FDFD-task2

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In [1]: import numpy as np
        import scipy as sp
        import scipy.sparse.linalg as linalg
        import matplotlib.pyplot as plt
       %matplotlib inline
In [3]: n=101
                       # number of grid nodes
       dx=1.0/n
                         # discretization step, domain size = 1
       eps1 = 1
                    # Layer 1
       eps2 = 13.0
                          # Layer 2
       ratio = 0.5
       nmodes = 6
                     # Solver for first # eigenmodes
        epscol = np.ones(n) # vectro representation
        epscol[:int(n*ratio)] = 1.0/eps1
        epscol[int(n*ratio):] = 1.0/eps2
       epsinv = sp.sparse.dia_matrix(([epscol], [0]), [n,n]) #convert to diagonal matrix
        # Prepare forward and backward difference operators
       diag = np.ones(n) * -1.0/dx+0.j
        up_diag = np.ones(n) * 1.0/dx+0.j
       Fd = sp.sparse.dia_matrix(([up_diag, diag], [1, 0]), [n,n])
       Bd = sp.sparse.dia_matrix(([diag, up_diag], [-1, 0]), [n,n])
        # Convert to lil_matrix to allow per element access.
       F = sp.sparse.lil_matrix(Fd)
       B = sp.sparse.lil_matrix(Bd)
       kk0 = 2*sp.pi*np.linspace(-0.5,0.5,301)+0.0j
       k = np.zeros((nmodes, kk0.size), dtype=complex)
        for ik in range(kk0.size):
           k0=kk0[ik]
           bc = np.exp(1.j*k0*1)/dx # to impose periodic boundary conditions
           F[n-1, 0] = bc
           B[0, n-1] = -np.conj(bc)
            q = -epsinv*B*F; # to assemble eigenmatrix
            # eigs can be provided a guess solution as a starting point,
            # use k=w/c by mean value of epsilon
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kt = k0/np.sqrt((eps1*ratio+eps2*(1-ratio)));
            k2, V = linalg.eigs(q, k=nmodes, M=None, sigma=kt**2)
            k[:,ik] = np.sqrt(k2) # k=w/c
        for i in range(nmodes):
            plt.hold(True)
            plt.plot(kk0/(2*sp.pi), np.real(k[i,:]/(2*sp.pi)) ,'-')
        plt.xlabel("k vector")
        plt.ylabel("frequency")
        plt.xlim([-0.5, 0.5])
        plt.ylim([-.0, 0.3])
Out[3]: (-0.0, 0.3)
            0.30
            0.25
            0.20
        frequency
            0.15
            0.10
            0.05
            0.00
                     -0.4
                                   -0.2
                                                              0.2
                                                 0.0
                                                                            0.4
```

k vector