Poisson Distribution:

The **Poisson distribution** is the probability distribution of independent event occurrences in an interval. If λ is the mean occurrence per interval, then the probability of having *x* occurrences within a given interval is:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad where\ x = 0, 1, 2, 3, \ldots$$

1. If there are twelve cars crossing a bridge per minute on average, find the probability of having seventeen or more cars crossing the bridge in a particular minute.

The probability of having sixteen or less cars crossing the bridge in a particular minute is given by the function ppois.

ppois(16, lambda=12)

Hence the probability of having seventeen or more cars crossing the bridge in a minute is in the upper tail of the probability density function.

ppois(17, lambda=12, lower=FALSE) or

1- ppois(17, lambda=12)

2. Suppose there are on average 10 asthmatic patients that come to a particular health clinic on a given day. What is the probability that only 4 will come to the clinic today?

ppois(4,lambda=10)

This is the probability that 4 or fewer will come to the clinic today. The probability of 5 or more coming to the clinic will be

ppois(4,lambda=10, lower=FALSE)

3. A producer of pins realized that on a normal 5% of his item is faulty. He offers pins in a parcel of 100 and insurances that not more than 4 pins will be flawed. What is the likelihood that a bundle will meet the ensured quality?

n=100, p= 5/100

np= 100*5/100 =5

Required probability = P [packet will meet the guarantee]

= P [packet contains up to 4 defectives]

= P (0) +P (1) +P (2) +P (3) +P (4)

$$= e^{-5}.\frac{5^0}{0!} + e^{-5}.\frac{5^1}{1!} + e^{-5}.\frac{5^2}{2!} + e^{-5}.\frac{5^3}{3!} + e^{-5}.\frac{5^4}{4!},$$

$$= e^{-5}[1 + \frac{5}{1} + \frac{25}{2} + \frac{125}{6} + \frac{625}{24}],$$

$$= 0.0067 \times 65.374 = 0.438$$

or ppois(4,lambda=5)

4. Consider a computer system with Poisson job-arrival stream at an average of 2 per minute. Determine the probability that in any one-minute interval there will be

(i) 0 jobs; **dpois(0, 2)**

(ii) exactly 2 jobs; **dpois(3, 2)**

(iii) at most 3 arrivals. **ppois(3,2)**

(iv) What is the maximum jobs that should arrive one minute with 90 % certainty? **qpois(.9, 2)**

- at least a 90% chance that the number of job submissions in any minute does not exceed 4.

5. A complex software system averages 7 errors per 5,000 lines of code. What is the probability of exactly 2 errors in 5,000 lines of randomly selected lines of code?

The probability of exactly 2 errors in 5,000 lines of randomly selected lines of code is ppois(2, lambda=7)

6. A random variable X has Poisson distribution with mean 7. Find the probability that

(i) X is less than 5 less or equal is:

```
> ppois(5,7)
[1]0.3007083
```

less than is

```
> ppois(4,7)
[1]0.1729916
```

(ii) X is greater than 10 (strictly)

```
> 1-ppois(10,7)
 [1] 0.0985208
```

(iii) X is between 4 and 16

```
 > ppois(16,7)-ppois(4,7)
 [1] 0.9172764
```

<u>Quantiles</u>

Poisson(λ) Distribution :

```
> y <- c(.01,.05,.1,.2,.5,.8,.95,.99)
> qpois(y,6)
[1]  1  2  3  4  6  8 10 12
```

<u>Random Variable generation</u>

Poisson(λ) Distribution :

```
> x <- rpois(20,6)
> x
[1]  2  8  7  5  5  5  3  8  5  5  1  8  5  5  5  4 10  7  3  4
```

Exercise Problems:

1. You have observed that the number of hits to your web site occur at a rate of 2 a day. Let 12 be be the number of hits in a day
2. You observe that the number of telephone calls that arrive each day on your mobile phone over a period of a year, and note that the average is 3. Let 12 be the number of calls that arrive in any one day.
3. Records show that the average rate of job submissions in a busy computer centre is 4 per minute. Let X (12) be the number of jobs arriving in any one minute.
4. Records indicate that messages arrive to a computer server at the rate of 6 per hour. Let X be the number of messages arriving in any one hour.
5. Customers call us at a rate of 12 per minute. “The boss” wants us to deliver excellent service and stay very productive. Our service will suffer if we get more than twenty calls in a minute. We’re going to look lazy if five or less calls arrive in a minute. What are the odds of getting in trouble with the boss?

Answers:

par(mfrow = c(2,2))

x<-0:12

1: plot (x, dpois(x, 2), xlab = "Number of Hits", ylab = "P(X = x)", type = "h", main= "Web Site Hits: Poisson(2)")

2: plot (x, dpois(x, 3), xlab = "Number of Calls", ylab = "P(X = x)", type = "h", main= "Calls to Mobile: Poisson(3)")

3: plot (x, dpois(x, 4), xlab = "Number of Submissions", ylab = "P(X = x)", type = "h", main= "Job Submissions: Poisson(4)")

4: plot (x, dpois(x, 6), xlab = "Number of Messages", ylab = "P(X = x)", type = "h", main= "Messages to Server: Poisson(6)")