Macroeconomics II Homework 3

Graduate School of Economics, The University of Tokyo

29–246029 Rin NITTA
29-246033 Rei HANARI
29–246004 Kosuke IGARASHI

December 11, 2024

Q1

(a)

The recursive formulation of a standard neoclassical growth model studied in class in Lecture 3 is

$$v(k) = \max_{0 \leq k' \leq f(k)} \{U(f(k) - k') + \beta v(k')\}.$$

Q2

Q3

Q4

(a)

Vacancies are filled at rate

$$\frac{m(u,v)}{v} = \frac{u}{v+u} = \frac{\frac{1}{\theta}}{1+\frac{1}{\theta}} = m\left(\frac{1}{\theta}, 1\right) \equiv q(\theta)$$

As we did in the lecture, suppose there is a continuum of workers with measure 1. Every period, $\lambda(1-u)$ workers enter unemployment, and $\theta q(\theta)u$ workers find a job.

$$\Delta u = \lambda(1-u) + \theta q(\theta)u$$

At the steady state, $\Delta u = 0$, so we have

$$\begin{aligned} u &= \frac{\lambda}{\lambda + \theta q(\theta)} \\ &= \frac{\lambda}{\lambda + \frac{1}{1+\frac{1}{\theta}}} \\ &= \frac{\lambda}{\lambda + \frac{\theta}{\theta+1}} \end{aligned}$$

(b)

At any period of steady state, $1-u$ workers are employed. For each employed worker, the probability of transitioning from employment to unemployment is λ, and the probability of being unemployed for n periods is $(1-\theta q(\theta))^n$. Therefore, for each $n = 1, 2, \cdots$,

$$u_n = \lambda(1-u)(1-\theta q(\theta))^n$$

$$\begin{aligned}
\sum_{n=1}^{\infty} u_n &= \frac{\lambda(1-u)}{1-(1-\theta q(\theta))} \\
&= \frac{\lambda\left(1-\frac{\lambda}{\lambda+\frac{\theta}{\theta+1}}\right)}{\theta q(\theta)} \\
&= \frac{\lambda}{\lambda+\frac{\theta}{\theta+1}} \\
&= u
\end{aligned}$$

(c)

(d)

$$\begin{aligned}
V &= -pc + \beta\{q(\theta)J + [1-q(\theta)]V\} \\
J &= p - w + \beta[\lambda V + (1-\lambda)J]
\end{aligned}$$

(e)

Workers' states are classified in 3 states: employed, unemployed for 1 period, and unemployed for 2 or more periods. Let U_1 be the value of unemployment for 1 period, U_2 be the value of unemployment for 2 or more periods, and W be the value of employment. Then,

$$\begin{aligned}
U_1 &= z + \beta\left\{\theta q(\theta)W + [1-\theta q(\theta)]U_2\right\} \\
U_2 &= 0 + \beta\left\{\theta q(\theta)W + [1-\theta q(\theta)]U_2\right\} \\
W &= w + \beta\left\{\lambda U_1 + (1-\lambda)W\right\}
\end{aligned}$$

(f)

Free entry implies $V = 0$. Then, solving the equations in (d) gives

$$p = w + \frac{\left(\frac{1-\beta}{\beta}+\lambda\right)pc}{q(\theta)} = w + \left(\frac{1-\beta}{\beta}+\lambda\right)pc(\theta+1)$$

Post vacancies up to the point where marginal product p equals wage w plus expected capitalizes value of hiring costs.