

Q1(1):

$$\begin{cases} \frac{a+b}{4} = \frac{b+c}{5} \\ \frac{b+c}{5} = \frac{c+a}{6} \end{cases} \iff \begin{cases} 5a + b = 4c \dots (1) \\ 5a - 6b = c \dots (2) \end{cases}$$

By (1)-(2), we have $b = \frac{3}{7}c$.

Substitute it into (1), we have $a = \frac{5}{7}c$.

Therefore, $a : b : c = 5 : 3 : 7 = 1 : \boxed{\frac{3}{5}} : \boxed{\frac{7}{5}}$.

Q1(2):

We have $a + b = 6$ and $ab = 3^2 - (2\sqrt{2})^2 = 1$.

Therefore, $a^2 + b^2 = (a + b)^2 - 2ab = 6^2 - 2(1) = \boxed{34}$.

Moreover, $\frac{a^2}{b} + \frac{b^2}{a} = \frac{(a+b)^3 - 3ab(a+b)}{ab} = \frac{6^3 - 3(1)(6)}{1} = \boxed{198}$.

Alternative Calculating directly, we have:

$$a^2 + b^2 = (3 + 2\sqrt{2})^2 + (3 - 2\sqrt{2})^2 = (17 + 12\sqrt{2}) + (17 - 12\sqrt{2}) = \boxed{34}.$$

$$\frac{a^2}{b} + \frac{b^2}{a} = \frac{17+12\sqrt{2}}{3-2\sqrt{2}} + \frac{17-12\sqrt{2}}{3+2\sqrt{2}} = \frac{(99+70\sqrt{2})+(99-70\sqrt{2})}{3^2-(2\sqrt{2})^2} = \boxed{198}.$$

Q1(3):

$$\cos 30^\circ \sin 45^\circ \tan 60^\circ + \cos 135^\circ \sin 120^\circ \tan 150^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)(\sqrt{3}) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{3}\right)$$

$$= \frac{3\sqrt{2}}{4} + \frac{\sqrt{2}}{4}$$

$$= \boxed{\sqrt{2}}$$

Q1(4):

$$(x+1)^2 + 9(x+1) + 20 = 0$$

$$(x+1+5)(x+1+4) = 0$$

$$x = \boxed{-5}, \boxed{-6}$$

Q1(5):

$$-ax^2 + bx + 4 \geq 0$$

$$ax^2 - bx - 4 \leq 0$$

$$\alpha \leq x \leq \beta$$

Where $\alpha \leq \beta$ are the two solutions of $-ax^2 + bx + 4 = 0$.

As $-\frac{1}{3} \leq x \leq 4$, we have:

$$\begin{cases} -a(\frac{1}{9}) - b(\frac{1}{3}) + 4 = 0 \\ -16a + 4b + 4 = 0 \end{cases} \iff \begin{cases} a + 3b = 36 \dots\dots (1) \\ 4a - b = 1 \dots\dots (2) \end{cases}$$

By $(1) + 3 \times (2)$, we have $13a = 19$, i.e. $a = \boxed{3}$.

Substitue it into (2), we have $b = 12 - 1 = \boxed{11}$.

Q1(6):

$$\begin{aligned} & \log_a b^a \times \log_b a^b \\ &= a \log_a b \times b \log_b a \\ &= a \frac{\log b}{\log a} \times b \frac{\log a}{\log b} \\ &= ab. \end{aligned}$$

When $a = 3, b = 2$, the expression = $\boxed{6}$.

Q1(7):

As $f(1) = 3$, we have $-1 - 2a + b = 3$, i.e. $2a - b = -4$(1).

By completing the square, we have $f(x) = -(x+a)^2 + a^2 + b$. As the maximum value is 4, we have $a^2 + b = 4$(2).

By (1)+(2), we have $a^2 + 2a = 0$, i.e. $a = \boxed{-2}$ as $a \neq 0$.

Substitute it into (1), we have $b = 2(-2) + 4 = \boxed{0}$.

Q1(8):

Note that $\{a_n\}$ is an arithmetic sequence with a common difference 3. Therefore, we have the general formula $a_n = 2 + 3(n - 1)$.

Solving $a_n > 100$ for $n \in \mathbb{N}$, we have $3n > 101$, i.e. $n > \frac{101}{3} > 33$.

Therefore, the minimum value of n satisfies it is $\boxed{34}$.

Q1(9):

(i): $f(2) = 2^3 - 2 \cdot 2 + 4 = \boxed{8}$.

(ii): As $f'(x) = 3x^2 - 2$, we have $f'(2) = 3(2)^2 - 2 = \boxed{10}$.

(iii): By testing among potential possible rational roots $\pm 1, \pm 2, \pm 4$ given by the rational root theorem, we have $x = -2$ is a rational root.

By the long division, we have $x^3 - 2x + 4 = (x + 2)(x^2 - 2x + 2)$.

Therefore, $f(x) = 0 \iff x = -2$ or $x^2 - 2x + 2 = 0$.

For the latter, as $\Delta = 4 - 4(1)(2) = -4 < 0$, the equation has no real roots.

Given the above, the only real value of x satisfying $f(x) = 0$ is $x = \boxed{-2}$.

(iv): $\int_0^2 f(x)dx = \int_0^2 (x^3 - 2x + 4)dx = \left[\frac{1}{4}x^4 - x^2 + 4x\right]_0^2 = \boxed{8}$.

Q2:

(1): The coordinates of the centre of $\triangle ABC$ is given by

$$\left(\frac{a-1+2}{3}, \frac{b+0+1}{3}\right) = \left(\frac{a+1}{3}, \frac{b+1}{3}\right).$$

If D is the centre, we have $\frac{a+1}{3} = 0$, i.e. $a = \boxed{-1}$ and $\frac{b+1}{3} = 2$, i.e. $b = \boxed{5}$.

(2): If ABCD is a parallelogram, then $AB \parallel CD$ and $AD \parallel BC$, i.e. the slope of AB is equal to that of CD and that of AD is equal to that of BC.

Therefore, we have $\frac{b-0}{a+1} = \frac{1-2}{2-0}$ and $\frac{b-2}{a-0} = \frac{0-1}{-1-2}$.

i.e. $a + 2b = -1 \dots (1)$ and $a - 3b = -6 \dots (2)$.

By $(1) - (2)$, we have $5b = 5$, i.e. $b = \boxed{1}$.

Substitue it into (1) , we have $a = -1 - 2 = \boxed{-3}$.

Alternative If ABCD is a parallelogram, then we have $\vec{BA} = \vec{CD}$, i.e.

$$\langle 0 - 2, 2 - 1 \rangle = \langle -2, 1 \rangle.$$

Therefore, $(a, b) = (-1 - 2, 0 + 1) = (\boxed{-3}, \boxed{1})$.

(3): If $\angle ABC = 90^\circ$, we have the slope of AB multiplied by the slope of BC is equal to -1, i.e. $(\frac{b-0}{a+1})(\frac{0-1}{-1-2}) = -1$, i.e. $3a + b = -3 \dots (1)$.

On the other hand, as D is lying on AC , we have the slope of AC is equal to the slope of DC , i.e. $\frac{b-1}{a-2} = \frac{1-2}{2-0}$, i.e. $a + 2b = 4 \dots (2)$.

By $(2) - 2 \times (1)$, we have $-5a = 10$, i.e. $a = \boxed{-2}$.

Substitue it into (1) , we have $b = -3 - 3(-2) = \boxed{3}$.

(4): $\vec{CA} = \langle a - 2, b - 1 \rangle$, $\vec{CB} = \langle -3, -1 \rangle$ and $\vec{CD} = \langle -2, 1 \rangle$.

If $\vec{CA} = 2\vec{CB} - 3\vec{CD}$, then $\langle a - 2, b - 1 \rangle = 2\langle -3, -1 \rangle - 3\langle -2, 1 \rangle = \langle 0, -5 \rangle$.

By solving, we have $a = \boxed{2}$ and $b = \boxed{-4}$.

(5): $\vec{BA} = \langle a + 1, b \rangle$, $\vec{BC} = \langle 3, 1 \rangle$ and $\vec{BD} = \langle 1, 2 \rangle$.

By $\vec{BA} \cdot \vec{BC} = -2$, we have $3(a + 1) + b = -2$, i.e. $3a + b = -5 \dots (1)$.

By $\vec{BA} \cdot \vec{BD} = 1$, we have $a + 1 + 2b = 1$, i.e. $a + 2b = 0 \dots (2)$.

By $(2) - 2 \times (1)$, we have $-5a = 10$, i.e. $a = \boxed{-2}$.

Substitue it into (1), we have $b = -5 - 3(-2) = \boxed{1}$.

Q3:

(1): As the region is on the left of the vertical line $x = 1$, we have $\boxed{x < 1}$.

Moreover, as the region is above the inclined line $y = x + 1$, we have $\boxed{y > x + 1}$.

(2): As the region is below the inclined line $y = x + 1$, we have $\boxed{y < x + 1}$.

As the region is above the parabola $y = x^2$, we have $\boxed{y > x^2}$.

(3): As the region is outside the region bounded by $|x| = 1$, we have $\boxed{|x| > 1}$.

As the region is above the parabola $y = x^2$, we have $\boxed{y > x^2}$.

(4): As the region is below the parabola $y = x^2$, we have $\boxed{y < x^2}$.

As the region is inside the circle $x^2 + y^2 = 1$, we have $\boxed{x^2 + y^2 < 1}$.

(5): As the region is inside the region bounded by $|x| = 1$, we have $\boxed{|x| < 1}$.

As the region is outside the circle $x^2 + y^2 = 1$, we have $\boxed{x^2 + y^2 > 1}$.