

Q1(1):

As the axis of symmetry of the quadratic function is $x = 1$, we have the equation of it is in a form of $y = a(x - 1)^2 + b$.

Now, substitute $(x, y) = (0, 2)$ and $(x, y) = (3, 5)$ into the equation respectively, we have:

$$a + b = 2 \text{ and } 4a + b = 5$$

By solving, we have $(a, b) = (1, 1)$.

Therefore, the function is $(x - 1)^2 + 1 = \boxed{1}x^2 + \boxed{-2}x + \boxed{2}$.

Q1(2):

Let θ be the angle between the side with length 5 and that with length 7.

Then, by the cosine formula, we have $\cos \theta = \frac{5^2 + 7^2 - 8^2}{2 \cdot 5 \cdot 7} = \frac{1}{7}$.

As $0 < \theta < 180^\circ$, we have $\sin \theta = \sqrt{1 - \cos^2 \theta} = \frac{4}{7}\sqrt{3}$.

Therefore, the area of the triangle is $\frac{1}{2}(5)(7)(\frac{4}{7}\sqrt{3}) = \boxed{10\sqrt{3}}$.

Moreover, let r be the radius of the inscribed circle, by considering the area of the triangle, we have $\frac{1}{2}(5)(r) + \frac{1}{2}(7)(r) + \frac{1}{2}(8)(r) = 10\sqrt{3}$. Therefore, $r = \boxed{\sqrt{3}}$.

(Note: Similarly, one can consider the other interior angles of the triangle instead.)

Alternative The area of the triangle can also be evaluated by the Heron's formula:

As $\frac{5+7+8}{2} = 10$, we have the area $= \sqrt{10(10-5)(10-7)(10-8)} = \boxed{10\sqrt{3}}$.

Q1(3):

The first element of the n th group is given by $(1+2+\dots+n-1)+1 = \frac{n(n-1)}{2} + 1$.

Solving $2675 < \frac{n(n-1)}{2} + 1$, i.e. $n(n-1) > 5342$ for the least natural number n :

By some trials, we know that when $n = 73$, $n(n-1) = 5256 < 5342$ and when $n = 74$, $n(n-1) = 5402 > 5342$. Therefore, the least natural number n satisfy the inequality is 74, and hence 2675 is in the $\boxed{73}$ th group.

Moreover, the first element of the 73th group is 2629.

As we have $2675 = 2629 + 46$, we have 2675 is No. $\boxed{47}$ in the group.

Q1(4):

By partial fraction, we have

$$\frac{1}{(3n-1)(3n+2)} = \frac{1}{3(3n-1)} - \frac{1}{3(3n+2)} = \frac{1}{3(3n-1)} - \frac{1}{3(3(n+1)-1)}.$$

Therefore, by the telescoping property, we have

$$\sum_{i=1}^n \left(\frac{1}{(3i-1)(3i+1)} \right) = \sum_{i=1}^n \left(\frac{1}{3(3n-1)} - \frac{1}{3(n+1)-1} \right) = \frac{1}{6} - \frac{1}{3(3n+2)} = \frac{\boxed{n}}{\boxed{2(3n+2)}}.$$

Q1(5):

We separate the first situation into two cases:

1: There is only 1 red ball at the end, i.e. there is one red ball at the end and 1 white ball at the end. The number of such a permutaion= $C_1^3 \cdot C_1^3 \cdot 2! \cdot 4! = 432$.

2: There are 2 red balls at the end. The number of such a permutation= $P_2^3 \cdot 4! = 144$.

Given the above, the total number of permutation= $432 + 144 = \boxed{576}$. For the second situation, we fix one red ball (or white ball, and follow respectively) as the top of the circle.

Then, the two balls next to it must be the white ball. We have $P_2^3 = 6$ possibilities. Moreover, the balls next to those two balls must be red, and we have $2! = 2$ possibilities.

And the remaining white ball will be placed at the bottom automatically.

Given the above, the number of permutations= $6 \cdot 2 = \boxed{12}$.

Alternative For the first situation, we can also calculate as the following:

The total number of permutations without constraints= $6! = 720$.

The number of permutations with two white balls at the end= $P_2^3 \cdot 4! = 144$.

Therefore, the required number of permutations= $720 - 144 = \boxed{576}$.

(Note: For the first part of the question, the official answer is wrong. The official answer is calculated by $6! - C_2^3 \cdot 4! = 648$, which assumed the two ends are indistinctive (i.e. a ball on the left end is the same case as the ball on the right end). However, then the total number of permutations without constraints will no longer be $6!$, but $\frac{6!}{2}$.)

Q1(6):

By the quadratic formula, the two solutions of $x^2 - 3x - 1 = 0$ are $x = \frac{3 \pm \sqrt{13}}{2}$.

As $3 = \sqrt{9} < \sqrt{13} < \sqrt{16} = 4$, we have $3 < x_1 < \frac{7}{2}$ and $-\frac{1}{2} < x_2 < 0$. Therefore, we have $m = 3$ and $n = -1$.

Q1(7):

$$\sin 2x > \sqrt{2} \cos\left(x + \frac{\pi}{4}\right) + \frac{1}{2}$$

$$2 \sin x \cos x > \sqrt{2} \left(\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4} \right) + \frac{1}{2}$$

$$2ab > b - a + \frac{1}{2}$$

$$4ab + 2a + (-2)b - 1 > 0$$

By factorising the L.H.S., we have

$$(2a - 1)(2b + 1) > 0$$

$$\begin{cases} a > \frac{1}{2} \\ b > -\frac{1}{2} \end{cases} \quad \text{or} \quad \begin{cases} a < \frac{1}{2} \\ b < -\frac{1}{2} \end{cases}$$

$$\begin{cases} \frac{\pi}{6} < x < \frac{5\pi}{6} \\ 0 \leq x < \frac{2\pi}{3} \text{ or } \frac{4\pi}{3} < x < 2\pi \end{cases} \quad \text{or} \quad \begin{cases} 0 \leq x < \frac{\pi}{6} \text{ or } \frac{5\pi}{6} < x < 2\pi \\ \frac{2\pi}{3} < x < \frac{4\pi}{3} \end{cases}$$

$$\frac{\pi}{6} < x < \frac{2\pi}{3} \text{ or } \frac{5\pi}{6} < x < \frac{4\pi}{3}$$

Q1(8):

By the long division,

$$x^4 - 8x^3 + 14x^2 + 8x - 1 = (x^2 - 5x - 2)(x^2 + \boxed{-3}x + \boxed{1}) + (\boxed{7}x + \boxed{1}).$$

Q1(9):

$$\text{As } y = \log_2\left(\frac{x}{2} + 3\right) \iff y = \log_2(x - (-6)) - 1.$$

The graph $y = \log_2 x$ is translated $\boxed{-6}$ units in x and $\boxed{-1}$ units in y .

Solving $\log_2 x = \log_2(\frac{x}{2} + 3)$, we have $x = 6$. By that time, $y = \log_2 6 = 1 + \log_2 3$.

Therefore, the point of intersection of two graphs is $(\boxed{6}, 1 + \log_2 \boxed{3})$.

Q1(10):

By $|\vec{a} + \vec{b}| = 1$, we have

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = 1^2$$

$$|\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = 1$$

$$1^2 + 2\vec{a} \cdot \vec{b} + 1^2 = 1$$

$$\vec{a} \cdot \vec{b} = \boxed{-\frac{1}{2}}$$

$$\text{Moreover, } |2\vec{a} + \vec{b}| = \sqrt{(2\vec{a} + \vec{b}) \cdot (2\vec{a} + \vec{b})}$$

$$= \sqrt{4|\vec{a}|^2 + 4\vec{a} \cdot \vec{b} + |\vec{b}|^2}$$

$$= \sqrt{4(1) + 4(-\frac{1}{2}) + 1}$$

$$= \boxed{\sqrt{3}}.$$

The angle between $2\vec{a} + \vec{b}$ and \vec{b} is given by

$$\begin{aligned}
& \arccos \frac{(2\vec{a}+\vec{b}) \cdot \vec{b}}{|2\vec{a}+\vec{b}| \cdot |\vec{b}|} \\
&= \arccos \frac{2\vec{a} \cdot \vec{b} + |\vec{b}|^2}{\sqrt{3} \cdot 1} \\
&= \arccos 0 \\
&= \boxed{90}^\circ.
\end{aligned}$$

Q2:

(1): As the means of A and B are equal, we have:

$$\begin{aligned}
\frac{c + a + b + a + b + a + b + c}{8} &= \frac{b + b + a + a + b + a + b + a}{8} \\
\frac{c}{4} &= \frac{a + b}{8} \\
c &= \boxed{\frac{a + b}{2}}
\end{aligned}$$

(2): The mean of A = the mean of B = $\frac{a+b}{2} = \boxed{c}$.

(3): The deviation of a datum x_i is defined by $(x_i - \bar{x})$.

The variance of a data set is given by the sum of square of deviations over the number of data.

As the deviation of c in data set A is 0, we have $s_A^2 = \frac{3((a-c)^2 + (b-c)^2)}{8}$.

On the other hand, we have $s_B^2 = \frac{4((a-c)^2 + (b-c)^2)}{8}$.

Therefore, $s_A^2/s_B^2 = \boxed{\frac{3}{4}}$, so that $s_A^2 \boxed{<} s_B^2$.

Q3:

(1): As $\frac{d}{dx}(f(x) + g(x)) = 2$, we have $f(x) + g(x) = 2x + \text{Constant}$.

As $f(1) = 2$ and $g(1) = 0$, we have $f(1) + g(1) = 2 + \text{Constant} = 2$, i.e. $\text{Constant} = 0$.

Therefore, $f(x) + g(x) = \boxed{2x}$.

$$\begin{aligned} (2): \quad & \frac{d}{dx}(f(x)^2 + g(x)^2) = \frac{d}{dx}((f(x) + g(x))^2 - 2f(x)g(x)) \\ & = 2(f(x) + g(x))\frac{d}{dx}(f(x) + g(x)) - 2\frac{d}{dx}(f(x)g(x)) \text{ by the chain rule.} \end{aligned}$$

As $\frac{d}{dx}(f(x)^2 + g(x)^2) = 4x$, we have $\frac{d}{dx}(f(x)g(x)) = -\frac{4x - 4(2x)}{2} = 2x$, i.e. $f(x)g(x) = x^2 + \text{Constant}$.

Moreover, $f(1)g(1) = 1 + \text{Constant} = (2)(0)$, i.e. $\text{Constant} = -1$.

Therefore, $f(x)g(x) = \boxed{x^2 - 1}$.

(3): $f(x)$ and $g(x)$ are solution to the equation

$$h(x)^2 - (f(x) + g(x))h(x) + f(x)g(x) = 0$$

$$h(x)^2 - (2x)h(x) + (x^2 - 1) = 0$$

$$(h(x) - (x - 1))(h(x) - (x + 1)) = 0$$

$$h(x) = x - 1 \text{ or } h(x) = x + 1$$

Therefore, $f(x) = x - 1$ and $g(x) = x + 1$ or $f(x) = x + 1$ and $g(x) = x - 1$.

As to satisfy the initial value, we have $f(x) = \boxed{x + 1}$ and $g(x) = \boxed{x - 1}$.