- ggplot(temp, aes(x = index(temp), y = temp)) +
    geom_line(aes(color = 'Volatility')) +
    geom_hline(aes(yintercept = temp_mean, color = 'Mean'),
               size=.5, linetype='dashed') +
    geom_text( aes( min(index(temp)) , temp_mean, label = round(temp_mean, 4), vjust = 2)) +
    labs(x = 'Date', y = y_axis_label, title = names(funds)[i]) +
    theme(plot.title = element_text(hjust = 0.5))

    suppressMessages(suppressWarnings(print(p)))
  }
}

visualize_funds_hist <- function(funds, x_axis_label = 'Return') {
  for (i in 1:ncol(funds)) {
      temp <- funds[,i]
      title <- paste(funds_names[i], 'returns')

      chart.Histogram(temp,
                methods = c('add.normal', 'add.density'),
                main = title)

      temp <- (temp - mean(temp, na.rm = T))/sd(temp, na.rm = T)
      title <- paste('Standardized', title)
```

```r
    chart.Histogram(temp,
                    methods = c('add.normal', 'add.density'),
                    main = title)
  }
}
```

```r
evaluate_garch <- function(fund, fund_vol) {

  # Models naming
   model_names <- c('Standard GARCH with normal distribution of errors',
                    'GJR GARCH with normal distribution of errors',
                    'Standard GARCH with skewed Student t distribution of errors',
                    'GJR GARCH with skewed Student t distribution of errors')

  short_model_names <- c('Normal GARCH', 'Normal GJR', 'Skewed t GARCH', 'Skewed t GJR')

  garch_fits <- get_garch_fits(fund)

  names(garch_fits) <- model_names

  visualize_residuals(garch_fits)

  test_garch_models(garch_fits)

  print_garch_coefficients(garch_fits)

  compare_garch_models(garch_fits, short_model_names)

  visualize_dependecy_ret_var(garch_fits, short_model_names)

  garch_vol <- visualizing_volatility(garch_fits, short_model_names, fund_vol)

  predict_volatility(garch_fits = garch_fits,
                     garch_vol = garch_vol,
                     fund_vol = fund_vol,
                     short_model_names = short_model_names)
}
```
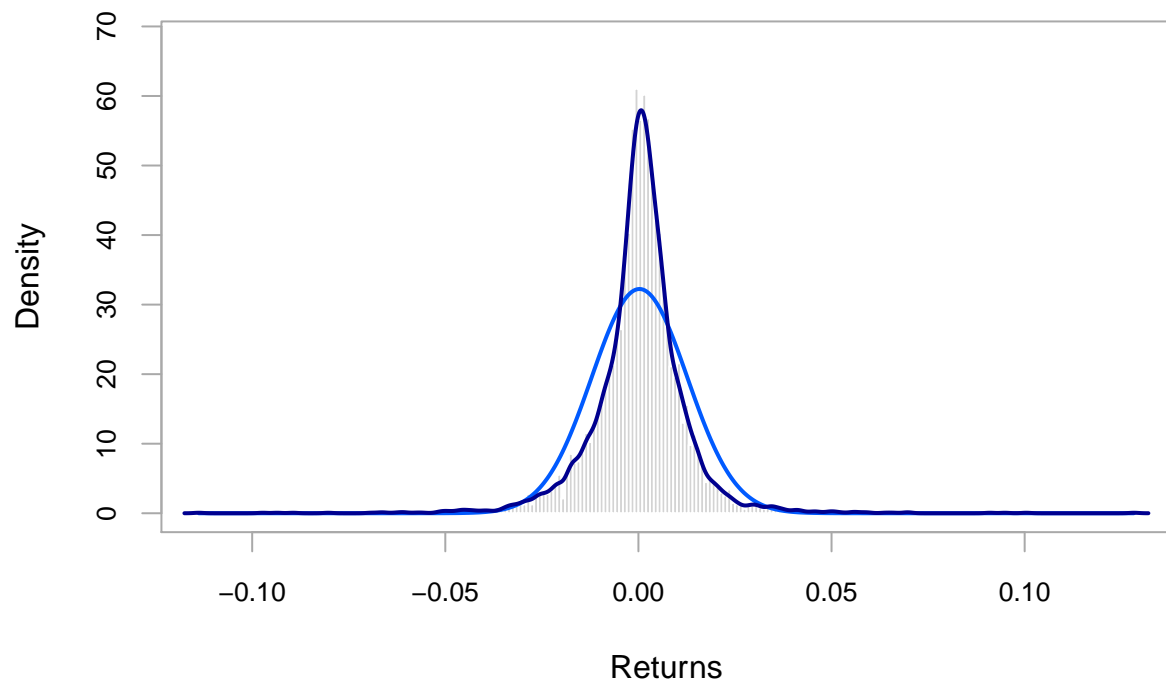
```r
vol_df <- get_volatiles(funds, funds_names =  funds_names)
vol_df <- na.omit(vol_df)
visualize_funds_lines(vol_df)
```

Vanguard

0.0102

Blackrock

0.0182

Statestreet

JPmorgan

0.019
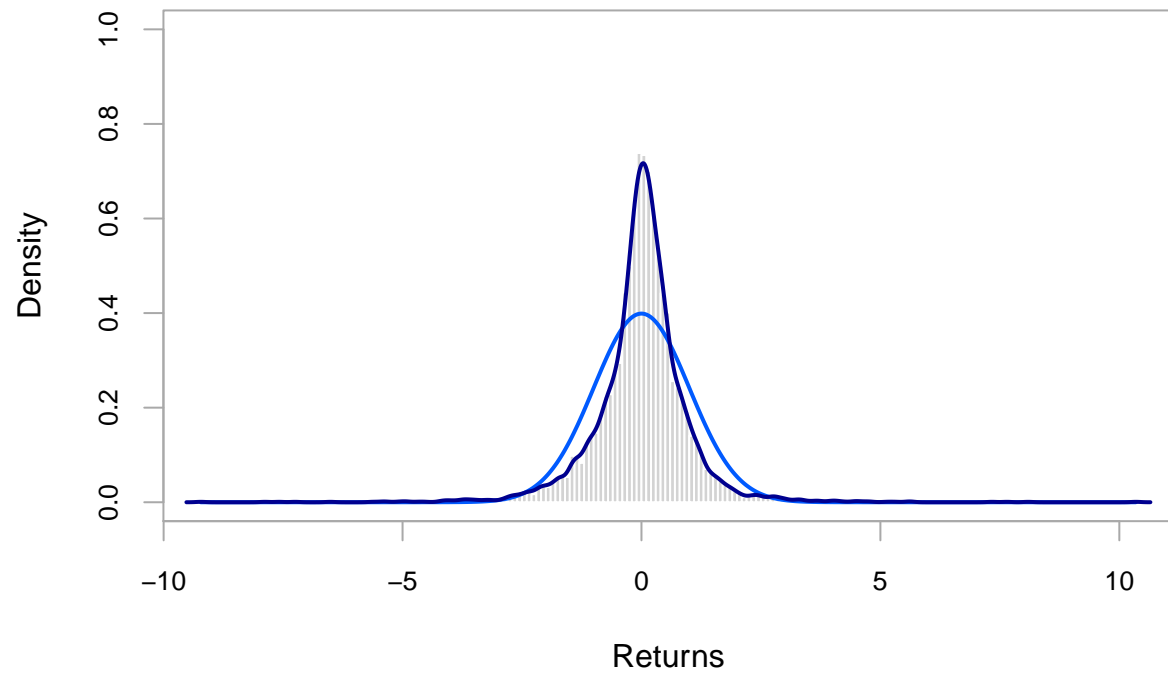
Bankmellon

0.0187

Allianz
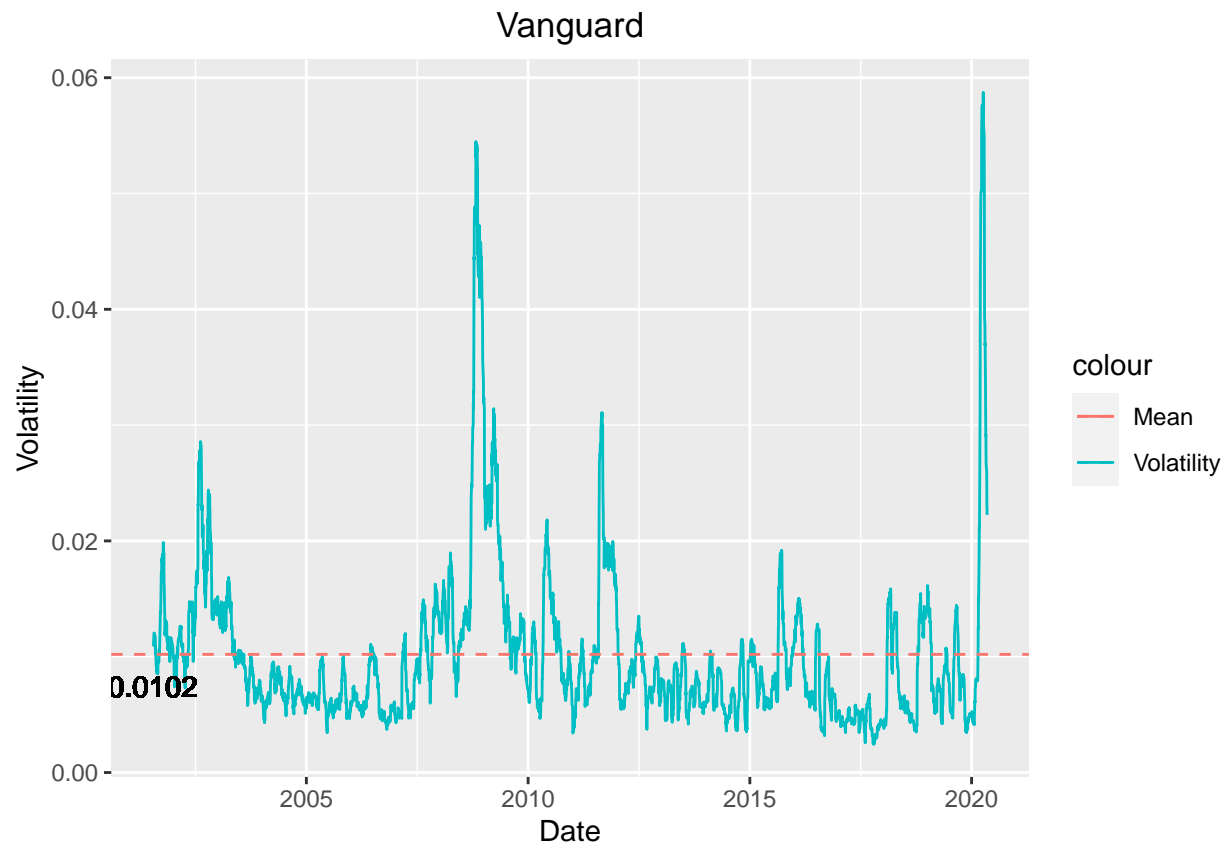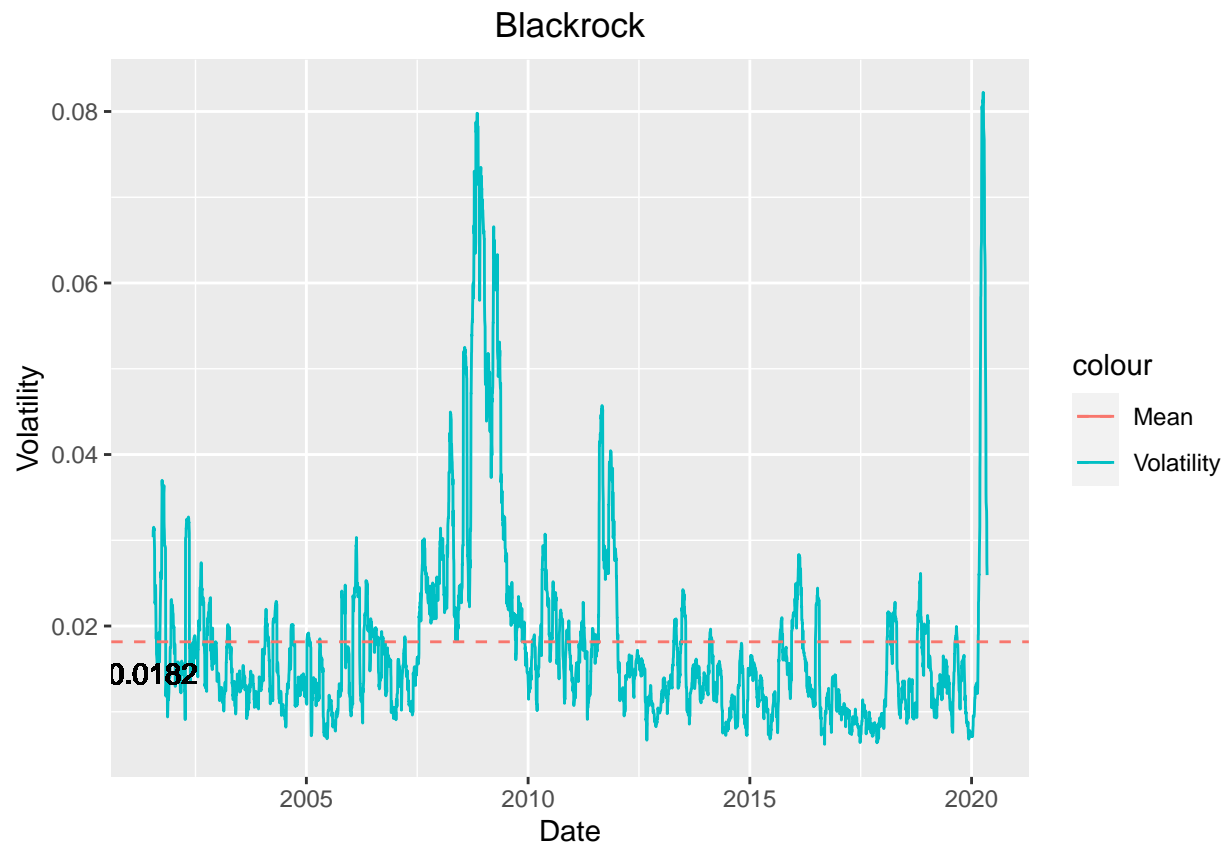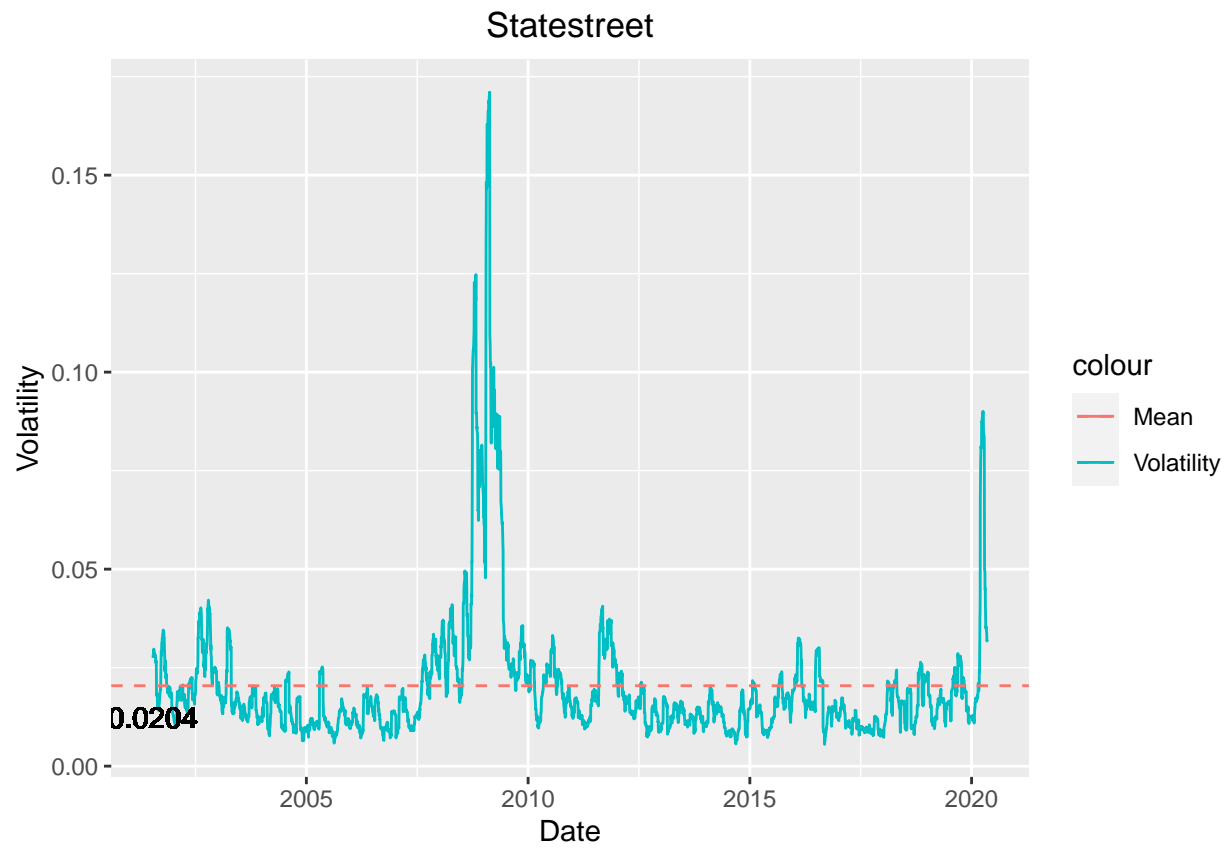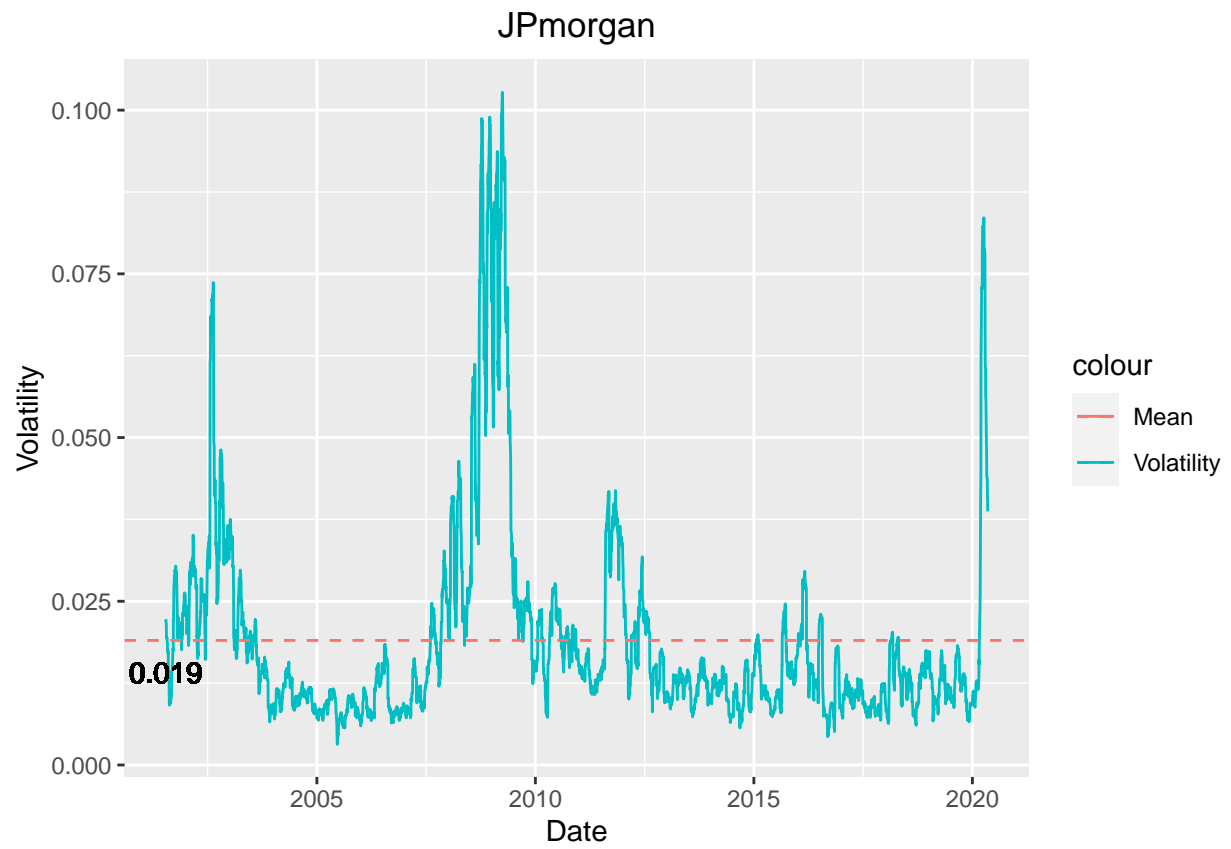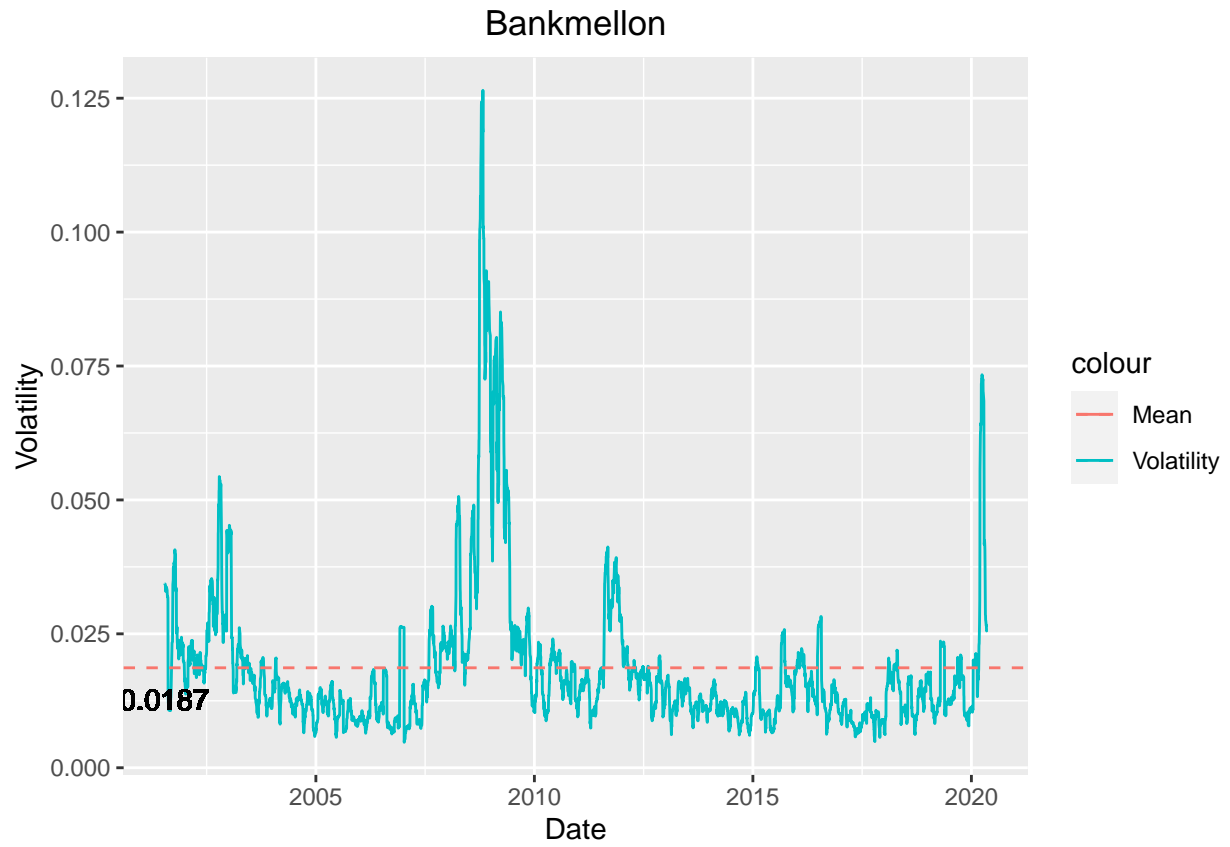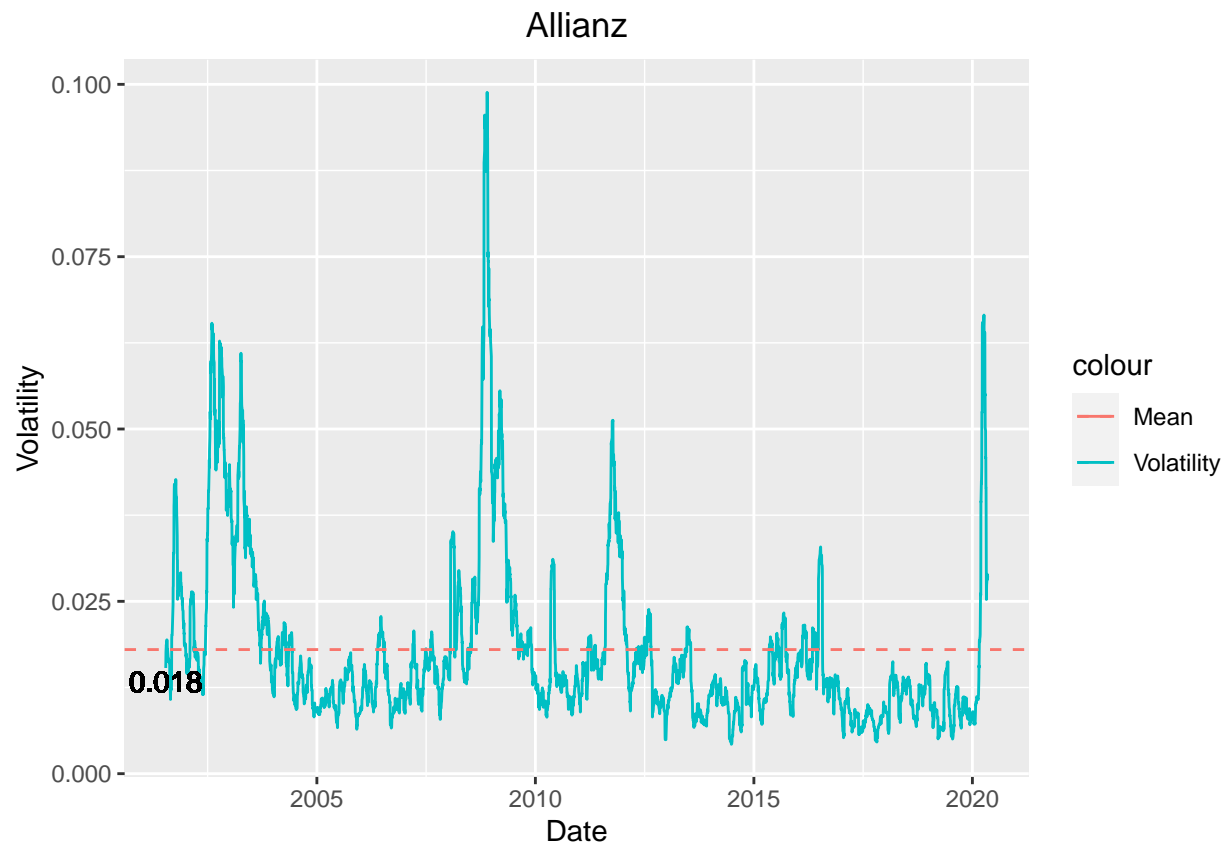

```
visualize_funds_hist(funds)
```
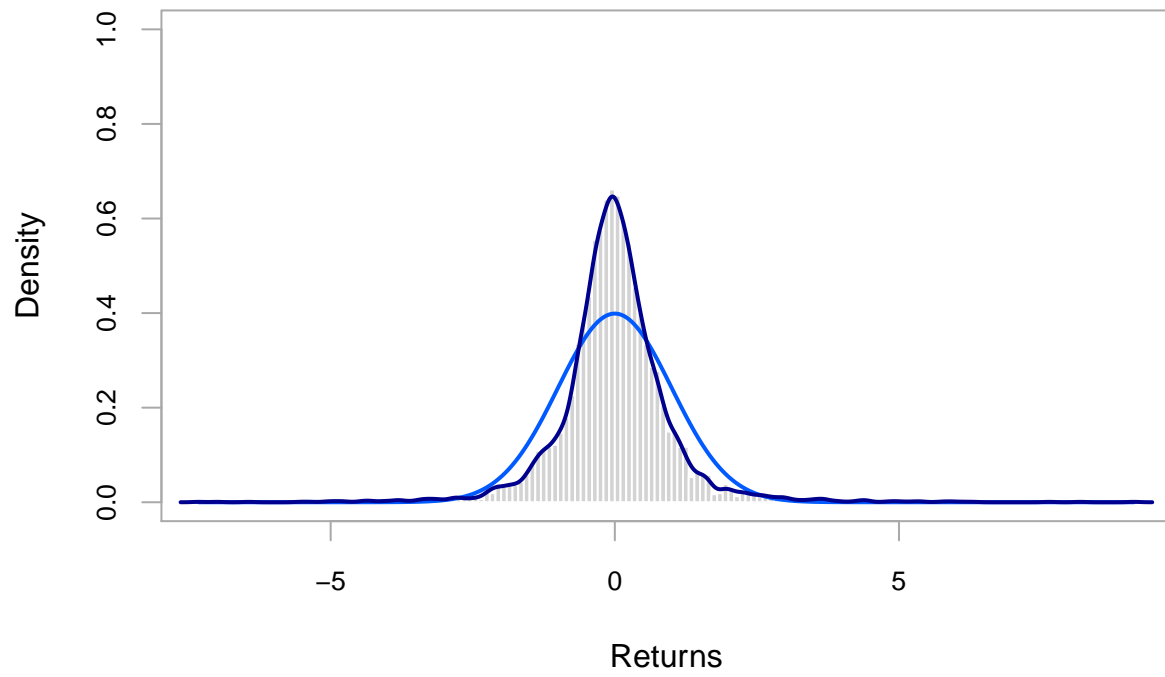
Vanguard returns
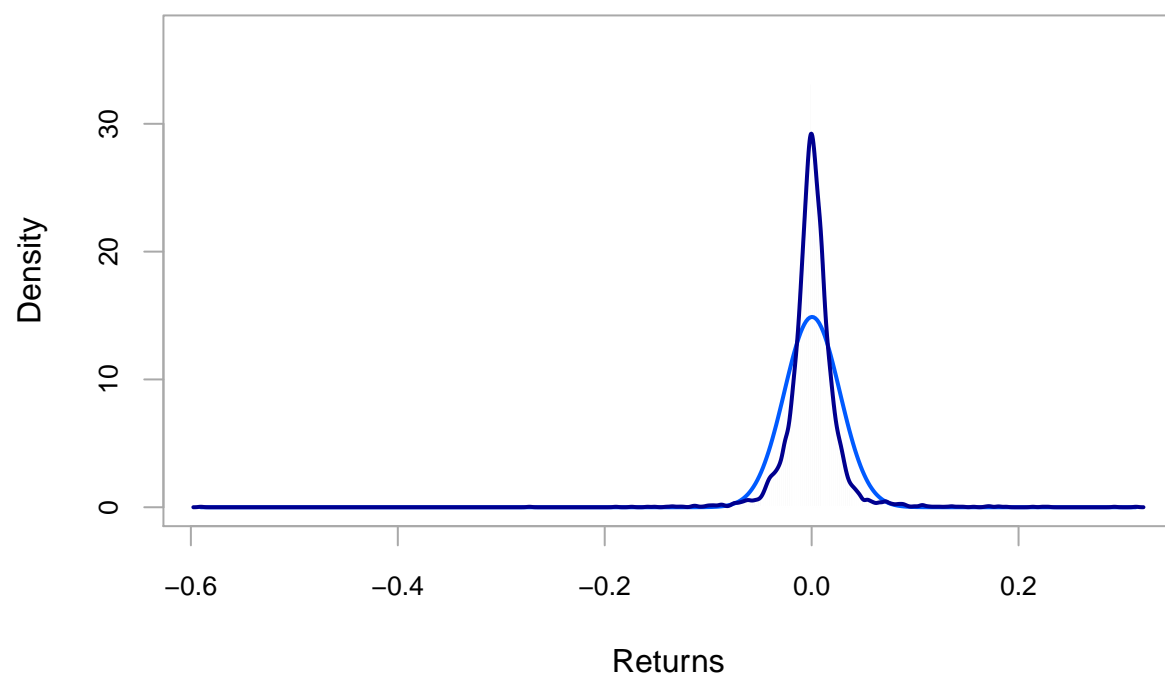

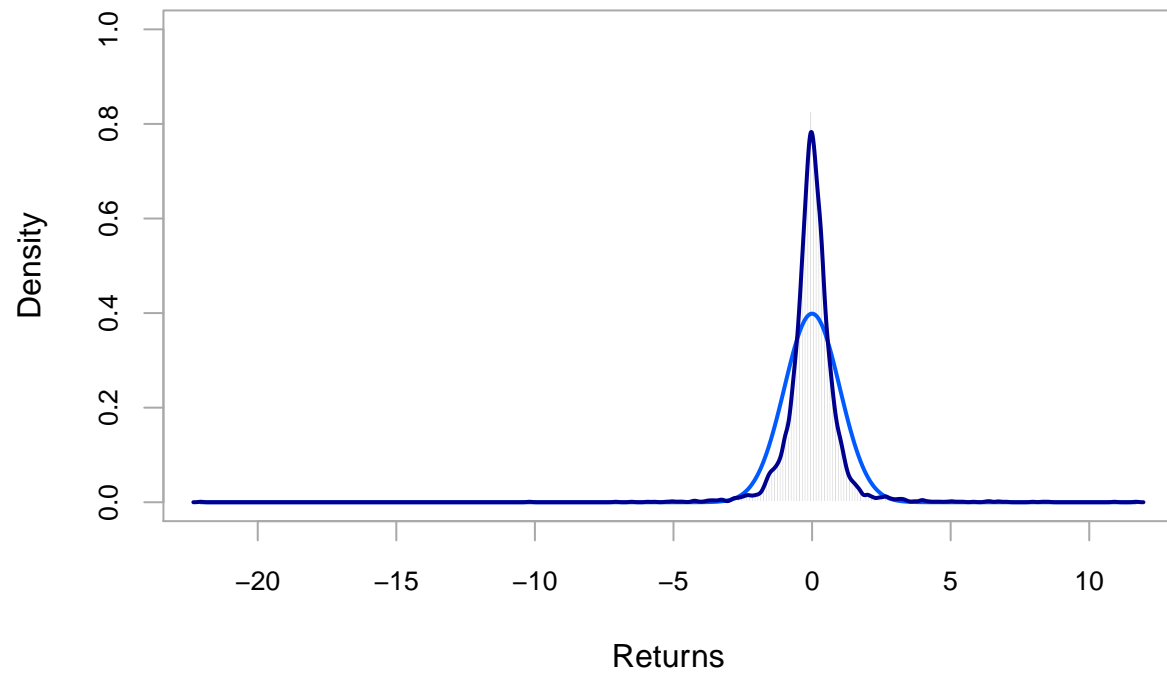Returns

Standardized Vanguard returns
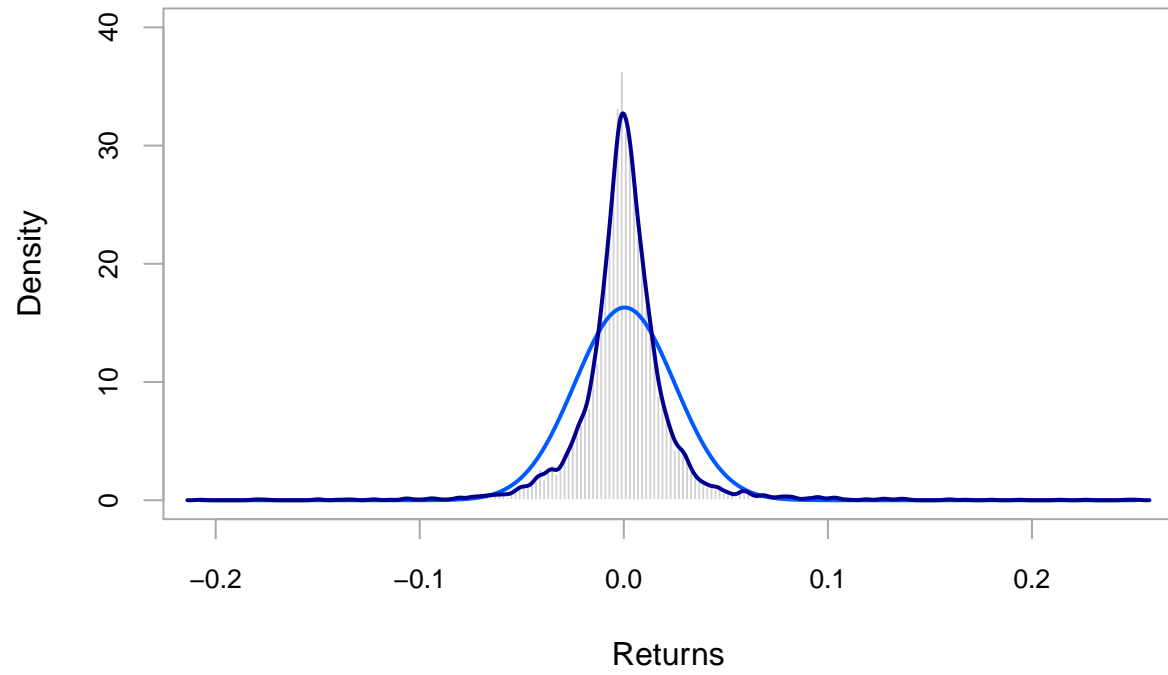
Blackrock returns
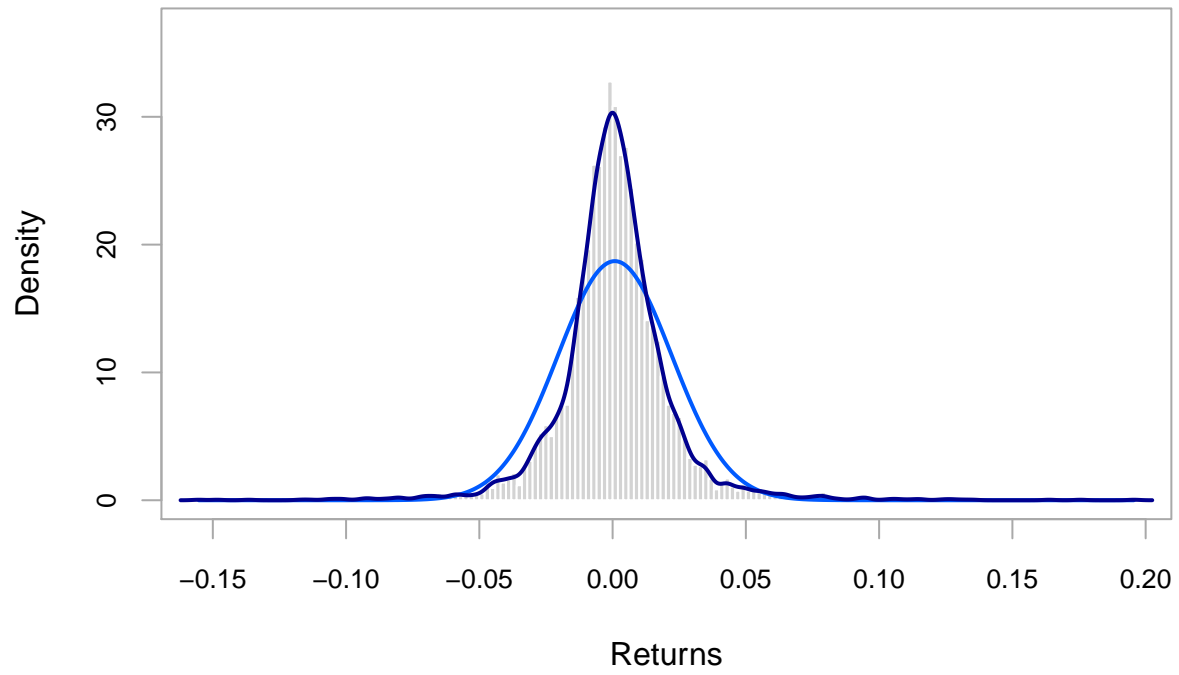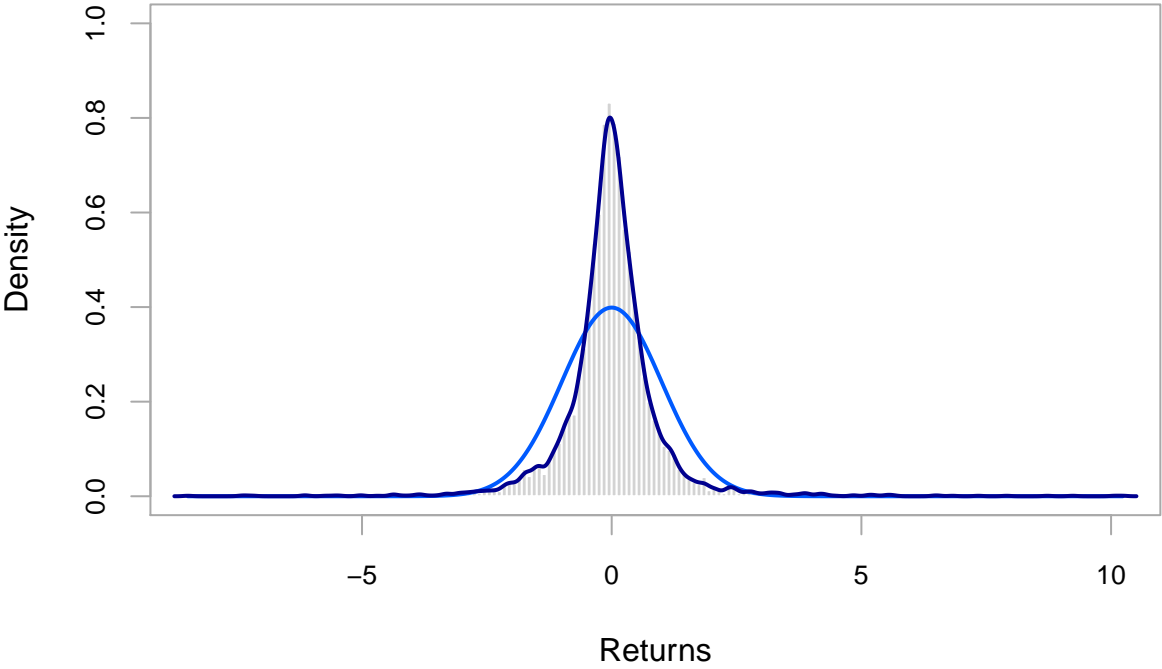
Standardized Blackrock returns

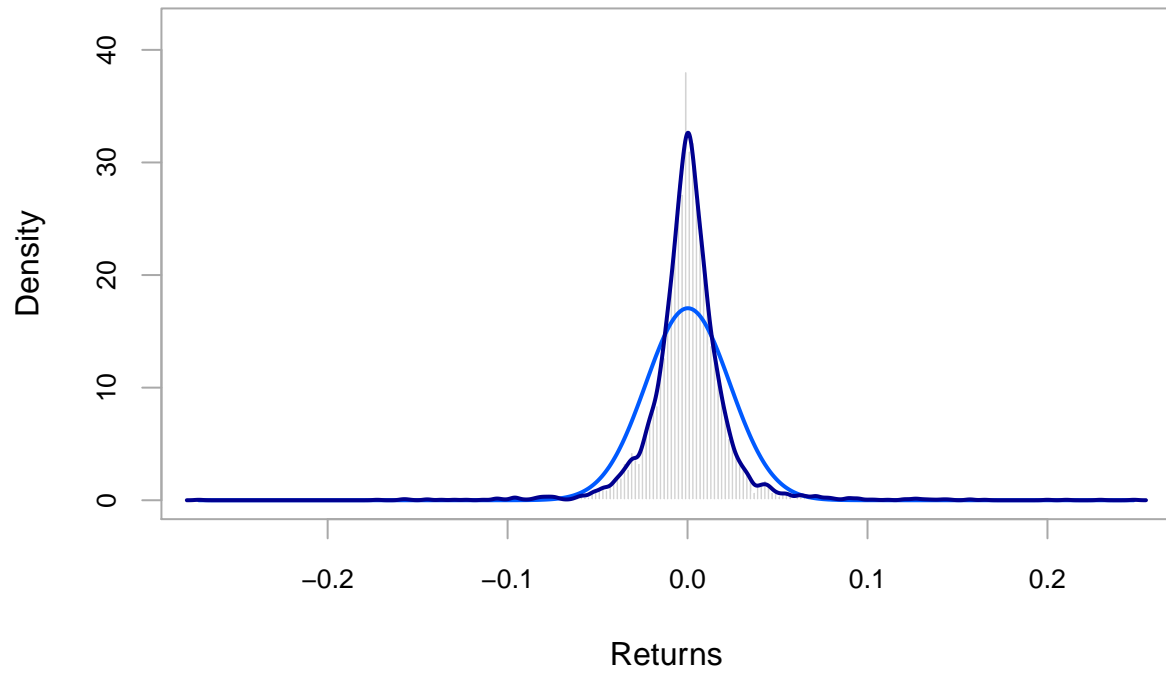# Statestreet returns

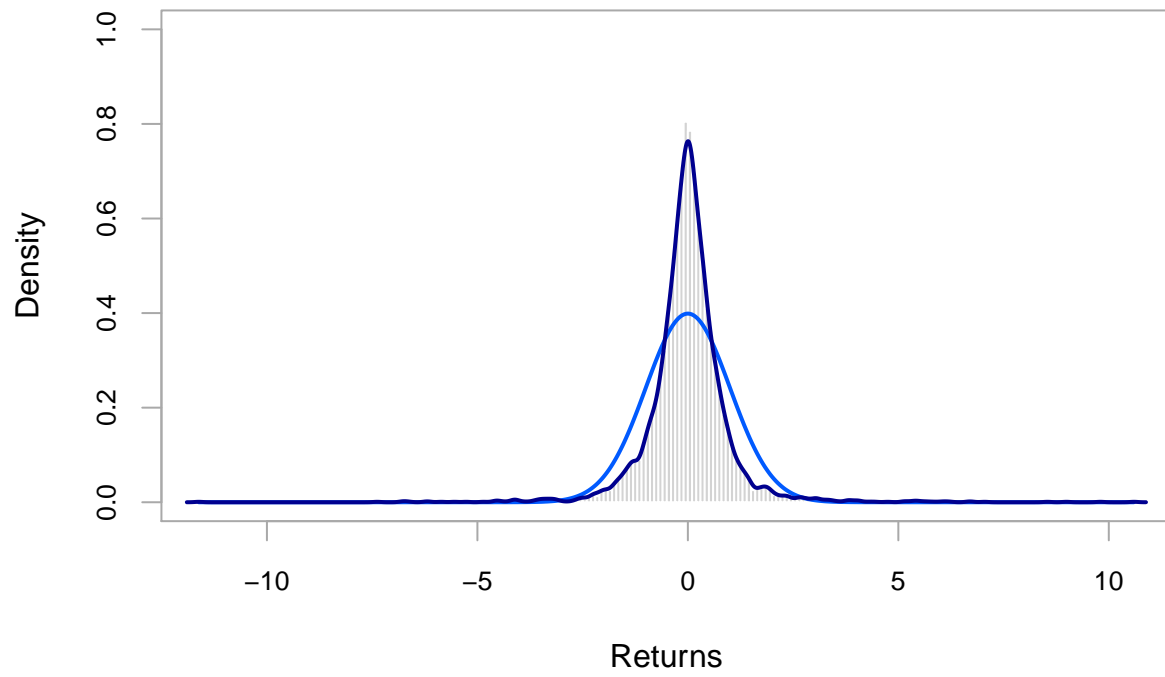## Standardized Statestreet returns
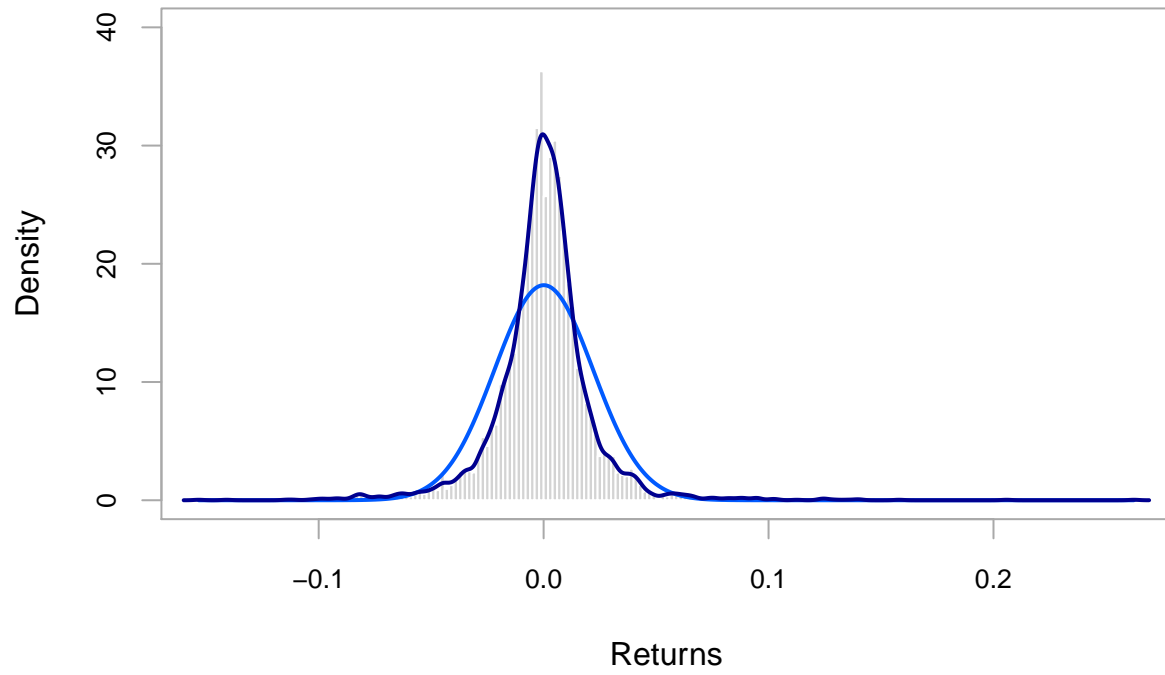
JPmorgan returns

Standardized JPmorgan returns

Bankmellon returns

Standardized Bankmellon returns

Allianz returns

Standardized Allianz returns
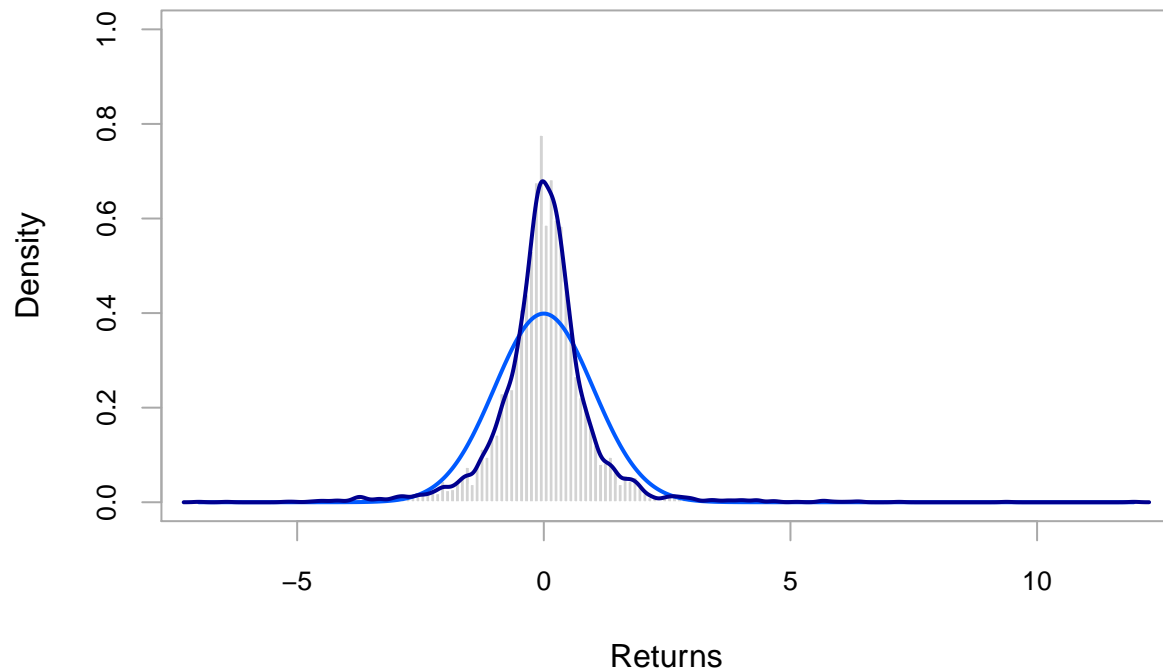

```r
fund_tail_volatility <- tail(vol_df$Vanguard, 10)

cat('Data up to 2020-04-30\n')
```
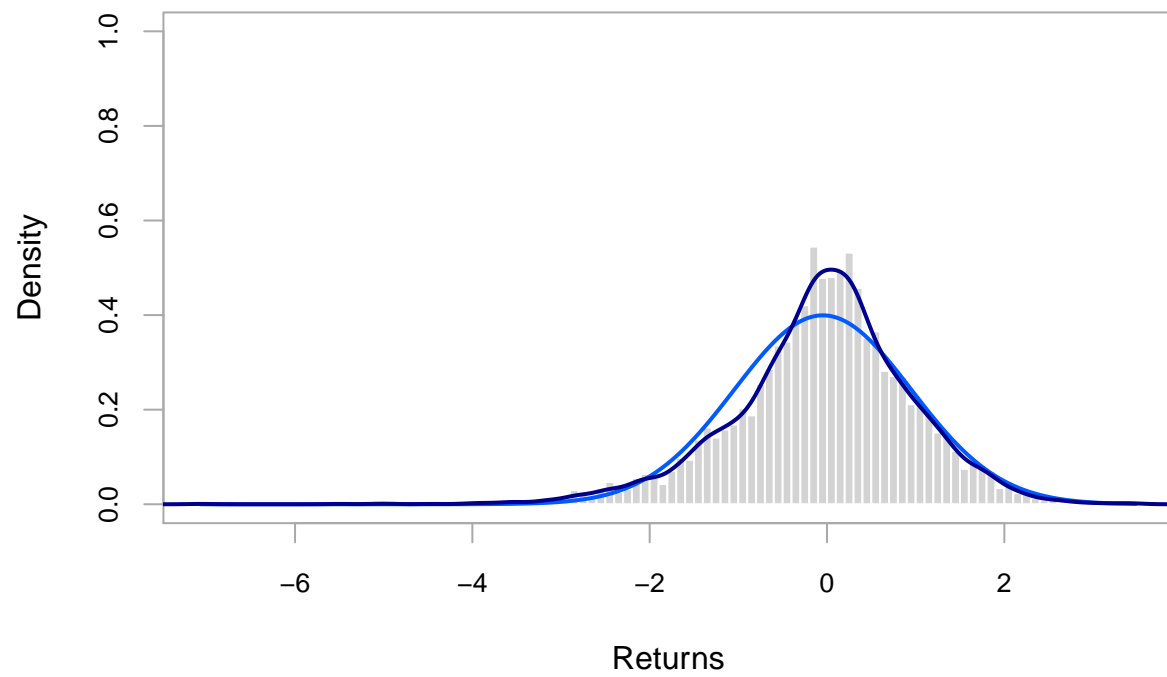
```
## Data up to 2020-04-30
```

```r
for (i in 1:ncol(funds_red)) {
    cat(colnames(vol_df)[i],'\n')
    evaluate_garch(funds_red[,i], vol_df[,i])
    cat('\n------------------------------\n')
}
```

```
## Vanguard
```

Standardized residuals of Standard GARCH with normal distribution of errors

Standardized residuals of GJR GARCH with normal distribution of errors

Standardized residuals of Standard GARCH with skewed Student t distribution of ϵ

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
## 
##  Standard GARCH with normal distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 34.349, df = 22, p-value = 0.04525
```

GJR GARCH with normal distribution of errors


```
##
##  GJR GARCH with normal distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 35.737, df = 22, p-value = 0.0324
```

Standard GARCH with skewed Student t distribution of errors


```
##
##   Standard GARCH with skewed Student t distribution of errors
##
##   Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 30.047, df = 22, p-value = 0.1173
```
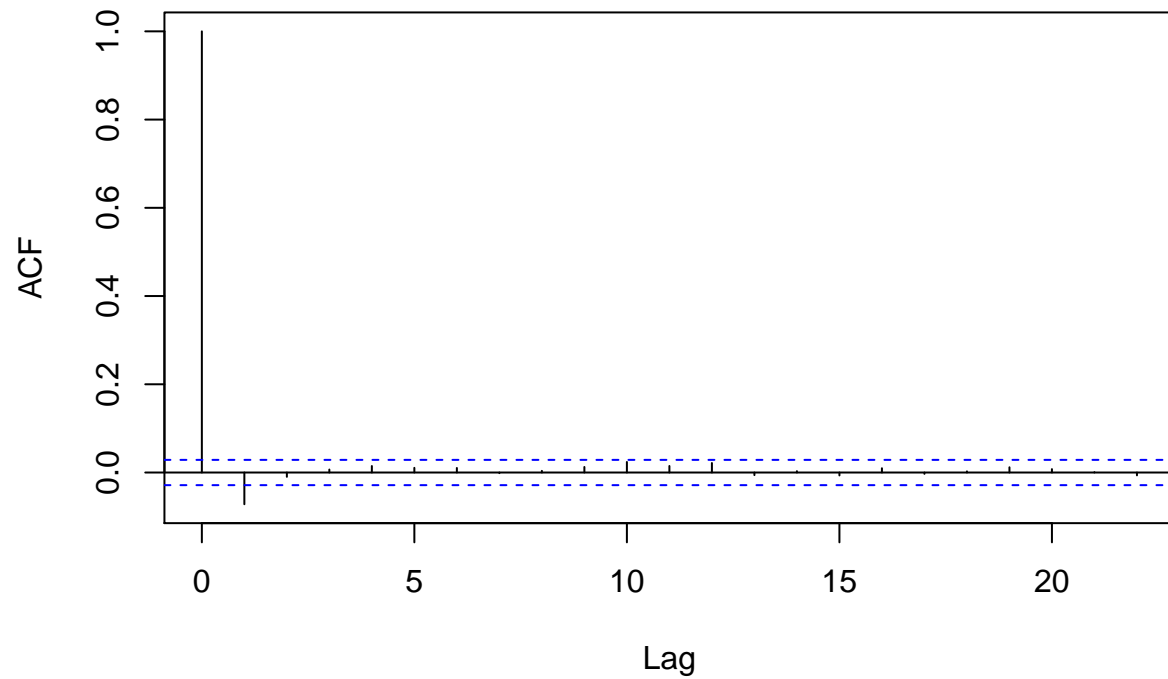
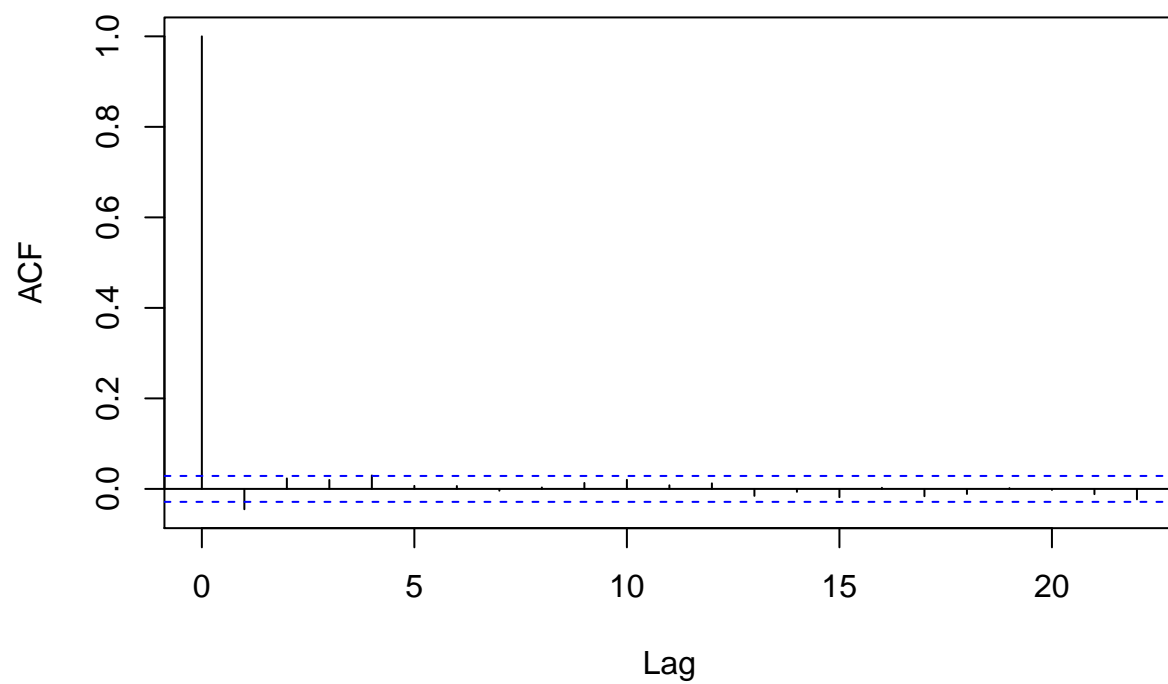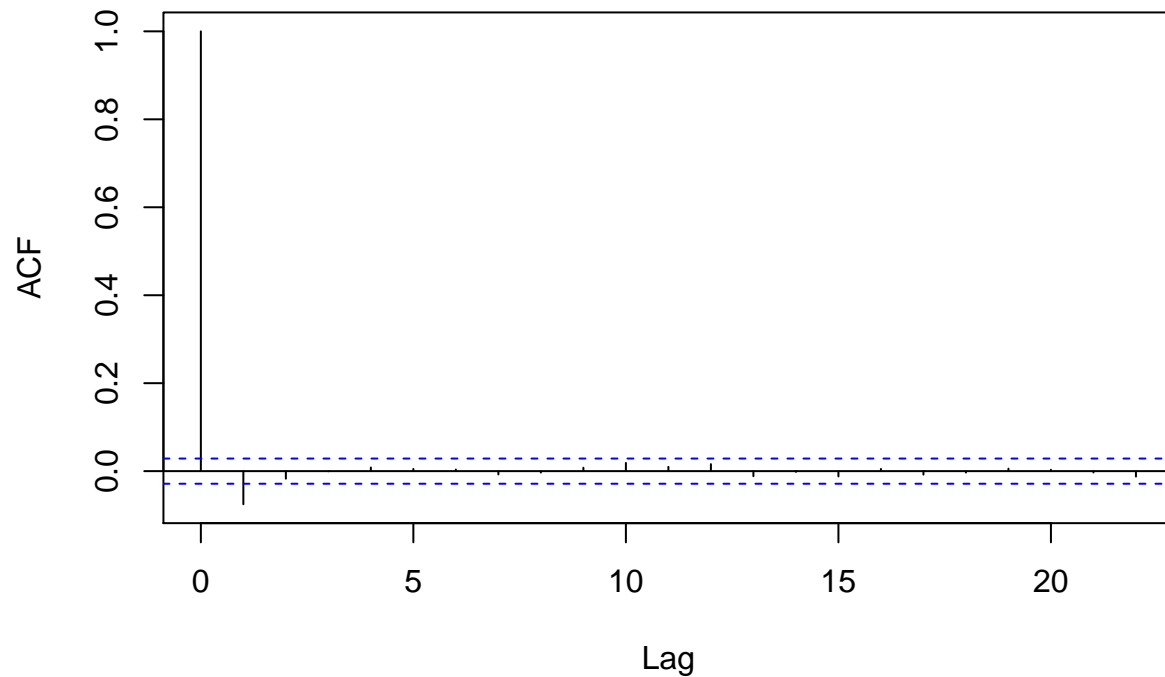## GJR GARCH with skewed Student t distribution of errors


```
## 
##  GJR GARCH with skewed Student t distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 35.065, df = 22, p-value = 0.03815
## 
## 
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0006524874 0.0001147633   5.685503 0.000000013
## omega  0.0000024508 0.0000007865   3.115893 0.001833889
## alpha1 0.1205992396 0.0098197314 12.281318 0.000000000
## beta1  0.8594536598 0.0106481285 80.714058 0.000000000
## 
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0006524874 0.0000965711   6.7565500 0.0000000000
## omega  0.0000024508 0.0000044008   0.5568893 0.5776030682
## alpha1 0.1205992396 0.0251184073   4.8012296 0.0000015769
## beta1  0.8594536598 0.0406037674 21.1668452 0.0000000000
## 
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate    Std. Error      t value    Pr(>|t|)
## mu     0.0002477111 0.0000998018 2.482030e+00 0.01306363
```

```
## omega  0.0000024813 0.0000002379 1.042998e+01 0.00000000
## alpha1 0.0000000542 0.0034471610 1.572800e-05 0.99998745
## beta1  0.8746538205 0.0066311516 1.319007e+02 0.00000000
## gamma1 0.2026440668 0.0127810580 1.585503e+01 0.00000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##              Estimate    Std. Error      t value     Pr(>|t|)
## mu     0.0002477111 0.0001308254 1.893449e+00 0.0582982283
## omega  0.0000024813 0.0000005808 4.272299e+00 0.0000193468
## alpha1 0.0000000542 0.0136585485 3.969500e-06 0.9999968328
## beta1  0.8746538205 0.0074170696 1.179244e+02 0.0000000000
## gamma1 0.2026440668 0.0315200843 6.429046e+00 0.0000000001
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error     t value      Pr(>|t|)
## mu     0.0005924393 0.0001184278  5.0025347 0.0000005658
## omega  0.0000013929 0.0000032444  0.4293241 0.6676873913
## alpha1 0.1172198092 0.0546264437  2.1458437 0.0318854521
## beta1  0.8761141511 0.0514355947 17.0332268 0.0000000000
## skew   0.8889153047 0.0217947432 40.7857663 0.0000000000
## shape  7.0598730007 1.6825791824  4.1958638 0.0000271834
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error     t value      Pr(>|t|)
## mu     0.0005924393 5.950877e-04 0.99554951 0.3194690792
## omega  0.0000013929 4.933230e-05 0.02823482 0.9774748630
## alpha1 0.1172198092 8.184706e-01 0.14321811 0.8861179292
## beta1  0.8761141511 7.726214e-01 1.13395013 0.2568154689
## skew   0.8889153047 1.978793e-01 4.49221016 0.0000070488
## shape  7.0598730007 2.298408e+01 0.30716364 0.7587188171
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error      t value    Pr(>|t|)
## mu     0.0002403827 0.0001172261 2.050589e+00 0.04030694
## omega  0.0000019857 0.0000010552 1.881878e+00 0.05985257
## alpha1 0.0000005843 0.0121633413 4.803760e-05 0.99996167
## beta1  0.8757172238 0.0111691773 7.840481e+01 0.00000000
## gamma1 0.2193849084 0.0305851524 7.172922e+00 0.00000000
## skew   0.8545123813 0.0170646651 5.007496e+01 0.00000000
## shape  8.2171916513 0.9868646400 8.326564e+00 0.00000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error      t value    Pr(>|t|)
## mu     0.0002403827 0.0002621922 9.168188e-01 0.35923761
## omega  0.0000019857 0.0000055388 3.585037e-01 0.71996641
## alpha1 0.0000005843 0.0401046660 1.456930e-05 0.99998838
## beta1  0.8757172238 0.0360296110 2.430549e+01 0.00000000
## gamma1 0.2193849084 0.1252958013 1.750936e+00 0.07995696
## skew   0.8545123813 0.0162712086 5.251684e+01 0.00000000
## shape  8.2171916513 1.0198432565 8.057308e+00 0.00000000
##              Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood   15225.702099 15327.477444   15339.237762 15433.032588
## Akaike          -6.505001    -6.548067      -6.552666    -6.592322
## Bayes           -6.499487    -6.541175      -6.544395    -6.582673
```
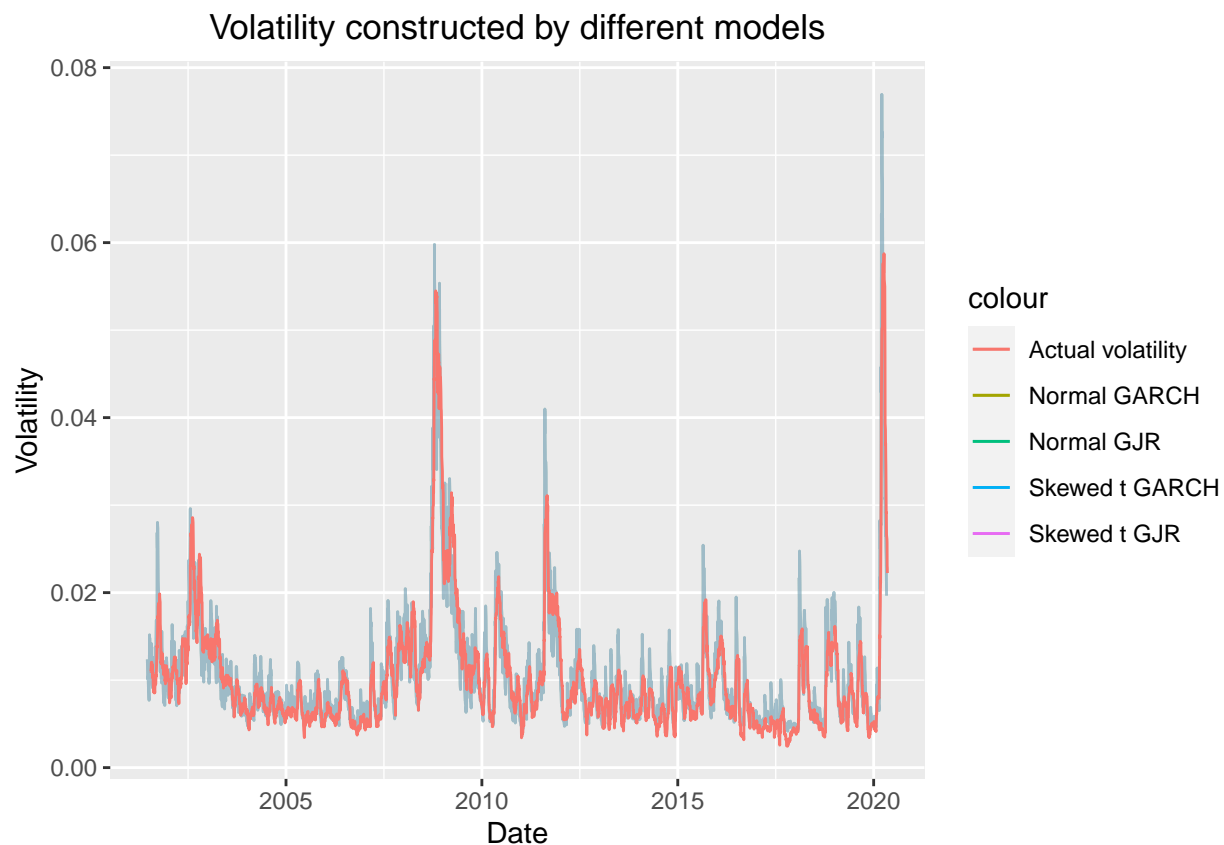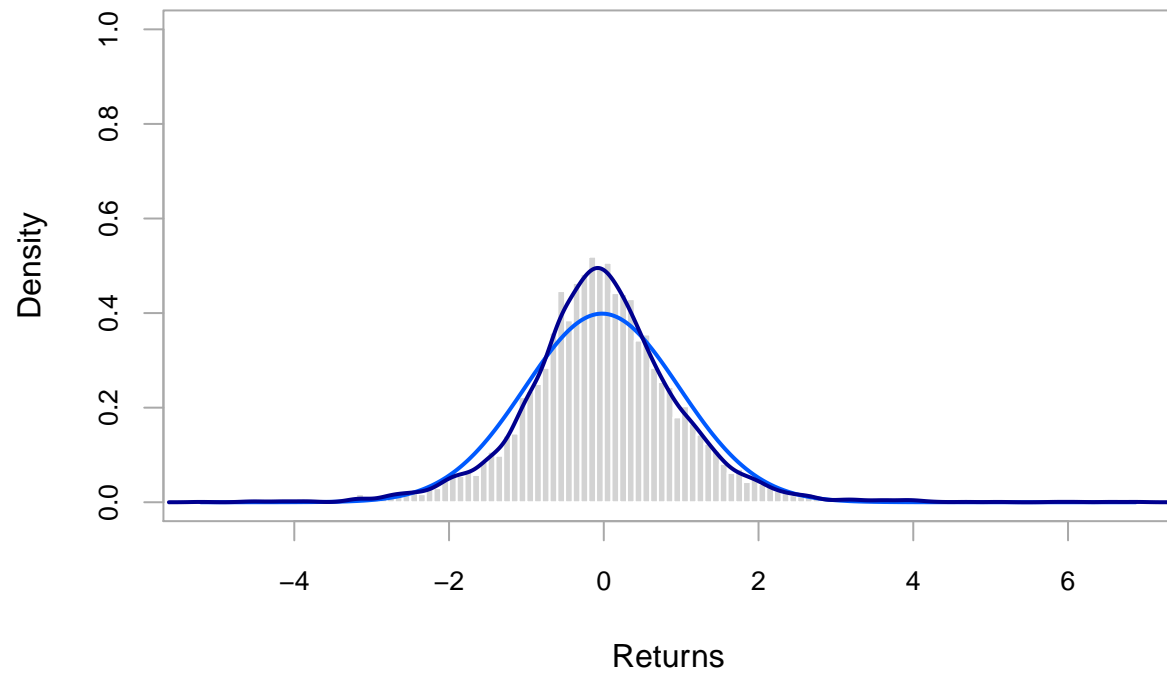
34

```
## Shibata          -6.505002    -6.548070       -6.552669    -6.592326
## Hannan-Quinn     -6.503062    -6.545644       -6.549757    -6.588928
```

Dependence of variance on errors in different models

Volatility constructed by different models


```
##    TES for Normal GARCH      TES for Normal GJR TES for Skewed t GARCH
##              0.05475380              0.08707727             0.02621860
##    TES for Skewed t GJR
##              0.07871238
##              Vanguard Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.03443334    0.02191476 0.01865495     0.02429064   0.01945951
## 2020-04-27 0.02908799    0.02175150 0.01849670     0.02423829   0.01930974
## 2020-04-28 0.02916100    0.02159030 0.01834093     0.02418618   0.01916192
## 2020-04-29 0.02756983    0.02143114 0.01818763     0.02413431   0.01901603
## 2020-04-30 0.02659937    0.02127399 0.01803674     0.02408267   0.01887205
## 2020-05-04 0.02609812    0.02111885 0.01788826     0.02403127   0.01872996
## 2020-05-05 0.02570555    0.02096569 0.01774215     0.02398010   0.01858974
## 2020-05-06 0.02301671    0.02081449 0.01759837     0.02392916   0.01845137
## 2020-05-07 0.02288516    0.02066523 0.01745691     0.02387845   0.01831483
## 2020-05-08 0.02224057    0.02051790 0.01731773     0.02382798   0.01818011
##
## --------------------------------
## Blackrock
```
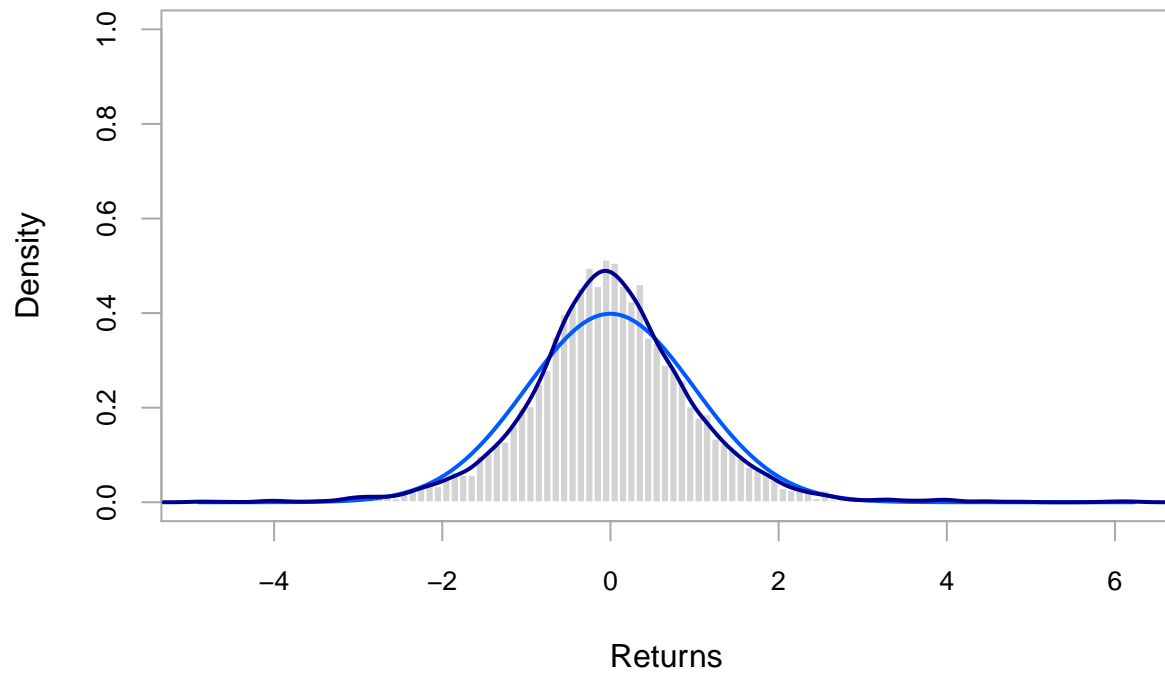
Standardized residuals of Standard GARCH with normal distribution of errors

Standardized residuals of GJR GARCH with normal distribution of errors

Standardized residuals of Standard GARCH with skewed Student t distribution of e

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
## 
##   Standard GARCH with normal distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 12.581, df = 22, p-value = 0.9442
```

GJR GARCH with normal distribution of errors


```
##
##   GJR GARCH with normal distribution of errors
##
##   Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 17.975, df = 22, p-value = 0.7075
```

Standard GARCH with skewed Student t distribution of errors


```
##
##  Standard GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 18.027, df = 22, p-value = 0.7044
```

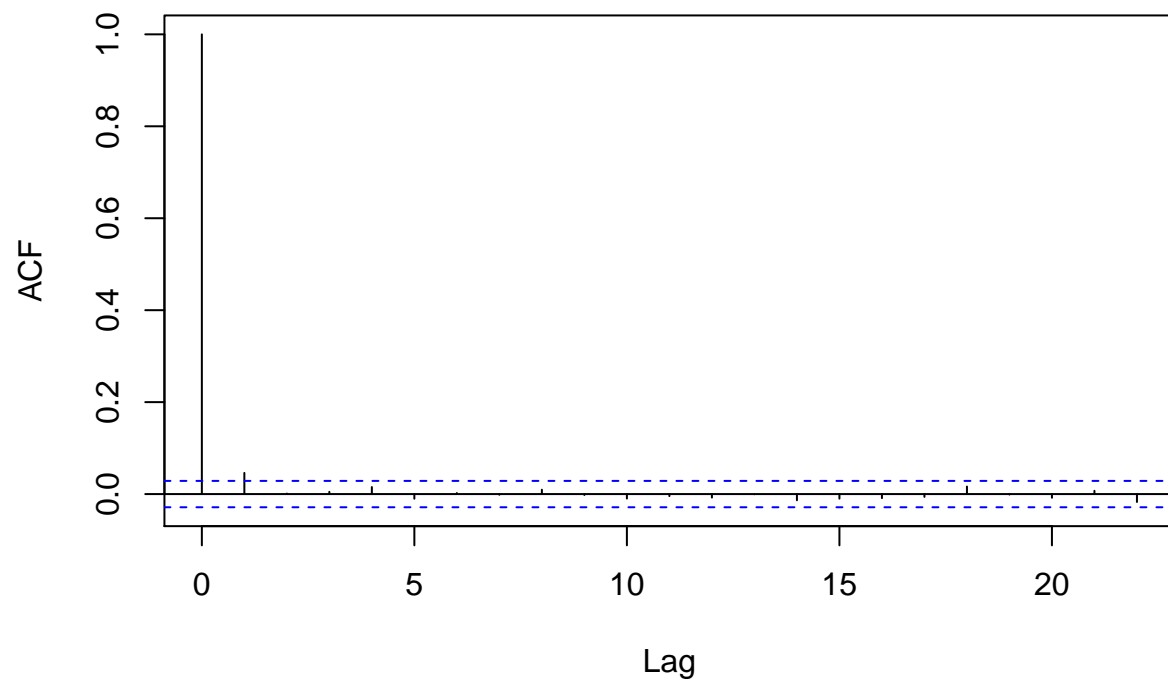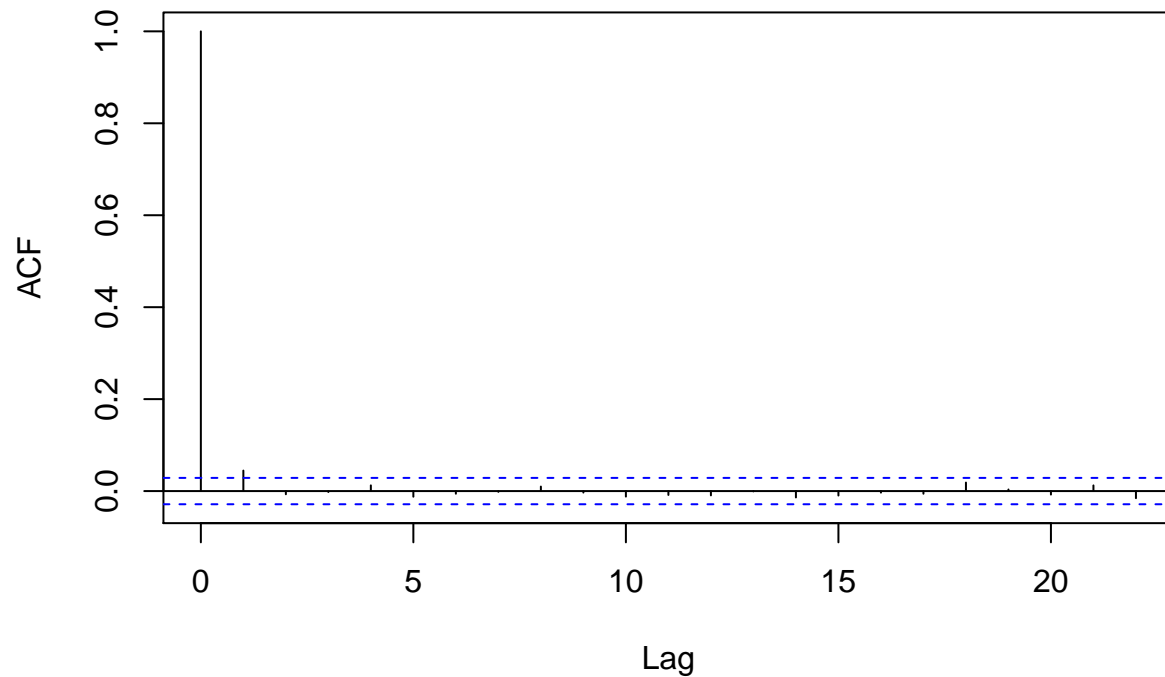**GJR GARCH with skewed Student t distribution of errors**


```
##
##  GJR GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 18.501, df = 22, p-value = 0.6759
##
##
## Coefficients of Standard GARCH with normal distribution of errors
##             Estimate  Std. Error     t value Pr(>|t|)
## mu     0.0011372119 2.19327e-04    5.185005 2.16e-07
## omega  0.0000098047 9.96100e-07    9.843453 0.00e+00
## alpha1 0.1055437293 6.30464e-03   16.740642 0.00e+00
## beta1  0.8699984930 7.50818e-03  115.873427 0.00e+00
##
## Robust coefficients of Standard GARCH with normal distribution of errors
##             Estimate   Std. Error    t value   Pr(>|t|)
## mu     0.0011372119 0.0002252054   5.049665 4.4260e-07
## omega  0.0000098047 0.0000020263   4.838728 1.3067e-06
## alpha1 0.1055437293 0.0098783468  10.684352 0.0000e+00
## beta1  0.8699984930 0.0129177290  67.349183 0.0000e+00
##
## Coefficients of GJR GARCH with normal distribution of errors
##             Estimate   Std. Error    t value    Pr(>|t|)
## mu     0.0007669611 0.0002043304   3.753534 0.0001743589
```
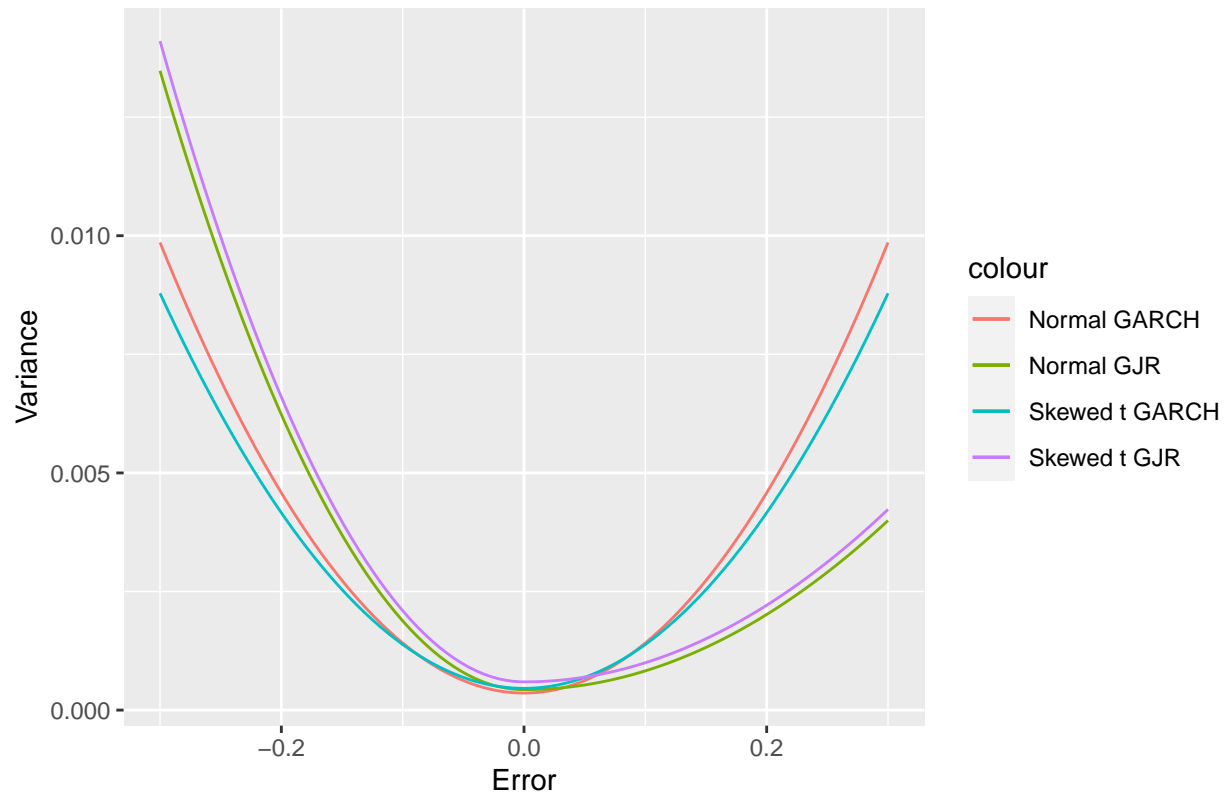
```
## omega  0.0000073817 0.0000013218    5.584698 0.0000000234
## alpha1 0.0396059054 0.0056308464    7.033739 0.0000000000
## beta1  0.8921657305 0.0063027440 141.551955 0.0000000000
## gamma1 0.1053526778 0.0136025021    7.745095 0.0000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##             Estimate    Std. Error   t value      Pr(>|t|)
## mu     0.0007669611 0.0002838199   2.702281 0.0068865594
## omega  0.0000073817 0.0000030137   2.449386 0.0143099804
## alpha1 0.0396059054 0.0090082599   4.396621 0.0000109949
## beta1  0.8921657305 0.0126930924 70.287500 0.0000000000
## gamma1 0.1053526778 0.0252650136   4.169904 0.0000304728
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error   t value      Pr(>|t|)
## mu     0.0009973209 0.0002181637   4.571435 0.000004844
## omega  0.0000066732 0.0000023472   2.843053 0.004468366
## alpha1 0.0925351283 0.0104546038   8.851137 0.000000000
## beta1  0.8942466400 0.0118573164 75.417287 0.000000000
## skew   1.0478669015 0.0204174758 51.322059 0.000000000
## shape  4.9496121142 0.3778308453 13.100074 0.000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error   t value      Pr(>|t|)
## mu     0.0009973209 0.0002026057   4.922473 0.0000008546
## omega  0.0000066732 0.0000044259   1.507756 0.1316170798
## alpha1 0.0925351283 0.0140043103   6.607618 0.0000000000
## beta1  0.8942466400 0.0169854754 52.647725 0.0000000000
## skew   1.0478669015 0.0210434652 49.795359 0.0000000000
## shape  4.9496121142 0.4063335792 12.181155 0.0000000000
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error   t value      Pr(>|t|)
## mu     0.0006914519 0.0002296984   3.010260 0.0026102427
## omega  0.0000068582 0.0000026221   2.615557 0.0089082163
## alpha1 0.0403951248 0.0104819645   3.853774 0.0001163109
## beta1  0.8935391870 0.0108783675 82.139088 0.0000000000
## gamma1 0.1097060616 0.0183454359   5.980019 0.0000000022
## skew   1.0332533449 0.0204951256 50.414589 0.0000000000
## shape  5.0961280193 0.4077562921 12.497975 0.0000000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error   t value    Pr(>|t|)
## mu     0.0006914519 0.0002738265   2.525146 0.01156501
## omega  0.0000068582 0.0000061876   1.108383 0.26769623
## alpha1 0.0403951248 0.0181985312   2.219691 0.02643972
## beta1  0.8935391870 0.0168245406 53.109277 0.00000000
## gamma1 0.1097060616 0.0256521486   4.276681 0.00001897
## skew   1.0332533449 0.0228895433 45.140846 0.00000000
## shape  5.0961280193 0.5466713919  9.322105 0.00000000
##             Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood   12353.595569 12386.926043   12546.367965 12570.031644
## Akaike          -5.277605    -5.291421      -5.359132    -5.368817
## Bayes           -5.272091    -5.284529      -5.350861    -5.359168
```
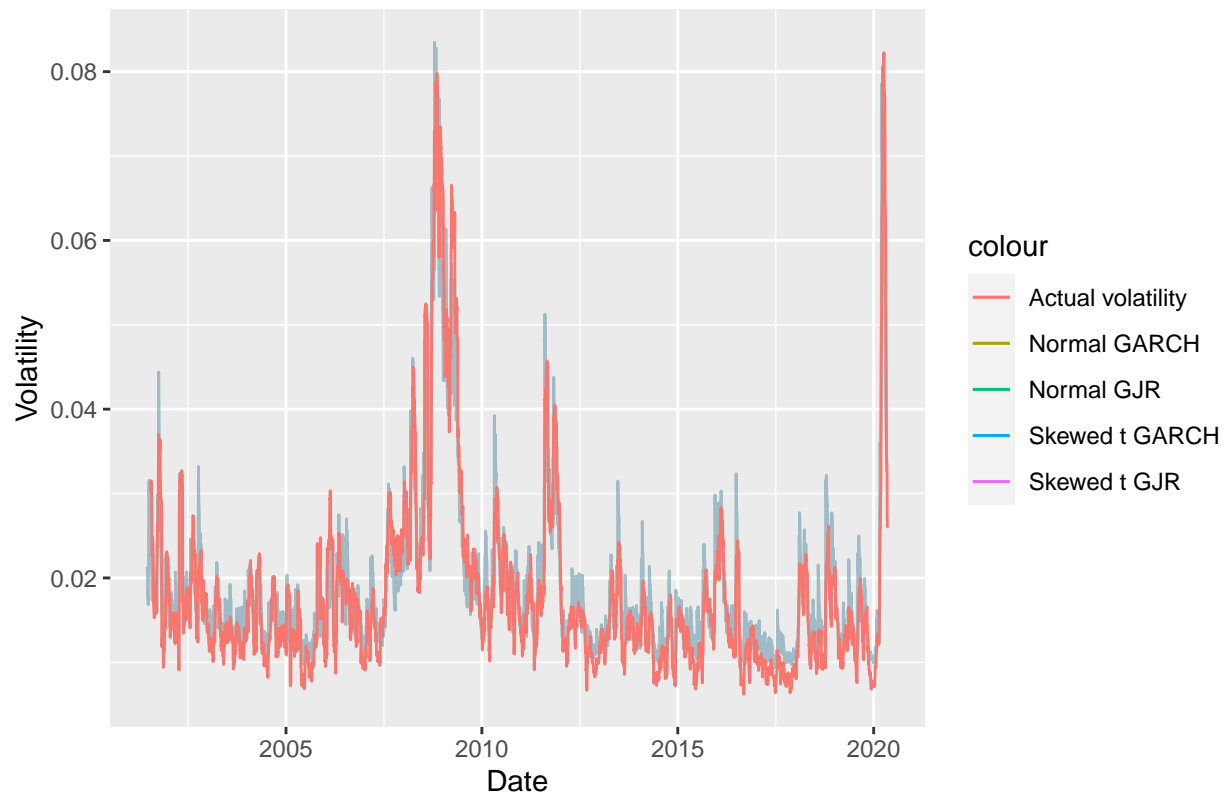
```
## Shibata          -5.277606    -5.291424    -5.359135    -5.368821
## Hannan-Quinn      -5.275666    -5.288998    -5.356223    -5.365424
```

Dependence of variance on errors in different models
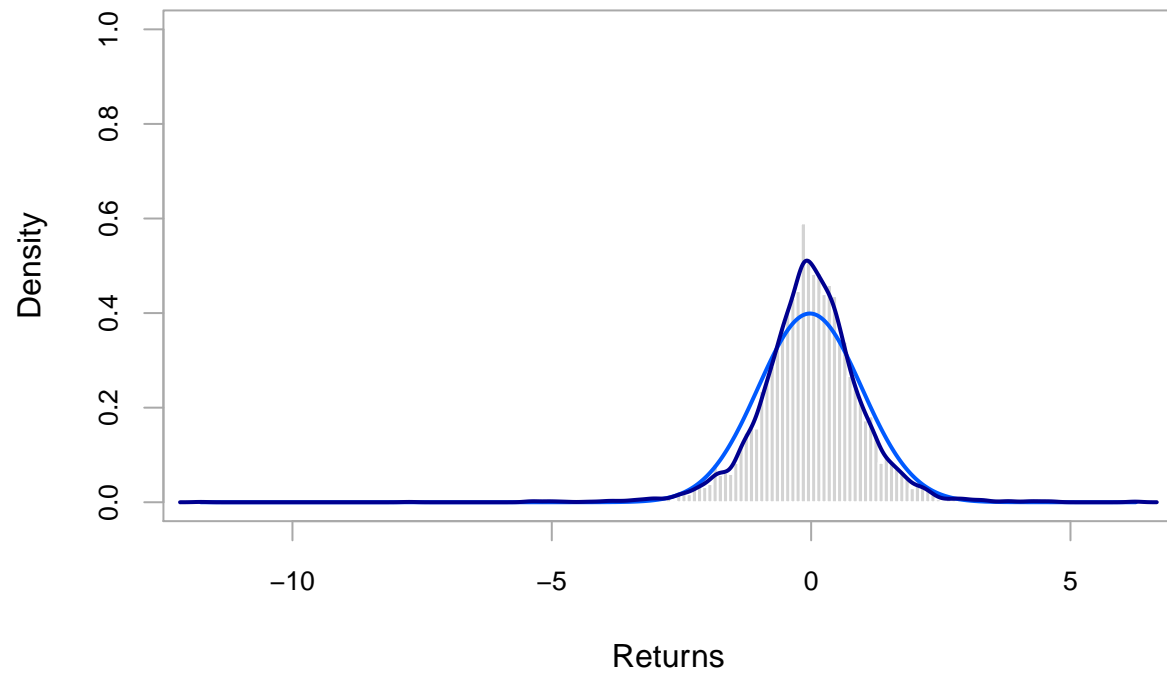
Volatility constructed by different models


```
##    TES for Normal GARCH    TES for Normal GJR TES for Skewed t GARCH
##            0.06055291              0.05150227              0.01125850
##    TES for Skewed t GJR
##            0.04087495
##             Blackrock Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.05190118    0.03047799 0.03100370     0.03509666   0.03178793
## 2020-04-27 0.04556408    0.03026539 0.03088142     0.03495950   0.03172997
## 2020-04-28 0.04334570    0.03005653 0.03076056     0.03482363   0.03167250
## 2020-04-29 0.03545849    0.02985138 0.03064112     0.03468903   0.03161554
## 2020-04-30 0.03441642    0.02964987 0.03052309     0.03455570   0.03155906
## 2020-05-04 0.03338657    0.02945197 0.03040643     0.03442362   0.03150308
## 2020-05-05 0.03214501    0.02925762 0.03029116     0.03429279   0.03144758
## 2020-05-06 0.02728841    0.02906676 0.03017725     0.03416320   0.03139256
## 2020-05-07 0.02675721    0.02887936 0.03006468     0.03403483   0.03133802
## 2020-05-08 0.02594207    0.02869537 0.02995346     0.03390769   0.03128396
##
## --------------------------------
## Statestreet
```
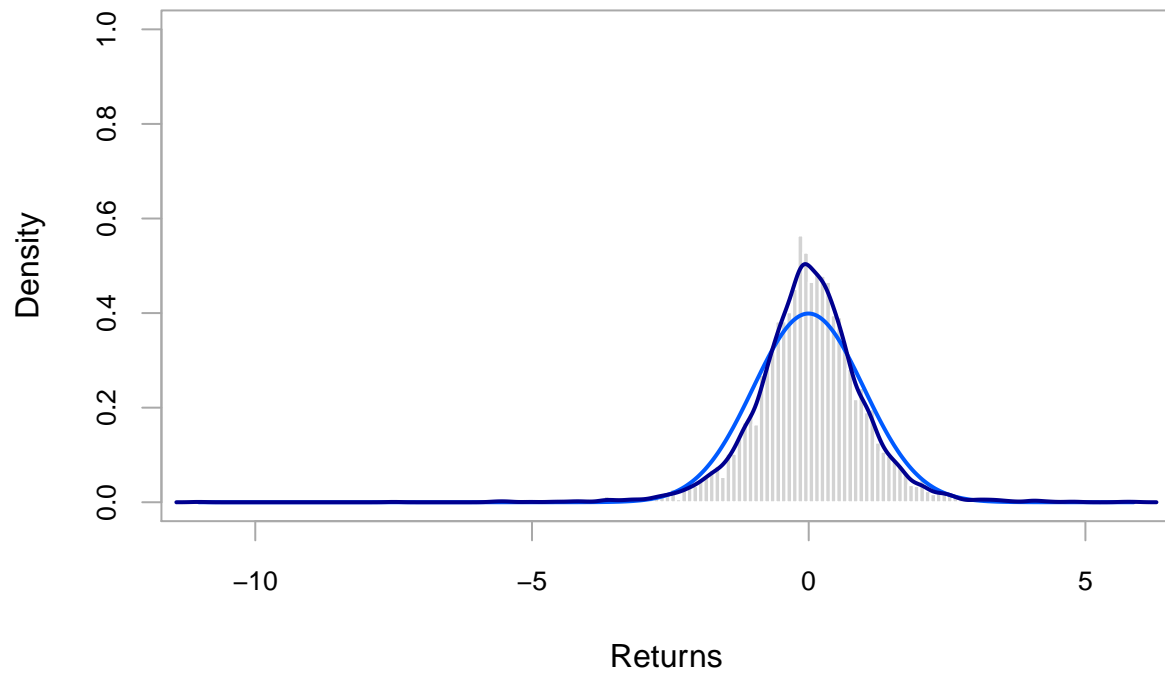
Standardized residuals of Standard GARCH with normal distribution of errors

Standardized residuals of GJR GARCH with normal distribution of errors

Standardized residuals of Standard GARCH with skewed Student t distribution of ε

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
## 
##  Standard GARCH with normal distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 38.431, df = 22, p-value = 0.01638
```

GJR GARCH with normal distribution of errors


```
## 
##  GJR GARCH with normal distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 35.918, df = 22, p-value = 0.03098
```

Standard GARCH with skewed Student t distribution of errors


```
## 
##  Standard GARCH with skewed Student t distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 32.885, df = 22, p-value = 0.0635
```
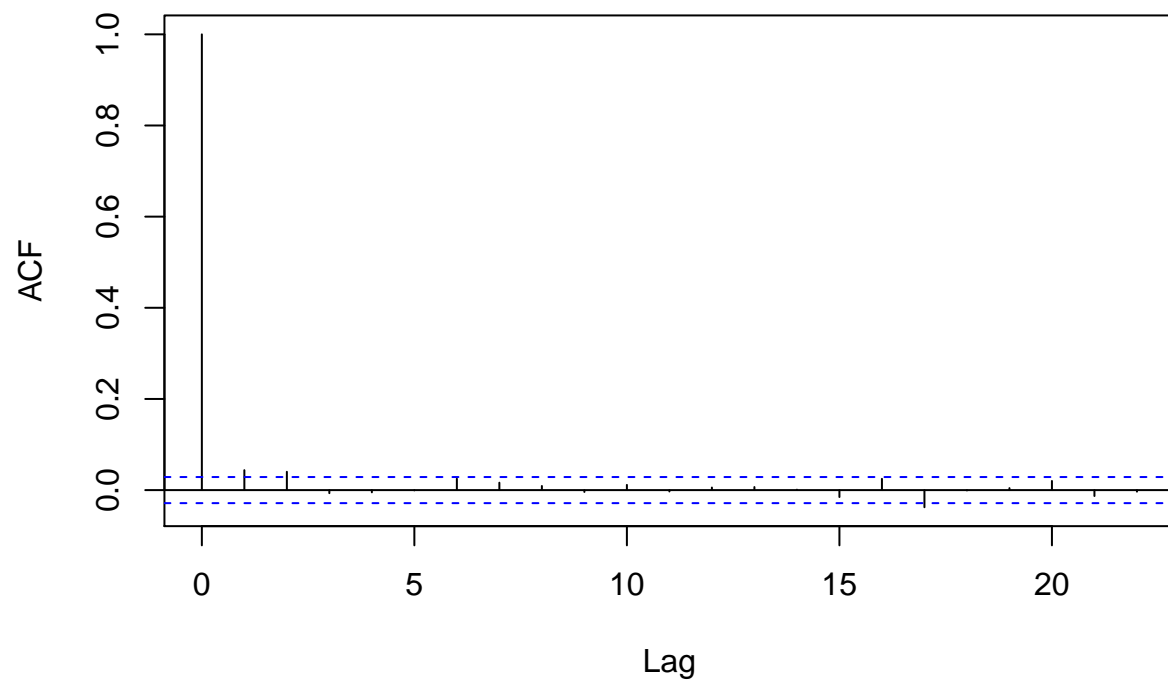
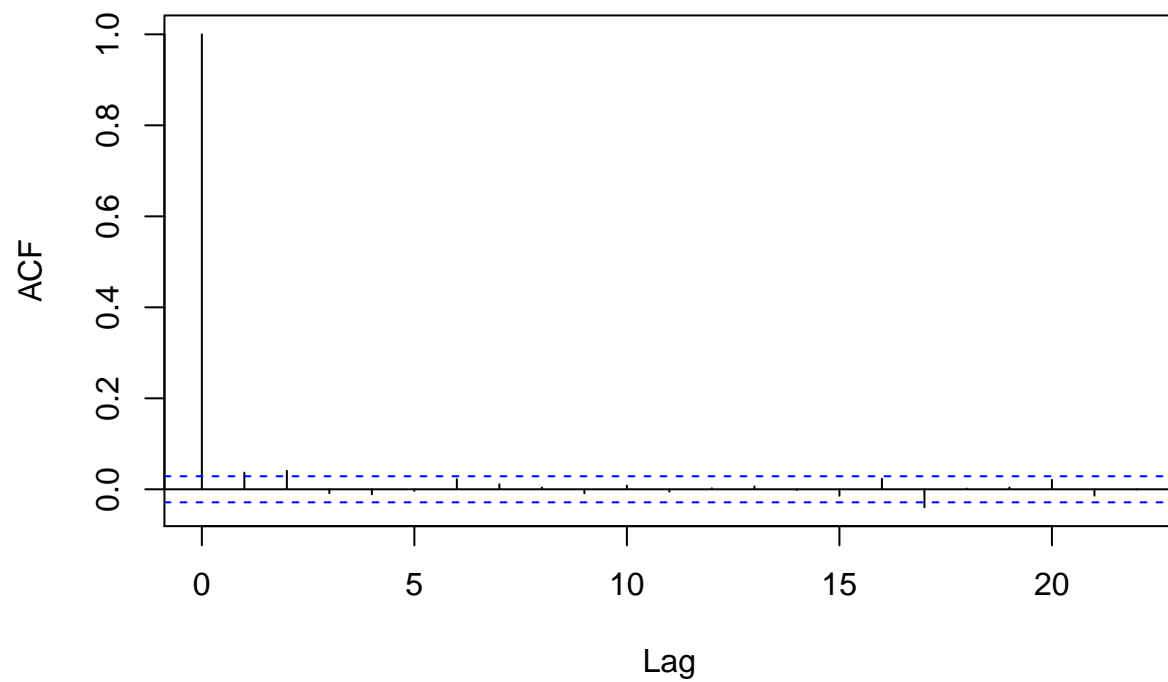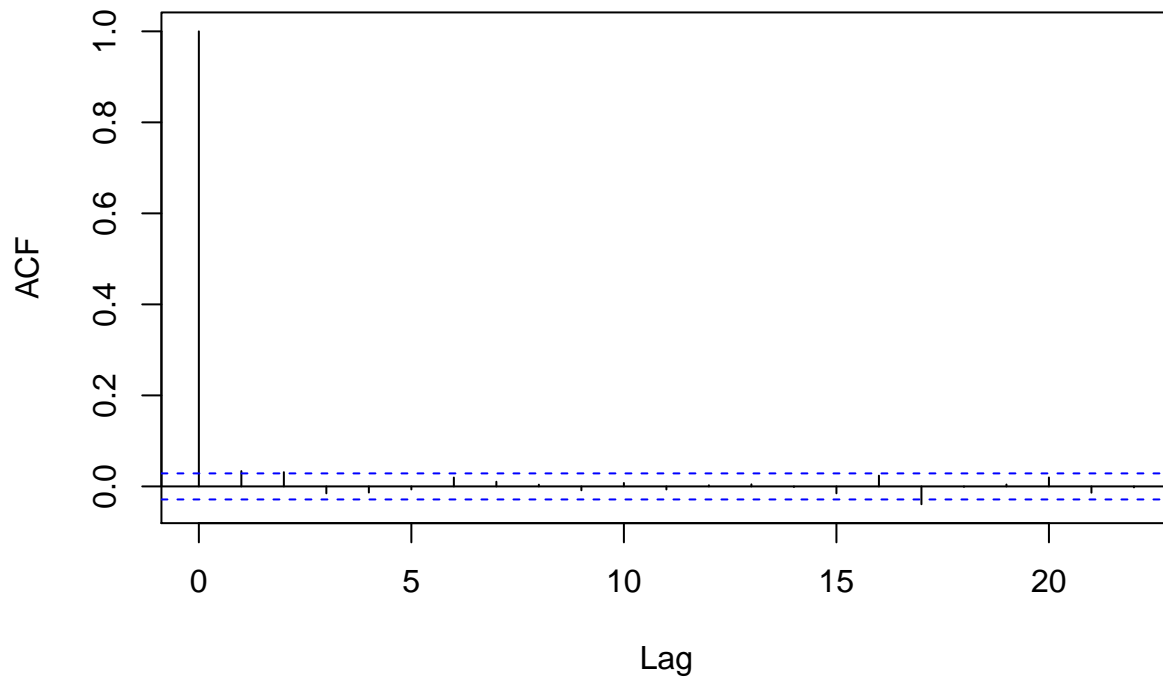## GJR GARCH with skewed Student t distribution of errors


```
## 
##  GJR GARCH with skewed Student t distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 29.104, df = 22, p-value = 0.1419
## 
## 
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error     t value     Pr(>|t|)
## mu      0.0006229046 0.0002336663    2.665787 0.007680828
## omega   0.0000040924 0.0000014199    2.882282 0.003948063
## alpha1  0.0678763140 0.0084295853    8.052153 0.000000000
## beta1   0.9265222666 0.0086931268  106.581014 0.000000000
## 
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error     t value     Pr(>|t|)
## mu      0.0006229046 0.0002366809    2.6318328 0.008492564
## omega   0.0000040924 0.0000090613    0.4516386 0.651529353
## alpha1  0.0678763140 0.0437535648    1.5513322 0.120822094
## beta1   0.9265222666 0.0524587509   17.6619201 0.000000000
## 
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate    Std. Error     t value     Pr(>|t|)
## mu      0.0002746120 0.0002367493    1.159927 0.246078453
```
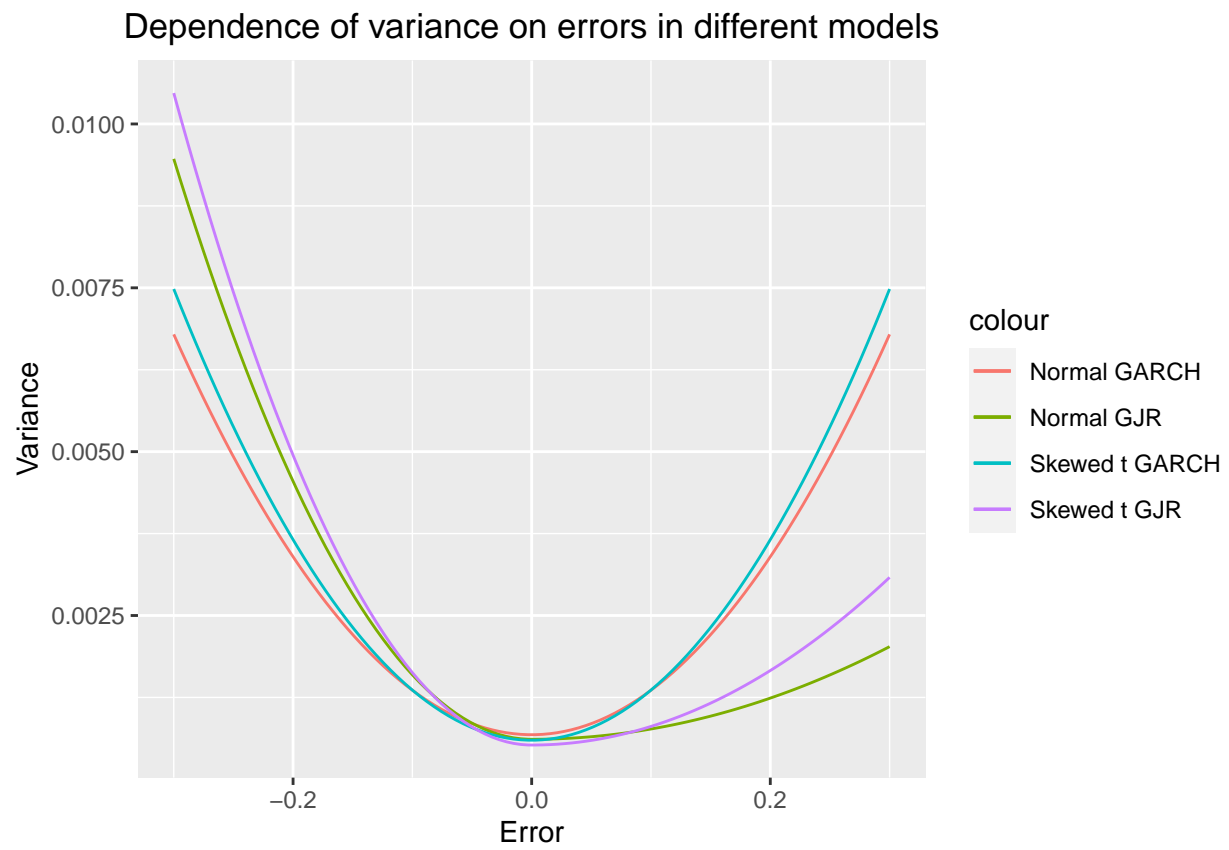
```
## omega  0.0000037187 0.0000013038   2.852206 0.004341696
## alpha1 0.0157118508 0.0055381410   2.837026 0.004553588
## beta1  0.9372042008 0.0071491406 131.093267 0.000000000
## gamma1 0.0826934991 0.0110567934   7.478977 0.000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##             Estimate    Std. Error    t value   Pr(>|t|)
## mu     0.0002746120 0.0003093397  0.8877361 0.37468271
## omega  0.0000037187 0.0000071647  0.5190367 0.60373516
## alpha1 0.0157118508 0.0144552270  1.0869321 0.27706680
## beta1  0.9372042008 0.0343471505 27.2862286 0.00000000
## gamma1 0.0826934991 0.0369137311  2.2401826 0.02507907
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error   t value      Pr(>|t|)
## mu     0.0005319452 0.0002266178  2.347323 0.0189088532
## omega  0.0000050417 0.0000021412  2.354596 0.0185428760
## alpha1 0.0764961144 0.0119287007  6.412778 0.0000000001
## beta1  0.9157055122 0.0130645483 70.090867 0.0000000000
## skew   0.9847034128 0.0195080660 50.476732 0.0000000000
## shape  4.4933912560 0.2987209745 15.042102 0.0000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error   t value   Pr(>|t|)
## mu     0.0005319452 0.0002056191  2.587043 0.009680361
## omega  0.0000050417 0.0000046219  1.090827 0.275349076
## alpha1 0.0764961144 0.0237620215  3.219260 0.001285221
## beta1  0.9157055122 0.0276363322 33.134119 0.000000000
## skew   0.9847034128 0.0196608988 50.084354 0.000000000
## shape  4.4933912560 0.3077369757 14.601402 0.000000000
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error   t value      Pr(>|t|)
## mu     0.0003108593 0.0002295581  1.354164 0.1756841237
## omega  0.0000048506 0.0000020210  2.400153 0.0163882417
## alpha1 0.0284228583 0.0088768782  3.201898 0.0013652534
## beta1  0.9223473218 0.0121751914 75.756289 0.0000000000
## gamma1 0.0821119888 0.0163685317  5.016454 0.0000005263
## skew   0.9761621443 0.0194613310 50.159064 0.0000000000
## shape  4.6039164960 0.3099168312 14.855329 0.0000000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error   t value   Pr(>|t|)
## mu     0.0003108593 0.0002306281  1.347881 0.177696534
## omega  0.0000048506 0.0000047520  1.020760 0.307368268
## alpha1 0.0284228583 0.0111510055  2.548905 0.010806160
## beta1  0.9223473218 0.0267371603 34.496832 0.000000000
## gamma1 0.0821119888 0.0314344004  2.612170 0.008996958
## skew   0.9761621443 0.0201394502 48.470149 0.000000000
## shape  4.6039164960 0.3271047777 14.074745 0.000000000
##             Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood   11962.325248 12004.024315   12269.313360 12288.489169
## Akaike          -5.110395    -5.127788      -5.240732    -5.248500
## Bayes           -5.104882    -5.120896      -5.232462    -5.238851
```
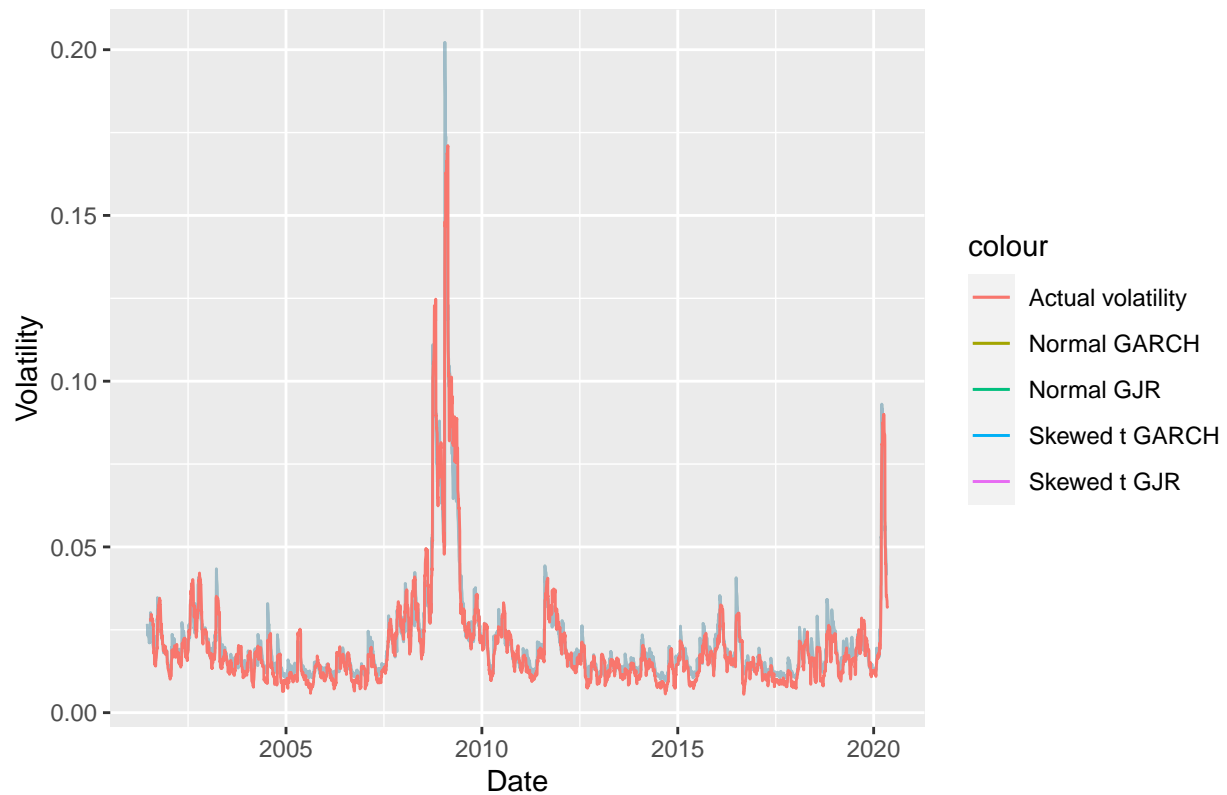
```
## Shibata           -5.110397      -5.127790         -5.240735      -5.248504
## Hannan-Quinn      -5.108456      -5.125364         -5.237824      -5.245106
```

Dependence of variance on errors in different models
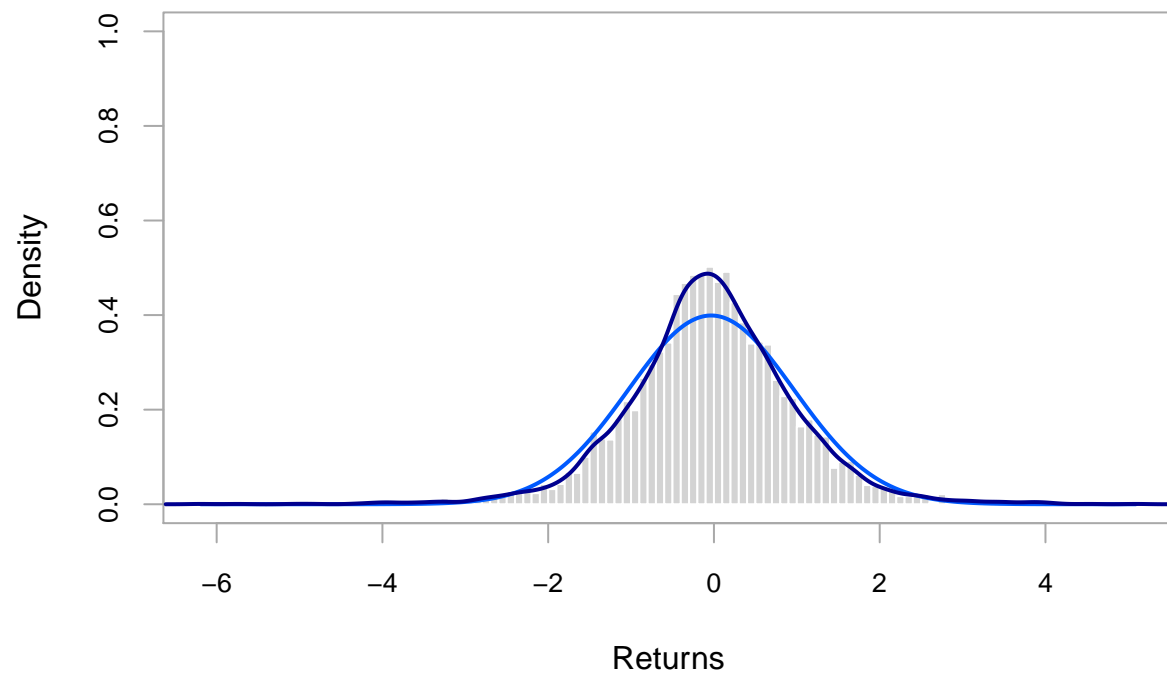
Volatility constructed by different models


```
##    TES for Normal GARCH    TES for Normal GJR TES for Skewed t GARCH
##           -0.09714092             -0.08877268             -0.07214436
##   TES for Skewed t GJR
##           -0.05564191
##            Statestreet Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24  0.04082046   0.04455275 0.04373417     0.04216634   0.04056256
## 2020-04-27  0.03499251   0.04447383 0.04365115     0.04206158   0.04044718
## 2020-04-28  0.03506750   0.04439521 0.04356845     0.04195737   0.04033247
## 2020-04-29  0.03509407   0.04431690 0.04348607     0.04185373   0.04021843
## 2020-04-30  0.03538565   0.04423888 0.04340401     0.04175064   0.04010504
## 2020-05-04  0.03390033   0.04416117 0.04332226     0.04164809   0.03999232
## 2020-05-05  0.03378543   0.04408375 0.04324083     0.04154610   0.03988026
## 2020-05-06  0.03202825   0.04400664 0.04315971     0.04144466   0.03976886
## 2020-05-07  0.03177526   0.04392982 0.04307891     0.04134376   0.03965810
## 2020-05-08  0.03202185   0.04385329 0.04299842     0.04124340   0.03954800
##
## --------------------------------
## JPmorgan
```
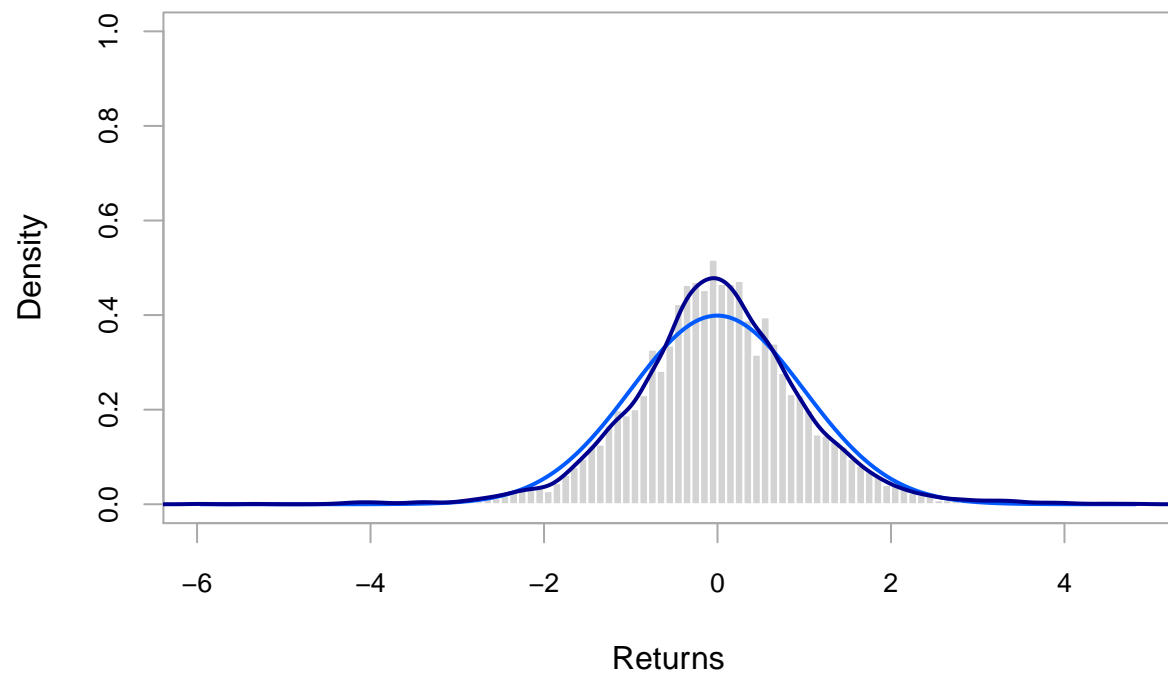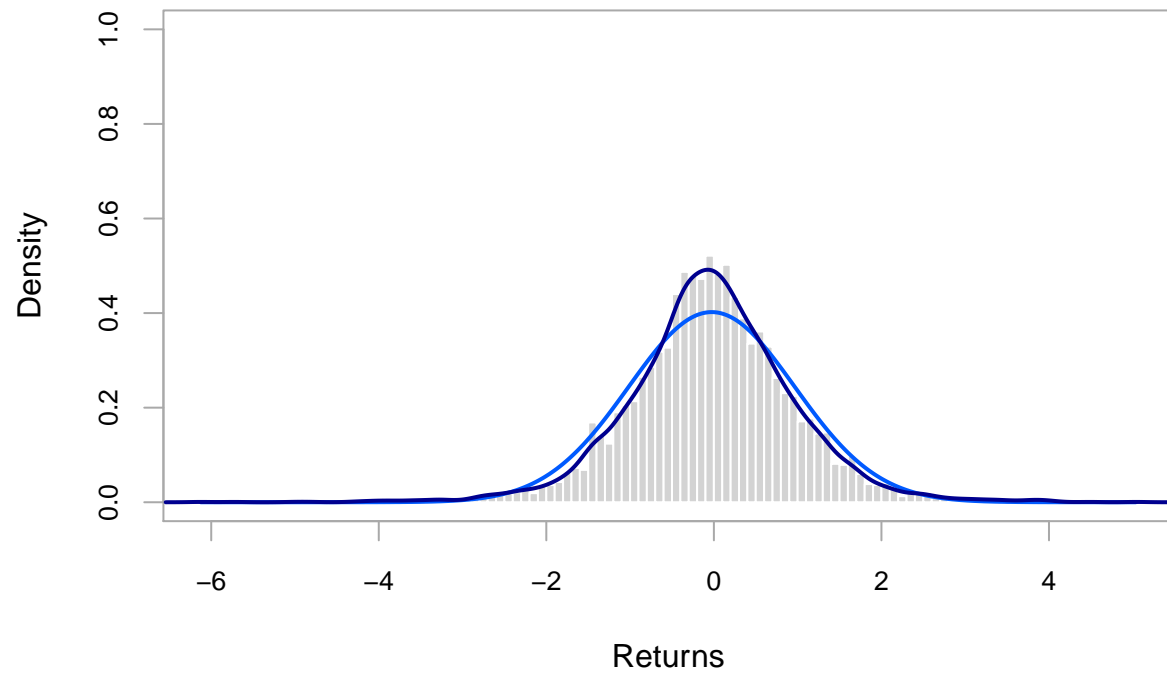
Standardized residuals of Standard GARCH with normal distribution of errors

Standardized residuals of GJR GARCH with normal distribution of errors

Standardized residuals of Standard GARCH with skewed Student t distribution of e

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
##
##  Standard GARCH with normal distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 24.517, df = 22, p-value = 0.3207
```

GJR GARCH with normal distribution of errors


```
## 
##  GJR GARCH with normal distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 18.568, df = 22, p-value = 0.6718
```

Standard GARCH with skewed Student t distribution of errors


```
## 
##  Standard GARCH with skewed Student t distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 25.874, df = 22, p-value = 0.2571
```
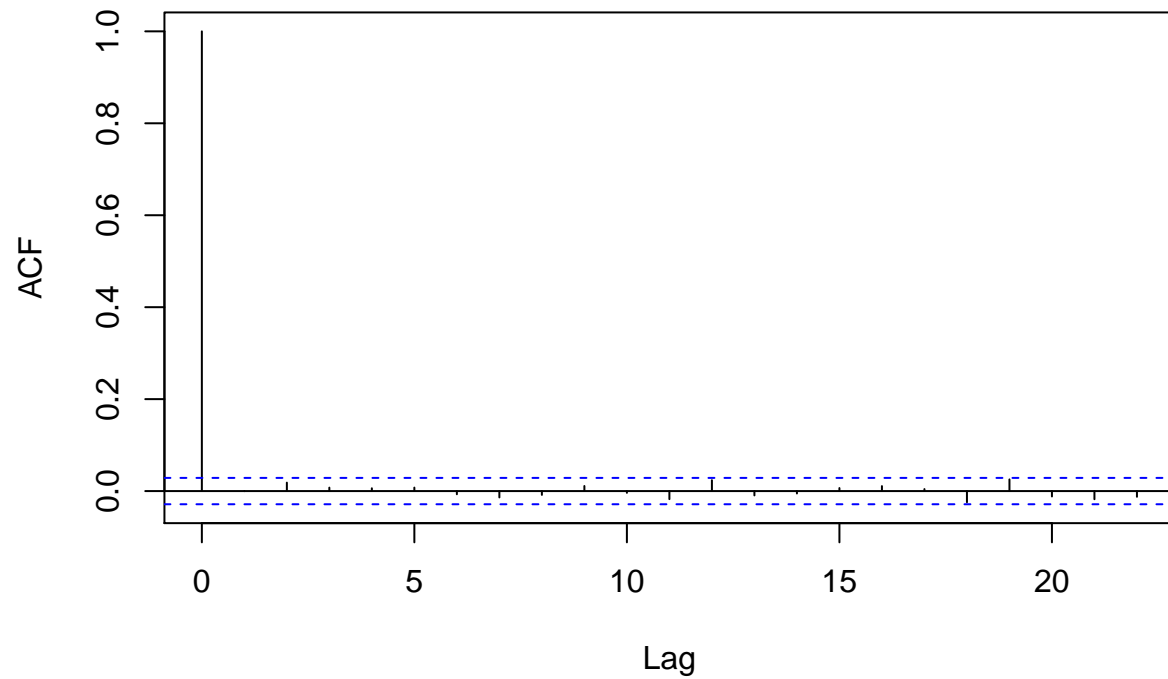
GJR GARCH with skewed Student t distribution of errors


```
##
##  GJR GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 19.644, df = 22, p-value = 0.6053
##
##
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value       Pr(>|t|)
## mu      0.0008025599 0.0001991405   4.030119 0.0000557487
## omega   0.0000037079 0.0000014476   2.561377 0.0104258206
## alpha1  0.0940084823 0.0103486903   9.084095 0.0000000000
## beta1   0.8992903956 0.0110887192  81.099573 0.0000000000
##
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value       Pr(>|t|)
## mu      0.0008025599 0.0002038019   3.9379412 0.0000821837
## omega   0.0000037079 0.0000053806   0.6891181 0.4907489737
## alpha1  0.0940084823 0.0324668126   2.8955255 0.0037852433
## beta1   0.8992903956 0.0371703513  24.1937556 0.0000000000
##
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate    Std. Error    t value       Pr(>|t|)
## mu      0.0003337115 0.0002109749   1.581759 0.1137045805
```
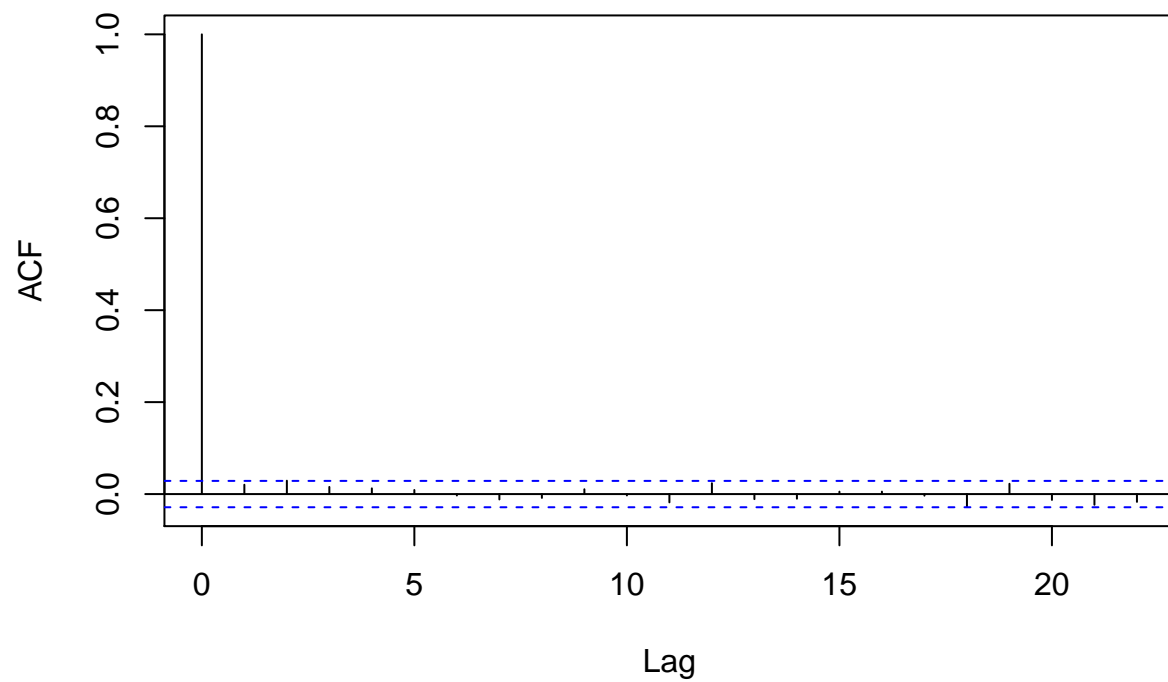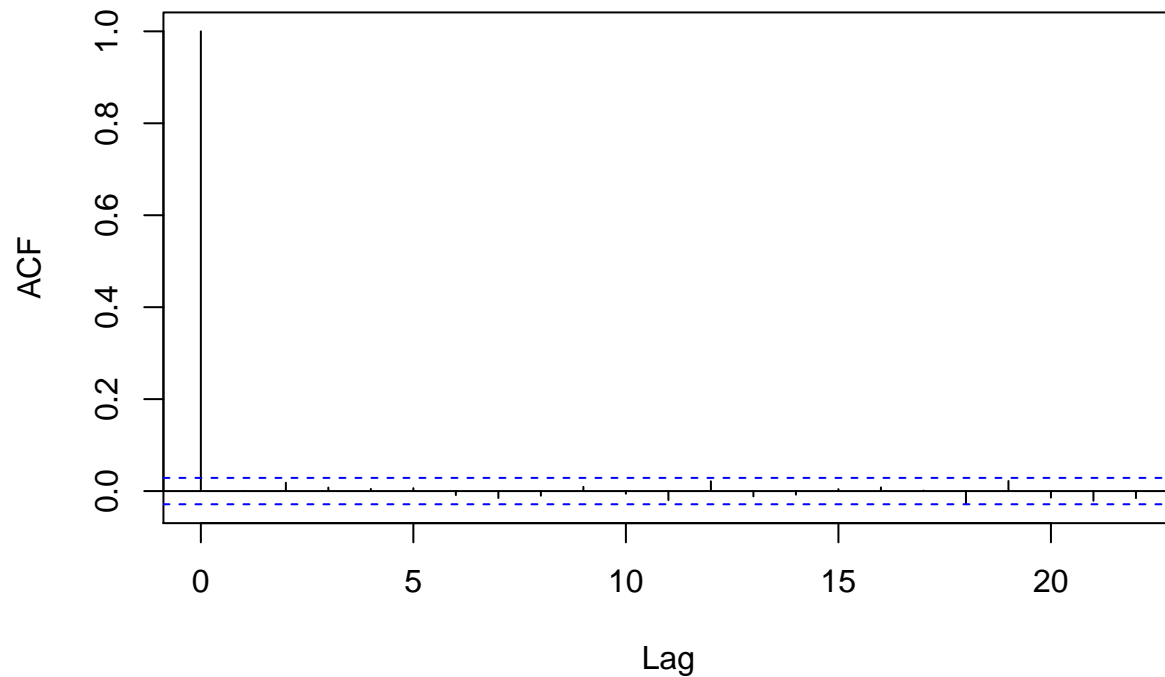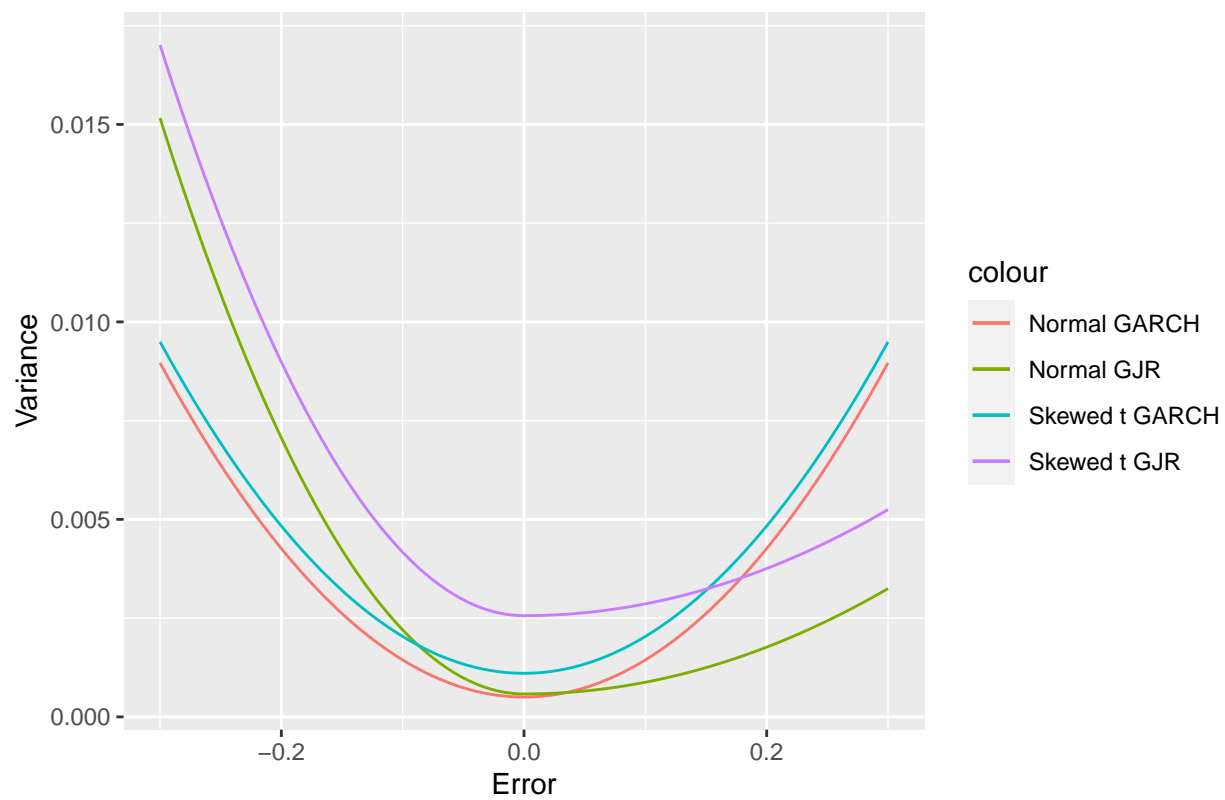
```
## omega  0.0000043912 0.0000015807   2.778102 0.0054677379
## alpha1 0.0296360574 0.0062706203   4.726176 0.0000022879
## beta1  0.8973292575 0.0096343825 93.138222 0.0000000000
## gamma1 0.1323862020 0.0161901970   8.176936 0.0000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0003337115 0.0003307577  1.0089303 0.3130080676
## omega  0.0000043912 0.0000059460  0.7385191 0.4601990446
## alpha1 0.0296360574 0.0097865457  3.0282449 0.0024597864
## beta1  0.8973292575 0.0283342262 31.6694464 0.0000000000
## gamma1 0.1323862020 0.0389134695  3.4020663 0.0006687842
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0006885213 0.0001963165  3.507200 0.0004528492
## omega  0.0000030960 0.0000025344  1.221573 0.2218690380
## alpha1 0.0932414078 0.0218512452  4.267098 0.0000198032
## beta1  0.9042134147 0.0212783064 42.494614 0.0000000000
## skew   1.0243320714 0.0201902157 50.734083 0.0000000000
## shape  5.3407621443 0.4675322590 11.423302 0.0000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0006885213 0.0001847397  3.7269809 0.0001937871
## omega  0.0000030960 0.0000107845  0.2870793 0.7740515726
## alpha1 0.0932414078 0.0897352308  1.0390725 0.2987710339
## beta1  0.9042134147 0.0882962523 10.2406772 0.0000000000
## skew   1.0243320714 0.0200539199 51.0788951 0.0000000000
## shape  5.3407621443 1.0032298544  5.3235678 0.0000001018
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0003553160 0.0001941626  1.829993 0.0672510562
## omega  0.0000034453 0.0000018022  1.911688 0.0559162145
## alpha1 0.0298318398 0.0093589493  3.187520 0.0014349844
## beta1  0.9031972239 0.0150478196 60.021800 0.0000000000
## gamma1 0.1306817031 0.0215986930  6.050445 0.0000000014
## skew   1.0154454501 0.0201420290 50.414258 0.0000000000
## shape  5.7459614158 0.4882562557 11.768331 0.0000000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0003553160 0.0001977740  1.7965762 0.072402926
## omega  0.0000034453 0.0000052368  0.6578953 0.510605417
## alpha1 0.0298318398 0.0179380462  1.6630484 0.096302740
## beta1  0.9031972239 0.0401706087 22.4840313 0.000000000
## gamma1 0.1306817031 0.0485359165  2.6924742 0.007092402
## skew   1.0154454501 0.0211991039 47.9003949 0.000000000
## shape  5.7459614158 0.5967003029  9.6295601 0.000000000
##              Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood   12504.555570 12569.633840   12658.494820 12700.275512
## Akaike           -5.342118    -5.369502      -5.407049    -5.424477
## Bayes            -5.336604    -5.362609      -5.398778    -5.414828
```
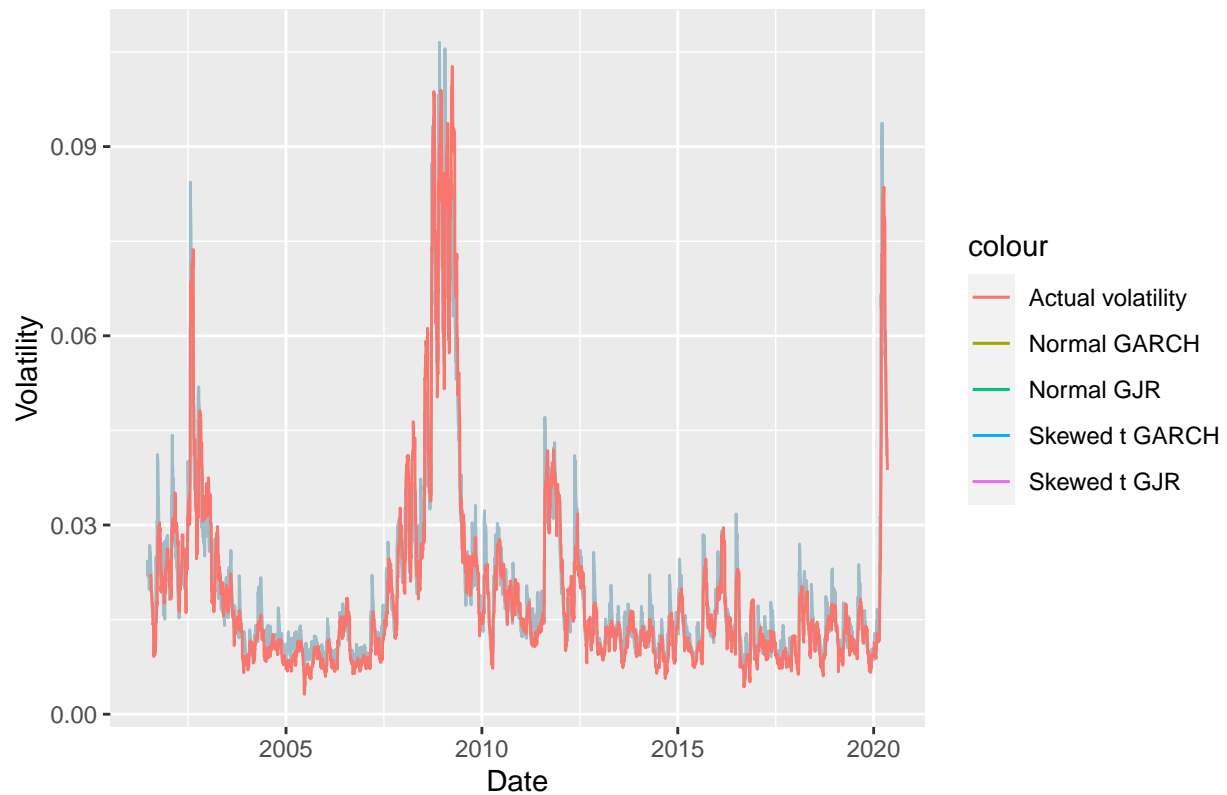
```
## Shibata          -5.342119    -5.369504        -5.407052    -5.424481
## Hannan-Quinn      -5.340179    -5.367078        -5.404141    -5.421084
```

Dependence of variance on errors in different models
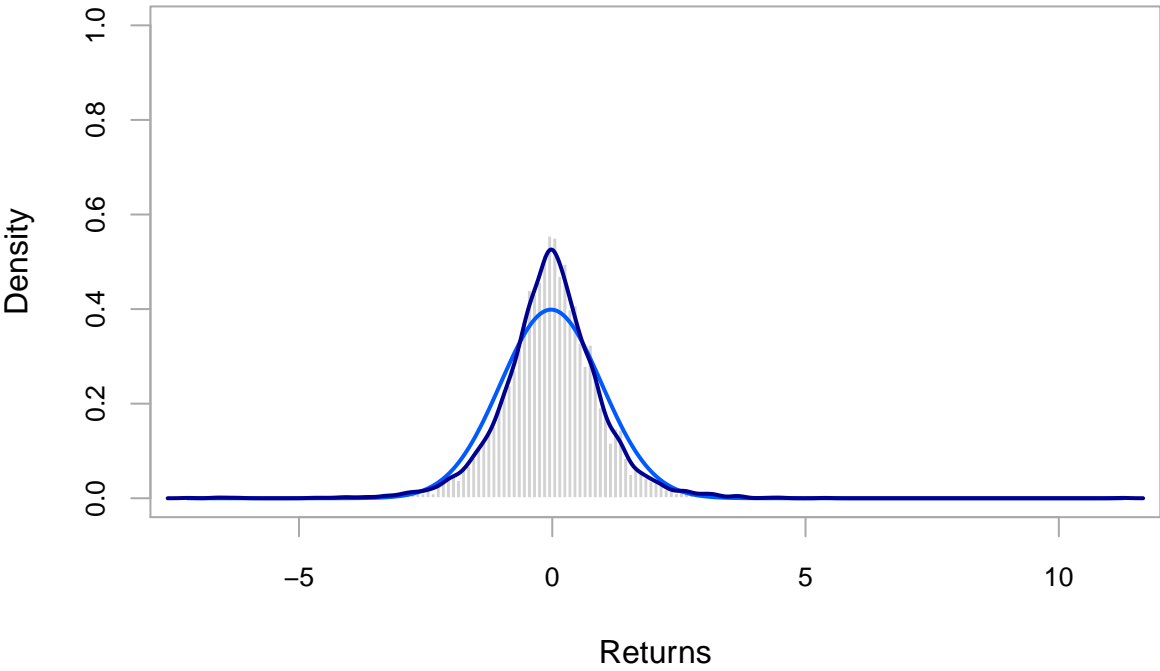
Volatility constructed by different models


```
##    TES for Normal GARCH      TES for Normal GJR TES for Skewed t GARCH
##               0.03222845              0.07211499             0.01223805
##    TES for Skewed t GJR
##               0.04774524
##               JPmorgan Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.05410294    0.04202937 0.03792954     0.04368805   0.03996284
## 2020-04-27 0.04898786    0.04193254 0.03785761     0.04366788   0.03998165
## 2020-04-28 0.04849763    0.04183615 0.03778604     0.04364776   0.04000042
## 2020-04-29 0.04660068    0.04174018 0.03771482     0.04362767   0.04001916
## 2020-04-30 0.04395919    0.04164463 0.03764395     0.04360763   0.04003787
## 2020-05-04 0.04384921    0.04154951 0.03757344     0.04358763   0.04005654
## 2020-05-05 0.04292573    0.04145480 0.03750328     0.04356767   0.04007519
## 2020-05-06 0.04055567    0.04136052 0.03743347     0.04354775   0.04009380
## 2020-05-07 0.04000345    0.04126666 0.03736400     0.04352788   0.04011238
## 2020-05-08 0.03873366    0.04117321 0.03729489     0.04350804   0.04013094
##
## -------------------------------
## Bankmellon
```
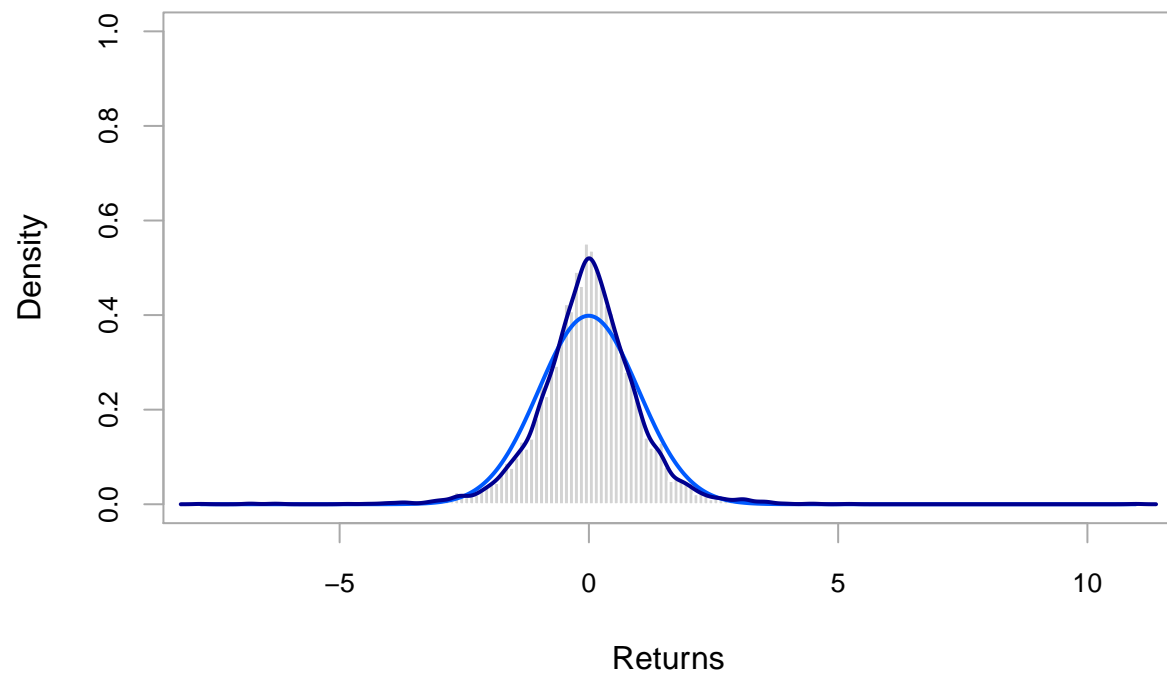
Standardized residuals of Standard GARCH with normal distribution of errors

Standardized residuals of GJR GARCH with normal distribution of errors

Standardized residuals of Standard GARCH with skewed Student t distribution of ε

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
## 
##   Standard GARCH with normal distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 13.069, df = 22, p-value = 0.9312
```

GJR GARCH with normal distribution of errors


```
##
##  GJR GARCH with normal distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 17.044, df = 22, p-value = 0.7609
```

Standard GARCH with skewed Student t distribution of errors


```
## 
##  Standard GARCH with skewed Student t distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 12.621, df = 22, p-value = 0.9432
```

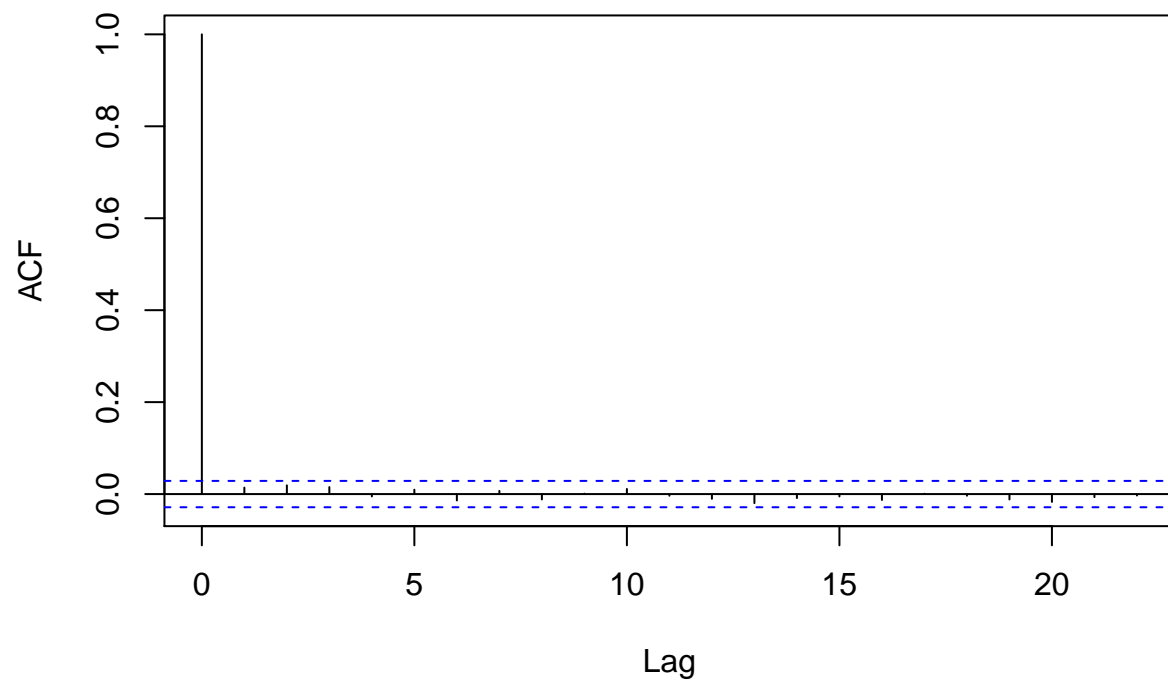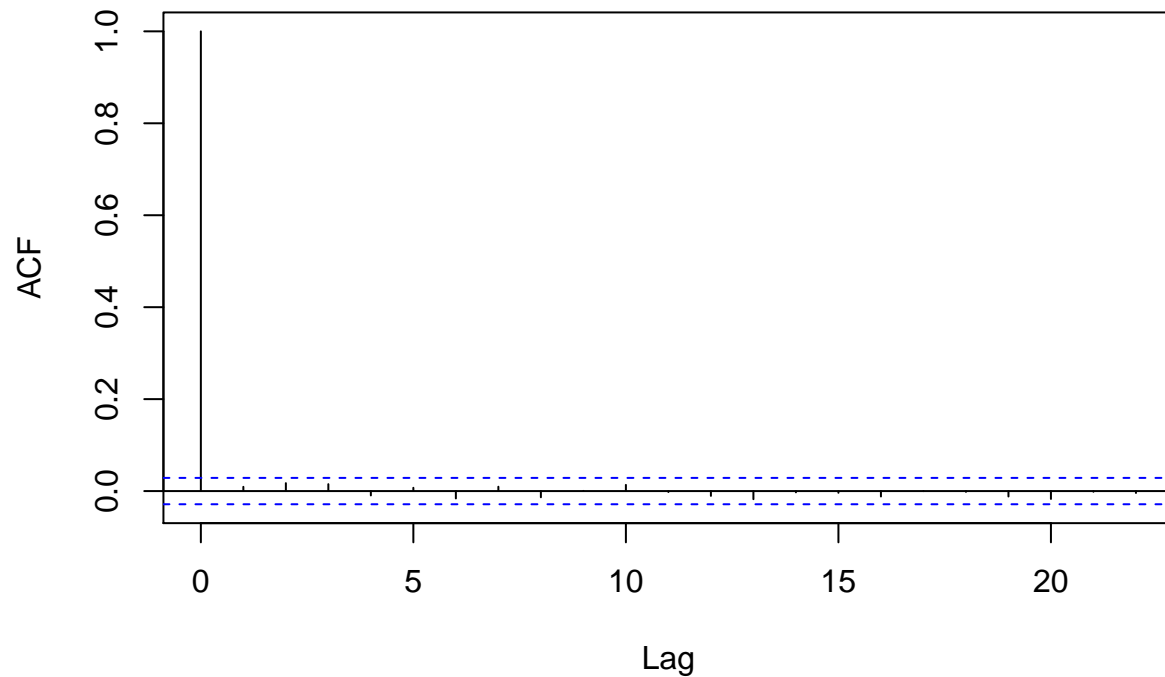## GJR GARCH with skewed Student t distribution of errors


```
## 
##  GJR GARCH with skewed Student t distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 11.804, df = 22, p-value = 0.9613
## 
## 
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value    Pr(>|t|)
## mu     0.0005565676 0.0002181872    2.550872 0.01074537
## omega  0.0000085033 0.0000012482    6.812440 0.00000000
## alpha1 0.0963925889 0.0042059078   22.918379 0.00000000
## beta1  0.8831197658 0.0066368376  133.063338 0.00000000
## 
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate Std. Error    t value     Pr(>|t|)
## mu     0.0005565676 0.000223894   2.485853 0.012924133
## omega  0.0000085033 0.000006642   1.280227 0.200465436
## alpha1 0.0963925889 0.020485651   4.705371 0.000002534
## beta1  0.8831197658 0.021855789  40.406676 0.000000000
## 
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate    Std. Error     t value     Pr(>|t|)
## mu     0.0001956997 0.0001957810    0.9995847 0.3175115124
```

```
## omega  0.0000070117 0.0000005894   11.8969912 0.0000000000
## alpha1 0.0236004194 0.0045543672    5.1819316 0.0000002196
## beta1  0.9067820336 0.0056798902 159.6478093 0.0000000000
## gamma1 0.1044815986 0.0124557037    8.3882533 0.0000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##              Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0001956997 0.0004354262   0.449444 0.6531114192
## omega  0.0000070117 0.0000024009   2.920467 0.0034950774
## alpha1 0.0236004194 0.0113369127   2.081732 0.0373669127
## beta1  0.9067820336 0.0131888498 68.753685 0.0000000000
## gamma1 0.1044815986 0.0301694167   3.463163 0.0005338655
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0005702877 0.0002076088   2.746934 0.006015527
## omega  0.0000056422 0.0000020209   2.791934 0.005239404
## alpha1 0.1010891051 0.0132544746   7.626791 0.000000000
## beta1  0.8914282214 0.0138565834 64.332469 0.000000000
## skew   1.0064232712 0.0193637701 51.974552 0.000000000
## shape  4.4455666007 0.2972196912 14.957174 0.000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0005702877 0.0001926085   2.960865 0.0030677653
## omega  0.0000056422 0.0000036281   1.555148 0.1199107788
## alpha1 0.1010891051 0.0196201877   5.152301 0.0000002573
## beta1  0.8914282214 0.0226249320 39.400261 0.0000000000
## skew   1.0064232712 0.0197306195 51.008194 0.0000000000
## shape  4.4455666007 0.3151782285 14.104929 0.0000000000
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0002367350 0.0002242989   1.055444 0.2912221503
## omega  0.0000056932 0.0000022647   2.513865 0.0119416049
## alpha1 0.0272174757 0.0123053541   2.211840 0.0269777156
## beta1  0.8993436776 0.0120065989 74.904116 0.0000000000
## gamma1 0.1306615680 0.0239456725   5.456584 0.0000000485
## skew   0.9910566542 0.0195837747 50.606008 0.0000000000
## shape  4.5824743849 0.3159590395 14.503381 0.0000000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0002367350 0.0003218238   0.7356044 0.46197150
## omega  0.0000056932 0.0000057529   0.9896239 0.32235796
## alpha1 0.0272174757 0.0302000487   0.9012395 0.36746101
## beta1  0.8993436776 0.0206846919 43.4787079 0.00000000
## gamma1 0.1306615680 0.0531746018   2.4572176 0.01400178
## skew   0.9910566542 0.0220970220 44.8502361 0.00000000
## shape  4.5824743849 0.3855923807 11.8842452 0.00000000
##               Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood    12345.345038 12377.211104   12630.718834 12657.961065
## Akaike           -5.274079    -5.287270      -5.395179    -5.406394
## Bayes            -5.268565    -5.280378      -5.386908    -5.396745
```
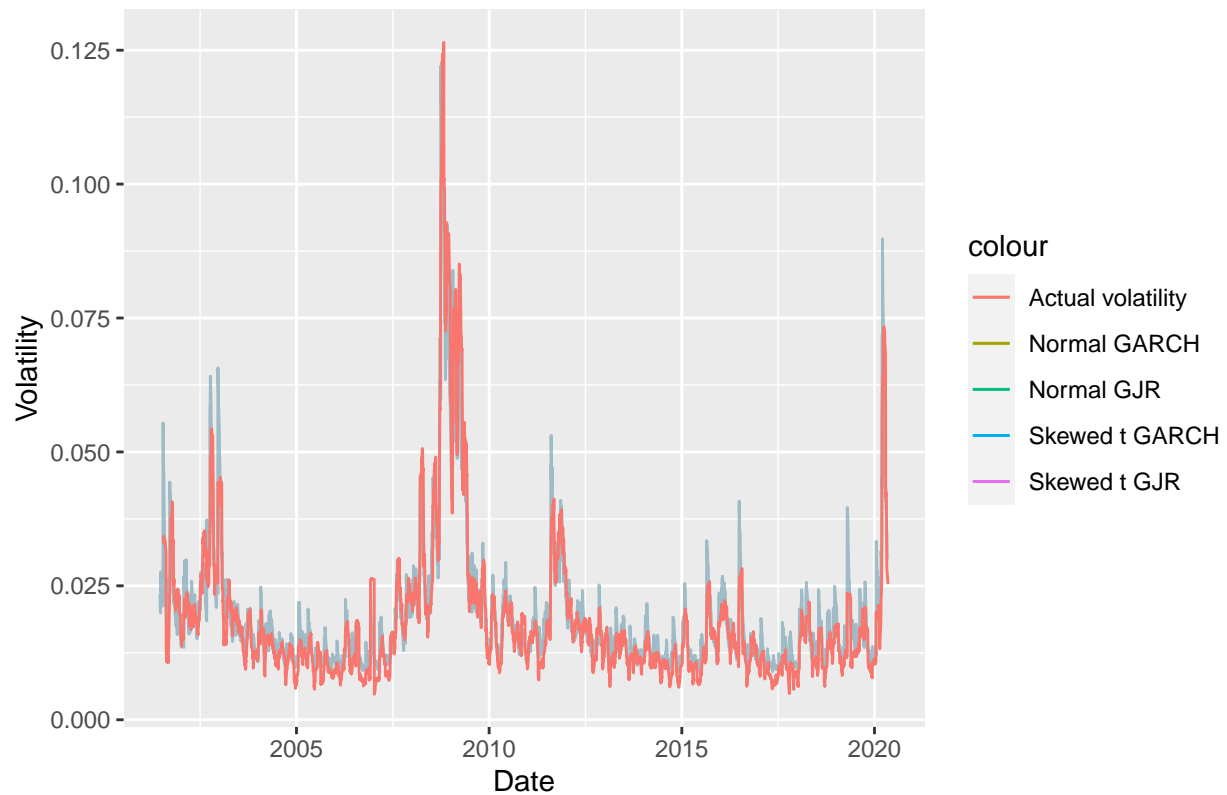
```
## Shibata          -5.274081    -5.287272       -5.395182    -5.406398
## Hannan-Quinn      -5.272140    -5.284846       -5.392271    -5.403000
```


Dependence of variance on errors in different models
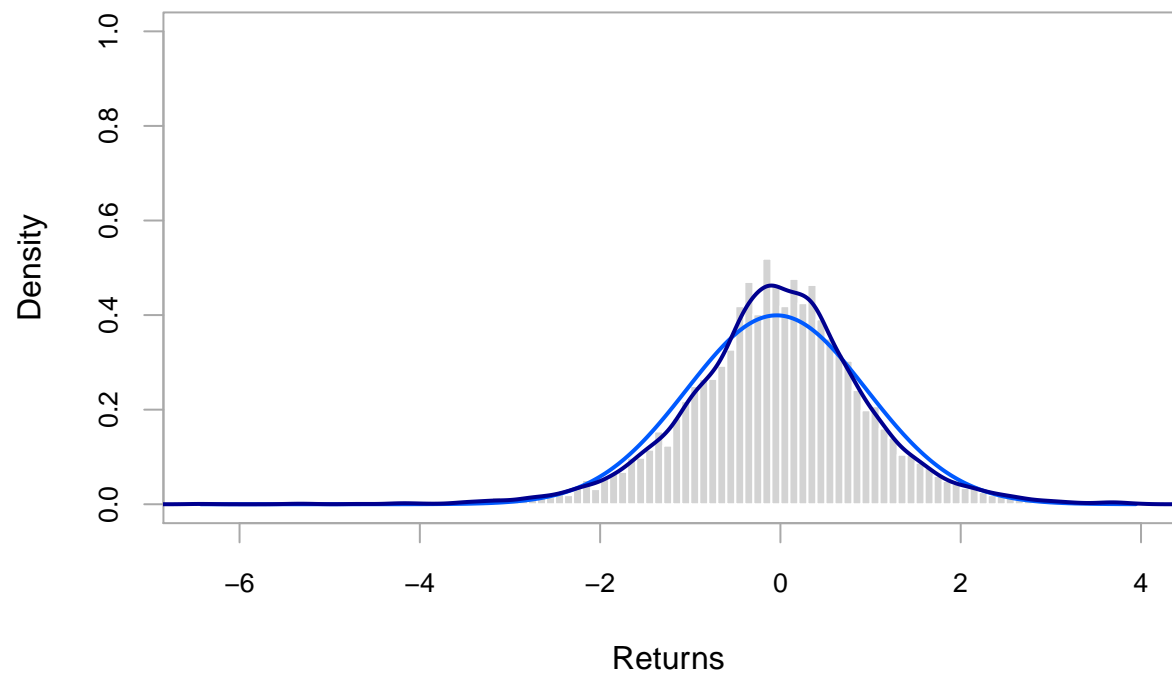
Volatility constructed by different models
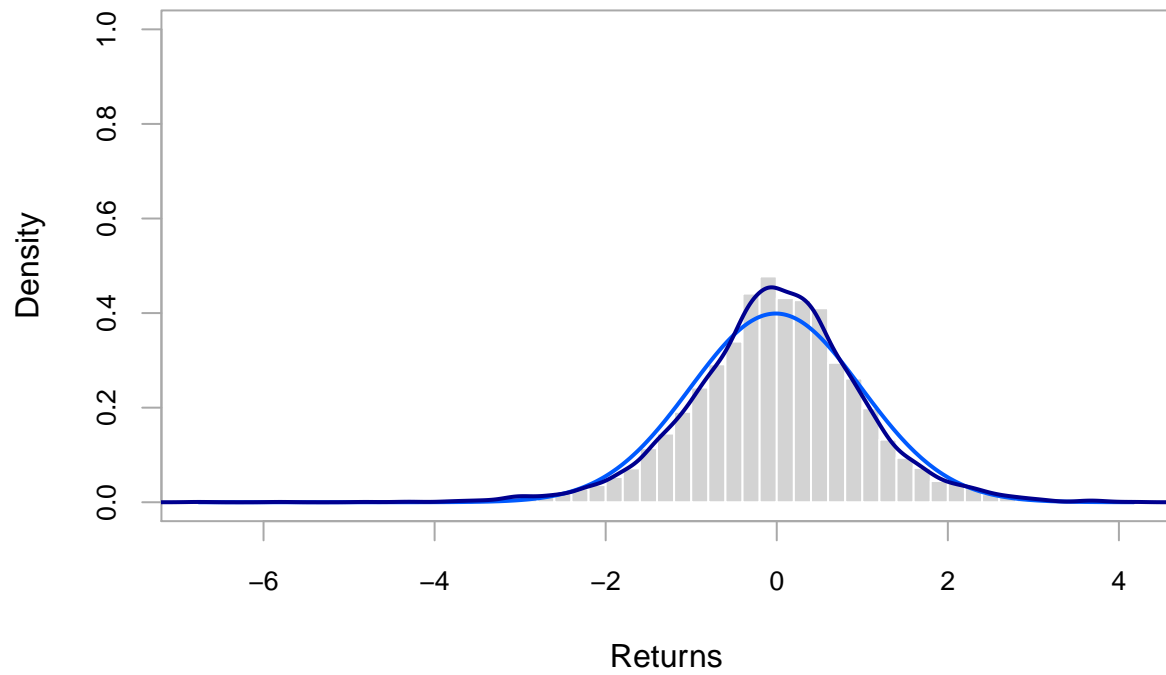


```
##   TES for Normal GARCH     TES for Normal GJR TES for Skewed t GARCH
##           0.007921520          -0.004295564          -0.020627739
##   TES for Skewed t GJR
##          -0.009151344
##            Bankmellon Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.03783187    0.02778325 0.02900754     0.03015433   0.02902535
## 2020-04-27 0.02854311    0.02765136 0.02887608     0.03013506   0.02900182
## 2020-04-28 0.02844841    0.02752156 0.02874631     0.03011593   0.02897846
## 2020-04-29 0.02750750    0.02739382 0.02861822     0.03009692   0.02895528
## 2020-04-30 0.02745840    0.02726812 0.02849180     0.03007805   0.02893227
## 2020-05-04 0.02653821    0.02714443 0.02836703     0.03005930   0.02890944
## 2020-05-05 0.02659099    0.02702272 0.02824389     0.03004069   0.02888678
## 2020-05-06 0.02600956    0.02690298 0.02812236     0.03002220   0.02886430
## 2020-05-07 0.02553621    0.02678517 0.02800243     0.03000384   0.02884198
## 2020-05-08 0.02559992    0.02666926 0.02788408     0.02998560   0.02881984
##
## --------------------------------
## Allianz
```
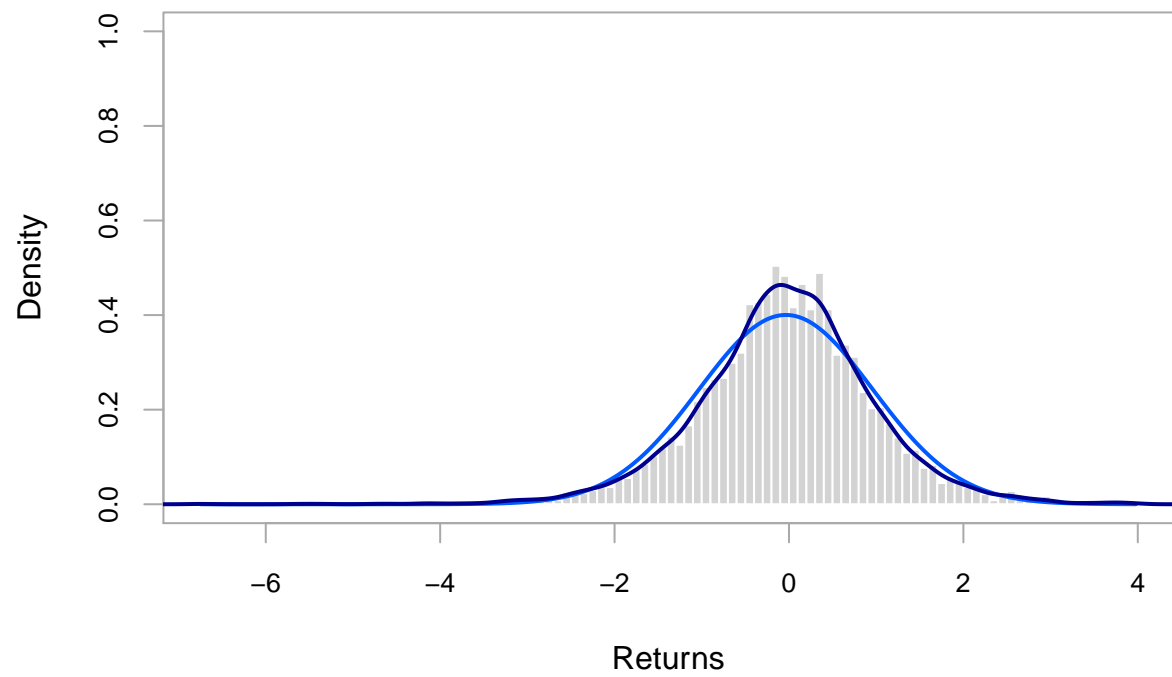
Standardized residuals of Standard GARCH with normal distribution of errors

Standardized residuals of GJR GARCH with normal distribution of errors

Standardized residuals of Standard GARCH with skewed Student t distribution of ε

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
##
##   Standard GARCH with normal distribution of errors
##
##   Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 33.345, df = 22, p-value = 0.05719
```

GJR GARCH with normal distribution of errors


```
##
##  GJR GARCH with normal distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 29.9, df = 22, p-value = 0.1209
```

Standard GARCH with skewed Student t distribution of errors


```
## 
##   Standard GARCH with skewed Student t distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 35.811, df = 22, p-value = 0.03181
```
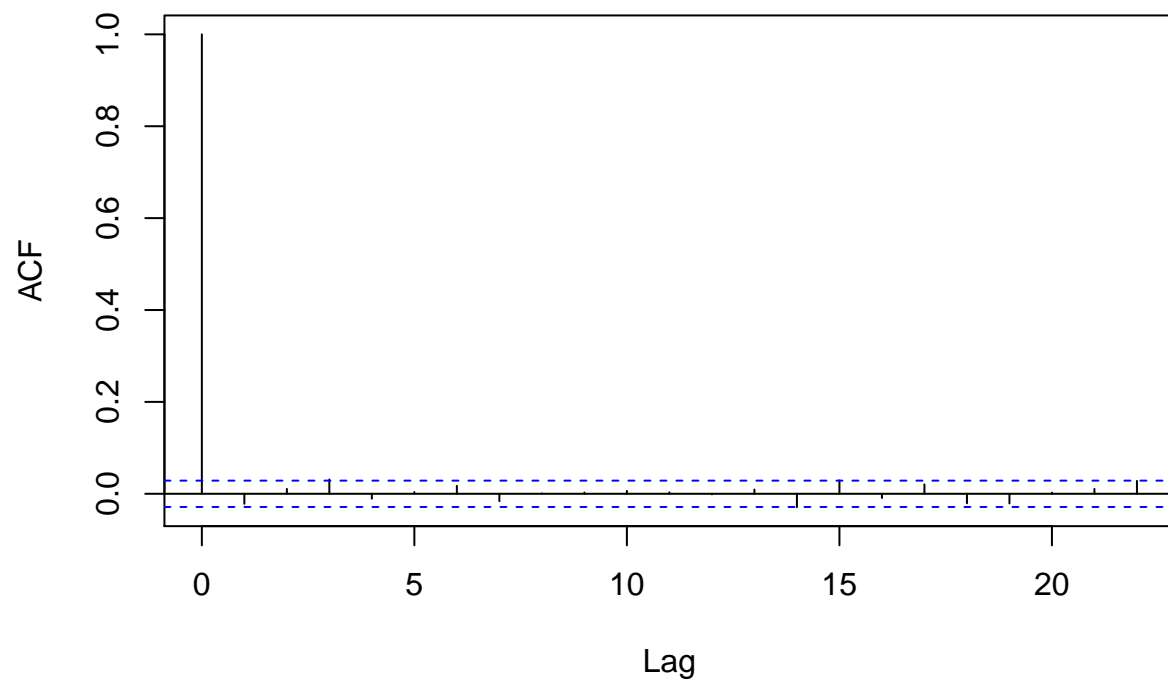
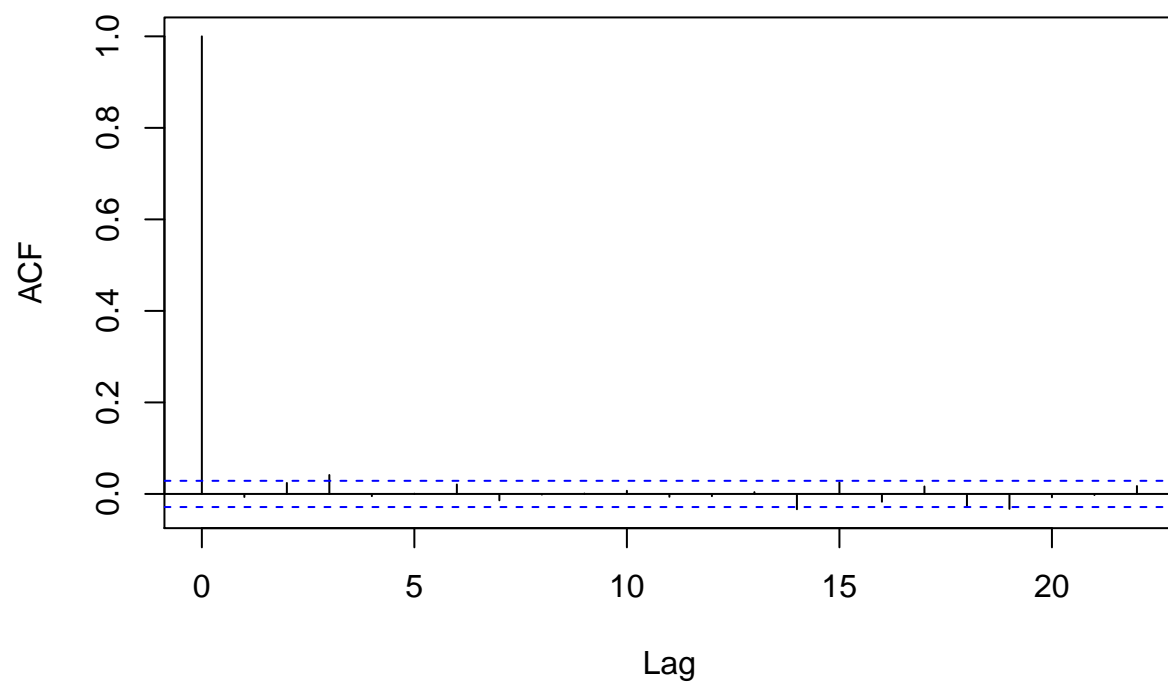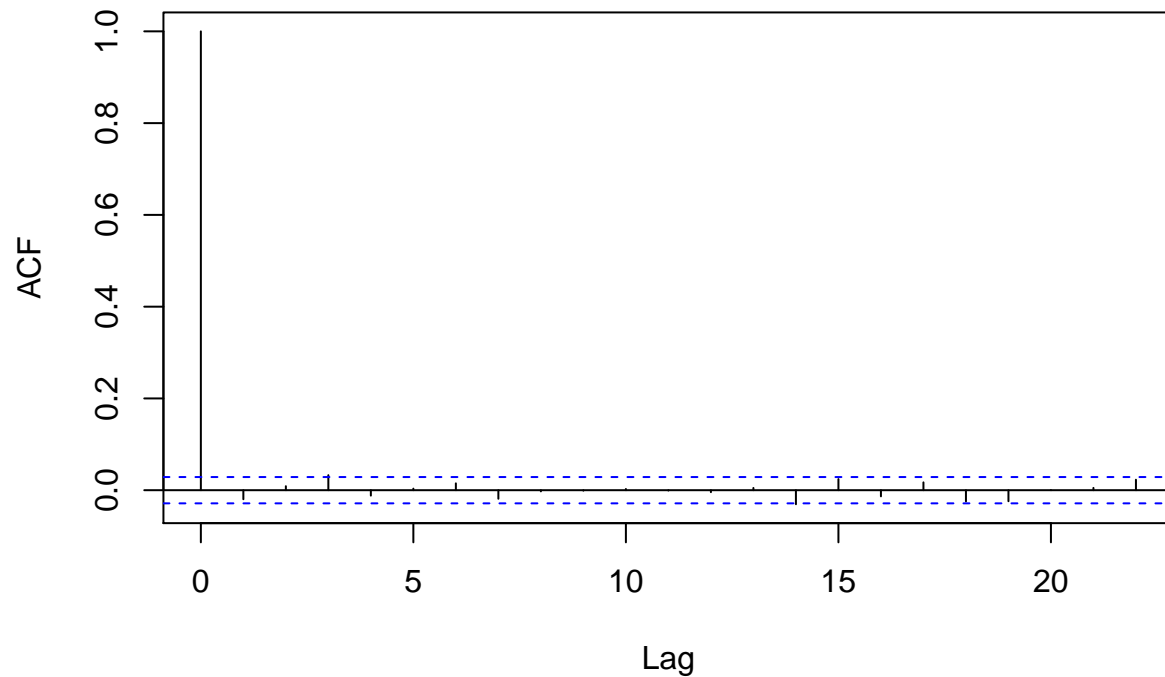## GJR GARCH with skewed Student t distribution of errors


```
## 
##   GJR GARCH with skewed Student t distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 28.397, df = 22, p-value = 0.1629
## 
## 
## Coefficients of Standard GARCH with normal distribution of errors
##             Estimate     Std. Error     t value        Pr(>|t|)
## mu      0.0008124093  0.0001990795    4.080829   0.0000448753
## omega   0.0000046129  0.0000012937    3.565761   0.0003628013
## alpha1  0.1017953831  0.0086051056   11.829649   0.0000000000
## beta1   0.8873064262  0.0091330058   97.153823   0.0000000000
## 
## Robust coefficients of Standard GARCH with normal distribution of errors
##             Estimate     Std. Error     t value        Pr(>|t|)
## mu      0.0008124093  0.0002056529    3.950390   0.0000780238
## omega   0.0000046129  0.0000038591    1.195337   0.2319554317
## alpha1  0.1017953831  0.0146803930    6.934105   0.0000000000
## beta1   0.8873064262  0.0185231852   47.902476   0.0000000000
## 
## Coefficients of GJR GARCH with normal distribution of errors
##             Estimate     Std. Error     t value        Pr(>|t|)
## mu      0.0003517914  0.0001275091    2.758952   0.0057987095
```
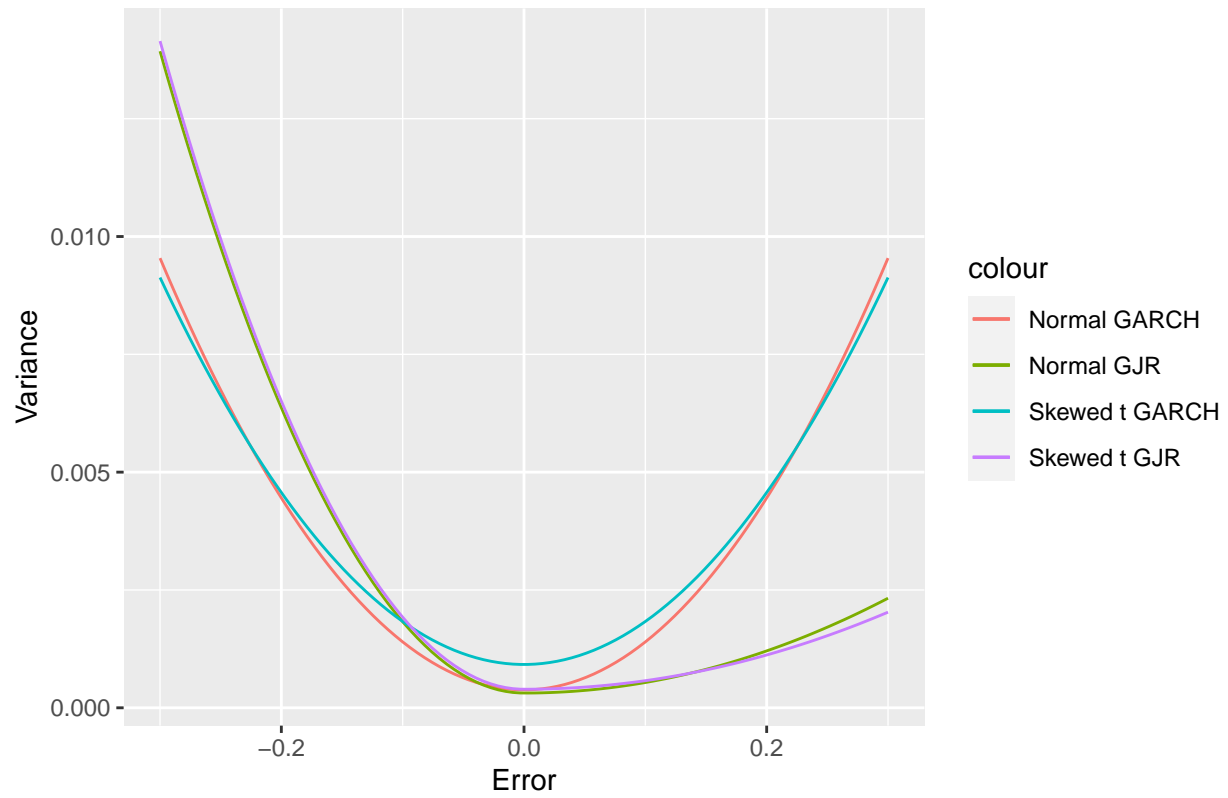
```
## omega  0.0000049747 0.0000012451   3.995440 0.0000645743
## alpha1 0.0223577799 0.0046269611   4.832066 0.0000013512
## beta1  0.8986410615 0.0064308270 139.739580 0.0000000000
## gamma1 0.1289652652 0.0077223953  16.700164 0.0000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0003517914 0.0005419566  0.6491137 0.516264874
## omega  0.0000049747 0.0000046637  1.0666868 0.286113299
## alpha1 0.0223577799 0.0288037894  0.7762097 0.437625197
## beta1  0.8986410615 0.0094469413 95.1250816 0.000000000
## gamma1 0.1289652652 0.0425442968  3.0313174 0.002434892
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0007065779 0.0001977419  3.5732339 0.0003525995
## omega  0.0000026154 0.0000028153  0.9290046 0.3528866962
## alpha1 0.0912238966 0.0243846588  3.7410364 0.0001832630
## beta1  0.9061908113 0.0241496180 37.5240225 0.0000000000
## skew   0.9761355956 0.0199434264 48.9452303 0.0000000000
## shape  6.3168681420 0.6568242403  9.6172884 0.0000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0007065779 0.0002195923  3.2176801 0.0012923185
## omega  0.0000026154 0.0000153230  0.1706859 0.8644707842
## alpha1 0.0912238966 0.1263401059  0.7220502 0.4702636246
## beta1  0.9061908113 0.1265759473  7.1592655 0.0000000000
## skew   0.9761355956 0.0247884939 39.3785761 0.0000000000
## shape  6.3168681420 1.8216935400  3.4675800 0.0005251673
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value    Pr(>|t|)
## mu     0.0003761577 0.0001942831  1.936132 0.05285151
## omega  0.0000035397 0.0000014421  2.454487 0.01410859
## alpha1 0.0181718320 0.0080697143  2.251856 0.02433139
## beta1  0.9072260117 0.0111648560 81.257296 0.00000000
## gamma1 0.1346447072 0.0189601593  7.101454 0.00000000
## skew   0.9638760358 0.0196474135 49.058673 0.00000000
## shape  6.8956694821 0.6733474222 10.240879 0.00000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0003761577 0.0002159396  1.7419582 0.0815157534
## omega  0.0000035397 0.0000038409  0.9215795 0.3567479301
## alpha1 0.0181718320 0.0082775957  2.1953032 0.0281418571
## beta1  0.9072260117 0.0221791180 40.9045127 0.0000000000
## gamma1 0.1346447072 0.0357464263  3.7666620 0.0001654447
## skew   0.9638760358 0.0194336547 49.5982898 0.0000000000
## shape  6.8956694821 0.7436921555  9.2722095 0.0000000000
##                Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood   12618.436112 12677.873610   12722.374662 12769.530904
## Akaike          -5.390785    -5.415758      -5.434348    -5.454073
## Bayes           -5.385271    -5.408866      -5.426078    -5.444424
```

```
## Shibata            -5.390786      -5.415760      -5.434351      -5.454078
## Hannan-Quinn       -5.388846      -5.413334      -5.431440      -5.450680
```
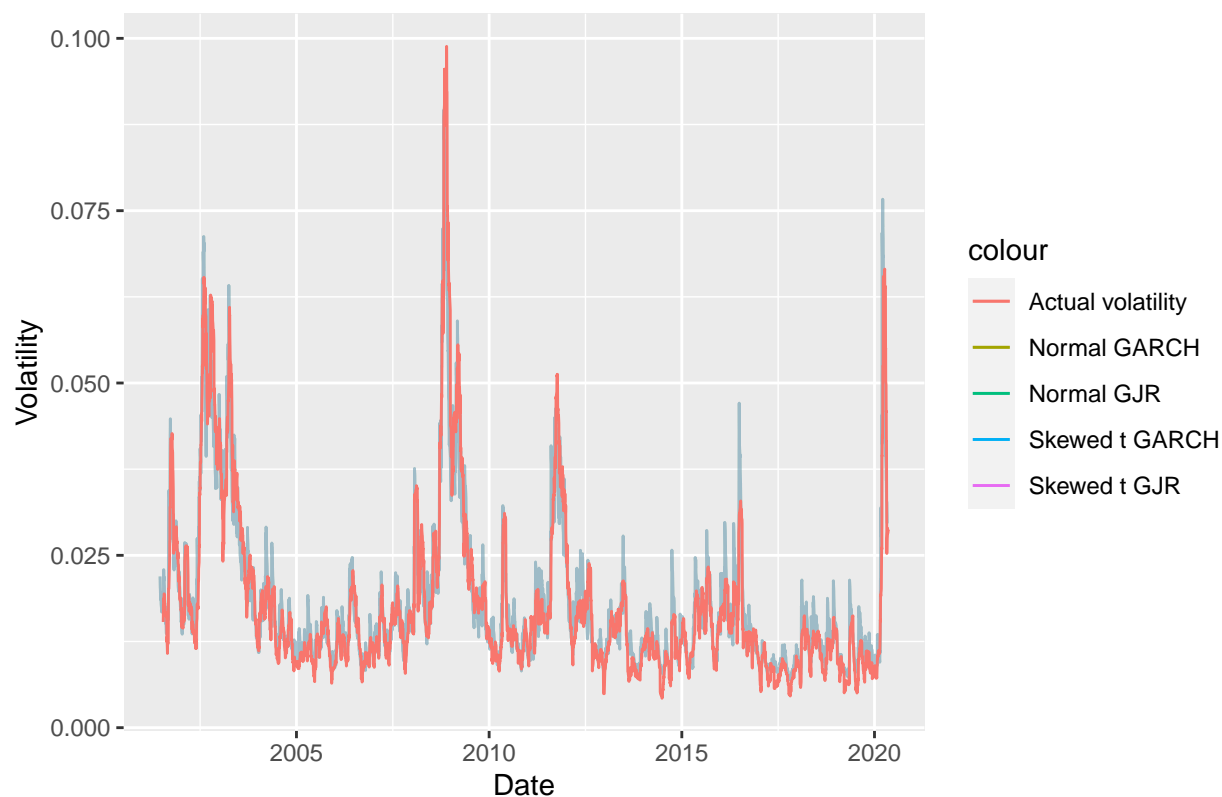
Dependence of variance on errors in different models

Volatility constructed by different models


```
##    TES for Normal GARCH       TES for Normal GJR TES for Skewed t GARCH
##           -0.029026922               0.005354399            -0.059270729
##    TES for Skewed t GJR
##           -0.013365337
##               Allianz Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.04147483    0.03246860 0.02911545     0.03506125   0.03074048
## 2020-04-27 0.02527030    0.03236254 0.02898925     0.03505322   0.03067146
## 2020-04-28 0.02640315    0.03225729 0.02886434     0.03504522   0.03060286
## 2020-04-29 0.02742532    0.03215285 0.02874072     0.03503723   0.03053466
## 2020-04-30 0.02765949    0.03204922 0.02861836     0.03502926   0.03046688
## 2020-05-04 0.02892837    0.03194638 0.02849727     0.03502131   0.03039951
## 2020-05-05 0.02846341    0.03184433 0.02837744     0.03501338   0.03033254
## 2020-05-06 0.02818765    0.03174308 0.02825884     0.03500547   0.03026598
## 2020-05-07 0.02869809    0.03164261 0.02814148     0.03499758   0.03019982
## 2020-05-08 0.02847229    0.03154292 0.02802534     0.03498970   0.03013406
##
## --------------------------------
```
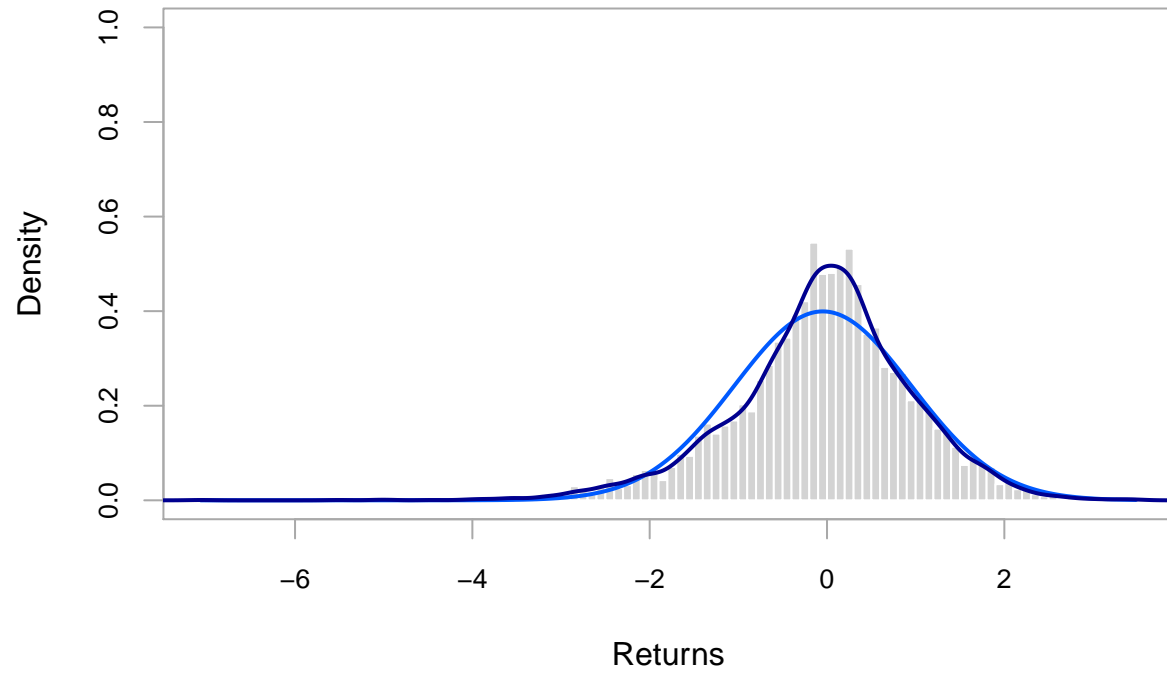
```r
cat('Data up to 2020-05-08\n')
```

```
## Data up to 2020-05-08
```
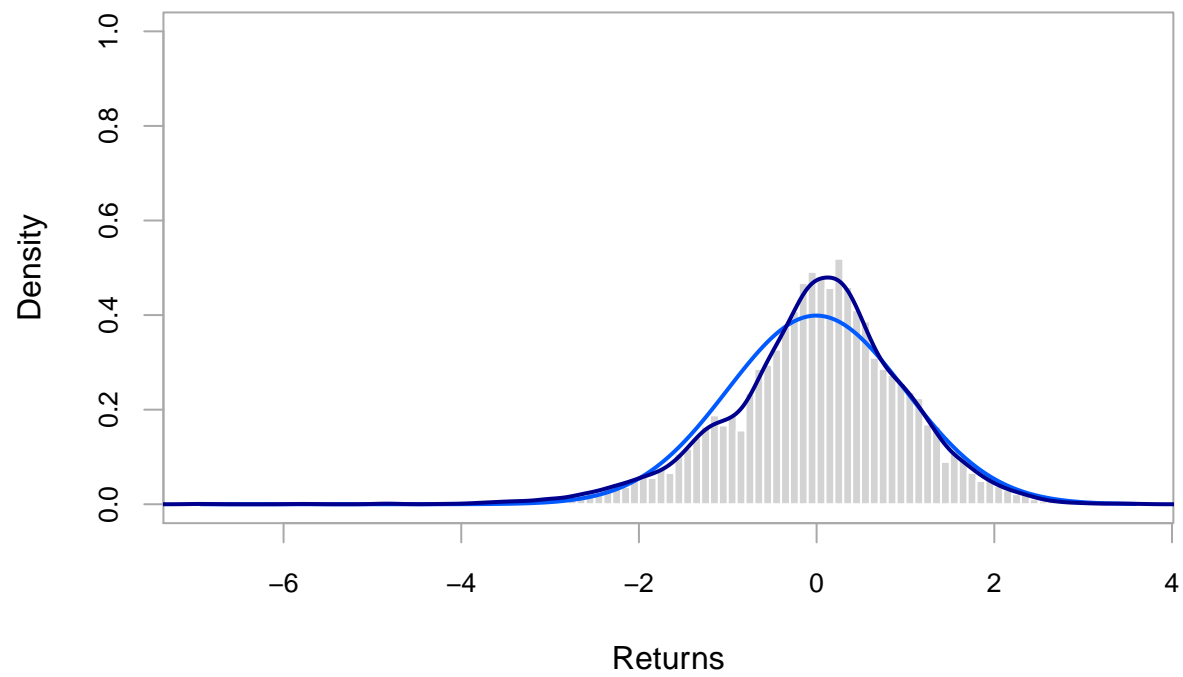
```r
for (i in 1:ncol(funds)) {
    cat(colnames(vol_df)[i],'\n')
    evaluate_garch(funds_red[,i], vol_df[,i])
    cat('\n----------------------------\n')
}
```
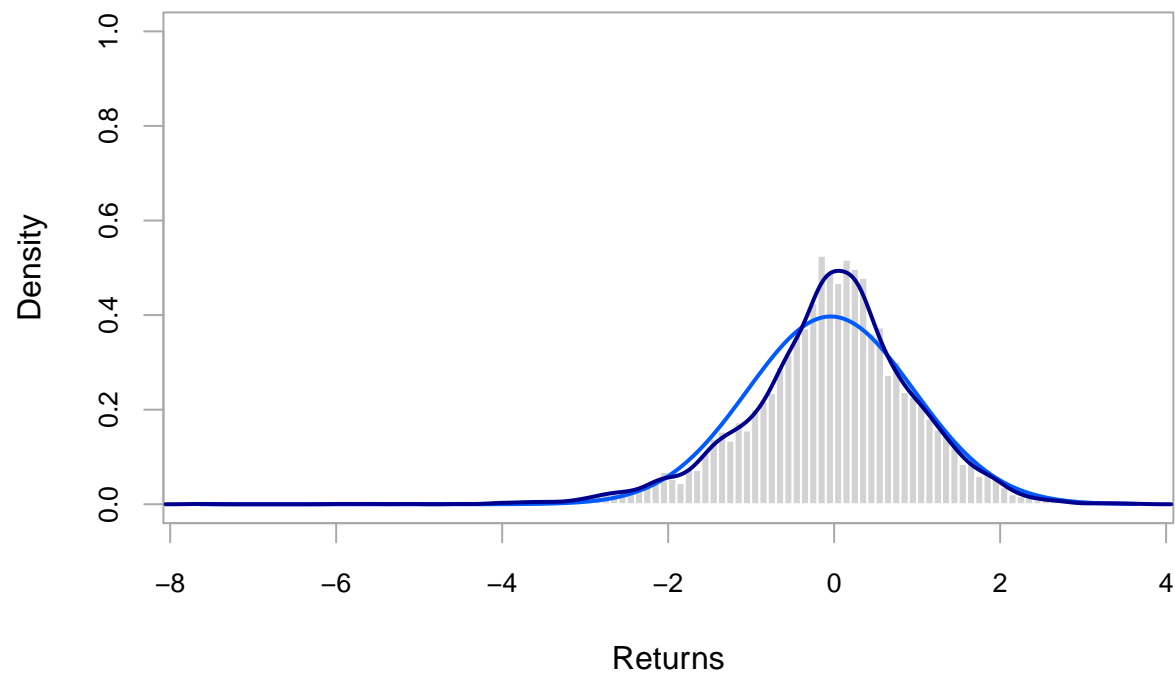
Standardized residuals of Standard GARCH with normal distribution of errors

Standardized residuals of GJR GARCH with normal distribution of errors

Standardized residuals of Standard GARCH with skewed Student t distribution of e

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
## 
##   Standard GARCH with normal distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 34.349, df = 22, p-value = 0.04525
```

GJR GARCH with normal distribution of errors


```
##
##  GJR GARCH with normal distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 35.737, df = 22, p-value = 0.0324
```

Standard GARCH with skewed Student t distribution of errors


```
## 
##   Standard GARCH with skewed Student t distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 30.047, df = 22, p-value = 0.1173
```
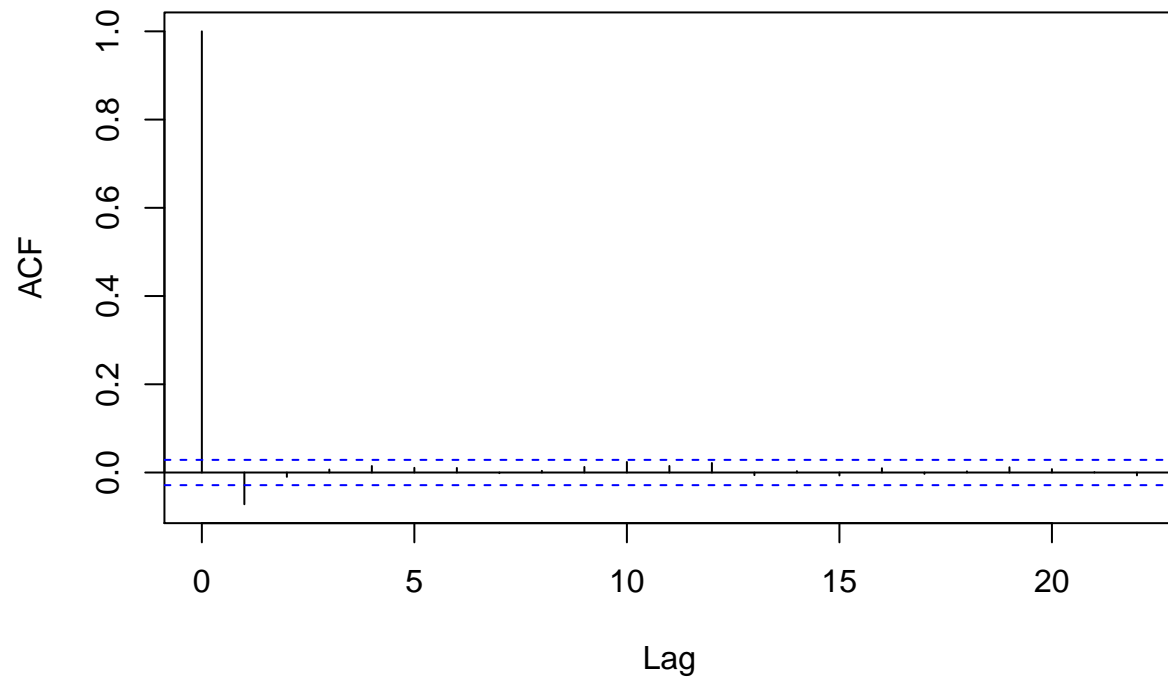
GJR GARCH with skewed Student t distribution of errors


```
##
##  GJR GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 35.065, df = 22, p-value = 0.03815
##
##
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value      Pr(>|t|)
## mu      0.0006524874 0.0001147633   5.685503 0.000000013
## omega   0.0000024508 0.0000007865   3.115893 0.001833889
## alpha1  0.1205992396 0.0098197314  12.281318 0.000000000
## beta1   0.8594536598 0.0106481285  80.714058 0.000000000
##
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value      Pr(>|t|)
## mu      0.0006524874 0.0000965711   6.7565500 0.0000000000
## omega   0.0000024508 0.0000044008   0.5568893 0.5776030682
## alpha1  0.1205992396 0.0251184073   4.8012296 0.0000015769
## beta1   0.8594536598 0.0406037674  21.1668452 0.0000000000
##
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate    Std. Error       t value      Pr(>|t|)
## mu      0.0002477111 0.0000998018  2.482030e+00 0.01306363
```
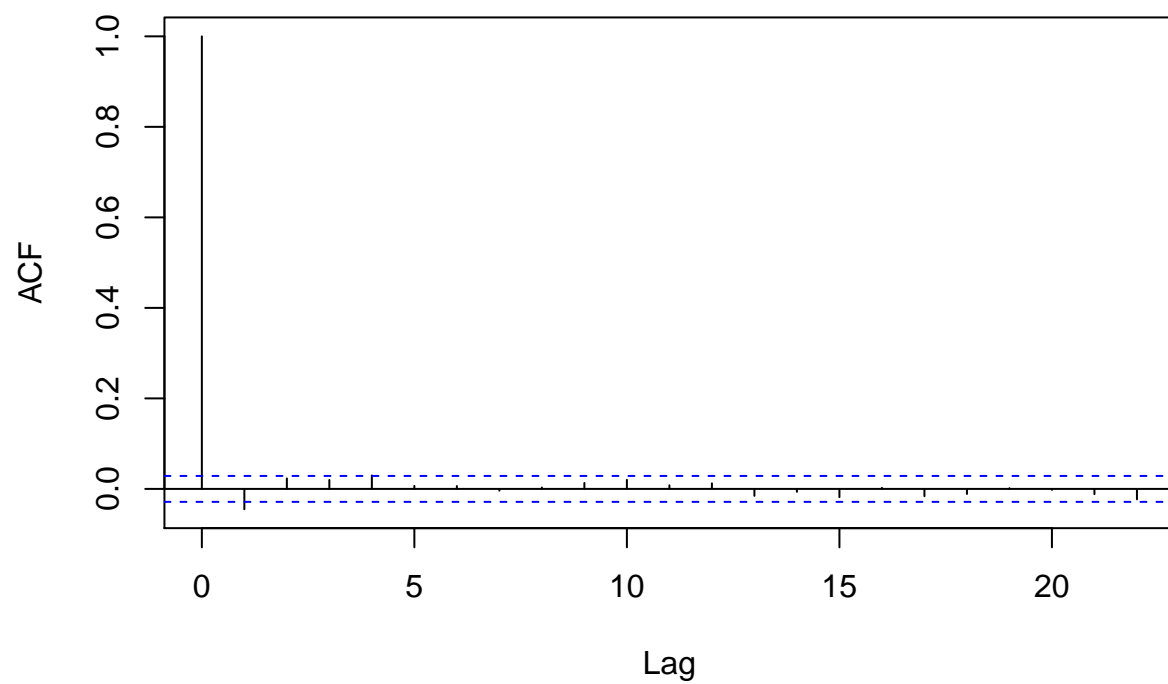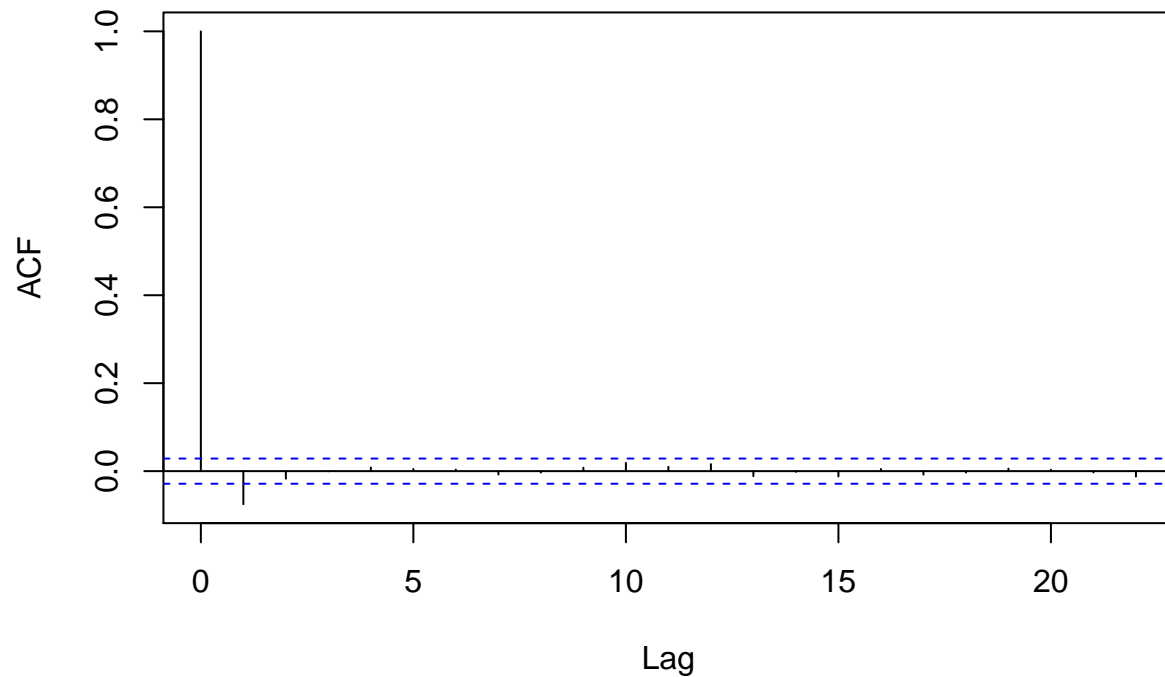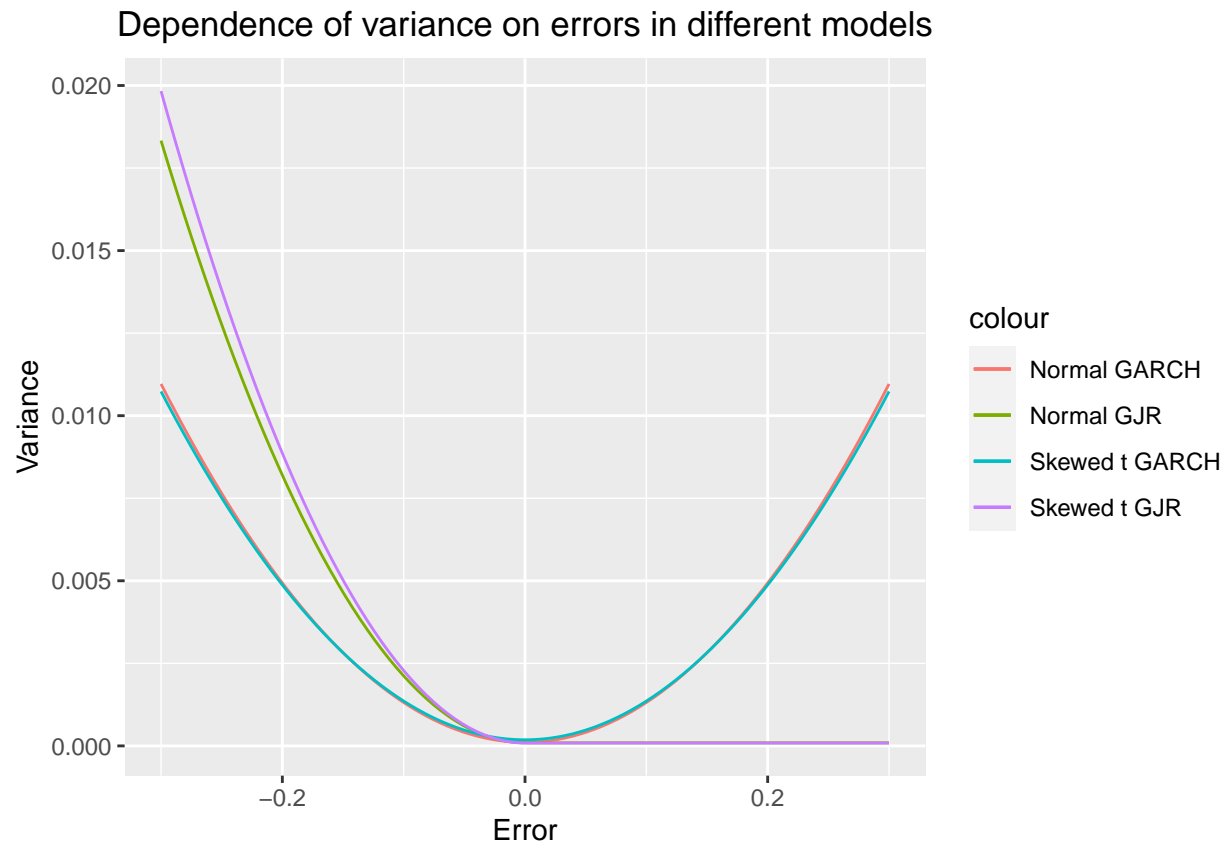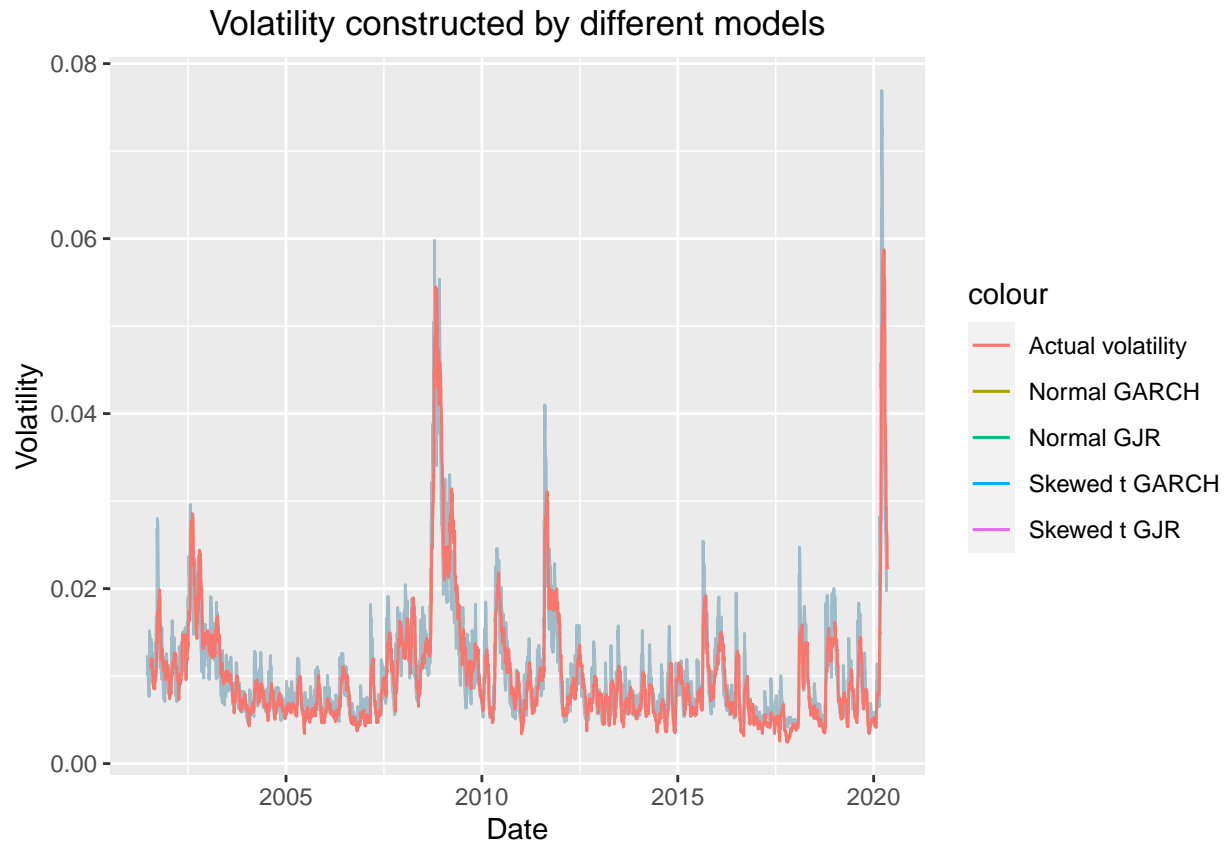
```
## omega  0.0000024813 0.0000002379 1.042998e+01 0.00000000
## alpha1 0.0000000542 0.0034471610 1.572800e-05 0.99998745
## beta1  0.8746538205 0.0066311516 1.319007e+02 0.00000000
## gamma1 0.2026440668 0.0127810580 1.585503e+01 0.00000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##              Estimate    Std. Error      t value      Pr(>|t|)
## mu     0.0002477111 0.0001308254 1.893449e+00 0.0582982283
## omega  0.0000024813 0.0000005808 4.272299e+00 0.0000193468
## alpha1 0.0000000542 0.0136585485 3.969500e-06 0.9999968328
## beta1  0.8746538205 0.0074170696 1.179244e+02 0.0000000000
## gamma1 0.2026440668 0.0315200843 6.429046e+00 0.0000000001
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0005924393 0.0001184278  5.0025347 0.0000005658
## omega  0.0000013929 0.0000032444  0.4293241 0.6676873913
## alpha1 0.1172198092 0.0546264437  2.1458437 0.0318854521
## beta1  0.8761141511 0.0514355947 17.0332268 0.0000000000
## skew   0.8889153047 0.0217947432 40.7857663 0.0000000000
## shape  7.0598730007 1.6825791824  4.1958638 0.0000271834
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0005924393 5.950877e-04 0.99554951 0.3194690792
## omega  0.0000013929 4.933230e-05 0.02823482 0.9774748630
## alpha1 0.1172198092 8.184706e-01 0.14321811 0.8861179292
## beta1  0.8761141511 7.726214e-01 1.13395013 0.2568154689
## skew   0.8889153047 1.978793e-01 4.49221016 0.0000070488
## shape  7.0598730007 2.298408e+01 0.30716364 0.7587188171
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error      t value    Pr(>|t|)
## mu     0.0002403827 0.0001172261 2.050589e+00 0.04030694
## omega  0.0000019857 0.0000010552 1.881878e+00 0.05985257
## alpha1 0.0000005843 0.0121633413 4.803760e-05 0.99996167
## beta1  0.8757172238 0.0111691773 7.840481e+01 0.00000000
## gamma1 0.2193849084 0.0305851524 7.172922e+00 0.00000000
## skew   0.8545123813 0.0170646651 5.007496e+01 0.00000000
## shape  8.2171916513 0.9868646400 8.326564e+00 0.00000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error      t value    Pr(>|t|)
## mu     0.0002403827 0.0002621922 9.168188e-01 0.35923761
## omega  0.0000019857 0.0000055388 3.585037e-01 0.71996641
## alpha1 0.0000005843 0.0401046660 1.456930e-05 0.99998838
## beta1  0.8757172238 0.0360296110 2.430549e+01 0.00000000
## gamma1 0.2193849084 0.1252958013 1.750936e+00 0.07995696
## skew   0.8545123813 0.0162712086 5.251684e+01 0.00000000
## shape  8.2171916513 1.0198432565 8.057308e+00 0.00000000
##              Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood   15225.702099 15327.477444   15339.237762 15433.032588
## Akaike           -6.505001    -6.548067      -6.552666    -6.592322
## Bayes            -6.499487    -6.541175      -6.544395    -6.582673
```
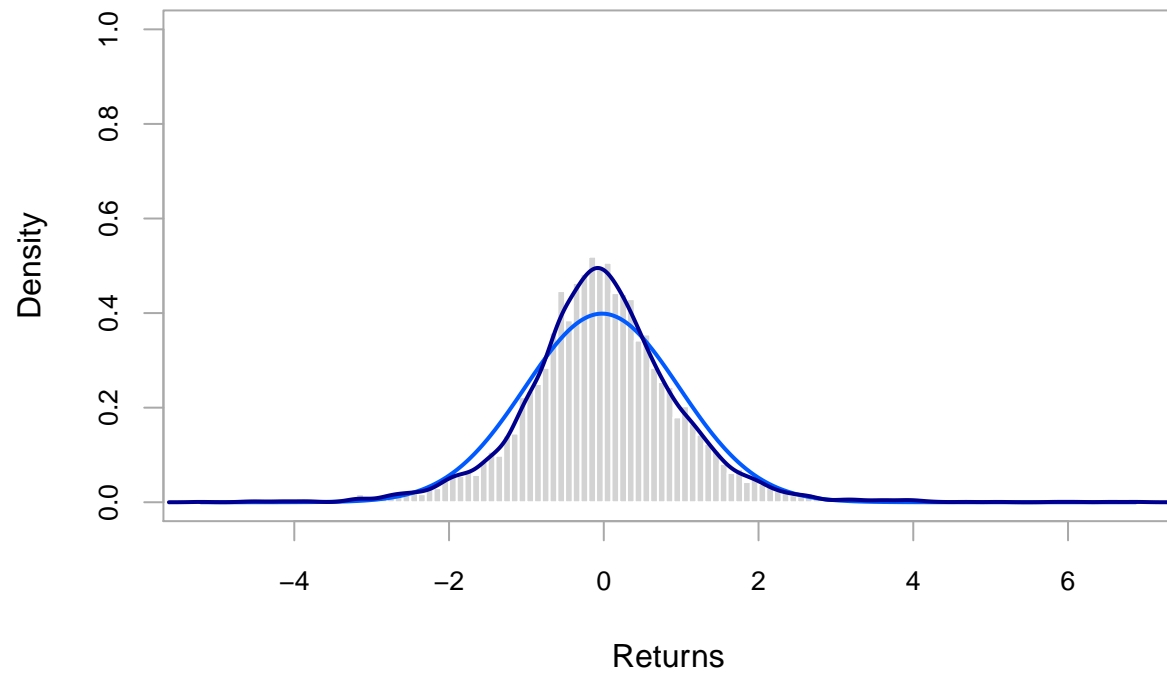
100

```
## Shibata          -6.505002    -6.548070      -6.552669    -6.592326
## Hannan-Quinn      -6.503062    -6.545644      -6.549757    -6.588928
```


Dependence of variance on errors in different models

Volatility constructed by different models


```
##   TES for Normal GARCH     TES for Normal GJR TES for Skewed t GARCH
##          0.05475380             0.08707727             0.02621860
##   TES for Skewed t GJR
##          0.07871238
##              Vanguard Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.03443334    0.02191476 0.01865495     0.02429064   0.01945951
## 2020-04-27 0.02908799    0.02175150 0.01849670     0.02423829   0.01930974
## 2020-04-28 0.02916100    0.02159030 0.01834093     0.02418618   0.01916192
## 2020-04-29 0.02756983    0.02143114 0.01818763     0.02413431   0.01901603
## 2020-04-30 0.02659937    0.02127399 0.01803674     0.02408267   0.01887205
## 2020-05-04 0.02609812    0.02111885 0.01788826     0.02403127   0.01872996
## 2020-05-05 0.02570555    0.02096569 0.01774215     0.02398010   0.01858974
## 2020-05-06 0.02301671    0.02081449 0.01759837     0.02392916   0.01845137
## 2020-05-07 0.02288516    0.02066523 0.01745691     0.02387845   0.01831483
## 2020-05-08 0.02224057    0.02051790 0.01731773     0.02382798   0.01818011
##
## -------------------------------
## Blackrock
```
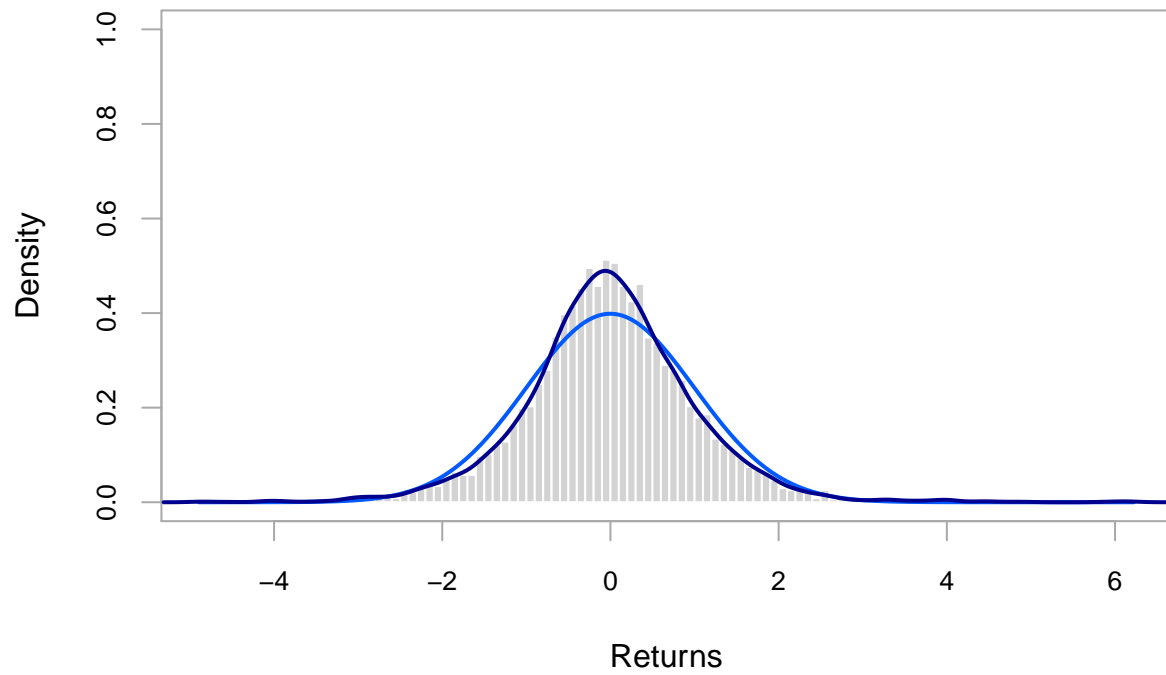
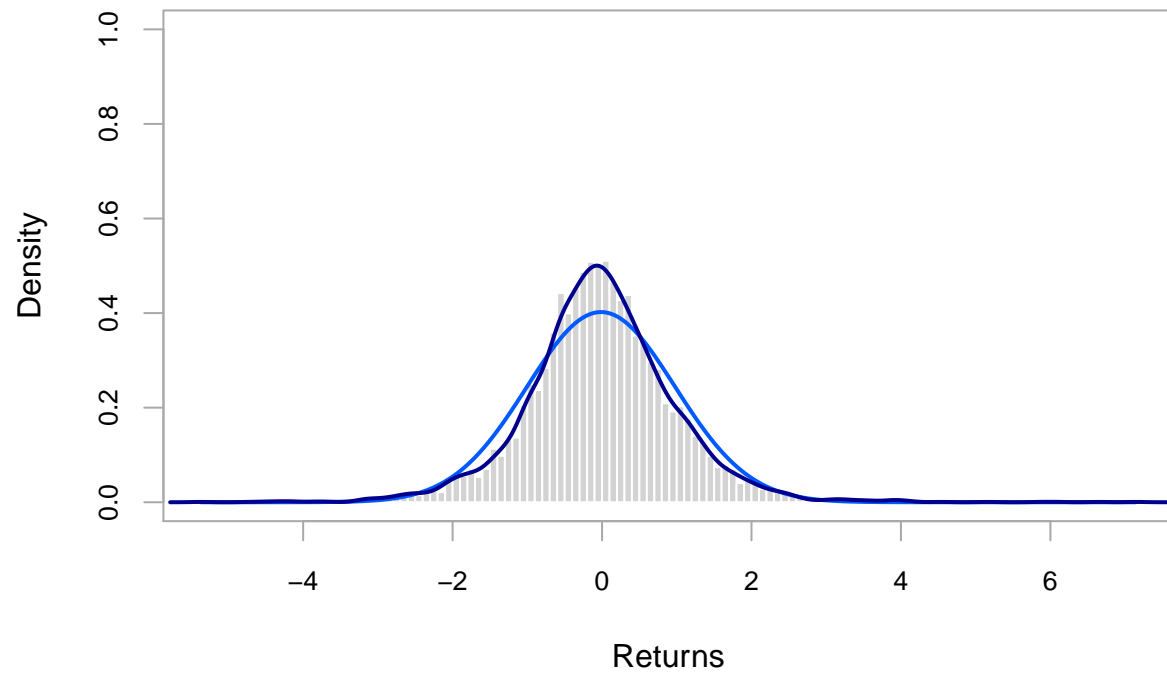**Standardized residuals of Standard GARCH with normal distribution of errors**

Standardized residuals of GJR GARCH with normal distribution of errors
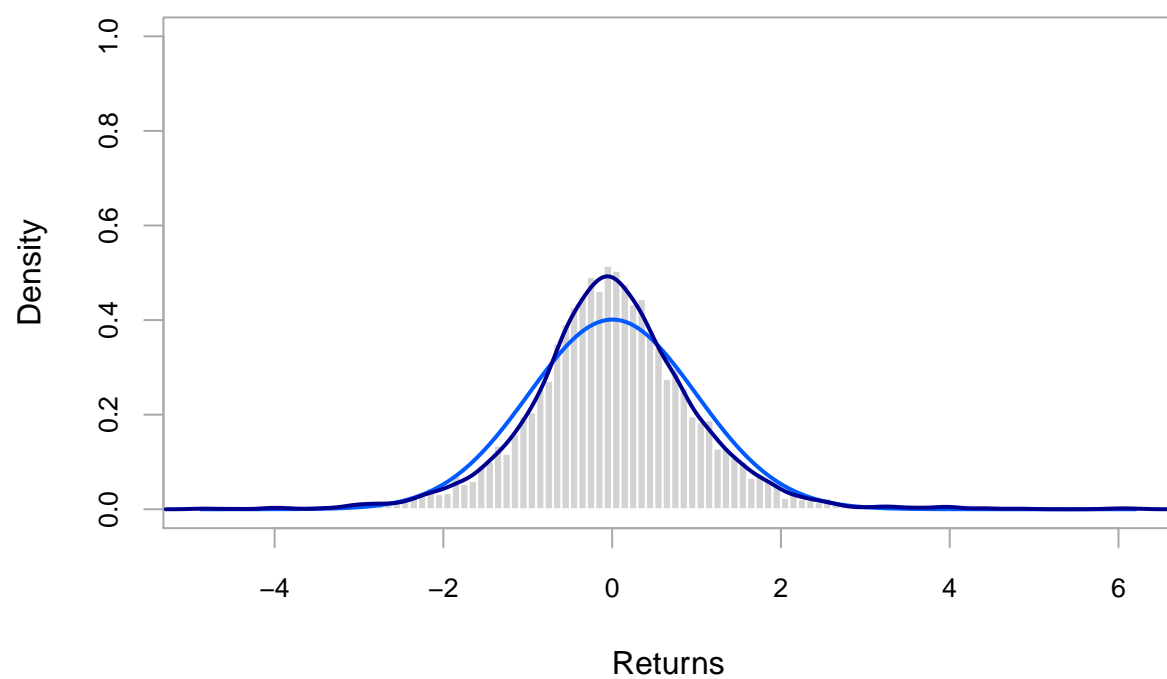
Standardized residuals of Standard GARCH with skewed Student t distribution of ε

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
##
##  Standard GARCH with normal distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 12.581, df = 22, p-value = 0.9442
```

GJR GARCH with normal distribution of errors


```
##
##  GJR GARCH with normal distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 17.975, df = 22, p-value = 0.7075
```

Standard GARCH with skewed Student t distribution of errors


```
##
##  Standard GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 18.027, df = 22, p-value = 0.7044
```
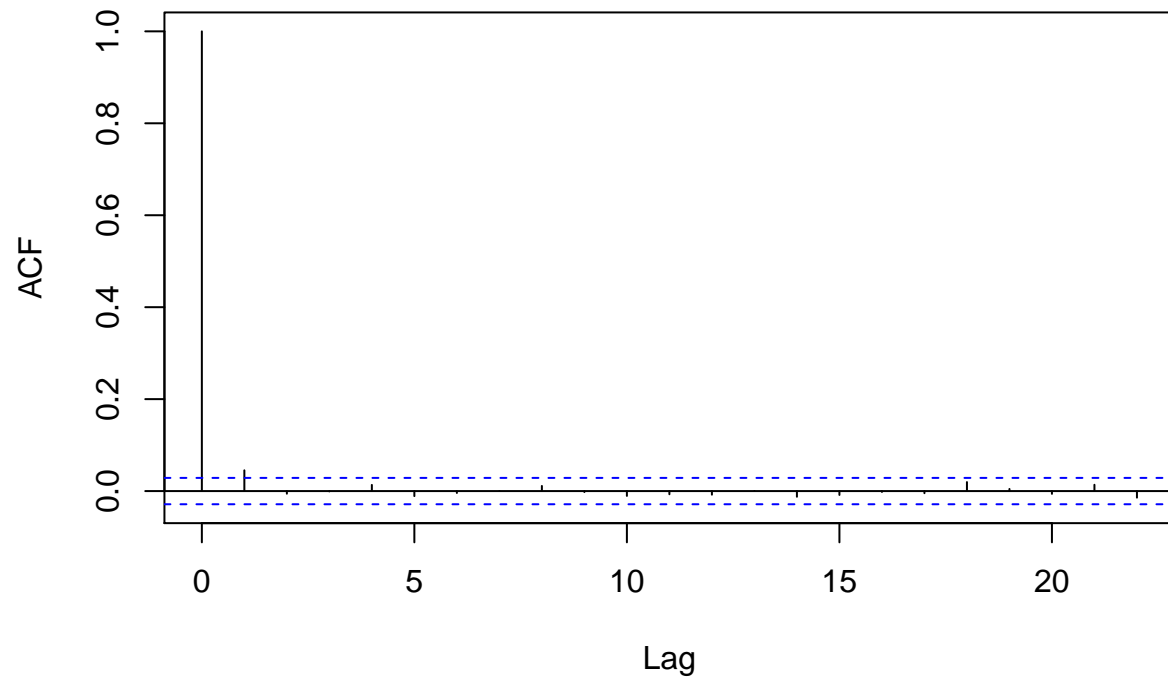
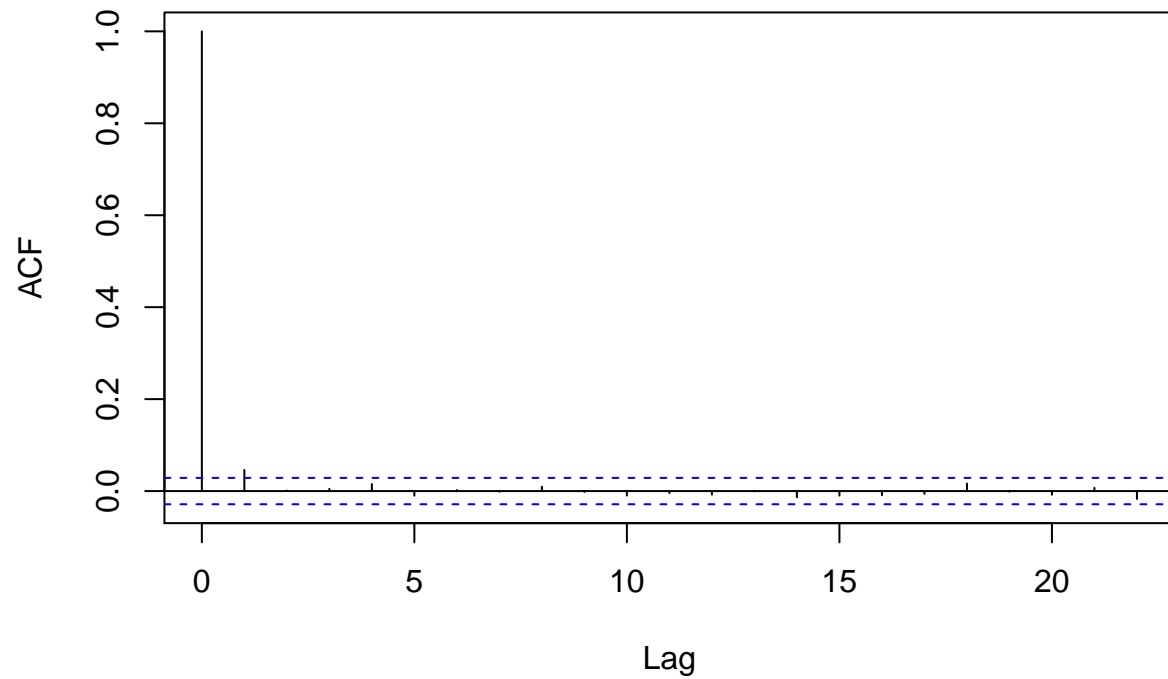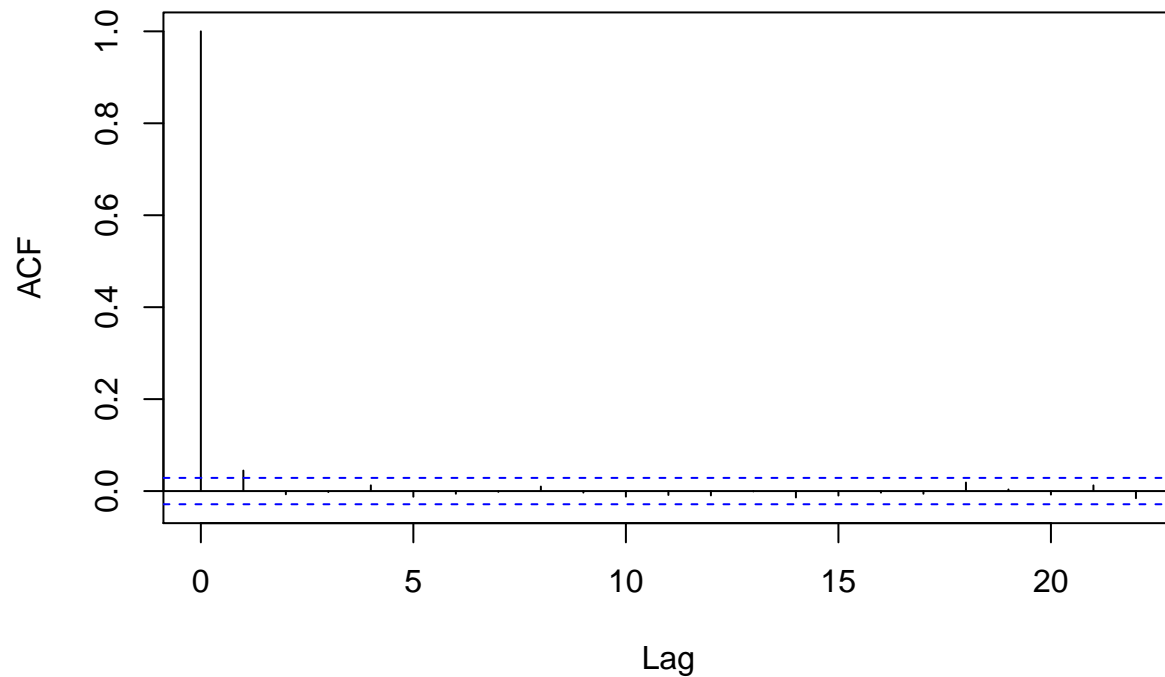## GJR GARCH with skewed Student t distribution of errors


```
##
##  GJR GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 18.501, df = 22, p-value = 0.6759
##
##
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate  Std. Error     t value Pr(>|t|)
## mu      0.0011372119 2.19327e-04    5.185005 2.16e-07
## omega   0.0000098047 9.96100e-07    9.843453 0.00e+00
## alpha1  0.1055437293 6.30464e-03   16.740642 0.00e+00
## beta1   0.8699984930 7.50818e-03  115.873427 0.00e+00
##
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate   Std. Error   t value   Pr(>|t|)
## mu      0.0011372119 0.0002252054  5.049665 4.4260e-07
## omega   0.0000098047 0.0000020263  4.838728 1.3067e-06
## alpha1  0.1055437293 0.0098783468 10.684352 0.0000e+00
## beta1   0.8699984930 0.0129177290 67.349183 0.0000e+00
##
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate   Std. Error   t value    Pr(>|t|)
## mu      0.0007669611 0.0002043304  3.753534 0.0001743589
```
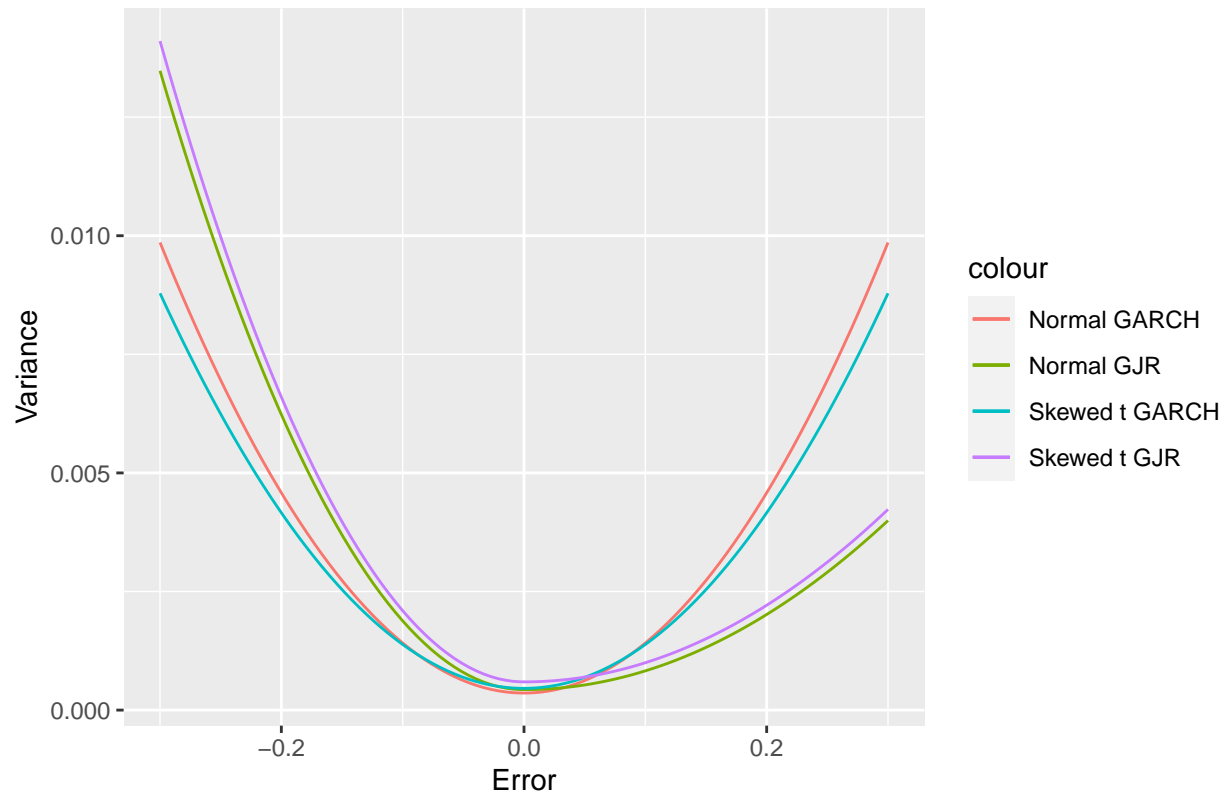
```
## omega  0.0000073817 0.0000013218     5.584698 0.0000000234
## alpha1 0.0396059054 0.0056308464     7.033739 0.0000000000
## beta1  0.8921657305 0.0063027440 141.551955 0.0000000000
## gamma1 0.1053526778 0.0136025021     7.745095 0.0000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##             Estimate   Std. Error   t value     Pr(>|t|)
## mu     0.0007669611 0.0002838199   2.702281 0.0068865594
## omega  0.0000073817 0.0000030137   2.449386 0.0143099804
## alpha1 0.0396059054 0.0090082599   4.396621 0.0000109949
## beta1  0.8921657305 0.0126930924 70.287500 0.0000000000
## gamma1 0.1053526778 0.0252650136   4.169904 0.0000304728
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate   Std. Error   t value     Pr(>|t|)
## mu     0.0009973209 0.0002181637   4.571435 0.000004844
## omega  0.0000066732 0.0000023472   2.843053 0.004468366
## alpha1 0.0925351283 0.0104546038   8.851137 0.000000000
## beta1  0.8942466400 0.0118573164 75.417287 0.000000000
## skew   1.0478669015 0.0204174758 51.322059 0.000000000
## shape  4.9496121142 0.3778308453 13.100074 0.000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate   Std. Error   t value     Pr(>|t|)
## mu     0.0009973209 0.0002026057   4.922473 0.0000008546
## omega  0.0000066732 0.0000044259   1.507756 0.1316170798
## alpha1 0.0925351283 0.0140043103   6.607618 0.0000000000
## beta1  0.8942466400 0.0169854754 52.647725 0.0000000000
## skew   1.0478669015 0.0210434652 49.795359 0.0000000000
## shape  4.9496121142 0.4063335792 12.181155 0.0000000000
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate   Std. Error   t value     Pr(>|t|)
## mu     0.0006914519 0.0002296984   3.010260 0.0026102427
## omega  0.0000068582 0.0000026221   2.615557 0.0089082163
## alpha1 0.0403951248 0.0104819645   3.853774 0.0001163109
## beta1  0.8935391870 0.0108783675 82.139088 0.0000000000
## gamma1 0.1097060616 0.0183454359   5.980019 0.0000000022
## skew   1.0332533449 0.0204951256 50.414589 0.0000000000
## shape  5.0961280193 0.4077562921 12.497975 0.0000000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate   Std. Error   t value    Pr(>|t|)
## mu     0.0006914519 0.0002738265   2.525146 0.01156501
## omega  0.0000068582 0.0000061876   1.108383 0.26769623
## alpha1 0.0403951248 0.0181985312   2.219691 0.02643972
## beta1  0.8935391870 0.0168245406 53.109277 0.00000000
## gamma1 0.1097060616 0.0256521486   4.276681 0.00001897
## skew   1.0332533449 0.0228895433 45.140846 0.00000000
## shape  5.0961280193 0.5466713919   9.322105 0.00000000
##              Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood   12353.595569 12386.926043   12546.367965 12570.031644
## Akaike          -5.277605    -5.291421      -5.359132    -5.368817
## Bayes           -5.272091    -5.284529      -5.350861    -5.359168
```
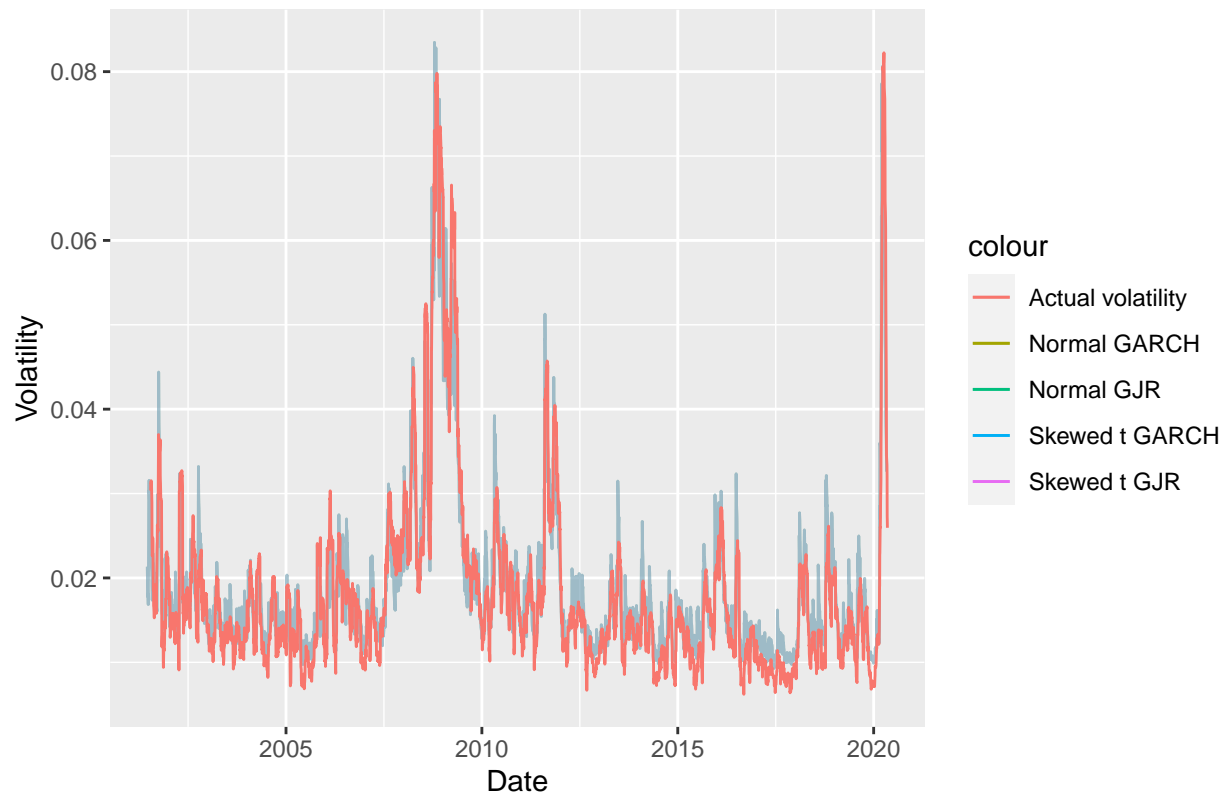
```
## Shibata          -5.277606    -5.291424        -5.359135    -5.368821
## Hannan-Quinn      -5.275666    -5.288998        -5.356223    -5.365424
```

Dependence of variance on errors in different models
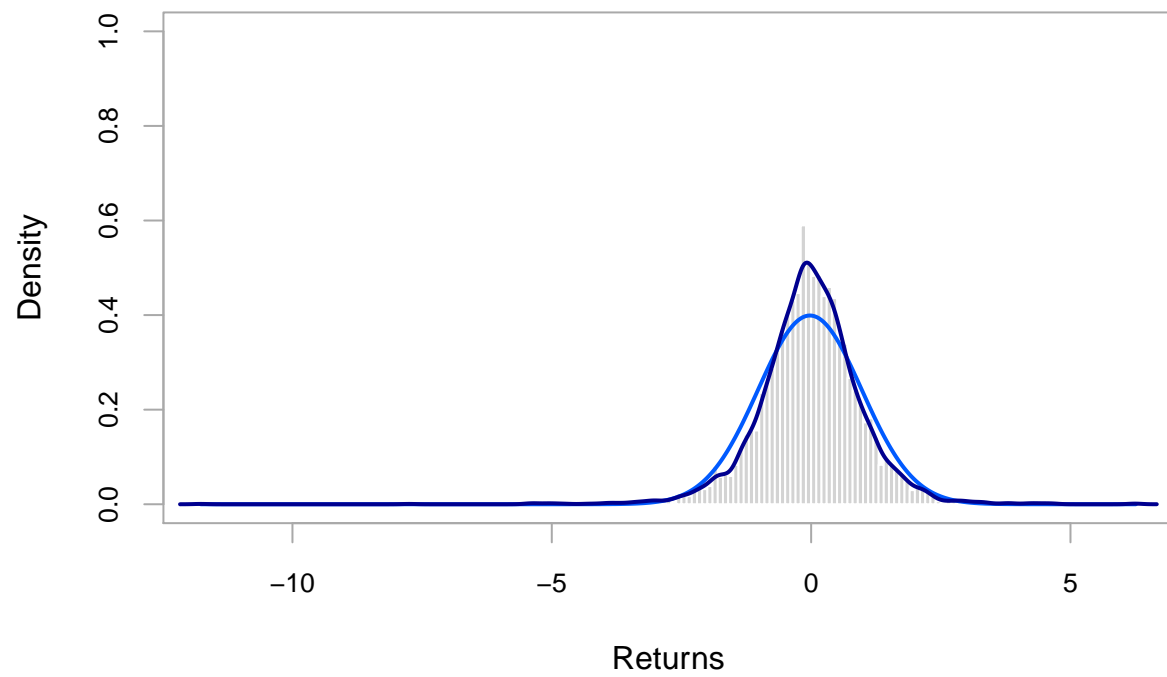
Volatility constructed by different models


```
##    TES for Normal GARCH      TES for Normal GJR TES for Skewed t GARCH
##             0.06055291              0.05150227             0.01125850
##   TES for Skewed t GJR
##             0.04087495
##             Blackrock Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.05190118    0.03047799 0.03100370     0.03509666   0.03178793
## 2020-04-27 0.04556408    0.03026539 0.03088142     0.03495950   0.03172997
## 2020-04-28 0.04334570    0.03005653 0.03076056     0.03482363   0.03167250
## 2020-04-29 0.03545849    0.02985138 0.03064112     0.03468903   0.03161554
## 2020-04-30 0.03441642    0.02964987 0.03052309     0.03455570   0.03155906
## 2020-05-04 0.03338657    0.02945197 0.03040643     0.03442362   0.03150308
## 2020-05-05 0.03214501    0.02925762 0.03029116     0.03429279   0.03144758
## 2020-05-06 0.02728841    0.02906676 0.03017725     0.03416320   0.03139256
## 2020-05-07 0.02675721    0.02887936 0.03006468     0.03403483   0.03133802
## 2020-05-08 0.02594207    0.02869537 0.02995346     0.03390769   0.03128396
##
## -------------------------------
## Statestreet
```
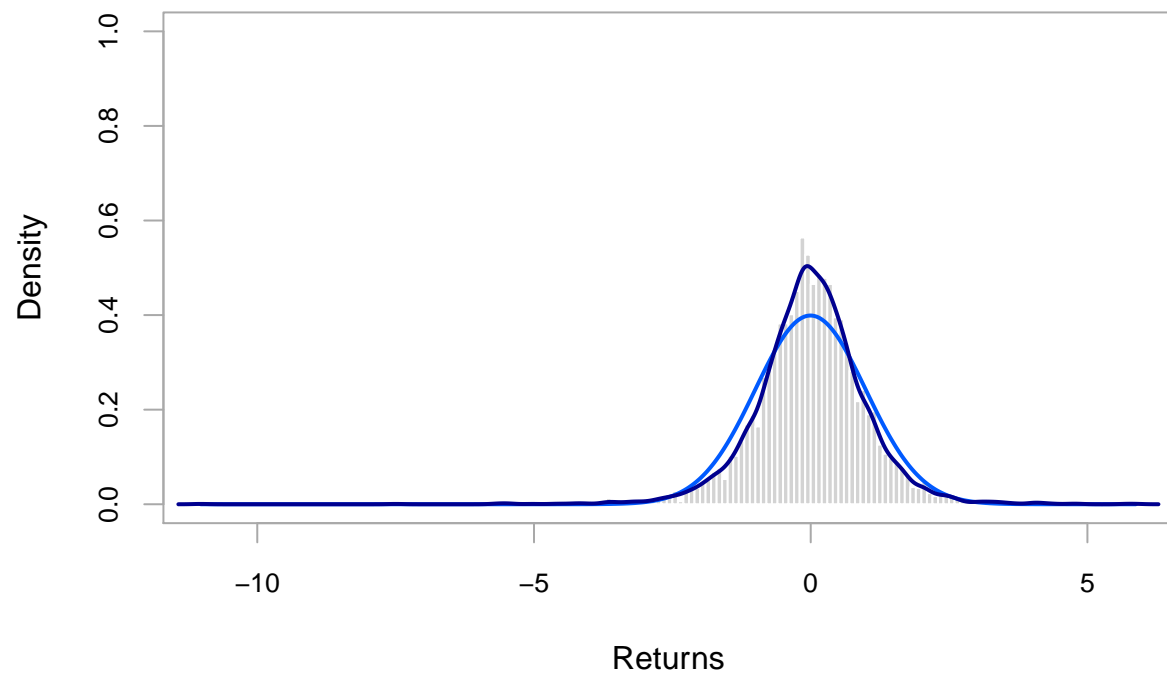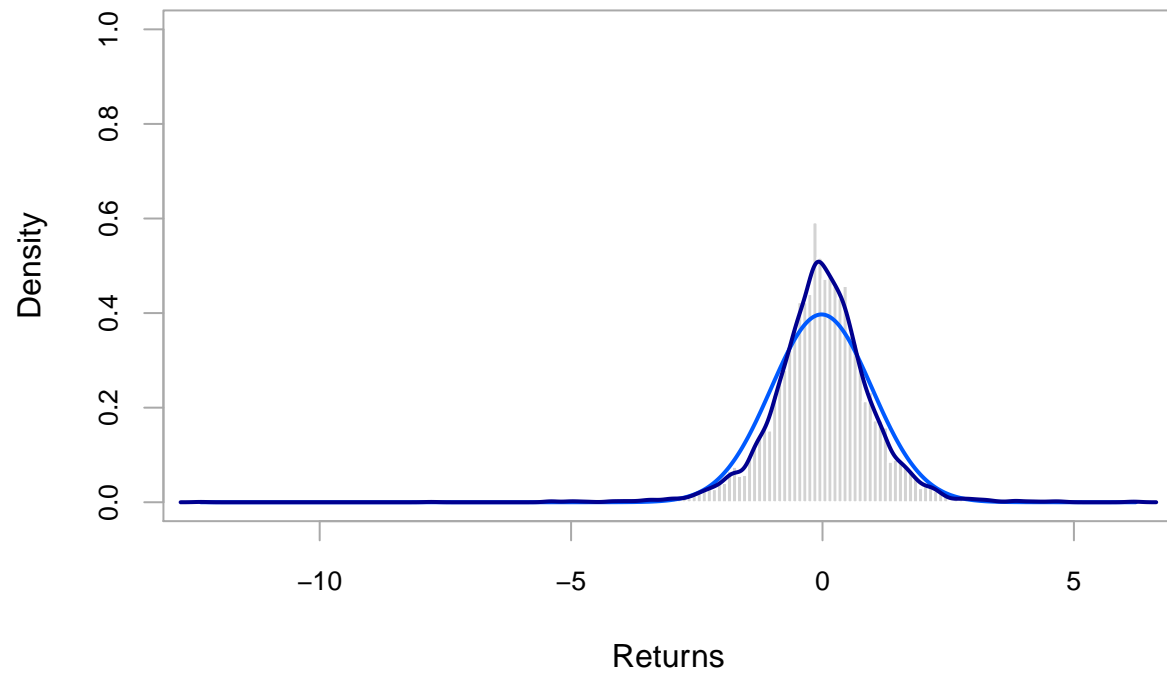
Standardized residuals of Standard GARCH with normal distribution of errors

Standardized residuals of GJR GARCH with normal distribution of errors

Density

Returns

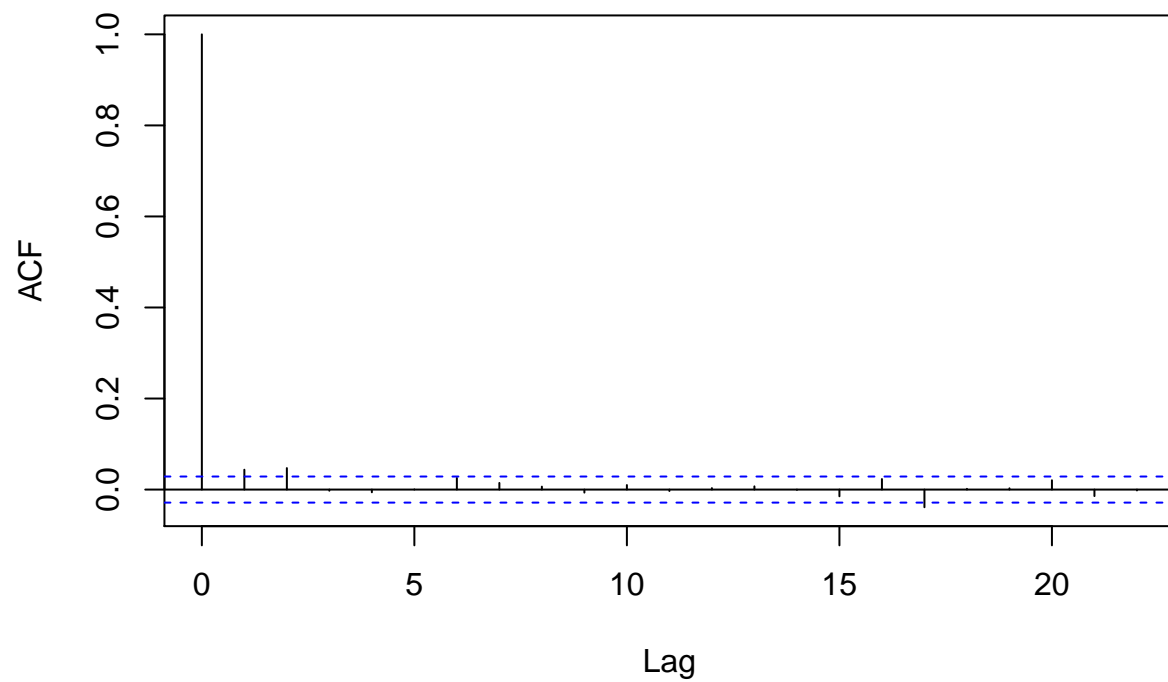**Standardized residuals of Standard GARCH with skewed Student t distribution of e**

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
## 
##   Standard GARCH with normal distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 38.431, df = 22, p-value = 0.01638
```

GJR GARCH with normal distribution of errors


```
##
##  GJR GARCH with normal distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 35.918, df = 22, p-value = 0.03098
```

Standard GARCH with skewed Student t distribution of errors


```
##
##  Standard GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 32.885, df = 22, p-value = 0.0635
```

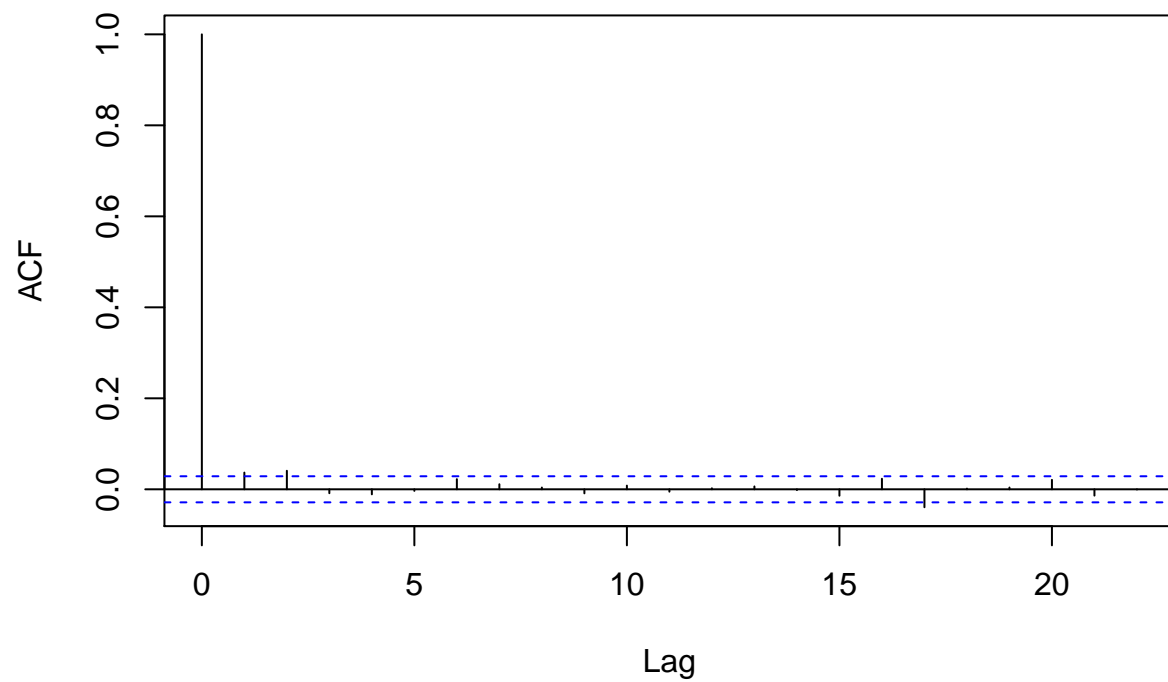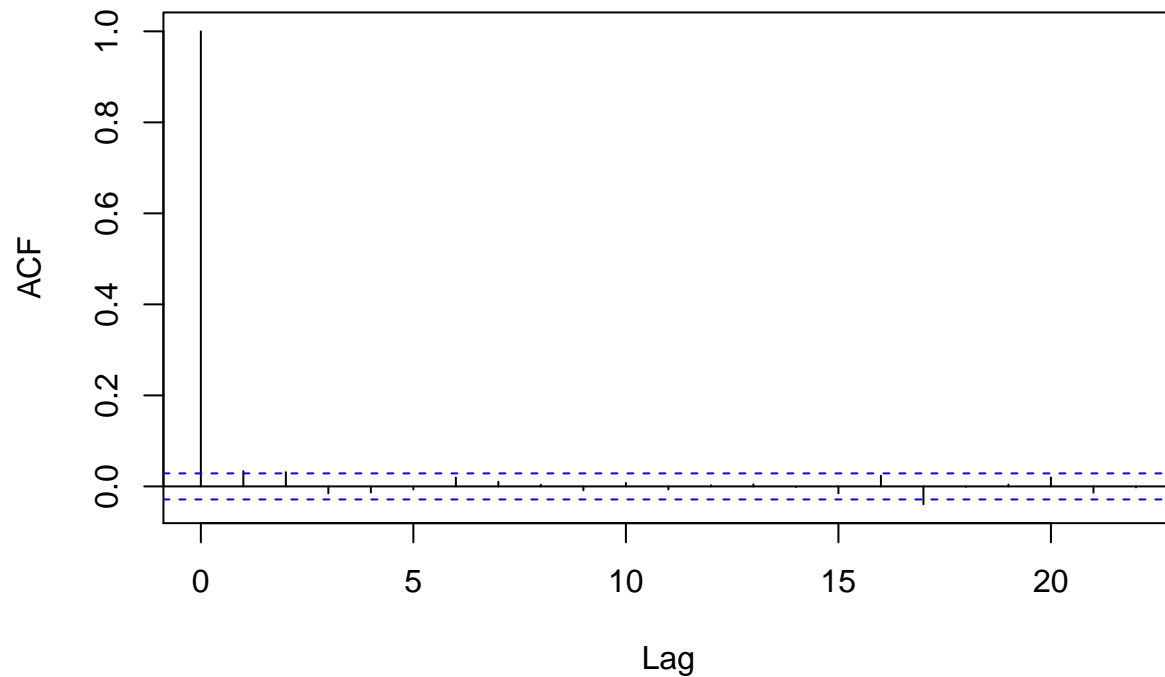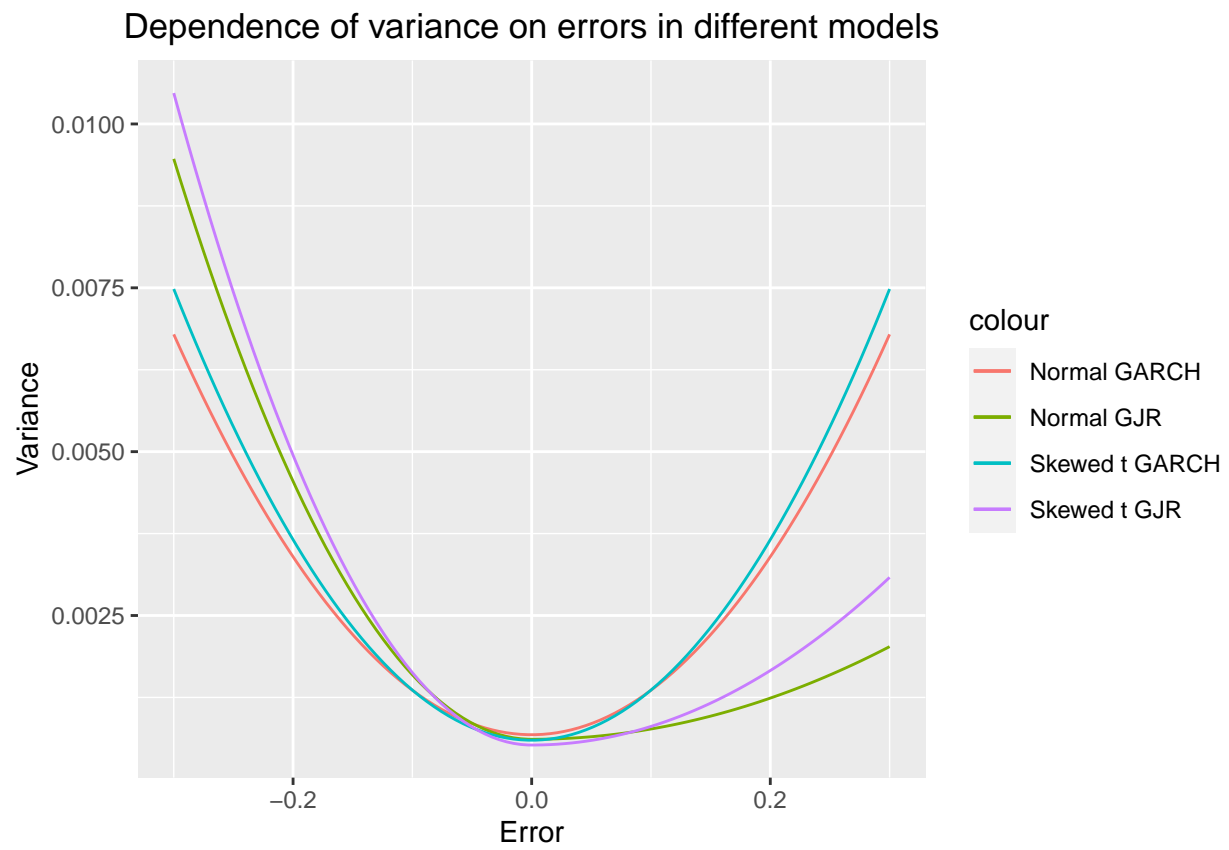## GJR GARCH with skewed Student t distribution of errors


```
##
##  GJR GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 29.104, df = 22, p-value = 0.1419
##
##
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error     t value     Pr(>|t|)
## mu     0.0006229046 0.0002336663    2.665787 0.007680828
## omega  0.0000040924 0.0000014199    2.882282 0.003948063
## alpha1 0.0678763140 0.0084295853    8.052153 0.000000000
## beta1  0.9265222666 0.0086931268 106.581014 0.000000000
##
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error     t value     Pr(>|t|)
## mu     0.0006229046 0.0002366809    2.6318328 0.008492564
## omega  0.0000040924 0.0000090613    0.4516386 0.651529353
## alpha1 0.0678763140 0.0437535648    1.5513322 0.120822094
## beta1  0.9265222666 0.0524587509   17.6619201 0.000000000
##
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate    Std. Error     t value     Pr(>|t|)
## mu     0.0002746120 0.0002367493    1.159927 0.246078453
```
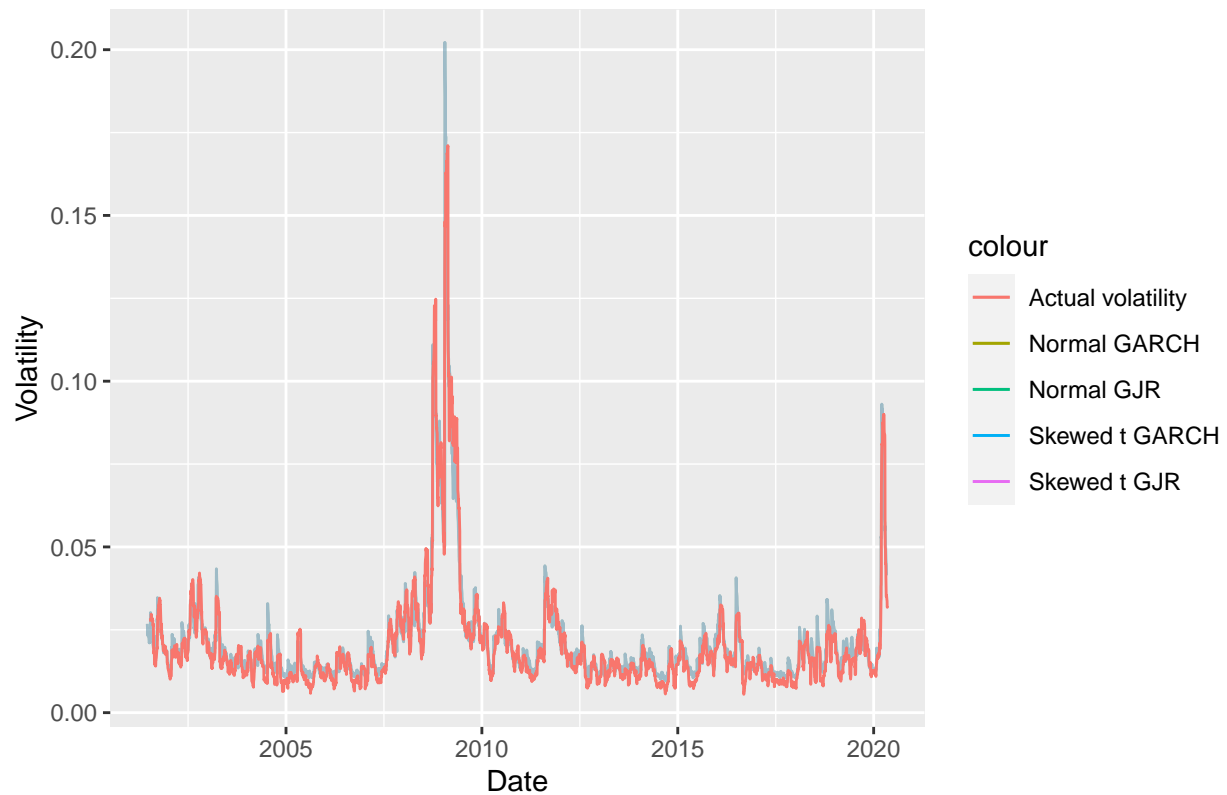
```
## omega  0.0000037187 0.0000013038    2.852206 0.004341696
## alpha1 0.0157118508 0.0055381410    2.837026 0.004553588
## beta1  0.9372042008 0.0071491406 131.093267 0.000000000
## gamma1 0.0826934991 0.0110567934    7.478977 0.000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##             Estimate    Std. Error    t value    Pr(>|t|)
## mu      0.0002746120 0.0003093397  0.8877361 0.37468271
## omega   0.0000037187 0.0000071647  0.5190367 0.60373516
## alpha1  0.0157118508 0.0144552270  1.0869321 0.27706680
## beta1   0.9372042008 0.0343471505 27.2862286 0.00000000
## gamma1  0.0826934991 0.0369137311  2.2401826 0.02507907
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu      0.0005319452 0.0002266178  2.347323 0.0189088532
## omega   0.0000050417 0.0000021412  2.354596 0.0185428760
## alpha1  0.0764961144 0.0119287007  6.412778 0.0000000001
## beta1   0.9157055122 0.0130645483 70.090867 0.0000000000
## skew    0.9847034128 0.0195080660 50.476732 0.0000000000
## shape   4.4933912560 0.2987209745 15.042102 0.0000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value    Pr(>|t|)
## mu      0.0005319452 0.0002056191  2.587043 0.009680361
## omega   0.0000050417 0.0000046219  1.090827 0.275349076
## alpha1  0.0764961144 0.0237620215  3.219260 0.001285221
## beta1   0.9157055122 0.0276363322 33.134119 0.000000000
## skew    0.9847034128 0.0196608988 50.084354 0.000000000
## shape   4.4933912560 0.3077369757 14.601402 0.000000000
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu      0.0003108593 0.0002295581  1.354164 0.1756841237
## omega   0.0000048506 0.0000020210  2.400153 0.0163882417
## alpha1  0.0284228583 0.0088768782  3.201898 0.0013652534
## beta1   0.9223473218 0.0121751914 75.756289 0.0000000000
## gamma1  0.0821119888 0.0163685317  5.016454 0.0000005263
## skew    0.9761621443 0.0194613310 50.159064 0.0000000000
## shape   4.6039164960 0.3099168312 14.855329 0.0000000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value    Pr(>|t|)
## mu      0.0003108593 0.0002306281  1.347881 0.177696534
## omega   0.0000048506 0.0000047520  1.020760 0.307368268
## alpha1  0.0284228583 0.0111510055  2.548905 0.010806160
## beta1   0.9223473218 0.0267371603 34.496832 0.000000000
## gamma1  0.0821119888 0.0314344004  2.612170 0.008996958
## skew    0.9761621443 0.0201394502 48.470149 0.000000000
## shape   4.6039164960 0.3271047777 14.074745 0.000000000
##              Normal GARCH    Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood   11962.325248 12004.024315   12269.313360 12288.489169
## Akaike          -5.110395     -5.127788      -5.240732    -5.248500
## Bayes           -5.104882     -5.120896      -5.232462    -5.238851
```

```
## Shibata          -5.110397    -5.127790       -5.240735    -5.248504
## Hannan-Quinn      -5.108456    -5.125364       -5.237824    -5.245106
```

Dependence of variance on errors in different models
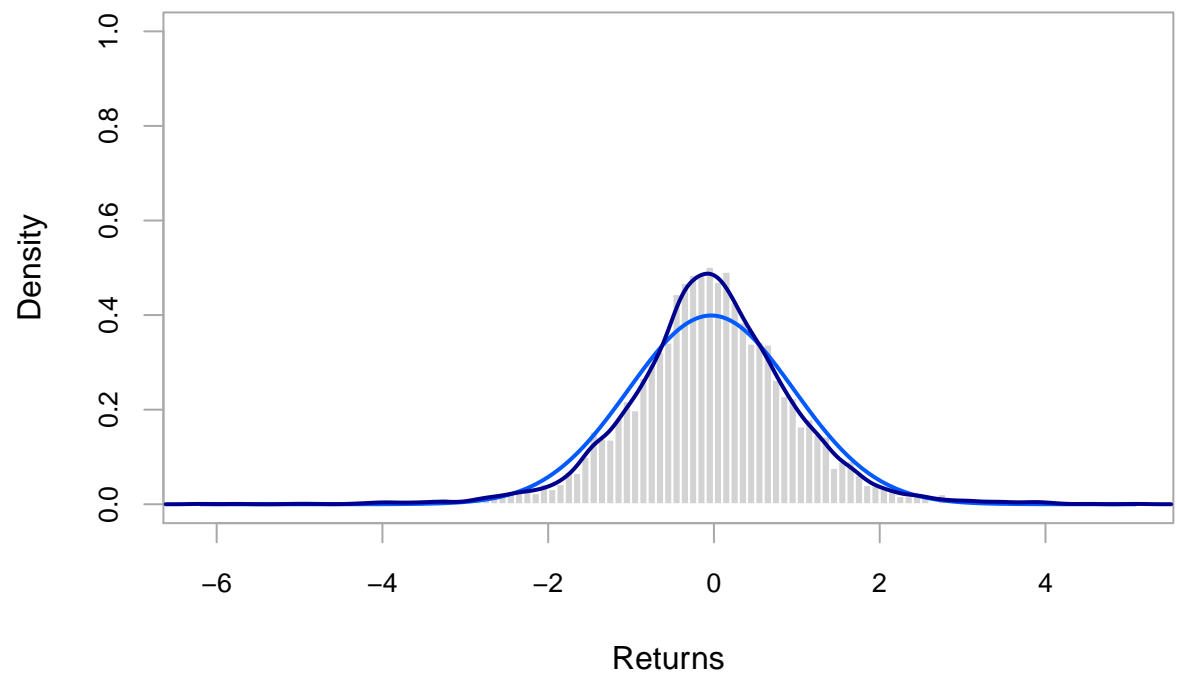
Volatility constructed by different models


```
##    TES for Normal GARCH    TES for Normal GJR TES for Skewed t GARCH
##           -0.09714092           -0.08877268            -0.07214436
##   TES for Skewed t GJR
##           -0.05564191
##             Statestreet Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24  0.04082046   0.04455275 0.04373417     0.04216634   0.04056256
## 2020-04-27  0.03499251   0.04447383 0.04365115     0.04206158   0.04044718
## 2020-04-28  0.03506750   0.04439521 0.04356845     0.04195737   0.04033247
## 2020-04-29  0.03509407   0.04431690 0.04348607     0.04185373   0.04021843
## 2020-04-30  0.03538565   0.04423888 0.04340401     0.04175064   0.04010504
## 2020-05-04  0.03390033   0.04416117 0.04332226     0.04164809   0.03999232
## 2020-05-05  0.03378543   0.04408375 0.04324083     0.04154610   0.03988026
## 2020-05-06  0.03202825   0.04400664 0.04315971     0.04144466   0.03976886
## 2020-05-07  0.03177526   0.04392982 0.04307891     0.04134376   0.03965810
## 2020-05-08  0.03202185   0.04385329 0.04299842     0.04124340   0.03954800
##
## --------------------------------
## JPmorgan
```
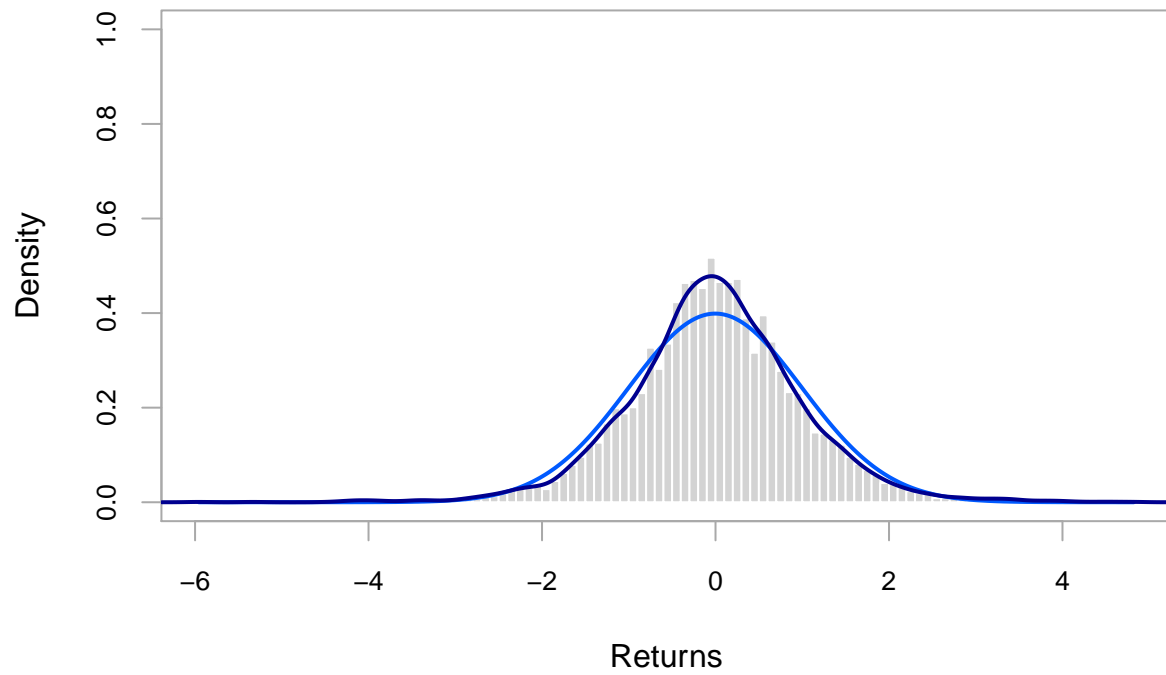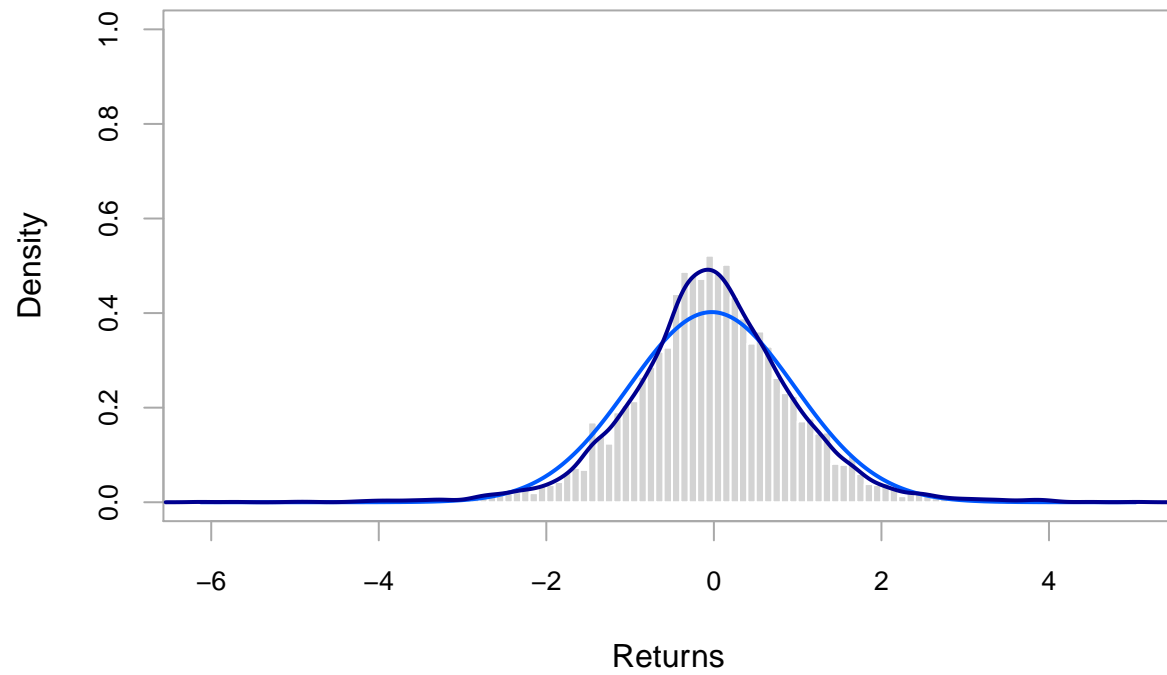
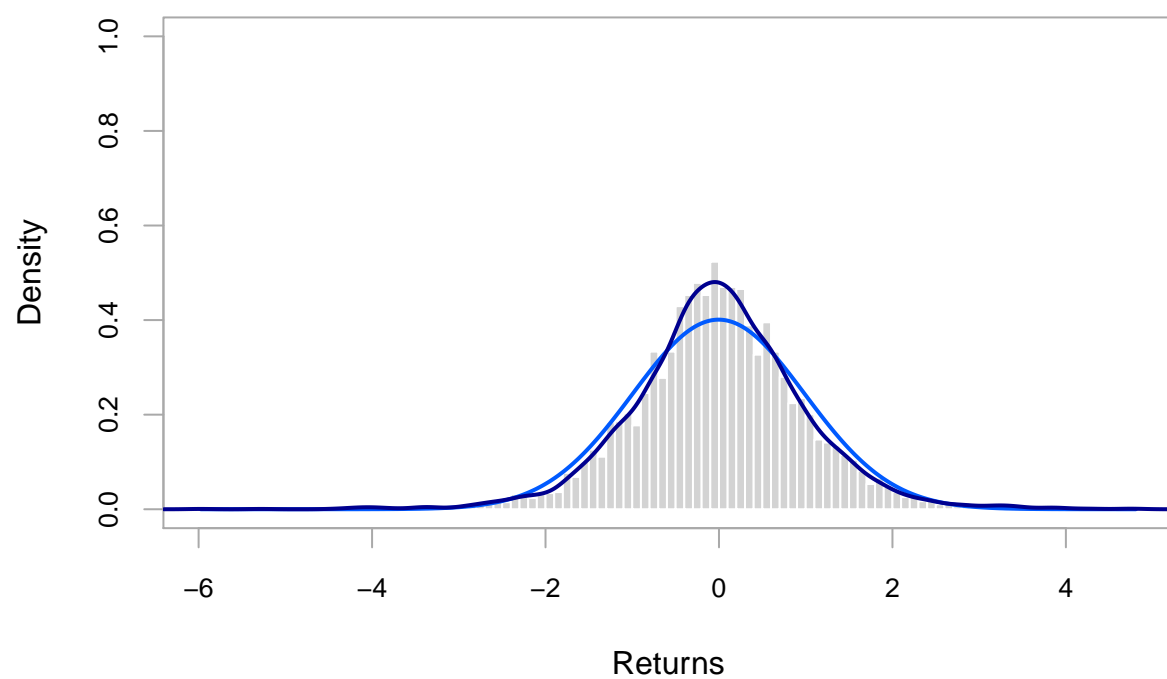**Standardized residuals of Standard GARCH with normal distribution of errors**

Standardized residuals of GJR GARCH with normal distribution of errors

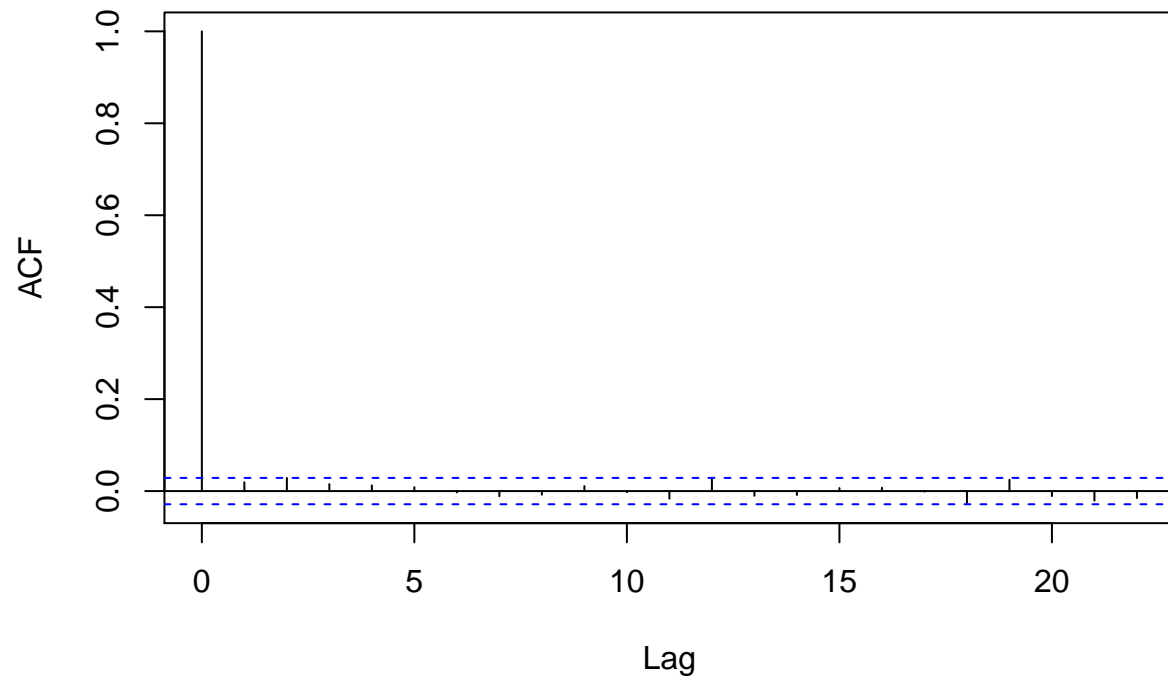**Standardized residuals of Standard GARCH with skewed Student t distribution of ε**

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
## 
##   Standard GARCH with normal distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 24.517, df = 22, p-value = 0.3207
```

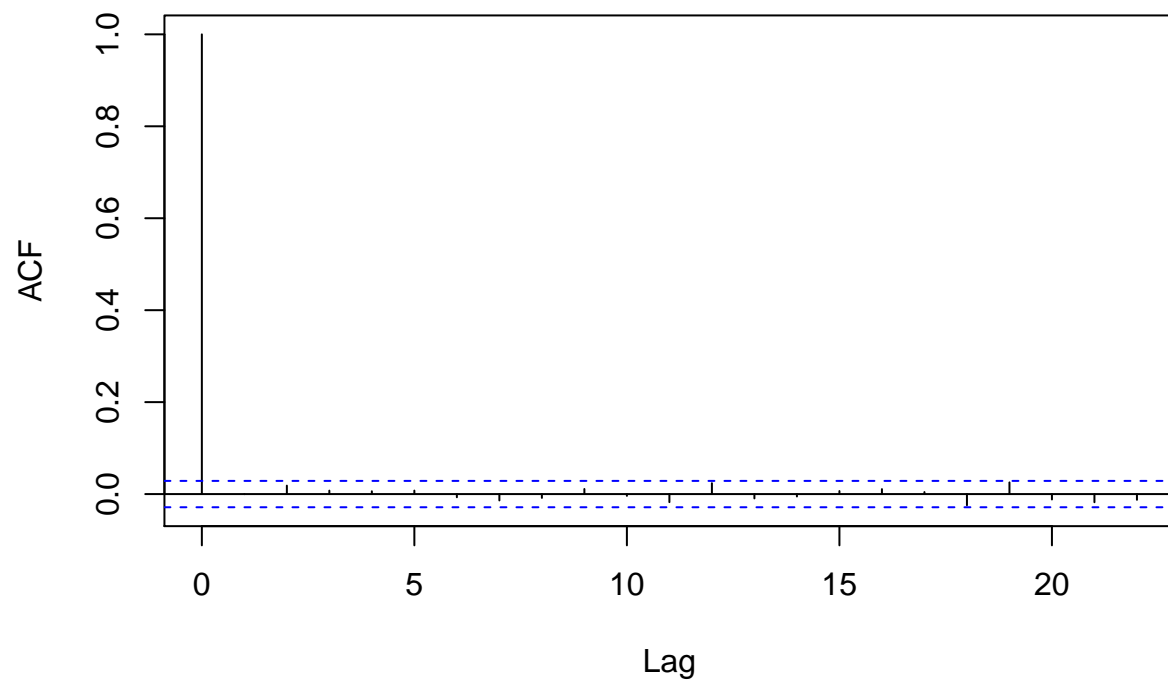**GJR GARCH with normal distribution of errors**


```
## 
##  GJR GARCH with normal distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 18.568, df = 22, p-value = 0.6718
```

Standard GARCH with skewed Student t distribution of errors


```
## 
##  Standard GARCH with skewed Student t distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 25.874, df = 22, p-value = 0.2571
```

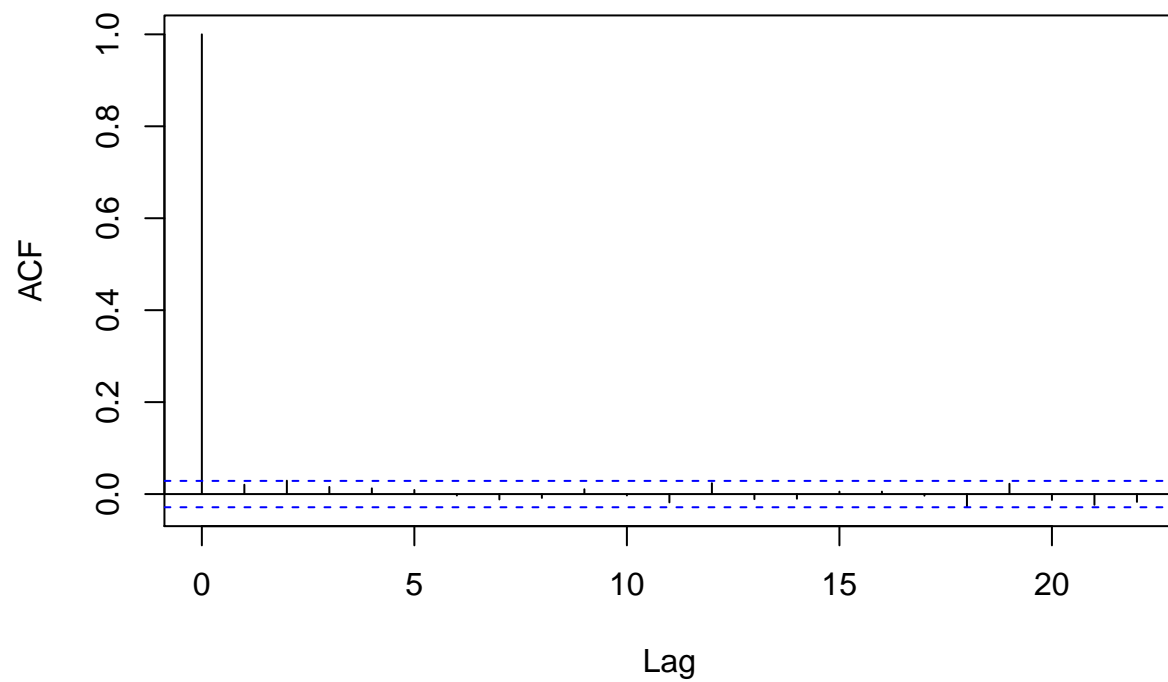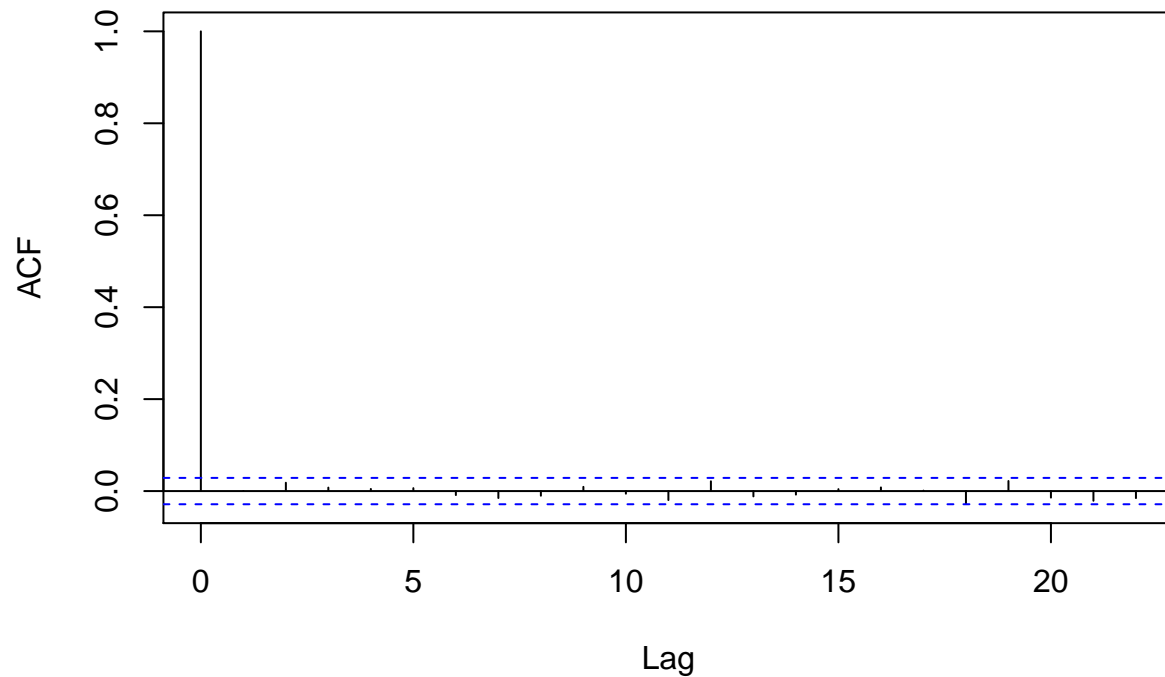## GJR GARCH with skewed Student t distribution of errors


```
##
##   GJR GARCH with skewed Student t distribution of errors
##
##   Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 19.644, df = 22, p-value = 0.6053
##
##
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value      Pr(>|t|)
## mu      0.0008025599 0.0001991405   4.030119 0.0000557487
## omega   0.0000037079 0.0000014476   2.561377 0.0104258206
## alpha1  0.0940084823 0.0103486903   9.084095 0.0000000000
## beta1   0.8992903956 0.0110887192  81.099573 0.0000000000
##
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value      Pr(>|t|)
## mu      0.0008025599 0.0002038019   3.9379412 0.0000821837
## omega   0.0000037079 0.0000053806   0.6891181 0.4907489737
## alpha1  0.0940084823 0.0324668126   2.8955255 0.0037852433
## beta1   0.8992903956 0.0371703513  24.1937556 0.0000000000
##
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate    Std. Error    t value      Pr(>|t|)
## mu      0.0003337115 0.0002109749   1.581759 0.1137045805
```
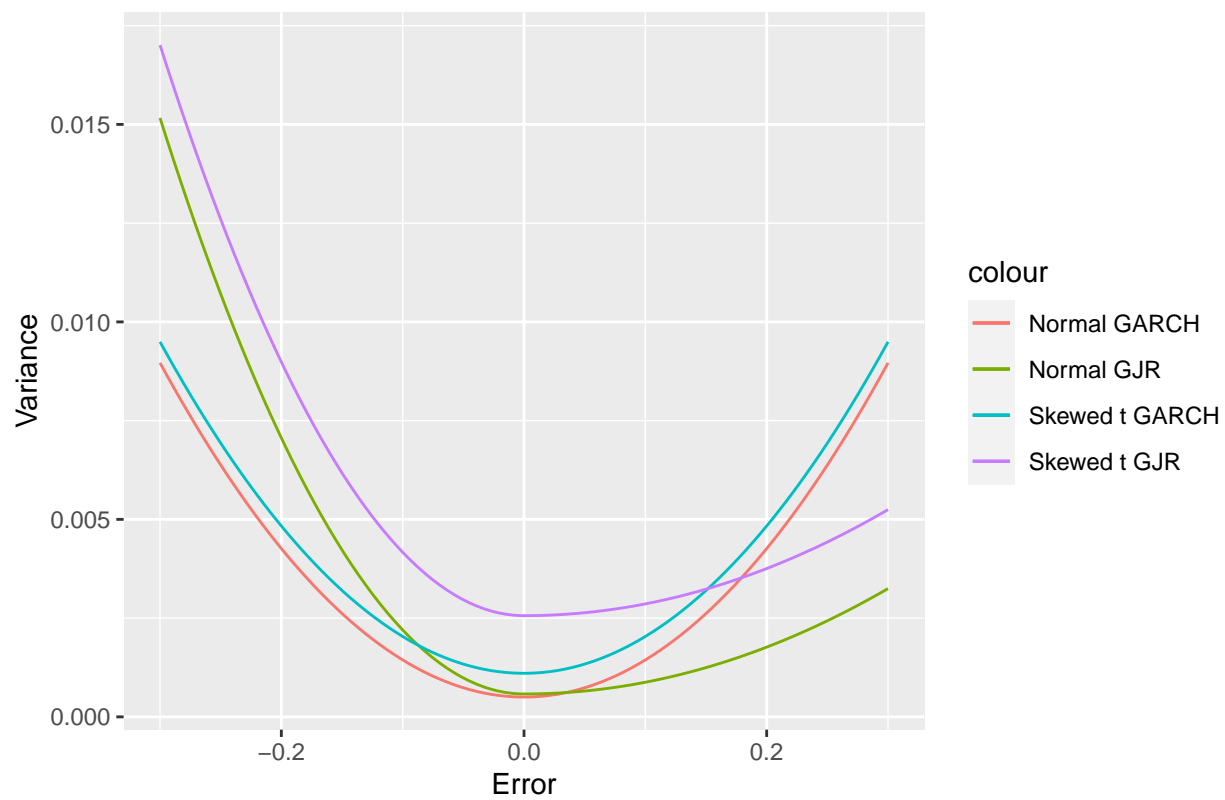
```
## omega  0.0000043912 0.0000015807   2.778102 0.0054677379
## alpha1 0.0296360574 0.0062706203   4.726176 0.0000022879
## beta1  0.8973292575 0.0096343825 93.138222 0.0000000000
## gamma1 0.1323862020 0.0161901970   8.176936 0.0000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##             Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0003337115 0.0003307577  1.0089303 0.3130080676
## omega  0.0000043912 0.0000059460  0.7385191 0.4601990446
## alpha1 0.0296360574 0.0097865457  3.0282449 0.0024597864
## beta1  0.8973292575 0.0283342262 31.6694464 0.0000000000
## gamma1 0.1323862020 0.0389134695  3.4020663 0.0006687842
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0006885213 0.0001963165  3.507200 0.0004528492
## omega  0.0000030960 0.0000025344  1.221573 0.2218690380
## alpha1 0.0932414078 0.0218512452  4.267098 0.0000198032
## beta1  0.9042134147 0.0212783064 42.494614 0.0000000000
## skew   1.0243320714 0.0201902157 50.734083 0.0000000000
## shape  5.3407621443 0.4675322590 11.423302 0.0000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0006885213 0.0001847397  3.7269809 0.0001937871
## omega  0.0000030960 0.0000107845  0.2870793 0.7740515726
## alpha1 0.0932414078 0.0897352308  1.0390725 0.2987710339
## beta1  0.9042134147 0.0882962523 10.2406772 0.0000000000
## skew   1.0243320714 0.0200539199 51.0788951 0.0000000000
## shape  5.3407621443 1.0032298544  5.3235678 0.0000001018
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value      Pr(>|t|)
## mu     0.0003553160 0.0001941626  1.829993 0.0672510562
## omega  0.0000034453 0.0000018022  1.911688 0.0559162145
## alpha1 0.0298318398 0.0093589493  3.187520 0.0014349844
## beta1  0.9031972239 0.0150478196 60.021800 0.0000000000
## gamma1 0.1306817031 0.0215986930  6.050445 0.0000000014
## skew   1.0154454501 0.0201420290 50.414258 0.0000000000
## shape  5.7459614158 0.4882562557 11.768331 0.0000000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0003553160 0.0001977740  1.7965762 0.072402926
## omega  0.0000034453 0.0000052368  0.6578953 0.510605417
## alpha1 0.0298318398 0.0179380462  1.6630484 0.096302740
## beta1  0.9031972239 0.0401706087 22.4840313 0.000000000
## gamma1 0.1306817031 0.0485359165  2.6924742 0.007092402
## skew   1.0154454501 0.0211991039 47.9003949 0.000000000
## shape  5.7459614158 0.5967003029  9.6295601 0.000000000
##              Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood   12504.555570 12569.633840   12658.494820 12700.275512
## Akaike          -5.342118    -5.369502      -5.407049    -5.424477
## Bayes           -5.336604    -5.362609      -5.398778    -5.414828
```
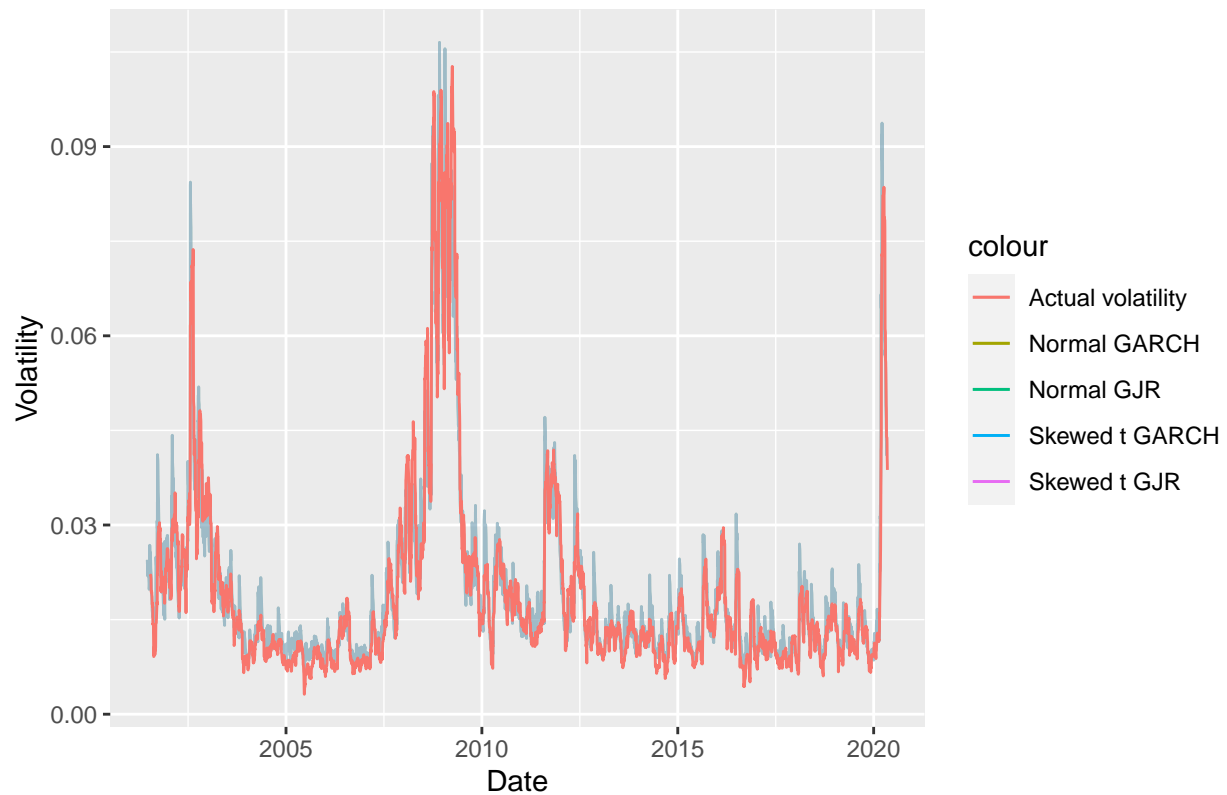
```
## Shibata          -5.342119    -5.369504         -5.407052    -5.424481
## Hannan-Quinn      -5.340179    -5.367078         -5.404141    -5.421084
```


Dependence of variance on errors in different models
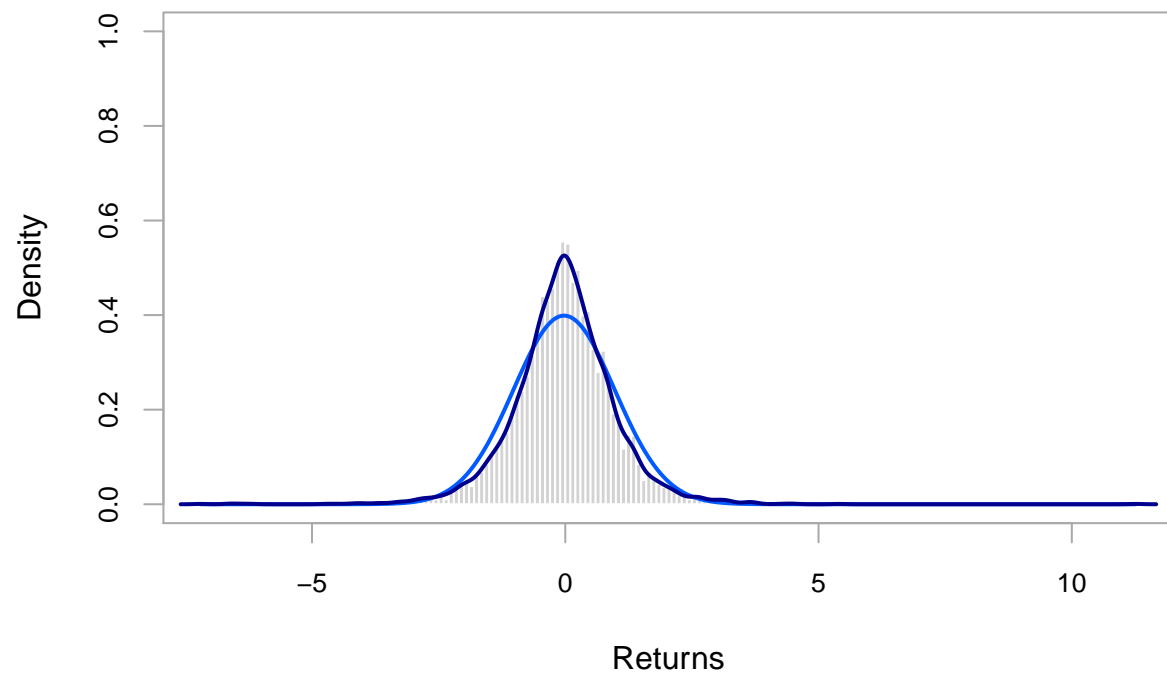
Volatility constructed by different models


```
##    TES for Normal GARCH      TES for Normal GJR TES for Skewed t GARCH
##              0.03222845              0.07211499             0.01223805
##    TES for Skewed t GJR
##              0.04774524
##               JPmorgan Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.05410294    0.04202937 0.03792954     0.04368805   0.03996284
## 2020-04-27 0.04898786    0.04193254 0.03785761     0.04366788   0.03998165
## 2020-04-28 0.04849763    0.04183615 0.03778604     0.04364776   0.04000042
## 2020-04-29 0.04660068    0.04174018 0.03771482     0.04362767   0.04001916
## 2020-04-30 0.04395919    0.04164463 0.03764395     0.04360763   0.04003787
## 2020-05-04 0.04384921    0.04154951 0.03757344     0.04358763   0.04005654
## 2020-05-05 0.04292573    0.04145480 0.03750328     0.04356767   0.04007519
## 2020-05-06 0.04055567    0.04136052 0.03743347     0.04354775   0.04009380
## 2020-05-07 0.04000345    0.04126666 0.03736400     0.04352788   0.04011238
## 2020-05-08 0.03873366    0.04117321 0.03729489     0.04350804   0.04013094
##
## ------------------------------
## Bankmellon
```
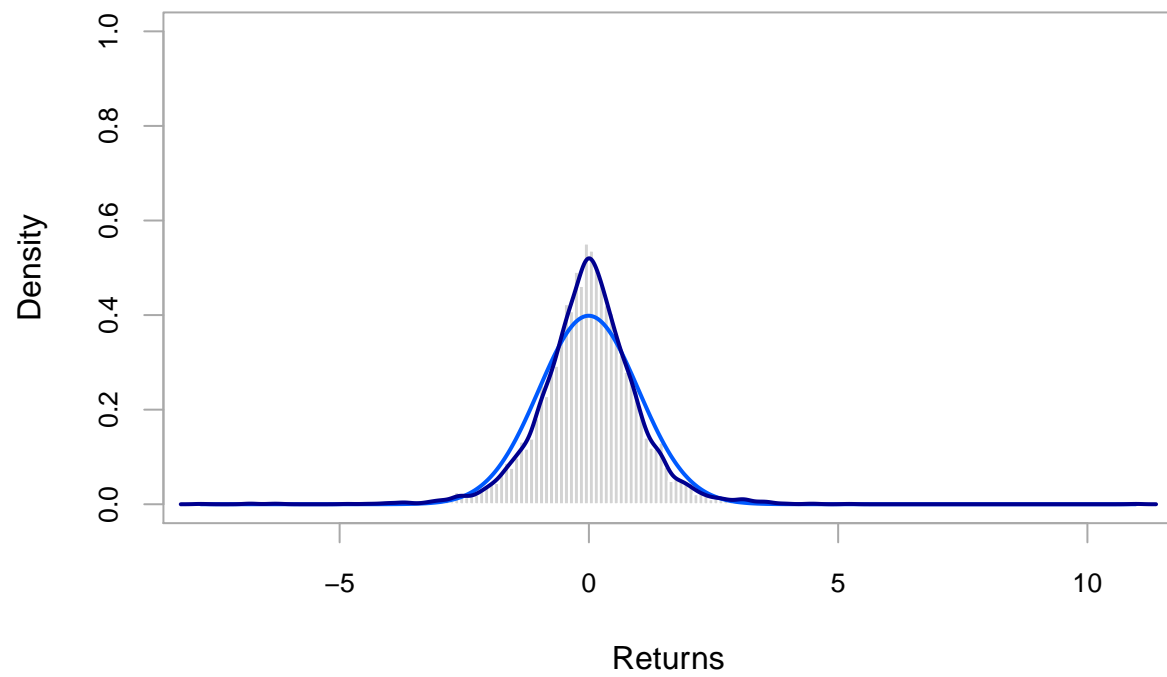
Standardized residuals of Standard GARCH with normal distribution of errors

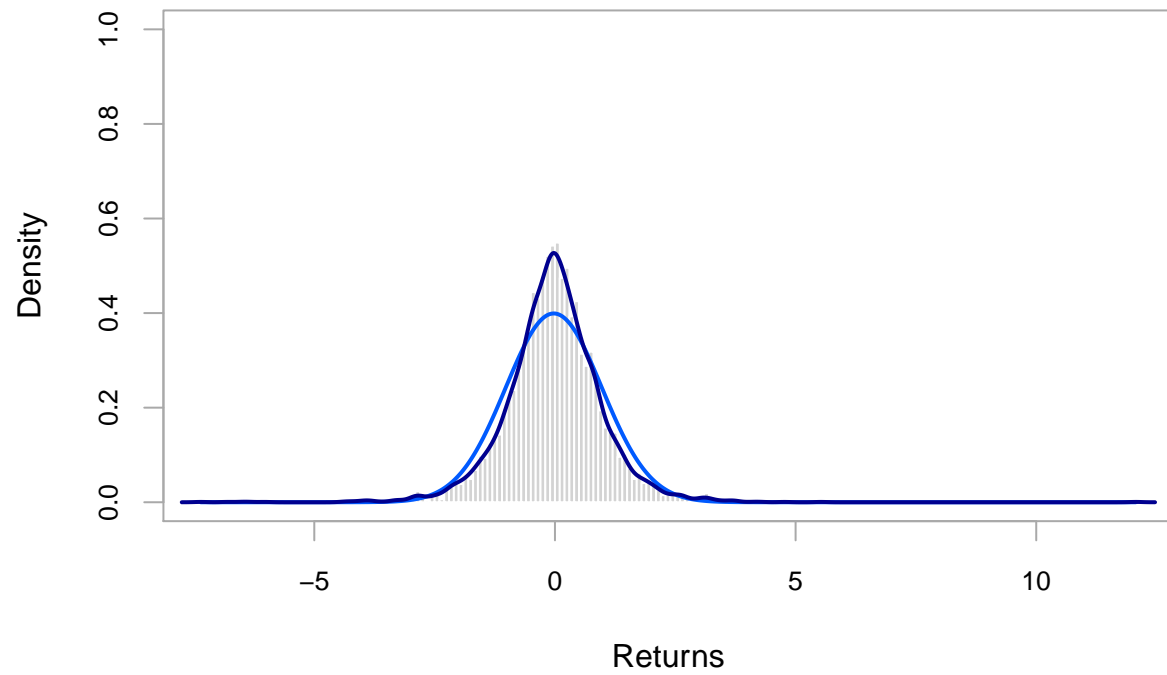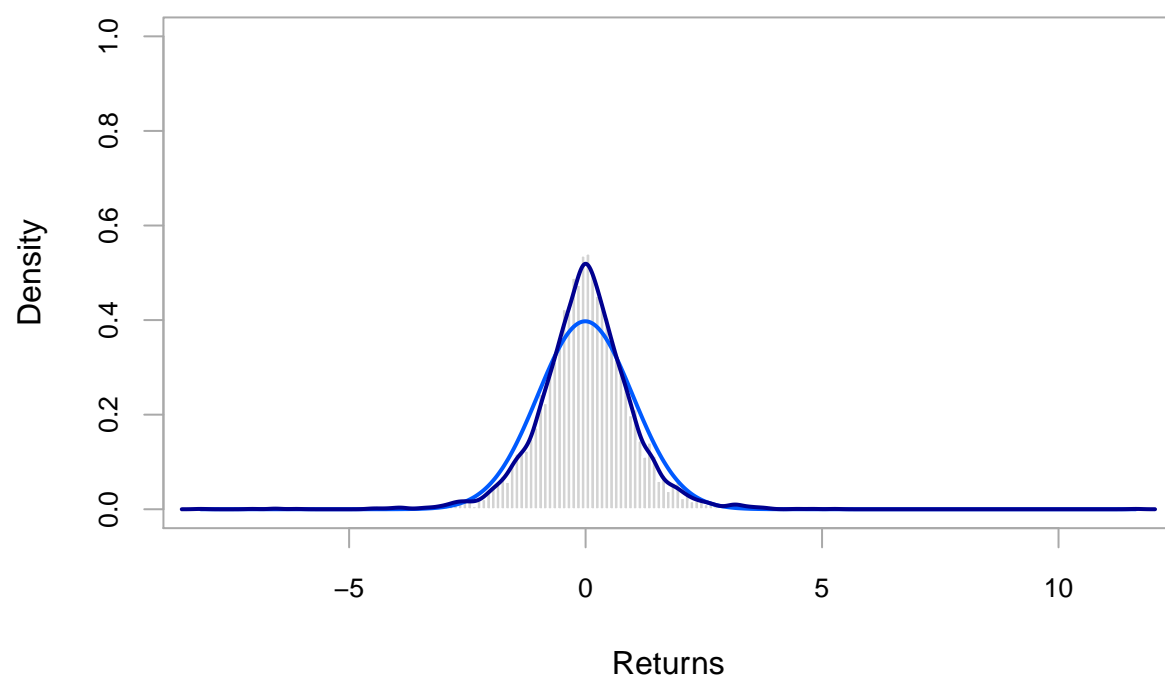**Standardized residuals of GJR GARCH with normal distribution of errors**

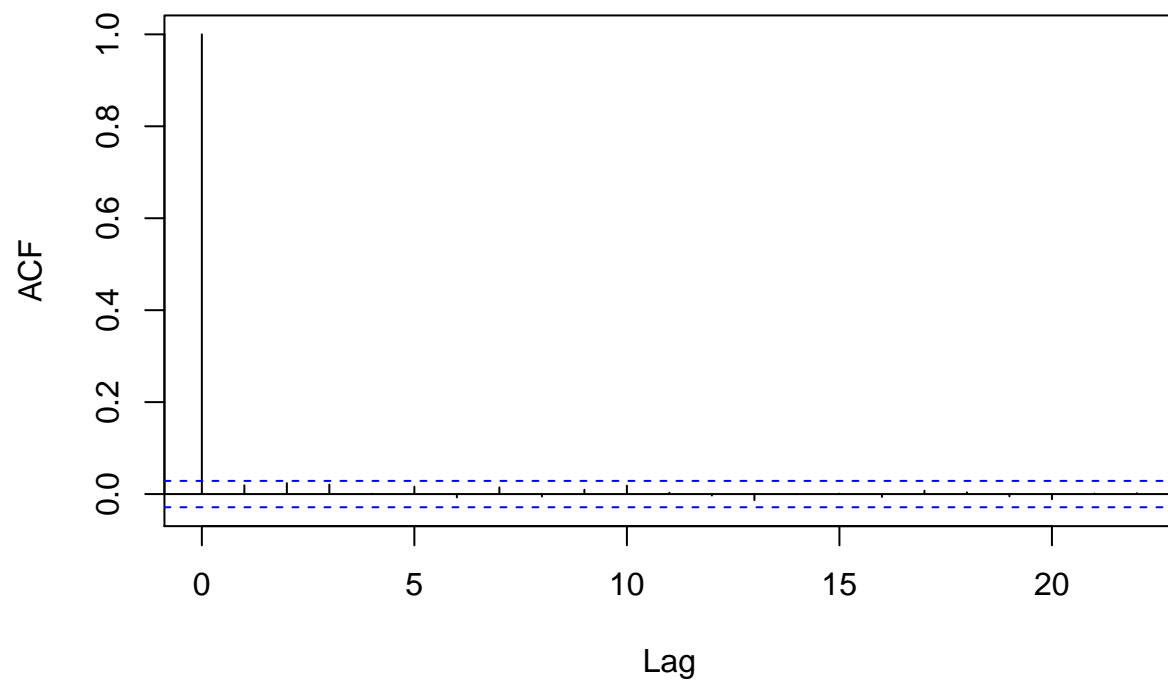**Standardized residuals of Standard GARCH with skewed Student t distribution of e**

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
## 
##  Standard GARCH with normal distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 13.069, df = 22, p-value = 0.9312
```

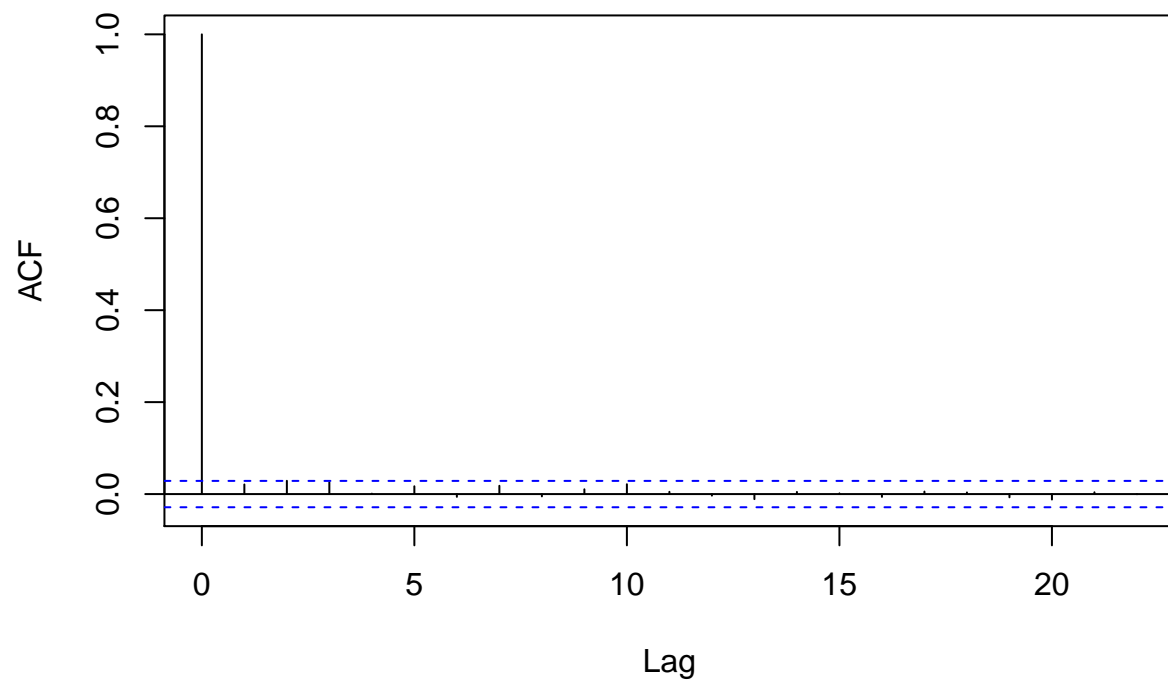## GJR GARCH with normal distribution of errors


```
##
##  GJR GARCH with normal distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 17.044, df = 22, p-value = 0.7609
```

Standard GARCH with skewed Student t distribution of errors


```
## 
##   Standard GARCH with skewed Student t distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 12.621, df = 22, p-value = 0.9432
```

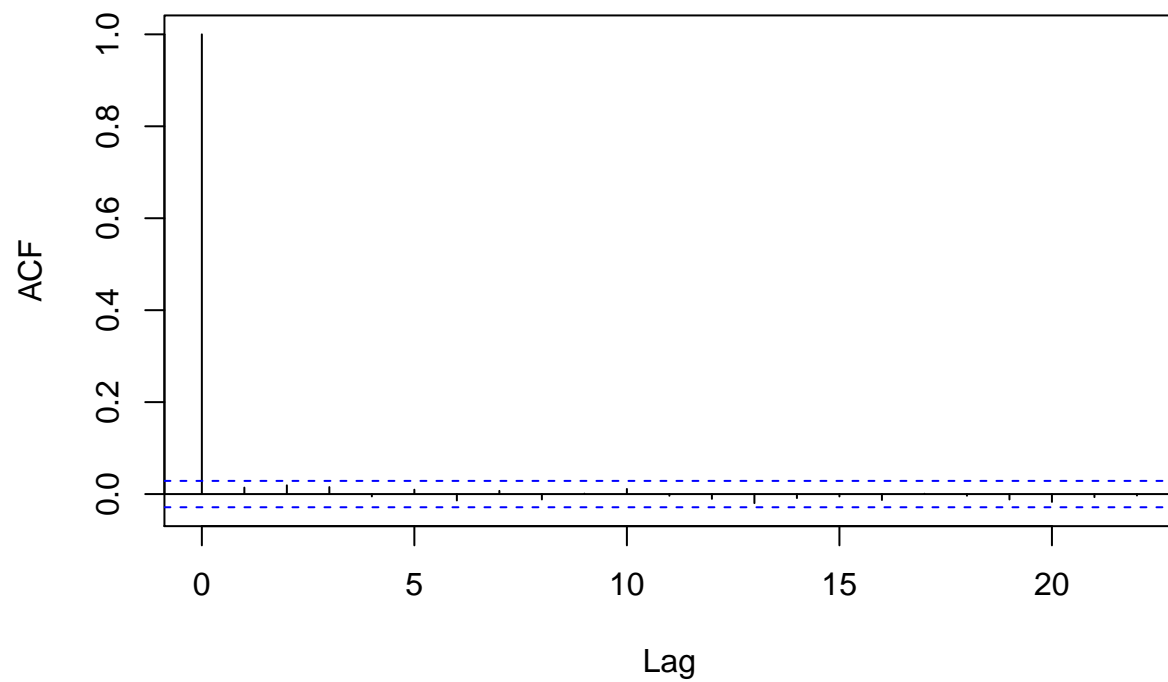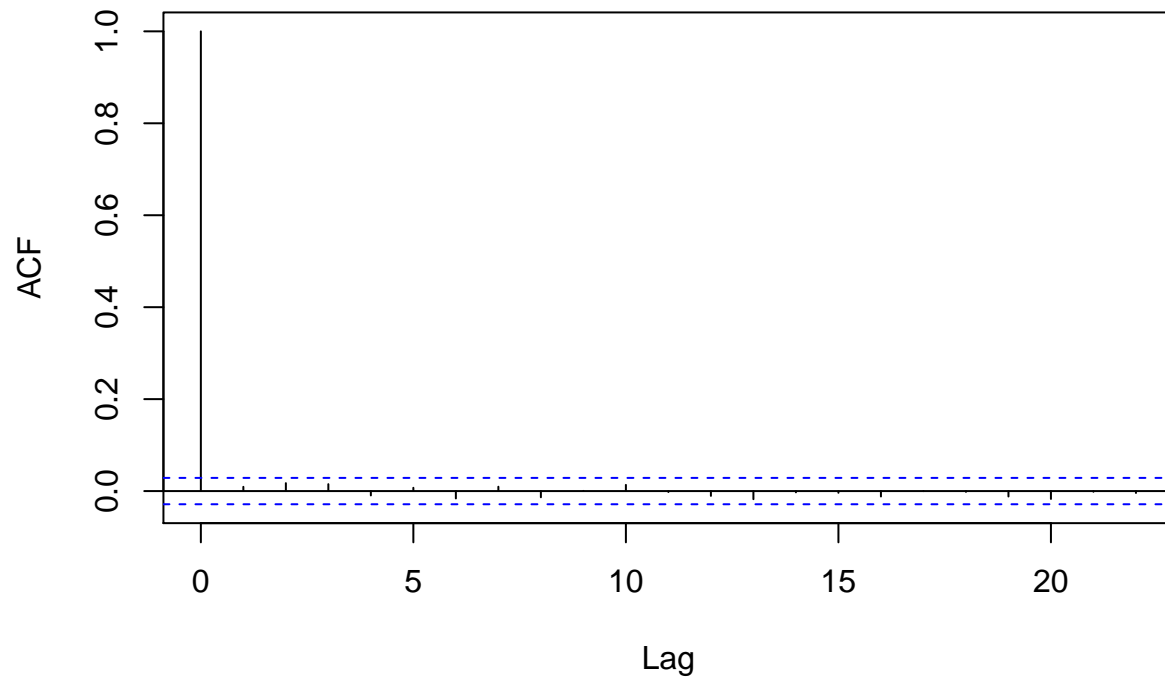## GJR GARCH with skewed Student t distribution of errors


```
##
##  GJR GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 11.804, df = 22, p-value = 0.9613
##
##
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value     Pr(>|t|)
## mu      0.0005565676 0.0002181872    2.550872  0.01074537
## omega   0.0000085033 0.0000012482    6.812440  0.00000000
## alpha1  0.0963925889 0.0042059078   22.918379  0.00000000
## beta1   0.8831197658 0.0066368376  133.063338  0.00000000
##
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error   t value     Pr(>|t|)
## mu      0.0005565676 0.000223894   2.485853  0.012924133
## omega   0.0000085033 0.000006642   1.280227  0.200465436
## alpha1  0.0963925889 0.020485651   4.705371  0.000002534
## beta1   0.8831197658 0.021855789  40.406676  0.000000000
##
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate    Std. Error    t value      Pr(>|t|)
## mu      0.0001956997 0.0001957810    0.9995847  0.3175115124
```
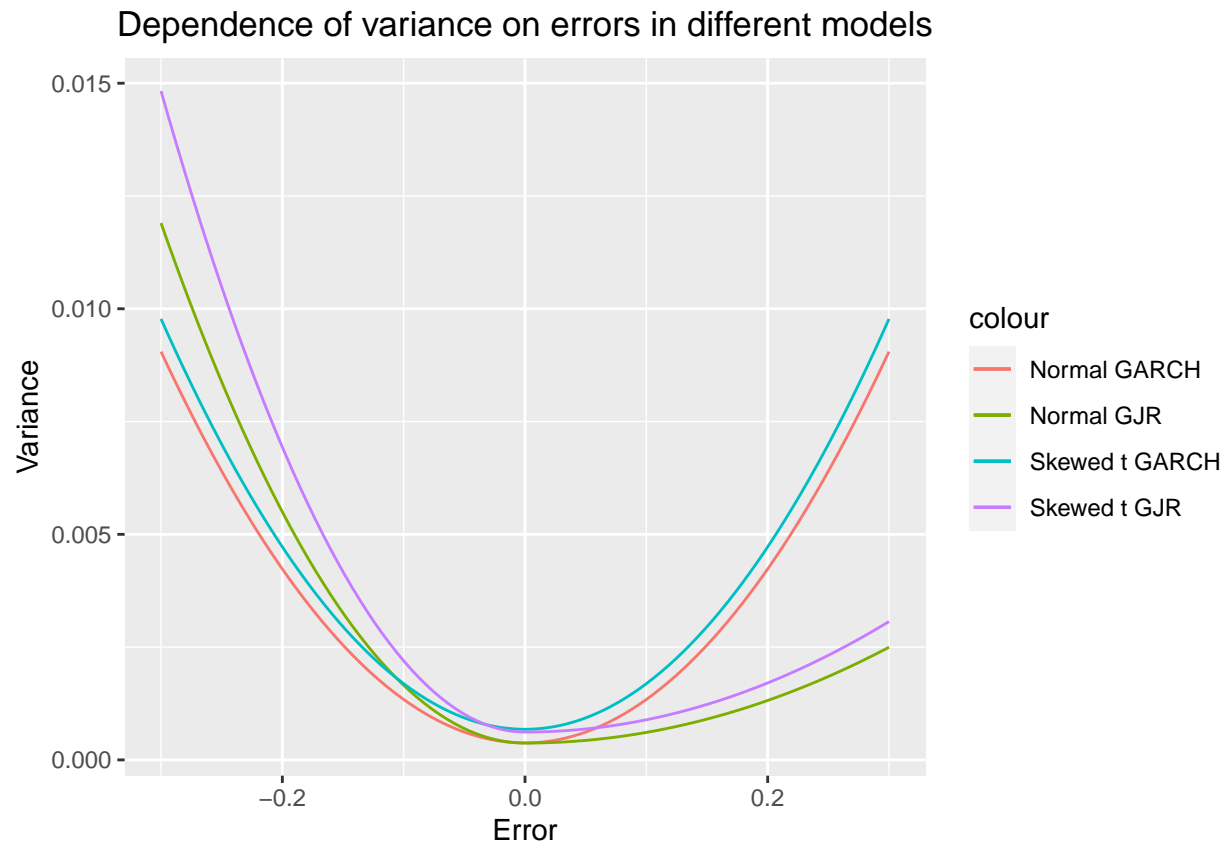
```
## omega  0.0000070117 0.0000005894  11.8969912 0.0000000000
## alpha1 0.0236004194 0.0045543672   5.1819316 0.0000002196
## beta1  0.9067820336 0.0056798902 159.6478093 0.0000000000
## gamma1 0.1044815986 0.0124557037   8.3882533 0.0000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##              Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0001956997 0.0004354262   0.449444 0.6531114192
## omega  0.0000070117 0.0000024009   2.920467 0.0034950774
## alpha1 0.0236004194 0.0113369127   2.081732 0.0373669127
## beta1  0.9067820336 0.0131888498  68.753685 0.0000000000
## gamma1 0.1044815986 0.0301694167   3.463163 0.0005338655
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0005702877 0.0002076088   2.746934 0.006015527
## omega  0.0000056422 0.0000020209   2.791934 0.005239404
## alpha1 0.1010891051 0.0132544746   7.626791 0.000000000
## beta1  0.8914282214 0.0138565834  64.332469 0.000000000
## skew   1.0064232712 0.0193637701  51.974552 0.000000000
## shape  4.4455666007 0.2972196912  14.957174 0.000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0005702877 0.0001926085   2.960865 0.0030677653
## omega  0.0000056422 0.0000036281   1.555148 0.1199107788
## alpha1 0.1010891051 0.0196201877   5.152301 0.0000002573
## beta1  0.8914282214 0.0226249320  39.400261 0.0000000000
## skew   1.0064232712 0.0197306195  51.008194 0.0000000000
## shape  4.4455666007 0.3151782285  14.104929 0.0000000000
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0002367350 0.0002242989   1.055444 0.2912221503
## omega  0.0000056932 0.0000022647   2.513865 0.0119416049
## alpha1 0.0272174757 0.0123053541   2.211840 0.0269777156
## beta1  0.8993436776 0.0120065989  74.904116 0.0000000000
## gamma1 0.1306615680 0.0239456725   5.456584 0.0000000485
## skew   0.9910566542 0.0195837747  50.606008 0.0000000000
## shape  4.5824743849 0.3159590395  14.503381 0.0000000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##              Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0002367350 0.0003218238   0.7356044 0.46197150
## omega  0.0000056932 0.0000057529   0.9896239 0.32235796
## alpha1 0.0272174757 0.0302000487   0.9012395 0.36746101
## beta1  0.8993436776 0.0206846919  43.4787079 0.00000000
## gamma1 0.1306615680 0.0531746018   2.4572176 0.01400178
## skew   0.9910566542 0.0220970220  44.8502361 0.00000000
## shape  4.5824743849 0.3855923807  11.8842452 0.00000000
##               Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood    12345.345038 12377.211104   12630.718834 12657.961065
## Akaike           -5.274079    -5.287270      -5.395179    -5.406394
## Bayes            -5.268565    -5.280378      -5.386908    -5.396745
```
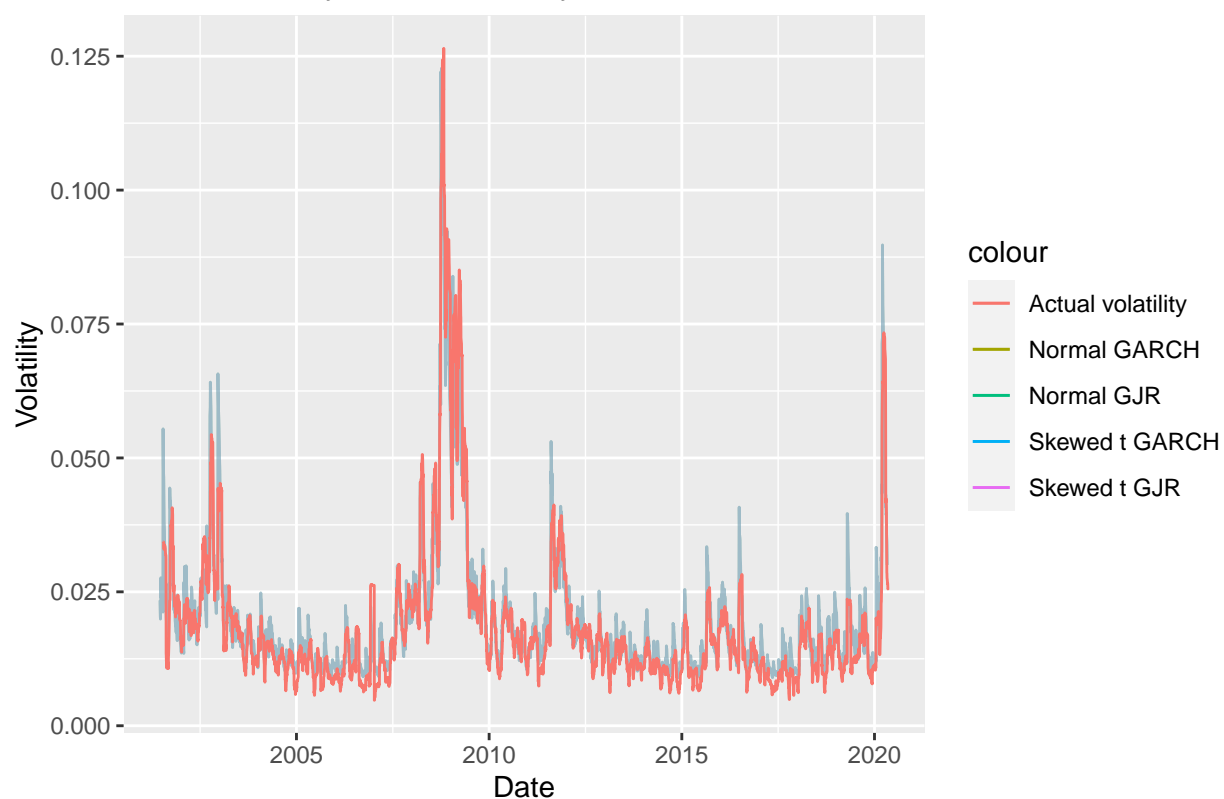
```
## Shibata          -5.274081    -5.287272      -5.395182    -5.406398
## Hannan-Quinn      -5.272140    -5.284846      -5.392271    -5.403000
```


Dependence of variance on errors in different models
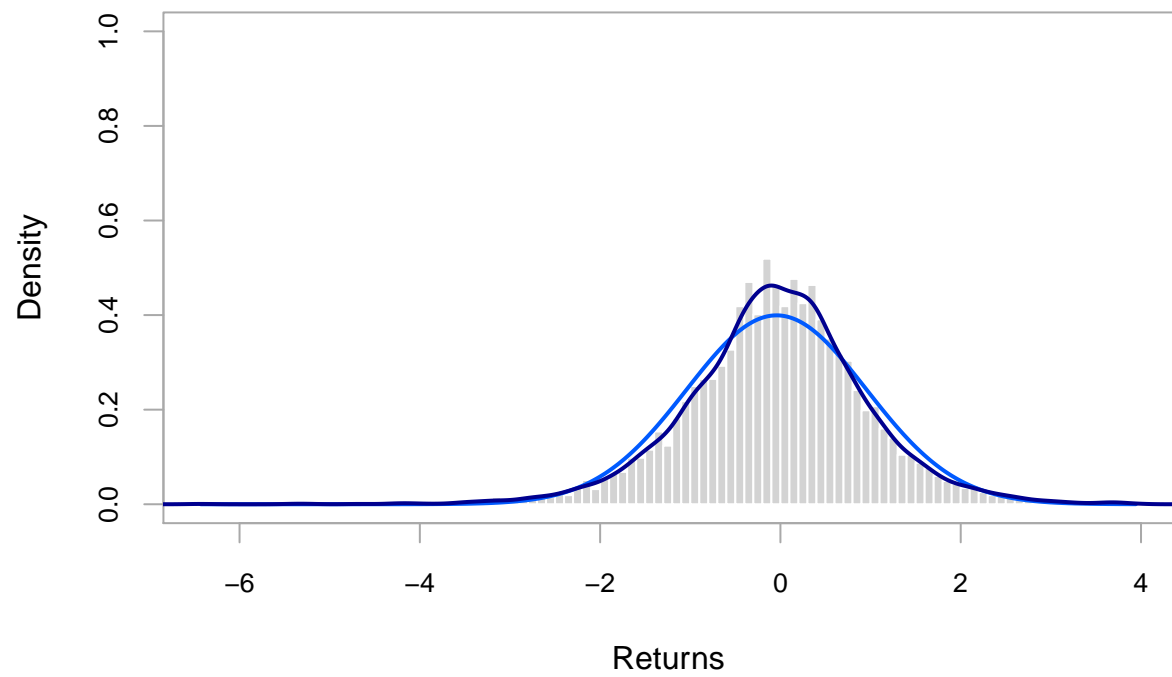
Volatility constructed by different models
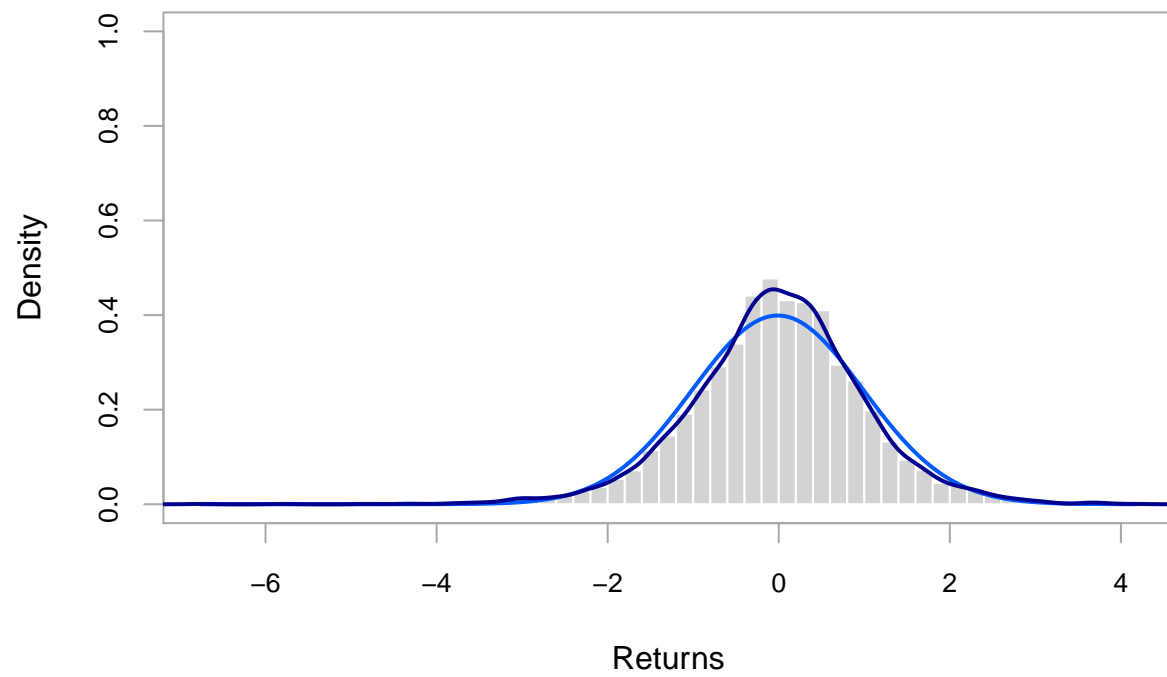
```
##   TES for Normal GARCH   TES for Normal GJR TES for Skewed t GARCH
##          0.007921520            -0.004295564           -0.020627739
##   TES for Skewed t GJR
##          -0.009151344
##            Bankmellon Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.03783187    0.02778325 0.02900754     0.03015433   0.02902535
## 2020-04-27 0.02854311    0.02765136 0.02887608     0.03013506   0.02900182
## 2020-04-28 0.02844841    0.02752156 0.02874631     0.03011593   0.02897846
## 2020-04-29 0.02750750    0.02739382 0.02861822     0.03009692   0.02895528
## 2020-04-30 0.02745840    0.02726812 0.02849180     0.03007805   0.02893227
## 2020-05-04 0.02653821    0.02714443 0.02836703     0.03005930   0.02890944
## 2020-05-05 0.02659099    0.02702272 0.02824389     0.03004069   0.02888678
## 2020-05-06 0.02600956    0.02690298 0.02812236     0.03002220   0.02886430
## 2020-05-07 0.02553621    0.02678517 0.02800243     0.03000384   0.02884198
## 2020-05-08 0.02559992    0.02666926 0.02788408     0.02998560   0.02881984
##
## -------------------------------
## Allianz
```
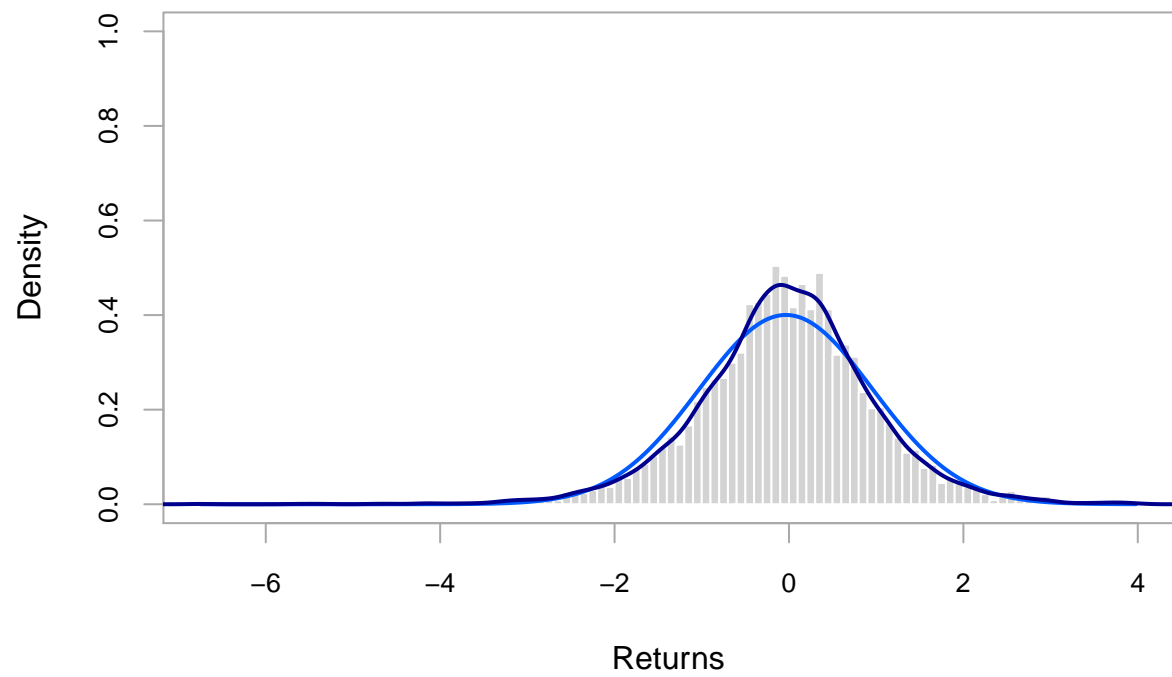
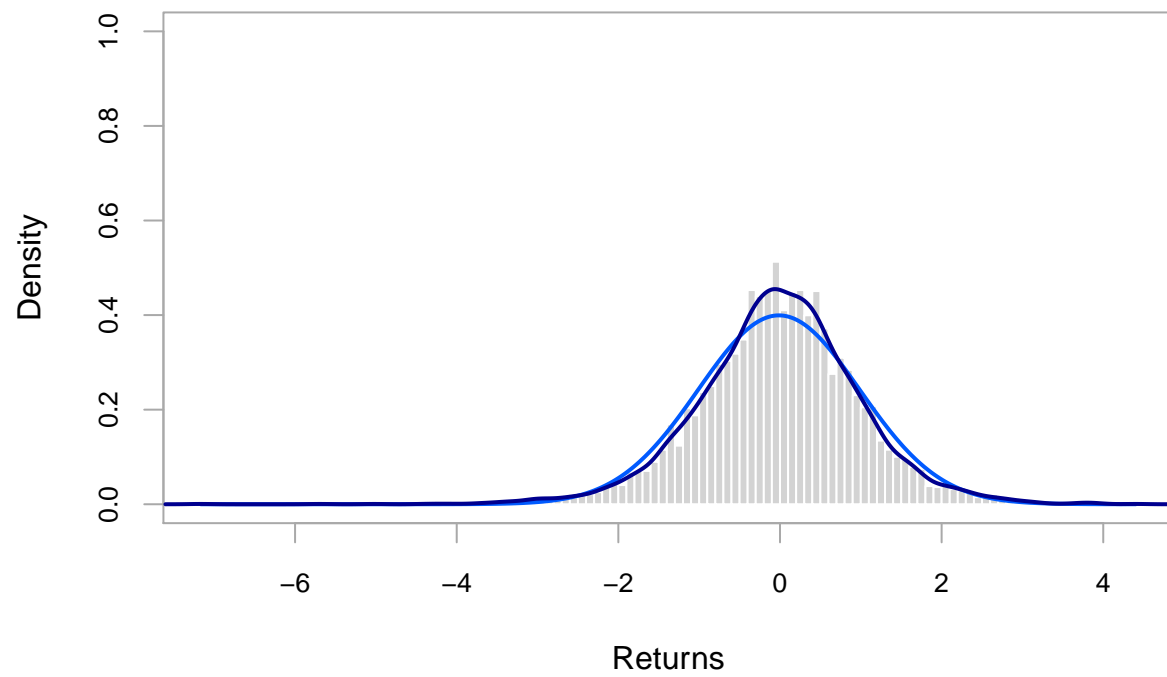**Standardized residuals of Standard GARCH with normal distribution of errors**

Standardized residuals of GJR GARCH with normal distribution of errors

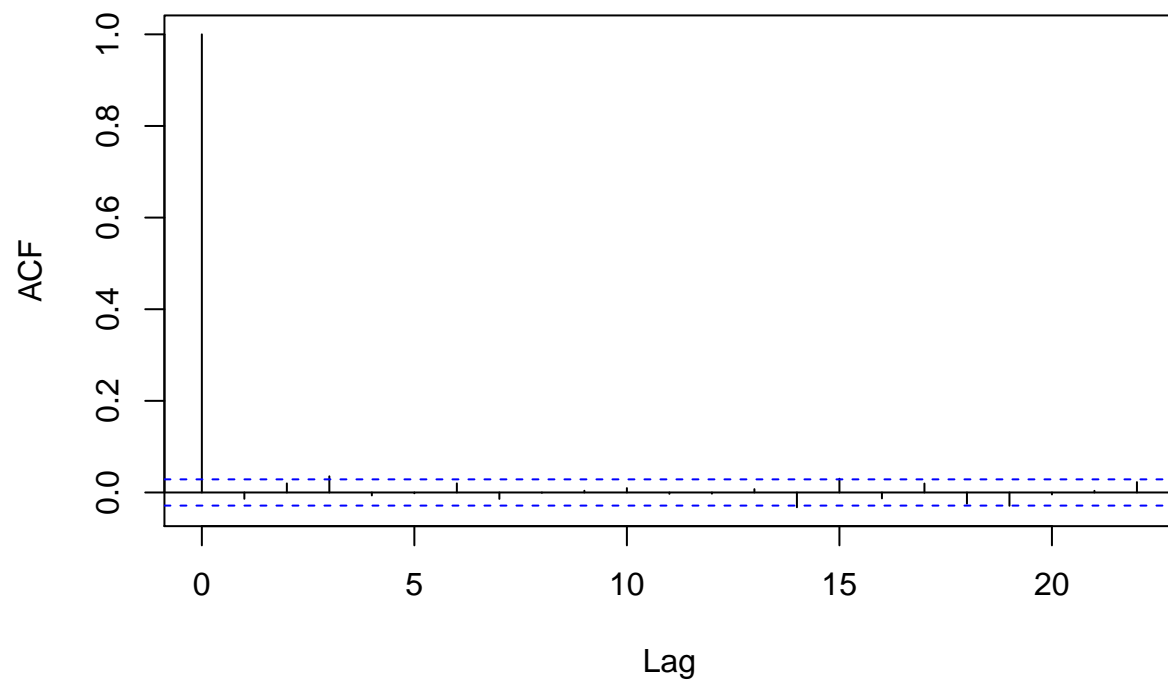**Standardized residuals of Standard GARCH with skewed Student t distribution of ε**

Standardized residuals of GJR GARCH with skewed Student t distribution of err

Standard GARCH with normal distribution of errors


```
## 
##   Standard GARCH with normal distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 33.345, df = 22, p-value = 0.05719
```
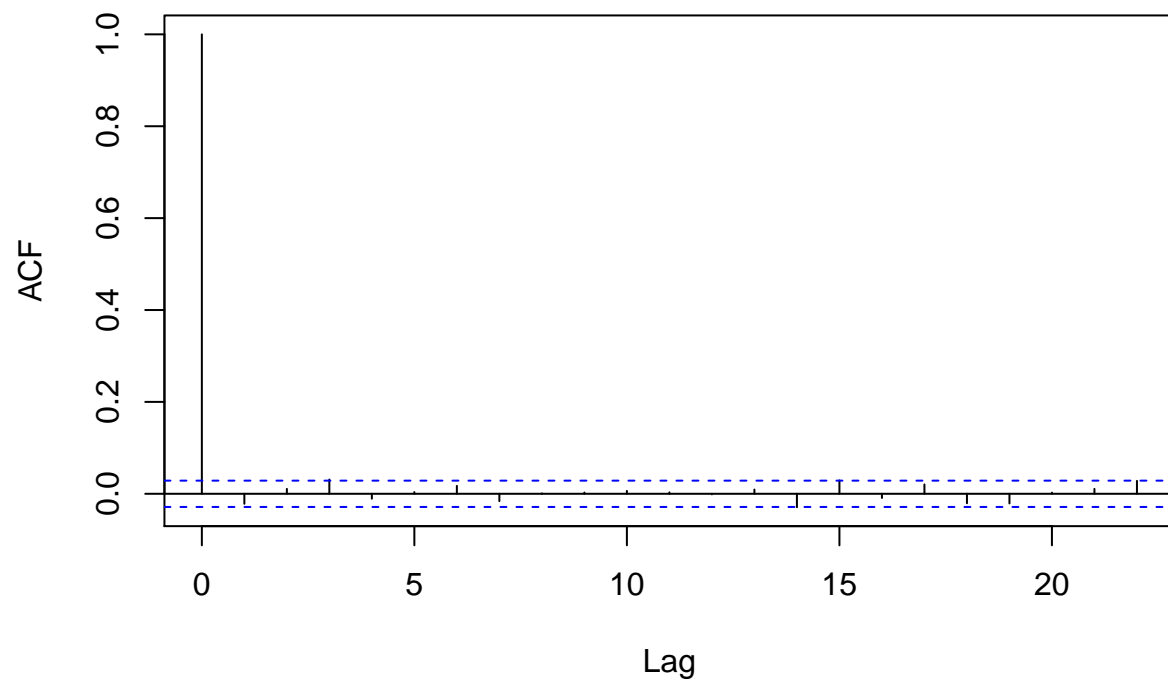
GJR GARCH with normal distribution of errors


```
## 
##  GJR GARCH with normal distribution of errors
## 
##  Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 29.9, df = 22, p-value = 0.1209
```

Standard GARCH with skewed Student t distribution of errors


```
## 
##   Standard GARCH with skewed Student t distribution of errors
## 
##   Box-Ljung test
## 
## data:  abs(standard_residuals)
## X-squared = 35.811, df = 22, p-value = 0.03181
```

GJR GARCH with skewed Student t distribution of errors


```
##
##  GJR GARCH with skewed Student t distribution of errors
##
##  Box-Ljung test
##
## data:  abs(standard_residuals)
## X-squared = 28.397, df = 22, p-value = 0.1629
##
##
## Coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value       Pr(>|t|)
## mu     0.0008124093 0.0001990795   4.080829 0.0000448753
## omega  0.0000046129 0.0000012937   3.565761 0.0003628013
## alpha1 0.1017953831 0.0086051056  11.829649 0.0000000000
## beta1  0.8873064262 0.0091330058  97.153823 0.0000000000
##
## Robust coefficients of Standard GARCH with normal distribution of errors
##            Estimate    Std. Error    t value       Pr(>|t|)
## mu     0.0008124093 0.0002056529   3.950390 0.0000780238
## omega  0.0000046129 0.0000038591   1.195337 0.2319554317
## alpha1 0.1017953831 0.0146803930   6.934105 0.0000000000
## beta1  0.8873064262 0.0185231852  47.902476 0.0000000000
##
## Coefficients of GJR GARCH with normal distribution of errors
##            Estimate    Std. Error    t value       Pr(>|t|)
## mu     0.0003517914 0.0001275091   2.758952 0.0057987095
```
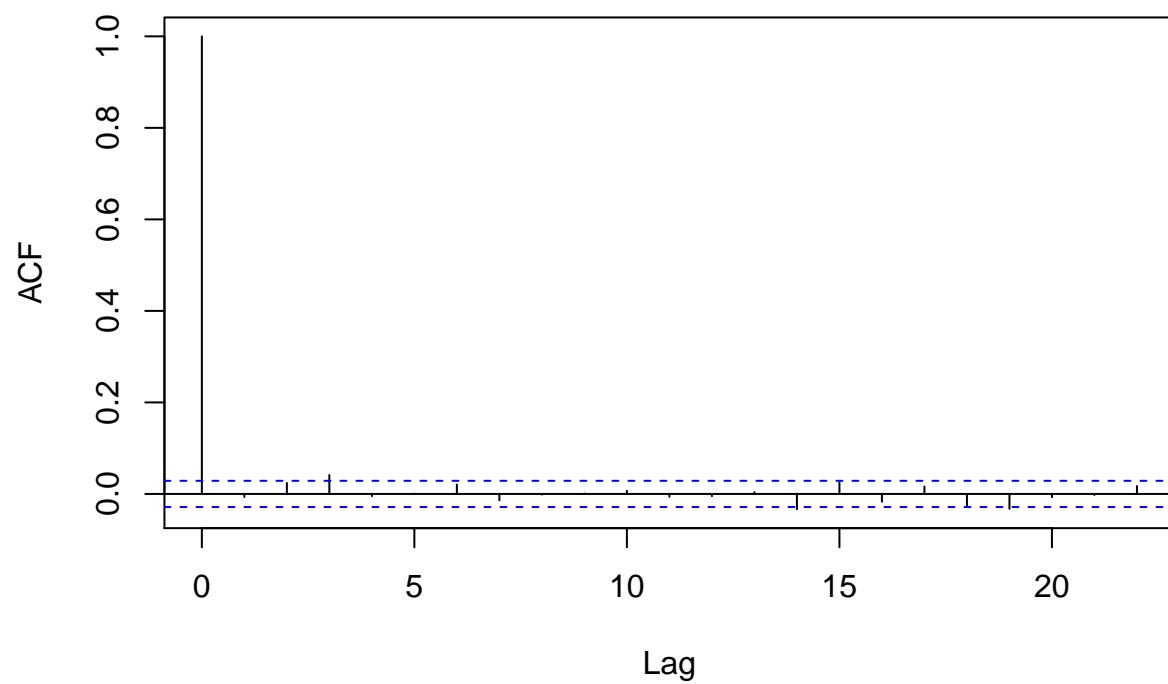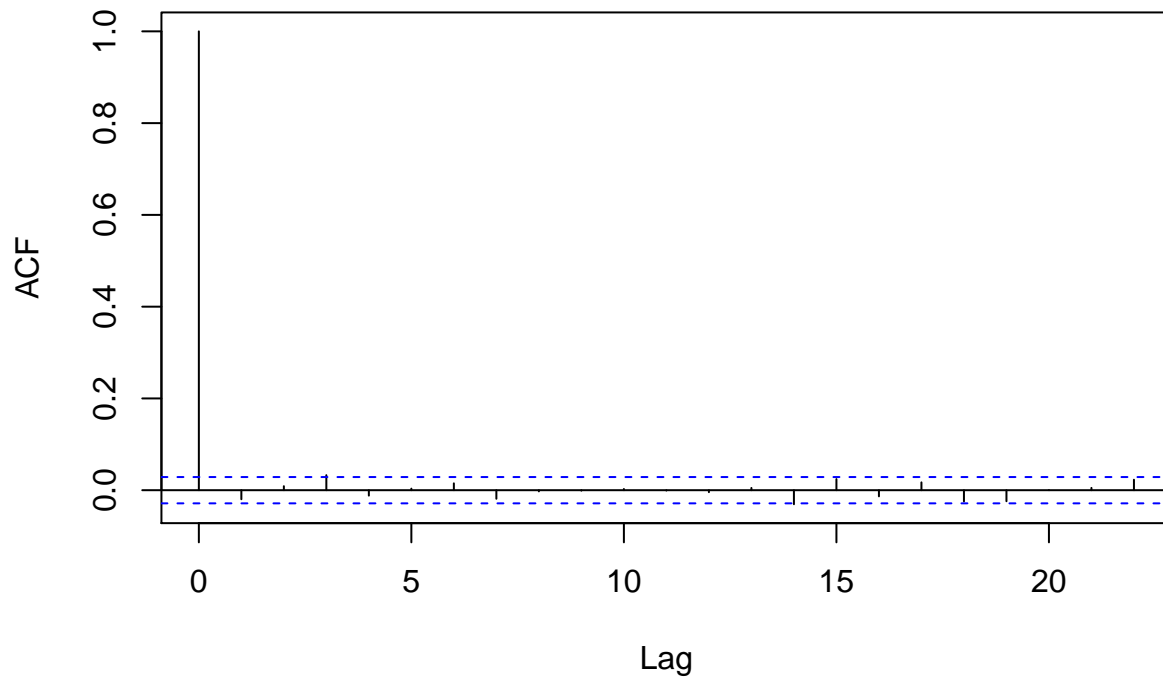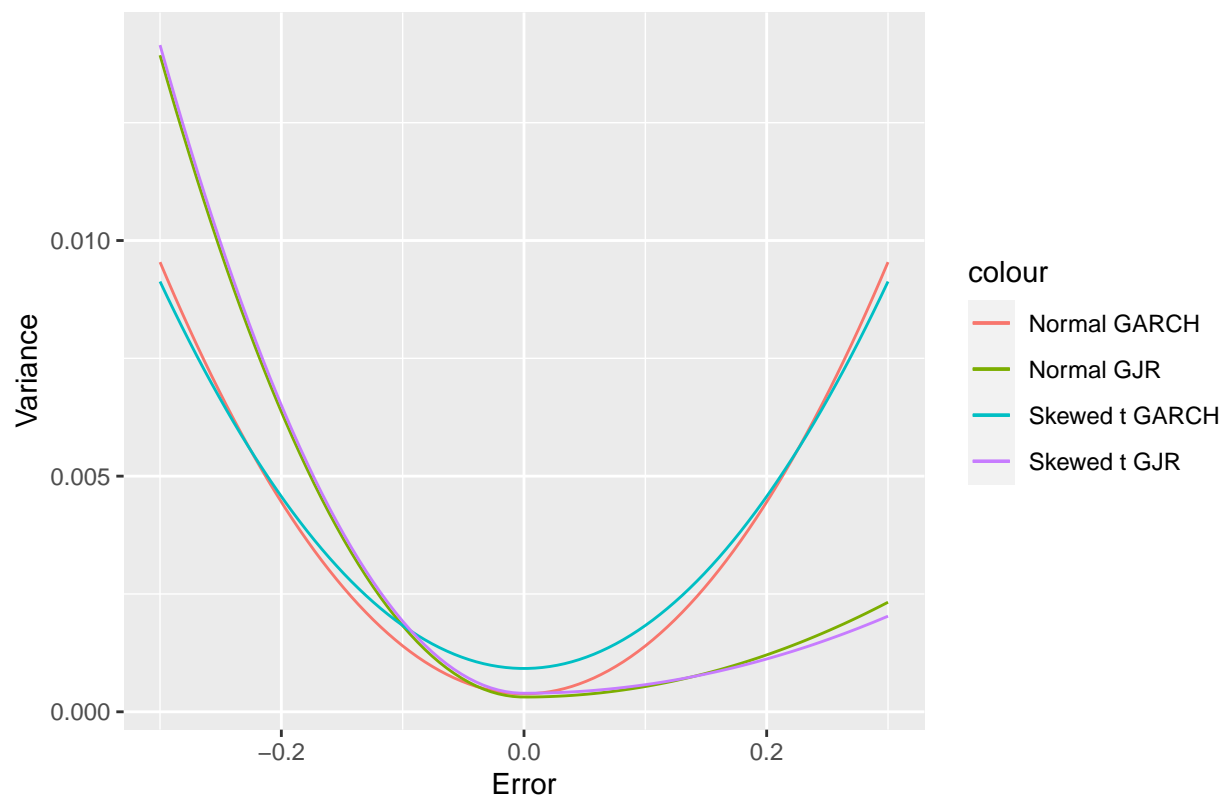
```
## omega  0.0000049747 0.0000012451    3.995440 0.0000645743
## alpha1 0.0223577799 0.0046269611    4.832066 0.0000013512
## beta1  0.8986410615 0.0064308270 139.739580 0.0000000000
## gamma1 0.1289652652 0.0077223953  16.700164 0.0000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0003517914 0.0005419566  0.6491137 0.516264874
## omega  0.0000049747 0.0000046637  1.0666868 0.286113299
## alpha1 0.0223577799 0.0288037894  0.7762097 0.437625197
## beta1  0.8986410615 0.0094469413 95.1250816 0.000000000
## gamma1 0.1289652652 0.0425442968  3.0313174 0.002434892
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0007065779 0.0001977419  3.5732339 0.0003525995
## omega  0.0000026154 0.0000028153  0.9290046 0.3528866962
## alpha1 0.0912238966 0.0243846588  3.7410364 0.0001832630
## beta1  0.9061908113 0.0241496180 37.5240225 0.0000000000
## skew   0.9761355956 0.0199434264 48.9452303 0.0000000000
## shape  6.3168681420 0.6568242403  9.6172884 0.0000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0007065779 0.0002195923  3.2176801 0.0012923185
## omega  0.0000026154 0.0000153230  0.1706859 0.8644707842
## alpha1 0.0912238966 0.1263401059  0.7220502 0.4702636246
## beta1  0.9061908113 0.1265759473  7.1592655 0.0000000000
## skew   0.9761355956 0.0247884939 39.3785761 0.0000000000
## shape  6.3168681420 1.8216935400  3.4675800 0.0005251673
##
## Coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value    Pr(>|t|)
## mu     0.0003761577 0.0001942831  1.936132 0.05285151
## omega  0.0000035397 0.0000014421  2.454487 0.01410859
## alpha1 0.0181718320 0.0080697143  2.251856 0.02433139
## beta1  0.9072260117 0.0111648560 81.257296 0.00000000
## gamma1 0.1346447072 0.0189601593  7.101454 0.00000000
## skew   0.9638760358 0.0196474135 49.058673 0.00000000
## shape  6.8956694821 0.6733474222 10.240879 0.00000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##             Estimate    Std. Error    t value     Pr(>|t|)
## mu     0.0003761577 0.0002159396  1.7419582 0.0815157534
## omega  0.0000035397 0.0000038409  0.9215795 0.3567479301
## alpha1 0.0181718320 0.0082775957  2.1953032 0.0281418571
## beta1  0.9072260117 0.0221791180 40.9045127 0.0000000000
## gamma1 0.1346447072 0.0357464263  3.7666620 0.0001654447
## skew   0.9638760358 0.0194336547 49.5982898 0.0000000000
## shape  6.8956694821 0.7436921555  9.2722095 0.0000000000
##               Normal GARCH   Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood    12618.436112 12677.873610   12722.374662 12769.530904
## Akaike           -5.390785    -5.415758      -5.434348    -5.454073
## Bayes            -5.385271    -5.408866      -5.426078    -5.444424
```
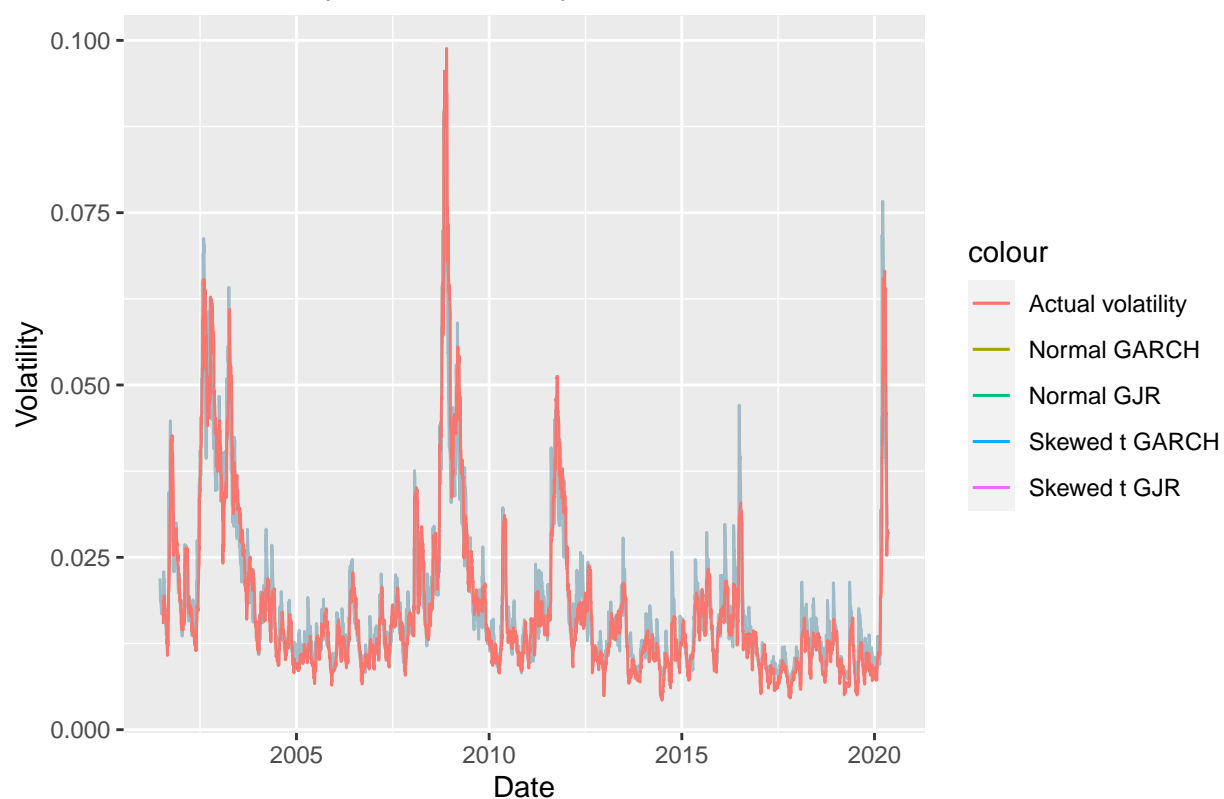
```
## Shibata          -5.390786    -5.415760      -5.434351    -5.454078
## Hannan-Quinn      -5.388846    -5.413334      -5.431440    -5.450680
```

Dependence of variance on errors in different models

Volatility constructed by different models


```
##    TES for Normal GARCH      TES for Normal GJR TES for Skewed t GARCH
##           -0.029026922               0.005354399            -0.059270729
##    TES for Skewed t GJR
##           -0.013365337
##               Allianz Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-24 0.04147483   0.03246860 0.02911545     0.03506125   0.03074048
## 2020-04-27 0.02527030   0.03236254 0.02898925     0.03505322   0.03067146
## 2020-04-28 0.02640315   0.03225729 0.02886434     0.03504522   0.03060286
## 2020-04-29 0.02742532   0.03215285 0.02874072     0.03503723   0.03053466
## 2020-04-30 0.02765949   0.03204922 0.02861836     0.03502926   0.03046688
## 2020-05-04 0.02892837   0.03194638 0.02849727     0.03502131   0.03039951
## 2020-05-05 0.02846341   0.03184433 0.02837744     0.03501338   0.03033254
## 2020-05-06 0.02818765   0.03174308 0.02825884     0.03500547   0.03026598
## 2020-05-07 0.02869809   0.03164261 0.02814148     0.03499758   0.03019982
## 2020-05-08 0.02847229   0.03154292 0.02802534     0.03498970   0.03013406
##
## --------------------------------
```