

14th EWGT & 26th MEC & 1st RH

A multimodal transport network model for advanced traveler information systems

Jianwei Zhang^a, Feixiong Liao^a, Theo Arentze^a, Harry Timmermans^{a,*}^a*Urban Planning Group, Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, The Netherlands*

Abstract

There is an application need for seamless multimodal advanced traveler information systems. Currently, no comprehensive network modeling approach exists to deal with routing queries for different private and public transport modes taking into account multiple attributes, dynamic travel times and time tables in large-scale transport networks. The goal of this paper is to develop and test a generic multimodal transport network model for ATIS applications. First, we model multimodal transport networks from an abstract point of view and categorize networks into private and public modes. Then we use a generic method to construct a multimodal transport network representation by using transfer links which is inspired by the so-called supernetwork technique. Among all modes, pedestrian networks play an important role in modeling transfer connections. We test our model and algorithm based on a case study in the Eindhoven region. The results indicate that our model and algorithms provide a suitable basis for ATIS applications. One current limitation is that much time is required for data reading and compiling. This can be solved by implementing existing computational strategies to increase efficiency.

© 2011 Published by Elsevier Ltd. Open access under [CC BY-NC-ND license](#).

Selection and/or peer-review under responsibility of the Organizing Committee.

Keywords: Multimodal transport, traveler information system, route planning, supernetwork

1. Introduction

The term intelligent transport system (ITS) refers to efforts to add information and communication technology to travelers, transport infrastructure and vehicles in an effort to improve safety and reduce vehicle wear, transportation times, and fuel consumption. Generally speaking, ITS can be classified into five categories: Advanced Travelers Information Systems (ATIS), Advanced Traffic Management Systems (ATMS), ITS-Enabled Transportation Pricing Systems, Advanced Public Transportation Systems (APTS), Vehicle-to-Infrastructure Integration (VII) and Vehicle-to-Vehicle Integration (V2V). As an integral important part of ITS, ATIS will provide travelers with pre-trip information about travel options as well as real-time advice on navigating through a dynamic transportation network, where conditions may change rapidly many times in the course of a typical day. Common services provided by ATIS include route planning, navigation, news on disruptions, real time information alerts and so on.

* Corresponding author. Tel.: +31-040-247-2274; fax: +31-040-243-8488.

E-mail address: h.j.p.timmermans@tue.nl.

In many countries, P&R facilities are introduced, which facilitate changes between private (e.g., car) and a public transport mode (e.g., train), to alleviate congestion problems in inner city areas. Therefore, the ability to model multi-modal trips that involves both private and public transport modes is increasingly relevant. The fundamental issues behind above mentioned services are how to model properly the multimodal transport network for ATIS and how to design the corresponding algorithms for supporting queries of travelers. To the best of the authors knowledge, no general multimodal transport network models (or seamless integrated models) and algorithms are available or suitable for large-scale ATIS applications that simultaneously consider private and public transport modes.

The purpose of this study is to develop and test a generic multimodal transport network model for ATIS application that can be used for large-scale transport systems. We propose and test a supernetwork approach where the networks for different modalities are integrated in a single network (Nagurney, 2001, 2002). The model takes into account both time tables of public transport services and dynamic travel times. Time is the only attribute of the model in this research, more attributes and performance optimization will be conducted in following research. The result of a test experiment in Eindhoven region verified the validity and feasibility of our model. The paper is structured as follows: some basic concepts will be introduced in section two; some related researches and applications will be introduced in section three; our model and algorithms will be presented in section four; test results will be discussed in section five; the final section will summarize the major conclusions.

2. Basic concepts

To model the multimodal transport network, it is helpful to take an abstract view at first. The multimodal network can be viewed from many aspects. From a physical point of view, it can be classified into road, rail, water and air. On the other hand, from a functional point of view, it can be classified into private modes (e.g. foot, bike and car) and public modes (e.g. bus, train, tram, metro). An advantage of the functional view is that it highlights the service provision to a traveler. Private networks offer continuous service at any time associated with both physical nodes and physical links. On the other hand, public transport networks offer discrete services according to time tables whereby physical nodes (e.g. stops, stations) are visible while physical links are usually invisible. Therefore, the functional view is suitable for modeling the multimodal transport network.

A second step for modeling involves finding a general representation that supports later multi-criteria evaluation of routes. For private transport networks, the physical nodes and links can be represented as such in a model. For public transport networks, it is more complex. In these networks, the time tables of services determine the transport links; physical links may sometimes even be unknown to the modeler (e.g. metro). Therefore, the goal for modeling multi-modal transport networks is to integrate all above factors together. Two available solutions are known in the literature: one is the time dependent approach where time table events are handled as properties of links (the link costs function); the other is the time expanded approach where time table events are separately represented as event nodes (i.e., arrivals and departures) (Pyrga, Schulz, Wagner & Zaroliagis 2007; Pajor, 2009). To create an integrated multi modal transport network, transfer links between different modes have to be added when all subnetworks are ready. The resulting integrated network is often referred to as a supernetwork. While routing in the supernetwork, the switch between modes occurs only when the transfer is possible. For example, from bus network to car network, there is a link representing picking-up the car only when car is available at the particular location.

The third step for modeling is elaborating the model to make it fully meet the multi-criteria measuring requirement where time, monetary cost, effort and comfort (e.g. quality of mode, transfer time) are all integrated in a generalized costs measure. For time attributes, the private network models (especially for car) can be further classified into three types of links: time independent (the costs of the link are static); time dependent (costs of link vary through time in a known way from history) or stochastic time dependent (both history and real-time information are considered). Public transport network links are always time dependent and possibly stochastic. A general solution is to allow all nodes in a multimodal network to have a timestamp and all links to have a time-depending travel time. Travel time of a next link in an evolving trip can then be determined during the search for an optimal path by keeping track of the time consumed up to the current node and retrieving the appropriate travel time depending on the current time. For monetary cost attributes, the ordinary method can be used where the total monetary costs of a trip are accumulated by money consumed in each fragment of the trip. This is correct in private networks (e.g. car network) but incorrect in some public transport cases. Given a trip composed by three linear ordered nodes A, B, C, the total monetary costs of going from A to C may not be equal to the sum of costs from A to

B and from B to C, for example, due to specific tariff structures. Thus, some extra measures may need to be added to handle this problem. For comfort attributes, the main factors are the quality of services or mode for the link, whereas the transfer and waiting effort should also be considered in the multimodal transport network. This means that transfer links should be explicitly represented in the network in some way (e.g. transfer node, transfer link). To integrate all these aspects in a measure of generalized costs, the key issue is how to judge the relative weights of the different attributes. In most existing approaches (Andre, 2007), one lets the user assign a weight value. A more advanced way is to use conjoint analysis (stated choice experiments) or estimate the weights based on observations of actual travel choices of a sample of individuals.

The final step involves selecting a proper algorithm for computing shortest paths. In the ideal case, a general shortest path algorithm (e.g. Dijkstra, A*) can be implemented directly. If this is not feasible given the network model, then incorporating some form of restriction checking into the algorithm may provide a solution.

3. Related work

Multimodal transport models and algorithms attract many researchers' interests. In Germany, Schultes (2008), Delleng (2009) and Pajor (2009) did extensive research to extend networks from single mode (mainly road network) to multimodal. Liu (2009) proposed a switch point approach to model multimodal transport networks. Mentz Company (Rehrl, 2007) developed a journey planner system and applied it to a regional scale with relatively high spatial resolution (e.g. 511 transit services in San Francisco, journey plan service for Transport for London). Other applications include Bahn (German national railways timetable). In the United States, Zhang (2011), Li (2010), Jariyasunant (2010) reported applications to support mobile multimodal ATIS in California for route planning. Peng (2008) proposed a distributed solution for planning of trips in a larger transport system. Companies like Trapeze, Jeppesen, Google also developed their product. In the United Kingdom, there are also many multimodal ATIS applications like Transport Direct, Journey plan and TFL. Related companies include Logica, Journeyplan and so on. In the Netherlands, Van Nes conducted an extensive (2002) research for designing multimodal transport networks. Beelen (2004) developed a personal intelligent travel assistant for public transport. A national service for public transport route planning in the Netherlands is 9292ov. In other countries and regions, Houda (2006) proposed a public transportation ontology. Ayed (2008) proposed a transfer graph approach for multimodal transport problems. Zografos (2008) described an algorithm for itinerary planning based on dynamic programming. He also reported work on the design and value of online passenger information systems. Wang (2008) did a study on handling times and fares in a routing algorithm for public transport. Su (2010) developed a multimodal trip planning system for intercity transportation in Taiwan. Kumar (2005) developed a multimodal transport system for Hyderabad city in India. All above works reflect some aspects of multimodal ATIS. However, there is no general representation solution for multimodal transport that can take into account a broad range of attributes and transport modes as well as time dependency of transport services. In many cases, the connections between different modes are not clearly described or are handled in an ad-hoc fashion. Besides that, to what extent they balance objectives of accuracy and efficiency, which is important in large-scale applications, is also unknown.

The so-called supernetwork is a network of networks for different modalities or activities. This concept is first introduced by Sheffi (1984) in his theory about urban transport network equilibrium analysis and then extended by Nagurney (2006) to include also non-transport activities (e.g. supply chains, financial networks). Carlier, Fiorenzo-Catalano, Lindveld and Bovy (2003) show how the approach can be used to model multi-modal networks that include both public and private modes. In their model, the networks involved are interconnected seamlessly by transfer links. This general and flexible feature makes models of a multimodal transport networks very suitable for ATIS, especially, because it allows representing transport mode choices as well as the effort involved in transfers from private to public modes. Arentze and Timmermans (2004) have developed a methodology to include also activities at locations and to specify generalized costs of links in a supernetwork as a function of an individual traveller's state which changes as execution of an activity schedule progresses. Liao et al. (2009) have extended this approach and more importantly have demonstrated the possibility of fast algorithms to find optimal paths in these complex networks. Although the supernetwork approach is not new, there is no explicit description for modeling processes and precision requirements for ATIS applications based on a supernetwork. Besides that, a novel element of the supernetwork approach is that the time-dependent (private vehicles) and time-expanded (public transport) methods are combined in a single network.

4. Method

First, from the abstract view, we would like to model the multimodal transport network distinguishing two types of networks: the private and the public. In the private transport network, only physical nodes are contained whereas in public transport network, both physical nodes and event nodes are included to account for time tables of public transport services. An abstract fragment of a private transport network is represented in figure 1. All nodes and links in a private transport network are physical links (road segments).

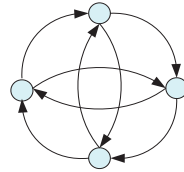


Figure 1 Abstract fragment for private transport network

An abstract fragment of a public transport network is represented in figure 2 which includes a stop (e.g., a bus stop or train station) and related events (Pajor, 2009). The top-level node is a physical node (the stop) while the other nodes are event nodes (arrivals or departures). Each event node has a link to the stop node and the direction is decided by event type. If the event type is arrival then the direction is from event node to stop node. If event type is departure, then the direction is reversed. Besides that, all event nodes are ordered in the way that a higher-level node refers to an earlier event. The directions of links between event nodes are from an earlier event to a later event. The latter links refer to either waiting or transferring. Another type of link is a trip sequence link which connects event nodes between stops from an earlier event to later event. These links represent movement of a public vehicle from one stop to another stop.

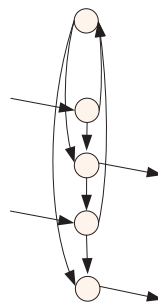


Figure 2 Abstract fragment of public transport network

Second, from the general view, we would like to have a general representation of a multimodal transport network where the basic elements are nodes and links. This is described in figure 3. As we mentioned before, there are two kinds of nodes in a supernetwork: physical nodes which represents locations and have X, Y coordinates as necessary attributes, and event nodes which represent the arrival and departure events at certain stops or stations. The latter nodes have event type, event time and service related factors (e.g. bus route, but trip, stop sequence) as necessary attributes. Only physical links have distance, time, speed, monetary costs, emission, quality and general cost as necessary or possible attributes.

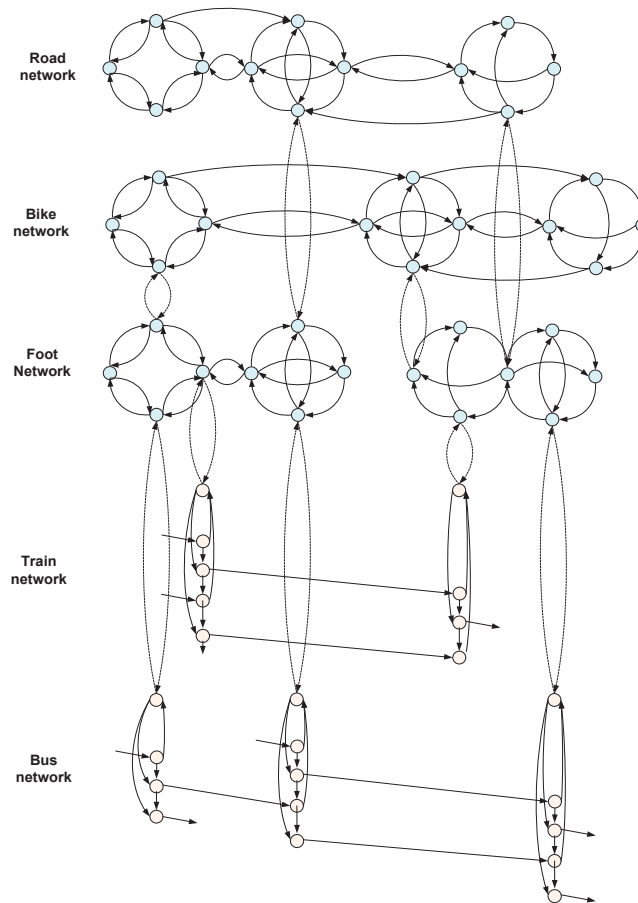


Figure 3 Representation of a multimodal transport network

It should be noticed that there are two kinds of transfers: one is within a same mode (e.g. transfer from one bus line to another), the other is between different modes (e.g., transfer from bus to train) which are represented by dashed lines in figure 3. The foot (pedestrian) network plays a key role in mode transfers: all transfer links are connected to this network, as walking is always involved in such transfers. There are several solutions for adding transfer links. A simple solution is that each node in a particular layer is linked to the nearest neighborhood node in the foot network. Another possible solution is to find the nearest *link* in the foot network and insert a transfer node at the intercept point. The latter one is more realistic, but requires more operations.

Different degrees of elaboration of a network exist. It is important to check whether a model meets the requirements for measuring and calculating all performance characteristics of routes that are considered important. If not, one has to modify or elaborate. In this study, we mainly focus on travel time. The model displayed in figure 3 is adequate for accurate time calculations in a multimodal transport network. This model may not be appropriate, however, for complex fare computations. Required extensions will be considered in future research.

To test whether this conceptual model works well, we program the algorithms to generate the structures using data of real networks and transport services. Thus, the algorithms needed are twofold: an algorithm for compiling the multimodal transport network based on data about road networks and public transport services and a routing algorithm that is able to find multi-modal routes as shortest paths through the network. Figure 4 shows a flow diagram of the compiling algorithm part.

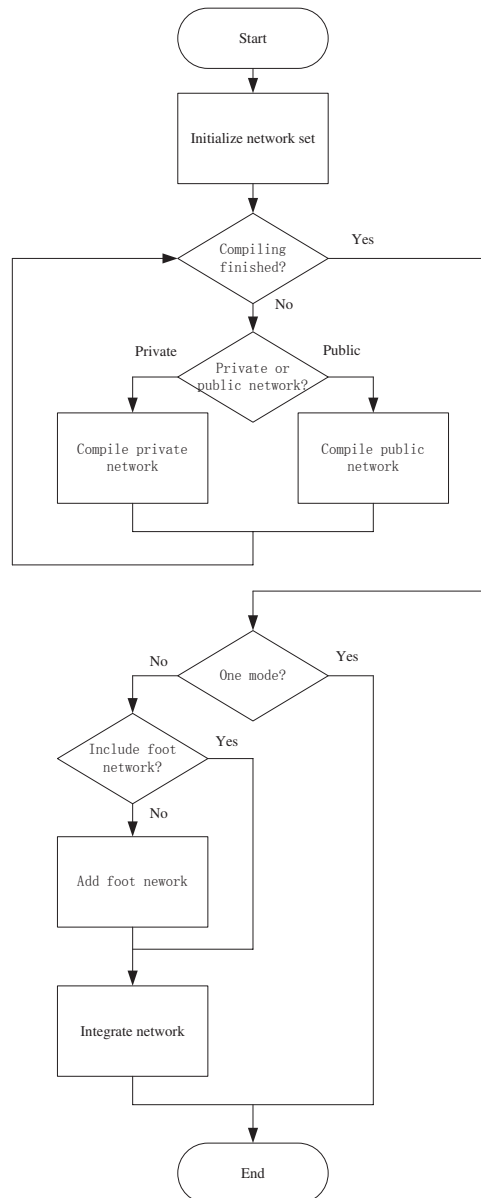


Figure 4. Flow diagram of the algorithm for compiling multimodal transport networks

The algorithm consists of three steps: 1) initialize individual networks; 2) compile each individual network and 3) integrate the individual networks into a single multimodal transport network. In the compilation step, there are two compilers corresponding to two kinds of abstract networks – private and public. In the integration step, which is relevant only if there is more than one mode, the foot network is crucial for determining transfer links. The foot network needs to be added and compiled also when it is not in the network mode set because this mode is always involved in transfers. In the integration step, for simplification, we choose to add transfer links through searching nearest nodes in the foot network. When a full multimodal transport network has been constructed in this way, we can use routing algorithms to check the consistency of the model. Commonly used algorithms include the Dijkstra algorithm and A*. In terms of calculating the time between two nodes, we distinguish four link cases: location node to location node; location node to event node; event node to location and event node to event node (figure 5).

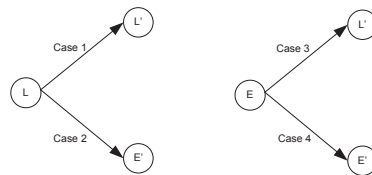


Figure 5 Four cases for calculation time on link

In above figure 5, L means location node and E means event node. To calculate the time spent on the link, the following rules can be used. If the origin node is a location node and the target node is also a location node, then time on the link equals the length of the link divided by speed. If the origin node is a location node and the target node is an event node, there are two cases. If the time stamp of the event node is later than the current time at the location node then the time on the link equals the difference, else it equals positive infinity (the node cannot be reached). If the origin node is an event node and the target node is a location node, then the time on the link equals the length of the link divided by speed. If the origin node is an event node and the target node is also an event node, then time on the link equals the difference between their timestamps (waiting time). Given the way we compile the whole network (as explained before), the timestamp of the target node will always be later than the origin node. In summary, we define the time on a link as follows:

$$\text{Time on link} = \begin{cases} \text{length} / \text{speed} & \text{case 1} \\ E'.\text{timestamp} - L.\text{timestamp} & \text{case 2a} \\ +\infty & \text{case 2b} \\ \text{length} / \text{speed} & \text{case 3} \\ E'.\text{timestamp} - E.\text{timestamp} & \text{case 4} \end{cases}$$

To accelerate the speed of path-search algorithms, strategies like hierarchical architecture, pre-processing data (combination of nodes) and bidirectional search, heuristic searching can also be incorporated. However, in the present study, we use the classic Dijkstra algorithm without any accelerating strategy.

We collected public transport and road network data for the Eindhoven region to test the algorithms. The data include car nodes, car links, bike nodes, bike links, foot nodes, foot links, bus stops and bus time tables. Accordingly, foot, bike, car and bus are selected as mode set. We assumed that the speed of walking is 4.5 km/h, the speed of cycling is 12 km/h, and the speed of car follows the maximum speed limitation for the road concerned. We consider a trip from TU/e Campus to Woensel shopping center. The start time of the trip is set as Feb. 11 2011, 18:10:00. Each single mode and mode combination will be considered as an individual test case. The foot network includes 4,654 nodes and 6,819 links. The bike network includes 4646 nodes and 6781 links. The car network includes 4755 nodes and 6957 links. In terms of the bus network, there are 1736 bus stations and 121584 arrival/departure events for one day, given the time tables of the bus lines in the city. The details of the test environment are as follows: CPU: Intel E8400 RAM: 2G (shared with graphic card); operation system: Windows XP SP3; database: MySQL 5.5; IDE: Netbeans 6.7.1; programming language: JAVA; algorithm: basic Dijkstra.

5. Results

Table 1 shows the computation times for different modes and mode combinations. Route calculation is based on the basic Dijkstra algorithm. “Data reading and compiling” refers to computation time involved in constructing a multimodal network from the database in computer RAM. As can be seen from the table, the computation time for route search is very short (< 0.01s); the bottle neck lies in the data reading and compiling. Although data reading and compiling time for a single mode is short, it increases dramatically when the bus mode is added or more than one mode is included in the mode set. The reason is that a lot of time is consumed in searching for neighborhood nodes in the foot network for adding transfer links. This computation time can be reduced to acceptable levels by using indexing or pre-compiling technologies. Also cache technology will greatly reduce the time for data reading and compiling.

Table 1 Algorithm efficiency results

Mode	Route calculation(s)	Data reading and compiling(s)	Total time(s)
Foot	0.005	0.341	0.346
Bike	0.005	0.337	0.342
Car	0.004	0.337	0.341
Bus	0.077	3.323	3.4
Foot +bike	0.008	73.863	73.871
Foot +car	0.007	75.499	75.506
Foot +bus	0.041	55.544	55.585
Foot +bike+car	0.009	149.685	149.694
Foot + bike+bus	0.065	198.272	198.337
Foot +car +bus	0.034	211.791	211.825
Foot+bike+car +bus	0.092	370.320	370.412

Figure 6 to figure 16 shows the route planning results of different mode sets. . As can be seen from the figures, the foot route is similar to the bike route whereas the car route is quite different. This is due to the fact that some of the roads have stricter speed limitations for car. For single bus mode, the result is not very reasonable as in reality a traveler can always walk to the destination with quite short time. With the foot mode excluded the model suggests (correctly) to wait twelve minutes and transfer to another bus for reaching the destination. This indicates the necessasity to include the foot mode always in combination with public transport mode. For foot + bike or foot + car mode, only the start node and end node are in foot mode; the rest trip uses bike or car as time is the only costs attribute considered and cycling or driving always costs less time than walking. For foot + bus, the result is very reasonable. It suggests to first walk ten minutes to a proper station and then wait one minute to take a bus to the destination. For foot + bike + car, bike has not appeared because of the presence of car. For foot + bike + bus, the result is also very reasonable implying that foot mode only appeared at the start and the end; bike is used to transit to the nearest proper bus station. For foot + car + bus and foot + bike + car + bus, the routes are identical to the car route as car is faster than any other mode and only time was taken into account.

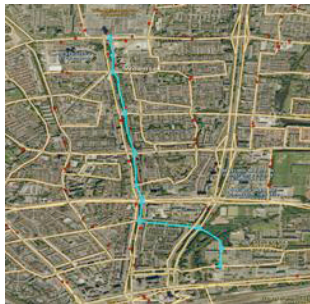


Figure 6 Foot route

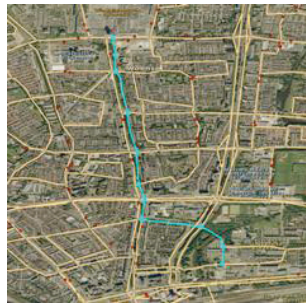


Figure 7 Bike route

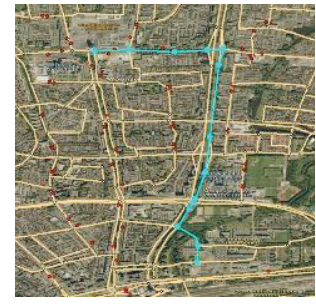


Figure 8 Car route

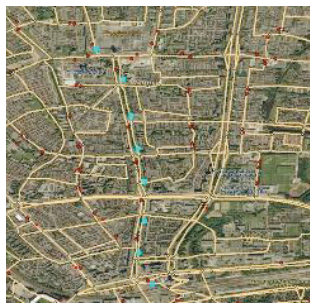


Figure 9 Bus route



Figure 10 Foot + bike route

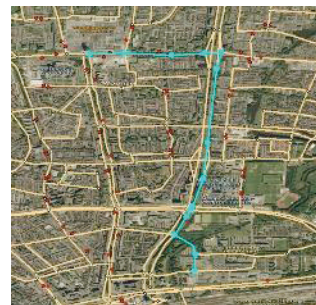


Figure 11 Foot + car route

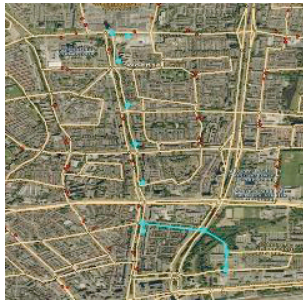


Figure 12 Foot + bus route

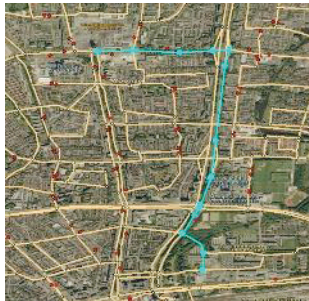


Figure 13 Foot + bike + car route

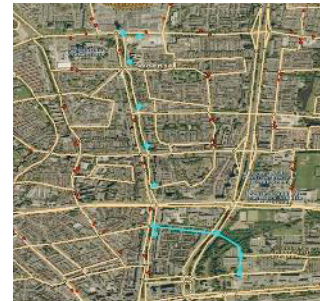


Figure 14 Foot + bike + bus route

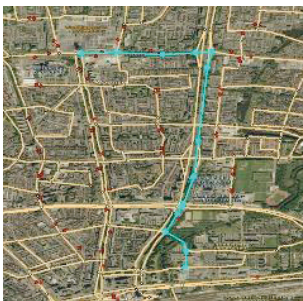


Figure 15 Foot + car + bus route

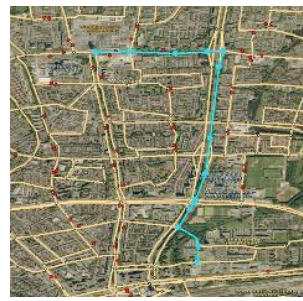


Figure 16 Foot + bike + bus + car route

6. Conclusion and discussion

The test algorithm and results verified the validity and feasibility of our proposed generic multimodal transport network model. The test results indicate that even the basic Dijkstra algorithm could be used to find high quality routes in short computation time for realistic networks. However, a limitation of the current model is that long computation time is needed to read and compile the integrated network. We will test several accelerating strategies in future research. Furthermore, we will extend the set of attributes included in generalized costs functions for links.

There is no generic multimodal transport network model existing for ATIS applications. In this study, we proposed an approach that is based on an abstract modeling view (private network and public transport) and is inspired by the supernetwork technique (a general way to construct an integrated multimodal transport network). Results of a test experiment in the Eindhoven region indicate that our model and algorithms provide a suitable basis for ATIS applications.

Acknowledgements

i-Tour project is conducted by a consortium including FORMIT Servizi S.p.A., Graphitech Foundation, University College of London, Eindhoven University of Technology, ULA S.r.l., ELASIS S.c.p.A., PTV AG, Cadzow Communications Consulting Ltd, FORMIT Foundation. The research leading to these results has received funding from the European Community's Seventh Framework Program (FP7/2007-2013) under the Grant Agreement number 234239. The authors are solely responsible for the information reported in this paper. It does not represent the opinion of the Community. The Community is not responsible for any use that might be made of the information contained in this paper.



References

- Andre, P., Wilson, M. L., Owens, A. & Smith, D. A. (2007). Journey planning based on user needs. *Computer/ Human Interaction 2007 Conference*, San Jose.
- Arentze, T., & Timmermans, H. J. P. (2004). Multistate supernetwork approach to modeling multi-activity, multimodal trip chains. *International Journal of Geographic Information Science*, 18(7), 631–651.
- Ayed, H., & Khadraoui, D. (2008). Transfer graph approach for multimodal transport problems. *Second International Conference MCO*, Metz, France – Luxembourg.
- Beele, M. (2004) *Personal intelligent travel assistant: a distributed approach*. Master's thesis of Delft University of Technology
- Carlier K., Fiorenzo-Catalano, S., Lindveld, C. & Bovy, P. (2003). A supernetwork approach towards multimodal travel modeling. *Transportation Research Board 2003 Annual Meeting*, Washington, D. C.
- Delling, D., Pajor T. & Wagner, D. (2009) Accelerating Multi-Modal Route Planning by Access-Nodes. *17th Annual European Symposium on Algorithms (ESA'09)*, Copenhagen, Denmark.
- Houda, M., Khemaja, M., Oliveira, K & Abed, M. (2010). A public transportation ontology to support user travel planning. *IEEE International Conference on Research Challenges in Information Science*, Nice, France.
- Jariyasunant, J. (2010). Mobile transit trip planning with real-time data. *Transportation Research Board 2010 Annual Meeting*, Washington, D. C.
- Kumar, P, Singh, V, Reddy, D. (2005). Advanced traveller information system for hyderabad city. *IEEE Transactions on Intelligent Transportation Systems*, 6(1), 26-37.
- Liao, F., Arentze, T., & Timmermans, H. J. P. (2010). A heuristic approach for constructing personalized transportation networks: a supernetwork approach. *Transportation Research Board 2010 Annual Meeting*, Washington, D. C.
- Li, J. Q., Zhou, K., Zhang, L. & P. Zhang, W. B., (2010) A multimodal trip planning system incorporating the park-and-ride mode and real-time traffic/transit information. *Transportation Research Board 2010 Annual Meeting*, Washington, D. C.
- Liu, L., Meng, L. (2009): Algorithms of multi-modal route planning based on the concept of switch point. *6th International Symposium on LBS & TeleCartography*, Nottingham, UK.
- Nagurney, A, Dong, J., Mokhtarian, P. L. (2001). Teleshopping versus shopping: a multicriteria network equilibrium framework. *Mathematical and Computer Modelling*, 34 (7-8), 783–798.
- Nagurney, A, Dong, J. (2002). *Supernetworks: decision-making for the information age*. Edward Elgar Publishing.
- Nagurney, A. (2006). *Supply chain network economics: dynamics of prices, flows, and profits*. Edward Elgar Publishing.
- Pajor, T. (2009). *Multi-modal route planning*. Master thesis of Karlsruhe Institute of Technology.
- Peng, Z. R., Kim, E. (2008). A standard-based integration framework of distributed transit trip planning systems. *Journal of the Intelligent Transportation Systems*, 12(1), 13-19.
- Pyrge, E., Schulz, F., Wagner, D. & Zaroliagis, C. (2007). Efficient Models for Timetable Information in Public Transportation Systems. *ACM Journal on Experimental Algorithms*, 12(2.4), 1--39.
- Rehrl, K., Brunsch, S., Mentz, H. J. (2007). Assisting multimodal travelers: design and prototypical implementation of a personal travel companion. *IEEE Transactions on Intelligent Transportation Systems*, 8(1), 31-41.
- Schultes, D. (2008). *Route planning in road networks*. Ph.D Thesis of Karlsruhe Institute of Technology.
- Sheffi, Y. (1985) *Urban transportation networks: equilibrium analysis with mathematical programming methods*, Prentice-Hall.
- Su, J.M. Chang, C.H., Ho, W.C., (2008) Development of trip planning systems on public transit in Taiwan, *IEEE International Conference on Networking, Sensing and Control*, Hainan, China.
- Transport for London from <http://www.tfl.gov.uk/>.
- van Nes, R. (2002) *Design of multimodal transport networks: a hierarchical approach*. Ph.D thesis of Delft University of Technology.
- Wang, M. (2008). *Optimal paths based on Time and fare in transit network*. Master thesis of National Cheng Kung University.
- Zhang, L., Li, J.Q., & Zhou, K.. (2011). Design and implementation of a traveler information tool with integrated real-time transit information and multi-modal trip planning. *Transportation Research Board 2011 Annual Meeting*, Washington, D. C.
- Zografos, K. G., & Androustopoulos, K.N. (2008). Algorithms for itinerary planning in multimodal transportation networks. *IEEE Transactions on Intelligent Transportation Systems*, 9 (1), 175-184.

511 transit from <http://www.511.org>.