Lecture-32

UNIT-3

Part A :- Sampling Distributions

Part-B :- Estimation

Part-C :- Test of Hypothesis

Part-d :- Small Samples.

Part-A :- Sampling distributions :-

Introduction :- The outcome (Result) of a statistical experiment may be recorded either as a Numerical Value (or) Descriptive presentation.

For example :- 1. A pair of dice are tossed, we will get numerical value.

2. The students of a certain school are given blood tests and the type of blood is a Descriptive type. i.e A person's blood can be classified into 8 types. i.e A, B, AB, O with +ve (or) -ve. Depending on the presence (or) absence of the Rh antigen. For classification of blood types. It may be convenient to use numbers from 1 to 8, to represent blood types. These data may be Sometimes Small (or) large (or) finite (or) Infinite.

In this chapter we are focusing Sample Me[an]
Sample Variance.

Imp points & Definitions:-

Size of the population:- The Number of observations in the population is S.P. It may be finite or Infinite.

It is denoted by N.

Random Sample:- Random Sample is one in which each member of the population has equal chance.

If R.S is finite. Then it is known as R.S of finite population.

If R.S is Infinite. Then, it is known as R.S of Infinite population.

UNIT-3 ① (P&S)

Total: 1to9 **Part-A (Sampling Distribution)**

Introduction:- ① In real life, we may have to take some samples to get the information about the population. The information taken from samples may be taken to represent the whole population.

② We can't take each and every element of the population to get the information. Therefore, we need "a sample".

Examples:- ① To test whether rice has been cooked or not. A housewife tests a small quantity of rice taken in a spoon.

② The goods manufactured in some factory should be tested before sending them to the market. If each and every items has to be tested, perhaps in some cases no goods can be sent to the market.

i.e In case of match boxes, a test would mean the burning of all the matches.

Quiz

1. The population measures Mean, S.D, Variance — = parameters
2. Sampling distribution " " " ... = Statistics

Sampling :- A method of selecting Sample from the population is called Sampling.

Sampling Distribution :- It is the frequency distribution of a statistic which is taken from the population. Example :- To examine the quality of a bag of rice, sugar, wheat ... first, we examine only a portion of it by taking a handful of it from the bag & decide to purchase it.

Variance :- Difference of objects to objects.

Difference b/w population measures & Sampling distribution measures :-

population measures	Sampling distribution measures
1. This is population	1. This is Sampling distribution
2. population measures called Parameters	2. Sampling distribution measures called Statistics
3. parameter are Mean, S.D, Variance, - - ..	3. Statistics are Mean, Variance, S.D - - - ..
4. Mean $= \mu$, Variance $= \sigma^2$, Standard deviation $= \sigma$ } Greek letters	4. Mean $= \bar{x}$, Variance $= S^2$, Standard Error (or) S.D $= S$ } Roman Letters

...ation :- population is the collection of objects.
...iverse

population may be finite (or) Infinite i.e According the number of objects in the population.

Examples :- 1. The Number of students in a College is a finite population

2. The goods prepared in a factory can be considered as an Infinite population.

3. population of the heights of Indian 4. The population of Nationalised Banks in India.
5. population of births, weights, prices of vegetables...

Batch (or) Sample :- A finite subset of a population is a Batch.

In some cases batches are taken from population and samples are taken from batches.

Batch Size :- The Number of items in a batch

Sample Size :- (or) batch Size The Number of items in a Sample is called Sample size

Small Samples :- A small with less than 30 items in it is called Small Sample.

Large Samples :- A small with greather than or equal to 30 items in it is called a Large Sample.

v.v.v.Imp ① Explain Types of Sampling?

(or)

Explain Sampling Distribution types?

Sampling distribution:- A method of selecting Sample the population is called Sampling distribution.

Sampling distribution

- Random Sampling distribution (or) Probability Sampling
  1. Simple
  2. Stratified
  3. Systematic
  4. cluster
  5. Quota
  6. Multi Stage
- Non-Random Sampling distribution
  1. purposive Sampling
  2. Judgement.

I - Random Sampling distribution:-

① Simple Sampling distribution:- This is the chance of member of the population for the Sample is independent of the previous selection.

This distribution is called Simple Sampling distribution

Example:-
1. Selecting randomly 20 words from a dictionary
2. A die or Coin — Simple Sampling
3. A packet Cards — Not a Simple Sampling.

Stratified Sampling:- (or) Sub-population Sampling:- } Sometimes (3) the heterogeneous population may be divided into sub-populations of strata, which are more homogeneous with in itself.

but Sampling distribution:-

Examples:- The population of Employees may be divided into strata rural employees and Urban employees. After dividing the population into strata, we select individuals at random.

This Sampling distribution is best. It is a good representive of the population.

(or) Quasi Random Sampling:-

(3) Systematic Sampling Distribution:- In this, we select units from the population "at Uniform" intervals of time.

(4) Quota Sampling distribution:- In this distribution, each method engaged in the primary collection of data is assigned a Certain Quota of investigations.

(5) Multi stage Sampling distribution:- In this distribution, Sampling is done at each stage starting from the Larger Units, intermediate Units and finally reaching the Ultimate Units of selection.

II – Non-Random Sampling distribution:-

① purposive Sampling distribution:-

(or)

Judgement Sampling distribution:-

If the Sample elements are selected with a definite purpose in mind, then, the Sample selected is called purposive Sampling distribution.

Example:- ① In a factory, Suppose there types of items are produced, Suppose there are some Complaints against a product. Then, they Consider only the products (or) the individual who have Complaints and ignore the others.

② 20 students to be selected from a class of 100 to analyse the extra curricular activities of the students. The Investigator would select 20 students who, in his judgement to represent the class.

② Explain Sampling distribution of the Mean re σ – known?

Suppose, we take Sample-1 of size n_1 from a population to test, mean is $\overline{x}_1$.

Suppose, we take Sample-2 of size n_2 from a population to test, mean is $\overline{x}_2$.

$$\boxed{\therefore \text{Mean} = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

Variance $s^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$ (4)

Example:- Consider Samples 5

Population:-

population = (1, 2, 3, 4, 5)

Mean of the population $\mu = \frac{1+2+3+4+5}{5} = 3$ [illegible]

Variance of the population $\sigma^2 = \frac{(1-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2}{5}$

$$\boxed{= \frac{10}{5}} = 2$$

Sampling distribution:-

25 - Samples are Have

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)

Means of Sampling distribution:-

1	1.5	2	2.5	3
1.5	2	2.5	3	3.5
2	2.5	3	3.5	4
2.5	3	3.5	4	4.5
3	3.5	4	4.5	5

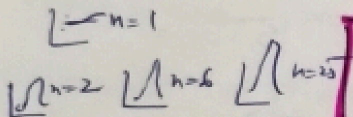
$\mu_{\bar{x}} = \frac{1+1.5+2+2.5+ \cdots + 4.5+5}{25} = 3$

$$\therefore \sigma^2_x = \frac{\Sigma(x_i - \bar{\mu}x)^2}{n} = 1$$

$$\boxed{\begin{array}{c} \therefore \sigma^2 \neq \sigma^2 x \\ \mu = \bar{\mu} x \end{array}}$$

Central Limit theorem:-

Statement:- "If $\bar{x}$ is the mean of a random sample of size n - taken from a population having the mean μ & finite Variance σ^2 "



when n increases it approached to Normality

$$\boxed{Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}$$

Then, this approaches to Standard Normal distribution as $n \to \infty$.

v.v.Imp **probable Error:-** "For large Sample size 'n' there is a fifty-fifty chance that the mean of a random Sample from an infinite population with the Standard deviation σ will differ from μ by less than $0.6745 \frac{\sigma}{\sqrt{n}}$. This is called "probable Error"."

If p denotes the probable Error

$$P = 0.6745 \frac{\sigma}{\sqrt{n}}$$

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Explain Standard Error ?

or

Explain S.E of a Statistic :-

The standard Error of a statistic t (i.e S.E of Sample Mean or Sample S.D) is the standard deviation of the sampling distribution of the statistic.

S.E of Saple Mean $= \bar{x}$.

Note : S.E of a statistic may be reduced by increasing Sample size $\boxed{n}$, but this results in corresponding increase in Cost, time and Labour ... e.tc.

Formulas :-

1. S.E of Sample Mean $\bar{x} = \frac{\sigma}{\sqrt{n}}$
2. " " " proportion $p = \sqrt{\frac{PQ}{n}}$ (then $Q = 1-p$)
3. " " " S.D $s = \frac{\sigma}{\sqrt{2n}}$
4. S.E of the difference of two Saple Means $\bar{x}_1$ & $\bar{x}_2$:-

   S.E of $(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
5. S.E of $P_1 - P_2 = \sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}$
6. S.E of $(s_1 - s_2) = \sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}$

7) Finite population of size N

A Sample is drawn without Replacement :-

1) S.E of Sample Mean $= \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$

2) S.E of Sample proportion $= \sqrt{\frac{PQ}{n}} \sqrt{\frac{N-n}{N-1}}$

for also same formulae.

V.I.S

Infinite population :- The Samples are drawn from an Infinite population i.e $N \to \infty$ (or)

Sampling is done with Replacement. Then,

The Mean of the Sampling distribution of Means $\underline{E(\bar{x}) = \mu}$

S.D of Mean $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$Var(\bar{x}) = \frac{\sigma^2}{n}$

Sample size large :- $Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

V.V.Imp

Finite population :-

1. $E(\bar{x}) = \mu$

2. $Var(\bar{x}) = \frac{\sigma^2}{n} \left(\frac{N-n}{N-1}\right)$

3. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$

Where $\frac{N-n}{N-1}$ = Correction factor.

ssification of Samples:-

Samples are classified into two types:-

① Large Sample (or) Non-Normal population

② Small Sample. (or) Normal population.

① Large Sample:- If the size of the Sample $\boxed{n \geq 30}$.
Then, it is called Large Sample.

② Small Sample:- (② Exact Sample:-) If the size of the Sample $\boxed{n < 30}$
Then, Sample is said to be Small Sample (or) Exact Sample.

Explain types of Sampling distribution? It is of two types ① with Replace ② without Replace

① Sampling with Replacement:-

Each member of the population may be chosen more than once. Since, the member is replaced in the population.

∴ Sampling from finite population with Replacement can be considered theoretically as a Sampling from Infinite population.

② Sampling Without Replacement:-

An element of population cannot be chosen more than once, as it is not replaced.

∴ Sampling from finite population without replacement can be considered theoretically as Sampling from finite population only.

Joint probability distribution:-

$f(x_1, x_2 \dots x_n) = f(x_1), f(x_2) \dots f(x_n)$.

Statistic:- It is a real valed function of the saple.

∴ Statistic is a function of one or More Random variables not involving any parameter.

∴ Statistic is a R.V & has a probability or frequecy distribution.

Parameter:- parameter is a statistical measure based on all the units or observations of a population

Sapling fluctuation:- If the Variation in the value of a statistic is called Sapling fluctuation.

"But A parameter has no Sapling fluctuation"

Saple Mean → $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$

Saple Variance →

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

Saple Standard Deviation = $S.D = \sqrt{S}$

Stand error !-

The standard deviation of the sampling distribution of a statistic is called known as standard error and is denoted by S.E.

Statistics	Standard Error
Means	$\sigma_{x} = \frac{\sigma}{\sqrt{n}}$
Standard deviations	$\sigma_s = \frac{\sigma}{\sqrt{2n}}$
varience	$\sigma_s^2 = \sigma^2 \cdot \sqrt{\frac{2}{n}}$
Correlation Coefficient	$\sigma_r = \frac{1-\rho^2}{\sqrt{n}}$
poportions	$\sigma_p = \frac{\sqrt{PQ}}{n}$

Note !- The no of Samples with replacement $= N^n$

" " " with out " $= N_{C_n}$

Correction factor $= \frac{N-n}{N-1}$

probable error !- for a large Sample of size n, there is 50% Chance that the mean of a random Sample from an infinite population with the standard deviation σ will differ from mean μ by less than $0.6745\frac{\sigma}{\sqrt{n}}$. This is called probable error. i.e $P = 0.6745\sigma$.

p is probable error $\sigma = \frac{P}{0.6745}$

$$\Rightarrow 3\sigma = \frac{3P}{0.6745} = 4.5P.$$

The probable error of Distribution is 0.6745 times of standard error.

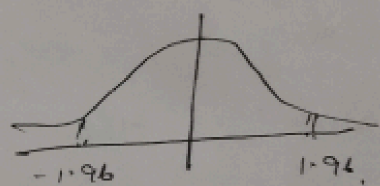
Sampling Distribution.

Let us consider a finite population of size ... all possible samples of size n. For any sample we can com... statistics such as mean, varience etc. If the sample is given by $\{x_1, x_2, \dots x_n\}$. Mean and varience. These statistics of different samples from the given population are not equal. Such a set of all statistics one for each sample is called sampling distribution of a statistic.

P. A research worker wishes to estimate mean of a population by using sufficiently large sample. The probability is 95%. that sample mean will not differ from the true mean by more than 25%. of the standard deviation. How large sample should be taken

Sol ~~The~~

Given probability is 95%. = 0.95



$0.95 \Rightarrow \frac{0.95}{2} = \pm 0.475$

$= \pm 1.96$ [from N.D tables

$z = \pm 1.96$

$|z| = 1.96 \Rightarrow \left|\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right| = 1.96 \Rightarrow |\bar{x} - \mu| = 1.96 \frac{\sigma}{\sqrt{n}} \rightarrow (1)$

given that sample mean will not differ from the true mean by more than 25%. of the S.D.

$|\bar{x} - \mu| = \frac{\sigma}{4} \longrightarrow (2)$

from (1) and (2)

$\therefore \sqrt{n} = \frac{1.96\sigma}{\sigma/4} = 4 \times 1.96 = 7.84$

$n = (7.84)^2 = 61.46 \simeq 62$

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...tem:-

...tement:- Sample Mean is Unbiased (not change) estimate of population Mean?

Let $E(X) = \mu$

$V(X) = \sigma^2$

$$E(\bar{X}) = \frac{E(X_1 + X_2 + \cdots + X_n)}{n}$$

$$= \frac{E(X_1) + E(X_2) + \cdots + E(X_n)}{n}$$

$$= \frac{\mu + \mu + \cdots + \mu}{n}$$

$$= \frac{n\mu}{n}$$

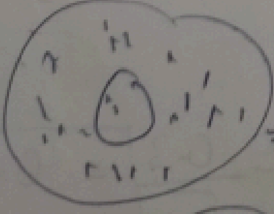
$$\boxed{\therefore E(\bar{X}) = \mu}$$

Sample Mean → population Mean

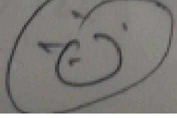
Note:- 1. ISU = Indian Statical Unit.

2. Test:- Eg:- Using to find the precisions of the goods

$$\text{precision (or) quality} = \frac{1}{\text{Variance (or) spreading}}$$



= precision is Low (i.e high = $\frac{1}{Low}$)

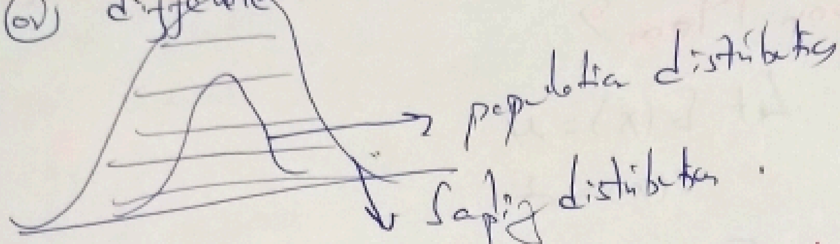


= precision is high.

(2) Variance :- Difference of objects to

(or) difference.

(3)



Problems Under Correction factor & Sample Error

(1) Find the value of the finite population Correction factor for n=10 & N=1000?

Correction factor for finite population $= \frac{N-n}{N-1}$

N = population items = 1000
n = Sample items = 10

$= \frac{1000-10}{1000-1}$

$= \frac{990}{999}$

$\boxed{= 0.99}$

(2) A Sample is collected from the items produced by a factory. The Sample Size is 81. The standard deviation of the population is 0.3. find the standard error of the Mean of Sampling distribution?

$n = 81$

$\sigma = 0.3$

$S.E = \frac{\sigma}{\sqrt{n}} = \frac{0.3}{\sqrt{81}}$ $\boxed{= 0.033}$

③ A Normal population has a s.d of 1. Sample of size 400 is collected for testing. find the standard error of the Mean of sampling distribution?

$\sigma = 1$

$n = 400$

$$S.E = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{400}} = \boxed{0.05}$$

④ The Variance of a population is 2. The size of the sample collected from the population is 169. What is the standard error of Mean?

$n = 169$

$\sigma = \sqrt{2}$

$$S.E = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{2}}{\sqrt{169}} = \boxed{0.1087}$$

⑤ A sample of size = 100 is taken from population whose S.D is 16. Find S.E & probable Error?

$\sigma = 16,\ n = 100$

$$S.E = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{100}} = 1.6$$

$\therefore$ probable Error $= 0.6745 \times S.E$

$= 0.6745 \times 1.6$

$$\boxed{PE = 1.0792}$$

H.W

⑥ A Sample is collected from the items produced by a factory. The Sample size is 81. The standard deviation is 0.3. What is the Probable error of Sampling distribution?

$P.E = 0.6745 \times S.E$

$= 0.6745 \times 0.033 = 0.022$

8

(7) If the population size is 6, Sample size = 3.
Find the Number of samples that can be drawn
(1) with Replacement (2) without Replacement

(1) with Replacement :- $N^n = 6^3 = 216$

(2) without Replacement :- $^{N}C_{n} = {}^{6}C_{3} = \frac{4 \times 5 \times 6}{1 \times 2 \times 3} = 20$

II - Problems Under Sampling Distribution