FMAMM w/ on-chain Batching

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \xrightarrow{\text{FMAMM}} \begin{bmatrix} x_0 + \Delta x \\ y_0 \cdot \dfrac{x_0 + \Delta x}{x_0 + 2\Delta x} \end{bmatrix} \text{ or } \begin{bmatrix} x_0 \cdot \dfrac{y_0 + \Delta y}{y_0 + 2\Delta y} \\ y_0 + \Delta y \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1/(y_0 + 2y_{in}) \\ 1/(x_0 + 2x_{in}) \end{bmatrix} (x_0 y_0 + x_0 y_{in} + x_{in} y_0)$$

How to implement On-chain Batching?

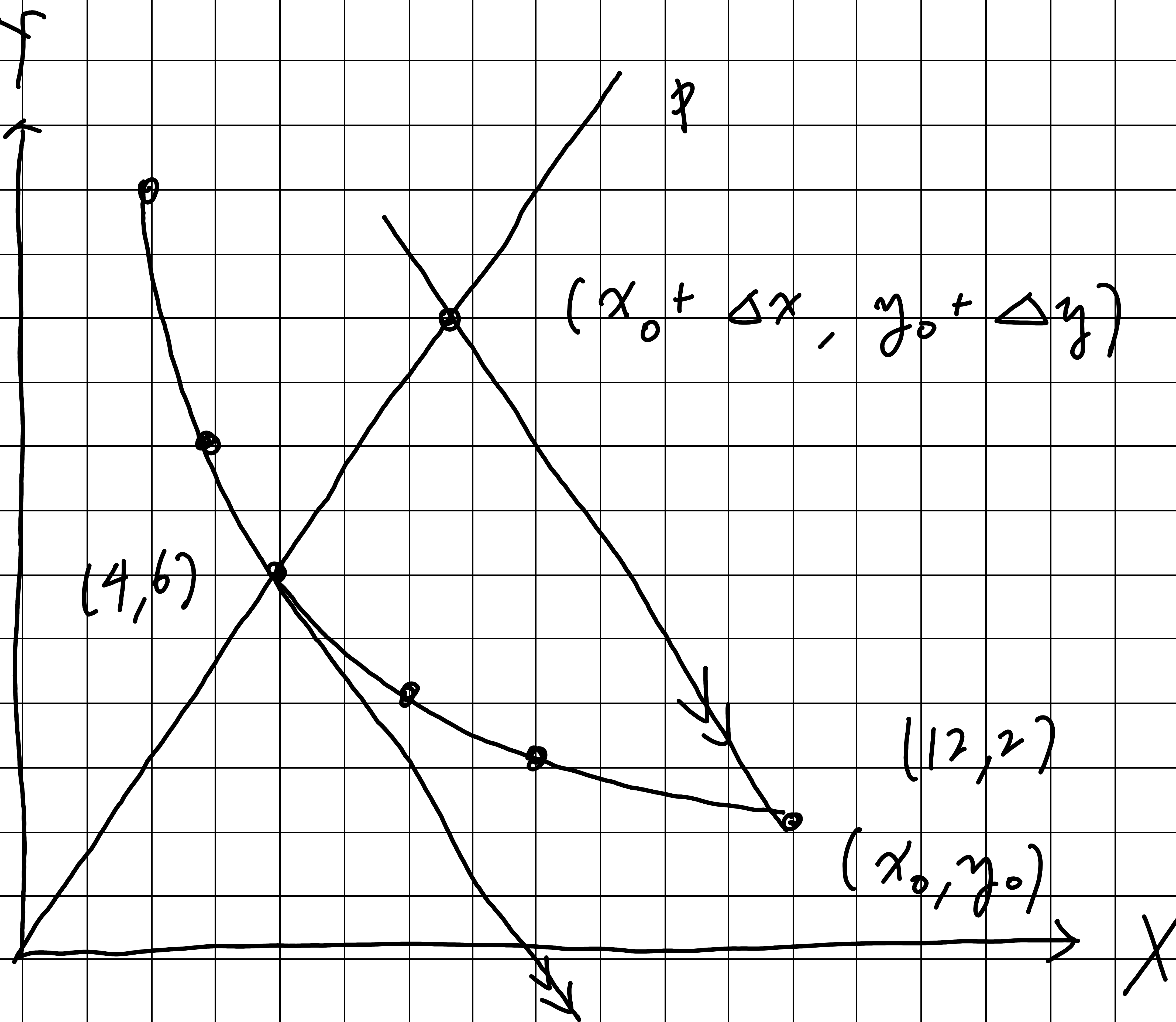
↳ at most 1 settlement per block.

: better than CPMM, but suboptimal.

↳ extend batch window for every order

: optimal but UX is terrible.

{ submit_order() clear() : once per block
cancel_order() ⟹ ↳ { burn ⟹ claim().
mint() swap
burn(). mint



$$\frac{y_0+\Delta y}{x_0+\Delta x} = -\frac{\Delta y}{\Delta x} \quad (=: p)$$

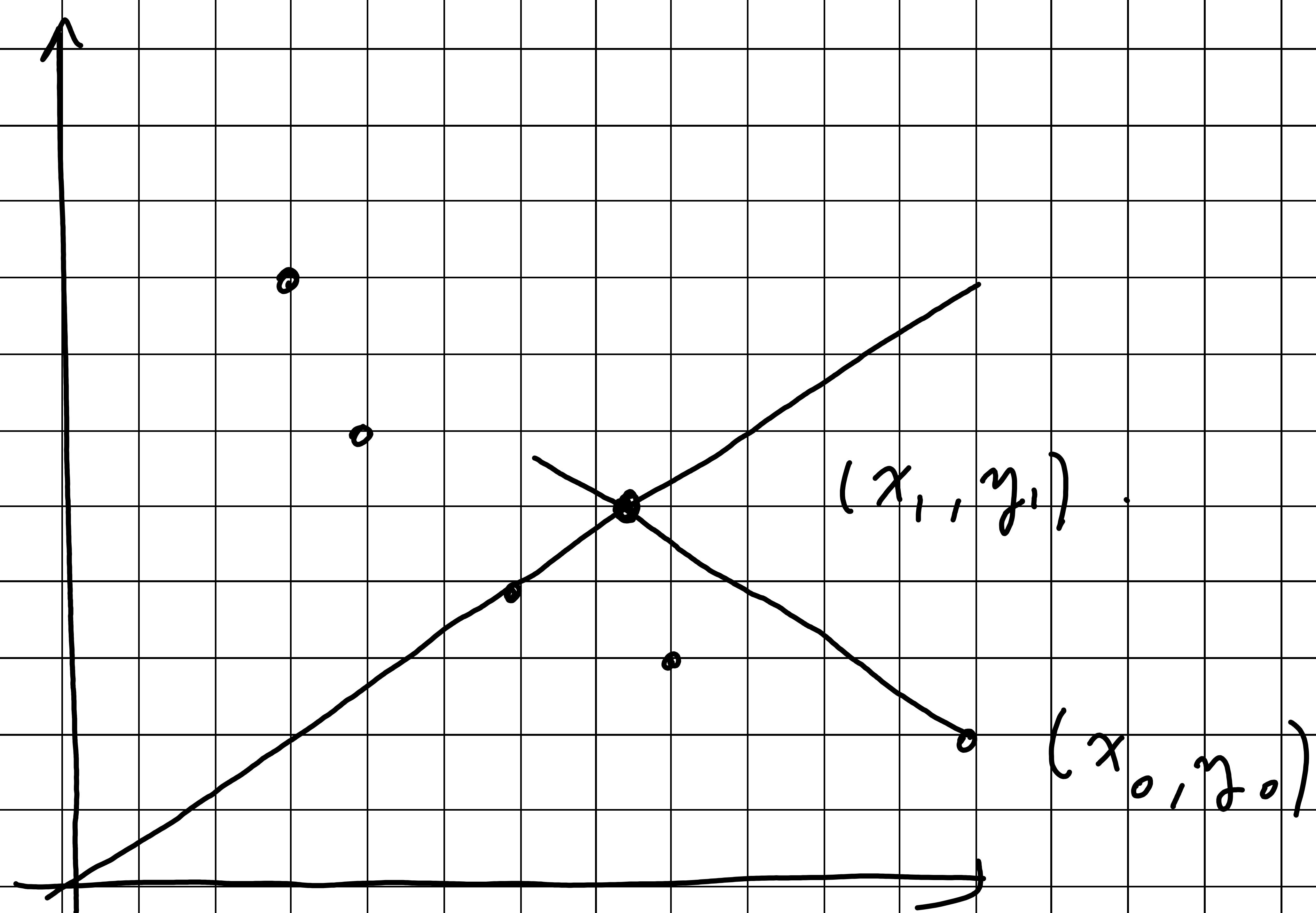
$$\Delta x\, y_0 + \Delta x\, \Delta y = -\Delta y\, x_0 - \Delta x\, \Delta y$$

$$\Delta y\,(x_0 + 2\Delta x) = -\Delta x\, y_0$$

$$(\Longrightarrow) \begin{cases} \Delta y = \dfrac{-y_0\,\Delta x}{x_0+2\Delta x} \\[2ex] \Delta x = \dfrac{-x_0\,\Delta y}{y_0+2\Delta y} \end{cases}$$

$$\begin{bmatrix} x_0 \\ \\ y_0 \end{bmatrix} \xrightarrow{\text{FMAMM}} \begin{bmatrix} x_0+\Delta x \\ \\ y_0 \cdot \dfrac{x_0+\Delta x}{x_0+2\Delta x} \end{bmatrix}$$

$(x_0, y_0) \xrightarrow[(x_{in},\, y_{in})]{} (x_1, y_1)$



$$\begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} = x_{in} \begin{bmatrix} 1 \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix}.$$

$$\begin{bmatrix} x_{in} \\ y_{in} - \alpha \end{bmatrix}, \begin{bmatrix} x_0 \cdot \dfrac{y_0 + \alpha}{y_0 + 2\alpha} \\ y_0 + \alpha \end{bmatrix} \in \operatorname{span}\left(\left\{ \begin{bmatrix} 1 \\ p \end{bmatrix} \right\}\right).$$

$$\Rightarrow \quad x_{in}\cancel{(y_0+\alpha)} = (y_{in}-\alpha)\cdot x_0 \cdot \frac{\cancel{y_0+\alpha}}{y_0+2\alpha}$$

$$x_{in}(y_0+2\alpha) = (y_{in}-\alpha)x_0$$

$$2x_{in}\alpha + x_{in}y_0 = -x_0\alpha + x_0 y_{in}$$

$$(2x_{in}+x_0)\alpha = x_0 y_{in} - x_{in}y_0$$

$$\alpha = \frac{x_0 y_{in} - x_{in}y_0}{x_0+2x_{in}}$$

$$\therefore \quad y_0+\alpha = \frac{x_0y_0 + 2y_0x_{in} + x_0y_{in} - x_{in}y_0}{x_0+2x_{in}}$$

$$= \frac{x_0y_0 + x_0y_{in} + x_{in}y_0}{x_0+2x_{in}}$$

$$y_0+2\alpha = \frac{x_0y_0 + 2y_0x_{in} + 2x_0y_{in} - 2x_{in}y_0}{x_0+2x_{in}}$$

$$= \frac{x_0y_0 + 2x_0y_{in}}{x_0+2x_{in}}$$

$$\Rightarrow x_0 \cdot \frac{y_0 + \alpha}{y_0 + 2\alpha}$$

$$= \cancel{x_0} \cdot \frac{x_0 y_0 + x_0 y_{in} + x_{in} y_0}{\cancel{x_0 + 2x_{in}}} \cdot \frac{\cancel{x_0 + 2x_{in}}}{\cancel{x_0}(y_0 + 2y_{in})}$$

$$= \frac{x_0 y_0 + x_0 y_{in} \; x_{in} y_0}{y_0 + 2y_{in}}$$

$$\begin{bmatrix} x_1 \\ \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 / (y_0 + 2y_{in}) \\ \\ 1 / (x_0 + 2x_{in}) \end{bmatrix} (x_0 y_0 + x_0 y_{in} + x_{in} y_0)$$

$$\phi_1 = \frac{y_1}{x_1} = \frac{1}{x_0 + 2x_{in}} \cdot \left(\frac{1}{y_0 + 2y_{in}} \right)^{-1}$$

$$= \frac{y_0 + 2y_{in}}{x_0 + 2x_{in}}$$

trader's PnL $:= p\cdot(x_{out} - x_{in}) + y_{out} - y_{in}$

$$\begin{bmatrix} x_{out} \\ y_{out} \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} - \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix}$$

$$- \begin{bmatrix} (x_0 y_0 + x_0 y_{in} + x_{in} y_0) / (y_0 + 2y_{in}) \\ (x_0 y_0 + x_0 y_{in} + x_{in} y_0) / (x_0 + 2x_{in}) \end{bmatrix}$$

$$= \begin{bmatrix} y_{in}(x_0 + 2x_{in}) / (y_0 + 2y_{in}) \\ x_{in}(y_0 + 2y_{in}) / (x_0 + 2x_{in}) \end{bmatrix}.$$

$$\begin{bmatrix} x_{out} - x_{in} \\ y_{out} - y_{in} \end{bmatrix} = (x_0 y_{in} - x_{in} y_0) \begin{bmatrix} \dfrac{1}{y_0 + 2y_{in}} \\ -\dfrac{1}{x_0 + 2x_{in}} \end{bmatrix}$$

$$\therefore \text{PnL} = (x_0 y_{in} - x_{in} y_0) \times \left(\frac{p}{y_0 + 2y_{in}} - \frac{1}{x_0 + 2x_{in}} \right).$$

i) Price goes up

$$PnL = (x_0 y_{in} - x_{in} y_0) \cdot \left(\frac{p}{y_0 + 2y_{in}} - \frac{1}{x_0 + 2x_{in}} \right)$$

Let $x_0 = \frac{L}{\sqrt{P_0}}$, $y_0 = L\sqrt{P_0}$, $P \geq P_0$.

$$PnL\Big|_{x_{in}=0} = x_0 y_{in} \cdot \left(\frac{p}{y_0 + 2y_{in}} - \frac{1}{x_0} \right).$$

$$\frac{\partial}{\partial y_{in}} \left(PnL\Big|_{x_{in}=0} \right) = x_0 \cdot \left(\frac{p}{y_0 + 2y_{in}} - \frac{1}{x_0} \right) + x_0 y_{in} \cdot -\frac{p}{(y_0 + 2y_{in})^2} \cdot 2$$

$$= \frac{-2Px_0 y_{in} + Px_0 (y_0 + 2y_{in})}{(y_0 + 2y_{in})^2} - 1$$

$$= Px_0 \cdot \frac{y_0}{(y_0 + 2y_{in})^2} - 1$$

local maximum

$$\Longleftrightarrow \quad Px_0 y_0 = (y_0 + 2y_{in})^2 = L^2 P.$$

$$\Longleftrightarrow \quad L\sqrt{P} = y_0 + 2y_{in} = L\sqrt{P_0} + 2y_{in}$$

$$\Longrightarrow \quad y_{in} = \frac{L(\sqrt{P} - \sqrt{P_0})}{2}$$

$$\therefore \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$= \begin{bmatrix} (x_0 y_0 + x_0 y_{in} + y_0 x_{in}) / (y_0 + 2y_{in}) \\ (x_0 y_0 + x_0 y_{in} + y_0 x_{in}) / (x_0 + 2x_{in}) \end{bmatrix}$$

$$= \begin{bmatrix} x_0 (y_0 + y_{in}) / (y_0 + 2y_{in}) \\ x_0 (y_0 + y_{in}) / x_0 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{L}{\sqrt{P_0}} \cdot \dfrac{\cancel{L}(\sqrt{P_0} + \sqrt{P})}{2} \cdot \dfrac{1}{\cancel{L}\sqrt{P}} \\ \dfrac{L}{\cancel{\sqrt{P_0}}} \cdot \dfrac{\cancel{L}(\sqrt{P_0} + \sqrt{P})}{2} \cdot \dfrac{\cancel{\sqrt{P_0}}}{\cancel{L}} \end{bmatrix}$$

$$= L \cdot \frac{\sqrt{P_0} + \sqrt{P}}{2} \begin{bmatrix} \dfrac{1}{\sqrt{PP_0}} \\ 1 \end{bmatrix}.$$

$$= L \cdot \frac{\sqrt{P_0} + \sqrt{P}}{2} \cdot (\sqrt{PP_0})^{-\frac{1}{2}} \begin{bmatrix} (\sqrt{PP_0})^{-\frac{1}{2}} \\ (\sqrt{PP_0})^{\frac{1}{2}} \end{bmatrix}.$$

$$\Rightarrow (L, P_0) \xrightarrow{Arb} \left(L \cdot \frac{AM(\sqrt{P}, \sqrt{P_0})}{GM(\sqrt{P}, \sqrt{P_0})}, \; GM(P, P_0) \right)$$

iii) price goes down

$$PnL = (x_0 y_{in} - x_{in} y_0) \cdot \left(\frac{P}{y_0 + 2y_{in}} - \frac{1}{x_0 + 2x_{in}} \right)$$

Let $x_0 = \frac{L}{\sqrt{P_0}}$, $y_0 = L\sqrt{P_0}$, $P \le P_0$.

$$PnL\Big|_{y_{in}=0} = -x_{in} y_0 \left(\frac{P}{y_0} - \frac{1}{x_0 + 2x_{in}} \right)$$

$$= x_{in} y_0 \left(\frac{1}{x_0 + 2x_{in}} - \frac{P}{y_0} \right)$$

$$\frac{\partial}{\partial x_{in}} \left(PnL\Big|_{y_{in}=0} \right) = y_0 \left(\frac{1}{x_0 + 2x_{in}} - \frac{P}{y_0} \right)$$

$$+ x_{in} y_0 \cdot \frac{-1}{(x_0 + 2x_{in})^2} \cdot 2$$

$$= y_0 \cdot \frac{x_0 + 2x_{in} - 2x_{in}}{(x_0 + 2x_{in})^2} - P$$

$$= y_0 \cdot \frac{x_0}{(x_0 + 2x_{in})^2} - P$$

local maximum

$$\Leftrightarrow \frac{x_0 y_0}{(x_0 + 2x_{in})^2} = P$$

$$\Leftrightarrow (x_0 + 2x_{in})^2 = \frac{L^2}{P}$$

$$\Rightarrow x_{in} = \frac{1}{2}\left(\frac{L}{\sqrt{P}} - \frac{L}{\sqrt{P_0}} \right) = \frac{L}{2}\left(\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{P_0}} \right).$$

$$\therefore \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$= \begin{bmatrix} (x_0 y_0 + x_0 y_{in} + y_0 x_{in}) / (y_0 + 2y_{in}) \\ (x_0 y_0 + x_0 y_{in} + y_0 x_{in}) / (x_0 + 2x_{in}) \end{bmatrix}$$

$$= \begin{bmatrix} (x_0 y_0 + y_0 x_{in}) / y_0 \\ (x_0 y_0 + y_0 x_{in}) / (x_0 + 2x_{in}) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{L}{2}\left(\frac{1}{\sqrt{P}} + \frac{1}{\sqrt{P_0}}\right) \\ \frac{L}{2}\left(\frac{1}{\sqrt{P}} + \frac{1}{\sqrt{P_0}}\right) \cdot \cancel{L}\sqrt{P_0} \cdot \frac{\sqrt{P}}{\cancel{L}} \end{bmatrix}$$

$$= \frac{L}{2} \cdot \left(\frac{1}{\sqrt{P}} + \frac{1}{\sqrt{P_0}}\right) \begin{bmatrix} 1 \\ \sqrt{PP_0} \end{bmatrix}.$$

$$= L \cdot \frac{\sqrt{P_0} + \sqrt{P}}{2} \cdot \left(\sqrt{PP_0}\right)^{-\frac{1}{2}} \begin{bmatrix} \left(\sqrt{PP_0}\right)^{-\frac{1}{2}} \\ \left(\sqrt{PP_0}\right)^{\frac{1}{2}} \end{bmatrix}$$

$$\Rightarrow (L, P_0) \xrightarrow{Arb} \left(L \cdot \frac{AM(\sqrt{P}, \sqrt{P_0})}{GM(\sqrt{P}, \sqrt{P_0})},\ GM(P, P_0) \right)$$

$$\Rightarrow (L_n, P_n) = \left(L_{n-1} \cdot \frac{AM(\sqrt{P}, \sqrt{P_{n-1}})}{GM(\sqrt{P}, \sqrt{P_{n-1}})}, GM(P, P_{n-1}) \right).$$

$$\begin{bmatrix} x_{out} \\ y_{out} \end{bmatrix} = \begin{bmatrix} y_{in}(x_0 + 2x_{in}) / (y_0 + 2y_{in}) \\ x_{in}(y_0 + 2y_{in}) / (x_0 + 2x_{in}) \end{bmatrix}.$$

$$x_{out}(y_0 + 2y_{in}) = y_{in}(x_0 + 2x_{in})$$

$$y_{out}(x_0 + 2x_{in}) = x_{in}(y_0 + 2y_{in}).$$

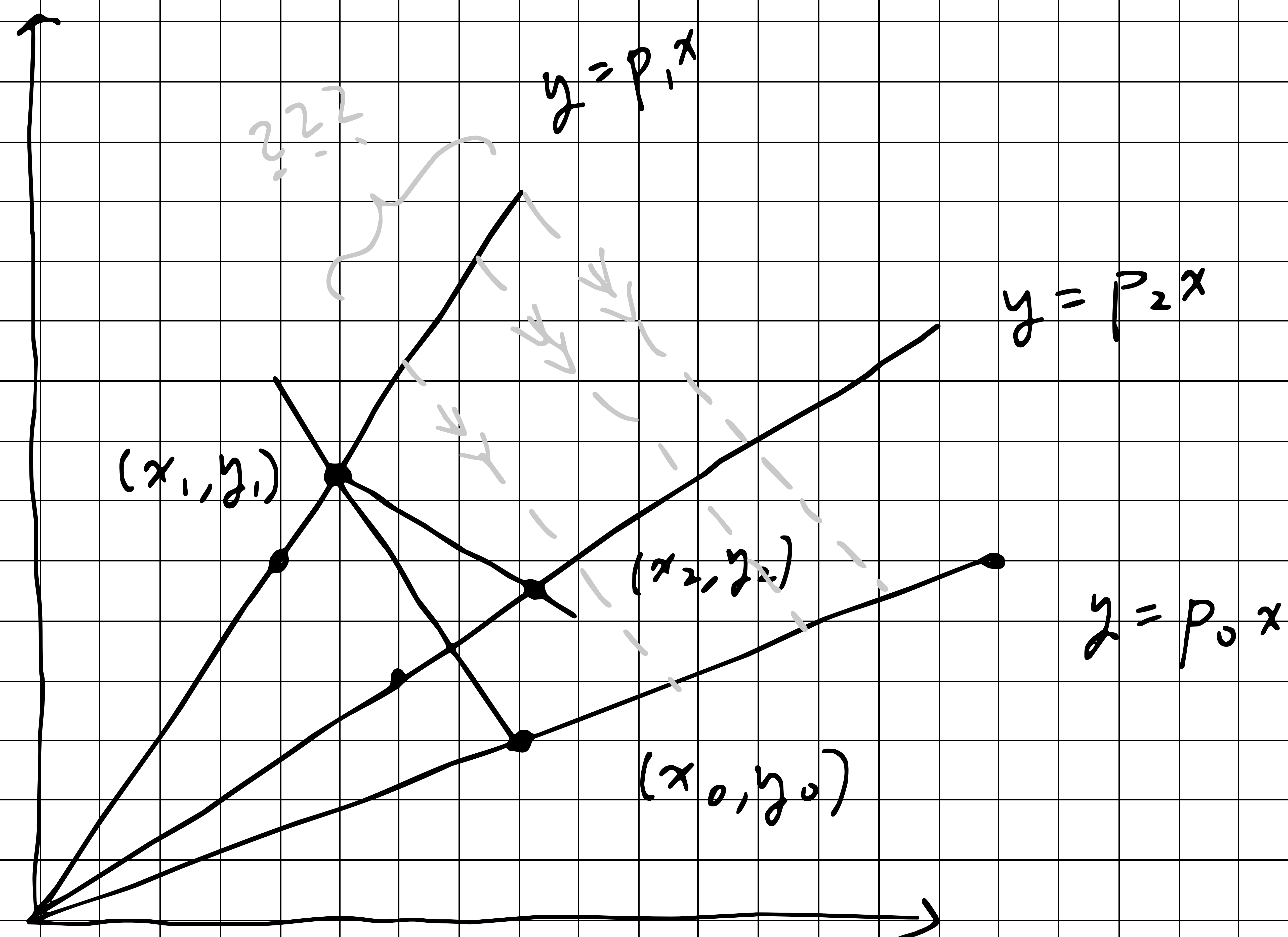
$$-y_{in} x_0 + x_{out} y_0 = 2x_{in} y_{in} - 2x_{out} y_{in}$$

$$y_{out} x_0 - x_{in} y_0 = 2x_{in} y_{in} - 2x_{in} y_{out}$$

$$\begin{bmatrix} -y_{in} & x_{out} \\ y_{out} & -x_{in} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = 2 \begin{bmatrix} x_{in} y_{in} - x_{out} y_{in} \\ x_{in} y_{in} - x_{in} y_{out} \end{bmatrix}$$

$$-y_{in} : x_{out} = y_{out} : -x_{in}.$$

$\Rightarrow$ singular $\Rightarrow$ not solvable.



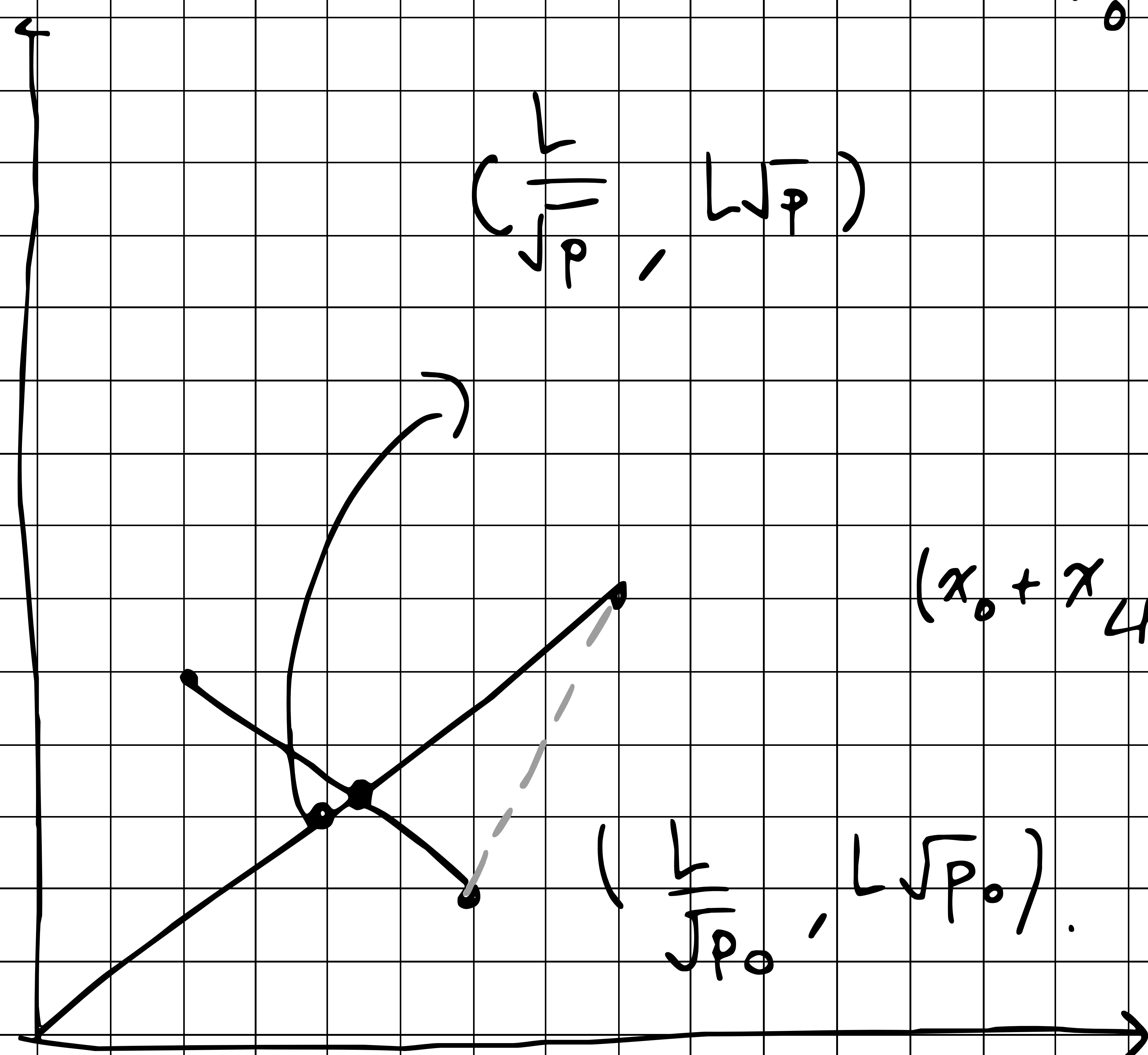
p_i : can be tracked.

(x_i, y_i) : cannot be tracked

if $(x_{mint}, y_{mint}, x_{in}, y_{in})^i$

is encrypted.

$x_0 + \frac{y_0}{p}$



$6 \times 2 \longrightarrow 8 \times 6.$

typical AMM : $L_0 = \sqrt{12}$

$L_1 = \sqrt{48}$.

$$xy = \frac{1}{4p}(y_0 + px_0)^2$$

FMAMM : $L_0 = \sqrt{12} \longrightarrow \sqrt{12} \times \alpha$

$L_1 = \sqrt{48}$.

$$\begin{cases} y = px \\ (-y + L\sqrt{p_0}) = +p\left(x - \frac{L}{\sqrt{p_0}}\right). \end{cases}$$

$$2\cancel{p}x = \left(L\frac{\sqrt{p_0}}{p} + L\cdot\frac{\cancel{p}}{\sqrt{p_0}}\right)$$

$$2y = L\left(\sqrt{p_0} + \frac{p}{\sqrt{p_0}}\right)$$

$$= y_0 + p\,x_0.$$

$$2x = \frac{y_0}{p} + x_0.$$

$$xy = \frac{1}{4p} \cdot (y_0 + p x_0)^2$$

$$= \frac{1}{4p} \cdot L_0^2 \left(\sqrt{p_0} + \frac{p}{\sqrt{p_0}} \right)^2$$

$$= \frac{1}{4 p p_0} \cdot L_0^2 (p + p_0)^2$$

$$= \left(\frac{p + p_0}{2} \right)^2 \cdot \frac{1}{p p_0} \cdot L_0^2$$

$$\Rightarrow L = L_0 \cdot \frac{AM(p, p_0)}{GM(p, p_0)}.$$

$$= L_0 \cdot \frac{1}{2} \cdot \left(\frac{y_0}{x_0} + \frac{y}{x} \right) \times \frac{1}{\sqrt{\frac{y_0}{x_0} \times \frac{y}{x}}}$$

$$= L_0 \cdot \frac{1}{2} \cdot \frac{x y_0 + x_0 y}{x x_0} \times \frac{\sqrt{x x_0}}{\sqrt{y y_0}}.$$

$$= L_0 \cdot \frac{1}{2} \cdot \frac{x y_0 + x_0 y}{\sqrt{x y x_0 y_0}}$$

$$L_0 \cdot \frac{AM(P_0, \sqrt{PP_0})}{GM(P_0, \sqrt{PP_0})}$$

$$= L_0 \cdot \frac{1}{2}\left(P_0 + \sqrt{PP_0}\right) \times \frac{1}{\sqrt{P_0 \cdot \sqrt{PP_0}}}$$

$$= L_0 \cdot \frac{1}{2}\sqrt{P_0}\left(\sqrt{P_0} + \sqrt{P}\right) \times \frac{1}{\sqrt{P_0} \cdot \sqrt{\sqrt{P} \cdot \sqrt{P_0}}}$$

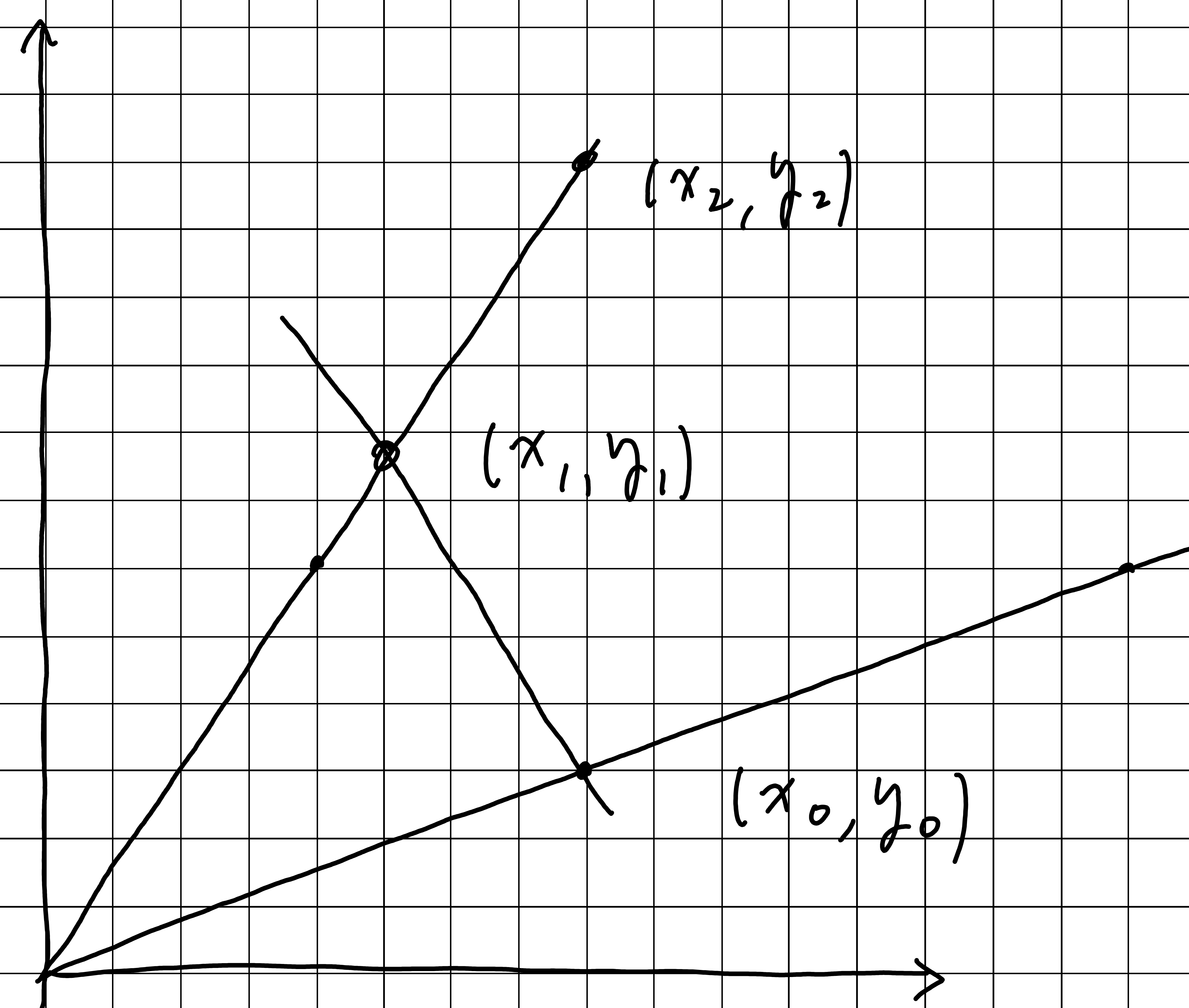
$$= L_0 \cdot \frac{1}{2}\left(\sqrt{P_0} + \sqrt{P}\right) \cdot \frac{1}{\sqrt{\sqrt{P} \cdot \sqrt{P_0}}}$$

$$= L_0 \cdot \frac{AM(\sqrt{P_0}, \sqrt{P})}{GM(\sqrt{P_0}, \sqrt{P})}.$$

Principle :

1. Batch the Actions that can move price.
2. immidiate otherwise.

Batch Mint & Swap



$$\begin{bmatrix} x_{mint} \\ y_{mint} \end{bmatrix} = x_{mint} \begin{bmatrix} 1 \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ 2\alpha \end{bmatrix} \left(\longrightarrow \left(x_{mint} + \frac{\alpha}{p}\right) \begin{bmatrix} 1 \\ p \end{bmatrix} \right).$$

$$\begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} = x_{in} \begin{bmatrix} 1 \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x_0 \cdot \dfrac{y_0 + \alpha + \beta}{y_0 + 2\alpha + 2\beta} \\ y_0 + \alpha + \beta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_{mint} \\ y_{mint} - 2\alpha \end{bmatrix} \parallel \begin{bmatrix} x_{in} \\ y_{in} - \beta \end{bmatrix} \parallel \begin{bmatrix} x_0 \cdot \dfrac{y_0 + \alpha + \beta}{y_0 + 2\alpha + 2\beta} \\ y_0 + \alpha + \beta \end{bmatrix}$$

$$\begin{bmatrix} x_{mint} \\ y_{mint} - 2\alpha \end{bmatrix} // \begin{bmatrix} x_{in} \\ y_{in} - \beta \end{bmatrix} // \begin{bmatrix} x_0 \cdot \cancel{\dfrac{y_0+\alpha+\beta}{y_0+2\alpha+2\beta}} \\ \cancel{y_0+\alpha+\beta} \end{bmatrix}$$

$$y_0+2\alpha+2\beta$$

$$\Rightarrow \begin{cases} x_{mint}(y_0+2\alpha+2\beta) = x_0(y_{mint}-2\alpha) \\ x_{in}(y_0+2\alpha+2\beta) = x_0(y_{in}-\beta) \end{cases}$$

$$\Rightarrow \begin{cases} (2x_{mint}+2x_0)\alpha + 2x_{mint}\beta = x_0 y_{mint} - x_{mint} y_0 \\ 2x_{in}\alpha + (2x_{in}+x_0)\beta = x_0 y_{in} - x_{in} y_0 \end{cases}$$

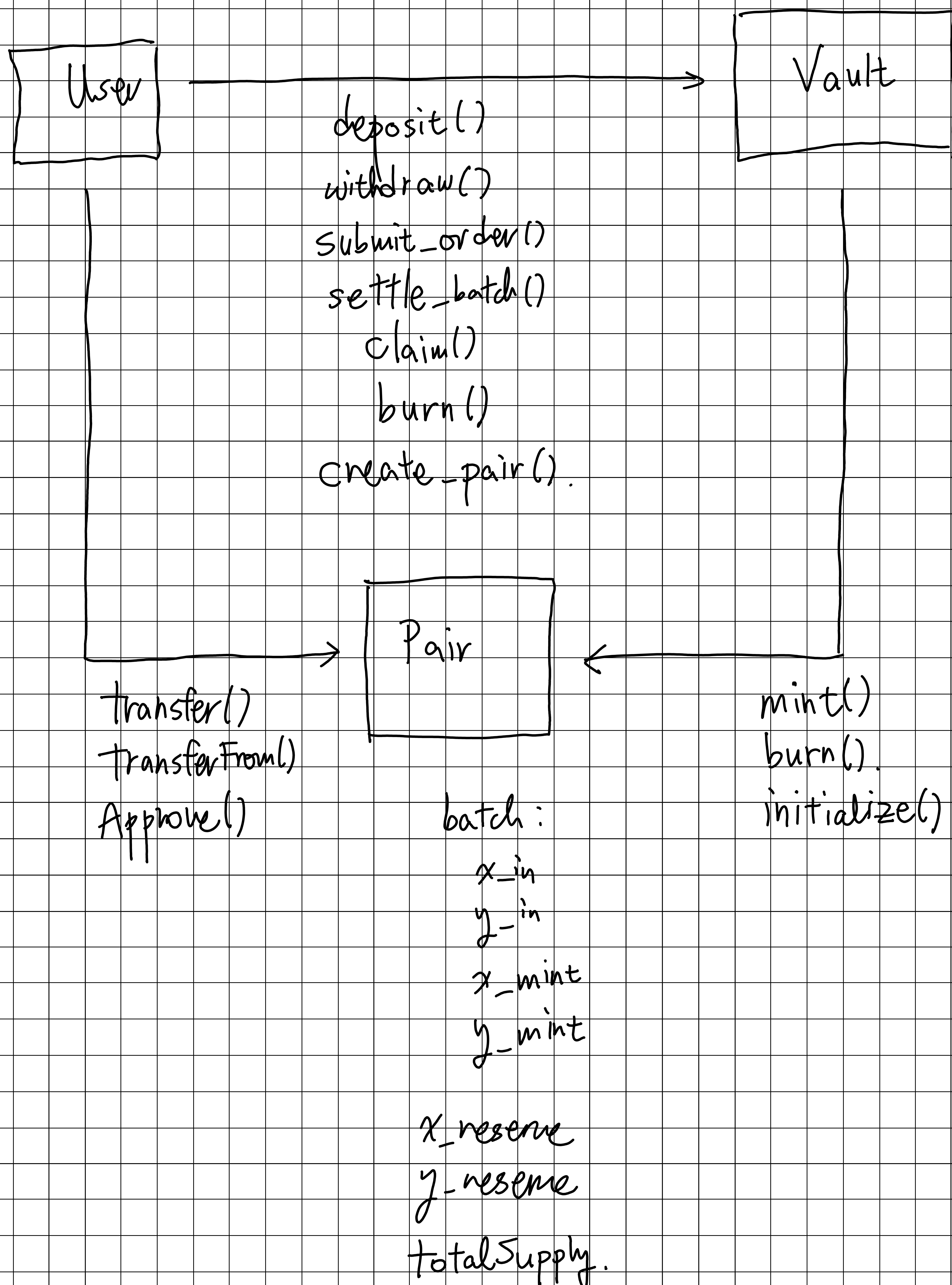
$$\Rightarrow \begin{bmatrix} 2x_0+2x_{mint} & 2x_{mint} \\ 2x_{in} & 2x_{in}+x_0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} x_0 y_{mint} - x_{mint} y_0 \\ x_0 y_{in} - x_{in} y_0 \end{bmatrix}$$

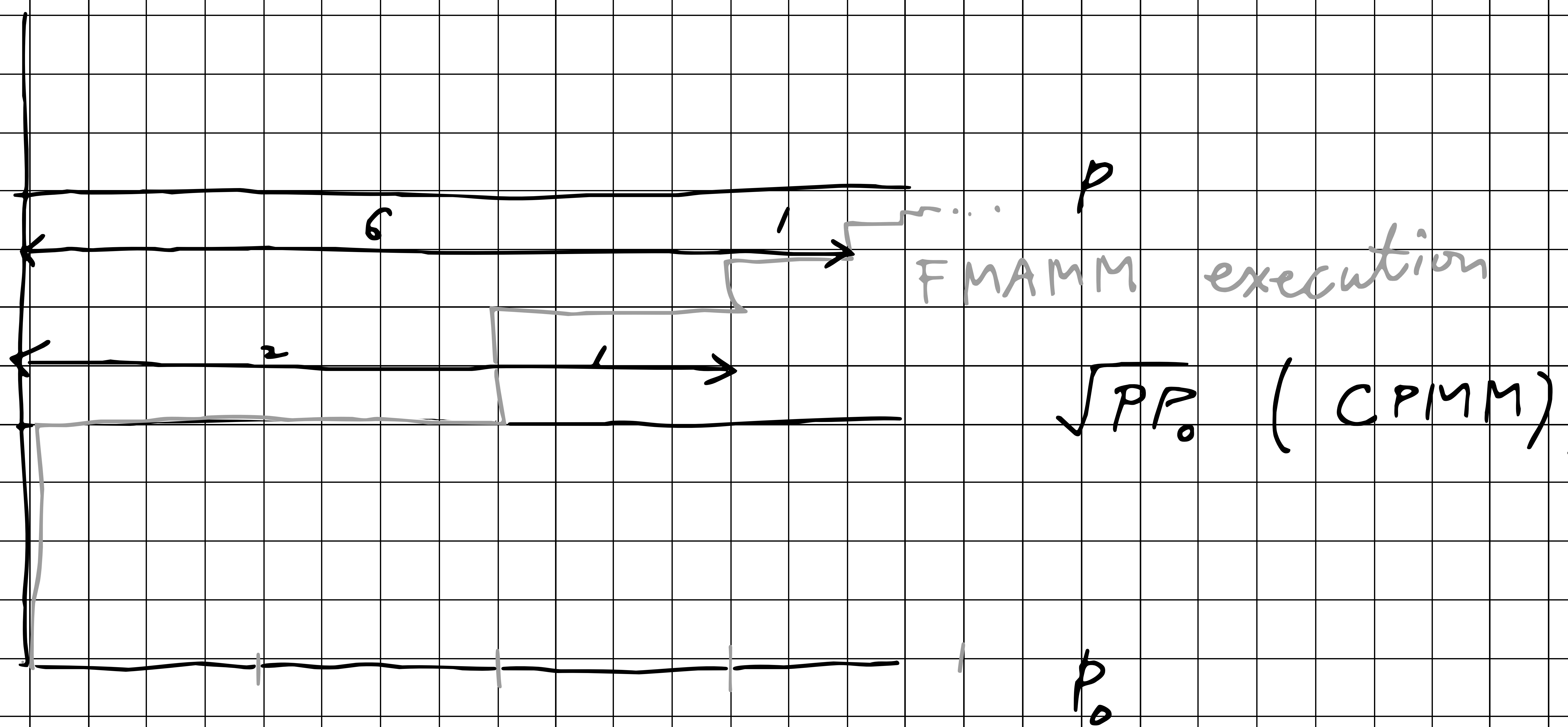
$$(=: M)$$

$$\det M = 2x_0^2 + 2x_0(x_{mint}+2x_{in}) > 0.$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \longrightarrow \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$$mintShare = \frac{p x_{user} + y_{user}}{p x_{mint} + y_{mint}} \times \left(\sqrt{\frac{x_2 y_2}{x_1 y_1}} - 1 \right) \times TotalSupply$$


User
Vault
deposit()
withdraw()
submit_order()
settle_batch()
claim()
burn()
create_pair().
Pair
transfer()
transferFrom()
Approve()
mint()
burn().
initialize()
batch:
x_in
y_in
x_mint
y_mint
x_reserve
y_reserve
totalSupply.



last_action + 1 < block number.