Abstract

A brassiere cup consists of several patterns. Traditionally, it is designed by repeatedly creating a paper cup model and verifying its three-dimensional (3D) shape. Reducing such trial and error will enhance the design efficiency. A method to partly automate the design of two-dimensional shapes in the patterns for a two-piece brassiere cup is proposed when the target 3D shape is given as a cloud of data points. It is assumed that the surface of the model is composed of several developable surfaces because the model is made of paper and not cloth. If two lines lying on a developable surface are given, the shape of the surface can be determined. Consequently, a two-piece brassiere cup can be designed by minimizing the error between the surface and the given data points. Here, we mathematically verify that our proposed method can reproduce the original developable surface.

Paper

DESIGN OF A TWO-PIECE BRASSIERE CUP TO FIT BREAST DATA POINTS TOWARD ITS AUTOMATION[*]

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A brassiere cup consists of several patterns. Traditionally, it is designed by repeatedly creating a paper cup model and verifying its three-dimensional (3D) shape. Reducing such trial and error will enhance the design efficiency. A method to partly automate the design of two-dimensional shapes in the patterns for a two-piece brassiere cup is proposed when the target 3D shape is given as a cloud of data points. It is assumed that the surface of the model is composed of several developable surfaces because the model is made of paper and not cloth. If two lines lying on a developable surface are given, the shape of the surface can be determined. Consequently, a two-piece brassiere cup can be designed by minimizing the error between the surface and the given data points. Here, we mathematically verify that our proposed method can reproduce the original developable surface.

INTRODUCTION

Developable surfaces can be unfolded into a plane without expanding or contracting. They are widely used in many industries from shipbuilding to manufacturing of clothing because they can represent surfaces made of leather, paper, or sheet metal. It is important to design two-dimensional (2D) shapes that form the required three-dimensional (3D) shape by bending and joining. In this paper, we focus on production of brassieres, which is related to design of two-dimensional shapes of plates. Brassieres are manufactured to meet various demands, such as to Herein we focus on the production of brassieres, which is related to the design of 2D shapes of plates. Brassieres are manufactured to meet various demands such as enhancing a woman ' s breast size, creating cleavage, or minimizing breast movement. Due to these diverse demands, the cup shape is a critical element when designing a brassiere. A brassiere cup is formed by several pieces of cloth called patterns and a wire. Figure 1 shows an example of a two-piece brassiere cup composed of an upper pattern, lower pattern, and lower line. The three resulting curves are important to the design. The wire line corresponds to the boundary between the breast and body (Fig. 2). The ridge line of the cup corresponds to the outline of a bust on a transverse plane, while the upper line connects the cup to the shoulder strap.

(a) upper pattern (b) lower pattern

(c) lower line

Fig. 1 Parts of a two-piece brassiere cup

In apparel industries, the form of a product is fixed by fashion designers and pattern makers. Fashion designers draw the product image, while pattern makers determine the pattern shapes to meet various demands of the ideal shape. Fig. 3 overviews the design process of a brassiere cup. Pattern makers first consider the functional requirement of a brassiere cup based on the breast shape, and then they determine the pattern shape. This design process is based on experience and intuition. Hence, pattern makers must verify the shapes with a paper model of the cup and repeatedly modify the shapes to realize the intended

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Fig. 2 Expression of a wire line, a ridge line and an upper line

function. This process may cause unnecessary repetition. To improve the design efficiency of brassieres, trial and error should be minimized. Herein we strive to automate the steps indicated by the dotted line in Fig. 3. A previous study on the influence of design on the function of a brassiere cup found that fitting the breast has the largest impact[1]. Hence, this study proposes a method to design the pattern shapes of a brassiere cup to fit the given breast shape. Here, the given 3D breast shape is assumed to be a cloud of data points, which can be obtained easily by measurements.

Fig. 3 Design process of a brassiere cup

Because the cup shape is made by bending and sewing each pattern and is assumed to be inextensible, a cup model consists of several developable surfaces. With respect to modeling of developable surfaces, previously proposed modeling methods have focused on the geodesic line[2], B-spline or NURBS surface[3,4], offsets of Bertrand curves, which coincide with each normal direction[5,6], and arbitrary curves[7]. Martin proposed a method to reconstruct a developable surface from its point clouds based on the Laguerre geometry[8]. Chen et al. proposed an algorithm to approximate a developable surface from its point cloud[9]. However, none of these methods reference the developed shape as they focus on a single developable surface. Manufacturing of a brassiere cup requires the pattern shapes (i.e., developed shapes of a cup) but not the 3D cup shape.

In a study to approximate developable surface, Yang et al. proposed a method to approximate surface with meshes composed of planar quadrilaterals[?] and Odeo et al. proposed a method to define the developability of triangle meshes and appoximate them by a set of developable surfaces[?], and Martin et al. proposed a method to reconstruct surfaces by curved forming[?]. But all of them only focus on approximating surface by developable surface, which means that they do not refer to the condition of a developable surface itself. When designing a brassiere, it is required to satisfy some conditons of a brassiere cup that designers intend.

In a study on brassieres, Wakamatsu et al. proposed a method to predict the 3D shape of a paper cup model when the 2D shape of a pattern is given[10,11]. Hence, patterns can be evaluated without actually creating a paper model. However, repetitive modification of the pattern is still required to obtain the target 3D shape of the cup. Ito et al. developed a paper model CAD system based on the theory of developable surfaces[12]. If a 3D curve, which means a sewn curve, and a 2D curve, which means a piece to be sewn, are given, this system can identify the feasible region where the sewn surface does not intersect itself. Unfortunately, it is difficult to design the pattern shapes. That is, determining the shape of 2D curves is challenging with this system. We previously proposed a method to design the pattern shape and its developable surface from two lines: a wire line and a ridge line. These lines are obtained by solving the optimization problem when the design parameters are the geodesic curvatures of a pattern shape and the objective function is the error between generatrices determined by the geodesic curvature of a lower edge and the wire line and generatrices determined by the geodesic curvature of an upper edge and the ridge line. However, the previously proposed method requires an iterative process to solve the optimization problem for the intended shape[14], which is time-consuming. Brassiere patterns are typically comprised of two or three pieces. Herein we focus on a two-piece brassiere cup, and we propose a design method to improve the design efficiency when a cloud of its data points is given.

MODELING OF DEVELOPABLE SURFACE AND PATTERN SHAPE

This section explains the numerical expressions of a developable surface. The curvature of a space curve on a surface can be divided into two: the normal curvature and geodesic curvature. The former is realized by deforming the surface, while the latter is realized by deforming a space curve. Deforming the surface does not change the geodesic curvature as long as the surface is not stretched. In the case of a brassiere cup, only the normal curvature can be manipulated by deforming the surface. Because the geodesic curvature cannot be manipulated by deforming the surface, it must be designed such that the space curve coincides with a boundary of the developable surface. Here, we propose a method to determine a developable surface using two curves that lie on the surface and design a pattern shape. Consider the case where the *lower edge* is the curve connecting the lower wire (Fig. (b)1(b)) and the *upper edge* is that combining the upper cup (Fig. (a)1(a)). Our method expresses the developable surface using the parameters of a space curve, the condition of two lines that lie on the developable surface, and their obtained pattern shape. Fig. 4 depicts the object coordinate system on the lower edge of the cup surface, where the ζ-axis always coincides with the tangential direction of the edge line and the η-axis always coincides with the surface normal direction. Deformation of the cup changes the posture of the object coordinate system. Thus, the infinitesimal displacement vector of each axial direction can be described by the infinitesimal rotational ratio vector $\boldsymbol{\omega} = [\omega_\xi \ \omega_\eta \ \omega_\zeta]^{\mathrm{T}}$ as

$$\left[\, \boldsymbol{\xi}' \ \boldsymbol{\eta}' \ \boldsymbol{\zeta}' \,\right] = \left[\, \boldsymbol{\xi} \ \boldsymbol{\eta} \ \boldsymbol{\zeta} \,\right] \boldsymbol{\Omega}(\boldsymbol{\omega}), \tag{1}$$

where a prime means a derivative of s, and $\boldsymbol{\Omega}(\boldsymbol{\omega})$ is represented as

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} 0 & -\omega_\zeta & \omega_\eta \\ \omega_\zeta & 0 & -\omega_\xi \\ -\omega_\eta & \omega_\xi & 0 \end{bmatrix}. \tag{2}$$

Then, the tangent vector is expressed as

$$\boldsymbol{\zeta}(s) = \boldsymbol{\zeta}_0 + \int_0^s (\omega_\eta \boldsymbol{\xi} - \omega_\xi \boldsymbol{\eta})\, ds, \tag{3}$$

where $\boldsymbol{\zeta}_0$ represents the tangent vector at $s = 0$. By integrating eq. (3), the position of the curve can be calculated as

$$\boldsymbol{x}(s) = \boldsymbol{x}_0 + \int_0^s \boldsymbol{\zeta}\, ds, \tag{4}$$

where $\boldsymbol{x}_0$ indicates the position at $s = 0$.

Fig. 4 Object coordinate system on the surface

First, consider the numerical expression of a developable surface constrained by a Gaussian curvature. In general, the normal curvature of a direction vector $\boldsymbol{d}_\theta = \boldsymbol{\zeta}\cos\theta + \boldsymbol{\xi}\sin\theta$ can be described using coefficients of the first and the second fundamental forms E, F, G, L, M and N as

$$\kappa_\theta = \frac{L\cos^2\theta + 2M\sin\theta\cos\theta + N\sin^2\theta}{E\cos^2\theta + 2F\sin\theta\cos\theta + G\sin^2\theta}. \tag{5}$$

Then the Gaussian curvature K and the mean curvature H, which characterize a surface, can be defined as extreme values of eq. (5): $\kappa_{\max}$ and $\kappa_{\min}$ as follows[**?**]

$$\begin{aligned} K &= \kappa_{\max}\kappa_{\min} = \frac{LN - M^2}{EG - F^2}, \\ H &= \frac{\kappa_{\max} + \kappa_{\min}}{2} = \frac{EN - 2FM + GL}{2(EG - F^2)}. \end{aligned} \tag{6}$$

Here, the coefficients of the first fundamental form are represented as

$$E = \boldsymbol{\zeta}\cdot\boldsymbol{\zeta} = 1, F = \boldsymbol{\zeta}\cdot\boldsymbol{\xi} = 0, G = \boldsymbol{\xi}\cdot\boldsymbol{\xi} = 1. \tag{7}$$

In addition, the coefficients of the second fundamental form are represented as

$$L = \boldsymbol{\zeta}'\cdot\eta = -\omega_\xi, M = \boldsymbol{\xi}'\cdot\eta = -\omega_\zeta. \tag{8}$$

Then Gaussian curvature K and the mean curvature H are described by the following equations

$$K = -\omega_\xi N - \omega_\zeta^2, \tag{9}$$

$$H = \frac{-\omega_\xi + N}{2}. \tag{10}$$

A developable surface is defined as a surface with a Gaussian curvature $K = 0$. That is $\kappa_{\min} = 0$. TThus, the mean curvature can be calculated as $2H = \kappa_{\max}$. The line direction coinciding with the direction $\boldsymbol{d}_{\min}$ remains straight after a deformation. This straight line is referred to as a generatrix. In this paper, the principal directions are described using the angle α as

$$\begin{aligned} \boldsymbol{d}_{\max} &= \boldsymbol{\zeta}\cos\alpha + \boldsymbol{\xi}\sin\alpha, \\ \boldsymbol{d}_{\min} &= -\boldsymbol{\zeta}\sin\alpha + \boldsymbol{\xi}\cos\alpha. \end{aligned} \tag{11}$$

The angle α is referred to as the *rib angle* in this paper. By solving $\kappa_\theta = 0$, the rib angle α can be calculated as

$$\tan\alpha = -\frac{\omega_\zeta}{\omega_\xi}. \tag{12}$$

From eq. (9), κ_1 is calculated as

$$\kappa_1 = -\frac{\omega_\xi^2 + \omega_\zeta^2}{\omega_\xi}. \tag{13}$$

From above, a developable surface can be determined by $\boldsymbol{\omega}$.

Next, consider the constraint of a developable surface when two curves lie on the surface. Let s_a and s_b be the arc length and $\boldsymbol{x}_a(s_a)$, $\boldsymbol{x}_b(s_b)$ be positions of each curve. We assume that these curves have C^2 community at least. In general, developable surfaces are a special kind of ruled surface. When a surface is a ruled surface, the generatrix $\boldsymbol{g}$ is described as

$$\boldsymbol{g} = \boldsymbol{x}_b(s_b) - \boldsymbol{x}_a(s_a). \tag{14}$$

Note that s_a can take an arbitrary value against

s_b. Therefore, the variation of a generatrix $\Delta\boldsymbol{g}$ is described by total differential form as

$$\Delta\boldsymbol{g} = -\boldsymbol{\zeta}_a \Delta s_a + \boldsymbol{\zeta}_b \Delta s_b, \tag{15}$$

where $\boldsymbol{\zeta}_a$ and $\boldsymbol{\zeta}_b$ are tangent vectors of $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$, respectively. Next, let Θ be the infinitesimal plane surrounded by four vectors $\boldsymbol{\zeta}_a\Delta s_a, \boldsymbol{\zeta}_b\Delta s_b, \boldsymbol{g}$, and $\boldsymbol{g}+\Delta\boldsymbol{g}$ (Fig. 5). When the ruled surface is a developable surface, the plane Θ is a tangent plane without twisting. Therefore, it leads to following equation:

$$\det(\boldsymbol{\zeta}_a, \boldsymbol{g}, \boldsymbol{g}+\Delta\boldsymbol{g}) = 0 \tag{16}$$

Solving eq. (16) gives

$$\det(\boldsymbol{\zeta}_a, \boldsymbol{\zeta}_b, \boldsymbol{g}) = 0 \tag{17}$$

Fig. 5 Definition of Θ

eq. (17) indicates that the arc length of each curve is subordination. When two curves are corresponded to edge lines and s_b can be expressed as $s_b(s_a)$, $\boldsymbol{d}_{\min} \equiv \dfrac{\boldsymbol{g}}{|\boldsymbol{g}|}$ is established. Then, a rib angle is calculated by eq. (11) as follows:

$$\alpha = -\sin^{-1}(\boldsymbol{d}_{\min}\cdot\boldsymbol{\zeta}_a). \tag{18}$$

Using $\boldsymbol{d}_{\min}$ and α, $\boldsymbol{\omega}$ can be represented as

$$\boldsymbol{\omega} = \begin{bmatrix} -\det(\boldsymbol{\zeta}'_a, \boldsymbol{\zeta}_a, \boldsymbol{d}_{\min}) \\ \boldsymbol{\zeta}'_a \cdot \boldsymbol{d}_{\min} \\ \det(\boldsymbol{\zeta}'_a, \boldsymbol{\zeta}_a, \boldsymbol{d}_{\min})\tan\alpha \end{bmatrix} \tag{19}$$

Therefore, when two curves that lie on the surface are given, a developable surface can be determined.

Now let ' s consider how to determine the pattern shape from a developable surface. The planar curve generated by developing the space curve $\boldsymbol{x}_a$ is defined as the lower curve, whereas that generated by developing the space $\boldsymbol{x}_b$ is defined as the upper curve. Fig. 6 shows the developed planar coordinate system vw so that v-axis always coincides the line AB. Let μ_a be the angle between the tangential direction of the lower edge and the v-axis. If the curvature of the lower curve ω_{η_a} is given, μ_a is obtained as

$$\mu_a = \mu_0 + \int_0^{s_a} \omega_{\eta_a} ds_a, \tag{20}$$

where μ_0 represents the angle of the lower edge at $s=0$, which is discussed later. From eq. (20), the planar position $\boldsymbol{x}_{ae}$ is described as

$$\boldsymbol{x}_{ae} = \int_0^{s_a} \begin{bmatrix} \cos\mu_a \\ \sin\mu_a \end{bmatrix} ds_a \tag{21}$$

With respect to eq. (20), let a,b be defined as follows:

$$a = \int_0^{L_a} \cos\left(\int_0^{s_a} \omega_{\eta_a} ds\right) ds_a, \tag{22}$$

$$b = \int_0^{L_a} \sin\left(\int_0^{s_a} \omega_{\eta_a} ds\right) ds_a. \tag{23}$$

Then, μ_0 can be calculated as follows:

$$\mu_0 = \tan^{-1}\frac{b}{a} \tag{24}$$

Let $\boldsymbol{x}_{be}$ be the planar position of an upper edge. Since the rib angle of a pattern α_a and the length of a generatrix of a pattern $|\boldsymbol{g}|$ do not change by being developed, $\boldsymbol{x}_{be}$ can be calculated as

$$\boldsymbol{x}_{be} = \boldsymbol{x}_{ae} + |\boldsymbol{g}| \begin{bmatrix} \cos(\mu_a + \pi/2 + \alpha_a) \\ \sin(\mu_a + \pi/2 + \alpha_a) \end{bmatrix} \tag{25}$$

From above, a developed shape can be obtained when two curves, which lie on the developable surface, are given.

Fig. 6 Description of pattern shape using geodesic curvature of the lower curve

EXPRESSION OF THE DESIGN PROCESS AS AN OPTIMIZATION PROBLEM

From section 2, the cup shape can be determined by three curves: a wire line, a ridge line, and an upper line. If the wire line is given, designing the parameters for the ridge and upper lines such that the degree of fitting to the 3D breast shape is minimized will enhance the design efficiency. This section formulates the optimization problem that our proposed method aims to solve.

Formulation of the error between a point and a surface

To formulate the degree of fitting to a 3D ideal shape, we formulate the error between a point and a surface. In general, the position $\boldsymbol{X}(s,t)$ of a developable surface S is expressed as

$$\boldsymbol{X}(s,t) = \boldsymbol{x}(s) + t\boldsymbol{d}_{\min}(s) \tag{26}$$

The distance $\varepsilon(\boldsymbol{p})$ between a point $\boldsymbol{p}$ and a developable surface is formulated by the difference vector $\boldsymbol{\delta} = \boldsymbol{p} - \boldsymbol{X}(s,t)$ as

$$\varepsilon(\boldsymbol{p}) = \min_{s,t} |\boldsymbol{\delta}| \tag{27}$$

When a set of surface parameters (s^*, t^*) satisfies to minimize eq. (27), $\boldsymbol{\delta}$ is parallel to $\boldsymbol{\eta}$. Therefore, the following equations are satisfied

$$\boldsymbol{d}_{\min} \cdot (\boldsymbol{p} - \boldsymbol{X}(s^*, t^*)) = 0 \tag{28}$$

$$\boldsymbol{d}_{\max} \cdot (\boldsymbol{p} - \boldsymbol{X}(s^*, t^*)) = 0 \tag{29}$$

By solving eq. (28), t^* is described as

$$t^* = \boldsymbol{d}_{\min} \cdot (\boldsymbol{p} - \boldsymbol{X}(s^*)) \tag{30}$$

By using eq. (30) and solving eq. (29), s^* can be determined.

Let s_w, s_r, s_u be the arc lengths of the lower wire, ridge line, and upper line and let $\boldsymbol{\omega}_W(s_w), \boldsymbol{\omega}_R(s_r), \boldsymbol{\omega}_U(s_u)$ be vectors to characterize each of them, respectively. From eq. (17), s_r, s_u can be expressed as functions of s_w: $s_r(s_w), s_u(s_w)$. As mentioned, a two-piece brassiere cup is composed of two developable surfaces S_L and S_U. Let D be a set of coordinates of points on a breast, $D_L = \{\boldsymbol{p}_{L,i} \in D\}$ be a set of points to evaluate the surface S_L, and $D_U = \{\boldsymbol{p}_{U,i} \in D\}$ be a set of points to evaluated the surface S_U. Note that $D_U = D \backslash D_L$. Whether a point $\boldsymbol{p}_k$ is classified to the set of D_U or the set of D_L can determined with the length of the nearest generatrix.

First, using eqs. (28)–(30), the nearest surface parameters (s_w^*, t_w^*) of the lower cup to the point $\boldsymbol{p}_k$ can be derived. Then, the nearest point $\boldsymbol{X}(s^*, t^*)$ on the lower cup surface to the point $\boldsymbol{p}_k$ can be derived. Let $t_{\max} = |\boldsymbol{x}_R(s_r(s_w^*)) - \boldsymbol{x}_W(s_w^*)|$ be the length of a generatrix on the wire line. If $t_{\max} > t_w^*(s_w^*)$, that is, if the perpendicular projection point of the point $\boldsymbol{p}_k$ to the surface S_L is included in the lower cup, the point $\boldsymbol{p}_k$ is classified to the set D_L. Otherwise, it is classified to the set D_U.

(a) $t^* \leq t_{\max}$ (b) $t^* > t_{\max}$

Fig. 7 Relationship between t^* and $t_{\max}$.

Thus, we can classify point data on a breast into two sets once two developable surfaces for the lower and the upper cup are determined.

From above, the error between the surface of a two-piece brassiere cup and its data points can be formulated:

$$\Lambda(D) = \sum_{i=1}^{N_L} \varepsilon(\boldsymbol{p}_{L,i}) + \sum_{i=1}^{N_U} \varepsilon(\boldsymbol{p}_{U,i}). \tag{31}$$

By finding two developable surfaces that minimize the error described by eq. (31), the appropriate pattern shapes can be determined.

Formulation of Objective Function and Conditions

To consider the optimization conditions, we first explain the conditions for the parameters of the ridge line and the upper line. ω_ζ does not affect the shape of the space curve. Therefore, the following conditions are added

$$\omega_{\zeta_R}(s_r(s_w)) = 0 \;\forall s_w \in [0, L_L], \tag{32}$$
$$\omega_{\zeta_U}(s_u(s_w)) = 0 \;\forall s_w \in [0, L_L]. \tag{33}$$

Then the following conditions must be satisfied to design the arc length

$$s_r(s_w) \geq 0 \;\forall s_w \in [0, L_L], \tag{34}$$

$$s_u(s_w) \geq 0 \;\forall s_w \in [0, L_L], \tag{35}$$

$$s_r'(s_w) \geq 0 \;\forall s_w \in [0, L_L], \tag{36}$$

$$s_u'(s_w) \geq 0 \;\forall s_w \in [0, L_L]. \tag{37}$$

In general, the end positions of the ridge and upper lines are aligned to the end point of the wire line. Therefore, the following equations must be satisfied

$$\boldsymbol{x}_R(s_r(s_w)) = \boldsymbol{x}_L(s_w), \tag{38}$$

$$\boldsymbol{x}_U(s_u(s_w)) = \boldsymbol{x}_L(s_w). \tag{39}$$

To guarantee that S_L and S_U are developable surfaces, the following equations must be satisfied

$$\int_0^{L_L} \det(\boldsymbol{\zeta}_R(s_r(s_w)), \boldsymbol{\zeta}_W(s_w), \boldsymbol{g}) ds_w = 0, \tag{40}$$

$$\int_0^{L_L} \det(\boldsymbol{\zeta}_U(s_u(s_w)), \boldsymbol{\zeta}_W(s_w), \boldsymbol{g}) ds_w = 0. \tag{41}$$

Next, consider a "fuzzy" condition for a brassiere cup. The upper line has a point where it is connected with the shoulder strap. Hereafter, this point is called the connection point. Typically, this point is given as a fuzzy condition in the design process. This can lead to an unexpected error in the optimization problem. Therefore, we propose the following equation to deal with this fuzzy condition $y \simeq Y$:

$$C(y, Y) = k_1 \exp(k_2 (y - Y)^2) \tag{42}$$

where k_1 and k_2 are the variables to adjust the range that satisfies this condition. By adding this equation to the objective function, the fuzzy condition can be handled. Here, we assume that the position $\boldsymbol{X}_C$ to connect the upper line and the shoulder strap is given. Then the objective function V is described as

Fig. 8 Expression about the connection point of a brassiere cup

$$V=\Lambda(\boldsymbol{D})+\sum_{i=0}^{2}C(\boldsymbol{x}_U\cdot\boldsymbol{e}_i,\boldsymbol{X}_C\cdot\boldsymbol{e}_i) \tag{43}$$

where $\boldsymbol{e}_0$,$\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are represented as unit vectors of x,y and z-axis. Solving this optimization under these constraints can provide the shape of the brassiere cup to fit the data points of a breast.

Procedure to Solve the Optimization Problem

Prior to explaining the procedure to solve the optimization problem, we explain how to eliminate the condition. To satisfy eqs. (34)–(37), $s_r(s_w)$,$s_u(s_w)$ are described using arbitrary functions $\upsilon_r(s_w)$,$\upsilon_u(s_w)$ as

$$s_r(s_w)=s_{r_0}+\int_0^{s_w}\upsilon_r^2 ds_w, \tag{44}$$

$$s_u(s_w)=s_{u_0}+\int_0^{s_w}\upsilon_u^2 ds_w. \tag{45}$$

The initial position is assumed to be the wire line. Then the ridge and upper lines are aligned such that $s_{r_0}=s_{u_0}=0$. Let a composite function of $s_r(s_w)$ and arbitrary function $g(s_r)$ be defined as $\tilde{g}(s_w)$, and a composite function of $s_u(s_w)$ and arbitrary function $g(s_w)$ be defined as $\hat{g}(s_w)$. From eq. (1), the object coordinate system of the ridge and upper lines can be expressed as

$$\left[\,\tilde{\boldsymbol{\xi}}'_R\ \tilde{\boldsymbol{\eta}}'_R\ \tilde{\boldsymbol{\zeta}}'_R\,\right]=s'_r\left[\,\tilde{\boldsymbol{\xi}}_R\ \tilde{\boldsymbol{\eta}}_R\ \tilde{\boldsymbol{\zeta}}_R\,\right]\Omega(\tilde{\boldsymbol{\omega}}_R) \tag{46}$$

$$\left[\,\hat{\boldsymbol{\xi}}'_U\ \hat{\boldsymbol{\eta}}'_U\ \hat{\boldsymbol{\zeta}}'_U\,\right]=s'_u\left[\,\hat{\boldsymbol{\xi}}_U\ \hat{\boldsymbol{\eta}}_U\ \hat{\boldsymbol{\zeta}}_U\,\right]\Omega(\hat{\boldsymbol{\omega}}_U) \tag{47}$$

and the position $\boldsymbol{x}_R(s_r)\equiv\tilde{\boldsymbol{x}}_R(s_w)$ and $\boldsymbol{x}_U(s_u)\equiv\hat{\boldsymbol{x}}_U(s_w)$ are expressed by the following equations

$$\tilde{\boldsymbol{x}}_R(s_w)=\int_0^{s_w}\tilde{\boldsymbol{\zeta}}_R s'_r ds_w \tag{48}$$

$$\hat{\boldsymbol{x}}_U(s_w)=\int_0^{s_w}\hat{\boldsymbol{\zeta}}_U s'_u ds_w \tag{49}$$

From above, a curve can be determined by ω_ξ,ω_η and υ. Let $\boldsymbol{\psi}$ be a set of functions of $[\omega_\xi\ \omega_\eta\ \upsilon]$. Then a set of functions of the ridge line $\boldsymbol{\psi}_R$ and a set of functions of the upper line $\boldsymbol{\psi}_U$ are parameterized by using Ritz method[14] as

$$\boldsymbol{\psi}_R=\left[\,\boldsymbol{a}_{\omega_{\xi_R}}\cdot\boldsymbol{e}(s_w)\ \boldsymbol{a}_{\omega_{\eta_R}}\cdot\boldsymbol{e}(s_w)\ \boldsymbol{a}_{\upsilon_R}\cdot\boldsymbol{e}(s_w)\,\right] \tag{50}$$

$$\boldsymbol{\psi}_U=\left[\,\boldsymbol{a}_{\omega_{\xi_U}}\cdot\boldsymbol{e}(s_w)\ \boldsymbol{a}_{\omega_{\eta_U}}\cdot\boldsymbol{e}(s_w)\ \boldsymbol{a}_{\upsilon_U}\cdot\boldsymbol{e}(s_w)\,\right] \tag{51}$$

where $\boldsymbol{e}_i(s)$ is composed of trigonometric functions with different periods. Let $\boldsymbol{\Phi}_0=[\boldsymbol{\xi}_0^{\mathrm{T}}\ \boldsymbol{\eta}_0^{\mathrm{T}}\ \boldsymbol{\zeta}_0^{\mathrm{T}}]$ be the initial basis of the objective coordinate system.

From above, the objective function, which is expressed as eq. (43), and the geometric constraints, which are expressed as eqs. (38)–(41) are described by the following total parameter vector

$$\boldsymbol{a}_{\mathrm{all}}=\left[\,\boldsymbol{a}_R\ \boldsymbol{a}_U\ \boldsymbol{\Phi}_{R0}\ \boldsymbol{\Phi}_{U0}\,\right]. \tag{52}$$

Consequently, this problem can be converted into a nonlinear programming problem of $\boldsymbol{a}$, which is solved using the multiplier method and Nelder-Mead method in this study. By solving the nonlinear programming problem, we can design the entire shape of a two-piece brassiere cup.

SIMULATION AND VERIFICATION

To demonstrate the utility of our method, we conducted simulations and a verification. For the verification, the infinitesimal rotational ratios vector of the wire line is given as

$$\boldsymbol{\omega}_W=\left[\,0\ 2.91\ 0\,\right]^{\mathrm{T}}, \tag{53}$$

while the connection point $\boldsymbol{X}_C$ is given as

$$\boldsymbol{X}_C=\left[\,0\ 0.34\ 0.34\,\right]^{\mathrm{T}}. \tag{54}$$

We prepared three examples of the simulation. Case (1) is a point cloud on a set of a developable surface by a uniform random number using eq. (26). Case (2) is a point cloud on a sphere whose radius is set on R by a uniform random number. Case (3) is a point cloud on a measured shape of a torso, which is a representative breast shape. The simulations aim to confirm that the propose method recreates a given shape and to verify whether a two-piece brassiere cup can be designed to approximate an undevelopable surface that manifests the function of " fitting to a breast ".

Fig. 9 shows the obtained shape and input data for Case (1), where the solid line represents the given wire line. Each dotted line denotes the calculated generatrices of the cup shape, and each dot represents a data point. Fig. 10 shows the obtained pattern where the solid and dotted lines denote the lower and upper edges, respectively. Next, a uniform random number was generated by numpy.random() of Python. Fig. 9 shows the obtained shape and input data for Case (2), and Fig. **?** shows the obtained shape and input data for Case (3) Table 1 shows the error and calculation time of each case.

From the result of case(1), we confirm that our proposed method can recreate a given shape as a two-piece brassire cup. And From the result of case(2) and (3), we confirm that our proposed method also can design a two-piece brassiere cup to approximate data points under the conditions.

Hence, we conclude our proposed method in this paper is useful for the efficient design of paper patterns of a two-piece brassiere cup.

	Error	Time[sec]
case(1)	0.10	1802
case(2)	0.12	740
case(3)	1.04	1682

Table 1 The error and calculation time of our proposed method in each case.

(a) zx-view

(b) zy-view

(c) xy-view

Fig. 9 Obtained shape and input data points in Case(1)

(a) LOWER PATTERN

(b) UPPER PATTERN

Fig. 10 Obtained patterns in Case(1)

CONCLUSION

Summary

Herein we propose a method to design the shape of a two-piece brassiere cup and its patterns to satisfy the function of "fitting to a breast shape" when the shape is given as a cloud of data points. In the typical design process, a cup model is made of paper to verify the pattern shape. Our model assumes that the model surface is comprised of several developable surfaces. First, we formulate the cup model based on differential geometry and hypothesize that a developable surface can be determined by two curves included in it. Hence, the design process of a two-piece brassiere cup can be converted into an optimization problem, whose objective function is the error between the cup shape and its data points. To calculate the error, first the error between a point and the surface from the geometry condition is formulated. Second, our proposed method divides the data cloud into two: the lower and upper cups. Then the cups are evaluated. To verify our proposed method, we assessed two cases. One is a non-developable surface, and the other is a developable surface. The results confirm that the proposed method can design a cup shape, which manifests the intended function. Consequently, our proposed method should be useful to efficiently design a two-piece brassiere cup.

Limitation and Future Work

Though we proposed a method to design the shape of a two-piece brassiere cup and its patterns to satisfy the function of "fitting to a breast shape" when the shape is given as a cloud of data points, there is still two questions from the point of view of real-world applications. One is whether the brassire cup should cover the breast shape. A brassire cup has several demands, which include the function to form the ideal breast shape. So the function of "fitting to a breast shape" does not only mean fitting to a breast shape itself, but also fitting to an ideal breast shape one wants. For future work, it is required to quantify how much fitting to a breast shape and design the brassire cup shape satisfying it.

References

[1] Yukiko Higuchi, *Sensory test of brassiere*, in The Japanese Journal of Ergonomics, Volume 20, Issue 2, 1984, pp.73-80, (Japanese).

[2] P.Bo and W.Wang, *"Geodesic-Controlled Developable Surfaces for Modeling Paper Bending"*, in

(a) zx-view

(b) zy-view

(c) zy-view

Fig. 11 Obtained shape and input data points in Case(2)

(a) LOWER PATTERN

(b) UPPER PATTERN

Fig. 12 Obtained patterns in Case(2)

Computer Graphics Forum, Volume 26, 2007, pp.365-374.

[3] H.Pottmann and J.Wallner, *"Approximation algorithms for developable surfaces"*, in Computer Aided Geometric Design, Volume 16, 1999, pp.539-556.

[4] K.Rose, A.Sheffer, J.Wither, M.Cani, and B.Thibert, *"Developable surfaces from arbitrary sketched boundaries"*, SGP '07 Proceedings of the fifth Eurographics symposium on Geometry processing, 2007, pp.163-172.

[5] G.Aumann, *"A simple algorithm for designing developable Bezier surfaces"*, in Computer Aided Geometrix Design Volume 20, 2003, pp.601-619.

[6] B.Ravani and T.S. Ku, *"Bertrand offsets of ruled and developable surfaces"* in Computer-Aided Design Volume 23, 1991, pp.145-152.

[7] R.M.C Bodduluri and B.Ravani, *"Design of developable surfaces using duality between plane and point geometries"*, in Computer Aided Design Volume 25, 1993, pp.621-632.

[8] M. Peternell, *"Developable surface fitting to point clouds"*, in Computer Aided Design Vol.21, 2004, pp.785-803.

[9] H.-Y. Chen, I.-K. Lee, S. Leopoldseder, H. Pottmann, T. Randrup , J. Wallner, *"On Surface Approximation using Developable Surfaces"*, Graphical Models and Image Processing, Volume 61, Issue2, 1999, pp.110-124

[10] H.Wakamatsu, Y.Ryu, E.Morinaga, E.Arai, and T. Kubo, M.Oyama, and T.Kishimoto, *"3D Shape Prediction of A Paper Model of Two Piece Brassiere Cup for Its Efficient Design"*, Proc.Int. Symp. Flexible Automation, 2016, pp.326-332

[11] H.Wakamatsu, E.Morinaga, E.Arai, and T. Kubo, *"A Virtual Paper Model of a Three Piece Brassiere Cup to Improve the Efficiency of Cup Design Process"*, Proc. 2017 IEEE Int. Conf. on Robotics and Automation, 2017.

[12] M.Itoh, and H.Imaoka, *"A Method of Prediction Sewn Shapes and a Possibility of Sewing by the Theory of Developable Surfaces"*, J. the Japan Research Association for Textile End-Uses, Volume 48, 2007, pp.42-51, (Japanese).

[13] Kotaro Yoshida, Hidefumi Wakamatsu, Eiji Morinaga, Eiji Arai, Seiichiro Tsutsumi, Takahiro Kubo,*"Pattern Shape Optimization of a Two-piece brassiere cup to improve its design efficiency"*, Trans. of the Institute of Systems, Control and Information Engineers, Volume 32, No. 5, 2019, pp. 192-202.

[14] Elsgolc L.E *Calculus of Variations*, Pergamon Press, 1961, pp. 144-157.

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