06|08|19

UNIT- II

Mathematical

If x is a discrete random variable taking the value of x from 0, 1, 2 .... n is denoted by E(x) and is defined as

$$E(x) = \sum_{x=0}^{n} x \cdot p(x)$$

similarly

$$E(x^r) = \sum_{x=0}^{n} x^r p(x)$$

Similarly, If 'x' is a continuous random variable taking x value in between $-\alpha$ & α i.e; $-\alpha \leq x \leq \alpha$ with f(x) as the probability density function, expectation of x is

$$E(x) = \int_{-\alpha}^{\infty} x \cdot f(x)\,dx$$

similarly

$$E(x^r) = \int_{-\alpha}^{\infty} x^r \cdot f(x)\,dx$$

Mean:-

E(x) is also called a mean of the x denoted by μ or $\bar{x}$ → to denote sample

↓

to denote population

Variance (σ^2):

If 'x' is random variable then variance of 'x' is denoted by σ^2 and is defined as

$$V(x) = \sigma^2 = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \mu^2$$

$$= E(x^2) - \bar{x}^2$$

Standand deviation (σ):

positive ~~sequence~~ square root of variance of x.

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$\sigma = \sqrt{V(x)} = \sqrt{E(x^2) - (E(x))^2}$

$E(x^2) = \sum_{x=0}^{n} x^2 \cdot P(x)$ for discrete random variable x

$= \int_{-\infty}^{\infty} x^2 f(x)\, dx$ for continuous random variable x

$V(x) = E(x^2) - [E(x)]^2$

$= \sum_{x=0}^{n} x^2 P(x) - \left[\sum_{x=0}^{n} x P(x)\right]^2$ for discrete random variable x

$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - \left[\int_{-\infty}^{\infty} x f(x)\, dx\right]^2$ for continuous random variable x

Co-Variance

$Cov(x, y) = E[(x - E(x))(Y - E(Y))]$

$= E[xY - xE(Y) - YE(x) + E(x)E(Y)]$

Properties of Expectations

1. x, Y are random variables and a, b are constants then

a) $E(a) = a$

b) $E(ax) = aE(x)$

c) $E(ax \pm b) = aE(x) \pm b$

d) $E(ax + bY) = aE(x) + bE(Y)$

Properties of variance

If x and Y are two random variables and a, b are constants then

① $V(x+a) = V(x)$

② $V(ax) = a^2 V(x)$

③ $V(a) = 0$

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4. $V(ax+b) = a^2V(x)$

5. $V(ax-b) = a^2V(x)$

6. $V(aX \pm bY) = a^2V(x) + b^2V(y)$ (if X and Y are independent random variables)

7. $V(X \pm Y) = V(X) + V(Y) \pm 2\,cov(X,Y)$

Covariance

$$cov(X,Y) = E[(X-E(X))(Y-E(Y))]$$

$$= E[XY - XE(Y) - YE(X) + E(X)\cdot E(Y)]$$

$$= E(XY) - E[XE(Y)] - E[YE(X)] + E(E(X)\cdot E(Y))$$

$$= E(XY) - E(Y)\cdot E(X) - E(X)\cdot E(Y) + E(X)E(Y)$$

$$cov(X,Y) = E(XY) - E(X)\cdot E(Y)$$

Addition Theorem of Expectation:

The mathematical expections of sum of random variables is equal to the sum of their individual expentqo-ns provied all the expections exist.

i.e. $E(X+Y+Z) = E(X) + E(Y) + E(Z)$

Multiplication Theorem of Expectation

The mathematical expectation of the product of a no. of independent random variables is equal to product of their expectation.

i.e., for X, Y are 2 independent random variables

$$E(XY) = E(X)\cdot E(Y)$$

Expectation of linear combination of random variab-les:

If $x_1, x_2 \ldots\ldots x_n$ be any n random variables and if $a_1, a_2 \ldots\ldots a_n$ are any 'n' constants then

$$E\left[\sum_{i=1}^{n} a_i x_i\right] = \sum_{i=1}^{n} a_i E(x_i)$$ provided All expectation exists

Proof:- $E\left[\sum_{i=1}^{n} a_i x_i\right]$

$$\sum_{i=1}^{n} a_i x_i = a_1x_1 + a_2x_2 + \dots + a_nx_n = Y$$

$$E(Y) = E(a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n)$$

$$= a_1E(x_1) + a_2E(x_2) + a_3E(x_3) + \dots + a_nE(x_n)$$

$$E\left[\sum_{i=1}^{n} a_i x_i\right] = \sum_{i=1}^{n} a_i E(x_i)$$

Variance of linear combination of random variables

If $x_1, x_2, \dots, x_n$ are in random variables and if $a_1, a_2, \dots, a_n$ are any 'n' constants, then

$$V\left[\sum_{i=1}^{n} a_i x_i\right] = \sum_{i=1}^{n} a_i^2 V(x_i) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j \operatorname{cov}(x_i, x_j)$$

Proof:-

$$V\left[\sum_{i=1}^{n} a_i x_i\right] = V(Y) = E\left[Y - E(Y)\right]^2 = E\left[Y^2 - 2YE(Y) + \left[E(Y)\right]^2\right]$$

$$= E(Y^2) - 2E(Y)E(Y) + \left[E(Y)\right]^2$$

$$= E(Y^2) - 2\left[E(Y)\right]^2 + \left[E(Y)\right]^2$$

$$= E(Y^2) - \left[E(Y)\right]^2$$

$$V(Y) = E(Y - E(Y))^2$$

$$= E\left[(a_1x_1 + a_2x_2 + \dots a_nx_n) - E(a_1x_1 + a_2x_2 \dots + a_nx_n)\right]^2$$

$$= E\left[a_1\left[x_1 - E(x_1)\right] + a_2(x_2 - E(x_2)\right] + a_3(x_3 - E(x_3)] + \dots$$
$$a_n\left[x_n - E(x_n)\right]\Big]^2$$

$$= E\left[a_1^2\left[x_1 - E(x_1)\right]^2 + a_2\left[x_2 - E(x_2)\right]^2 + a_3^2\left[x_3 - E(x_3)\right]^2 + \dots\right.$$
$$a_n^2\left[x_n - E(x_n)\right]^2 + 2a_1a_2(x_1 - E(x_1)) + 2a_1a_3(x_1 - E(x_1))$$
$$\left.(x_3 - E(x_3)) + \dots\dots\right]$$

$$= a_1^2 E\left[x_1 - E(x_1)\right]^2 + a_2^2 E(x_2 - E(x_2))^2 + \dots + a_n^2 E(x_n - E(x_n))^2$$
$$+ 2a_1a_2 E\left[(x_1 - E(x_1))(x_2 - E(x_2))\right] + \dots$$

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$= a_1^2 V(x_1) + a_2^2 V(x_2) + \dots + a_n^2 V(x_n) + 2a_1 a_2 \, cov(x_1, x_2) + 2a_1 a_3$

$cov(x_1, x_3) + \dots$

$\therefore V\left[\sum_{i=1}^{n} a_i x_i\right] = \sum_{i=1}^{n} a_i^2 V(x_i) + 2\sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \, cov(x_i, x_j)$

Results

1. if all $a_i = 1$ then

$V\left[\sum_{i=1}^{n} x_i\right] = \sum_{i=1}^{n} V(x_i) + 2\sum_{i=1}^{n} \sum_{j=1}^{n} cov(x_i, x_j)$

if $a_1 = a_2 = 1$, x_1, x_2 and then random variables

$V[x_1 + x_2] = V(x_1) + V(x_2) + 2\,Cov(x_1, x_2)$

if $a_1 = a_2 = a_3 = 1$, x_1, x_2, x_3 are random variables

$V(x_1 + x_2 + x_3) = V(x_i) + V(x_2) + V(x_3) + 2\,Cov(x_1, x_2) + 2\,Cov(x_1, x_3) +$

$2\,cov(x_2, x_3)$

2. If $x_1, x_2, x_3 \dots\dots x_n$ are pairwise independent random variables

07/08/19

Problems on Expectations, Variance

1. Given the following probability distribution of x compute

i) $E(x)$ ii) $E(2x \pm 3)$ iii) $E(x^2)$ iv) $V(x)$ v) $V(2x \pm 3)$

x	-3	-2	-1	0	1	2	3
$P(x)$	0.05	0.10	0.30	0	0.30	0.15	0.10

i) $E(x) = \sum_{x=-3}^{3} x\,P(x)$

$= -3 \cdot P(-3) + (-2)P(-2) + (-1)P(-1) + 0 \cdot P(0) + 1P(1) + 2 \cdot P(2) + 3 \cdot P(3)$

$= -3 \cdot (0.05) + (-2)(0.10) + (-1)(0.30) + 0(0) + 1 \cdot (0.30) + 2(0.15) +$

$3(0.10)$

$= -0.15 - 0.20 - 0.30 + 0 + 0.30 + 0.30 + 0.30$

$= 0.25$

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iii) $E(2x \pm 3)$

$\Rightarrow E(2x+3) = 2E(x)+3 = 2(0.25)+3$

$= 3.5$

$E(2x-3) = 2E(x)-3 = -2.50$

iii) $E(x^2) = \sum_{x=-3}^{3} x^2 p(x) = (-3)^2(0.05)+(-2)^2(0.10)+(-1)^2(0.30)+0^2(0)+1^2$

$(0.30)+2^2(0.15)+3^2(0.10)$

$= 9(0.05)+4(0.10)+1(0.30)+1(0.30+4(0.15)$

$+9(0.10)$

$= 0.45+0.40+0.30+0.30+0.60+0.90$

$= 2.95$

iv) $V(x) = E(x^2) - [E(x)]^2$

$= 2.95 - (0.25)^2$

$= 2.95 - 0.0625$

$= 2.8875$.

v) $V(2x \pm 3)$

$V(2x+3) = 2^2 V(x) = 4(2.8875) = 11.55$

$V(2x-3) = 2^2 V(x)$

$= 4(2.8875)$

$= 11.55$

2. a random variable x has the following probability distribution

x	-2	-1	0	1	2	3
$P(x)$	0.1	K	0.2	2K	0.3	K

find i) the value of K ii) mean iii) variance

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Sol:- $\sum_{x=-2}^{3} P(x) = 1$

$\Rightarrow 0.1+k+0.2+2k+0.3+k=1$

$\Rightarrow 4k+0.6=1$

$4k = 1-0.6$

$k = \frac{0.4}{4}$

$=0.1$

now probability distribution become

x	-2	-1	0	1	2	3
P(x)	0.1	0.1	0.2	0.2	0.3	0.1

mean $= E(x) = \sum_{x=-1}^{3} x p(x) = -2(0.1)+(-1)(0.1)+0(0.2)+1(0.2)+2(0.3)+3(0.1)$

$=-0.2-0.1+0+0.2+0.6+0.3$

$=0.8$

Variance $(\sigma^2) = E(x^2)-[E(x)]^2$

$E(x^2) = \sum_{x=-2}^{3} x^2 p(x) = (-2)^2(0.1)+(-1)^2(0.1)+0^2(0.2)+1^2(0.2)+2^2(0.3)+3^2(0.1)$

$= 0.4+0.1+0+0.2+1.2+0.9$

$=2.8$

$V(x) = E(x^2)-[E(x)]^2 = 2.8-(0.8)^2$

$=2.8-0.64$

$=2.16$

3) If x_1 and x_2 are two independent r.v's such that Mean of x_1 is 4 and that of x_2 is 2 then $E(2x_1+3x_2+2)$

Sol:- Given that $E(x_1)=4$, $E(x_2)=2$

$E(2x_1+3x_2+2) = 2E(x_1)+3E(x_2)+2$

$E(ax+b) = aE(x)+b$

$E(ax+bY) = aE(x)+bE(Y)$

$= 2\times4+3(2)+2$

$= 8+6+2$

$= 16$

4. If x_1 and x_2 are two independent random variable such that variance of x_1 is 5 and that of x_2 is 3 then i) $V(2x_1+3x_2+4)$

ii) $V(2x_1-3x_2+3)$

sol:- Given that $V(x_1)=5$, $V(x_2)=3$

$V(2x_1+3x_2+4) = 2^2V(x_1)+3^2V(x_2)$

$= 4\times5+9\times3$

$= 47$

$V(2x_1-3x_2+3) = 2^2V(x_1)+3^2V(x_2)$

$= 4\times5+9\times3$

$= 47.$

5. a fair coin is tossed until a head or five tails occurs Find the expected number E of tosses of the coin.

sol:- x - number tosses required

$P(1) = P(\text{success in the } 1^{st} \text{ toss}) = P(H) = \frac{1}{2}$

$P(2) = P(\text{success in the } 2^{nd} \text{ toss}) = P(TH) = P(T)\cdot P(H)$

$= \frac{1}{2}\cdot\frac{1}{2}$

$= \frac{1}{4}$

$P(3) = P(\text{success in the } 3^{rd} \text{ toss}) = P(TTH) = \frac{1}{8}$

$P(4) = P(\text{success in the } 4^{th} \text{ toss}) = P(TTTH) = \frac{1}{16}$

$P(5) = P(\text{success in the } 5^{th} \text{ toss}) = P(TTTTH \cup TTTTT)$

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$= P(TTTTH) + P(TTTTT)$

$= \frac{1}{32} + \frac{1}{32}$

$= \frac{1}{16}$

probability distribution of x

x	1	2	3	4	5
P(x)	1/2	1/4	1/8	1/16	1/16

$E(x) = \sum_{x=1}^{5} x P(x)$

$= 1 \cdot \frac{1}{2} + 2 \times \frac{1}{4} + 3 \cdot \frac{1}{8} + 4 \times \frac{1}{16} + 5 \times \frac{1}{16}$

$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \frac{5}{16}$

$= 1.9375$

6. A pair of fair die is tossed. let x denotes the maximum of the number appearing i.e $x(a,b) = \max(a,b)$ of the number appearing (a,b) and let Y denotes the sum of the number appearing i.e $Y(a,b) = a+b$ then find the variance and standard deviation of x and Y, x+Y

$V(x) = E(x^2) - [E(x)]^2$

sol:- $x(a,b) = \max(a,b)$

$V(Y) = E(Y^2) - [E(Y)]^2$

x	1	2	3	4	5	6
P(x)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$V(x+Y) = V(x) + V(Y)$

$E(x) = \sum_{x=1}^{6} x P(x)$

$= 1 \times \frac{1}{36} + \frac{2 \times 3}{36} + \frac{3 \times 5}{36} + \frac{4 \times 7}{36} + \frac{5 \times 9}{36} + \frac{6 \times 11}{36}$

$= \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36} = \frac{161}{36}$

$= 4.4722$

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$$E(x^2) = \sum_{x=1}^{6} x^2 P(x) = 1^2 \times \frac{1}{36} + 2^2 \times \frac{3}{36} + 3^2 \times \frac{5}{36} + 4^2 \times \frac{7}{36} + 5^2 \times \frac{9}{36} + 6^2 \times \frac{11}{36}$$

$$= \frac{1}{36} + \frac{12}{36} + \frac{45}{36} + \frac{112}{36} + \frac{225}{36} + \frac{396}{36}$$

$$= \frac{791}{36}$$

$$= 21.97$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= 21.97 - (4.47)^2$$

$$= 21.97 - 19.98$$

$$= 1.99$$

Y	2	3	4	5	6	7	8	9	10	11	12
P(Y)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$$E(x) = \sum_{x=2}^{12} YP(Y) = 2 \times \frac{1}{36} + 3 \times \frac{2}{36} + 4 \times \frac{3}{36} + 5 \times \frac{4}{36} + 6 \times \frac{5}{36} + 7 \times \frac{6}{36}$$

$$+ 8 \times \frac{5}{36} + 9 \times \frac{4}{36} + 10 \times \frac{3}{36} + 11 \times \frac{2}{36} + 12 \times \frac{1}{36}$$

$$= \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$$

$$= \frac{252}{36}$$

$$= 7$$

$$E(x^2) = \sum_{x=2}^{12} Y^2 P(Y) = 2^2 \times \frac{1}{36} + 3^2 \times \frac{2}{36} + 4^2 \times \frac{3}{36} + 5^2 \times \frac{4}{36} + 6^2 \times \frac{5}{36} + 7^2 \times \frac{6}{36}$$

$$8^2 \times \frac{5}{36} + 9^2 \times \frac{4}{36} + 10^2 \times \frac{3}{36} + 11^2 \times \frac{2}{36} + 12^2 \times \frac{1}{36}$$

$$= \frac{4}{36} + \frac{18}{36} + \frac{48}{36} + \frac{100}{36} + \frac{180}{36} + \frac{294}{36} + \frac{320}{36}$$

$$+ \frac{324}{36} + \frac{300}{36} + \frac{242}{36} + \frac{144}{36}$$

$$= \frac{1974}{36}$$

$$= 54.83$$

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$V(Y) = E(Y^2) - [E(Y)]^2$

$= 54.83 - 49$

$= 5.83$

$V(X+Y) = V(X) + V(Y)$

$= 1.99 + 5.83$

$= 7.82$

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9/08/19

Problems on continuous random variables

1. let a continuous random variable x has the p.d.f

$$f(x) = \begin{cases} \frac{x+1}{2} & , \text{if } -1 < x < 1 \\ 0 & , \text{otherwise} \end{cases}$$

Sol:- $\mu = \text{Mean} = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_{-\infty}^{-1} x f(x)\,dx + \int_{-1}^{1} x f(x)\,dx + \int_{1}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{-1} x(0)\,dx + \int_{-1}^{1} x\left(\frac{x+1}{2}\right)dx + \int_{1}^{\infty} x(0)\,dx$$

$$= 0 + \int_{-1}^{1}\left(\frac{x^2+x}{2}\right)dx + 0$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\left(\frac{1^3}{3} + \frac{1^2}{2}\right) - \left(\frac{-1}{3} + \frac{1}{2}\right)\right]$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right)$$

$$= \frac{1}{3}$$

Variance $v(x) = E(x^2) - [E(x)]^2$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_{-\infty}^{-1} x^2 f(x)\,dx + \int_{-1}^{1} x^2 f(x)\,dx + \int_{1}^{\infty} x^2 f(x)\,dx$$

$$= 0 + \int_{-1}^{1} x^2\left(\frac{x+1}{2}\right)dx + 0$$

$$= \int_{-1}^{1} \frac{1}{2}(x^3+x^2)\,dx$$

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$$= \frac{1}{2}\left|\frac{x^4}{4} + \frac{x^3}{3}\right|_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{4}+\frac{1}{3} - \left(\frac{1}{4} + \frac{-(1)}{3}\right)\right]$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right)$$

$$= \frac{1}{3}$$

$$V(x) = E(x^2) - [E(x)]^2 = \frac{1}{3} - \left(\frac{1}{3}\right)^2 = \frac{3-1}{9} = \frac{2}{9}$$

standard deviation $(\sigma) = \sqrt{V(x)}$

$$= \sqrt{\frac{2}{9}}$$

$$= \sqrt{2}/3$$

2. let a continuous random variable x has the p.d.f

$f(x) = kx^2e^{-x}$, when $x \geq 0$

$= 0$ otherwise

find i) k ii) Mean iii) variance

sol:- We know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow 0 + \int_0^{\infty} kx^2e^{-x}\,dx = 1$$

$$\Rightarrow k\cdot\int_0^{\infty} x^2e^{-x}\,dx = 1 \qquad \int uv\,dx = uv_1 - u_1v_2 + u_2v_3 \ldots$$

$$\Rightarrow k\left|x^2\left(\frac{e^{-x}}{-1}\right) - 2x\cdot(e^{-x}) + 2\cdot1\left(\frac{e^{-x}}{-1}\right)\right|_0^{\infty} = 1$$

$$k[(0-0+0) - (0-0-2e^{-0})] = 1$$

$$k(2\cdot1) = 1$$

$$k = 1/2$$

$$\text{Mean} = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^\infty x\cdot(kx^2e^{-x})\,dx$$

$$= \frac{1}{2}\int_0^\infty x^3e^{-x}\,dx$$

$$= \frac{1}{2}\left[x^3\left(\frac{e^{-x}}{-1}\right) - 3x^2\left(e^{-x}\right) + 6x\left(\frac{e^{-x}}{-1}\right) - 6\cdot1\cdot e^{-x}\right]_0^\infty$$

$$= \frac{1}{2}\left[(0-0+0+0) - (0+0+0-6)\right]$$

$$= \frac{6}{2}$$

$$= 3$$

3. Variance $V(x) = E(x^2) - [E(x)]^2$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_0^\infty x^2 f(x)\,dx = \int_0^\infty x^2\cdot(k\cdot x^2e^{-x})\,dx$$

$$= \frac{1}{2}\int_0^\infty x^4e^{-x}\,dx$$

$$= \frac{1}{2}\left[x^4\left(\frac{e^{-4}}{-1}\right) - 4x^3\left(e^{-x}\right) + 12x^2\left(\frac{e^{-x}}{-1}\right) - 24x\left(e^{-x}\right) + 24\left(\frac{e^{-x}}{-1}\right)\right]_0^\infty$$

$$= \frac{1}{2}\left[-(-24)\right] = 12$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= 12 - 3^2$$

$$= 3$$

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13/08/19

chebyshev's Inequality

If we know the probability distribution of a random variable 'x' we can get ~~new~~ μ and σ^2 but if μ and σ^2 are known it is not possible to construct the probability of 'x'. To Obtain the probability of $P\{|x-\mu| \le k\}$ distribution

several approximation techniques are available to get upper & lower bounds to such probabilities, the most important technique is chebyshev's inequality. This can be applied to any discrete or continuous random variable

Theoream:-

If a probability distribution has mean μ and standard deviation σ. then probability of getteing a value which deviates from μ by atleast $k\sigma$ is at most $\frac{1}{k^2}$

i.e $P(|x-\mu| \ge k\sigma) \le \frac{1}{k^2}$

proof:

case(i): If x is a continuous random variable

By defination

$$\text{Variance} = \sigma^2 = E[x - E(x)]^2 = E(x-\mu)^2$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

$-\infty$ $\mu-k\sigma$ $\mu+k\sigma$ ∞

$$= \int_{-\infty}^{\mu-k\sigma} (x-\mu)^2 f(x)\,dx + \int_{\mu-k\sigma}^{\mu+k\sigma} (x-\mu)^2 f(x)\,dx + \int_{\mu+k\sigma}^{\infty} (x-\mu)^2 f(x)\,dx$$

$$\sigma^2 \ge \int_{-\infty}^{\mu-k\sigma} (x-\mu)^2 f(x)\,dx + \int_{\mu+k\sigma}^{\infty} (x-\mu)^2 f(x)\,dx \quad \left(\because \int_{\mu-k\sigma}^{\mu+k\sigma} (x-\mu)^2 f(x)\,dx \ge 0\right)$$

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for the first integral $x \leq \mu - k\sigma$

$$\Rightarrow k\sigma \leq \mu - x$$

$$\Rightarrow k\sigma \leq -(x-\mu) \quad \text{--- (1)}$$

for the second integral $x \geq \mu + k\sigma$

$$x - \mu \geq k\sigma \quad \text{--- (2)}$$

$$\sigma^2 \geq \int_{-\infty}^{\mu-k\sigma} k^2\sigma^2 \cdot f(x)\,dx + \int_{\mu+k\sigma}^{\infty} k^2\sigma^2 f(x)\,dx$$

$$\sigma^2 \geq k^2\sigma^2 \left[\int_{-\infty}^{\mu-k\sigma} f(x)\,dx + \int_{\mu+k\sigma}^{\infty} f(x)\,dx \right]$$

$$\frac{1}{k^2} \geq \int_{-\infty}^{\mu-k\sigma} f(x)\,dx + \int_{\mu+k\sigma}^{\infty} f(x)\,dx$$

$$\frac{1}{k^2} \geq P(-\infty \leq x \leq \mu - k\sigma) + P(\mu + k\sigma \leq x \leq \infty)$$

$$\frac{1}{k^2} \geq P(x \leq \mu - k\sigma) + P(\mu + k\sigma \leq x)$$

from (1) & (2)

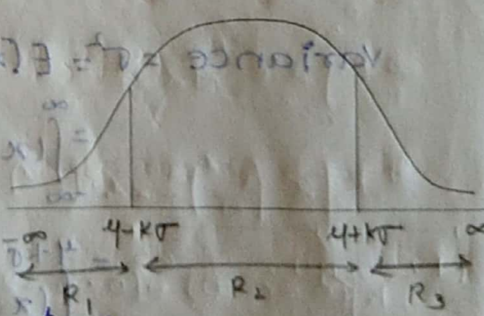
$$\frac{1}{k^2} \geq P\{|x-\mu| \geq k\sigma\}$$

$$\Rightarrow P\{|x-\mu| \geq k\sigma\} \leq \frac{1}{k^2}$$

Case 2) When x is a discreate random variable

$$\sigma^2 = E(x-\mu)^2$$

$$\sigma^2 = \sum_{\text{all } x} (x-\mu)^2 P(x)$$



$$\sigma^2 = \sum_{R_1} (x-\mu)^2 P(x) + \sum_{R_2} (x-\mu)^2 P(x) + \sum_{R_3} (x-\mu)^2 p(x)$$

Where R_1 is the region for which $x \leq \mu - k\sigma$

R_2 is the region for which $\mu - k\sigma \leq x \leq \mu + k\sigma$

R_3 is the region for which $x \geq \mu + k\sigma$

since $(x-\mu)^2 p(x)$ cannot be negative

$\therefore$ The above sum over the region R_2 is non-negative and without it the sum of the sumations over R_1 & R_2 is $\leq \sigma^2$

i.e $\sigma^2 \geq \sum_{R_1} (x-\mu)^2 p(x) + \sum_{R_3} (x-\mu)^2 p(x)$

but

$x \leq \mu - k\sigma$

$\Rightarrow x-\mu \leq -k\sigma$ in R_1

and $x-\mu \geq k\sigma$ in R_3

$|x-\mu| \geq k\sigma$ in both region

$\therefore (x-\mu)^2 \geq k^2\sigma^2$

$\therefore \sigma^2 \geq \sum_{R_1} k^2\sigma^2 p(x) + \sum_{R} k^2\sigma^2 p(x) \Rightarrow \frac{1}{k^2} \geq \sum_{R_1} p(x) + \sum_{R_2} p(x)$ —⊛

$\sum_{R_1} p(x) + \sum_{R_2} p(x)$

Since (sumation over R_1 $p(x)$ sumation over R_2 $p(x)$) represent the probabilities assigned to the region $R_1 \cup R_3$

$\therefore P\{|x-\mu| \geq k\sigma\} \leq \frac{1}{k^2}$

Note: 1. Another form of chebyshev's inequality

$P\{|x-\mu| < k\sigma\} \geq 1 - \frac{1}{k^2}$

$\Rightarrow P(|x-\mu| \geq k\sigma) + P(|x-\mu| < k\sigma) = 1$

$\Rightarrow P(|x-\mu| < k\sigma) = 1 - P(|x-\mu| \geq k\sigma)$

$P(|x-\mu| < k\sigma) = 1 - \frac{1}{k^2}$

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2. In most of the problem we take

$k\sigma = c (>0)$

in Chebycher's equality

$P(|x-\mu| \geq c) \leq \frac{\sigma^2}{c^2}$

or

$P(|x-\mu| < c) \geq 1 - \frac{\sigma^2}{c^2}$

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3. $P(|x-\mu| \geq k\sigma)$ is the probability that the r.v x lies outside the limits $(\mu-k\sigma, \mu+k\sigma)$ $P(|x-\mu| < k\sigma)$ is the probability that the r.v x lies inside the limits

$(\mu-k\sigma, \mu+k\sigma)$

Q. If x is the number appearing on a die when it is thrown show that the chebychew's theorem give $P(|x-3.5| \geq 2.5) < 0.47$ while the actual prob. is zero

sol:- x: Number on the die.

prob. dist. function.

x:	1	2	3	4	5	6
$P(x)$:	1/6	1/6	1/6	1/6	1/6	1/6

$E(x) = \text{mean} = \Sigma x P(x)$

$= 1\cdot\frac{1}{6} + 2\cdot\frac{1}{6} + 3\cdot\frac{1}{6} + 4\cdot\frac{1}{6} + 5\cdot\frac{1}{6} + 6\cdot\frac{1}{6}$

$= 21/6$

$= 7/2$

$= 3.5$

$V(x) = \sigma^2 = E(x^2) - [E(x)]^2$

$E(x^2) = \Sigma x^2 P(x) = 1^2\cdot\frac{1}{6} + 2^2\cdot\frac{1}{6} + 3^2\cdot\frac{1}{6} + 4^2\cdot\frac{1}{6} + 5^2\cdot\frac{1}{6} + 6^2\cdot\frac{1}{6}$

$= \frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2)$

$= \frac{91}{6}$

$= 15.16$

$V(x) = 15.16 - (3.5)^2$

$= 15.16 - 12.25$

$= 2.91$

$\sigma = \sqrt{V(x)} = \sqrt{2.91} = 1.705$

cheby chew's theorem

$P(|x-\mu| \geq k\sigma) < \frac{1}{k^2}$

$P(|x-3.5| \geq k\sigma) < \frac{1}{(1.466)^2}$

$P(|x-3.5| \geq 2.5) < 0.465$

$k\sigma = 2.5$

$k = \frac{2.5}{\sigma}$

$= \frac{2.5}{1.705}$

$= 1.466$

Actual probability:-

$P(|x-\mu| \geq k\sigma)$ is the probability that x lies outside the limit

$(\mu-k\sigma, \mu+k\sigma) = (3.5-2.5, 3.5+2.6)$

$= (1, 6)$

from position table,

i.e, no probability is assigned to the values below 1 and above 6

2. The no.of costamers who visits a car dealer's show room on a saturday morning is a r.v with $\mu = 18$ and $\sigma = 2.5$ what probability can we assent that there will be b/w 8 and 28 costumers.

Sol:- Given that,

No of costumers who visit a car dealer's show room on saturday morning with $\mu = 18$ and $\sigma = 2.5$

According to chebychew's inequality

$P(|x-\mu| < k\sigma)$ is the probability that x lies b/w the limits

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$(\mu - k\sigma, \mu + k\sigma)$

$k\sigma = x - \mu$ $\quad x \leq \mu - k\sigma \Rightarrow x - \mu \leq -k\sigma$

$x \geq \mu + k\sigma \Rightarrow -(x-\mu) \geq k\sigma$

$x - \mu \geq k.\sigma \Rightarrow |x-\mu| \geq k\sigma$

$$k = \frac{|x-\mu|}{\sigma}$$

$$k = \frac{-(x-\mu)}{\sigma} = \frac{x-\mu}{\sigma}$$

Where, $x = 8$ $\quad k = \frac{-(8-18)}{2.5}$

$$= \frac{-8+18}{2.5}$$

$$= 4$$

$x = 28 \Rightarrow k = \frac{28-18}{2.5}$

$$= \frac{10}{2.5}$$

$$= 4$$

$P(8 \leq x \leq 28) = P(|x-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

$$P(|x-18| < 10) \geq 1 - \frac{1}{16}$$

$$P(|x-18| < 10) \geq \frac{15}{16}$$

$\therefore$ minimum probability is $\frac{15}{16}$

3. Two unbiased dice are thrown if x is the sum of ~~men~~ numbers showing prove that the $P\left(|x-7| \leq \frac{35}{54}\right)$ compare with actual probability

sol:- Given that

x = sum of numbers showing

probability distribution x is

x:	2	3	4	5	6	7	8	9	10	11	12
P(x):	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$$\text{mean}(\mu) = E(x) = \sum x P(x) = 2\cdot\frac{1}{36} + 3\cdot\frac{2}{36} + 4\cdot\frac{3}{36} + 5\cdot\frac{4}{36} + 6\cdot\frac{5}{36}$$

$$+ 7\cdot\frac{6}{36} + 8\cdot\frac{5}{36} + 9\cdot\frac{4}{36} + 10\cdot\frac{3}{36} + 11\cdot\frac{2}{36} + 12\cdot\frac{1}{36}$$

$$= \frac{252}{36}$$

$$= 7$$

$$E(x^2) = \sum x^2 P(x)$$

$$= 2^2\cdot\frac{1}{36} + 3^2\cdot\frac{2}{36} + 4^2\,\frac{3}{36} + 5^2\cdot\frac{4}{36} + 6^2\cdot\frac{5}{36} + 7^2\cdot\frac{6}{36} + 8^2\cdot\frac{5}{36} + 9^2\cdot\frac{4}{36}$$

$$+ 10^2\cdot\frac{3}{36} + 11^2\cdot\frac{2}{36} + 12^2\cdot\frac{1}{36}$$

$$= \frac{329}{6}$$

$$(\sigma^2) = V(x) = E(x^2) - [E(x)]^2$$

$$= \frac{329}{6} - (7)^2$$

$$= 5.833$$

By chebychev's inequality

for $c>0$ we have

$$P(|x-\mu| \geq c) \leq \frac{\sigma^2}{c^2}$$

Comparing

$$\mu = 7, c = 3$$

$$= P(|x-7| \geq 3) \leq \frac{\frac{35}{6}}{9}$$

$$= P(|x-7| \geq 3) \leq \frac{35}{54}.$$

O.T.4

17/08/19

Probability distribution

1. Discrete probability
   a. binomial distribution
   b. poisson distribution
   c. Geometric distribution

2. continuous probability distⁿ
   a. Normal distribution
   b. Exponential distribution
   c. Gamma distribution

Tossing a single coin single time

H, T   X: no. of heads (success), (P)

$T \rightarrow$ Failure (q)

x: no. of success

x :	0	1
$P(x)$:	$1/2$	$1/2$

Tossing a coin two times

HH, HT, TH, TT   x: no. of heads (success)

x :	0	1	2
$P(x)$:	$1/4$	$2/4$	$1/4$

Tossing a coin 3 times

HHH, HHT HTH THH TTH THT HTT TTT

x :	0	1	2	3
$P(x)$:	$1/8$	$3/8$	$3/8$	$1/8$

$P(x=3) = P(HHH) = P(H)P(H)P(H) = PPP = 3C_0 P^3$

$P(x=2) = P(HHT \cup HTH \cup THH)$

$= P(HHT) + P(HTH) + P(THH)$

$= P(H)P(H)P(T) + P(H)P(T)P(H) + P(T)P(H)P(H)$

$= ppq + pqp + qpp$

$= 3C_1 p^2 q$

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$P(x=1) = P(TTH \cup THT \cup HTT)$

$= P(TTH) + P(THT) + P(HTT)$

$= q \cdot q \cdot p + q \cdot p \cdot q + p \cdot q \cdot q$

$= 3C_2 q^2 p$

$P(x=0) = P(TTT)$

$= 3C_3 q^3$

$\sum_{x=0}^{3} P(x=x) = P[x=0 \cup x=1 \cup x=2 \cup x=3]$

$= P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= 3C_0 q^3 + 3C_1 p q^2 + 3C_2 p^2 q + 3C_3 p^3$

$= (q+p)^3 = 1$

n ~~toss~~ trails

$(q+p)^n$ $\qquad P(x=x) = nC_x p^x q^{n-x}$

$(q+p)' = 1$

$q + p = 1$

$p = 1 - q$

$(q+p)^2 = 2C_0 q^2 + 2C_1 pq + 2C_2 p^2$

Trial satisfying the following Assumptions are reffered as Bernoulis.

1. There are only 2 possible outcomes for each trial denoted as sucess and failure.

2. The probobability of success (P) is the same for each trial.

3. There are n independent trial where n is a constant

Random experiment consisting of n repeated trials such that

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1. The trials are independent

2. Each trial results in only 2 possible outcomes labelled as sucess and failures

3. The probability of sucess in each trial denoted by small p remains constant is called as Binomial Experiment.

BINOMIAL DISTRIBUTION

A Random variable x is said to follow binomial distribution of its probability mass function is defined as

$$P(X=x) = nC_x p^x q^{n-x} \qquad x = 0, \dots n$$
$$= 0 \qquad \text{otherwise}$$

$$(q+p)^n = 1 = nC_0 p^0 q^n + nC_1 p q^{n-1} + nC_2 p^2 q^{n-2} + \dots + nC_r p^r q^{n-r} \dots + nC_n p^n . q^{n-n}$$

Constants of Binomial distribution

1. Mean $(\mu) = E(x) = \sum x P(x) = \sum_{x=0}^{n} x . p(X=x)$

$n! = n(n-1)!$

$= n(n-1)(n-2)!$

$p^x = p^{x+1-1}$

$= p^1 . p^{x-1}$

$n-x = n-x+1-1$

$= (n-1)-(x-1)$

let $x-1 = y$

$$= \sum_{x=0}^{n} x . nC_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x . \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{(n-x)! x(x-1)!} p^x q^{n-x}$$

$$= \sum \frac{n(n-1)!}{[(n-1)-(x-1)]!(x-1)!} p . p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{y=0}^{n} \frac{(n-1)!}{[(n-1)-y]!y!} . p^y . q^{(n-1)-y}$$

$$= np \sum_{y=0}^{n} {}^{n-1}c_y \, p^y . q^{(n-1)-y}$$

$$= np . (q+p)^{n-1}$$

$$= np . (1)^{n-1}$$

$$E(x) = np$$

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2) $V(x) = E(x^2) - [E(x)]^2$

$$E(x^2) = \sum_{x=0}^{n} x^2 P(X=x)$$

$$= \sum_{x=0}^{n} [x(x-1)+x] P(X=x)$$

$$= \sum_{x=0}^{n} x(x-1) P(X=x) + \sum_{x=0}^{n} x P(X=x)$$

now

$$\sum_{x=0}^{n} x(x-1) P(X=x)$$

$$= \sum_{x=0}^{n} x(x-1)\, nC_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{\cancel{x(x-1)}\, n(n-1)(n-2)!}{[(n-2)-(x-2)]!\,\cancel{x(x-1)}(x-2)!} p^2 \cdot p^{x-2} q^{(n-2)-(x-2)}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{[(n-2)-(x-2)]!\,(x-2)!} p^{x-2} q^{(n-2)-(x-2)}$$

$$= n(n-1)p^2 (q+p)^{n-2}$$

$$\sum_{x=0}^{n} x(x-1) P(X=x) = [n^2p^2 - np^2] \cdot 1$$

$$E(x^2) = n^2p^2 - np^2 + np$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= n^2p^2 - np^2 + np - (np)^2$$

$$= np(1-p)$$

$$= npq$$

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Mode of Binomial Distribution

Mode is the value of x for which $P(x)$ is maximum

$P(x) \geq P(x-1)$ —(1) $P(x) \geq P(x+1)$ —(2)

from ①

$P(x) \geq P(x-1)$

$P(X=x) = nC_x p^x q^{n-x}$

$P(X=x-1) = nC_{x-1} p^{x-1} q^{n-(x-1)}$

$$\frac{n!}{(n-x)!\,x!} p^x q^{n-x} \geq \frac{n!}{[n-(x-1)]!\,(x-1)!} p^{x-1} q^{n-x+1}$$

$$\frac{\cancel{p^{x-1}}\cdot p \cdot \cancel{q^{n-x}}}{\cancel{(n-x)!}\, x \cancel{(x-1)!}} \geq \frac{\cancel{p^{x-1}} \cdot \cancel{q^{n-x}} \cdot q}{(n-x+1)\cancel{(n-x)!}\,\cancel{(x-1)!}}$$

$$\frac{p}{x} \geq \frac{\cancel{p}q}{n-x+1}$$

$p(n-x+1) \geq qx$

$p(n-x+1) \geq (1-p)x$

$pn - \cancel{px} + p \geq x - \cancel{px}$

$(n+1)p \geq x$ —(i)

from 2

$P(x) \geq P(x+1)$

$nC_x p^x q^{n-x} \geq nC_{x+1} p^{x+1} q^{n-(x+1)}$

$$\frac{\cancel{n!}}{(n-x)!\,x!} \cancel{p^x} q^{n-x} \geq \frac{\cancel{n!}}{(n-x-1)!\,(x+1)!} \cancel{p^x}\cdot p\, q^{n-x-1}\cdot q$$

$$\frac{\cancel{q^{n-x-1}}\cdot q}{(n-x)\cancel{(n-x-1)!}\,\cancel{x!}} \geq \frac{p\cdot \cancel{q^{n-x-1}}}{\cancel{(n-x-1)!}\,(x+1)\cancel{x!}}$$

$$\frac{q}{n-x} \geq \frac{p}{x+1}$$

$(1-p)(x+1) \geq (n-x)p$

$x+1-px-1 \geq np-xp$

$x \geq np+p-1$

$x \geq p(n+1)-1$ — ②

from ① & ②

$(n+1)p-1 \leq x \leq (n+1)p$

Here x depends upon $(n+1)p$

Case(i) If $(n+1)p$ is an integer

$\Rightarrow (n+1)p-1$ also an integer

In this case distribution will have 2 modes i.e distribution is bimodel modes are $(n+1)p-1$ and $(n+1)p$

case ii) If $(n+1)p$ is not an integer then mode is the integral put of $(n+1)p$.

Note:

Frequency funtion $F(x)$ = Expected frequency

$= N \cdot P(X=x)$

1. 10 coins are tossed simultaneously. Find the probablility of getting atleast 7 heads.

Sol:- given that

n = no. of trials = 10

p = probability of success = getting head = $\frac{1}{2}$

$q = 1-p$ = probability of failure = $1-\frac{1}{2} = \frac{1}{2}$

x = no of success

Req. probability

$P(X \geq 7) = P(x=7)+P(x=8)+P(x=9)+P(x=10)$

$= 10C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7} + 10C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} + 10C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{10-9}$

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$+{}^{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{10-10}$

$=120\left(\frac{1}{2}\right)^{10}+45\left(\frac{1}{2}\right)^{10}+10\left(\frac{1}{2}\right)^{10}+\left(\frac{1}{2}\right)^{10}$

$=(120+45+10+1)\left(\frac{1}{2}\right)^{10}=\frac{176}{1024}=0.171$

(or)

$P(x\geq 7)=1-P(x<7)$

$=1-\{P(x=0)+\cdots P(x=6)\}$

2. A discrete random variable x has the mean 6 and variance 2. If It is assumed that the distribution is binomial. Find the probability that $5\leq x\leq 7$

sol:- Mean $=np=6$ —— ①

variance $=npq=2$ —— ②

$\frac{②}{①}=\frac{2}{6}=\frac{\cancel{np}q}{\cancel{np}}$ from ① $np=6$

$q=\frac{1}{3}$ $n\left(\frac{2}{3}\right)=6$

$p=1-q$ $n=9$

$p=1-\frac{1}{3}$

$=\frac{2}{3}$

Here n, p are known as parameters of Binomial distribution

$P(5\leq x\leq 7)=P(x=5)+P(x=6)+P(x=7)$

$={}^9C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^4+{}^9C_6\left(\frac{2}{3}\right)^6\left(\frac{1}{9}\right)^3+{}^9C_7\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^2$

$(1/3)^4$ $(1/3)^3$

$=126\left(\frac{2}{3}\right)^5+84\left(\frac{2}{3}\right)^6+36\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^2$

$=$

$=0.712$

3. The mean and variance of binomal distribution are 4 and $\frac{4}{3}$ respectively find $P(X\geq 1)$

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Sol:- $4 = \text{mean} = np\cancel{q}$ — ①

$npq = \text{variance} = \frac{4}{3}$ — ②

$\frac{②}{①} = \frac{4/3}{4}$

$q = \frac{1}{3}$

$p = 1 - q$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

$np = 4$

$n\left(\frac{2}{3}\right) = 4$

$n = 6$

$P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - {}^{6}C_{0}\left(\frac{2}{3}\right)^{0}\left(\frac{1}{3}\right)^{6}$

$= 1 - \frac{1}{721}$

$= \frac{720}{721}$

$= 0.999$

4. Detemine the Binomial distribution ~~for~~ which the mean is 4 and variance 3 Also find its mode

Sol: $np = 4$

$npq = 3$

$q = \frac{3}{4}$

$p = 1 - \frac{3}{4} = 1/4$

$np = 4$

$n \cdot \frac{1}{4} = 4$

$n = 16$

Binomial distribution is

$= (q+p)^{n}$

$= \left(\frac{3}{4} + \frac{1}{4}\right)^{16}$

$P(X = x) = 16C_{x}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{4}\right)^{16-x}$

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5. Comment on the following

"the mean of B.D is and varience is 4"

Sol:- $np = 3$ — ① $\frac{②}{①} \Rightarrow q = \frac{4}{3} > 1$

$npq = 4$ — ②

It is impossible. The prob. cannot exceed 1

6. An irregular six faced die is thrown and the expalection that in 10 throws it will give five even number is twice the expection that it will give four even number. How many time in 10,000 sets of 10 throws would you expect it to give no even number.

Sol:- n = no. of times the die is throw n = 10

p = probability of getting no. of even numbers =

q = probability of getting even number

Given that

$P(x=5) = 2P(x=4)$

$10C_5 (p)^5 (q)^{10-5} = 2\left[10C_4 (p)^4 q^{10-4}\right]$

$\cancel{252} \cdot p^5 q^5 = \cancel{2}(\cancel{210})\, p^4 q^6$

$3p = 5q$

$3p = 5q$

$3p = 5(1-p)$

$8p = 5$

$p = 5/8$

$q = 3/8$

Expected no. of times no even number

$F(0) = N \times P(x=0)$

$= 10{,}000 \times P(x=0)$

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$= 10{,}000\ {}^{10}C_0 \left(\frac{5}{8}\right)^0 \left(\frac{3}{8}\right)^{10}$

$= (10{,}000)(1)(1)(54993) \times 10^{-5}$

$= 0.54993$

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1. For the density function $f(x) = 6x(1-x)$, $0 \le x \le 1$ find mean (hint $E(x) = \int_0^1 x\,6x(1-x)dx$) $= 0$ otherwise

2. If mean of the B.D is 4 and variance is 2 then find p. ~~4 items is~~

3. From a lot of, 10 items containing 3 defective a simple of 4 items is drawn at random let the r.v x denote the no. of defective items in the sample. Find the prob. distn of x. when the sample is drawn without replacement.

4. The mean and variance of B.D are 4 & 4/3 respectively. find $p(x \ge 1)$

5. The p.d.f $f(x)$ of a cont r.v is $f(x) = kx^3$, $0 < x \le 1$
$= 0$ otherwise
find k, and the prob. that the r.v takes on a variate b/w 1/4 and 3/4

FITTING OF BINOMIAL DISTRIBUTION

1. Fit a Binomal distribution of the following data.

x	0	1	2	3	4	5
f	38	144	342	287	164	25

$= \Sigma f = N = 1000$

sol:- $n = 5$ (from given data)

$$np = \text{mean} = \frac{\Sigma fx}{\Sigma f} = \frac{\Sigma fx}{N} = \frac{0\times38 + 1\times144 + 2\times342 + 3\times287 + 4\times164 + 5\times25}{1000}$$

$$= 2470/1000 = 2.47$$

$np = 2.47$

$5p = 2.47$

$p = \frac{2.47}{5}$

$= 0.494$

$q = 1 - p$

$= 1 - 0.494$

$= 0.506$

Expected frequencies

$F(x) = N \cdot P(x) = N \times {}^nC_x \, p^x q^{n-x}$

$F(0) = \cancel{N \cdot P(} 1000 \times 5C_0 (0.494)^0 (0.506)^{5-0} = 33.170 \cong 33$

$F(1) = 1000 \times 5C_1 (0.494)^1 (0.506)^4 = 161.919 \cong 162$

$F(2) = 1000 \times 5C_2 (0.494)^2 (0.506)^3 = 316.158 \cong 316$

$F(3) = 1000 \times 5C_3 (0.494)^3 (0.506)^2 = 308.661 \cong 309$

$F(4) = 1000 \times 5C_4 (0.494)^4 (0.506)^1 = 150.670 \cong 151$

$F(5) = 1000 \times 5C_5 (0.494)^5 (0.506)^0 = 29.419 \cong 29$

Required Binomial distribution is

x	0	1	2	3	4	5
f	38	144	342	287	164	25
$F(x)$	33	162	316	309	151	29

POISSION DISTRIBUTION

→ It is discovered by French mathematician simen Denis poission in 1873

Poisson $dist^n$ as a limiting case of Biomial distributi-on. (Poisson appoximation to Binomial distribution) under the following condition, we get the possion distribution from Binomial Distribution

conditions are

i) as number of trials increses to infinite i.e as $n \to \infty$

ii) If the probability of sucess or failure is very so small

i.e $p \to 0$

iii. $np = \lambda$

$\Rightarrow p = \frac{\lambda}{n}$

Proof:

$$P(X=x) = nC_x p^x q^{n-x}$$

$$= \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

$$= \frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x}$$

$$= \frac{n(n-1)(n-2)\cdots\cdots[n-(x-1)]\cancel{(n-x)!}}{\cancel{(n-x)!}\,x!} \cdot p^x (1-p)^{n-x}$$

$$= \frac{n(n-1)(n-2)\cdots\cdots(n-(x-1))}{x!} \cdot \left(\frac{\lambda}{n}\right)^x \left(1-\frac{\lambda}{n}\right)^{n-x} \quad \left(\because p=\frac{\lambda}{n}\right)$$

$$= \frac{\cancel{(n.n.n\cdots\cdots n)}\ (x\text{ times common}) \cdot \left[1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\left(\frac{x-1}{n}\right)\right)\right]}{x!} \cdot \frac{\lambda^x}{\cancel{n^x}}\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\left(1-\frac{3}{n}\right)\cdots\left(1-\left(\frac{x-1}{n}\right)\right)}{x!} \lambda^x \cdot \left(1-\frac{\lambda}{n}\right)^{n-x}$$

as $n \to \infty$

$$\underset{n\to\infty}{\text{lt}}\ P(X=x) = \underset{n\to\infty}{\text{Lt}} \left[\frac{1-\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\left(\frac{x-1}{n}\right)\right)}{x!} \cdot \lambda^x \cdot \left(1-\frac{\lambda}{n}\right)^{n-x}\right]$$

$$= \frac{\lambda^x}{x!}\left(\underset{(x\text{ times})}{1.1.1\cdots\cdots1}\right)\underset{n\to\infty}{\text{Lt}}\left(1-\frac{\lambda}{n}\right)^{n-x} \qquad \text{as } n\to\infty,\ \frac{1}{n}\to 0$$

$$= \frac{\lambda^x}{x!} \cdot \frac{\underset{n\to\infty}{\text{lt}}\left(1-\frac{\lambda}{n}\right)^n}{\underset{n\to\infty}{\text{lt}}\left(1-\frac{\lambda}{n}\right)^x} \qquad \underset{n\to\infty}{\text{Lt}}\left(1-\frac{\lambda}{n}\right)^n = \underset{n\to\infty}{\text{Lt}}\left[\left(1-\frac{\lambda}{n}\right)^{-n/\lambda}\right]^{-\lambda} = e^{-\lambda}$$

$$\underset{n\to\infty}{\text{Lt}}\left(1-\frac{\lambda}{n}\right)^x = 1$$

$$= \frac{\lambda^x}{x!} \cdot \frac{e^{-\lambda}}{1} = \frac{e^{-\lambda}\lambda^x}{x!},\quad x = 0,1,2\cdots\cdots\infty$$

probability mass funtion of poisson distribution

Definition

A random variable x is said to fallow poisson distribution if its probability mass funtion is given by

$p(x=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, x=0,1,2 \dots \infty$

$= 0$, otherwise

Here λ is known as parameter of poisson dist^n

We use the notation to express the r.v. x which follows poisson distribution as $x \sim P(\lambda)$

Mean and variance of poisson distribution

$$\text{Mean} = M = E(x) = \sum_{x=0}^{\infty} x\, P(x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= \sum_{x=1}^{\infty} \frac{x \cdot e^{-\lambda}\lambda^{x}}{x(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1} \cdot \lambda}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \left[1 + \frac{\lambda^{2-1}}{1!} + \frac{\lambda^{3-1}}{2!} + \frac{\lambda^{4-1}}{3!} + \dots\right]$$

$$= \lambda e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \dots\right]$$

$$= \lambda e^{-\lambda} \left[e^{\lambda}\right]$$

$$= \lambda \cdot 1$$

$$= \lambda$$

Variance of poisson distribution

$$V(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum x^2 p(x) = \sum_{x=0}^{\infty} [x(x-1)+x]\, p(x)$$

$$= \sum_{x=0}^{\infty} x(x-1)P(x) = \sum_{x=0}^{\infty} x \cdot P(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda}\lambda^x}{x!} + \lambda$$

$$= \sum_{x=0}^{\infty} \frac{\cancel{x(x-1)} e^{-\lambda}\lambda^x}{\cancel{x(x-1)}(x-2)!} + \lambda$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^{x-2}\lambda^2}{(x-2)!} + \lambda$$

$$= e^{-\lambda}\lambda^2 \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= e^{-\lambda}\lambda^2 \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right] + \lambda$$

$$= e^{-\lambda}\lambda^2 (e^{\lambda}) + x$$

$$= \lambda^2 + \lambda$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \lambda^2 + \lambda - (\lambda)^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$\therefore V(X) = \lambda.$$

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Mode of poisson distribution

Mode is the value of 'x' for which the P(x) is maximum

$P(x) \geq P(x-1)$

$$\frac{e^{-\lambda}\lambda^x}{x!} \geq \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}$$

$$\frac{\cancel{\lambda^{x-1}}\cdot\lambda}{x\cancel{(x-1)!}} \geq \frac{\cancel{\lambda^{x-1}}}{\cancel{(x-1)!}}$$

$\lambda \geq x$ —(i)

$P(x) \geq P(x+1)$

$$\frac{\cancel{e^{-\lambda}}\lambda^x}{x!} \geq \frac{\cancel{e^{-\lambda}}\lambda^{x+1}}{(x+1)!}$$

$$\frac{\cancel{\lambda^x}}{\cancel{x!}} \geq \frac{\cancel{\lambda^x}\lambda}{(x+1)\cancel{x!}}$$

$x+1 \geq \lambda$

$x \geq \lambda - 1$ —(ii)

from ① & ②

$\lambda - 1 \leq x \leq \lambda$

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case ① If λ is an integer then $\lambda, \lambda-1$ are the modes of poisson distⁿ

case ⓘⓘ If λ is not an integer then integral part of λ is the mode

1. Given that $P(x=2) = 45\,P(x=6) - 3P(x=4)$ for a possion variate x. Find the probabilities of i) $x \geq 1$ ii) $x < 2$

Sol: Given that

$P(x=2) = 45P(x=6) - 3P(x=4)$

$$\frac{\cancel{e^{-\lambda}}\lambda^2}{2!} = 45 \cdot \frac{\cancel{e^{-\lambda}} \cdot \lambda^6}{6!} - 3\frac{\cancel{e^{-\lambda}}\lambda^4}{4!}$$

$$\frac{\lambda^2}{2} = \frac{\cancel{45}^{\,1}}{\cancel{720}_{\,16}}\lambda^6 - \frac{\cancel{3}\lambda^2}{\cancel{24}_{\,8}}$$

$$\frac{\lambda^2}{2} = \frac{\lambda^2}{2}\left(\frac{\lambda^4}{8} - \frac{\lambda^2}{4}\right)$$

$$\Rightarrow \frac{\lambda^4}{8} - \frac{\lambda^2}{4} = 1$$

$\lambda^4 - 2\lambda^2 - 8 = 0$

$\lambda^4 - 4\lambda^2 + 2\lambda^2 - 8 = 0$

$\lambda^2(\lambda^2-4) + 2(\lambda^2-4) = 0$

$(\lambda^2+2)(\lambda^2-4) = 0$

$\lambda^2+2 = 0, \quad \lambda^2 - 4 = 0$

$\lambda^2 = -2 \qquad \lambda^2 = 4$

$\lambda = \pm\sqrt{2}i$ ✗ $\quad \lambda = \pm 2$

$\lambda = +2 \quad \lambda = -2$ ✗

$\therefore \lambda = 2$

i) $P(x \geq 1) = 1 - P(x<1)$

$= 1 - P(x=0)$

$$= 1 - \frac{e^{-2}(2)^0}{0!}$$

$= 1 - e^{-2}$

$= 1 - 0.136 = 0.864$

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ii) $P(x<2) = P(x=0) + P(x=1)$

$$= \frac{e^{-2}2^0}{0!} + \frac{e^{-2}(2)}{1!}$$

$$= e^{-2}(1+2)$$

$$= 3e^{-2} = 3 \times 0.136 = 0.408$$

1. The probability of a poisson variate taking the value 1 and 2 are equal calculate the probabilities of the variate taking the values 0 an 3

Sol:- Given that

$$P(x=1) = P(x=2)$$

$$\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\frac{\lambda}{1} = \frac{\lambda^2}{2}$$

$\therefore \lambda = 2$

$$P(x=0) = \cancel{\frac{e^{-2}\lambda^0}{0!}} = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-2} = 0.136$$

$$P(x=3) = \frac{e^{-2}2^3}{3!} = \frac{e^{-2}(8)}{6} = 0.181$$

3. If for a possion variate $2P(x=0) = P(x=2)$ find i) $P(x\leq 3)$ ii) $P(2<x\leq 5)$ iii) $P(x\geq 3)$ iv) mean

Sol:- Given that

$$2P(x=0) = P(x=2)$$

$$2 \cdot \frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\lambda^2 = 4$$

$$\lambda = \pm 2$$

$$\lambda = 2$$

i) $P(x\leq 3)$

$$= P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

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$$= \frac{e^{-2}(2)^0}{0!} + \frac{e^{-2}(2)^1}{1!} + \frac{e^{-2}(2)^2}{2!} + \frac{e^{-2}(2)^3}{3!}$$

$$= e^{-2}\left(1+2+2+\frac{8}{3}\right)$$

$$= e^{-2}\left[\frac{15+4}{3}\right]$$

$$= \frac{19}{3}e^{-2} = \frac{19}{3}(0.136)$$

$$= 0.0861$$

ii) $p(2 < x \leq 5)$

$$= P(x=3) + P(x=4) + P(x=5)$$

$$= \frac{e^{-2}(2)^3}{3!} + \frac{e^{-2}(2)^4}{4!} + \frac{e^{-2}2^5}{5!}$$

$$= e^{-2}\left[\frac{8}{6} + \frac{16}{24} + \frac{32}{120}\right]$$

$$= e^{-2}\left[\frac{4}{3} + \frac{2}{3} + \frac{4}{15}\right]$$

$$= e^{-2}\left(\frac{20+10+4}{15}\right)$$

$$= \frac{34}{15}e^{-2}$$

$$= \frac{34}{15}(0.136)$$

$$= 0.0308$$

5. 2% of the items of a factory are defective. The items are packed in boxes. what is the probability that there will be

i) 2 defective items

ii) Atleast 3 defective items

iii) 2 < defectives items < 5 in a box of 100 items

probability of defective items = P = 2%

$$= \frac{2}{100}$$

$$= 0.02$$

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i) $P(X=2)$ (using poisson approximation as $n=100$ $p=0.02<0.1$)

$= \frac{e^{-2}(2)^2}{2!}$

$= e^{-2}(2)$

$= 2(0.136)$

$= 0.272$

ii) $P(X \geq 3) = 1 - P(X<3)$

$= 1 - \{P(X=0) + P(X=1) + P(X=2)\}$

$= 1 - \left\{ \frac{e^{-2}(2)^0}{0!} + \frac{e^{-2}(2)!}{1!} + \frac{e^{-2}(2)^2}{2!} \right\}$

$= 1 - e^{-2}\{1+2+2\}$

$= 1 - 5e^{-2}$

$= 1 - (0.136)5$

$= 0.32$

iii) $P(2<X<5)$

$= P(X=3) + P(X=4)$

$= \frac{e^{-2}(2)^3}{3!} + \frac{e^{-2}(2)^4}{4!}$

$= \frac{e^{-2}(\cancel{8})^4}{\cancel{6}_3} + \frac{e^{-2}(\cancel{16})^2}{\cancel{24}_3}$

$= e^{-2}\left(\frac{4}{3} + \frac{2}{3}\right)$

$= e^{-2}\left(\frac{\cancel{6}^2}{\cancel{3}}\right)$

$= 2e^{-2}$

$= 2(0.136)$

$= 0.272$

5. Average number of accidents on any day on a national high way is 1.8. Determine the probability that the no. of accidents are

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i) atleast one ii) At most one

sol:- $\lambda = 1.8$

i) $P(X \geq 1)$

$= 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - \frac{e^{-1.8} (1.8)^0}{0!}$

$= 0.8347$

ii) $P(X \leq 1) = P(X=0) + P(X=1)$

$= \frac{e^{-1.8} \cdot (1.8)^0}{0!} + \frac{e^{-1.8} (1.8)^1}{1!}$

$= 0.4628$

poisson variate has a double mode at $x = 2$ and $x = 3$ find the maximum probability and also find $P(x \geq 2)$

sol:- $P(x=2) = P(x=3)$

$\frac{e^{-\lambda} \lambda^2}{2!} = \frac{e^{-\lambda} \lambda^3}{3!}$

$\frac{6}{2} = \lambda$

$\lambda = 3$

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[\frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!}\right]$

$= 1 - e^{-3}(1+3)$

$= 1 - 4e^{-3}$

$= 0.8008$

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Fitting of poisson distribution

Fit a poisson distribution to the following data

x	0	1	2	3	4	
f	109	69	45	23	9	$= N = \Sigma f = 255$

sol: Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{\Sigma fx}{N} = \frac{109\times0+69\times1+45\times2+23\times3+9\times4}{109+69+45+23+9}$

$$= \frac{0+69+90+69+36}{255}$$

$$= 1.035 = \lambda$$

Expected frequencies

$F(x) = N\cdot P(X=x)$

$F(0) = 255\cdot P(0)$

$$= 255\cdot \frac{e^{-1.035}(1.035)^0}{0!} = 255\cdot e^{-1.035} = 90.582 \cong 91$$

$F(1) = 255\cdot P(1)$

$$= 255\cdot \frac{e^{-1.035}(1.035)^1}{1!} = 255(1.035)\cdot e^{-1.035} = 93.753 \cong 94$$

$F(2) = 255\cdot P(2)$

$$= 255\cdot \frac{e^{-1.035}(1.035)^2}{2!} = \cancel{255P(3)}\ 48.517 \cong 49$$

$= 255\, P(3)$

$$F(3) = \frac{255(e^{-1.035})(1.035)^3}{3!} = 16.738 \cong 17$$

$F(4) = 255\, P(4)$

$$= \frac{255(e^{(-1.035)})\cdot(1.035)^4}{4!} = 4.331 \cong 5$$

$\therefore$ Req. poisson distribution is

x	0	1	2	3	4
f	109	69	45	23	9
F	91	94	49	17	5

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Geometric Distribution

A random variable x is said to follow geometric distribution if it assumes only non-negative value and its probability mass function is given by

$$P(X=x) = q^{x-1} \cdot P \; ; \; x = 1, 2 \ldots \infty$$
$$= 0 \quad \text{otherwise}$$

where p = probability of success

q = probability of failure

x = no. of trails required to get success

Any variable which follows geometric distribution is known as, Geometric variate.

here 'p' is the only parameter required for geometric distribution.

$P(X=x)$

$P(x=1) = P(H) = \frac{1}{2} = P$

$P(X=2) = P(TH) = P(T)\,P(H) = \frac{1}{2}\cdot\frac{1}{2} = q \cdot P$

$P(x=3) = P(TTH) = P(T)\,P(T)\,P(H) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = q^2 P$

⋮

$P(X=x) = P(\underbrace{TTT \cdots T}_{(x-1)}H) = q^{x-1} P$

condition of Geometric distribution

- No. of trials are not fixed (infinite)
- Each trial is independent of each other
- every trial results in only 2 outcomes
- probability of success is constant.
- x : no. of trials required to get the first success.

$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots$

$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots$

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$$(1-x)^{-2} = 1+2x+3x^2+4x^3+\cdots$$

$$(1+x)^{-3} = 1-3x+6x^2-10x^3+\cdots$$

$$(1-x)^{-3} = 1+3x+6x^2+10x^3+\cdots$$

$$\sum_{x=1}^{\infty} P(x) = \sum_{x=1}^{\infty} q^{x-1} p = p+qp+q^2p+\cdots$$

$$= p[1+q+q^2+q^3+\cdots]$$

$$= p(1-q)^{-1}$$

$$= \frac{p}{(1-q)}$$

$$= \frac{p}{p}$$

$$= 1$$

Constants of Geometric distribution

1. Mean $= E(x)$

$$= \sum_{x=1}^{\infty} x P(x)$$

$$= \sum_{x=1}^{\infty} x \cdot q^{x-1} \cdot p$$

$$= 1\cdot q^0 \cdot p + 2\cdot q^{2-1}\cdot p + 3\cdot q^{3-1}\cdot p + \cdots$$

$$= p+2qp+3q^2p+\cdots$$

$$= p[1+2q+3q^2+\cdots]$$

$$= p(1-q)^{-2}$$

$$= \frac{p}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p}$$

$$V[x] = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum_{x=1}^{\infty} x^2 P(x) = \sum_{x=1}^{\infty} x^2 q^{x-1} p$$

$$= 1^2 q^0 p + 2^2 \cdot q^{2-1} \cdot p + 3^2 \cdot q^{3-1} \cdot p + 4^2 \cdot q^{4-1} \cdot p + \cdots$$

$$= p+4pq+9q^2p+16q^3p+\cdots$$

$$= p[1+4q+9q^2+16q^3+\cdots]$$

$$= p[1+q+3q+3q^2+6q^2+6q^3+10q^3+\cdots]$$

$= p[(1+q)+3q(1+q)+6q^2(1+q)+10q^2(1+q)+\cdots]$

$= p(1+q)\,[1+3q+6q^2+10q^3+\cdots]$

$= p(1+q)(1-q)^{-3}$

$= \frac{p(1+q)}{p^3}$

$= \frac{1+q}{p^2}$

$= \frac{1+q-p}{p^2}$

$= \frac{2-p}{p^2}$ or $\frac{p+2q}{p^2}$

$V(x) = E(x^2) - [E(x)]^2$

$= \frac{1+q}{p^2} - \left(\frac{1}{p}\right)^2$

$= \cancel{\frac{1}{p^2}} + \frac{q}{p^2} - \cancel{\frac{1}{p^2}}$

$= q/p^2$

Note:- Geometric probabilities always decreases as x increases.

$p(x) = q^{x-1}p$

$p(1) = \frac{1}{2}$

$p(2) = 1/4$

$p(3) = 1/8$

1. A die is tossed until 6 appears find the probability that it must be cast more than 5 times.

Sol:- P = probab. of success = prob. of getting 6

$= 1/6$

$q = \cancel{1}\ 1-p = 1-\frac{1}{6} = 5/6$

x: no of trials required for getting first success

required prob =

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$P(x>5) = 1 - P(x \le 5)$

$= 1 - \{P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)\}$

$= 1 - \{p + qp + q^2p + q^3p + q^4p\}$

$= 1 - \{\frac{1}{6} + \frac{5}{6}\cdot\frac{1}{6} + (\frac{5}{6})^2\frac{1}{6} + (\frac{5}{6})^3\frac{1}{6} + (\frac{5}{6})^4\frac{1}{6}\}$

$= 1 - \{\frac{1}{6} + \frac{5}{36} + \frac{25}{36}\cdot\frac{1}{6} + \frac{125}{216}(\frac{1}{6}) + \frac{625}{1296}(\frac{1}{6})\}$

$= 1 - 0.5981$

$= 0.405$

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Q. Find the mean and variance of the geometrical distribution given by $P(x) = 2^{-x}, x = 1, 2, \ldots \infty$

Sol:- $E(x) = \text{mean} = \mu = \sum x\, p(x)$

$= \sum_{x=1}^{\infty} x \cdot \frac{1}{2^x}$

$= \frac{1}{2} + \frac{1}{2^2}\cdot 2 + 3\cdot\frac{1}{2^3} + 4\cdot\frac{1}{2^4} + \cdots$

$= \frac{1}{2}\left(1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2^2}\right) + 4\left(\frac{1}{2^3}\right) + \cdots\right)$

$= \frac{1}{2}\left[1 - \frac{1}{2}\right]^{-2}$

$= \frac{1}{2}\left(\frac{1}{2}\right)^{-2}$

$= \frac{1}{2}\left(\frac{1}{(1/2)^2}\right)$

$= \frac{2^2}{2}$

$= 2$

variance $V(x) = \sigma^2 = E(x^2) - [E(x)]^2$

$E(x^2) = \sum_{x=1}^{\infty} x^2 \cdot P(x)$

$= \sum_{x=1}^{\infty} x^2 \cdot \frac{1}{2^x}$

$= 1^2\cdot\frac{1}{2^1} + 2^2\cdot\frac{1}{2^2} + 3^2\cdot\frac{1}{2^3} + 4^2\cdot\frac{1}{2^4} + \cdots$

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$$= \frac{1}{2}\left[1+\frac{4}{2}+\frac{9\cdot 1}{2^2}+16\cdot\frac{1}{2^3}+\cdots\right]$$

$$= \frac{1}{2}\left[1+\frac{1}{2}+1+3\cdot\frac{1}{2}+3\cdot\frac{1}{2^2}+6\cdot\frac{1}{2^2}+6\cdot\frac{1}{2^3}+10\cdot\frac{1}{2^3}\cdots\right]$$

$$= \frac{1}{2}\left[\left(1+\frac{1}{2}\right)+3\left(\frac{1}{2}\right)\left(1+\frac{1}{2}\right)+6\cdot\left(\frac{1}{2^2}\right)\left(1+\frac{1}{2}\right)+10\left(\frac{1}{2^3}\right)\left(1+\frac{1}{2}\right)+\cdots\right]$$

$$= \frac{1}{2}\left(1+\frac{1}{2}\right)\left[1+3\left(\frac{1}{2}\right)+6\left(\frac{1}{2^2}\right)+10\left(\frac{1}{2^3}\right)+\cdots\right]$$

$$= \frac{1}{2}\left(1+\frac{1}{2}\right)\left(1-\frac{1}{2}\right)^{-3}$$

$$= \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)^{-3}$$

$$= \frac{1}{2}\left(\frac{3}{2}\right)\cdot\frac{1}{\left(\frac{1}{2^3}\right)}$$

$$= \frac{3}{4}\times 2^3$$

$$= \frac{24}{4} = 6$$

variance $V(X) = E(X^2) - [E(X)]^2$

$$= 6-(2)^2$$

$$= 6-4$$

$$= 2.$$

Verification

Mean $= \frac{1}{p}$ $\qquad V(X) = \frac{q}{p^2} = (1/2)/(1/2)^2 = \frac{1}{(1/2)} = 2$

$$= \frac{1}{(1/2)} = 2$$

3. For a geometric distribution $f(x) = \frac{1}{2^x}$ where $x = 1,2,3\cdots$ prove that chebyscher's in equality given $P(|x-2|<2) \geq \frac{1}{2}$ while the actual probability is $15/16$.

using Chebyshev's inequality

We $P(|x-\mu| < k\sigma) \geq 1-\frac{1}{k^2}$ $\qquad \sigma^2 = 2$

$\sigma = \sqrt{2}$

we have to show that $P(|x-2|<2) \geq 1-\frac{1}{(\sqrt{2})^2}$ $\qquad k\sigma = 2$

$k\cdot\sqrt{2} = 2$

$\geq 1-\frac{1}{2}$ $\qquad k = \sqrt{2}$

$\geq \frac{1}{2}$

Probability of x lies b/w $\mu - k\sigma$ and $\mu + k\sigma$ is $P(|x-\mu| < k\sigma)$

$\Rightarrow P(2-2 \le x \le 2+2) = P(0 \le x \le 4)$

$= P(x=0) + P(x=1) + P(x=3) + P(x=4)$

$= 0 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}$

$= \frac{0+8+4+2+1}{16}$

$= 15/16$

4. for a certain manufacturing process. It is known that on the average, one in every 100 items is defective. what is that probability that fifth item inspected is that first defective item found?

Sol:- probability of defective item $= P = x/n$

$= \frac{\text{number of defectives found}}{\text{Total no. of items inspected}} = \frac{1}{100} = 0.01$

$P = 0.01 \quad q = 1 - P = 1 - 0.01 = 0.99$

$P(x=x) = q^{x-1} \cdot p; \; x = 1, 2, 3 \ldots$

$P(x=5) = (0.99)^{5-1}(0.01)$

$= 0.00960$

5. At a "busy time" a telephone exchange is very here capacity, so callers have difficulty placing their cells. It may be of interest to known the namber of attempts necessary in order to make a connection suppose that we let $P = 0.05$ be the probabability of a connection during a busy time. what is the probability that 5 attempts are necessary fo a successful call.

$p = 0.005$

$P(x=5) = (1-0.05)^{5-1}(0.05)$

$= 0.040$

6. A fair die is tossed 720 times use chebyshev's inequlity to find a lower bound for the probability of getting 100 to 140 sixe

sol:- prob. that x lies b/w $\mu - k\sigma$ and $\mu + k\sigma$ is

$P(|x-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

given that $n = 720$

p = getting a six $= \frac{1}{6}$

Here x follows (~) Binomial distributⁿ (n,p)

mean $\mu = np = 720 \cdot \frac{1}{6} = 120.$

$\sigma^2 = npq = 720 \cdot \frac{1}{6} \cdot \frac{5}{6}$

$= 100$

$\sigma = 10$ $\quad k\sigma = 10k$

$P(\mu - k\sigma \leq x \leq \mu + k\sigma) \leq 1 - \frac{1}{k^2}$

$P(100 \leq x \leq 140)$

Now, $\mu - k\sigma = 100$ (or) $\mu + k\sigma = 140.$

$120 - k \cdot 10 = 100$

$10k = 20$

$k = 2$

$P(100 \leq x \leq 140) \leq 1 - \frac{1}{4}$

$P(100 \leq x \leq 140) \leq 3/4$

lower bound $= 3/4$.

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31/08/19 UNIT-III

Continuous probability distributions

1. Uniform distribution
2. Normal distribution
3. Gamma distribution
4. Exponential distribution

1. Continuous uniform distribution [Rectangular distribution]

A random variable 'x' is said to follow a continuous uniform distribution over an interval (a,b). probability distribution function (Pdf) is constant(k) over the entire range of 'x'

$$f(x) = k, \quad a < x < b$$
$$= 0, \text{ otherwise}$$

Now $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_{-\infty}^{a} 0\,dx + \int_{a}^{b} k\,dx + \int_{b}^{\infty} 0\cdot dx = 1$$

$$\Rightarrow \int_{a}^{b} k\,dx = 1$$

$$\Rightarrow k[x]_a^b = 1$$

$$\Rightarrow k(b-a) = 1$$

$$k = \frac{1}{b-a}$$

now p.d.f of uniform distribution becomes

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

Here a,b are known as parameters of uniform distn (a<b)

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Constants of uniform distribution

1. Mean $(\mu) = E(x)$

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$= \int_a^b x \cdot \left(\frac{1}{b-a}\right) dx$$

$$= \frac{1}{b-a} \int_a^b x\, dx$$

$$= \frac{1}{b-a} \left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{2(b-a)} (b^2 - a^2)$$

$$= \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}}$$

$$= \frac{b+a}{2}$$

2. Variance $(\sigma^2) = V(x) = E(x^2) - [E(x)]^2$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_a^b x^2 \left(\frac{1}{b-a}\right) dx$$

$$= \frac{1}{b-a} \left[\frac{x^3}{3}\right]_a^b$$

$$= \left(\frac{1}{b-a}\right)\left(\frac{b^3 - a^3}{3}\right)$$

$$= \frac{1}{\cancel{(b-a)}}\left(\frac{\cancel{(b-a)}(b^2+ab+a^2)}{3}\right)$$

$$= \frac{b^2+ab+a^2}{3}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \frac{b^2+ab+a^2}{3} - \frac{(b+a)^2}{4}$$

$$= \frac{4(b^2+ab+a^2) - 3(b^2+2ab+a^2)}{12}$$

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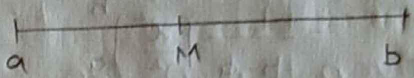
$$= \frac{4b^2+4ab+4a^2-(3b^2+6ab+3a^2)}{12}$$

$$= \frac{b^2-2ab+a^2}{12}$$

$$= \frac{(b-a)^2}{12}$$

3. Median

Median is the value of x which lies in the middle of the distribution.



$$\int_a^M f(x)\,dx = \int_M^b f(x)\,dx$$

$$\int_a^M \frac{1}{b-a}\,dx = \int_M^b \frac{1}{b-a}\,dx$$

$$\cancel{\frac{1}{b-a}} \int_a^M 1\,dx = \cancel{\frac{1}{b-a}} \int_M^b 1\,dx$$

$$(x)_a^M = (x)_M^b$$

$$M-a = b-M$$

$$2m = b+a$$

$$M = \frac{b+a}{2}$$

for a uniform distⁿ

$$\text{Mean} = \text{mediam} = \frac{b+a}{2}$$

$\Rightarrow$ The distⁿ is symmetrical

Note ① a and b, $(a<b)$ are the two parameters of the uniform distribution on interval (a, b)

2. The distribution is also known as rectangular distribution since the curve $y=f(x)$ describes a rectangle over the x-axis and b/w the ordinates $x=a$ & $x=b$

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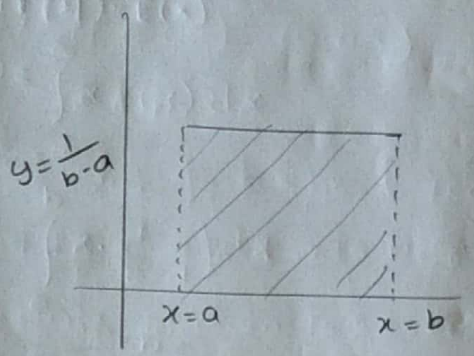
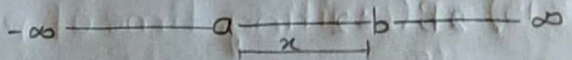


3. The distribution $F(x) = P(X \leq x)$ is given by



$$F(x) = \begin{cases} 0, & \text{if } -\infty < x < a \\ \frac{x-a}{b-a}, & \text{if } a < x < b \\ 1, & \text{if } b < x < \infty \end{cases}$$

4. For a rectangular variate x in $(-a, a)$ the p.d.f

is $f(x) = \begin{cases} \frac{1}{(a+a)}, & -a < x < a \\ 0, & \text{otherwise} \end{cases} \Rightarrow f(x) = \begin{cases} \frac{1}{2a}, & -a < x < a \\ 0, & \text{otherwise} \end{cases}$

Problems on uniform distribution

1. If x is uniformly distributed with mean $=1$ and variance $= 4/3$, find $P(x<0)$, ii) $P(x>0)$

Sol:- Given that mean $= 1 = \frac{b+a}{2}$ — ①

variance $= \frac{4}{3} = \frac{(b-a)^2}{12}$ — ②

From ①

$b+a=2$

from ② $(b-a)^2 = \frac{4 \times \cancel{12}^{4}}{\cancel{3}}$

$(b-a)^2 = 16$

$b-a = \pm 4 \Rightarrow b-a=4$ — ③

$b-a=-4$ — ④

from ① & ③ $b+\cancel{a} = 2$

$b-\cancel{a} = 4$

$2b = 6 \Rightarrow b=3$

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$① \Rightarrow b+a=2$

$$a = 2-b$$
$$= 2-3$$
$$= -1$$

$a=-1, b=3$

$(b>a)$

from ① & ④

$$b+a=2$$
$$b-a=-4$$
$$\overline{2b=-2}$$
$$b=-1$$

from $① \Rightarrow b+a=2$

$$-1+a=2$$
$$a=3$$

$a>b$ ✗

$\therefore a=-1, b=3$ are parameters of uniform $dist^n$.

$$P(x<0) = \int_{-\infty}^{0} f(x)dx$$

$$= \int_{-\infty}^{-1} f(x)dx + \int_{-1}^{0} f(x)dx$$

$$= 0 + \int_{-1}^{0} \frac{1}{3-(-1)} dx$$

$$= \frac{1}{4}\int_{-1}^{0} 1 dx$$

$$= \frac{1}{4}[x]_{-1}^{0}$$

$$= \frac{1}{4}(0+1)$$

$$= 1/4$$

$$P(x>0) = \int_{0}^{\infty} f(x)dx$$

$$= \int_{0}^{3} f(x)dx + \int_{3}^{\infty} f(x)dx$$

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$$= 0.4\int_0^3 \frac{1}{3-(-1)}\,dx + 0 \qquad \text{or } P(x>0) = 1 - P(x<0)$$

$$= 1 - 1/4$$

$$= \frac{1}{4}\int_0^3 1\,dx \qquad = 3/4$$

$$= \frac{1}{4}[x]_0^3\,dx$$

$$= \frac{1}{4}(3-0)$$

$$= 3/4$$

2. A bus company schedules a north bound bus every 30 minutes at a certain busstop. A man comes to the stop at a random time let the random x count the no. of minutes he has to wait for the next bus. Assumme x has a uniform distⁿ over the interval (0,30). compute the probability that he has to wait atleast k minutes for the next bus [use k=5, 10, 15, 20, 25, 30]

Sol:- Here $a=0, b=30$

$$f(x) = \begin{cases} \frac{1}{b-a}, & a<x<b \\ 0, & \text{otherwise} \end{cases} \Rightarrow f(x) = \begin{cases} \frac{1}{30-0} & 0<x<30 \\ 0 & \text{otherwise} \end{cases}$$

1) If $k=5$

$P(x\geq 5)$ = P(waiting time of passanger more than 5 min)

$$= \int_5^{30} f(x)\,dx$$

$$= \int_5^{30} \frac{1}{30}\,dx$$

$$= \frac{1}{30}[x]_5^{30} = \frac{30-5}{30} = \frac{25}{30}$$

$P(x\geq 10)$ = P(waiting time of passanger more than 10 min)

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$$= \int_{10}^{30} \frac{1}{30} dx$$

$$= \frac{1}{30} [x]_{10}^{30}$$

$$= \frac{30-10}{30}$$

$$= \frac{20}{30} = 2/3$$

If k = 15)

P(x ≥ 15) = P(waiting time of passenger more than 15 min)

$$= \int_{15}^{30} f(x) dx$$

$$= \int_{15}^{30} \frac{1}{30} dx$$

$$= \frac{1}{30} [x]_{15}^{30} = \frac{30-15}{30} = \frac{15}{30} = 1/2$$

If (k = 20)

$$P(x \geq 20) = \int_{20}^{30} f(x) dx$$

$$= \int_{20}^{30} \frac{1}{30} dx$$

$$= \frac{1}{30} [x]_{20}^{30} = \frac{30-20}{30} = \frac{10}{30} = 1/3$$

If (k = 25)

$$P(x \geq 25) = \int_{25}^{30} f(x) dx$$

$$= \int_{25}^{30} \frac{1}{30} dx$$

$$= \frac{1}{30} [x]_{25}^{30} = \frac{30-25}{30} = \frac{5}{30}$$

$$= 1/6$$

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If (k = 30)

$$p(x \geq 30) = \int_{30}^{30} f(x)\,dx$$

$$= \int_{30}^{30} \frac{1}{30}\,dx$$

$$= \frac{30-30}{30}$$

$$= 0$$

3/09/'19

Gamma distribution:

A random variable x is said to have Gamma distrib-ution with parameters λ and a if its p.d.f is given by

$$f(x) = \frac{e^{-\lambda x} \cdot x^{a-1}}{(\Gamma a/\lambda^a)} \quad , \lambda > 0 \quad 0 \leq x < \infty \quad a > 0$$

gamma

$= 0$ otherwise

We use notation $x \sim \gamma(\lambda\, a)$

Gamma function

We known that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{\infty} \frac{e^{-\lambda x}\, x^{a-1}}{(\Gamma a/\lambda^a)}\,dx = 1$$

$$\Rightarrow \int_{0}^{\infty} e^{-\lambda x} \cdot x^{a-1}\,dx = \frac{\Gamma a}{\lambda^a}$$ → This is gamma function of 2 parameters

If $\lambda = 1$

$$\int_{0}^{\infty} e^{-x} \cdot x^{a-1}\,dx = \Gamma a$$

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1. Mean $(\mu) = E(x) = \int_0^\infty x f(x)\,dx$

$$= \int_0^\infty x \cdot \frac{e^{-\lambda x} x^{a-1}}{\left(\Gamma a / \lambda^a\right)}\,dx$$

$$= \frac{\lambda^a}{\Gamma a}\int_0^\infty x^{(a+1)-1} \cdot e^{-\lambda x}\,dx$$

$$= \frac{\lambda^a}{\Gamma a}\left[\frac{\Gamma a+1}{\lambda^{a+1}}\right] \qquad \because \Gamma n+1 = n\Gamma n \qquad \because \Gamma \tfrac{1}{2} = \sqrt{\pi}$$

$$= \frac{\cancel{\lambda^a}}{\cancel{\Gamma a}} \cdot \frac{a\cancel{\Gamma a}}{\cancel{\lambda^a} \cdot \lambda} = \frac{a}{\lambda}$$

If $\lambda = 1 \Rightarrow E(x) = a$

variance $(\sigma^2) = E(x^2) - [E(x)]^2$

$$E(x^2) = \int_0^\infty x^2 f(x)\,dx$$

$$= \int_0^\infty x^2 \cdot \frac{e^{-\lambda x} \cdot x^{a-1}}{\left(\Gamma a / \lambda^a\right)}\,dx$$

$$= \frac{\lambda^a}{\Gamma a}\int_0^\infty e^{-\lambda x} \cdot x^{(a+2)-1}\,dx$$

$$= \frac{\lambda^a}{\Gamma a} \cdot \frac{\Gamma a+2}{\lambda^{a+2}}$$

$$= \frac{\lambda^2}{\Gamma a} \cdot \frac{(a+1)a\Gamma a}{\lambda^a \cdot \lambda^2} = \frac{a^2+a}{\lambda^2}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \frac{a^2+a}{\lambda^2} - \left(\frac{a}{\lambda}\right)^2$$

$$= \cancel{\frac{a^2}{\lambda^2}} + \frac{a}{\lambda^2} - \cancel{\frac{a^2}{\lambda^2}}$$

$$= \frac{a}{\lambda^2}$$

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If $\lambda=1$

$\Rightarrow V(X)=a$

Hence for $\lambda=1$

Mean = variance = a

Exponential distribution:

A r.v. X is said to follow an exponential distribution with parameter $\theta>0$, if its p.d.f is given by

$$f(x;\theta)=\begin{cases}\theta e^{-\theta x}, & x\geq 0\\ 0, & \text{otherwise}\end{cases}$$

Mean

$$E(X)=\int_0^\infty x\cdot f(x)\,dx=\int_0^\infty x\cdot\theta\cdot e^{-\theta x}\,dx$$

$$=\theta\int_0^\infty x\cdot e^{-\theta x}\,dx$$

$$=\theta\left[x\cdot\frac{e^{-\theta x}}{-\theta}-1\cdot\frac{e^{-\theta x}}{\theta^2}\right]_0^\infty$$

$$=\frac{\cancel{\theta}}{\cancel{\theta}}\left[-xe^{-\theta x}-\frac{e^{-\theta x}}{\theta}\right]_0^\infty$$

$$=-\left(-\frac{1}{\theta}\right)=\frac{1}{\theta}$$

$$E(X^2)=\int_0^\infty x^2 f(x)\,dx=\int_0^\infty x^2\cdot\theta\cdot e^{-\theta x}\,dx$$

$$=\theta\int_0^\infty x^2 e^{-\theta x}\,dx$$

$$=\theta\left[x^2\left(\frac{e^{-\theta x}}{-\theta}\right)-(2x)\left(-\frac{1}{\theta}\cdot\frac{e^{-\theta x}}{\theta}\right)+2\left(\frac{1}{\theta^2}\frac{e^{-\theta x}}{-\theta}\right)\right]_0^\infty$$

$$=\theta\left(+\frac{1}{\theta^3}\right)=\frac{2}{\theta^2}$$

$$\text{variance }(\sigma^2)=E(X^2)-(E(X))^2$$

$$=\frac{2}{\theta^2}-\left(\frac{1}{\theta}\right)^2$$

$$=1/\theta^2$$

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Note:- i) variance > mean , if $0 < \theta < 1$

ii) If variance = mean , if $\theta = 1$

iii) variance < mean , if $\theta > 1$

04/09/19

NORMAL DISTRIBUTION

A continuous random variable x is said to follow a normal distribution with parameters a, b if its p.d.f is given by

$$f(x) = c \cdot e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}, \quad -\infty \le x \le \infty, -\infty < a < \infty, b > 0$$

c is constant

$= 0$ otherwise

To find c

since f(x) is p.d.f

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} c \cdot e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2} dx = 1$$

$$\Rightarrow c\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2} dx = 1$$

$$c\int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2}\, b\,dz = 1$$

Put $\frac{x-a}{b} = z$

$x - a = bz$

$x = a + bz$

$dx = b\,dz$

When $x = -\infty \Rightarrow z = -\infty$

$x = \infty \Rightarrow z = \infty$

let $\phi(z) = e^{-z^2/2}$

$f(-x) = f(x) \Rightarrow$ even function

$f(-x) = -f(x) \Rightarrow$ odd function

$\phi(-z) = e^{-\frac{(-z)^2}{2}} = e^{-\frac{1}{2}z^2} = \phi(z)$

$\therefore \phi(z)$ is even function

$$\int_{-\infty}^{\infty} \phi(z)\,dz = 2\int_{0}^{\infty} \phi(z)\,dz$$

let $\frac{z^2}{2} = t$

$\Rightarrow z = \sqrt{2t}$

$\cancel{2} z\,dz = \cancel{2}\,dt$

$dz = \frac{dt}{\sqrt{2t}}$

when $z = 0 \Rightarrow t = 0$

$z = \infty \Rightarrow t = \infty$

($\because$ Gama function for one variable $\int_{0}^{\infty} e^{-x} \cdot x^{a-1}\,dx = \Gamma a$)

$$\Rightarrow 2bc\int_0^\infty e^{-t}\frac{dt}{\sqrt{2t}} = 1$$

$$\Rightarrow \sqrt{2}\,bc\int_0^\infty e^{-t}\cdot t^{-1/2}\,dt = 1$$

$$\Rightarrow \sqrt{2}\,bc\int_0^\infty e^{-t}\,t^{1/2-1}\,dt = 1$$

$$\Rightarrow \sqrt{2}\,bc\cdot\sqrt{\tfrac{1}{2}} = 1$$

$$\sqrt{2}\,bc\,\sqrt{\pi} = 1$$

$$c = \frac{1}{\sqrt{2\pi}\,b}$$

Now the p.d.f of normal distribution becomes

$$f(x) = \frac{1}{\sqrt{2\pi}\,b}e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}, \quad -\infty\leq x\leq\infty,\ -\infty\leq a\leq\infty,\ b>0$$

$$= 0 \quad \text{otherwise}$$

Mean of Normal distribution

$$M = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} x\cdot\frac{1}{\sqrt{2\pi}\,b}\cdot e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}dx$$

Put $\frac{x-a}{b} = z$

$x = a+bz$

$dx = bdz$

$x = -\infty \Rightarrow z = -\infty$

$x = +\infty \Rightarrow z = +\infty$

$$= \int_{-\infty}^{\infty}(a+bz)\frac{1}{\sqrt{2\pi}\,b}\cdot e^{-z^2/2}\cdot bdz$$

$$\Rightarrow \int_{-\infty}^{\infty} a\cdot\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz + \int_{-\infty}^{\infty} bz\cdot\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz$$

$$\Rightarrow \frac{a}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-z^2/2}dz + \frac{b}{\sqrt{2\pi}}\int_{-\infty}^{\infty} z\cdot e^{-z^2/2}dz$$

$$\Rightarrow \frac{2a}{\sqrt{2\pi}}\int_0^\infty e^{-z^2/2}dz + \frac{b}{\sqrt{2\pi}}(0)$$

using Gamma function

let $\frac{z^2}{2} = t$

$2zdz = 2dt$

$dz = \frac{dt}{\sqrt{2t}}$

$$= \frac{\cancel{2}a}{\cancel{\sqrt{2}}\sqrt{\pi}}\int_0^\infty e^{-t}\frac{dt}{\cancel{\sqrt{2}}\sqrt{t}}$$

$$= \frac{a}{\sqrt{\pi}}\int_0^\infty e^{-t}\cdot t^{-1/2}dt$$

$z = 0 \Rightarrow t = 0$

$z = \infty \Rightarrow t = \infty$

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$$= \frac{a}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{1/2 - 1} dt$$

$$= \frac{a}{\sqrt{\pi}} \cdot \sqrt{\pi} = a$$

2. Variance of Normal distribution

$$\sigma^2 = E(x - E(x))^2 = E(x-\mu)^2$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x) dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi} b} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{b}\right)^2} dx$$

$$= \frac{1}{\sqrt{2\pi} b} \int_{-\infty}^{\infty} (x-\mu)^2 e^{-1/2\left(\frac{x-\mu}{b}\right)^2} dx$$

let $\frac{x-\mu}{b} = z$

$dx = b\,dz$

$x - \mu = bz$

$x = -\infty \Rightarrow z = -\infty$

$x = +\infty \Rightarrow z = +\infty$

$$= \frac{1}{\sqrt{2\pi} b} \int_{-\infty}^{\infty} (bz)^2 e^{-\frac{1}{2}z^2} b\,dz$$

$$= \frac{b^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 \cdot e^{-\frac{1}{2}z^2} dz$$

$$= \frac{2b^2}{\sqrt{2\pi}} \int_0^\infty z^2 e^{-1/2 z^2} dz$$

$$= \frac{2b^2}{\sqrt{2\pi}} \int_0^\infty 2t \cdot e^{-t} \cdot \frac{dt}{\sqrt{2t}}$$

$$= \frac{2b^2}{\sqrt{\pi}} \int_0^\infty t^{1/2} e^{-t} dt$$

$$= \frac{2b^2}{\sqrt{\pi}} \int_0^\infty e^{-t} t^{3/2 - 1} dt$$

$$= \frac{2b^2}{\sqrt{\pi}} \Gamma\frac{3}{2}$$

$(\because \Gamma n+1 = n\Gamma n)$

$$\Gamma\frac{3}{2} = \Gamma\frac{1}{2}+1 = \frac{1}{2}\Gamma\frac{1}{2} = \frac{1}{2}\sqrt{\pi}$$

$$\Rightarrow \sigma^2 = \frac{2b^2}{\sqrt{\pi}} \Gamma\frac{3}{2}$$

$$= \frac{2b^2}{\sqrt{\pi}} = \frac{\sqrt{\pi}}{2}$$

$$\sigma^2 = b^2$$

$$\Rightarrow \sigma = b$$

P.d.f becomes

$$\therefore f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \sigma > 0$$

$$-\infty < \mu < \infty$$

$$= 0 \quad \text{otherwise} \quad -\infty < x < \infty$$

06/09/'19

Mode of Normal distⁿ

Mode is the value of x for which $f(x)$ is maximum

$\therefore f'(x) = 0$ and $f''(x) < 0$ at the value of x

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$f'(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x+1}{\sigma}\right)^2} \cdot \left\{-\frac{1}{2} \cdot 2\left(\frac{x-\mu}{\sigma}\right) \cdot \frac{1}{\sigma}\right\}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \left\{-\frac{x-\mu}{\sigma^2}\right\} = f(x)\left\{-\frac{x-\mu}{\sigma^2}\right\}$$

$$f'(x) = 0 \text{ if } \frac{x-\mu}{\sigma^2} = 0$$

$$\Rightarrow x - \mu = 0$$

$$x = \mu$$

$$f'(x) = f(x) \cdot \left\{-\frac{x-\mu}{\sigma^2}\right\}$$

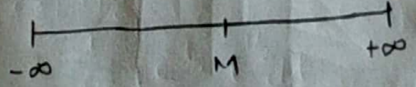
$$f''(x) = f(x) \cdot \left\{-\frac{1}{\sigma^2}(1)\right\} + -\left[\frac{x-\mu}{\sigma^2}\right] \cdot f'(x)$$

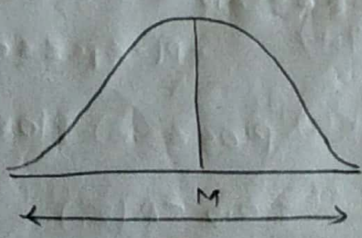
check at $x = \mu$ f'' is +ve or −ve

$$f''(x)\Big|_{\text{at } x=\mu} = -\frac{1}{\sigma^2} f(x) < 0$$

$\therefore x = \mu$ is mode of normal distribution

Median of Normal distribution





$\Rightarrow$ If M is the median

then $\int_{-\infty}^{M} f(x)\,dx = \int_{M}^{\infty} f(x)\,dx = \frac{1}{2}$

I II

consider

$$\int_{M}^{\infty} f(x)\,dx = \frac{1}{2}$$

$$\int_{M}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{1}{2}$$

let μ which lies in between M & ∞

$$\int_{M}^{\mu} \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx + \int_{\mu}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{1}{2}$$

consider

$$\int_{\mu}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

put $\frac{x-\mu}{\sigma} = z$

$x = \mu + \sigma z$

$dx = \sigma\,dz$

$x = \mu \Rightarrow z = 0$

$x = \infty \Rightarrow z = \infty$

$\frac{z^2}{2} = t \Rightarrow z = \sqrt{2t}$

$2z\,dz = 2\,dt$

$\sqrt{2t}\,dz = dt$

$dz = \frac{dt}{\sqrt{2t}}$

$$= \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}z^2} \sigma\,dz$$

$$= \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t} \frac{dt}{\sqrt{2t}}$$

$$= \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t} \frac{t^{-1/2}}{\sqrt{2}} dt$$

$$= \frac{1}{2\sqrt{\pi}} \int_{0}^{\infty} e^{-t} \cdot e^{t-1}\,dt$$

$$= \frac{1}{2\sqrt{\pi}} \sqrt{\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{\pi}} \sqrt{\pi}$$

$$= \frac{1}{2}.$$

(*) $\int_{M}^{\mu} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx + \frac{1}{2} = \frac{1}{2}$

$\Rightarrow \int_{M}^{\mu} \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 0$ $\quad f(x) \geq 0$

$\Rightarrow$ If $M=\mu$ then the above is possible

$\therefore$ Median of normal distribution is μ

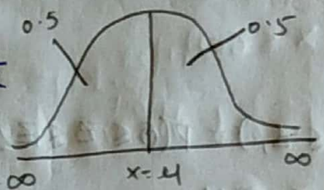
Hence for normal distribution

Mean = Median = Mode = μ

$\Rightarrow$ normal distribution is a symmetrical distribution

$\therefore P(-\infty \leq x \leq \mu) = P(\mu \leq x \leq \infty) = 0.5 = \frac{1}{2}$

(or) $P(-\infty \leq z \leq 0) = P(0 \leq z \leq \infty) = 0.5 = \frac{1}{2}$



Area Property [standard normal form Normal distribution]

$x \rightarrow$ normal variate

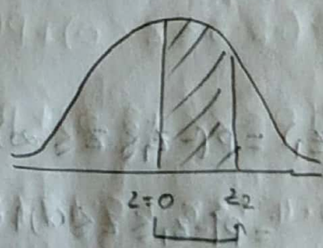
$\downarrow$

$z = \frac{x-\mu}{\sigma} \rightarrow$ standard normal variate

$P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f(x)\,dx = \int_{z_1}^{z_2} f(z)\,dz = P(z_1 \leq z \leq z_2)$

1. $P(0 \leq z \leq z_2) =$

$P(0 \leq z \leq 2.3)$



Areas under the std normal distribution from $z=0$ to $z=z$

$z = \frac{x-\mu}{\sigma}$ μ = Mean, σ = Std. of

Case 1:-

$$P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$$

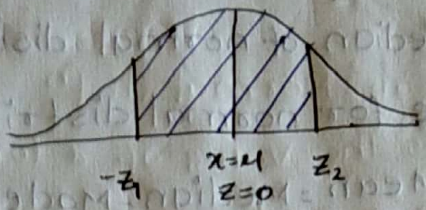
$$\int_{x_1}^{x_2} f(x)\,dx = \int_{z_1}^{z_2} \phi(z)\,dz$$

$$P(-z_1 \le z \le z_2)$$

$$= P(-z_1 \le z \le 0) + P(0 \le z \le z_2)$$

$$= P(0 \le z \le z_1) + P(0 \le z \le z_2)$$

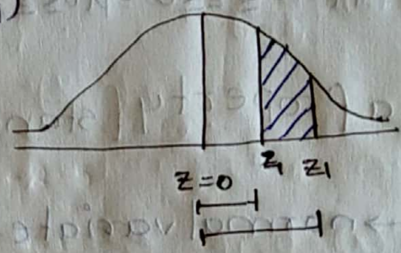
$$= A(z_1) + A(z_2)$$



Case 2:

$$P(z_1 \le z \le z_1) = P(0 \le z \le z_2) - P(0 \le z \le z_1)$$

$$= A(z_2) - A(z_1)$$

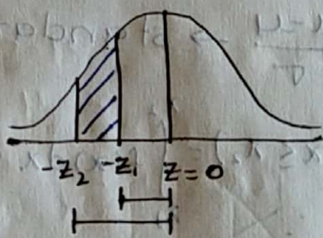


Case 3:

$$P(-z_1 \le z \le -z_2) = P(-z_2 \le z \le 0) - P(-z_1 \le z \le 0)$$

$$= P(0 \le z \le z_2) - P(0 \le z \le z_1)$$

$$= A(z_2) - A(z_1)$$



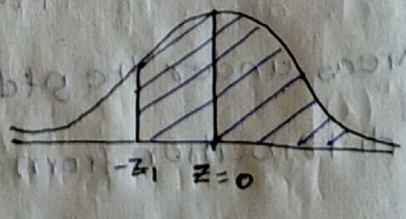
Case 4:

$$P(z \ge -z_1) = P(-z_1 \le z \le \infty)$$

$$= P(-z_1 \le z \le 0) + P(0 \le z \le \infty)$$

$$= P(0 \le z \le z_1) + 0.5$$

$$= 0.5 + A(z_1)$$

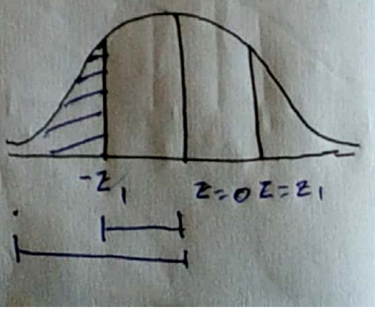


Case 5:

$$P(z \le -z_1) = P(-\infty \le z \le -z_1) = P(-\infty \le z \le 0) - P(-z_1 \le z \le 0)$$

$$= P(0 \le z \le \infty) - P(0 \le z \le z_1)$$

$$= 0.5 - A(z_1)$$



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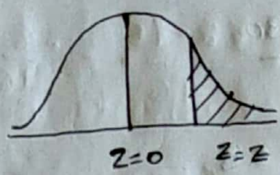
Case 6:-

$p(z \geq z_1) = P(z_1 \leq z \leq \infty)$

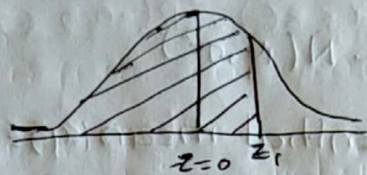


$= P(0 \leq z \leq \infty) - P(0 \leq z \leq z_1)$

$= 0.5 - A(z_1)$

Case 7:-

$p(z \leq z_1) = P(-\infty \leq z \leq 0) + P(0 \leq z \leq z_1)$



$= 0.5 + A(z_1)$

1. If x is a normal variate with mean 30 and standard deviation 5 find the probabilities that (i) $26 \leq x \leq 40$
(ii) $x \geq 45$

Sol:- $X \sim N(\mu, \sigma^2)$

Given that mean $(\mu) = 30$

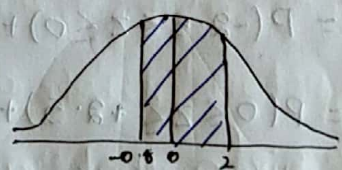
standard devation $(\sigma) = 5$

i) $P(26 \leq x \leq 40)$

standard normal variate

$z = \frac{x - \mu}{\sigma}$

When $x_1 = 26 \Rightarrow z = \frac{26-30}{5} = \frac{-4}{5} = -0.8 = -z_1$



$x_2 = 40 \Rightarrow z = \frac{40-30}{5} = \frac{10}{5} = 2 = z_2$

$P(26 \leq x \leq 40) = P(-0.8 \leq z \leq 2) = P(-0.8 \leq z \leq 0) + P(0 \leq z \leq 2)$

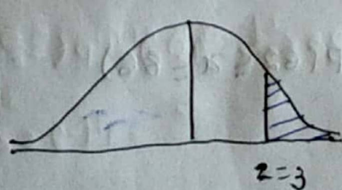
$= P(0 \leq z \leq 0.8) + P(0 \leq z \leq 2)$

$= 0.2881 + 0.4772$

$= 0.7653$

ii) $p(x \geq 45) = p(z \geq 3)$

$= p(0 \leq z \leq \infty) - p(0 \leq z \leq 3)$



$= 0.5 - A(3)$

$= 0.5 - 0.4987$

$= 0.0013$

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$x = 45$

$$z = \frac{45-30}{5}$$

$= 15/5 = 3$

2. Find the probability b/w $x=15$ and 60. Given that mean $\mu=40$ and standard devation 10

Sol:- $x \sim N(\mu, \sigma^2)$

Giventhat mean$(\mu) = 40$

standard devitation$(\sigma) = 10$

i) $P(15 \leq x \leq 60)$

standard normal variate

$$z = \frac{x-\mu}{\sigma}$$

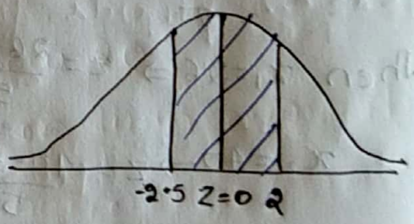
when $x_1 = 15 \Rightarrow z = \frac{15-40}{10} = \frac{-25}{10} = -2.5 = z_1$

when $x_2 = 60 \Rightarrow z = \frac{60-40}{10} = \frac{20}{10} = 2 = z_2$

$P(15 \leq x \leq 60) = P(-2.5 \leq z \leq 2)$

$= P(-2.5 \leq z \leq 0) + P(0 \leq z \leq 2)$

$= P(0 \leq z \leq +2.5) + P(0 \leq z \leq 2)$



$= 0.4938 + 0.4772$

$= 0.9710$

3. If mean $= 70$, standard deviation is 16 find

i) $P(38 \leq x \leq 46)$

ii) $P(82 \leq x \leq 94)$

iii) $P(62 \leq x \leq 86)$

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Sol:- $x \sim N(\mu, \sigma^2)$

Given that mean$(\mu) = 70$

standard deviation$(\sigma) = 16$

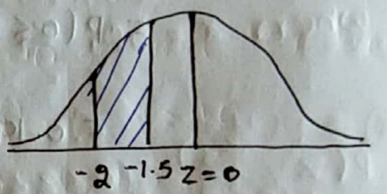
standard normal variante

$$Z = \frac{x-\mu}{\sigma}$$

i) $P(38 \leq x \leq 46)$

when $x_1 = 38 \Rightarrow z = \frac{38-70}{16} = \frac{-32}{16} = -2 = -z_1$

when $x_2 = 46 \Rightarrow z = \frac{46-70}{16} = \frac{-24}{16} = -1.5 = -z_2$



$P(38 \leq x \leq 46) = P(-2 \leq z \leq -1.5)$

$\cancel{= P(-1.5 \leq z \leq 0) - P(-2 \leq z \leq 0)}$

$\cancel{= P(0 \leq z \leq +1.5) - P(0 \leq z \leq +2)}$

$\cancel{= P(1.5 \leq z \leq 0) - P(2 \leq z}$

$= P(-2 \leq z \leq 0) - P(-1.5 \leq z \leq 0)$

$= P(0 \leq z \leq 2) - P(0 \leq z \leq 1.5)$

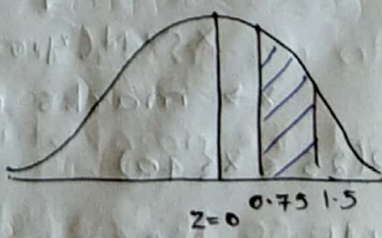
$= 0.4772 - 0.4332$

$= 0.044$

ii) $P(82 \leq x \leq 94)$

$x_1 = 82 \Rightarrow z = \frac{82-70}{16} = \frac{12}{16} = 0.75 = z_1$

$x_2 = 94 \Rightarrow z = \frac{94-70}{16} = \frac{24}{16} = 1.5 = z_2$



$P(82 \leq x \leq 94) = P(0.75 \leq z \leq 1.5)$

$= P(0 \leq z \leq 1.5) - P(0 \leq z \leq 0.75)$

$= 0.4332 - 0.2734$

$= 0.1598$

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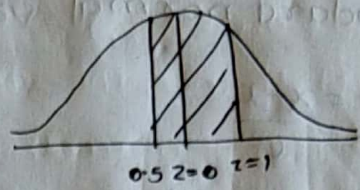
iii) $P(62 \le x \le 86)$

$x = 62 \quad z = \dfrac{62-70}{16} = \dfrac{-8}{16} = -0.5 = z_1$

$x = 86 \quad z = \dfrac{86-70}{16} = \dfrac{16}{16} = 1 = z_2$



$P(62 \le x \le 86) = P(-0.5 \le z \le 1)$

$= P(-0.5 \le z \le 0) + P(0 \le z \le 1)$

$= P(0 \le z \le 0.5) + P(0 \le z \le 1)$

$= 0.1915 + 0.3413$

$= 0.5328$

4. 1000 students had written an examination the mean of test is 35 and standard deviation is 5. Assuming the distribution is to be normal find

i) how many students marks lies b/w 25 and 40

ii) " " " get more than 40

iii) " " " " below 20

iv) " " " " more than 50.

sol:- Given that $\mu = 35 \quad \sigma = 5$

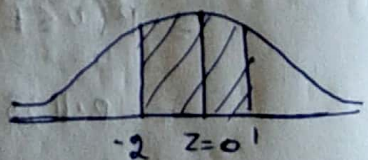
$x \sim N(\mu, \sigma^2)$

$x \to$ marks obtained by students

i) $P(25 \le x \le 40)$

When $x = 25 \quad z = \dfrac{25-35}{5} = -2 = -z_1$

$x = 40 \quad z = \dfrac{40-35}{5} = 1 = z_2$



$P(25 \le x \le 40) = P(-2 \le z \le 1)$

$= P(-2 \le z \le 0) + P(0 \le z \le 1)$

$= P(0 \le z \le 2) + P(0 \le z \le 1)$

$= 0.3413$

$+ 0.4772$

$= 0.8185$

No. of students whose marks b/w 25 and 40 is

$= N \times P(25 \leq x \leq 40)$

$= 1000 \times (0.8185)$

$= 818.5 \cong 819$ students

ii) $P(x \geq 40)$

when $x = 40$ $z = \frac{40-35}{5} = \frac{5}{5} = 1$

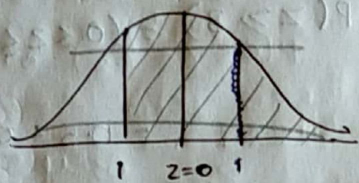
$= z_1$

~~$P(x \geq 40) = P(z \geq 1)$~~

~~$= P(-1 \leq z \leq 0) + P(0 \leq z \leq \infty)$~~

$=$ ~~$P(0 \leq z \leq 1) + 0.5$~~

~~$= 0.5 +$~~

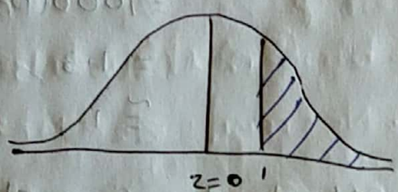


$P(x \geq 40) = P(z \geq 1)$

$= P(0 \leq z \leq \infty) - P(0 \leq z \leq z_1)$

$= 0.5 - 0.3413$

$= 0.1587$



No. of students whose marks more than 40

$= N \times P(x > 0)$

$= 1000 \times 0.1587$

$= 158.7$

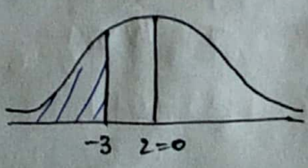
$= 159$

iii) $P(x \leq 20)$

when $x = 20$ $Z = \frac{20-35}{5} = \frac{-15}{5} = -3 = -z_1$

$P(z \leq -3)$

$= P(-\infty \leq z \leq 0) - P(-3 \leq z \leq 0)$



$= P(0 \leq z \leq \infty) - P(0 \leq z \leq 3)$

$= 0.5 - 0.4987$

$= 0.0013$

No. of students below 20

$= N \times P(x \leq 20)$

$= 1000(0.0013)$

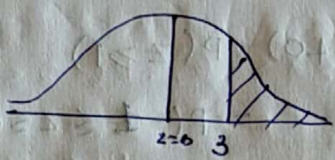
$= 1.3$

$= 1$

iv) $P(x \geq 50)$

when $x = 50$, $z_1 = \frac{50-35}{5} = \frac{15}{5} = 3$

$P(z \geq 3) = (0 \leq z \leq \infty) - (0 \leq z \leq 3)$



$= 0.5 - 0.4987$

$= 0.0013$

No. of students more than 50

$= N \times P(x \geq 50)$

$= 1000(0.0013)$

$= 1.3$

$\simeq 1$

5. Students of a class were given an examination. Their marks were found to be normally distributed with mean 55 marks and standard deviations 5. Find the no. of students who get the marks more than 60 if 500 students were written the exam.

Sol:- $N = 500$

$\mu = \text{mean} = 55$

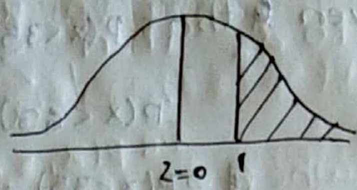
$\sigma = 5$

i) $P(x \geq 60)$

when $x = 60$, $z_1 = \frac{60-55}{5} = \frac{5}{5} = 1$

$z_1 = 1$

$P(z \geq 1) = (0 \leq z \leq \infty) - (0 \leq z \leq 1)$



$= 0.5 - 0.3413$

$= 0.1587$

No. of students more than 60

$= N \times P(X \geq 60)$

$= 500 \times 0.1567$

$= 79.35$

$= 79.$

18/09/19

let x be a random variable with the standard normal distribution determine the value of z_1

i. $P(0 \leq z \leq z_1) = 0.4938$ ii) $P(z \leq z_1) = 0.834$ iii) $P(z_1 \leq z \leq 2) = 0.2857$

i) $P(0 \leq z \leq z_1) = 0.4936$

$A(z_1) = 0.4938$

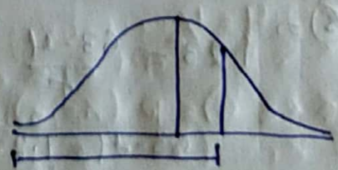
$z_1 = 2.5$

ii) $P(z \leq z_1) = 0.834$

$P(-\infty \leq z \leq 0) + P(0 \leq z \leq z_1) = 0.5 + 0.334$

$A(z) = 0.334$

$z_1 = 0.97$



iii) $P(z_1 \leq z \leq 2) = 0.2857$

$A(2) - A(z_1) = 0.2857$

$0.4772 - A(z_1) = 0.2857$

$A(z) = 0.4772 - 0.2857$

$= 0.1915$

from table $z_1 = 0.5$

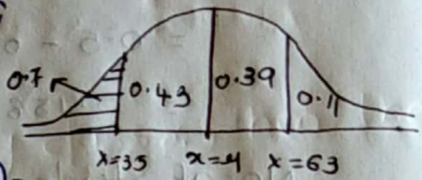
Area 0.2857 which is less than 0.5

$\therefore z_1$ & 2 both are +ve number

1. In a distribution exactly normal 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the dist?

Given that $p(x<35) = 7\% = 0.07$ — ①

$p(x<63) = 89\% = 0.89$ — ②



~~① ⇒ p(x<35) = 0.07~~

we have $p(-z_1 \le z \le 0) = 0.43$ — ③

When $x = 35$ $-z_1 = \frac{35-\mu}{\sigma}$ — ④

(since 35 lies in the left side of $\lambda = \mu$ hence z is -ve)

② ⇒ $p(\mu < x < 63) = 0.39$

$p(0 \le z \le z_2) = 0.39$ — ⑤

Where

$z_2 = \frac{63-\mu}{\sigma}$ — ⑥

using the table

③ ⇒ $A(z_1) = 0.4300$

$z_1 = 1.48$

⑤ ⇒ $A(z_2) = 0.39$

$z_2 = 1.23$

④ ⇒ $-1.48 = \frac{35-\mu}{\sigma}$

⇒ $35-\mu = -1.48\sigma$ — ⑦

⑥ ⇒ $1.23 = \frac{63-\mu}{\sigma}$

$63-\mu = 1.23\sigma$ — ⑧

solving ⑦ & ⑧ ⇒ $35-\mu = -1.48\sigma$

$\underline{\begin{array}{r} 63-\mu = 1.23\sigma \\ - \quad + \quad\quad - \end{array}}$

$-28 = -2.71\sigma$

$\sigma = \frac{28}{2.71} = 10.33$

we get

$\mu = 50.3$

$\sigma = 10.3$

$35-\mu = -1.48$ (10.33)

$-\mu = -15.288$

$-\mu = -50.28$

$-\mu = -50.3$

$\mu = 50.3$

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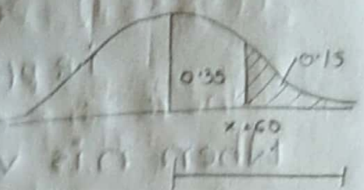
2) The marks obtain in statistics in a certain Examination are found to be normally distributed. If 15% of candidates got ≥ 60 marks, 40% got <30 marks. Find the mean and standard deviation of marks.

Sol:- We have

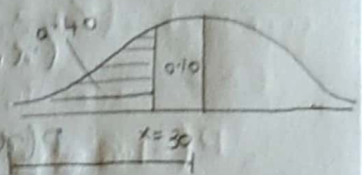
$P(x \geq 60) = 15\%$

$P(x < 30) = 40\%$



figure② $P(-z_1 \leq z \leq 0) = 0.10$

$-z_1 = \frac{30-\mu}{\sigma}$ — ① $z_1 = 0.26$ (from table)



fig① $P(0 \leq z \leq z_2) = 0.35$ hence $z_2 = 1.04$ (from table)

$z_2 = \frac{60-\mu}{\sigma}$ — ②

① $\Rightarrow -0.26 = \frac{30-\mu}{\sigma} \Rightarrow 30-\mu = -0.26\sigma$

② $\Rightarrow 1.04 = \frac{60-\mu}{\sigma} \Rightarrow 60-\mu = 1.04\sigma$

solving we get

$$30 - \mu = -0.26\sigma$$
$$60 - \mu = 1.04\sigma$$
$$- \quad + \quad -$$
$$-30 = -1.30\sigma$$

$$\sigma = \frac{30}{1.30}$$

$$= 23.076$$

$30 - \mu = (-0.26)(23.076)$

$30 - \mu = -5.999$

$\mu = 35.99$

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20/09/19

Normal Approximation to binomial distribution:

Suppose the number of successes ranges from x_1 to x_2 then $\sum_{x=x_1}^{x_2} p(x) = \sum_{x=x_1}^{x_2} nC_x p^x q^{n-x}$

i.e $p(x_1 \leq x \leq x_2) =$

When n is very large of any class x, then the real class interval is

$$(x_1, x_2) \cong (x_1 - 1/2, x_2 + 1/2)$$

$$P(x_1 \leq x \leq x_2) \Rightarrow P(x_1 - \tfrac{1}{2} \leq X \leq x_2 + 1/2) =$$

$$P(z_1 \leq X \leq z_2) = \int_{z_1}^{z_2} \phi(z)\,dz$$

Where $z_1 = \frac{(x_1 - 1/2) - \mu}{\sigma}$ $\mu = np$

$z_2 = \frac{(x_2 + 1/2) - \mu}{\sigma}$ $\sigma = \sqrt{npq}$

1. If p is the probability of getting head in tossing of a coin is tossed 12 times. find $P(3 \leq x \leq 6)$ using
a. binomial distribution
b. Normal distribution

① $P(3 \leq x \leq 6) = ?$

$n = 12$, p = prob. of getting head $= 1/2$

$q = 1/2$

$$P(3 \leq x \leq 6) = P(x=3) + P(x=4) + P(x=5) + P(x=6)$$

$$= 12C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^{12-3} + 12C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^{12-4} + 12C_5 \left(\tfrac{1}{2}\right)^5 \left(\tfrac{1}{2}\right)^{12-5} + 12C_6 \left(\tfrac{1}{2}\right)^6 \left(\tfrac{1}{2}\right)^{12-6}$$

$$= \left(\tfrac{1}{2}\right)^{12} (220 + 495 + 792 + 924)$$

$$= \frac{2431}{4096} = 0.6$$

b) mean $(\mu) = np = 12 \cdot \frac{1}{2} = 6$

variance $(\sigma^2) = npq = 12 \cdot \frac{1}{2} \cdot \frac{1}{2} = 3$

$$\sigma = \sqrt{npq} = \sqrt{3}$$

$P(3 \leq x \leq 6) \cong P(3-0.5 \leq x \leq 6+0.5)$ (or)

$$= P(2.5 \leq x \leq 6.5)$$

$$P(3 \leq x \leq 6) = P(z_1 \leq z \leq z_2) = P(-2.02 \leq z \leq 0.29)$$

$$= A(-2.02) + A(0.29)$$

$$= 0.4783 + 0.1141 = 0.5924$$

When $z_1 = \frac{x_1 - 1/2 - \mu}{\sigma} = \frac{(3-1/2)-6}{\sqrt{3}} = \frac{2.5-6}{1.732} = -2.02$

$$z_2 = \frac{(x_2 + 1/2) - \mu}{\sigma} = \frac{(6+1/2)-6}{\sqrt{3}} = \frac{6.5-6}{1.732} = 0.29$$

2. Find the probability of getting 1 or 4 or 5 or 6 in throwing a die 5 to 7 times among 9 trials using i) binomial distribution ii) normal distribution.

$n = 9$

$p = P(1 \text{ or } 4 \text{ or } 5 \text{ or } 6)$

$= P(1) + P(4) + P(5) + P(6)$

$= 1/6 + 1/6 + 1/6 + 1/6 = 4/6$

$q = 1 - p = 1 - \frac{4}{6} = \frac{2}{6}$

$P(5 \leq x \leq 7) = P(x=5) + P(x=6) + P(x=7)$

$$= 9C_5 \left(\frac{4}{6}\right)^5 \left(\frac{2}{6}\right)^4 + 9C_2 \left(\frac{4}{6}\right)^6 \left(\frac{2}{6}\right)^3 + 9C_7 \left(\frac{4}{6}\right)^7 \left(\frac{2}{6}\right)^2$$

$=$

b. mean $\mu = np = 9 \cdot \frac{4}{6} = 6$

Variance $(\sigma^2) = npq = 9 \cdot \frac{4}{6} \cdot \frac{2}{6} = 2$

$\sigma = \sqrt{npq} = \sqrt{2}$

$P(5 \leq X \leq 7) = P(z_1 \leq Z \leq z_2)$

where $z_1 = \frac{x_1 - \frac{1}{2} - \mu}{\sigma} = \frac{(5 - 1/2) - 6}{\sqrt{2}} = -1.06$

$z_2 = \frac{(x + \frac{1}{2}) - \mu}{\sigma} = \frac{(7 + 1/2) - 6}{\sqrt{2}} = 1.06$

$P(-1.06 \leq Z \leq 1.06)$

$= A(-1.06) + A(1.06) = 0.3554 + 0.3554$

3. Ten cards are drawn from a deck of 52 cards. Find the probability of getting 2 to 5 diamonds using normal distn.

p = probability of getting diamonds $= \frac{13}{52} = 1/4$

$q = 1 - 1/4 = 3/4$

$n = 10$

Mean $(\mu) = np = 10 \times \frac{1}{4} = 2.5$

variance $(\sigma^2) = npq = 10 \times \frac{1}{4} \times \frac{3}{4} = 7.5$

$\sigma = \sqrt{7.5} =$

$P(2 \leq x \leq 5) = P(z_1 \leq z \leq z_2)$

where $z_1 = \frac{(x - \frac{1}{2}) - \mu}{\sigma} = \frac{(2 - \frac{1}{2}) - 2.5}{2.73} = -0.36$

$z_2 = \frac{(x_2 + 1/2) - \mu}{\sigma} = \frac{(5 + 1/2) - 2.5}{2.73} = 1.09$

$P(-0.36 \leq x \leq 1.09$

$A(-0.36) + A(1.09)$

$0.1406 + 0.3621$

$= 0.1406 + 0.3621$

21/09/19

Sampling distributions

1. Collection of data
2. presentation of data
3. Analysis
4. making decisions

The process of drawing samples is called sampling.

population and sample:-

population:- In a statical study, the population is the set or collection of observations above which inferences are to be drawn.

Population consist of sets of numbers, measurements or observation.

population (N):- The no. of objects in the populati-on is known as population size. It is denoted by 'N'. It may be finite (or) infinite depending upon size of N.

Sample:- A finite subset of population is known as sample.

sample size(n):-

It is denoted by 'n'. It is the no. of objects or observation in the sample.

ex:- engineering graduates in Telangana (population)

engineering graduates in college (sample).

2). Total production of items in a month (population)

Total production of items in day (sample).

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Population parameters and sample statistics.

A population parameter is a statistical ~~meth~~ a mesure or a constant obtained from the population

Ex:- population mean(μ)

population variance(σ^2)

Sample statistics:-

A sample statistics is a statistical measure computed from sample observations.

Ex:- sample mean($\bar{x}$)

sample variance (s^2)

We use the notations μ, σ, P to represent the population mean, standard deviation, population proportion

similarily $\bar{x}, s, p$ used to denote sample mean, sample standard deviation, sample proportion.

samplings-

The process of drawing (or) obtaining sample is called sampling

Types are samplings:-

1. large sampling
2. small sampling

1. large sampling:-

If $n \geq 30$ then the sampling is known as large sampling.

2. Small sampling:-

If $n < 30$ the sampling is known as small

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sampling or exact sampling.

simple random sampling:-

Random sampling is one in which each member of the population has the equal chances or probabilites of being included in the sample the sample obtain by this method is termed as random sample.

Finite and Infinite population

If the no. of items or observation constituting the population is fixed and limited it is known as finite population.

ex:- The workers in a factory, students in a college

If the population consists of an infinite no. of items, it is known as infinite population.

ex:- The population of stars in sky. the population of ~~sampl~~ all real no.s laying b/w 2 and 3

sampling with replacement

If the items are selected are drawn one by one in such a way that an item drawn at a item is replaced back to the population before the next item or subsequent draw. The process is known as sampling with replacement.

In this type of sampling with population size N the probability of selecting a unit in each draw is equal to $1/N$

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∴ Sampling from finite population with replacement can be considered theoretically as sampling from infinite population

In this case, there will be N^n (no. of samples).

Without replacement

The item of the population which can't be choosen more than once as it is not replaced in the sampling without replacement process.

Hence the probability of drawing a unit from population of N items

at 1^{st} draw is $\frac{1}{N}$

2^{nd} draw is $\frac{1}{N-1}$

3^{rd} draw is $\frac{1}{N-1}$

⋮

r^{th} draw is $\frac{1}{N-(r-1)} = \frac{1}{N-r+1}$

In sampling with out replacement there will be N_{C_n} possible samples

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