Exponential Distribution:

The exponential distribution describes the arrival time of a randomly recurring independent event sequence. If μ is the mean waiting time for the next event recurrence, its probability density function is:

$$f(x) = \begin{cases} \frac{1}{\mu} e^{-x/\mu} & when\ x \geq 0 \\ 0 & when\ x < 0 \end{cases}$$

The exponential distribution is known to have mean $\mu = 1/\lambda$ and standard deviation $\sigma = 1/\lambda$.

Suppose that we set $\lambda = 1$. Then the mean of the distribution should be $\mu = 1$ and the standard deviation should be $\sigma = 1$ as well. We could use the formula in Figure 6 to produce values of the probability density function, but as we've seen before, it will be more efficient to use the dexp command**.**

```
x=seq(0,4,length=200)
y=dexp(x,rate=1)
plot(x,y,type="l",lwd=2,col="red",ylab="p")
```

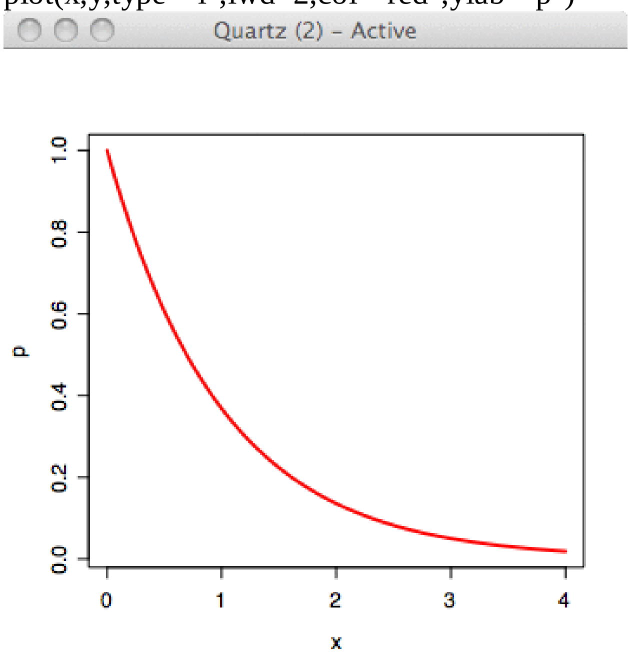

Fig: The exponential probability density function is shown on the interval [0,4]

Using pexp

Suppose that we want to find the probability that *x &le 1*. We would shade the area under the exponential probability density function to the left of 1, as shown in Figure

```
x=seq(0,4,length=200)
y=dexp(x,rate=1)
plot(x,y,type="l",lwd=2,col="red",ylab="p")
x=seq(0,1,length=200)
y=dexp(x,rate=1)
polygon(c(0,x,1),c(0,y,0),col="lightgray")
```

Fig:*Shading the region under the density curve to the left of x = 1*

By using the **pexp** command to compute the area of the shaded region in above Figure:

pexp(1,rate=1)

[1] 0.6321206

Second example, suppose that we want to find the probability that *x* lies between 1 and 2.

```
x=seq(0,4,length=200)
y=dexp(x,rate=1)
plot(x,y,type="l",lwd=2,col="red",ylab="p")
x=seq(1,2,length=200)
y=dexp(x,rate=1)
polygon(c(1,x,2),c(0,y,0),col="lightgray")
```

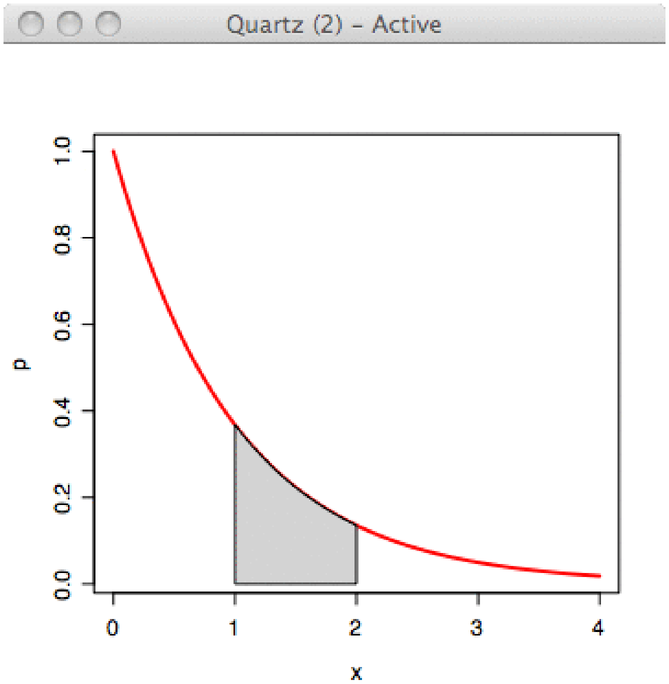

Fig:*Calculating the probability that x lies between 1 and 2.*

To find the shaded area in above Figure, we must subtract the area to the left of *x = 1* from the area to the left of *x = 2*.

```
pexp(2,rate=1)-pexp(1,rate=1)
[1] 0.2325442
```

Finally, if we need to find the area to the right of a given number, simply subtract the area to the left of the given number from the total area. For example, to find the probability that $x > 3$, subtract the probability that $x \leq 3$ from 1.

```
1-pexp(3,rate=1)
[1] 0.04978707
```

Using qexp

The command **qexp** will find quantiles for the exponential distribution in the same way as we saw the **qunif** find quantiles for the uniform distribution. Thus, to find the 50th percentile for the exponential distribution on the interval, we execute the following code.

```
qexp(0.50,rate=1)
[1] 0.6931472
```

Example:

Suppose the mean checkout time of a supermarket cashier is three minutes. Find the probability of a customer checkout being completed by the cashier in less than two minutes. The checkout processing rate is equals to one divided by the mean checkout completion time. Hence the processing rate is 1/3 checkouts per minute. We then apply the function pexp of the exponential distribution with rate=1/3.

pexp(2, rate=1/3)

If jobs arrive every 15 seconds on average, λ = 4 per minute, what is the probability of waiting less than or equal to 30seconds, i.e .5 min?

pexp(.5,4)

What is the maximum waiting time between two job submissions with 95% confidence?

qexp(.95, 4)