FuGaZi

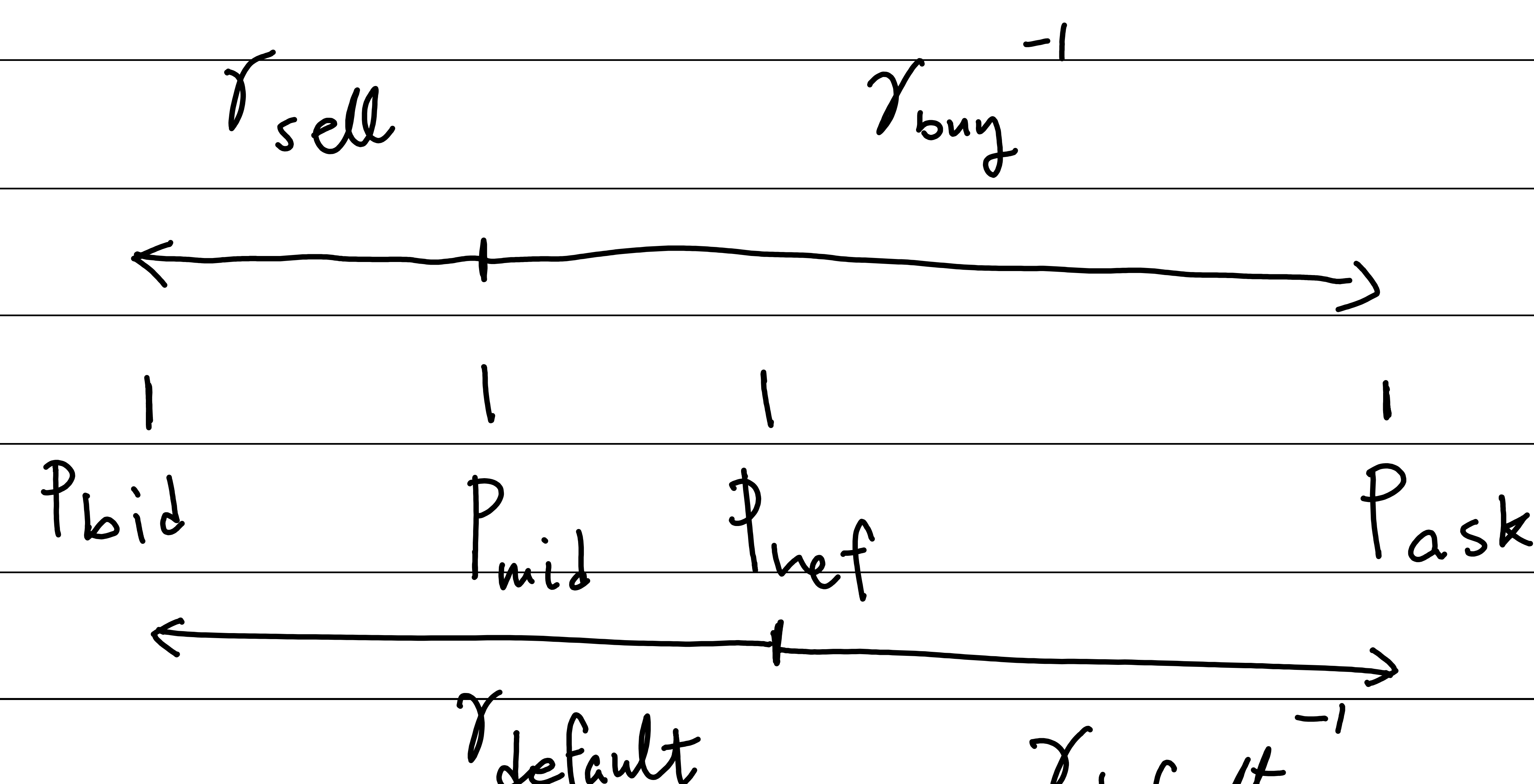
Optimal input routing:

$$\delta_i = \frac{L_i}{\sqrt{r_i}} \cdot \frac{\Delta + \sum \frac{X_j}{r_j}}{\sum \frac{L_j}{\sqrt{r_j}}} - \frac{X_i}{r_i}$$

Optimal output routing:

$$\delta_i = Y_i - \frac{L_i}{\sqrt{r_i}} \cdot \frac{\sum Y_j - \Delta}{\sum \frac{L_j}{\sqrt{r_j}}}$$

Dynamic Fee $(\gamma_{buy}, \gamma_{sell})$



Leave-Or-Take-It Offer

$$\text{Fee} = \underset{x}{\operatorname{argmax}} \ \mathbb{P}(LVR \geq x) \cdot x$$

$$LVR = L\sqrt{p}\left(\sqrt{\frac{p_t}{p}} - 1\right)^2.$$

$$\mathbb{P}(LVR \geq x)$$

$$= \mathbb{P}\left(\left(\sqrt{\frac{p_t}{p}} - 1\right)^2 \geq \frac{x}{L\sqrt{p}}\right)$$

$$= 1 - \mathbb{P}\left(-\sqrt{\frac{x}{L\sqrt{p}}} \leq \sqrt{\frac{p_t}{p}} - 1 \leq \sqrt{\frac{x}{L\sqrt{p}}}\right)$$

$$= 1 - \mathbb{P}\left(1 - \sqrt{\frac{x}{L\sqrt{p}}} \leq \sqrt{\frac{p_t}{p}} \leq 1 + \sqrt{\frac{x}{L\sqrt{p}}}\right)$$

$$= 1 - \mathbb{P}\left(2\log\left(1 - \sqrt{\frac{x}{L\sqrt{p}}}\right) \leq \log\frac{p_t}{p} \leq 2\log\left(1 + \sqrt{\frac{x}{L\sqrt{p}}}\right)\right)$$

$$= 1 - \int_{2\log\left(1 - \sqrt{\frac{x}{L\sqrt{p}}}\right)}^{2\log\left(1 + \sqrt{\frac{x}{L\sqrt{p}}}\right)} f_{\log p_t}(s)\, ds. \quad \cdots (*)$$

$$dP = p \cdot \mu \, dt + p \cdot \sigma \, dW$$

$$\Rightarrow (*) \approx 1 - 4\sqrt{\frac{x}{L\sqrt{p}}} \cdot \frac{1}{\sigma\sqrt{2\pi}}$$

$$\therefore \mathbb{P}(LVR \geq x) \cdot x$$

$$= \left(1 - 4\sqrt{\frac{x}{L\sqrt{p}}} \cdot \frac{1}{\sigma\sqrt{2\pi}}\right) x$$

$$\frac{d}{dx}\left(\mathbb{P}(LVR \geq x) \cdot x\right)$$

$$= \left(1 - 4\sqrt{\frac{x}{L\sqrt{p}}} \cdot \frac{1}{\sigma\sqrt{2\pi}}\right)$$

$$+ x \cdot (-4) \cdot \frac{1}{2}\left(\frac{x}{L\sqrt{p}}\right)^{-\frac{1}{2}} \cdot \frac{1}{L\sqrt{p}} \cdot \frac{1}{\sigma\sqrt{2\pi}}$$

local maximum $\Leftrightarrow$ $1 - 6\sqrt{\frac{x}{L\sqrt{p}}} \cdot \frac{1}{\sigma\sqrt{2\pi}} = 0$.

$$\frac{\sigma^2 \cdot \cancel{2}\pi}{36} = \frac{x}{\cancel{2}L\sqrt{p}} = \frac{x}{V(p)}$$

$x = V(p) \cdot \frac{\sigma^2 \pi}{36}$. (Optimal Reserve Price)