Macroeconomics II Homework 3

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Q1

(a)

A recursive problem of household is the Bellman equation with an offer w in hand as below. Let $v(w)$ denote the reservation wage.

$$v(w) = \max_{accept,reject} \left\{ \frac{w}{1-\beta}, c + \beta \int_0^B v(w')dF(w') \right\}$$

(b)

Let $\bar{w}$ denote the reservation wage.

$$v(w) = \begin{cases} \max_{accept,reject}\{\frac{w}{1-\beta}, c + \beta \int_0^B v(w')dF(w')\} & \text{if } w \leq \bar{w} \\ \frac{w}{1-\beta} & \text{if } w \geq \bar{w} \end{cases}$$

Now,

$$v(\bar{w}) = \frac{\bar{w}}{1-\beta} = c + \beta \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF(w') + \beta \int_{\bar{w}}^B \frac{w'}{1-\beta} dF(w')$$

Then, we get $\bar{w} - c = \frac{\beta}{1-\beta} \int_{\bar{w}}^B (w' - w)dF(w')$.
Define the RHS as

$$h(w) = \frac{\beta}{1-\beta} \int_{\bar{w}}^B (w' - w)dF(w')$$

Note that

$$\begin{aligned} &h(0) = E[w]\frac{\beta}{1-\beta} \\ &h(B) = 0 \\ &h'(w) = \frac{\beta}{1-\beta}[-(w'-w)f(w) + \int_w^\beta (-1)dF(w')] = -\frac{\beta}{1-\beta}[1 - F(w)] < 0 \\ &h''(w) = \frac{\beta}{1-\beta}F'(w) > 0 \end{aligned}$$

So the relationship between cost and benefit is described as below.
Therefore, a rise in β leads to a rise in $\bar{w}$.

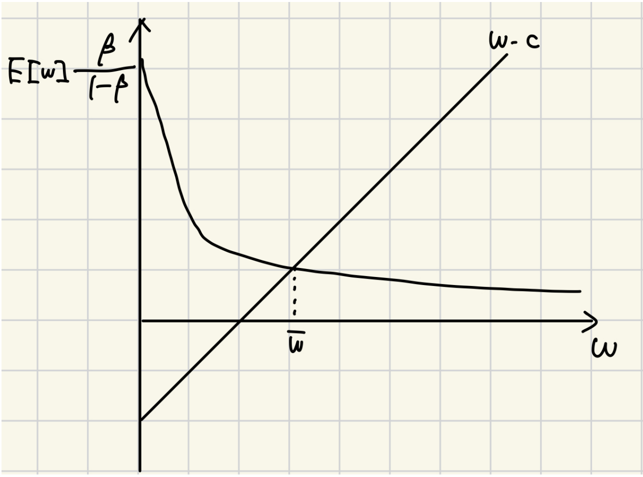

Figure1: Relationship between cost and benefit

An increase in β means that people become more patient, valuing future benefits more highly and thus aspiring to wait for higher wages.
This aligns with the results derived from the equation.

(c)

From the graph, a rise in c leads to a rise in $\bar{w}$.
An increase in c means that the compensation for rejecting an offer increases, leading job seekers to pursue employment with higher offers. This is consistentwith the results derived from the equation.

Q2

Let the consider the value function for employed and unemployed workers.
Let $\bar{w}$ denote the reservation wage.
(i) Employed workers:
The budget constraint of an unemployed worker at time t is

$$a_{t+1} \leq R_t(a_t + w_t n_t - c_t)$$

When this constraint binds, $c_t = a_t + w_t n_t - \frac{a_{t+1}}{R_t}$.
So the value function is

$$v(w) = \max_{n_t}\{p(a_t + w_t n_t - \frac{a_{t+1}}{R_t}, 1 - n_t) + \beta \int_R v(w) dH(R)\}$$

(ii) Unemployed Workers:
The budget constraint of an unemployed worker at time t is

$$a_{t+1} \leq R_t(a_t + z - c_t)$$

When this constraint binds, $c_t = a_t + z - \frac{a_{t+1}}{R_t}$.
So the value function is

$$v(w) = \max_{n_t}\{z + \beta \int_0^B \int_R v(w') dF(w) dH(R)\}$$

Q3

Q4

(a)

Vacancies are filled at rate

$$\frac{m(u,v)}{v} = \frac{u}{v+u} = \frac{\frac{1}{\theta}}{1+\frac{1}{\theta}} = m\left(\frac{1}{\theta}, 1\right) \equiv q(\theta)$$

As we did in the lecture, suppose there is a continuum of workers with measure 1. Every period, $\lambda(1-u)$ workers enter unemployment, and $\theta q(\theta)u$ workers find a job.

$$\Delta u = \lambda(1-u) + \theta q(\theta) u$$

At the steady state, $\Delta u = 0$, so we have

$$\begin{aligned} u &= \frac{\lambda}{\lambda + \theta q(\theta)} \\ &= \frac{\lambda}{\lambda + \frac{1}{1+\frac{1}{\theta}}} \\ &= \frac{\lambda}{\lambda + \frac{\theta}{\theta+1}} \end{aligned}$$

(b)

At any period of steady state, $1-u$ workers are employed. For each employed worker, the probability of transitioning from employment to unemployment is λ, and the probability of being unemployed for n periods is $(1-\theta q(\theta))^n$. Therefore, for each $n = 1, 2, \cdots$,

$$u_n = \lambda(1-u)(1-\theta q(\theta))^n$$

$$\begin{aligned} \sum_{n=1}^{\infty} u_n &= \frac{\lambda(1-u)}{1-(1-\theta q(\theta))} \\ &= \frac{\lambda\left(1 - \frac{\lambda}{\lambda + \frac{\theta}{\theta+1}}\right)}{\theta q(\theta)} \\ &= \frac{\lambda}{\lambda + \frac{\theta}{\theta+1}} \\ &= u \end{aligned}$$

(c)

At each period, the government's total tax revenew is $(1-v)\tau$, and the government's total unemployment benefit is $u_1 z$, where u_1 is the number of workers who have been unemployed for 1 period. Then the government's budget constraint for each period is

$$(1-v)\tau = u_1 z$$

(d)

$$
\begin{aligned}
V &= -pc + \beta\{q(\theta)J + [1 - q(\theta)]V\} \\
J &= p - w + \beta[\lambda V + (1-\lambda)J]
\end{aligned}
$$

(e)

Workers' states are classified in 3 states: employed, unemployed for 1 period, and unemployed for 2 or more periods. Let U_1 be the value of unemployment for 1 period, U_2 be the value of unemployment for 2 or more periods, and W be the value of employment. Then,

$$
\begin{aligned}
U_1 &= z + \beta\left\{\theta q(\theta)W + [1 - \theta q(\theta)]U_2\right\} \\
U_2 &= 0 + \beta\left\{\theta q(\theta)W + [1 - \theta q(\theta)]U_2\right\} \\
W &= w + \beta\left\{\lambda U_1 + (1-\lambda)W\right\}
\end{aligned}
$$

(f)

Free entry implies $V = 0$. Then, solving the equations in (d) gives

$$
p = w + \frac{\left(\frac{1-\beta}{\beta} + \lambda\right) pc}{q(\theta)} = w + \left(\frac{1-\beta}{\beta} + \lambda\right) pc(\theta + 1)
$$

Post vacancies up to the point where marginal product p equals wage w plus expected capitalizes value of hiring costs.