Ecomonmetrics II HW1

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1 Exercise 14.14

Assume e_t and u_t is white noise process
We have

$$\begin{aligned} Y_t &= X_t + e_t \\ &= \alpha X_{t-1} + u_t + e_t \\ &= \alpha(Y_{t-1} - e_{t-1}) + u_t + e_t \\ &= \alpha Y_{t-1} + u_t + e_t - \alpha e_{t-1} \\ &= \alpha Y_{t-1} + w_t \end{aligned}$$

where $w_t = u_t + e_t - \alpha e_{t-1}$
Because e_t and u_t are mutually independent i.i.d, w_t is strictly stationary.
Recall e_t and u_t is white noise process

$$\begin{aligned} v\left(w_t\right) =& v\left(u_t + e_t - \alpha e_{t-1}\right) \\ =& v\left(u_t\right) + v\left(e_t\right) + \alpha^2 v\left(e_{t-1}\right) + 2\operatorname{cov}\left(u_t, e_t\right) \\ & - 2\alpha \operatorname{cov}\left(e_t, e_{t-1}\right) - 2\alpha \operatorname{cov}\left(u_t, e_{t-1}\right) \\ =& v\left(u_t\right) + v\left(e_t\right) + \alpha^2 v\left(e_{t-1}\right) < \infty \end{aligned}$$

$$\operatorname{cov}\left(w_t, w_{t-s}\right) = \begin{cases} -\alpha v\left(e_{t-1}\right) & \neq 0 \quad s = 1 \\ 0 & \quad\quad s \geq 2 \end{cases}$$

Therefore w_t is white noise. Thus we have Y_t is an ARMA(1,1) process.

2 Exercise 14.15

Consider the AR(1) process $Y_t = \alpha_0 + \alpha_1 Y_{t-1} + e_t$. Since this is transformed into $(1-\alpha_1 L)Y_t = \alpha_0 + e_t$, we get the MA representation of Y_t as

$$Y_t = (1-\alpha_1 L)^{-1}(\alpha_0 + e_t) = \frac{\alpha_0}{1-\alpha_1} + \sum_{j=0}^{\infty} \alpha_1^j e_{t-j}.$$

Because $|\alpha_1| < 1$, the coefficients satisfy $\sum_{j=0}^{\infty} \alpha_1^{2j} < \infty$. Hence

$$E[Y_t] = E\left[\frac{\alpha_0}{1-\alpha_1} + \sum_{j=0}^{\infty} \alpha_1^j e_{t-j}\right] = \frac{\alpha_0}{1-\alpha_1} + \sum_{j=0}^{\infty} \alpha_1^j E[e_{t-j}] = \frac{\alpha_0}{1-\alpha_1},$$

$$Var(Y_t) = Var\left(\sum_{j=0}^{\infty} \alpha_1^j e_{t-j}\right) = \sum_{j=0}^{\infty} \alpha_1^{2j} Var(e_{t-j}) = \sigma^2 \sum_{j=0}^{\infty} \alpha_1^{2j} = \frac{\sigma^2}{1-\alpha_1^2}.$$

Define the moment–generating function (MGF)

$$M_S(t) \;:=\; E\big[e^{tS}\big], \quad t \in \mathbb{R}.$$

Using independence, factor the expectation:

$$M_S(t) = E\Big[\exp\{t\sum_{j=0}^{\infty} \alpha_1^j e_{t-j}\}\Big] = \prod_{j=0}^{\infty} E\big[e^{\,t\alpha_1^j e_{t-j}}\big].$$

Each term is the MGF of a normal variable with mean 0 and variance $\alpha_1^{2j}\sigma^2$:

$$E\big[e^{\,t\alpha_1^j e_{t-j}}\big] = \exp\Big\{\tfrac{1}{2}t^2\alpha_1^{2j}\sigma^2\Big\}.$$

Hence

$$M_S(t) = \prod_{j=0}^{\infty} \exp\Big\{\tfrac{1}{2}t^2\alpha_1^{2j}\sigma^2\Big\} = \exp\Big\{\tfrac{1}{2}t^2\sigma^2 \sum_{j=0}^{\infty} \alpha_1^{2j}\Big\}.$$

Because $|\alpha_1| < 1$, the geometric series converges: $\sum_{j=0}^{\infty} \alpha_1^{2j} = 1/(1-\alpha_1^2)$. Thus

$$M_S(t) = \exp\Big\{\tfrac{1}{2}t^2\sigma^2/(1-\alpha_1^2)\Big\},$$

which is precisely the MGF of the normal distribution $N\big(0, \sigma^2/(1-\alpha_1^2)\big)$. Therefore

$$S \;\sim\; N\Big(0,\; \sigma^2/(1-\alpha_1^2)\Big).$$

Finally,

$$Y_t = \frac{\alpha_0}{1-\alpha_1} + S \;\sim\; N\Big(\tfrac{\alpha_0}{1-\alpha_1}, \tfrac{\sigma^2}{1-\alpha_1^2}\Big),$$

so the marginal distribution of the stationary AR(1) process is Gaussian with the stated mean and variance.

3 Exercise 14.18

Attached at the end of this document.

4 Exercise 16.1

(a)

$$\begin{aligned} S_t &= e_t + e_{t-1} + \cdots + e_1 + S_0 \\ &= e_t + e_{t-1} + \cdots + e_1. \end{aligned}$$

Since e_t is i.i.d, $E[e_t] = 0$ and $V(e_t) = \sigma^2$,

$$\begin{aligned} E[S_t] &= E[e_t + t_t - 1 + \cdots + e_1] = 0 \\ V(S_t) &= V(e_t + t_t - 1 + \cdots + e_1) = t\sigma^2 \end{aligned}$$

(b)

$$Y_t = \frac{S_t - E[S_t]}{\sqrt{V(S_t)}} = \frac{S_t}{\sigma\sqrt{t}}$$

Therefore,

$$\begin{aligned}
Cov(T_t, Y_{t-j}) &= Cov(\frac{S_t}{\sigma\sqrt{t}}, \frac{S_{t-j}}{\sigma\sqrt{t-j}}) \\
&= E[\frac{S_t S_{t-j}}{\sigma^2\sqrt{t(t-j)}}] \quad (\because E[S_t]=0) \\
&= \frac{E[(e_t+\cdots+e_1)(e_{t-j}+\cdots+e_1)]}{\sigma^2\sqrt{t(t-j)}} \\
&= \frac{(t-j)\sigma^2}{\sigma^2\sqrt{t(t-j)}} \quad (\because E[e_t e_s]=0, \forall t\neq s) \\
&= \sqrt{\frac{t-j}{t}}
\end{aligned}$$

Thus Y_t is not stationary since this depends on t.

(c) Assume $\delta > 0$.

$$Y_{\lfloor nr\rfloor} = \frac{S_{\lfloor nr\rfloor}}{\sigma\sqrt{\lfloor nr\rfloor}} = \frac{\sum_{s=1}^{\lfloor nr\rfloor} e_s}{\sigma\sqrt{\lfloor nr\rfloor}}$$

Since $\lfloor nr\rfloor \to \infty$ as $n\to\infty$ because $r\in[\delta,1]$, and e_t follows i.i.d $(0,\sigma^2)$, by CLT

$$\frac{\sum_{s=1}^{\lfloor nr\rfloor} e_s}{\sqrt{\lfloor nr\rfloor}} \xrightarrow{\text{d}} N(0,\sigma^2)$$

and then,

$$Y_{\lfloor nr\rfloor} = \frac{\sum_{s=1}^{\lfloor nr\rfloor} e_s}{\sigma\sqrt{\lfloor nr\rfloor}} \xrightarrow{\text{d}} N(0,1).$$

5 Exercise 16.4

5.1 (a)

Since e_t is i.i.d, $Y_t = e_t$ is i.i.d and that stationary process. In addition, $\forall j>0, \gamma_j = 0$ holds. Thus, as $n\to\infty$, the long run variance is

$$\begin{aligned}
nVar(\frac{1}{n}\sum_{t=1}^{n} Y_t) &\to \gamma_0 + 2\sum_{j=1}^{\infty}\gamma_j \\
&= \gamma_0 > 0
\end{aligned}$$

Hence, Y_t is $I(0)$ process.

5.2 (b)

Let $\phi(Y_t, Y_{t-1}) = Y_t - Y_{t-1}$. Because Y_t is i.i.d, it is also strictly stationary process. Since ϕ is measurable and $X_t = \phi(Y_t, Y_{t-1})$, X_t is also strictly stationary process. Moreover, because e_t is i.i.d, we have $\forall t\neq j,\ E(e_t) = E(e_j), Var(e_t) = Var(e_j)$, and

$$\begin{aligned}
E[X_t] &= E[e_t] + E[e_{t-1}] = 0 \\
Cov(X_t, X_{t-j}) &= Cov(e_t - e_{t-1}, e_{t-j} - e_{t-j-1}) \\
&= E[(e_t - e_{t-1})(e_{t-j} - e_{t-j-1})] \\
&= \begin{cases} 2(E[e_t^2] - E[e_t]^2) = 2Var(e_t) & \text{when } j=0, \\ E[e_t]^2 - E[e_t^2] = -Var(e_t) & \text{when } j=1, \\ 0 & \text{when } j\geq 2 \end{cases}
\end{aligned}$$

Since $E(X_t)$ nor $\forall j > 0, Cov(X_t, X_{t-j})$ doesn't depends on t, X_t is weekly stationary process. The long run variance is

$$
\begin{aligned}
nVar\left(\frac{1}{n}\sum_{t=1}^{n} X_t\right) &\to Var(X_t) + 2\sum_{j=1}^{\infty} Cov(X_t, X_{t-j}) \\
&= 2Var(e_t) - 2Var(e_t) = 0
\end{aligned}
$$

as $n \to \infty$. Therefore, X_t is NOT $I(0)$.

6 Exercise 16.9

Yes.
To test the hypothesis of unit root, we need to use ADF t-stat instead of ordinary t-stat since the distribution is no longer the normal when $\alpha = 1$. Then, ADF t-stat is $\frac{\hat{\alpha}-1}{s(\hat{\alpha})} = -2.5$, which is larger than -2.86. Hence, we cannot reject the hypothesis of the unit root.

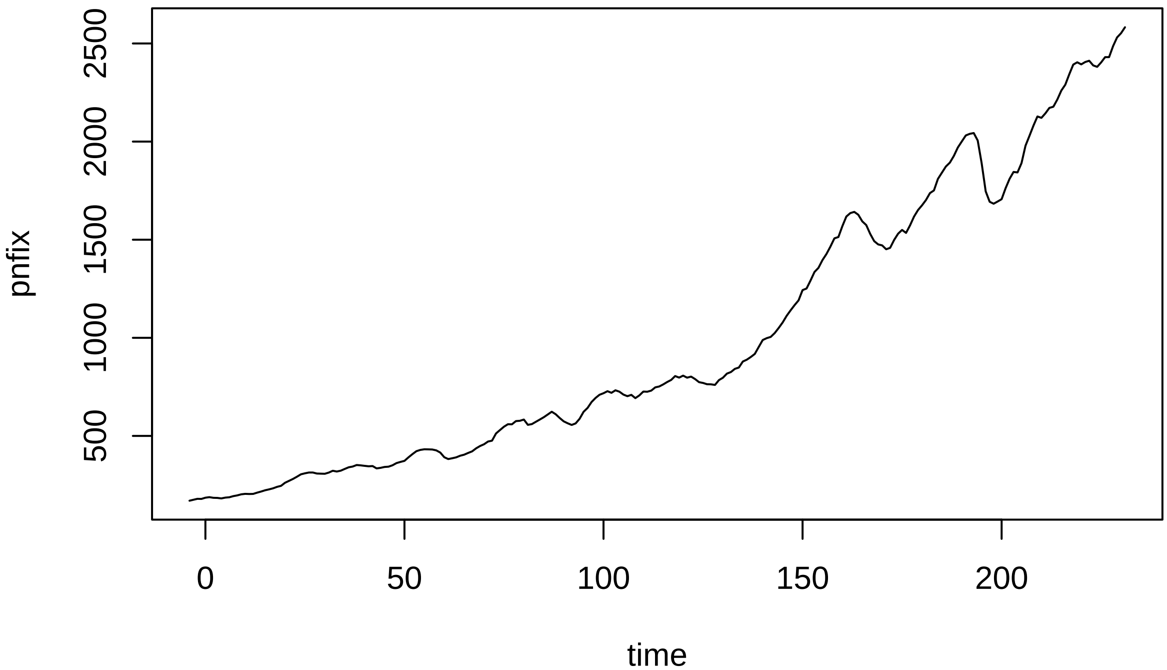
7 Exercise 16.12

Attached at the end of this document.

14.18

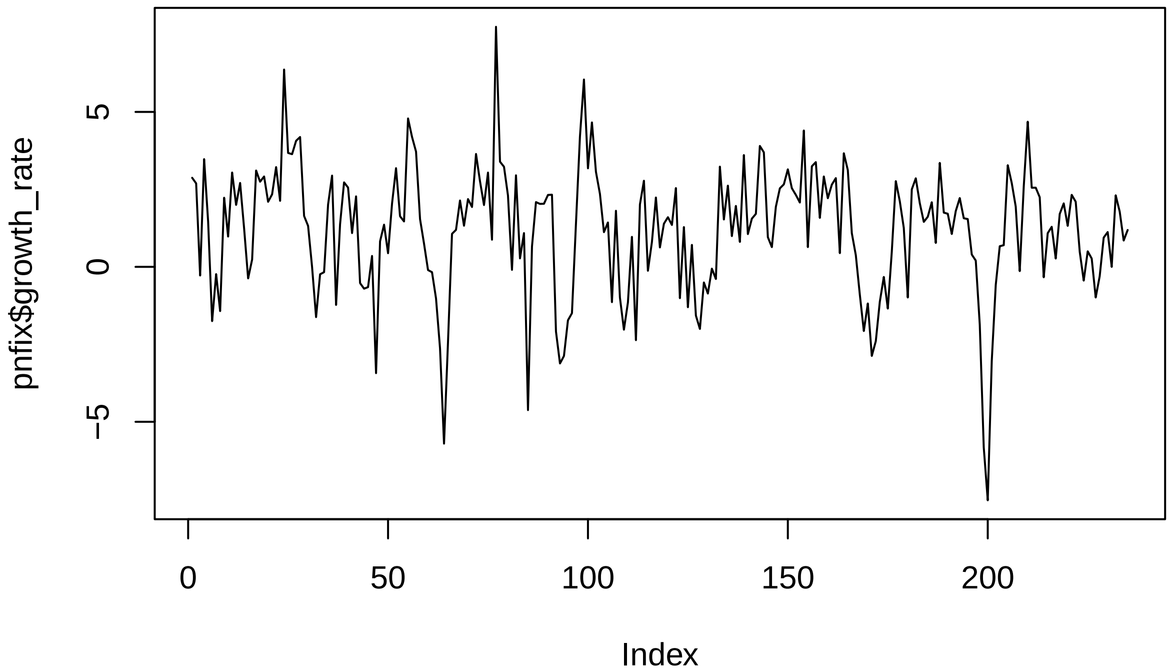
```
library(pacman)
p_load(tidyverse, estimatr, sandwich, lmtest, haven)
```

```
data <- read_dta("FRED-QD.dta")
pnfix <- data %>% select(time, pnfix)
plot(pnfix, type = "l")
```



(a) Transform the series into quarterly growth rates.

```
pnfix <- mutate(pnfix, growth_rate = (pnfix / lag(pnfix)- 1) * 100)
pnfix <- na.omit(pnfix)
plot(pnfix$growth_rate, type="l")
```



(b) Estimate an AR(4) model. Report using heteroskedastic-consistent standard errors.

```
model_ar4_rb <- lm_robust(growth_rate~ lag(growth_rate, 1)
                          + lag(growth_rate, 2) + lag(growth_rate, 3)
                          + lag(growth_rate, 4),
                          data = pnfix,
                          se_type = "HC2")

summary(model_ar4_rb)
```

```
##
## Call:
## lm_robust(formula = growth_rate ~ lag(growth_rate, 1) + lag(growth_rate,
##     2) + lag(growth_rate, 3) + lag(growth_rate, 4), data = pnfix,
##     se_type = "HC2")
##
## Standard error type:  HC2
##
## Coefficients:
##                       Estimate Std. Error t value  Pr(>|t|) CI Lower CI Upper  DF
## (Intercept)            0.49115    0.14848  3.3077 1.094e-03  0.19856  0.78374 226
## lag(growth_rate, 1)  0.50161    0.07670  6.5402 4.050e-10  0.35048  0.65274 226
## lag(growth_rate, 2)  0.16832    0.07068  2.3813 1.808e-02  0.02904  0.30761 226
## lag(growth_rate, 3) -0.02618    0.06296 -0.4159 6.779e-01 -0.15024  0.09787 226
## lag(growth_rate, 4) -0.06827    0.05269 -1.2956 1.964e-01 -0.17210  0.03556 226
##
## Multiple R-squared:  0.3431 ,    Adjusted R-squared:  0.3314
## F-statistic: 24.79 on 4 and 226 DF,  p-value: < 2.2e-16
```

(c) Repeat using the Newey-West standard errors, using M = 5.

```
model_ar4 <- lm(growth_rate~ lag(growth_rate, 1) + lag(growth_rate, 2)
                + lag(growth_rate, 3) + lag(growth_rate, 4),
                data = pnfix)
nw_se <- NeweyWest(model_ar4, lag = 5)
coeftest(model_ar4, vcov = nw_se)
```

```
##
## t test of coefficients:
##
##                       Estimate Std. Error t value  Pr(>|t|)
## (Intercept)           0.491146   0.145256  3.3813 0.0008501 ***
## lag(growth_rate, 1)  0.501609   0.084749  5.9188 1.194e-08 ***
## lag(growth_rate, 2)  0.168321   0.069927  2.4071 0.0168832 *
## lag(growth_rate, 3) -0.026183   0.064694 -0.4047 0.6860627
## lag(growth_rate, 4) -0.068269   0.050779 -1.3444 0.1801588
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(d) Comment on the magnitude and interpretation of the coefficients.

The coefficients indicate how past quarterly growth rates affect the current growth rate: Lag 1 Coefficient: Reflects the immediate past quarter's influence. Lag 2 Coefficient: Shows the impact from two quarters ago, indicating the persistence of growth effects. Lag 3 Coefficient: Captures the influence from three quarters ago, indicating longer-term effects. Lag 4 Coefficient: Represents the effect from a year ago, indicating seasonal or annual patterns. The magnitude of these coefficients show the persistence growth rates over time, showing the persistence or decay of economic shocks.

16.12

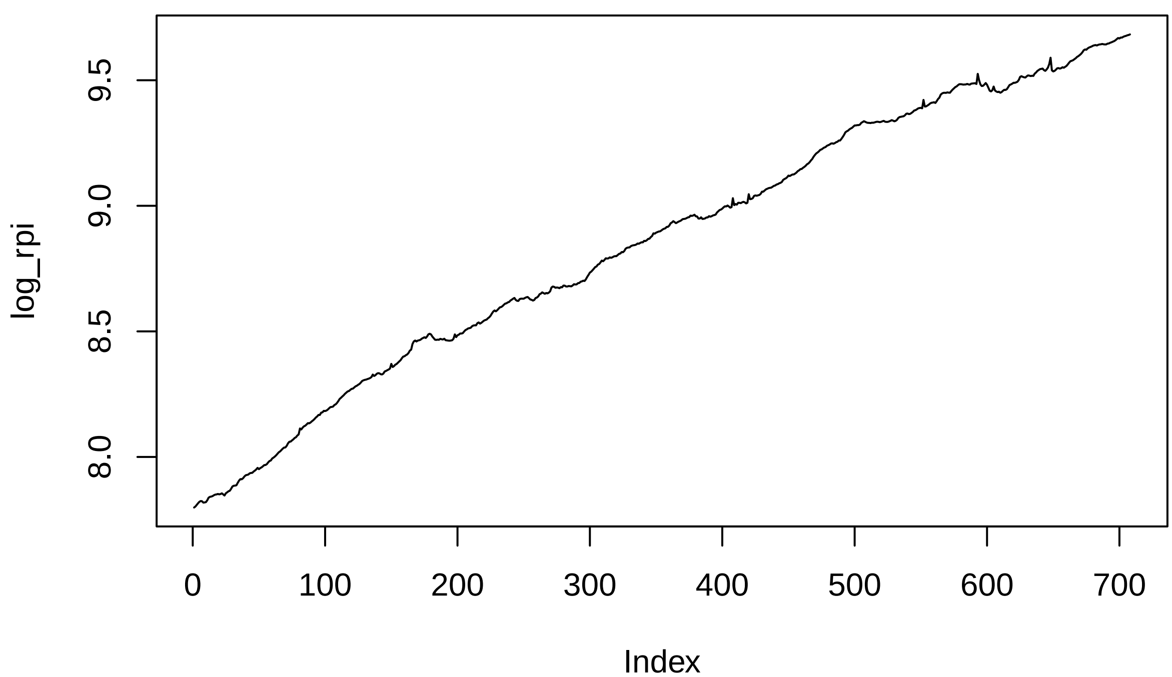
```r
library(pacman)
p_load(haven, urca)
```

```r
data <- read_dta("FRED-MD.dta")
log_rpi <- log(data$rpi)
indpro <- data$indpro
houst <- data$houst
hwi <- data$hwi
clf16ov <- data$clf16ov
claims <- data$claimsx
ipfuels <- data$ipfuels
```

(a)

Because the series has a drift, we use adf test with drift in the regression.

```r
plot(log_rpi, type="l")
```




```r
adf_test <- ur.df(log_rpi, type="drift", lags = 12, selectlags="AIC")
summary(adf_test)
```

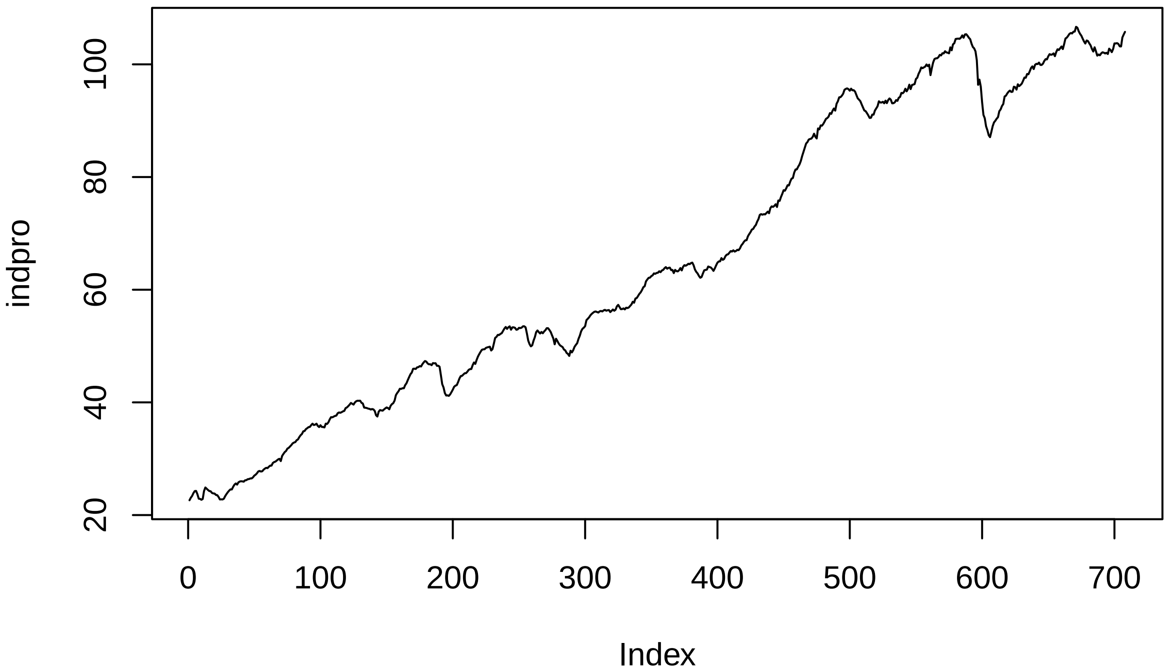
```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
```

```
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.047493 -0.002238  0.000108  0.002280  0.038382
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.0095334  0.0038057   2.505 0.012477 *
## z.lag.1      -0.0008597  0.0004146  -2.074 0.038470 *
## z.diff.lag1  -0.1240266  0.0379358  -3.269 0.001132 **
## z.diff.lag2  -0.0552146  0.0381796  -1.446 0.148587
## z.diff.lag3  -0.0503582  0.0382259  -1.317 0.188152
## z.diff.lag4   0.0244878  0.0382450   0.640 0.522202
## z.diff.lag5   0.0971534  0.0380704   2.552 0.010930 *
## z.diff.lag6   0.0954644  0.0380887   2.506 0.012430 *
## z.diff.lag7   0.0502831  0.0380583   1.321 0.186874
## z.diff.lag8   0.0897763  0.0379406   2.366 0.018249 *
## z.diff.lag9   0.0281037  0.0380953   0.738 0.460939
## z.diff.lag10  0.0089733  0.0380540   0.236 0.813656
## z.diff.lag11 -0.0192896  0.0379936  -0.508 0.611822
## z.diff.lag12  0.1334783  0.0377433   3.536 0.000433 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.005432 on 681 degrees of freedom
## Multiple R-squared:  0.07206,    Adjusted R-squared:  0.05434
## F-statistic: 4.068 on 13 and 681 DF,  p-value: 1.756e-06
##
##
## Value of test-statistic is: -2.0738 11.7641
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

(b)

Because this series also has a drift, we use adf test with drift in the regression.

```r
plot(indpro, type="l")
```




```r
adf_test <- ur.df(indpro, type="drift", lags = 12, selectlags="AIC")
summary(adf_test)
```

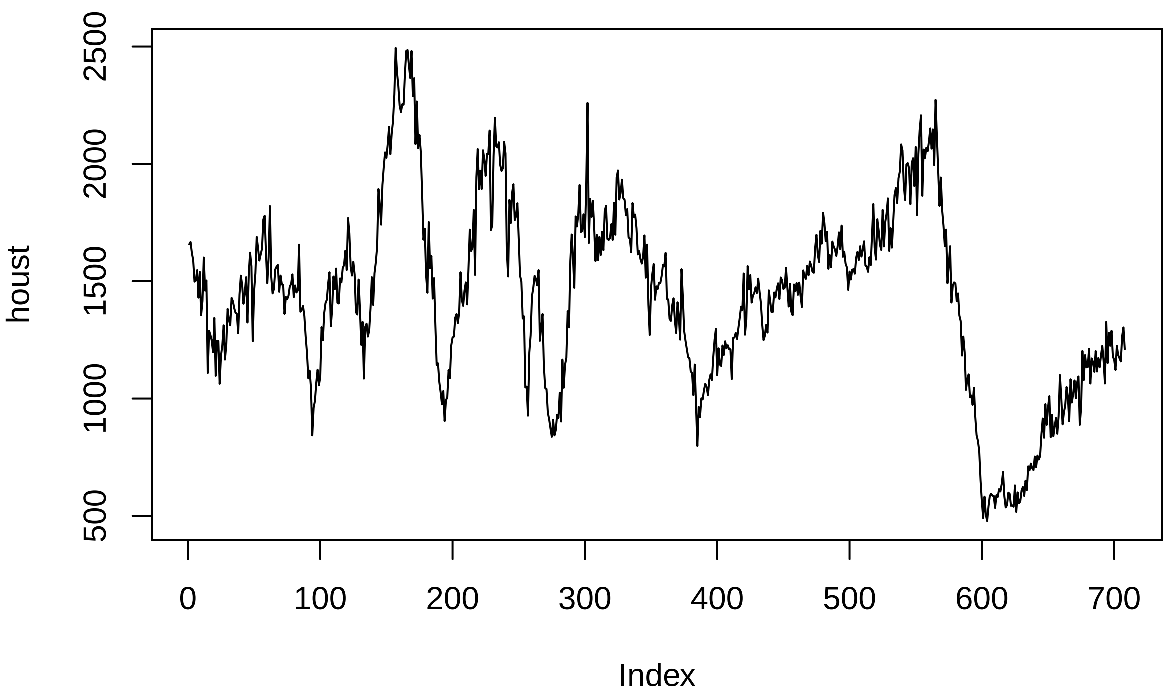
```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.9711 -0.2349 -0.0170  0.1992  1.8865
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0651440  0.0464170   1.403 0.160933
## z.lag.1     -0.0002812  0.0006459  -0.435 0.663379
## z.diff.lag1  0.1490367  0.0377619   3.947 8.73e-05 ***
## z.diff.lag2  0.1353190  0.0373129   3.627 0.000308 ***
## z.diff.lag3  0.1874535  0.0375893   4.987 7.77e-07 ***
## z.diff.lag4  0.1283488  0.0378050   3.395 0.000726 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4421 on 689 degrees of freedom
## Multiple R-squared:  0.1655, Adjusted R-squared:  0.1594
## F-statistic: 27.33 on 5 and 689 DF,  p-value: < 2.2e-16
##
```

```
##
## Value of test-statistic is: -0.4354 3.4986
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

(c)

Because the series does not have trend or drift, we do not use drift or trend in the regression.

```
plot(houst, type="l")
```




```
adf_test <- ur.df(houst, type="none", lags = 12, selectlags="AIC")
summary(adf_test)
```

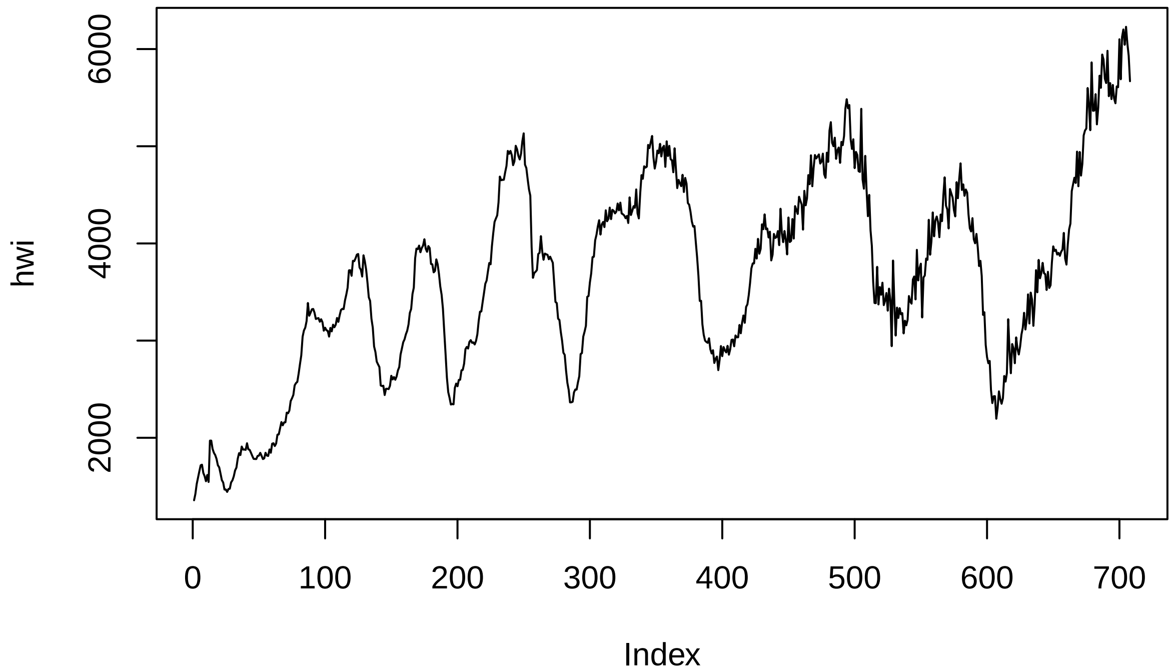
```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -442.57  -57.14    3.04   65.52  426.69
##
## Coefficients:
```

```
##               Estimate Std. Error t value Pr(>|t|)
## z.lag.1      -0.002315   0.002722  -0.850  0.39546
## z.diff.lag1  -0.368713   0.038031  -9.695  < 2e-16 ***
## z.diff.lag2  -0.143306   0.040503  -3.538  0.00043 ***
## z.diff.lag3   0.021814   0.040858   0.534  0.59358
## z.diff.lag4   0.118173   0.040814   2.895  0.00391 **
## z.diff.lag5   0.106831   0.041060   2.602  0.00947 **
## z.diff.lag6   0.112003   0.041198   2.719  0.00672 **
## z.diff.lag7   0.052024   0.041210   1.262  0.20723
## z.diff.lag8   0.008387   0.041051   0.204  0.83817
## z.diff.lag9   0.041162   0.040802   1.009  0.31342
## z.diff.lag10 -0.042851   0.040842  -1.049  0.29446
## z.diff.lag11 -0.017192   0.040475  -0.425  0.67114
## z.diff.lag12 -0.115831   0.037894  -3.057  0.00233 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 106.4 on 682 degrees of freedom
## Multiple R-squared:  0.1527, Adjusted R-squared:  0.1366
## F-statistic: 9.455 on 13 and 682 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -0.8503
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

(d)

Because the series does not have trend or drift, we do not use drift or trend in the regression.

```r
plot(hwi, type="l")
```




```r
adf_test <- ur.df(hwi, type="none", lags = 12, selectlags="AIC")
summary(adf_test)
```

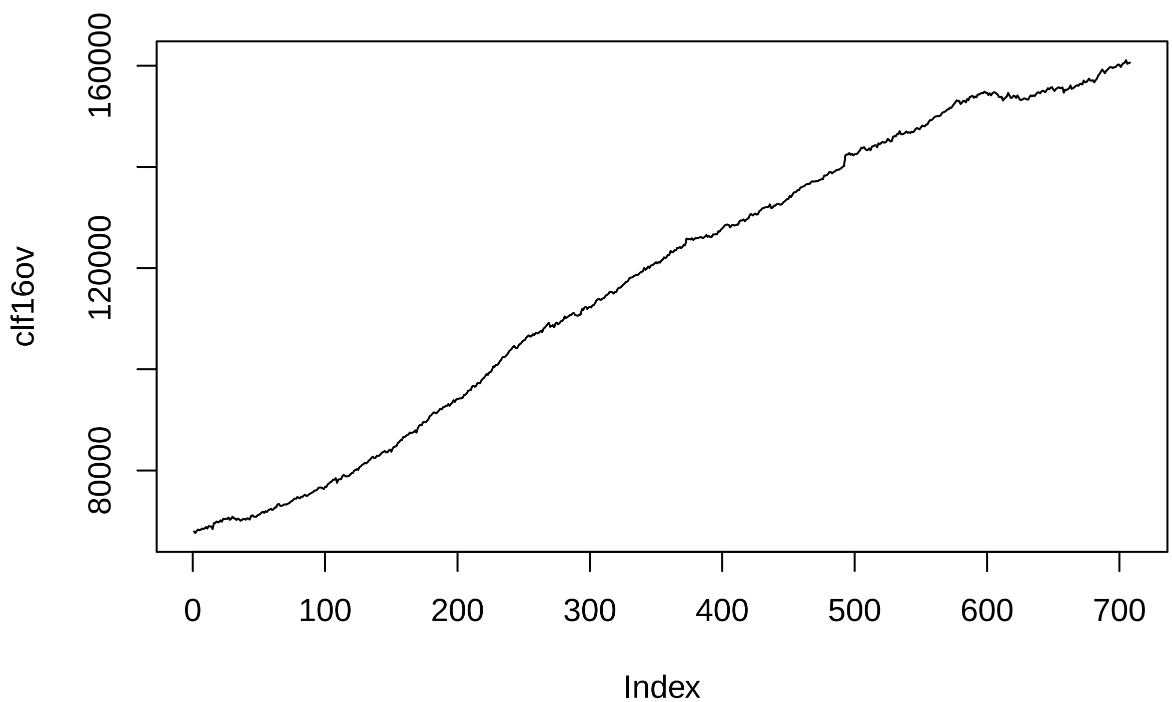
```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -521.29  -72.49    9.05   76.40  681.09
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## z.lag.1      -0.0006096  0.0014554  -0.419   0.6755
## z.diff.lag1  -0.2852276  0.0376042  -7.585 1.09e-13 ***
## z.diff.lag2   0.0630620  0.0391831   1.609   0.1080
## z.diff.lag3   0.2972485  0.0392373   7.576 1.17e-13 ***
## z.diff.lag4   0.0971186  0.0407706   2.382   0.0175 *
## z.diff.lag5   0.0544146  0.0408294   1.333   0.1831
## z.diff.lag6   0.0728151  0.0408551   1.782   0.0751 .
## z.diff.lag7   0.0118273  0.0409623   0.289   0.7729
## z.diff.lag8  -0.0818484  0.0409570  -1.998   0.0461 *
## z.diff.lag9   0.0502567  0.0410568   1.224   0.2213
## z.diff.lag10 -0.0665876  0.0395576  -1.683   0.0928 .
## z.diff.lag11  0.0567797  0.0396084   1.434   0.1522
## z.diff.lag12  0.1816545  0.0381966   4.756 2.41e-06 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 143.8 on 682 degrees of freedom
## Multiple R-squared:  0.2523, Adjusted R-squared:  0.2381
## F-statistic: 17.71 on 13 and 682 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -0.4189
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

(e)

Because the series has a drift, we use adf test with drift in the regression.

```r
plot(clf16ov, type="l")
```




```r
adf_test <- ur.df(clf16ov, type="drift", lags = 12, selectlags="AIC")
summary(adf_test)
```

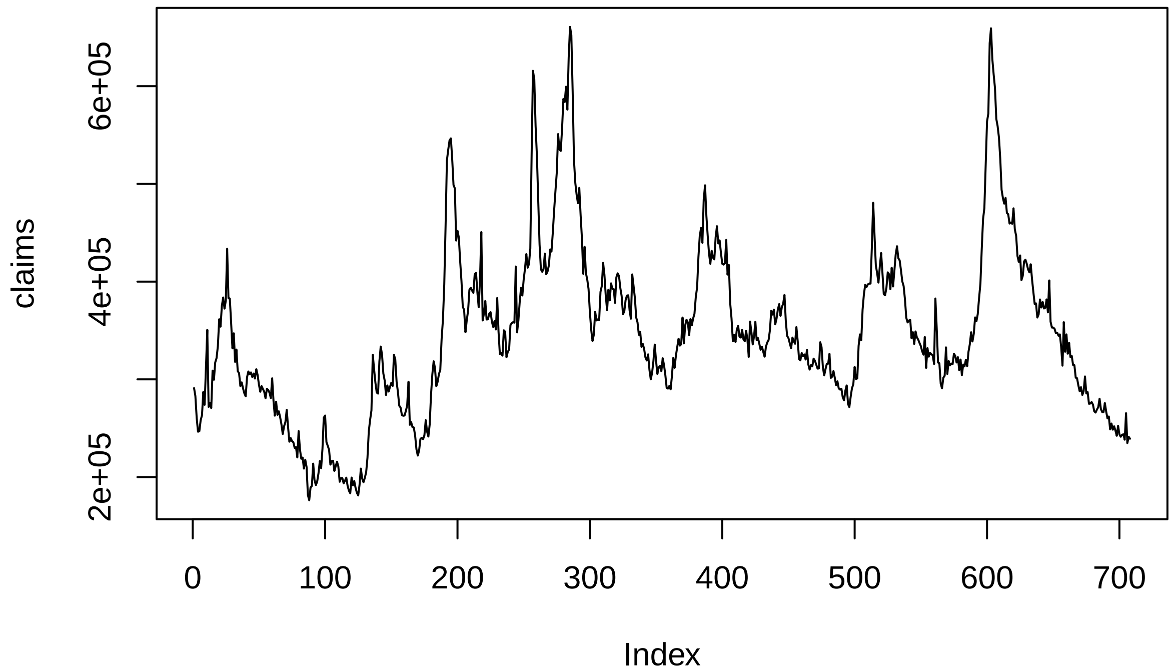
```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
```

```
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1063.04  -161.24    12.96   159.77  1992.85
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.087e+02  5.373e+01   3.885 0.000112 ***
## z.lag.1      -6.554e-04  3.765e-04  -1.741 0.082208 .
## z.diff.lag1  -2.301e-01  3.812e-02  -6.035 2.61e-09 ***
## z.diff.lag2  -5.359e-02  3.906e-02  -1.372 0.170543
## z.diff.lag3   4.255e-03  3.915e-02   0.109 0.913496
## z.diff.lag4  -4.055e-02  3.915e-02  -1.036 0.300665
## z.diff.lag5  -7.323e-02  3.895e-02  -1.880 0.060484 .
## z.diff.lag6   4.612e-02  3.901e-02   1.182 0.237607
## z.diff.lag7   1.108e-01  3.896e-02   2.844 0.004582 **
## z.diff.lag8   2.397e-02  3.927e-02   0.610 0.541851
## z.diff.lag9   7.896e-02  3.926e-02   2.011 0.044714 *
## z.diff.lag10  5.439e-02  3.934e-02   1.383 0.167252
## z.diff.lag11  8.490e-02  3.839e-02   2.212 0.027325 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 284.7 on 682 degrees of freedom
## Multiple R-squared:  0.08091,    Adjusted R-squared:  0.06474
## F-statistic: 5.003 on 12 and 682 DF,  p-value: 5.302e-08
##
##
## Value of test-statistic is: -1.7406 16.679
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

(f)

Because the series does not have trend or drift, we do not use drift or trend in the regression.

```r
plot(claims, type="l")
```




```r
adf_test <- ur.df(claims, type="none", lags = 12, selectlags="AIC")
summary(adf_test)
```

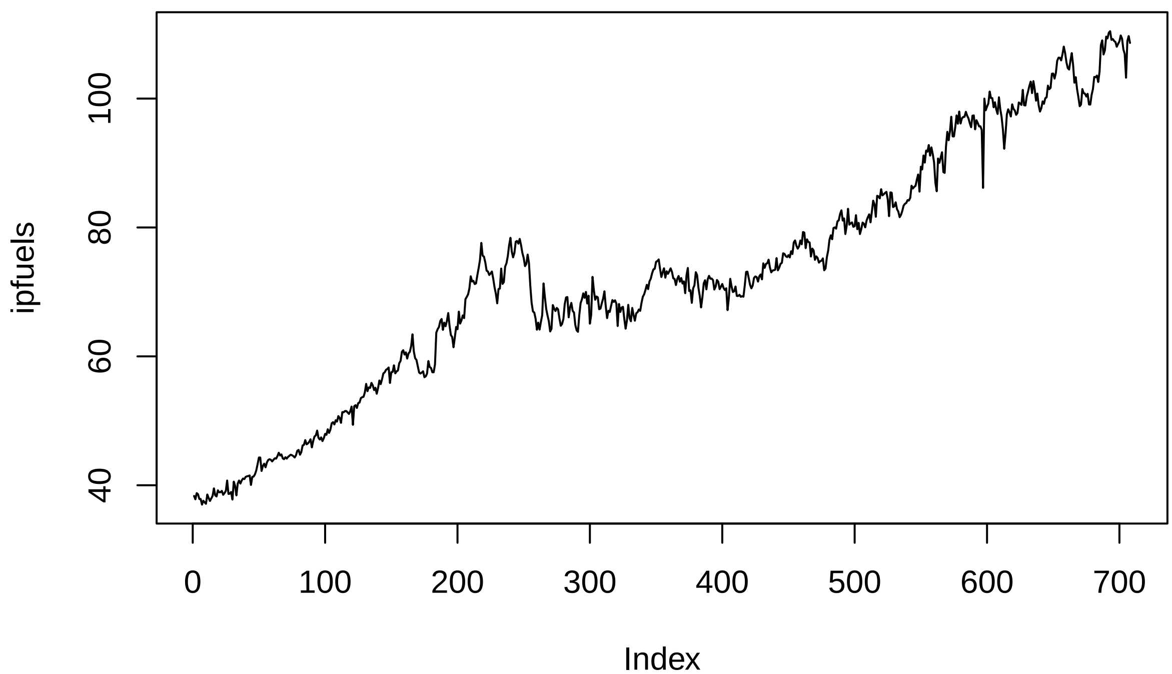
```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -95400  -8989   -124   9348  95146
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1     -0.001565   0.001922  -0.814   0.4157
## z.diff.lag1  0.079220   0.037924   2.089   0.0371 *
## z.diff.lag2  0.061594   0.037420   1.646   0.1002
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18100 on 692 degrees of freedom
## Multiple R-squared:  0.01172,    Adjusted R-squared:  0.007436
## F-statistic: 2.736 on 3 and 692 DF,  p-value: 0.04273
##
##
## Value of test-statistic is: -0.8143
##
```

```
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

(g)

Because the series has a drift, we use adf test with drift in the regression.

```
plot(ipfuels, type="l")
```




```
adf_test <- ur.df(ipfuels, type="drift", lags = 12, selectlags="AIC")
summary(adf_test)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.2703 -0.7495  0.0221  0.7113 11.4321
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.291881   0.218562   1.335  0.18217
## z.lag.1     -0.001815   0.002911  -0.624  0.53308
## z.diff.lag1 -0.233333   0.038094  -6.125 1.53e-09 ***
```

```
## z.diff.lag2  -0.167165   0.039135  -4.271 2.22e-05 ***
## z.diff.lag3  -0.051257   0.039905  -1.284  0.19942
## z.diff.lag4  -0.059250   0.040081  -1.478  0.13980
## z.diff.lag5  -0.055301   0.040133  -1.378  0.16868
## z.diff.lag6   0.057359   0.040233   1.426  0.15443
## z.diff.lag7   0.055134   0.040240   1.370  0.17110
## z.diff.lag8   0.047012   0.040233   1.169  0.24301
## z.diff.lag9   0.001658   0.040206   0.041  0.96712
## z.diff.lag10 -0.029575   0.040166  -0.736  0.46179
## z.diff.lag11 -0.010903   0.039680  -0.275  0.78358
## z.diff.lag12 -0.118107   0.038645  -3.056  0.00233 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.411 on 681 degrees of freedom
## Multiple R-squared:  0.08858,    Adjusted R-squared:  0.07118
## F-statistic: 5.091 on 13 and 681 DF,  p-value: 1.148e-08
##
##
## Value of test-statistic is: -0.6236 4.1876
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```