MSF

BVRIT H

CSE AIML

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UNIT-1 Formulae [Probability & Random Variables]

S.No	Name	Formulae
1	RV: DRV (Discrete Random Variable)	$\sum p(x)$
	RV: CRV (Continuous Random Variable)	$\int_{-\infty}^{+\infty} f(x)\,dx$
2	Mathematical Expectation (or) E(X): DRV	$\sum X\,p(x)$
	Mathematical Expectation (or) E(X): CRV	$\int_{-\infty}^{+\infty} X\,f(x)\,dx$
3	Variance (or) $V(X)$ (or) σ^2: DRV	$\sum (x-\mu)^2 p(x)$ (or) $\sum x^2 p(x) - [\mu]^2$
	Variance (or) $V(X)$ (or) σ^2: CRV	$\int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\,dx$ (or) $\int_{-\infty}^{+\infty} x^2 f(x)\,dx - [\mu]^2$
4	Standard Deviation [S.d] σ	$\sigma = \sqrt{V(X)}$
5	Moments [mr] are 2 types (General): μ_r' origin	Moments about Origin [μ_r'] 1. μ_1' = 1st Moment about origin = $E(X)$ 2. μ_2' = 2nd Moment about origin = $E(X^2)$ 3. μ_3' = 3rd Moment about origin = $E(X^3)$
	Mean μ_r	Moments about Mean [μ_r] 1. μ_1 = 1st Moment about Origin = $\mu_1 = 0$

Based on RV's:-

mr: DRV — $\sum x^r p(x)\,dx$

mr: CRV — $\int_{-\infty}^{+\infty} (x-\bar{x})^r f(x)\,dx$

2. μ_2 = 2nd Moment about Mean = Variance

$$\boxed{\therefore V(x) = \mu_2' - (\mu_1')^2}$$

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3. μ_3 = 3rd Moment about Mean = Skewness

$$\boxed{\mu_3 =}$$

4. μ_4 = 4th Moment about Mean = peakedness (or) kurtosis

$$\boxed{\mu_4 =}$$

6. Moment generating function

MGF (or) Mgf

Representation: $M_x(t)$ (or) $M(t)$

2 types

DRV → $M_x(t) = E[e^{tx}] = \sum e^{tx} f(x)$

CRV → $M_x(t) = E[e^{tx}] = \int_{-\infty}^{+\infty} e^{tx} f(x)\,dx$

9. DRV problems Identification — table (or) probability (or) Some values 1 & −1, $x \le 6$, $x > 1$, probability Mass fuction, Σ, $\Sigma p(x)$

CRV problems Identification — $\int_{-\infty}^{+\infty}$ (or) function (or) probability density fn (or) $\int_a^b$ (or) Limits/End points (or) Cummulative (or) frequency distribution.

Properties :—

Cumulative distributive fn (or)

Probability Mass function [Pmf]	Probability density function [Pdf]	Probability Distribution function [PDF]
DRV	CRV	DRV & CRV
1. $p(x) \geq 0$	1. $f(x) \geq 0$ (Always +ve)	1. $F(x) \geq 0$
2. $\sum p(s) = 1$ (or) $p(x) = 1$	2. $\int_{-\infty}^{+\infty} f(x)\, dx = 1$	2. $F(-\infty) = 0$ $F(\infty) = 1$
	3. $f(x) = F'(x)$	3. $p(a \leq X \leq b) = \int_a^b f(x)\, dx$ (or) $p(a \leq X \leq b) = F(b) - F(a)$
		4. $F'(x) = f(x)$
$p(x)$ is used in pmf	$f(x)$ is used in pdf	$F(x)$ is used in this distribution

II: When ever, we want to find k (or) Any Constant Value :—

DRV :— $\sum k + 2k + \cdots + = 1$

CRV :— $\int_{-\infty}^{+\infty} k + 2k + \cdots = 1$

ie Total probability $= 1$

12. Measures of Central tendency :-

[Mean, Median, Mode, Variance, S.d]

DRV	CRV
1. Mean $= E(x) = \sum x \cdot p(x)$	1. Mean $= E(x) = \int_{-\infty}^{+\infty} x \cdot f(x)\, dx$
★ (M) 2. **Median** :- Median is the point which divides the entire distribution into two equal parts. Where M = Median	★ (M) 2. **Median** :- Median is the point which divides the total Area into two equal parts. Where M = Median
Right now in our syllabus $\boxed{\text{No formula here}}$	$$\boxed{\int_a^M f(x)\, dx = \int_M^b f(x)\, dx = \frac{1}{2}}$$
3. **Mode** :- — No formula.	3. **Mode** :- Mode is the value of X for which f(x) is Maximum. 1st Condition — $f'(x) = 0$; 2nd Condition — $f''(x) < 0$

4. Mean Deviation:-

No formula in your syllabus

5. Variance [$V(x)$ or σ^2]:-

$$\boxed{\sigma^2 = \sum x^2 p(x) - \mu^2}$$

or

$$\boxed{\sigma^2 = \sum (x-\mu)^2 p(x)}$$

6. Standard Deviation :-

S.d or σ

It is the positive square Root of the Variance

$$\boxed{\sigma = \sqrt{V(x)}}$$

S.d here

4. Mean Deviation:-

$$\boxed{\int_{-\infty}^{+\infty} |x-\mu| \cdot f(x)\, dx}$$

5. Variance [$V(x)$ or σ^2]:-

$$\boxed{\sigma^2 = \int_{-\infty}^{+\infty} x^2 f(x)\,dx - \mu^2}$$

or

$$\boxed{\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\, dx}$$

6. Standard Deviation:-

S.d or σ :-

It is the positive square root of the Variance

$$\boxed{\sigma = \sqrt{V(x)}}$$

here S.d

13. Mean properties

1. $E(X+k) = E(X) + k$

2. $E(k) = k$
   where k is a Constant

3. $E(kX) = k.E(X)$

4. $E(aX+b) = aE(X)+b$

5. If X & Y are two discrete Random Variables. Then
   $E(X+Y) = E(X) + E(Y)$
   lly $E(X+Y+Z) = E(X)+E(Y)+E(Z)$

6. $E(X-\bar{X}) = 0$

7. $E(XY) = E(X) \cdot E(Y)$
   here X, Y = Independent Variables.

8. Any function of Random Variable
   $$E[g(x)] = \sum g(x) \cdot f(x)$$

Variance properties

1. $V(X+k) = V(X) + 0$

2. $V(k) = 0$
   where k is a Constant

3. $V(kX) = k^2 V(X)$

4. $V(aX+b) = a^2 V(X) + 0$

5. $V(X+Y) = V(X) + V(Y)$

   lly $V(X+Y+Z) = V(X)+V(Y)+V(Z)$

6. ——

7. $V(XY) = V(X) \cdot V(Y)$

8. ——

Mgf in Discrete Distribution:—

There are 2 types 1. Binomial Distribution
2. Geometric Distribution

Mgf in B.D	Mgf in G.D
1. Mean $= np$	1. Mean $= \frac{1}{p}$
2. Varince $= npq$	2. Variance $= \frac{1-p}{p^2}$
3. Mgf in B.D function $\boxed{M_x(t) = (q+pe^t)^n}$	3. Mgf in G.D function $\boxed{M_x(t) = \frac{pe^t}{1-q \cdot e^t}}$

15. Mgf in Continuous Distribution:—

1. Normal Distribution.

1. Mean $= \mu$

2. Variance $= \sigma^2$

3. Mgf in N.D function $\boxed{M_x(t) = \int_{-\infty}^{+\infty} e^{tx} \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}}$

$$M_x(t) = \mu t + \frac{t^2\sigma^2}{2} \quad \text{or} \quad e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

16. Distribution Analysis:-

Discrete Distribution — 2 types

1. B.D
2. G.D
3. P.D

Continuous Distribution

1. N.D

Distribution Name	Range	Parameters	PDF/PMF	E(X)	V(X)
B.D Binomial Distribution	$x=0,1 \ldots n$	$0 \le p \le 1$ $\boxed{n, p}$	${}^{n}C_{x}\, p^{x} q^{n-x}$	np	npq
G.D Geometric Distribution	$x=1,2 \ldots \infty$	$0 \le p \le 1$ $\boxed{p}$	$p(q)^{x-1}$	$\frac{1}{p}$	$\frac{q}{p^2}$
N.D Normal Distribution	$(-\infty$ to $\infty)$	$\mu, \sigma^2 > 0$ $\boxed{(\mu, \sigma)}$	$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$	μ	σ^2
P.D Poisson Distribution					