Numerical Methods

Introduction

1. Objectives

This course is of interest to all areas of engineering where numerical techniques can be used to obtain reliable and efficient solutions to modelling problems. It examines briefly the theoretical basis of ordinary and partial differential equations and discusses that algorithms to solve them.

2. Syllabus

Revision of linear systems (1st 2nd weeks)

Linear systems, direct methods (Gaussian), tridiagonal systems (LU decomposition, Crout's), indirect methods (Jacobi, Gauss-Zeidel). Eigenvalues, spectral radius, stability.

Quadrature (3rd week)

Newton-Cotes formulae and derivations (exact matching and Lagrange polynomials), error analysis and adaptive method.

Initial value problems for ordinary differential equations (ODE) (4th and 5th weeks) Basic theory, one step methods (Euler, Runge-Kutta, derivation, error analysis and control), multi-step method (Adam-Beshford, Adam-Moulton, derivation, predict-corrector, error analysis: truncation, convergence, stability). High-order ODE and system of ODEs. Stiff equations.

Boundary value problems for ODE (6th week)

Second order, linear shooting, finite difference, Rayleigh-Ritz.

Partial Differential Equations (PDE) (7th - 10th weeks)

Basic theory, simple PDEs (Poisson eq., heat eq., wave eq.). Solutions of Elliptic PDEs (finite differences, relabelling points, error analysis, maximum principal), parabolic PDEs (forward, backward, Richardson, Crank-Nicolson, error analysis), hyperbolic PDEs (finite differences, error analysis). Heat and wave equations in two variables. Quasi-linear PDEs and classification, characteristic curves

Matlab

Introduction, commands to solve integration problems and ODEs and PDEs. Basic programming techniques.

Recommended Reading

R. Burden and J. Faires, Brooks Cole (2001), *Numerical Analysis*

D. Kincaid and W. Cheney, Brooks Cole (1996), *Numerical Analysis*

3. Assessment

2-hour closed book examination (70%)

4 coursework assignments (for a total of 30%)

1 coursework - linear systems and quadrature

Hand-out day - 29 October 2001

Hand-in day - 12 November 2001

2 coursework - initial value problems for ODEs

Hand-out day - 12 November 2001

Hand-in day - 26 November 2001

3 coursework - boundary value problems for ODEs and elliptic PDEs

Hand-out day - 26 November 2001

Hand-in day - 10 December 2001

4 coursework - parabolic and hyperbolic PDEs

Hand-out day - 14 December 2001

Hand-in day - 11 January 2002

Penalty for late hand-ins: 40% of the marks will be deducted before marking if handed in up to one week after the deadline. If handed-in later than one week after the deadline a zero mark will be given