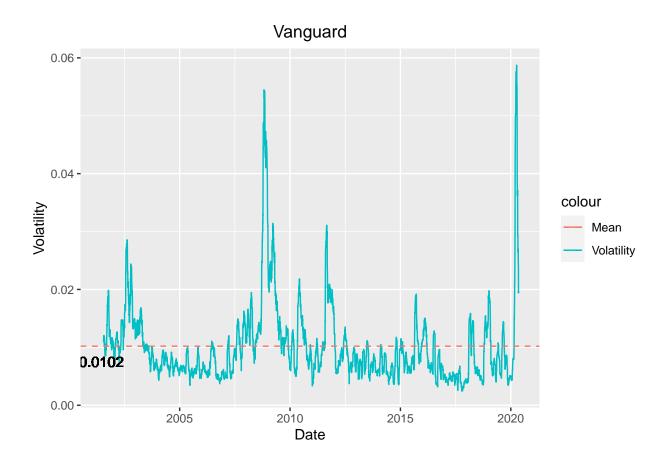
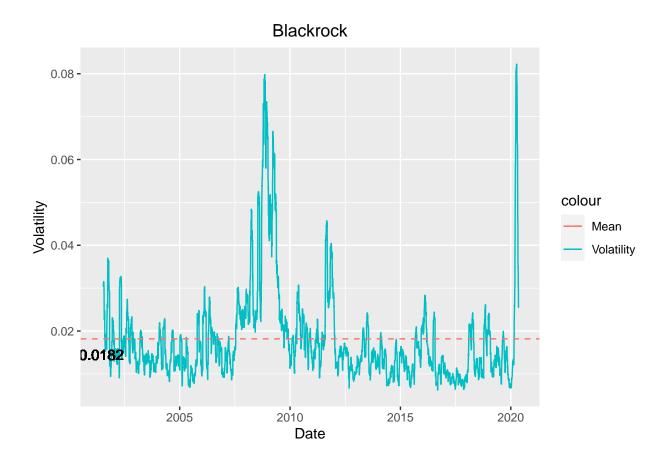
R. Notebook

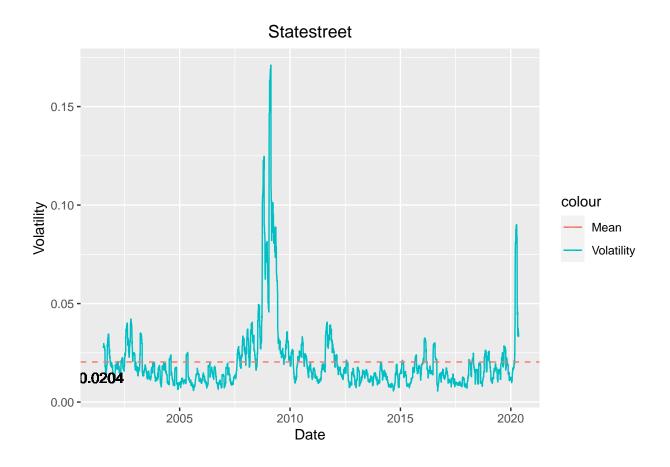
```
require(dplyr)
## Loading required package: dplyr
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##
       filter, lag
## The following objects are masked from 'package:base':
##
##
       intersect, setdiff, setequal, union
require(ggplot2)
## Loading required package: ggplot2
require(xts)
## Loading required package: xts
## Loading required package: zoo
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
       as.Date, as.Date.numeric
##
## Attaching package: 'xts'
## The following objects are masked from 'package:dplyr':
##
##
       first, last
require(rugarch)
## Loading required package: rugarch
## Loading required package: parallel
##
## Attaching package: 'rugarch'
## The following object is masked from 'package:stats':
##
##
       sigma
require(PerformanceAnalytics)
## Loading required package: PerformanceAnalytics
```

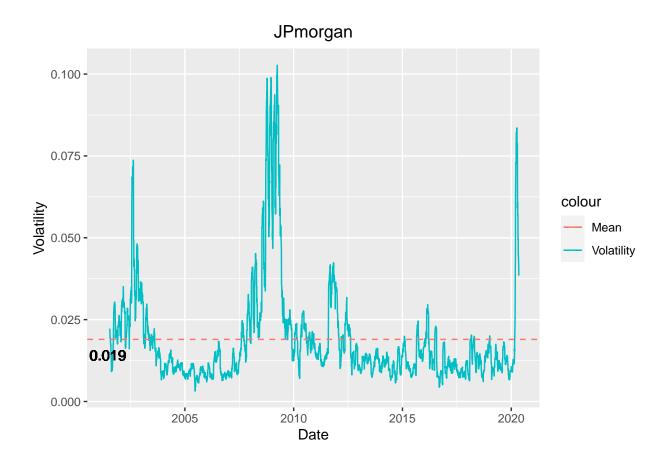
```
##
## Attaching package: 'PerformanceAnalytics'
## The following object is masked from 'package:graphics':
##
       legend
require(quantmod)
## Loading required package: quantmod
## Loading required package: TTR
## Registered S3 method overwritten by 'quantmod':
##
     method
                         from
     as.zoo.data.frame zoo
## Version 0.4-0 included new data defaults. See ?getSymbols.
library(skewt)
read_funds <- function(lf) {</pre>
 dfs <- list()</pre>
browser()
  for (f in 1:length(lf)) {
      cond <- substr(lf[f], nchar(lf[f])-3, nchar(lf[f])) == '.csv'</pre>
      if (cond) {
      temp <- data.frame(read.csv(lf[f], stringsAsFactors = FALSE))</pre>
      temp$Date <- as.Date(temp$Date)</pre>
      temp$Close <- as.numeric(temp$Close)</pre>
      dfs[[f]] \leftarrow temp[,c(1,5)]
  }
 return(dfs)
}
funds_names <- c("Vanguard", "Blackrock", "Statestreet",</pre>
                       "JPmorgan", "Bankmellon", "Allianz")
# Data loading
df <- read.csv('Funds.csv', stringsAsFactors = F)</pre>
df < - df[,-1]
df$Date <- as.Date(df$Date)</pre>
# Transforming data from df to xts
funds <- xts(df[,2:ncol(df)], order.by = df$Date)</pre>
rm(df)
# Subseting original data for April of 2020 year
funds red <- funds['/202004']</pre>
get_volatiles <- function(funds, width = 22, time_scale = 1, funds_names) {</pre>
    vol_df <- data.frame()</pre>
    for (i in 1:ncol(funds)) {
      fund <- funds[,i]</pre>
```

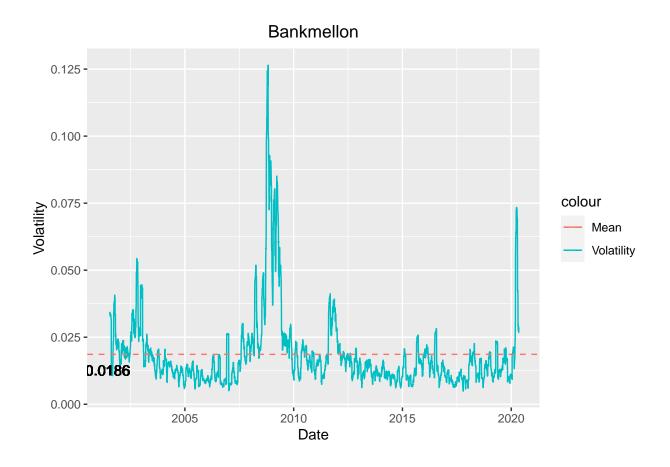
```
temp <- rollapply(data = fund, width = 22, FUN = 'sd.annualized', scale = time_scale)
      if (nrow(vol_df) == 0) {
        vol_df <- temp</pre>
      } else {
        vol_df <- cbind(vol_df, temp)</pre>
    }
    names(vol_df) <- funds_names</pre>
    return(vol_df)
}
visualize_funds_lines <- function(funds, y_axis_label = 'Volatility') {</pre>
  for (i in 1:ncol(funds)) {
    temp <- funds[,i]</pre>
    temp_mean <- mean(temp, na.rm = TRUE)</pre>
    p <- ggplot(temp, aes(x = index(temp), y = temp)) +</pre>
    geom_line(aes(color = 'Volatility')) +
    geom_hline(aes(yintercept = temp_mean, color = 'Mean'),
               size=.5, linetype='dashed') +
    geom_text( aes( min(index(temp)) , temp_mean, label = round(temp_mean, 4), vjust = 2)) +
    labs(x = 'Date', y = y_axis_label, title = names(funds)[i]) +
    theme(plot.title = element_text(hjust = 0.5))
    suppressMessages(suppressWarnings(print(p)))
 }
}
visualize_funds_hist <- function(funds, x_axis_label = 'Return') {</pre>
  for (i in 1:ncol(funds)) {
      temp <- funds[,i]</pre>
      title <- paste(funds_names[i], 'returns')</pre>
      chart.Histogram(temp,
                 methods = c('add.normal', 'add.density'),
                 main = title)
      temp <- (temp - mean(temp, na.rm = T))/sd(temp, na.rm = T)
      title <- paste('Standardized', title)</pre>
      chart.Histogram(temp,
                       methods = c('add.normal', 'add.density'),
                       main = title)
 }
vol_df <- get_volatiles(funds, funds_names = funds_names)</pre>
visualize funds lines(vol df)
```

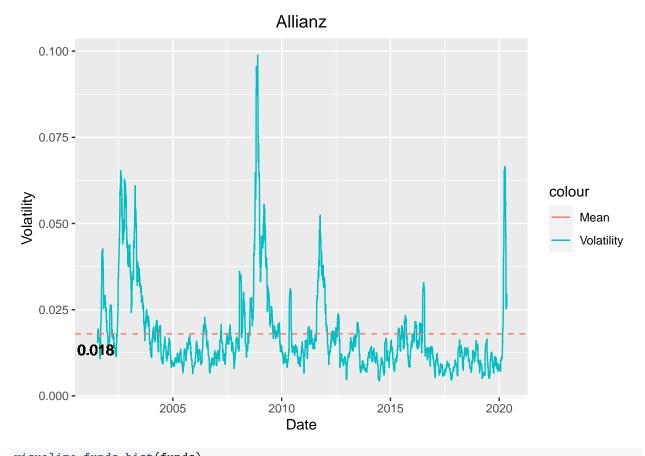






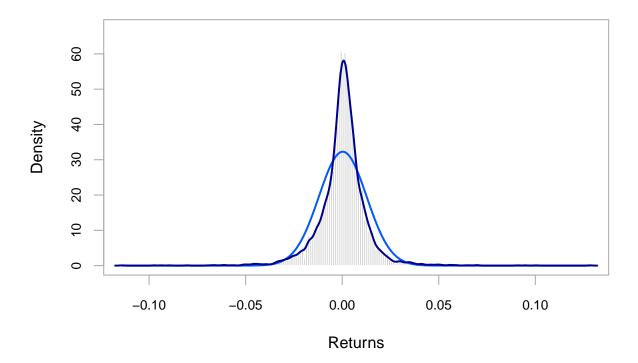




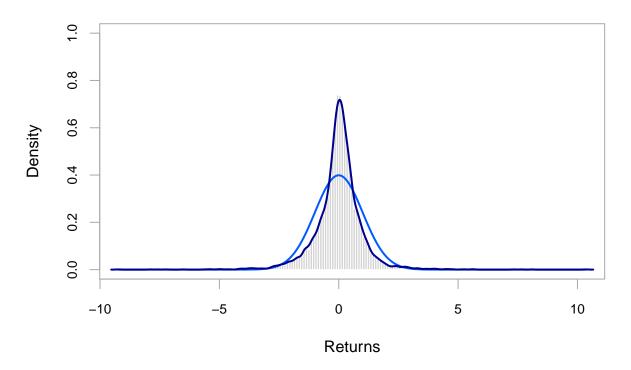


visualize_funds_hist(funds)

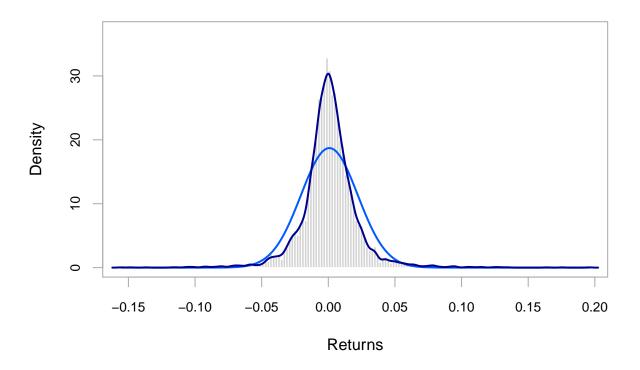
Vanguard returns



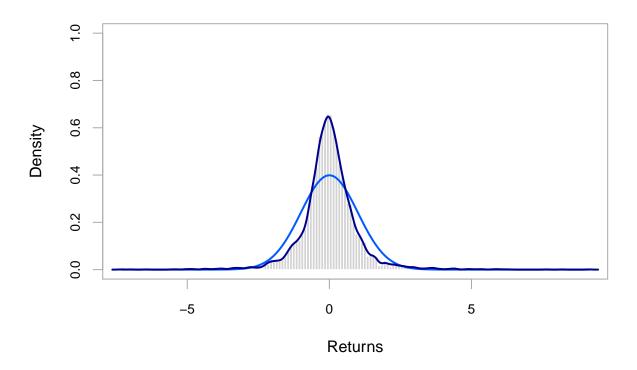
Standardized Vanguard returns



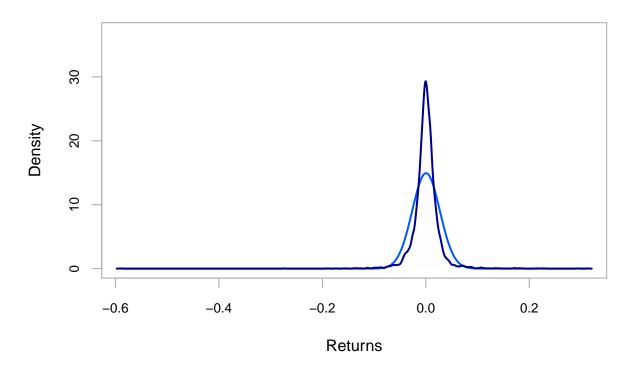
Blackrock returns



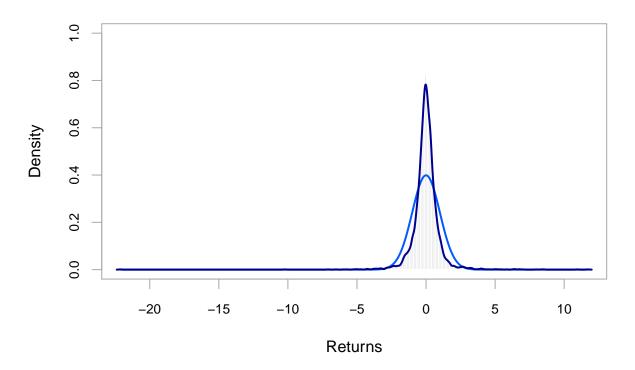
Standardized Blackrock returns



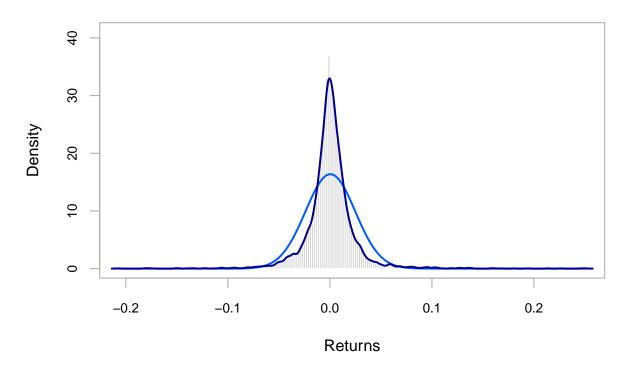
Statestreet returns



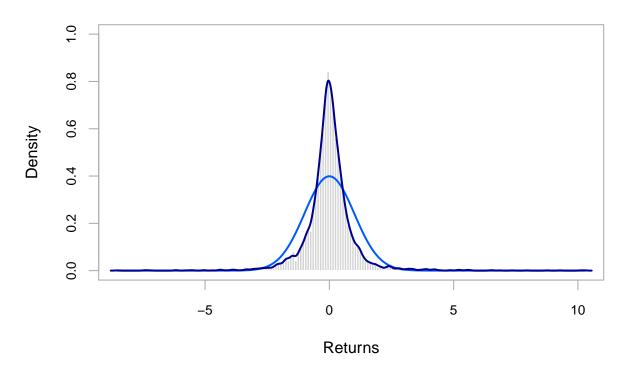
Standardized Statestreet returns



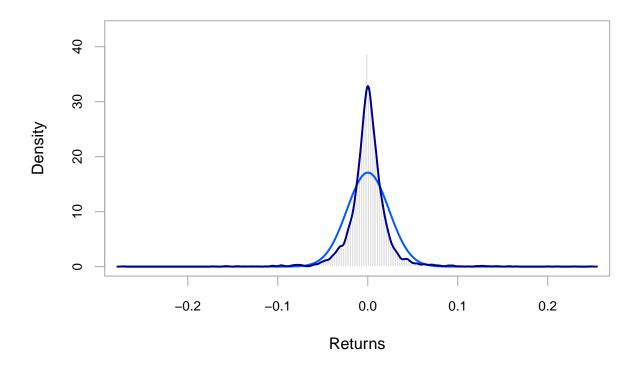
JPmorgan returns



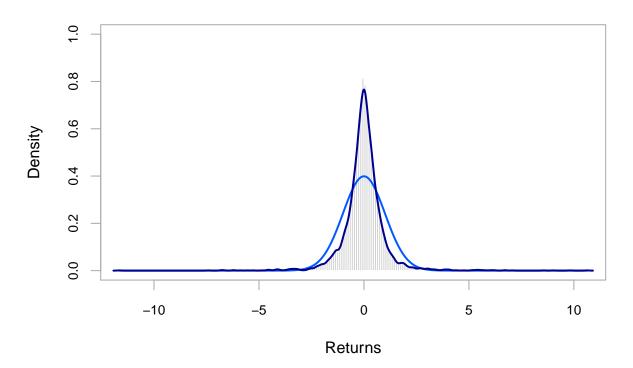
Standardized JPmorgan returns



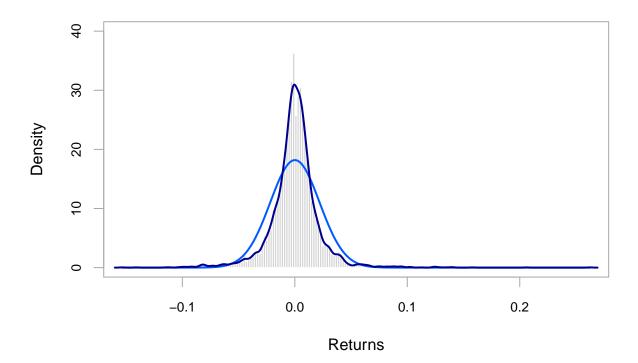
Bankmellon returns



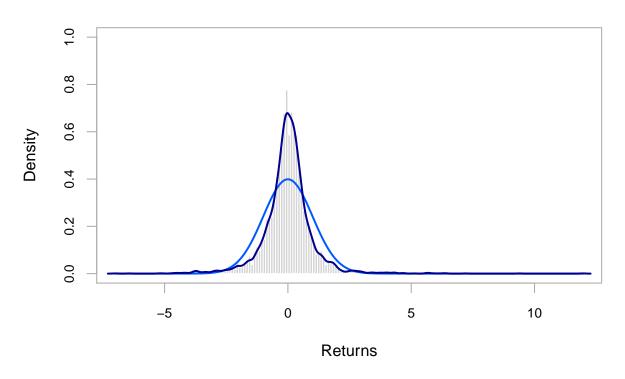
Standardized Bankmellon returns



Allianz returns



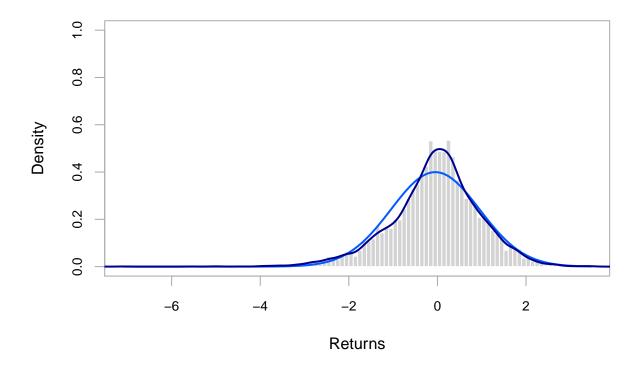
Standardized Allianz returns



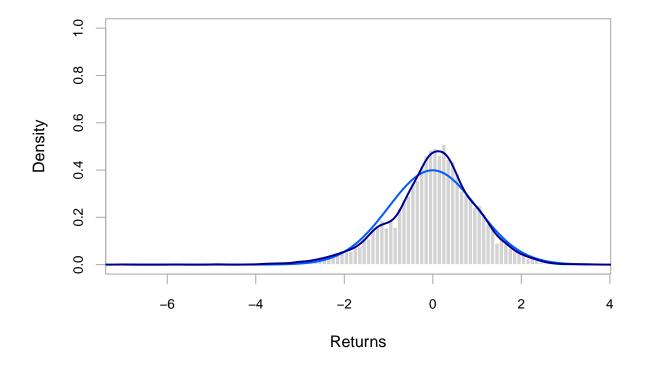
```
fund_tail_volatility <- tail(vol_df$Vanguard, 10)</pre>
plot_acf <- function(fund) {</pre>
  f_mean <- mean(fund)</pre>
  acf(abs(f mean))
}
fund <- funds$vanguard_return</pre>
# Forming set of parameters for different GARCH model
# The most important is distribution.model - we are specifying type of distribution
# Standard GARCH with normal distribution of errors
norm_garch_spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),</pre>
                        variance.model = list(model = 'sGARCH'),
                        distribution.model = 'norm')
# GJR GARCH with normal distribution of errors
norm_gjr_spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),</pre>
                        variance.model = list(model = 'gjrGARCH'),
                        distribution.model = 'norm')
\# Standard GARCH with skewed Student t distribution of errors
sstd_garch_spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),</pre>
                        variance.model = list(model = 'sGARCH'),
                        distribution.model = 'sstd')
```

```
# GJR GARCH with skewed Student t distribution of errors
sstd_gjr_spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),</pre>
                        variance.model = list(model = 'gjrGARCH'),
                        distribution.model = 'sstd')
garch_specs <- list(norm_garch_spec, norm_gjr_spec,</pre>
                      sstd_garch_spec, sstd_gjr_spec)
rm(norm_garch_spec, norm_gjr_spec, sstd_garch_spec, sstd_gjr_spec)
# Models naming
model_names <- c('Standard GARCH with normal distribution of errors',</pre>
                          'GJR GARCH with normal distribution of errors',
                          'Standard GARCH with skewed Student t distribution of errors',
                          'GJR GARCH with skewed Student t distribution of errors')
short_model_names <- c('Normal GARCH', 'Normal GJR', 'Skewed t GARCH', 'Skewed t GJR')</pre>
names(garch specs) <- model names</pre>
# Apply GARCH model to our data
garch_fits <- list()</pre>
for (s in 1:length(garch_specs)) {
  suppressWarnings(garch_fits[[s]] <- ugarchfit(data = fund,</pre>
                                                spec = garch_specs[[s]]))
}
rm(s)
names(garch_fits) <- model_names</pre>
# Visualizing standardized residuals for models
for (f in 1:length(garch_fits)) {
  chart.Histogram(residuals(garch_fits[[f]], standardize = T),
                methods = c('add.normal', 'add.density'),
                main = paste('Standardized residuals of', model_names[f]))
}
```

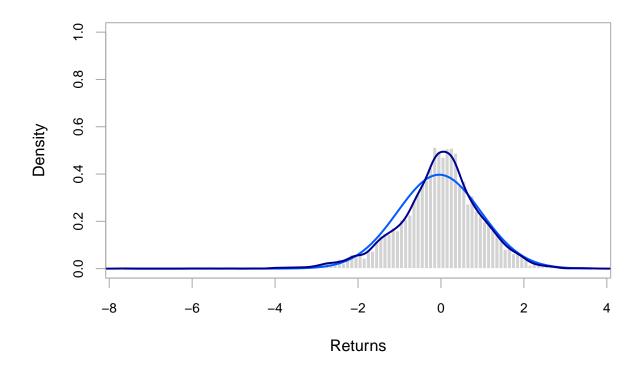
Standardized residuals of Standard GARCH with normal distribution of errors



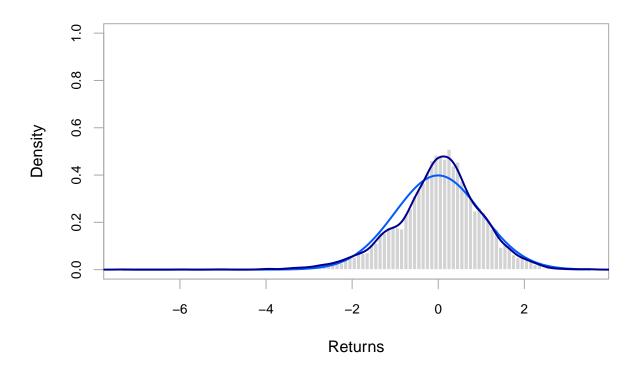
Standardized residuals of GJR GARCH with normal distribution of errors



Standardized residuals of Standard GARCH with skewed Student t distribution of $\boldsymbol{\varepsilon}$

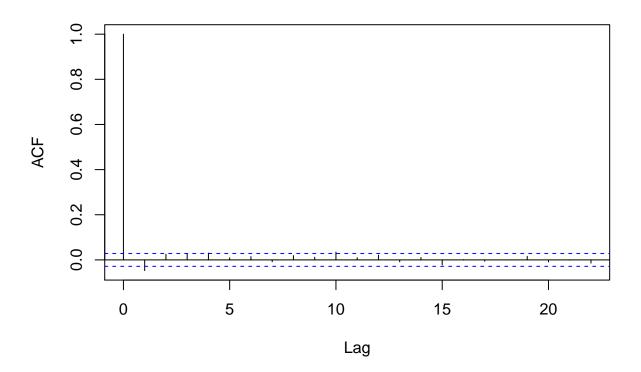


Standardized residuals of GJR GARCH with skewed Student t distribution of erro



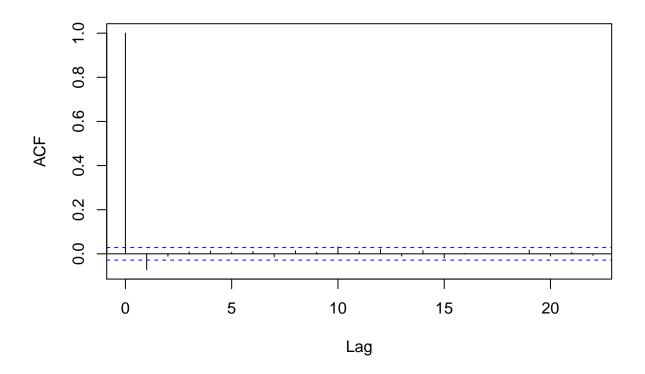
```
# Models validation
for (f in 1:length(garch_fits)) {
    standard_residuals <- residuals(garch_fits[[f]], standardize = T)
    p <- acf(abs(standard_residuals),22, plot = F)
    plot(p, main = names(garch_fits)[f])
    cat('\n', names(garch_fits)[f],'\n')
    print(Box.test(abs(standard_residuals), 22, type = 'Ljung-Box'))
}</pre>
```

Standard GARCH with normal distribution of errors



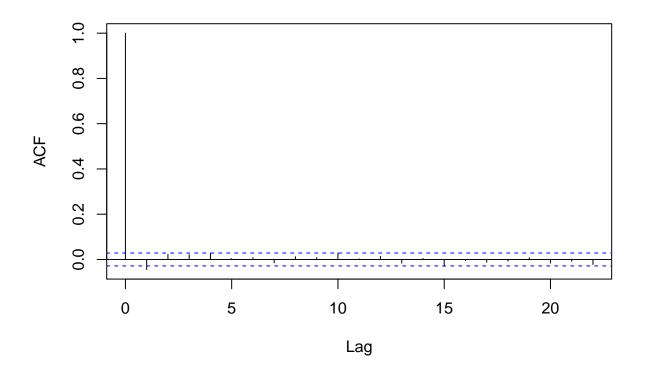
```
##
## Standard GARCH with normal distribution of errors
##
## Box-Ljung test
##
## data: abs(standard_residuals)
## X-squared = 39.552, df = 22, p-value = 0.01219
```

GJR GARCH with normal distribution of errors



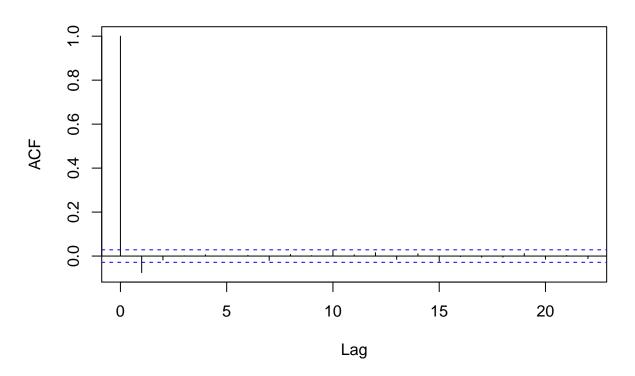
```
##
## GJR GARCH with normal distribution of errors
##
## Box-Ljung test
##
## data: abs(standard_residuals)
## X-squared = 41.362, df = 22, p-value = 0.00746
```

Standard GARCH with skewed Student t distribution of errors



```
##
## Standard GARCH with skewed Student t distribution of errors
##
## Box-Ljung test
##
## data: abs(standard_residuals)
## X-squared = 36.032, df = 22, p-value = 0.03013
```

GJR GARCH with skewed Student t distribution of errors

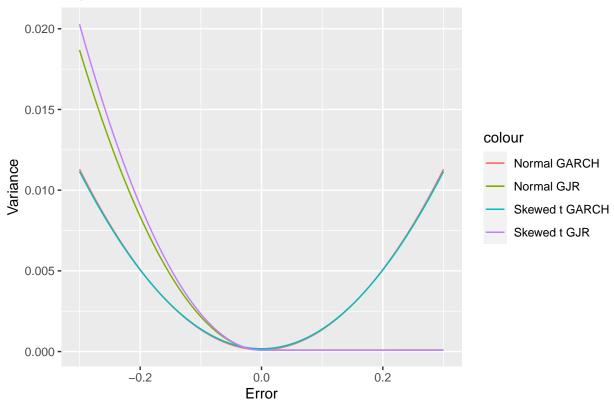


```
##
    GJR GARCH with skewed Student t distribution of errors
##
##
##
   Box-Ljung test
##
## data: abs(standard_residuals)
## X-squared = 42.602, df = 22, p-value = 0.00528
rm(standard_residuals, p)
#Coefficients
for (f in 1:length(garch_fits)) {
  cat('\nCoefficients of', names(garch_fits)[f], '\n')
  print(round(garch_fits[[f]]@fit$matcoef,10))
  cat('\nRobust coefficients of', names(garch_fits)[f], '\n')
  print(round(garch_fits[[f]]@fit$robust.matcoef,10))
}
##
## Coefficients of Standard GARCH with normal distribution of errors
              Estimate
##
                         Std. Error
                                      t value
## mu
          0.0006678893 0.0001133821 5.890605 0.0000000038
## omega 0.0000025218 0.0000007821 3.224299 0.0012628155
## alpha1 0.1243765581 0.0097966632 12.695808 0.0000000000
## beta1 0.8552415698 0.0105548476 81.028320 0.0000000000
##
```

```
## Robust coefficients of Standard GARCH with normal distribution of errors
##
                         Std. Error
                                      t value
              Estimate
                                                   Pr(>|t|)
         0.0006678893 0.0000947508 7.0489065 0.0000000000
## mu
## omega 0.0000025218 0.0000042624 0.5916353 0.5540948315
## alpha1 0.1243765581 0.0228482046 5.4436031 0.0000000522
## beta1 0.8552415698 0.0383271265 22.3142627 0.0000000000
## Coefficients of GJR GARCH with normal distribution of errors
##
              Estimate
                        Std. Error
                                        t value
                                                    Pr(>|t|)
          0.0002606009 0.0000979826 2.659665e+00 0.007821837
## mu
## omega 0.0000024778 0.0000002397 1.033524e+01 0.000000000
## alpha1 0.0000209156 0.0033010953 6.335971e-03 0.994944661
## beta1 0.8729056647 0.0066451863 1.313591e+02 0.000000000
## gamma1 0.2065843904 0.0129819001 1.591326e+01 0.000000000
##
## Robust coefficients of GJR GARCH with normal distribution of errors
##
                        Std. Error
                                        t value
              Estimate
                                                     Pr(>|t|)
## mu
          0.0002606009 0.0001316477 1.979533e+00 0.0477560492
## omega 0.0000024778 0.0000005923 4.183009e+00 0.0000287676
## alpha1 0.0000209156 0.0140679940 1.486754e-03 0.9988137426
## beta1 0.8729056647 0.0072181000 1.209329e+02 0.0000000000
## gamma1 0.2065843904 0.0314439994 6.569915e+00 0.0000000001
##
## Coefficients of Standard GARCH with skewed Student t distribution of errors
##
             Estimate Std. Error
                                    t value
                                                  Pr(>|t|)
          0.0006064283 0.000111164 5.4552561 0.0000000489
## omega 0.0000015080 0.000001644 0.9172266 0.3590239141
## alpha1 0.1220516502 0.027618551 4.4191910 0.0000099071
## beta1 0.8703537793 0.026023588 33.4448033 0.0000000000
## skew
          0.8881626235 0.018172634 48.8736316 0.0000000000
## shape 7.1207987301 1.039608073 6.8495031 0.0000000000
##
## Robust coefficients of Standard GARCH with skewed Student t distribution of errors
                        Std. Error
                                      t value
                                                  Pr(>|t|)
             Estimate
         0.0006064283 0.0001297724 4.6730161 0.0000029681
## omega 0.0000015080 0.0000119600 0.1260838 0.8996656146
## alpha1 0.1220516502 0.1884915577 0.6475179 0.5172968387
## beta1 0.8703537793 0.1798182388 4.8401863 0.0000012972
## skew
          0.8881626235 0.0435211859 20.4075924 0.0000000000
## shape 7.1207987301 5.2846067667 1.3474605 0.1778319757
## Coefficients of GJR GARCH with skewed Student t distribution of errors
                         Std. Error
              Estimate
                                        t value
                                                  Pr(>|t|)
         0.0002498529 0.0001176175 2.124284e+00 0.03364643
## mu
## omega 0.0000020128 0.0000010605 1.897992e+00 0.05769708
## alpha1 0.0000000821 0.0127379255 6.447100e-06 0.99999486
## beta1 0.8732435063 0.0108249601 8.066944e+01 0.00000000
## gamma1 0.2244263082 0.0306579443 7.320331e+00 0.00000000
         0.8542197386 0.0168737583 5.062415e+01 0.00000000
## shape 8.2660071635 0.9890413211 8.357595e+00 0.00000000
##
## Robust coefficients of GJR GARCH with skewed Student t distribution of errors
##
             Estimate
                        Std. Error
                                        t value
                                                  Pr(>|t|)
         0.0002498529 0.0002844967 8.782277e-01 0.37982015
## mu
```

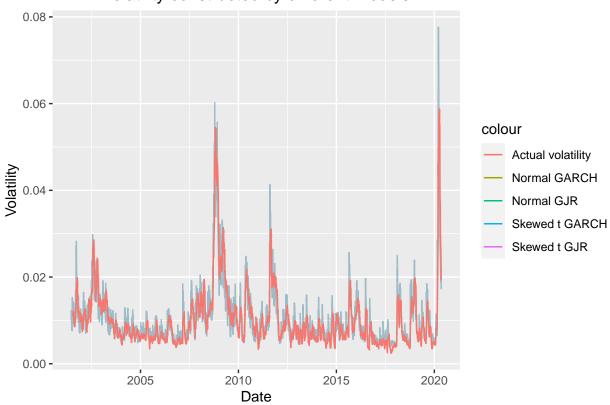
```
## omega 0.0000020128 0.0000055599 3.620113e-01 0.71734359
## alpha1 0.0000000821 0.0442751785 1.854800e-06 0.99999852
## beta1 0.8732435063 0.0324759355 2.688894e+01 0.00000000
## gamma1 0.2244263082 0.1250738277 1.794351e+00 0.07275721
         0.8542197386 0.0165412469 5.164180e+01 0.00000000
## shape 8.2660071635 1.0141193171 8.150922e+00 0.00000000
# Models comparing
model_comparison <- data.frame()</pre>
for (f in 1:length(garch_fits)) {
    temp <- data.frame()</pre>
    temp[1,1] <- likelihood(garch_fits[[f]])</pre>
    inf_criterion <- infocriteria(garch_fits[[f]])</pre>
    temp <- rbind(temp, inf_criterion)</pre>
    model_comparison <- c(model_comparison, temp)</pre>
}
model_comparison <- as.data.frame(model_comparison)</pre>
rownames(model_comparison) <- c('Likelihood',rownames(inf_criterion))</pre>
colnames(model_comparison) <- short_model_names</pre>
model_comparison
##
                Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## Likelihood 15464.113912 15571.523249 15577.701243 15677.651493
## Akaike
                  -6.505413 -6.550189
                                               -6.552367 -6.594004
## Bayes
                  -6.499971 -6.543386
                                               -6.544204 -6.584481
## Shibata
                  -6.505414 -6.550191
                                               -6.552371
                                                             -6.594009
## Hannan-Quinn
                   -6.503501
                                -6.547798
                                                -6.549499
                                                              -6.590658
rm(f, temp, inf_criterion, model_comparison)
# Visualizing impact of negative previous return on variance
p <- ggplot()</pre>
for (f in 1:length(garch_fits)) {
  garch_news <- as.data.frame(newsimpact(garch_fits[[f]])[1:2])</pre>
 model name <- short model names[f]</pre>
 model_name <- enquo(model_name)</pre>
 p <- p + geom_line(data = garch_news,</pre>
            aes(x = zx, y = zy, color = !!model_name))
}
p <- p + labs(x = 'Error', y = 'Variance',</pre>
       title = 'Dependence of variance on errors in different models') +
  theme(plot.title = element_text(hjust = 0.5))
print(p)
```

Dependence of variance on errors in different models



```
rm(p, model_name, garch_news)
# norm_garch_news <- as.data.frame(newsimpact(norm_garch_fit)[1:2])</pre>
# norm_gjr_news <- as.data.frame(newsimpact(norm_gjr_fit) [1:2])</pre>
# sstd_garch_news <- as.data.frame(newsimpact(sstd_garch_fit)[1:2])</pre>
# sstd_gjr_news <- as.data.frame(newsimpact(sstd_gjr_fit)[1:2])</pre>
# ggplot() +
   geom_point(data = norm_garch_news,
#
                aes(x = zx, y = zy, color = 'Normal GARCH')) +
#
   geom_line(data = norm_gjr_news,
#
               aes(x = zx, y = zy, color = 'Normal GJR')) +
#
   geom_line(data = sstd_garch_news,
#
               aes(x = zx, y = zy, color = 'Skewed t GARCH')) +
#
    geom_line(data = sstd_gjr_news,
               aes(x = zx, y = zy, color = 'Skewed t GJR')) +
# Visualizing volatility
p <- ggplot()</pre>
garch_vol <- list()</pre>
for (f in 1:length(garch_fits)) {
  garch_vol[[f]] <- sigma(garch_fits[[f]])</pre>
  model_name <- short_model_names[f]</pre>
  model_name <- enquo(model_name)</pre>
```

Volatility constructed by different models



```
names(garch_sst)[f] <- paste('TES for', short_model_names[f])</pre>
}
names(predict_results)[2:ncol(predict_results)] <- short_model_names</pre>
# Total error sum for models
garch_sst
##
     TES for Normal GARCH
                              TES for Normal GJR TES for Skewed t GARCH
##
               0.08050812
                                      0.09724891
                                                             0.06188808
##
     TES for Skewed t GJR
##
               0.09188936
#Comparing predicted volatility for models with actual one
predict_results
##
                Vanguard Normal GARCH Normal GJR Skewed t GARCH Skewed t GJR
## 2020-04-27 0.02937419
                          0.01723897 0.01557547
                                                     0.01879428
                                                                  0.01610753
                          0.01713613 0.01546945
## 2020-04-28 0.02737708
                                                     0.01876301
                                                                  0.01600320
## 2020-04-29 0.02656118 0.01703477 0.01536525
                                                     0.01873192 0.01590036
## 2020-04-30 0.02637500 0.01693490 0.01526283
                                                     0.01870101
                                                                  0.01579900
## 2020-05-01 0.02699270 0.01683649 0.01516219
                                                     0.01867029 0.01569910
## 2020-05-04 0.02449756 0.01673952 0.01506329
                                                     0.01863975 0.01560065
## 2020-05-05 0.02430946 0.01664398 0.01496611
                                                    0.01860940 0.01550362
## 2020-05-06 0.02389937 0.01654985 0.01487063
                                                   0.01857922 0.01540801
## 2020-05-07 0.01941791 0.01645712 0.01477682
                                                    0.01854923 0.01531380
## 2020-05-08 0.01964116 0.01636577 0.01468467
                                                     0.01851942 0.01522096
# garch_for_funds <- function(funds) {</pre>
   for (i in 1:ncol(funds)) {
#
      return()
#
    7
# }
# vanguard_std <- (funds$vanguard_return - mean(funds$vanguard_return))/sd(funds$vanguard_return)</pre>
\# ggplot(data = data.frame(x = c(-10, 10)), aes(x)) +
   stat_function(fun = dnorm, n = 4800, args = list(mean = 0, sd = 3)
                  , aes(x = x, colour = 'Normal')) + ylab("") +
#
#
  scale_y_continuous(breaks = NULL) +
#
  stat_function(fun = dskt, n = 4800, args = list(df = 100, gamma = 1.2),
                  aes(colour = 'Skewed Student t')) +
    geom\_histogram(data = vanguard\_std, aes(x = vanguard\_std, y = ..density..), bins = 100, alpha = 0.3)
#
```