Uniform Distribution

Uniform distribution may refer to discrete uniform distribution or continuous uniform distribution. It is a distribution that has constant probability and is known as a rectangular distribution. The distribution is abbreviated as U(a, b).When the distribution is equally spaced and when the probability density is same at any point, the uniform distribution is called 'discrete uniform distribution'. The other type of continuous distribution is a case where the variable can be assigned with infinite number of values known as continuous uniform distribution.

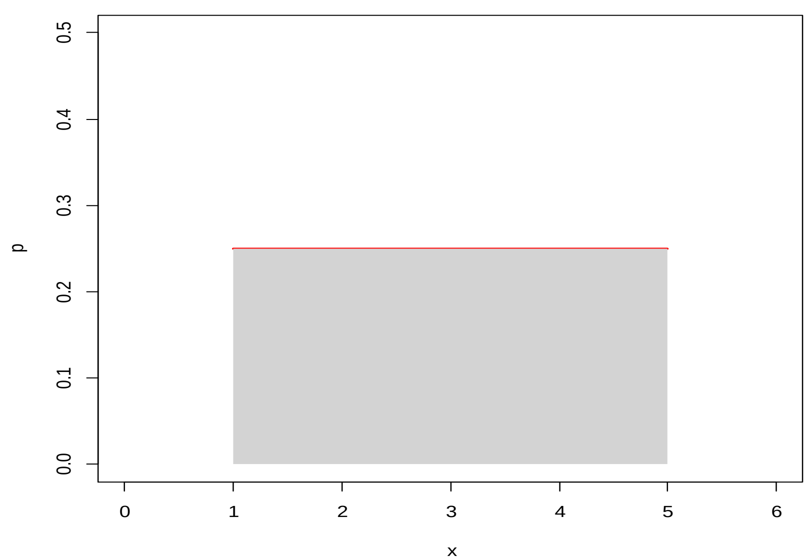
The Uniform Distribution is defined on an interval [a, b]. The idea is that any number selected from the interval [a, b] has an equal chance of being selected. Therefore, the probability density function must be a constant function. Because the total are under the probability density curve must equal 1 over the interval [a, b], it must be the case that the probability density function is defined as follows:

$$f(x) = \frac{1}{b-a}$$

For example, the uniform probability density function on the interval [1,5] would be defined by f(x) = 1/(5-1), or equivalently, f(x) = 1/4. The following code sketches this uniform probability density function over the interval [1,5].

```
x=seq(1,5,length=200)
y=rep(1/4,200)
plot(x,y,type="l",xlim=c(0,6),ylim=c(0,0.5),lwd=2,col="red",ylab="p")
polygon(c(1,x,5),c(0,y,0),col="lightgray",border=NA)
lines(x,y,type="l",lwd=2,col="red")
```

Fig: *The shaded area under the uniform density function is 1*



Alternatively, we could have used **dunif**, which will produce density values for the uniform distribution.

```
x=seq(1,5,length=200)
y=dunif(x,min=1,max=5)
plot(x,y,type="l",xlim=c(0,6),ylim=c(0,0.5),lwd=2,col="red",ylab="p")
polygon(c(1,x,5),c(0,y,0),col="lightgray",border=NA)
lines(x,y,type="l",lwd=2,col="red")
```

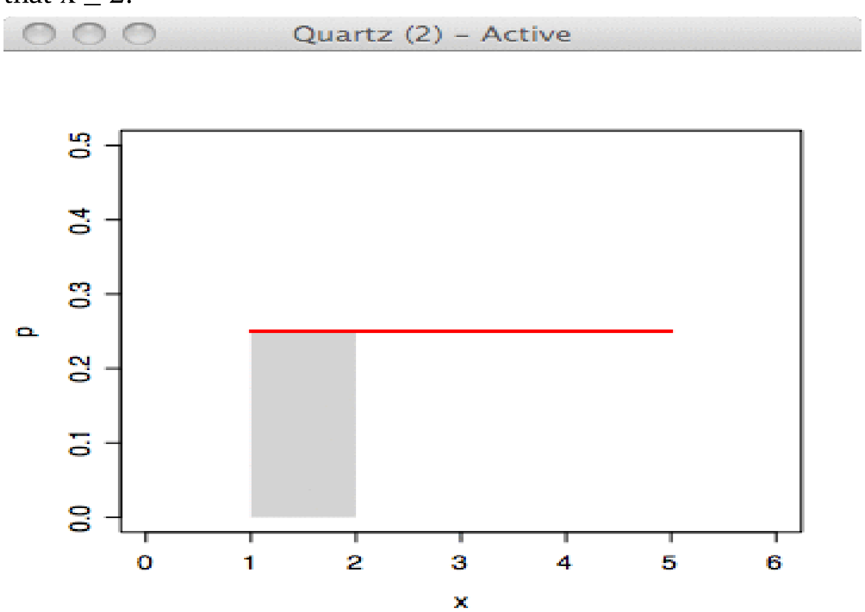
Note that the arguments **min=1** and **max=5** provide the endpoints of the interval [1,5] on which the uniform probability density function is defined.

Using punif

Suppose that we would like to find the probability that the random variable *X* is less than or equal to 2. To calculate this probability, we would shade the region under the density function to the left of and including 2, then calculate its area.

```
x=seq(1,5,length=200)
y=dunif(x,min=1,max=5)
plot(x,y,type="l",xlim=c(0,6),ylim=c(0,0.5),lwd=2,col="red",ylab="p")
x=seq(1,2,length=100)
y=dunif(x,min=1,max=5)
polygon(c(1,x,2),c(0,y,0),col="lightgray",border=NA)
x=seq(1,5,length=200)
y=dunif(x,min=1,max=5)
lines(x,y,type="l",lwd=2,col="red")
```

Fig: The shaded area under the uniform probability density function represents the probability that $x \leq 2$.



In above Figure, note that the width of the shaded area is 1, the height is 1/4, so the area of the shaded region is 1/4. Thus, the probability that $x \leq 2$ is 1/4.

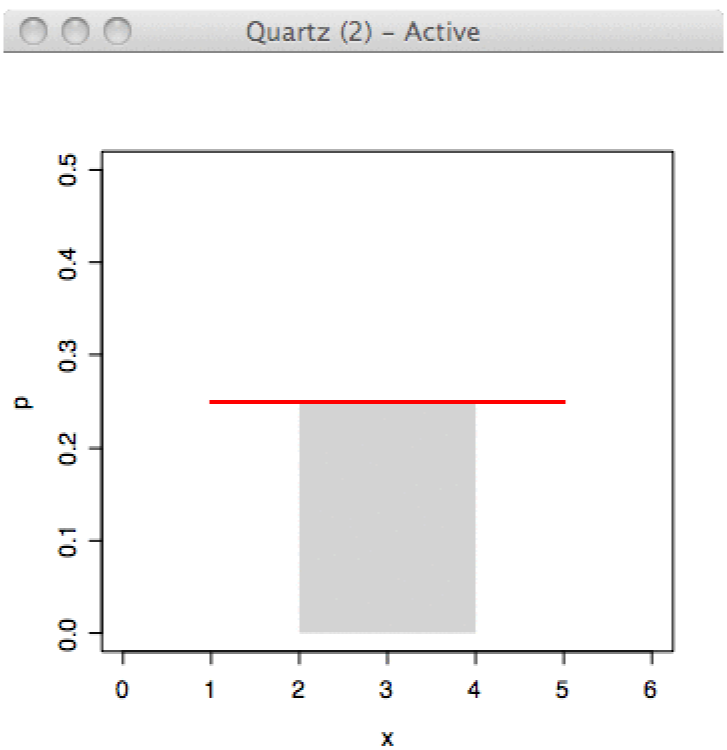
We can use the **punif** command to compute the probability that $x \leq 2$.

```
>punif(2,min=1,max=5)
[1] 0.25
```

Let's look at another example. Suppose that we wanted to find the probability that *x* lies between 2 and 4. We could draw the uniform distribution, then shade the area under the curve between 2 and 4.

```
x=seq(1,5,length=200)
y=dunif(x,min=1,max=5)
plot(x,y,type="l",xlim=c(0,6),ylim=c(0,0.5),lwd=2,col="red",ylab="p")
x=seq(2,4,length=100)
y=dunif(x,min=1,max=5)
polygon(c(2,x,4),c(0,y,0),col="lightgray",border=NA)
x=seq(1,5,length=200)
y=dunif(x,min=1,max=5)
lines(x,y,type="l",lwd=2,col="red")
```

fig: The shaded area under the uniform probability density function represents the probability that 2 < x < 4.



Note that the width of the shaded area is 2, the height is 1/4, so the area is 1/2. That is, the probability that $2 < x < 4$ is 1/2.

We can use **punif** to arrive at the same conclusion. To find the area between 2 and 4, we must subtract the area to the left of 2 from the area to the left of 4

```
>punif(4,min=1,max=5)-punif(2,min=1,max=5)
[1] 0.5
```

Finally, if we need to find the area to the right of a given number, simply subtract the area to the left of the given number from the total area; i.e., subtract the area from the left of a given number from 1. So, the following calculation will find the probability that $x > 4$.

```
> 1-punif(4,min=1,max=5)
[1] 0.25
```

Using qunif

The command **qunif** will find quantiles for the uniform distribution in the same way as we saw the **qnorm** find quantiles in the activity **The Normal Distribution**. Thus, to find the 25th percentile for the uniform distribution on the interval [1,5], we execute the following code.

```
>qunif(0.25,min=1,max=5)
[1] 2
```

Exercise

Consider the uniform probability distribution given by X∼U(−3,5.5)X∼U(−3,5.5).

Question 1

What is the mean, μμ, for the given uniform distribution

```
unif.mean <- (-3+5.5)/2
unif.mean
## [1] 1.25
```

The mean, μμ, for the uniform probability distribution given by X∼U(−3,5.5)X∼U(−3,5.5) is 1.25.

Question 2

What is value of xx corresponding to the value that divides the given uniform distribution into two equal parts, or written more formally P(X<?)=0.5P(X<?)=0.5.

```
px.0.5  <- qunif(0.5, min = -3, max = 5.5)
px.0.5
## [1] 1.25
```