Q1(1):

$\log_{10}(ax)\log_{10}(bx)+1=0$

$\iff (\log_{10}a+\log_{10}x)(\log_{10}b+\log_{10}x)+1=0$

$\iff (\log_{10}x)^2+(\log_{10}a+\log_{10}b)(\log_{10}x)+(1+\log_{10}a\log_{10}b)=0$

Regard it as a quadratic equation for $\log_{10}x$, we have

$$\Delta=(\log_{10}a+\log_{10}b))^2-4(1)(1+\log_{10}a\log_{10}b)\geq 0$$

$$(\log_{10}a-\log_{10}b)^2\leq 4$$

$$\log_{10}\frac{a}{b}\leq -2 \text{ or } \log_{10}\frac{a}{b}\geq 2$$

$$\frac{a}{b}\leq 10^{-2} \text{ or } \frac{a}{b}\geq 10^2$$

For the former case, we have also the constrain $\frac{a}{b}>0$ and $a,b>0$.

Therefore, we have $\frac{a}{b}\geq\boxed{100}$ or $\boxed{\dfrac{1}{100}}>\frac{a}{b}>\boxed{0}$.

Q1(2):

We have $\sin 2\theta=2\sin\theta\cos\theta$

$=-2\sqrt{(\frac{1}{2}+\frac{1}{2\sqrt{2}})(\frac{1}{2}-\frac{1}{2\sqrt{2}})}$

$=-2\sqrt{(\frac{1}{4}-\frac{1}{8})}$

$=-\frac{1}{\sqrt{2}}$.

Also, $\cos 2\theta=2\cos^2\theta-1$

$=2(\frac{1}{2}+\frac{1}{2\sqrt{2}})-1$

$=\frac{1}{\sqrt{2}}$.

As $\sin 2\theta<0$ and $\cos 2\theta>0$, we have 2θ lies on quadrant IV. Therefore, for

$0 \leq 2\theta < 4\pi$, we have $2\theta = \frac{7\pi}{4}$ or $\frac{15\pi}{4}$.

Moreover, as $\sin\theta < 0$ and $\cos\theta > 0$, we have θ also lies on quadrant IV. Therefore, we have only $2\theta = \boxed{\dfrac{15\pi}{4}}$.

(Note: One must evaluate at least two among those trigonometric ratios of 2θ so as to reject one possible value of 2θ.)

Q1(3):

$2^y - 2^{-y} = x + \sqrt{x^2+1} + \frac{1}{x+\sqrt{x^2+1}}$

$= x + \sqrt{x^2+1} + x - \sqrt{x^2-1}$ (rationalised the latter part)

$= \boxed{2}x$.

Q1(4):

As the domain for $\log_2(x)$ is $x \in (0, +\infty)$, we have:

$$\log_3(\log_2(\log_3(\log_2 x))) > 0$$

$$\log_2(\log_3(\log_2 x)) > 1$$

$$\log_3(\log_2 x) > 2$$

$$\log_2 x > 9$$

$$x > 2^9 = \boxed{512}.$$

Q1(5):

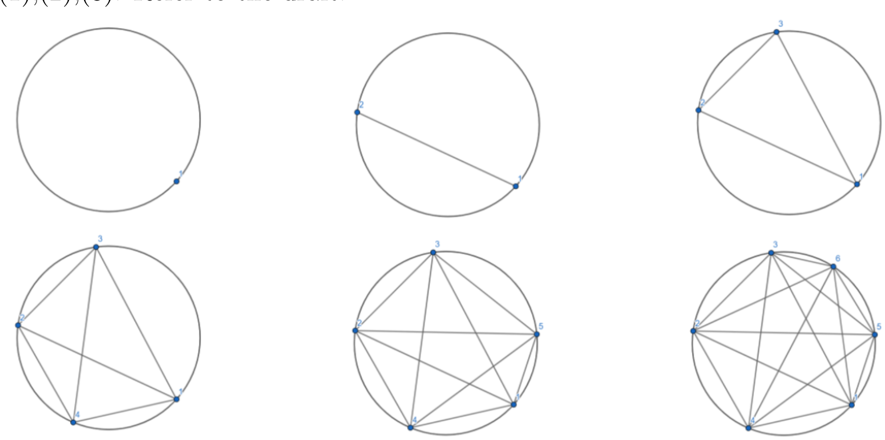
The only case that two digits 11 are remained after borrowing happened in the substraction is $20 - 9$. However, this case is prohibited as 0 is not allowed to use. Therefore, no borrowing is allowed in the subtraction.

Then, the leading digit of the five-digit number will be 1, and other digits of the five-digit number will be in a pair of $(9,8), (7,6), (5,4)$ or $(3,2)$ with the corresponding digits of the four-digit number.

Therefore, the total number of such five-digit number and four-digit number will be the number of permutations of that four pairs, i.e. $4! = \boxed{24}$.

Q2:

(1),(2),(3): Refer to the draft:



By counting directly, we have:

$c_1, ..., c_6 : \boxed{0, 1, 3, 6, 10, 15}$

$i_1, ..., i_6 : \boxed{0, 0, 0, 1, 5, 15}$

$r_1, ..., r_6 : \boxed{1, 2, 4, 8, 16, 31}$

(2): As two points can join a line, we have $c_n = \boxed{\binom{n}{2}}$.

As four distanct points guarantees an intersection, we have $i_n = \boxed{\binom{n}{4}}$.

Observe that each chord increases the number of regions by 1 and each intersection increases further the number of regions by 1. With the original 1 region of the circle, we have $r_n = \boxed{1 + \binom{n}{2} + \binom{n}{4}}$.

(Note: The cases $n = 5, 6$ are a bit complicated and the corresponding values might be difficult to count. It is recommended to find out the general expression first and then calculate the value rather than counting.)

Q3:

(1): Firstly, as 3 is a x-intercept of C, we have $27-36+3a+b = 0$, i.e. $3a+b = 9$. Also, the slope of the tangent to C is given by $y' = 3x^2-8x+a$. As C is tangent to the x-axis (whose slope is 0) at $x = 3$, we have $27-24+a = 0$, i.e. $a = \boxed{-3}$. Substitue $a = -3$ into the former equation, we have $b = 9 + 9 = \boxed{18}$.

(2): We are going to solve $x^3 - 4x^2 - 3x + 18 = 0$. As we know, $x = 3$ is a solution. Therefore, we can do the factorisation:

$$(x-3)(x^2 - x - 6) = 0$$

$$(x-3)^2(x+2) = 0$$

$$x = \boxed{-2, 3}$$

(3): The area$=\int_{-2}^{3}(x^3 - 4x^2 - 3x + 18)dx$

$= [\frac{x^4}{4} - \frac{4x^3}{3} - \frac{3x^2}{2} + 18x]_{-2}^{3}$

$= \frac{81}{4} - 36 - \frac{27}{2} + 54 - 4 - \frac{32}{3} + 6 + 36$

$= \boxed{\dfrac{625}{12}}.$

Alternative To simplify the calculation, we can compute the integral in the following way instead:

$\int_{-2}^{3}(x-3)^2(x+2)dx$

$= \frac{1}{3}\int_{-2}^{3}(x+2)d((x-3)^3)$

$= \frac{1}{3}(x-3)^3(x+2)|_{-2}^{3} - \frac{1}{3}\int_{-2}^{3}(x-3)^3 d(x+2)$

$= -\frac{1}{3}\int_{-2}^{3}(x-3)^3 d(x-3)$

$= -\frac{1}{12}(x-3)^4|_{-2}^{3}$

$= \frac{5^4}{12}$

$= \boxed{\dfrac{625}{12}}.$