Q1(1):

$|2x-1| < x+2$

$\iff -(x+2) < 2x-1 < x+2$

$\iff \begin{cases} -x-2 < 2x-1 \\ 2x-1 < x+2 \end{cases}$

$\iff \begin{cases} 3x > -1 \\ x < 3 \end{cases}$

$\iff \begin{cases} x > -\frac{1}{3} \\ x < 3 \end{cases}$

$\iff \boxed{-\dfrac{1}{3} < x < 3}$

Q1(2):

The necessary and sufficient condition for the graph of $y = x^2 + ax + 1$ touches the x-axis is that the equation $x^2 + ax + 1 = 0$ has only one solution, i.e.

$$\Delta = a^2 - 4(1)(1) = 0$$

$$a = \boxed{\pm 2}$$

Q1(3):

$f(x) = (\log_2 x)^2 + \log_4 x + 1 = (\log_2 x)^2 + \frac{1}{2}\log_2 x + 1.$

By completing the square, we have $f(x) = (\log_2 x + \frac{1}{4})^2 - (\frac{1}{4})^2 + 1 = (\log_2 x + \frac{1}{4})^2 + \frac{15}{16}$. As $(\log_2 x + \frac{1}{4})^2 \geq 0$, we have $f(x) \geq \frac{15}{16}$. Hence the minimum value of $f(x)$ is $\boxed{\dfrac{15}{16}}$.

Q1(4):

Denote the three points as $A(1, 2, 4)$, $B(2, 5, 6)$ and $C(n, m, 10)$ respectively. The three points are colinear if and only if $\vec{AB} // \vec{BC}$. As $\vec{AB} = < 1, 3, 2 >$ and $\vec{BC} = < n-2, m-5, 4 >$. In that case, we have $\frac{n-2}{1} = \frac{m-5}{3} = \frac{4}{2}$. Therefore, $n = 2 + 2 = \boxed{4}$ and $m = 2 \cdot 3 + 5 = \boxed{11}$.

Alternative: When the three points are colinear in the xyz-space, their projections on the xz-plane and the yz-plane should also be colinear (and hence the slopes between any two points are the same). Therefore, we have the slope of AB in the xz-plane=the slope of BC in the xz-plane, i.e. $\frac{6-4}{2-1} = \frac{10-6}{n-2}$, i.e. $n = \boxed{4}$. Also, the slope of AB in the yz-plane=the slope of BC in the yz-plane, i.e. $\frac{6-4}{5-2} = \frac{10-6}{m-5}$, i.e. $m = \boxed{11}$.

Q1(5):

$\int_0^{\frac{\pi}{2}} x \sin x dx$

$= \int_0^{\frac{\pi}{2}} x d(\cos x)$

$= x \cos x|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \cos x dx$

$= \frac{\pi}{2}\cos\frac{\pi}{2} - 0\cos 0 + \sin x|_0^{\frac{\pi}{2}}$

$= \sin\frac{\pi}{2} - \sin 0$

$= \boxed{1}$

Q2:

(1): $\triangle OAB : \triangle OBC = 1 : 2$

$\frac{1}{2}(OA)(OB)\sin\alpha : \frac{1}{2}(OB)(OC)\sin\beta = 1 : 2$

$\sin\alpha : \sin\beta = \boxed{1 : 2}$

(2): As AOD is a straight line, we have $\angle COD = \pi - \alpha - \beta$. Moreover, as $\triangle OBC : \triangle OCD = 2 : 2 = 1 : 1$, similar to that in part (1), we have

$$\sin\beta : \sin(\pi - \alpha - \beta) = 1 : 1$$

$$\sin\beta = \sin(\pi - \alpha - \beta)$$

$$\sin\beta = \sin(\alpha + \beta)$$

$$\sin\beta = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

Let $x = \sin\alpha$, by part (1), we have $\sin\beta = 2\sin\alpha = 2x$. Moreover, by the identity $\sin^2\theta = 1 - \cos^2\theta$, we have $\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - x^2}$ and $\cos\beta = \sqrt{1 - \sin^2\beta} = \sqrt{1 - 4x^2}$ (as α and β are less than $\frac{\pi}{2}$, $\cos\alpha$ and $\cos\beta$ are taking positive sign). Substitue the above into the equation, we have

$$2x = x\sqrt{1 - 4x^2} + 2x\sqrt{1 - x^2}$$

$$4 = (1 - 4x^2) + 4\sqrt{(1 - x^2)(1 - 4x^2)} + 4(1 - x^2) \text{ (as } x \neq 0)$$

$$64x^4 - 16x + 1 = 16(1 - 5x^2 + 4x^4)$$

$$64x^2 = 15$$

$$x = \frac{\sqrt{15}}{8} \text{ (as } \sin\alpha > 0)$$

Therefore, $\sin\alpha = x = \frac{\sqrt{15}}{8}$. We have $\triangle OAB = \frac{1}{2}(1)(1)\sin\alpha = \frac{\sqrt{15}}{16}$. The area of $ABCD = \triangle OAB + \triangle OBC + \triangle OCD = \triangle OAB + 2\triangle OBC + 2\triangle OCD = 5\triangle OAB = \boxed{\frac{5\sqrt{15}}{16}}$.

Q3:

(1): Let the point of tangent of l_2 to C be $Q(q, 2q^3)$ $(q < 0)$. Consider the derivative of C, $\frac{dy}{dx} = 6x^2$, we have the slope of the tangent to C at Q (i.e. l_2)$=\frac{dy}{dx}|_{x=q} = 6q^2$......(1).

On the other hand, as l_2 passes through P, we have the slope of $l_2 = \frac{2p^3-2q^3}{p-q} = \frac{2(p-q)(p^2+pq+q^2)}{p-q} = 2(p^2 + pq + q^2)$......(2).

Combine (1) and (2), we have

$$6q^2 = 2(p^2 + pq + q^2)$$

$$2q^2 - pq - p^2 = 0$$

$$(2q + p)(q - p) = 0$$

$$q = -\frac{p}{2} \text{ or } q = p\text{(rejected)}$$

Therefore, the slope of $l_2 = 6q^2 = 6(-\frac{p}{2})^2 = \boxed{\frac{3}{2}p^2}$.

(2): Note that for ϕ, the angle between a straight line and the positive x-axis, we have $\tan\phi$ =the slope of the line.

Let θ_1 and θ_2 be the angle between line l_1 (l_2) and the positive x-axis respectively, we have $\tan\theta_1$ =slope of $l_1 = 6p^2$ and $\tan\theta_2$ =slope of $l_2 = \frac{3}{2}p^2$. Then, we have:

$\tan\theta$

$= \tan(\theta_1 - \theta_2)$

$= \frac{\tan\theta_1 - \tan\theta_2}{1+\tan\theta_1\tan\theta_2}$

$= \frac{6p^2 - \frac{3}{2}p^2}{1+(6p^2)(\frac{3}{2}p^2)}$

$= \boxed{\dfrac{9p^2}{2+18p^4}}$

(3): We have $\frac{d(\tan\theta)}{dp} = \frac{18p(2+18p^4)-72p^3(9p^2)}{(2+18p^4)^2}$. Then, we have $\frac{d(\tan\theta)}{dp}$ if and only if

$$\frac{18p(2+18p^4)-72p^3(9p^2)}{(2+18p^4)^2} = 0$$

$$2+18p^4 - 36p^4 = 0 \text{ (as } p \neq 0)$$

$$p^4 = \frac{1}{9}$$

$$p = \frac{1}{\sqrt{3}} \text{ (as } p > 0)$$

.

The table of first derivative test:

p	$(0, \frac{1}{\sqrt{3}})$	$(\frac{1}{\sqrt{3}}, +\infty)$
$\frac{d(\tan\theta)}{dp}$	$+$	$-$
$\tan\theta$	$\nearrow$	$\searrow$

Therefore, $\tan\theta$ attains to its maximum when $p = \frac{1}{\sqrt{3}}$ and the corresponding value of $\tan\theta = \frac{9(\frac{1}{\sqrt{3}})^2}{2+18(\frac{1}{\sqrt{3}})^4} = \frac{3}{2+2} = \boxed{\frac{3}{4}}$.