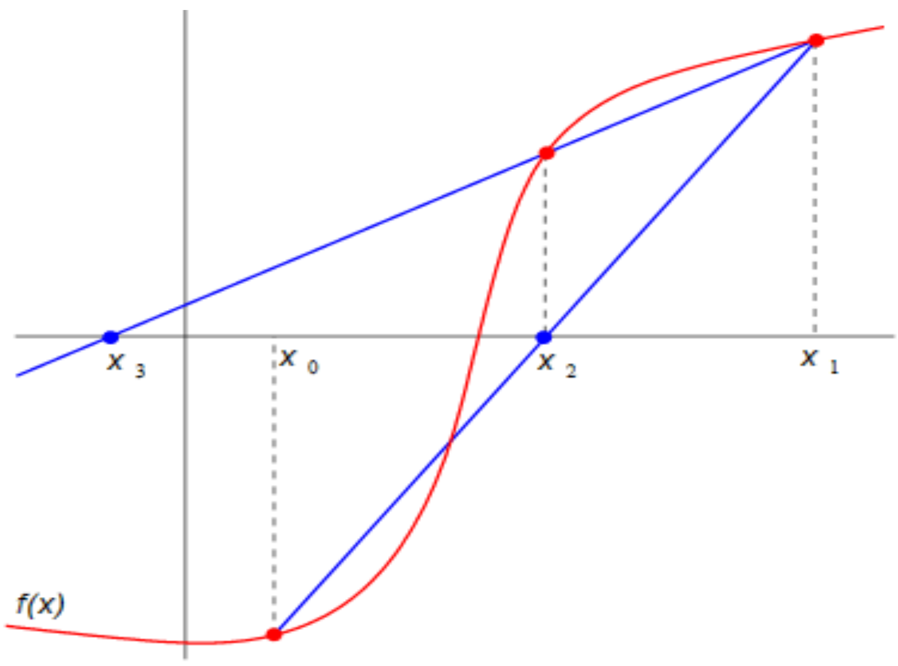
Secant Method

The Newton-Raphson algorithm requires the evaluation of two functions (the function and its derivative) per each iteration. If they are complicated expressions it will take considerable amount of effort to do hand calculations or large amount of CPU time for machine calculations. Hence it is desirable to have a method that converges as fast as Newton's method yet involves only the evaluation of the function.

Let x_0 and x_1 are two initial approximations for the root of $f(x) = 0$. If x_2 is the point of intersection of x-axis and the line-joining the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$ then x_2 is closer to the root than x_0 and x_1. The equation relating x_0, x_1 and x_2 is found by considering the slope 'm'



$$m = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{0 - f(x_1)}{x_2 - x_1}$$

$$x_2 - x_1 = \frac{- f(x_1)\ (x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = x_1 - \frac{f(x_1)\ (x_1 - x_0)}{f(x_1) - f(x_0)}$$

In general the iterative process can be written as

$$x_{n+1} = x_n - \frac{f(x_n)\ (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

This formula is similar to Regula-falsi scheme of root bracketing methods but differs in the implementation. The Regula-falsi method begins with the two initial approximations 'a' and 'b' such that $a \leq s \leq b$ where s is the root of $f(x) = 0$. It proceeds to the next iteration by calculating c using the above formula and then chooses one of the interval (a,c) or (c,b) depending on $f(a) * f(c) < 0$ or > 0 respectively.

On the other hand secant method starts with two initial approximation x_0 and x_1 (they may not bracket the root) and then calculates the x_2 by the same formula as in Regula-falsi method but proceeds to the next iteration without bothering about any root bracketing.

Secant Method:

Algorithm:

Given an equation $f(x) = 0$

Step1. Let the initial guesses be x_0 and x_1

Step2. Calculate

$$\mathbf{x_{n+1} = x_n - \frac{f(x_n)\ (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}}$$

For n=1,2,3…

Example 1: Use Secant method to find a root of the equation $\cos(x) + 2\sin(x) + x^2 = 0$ correct to three decimal places.(with initial approximations $x_0 = 0$ and $x_1 = -0.1$)

Solution: $f(x) = \cos(x) + 2\sin(x) + x^2$

Calculate:

$$x_{n+1} = x_n - \frac{f(x_n)\,(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

For n=1,2,3…

Iteration	x_n
0	0
1	- 0.1
2	-0.5136
3	-0.6100
4	-0.6514
5	-0.6582
6	-0.6598
7	-0.6595

First three decimal places have been stabilized; hence **-0.6595** is the real root correct to three decimal places.

Example 2: Use Secant method to find a root of the equation $\mathbf{x^4-x-10 = 0}$ correct to four decimal places.(with initial approximations $x_0= 1$ and $x_1 = 2$)

Solution: $\boldsymbol{f(x) = x^4-x-10 = 0}$

Calculate:

$$x_{n+1} = x_n - \frac{f(x_n)\ (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

For n=1,2,3…

Iteration	x_n
0	1
1	2
2	1.7149
3	1.83853
4	1.85778
5	1.85555
6	1.85558
7	1.85558

First four decimal places have been stabilized; hence 1.85558 is the real root correct to four decimal places.

Exercise Problems:

1. Calculate the first 3 iterations of secant method on $x^3 - 7x^2 + 14x - 6 = 0$ with the following initial approximation
i) $x_0 = 0.5$ and $x_1 = 0.8$
ii) $x_0 = 2.8$ and $x_1 = 3.8$
iii) $x_0 = 3.5$ and $x_1 = 3.8$

2. Apply secant method to find a root of the equation $xe^x = 1$ correct to three decimal places (with initial approximation ($x_0 = 0.5$ and $x_1 = 0.8$))

3. Calculate the first 5 iterations of the secant method on $f(x) = xe^x - 2 = 0$ (with initial approximation ($x_0 = 3.5$ and $x_1 = 3.8$))

4. Do 3 iterations of secant method on $f(x) = x^2 - 5$ with initial approximation ($x_0 = 2.5$ and $x_1 = 3.8$).
Or
Find the approximate value of square root of 5 using secant method .
(Do 3 iterations with initial approximation ($x_0 = 2.5$ and $x_1 = 3.8$).)

For more solved problem
https://mat.iitm.ac.in/home/sryedida/public_html/caimna/transcendental/iteration%20methods/secant/examples.html#exp2