Capital Growth Model

Introduction

The capital growth model describes how capital accumulates over time through investment and depreciation, while labor grows at a constant rate. It uses a system of differential equations with a production function to analyze long-term economic growth.

Model

The capital growth model consists of two key equations:

$$\frac{dK}{dt} = s(AK + BL) - \delta K$$

- K(t) - the capital stock at time t.
- s - the savings rate.
- A - Productivity coefficient for capital.
- B - Productivity coefficient for labor.
- δ - Depreciation rate of capital.

$$\frac{dL}{dt} = nL$$

- L(t) - the labor force at time t.
- n - the labor growth rate.

The capital stock grows due to savings from output, which is a function of both capital and labor, while it also depreciates over time. The labor force grows exponentially at a constant rate, reflecting demographic changes. This system captures the dynamics of capital accumulation and labor force growth, helping to model long-term economic development

Numerical Solution

Solution of a system of ordinary differential equations involves approximating the values of the dependent variables (in this case, capital K and labor L) over time using discrete steps. Instead of solving the ODEs analytically, which might not always be possible, we use numerical methods to estimate the solution.
Apply a numerical method (like
Euler's method) to approximate the values of K(t) and L(t) at discrete time points. In **Euler's method**, the update is based on the derivative at the current point

$$K(t+h) = K(t) + h \cdot \frac{dK}{dt}$$

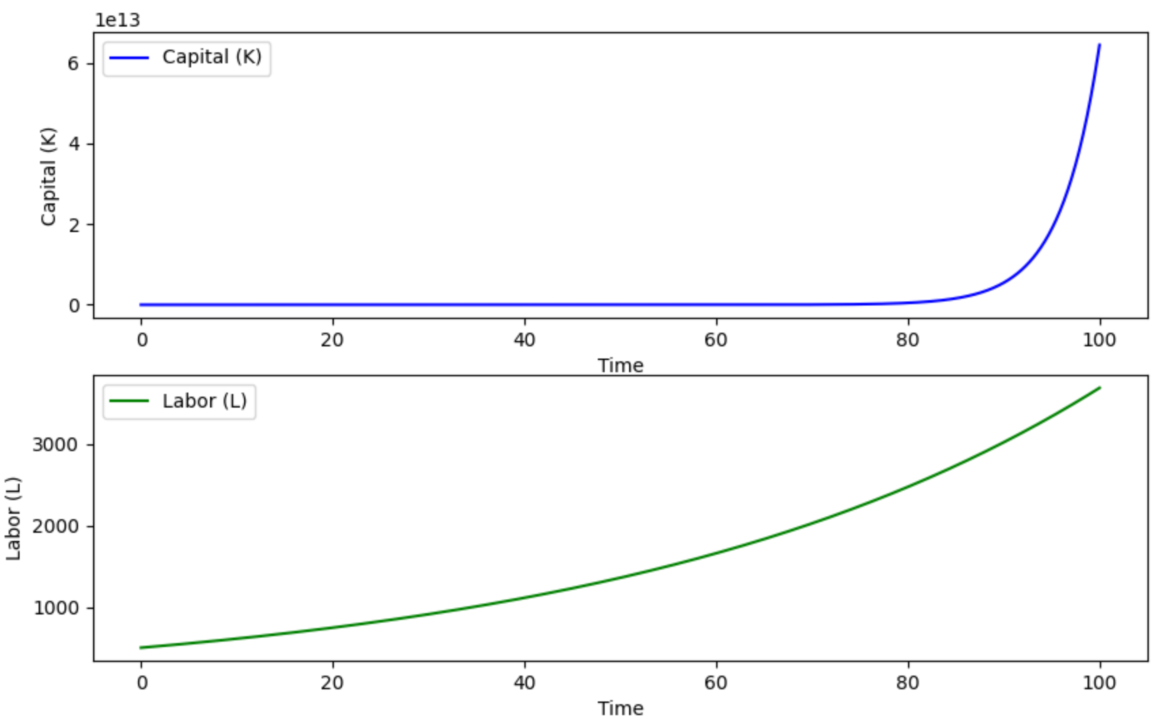
$$L(t+h) = L(t) + h \cdot \frac{dL}{dt}$$

Implementation

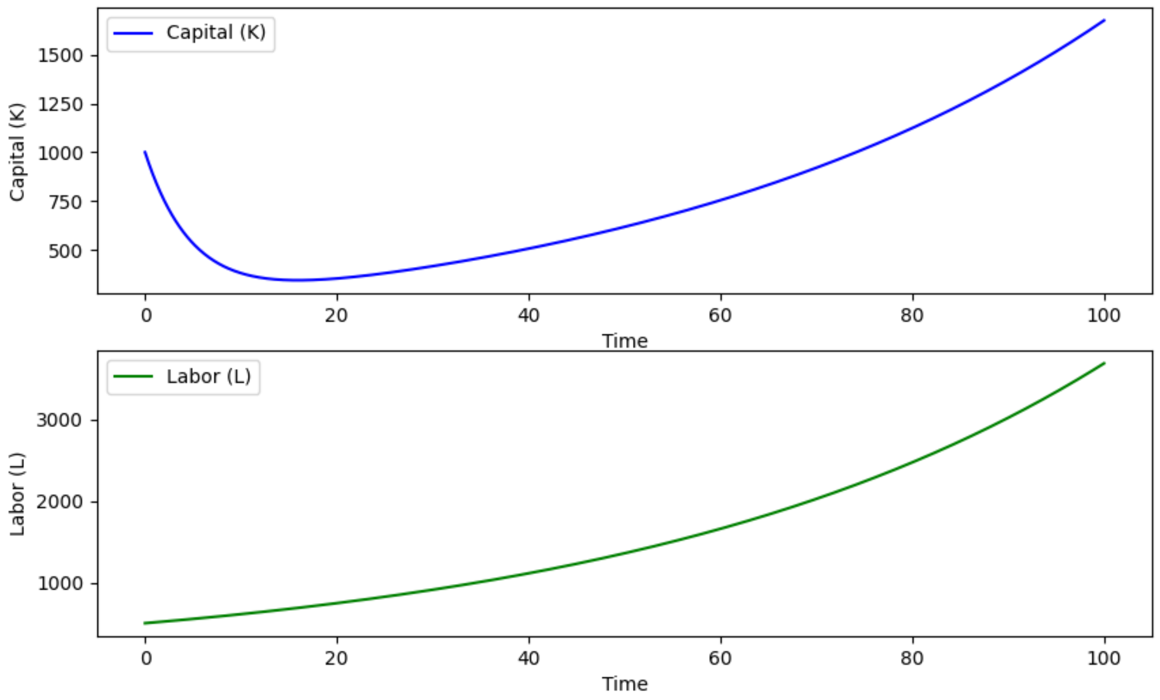
```
# Euler's Method for solving the system of ODEs
def euler_method(K0, L0, s, A, B, delta, n, t0, tf, h):
    # Number of steps
    steps = int((tf - t0) / h)

    # Arrays to store the values of K, L, and time
    K = np.zeros(steps + 1)
    L = np.zeros(steps + 1)
    t = np.linspace(t0, tf, steps + 1)

    # Initial conditions
    K[0] = K0
    L[0] = L0

    # Euler's method loop
    for i in range(steps):
        # Compute derivatives
        f_K = dK_dt(K[i], L[i], s, A, B, delta)
        f_L = dL_dt(L[i], n)

        # Update values using Euler's method
        K[i + 1] = K[i] + h * f_K
        L[i + 1] = L[i] + h * f_L

    return t, K, L
```

Visualization

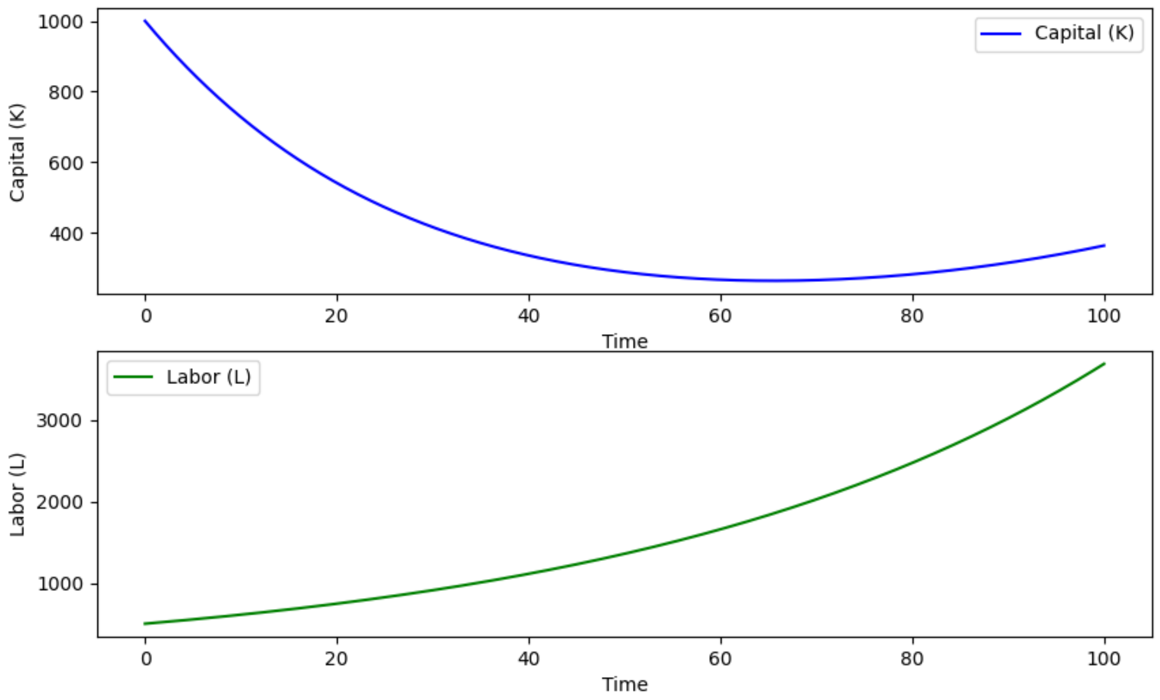
- `K = 1000`, `L = 500`, `s = 0.2`, `A = 1.5`, `B = 0.5`, `delta = 0.05`, `n = 0.02`



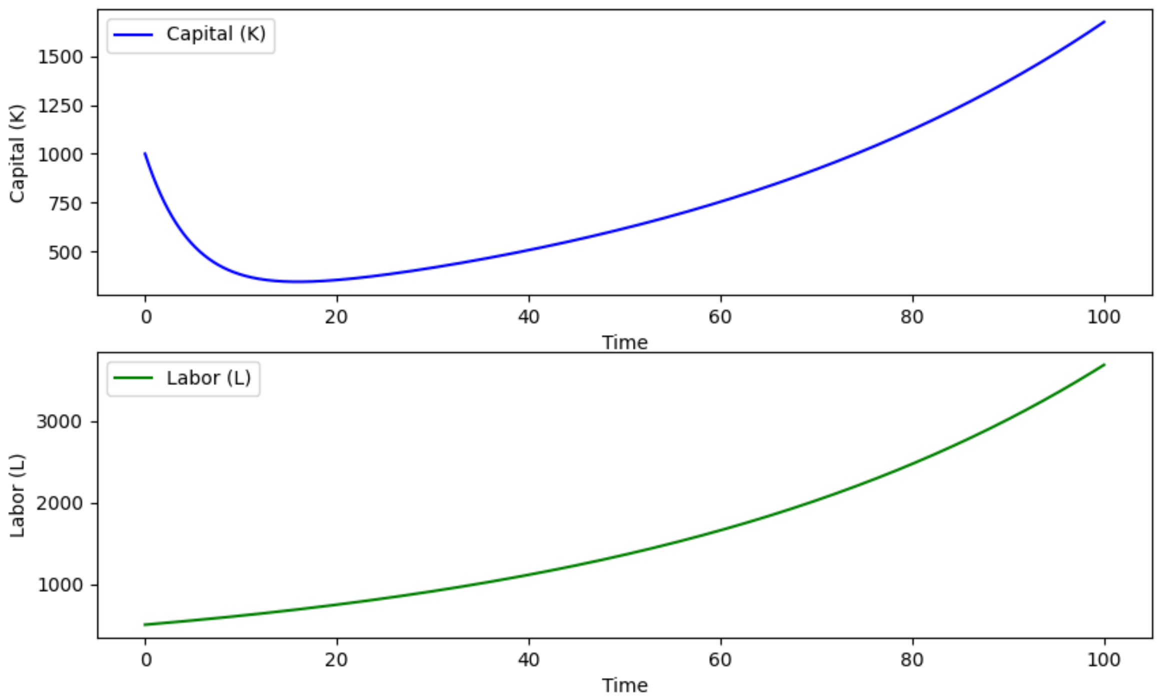
- `K = 1000`, `L = 500`, `s = 0.2`, `A =` 4, `B = 0.5`, `delta = 0.05`, `n = 0.02`



- `K = 1000`, `L = 500`, `s = 0.01`, `A = 1.5`, `B = 0.5`, `delta = 0.05`, `n = 0.02`



- `K = 1000`, `L = 500`, `s = 0.2`, `A = 1.5`, `B = 0.5`, `delta = 0.5`, `n = 0.02`



Conclusion

In conclusion, the relationships between the parameters and variables are as follows:

- **Savings rate (s)**: Increasing s accelerates capital accumulation (K).
- **Productivity coefficient for capital (A)**: Higher A boosts capital growth and may increase labor demand.
- **Productivity coefficient for labor (B)**: Increasing B enhances labor productivity, potentially raising both capital and labor demand.
- **Depreciation rate of capital (δ)**: A higher δ reduces capital growth by increasing capital loss through depreciation.
- **Labor growth rate (n)**: Increasing n speeds up labor force growth, influencing capital and output dynamics.

These interactions help optimize strategies in economic modeling and resource management.