Interesting Problems from Various Fields

MAS491 Final Presentation

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Table of Contents

Generalization of Heat Equation

Problem

- Consider following generalization of the heat equation

$$\frac{\partial u}{\partial t} = \nabla \cdot (C(x,t)\nabla u) \tag{1}$$

  where $C(x,t)$ is symmetric, its eigenvalues are all contained in $[a,b], a > 0$ for any x, t, and each C_{ij} are measurable.
- Existence and uniqueness of solution?
- Regularity of solution?

Generalization of Heat Equation

Solution

- Solved by John F. Nash in 1958.
- He assumed stronger condition on $C(x,t)$, and proved existence and uniqueness of global solution ($t > 0$).
- For the regularity he obtained hölder continuity of solution, where the exponent depends only on eigenvalue bounds a, b and dimension n.
- Lastly he extended the result to when $C(x,t)$ is measurable through taking limit.
- Interestingly, while it was him who came with outline of overall proof, most of critical inequalities such as entropy inequalities, were obtained by his collegues without knowing the whole picture.

Applications in Economics: Frequent Batch Auctions

High-frequency Trading Arms Race

Applications in Economics: Loss Versus Rebalancing

Loss Versus Rebalancing (LVR)