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Diagramatic Representation of Transition probabilities.

1. Transition Diagram
2. probability tree diagram

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Explain Diagramatic Representation of Transition probabilities?

Diagramatic Representation of Transition Probabilities;—

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The transition probabilities can also be represented by two types of diagrams.

(1) Transition Diagrams

(2) Probability tree diagram.

(5) Explain Transition Diagram ? 23

This is one of the Diagrammatic representation type. L-6

Two transition diagrams are shown in figures. It shows the transition probabilities. The states are vertices in the diagram & probability $\boxed{p_{ij}}$ is denoted by arrow (edge) from state $\boxed{b_i}$ to the state $\boxed{b_j}$.

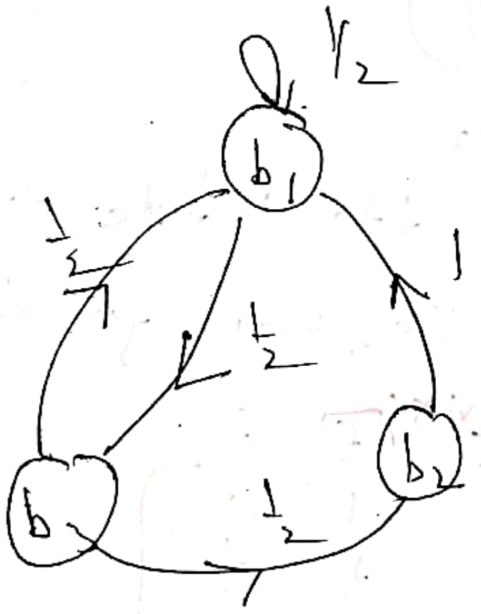


Diagram - a

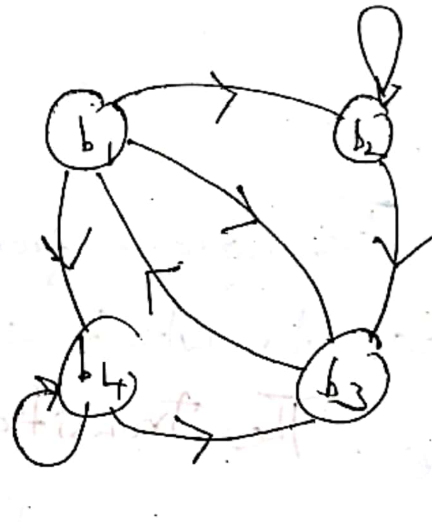


Diagram b.

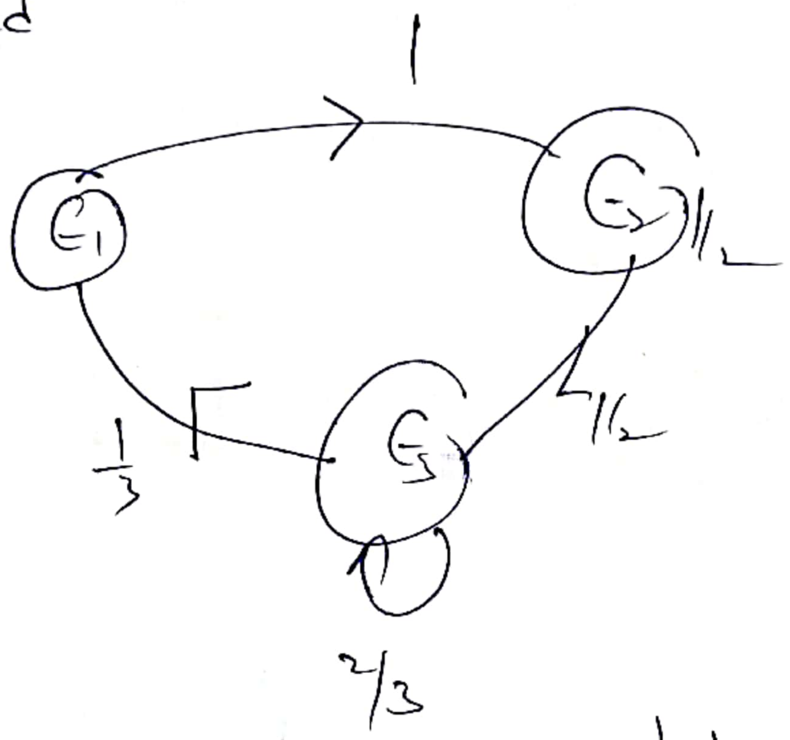
Transition matrix - a :-

$$P = \begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \end{array} \begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \end{array}$$

Transition matrix - b :-

$$P = \begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix} \end{array}$$

In transition diagram shows the transition probabilities or shifts that can occur in any particular situation. Such diagram is represented



The arrows from each state indicate, the possible states to which process can move from the given state. The transition Matrix;

$$q = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{2}{3} \end{bmatrix} \end{array}$$

Zero element in the above matrix indicates that the transition is not possible.

Probability tree diagram :-

"This diagram emphasizes the probabilities and their movement from one step to another, along with all possible branches or paths that may connect the outcomes over a period of time.

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probability Vector

Stochastic Matrix

Transition Matrix

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(6) Probability Vector:-

M. Shanmugh.. L-68

A vector $e = \{e_1, e_2 \ldots e_n\}$ is called a probability vector, if its all elements are positive and their sum is one.

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$\boxed{\begin{array}{l} \text{(1) Each } e_i \geq 0 \\ \text{(2) } e_1 + e_2 + \cdots + e_n = 1 \end{array}}$

Stochastic Matrix:-

A square matrix $\boxed{P = [p_{ij}]}$ is called a stochastic matrix, if each row of P is a probability vector.

Regular Stochastic Matrix:-

A stochastic matrix $\boxed{P}$ is said to be regular, if all the elements of some power $\boxed{P^k}$ are positive.

Explain Transition Matrix?

(7) Transition Matrix:-

A Markov process (or) Markov chain consists of a sequence of repeated trials of an experiment with the property that

① "In the experiment of repeated each out come belongs to a finite set $\{b_1, b_2 \cdots b_n\}$ called the State space of System."

② In the experiment of repeated trials, the out come of any future trial depends only on the outcome of the present [preceding] trial and not on any other previous (or) past trial.

For the given probability $\boxed{P_{ij}}$. If $\boxed{b_j}$ occurs immediately after $\boxed{b_i}$.

The following n-square Matrix

$$P = \begin{array}{c} \\ b_1 \\ b_2 \\ \vdots \\ b_n \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & \cdots & b_n \end{array} \\ \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} \end{array}$$

This Matrix P is called transition Matrix.

"Transition Matrix is a rectangular array which consist the transition probabilities for a given Markov process"

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In transition Matrix row = Current state of the system.

Columns = alternative states to which the system can move.

→ In Transition Matrix $\boxed{P_{ij} = 0}$

i.e

[No transition occurs i.e Transition is impossible]

$\boxed{P_{ij} = 1}$ → When the system is moving from i to j.

Problems

L-69

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① Show that the non-Zero Vector $e = [2, 5, 0, 1]$ is the probability Vector?

$\boxed{e = [2, 5, 0, 1]}$ is not a probability Vector.

For any Vector to be a probability Vector, it should follow two Conditions

① All entries should be non-negative $e_i \geq 0$

② Addition of all entries should be One. $e_1 + e_2 + \dots = 1$

① $2 \geq 0$, $5 \geq 0$, $0 = 0$, $1 \geq 0$

② $2 + 5 + 0 + 1 = 8 \neq 1$

∴ It satisfies only ①

Does not satisfies ②

∴ The given Vector is $e = [2, 5, 0, 1]$ is not a probability Vector.

② In the previous example for the non-Zero Vector $e = [2, 5, 0, 1]$ find probability Vector?

$e = [2, 5, 0, 1]$

There is a Unique probability Vector $= P$

P is a Scalar multiple of e.

∴ The probability vector [P], can be obtained by multiplying e. (i.e by the reciprocal of the addition of its entries].

$$P = \frac{1}{8} e$$

$$= \frac{1}{8} [2, 5, 0, 1]$$

$$\boxed{P = \left[\frac{2}{8}, \frac{5}{8}, \frac{0}{8}, \frac{1}{8}\right]}$$

(7) Check the following stochastic matrix is regular (or) not ?

$$A = \begin{bmatrix} 1 & 0 \\ \frac{7}{8} & \frac{1}{8} \end{bmatrix}$$

Sol:

We know that, if we have to prove stochastic matrix is regular.

i.e we have to show all the entries (or) elements of A^k should be positive.

$$A^2 = \begin{bmatrix} 1 & 0 \\ \frac{63}{64} & \frac{1}{64} \end{bmatrix} \longrightarrow \text{ఉంది.}$$

0, 1 ఉండాలి ఉండకూడదు.

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ \frac{4095}{4096} & \frac{1}{4096} \end{bmatrix} \longrightarrow \text{ఉంది కాబట్టి.}$$

$\boxed{\therefore A \text{ is not regular matrix}}$

4) Suppose $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Square stochastic matrix & $e = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is a probability Vector then show that eA is also a probability Vector? 28

$$eA = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

1) $\frac{1}{4} \geq 0 \quad \frac{3}{4} \geq 0$

2) $\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

$\therefore$ 1 & 2 Conditions are satisfied.

$\boxed{\therefore eA \text{ is a probability Vector}}$

5) Determine which of the following stochastic matrices are regular a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

a) $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

0, 1 تكرار

$\therefore A$ is not Regular

b) $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

$A^2 = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 1 & 0 \\ \frac{3}{8} & \frac{9}{16} & \frac{1}{16} \end{bmatrix}$

0, 1 تكرار

$\therefore A$ is not regular

check (a) not (b)

(6) Given $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ $U = [1 \quad 3 \quad -2]$ find

$$UA = [1 \quad 3 \quad -2] \cdot \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = [5 \quad 4 \quad 3]$$

(7) Find the unique fixed probability Vector S of each matrix (a) $A = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$ $B = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

(a) $S = [x \quad y]$

$\boxed{S = SA} \Rightarrow [x \quad y] = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix} [x \quad y]$

$[x \quad y] = [0.3x + 0.5y \quad 0.4x + 0.5y]$

$x = 0.3x + 0.5y$

$y = 0.4x + 0.5y$

$y = -0.1x$

$x = 0.3x + 0.5y$

$= 0.3x + 0.5(-0.1x)$

$x = 0.25x$

$\therefore y = 0.4(0.25) + 0.5y$

$\therefore S = [x \quad y]$

$= [\quad -0.]$

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(8) check the following stochastic matrix is regular (or) not?

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$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{16}{75} & \frac{37}{75} & \frac{22}{75} \\ \frac{24}{135} & \frac{26}{45} & \frac{33}{135} \end{bmatrix}$$

In this No 0, 1

$\therefore$ $\boxed{\text{A is a Regular}}$

(9) Find the Unique fixed probability Vector S.of the following regular stochastic Matrix?

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$\Rightarrow S = SP$

$$\Rightarrow [x\ y\ z] = [x\ y\ z] \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x &= \frac{1}{5}y \\ y &= x + \frac{2}{5}y + \frac{1}{2}z \\ z &= \frac{1}{5}y + \frac{z}{2} \end{aligned}\right\} \text{Solv.}$$

$$x = \frac{y}{5}$$

$$y = \frac{5}{4}z$$

$\therefore$ If we have to solve non-zero solution.

$$\boxed{x=1}$$

$\therefore$ $y=5$

$z=4$

$$\therefore S = \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 1 & 5 & 4 \end{bmatrix}$$

10) Suppose $e_0 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is the initial state distribution for a markov process with the following transition Matrix? $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$.

formulas

$e_1 = e_0 P$

$e_2 = e_1 P$

$e_3 = e_2 P$

$$\boxed{e_1 = e_0 P}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

$$\boxed{e_2 = e_1 P} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix}$$

$$\boxed{e_3 = e_2 P} = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{11}{16} & \frac{5}{16} \end{bmatrix}$$

Padma
given first

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(11) A person A goes to Tirupati temple Padmavathi temple & Srikalahasti. A always goes first. Tirupathi - Then Padmavathi temple. For Srikalahasti. If A first goes to Srikalahasti, he is just as likely to go Tirupatis padmavathi temple. Find the transition matrix of this Markov process?

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Sol:-

Tirupati temple = T

P T = P

S T = S

$$T = \begin{array}{c|ccc} & T & P & S \\ \hline T & 0 & 1 & 0 \\ P & 0 & 0 & 1 \\ S & \frac{1}{2} & \frac{1}{2} & 0 \end{array}$$

$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & \frac{2}{3} & [illegible] & \frac{2}{3} \\ C & \frac{2}{3} & \frac{1}{3} & 0 \end{array}$$

(12) A student tries to take admission in only three college A, B & C - first he goes to college A, second day to college B. Then third day to C - He never goes to same college in two continuous days. But if he goes either B or C. Then, the next day, he is twice as likely to go to college A. Find at how often in the long run he tries to each college?

Transition Matrix $P = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & \frac{2}{3} & 0 & \frac{1}{3} \\ C & \frac{2}{3} & \frac{1}{3} & 0 \end{array}$

fixed Vector $S = [x \ y \ z]$

$\boxed{S_1 = S P}$

$$= (x\ y\ z)\begin{bmatrix} 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x &= \frac{2}{3}y + \frac{2}{3}z \\ y &= x + \frac{z}{3} \\ z &= \frac{y}{3} \end{aligned}\right\} \text{Soly}$$

$\boxed{z = 1}$ → does solve Jacobi any of the answers to be $z = 1$

$y = 3$

Then

$x = \frac{8}{3}$

$$S = [x\ y\ z] = \left[\frac{8}{3}\ 3\ 1\right]$$

Unique fixed probability Vector $= \frac{8}{3} + 3 + 1 = \frac{26}{3}$

$$S = \left[\frac{8}{3} \times \frac{3}{26} \quad \frac{3}{\frac{26}{3}} \quad \frac{1}{\frac{26}{3}}\right] = [0.40 \quad 0.45 \quad 0.15]$$

A students lies 40% of time in college A, 45% of them in B, 15% of time in college C.

Transition Matrix

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix} \end{array}$$

fixed Vector $s = [x \; y \; z]$

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$$\boxed{s = sP}$$

$$= (x \; y \; z) \begin{bmatrix} 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x &= \tfrac{2}{3}y + \tfrac{2}{3}z \\ y &= x + \tfrac{z}{3} \\ z &= \tfrac{y}{3} \end{aligned}\right\} \text{Solving}$$

$\boxed{z = 1}$ → Lets solve since one of the answers to be $z = 1$

$y = 3$

Then

$x = \frac{8}{3}$

$$s = [x \; y \; z] = \left[\tfrac{8}{3} \;\; 3 \;\; 1\right]$$

Unique fixed probability Vector $= \frac{8}{3} + 3 + 1 = \frac{20}{3}$

$$s = \left[\frac{8/3}{20/3} \quad \frac{3}{20/3} \quad \frac{1}{20/3}\right] = [0.40 \;\; 0.45 \;\; 0.15]$$

A student lies 40% of time in college A, 45% of time in B, [illegible] in college C.

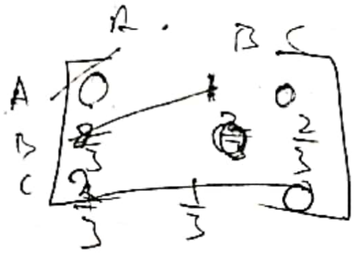
(11) A person A goes to Tirupati temple, Padmavathi temple & Srikalahasti. A always goes first Tirupathi - Then Padmavathi temple - Then Srikalahasti - If A first goes to srikalahasti, he is just as likely to go Tirupati or padmavathi temple. Find the transition matrix of this Markov process?

Sol:- Tirupati temple = T

P T = P

S T = S

$$T = \begin{array}{c} \\ T \\ P \\ S \end{array} \begin{array}{c} \begin{array}{ccc} T & P & S \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \end{array}$$



(12) A student tries to take admission in only three college A, B & C - first he goes to college A, Second day to college B. Then third day to C - He never goes to same college in two continuous days.

But If he goes either B or C. Then, then next day, he is twice as likely to go to college A. Find at how often in the long run he tries to each college?

6
problems

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No

(13) The School of International Studies for population found out by its survey that the mobility of the population of a state to village, town & city is in the following percentage

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From \ To	Village	town	City
Village	30%	20%	50%
town	30%	50%	20%
City	10%	40%	50%

What will be the proportion of population in village, town & city after 2 years? Present population has proportion of 0.4, 0.3 & 0.3 village, town & city respectively. Find the proportions in the long run?

Sol:

To

from

The above data can be written as P

$$\text{Transition Matrix } (P) = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

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By data present proportions $S_1 = \begin{bmatrix} 0.4 & 0.3 & 0.3 \end{bmatrix}$

Case-i :- proportion of population in V, T, City

After one year :-

$$S_2 = S_1 P$$

$$= \begin{bmatrix} 0.4 & 0.3 & 0.3 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0.24 & 0.35 & 0.41 \end{bmatrix}$$

Case-2 :- " " " After 2 years :-

$$S_3 = S_2 P$$

$$= \begin{bmatrix} 0.24 & 0.35 & 0.41 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.218 & 0.387 & 0.395 \end{bmatrix}$$

The proportion of population in village, town & city in the long Run $\Bigg\}$ = i.e Steady-state distribution of the chain

$$\boxed{\begin{aligned} &[x_1 \quad x_2 \quad x_3] = [x_1 \quad x_2 \quad x_3] \\ &\& \quad x_1 + x_2 + x_3 = 1 \end{aligned}}$$

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$$[x_1 \quad x_2 \quad x_3] = [x_1 \quad x_2 \quad x_3] \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

$P = [x_1 \quad x_2 \quad x_3]$

Get $x_1 = [\ ]$

1st column matrix

$$x_1 = 0.3x_1 + 0.3x_2 + 0.1x_3$$

$x_1 = 0.4$; so, 0.4

(14) With reference to the stochastic matrix A

$$A = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

. Verify the property that the sequence A^2, A^3, A^4 approaches the matrix, whose rows are each the fixed probability vector? ($\because$ fixed probability vector $S@V = \begin{bmatrix} 2/3 & \frac{1}{3} \end{bmatrix}$)

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$$\therefore A = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

$$V@S = (2/3 \quad 1/3)$$

Let B = The Matrix whose rows are each as $V@S$:

$$B = \begin{bmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$$

$$\therefore A = \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$$

$$A^2 = \frac{1}{16}\begin{bmatrix} 11 & 5 \\ 10 & 6 \end{bmatrix} \text{ or } \begin{bmatrix} 0.6875 & 0.3125 \\ 0.625 & 0.1375 \end{bmatrix}$$

$$A^3 = \frac{1}{64}\begin{bmatrix} 43 & 21 \\ 42 & 22 \end{bmatrix} \text{ or } \begin{bmatrix} 0.6718 & 0.328 \\ 0.656 & 0.343 \end{bmatrix}$$

$$A^4 = \frac{1}{256}\begin{bmatrix} 171 & 85 \\ 170 & 86 \end{bmatrix} \text{ or } \begin{bmatrix} 0.67 & 0.33 \\ 0.66 & 0.34 \end{bmatrix}$$

$$\boxed{\therefore \text{Each row of } A^4 \text{ is approaching } S@V = (2/3 \quad 1/3) \simeq (0.67 \quad 0.33)}$$

Note:- Higher transition probabilities:-

The first entry P_{ij} (in the transition matrix - P) of a Markov chain ⇒ scan In this the system changes from state a_i to a_j (in a single step).

$$a_i \quad - \quad - \quad - \quad - \longrightarrow a_j$$

The arrangement of the values in $P_{ij}^{(n)}$ in a Matrix is called n-step transition Matrix

Represented by $\underline{P^n}$

Initial probability distribution $\Rightarrow P^{(0)} = \left(P_1^{(0)}, P_2^{(0)} \ldots P_m^{(0)}\right)$

n^{th}-step probability distribution $\Rightarrow P^{(n)} = \left(P_1^{(n)}, P_2^{(n)} \ldots P_m^{(n)}\right)$

Then

$$P^{(1)} = P^{(0)} \cdot P$$

$$P^{(2)} = P^{(1)} \cdot P$$

$$- \quad - \quad -$$

$$P^{(n)} = P^{(0)} \cdot P^n$$

This implies that "the probability distributions of a Markov chain (Homogeneous) are Completely determined from One step transition probability matrix $\boxed{P}$ & Initial probability Vector $p^{(0)}$.

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(2) Actually, classification of states in Markov chain:-

(1) Transient

One state is every will not return to that previous state

$\boxed{\text{transient} = \text{Non-recurrent}}$

(2) Recurrent

if the process starting from the state i return to the state i with probability $=1$

(3) Absorbing

if the process reaches to a Certain state after, which it remains in the same state,

$\boxed{P_{ii} = 1 \text{ i.e. } \& \text{ probability matrix} = \text{Unit Matrix}}$

Irreducible

if every state can be reached from every other state in g finite No. of steps.

15) The transition matrix of a Markov chain is P is given below with Initial probability distribution $P^{(0)} = \left(\frac{1}{4} \ \frac{3}{4}\right)$. Define & find ① $P_{21}^{(2)}$ ② $P_{12}^{(2)}$ ③ $P^{(2)}$

④ $P_1^{(2)}$, when $P = \begin{bmatrix} 1/2 & 1/2 \\ 3/4 & 1/4 \end{bmatrix}$

sol:- ① $P_{21}^{(2)}$ = probability of moving from state a_2 to state a_1 in 2-steps.

∴ Here, How many steps = 2

∴ we have go for P^2

$$P^2 = [\ ][\ ] = \begin{bmatrix} 5/8 & 3/8 \\ 9/16 & 7/16 \end{bmatrix} = \begin{bmatrix} P_{11}^{(2)} & P_{12}^{(2)} \\ P_{21}^{(2)} & P_{22}^{(2)} \end{bmatrix}$$

$$\boxed{\therefore P_{21}^{(2)} = \frac{9}{16}}$$

② $p_{12}^{(2)}$ = probability of moving from state a_1 to state a_2 in 2-steps

$$\therefore p_{12}^{(2)} = \frac{3}{8}$$

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③ p^{2} = probability distribution of the system after 2-steps

$$\therefore p^{2} = p^{(0)} \cdot p^{2}$$

$$= \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} 5/8 & 3/8 \\ 9/16 & 7/16 \end{bmatrix} = \begin{bmatrix} \frac{37}{64} & \frac{27}{64} \end{bmatrix}$$

④ $p_1^{(2)}$ = probability that the process is in state a_1 after 2-steps

$$\boxed{\therefore p_1^{(2)} = \frac{37}{64}}$$

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(16) A habitual Gambler is a member of two clubs A & B. He visits either of the clubs every day for playing cards. He never visits club A on two consecutive days. But, if he visits club B on a particular day, then, the next day he is as likely to visit club B or club A. Find the transition matrix of this Markov chain. Also.
(1) Show that this matrix is a regular stochastic matrix and also find the unique fixed probability vector?
(2) If the person had visited club B on Monday. Find the probability that he visits club A on Thursday?

Habitual gambler $\xrightarrow{\text{member of}}$ two clubs = A, B

He visits a club — Every day — for playing cards.

He never visits club for $\boxed{\text{two consecutive days}}$ $\Rightarrow$ $\begin{array}{c|cc} & A & B \\ \hline A & 0 & \\ B & & \end{array}$

But, he visits club B on a particular day, then next day $\Rightarrow$ $\begin{array}{c|cc} & A & B \\ \hline A & 0 & 1 \\ B & & \end{array}$

Then, He, Next day
he is as likely to visit
club - B or club A $\Rightarrow$

$$\begin{array}{c|cc} & A & B \\ A & & \\ B & 1/2 & 1/2 \end{array}$$

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B or A
$1/2$ $1/2$, So

we have to keep in our mind, when
we are making stochastic Matrix. It should
follow $0+1=1$, $1/2+1/2=1$. Then, our

Matrix is correct.

$$P = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} A \quad B \\ \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} \end{array}$$

(a) To show that Regular Stochastic Matrix;
i.e we have to prove that Regular

$$P^2 = [\ ][\ ] = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$$

$\therefore$ All, elements are +ve.

$\therefore$ P is a Regular

To P.T Unique fixed probability Vector:-

$$\boxed{SP = S, \quad S = (x, y) \text{ where } x + y = 1}$$

$$[x, y]\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} = [x \ y]$$

$$\left[\frac{y}{2}, \ x + \frac{y}{2}\right] = [x, y]$$

$$\frac{y}{2} = x, \quad x + \frac{y}{2} = y$$

we know that $y = 1 - x$

$$\frac{y}{2} = x \Rightarrow \frac{1-x}{2} = x \Rightarrow x = \frac{1}{3}$$

$$\therefore y = \frac{2}{3}$$

$\therefore S = \left(\frac{1}{3} \ \ \frac{2}{3}\right)$ is a Unique fixed probability Vector.

(b) B:- club. Let, Alger visit Monday = 1

Thursday - will be 3 days after.

upto we have to calculate $P^n = P^3$

$$\therefore P^3 = P^2 \cdot P = [P^2][P] = \begin{bmatrix} 1/4 & 3/4 \\ 3/8 & 5/8 \end{bmatrix}$$

$$\boxed{\therefore q_{21}^{(3)} = 3/8}$$

D

Folla

Zoi

(17) A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study?

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The state space of the system $= [A, B]$

ie A = Studying

B = Not Studying

Transition Matrix } $P = \begin{array}{c|c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$

To find the Happening - Long run :-

ie we have to find Unique fixed probability Vector $= S$ (or) V

$\therefore S = (x, y)$: $SP = S$ where $x + y = 1$

$$[x \quad y] \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} = [x \quad y]$$

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$$P = \left[0.3x + 0.4y,\ 0.7x + 0.6y\right] = \left[x\ y\right]$$

$$\therefore \left.\begin{aligned} 0.3x + 0.4y &= x \\ 0.7x + 0.6y &= y \end{aligned}\right\} \longrightarrow (1)$$

&

$$x + y = 1 \Rightarrow y = 1 - x \longrightarrow (2)$$

Using (1) & (2)

$$0.3x + 0.4(1-x) = x \Rightarrow \boxed{x = \frac{4}{11}}$$

$$\therefore y = \frac{7}{11}$$

$$\boxed{\therefore S = (x, y) = \left(\frac{4}{11}, \frac{7}{11}\right)}$$

Conclusion:-

$\therefore$ We conclude that in the long run, the student will study $\frac{4}{11} = 36.36\%$ of the time.

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R13 H.W

17 (a) May 2015

If the transition probability matrix of Market shares of three brands A, B & C is

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.8 & 0.1 & 0.1 \\ 0.35 & 0.25 & 0.4 \end{bmatrix}$$

and Initial market shares are 50%, 25% & 25%.

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Find (a) The Market shares in second and third periods

(b) The Limiting probabilities?

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(18)

A man's smoking habits are as follows. If he smokes filter cigarettes one week, he switches to non filter cigarettes the next week with probability 0.2. On the other hand, if he smokes non filter cigarettes one week there is a probability of 0.7 that he will smoke non filter cigarettes the next week as well. In the long run how often does he smoke filter cigarettes?

Sol:- The state space of the system is A, B i.e., smoking filter cigarettes and smoking non filter cigarettes.

A: Smoking filter cigarettes

B: Smoking non filter cigarettes

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the transition matrix is formulated as below:

$$P = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix} \end{array} = \begin{bmatrix} 4/5 & 1/5 \\ 3/10 & 7/10 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 8 & 2 \\ 3 & 7 \end{bmatrix}$$

we have to find the unique fixed probability vector, SA=S

$V = (x, y)$ such that $VP = V$ or $SP = S$ where $x + y = 1$

SA=S

SA=S

i.e., $[x, y] \cdot \frac{1}{10}\begin{bmatrix} 8 & 2 \\ 3 & 7 \end{bmatrix} = [x, y]$

i.e., $[8x+3y, 2x+7y] = [10x, 10y]$

$\Rightarrow 8x+3y = 10x,$

$2x+7y = 10y$

Using $y = 1-x$ in the first eq^n, we get

$8x+3(1-x) = 10x$

$x = 3/5$

$\therefore y = 2/5$ $\quad$ [$\therefore V = [x, y] = [3/5, 2/5]$]

(19) Each year a man trades his car for a new car in 3 brands of the popular company Maruti Udyog limited. If he has a 'Standard' he trades it for 'Zen'. If he has a 'Zen' he trades it for a 'Esteem'. If he has a 'Esteem' he is just as likely to trade it for a new Esteem or for a Zen or a 'Standard' one. In 1996 he brought his first car which was Esteem.

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(i.) Find the probability that he has

(a.) 1998 Esteem (b) 1998 Standard

(c.) 1999 zen (d.) 1999 Esteem.

(ii) In the long run, how often will he have a Esteem.

Sol:- The state space of the system is A, B, C i.e, 3 brands

where A: Standard

B: Zen

C: Esteem

$$P = \begin{array}{c|ccc} & \text{standard} & \text{zen} & \text{esteem} \\ \text{Standard} & 0 & 1 & 0 \\ \text{zen} & 0 & 0 & 1 \\ \text{Esteem} & 1/3 & 1/3 & 1/3 \end{array}$$

(i.) with 1996 as the first year, 1998 is to be regarded as 2 years after and 1999 as 3 years after

we need to compute P^2 and P^3

$$P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix} \quad \text{—} (1)$$

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$$P^3 = P^2 \cdot P$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \\ 4/27 & 7/27 & 16/27 \end{bmatrix}$$ The

(a.) 1998 Esteem $= a_{33}^{(2)} = 4/9$
(b.) 1998 Standard $= a_{31}^{(2)} = 1/9$

with reference to P^2 from ①

(c.) 1999 Zen $= a_{32}^{(3)} = 7/27$
(d.) 1999 Esteem $= a_{33}^{(3)} = 16/27$

with reference to P^3 from ②

(ii.)

we have to find the unique fixed probability vector $V = (x, y, z)$ such that $VP = V$ (or) $SP = S$ where $x + y + z = 1$

i.e., $[x, y, z] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} = [x, y, z]$

i.e., $[x, y, z] \cdot \frac{1}{3} \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix} = [x, y, z]$

i.e., $[z, 3x + z, 3y + z] = [3x, 3y, 3z]$

$\Rightarrow z = 3x; \quad 3x + z = 3y; \quad 3y + z = 3z$

consider $3x + z = 3y$; Using $z = 3x$ and $y = 1 - x - z$ we get

$6x = (3 - 3x - 3z)$

or $18x = 3$

$\therefore x = 1/6$

Hence, we obtain

$y = 1/3, \; z = 1/2$

$\therefore V = [x, y, z] = [1/6, 1/3, 1/2]$

In the long run, probability of having Esteem $= 1/2$ i.e., 50% of the time he will have Esteem ~~i.e., 50% of the time~~

20 53

The price of an equity share of a company may increase, decrease or remain constant on any given day. It is assumed that the change in price on any day affects the change on the following day as described by the following transition matrix:

$$\text{chang today} \quad \begin{array}{c} \\ \text{Increase} \\ \text{Decrease} \\ \text{Unchanged} \end{array} \begin{array}{c} \text{change tomorrow} \\ \begin{array}{ccc} \text{Increase} & \text{Decrease} & \text{Unchanged} \end{array} \\ \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.7 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.1 \end{bmatrix} \end{array}$$

(i.) If the price of share increased today, what are changces that it will increase, decrease or remain unchanged tomorrow?

(ii.) If the prices of the share decreased today, what are the chances that it will increase tomorrow?

(iii.) If the price of the share remained unchanged today, what are the changces that it will increase, decrease or remain unchanged day after tomorrow?

Sol:- price share is increased today = 1

price shared, decreased or Remaining Unchanged = 0 or 0

$$\therefore p^{(0)} = [1 \quad 0 \quad 0]$$

$$p^{(1)} = p^{(0)} p$$

$$= [1 \quad 0 \quad 0] \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.7 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.1 \end{bmatrix}$$

$= [0.7 \quad 0.1 \quad 0.2]$

$$\begin{bmatrix} \text{Today's} \\ \text{preportion} \end{bmatrix} \times \begin{bmatrix} \text{Transition} \\ \text{Matrix} \end{bmatrix} = \begin{bmatrix} \text{Tomorrows} \\ \text{proportion} \end{bmatrix}$$

(2) Let probability of decrease = 1

$$\left.\begin{array}{l}\text{probability of other two} \\ \text{Events is Zero}\end{array}\right\} = \text{ie } P^{(0)} = [0 \quad 1 \quad 0]$$

$$[0 \quad 1 \quad 0] \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.7 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.1 \end{bmatrix} = [0.7 \quad 0.1 \quad 0.2]$$

(3) Again, probability of prices remain Unchanged today $\}$ = 1

probability of other two Events = 0

be

$$P^{(0)} = [0 \quad 0 \quad 1]$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.7 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.5 & 0.1 \end{bmatrix}$$

$$\left[\text{Today's propotion}\right] \times \left[\text{Transition Matrix}\right] = \left[\text{Tomorrows propotion}\right]$$

&

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$$\begin{bmatrix} 0.4 & 0.5 & 0.1 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.7 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.59 & 0.18 & 0.23 \end{bmatrix}$$

$$\left[\text{Tomorrow propotion}\right] \times \left[\text{Transition Matrix}\right] = \left[\text{Day-after tomorrow's propotion}\right]$$

∴ This concludes that "the chances that the price will increase, decrease or remain unchanged day-after tomorrow are 59%, 18%, 23%.

(21) A Gambler has Rs. 2. He bets Rs. 1 at a time and wins Rs. 1 with probability 0.5. He stops playing if he losses ₹2 or wins ₹4.

(a) what is the transition probability matrix of the related markov chain?

(b) what is the probability that he has lost his money at the end of 5 plays?

Sol:-

Let x_n be the amount with the player at the end of plays.

If S = the state of space of X_n {0, 1, 2, 3, 4, 5, 6} as the game ends, if the player ~~loos~~ loses all the money or wins ₹4 i.e., has ₹6 ($x_n = 6$). It is important to note that gambler has ₹2 at the start of the game and he stops play the game if he wins ₹2 or loses ₹2. The given problem is similar to random walk and gambler's ruin problem. Hence the transition probability matrix is obtained as follows

$$P_{ij} = \begin{cases} p, & \text{if } j = i+1 \\ q, & \text{if } j = i-1 \\ 0, & \text{else where} \end{cases}$$

Here, $p = 1/2$, which is the probability that the game will win and $q = (1-p)$, that he will lose $i, j = 0, 1, 2, \dots a$; $P_{00} = 1$, $P_{aa} = 1$ the states '0' and 'a' are called absorbing states.

$$P = \begin{array}{c|ccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

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This Markov chain is called Random walk with absorbing barriers at states 0 & 6. Because the chain cannot leave the states 0 & 6 once it reaches them. i.e The Game ends, when the chain reaches any of the two states 0 & 6.

15 Since, the gambler has initially Rs. 2. The Initial probability distribution is given by

$$p(0) = \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ (0 & 0 & 1 & 0 & 0 & 0 & 0) \end{array}$$

The probability distribution after one playing

$$p(1) = p(0) \cdot P$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$P(1) = \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{bmatrix}$$

P.D. after 2 plays:-

$$P(2) = P(1) \cdot P$$

$$= \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P(2) = \begin{bmatrix} \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \end{bmatrix}$$

The probability distribution after 3 plays :-

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$$p(3) = p(2) \cdot P$$

$$= \begin{bmatrix} \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$p(3) = \begin{bmatrix} 1/4 & 1/4 & 0 & \frac{3}{8} & 0 & \frac{1}{8} & 0 \end{bmatrix}$$

The probability distribution after 4 plays :-

$$p(4) = p(3) \cdot P$$

$$= \begin{bmatrix} p(3) \end{bmatrix} \begin{bmatrix} P \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & 0 & \frac{5}{10} & 0 & \frac{1}{4} & 0 & \frac{1}{16} \end{bmatrix}$$

The probability distribution after 5 plays :-

$$p(5) = p(4) \cdot P$$

$$= \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{8} & \frac{5}{32} & 0 & \frac{9}{32} & 0 & \frac{1}{8} & \frac{1}{16} \end{bmatrix}$$

$\therefore$ The probability that the Gambler has lost his money at the money at the end of 5 plays is given by

$P[X_5 = 0]$ = The entry corresponding to state 0 is $P(5)$

$$\boxed{P[X_5 = 0] = \frac{3}{8}}$$

(22)

On January 1 (this year), Bakery A had 40% of its local market share while the other two bakeries B and C had 40% and 20% respectively of the market share. Based upon a study by a marketing research firm, the following facts were compiled. Bakery A retains 90% of its customers while gaining 5% of competitor B's customers and 10% of C's customers. Bakery B retains 85% of its customers while gaining 5% of A's customers and 7% of C's customers. Bakery C retains 83% of its customers and gains 5% of A's customers and 10% of B's customers. What will each firm's share be on January 1, (next year) and what will each firm's market share be at equilibrium?

Sol:-

From the information given in the problem, we formulate the state-transition matrix is below:

$$P = \text{Bakery} \begin{array}{c} \text{Bakery} \\ \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.85 & 0.10 \\ 0.10 & 0.07 & 0.83 \end{bmatrix} \end{array} \text{Rentention and Gain}$$

→ Retention and Loss

On January 1,

the market shares of the three Bakeries are 40%, 40% and 20% respectively.

The management of three Bakeries is interested in knowing their market shares on January 1 (next year).

(22)

On January 1 (this year), Bakery A had 40% of its local market share while the other two bakeries B and C had 40% and 20% respectively of the market share. Based upon a study by a marketing research firm, the following facts were compiled. Bakery A retains 90% of its customers while gaining 5% of competitor B's customers and 10% of C's customers. Bakery B retains 85% of its customers while gaining 5% of A's customers and 7% of C's customers. Bakery C retains 83% of its customers and gains 5% of A's customers and 10% of B's customers. What will each firm's share be on January 1, (next year) and what will each firm's market share be at equilibrium?

5

Sol:- From the information given in the problem, we formulate the state-transition matrix is below:

$$P = \text{Bakery} \begin{array}{c} \text{Bakery} \\ \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.85 & 0.10 \\ 0.10 & 0.07 & 0.83 \end{bmatrix} \end{array} \text{ Rentention and Gain}$$

$\longrightarrow$ Retention and Loss

On January 1, the market shares of the three Bakeries are 40%, 40% and 20% respectively.

The management of three Bakeries is interested in knowing their market shares on January 1 (next year).

The expected market shares for Bakeries A, B, C On January 1 next year are determined as below:

$$\begin{bmatrix} \text{market share} \\ \text{On January 1} \\ \text{this year} \end{bmatrix} \times \begin{bmatrix} \text{Share} \\ \text{State-} \\ \text{transition} \\ \text{matrix} \end{bmatrix} = \begin{bmatrix} \text{Expected market} \\ \text{share on} \\ \text{January 1 next} \\ \text{year} \end{bmatrix}$$

$$(0.40 \quad 0.40 \quad 0.20) \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.85 & 0.10 \\ 0.10 & 0.07 & 0.83 \end{bmatrix} = (0.400 \quad 0.374 \quad 0.226)$$

So, the market shares of Bakeries A, B and C on January 1 next year will be 40%, 37.4%, 22.6% respectively.

Equilibrium market shares x, y and z for three Bakeries must satisfy the equation

$$(x \quad y \quad z) \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.85 & 0.10 \\ 0.10 & 0.07 & 0.83 \end{bmatrix} = (x \quad y \quad z)$$

where $x+y+z=1$ and $x, y, z \geq 0$

This gives us
$$x = 0.9x + 0.05y + 0.10z$$
$$y = 0.05x + 0.85y + 0.07z$$
$$z = 0.04x + 0.10y + 0.83z$$

we may rewrite

$-0.10x + 0.05y + 0.10z = 0$; $0.05x - 0.15y + 0.07z = 0$;
$0.05x + 0.10y - 0.17z = 0$

and $x+y+z=1$

So, we get $x = 0.43$, $y = 0.28$, $z = 0.29$.

the three firm's market shares at equilibrium: Bakery A : 43% of total market / Bakery B : 28% of " " / Bakery C : 29% of market sh.

3- A house wife buys (23) three kinds of cereals : A, B and C. She never buys the same cereal on successive weeks. If she buys cereal A, then the next week she buys cereal B. However, if she buys either B or C, then the next week she is three-times as likely to buy A as the other brand. Obtain the transition probability matrix and determine how often she would buy each of the cereals in the long run.

Sol:- we present the information in the following table in the form of a transition probability matrix:-

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Cereals	Cereals		
	A	B	C
A	0	1	0
B	0.75	0	0.25
C	0.75	0.25	0

$$[x \; y \; z] \times \begin{bmatrix} 0 & 1 & 0 \\ 0.75 & 0 & 0.25 \\ 0.75 & 0.25 & 0 \end{bmatrix} = [x \; y \; z]$$

where,

$$x+y+z=1 \quad \text{and} \quad x \geq 0,\; y \geq 0,\; z \geq 0$$

from this, we get

$$0.75y + 0.75z = x \; ;$$

$$x + 0.25z = y$$

$$0.25y = z \quad \text{with } x+y+z=1$$

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these eq^n's can be expressed as follows:

$$x - 0.75y - 0.75z = 0$$

$$x - y + 0.25z = 0$$

$$x + y + z = 1$$

and

Solving these eq^n's, we get

$$x = 0.428; \quad y = 0.457; \quad z = 0.114$$

Hence, in the long run, the house-wife would buy three cereals A, B and C with a frequency of 42.8%, 45.7% and 11.4% respectively.

Test mark

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24

Test the following transition matrix to see if the markov chain is regular and ~~regular~~ ergodic where x is some +ve P_{ij} value.

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$$P = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & x & x & 0 \\ 2 & x & 0 & 0 & x \\ 3 & x & 0 & 0 & x \\ 4 & 0 & x & x & 0 \end{array}$$

Sol:-

$$P^2 = [P][P] = \begin{bmatrix} x & 0 & 0 & x \\ 0 & x & x & 0 \\ 0 & x & x & 0 \\ x & 0 & 0 & x \end{bmatrix}$$

$$P^4 = [P^2][P^2] = \begin{bmatrix} x & 0 & 0 & x \\ 0 & x & x & 0 \\ 0 & x & x & 0 \\ x & 0 & 0 & x \end{bmatrix}$$

$$P^8 = [P^4][P^4] = \begin{bmatrix} x & 0 & 0 & x \\ 0 & x & x & 0 \\ 0 & x & x & 0 \\ x & 0 & 0 & x \end{bmatrix}$$

$\therefore$ from the above P is raised to an even number power $\Rightarrow$ ~~gN~~ ~~Very~~

P is raised to odd also = Original Matrix

∴ all elements are Non-Zero

+ve

∴ P is not Regular

∴ But it is Ergodic

Since possible to go state-1 to 2

or

state-3 to state 2 to

state-1

or state-4

from state-2 to ~~1~~.

from 3 to 1

from 4 to 2 or 1.

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