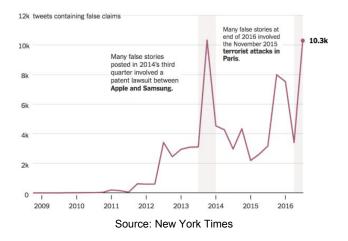
Sources and articles, Nash equilibria in quality journalism.

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Abstract: Motivated by the constant rising of fake news and lack of true quality journalism, it is presented a model that establish and describe an equilibria between opinion based and factual journalism. This model propose a generalized Nash equilibria and discuss the application of a third player inside a closed market. By means of a Markov chain is shown how to implement a stable indicization process without using SEO linking model.

Introduction

Since 2016 the term 'fake news' spread, with increasing occurrence, all across mass media news. It describe the issue of factually incorrect and misleading articles. But fake news are much older than that: let's quote the notorious case of Charles de Berenger, who in 1814 disquised himself as a Bourbon officer and appeared in Dover to announce that Napoleon had been killed by the Prussians. He sent a semaphore telegraph to the Admiralty in London, knowing it would find its way into the press. The price of British government bonds rose on the news, prompting De Berenger and co-conspirators to sell gilts they had already bought¹. While the creation of fake news is difficult to be correctly modeled, the transmissions is due to real people. One thing about social media that makes humans more susceptible to fake news is the popularity indicators that social network sites provide, that people use to signal approval for a message to other users. Research shows that, psychologically, we are not kin to check our source when the news has already been approved and shared by lot of people². Going further, we cannot mention Carrino et al. work: even an expert knowledge on the subject can eventually be deceived by a misleading news³. Thus can be better seen in NewYork Times research, where is shown an increasing number of fake news trends, spacing not only in politics and daily matter. Fake news directly affect trust and brand reputation of each press agency, which must act in order to raise the bar and produce a high end quality journalism.



The System Model

In this section it is presented a mathematical formulation for a correlation between journalistic articles and possible sources. This model is designed as a decentralized mechanism involving Nash game algebra. Let denote by $I = \{1, \ldots, I\}$, $I \in \mathbb{N}$ the set of articles. Let denote by $J = \{1, \ldots, J\}$, with $J \in \mathbb{N}$ the set of sources. The correlation between article i and source j is captured by matrix $C = I \times J$, where each entry $C_{ij} \in \{0, 1\}$. $C_{ij}=1$ implies that article i contain source j, while $C_{ij}=0$ implies the opposite. By means of the parameter $C_{ij} \in [0, 1]$ let represent the objectivity of an article i to the source i, where i is captured by means of the parameter i is i to the source i in the source i is the source i in the source i is i to the source i in the source i in the source i is i to the source i in the source i in the source i is i to the source i in the source i in the source i in the source i is i to the source i in the sourc

¹ "Fake news of Napoleon's death cited in guidance to help traders." . https://www.reuters.com/article/us-britain-markets-regulations/fake-news-of-napoleons-death-cited-in-guidance-to-help-traders-idUSKBN1KH0T3.

² E. C. Tandoc, Z. W. Lim, and R. Ling, "Defining 'Fake News': A Typology of Scholarly Definitions," Digital Journalism, vol. 6, no. 2, pp. 137–153, Feb. 2018.

³ The history to the test of the like. The case of media and digital neoborbonism. A. Carrino, GL Fruci,C Calefati, A Fiorio, MT Milicia, F Palmieri, C Pinto, S Sonetti, C Villani, AIUCD 2018.

Each press agency has the capacity of distinguish between factual/opinion based articles and misleading one. Hence, each pressa agency can select an action corresponding to a filter $a_i \in \{0, 1\}$. Motivated by "The impact of content, context, and creator on user engagement in social media marketing", it is clear the increasing margins for each press agency to depend on the engagement e_{ij} generated from opinion based articles. The total generated engagement thus reflect the potential revenue of the agency (both in sense of money and in reputation). It is reasonable to assume that filter a_i cannot effectively predict the revenue stream and the correct balance between trending articles and qualities one. Thus, to objectively quantify the impact of filtering, each press agency can be modeled as function v_i defined as:

$$v_i(e_i, a_k, x_j : k \in I, j \in J) : \mathbb{R}_{>0} \times A \times X \to \mathbb{R}_{>0}$$

Assumption 1

Each press agency, by means of its articles $I \in \mathbb{N}$, behaves in order to maximize their owns utility both in terms of revenue stream and brand consistency. In addition, function (1) must be concave, decreasing and differentiable with respect to the filter a_i .

This assumption it is better analyzed and presented in the last LSE report, where articles containing a certain number of trusted sources can appeal to a larger number of trending path. This dynamics of possible path can be defined through the function h_i :

$$h_i(a_i, x_j, j \in J) : A_i \times X \to [0, 1],$$

$$\frac{\partial h_i}{\partial a_i} > 0$$

Assumption 2

The function h_i is assumed to be concave, differentiable with respect to filter a_i . Assumption 1 is based on the hypothesis that filtering on the relation source/articles or opinion lead to an increase of engagement toward the press agency. In addition, at least a second strategic agent must exist in order to guarantee the right order of truth and opinion. The set of all strategic player is denoted by $D=I\cup 0$, where the action of strategic agent $a_0 \in \{0, 1\}$, is to select a lower bound threshold for the average appeal path $h_i(a_i, x_i : j \in J)$ for each article. Let the fraction of $i \in I$ be:

$$n_i = \frac{|J_i|}{\sum_{i \in I} |J_i|}$$

The fraction n_i represent the contribution of the average trust of the articles topic towards the average trust. The lower bound a_0 is set to be generated by the average common sense of the strategic agents. It must hold:

$$a_0 - \sum_{i \in I} n_i \cdot h_i(a_i, x_j : j \in J_i) \leqslant 0$$

The strategic agent valuation is assumed to be concave and monotonic increasing function: $v_0(a_0):[0,1]\to\mathbb{R}_\geqslant 0$, it's exact description goes beyond the scope of this print. Furthermore, the strategic agent has limited resources, both in time and monetary budget, to dedicate in the problem of filtering incentivization of the press agency.

Problem Statement

The problem of filtering incentivization between press agencies is handled by a return of investment given by the democratic market of news. This said, is then assumed that press agency and strategic reader statistically distribute over the whole spectrum of opinion bias. This is the real core of this formulation, in the presence of asymmetric information, it is clearly visible that an equilibrium between opinion based journalism (fake news are often part of this set) and quality journalism is formed.

An external player (more about in the last part) is necessary to define an appropriate incentive $i \in I$ for the press agency and a substantial investment $t_0 \in \mathbb{R}_{\geqslant 0}$ for the strategic agents. The utility of each press agency $i \in I$ is $u_i = v_i - t_0$ and the utility of the i-strategic agent is $u_0 = v_0 - t_0$. Ideally this mechanism must be designed with a null behaviour.

In view of the optimization problem, it is import to remark that each press agency filtering function h_i where $i \in I$ is not known to the other player of this equilibria. However, the action of function h_i can be a posteriori monitored since it represent the interaction of the press agency with the democratic reader market. The optimization problem to solve is:

$$max_{a,t}(v_0 - t_0 + \sum_{i \in I} v_i - t_i)$$

where the filter limits of each press agency gives $0 \le a_i \le 1$ and

$$a_0 - \sum_{i \in I} n_i \cdot h_i(a_i, x_i : j \in J_i) \le 0$$

Problem Design

The problem is designed by the function $\langle M,g(\cdot)\rangle$, where $M=M_0\times M_1\times\cdots M_I$ is the complete interaction space between press agency and strategic agent. $g(\cdot):M\to O$ is a map function wich create a one-to-one correspondence with the set of possible outcome O. Each article published by each press agency is denoted by $m_i\in M_i$, the outcome function g(m) determines the correlation of any article m_i with the set $m=(m_0,m_1,\cdots,m_I)\in M$. The revenue stream is defined for $t_i:M\to\mathbb{R}$ and the fraction generated by article m is:

$$m_i = (\widetilde{c}_i \widetilde{\pi}_i \widetilde{a}_i)$$

where $\widetilde{c}_i \in (R)_{\geq 0}$ is the minimum factualness an i-press agency want to achieve through filtering; $\widetilde{\pi}_i \in (R)_{\geq 0}$ is the collection of returns that the i-press agency is willing to "pay" in exchange for actions in the private filter, and $\widetilde{a}_i = (\widetilde{a}_0, \cdots, \widetilde{a}_I), \widetilde{a}_i \in (R)$.

In analogous way, the interaction of the strategic agents is given by:

$$m_i = (\widetilde{\pi_i}\widetilde{a_i})$$

where $\widetilde{\pi_i} \in (R)_{\geq 0}$ is the price, both in terms of money and time, a strategic agent is willing to pay in exchange for filter $\widetilde{a_0} \in \mathbb{R}$. Eg. A typical citizen is willing to pay for a printed copy of a local newspaper or be subjected to a certain number of ads for being able to read that opinion.

By means of matrix M it is now possible to clearly establish an asymmetric link between strategic agents and press agencies. Let's now introduce a former filter:

$$\varphi_i(m) = n_i \cdot \widetilde{c_i} \cdot \frac{\sum_{k \in I} \frac{\widetilde{a_i^k}}{|I|}}{\sum_{k \in I} n_k \cdot \widetilde{c_k}}$$

where the numerator represents the average filtering action undertake by the strategic agents and the denominator represent the whole set of *** . Each press-agency to increase their own profit will tend to select a filter a_i such that $n_i \cdot h_1(a_i, x_j : j \in J) \geq \varphi_i(m)$, recalling that function h_i is known only to the i-press agency.

Nash Equilibria and Induced Game

Given the action of filter $\varphi_i(m)$, the utility function $u_i: i \in I$ and the mechanism $\langle M, g(\cdot) \rangle$ a game is induced*. Each press agency is free to choose its own filter a_i and applying on each article m_i , with the constraint that $f_i \in S_i(m)$, where:

$$S_i(m) = \{a_i \in A_i, n_i \cdot h_i(a_i, x_i : j \in J_i)\} \ge \varphi_i(m)$$

Thus, the set of feasible outcomes $S_i(m)$ is a function strictly depending on the article m.

Eg. This dependence underline how certain topics are more likely to be subjected to higher flux in terms of reader distribution. The correct solution of this game is found to be in a generalized Nash equilibrium (GNE). If each $i \in I$, then the strategy profile for GNE is $m^* = (m_i^* : i \in D)$.

$$u_i(f_i(m_i^*, m_{-i}^*), \cdots, f_I(m_i^*, m_{-i}^*)) \ge u_i(f_i(m_i^*, m_{-i}^*), \cdots, f_I(m_i^*, m_{-i}^*))$$

The optimization of the induced game can be performed by an iterative process where for each $i \in I$

$$\max_{m_i \in M_i} \{v_1(f_1(m), \cdots, f_I(m)) - t_i(m)\}$$

where:

$$t_i = -\widetilde{\pi}_0 \cdot \varphi_i(m) + \sum_{l \in D} \widetilde{\pi}_l^i \cdot (\widetilde{a}_l^i - \widetilde{a}_l^{-i})^2$$

$$0 \leq f_i(m) \leq 1$$

$$\varphi_i(m) - n_i \cdot h_i(f_i(m)) \leq 0$$

Previously mentioned function t_i , now receive an explicit definition: given the filter parameter $\varphi_i(m)$, the revenue from a certain article must be weighted by the demand l on tha peculiar topic. A geometric mean is then added to guarantee and exclude the cases violating assumption 1. By imposing a value for parameter $\varphi_i(m)$ and monitoring the public filter of all strategic agents, it is clear how a third external player can recover information about the filtering function of press agency h_i . These third player can also assign a private rank about the article publish by each press agency without even know what those article contains, leading to the important and final remark of this discussion.

Bias Ranking

Recall the set of possible outcome O. Let define R_i be the page rank of a single article $i \in I$. We are going to model the network of Web pages by means of Markov Chain. Following assumption 1, it is possible to define a finite number of possible states in a finite state machine. Intuitively, the conclusion will O. Let define R_i be the page rank of a single article $i \in I$. We are going to model the network of Web pages by means of Markov Chain. Following assumption 1, it is possible to define a finite number of possible states in a finite state machine. Intuitively, the articles mutual ranks are proportional to the stationary states in the Markov chain. That is, the probability of relate article and respective source can be dictated by the filter h_i of the press agency, after a long time the probability describing the relation converges to a stationary distribution whatever is your starting point. The scaling factor between the rank of the article and the probability is

to not following the right path in terms of correlation source/article.

$$\frac{I}{D}$$

where $0 \le D \le 1$ is a chosen constant related to the restart due the wrong path source/article. Let's formally define Markov Chain with the following constraints:

- 1. Each published article is part of the set ${\cal M}$, and all the possible configuration state are contained in set ${\cal O}$.
- 2. d_0 is a special state that admits the restart state.
- 3. The transition probabilities between states are associated to filtering matrix $\varphi_i(m)$, in the following way:
 - a. Every state $i \in I$ has the probability 1 d to go to the Restart state d_0
 - b. The probability of changing state is defined as:

$$p_{d_0,\;i} = \frac{d}{h_i} \cdot I_{ij}$$
 , where $i \in I$

$$I_{i,j} = \begin{cases} 1 & if \ \varphi_i(m_j) \ge n_i \cdot h_i(f_i(m)) \\ 0 & otherwise \end{cases}$$

c. The system graph generates an ergodic Markov Chain and Perron-Frobenius theorem can be applied.

Let's define δ_{j}^{eq} as the asymptotic equilibrium of state i.

$$\delta_j^{eq} \ = \ \sum_{j=0}^I p_{i,j} \delta_j^{eq} \qquad \qquad \sum_{j=0}^I \delta_j^{eq} \ = \ 1$$

In this specific model:

$$\delta_0^{eq} = \sum_{j=0}^{I} p_{0,j} \delta_j^{eq}$$
$$= (1-d) \sum_{j=0}^{I} \delta_j^{eq}$$
$$= (1-d)$$

$$\begin{split} \delta_i^{eq} &= \sum_{j=0}^I p_{i,j} \delta_j^{eq} \\ &= p_{0,j} \delta_0^{eq} + \sum_{j=1}^I p_{i,j} \delta_j^{eq} \\ &= (1-d) \cdot \frac{d}{I} + \sum_{\varphi_i(m) \geq n_i \cdot h_i(f_i(m))}^I \frac{d}{h_i} \delta_j^{eq} \\ &\frac{I}{d} \cdot \delta_i^{eq} = (1-d) \cdot \frac{d}{I} + \sum_{\varphi_i(m) \geq n_i \cdot h_i(f_i(m))}^I \frac{d}{h_i} \delta_j^{eq} \cdot \frac{I}{d} \end{split}$$

Leading to the definition of rank R_i :

$$R_i := \frac{I}{d} \cdot \delta_i^{eq}$$

and a stable equilibrium in:

$$R_i = (1 - d) + d \cdot \sum_{\varphi_i(m) \ge n_i \cdot h_i(f_i(m))}^{I} \frac{R_i}{h_i}$$

Concluding Remark

This discussion leads to the definition of an induced Nash games and a direct application, an indexing model capable of determine a stable equilibria. Main feature of this system is the recovery of a complete map between article and sources by only observing the filter function imposed by each press agency. Future application will make use of Mixed Integer Programming techniques for efficiently solve the maximization problem.