EXERCISES: SESSION 1

1. Consider the following vectors of $\mathbb{R}^2$: $u = (1, 2)$, $v = (1, 3)$, $w = (-1, 2)$.
   (1) Compute $a = u + v$, $b = u + 3v$, $c = 2v - w$
   (2) Represent u, v and w geometrically.
   (3) Represent a, b, c alongside their relationship with u, v and w geometrically.
2. Justify whether in each case $S \subset \mathbb{R}^3$ is a vector subspace:
   (1) $S = \{(0, 0, 0), (1, 0, 0)\}$
   (2) S is any finite collection of vectors of V
   (3) $S = \{(x, y, z) \mid x + y = 0\}$
   (4) $S = \{(x, y, z) \mid x + y - 1 = 0\}$
   (5) $S = \{v + w \mid v \in \text{span}\{(1, 0, 0), (0, 1, 0)\}, w \in \text{span}\{(1, 1, 1)\}\}$
3. Consider the following vectors of $\mathbb{R}^2$: $u = (1, 3)$, $v = (2, 7)$, $w = (1, 5)$.
   (1) Are they linearly independent?
   (2) Do they span $\mathbb{R}^2$?
   (3) Compute two different linear combinations of u, v, w that produce the same vector $b = (0, 1)$.
   (4) Are there more than one solution to the previous question?
4. Justify why the vectors $(1, 0, 0), (0, 1, 1), (1, 0, 1)$ are a basis of $\mathbb{R}^3$.