Q1(1):

Note that $7^{2677}$ can be written as $a \cdot 10^n$, where $n \in \mathbb{N}$ and $a \in \mathbb{Q}$ such that $0 < a < 10$, then, the number of digits of $7^{2677}$ will be $n+1$.

As $\log_{10} 7^{2677} \approx 2677(0.8451) \approx 2262.3327 < 2263$, we have $n+1 = 2263$ and hence the number of digits is $\boxed{2263}$.

On the other hand, we observe the pattern of the last digit of $7^n$:

$n = 1 :\ 7$

$n = 2 :\ 9$

$n = 3 :\ 3$

$n = 4 :\ 1$

$n = 5 :\ 7$

$n = 6 :\ 9$

$\vdots$

We can see that the last digit of $7^{4k}$.

As $7^{2677} = 7^{667} \cdot 7$, the last digit of it will be 7.

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Q1(2):

By rationalisation, we have $\frac{1}{\sqrt{n}+\sqrt{n+1}} = \sqrt{n} - \sqrt{n-1}$.

Note that the expression$= \sum_{n=1}^{4} \frac{1}{\sqrt{n}+\sqrt{n+1}} = \boxed{1-\sqrt{5}}$ by the telescoping property.

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Q1(3):

As $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta} = \frac{2\tan\theta}{\tan^2\theta+1}$.

If $\sin 2\theta = \frac{1}{4}$, we have

$$\frac{2\tan\theta}{\tan^2\theta+1} = \frac{1}{4}$$

$$\tan^2\theta - 8\tan\theta + 1 = 0$$

$$\tan\theta = 4 \pm \sqrt{15}$$

On the other hand, $\frac{\sin\theta+\cos\theta}{-\sin\theta+\cos\theta} = \frac{\tan\theta+1}{-\tan\theta+1}$.

If $\tan\theta = 4+\sqrt{15}$, the expression will valued negative, which contradicts the condition $0 < \theta < \frac{\pi}{4}$.

Therefore, the expression=$\frac{5-\sqrt{15}}{\sqrt{15}-3} = \boxed{\dfrac{\sqrt{15}}{3}}$.

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Q1(4):

A triangle can be formed if $i, j, k$ are mutually different.

Then, the probability=$\frac{P^6_3}{6^3} = \frac{\boxed{5}}{9}$.

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Q1(5):

$$4^x - 2^x - 12 = 0$$

$$(2^x)^2 - 2^x - 12 = 0$$

$$(2^x - 4)(2^x + 3) = 0$$

$$2^x = 4$$

$$x = \boxed{2}$$

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Q1(6):

The position vector of the centroid of a triagnle with position vector $\vec{a}, \vec{b}, \vec{c}$ is given by $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$.

Therefore, we have $\vec{OF} = \frac{\vec{OA}+\vec{OB}}{3}$, $\vec{OG} = \frac{\vec{OB}+\vec{OC}}{3}$, and $\vec{OH} = \frac{\vec{OC}+\vec{OA}}{3}$.

Moreover, $\vec{OP} = \frac{\vec{OF}+\vec{OG}+\vec{OH}}{3} = \frac{2}{\boxed{9}}(\vec{OA}+\vec{OB}+\vec{OC})$.

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Q1(7):

Let the point on $OA$ be $P(p, 0)$ and the point on $OB$ be $Q(q, q)$.

When the length of the path is minimised, $CP$ reflected along the normal of $OA$ to reach $Q$, and the same for $PQ$*.

As $C$ becomes $(2, -1)$ (denote it as $C'$) after reflected along $OA$, we have the equation of the reflected line $CP$, $y + 1 = \frac{-1}{2-p}(x-2)$. As the line reaches $Q$, we have $q + 1 = \frac{q-1}{p-2}$.

Moreover, $C'$ becomes $(-1, 2)$ after reflected along $OB$, we have the equation of the reflected line $QD$, $y - 2 = \frac{2-q}{-1-q}(x+1)$. Now, substitue the coordinates of $D$ inside the equation, we have $-1 = \frac{q-2}{1+q}(4)$, i.e. $q = \frac{7}{5}$.

Now, substitue it into the former equation of $p, q$, we have $\frac{7}{5} + 1 = \frac{\frac{7}{5}-2}{p-2}$, i.e. $p = \frac{7}{4}$.

Therefore, the point on $OA$ is $(\boxed{\frac{7}{4}}, \boxed{0})$ and the point on $OB$ is $(\boxed{\frac{7}{5}}, \boxed{\frac{7}{5}})$.

Moreover, the length of the path=

$\sqrt{(2-\frac{7}{4})^2+(1-0)^2}+\sqrt{(\frac{7}{4}-\frac{7}{5})^2+(0-\frac{7}{5})^2}+\sqrt{(\frac{7}{5}-3)^2+(\frac{7}{5}-1)^2}$

$= \boxed{\sqrt{17}}$.

(*: To prove it, see the Fermat's principle.)

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Q1(8):

We plot the graph of $2|m|+3|n-1|=7$ on the $m,n$-plane first:

Note that the turning points are $(0, \pm\frac{7}{3}+1)$ and $(\pm\frac{7}{2}, 1)$.

Joining the turning points with straight line, we get the graph of the equation, a parallelogram.

Therefore, the region of $2|m|+3|n-1| \leq 7$ will be the region inside the parallelogram.

Obviously $m+n$ won't attain to maximum when either $m$ or $n$ is negative. Therefore, we are going to exhaust those lattices point inside the region in the Quadrant $I$.

Hence, we can see that $m+n$ attains to its maximum when $(m,n) = (3, \boxed{1})$ or $(\boxed{2}, \boxed{2})$, and the maximum value is $\boxed{4}$.

(Note: Using the methodology of liinear programming by graph, we should have the maximum value taken when $(m,n) = (3,1)$. The maximum value also taken when $(m,n) = (2,2)$ is just an coincidence that that point, which is lying inside the region, also lies on the line $x+y=4$. Therefore, using the methode of exhaustion will be more situable here than linear programming.)

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Q1(9):

As $f(x)$ is maximised at $(1, 5)$, we have $f(x)$ is in a form of $a(x-1)^2+5$.

Moreover, by $f(-2) = -22$, we can solve $a = -3$.

Therefore, we have $f(x) = -3(x-1)^2 + 5 = \boxed{-3}x^2 + \boxed{6}x + \boxed{2}$.

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Q1(10):

As for a geometric series, we have $\sum_{l=k}^{n} 2^l = 2^{\boxed{n+1}} - 2^{\boxed{k}}$.

Therefore, $\sum_{k=1}^{n} k2^k = \sum_{k=1}^{n}\sum_{l=k}^{n} 2^l$

$= \sum_{k=1}^{n} (2^{n+1} - 2^k)$

$= n2^{n+1} - (2^{n+1} - 2)$

$= (\boxed{n-1})2^{\boxed{n+1}} + 2$.

---

Q1(11):

As we have:

$123456 = 3 \cdot 41152$

$41152 = 3 \cdot 13717 + 1$

$13717 = 3 \cdot 4572 + 1$

$4572 = 3 \cdot 1524$

$1524 = 3 \cdot 508$

$508 = 3 \cdot 169 + 1$

$169 = 3 \cdot 56 + 1$

$56 = 3 \cdot 18 + 2$

$18 = 3 \cdot 6$

$6 = 2 \cdot 3$,

we have $123456 = 3 \cdot (3 \cdot 13717 + 1)$

$= 3 \cdot (3 \cdot (3 \cdot 4572 + 1) + 1)$

$\vdots$

$= 2 \cdot 3^{10} + 2 \cdot 3^7 + 3^6 + 3^5 + 3^2 + 3$

$= \boxed{20021100110}_2$.

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Q2:

(1): $f'(x) = 3x^2 - 6ax + 3b$.

As $x = 1, 3$ are extremum points, we have $f'(1) = f'(3) = 0$, i.e. $3 - 6a + 3b = 0$ and $27 - 18a + 3b = 0$.

By solving, we have $a = \boxed{2}$ and $b = \boxed{3}$.

Now, solving $f(x) = 0$, by testing among possible rational roots, we have $x = 2$ is a root*.

Then, by long division, we can factorise

$$x^3 - 6x^2 + 9x - 2 = 0$$

$$(x - 2)(x^2 - 4x + 1) = 0$$

$$x = \boxed{2-\sqrt{5}}, \boxed{2}, \boxed{2+\sqrt{5}}$$

(2): If $f(x)$ is monotonously increasing, then $f'(x) \geq 0$.

By completing the square, $f(x) = 3(x-a)^2 + 3a - 3a^2$, we have $f'(x) \geq 0$ if and only if

$$3a - 3a^2 \geq 0$$

$$a(a-1) \leq 0$$

$$\boxed{0} \leq a \leq \boxed{1}$$

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Q3:

(1): We consider the horizontal (parallel to the xy-plane) cross section of $B$.

For a fixed $z$, say $z = t$, we have $x^2 + y^2 \leq t^4$, which is a circle with radius $t^2$.

Hence, we have the cross section area of $B$ is $\pi t^4$.

Now, the volume of $B$ is $\int_0^1 (\pi t^4) dt = \pi[\frac{1}{5}t^5]_0^1 = \boxed{\frac{\pi}{5}}$.

(2): As $\frac{1}{9}x^2 + \frac{1}{4}y^2 \leq z^4 \iff (\frac{x}{3})^2 + (\frac{y}{2})^2 \leq z^2$, we have the solid $A$ is given by elongating the solid $B$ $\boxed{3}$ times in the x-axis direction and $\boxed{2}$ times in the y-axis direction.

(3): By (2), the volume of $A$ is $2 \cdot 3 = \boxed{6}$ times as large as that of $B$.

(4): By (3), the volume of $A$ is $6 \cdot \frac{\pi}{5} = \boxed{\frac{6\pi}{5}}$.