Q1(1):

Let $n(X)$ be the number of element of set X.

The provided information are:

$C \subset A$

$n(A) = 66$

$n(A \cap \overline{C}) = 47$

$n(B \cap \overline{C}) = 42$

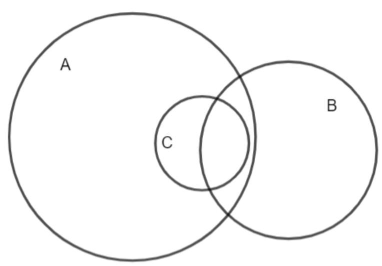
$n(C \cap \overline{B}) = 8$

$n(A \cap (\overline{B \cup C})) = 31$.

We are going to find $n(A \cup B \cup C)$.

As $C \subset A$, we have $n(C) = n(A) - n(A \cap \overline{C}) = 66 - 47 = 19$.

By the Venn diagram:



We have $n(A \cup B \cup C) = n(A \cap (\overline{B \cup C})) + n(B \cap \overline{C}) + n(C)$

$= 31 + 42 + 19$

$= \boxed{92}$.

Alternative By De Morgan theorem, $\overline{B \cup C} = \overline{B} \cap \overline{C}$.

Note that $(A \cap (\overline{B \cup C})) \cup (B \cap \overline{C}) = (A \cap (\overline{B} \cap \overline{C})) \cup (B \cap \overline{C})$

$= (A \cup B) \cap \overline{C}$.

Moreover, $(A \cap (\overline{B \cup C})) \cap (B \cap \overline{C}) = \emptyset$.

Therefore, $n(A \cup B \cup C) = n((A \cup B) \cap \overline{C}) + n(C)$

$= n(A \cap (\overline{B \cup C})) + n(B \cap \overline{C}) + n(C)$

$= 31 + 42 + 19$

$= \boxed{92}$.

Q1(2):

Let $P(p, p^2)$ be a point on $y = x^2$. Then the distance between it and $(0, 4)$ is $\sqrt{p^2 + (p^2 - 4)^2} = \sqrt{p^4 - 7p^2 + 16}$.

By completing the square, the distance$=\sqrt{(p^2 - \frac{7}{2})^2 + \frac{15}{4}}$.

As $(p^2 - \frac{7}{2})^2 \geq 0$, the distance$\geq \sqrt{\frac{15}{4}} = \frac{\sqrt{15}}{2}$.

Therefore, the minimum distance$=\boxed{\dfrac{\sqrt{15}}{2}}$.

Q1(3):

After the first symmetric transformation, the point (x, y) is transferred to (x', y).

Note that 2 lies on the middle of x and x', i.e. $\frac{x+x'}{2} = 2$, or $x' = 4 - x$.

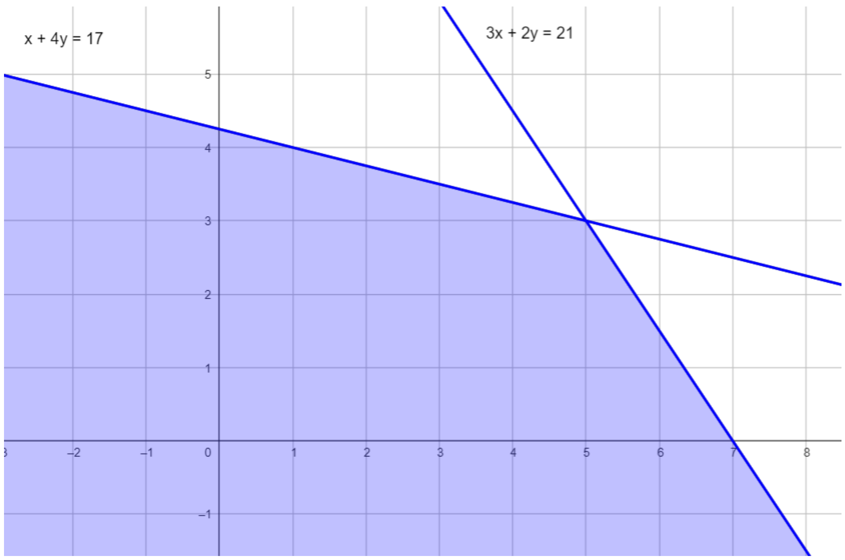
Therefore, the equation of the graph after the first transformation is $y = 2(4 - x)^2 - 6(4 - x) + 2$, i.e. $y = 2x^2 - 10x + 10$.

Similarly, after the second symmetric transformation, the point (x, y) is transferred to $(x, 6 - y)$.

Therefore, the equation of the graph after the second transformation is $6-y=2x^2-10x+10$, i.e. $y=\boxed{-2}x^2+\boxed{10}x+\boxed{-4}$.

Q1(4):

The region satisfies $\begin{cases} x+4y\le 17 \\ 3x+2y\le 21 \end{cases}$ is sketched:



Note that $x+2y$ attains to its extremum when it passes through the vertex of the region.

The region has only one vertex, which is the point of intersection of the two lines.

By solving the system of equation, we have the coordinates of it are $(5,3)$.

Let $x+2y=k$. When compared with the case $x+2y=0$, k need to be moved upwards so that the line $x+2y=k$ passes through the point $(5,3)$. Therefore, $x+2y$ attains to its maximum when $(x,y)=(5,3)$.

As both 5 and 3 are integers and $5+3=8$ is a multiple of 2, $(x,y)=(5,3)$ statiisfies all the constrains. Therefore, $x+2y$ is maximum when $x=\boxed{5}$ and $y=\boxed{3}$ and the maximum value is $5+6=\boxed{11}$.

(Note: For more detailed elaboration, see "linear programming by graphical method".)

Q1(5):

As $y=a_1x+a_0$ is tangent to both parabolas, we have both the equations $\frac{1}{8}x^2-2=a_1x+a_0$ and $\frac{1}{2}x^2-8=a_1x+a_0$ has only one solution. Then their decrimenants (respectively) are equal to 0:

$a_1^2-4(\frac{1}{8})(-2-a_0)=0$ and $a_1^2-4(\frac{1}{2})(-8-a_0)=0$

$a_1^2=-\frac{1}{2}(a_0+2)$ and $a_1^2=-2(a_0+8)$

Therefore, we have $-\frac{1}{2}(a_0+2)=-2(a_0+8)$, which gives $a_0=\boxed{-10}$.

Substitue $a_0=-10$ into either equation, we have $a_1=\pm 2$.

As both parabolas convex downwards, with the fact that a_1 represents the slope of tangent, we should have $a_1>0$ such that the x-coordinate of both tangential points are positive. Therefore, $a_1=\boxed{2}$.

Alternative (with calculus) Let $P(p,\frac{1}{8}p^2-2)$ and $Q(q,\frac{1}{2}q^2-8)$, where $p,q>0$, be the two tangential points.

Then, the slope of the common tangent will be $\frac{\frac{1}{2}q^2-8-\frac{1}{8}p^2+2}{q-p}$.

Moreover, considering the derivative, the slope of tangent at P is $\frac{1}{4}p$ and that at Q is q. As the tangent is common, we have $\frac{\frac{1}{2}q^2-8-\frac{1}{8}p^2+2}{q-p} = \frac{1}{4}p = q$.

Substitue the relation $p = 4q$ into $\frac{\frac{1}{2}q^2-8-\frac{1}{8}p^2+2}{q-p} = q$, we have

$$\frac{\frac{1}{2}q^2 - 8 - \frac{1}{8}(4q)^2 + 2}{q - 4q} = q$$

$$-\frac{3}{2}q^2 - 6 = -3q^2$$

$$q = 2$$

Then, the coordinates of Q are $(2, -6)$ and the slope of tangent=$q = 2$. By using the point-slope form of straight line, we have the equation of tangent is $y + 6 = 2(x - 2)$, i.e. $y = \boxed{2}x + \boxed{-10}$.

Q1(6):

$f(x) = \cos 2x + \cos 3x$

$= 2\cos^2 x - 1 + \cos 2x \cos x - \sin 2x \sin x$

$= 2\cos^2 x - 1 + 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$

$= 4\cos^3 x + 2\cos^2 x - 2\cos x - 1$

$= \boxed{4}t^3 + \boxed{2}t^2 + \boxed{-3}t + \boxed{-1}$.

Q1(7):

$P(C \text{ is odd}) = P(B_i \neq 2 \text{ for } i = 1, 2, 3)$

$= (\frac{2}{3})^3$

$= \frac{\boxed{8}}{27}$.

$P(C \text{ is a multiple of } 5) = 1 - P(C \text{ is not a multiple of } 5)$

$= 1 - P(B_i \neq 5 \text{ for } i = 1, 2, 3)$

$= 1 - (\frac{2}{3})^3$

$= \frac{\boxed{19}}{27}$.

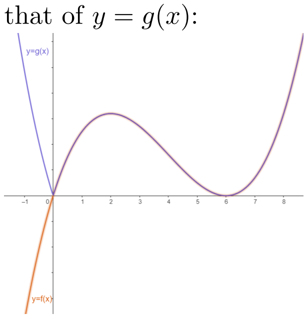
Q1(8):

$f'(x) = (x-6)^2 + 2x(x-6) = 3(x-6)(x-2)$.

To find the extremum of $f(x)$, we set $f'(x) = 0$. Then, we have $f(x)$ attains to extremum when $x = \boxed{2}$ and $x = \boxed{6}$.

$f''(x) = 6(x-4)$. We have $f''(6) > 0$ and hence $f(x)$ attains to minimum when $x = 6$, by that time, $f(x) = 0$.

With the above information, we can sketch the graph of $y = f(x)$ and hence that of $y = g(x)$:



Therefore, the maximum number of real solutions is $\boxed{4}$, which is obtained when a lies between 0 and the local maximum of $f(x)$.

Q1(9):

Mean=$\frac{1+1+3+5+6+8+9+15}{8} = \boxed{6}$.

Sum of squares of deviation= $(1-6)^2 + (1-6)^2 + (3-6)^2 + (5-6)^2 + (6-6)^2 + (8-6)^2 + (9-6)^2 + (15-6)^2$

$= 25 + 25 + 9 + 1 + 0 + 4 + 9 + 81$

$= \boxed{154}$.

Mean of squares of deviation= $\frac{154}{8} = \boxed{19.25}$.

Alternative The mean of squares of deviation is equal to the variance.

By $Var(X) = E(X)^2 - E(X^2)$, we have

Mean of squares of deviation=$\frac{1^2+1^2+3^2+5^2+6^2+8^2+9^2+15^2}{8} - 6^2$

$= \frac{1+1+9+25+36+64+81+225}{8} - 36$

$= \frac{154}{8}$

$= \boxed{19.25}$.

And the sum of squares of deviation=$8 \cdot \frac{154}{8} = \boxed{154}$.

Q2:

(1): By the angle bisector theorem, $OA : AB = OD : DB$,

i.e. $OA = AB(\frac{y}{x}) = \boxed{\dfrac{2y}{x}}$.

Alternative Consider the ratio of areas of $\triangle AOD$ and $\triangle ADB$. As they share the common altitude when regarding OD and DB as their bases, we have the ratio of their areas=$OD : DB$.

On the other hand, the ratio of their area=$(\frac{1}{2}(OA)(AD)\sin\angle DAO) : (\frac{1}{2}(AB)(AD)\sin\angle DAB) =$

$OA : AB$ as $\angle DAO = \angle DAB$.

Given the above, we have $OA : AB = OD : DB$, i.e. $OA = AB(\frac{y}{x}) = \boxed{\dfrac{2y}{x}}$.

(2): Similar to (1), we have $OC = \frac{y}{x}$.

As OB and OC are the radii of the circle, we have $OC = OB = x + y$.

Therefore, we have

$$x + y = \frac{y}{x}$$

$$x^2 = (1 - x)y$$

$$y = \boxed{\frac{x^2}{1-x}}$$

(3): As AB is tangent to the circle, we have $\angle OBA = 90^\circ$ and hence

$AB^2 + BO^2 = AO^2$ by Pythagoras' theorem.

Combine all the above results, we have

$$2^2 + (x + \frac{x^2}{1-x})^2 = (\frac{2(\frac{x^2}{1-x})}{x})^2$$

$$4 + (\frac{x}{1-x})^2 = 4(\frac{x}{1-x})^2$$

$$\frac{x}{1-x} = \frac{2}{\sqrt{3}} \text{ or } -\frac{2}{\sqrt{3}}(\text{rejected as } x < BC = 1 \text{ and hence } \frac{x}{1-x} > 0)$$

$$2 - 2x = \sqrt{3}$$

$$x = \frac{2}{\sqrt{3}+2} = 4 - \boxed{2\sqrt{3}}$$

And, $y = \frac{x^2}{1-x} = \frac{(4-2\sqrt{3})^2}{1-4+2\sqrt{3}} = \frac{28-16\sqrt{3}}{2\sqrt{3}-3} = \frac{\boxed{8\sqrt{3}}}{3} - 4$.

Q3:

(1): There are totally $1 + 3 + ... + (2(n-1)-1) = (n-1)^2$ elements from the 1st group to the $(n-1)$th group.

Therefore, the last element of the $(n-1)$th group is $(n-1)^2 = n^2 - 2n + 1$.

As a_n is the next element following it, we have $a_n = \boxed{0}n^3 + \boxed{1}n^2 + \boxed{-2}n + \boxed{2}$.

Alternative (cheating!) As the question provided, $a_n = An^3 + Bn^2 + Cn + D$.

We are going to find the values of A, B, C and D:

$a_1 = A + B + C + D = 1$......(1)

$a_2 = 8A + 4B + 2C + D = 2$......(2)

$a_3 = 27A + 9B + 3C + D = 5$......(3)

$a_4 = 64A + 16B + 4C + D = 10$......(4)

By (4)-(3), (3)-(2) and (2)-(1), we have:

$7A + 3B + C = 1$......(5)

$19A + 5B + C = 3$......(6)

$37A + 7B + C = 5$......(7)

By (7)-(6) and (6)-(5), we have:

$11A + 2B = 2$

$18A + 2B = 2$

Then, we have $(A, B, C, D) = (\boxed{0}, \boxed{1}, \boxed{-2}, \boxed{2})$.

(2): Find the greatest integer n satisfying $a_n \leq 2678$:

$$n^2 - 2n + 2 \leq 2678$$

$$n^2 - 2n \leq 2676$$

By some trials, we know that when $n = 52$, $n^2 - 2n = 2600 < 2676$ and when $n = 53$, $n^2 - 2n = 2703 > 2676$. Hence our demanding value of n is 52 and we have 2678 is in the $\boxed{52}$-th group.

On the other hand, $a_{52} = 2602$ as we have calculated. As $2602+(77-1) = 2678$, we have 2678 is the $\boxed{77}$-th term in the group.

(3): $S_n = \sum_{i=1}^{2n-1} (a_n + (i-1))$

$= \sum_{i=1}^{2n-1} (n^2 - 2n + 1 + i)$

$= (n^2 - 2n + 1)(2n - 1) + \frac{(2n)(2n-1)}{2}$

$= \boxed{2}n^3 + \boxed{-3}n^2 + \boxed{3}n + \boxed{-1}$.