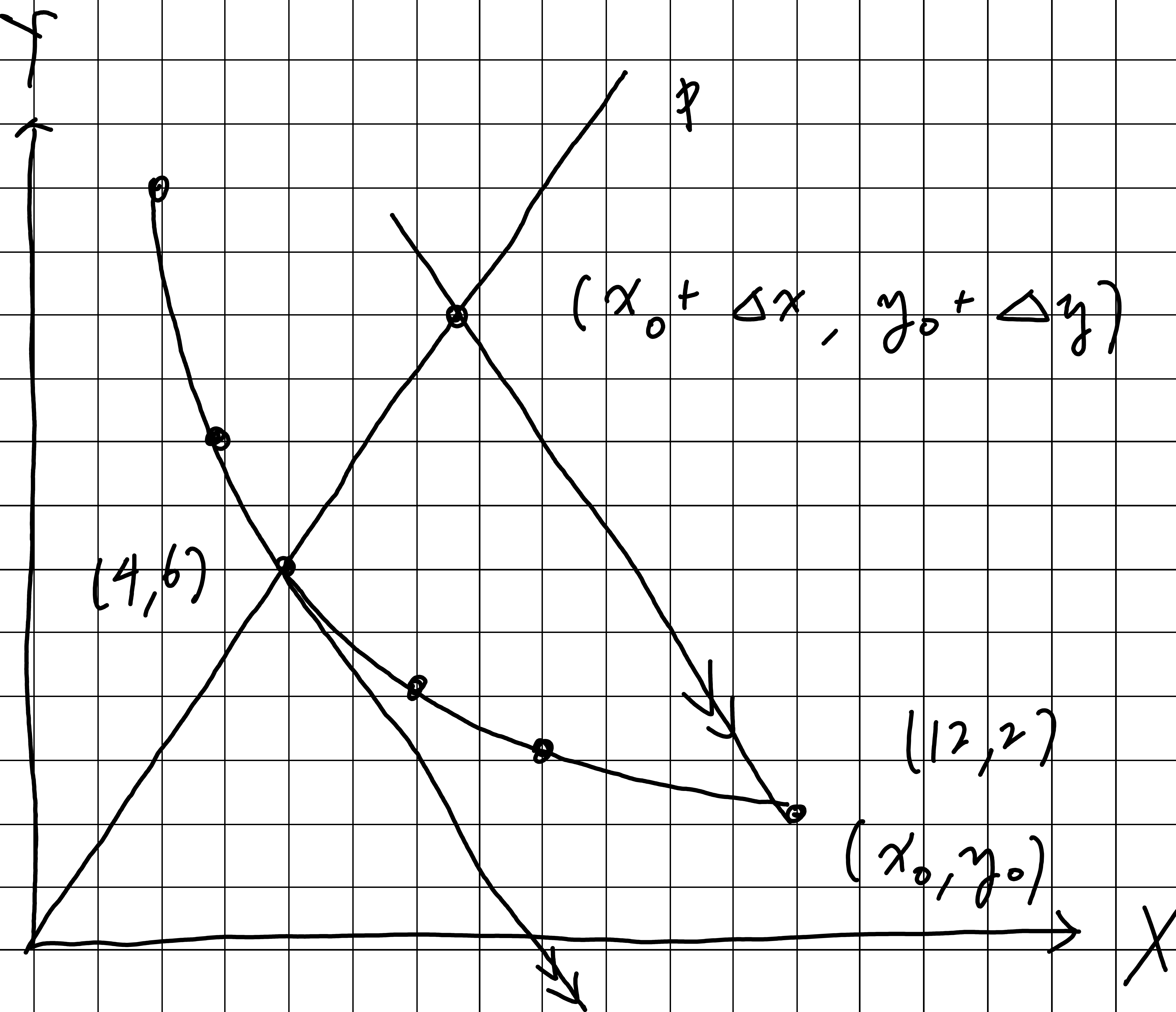


$$\frac{y_0+\Delta y}{x_0+\Delta x} = -\frac{\Delta y}{\Delta x} \quad (=: p)$$

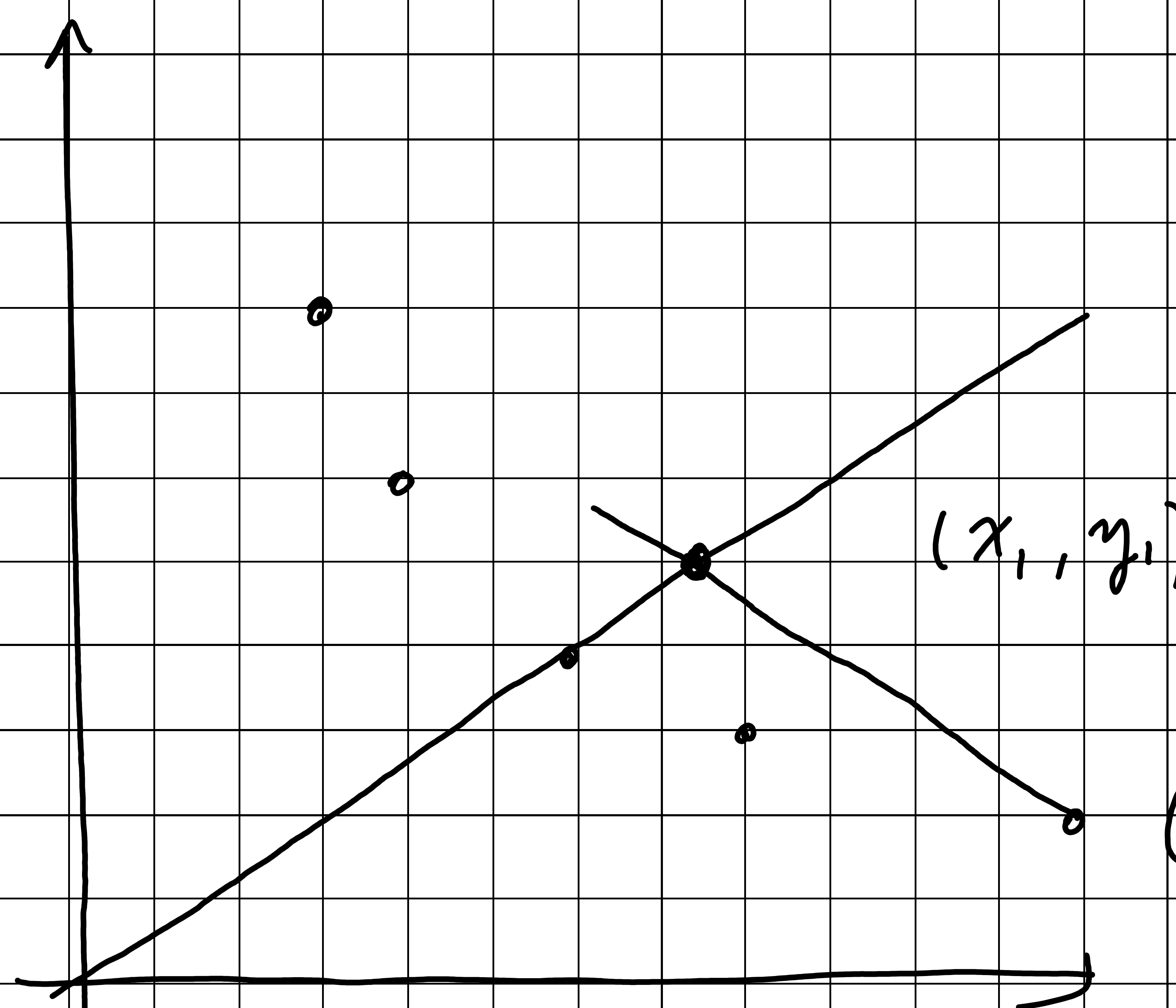
$$\Delta x\, y_0 + \Delta x\, \Delta y = -\Delta y\, x_0 - \Delta x\, \Delta y$$

$$\Delta y\,(x_0 + 2\Delta x) = -\Delta x\, y_0$$

$$(\Longrightarrow) \begin{cases} \Delta y = \dfrac{-y_0 \Delta x}{x_0 + 2\Delta x} \\ \Delta x = \dfrac{-x_0 \Delta y}{y_0 + 2\Delta y} \end{cases}$$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \xrightarrow{\text{FMAMM}} \begin{bmatrix} x_0 + \Delta x \\ y_0 \cdot \dfrac{x_0+\Delta x}{x_0+2\Delta x} \end{bmatrix}$$

$(x_0, y_0) \xrightarrow{\quad (x_{in}, y_{in}) \quad} (x_1, y_1)$



$$\begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} = x_{in} \begin{bmatrix} 1 \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix}.$$

$$\begin{bmatrix} x_{in} \\ y_{in} - \alpha \end{bmatrix}, \begin{bmatrix} x_0 \cdot \dfrac{y_0 + \alpha}{y_0 + 2\alpha} \\ y_0 + \alpha \end{bmatrix} \in \operatorname{span}\left(\left\{ \begin{bmatrix} 1 \\ p \end{bmatrix} \right\}\right).$$

$$\Rightarrow \quad x_{in} \cancel{(y_0 + \alpha)} = (y_{in} - \alpha) \cdot x_0 \cdot \frac{\cancel{y_0 + \alpha}}{y_0 + 2\alpha}$$

$$x_{in}(y_0 + 2\alpha) = (y_{in} - \alpha) x_0$$

$$2 x_{in} \alpha + x_{in} y_0 = - x_0 \alpha + x_0 y_{in}$$

$$(2 x_{in} + x_0)\alpha = x_0 y_{in} - x_{in} y_0$$

$$\alpha = \frac{x_0 y_{in} - x_{in} y_0}{x_0 + 2 x_{in}}$$

$$\therefore \quad y_0 + \alpha = \frac{x_0 y_0 + 2 y_0 x_{in} + x_0 y_{in} - x_{in} y_0}{x_0 + 2 x_{in}}$$

$$= \frac{x_0 y_0 + x_0 y_{in} + x_{in} y_0}{x_0 + 2 x_{in}}$$

$$y_0 + 2\alpha = \frac{x_0 y_0 + 2 y_0 x_{in} + 2 x_0 y_{in} - 2 x_{in} y_0}{x_0 + 2 x_{in}}$$

$$= \frac{x_0 y_0 + 2 x_0 y_{in}}{x_0 + 2 x_{in}}$$

$$\Rightarrow x_0 \cdot \frac{y_0 + \alpha}{y_0 + 2\alpha}$$

$$= \cancel{x_0} \cdot \frac{x_0 y_0 + x_0 y_{in} + x_{in} y_0}{\cancel{x_0 + 2x_{in}}} \cdot \frac{\cancel{x_0 + 2x_{in}}}{\cancel{x_0}(y_0 + 2y_{in})}$$

$$= \frac{x_0 y_0 + x_0 y_{in}\ x_{in} y_0}{y_0 + 2y_{in}}$$

$$\therefore \begin{bmatrix} x_1 \\ \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 / (y_0 + 2y_{in}) \\ \\ 1 / (x_0 + 2x_{in}) \end{bmatrix} (x_0 y_0 + x_0 y_{in} + x_{in} y_0)$$

$$p_1 = \frac{y_1}{x_1} = \frac{1}{x_0 + 2x_{in}} \cdot \left(\frac{1}{y_0 + 2y_{in}} \right)^{-1}$$

$$= \frac{y_0 + 2y_{in}}{x_0 + 2x_{in}}$$

trader's PnL $:= p \cdot (x_{out} - x_{in}) + y_{out} - y_{in}$

$$\begin{bmatrix} x_{out} \\ y_{out} \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} - \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} - \begin{bmatrix} (x_0 y_0 + x_0 y_{in} + x_{in} y_0) / (y_0 + 2y_{in}) \\ (x_0 y_0 + x_0 y_{in} + x_{in} y_0) / (x_0 + 2x_{in}) \end{bmatrix}$$

$$= \begin{bmatrix} y_{in} (x_0 + 2x_{in}) / (y_0 + 2y_{in}) \\ x_{in} (y_0 + 2y_{in}) / (x_0 + 2x_{in}) \end{bmatrix}.$$

$$\begin{bmatrix} x_{out} - x_{in} \\ y_{out} - y_{in} \end{bmatrix} = (x_0 y_{in} - x_{in} y_0) \begin{bmatrix} \dfrac{1}{y_0 + 2y_{in}} \\ -\dfrac{1}{x_0 + 2x_{in}} \end{bmatrix}$$

$$\therefore \text{PnL} = (x_0 y_{in} - x_{in} y_0) \times \left(\frac{p}{y_0 + 2y_{in}} - \frac{1}{x_0 + 2x_{in}} \right).$$

i) Price goes up

$$\mathrm{PnL} = (x_0 y_{in} - x_{in} y_0) \cdot \left(\frac{P}{y_0 + 2y_{in}} - \frac{1}{x_0 + 2x_{in}} \right)$$

Let $x_0 = \frac{L}{\sqrt{P_0}}$, $y_0 = L\sqrt{P_0}$, $P \geq P_0$.

$$\mathrm{PnL}\Big|_{x_{in}=0} = x_0 y_{in} \cdot \left(\frac{P}{y_0 + 2y_{in}} - \frac{1}{x_0} \right).$$

$$\frac{\partial}{\partial y_{in}} \left(\mathrm{PnL}\Big|_{x_{in}=0} \right) = x_0 \cdot \left(\frac{P}{y_0 + 2y_{in}} - \frac{1}{x_0} \right) + x_0 y_{in} \cdot - \frac{P}{(y_0 + 2y_{in})^2} \cdot 2$$

$$= \frac{-2P x_0 y_{in} + P x_0 (y_0 + 2y_{in})}{(y_0 + 2y_{in})^2} - 1$$

$$= P x_0 \cdot \frac{y_0}{(y_0 + 2y_{in})^2} - 1$$

local maximum

$$\Leftrightarrow \quad P x_0 y_0 = (y_0 + 2y_{in})^2 = L^2 P.$$

$$\Leftrightarrow \quad L\sqrt{P} = y_0 + 2y_{in} = L\sqrt{P_0} + 2y_{in}$$

$$\Rightarrow \quad y_{in} = \frac{L(\sqrt{P} - \sqrt{P_0})}{2}$$

iii) price goes down

$$PnL = (x_0 y_{in} - x_{in} y_0) \cdot \left(\frac{P}{y_0 + 2y_{in}} - \frac{1}{x_0 + 2x_{in}} \right)$$

Let $x_0 = \frac{L}{\sqrt{P_0}}$, $y_0 = L\sqrt{P_0}$, $P \le P_0$.

$$PnL\Big|_{y_{in}=0} = -x_{in} y_0 \left(\frac{P}{y_0} - \frac{1}{x_0 + 2x_{in}} \right)$$

$$= x_{in} y_0 \left(\frac{1}{x_0 + 2x_{in}} - \frac{P}{y_0} \right)$$

$$\frac{\partial}{\partial x_{in}} \left(PnL\Big|_{y_{in}=0} \right) = y_0 \left(\frac{1}{x_0 + 2x_{in}} - \frac{P}{y_0} \right)$$

$$+ x_{in} y_0 \cdot \frac{-1}{(x_0 + 2x_{in})^2} \cdot 2$$

$$= y_0 \cdot \frac{x_0 + 2x_{in} - 2x_{in}}{(x_0 + 2x_{in})^2} - P$$

$$= y_0 \cdot \frac{x_0}{(x_0 + 2x_{in})^2} - P$$

local maximum

$$\iff \frac{x_0 y_0}{(x_0 + 2x_{in})^2} = P$$

$$\iff (x_0 + 2x_{in})^2 = \frac{L^2}{P}$$

$$\Rightarrow x_{in} = \frac{1}{2}\left(\frac{L}{\sqrt{P}} - \frac{L}{\sqrt{P_0}} \right) = \frac{L}{2}\left(\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{P_0}} \right).$$

$$(L_0, P_0) \xrightarrow[P_{new}]{} (L_1, P_1) \longrightarrow \cdots$$

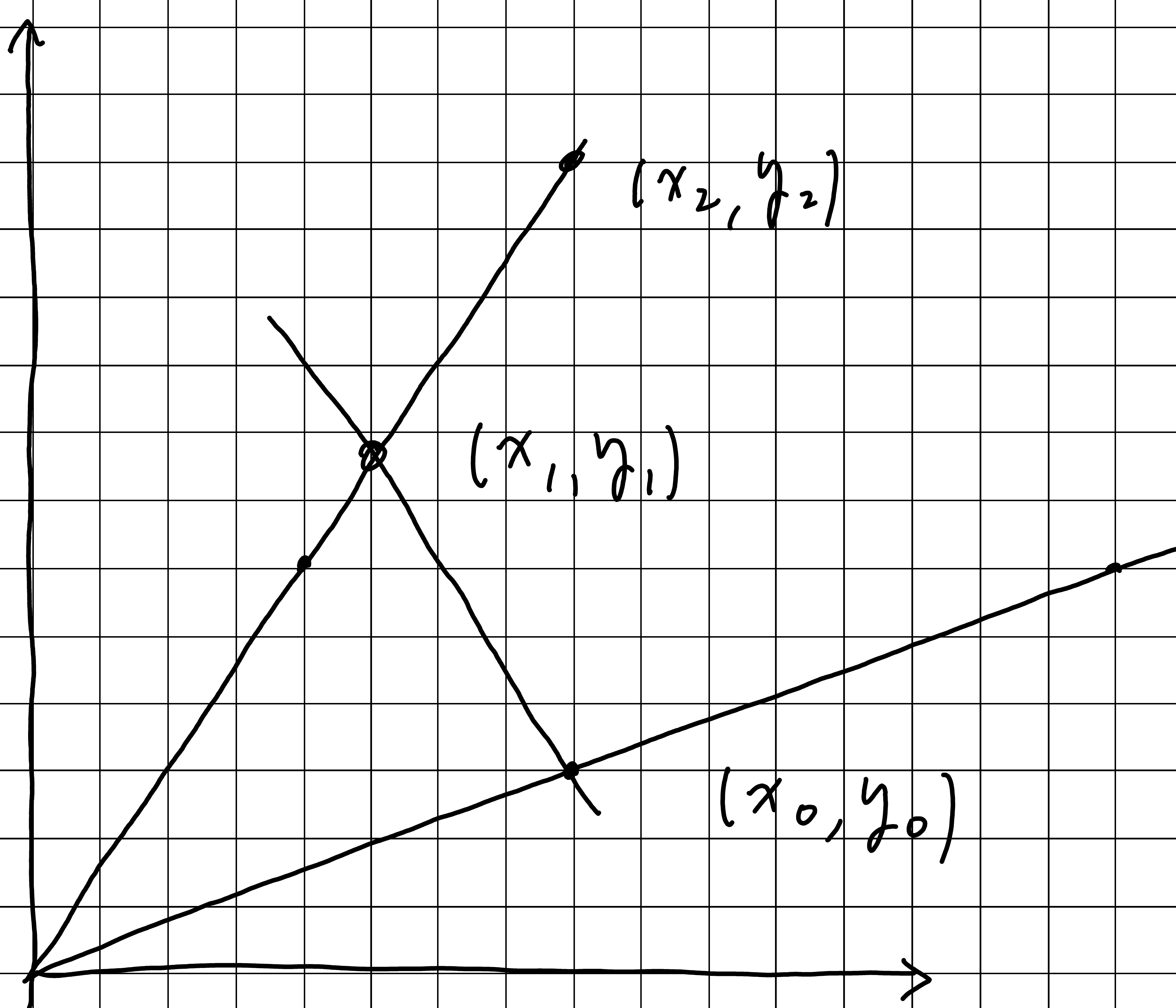
$$P_n = GM(P_{n-1}, P_{new})$$

$$L_n = L_{n-1} \cdot \frac{AM(P_{n-1}, P_n)}{GM(P_{n-1}, P_n)}$$

Principle :

1. Batch the Actions that can move price.
2. immidiate otherwise.

Batch Mint & Swap



$$\begin{bmatrix} x_{mint} \\ y_{mint} \end{bmatrix} = x_{mint} \begin{bmatrix} 1 \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ 2\alpha \end{bmatrix} \quad \left(\longrightarrow \left(x_{mint} + \frac{\alpha}{p}\right) \begin{bmatrix} 1 \\ p \end{bmatrix} \right).$$

$$\begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} = x_{in} \begin{bmatrix} 1 \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x_0 \cdot \dfrac{y_0 + \alpha + \beta}{y_0 + 2\alpha + 2\beta} \\ y_0 + \alpha + \beta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_{mint} \\ y_{mint} - 2\alpha \end{bmatrix} \,//\, \begin{bmatrix} x_{in} \\ y_{in} - \beta \end{bmatrix} \,//\, \begin{bmatrix} x_0 \cdot \dfrac{y_0 + \alpha + \beta}{y_0 + 2\alpha + 2\beta} \\ y_0 + \alpha + \beta \end{bmatrix}$$

$$\begin{bmatrix} x_{mint} \\ y_{mint} - 2\alpha \end{bmatrix} \parallel \begin{bmatrix} x_{in} \\ y_{in} - \beta \end{bmatrix} \parallel \begin{bmatrix} x_0 \cdot \cancel{\dfrac{y_0 + \alpha + \beta}{y_0 + 2\alpha + 2\beta}} \\ \cancel{y_0 + \alpha + \beta} \\ y_0 + 2\alpha + 2\beta \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_{mint}(y_0 + 2\alpha + 2\beta) = x_0 (y_{mint} - 2\alpha) \\ x_{in}(y_0 + 2\alpha + 2\beta) = x_0 (y_{in} - \beta) \end{cases}$$

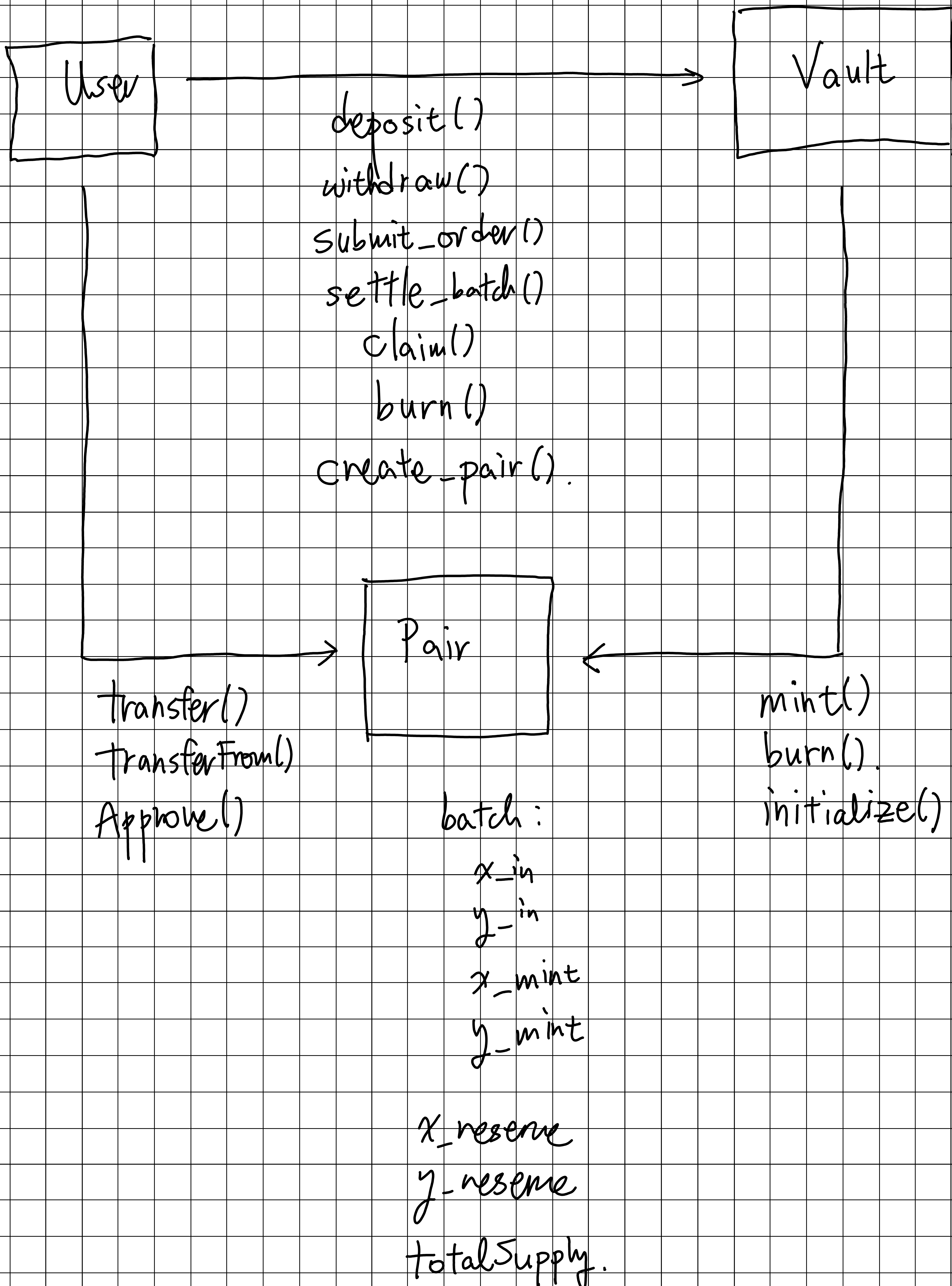
$$\Rightarrow \begin{cases} (2x_{mint} + 2x_0)\alpha + 2x_{mint}\beta = x_0 y_{mint} - x_{mint} y_0 \\ 2x_{in}\alpha + (2x_{in} + x_0)\beta = x_0 y_{in} - x_{in} y_0 \end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 2x_0 + 2x_{mint} & 2x_{mint} \\ 2x_{in} & 2x_{in} + x_0 \end{bmatrix}}_{(=:M)} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} x_0 y_{mint} - x_{mint} y_0 \\ x_0 y_{in} - x_{in} y_0 \end{bmatrix}$$

$$\det M = 2x_0^2 + 2x_0(x_{mint} + 2x_{in}) > 0.$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} \xrightarrow{\text{swap}} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \xrightarrow{\text{mint}} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$$\text{mintShare} = \frac{p x_{user} + y_{user}}{p x_{mint} + y_{mint}} \times \left(\sqrt{\frac{x_2 y_2}{x_1 y_1}} - 1 \right) \times \text{TotalSupply}$$


User
Vault
deposit()
withdraw()
submit_order()
settle_batch()
claim()
burn()
create_pair().
Pair
transfer()
transferFrom()
Approve()
mint()
burn().
initialize()
batch:
x_in
y_in
x_mint
y_mint
x_reserve
y_reserve
totalSupply.

trader's PnL w/ fee := $(1-r)(p \cdot x_{out} + y_{out}) - (1+r)(p \cdot x_{in} + y_{in})$

$$\begin{bmatrix} x_{out} \\ y_{out} \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} - \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} - \begin{bmatrix} (x_0 y_0 + x_0 y_{in} + x_{in} y_0) / (y_0 + 2y_{in}) \\ (x_0 y_0 + x_0 y_{in} + x_{in} y_0) / (x_0 + 2x_{in}) \end{bmatrix}$$

$$= \begin{bmatrix} y_{in}(x_0 + 2x_{in}) / (y_0 + 2y_{in}) \\ x_{in}(y_0 + 2y_{in}) / (x_0 + 2x_{in}) \end{bmatrix}.$$

PnL

$$= \left(p\, y_{in} \cdot \frac{x_0 + 2x_{in}}{y_0 + 2y_{in}} + x_{in} \cdot \frac{y_0 + 2y_{in}}{x_0 + 2x_{in}} \right) \times (1-r)$$

$$- (p x_{in} + y_{in}) \times (1+r)$$

PnL w/ fee, price goes up.

$$= \left(p\, y_{in} \cdot \frac{x_0 + 2x_{in}}{y_0 + 2y_{in}} + x_{in} \cdot \frac{y_0 + 2y_{in}}{x_0 + 2x_{in}}\right) \times (1-r)$$

$$- (p x_{in} + y_{in}) \times (1+r)$$

$$PnL\Big|_{x_{in}=0} = (1-r) \cdot p y_{in} \cdot \frac{x_0}{y_0 + 2y_{in}} - (1+r)\, y_{in}$$

$$= y_{in} \left((1-r) p \cdot \frac{x_0}{y_0 + 2y_{in}} - (1+r) \right).$$

$$\frac{\partial}{\partial y_{in}} \left(PnL\Big|_{x_{in}=0} \right) \qquad x_0 y_0 = L^2$$

$$= (1-r) p \cdot \frac{x_0}{y_0 + 2y_{in}} - (1+r)$$

$$+ y_{in} \cdot (1-r) p \cdot \frac{-x_0}{(y_0 + 2y_{in})^2} \cdot 2$$

$$= (1-r) p \cdot \frac{x_0 y_0 \cancel{+ 2x_0 y_{in} - 2x_0 y_{in}}}{(y_0 + 2y_{in})^2} - (1+r) = 0.$$

$$(1-r) p \cdot \frac{x_0 y_0}{(y_0 + 2y_{in})^2} = (1+r)$$

$$L^2 p \cdot \frac{1-r}{1+r} = (y_0 + 2y_{in})^2$$

$$L \sqrt{\frac{1-r}{1+r} p} = y_0 + 2y_{in}, \quad x_0 + 2x_{in} = \frac{L}{\sqrt{p_0}}.$$

$$\text{fee} = \gamma \begin{bmatrix} x_{in} + x_{out} \\ y_{in} + y_{out} \end{bmatrix}$$

$$= \gamma \begin{bmatrix} x_{out} \\ y_{in} \end{bmatrix} = \gamma\, y_{in} \begin{bmatrix} \frac{1}{P_{new}} \\ 1 \end{bmatrix} = \gamma \cdot \frac{y_{in}}{\sqrt{P_{new}}} \begin{bmatrix} \frac{1}{\sqrt{P_{new}}} \\ \sqrt{P_{new}} \end{bmatrix}$$

$$= \gamma \cdot \frac{1}{\sqrt{P_{new}}} \cdot \frac{L}{2} \left(\sqrt{\frac{1-\gamma}{1+\gamma} P} - \sqrt{P_0} \right) \begin{bmatrix} \frac{1}{\sqrt{P_{new}}} \\ \sqrt{P_{new}} \end{bmatrix}.$$

$$(L_0, P_0)$$

$$\Big\downarrow \text{swap}$$

$$\left(L_0 \cdot \frac{AM(P_0, P_{new})}{GM(P_0, P_{new})},\ P_{new} := GM\left(P_0, \frac{1-\gamma}{1+\gamma} P\right) \right)$$

$$\Big\downarrow \text{fee.}$$

$$L_0 \cdot \left(\frac{AM(P_0, P_{new})}{GM(P_0, P_{new})} + \frac{\gamma}{2\sqrt{P_{new}}} \left(\sqrt{\frac{1-\gamma}{1+\gamma} p} - \sqrt{P_0} \right) \right)$$

PnL w/ fee, price goes down.

$$= \left(p\, y_{in} \cdot \frac{x_0 + 2x_{in}}{y_0 + 2y_{in}} + x_{in} \cdot \frac{y_0 + 2y_{in}}{x_0 + 2x_{in}} \right) \times (1-\gamma)$$

$$- (p x_{in} + y_{in}) \times (1+\gamma)$$

$$\text{PnL}\Big|_{y_{in}=0} = \frac{y_0}{x_0 + 2x_{in}} \times (1-\gamma)\, x_{in}$$

$$- (1+\gamma)\, p\, x_{in}$$

$$= x_{in} \left((1-\gamma) \cdot \frac{y_0}{x_0 + 2x_{in}} - (1+\gamma) p \right)$$

$$(1-\gamma) \cdot \frac{y_0}{x_0 + 2x_{in}} - (1+\gamma) p$$

$$+ x_{in} \cdot (1-\gamma) \cdot \frac{-y_0}{(x_0 + 2x_{in})^2} \times 2$$

$$= -(1+\gamma) p + (1-\gamma) \cdot \frac{x_0 y_0 + \cancel{2x_{in} y_0} - \cancel{2x_{in} y_0}}{(x_0 + 2x_{in})^2}$$

$$(1+\gamma) p = \frac{L^2 (1-\gamma)}{(x_0 + 2x_{in})^2}$$

$$x_0 + 2x_{in} = L \cdot \sqrt{\frac{1-\gamma}{1+\gamma} \cdot \frac{1}{p}}$$

$$\text{fee} = \gamma \begin{bmatrix} x_{in} + x_{out} \\ y_{in} + y_{out} \end{bmatrix}$$

$$= \gamma \begin{bmatrix} x_{in} \\ y_{out} \end{bmatrix} = \gamma\, x_{in} \sqrt{P_{new}} \begin{bmatrix} \frac{1}{\sqrt{P_{new}}} \\ \sqrt{P_{new}} \end{bmatrix}$$

$$= \frac{L}{2} \cdot \gamma \cdot \sqrt{P_{new}} \cdot \left(\frac{1}{\sqrt{\frac{1+\gamma}{1-\gamma} P}} - \frac{1}{\sqrt{P_0}} \right) \cdot \begin{bmatrix} \frac{1}{\sqrt{P_{new}}} \\ \sqrt{P_{new}} \end{bmatrix}.$$

$$(L_0, P_0)$$

$$\Big\downarrow \text{swap}$$

$$\left(L_0 \cdot \frac{AM(P_0, P_{new})}{GM(P_0, P_{new})},\ P_{new} := GM\left(P_0, \frac{1-\gamma}{1+\gamma} P\right) \right)$$

$$\Big\downarrow \text{fee}$$

$$L_0 \cdot \left(\frac{AM(P_0, P_{new})}{GM(P_0, P_{new})} + \frac{\gamma}{2} \cdot \sqrt{P_{new}} \cdot \left(\frac{1}{\sqrt{\frac{1+\gamma}{1-\gamma} P}} - \frac{1}{\sqrt{P_0}} \right) \right).$$