Regula-Falsi method

Regula-Falsi method:

Regula-Falsi method is also known as method of false position as false position of curve is taken as initial approximation. Let $y = f(x)$ be represented by the curve AB.The real root of equation $fx = 0$ is α as shown in adjoining figure. The false position of curve AB is taken as chord AB and initial approximation x_0 is the point of intersection of chord AB with x-axis. Successive approximations x_1, x_2, …are given by point of intersection of chord $A'B$, $A''B$, …with $x-$ axis, until the root is found to be of desired accuracy.

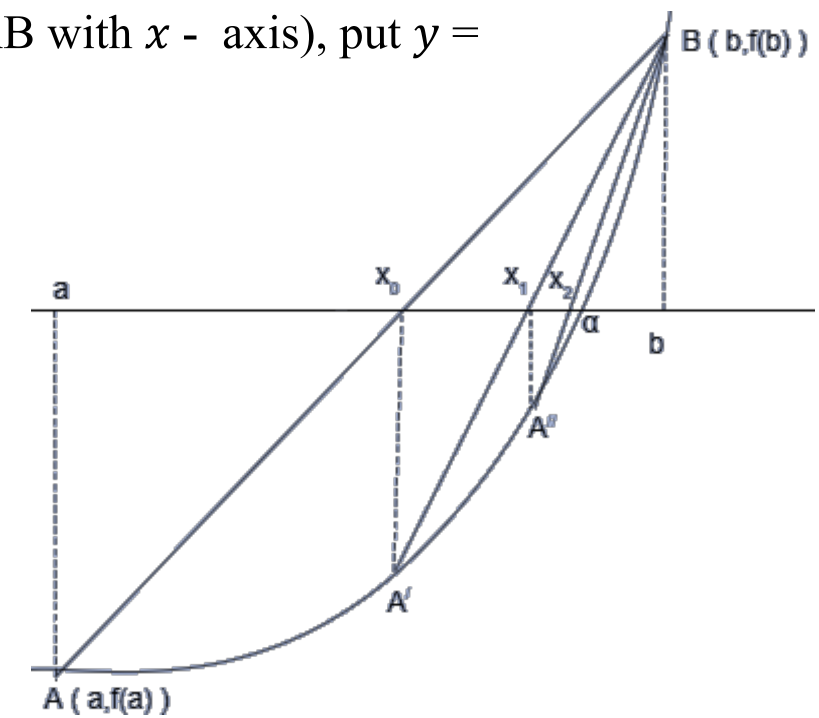
Now equation of chord AB in two-point form is given by:

$(y\text{-}\ f(\text{a}))(\text{b-a}) = (f(\text{b})\text{-}\ f(\text{a}))(x\text{-a})$

To find x_0 (point of intersection of chord AB with x - axis), put $y =$ 0.

$$x_0 = \text{a} - \frac{(\text{b-a})f(\text{a})}{f(\text{b}) - f(\text{a})}$$

Repeat the procedure until the root is found to the desired accuracy.



Regula-Falsi method:

Algorithm:

Let $f(x)$ be a continuous function in the interval $[a, b]$, such that $f(a)$ and $f(b)$ are of opposite signs, i.e. $f(a)\ f(b) < 0$.

Step1. Take the initial approximation as $x_0 = a - (b - a)f(a)/f(b) - f(a)$, one of the three conditions arises for finding the 1st approximation x_1

1. $f(x_0) = 0$, we have a root at x_0.

2. If $f(a)f(x_0) < 0$, the root lies between a and x_0

$\therefore\ x_1 = a - (x_0 - a)f(\mathrm{a})/f(x_0) - f(a)$.

3. If $f(b)f(x_0) < 0$, the root lies between x_0 and b

$\therefore\ x_1 = x_0 - (b - x_0)f(x_0)/f(b) - f(x_0)$.

4. Continue the process until root is found to be of desired accuracy.

Remarks:

Rate of convergence is much faster than bisection method.

Unlike bisection method, one end point will converge to the actual root a , whereas the other end point always remains fixed. As a result Regula- Falsi method **has linear convergence**.

Example 1: Apply Regula-Falsi method to find a root of the equation $x^3 + x - 1 = 0$ correct to two decimal places.

Solution: $f(x) = x^3 + x - 1$

Here $f(0) = -1$ and $f(1) = 1 \Rightarrow f(0)f(1) < 0$

Also fx is continuous on [0,1], $\therefore$ atleast one root exists in [0,1]

Iteration	a	b	x_n	$f(x_n)$
0	0	1	0.5	−0.375
1	0.5	1	0.636	−0.107
2	0.636	1	0.6711	−0.0267
3	0.6711	1	0.6796	-0.0065

First two decimal places have been stabilized; hence **0.6796** is the real root correct to two decimal places.

Example 2: Use Regula-Falsi method to find a root of the equation xlog(x) -1.2= 0 correct to two decimal places.

Solution: $f(x) = x\log(x) - 1.2$

Here e $f(2) = -0.5979$ and $f(3) = 0.2314 \Rightarrow f(2)f(3) < 0$

Also fx is continuous on [2,3], $\therefore$ atleast one root exists in [2,3]

Iteration	a	b	x_n	$f(x_n)$
0	2	3	2.721	−0.0171
1	2.721	3	2.7402	−0.0004
2	2.7402	3	2.7407	0.00005

First two decimal places have been stabilized; hence **2.7407** is the real root correct to two decimal places.

Example 3: Use Regula-Falsi method to find a root of the equation $\tan x + \tanh x = 0$ upto three iterations only.(in radian)

Solution: $f(x) = \tan x + \tanh x$

Here e $f(2) = -1.2210$ and $f(3) = 0.8525 \Rightarrow f(2)f(3) < 0$

Also $f x$ is continuous on $[2,3]$, $\therefore$ atleast one root exists in $[2,3]$

Iteration	a	b	x_n	$f(x_n)$
0	2	3	2.5889	0.3720
1	2	2.5889	2.4514	0.1596
2	2	2.4514	2.3992	0.0662
3	2	2.3992	2.3787	0.0269

Hence **2.3787** is the real root in the third iteration.

Exercise Problems:

1. Calculate the first 3 iterations of the Regula-Falsi method on $x^3 - 7x^2 + 14x - 6 = 0$ with the following starting intervals
i)[0,1]
ii)[2.5,3.2]
iii)[3.2,4]

2. Apply Regula-Falsi method to find a root of the equation $xe^x = 1$ correct to three decimal places

3. Calculate the first 5 iterations of the Regula-Falsi method on f(x) = xe^x -2 = 0 with interval [3, 4].

4. Do 3 iterations of the Regula-Falsi method on f(x) = x^2 -5 with interval [2,3].
Or
Find the approximate value of square root of 5 using Bisection method .
(Do 3 iterations with interval [2,3]).