Q1(1):

Sum of roots$=\alpha+\beta=-\frac{-1}{3}=\frac{1}{3}$ and product of roots$=\alpha\beta=\frac{-3}{3}=-1$.

Then, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(\frac{1}{3})^2-2(-1)=\boxed{\dfrac{19}{9}}$.

Alternative As $3x^2-x-3=0$, we have $x^2=\frac{x+3}{3}$.

Then, $\alpha^2+\beta^2=\frac{\alpha+3}{3}+\frac{\beta+3}{3}=\frac{(\alpha+\beta)+6}{3}=\frac{\frac{1}{3}+6}{3}=\boxed{\dfrac{19}{9}}$.

Q1(2):

$-x<x^2<2x+1$

$-x<x^2$ and $x^2<2x+1$

$x(x+1)>0$ and $(x-\frac{2+\sqrt{2^2-4(1)(-1)}}{2})(x-\frac{2-\sqrt{2^2-4(1)(-1)}}{2})<0$

$(x<-1$ or $x>0)$ and $1-\sqrt{2}<x<1+\sqrt{2}$

$\boxed{0<x<1+\sqrt{2}}$

Q1(3):

For $0<\alpha,\beta<90°$, we have $\cos\alpha>0$ and $\sin\beta>0$.

Then, by the identity $\sin^2\theta+\cos^2\theta=1$, we have:

$\cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-(\frac{1}{\sqrt{5}})^2}=\sqrt{\frac{4}{5}}=\frac{2}{\sqrt{5}}$

$\sin\beta=\sqrt{1-\cos^2\beta}=\sqrt{1-(\frac{3}{\sqrt{10}})^2}=\sqrt{\frac{1}{10}}=\frac{1}{\sqrt{10}}$

Then, $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$

$=(\frac{1}{\sqrt{5}})(\frac{3}{\sqrt{10}})+(\frac{1}{\sqrt{10}})(\frac{2}{\sqrt{5}})$

$= \frac{3}{5\sqrt{2}} + \frac{2}{5\sqrt{2}}$
$= \boxed{\dfrac{1}{\sqrt{2}}}$.
(Note: Rationalisation is not necessary. After retionalisation, the answer will become $\frac{\sqrt{2}}{2}$)

Q1(4):

$3^n < 2^{100} < 3^{n+1} \iff n < \log_3 2^{100} < n+1 \iff n < 100(0.631) < n+1 \iff 62.1 < n < 63.1$

As n is a natural number, we have only $n = \boxed{63}$ satisfies the above inequality.

Q1(5):

$$x^2 - 4xy + 5y^2 + 2y - 4 = 0$$

$$(x - 2y)^2 + y^2 + 2y - 4 = 0$$

$$(x - 2y)^2 + (y + 1)^2 = 5$$

When x, y are integers, $x - 2y$ and $y + 1$ are also integers, and the solutions are:

$(x - 2y, y + 1) = (\pm 1, 2), (\pm 1, -2), (\pm 2, 1)$ and $(\pm 2, -1)$.

Note that for each solution, y obtains an integer value and so do x. Moreover, there are no duplicated solutions. Therefore, there are totally $\boxed{8}$ pairs of (x, y).

Q2:

(1): As $0 \le a \le 2$, $f(a)$ can be rewritten as

$f(a) = \int_0^a |x(x-a)|dx + \int_a^2 |x(x-a)|dx$.

As for the interval $(0, a)$, $x - a < 0$, we have

$\int_0^a |x(x-a)|dx = \int_0^a x(a-x)dx = \int_0^a (ax - x^2)dx = \frac{a}{2}x^2 - \frac{1}{3}x^3|_0^a = \frac{a^3}{2} - \frac{a^3}{3} = \frac{a^3}{6}$

On the other hand, as for the interval $(a, 2)$, $x - a > 0$, we have

$\int_a^2 |x(x-a)|dx = \int_a^2 x(x-a)dx = \int_a^2 (x^2 - ax)dx = \frac{1}{3}x^3 - \frac{a}{2}x^2|_a^2$

$= \frac{8}{3} - 2a - \frac{a^3}{3} + \frac{a^3}{2} = \frac{a^3}{6} - 2a + \frac{8}{3}$.

Combine the above, we have $f(a) = \int_0^a |x(x-a)|dx + \int_a^2 |x(x-a)|dx$

$= \frac{a^3}{6} + \frac{a^3}{6} - 2a + \frac{8}{3} = \boxed{\frac{a^3}{3} - 2a + \frac{8}{3}}$.

(2): $f'(a) = a^2 - 2$, where $0 \le a \le 2$.

To find the extremum of $f(a)$, we set $f'(a) = 0$, then $a = \sqrt{2}$.

On the other hand, $f''(a) = 2a$.

Conduct the second derivative test: $f''(\sqrt{2}) = 2\sqrt{2} > 0$. Hence, $f(a)$ attains to its minimum when $a = \sqrt{2}$ and by that time, the minimum value of $f(a)$ is $f(\sqrt{2}) = \frac{(\sqrt{2})^3}{3} - 2\sqrt{2} + \frac{8}{3} = \boxed{\frac{8 - 4\sqrt{2}}{3}}$.

Alternative to (2) See MEXT's official solution, which used the first derivative test instead.

Q3:

(1): By the recurrence, we have:

$a_2 = |a_1| - 1 = |a| - 1 = a - 1$

$a_3 = |a_2| - 1 = |a - 1| - 1 = a - 2$

$a_4 = |a_3| - 1 = |a - 2| - 1 = 2 - a - 1 = \boxed{1 - a}$ (as $a - 2 < 0$)

$a_5 = |a_4| - 1 = |1 - a| - 1 = a - 1 - 1 = \boxed{a - 2}$ (as $1 - a < 0$)

$a_6 = |a_5| - 1 = |a - 2| - 1 = 2 - a - 1 = \boxed{1 - a}$

$a_7 = |a_6| - 1 = |1 - a| - 1 = a - 1 - 1 = \boxed{a - 2}$

(2): Calculate directly, we have:

$S_2 = a_1 + a_2 = a + a - 1 = \boxed{2a - 1}$

$S_4 = S_2 + a_3 + a_4 = 2a - 1 + a - 2 + 1 - a = \boxed{2a - 2}$

$S_6 = S_4 + a_5 + a_6 = 2a - 2 + a - 2 + 1 - a = \boxed{2a - 3}$

(3): Note that for an integer $k > 2$, we have $a_{2k-1} = a - 2$ and $a_{2k} = 1 - a$.

Therefore, $a_{2k-1} + a_{2k} = a - 2 + 1 - a = -1$ for all integers $k > 2$.

$S_n = S_{2m}$

$= S_2 + (a_3 + a_4) + (a_5 + a_6) + ... + (a_{2m-1} + a_{2m})$

$= (2a - 1) + \underbrace{(-1) + (-1) + ... + (-1)}_{\text{m-1 terms}}$

$= 2a - 1 - (m - 1)$

$= \boxed{2a - m}$.

(4): $S_n = S_{2m+1}$

$= S_{2m} + a_{2m+1}$

$= (2a - m) + (a - 2)$

$= \boxed{3a - m - 2}$.