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①

UNIT-5

Stochastic Processes

Random processes

① Write short Notes on Stochastic process - Explain?

Introduction:-

1. An electrical Communication system is used for the transfer of message signals over a Communication channel to a receiver end using electrical waveforms.

Consider a simple Communication system, Consisting of an information source, transmitter, channel (through with information is sent) Receiver and destination.

In such Communication system, we encounter a number of signals that are Unpredictable.

i.e Random Signals

The signals are Random in the sense that before Conducting an experiment. It is not possible to describe exactly the wave forms that will be observed

The Signal at the output of the Source is a Random Signal. The Audio Signal that is transmitted over the channel is a Random Signal the noise voltages generated in the resistors of the receiver are Random Signals.

All such Random Signals are functions of time

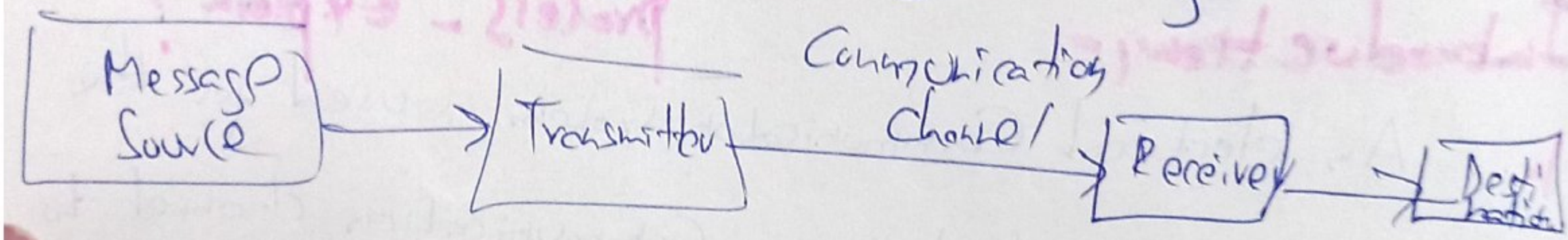

Figure: A Simple Communication System

Random process:-

i.e The Random Variable is a rule for assigning a real Number $X(\lambda)$ to each & every outcome (λ) of an experiment."

"i.e A Random process as a rule for assigning a Real valued function of time $X(t, \lambda)$ to every outcome λ of the experiment. Thus, for every outcome λ, one function will be assigned. This family of functions is called a Random process."

Definition:- It is an ensemble (or) Collection of functions of time, with a probability measure associated.

It is usually denoted by $X(t, \lambda)$ (or) $X(t)$.

Interpretations:-

① The Random process is family of functions $X(t, \lambda)$.

where t & λ are Variables.

② **If λ is fixed:-**

"The Random process is a function of time only. It is called <u>Single time function</u>."

③ **If t is fixed:-**

"The Random process is a function of λ only and hence the Random process will represent a Random Variable at time t."

④ If **both t & λ are fixed**, then $X(t, \lambda)$ is merely a number.

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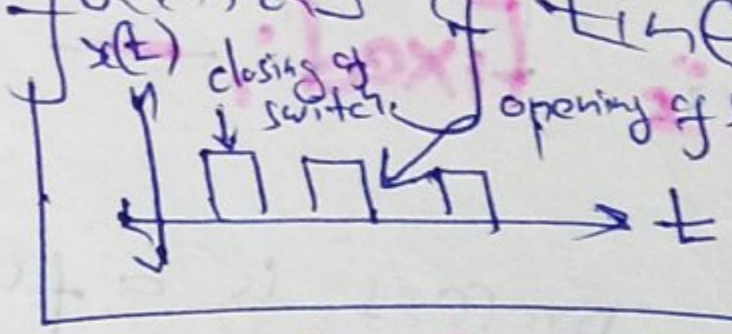
ie A stochastic process is a sequence of $\{X_t\}$. where $t \in T$ is time. The Range space X_t may be discrete (or) Continuous. The Analysis by Mathematical Model. We require not only several Random Variables, but also several different families of R.Vs which are functions of time.

Example:- In the Queueing System, waiting in queue $W(t)$ is a Random function of time. Such Random functions of time are called stochastic processes



Ex:- The voltage available at the one end of the switch because of Random opening & Closing of the switch is a discrete Random process.

Types of stochastic processes:-

There are 4 types

① Discrete S.P:- If state values are finite (or) Countable. Then it is called discrete state process.

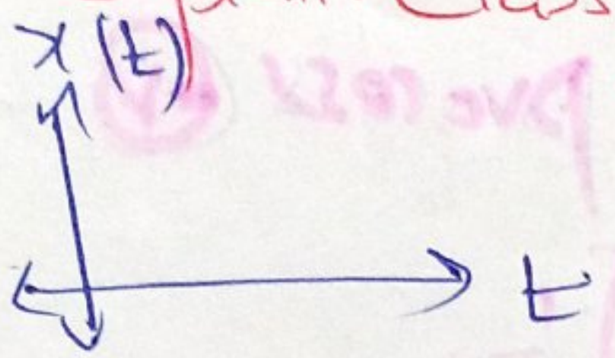
Ex:- No. of Customers in Queuing System

② Continuous state stochastic process:- " " Infinite

Ex:- ① The waiting time $W(t)$ may be Infinite.
② Dissolving of Sugar Crystals in Coffee. Continuous

⑤ fluctuations in air temperature & air pressure --- etc

Explain classification of Random Processes? L-64



see prev pages

③ Stationary Stochastic process (or) SSS (or) Strict Sense Stationary

Evolutionary Stochastic process:- (or) Non Stationary S.P

If the joint distribution of X_{t_1} & X_{t_2} depends only on $t_1 - t_2$. Then, the process is called Stationary.

other Evolutionary (or) Non-Stationary

④ Gaussian (or) Normal stochastic Process:-

If the joint distribution of any Number of Variables X_{t_1}, X_{t_2} ..... is a Normal distribution. Then, it is called Gaussian (or) Normal stochastic process.

Note:- Simulation Model

① ② ③ ④ - depending on time

⑤ & ⑥ = depending Sample functions.

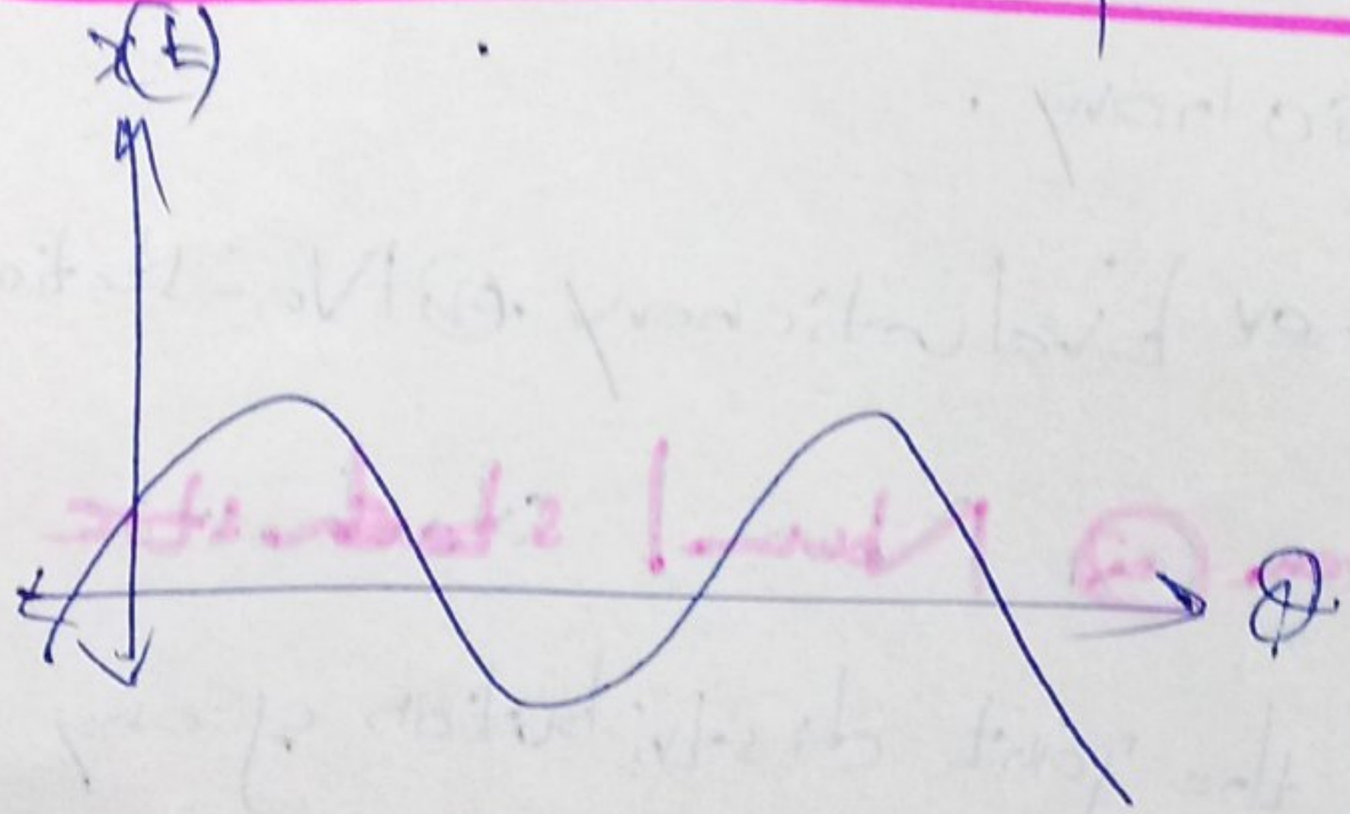
Queueing Model

Stochastic Model

5 Deterministic stochastic process

Deterministic Random process :-

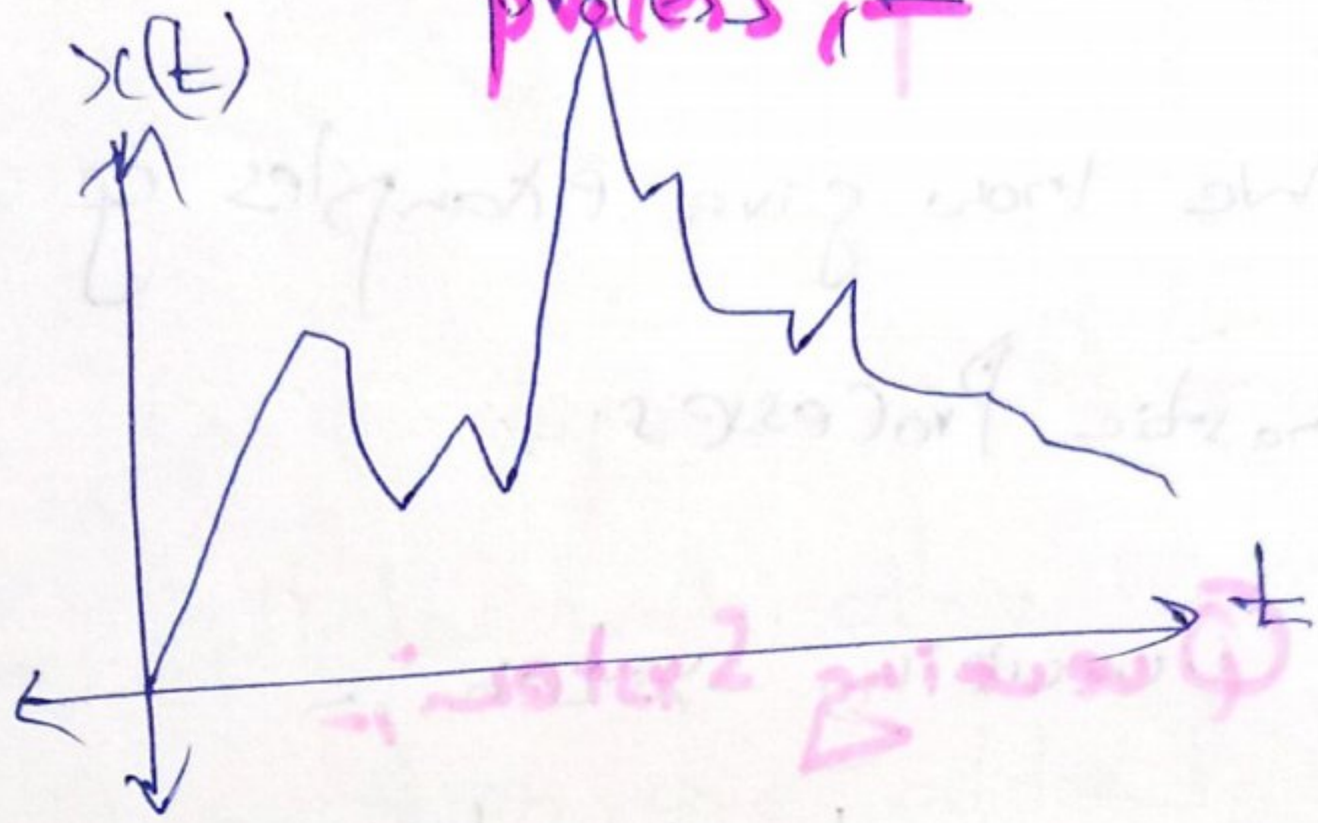
If the future values of any sample function can be predicted from a knowledge of the past values, then the Random process is called Deterministic Random process.



Ex:- $X(t) = A\cos(\omega t + \theta)$ → It consisting of a family of pure sine waves and it is completely specified in terms of the Random Variables A and θ.

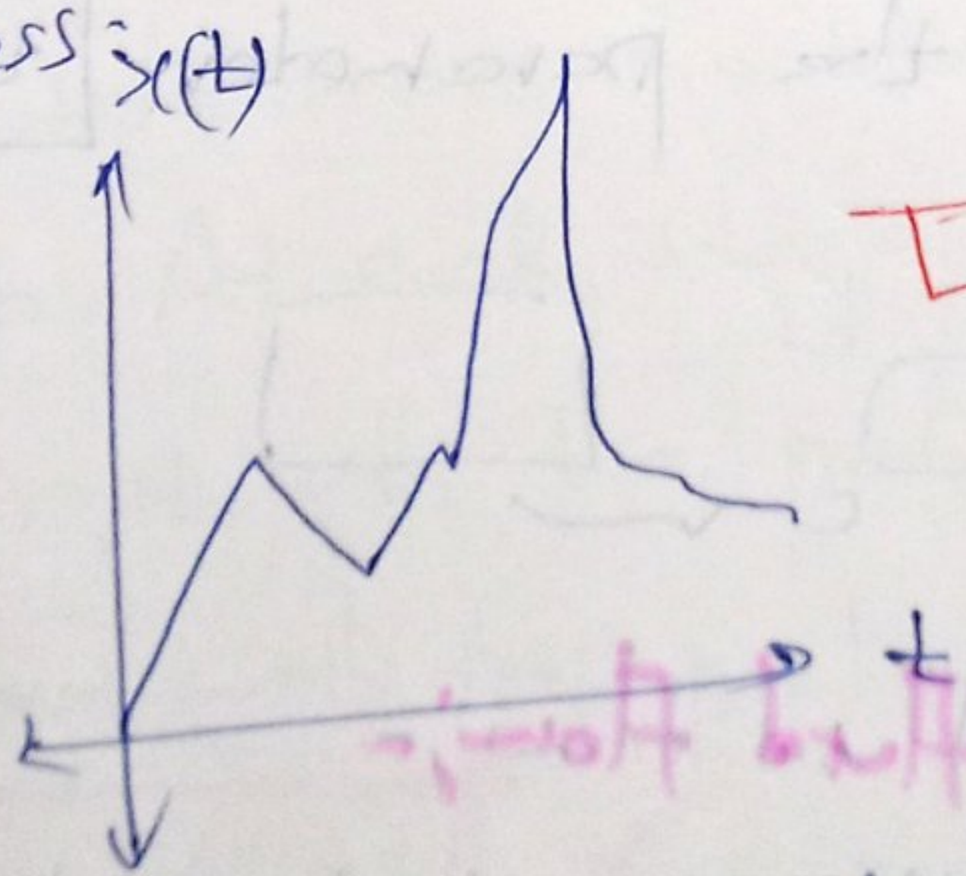
Separation of Salt & Rice which is predicable.

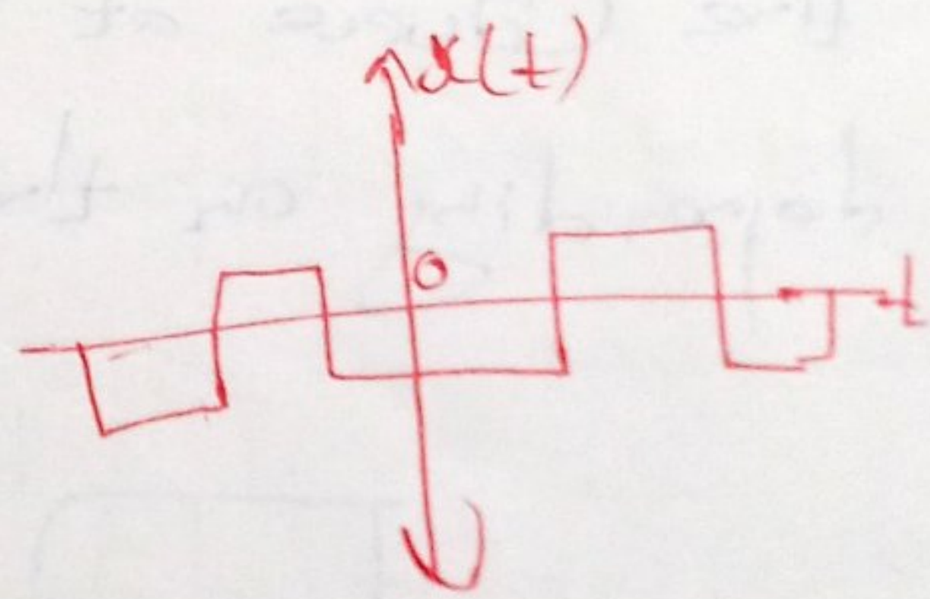
⑥ Non-Deterministic Random process :-



. If the future values can't be predicted from the knowledge of the past values, The Random process is called Non-deterministic

Random process :-





Ex:- Dissolving of Sugar Crystals in Coffee.

Note:- ① If any two stochastic process Equal.

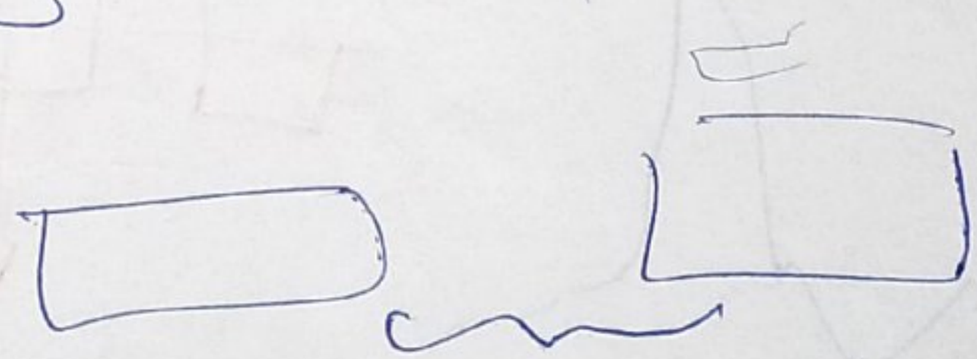
$$E\{|X(t)-Y(t)|^2\}=0 \text{ for every } t.$$

Explain Examples in Stochastic Process

Ans:- We now give examples of some stochastic Processes.

① A Queueing System:-

In this System, the Number of persons joining the Queue, the No. of persons being serviced in a time interval $\boxed{t}$ and the No. of persons in the Queue at time $\boxed{t}$ are all Random Variable depending on the parameter $\boxed{t}$.



② Turbulent fluid flow:-

In the turbulent fluid flow, the velocity Components u, v, w of the fluid are R.Vs depending on the space Co-ordinates x, y, z and time $\boxed{t}$.

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③ Movement of molecules of a gas (or) Liquid :-

In a gas (or) Liquid at Random instants the molecule collides with other molecules. Thus, its velocity and position are altered. Thus, the state of the molecule is subjected to Random changes at every instant of time.

④ A Random walk Model :-

A particle moves in a straight Line in steps of unit length. At each stage it can move one step to the right with the probability $\boxed{p}$ (or) one step to the left with the probability q.

$\boxed{\text{where } p+q=1}$. $\left(\text{ie } p=q=\frac{1}{2}\right)$

If the particles starts from Origin its position after n-moments is a R.V, which depends on discrete parameter $\boxed{n}$.

$p > \frac{1}{2}$ ⇒ drift towards the right. ($q > \frac{1}{2}$ ⇒ drift towards the left)

If walk = unrestricted ⇒ It goes to any point on the Line

5) Communication Process :-

The amplitude of the signals to be transmitted and amplitude of the noise power ced in the channel depending on time are both R.V's

6) Gamblers Ruin Problem (or) classical Ruin Problem :-

Suppose a Gambler wins (or) looses a unit (Rupee (or) dollar (or) pound) with probabilities p & q.

Let his initial capital is z & his opponents initial capital is $(a - z)$. So that the combined capital is $\boxed{a}$. The game continues until the gamblers capital is either reduced to zero (or) increased to a. ie until

one of the two players is ruined. We are interested in the probability of the gambler's ruin and the probability distribution of the duration of the game.

ie We can understand that the capital of the gambler after n-stages is a R.V.

ruin = दिवालिया

Hence probability that game never ends is $\underline{0}$.

Gambler's Ruin with Correlation

p_1, p_2 = winning trail probabilities

q_1, q_2 = Loosing " "

$+1$ = gains, -1 = Loosing

$x=1, y=1$ with probability $\frac{1}{2}p$

$x=1, y=-1$ " " $\frac{1}{2}q$

$x=-1, y=-1$ " " $\frac{1}{2}p$

$x=-1, y=1$ " " $\frac{1}{2}q$.

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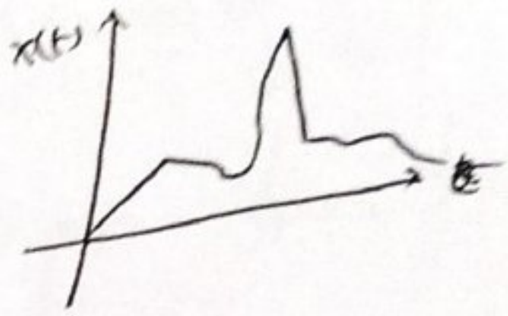
. Correlation Coefficient is R_{xy} is

$$R_{xy} = \frac{P_1 - q_2}{\sqrt{(P_1+q_2)(P_2+q_1)}}$$

If $R_{xy} = 0 \Rightarrow$ trials are Independent

deterministic Random process:-

If the future values can't be predicted from the knowledge of the past values, the random process is called non-deterministic random process.



Eg:- Dissolving of sugar crystals in coffee.

Statistics of Random process:-

Distrubution and density functions.

Given time t_1, the distribution function associated with a random process x, is given by

$$F_x(x_1, t_1) = P\{x(t_1) \le x_1\}$$

$\Rightarrow$ Distrubution function = probability of the event $\{x(t_1) < x\}$ consisting of all out comes λ_i such that, at the specific time t_1, the samples $x(t, \lambda_i)$ of the given process do not exceed the number x_1. This is called as first order distribution function.

First order density function

$$f_x(x_1, t_1) = \frac{d F_x(x_1, t_1)}{dx}$$

Second order joint distribution and density functions

$$F_x(x_1, x_2, t_1, t_2) = P\{x(t_1) \le x_1, x(t_2) \le x_2\}$$

$$f_x(x_1, x_2; t_1, t_2) = \frac{\partial^2 F_x(x_1, x_2; t_1, t_2)}{\partial x_1 \partial x_2}$$

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Write Short Notes on Markov Process?

(4)

Which is a special type of stochastic process called a Markov Process. If the future state of a process depends only on the present and is independent of the past. Then the process is called a Markov process.

In Markov process, next state can be analyzed based on the present state. A discrete state Markov process is called a Markov chain.

Ex:- A sequence of repeated trials of an experiment in which the outcomes at any step in the sequence depends at most on the outcomes of the preceding step and not on any other previous outcome. Such sequence is called a Markov chain (or) Markov process.

Types:- There are 2 types.

(1) Discrete — Time Markov chains

(2) Continuous — Time Markov chains

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Ex:-

① A man either eats idly or Vada each day. He never eats Idly for continuous 2 days, but if he eats Vada - Then, the next day he is just as likely to eat it again as he is to eat idly.

② A man goes to office by either Bus or Taxi. If he goes one day by Bus he is 70% sure to go by Taxi next day, but if he goes by Taxi one day he is 60% sure to go by Taxi Next day.

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③ Explain the classification of states in the Markov process?

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⑨

In Markov process, the states are classified in to 3 states. "for to find the Communicating classes."

ie Two states Communicate $\overset{\text{if \& only if}}{\Leftrightarrow}$ it is possible to go from each to other.

ie states A & B Communicate $\Leftrightarrow$ if it is possible to go from A to B & B to A.

There are 3 classifications of states

① Transient

② Periodic

③ Ergodic

① Transient:- A state is said to be transient, if it is possible to leave the state & never return back.

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(2) Periodic :-

A state is said to be periodic, if it is not transient & that state is returned to only on multiples of some positive integer > 1.

This integer is known as the period of the state

(3) Ergodic :-

A state is said to be ergodic if it is neither transient nor periodic.

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(N)

Markov process

occurence of an event at a specified time period - n

Current state E_n after a passage of unit time another event (period - n+1)

The system moved $E_n \xrightarrow{to} E_{n+1}$

* The probability of moving from one state to another (or) remaining in the same state in the single time period is called a one stage transition probability.

transition probability P_{ij}

state transition Matrix.

A state - transition matrix is a square matrix that summarizes the transition probabilities for a given Markov process. In such a matrix the row identify the current state of the system being studied and columns identify the alternatives states to which the system can move. Here P_{ij} denotes the transition probability of moving from state E_i to state E_j in one one step.

$$
\begin{array}{c}
 \\ E_1 \\ E_2 \\ \vdots \\ E_j \\ \vdots \\ E_m
\end{array}
\begin{array}{c}
\begin{array}{ccccc} E_1 & E_2 & \cdots E_j & \cdots & E_m \end{array} \\
\begin{bmatrix}
P_{11} & P_{12} & \cdots P_{1j} & \cdots & P_{1m} \\
P_{21} & P_{22} & \cdots P_{2j} & \cdots & P_{2m} \\
\vdots & & & & \\
P_{j1} & P_{j2} & \cdots P_{jj} & \cdots & P_{jm} \\
\vdots & & & & \\
P_{m1} & P_{m2} & \cdots P_{mj} & \cdots & P_{mm}
\end{bmatrix}
\end{array}
$$

$0 \le P_{ij} < 1$

$\sum_{j=1}^{m} P_{ij} = 1$

$i, j = 1, 2 \cdots m$

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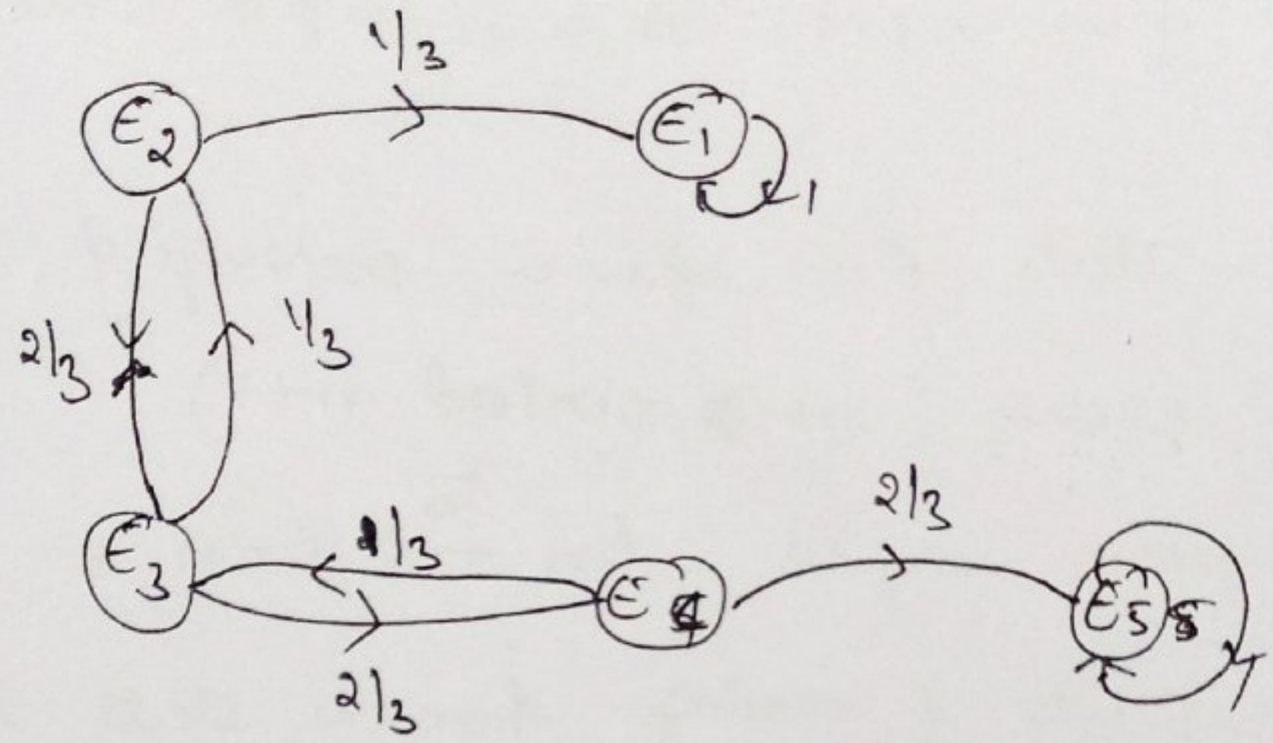

$$
P : \begin{array}{c|cccc}
 & 0 & 1 & 2 & 3 \\
\hline
0 & P_{00} & P_{01} & P_{02} & P_{03} \\
1 & P_{10} & P_{11} & P_{12} & P_{13} \\
2 & P_{20} & P_{21} & P_{22} & P_{23} \\
3 & P_{30} & P_{31} & P_{32} & P_{33}
\end{array}
$$

Markov process

A random process $x(t)$ is called a Markov process if

$$P\{x(t_{n+1}) \le x_{n+1} \mid x(t_n) = x_n, x(t_{n-1}) = x_{n-1}, \dots, x(t_1) = x_1, x(t_0) = x_0\}.$$

$$= P\{x(t_{n+1}) \le x_{n+1} \mid x(t_n) = x_n\}.$$

for all $t_0 < t_1 < t_2 \dots < t_n < t_{n+1}$ i.e $x(t_{n+1})$ is a Markov process if the past values of the sequence do not have any. If the future behaviour $x(t_{n+1})$ of the process depends only on the present value $x(t_n) = x_n$; but not on the past values $x_{n-1}, x_{n-2} \dots x_0$. then the process is said to be a markov process. The values $x_0, x_1, \dots x_n, x_{n+1}$ are called the states of the process.

Markov process can be classified as follows.

(i) Continuous parameter Markov process - t_i and $x(t_i)$ both are continuous

(ii) Continuous parameter Markov Chain - t_i continuous & $x(t_i)$ discrete.

(iii) Discrete parameter Markov process - t_i discrete & $x(t_i)$ continuous

(iv) Discrete parameter Markov chain - t_i discrete & $x(t_i)$ discrete.

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Markov chain

A Markov process $x(t)$ taking only discrete values $\{x_n\}$ for continuous or discrete t is called a Markov chain.

A sequence of states $\{x_n\}$ is defined as a Markov chain if each x_n is a random variable and if

$$P\{x_{n+1} = a_{n+1} \mid x_n = a_n, x_{n-1} = a_{n-1} \cdots x_1 = a_1, x_0 = a_0\}.$$

$$= P\{x_{n+1} = a_{n+1} \mid x_n = a_n\}.$$

where $a_1, a_2 \cdots a_n$ are called the states of the Markov chain.

Transition probabilities:-

The joint probability of a Markov chain is given by.

$$P\{x_{n+1} = a_{n+1}, x_n = a_n \cdots x_0 = a_0\} = P\{x_{n+1} = a_{n+1} \mid x_n = a_n\}$$

$$\times P\{x_n = a_n \mid x_{n-1} = a_{n-1}\} \cdots P\{x_1 = a_1 \mid x_0 = a_0\}.$$

$$P\{x_0 = a_0\}$$

$$= P\{x_0 = a_0\} \prod_{i=1}^{n+1} P\{x_i = x_i \mid x_{i-1} = x_{i-1}\}.$$

i.e The joint probability is the product of the conditional probabilities and the initial probability. These conditional probabilities are called a single-step transition probabilities.

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(4.) Explain the Markov chains with Examples.

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Markov Chain:-

Consider a system, which can be in any one of finite No of states $E_1, E_2 \ldots E_n$ also assume that the probability of the system being in a given state at the next trial depends only on its present state & not upon the states it may have been in earlier times.

If at any time, the system is in state E_i, the probability of it being in the state E_j at the next trials is $\boxed{P_{ij}}$.

This process is called Markov chain.

This property of the process is called Markov property.

General Equation:-

$$\boxed{P\{(E_i, E_j, E_k, E_v)\} = a_i P_{ij} P_{jk} P_{kv}}$$

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Definition:-

A sequence of trials with possible outcomes $E_1, E_2 \ldots$ is called a Markov chain if the probabilities of a sample sequence are defined by

$$P\{(E_{i_0}, E_{i_1} \ldots E_{i_n})\} = a_{i_0} P_{i_0 i_1} \cdots P_{i_{n-1} i_n} \ldots$$

in terms of a probability distribution $\{a_i\}$ for E_i at the initial (or zeroth) trial & fixed conditional probabilities $\boxed{P_{ij}}$ of E_j given that E_i has occurred at the preceding trial.

The possible outcomes $\boxed{E_j}$ are usually referred to as possible states of the system.

$\boxed{P_{ij}}$ is called the probability of a transition from E_i to E_j.

Problems

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(1) Show that the non-Zero Vector $e = [2, 5, 0, 1]$ is the probability Vector?

$e = [2, 5, 0, 1]$ is not a probability Vector.

For any Vector to be a probability Vector, it should follow two Conditions

① All entries should be non-negative $(e_i \geq 0)$

② Addition of all entries should be One. $e_1 + e_2 + \dots = 1$

① $2 \geq 0$, $5 \geq 0$, $0 = 0$, $1 \geq 0$

② $2 + 5 + 0 + 1 = 8 \neq 1$

$\therefore$ It satisfies only ①

Does not Satisfies ②

$\therefore$ The given Vector is $e = [2, 5, 0, 1]$ is not a probability Vector.

(2) In the previous example for the non-Zero Vector $e = [2, 5, 0, 1]$ find probability Vector?

$e = [2, 5, 0, 1]$

There is a Unique probability Vector $= p$

p is a Scalar Multiple of e.

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∴ The probability Vector [P], can be obtained by multiplying e. (i.e by the reciprocal of the addition of its entries].

$$P = \frac{1}{8} e$$

$$= \frac{1}{8}\left[2, 5, 0, 1\right]$$

$$\boxed{P = \left[\frac{2}{8}, \frac{5}{8}, \frac{0}{8}, \frac{1}{8}\right]}$$

(7) Check the following stochastic matrix is regular or not?

$$A = \begin{bmatrix} 1 & 0 \\ \frac{7}{8} & \frac{1}{8} \end{bmatrix}$$

Sol:

We know that, if we have to prove stochastic matrix is regular.

i.e we have to show all the entries (or) elements of p^k should be positive.

$$A^2 = \begin{bmatrix} 1 & 0 \\ \frac{63}{64} & \frac{1}{64} \end{bmatrix} \longrightarrow \text{cofac.}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ \frac{4095}{4096} & \frac{1}{4096} \end{bmatrix} \longrightarrow \text{cofac size.}$$

∴ A is not regular matrix

0, factorial 8!

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(4) Suppose $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Square stochastic matrix & $e = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is a probability Vector then show that eA is also a probability Vector?

$$eA = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

(1) $\frac{1}{4} \geq 0 \quad \frac{3}{4} \geq 0$

(2) $\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

$\therefore$ (1) & (2) Conditions are satisfied.

$$\boxed{\therefore eA \text{ is a probability Vector}}$$

(5) Determine which of the following stochastic matrices are regular (a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

(a) $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

0, 1 alternate

$\therefore$ A is not Regular

(b) $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

$A^2 = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 1 & 0 \\ \frac{3}{8} & \frac{9}{16} & \frac{1}{16} \end{bmatrix}$

0, 1 alternate

$\therefore$ A is not regular

(6) Given $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 1 & 3 & -2 \end{bmatrix}$ find UA

$$UA = \begin{bmatrix} 1 & 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 3 \end{bmatrix}$$

(7) find the unique fixed probability vector S of each matrix (a) $A = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$ $B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

(a) $S = \begin{bmatrix} x & y \end{bmatrix}$

$\boxed{S = SA} \Rightarrow \cancel{\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix}}$ $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$

$\cancel{\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0.3x + 0.5y & 0.4x + 0.5y \end{bmatrix}}$ $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0.3x + 0.5y & 0.4x + 0.5y \end{bmatrix}$

$\cancel{x = 0.3x + 0.5y}$

$\cancel{y = 0.4x + 0.5y}$

$\cancel{y = -0.1x}$

use calsi mode setup

(1)

$x = 0$

$y = 2$

$\cancel{x = 0.3x + 0.5y}$

$\cancel{= 0.3x + 0.5(-0.1x)}$

$\cancel{x = 0.25x}$

$\therefore y = 0.4(0.25) + 0.5y$

$S = \begin{bmatrix} x & y \end{bmatrix}$

$= \begin{bmatrix} & -0. \end{bmatrix}$

8 check the following stochastic matrix is regular or not?

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

L-70 (2)

$$A^2 = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{16}{75} & \frac{37}{75} & \frac{22}{75} \\ \frac{34}{135} & \frac{26}{45} & \frac{33}{135} \end{bmatrix}$$

In this No 0, 1

$\therefore$ $\boxed{A \text{ is a Regular}}$

9 Find the unique fixed probability vector S of the following regular stochastic Matrix?

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$\Rightarrow S = SP$

$$\rightarrow [x \; y \; z] = [x \; y \; z] \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x &= \tfrac{1}{5}y \\ y &= x + \tfrac{2}{5}y + \tfrac{1}{2}z \\ z &= \tfrac{2}{5}y + \tfrac{z}{2} \end{aligned}\right\} \text{solv.}$$

by solving $x = 0$, $y = 5$, $z = -2$

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$x = \frac{7}{5}$

$y = \frac{5}{4} z$

- If we have to solve non-zero solution.

$\boxed{x = 1}$

$\therefore y = 5$

$z = 4$

$\therefore S = [x \quad y \quad z] = [1 \quad 5 \quad 4]$

(10) Suppose $e_0 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is the initial state distribution for a Markov process with the following transition Matrix? $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$

find e_1 e_2 e_3 ?

$$\boxed{\begin{array}{c} \underline{\text{formulas}} \\ e_1 = e_0 P \\ e_2 = e_1 P \\ e_3 = e_2 P \end{array}}$$

$e_1 = e_0 P$

$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix}$

$e_2 = e_1 P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix}$

$e_3 = e_2 P = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{11}{16} & \frac{5}{16} \end{bmatrix}$

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Ex-3:- Random Walks Revisited:

In this state space is genuinely infinite. To make it easier to write down P - Lets impose the initial condition:

$$P_{01} = 1 \quad \text{for } i \geq 0.$$

The only non-zero transition probabilities are

$$P_{i,i+1} = p \;, \quad P_{i,i-1} = 1-p$$

Transition Matrix:-

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 1-p & 0 & p & 0 & \cdots \\ 0 & 1-p & 0 & p & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & q & 0 & q-1 \\ 0 & q & 0 & q-1 & 0 \\ q & 0 & q-1 & 0 & 0 \\ 1 & q-1 & 0 & 0 & 0 \end{bmatrix}$$

Ex-4:- A Gambling Model:-

Consider a gambler wins \$1 with probability p & loses with probability $(1-p)$ as long as he plays.

Consider he gives up either when he goes broke & attains a fortune of \$N.

We can Model this as a Markov chain with state space $S = \{0, 1, \ldots, N\}$.

Transition probabilities $\boxed{P_{00} = P_{NN} = 1}$

$$\boxed{P_{i,i+1} = p, \quad P_{i,i-1} = 1-p}$$

This is same as prev ex

This is finite state Random walk.

States 0 & N are called absorbing as once you enter there, you can never leave.

When $N=4$. $\therefore$ Transition Matrix

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1-p & 0 & p & 0 & 0 \\ 0 & 1-p & 0 & p & 0 \\ 0 & 0 & 1-p & 0 & p \\ 0 & 0 & 0 & 1-p & 1 \end{bmatrix}$$

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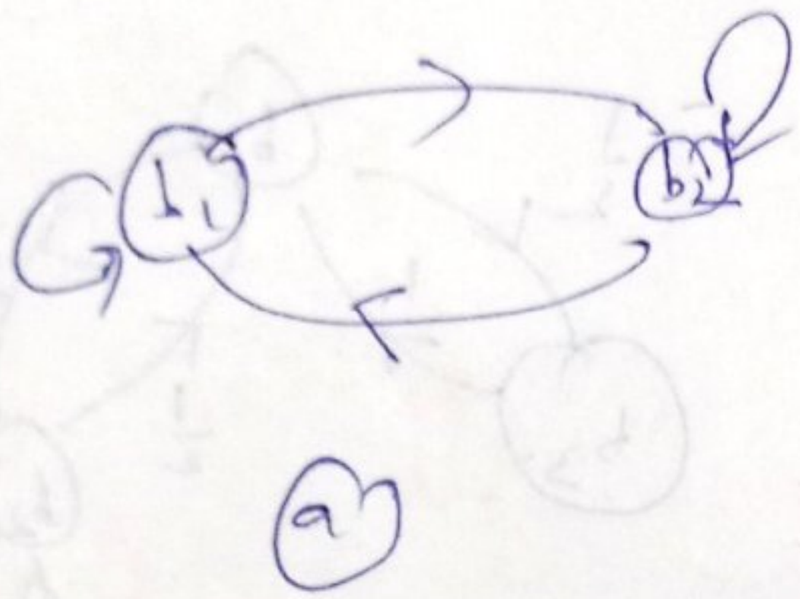
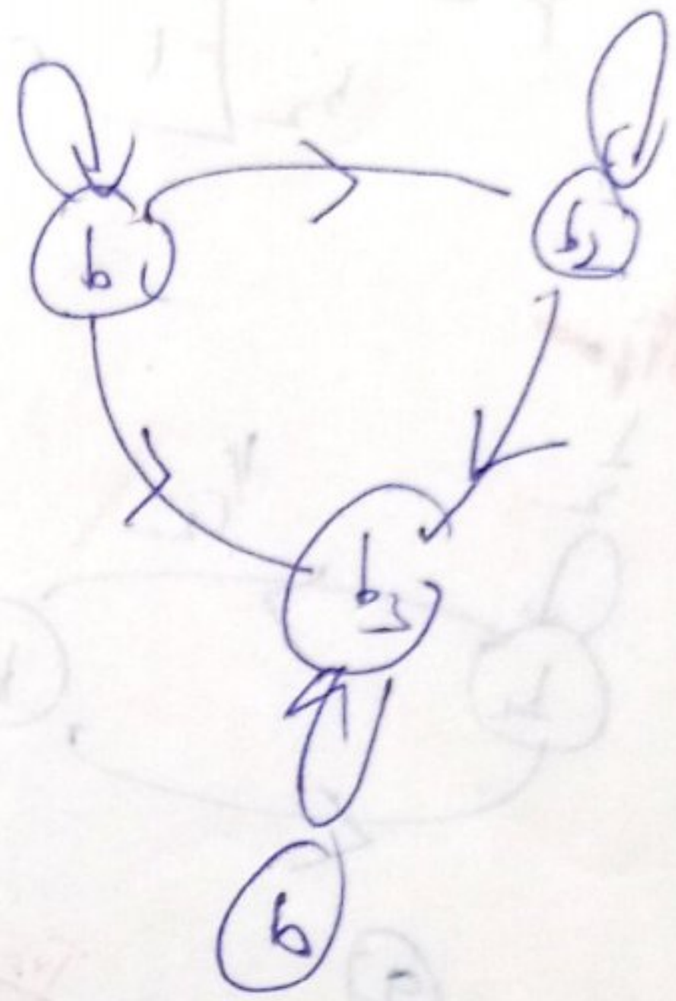
Problems

① Find the Transition Matrix Corresponding to each transition Diagram which are gives below?





Sol

$$P = \begin{array}{c|cc} & b_1 & b_2 \\ \hline b_1 & \frac{1}{2} & \frac{1}{2} \\ b_2 & \frac{1}{2} & \frac{1}{2} \end{array}$$

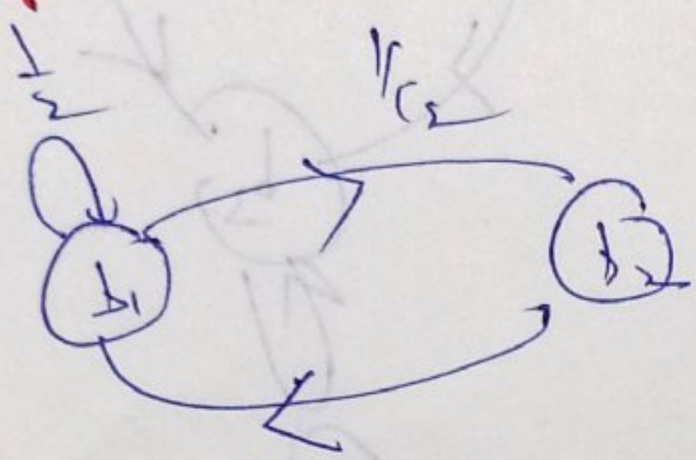
$$P = \begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline b_1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ b_2 & 0 & \frac{1}{2} & \frac{1}{2} \\ b_3 & 0 & 0 & 1 \end{array}$$

② Draw a transition diagram for each transition matrix which are given below

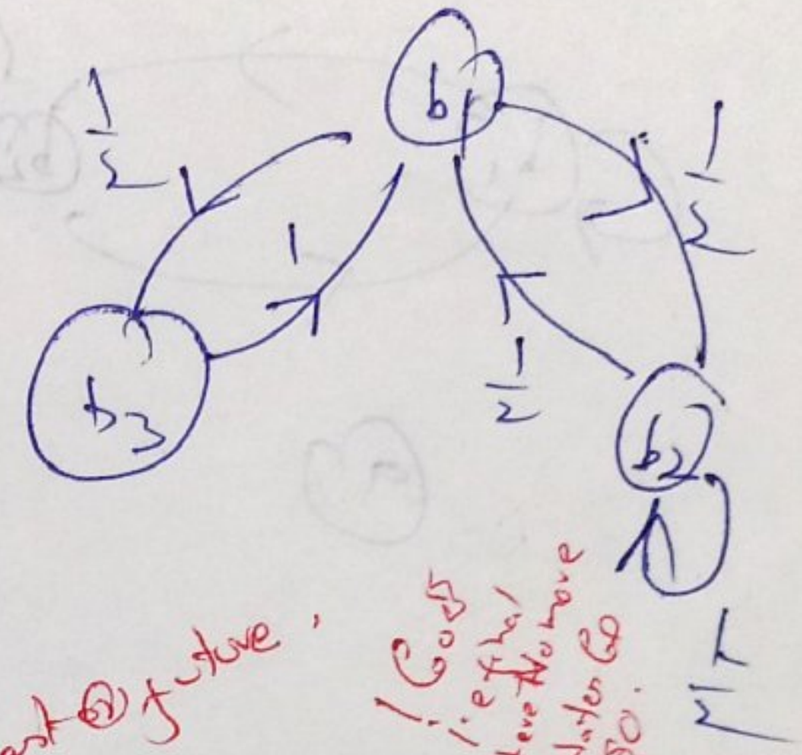
(a) $$P = \begin{array}{c|cc} & b_1 & b_2 \\ \hline b_1 & \frac{1}{2} & \frac{1}{2} \\ b_2 & 1 & 0 \end{array}$$

(b) $$P = \begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline b_1 & 0 & \frac{1}{2} & \frac{1}{2} \\ b_2 & \frac{1}{2} & \frac{1}{2} & 0 \\ b_3 & 1 & 0 & 0 \end{array}$$

Solⁿ



(a)



Transition Matrix
Only present state
Concentrate
not the past & future.

! Goes
i.e. that
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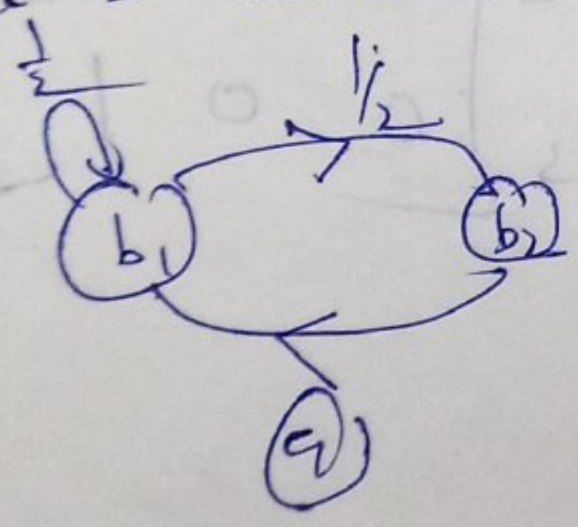
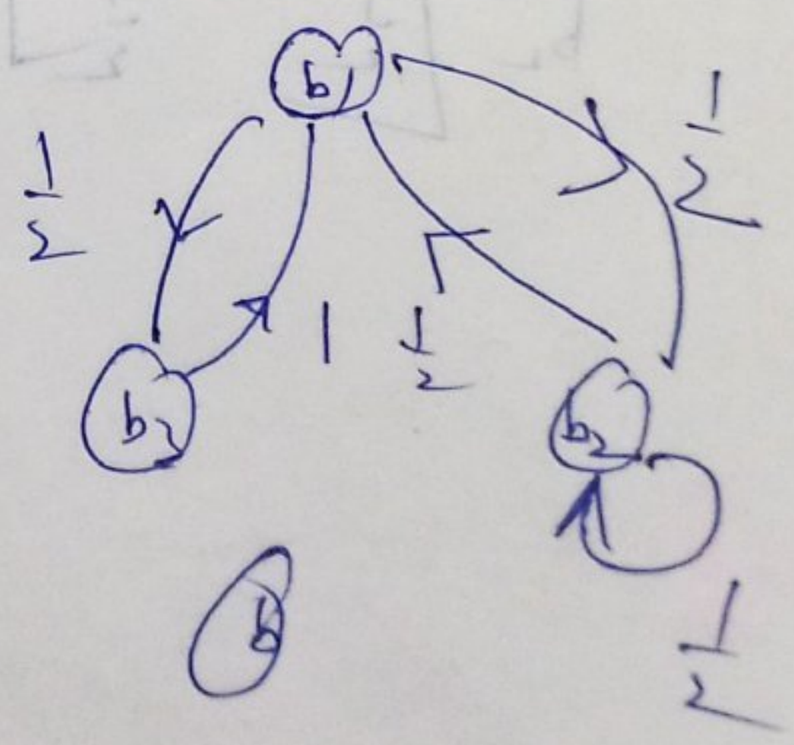
States to
Go.

③ Find the transition matrix corresponding to transition Diagram which is given below





Solⁿ

(a) $$P = \begin{array}{c|cc} & b_1 & b_2 \\ \hline b_1 & \frac{1}{2} & \frac{1}{2} \\ b_2 & 1 & 0 \end{array}$$

(b) $$P = \begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline b_1 & 0 & \frac{1}{2} & \frac{1}{2} \\ b_2 & \frac{1}{2} & \frac{1}{2} & 0 \\ b_3 & 1 & 0 & 0 \end{array}$$

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5) Explain Transition Diagram ? (10)

L-67

Two transition diagrams are shown in figures. It shows the transition probabilities. The states are Vertices in the diagram & probability $\boxed{P_{ij}}$ is denoted by arrow (edge) from State $\boxed{b_i}$ to the state $\boxed{b_j}$.

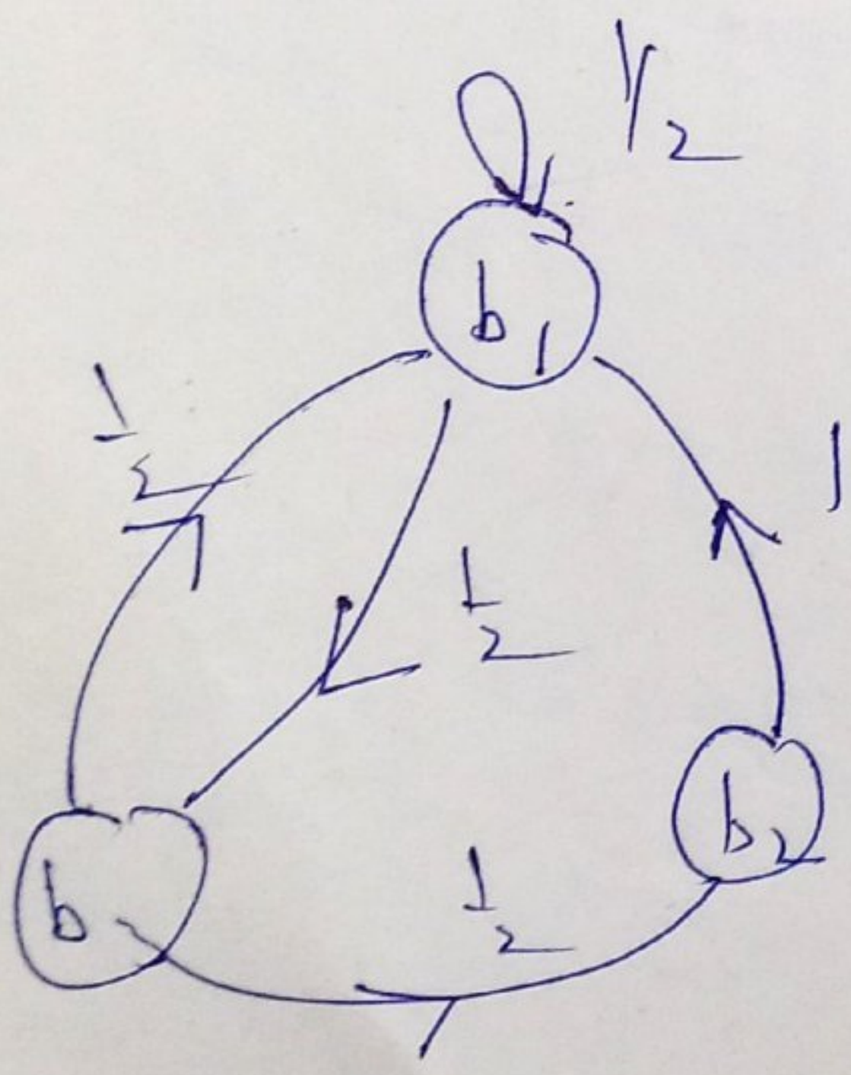

Diagram - a

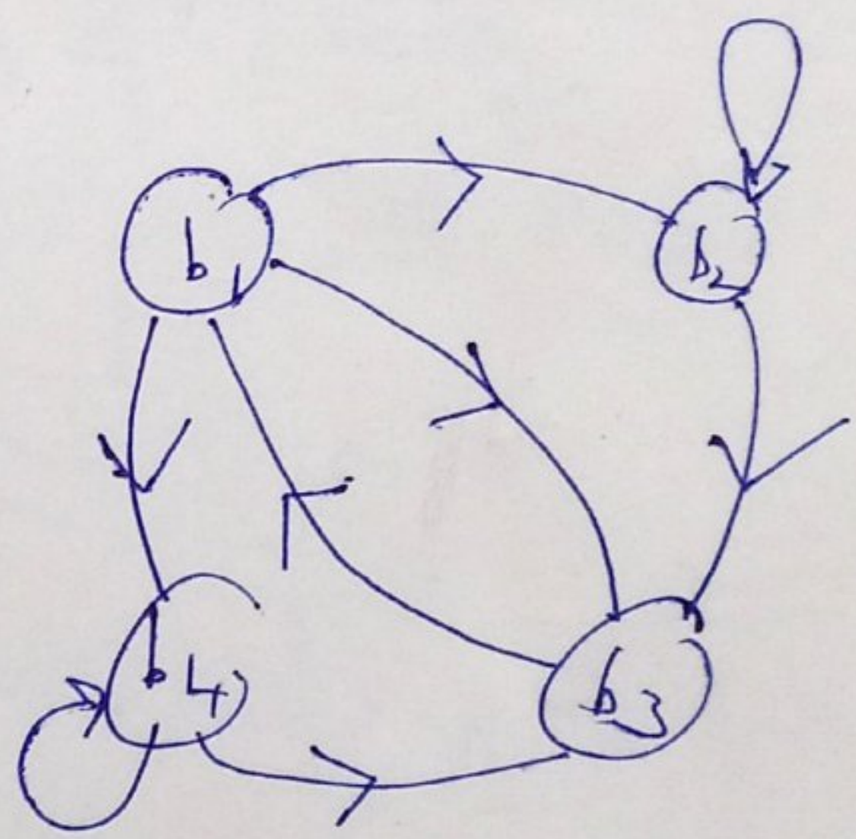

Diagram - b.

Transition matrix - a_i -

$$P = \begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \end{array} \begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \end{array}$$

Transition matrix - b_i

$$P = \begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array}$$

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4 - Explain the Markov chains? with at least Three Examples? L-66 3

Markov chain:-

A stochastic process $X(n)$ where $n = 0, 1, \ldots$ is said to be Markov chain if

$$\boxed{P(X(n+1) = k_{n+1} \mid X(n) = k_n) \quad \forall\, k_0, k_1, \ldots k_{n+1} \in S \;\&\; n \in N}$$

Example:- Transition probability Matrix:-

All the transition probabilities can be collected into matrix $\boxed{P = P_{ij}}$ is calle TPM

$$P = \begin{bmatrix} P_{00} & P_{01} & P_{02} & \cdots \\ P_{10} & P_{11} & P_{12} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ P_{i0} & P_{i1} & P_{i2} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

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Ex-2:- A Communication channel:-

Starting at time Zero,

A binary symbol (0 & 1) is transmitted every second across a simple Communication channel to a receiver.

The probability of successful transmission (ie $0 \to 0$ & $1 \to 1$) is $(1-p)$,

while probability of error ($0 \to 1$ & $1 \to 0$) is p - where $\boxed{0 \le p \le 1}$.

We can model this discrete time Markov chain with state space $S = \{0, 1\}$.

$\therefore$ Transition Matrix $P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}$

$\therefore P = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$

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(6) Probability Vector:-

L-68 (8)

A vector $e = \{e_1, e_2 \cdots e_n\}$ is called a probability vector, if its all elements are positive and their sum is one.

$$\boxed{\begin{array}{l} (1)\ \text{Each } e_i \geq 0 \\ (2)\ e_1 + e_2 + \cdots + e_n = 1 \end{array}}$$

Stochastic Matrix:-

A square matrix $P = [p_{ij}]$ is called a stochastic matrix, if each row of P is a probability vector.

$$\begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \end{bmatrix} \begin{array}{l} \text{probability vector} \Rightarrow e_i \geq 0,\ e_1 + e_2 = 1 \\ \text{p.v} \\ \text{p.v} \end{array}$$

Regular Stochastic Matrix:-

A stochastic matrix $\boxed{P}$ is said to be regular, if all the elements of some power $\boxed{P^k}$ are positive.

(7) Explain Transition Matrix?

Transition Matrix:-

A Markov process (or) Markov chain consists of a sequence of repeated trials of an experiment with the property that

① "In the experiment of repeated trials each out come belongs to a finite set $\{b_1, b_2 \cdots b_n\}$ called the <u>state space of the system</u>."

② In the experiment of repeated trials, the out come of any future trial depends only on the outcome of the "present" [preceding] trial and not on any other previous (or) past trial.

For the given probability $\boxed{P_{ij}}$. If $\boxed{b_j}$ occurs immediately after $\boxed{b_i}$.

The following n-square Matrix

Transition Matrix:-

$$\begin{array}{c|c} & b_1 \quad b_2 \cdots b_n \\ \hline \begin{array}{c} b_1 \\ b_2 \\ \vdots \\ b_n \end{array} & \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} \end{array}$$

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This Matrix P is called transition Matrix

∴ " Transition Matrix is a rectangular array which consist the transition probabilities for a given Markov process.

In transition Matrix row = Current state of the system

Columns = alternative states ~~to~~ which the system can move.

objective bit

→ In Transition Matrix $\boxed{P_{ij} = 0}$

i.e

No transition occurs

i.e Transition is impossible

$\boxed{P_{ij} = 1}$ — when the system is moving from "i to j".

(11) A person A goes to Tirupati temple Padmavathi temple & Srikalahasti. A always goes first Tirupathi - Then Padmavathi temple. Then Srikalahasti. If A first goes to Srikalahasti, he is just as likely to go Tirupatis padmavathi temple. Find the transition matrix of this Markov process?

Tirupati temple = T

P T = P

S T = S

$$T = \begin{array}{c|ccc} & T & P & S \\ \hline T & 0 & 1 & 0 \\ P & 0 & 0 & 1 \\ S & \frac{1}{2} & \frac{1}{2} & 0 \end{array}$$

$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & \frac{2}{3} & 0 & \frac{2}{3} \\ C & \frac{1}{3} & \frac{1}{3} & 0 \end{array}$$

(12) A student tries to take admission in any three college A, B & C - first he goes to college A, second day to college B. Then third day to C - He never goes to same college in two continuous days.

But if he goes either B or C. Then, then next day, he is twice as likely to go to college A. Find out how often in the long run, he tries to each college?

Transition Matrix $P = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & \frac{2}{3} & 0 & \frac{1}{3} \\ C & \frac{2}{3} & \frac{1}{3} & 0 \end{array}$

fixed vector $S = [x \;\; y \;\; z]$

$S = SP$

$= (x, y, z) \begin{bmatrix} 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$

$$\Rightarrow \left.\begin{aligned} x &= \tfrac{2}{3}y + \tfrac{2}{3}z \\ y &= x + \tfrac{z}{3} \\ z &= \tfrac{y}{3} \end{aligned}\right\} \text{sly}$$

$\boxed{z = 1}$ → then solve Jabre equations

$y = 3$

$x = \frac{8}{3}$

$S = [x \;\; y \;\; z] = \left[\frac{8}{3} \;\; 3 \;\; 1\right]$

$\therefore$ Unique fixed probability Vector $= \frac{8}{3} + 3 + 1 = \frac{20}{3}$

$\therefore S = \left[\frac{8}{3} \Big/ \frac{20}{3} \;\; \frac{3}{\frac{20}{3}} \;\; \frac{1}{\frac{20}{3}}\right] = [0.40 \;\; 0.45 \;\; 0.15]$

$\therefore$ Asbab lies 40% of time in college A, 45% of time in B, 15% of time in college C.

(5) Explain Transition Diagram ?

16

Two transition diagrams are shown in figures. It shows the transition probability. The states are vertices in the diagram & probability $\boxed{P_{ij}}$ is denoted by arrow (edge) from state $\boxed{b_i}$ to the state $\boxed{b_j}$.

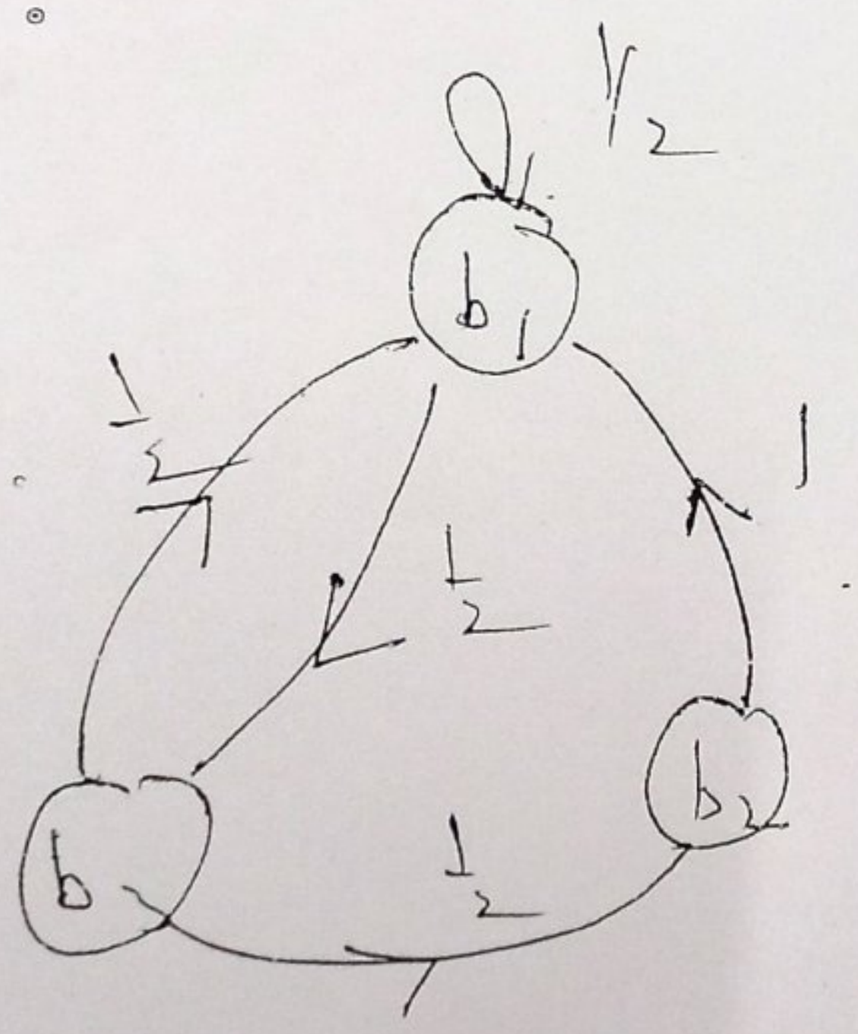
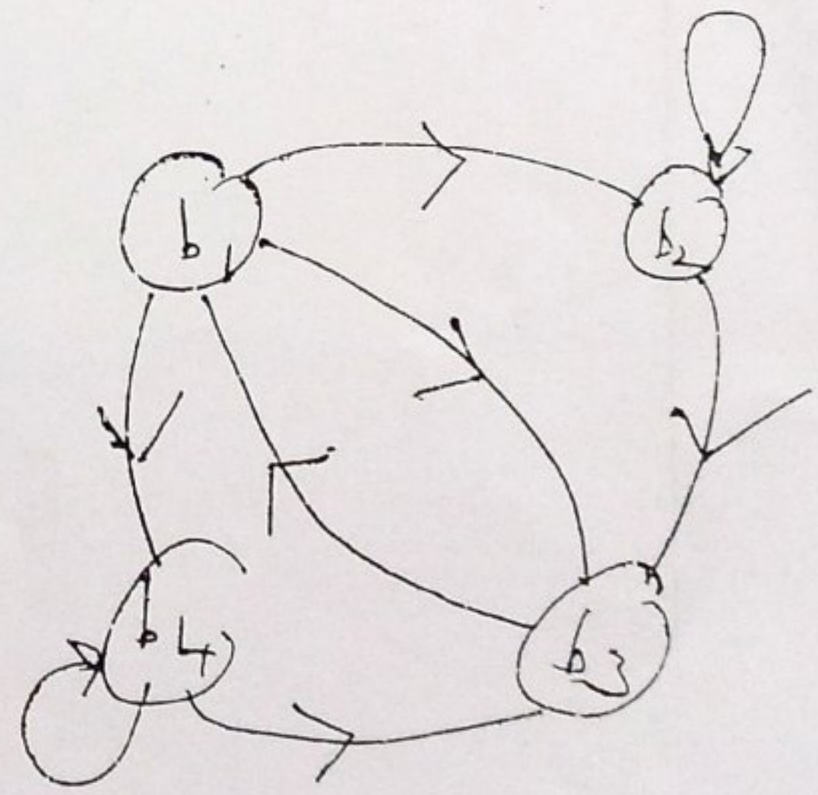


Diagram - a



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Transition matrix - b_i

$$P = \begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array}$$

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17

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We can model this discrete time Markov chain with state space $S = \{0, 1\}$.

$\therefore$ Transition Matrix $P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}$

$\therefore P = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$

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Ex-3:- Random Walks Revisited:

18

In this state space is genuine infinite. To make it easier to write down P - Lets impose the initial condition

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$\therefore$ Transition Matrix:-

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots \\ 1-p & 0 & p & 0 & \cdots \\ 0 & 1-p & 0 & p & \cdots \\ \vdots & \vdots & & \vdots & \vdots \end{pmatrix}$$

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Consider a gambler wins \$1 with probability p & loses with probability $(1-p)$ as long as he plays.

Consider he gives up either when he goes broke & attains a fortune of \$N.

We can Model this as a Markov chain with state space $S = \{0, 1 \ldots N\}$.

Transition probabilities $\boxed{P_{00} = P_{NN} = 1}$.

$$\boxed{P_{i,i+1} = p, \quad P_{i,i-1} = 1-p}$$

This is sum to prob.

This is finite state Random walk.

States 0 & N are called absorbing as once you enter there, you can never leave.

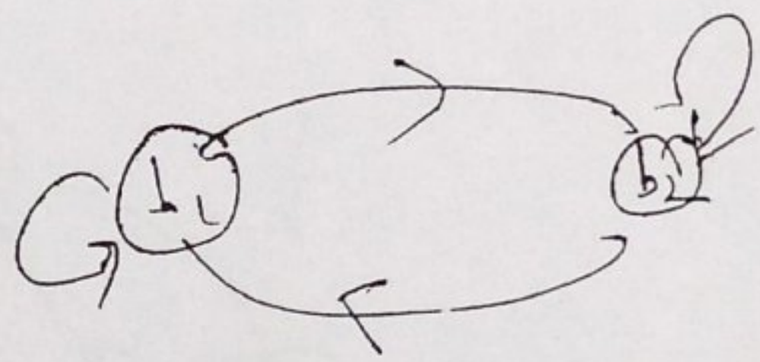
When $N=4$. $\therefore$ Transition Matrix:-

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1-p & 0 & p & 0 & 0 \\ 0 & 1-p & 0 & p & 0 \\ 0 & 0 & 1-p & 0 & p \\ 0 & 0 & 0 & 1-p & 1 \end{pmatrix}$$

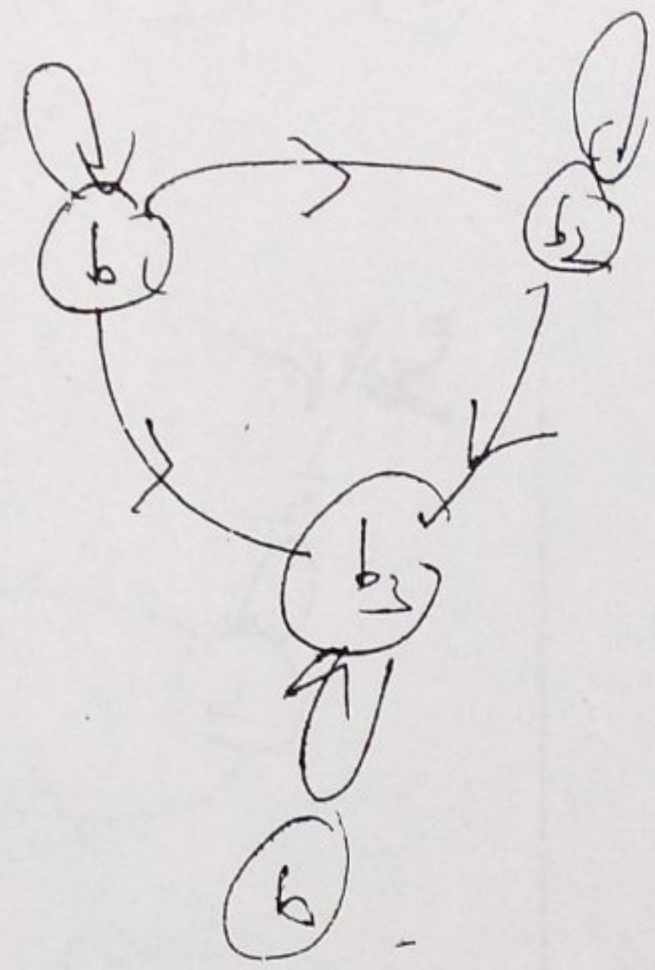
The Ideas

19

① Find the Transition Matrix Corresponding to each transition Diagram which are given below?



(a)



(b)

Ans

$$P = \begin{array}{c|cc} & b_1 & b_2 \\ \hline b_1 & \frac{1}{2} & \frac{1}{2} \\ b_2 & \frac{1}{2} & \frac{1}{2} \end{array}$$

$$P = \begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline b_1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ b_2 & 0 & \frac{1}{2} & \frac{1}{2} \\ b_3 & 0 & 0 & 1 \end{array}$$

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② Draw a transition diagram for each transition matrix which are given below

(a) $$P = \begin{array}{c|cc} & b_1 & b_2 \\ \hline b_1 & \frac{1}{2} & \frac{1}{2} \\ b_2 & 1 & 0 \end{array}$$

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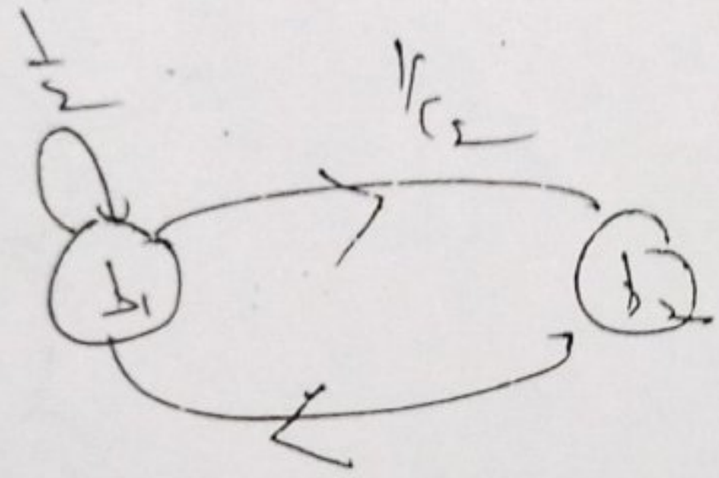
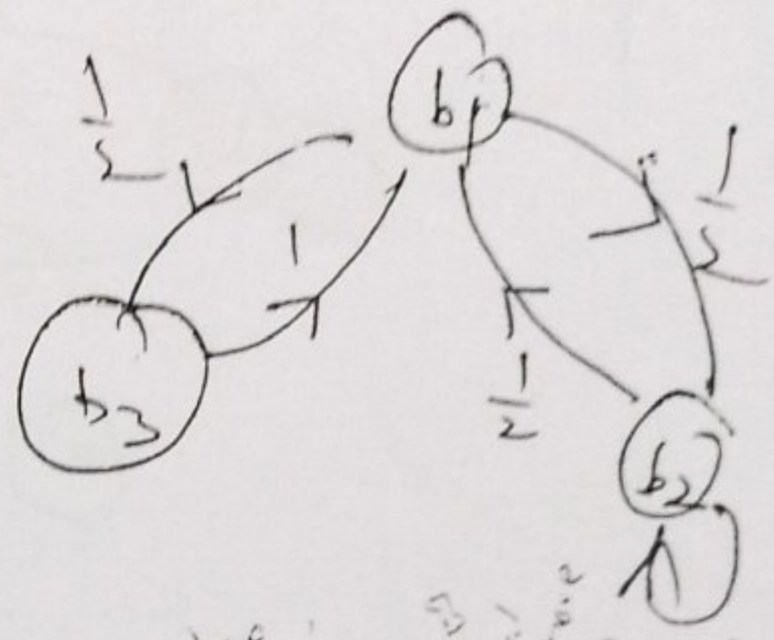
Solⁿ



(a)



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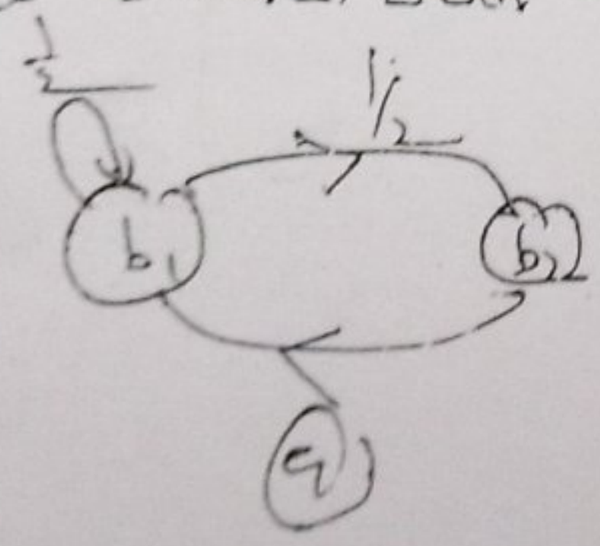
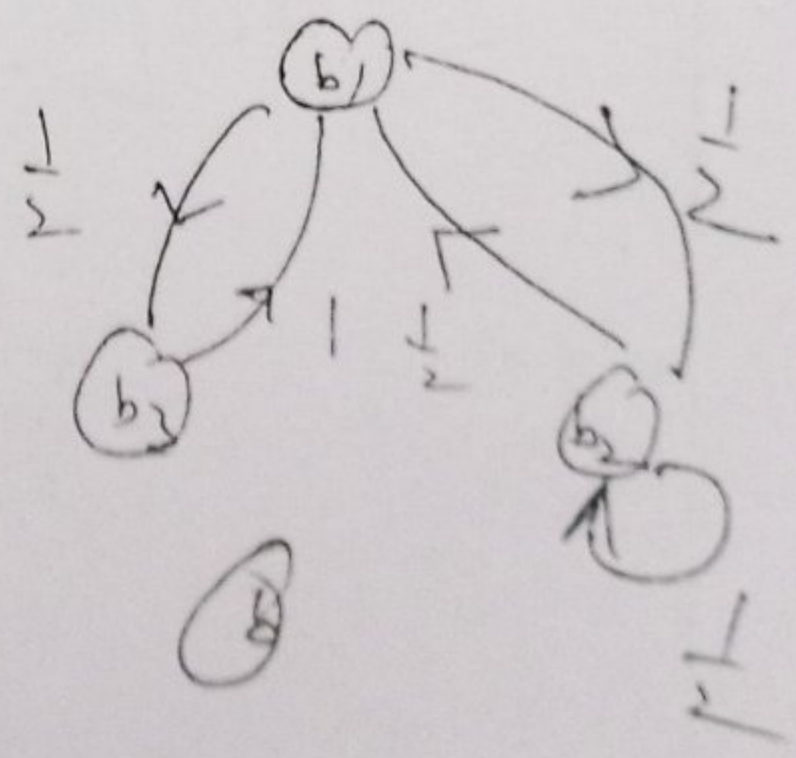
not the past & future

③ Find the transition matrix corresponding to transition Diagram which is given below





Solⁿ

(a)

$$P = \begin{array}{c|cc} & b_1 & b_2 \\ \hline b_1 & \frac{1}{2} & \frac{1}{2} \\ b_2 & 1 & 0 \end{array}$$

(b)

$$P = \begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline b_1 & 0 & \frac{1}{2} & \frac{1}{2} \\ b_2 & \frac{1}{2} & \frac{1}{2} & 0 \\ b_3 & 1 & 0 & 0 \end{array}$$

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(6) Probability Vector:-

A vector $e = \{e_1, e_2 \dots e_n\}$ is called probability vector, if its all elements are positive and their sum is one.

$$\boxed{\begin{array}{l} (1)\ \text{Each } e_i \geq 0 \\ (2)\ e_1 + e_2 + \dots + e_n = 1 \end{array}}$$

Stochastic Matrix:-

A square Matrix $P = [P_{ij}]$ is called Stochastic Matrix, if each row of P is a probability vector.

$$\begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \end{bmatrix} \begin{array}{l} \text{prob. vector} \\ P \checkmark \\ P \checkmark \end{array}$$

Regular stochastic Matrix:-

A stochastic Matrix $\boxed{P}$ is said to regular, if all the elements of some power $\boxed{P}$ are positive.

(7) Explain Transition Matrix?

Transition Matrix:-

A Markov process or Markov chain consists of a sequence of repeated trials of an experiment with the property that

① "In the experiment of repeated trials each out come belongs to finite set $\{b_1, b_2 \cdots b_n\}$ called the <u>state space of the system</u>."

② In the experiment of repeated trials, the out come of any future trial depends only on the outcome of the "present" [preceding] trial and not on any other previous (or) past trial.

For the given probability $\boxed{P_{ij}}$. If $\boxed{b_j}$ occurs immediately after $\boxed{b_i}$.

The following n-square Matrix

Transition Matrix:-

$$P = \begin{array}{c} \\ b_1 \\ b_2 \\ \vdots \\ b_n \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & \cdots & b_n \end{array} \\ \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} \end{array}$$

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∴ The probability Vector $[P]$, can be obtained by multiplying $e \cdot$ (i.e by the reciprocal of the addition of its entries].

$$P = \frac{1}{8} e$$

$$= \frac{1}{8} [2, 5, 0, 1]$$

$$\boxed{P = \left[\frac{2}{8}, \frac{5}{8}, \frac{0}{8}, \frac{1}{8}\right]}.$$

(7) Check the following stochastic matrix is regular or not?

$$A = \begin{bmatrix} 1 & 0 \\ \frac{7}{8} & \frac{1}{8} \end{bmatrix}$$

Sol:-

We know that, if we have to prove stochastic matrix is regular.

i.e We have to show all the entries of P^k should be positive

$$A^2 = \begin{bmatrix} 1 & 0 \\ \frac{63}{64} & \frac{1}{64} \end{bmatrix} \longrightarrow \text{ofcd.}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ \frac{4095}{4096} & \frac{1}{4096} \end{bmatrix} \longrightarrow \text{ofcd ఇచ్చే.}$$

∴ A is not regular matrix

Problems

L-69 (8)

(1) Show that the non-Zero Vector $e = [2, 5, 0, 1]$ is the probability Vector?

$e = [2, 5, 0, 1]$ is not a probability Vector.

For any Vector to be a probability Vector, it should follow two Conditions

(1) All entries should be non-negative ($e_i \geq 0$)

(2) Addition of all entries should be One. $e_1 + e_2 + \ldots =$

(1) $2 \geq 0$, $5 \geq 0$, $0 = 0$, $1 \geq 0$

(2) $2 + 5 + 0 + 1 = 8 \neq 1$

$\therefore$ It satisfies only (1)

Does not Satisfies (2)

$\therefore$ The given Vector is $e = [2, 5, 0, 1]$ is not a probability Vector.

(2) In the previous example for the non-Zero Vector $e = [2, 5, 0, 1]$ find probability Vector?

$e = [2, 5, 0, 1]$

There is a Unique probability Vector $= p$

p is a Scalar multiple of e.

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(4) Suppose $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Square stochastic matrix & $e = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is a probability Vector then show that eA is also a probability Vector?

$$eA = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

(1) $\frac{1}{4} \geq 0 \quad \frac{3}{4} \geq 0$

(2) $\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

$\therefore$ (1) & (2) Conditions are satisfied

$\boxed{\therefore eA \text{ is a probability Vector}}$

(5) Determine which of the following stochastic matrice are regular (a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

(a) $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

0, 1 are fixed

$\boxed{\therefore A \text{ is not Regular}}$

(b) $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

$A^2 = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 1 & 0 \\ \frac{3}{8} & \frac{9}{46} & \frac{1}{16} \end{bmatrix}$

0, 1 are fixed

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Given $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 1 & 3 & -2 \end{bmatrix}$ find UA?

$$UA = \begin{bmatrix} 1 & 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 3 \end{bmatrix}$$

find the unique fixed probability vector S of each matrix (a) $A = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$ $B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

(a) $S = \begin{bmatrix} x & y \end{bmatrix}$

$\boxed{S = SA} \Rightarrow \cancel{\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix}}$ $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$

$\cancel{\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0.3x + 0.5y & 0.4x + 0.5y \end{bmatrix}}$ $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0.3x + 0.5y & 0.4x + 0.5y \end{bmatrix}$

$\cancel{x = 0.3x + 0.5y}$

$\cancel{y = 0.4x + 0.5y}$

$\cancel{y = -0.1x}$

use calsi mode setup

(1)

$x = 0$
$y = 2$

$\cancel{x = 0.3x + 0.5y}$

$\cancel{= 0.3x + 0.5(-0.1x)}$

$\cancel{x = 0.25x}$

$\cancel{y = 0.4(0.25) + 0.5y}$

$\cancel{S = \begin{bmatrix} x & y \end{bmatrix}}$

$= \begin{bmatrix} & -0. \end{bmatrix}$

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check the following stochastic matrix is regl or not?

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{16}{75} & \frac{37}{75} & \frac{22}{75} \\ \frac{24}{135} & \frac{26}{45} & \frac{33}{135} \end{bmatrix}$$

In this No 0, 1

$\therefore$ $\boxed{A \text{ is a Regular}}$

Find the unique fixed probability vector S of the following regular stochastic matrix?

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$\Rightarrow S = SP$

$$\Rightarrow [x \; y \; z] = [x \; y \; z] \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x &= \tfrac{1}{5}y \\ y &= x + \tfrac{2}{5}y + \tfrac{1}{2}z \\ z &= \tfrac{2}{5}y + \tfrac{z}{2} \end{aligned}\right\} \text{solv}$$

by solving $x = 0$
$y = 5$

$x = \frac{y}{5}$

$y = \frac{5}{4} z$

$\therefore$ If we have to solve non-zero solution.

$\boxed{x = 1}$

$\therefore \quad y = 5$

$z = 4$

$\therefore S = [x \quad y \quad z] = [1 \quad 5 \quad 4]$

Suppose $e_0 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is the initial state distribution for a Markov process with the following transition Matrix? $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$

find $e_1 \; e_2 \; e_3$?

$$\boxed{\begin{array}{c} \underline{\text{formulas}} \\ e_1 = e_0 P \\ e_2 = e_1 P \\ e_3 = e_2 P \end{array}}$$

$e_1 = e_0 P$

$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix}$

$e_2 = e_1 P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix}$

$e_3 = e_2 P = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{11}{16} & \frac{5}{16} \end{bmatrix}$

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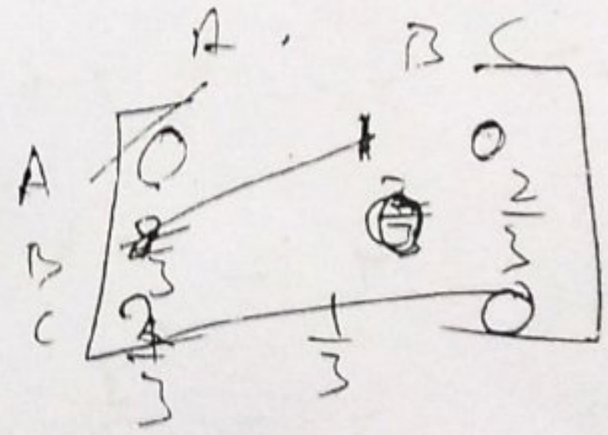
11) A person H goes to Tirupati temple, Padmavathi temple & Srikalahasti. A always goes first Tirupathi - Then Padmavathi temple then Srikalahasti. If A first goes to Srikalahasti, he is just as likely to go Tirupatis padmavathi temple. Find the transition matrix of this Markov process?

Tirupati temple = T

P T = P

S T = S

$$T = \begin{array}{c} \\ T \\ P \\ S \end{array} \begin{array}{c} \begin{array}{ccc} T & P & S \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \end{array}$$



12) A student tries to take admission in only three college A, B & C. first he goes to college A, second day to college B. Then third day to C. He never goes to same college in two continuous days.

But if he goes either B or C. Then, next day, he is twice as likely to go to college A. Find out how often in the long run, he tries to each college?

Transition Matrix

$$P = \begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

$\therefore$ fixed vector $s = [x \;\; y \;\; z]$

$s = s P$

$$= (x \;\; y \;\; z) \begin{bmatrix} 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x &= \tfrac{2}{3}y + \tfrac{2}{3}z \\ y &= x + \tfrac{z}{3} \\ z &= \tfrac{y}{3} \end{aligned}\right\} \text{solve}$$

$\boxed{z = 1}$ $\rightarrow$ also solve 3 equations we get $z = 1$

$y = 3$

Then

$x = \frac{8}{3}$

$$s = [x \;\; y \;\; z] = \left[\tfrac{8}{3} \;\; 3 \;\; 1\right]$$

$\therefore$ Unique fixed probability vector $= \frac{8}{3} + 3 + 1 = \frac{20}{3}$

$$\therefore s = \left[\frac{8}{3}\Big/\frac{20}{3} \;\; \frac{3}{20} \;\; \frac{1}{20}\right] = [0.40 \;\; 0.45 \;\; 0.15]$$

$\therefore$ A student spends 40% of his time in college A, 45% of time in B, 15% of time in college C.

Suppose $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Square stochastic matrix & $e = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is a probability Vector then show that eA is also a probability Vector?

$$eA = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

(1) $\frac{1}{4} \geq 0 \quad \frac{3}{4} \geq 0$

(2) $\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

$\therefore$ (1) & (2) Conditions are satisfied

$\boxed{\therefore eA \text{ is a probability Vector}}$

(5) Determine which of the following stochastic matrices are regular (a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

(a) $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

0, 1 are zero

$\boxed{\therefore A \text{ is not Regular}}$

(b) $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

$A^2 = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 1 & 0 \\ \frac{3}{8} & \frac{9}{46} & \frac{1}{16} \end{bmatrix}$

0, 1 are zero

$\boxed{\therefore A \text{ is not regular}}$

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Given $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 1 & 3 & -2 \end{bmatrix}$ fixed

$$UA = \begin{bmatrix} 1 & 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 3 \end{bmatrix}$$

find the Unique fixed probability Vector S of each matrix (a) $A = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$ $B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

(a) $S = \begin{bmatrix} x & y \end{bmatrix}$

$\boxed{S = SA} \Rightarrow \cancel{\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix}}$ $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$

$\cancel{\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0.3x + 0.5y & 0.4x + 0.5y \end{bmatrix}}$ $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0.3x + 0.5y & 0.4x + 0.5y \end{bmatrix}$

$x = 0.3x + 0.5y$

$y = 0.4x + 0.5y$

$y = -0.1x$

use calsi mode setup

(1)

$x = 0$
$y = 2$

$x = 0.3x + 0.5y$

$= 0.3x + 0.5(-0.1x)$

$x = 0.25x$

$y = 0.4(0.25) + 0.5y$

$S = \begin{bmatrix} x & y \end{bmatrix}$

$= \begin{bmatrix} & -0. \end{bmatrix}$

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(8) check the following stochastic matrix is regular or not ?

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{16}{75} & \frac{37}{75} & \frac{22}{75} \\ \frac{24}{135} & \frac{26}{45} & \frac{33}{135} \end{bmatrix}$$

In this No 0, 1

$\therefore$ $\boxed{A \text{ is a Regular}}$

(9) Find the Unique fixed probability vector S of the following regular stochastic Matrix ?

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$\Rightarrow S = SP$

$$\Rightarrow [x \; y \; z] = [x \; y \; z] \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x &= \tfrac{1}{5}y \\ y &= x + \tfrac{2}{5}y + \tfrac{1}{2}z \\ z &= \tfrac{2}{5}y + \tfrac{2}{2} \end{aligned}\right\} \text{Solv.}$$

by solving $x = 0$, $y = 5$.

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$$x = \frac{7}{5}$$

$$y = \frac{5}{4} z$$

$\therefore$ If we have to solve non-zero solution. get

$$\boxed{x = 1}$$

$\therefore$ $y = 5$

$z = 4$

$$\therefore S = [x \quad y \quad z] = [1 \quad 5 \quad 4]$$

10. Suppose $e_0 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is the initial state distribution for a Markov process with the following transition Matrix? $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$

find e_1 e_2 e_3 ?

formulas

$e_1 = e_0 P$

$e_2 = e_1 P$

$e_3 = e_2 P$

$$e_1 = e_0 P$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

$$e_2 = e_1 P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix}$$

$$e_3 = e_2 P = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{11}{16} & \frac{5}{16} \end{bmatrix}$$

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A person A goes to Tirupati temple

Padmavathi temple & Srikalahasti. A always

goes first Tirupathi - Then Padmavathi temple then

Srikalahasti. If A first goes to Srikalahasti, he

is just as likely to go Tirupatis Padmavati temple.

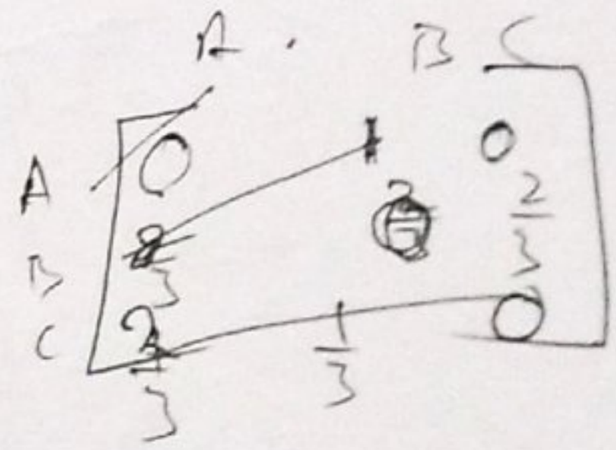
Find the transition matrix of this Markov process?

Tirupati temple = T

P T = P

S T = S

$$T = \begin{array}{c} \\ T \\ P \\ S \end{array} \begin{array}{c} \begin{array}{ccc} T & P & S \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \end{array}$$



(12) A student tries to take admission in any

three college A, B & C. first he goes to college A,

second day to college B. Then third day to C. He never

goes to same college in two continuous days.

But if he goes either B or C. Then,

then next day, he is twice as likely to go to college

A. find out how often in the long run, he visits each

college?

Transition Matrix

$$P = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & \frac{2}{3} & 0 & \frac{1}{3} \\ C & \frac{2}{3} & \frac{1}{3} & 0 \end{array}$$

$\therefore$ fixed vector $S = [x \;\; y \;\; z]$

$S = SP$

$$= (x \;\; y \;\; z) \begin{bmatrix} 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

$$\left.\begin{aligned} \Rightarrow \quad x &= \tfrac{2}{3}y + \tfrac{2}{3}z \\ y &= x + \tfrac{z}{3} \\ z &= \tfrac{y}{3} \end{aligned}\right\} \text{Sly}$$

$\boxed{z = 1}$ $\rightarrow$ also solve Jabre eq & the assume to be $z = 1$

$y = 3$

Then

$x = \frac{8}{3}$

$$S = [x \;\; y \;\; z] = \left[\tfrac{8}{3} \;\; 3 \;\; 1\right]$$

$\therefore$ Unique fixed probability vector $= \frac{8}{3} + 3 + 1 = \frac{20}{3}$

$$\therefore S = \left[\frac{8}{3}\Big/\frac{20}{3} \quad \frac{3}{\frac{20}{3}} \quad \frac{1}{\frac{20}{3}}\right] = [0.40 \;\; 0.45 \;\; 0.15]$$

$\therefore$ Ashok spends 40% of his time in college A, 45% of time in B, 15% of time in college C.

UNIT - 5

Stochastic Process

1

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Stochastic Process (or) Random Process.

Introduction;-

R. V or Continuous parameter

(1)

We consider Random functions of a Continuous parameter. This $\boxed{\text{Continuous parameter}}$ generally, We consider as $\boxed{\text{time}}$, but it could equally will be position (or) angle (or) Some other Continuous parameters.

↓

ie The collection of all these Continuous time functions is called a Random process $\boxed{\text{or}}$ Stochastic Process.

It is represented by $X(t, \lambda)$. X = Random Variable, λ = The possible outcomes of an experiment.

(or)

A stochastic process with State Space $\boxed{S}$.

S is the set of all possible Random Variables indexed by t (where $t \geq 0$)."

(or)

"The Random Variables whose outcomes Continuously changes with time."

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Applications of Stochastic Process

Deterministic Signals

1. Most Common Use is "As a Model in physical systems."

ie In this Model, Noise being the necessary first step in deciding on the best way to mitigate its -Ve Effects (Negative Effects)

② In Electrical Systems Voltage (or) Current Waveform :-

In this Model, Wave forms are used as "Signals for Collecting, transmitting (or) processing information as well as for Controlling and providing power to a Variety of devices...

→ These Signals (Voltage (or) Current Waveforms) are functions of time

Signals
2 types

Deterministic Signals

It can be described by Mathematical functions within time $\boxed{t}$.
(where t = Independent Variable)

Random Signals

It can be described by "In terms of its average properties."

Random Variable (RV):-

RV is a rule or a function that assigns a real number to every outcome of a Random Experiment.

Random process:- Random process is a rule or function that assigns a time function to every outcome of a Random Experiment.

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Process, — Markov Process is a stochastic process which has the property that the probability of transition from a given state to any future state depends only on the present state not on the manner in which it was reached.

$E_n \xrightarrow{t_0} E_{n+1}$

3

ఇది Markov chain కి సరిగా applicable states కి meaning = possible outcomes కి.

Ex: Car - Rental Company :-

It is running agencies in different cities. Suppose A car sent to one city may return to any city where the Company's agency is available.

ఇలా this situation is considered as Markov process.

i.e Different cities కి = states కింద లెక్కిస్తారు.

P_{ij} = probability that a car in city-i would rent a car in city-j where j may be equal to i

In this way we will calculate

1. Average (av) Expected long term profit of cars at each city.
2. Mean Number of trips a car (Cars stay put where ending point is).

2. Markov process is widely used in examining and predicting "the behaviour of consumers in terms of their brand loyalty and their switching patterns to others brands"

These processes are also used in the of equipment maintenance & failure problems. Analysing accounts receivable that will ultimately become bad debts.

4 It is also used to study the stock market price movements. 4

Note:- Suppose Board Cost A System is said to be Discrete in time. Cost if it is examined at Regular intervals.

Example:- Daily, weekly, Monthly or yearly.

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Characteristics of Markov Process :-

1. The states are both collectively exhaustive & mutually exclusive.

2. The transition probabilities are stationary (i.e. constant)

3. The probability of moving from one state to another depends only on the immediately preceding state

4. The transition probability of moving to alternative states in the next time period, gives a state in the current time period must sum to unity

i.e Total probability = 1 (you consider all the states = 1)

5. The process has a set of initial probabilities that may be either given or determined.

(or you can give Initial probabilities 2 ways or determined so dynamically determined so forward.)