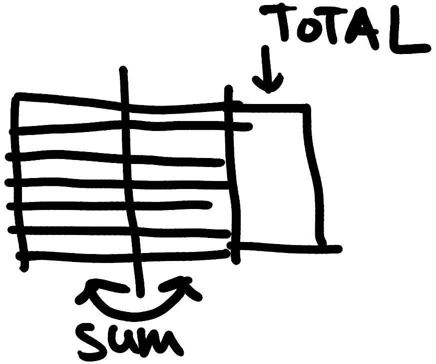
- Welcome Session :)
- How the course will work

Vectors & Vector Spaces

$$v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

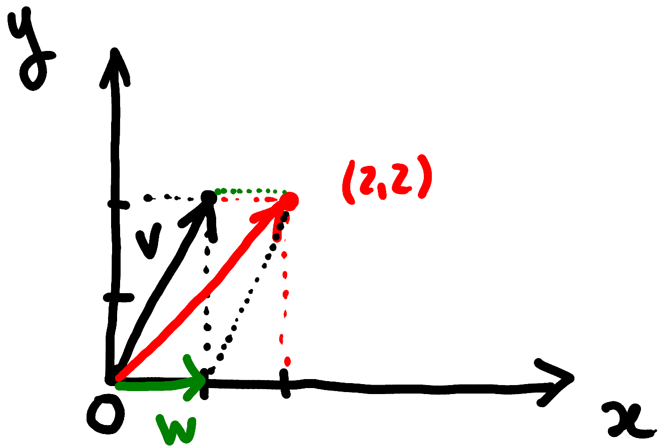
??

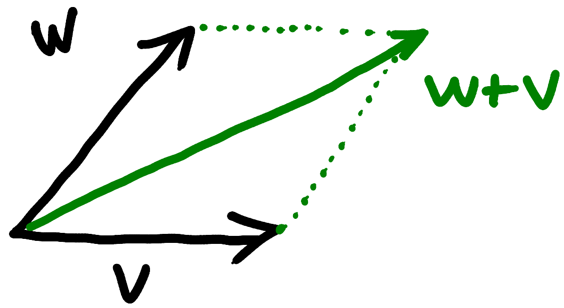
$$s = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \qquad t = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

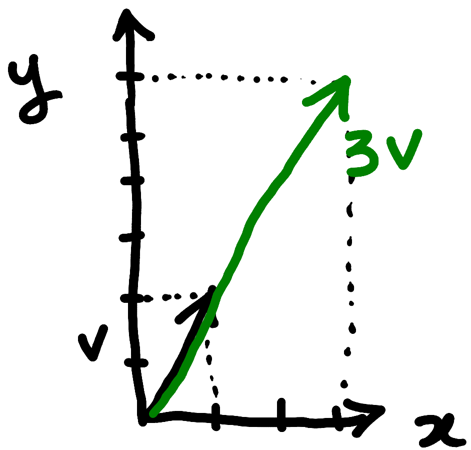


$$\rightarrow \; v+w = \begin{bmatrix} 1+1 \\ 2+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\rightarrow \; 3v = \begin{bmatrix} 3\cdot 1 \\ 3\cdot 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$







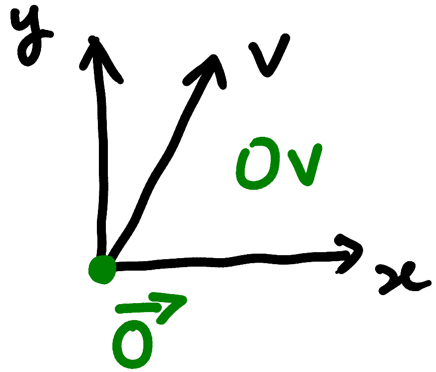
scalar

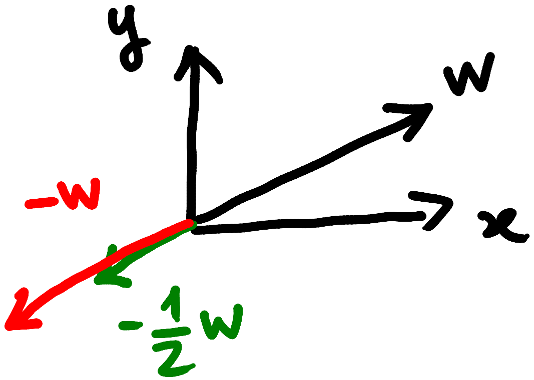
$$3v = 3\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}3\\6\end{bmatrix}$$

$$\text{length}(v) = \sqrt{1^2+2^2} = \sqrt{5}$$

$$\text{length}(3v) = \sqrt{3^2+(3\cdot 2)^2} = \sqrt{3^2(1^2+2^2)}$$

$$= \sqrt{3^2}\sqrt{1^2+2^2} = 3\cdot\sqrt{5}$$





$$0v = 0\begin{bmatrix}v_1\\v_2\end{bmatrix} = \begin{bmatrix}0\cdot v_1\\0\cdot v_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} = \vec{0}$$

$$w + \vec{0} = w$$

$$\left(-\frac{1}{2}\right)w = \begin{bmatrix}-\frac{1}{2}w_1\\-\frac{1}{2}w_2\end{bmatrix}$$

$$(-1)w = -w$$

$$w + (\underbrace{-w}_{\text{opposite of } w}) = \vec{0}$$

$$\begin{bmatrix}w_1\\w_2\end{bmatrix} + \underbrace{(-1)\begin{bmatrix}w_1\\w_2\end{bmatrix}} = \begin{bmatrix}w_1\\w_2\end{bmatrix} + \begin{bmatrix}-w_1\\-w_2\end{bmatrix} =$$

$$= \begin{bmatrix}w_1+(-w_1)\\w_2+(-w_2)\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} = \vec{0}$$

For any $w \in \mathbb{R}^n$ $\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = w$ $w_i \in \mathbb{R}$

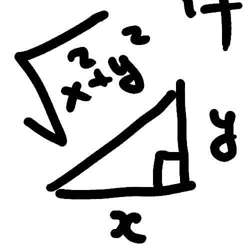
real numbers

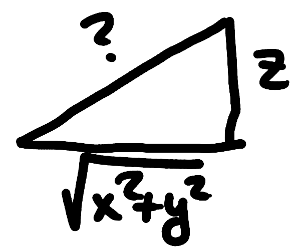
picking w from the collection of all vectors with n components

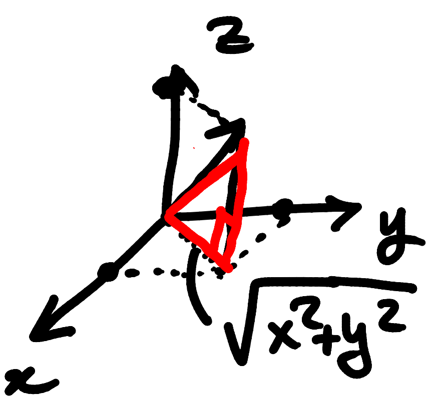
$$\text{length}(w) = \sqrt{w_1^2 + w_2^2 + \ldots + w_n^2}$$

Exercise

Proof that $\text{length}(w) = \sqrt{w_1^2 + w_2^2 + w_3^2}$ if $w \in \mathbb{R}^3$







$$\text{length}(\alpha w) = \underbrace{|\alpha|}_{\text{absolute value of } \alpha} \text{length}(w)$$

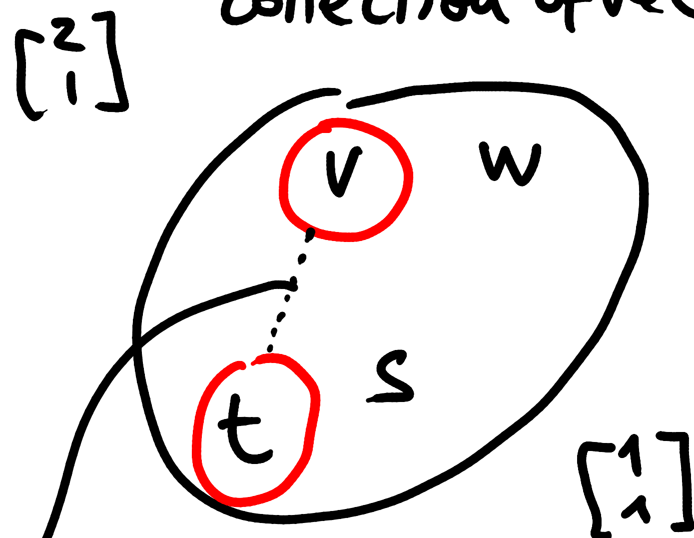
$\alpha \in \mathbb{R}$

$w \in \mathbb{R}^n$

Vector space

1) sum of vectors
2) mult. by scalars

collection of vectors



Can any pair of vectors be summed together? ✓

can any vector be multiplied by scalar? ✓

Corrected typo

Does the result already belong to the collection? ✓

VECTOR SPACE

Definition :

A vector space is a collection of vectors that satisfies :

1) vector operations are defined
2) closed w.r.t. these operation

with respect to

Example

$$\mathbb{R}^2,\ \mathbb{R}^3,\ \mathbb{R}^4,\ \dots$$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 1 \end{bmatrix} \dots$

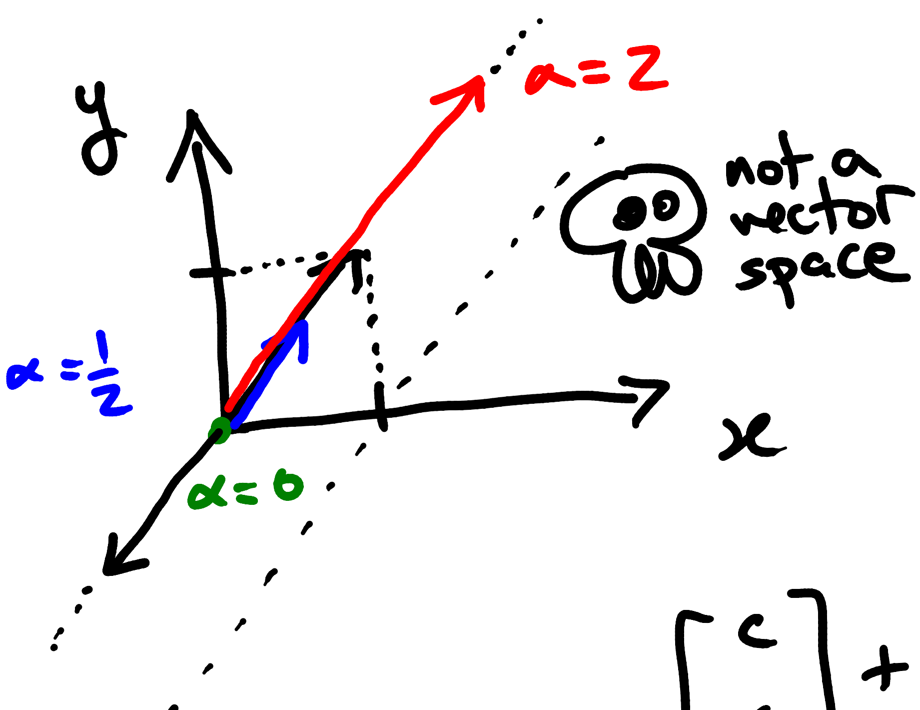
VECTOR SUBSPACE (of another space) $\rightarrow$ vector space that sits within another vector space

Example:

$S \subset \mathbb{R}^2$ "S is a collection within $\mathbb{R}^2$"

$$S = \left\{ \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} \;\middle|\; \alpha \in \mathbb{R} \right\}$$

↳ collection of all vector that arise after multiplying $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ by all scalars



1) S is not filling out all of $\mathbb{R}^2$

2) S is a vector subspace of $\mathbb{R}^2$

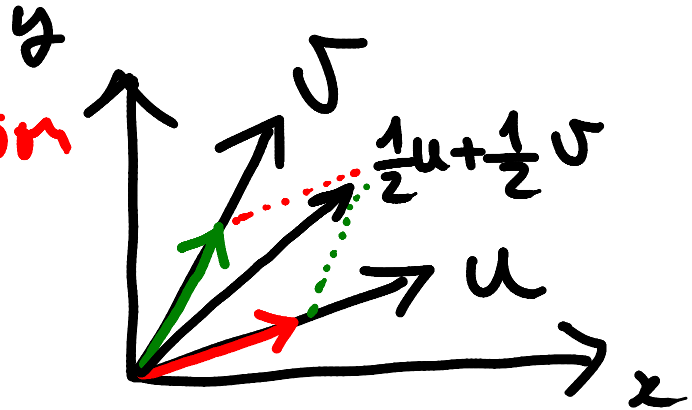
$$\begin{bmatrix} c \\ c \end{bmatrix} + \begin{bmatrix} d \\ d \end{bmatrix} = \begin{bmatrix} c+d \\ c+d \end{bmatrix} = \underbrace{(c+d)\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

Vector subspace spanned by a set of vectors

$$\text{span}\{u, v\} \qquad u, v \in \mathbb{R}^2$$

$$= \{\alpha u + \beta v \mid \alpha, \beta \in \mathbb{R}\}$$

linear combination of u and v



Definition

$$\mathrm{Span}\{u_1, u_2, \dots, u_k\} \quad u_i \in \mathbb{R}^n$$

$$\| $$

$$\{\lambda_1 u_1 + \dots + \lambda_k u_k \mid \lambda_i \in \mathbb{R}\} \subset \mathbb{R}^n$$

"collection of all the vectors that can arise as a linear combination of $u_1, \dots, u_k$"

Remark: $\vec{0}$ is always a member of any vector space.

$u \in V$

"pick u from a vector space V"

$$u \overset{(-1)u}{\rightsquigarrow} -u \rightsquigarrow u + (-u) = \vec{0} \in V$$

vector operations defined

$-u \in V$

V closed w.r.t. vector operations