EXERCISES: SESSION 2

1. Consider the following 3×3 matrix:

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

Multiply A with each of the following matrices in all the feasible orders.

(1)

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(2)

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

(3)

$$L = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(4)

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(5)

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

2. Given the matrix:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 7 & 5 \end{bmatrix}$$

(1) Provide a basis for the column space of A
(2) Provide a basis for the row space of A
(3) Provide a basis for the null space of A
(4) Check the dimensions of the respective spaces

3. Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$$

(1) What is the rank of A?
(2) Give two column vectors u and v such that $uv^t = A$.
(3) Compute a basis for the null space of A.

4. Given the matrix

$$A = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(1) What is the rank of A?
(2) Give two column vectors u and v such that $uv^t = A$.
(3) Compute a basis for the null space of A.

5. The transpose matrix of $A = (a_{ij}) \in \mathbb{R}^{n\times m}$ is another matrix that has as rows the columns of A: it is denoted A^t. In particular, notice that $A^t \in \mathbb{R}^{m\times n}$. For example:

$$A = \begin{bmatrix} -1 & 1 \\ 2 & -1 \\ 1 & 1 \end{bmatrix} \quad A^t = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

Taking the matrix A in the example:

(1) Can you deduce the sizes of AA^t and A^tA without doing any computation?
(2) Compute AA^t and A^tA.
(3) We say that a square matrix M is "symmetric" if $A^t = A$. Are AA^t and A^tA symmetric?

6. Consider two matrices $A, B \in \mathbb{R}^{m\times n}$. How would you manufacture two matrices $C \in \mathbb{R}^{m\times 2n}$ and $D \in \mathbb{R}^{2n\times n}$ using A and B as input, such that $A + B = CD$?