MA677 Final Project

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Planning

For this final project, first I started reading and tried to understand the relevant chapters in "In All Likelihood". However, as I did not have any math background, I expected it to be really hard to understand the contents by myself, and it was, so I also asked my classmates, especially Yuli, to help me understand them as well as to give some tips to solve the problems.
The results and the deriverable of my work are shown below. It was really fun to understand and get to know what I didn't know and solve these problems, and I also appreciate my classmates' kind help!

4.25: Approximation of the median using the distribution of median statistics

Based on the Example 4.8 of the textbook "In All Likelihood", the approximation of median of Ui is accurate. I will compare the values from the approximation with the value of expectation obtained from the distribtuion of the order statistics as below. [1]

```
#Create pdf and cdf of uniform distribution(0,1)
a <- 0
b <- 1
pdf <- function(x) dunif(x, a,b)
cdf <- function(x) punif(x, a,b, lower.tail=FALSE)

#Calculate the distribution of the order statistics
integrand <- function(x,r,n) {x * (1 - cdf(x))^(r-1) * cdf(x)^(n-r) * pdf(x)}

#Calculate the expectation from the above distribution
Expect <- function(r,n) {
  (1/beta(r,n-r+1)) * integrate(integrand,-Inf,Inf, r, n)$value
  }

#Create approximation function from the text
Approx <- function(k,n){
  m<-(k-1/3)/(n+1/3)
  return(m)
}

#Create table to compare approximation with expectation for n=5 and n=10 respectively
Approximation5<- vector()
Expectation5 <- vector()
Approximation10<- vector()
Expectation10 <- vector()

for (i in 1:5) {
```

```
  Approximation5 <- c(Approximation5, Approx(i,5))
  Expectation5 <- c(Expectation5,Expect(i,5))
}
table1 <- data.frame(Approximation5, Expectation5)

for (i in 1:10) {
  Approximation10 <- c(Approximation10, Approx(i,10))
  Expectation10 <- c(Expectation10,Expect(i,10))
}
table2 <- data.frame(Approximation10, Expectation10)

kable(table1,caption = "Comparison of Approximation with Expectation (n=5)") %>%
  kable_styling(position = "center") %>%
  kable_styling(latex_options = "HOLD_position")
```

Table 1: Comparison of Approximation with Expectation (n=5)

Approximation5	Expectation5
0.1250	0.1666667
0.3125	0.3333333
0.5000	0.5000000
0.6875	0.6666667
0.8750	0.8333333

```
kable(table2,caption = "Comparison of Approximation with Expectation (n=10)") %>%
  kable_styling(position = "center") %>%
  kable_styling(latex_options = "HOLD_position")
```

Table 2: Comparison of Approximation with Expectation (n=10)

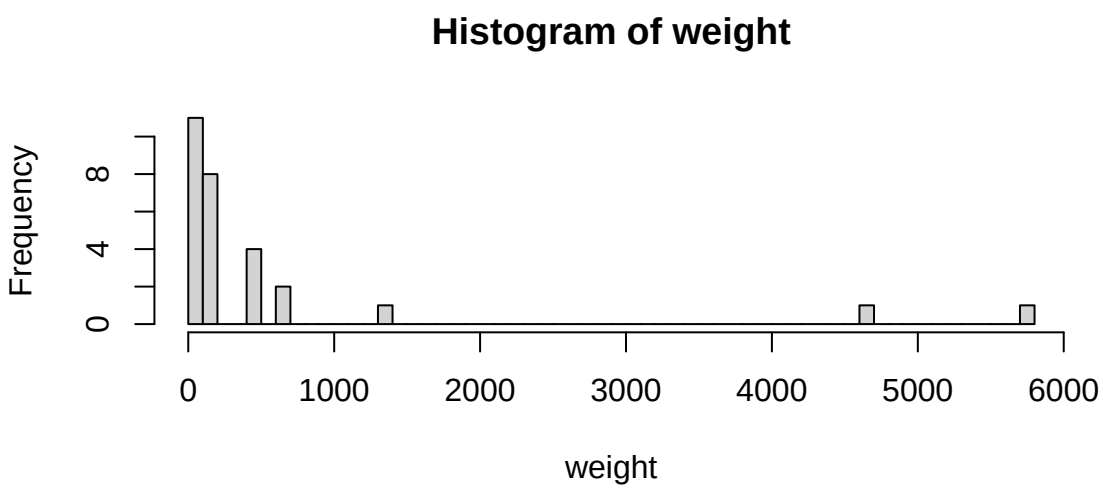
Approximation10	Expectation10
0.0645161	0.0909091
0.1612903	0.1818182
0.2580645	0.2727273
0.3548387	0.3636364
0.4516129	0.4545455
0.5483871	0.5454546
0.6451613	0.6363636
0.7419355	0.7272727
0.8387097	0.8181818
0.9354839	0.9090909

As you can see above tables, the smaller the value of n and the more extreme the value of i, the more the approximation tends to depart from the expectation. However, as already mentioned in the textbook, this approximation remains accurate to some extent even when n is small, especially when i is closer to the middle value.

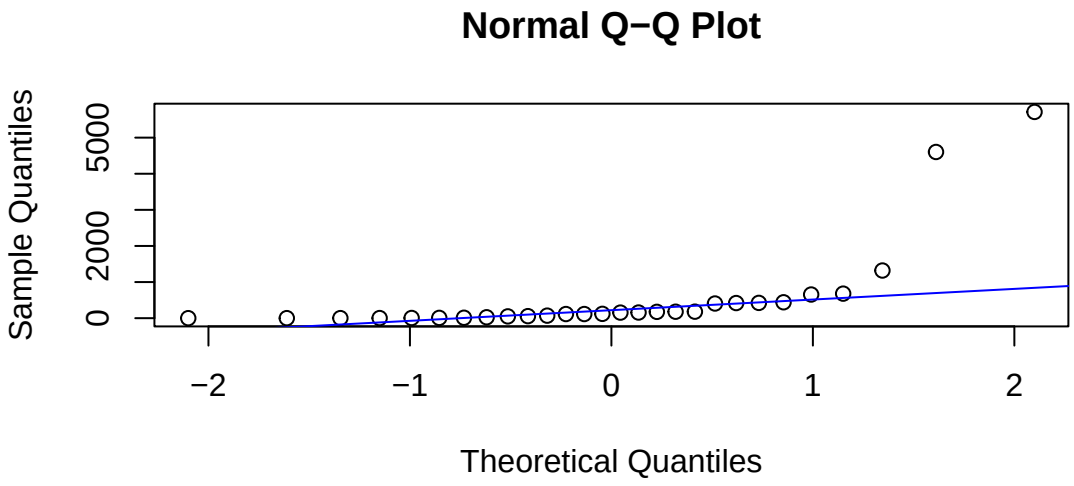
4.39:

First I entered the weight data and plot the histogram of the data as below.

```r
weight<-c(0.4, 1.0, 1.9, 3.0, 5.5, 8.1, 12.1, 25.6, 50.0, 56.0, 70.0, 115.0, 115.0, 119.5, 154.5, 157.0

#Histogram of the data
hist(weight, breaks = 50)
```

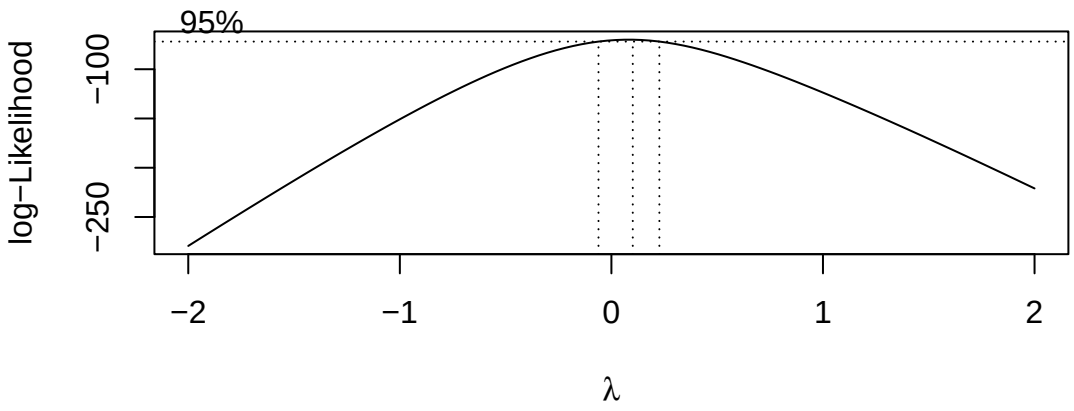



```r
#QQplot of the data
qqnorm(weight)
qqline(weight, col = "blue")
```



As you can see, the distribution is skewed to the right, so I could expect Box-Cox transformation similar to log-transform would work well (lambda=0). So first I looked at profile of likelihood of lambda of Box-Cox transformation family, and chose the lambda with maximum likelihood, which was expected to transform the distribution into approximate normal distribution. [2]

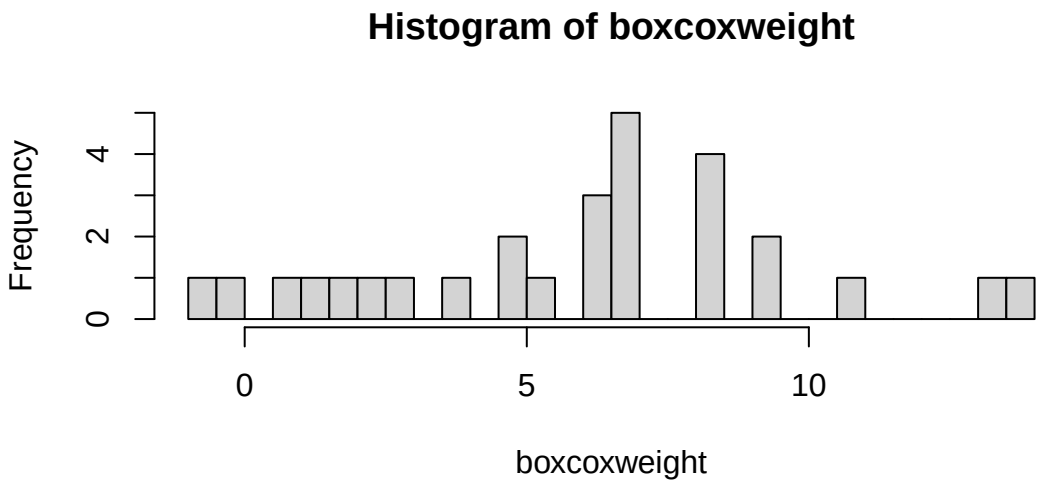
```r
#Create the profile of likelihood of lambda of Box-Cox
b <- boxcox(lm(weight ~ 1))
```



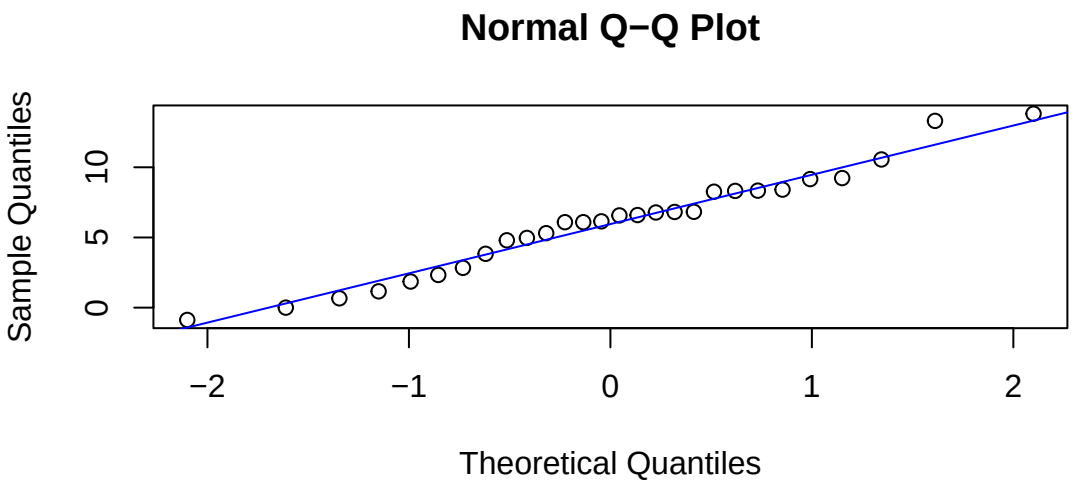

```
#Choose the lambda with maximum likelihood
lambda <- b$x[which.max(b$y)]
```

As we expected, the lambda with maximum likelihood is 0.1010101, which is around 0. Then I did Box-Cox transformation with the lambda value as below.

```
# Exact lambda
boxcoxweight <- (weight ^ lambda - 1) / lambda

# The histogram of data transformed and its QQplot
hist(boxcoxweight, breaks = 50)
```




```
qqnorm(boxcoxweight)
qqline(boxcoxweight, col = "blue")
```



After the Box-Cox transformation, the histogram and QQplot of the data show approximate normal.

4.27: The amount of rainfall in Valencia

(a)

```
#Set the dataset
Jan<-c(0.15, 0.25, 0.10, 0.20, 1.85, 1.97, 0.80, 0.20, 0.10, 0.50, 0.82, 0.40,
  1.80, 0.20, 1.12, 1.83, 0.45, 3.17, 0.89, 0.31, 0.59, 0.10, 0.10, 0.90,
  0.10, 0.25, 0.10, 0.90)
Jul<-c(0.30 ,0.22 ,0.10 ,0.12 ,0.20 ,0.10 ,0.10 ,0.10 ,0.10 ,0.10 ,0.10 ,0.17,
  0.20 ,2.80 ,0.85 ,0.10 ,0.10 ,1.23 ,0.45 ,0.30 ,0.20 ,1.20 ,0.10 ,0.15,
  0.10 ,0.20 ,0.10 ,0.20 ,0.35 ,0.62 ,0.20 ,1.22 ,0.30 ,0.80 ,0.15 ,1.53,
  0.10 ,0.20 ,0.30 ,0.40 ,0.23 ,0.20 ,0.10 ,0.10 ,0.60 ,0.20 ,0.50 ,0.15,
  0.60 ,0.30 ,0.80 ,1.10 ,0.20 ,0.10 ,0.10 ,0.10 ,0.42 ,0.85 ,1.60 ,0.10,
  0.25 ,0.10 ,0.20 ,0.10)

#Get summary statistics
kable(describeBy(Jan),caption = "Summary statistics of rainfall in January") %>%
  kable_styling(latex_options="scale_down") %>%
  kable_styling(latex_options = "HOLD_position")
```

Table 3: Summary statistics of rainfall in January

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
X1	1	28	0.7196429	0.7650586	0.425	0.6170833	0.481845	0.1	3.17	3.07	1.471453	1.627648	0.1445825

```
kable(describeBy(Jul),caption = "Summary statistics of rainfall in July") %>%
  kable_styling(latex_options="scale_down") %>%
  kable_styling(latex_options = "HOLD_position")
```

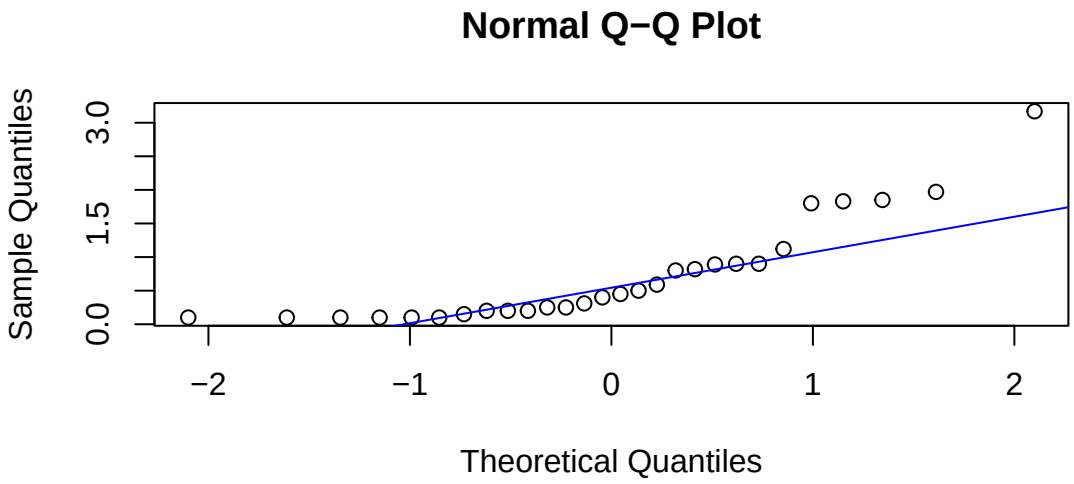
Table 4: Summary statistics of rainfall in July

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
X1	1	64	0.393125	0.4816733	0.2	0.2880769	0.14826	0.1	2.8	2.7	2.63659	8.414384	0.0602092

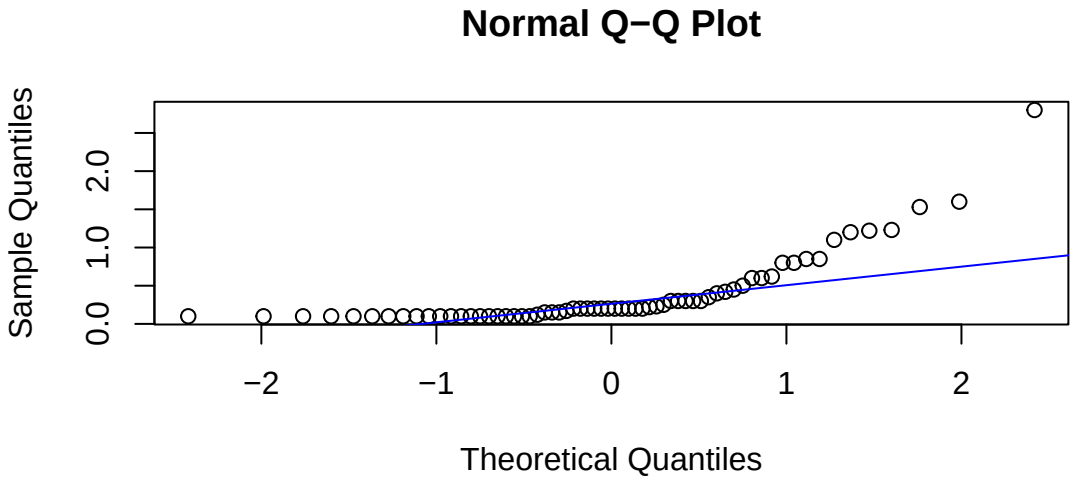
Based on the summary statistics above, we can see that the amount of rainfall in July is less than that in January in terms of mean and maximum. We can also see that we have more data in July so that the standard deviation is less in July than in January.

(b)

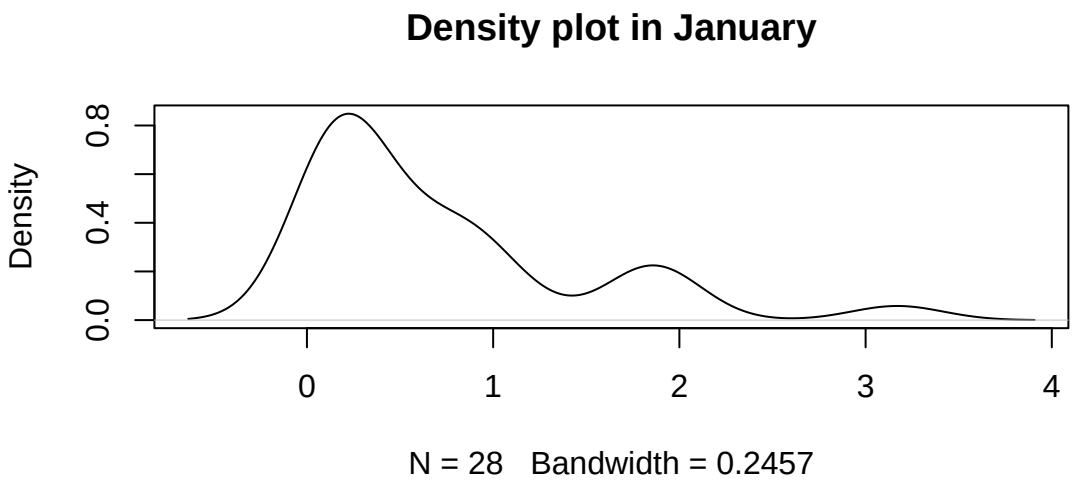
```
#Create QQplots
qqnorm(Jan)
qqline(Jan, col = "blue")
```



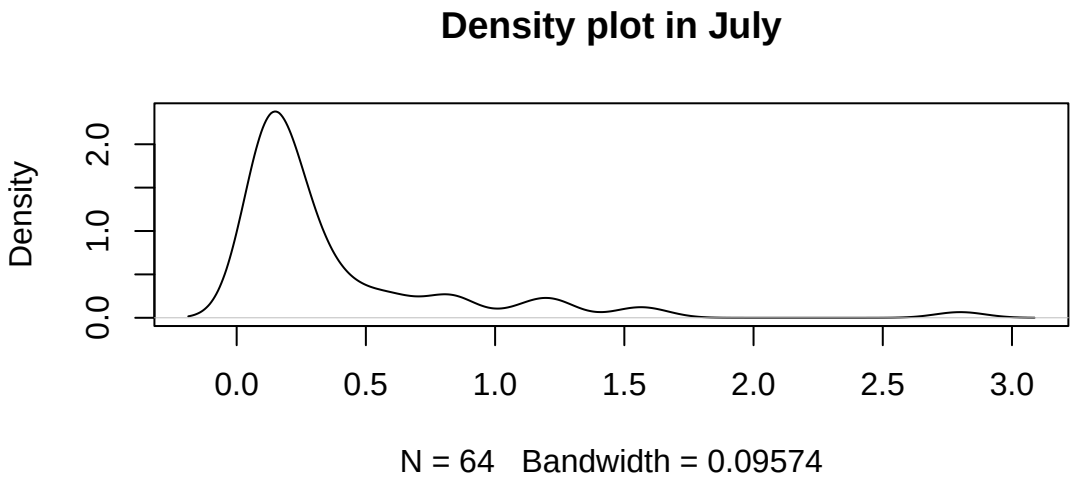

```
qqnorm(Jul)
qqline(Jul, col = "blue")
```




```
#Create Density plots
plot(density(Jan),main="Density plot in January")
```

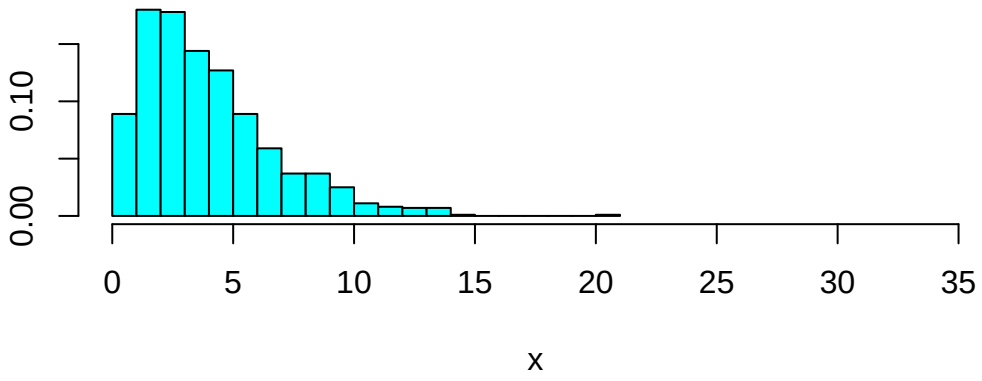



```
plot(density(Jul),main="Density plot in July")
```



Based on the QQplots and density plots, both data sets of rainfall in January and July seem to follow gamma distribution. Just to compare the distribution, I draw the gamma distribution with K=2 and theta=0.5 as below, which looks very similar to distributions of two data sets.[4]

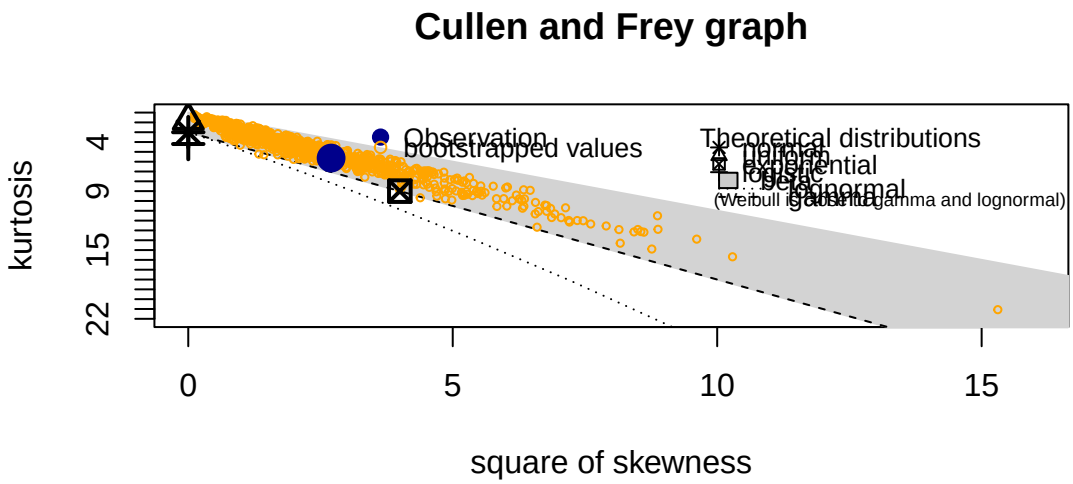
```
set.seed(2022)
x <- rgamma(1000, 2, 0.5)
truehist(x, xlim = c(0, 35))
```




```
rm(x)
```

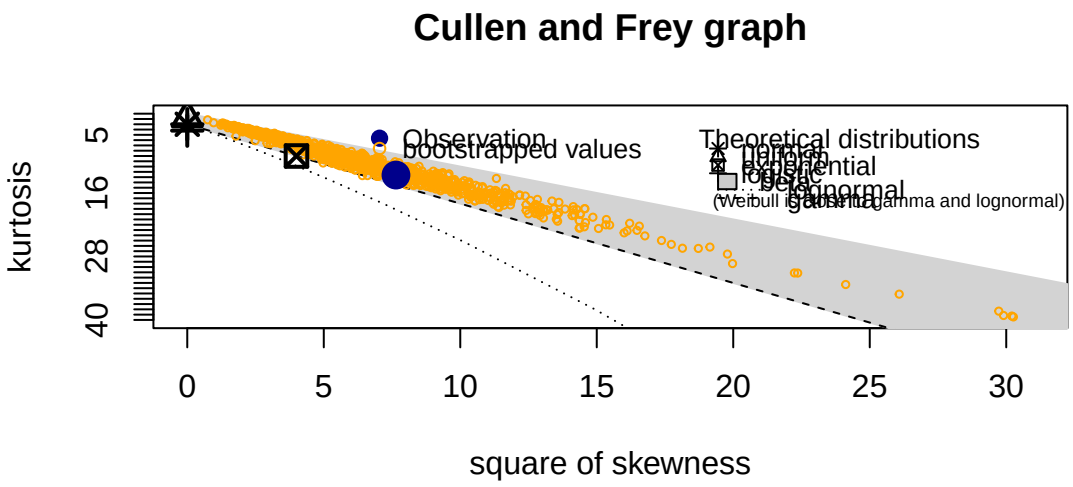
I also draw skewness-kurtosis plots and plot bootstrap samples as well as the true observations. Then we can see that both of the observations in January and July are very close to gamma distribution.[3]

```
descdist(Jan, boot = 1000)
```




```
## summary statistics
## ------
## min:  0.1   max:  3.17
## median:  0.425
## mean:  0.7196429
## estimated sd:  0.7650586
## estimated skewness:  1.643332
## estimated kurtosis:  5.630496
```

```
descdist(Jul, boot = 1000)
```




```
## summary statistics
## ------
## min:  0.1   max:  2.8
## median:  0.2
## mean:  0.393125
## estimated sd:  0.4816733
## estimated skewness:  2.764842
## estimated kurtosis:  12.60617
```

(c)

First I fit a gamma model to the data from each month as below.[3]

```
#Fit the gamma distribution to the data
fitg1 <- fitdist(Jan, "gamma", method = "mle")
fitg1
```

```
## Fitting of the distribution ' gamma ' by maximum likelihood
## Parameters:
##       estimate Std. Error
## shape 1.056222  0.2497495
## rate  1.467650  0.4396202
```

```
fitg2 <- fitdist(Jul, "gamma", method = "mle")
fitg2
```

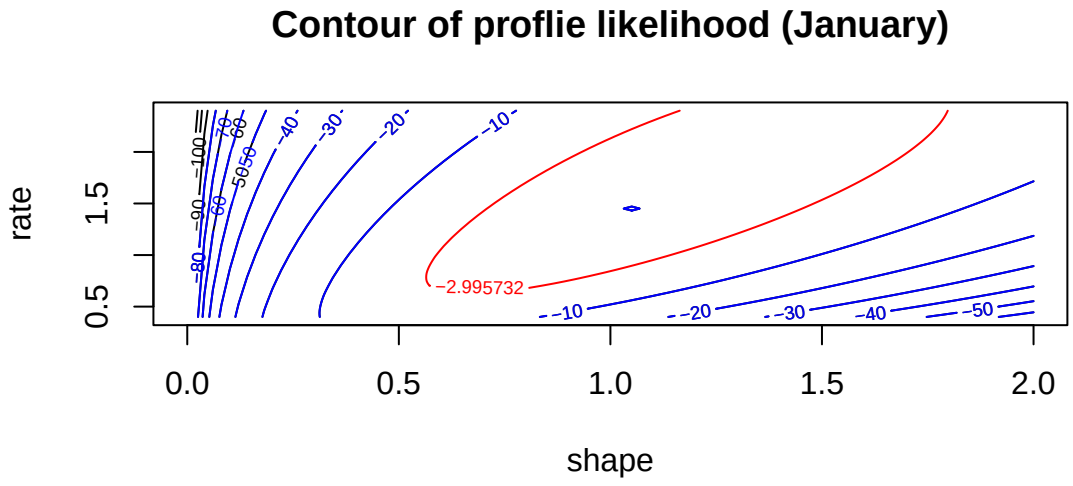
```
## Fitting of the distribution ' gamma ' by maximum likelihood
## Parameters:
##       estimate Std. Error
## shape 1.196419  0.1891196
## rate  3.043403  0.5936302
```

MLE and its standard error for shape parameter in January are 1.0562223 and 0.2497495 respectively. MLE and its standard error for rate parameter in January are 1.4676504 and 0.4396202 respectively. MLE and its standard error for shape parameter in July are 1.1964187 and 0.1891196 respectively. MLE and its standard error for rate parameter in January are 3.0434027 and 0.5936302 respectively.

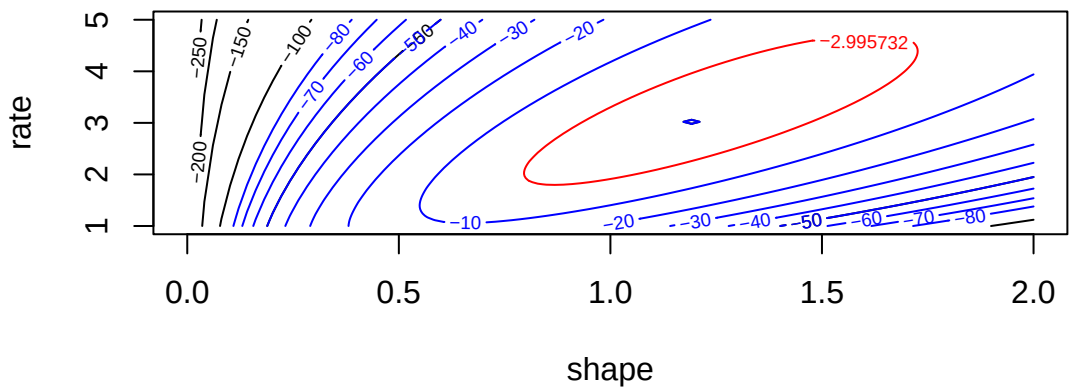
Then I drew profile likelihood with contour based on the results of fitting gamma model to each data set as

below.[4]

```
#Draw contour in January
x <- Jan
NxPts <- 100
NyPts <- 100

xGridPts <- seq(from = 0, to = 2, length = NxPts)
yGridPts <- seq(from = 0.4, to = 2.4, length = NyPts)

xypairs <- expand.grid(xGridPts, yGridPts)
# coordinates of all gridpoints, rows of an N x 2 matrix
# N = NxPts * NyPts
LogLikFun <- function(shape, rate) sum(dgamma(x, shape = shape, rate = rate, log = TRUE))

LogLikOnGrid <- mapply(LogLikFun, xypairs[ , 1], xypairs[ ,2]) # another long vector

## There is an explicit "for loop" going on inside of "mapply".
## There ought to be a "vectorised" way to do this.

dim(LogLikOnGrid) <- c(NxPts, NyPts) # convert to matrix

## I'll subtract the maximum from the log likelihood function.
## We are only interested in *differences*, not in its absolute
## value. Now I know the maximum of the surface is at height z = 0.
## Also I reshape as a matrix.

MaxLogLik <- max(LogLikOnGrid)
LogLikOnGrid <- matrix(LogLikOnGrid, NxPts, NyPts) - max(LogLikOnGrid)

#Draw contour of proflie likelihood

contour(x = xGridPts, y = yGridPts, z = LogLikOnGrid)

contour(x = xGridPts, y = yGridPts, z = LogLikOnGrid,
        levels = c( (0:-8)*10), col = "blue", add = TRUE)

contour(x = xGridPts, y = yGridPts, z = LogLikOnGrid,
        levels = - qchisq(0.95, 2)/2, col = "red", add = TRUE)
title("Contour of proflie likelihood (January)", xlab = "shape", ylab = "rate", font = 4)
```




```
#Draw contour in July (repeat the same process above)
x <- Jul
xGridPts <- seq(from = 0, to = 2, length = NxPts)
yGridPts <- seq(from = 1, to = 5, length = NyPts)
xypairs <- expand.grid(xGridPts, yGridPts)
LogLikFun <- function(shape, rate) sum(dgamma(x, shape = shape, rate = rate, log = TRUE))
LogLikOnGrid <- mapply(LogLikFun, xypairs[ , 1], xypairs[ ,2]) # another long vector
dim(LogLikOnGrid) <- c(NxPts, NyPts)
MaxLogLik <- max(LogLikOnGrid)
LogLikOnGrid <- matrix(LogLikOnGrid, NxPts, NyPts) - max(LogLikOnGrid)
contour(x = xGridPts, y = yGridPts, z = LogLikOnGrid)
contour(x = xGridPts, y = yGridPts, z = LogLikOnGrid,
        levels = c( (0:-8)*10), col = "blue", add = TRUE)
contour(x = xGridPts, y = yGridPts, z = LogLikOnGrid,
        levels = - qchisq(0.95, 2)/2, col = "red", add = TRUE)
title("Contour of proflie likelihood (July)", xlab = "shape", ylab = "rate", font = 4)
```



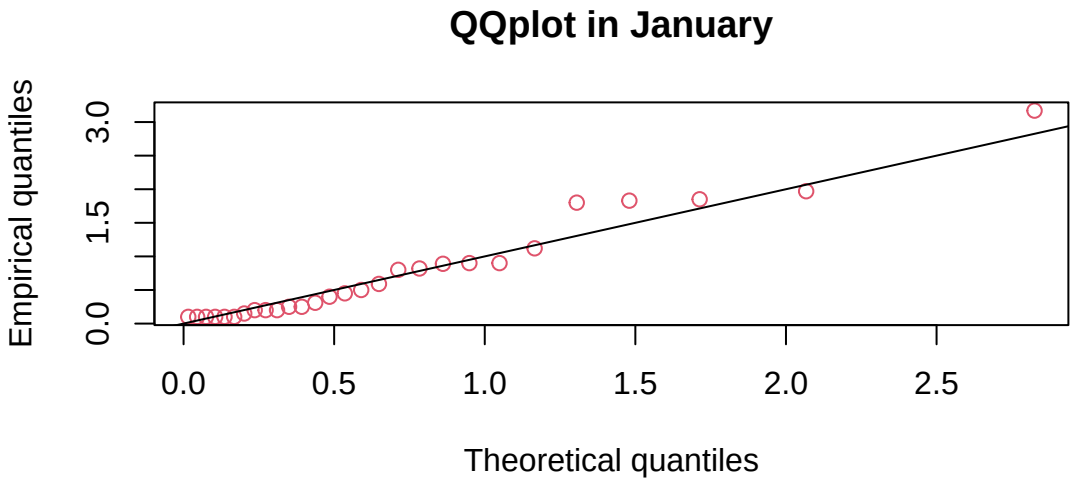
```
# Expectation and variance are calculated by parameters as follows:
# E(x)=hape/rate, Var(x)=shape/(rate^2)
Ex1 <- as.numeric(fitg1$estimate[1]/fitg1$estimate[2])
Var1 <- as.numeric(fitg1$estimate[1]/(fitg1$estimate[2]^2))
Ex2 <- as.numeric(fitg2$estimate[1]/fitg2$estimate[2])
Var2 <- as.numeric(fitg2$estimate[1]/(fitg2$estimate[2]^2))
```

Finally, I compared the parameters from the both data sets. Basically the parameters just decide the shape of gamma distribution, and what's important is we can calculate its expectation (rainfall) and variance based on the parameters.[5] Based on the parameters, the expectation and its variance of January was 0.7196689 and 0.4903544, and those of July was 0.3931188 and 0.1291708. These are quite similar to the mean and variance of the observations. So we can believe this gamma model fit well. We can also see that the expectation of rainfall in January is higher than that in July based on the results.

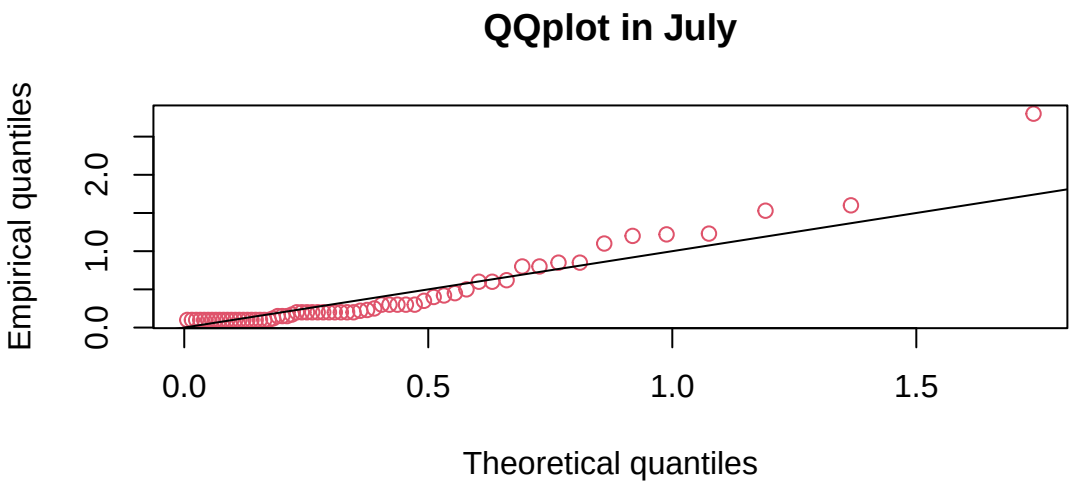
(d)

Lastly, I drew QQplots to check if gamma model is adequate.

```
qqcomp(fitg1, addlegend=FALSE, main="QQplot in January")
```




```
qqcomp(fitg2, addlegend=FALSE, main="QQplot in July")
```



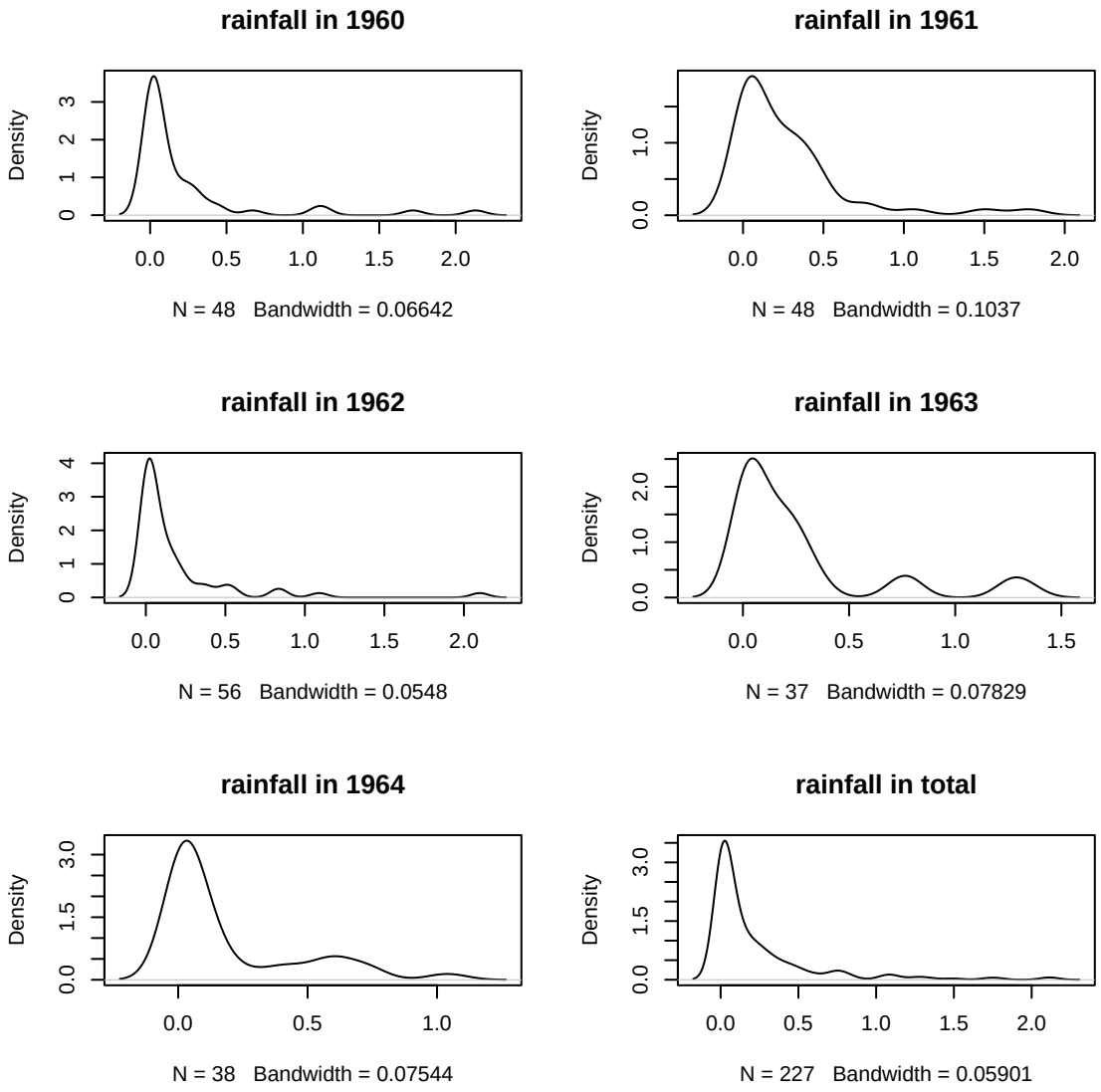
As you can see above, bot QQ-plots show adequate fit on the theoretical quantile line, which suggests this gammma models fit well.

The rainfall in Illinois

Identify the distribution of rainfall and Estimate the parameters of the distribution

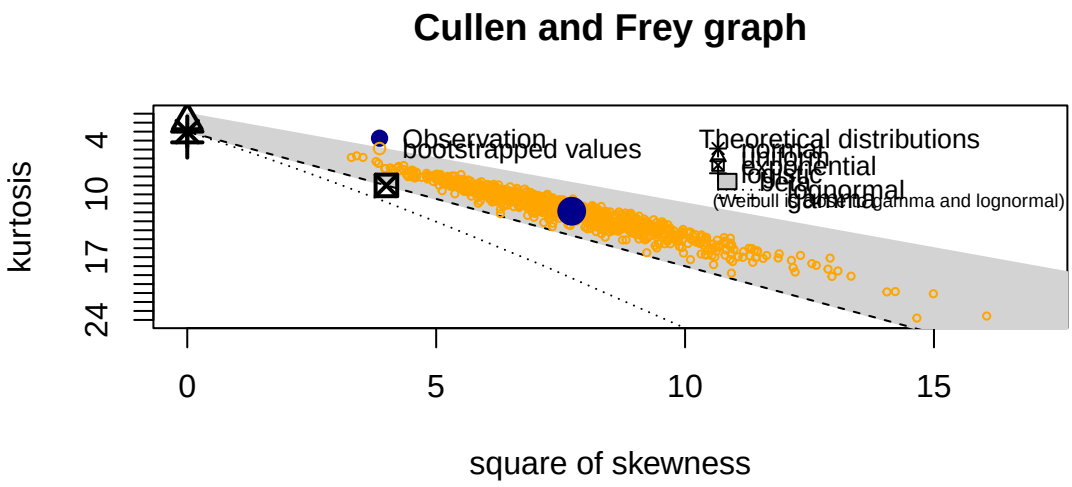
First, I drew density plots for each year as well as the on in total as below.

```
rain=read.xlsx('Illinois_rain_1960-1964.xlsx')
par(mfrow = c(3, 2))
plot(density(na.omit(rain$`1960`)),main="rainfall in 1960")
plot(density(na.omit(rain$`1961`)),main="rainfall in 1961")
plot(density(na.omit(rain$`1962`)),main="rainfall in 1962")
plot(density(na.omit(rain$`1963`)),main="rainfall in 1963")
plot(density(na.omit(rain$`1964`)),main="rainfall in 1964")
plot(density(na.omit(unlist(rain))),main="rainfall in total")
```



Then, I drew kurtosis-skewness plots for the total dataset to see which distribution should be fitted.

```
descdist(as.numeric(na.omit(unlist(rain))), boot = 1000)
```




```
## summary statistics
## ------
## min:  0.001   max:  2.13
## median:  0.07
## mean:  0.2243921
## estimated sd:  0.3658212
## estimated skewness:  2.778925
## estimated kurtosis:  11.87935
```

Based on the plot above, I thought gamma plot could be fitted to this data sets again, and I fitted the gamma model to the data set as below.

```
x <- as.numeric(na.omit(unlist(rain)))
fit <- fitdist(x, "gamma",method="mle")
fit
```

```
## Fitting of the distribution ' gamma ' by maximum likelihood
## Parameters:
##        estimate Std. Error
## shape 0.4408386  0.0337663
## rate  1.9648409  0.2474440
```

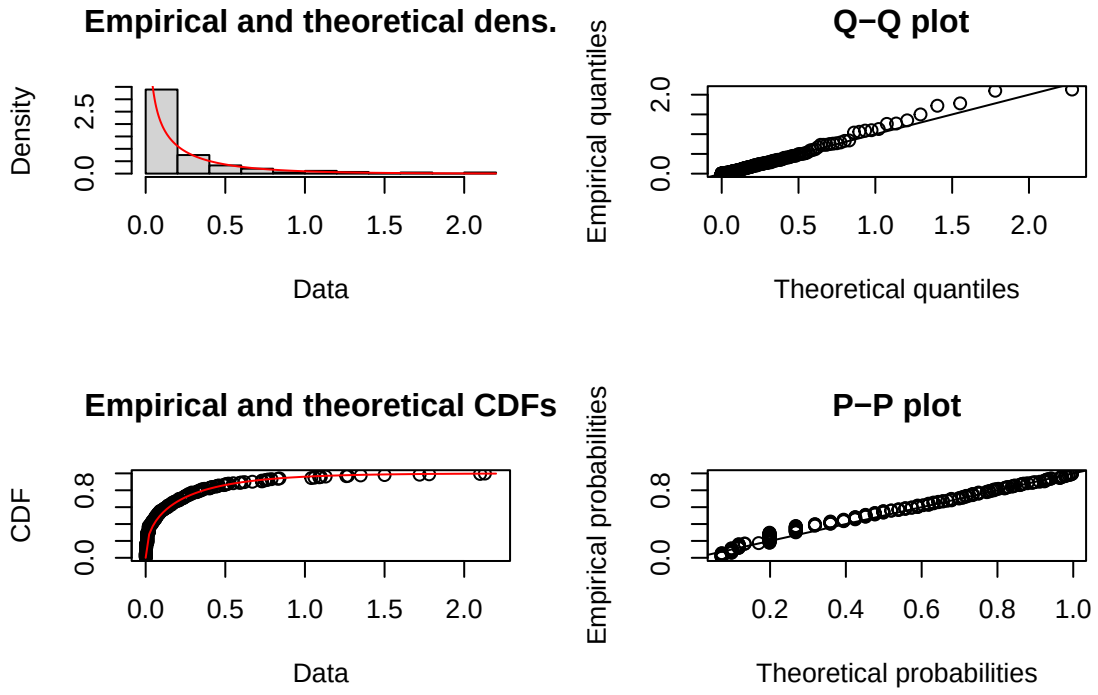
```
set.seed(2022)
summary(bootdist(fit)) #boot get confidence interval
```

Table 5: Parametric bootstrap median and 95%CIs for the MLE parameters

	Median	2.5%	97.5%
shape	0.4453178	0.3807186	0.5163997
rate	1.9989008	1.5697775	2.5787418

The median and 95% CIs for MLEs based on bootstrap samples are shown above. We can also see how the gamma model fit the data by plotting empirical and theoretical density, QQ plot, CDFs, and P-P plot as below. Based on these plot as well as 95%CIs

```
plot(fit)
```



Identify wet and dry years

First I identify wet and dry years by comparing total amount of rainfall in each year with average rainfall across 1960-1961. I also compare the number of storms and the average rainfall individual storm produced in each year with each other to look for the reason of the wet and dry years.

```
totalrain <- apply(rain,2,sum,na.rm=TRUE)
totalave <- mean(totalrain)
individualstorm <- apply(rain,2,mean,na.rm=TRUE)%>%round(5)
indstormave <- round(fit$estimate[1]/fit$estimate[2],digits = 5)
numstorm <- c(nrow(rain)-apply(is.na(rain),2,sum))
avenumstorm <- mean(numstorm)
```

	1960	1961	1962	1963	1964	Average
total rain	10.574	13.197	10.346	9.710	7.110	10.1874
individual storm rain	0.22029	0.27494	0.18475	0.26243	0.18711	0.22436
number of storms	48	48	56	37	38	45.4

As you can see above, we can say that 1960,1961, and 1962 are the wet years and 1963 and 1964 are dry years in terms of total amount of rain. For 1960 and 1962, the average rainfall individual storm produced was less than average of 5 years, but they have more storms, which caused the wet years. For 1961, the average rainfall individual storm produced was higher than average of 5 years and the number of storms was also more than average of 5 years. For 1963 and 1964, they have less storms than average, which seemed to cause dry years, even though the average rainfall individual storm produced in 1963 was higher than average of 5 years.

Generalizability

The cause of wet and dry years were different in each year, sometimes it was the number of storms and sometimes it was average rainfall individual storm produced. So we cannot generalize this assessment to other years, and we need more consistent results by taking more records from other years.

What I learned

From this project, I learned how I could look for what I didn't know totally as well as how to fit the distribution to some data and utilize the fitted distribution. As I have no math background and have a lot of difficulty in completion of this project, but I found I can usually refere to online tips as well as lots of my classmates. This exprience itself is my precious deriveralbe in this project.

Reference

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