Mechanics

(1) Motion

-2006 3(5)

An object is moving along a sphere. At a certain moment, it jumped out into the air and moved along a certain path. The frictions between the object and the sphere and between the object and the air are negligible. What is the path that it moved along?

(a) Straight line (with constant velocity)

(b) Straight line (with acceleration)

(c) Parabola (with constant speed)

(d) Parabola (with acceleration)

(e) Parabola (with deceleration)

-2008 1(1)

(1) A ball is thrown at angles 45° and 30° above the horizontal with the same initial speed. What multiple of the horizontal range at angle 30° is that at angle 45° ?

(a) $\sqrt{3}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

(e) $\frac{1}{2}$ (f) $\frac{2}{\sqrt{3}}$ (g) $\sqrt{\frac{2}{3}}$ (h) $\frac{1}{\sqrt{3}}$

3 A point object P of mass m is sliding along a frictionless wall of a radius of curvature, R_0, in a vertical plane as shown in Fig. 2. Assume that the point object is initially at rest at the point A and starts to move toward the bottom B. The angle between the normal (to the wall) at A and the vertical line is θ_0 radians, and assumed to be small.

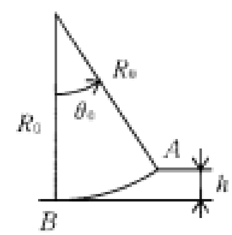


Fig. 2

(1) The elevation h between the points A and B is approximately

(a) $R_0\theta_0$ (b) $R_0\theta_0{}^2$ (c) $\frac{1}{2}R_0\theta_0{}^2$ (d) $\frac{1}{2}R_0\theta_0$

where $\sin\theta_0 \approx \theta_0$, $\cos\theta_0 \approx 1-\frac{1}{2}\theta_0{}^2$.

(2) The speed of the point object at B, v_{max}, is given by

(a) $\sqrt{gh}$ (b) $\sqrt{2gh}$ (c) $2\sqrt{gh}$

where g is an acceleration constant due to gravity.

(3) The arc length $\widehat{AB}$ is given by

(a) $\frac{1}{2}R_0\theta_0$ (b) $R_0\theta_0$ (c) $2R_0\theta_0$ (d) $R_0\theta_0{}^2$

(4) The time $\bar{t}$ which is defined as $\bar{t} = \widehat{AB}\,/\,v_{max}$ is given by

(a) $\sqrt{0.5R_0/g}$ (b) $\sqrt{R_0/g}$ (c) $\sqrt{2R_0/g}$ (d) $2\sqrt{R_0/g}$

(5) The ratio of the actual time t reaching B from A to $\bar{t}$, $t/\bar{t}$, satisfies

(a) $t/\bar{t} > 2$ (b) $t/\bar{t} = 2$ (c) $t/\bar{t} = \pi/2$

(d) $t/\bar{t} = 1$ (e) $t/\bar{t} < 1$

-2009 3(4)(5)

3 A point object of mass m is connected to an inertialess string of length L, the other end of which is fixed to a point O. At time $t = 0$, the object is assumed to begin to move horizontally in a vertical plane from the bottom point A ($\overline{OA} = L$) in the clockwise direction with an initial speed v_0 as shown in Fig. 5.

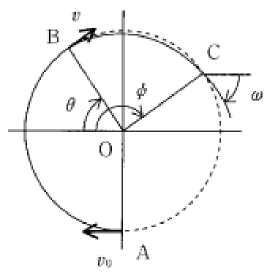


Fig. 5

If $\sqrt{2gL} < v_0 < \sqrt{5gL}$ (g : acceleration due to gravity), then at a point B (the angle between $\vec{OB}$ and the horizontal direction is designated θ as in Fig. 5) the magnitude of the force acting on the object from the string becomes zero, where $\overline{OB} = L$ and the velocity of the object is perpendicular to $\vec{OB}$, v being the magnitude of the velocity vector. We restrict ourselves to the case $0 < \theta < \pi/2$.

v is given by $\sqrt{gL \sin\theta}$.

From the point B, for a while, the object takes a parabolic orbit till a point C, where OC=L.

In the case $\theta = \pi/3$, the angle ϕ, measured as in Fig. 5 for specifying the point C, becomes

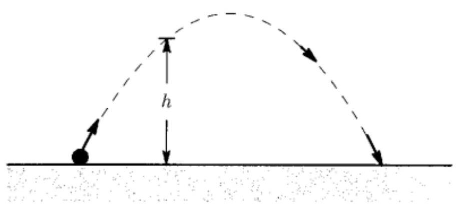
(4) (a) $\pi/2$, (b) $(2/3)\pi$, (c) $(5/6)\pi$, (d) π

and finally the angle ω, the angle between the object velocity at the point C and the horizontal dirction, is

(5) (a) $\pi/2$, (b) $\pi/3$, (c) $\pi/4$, (d) $\pi/6$

-2010 IC

C As shown in the figure below, a ball is launched diagonally upward from a horizontal ground. The time at launch is 0. After passing through a point at height h at time t, the ball lands at time T.



Q3 What is the height h? From ①-⑤ below choose the correct answer. $\boxed{3}$

① $\frac{1}{2}gt^2$ ② $\frac{1}{2}gtT$ ③ $\frac{1}{2}gT(T-t)$

④ $\frac{1}{2}gt(T-t)$ ⑤ $\frac{1}{2}g(T-t)^2$

-2013 1(1)

(1) There is a river with a current speed of V. A boat which is capable of sailing at the speed of $2V$ in still water is used to cross the river from point A to point B, as shown in Fig.1-1. The width of the river is L. Find the time the boat needs to cross the river when it goes from A to B along a straight line perpendicular to the river.

(a) $\frac{L}{V}$ (b) $\frac{2L}{V}$ (c) $\frac{L}{2V}$

(d) $\frac{\sqrt{3}L}{V}$ (e) $\frac{L}{\sqrt{3}V}$ (f) $\frac{\sqrt{5}L}{V}$

(g) $\frac{L}{\sqrt{5}V}$

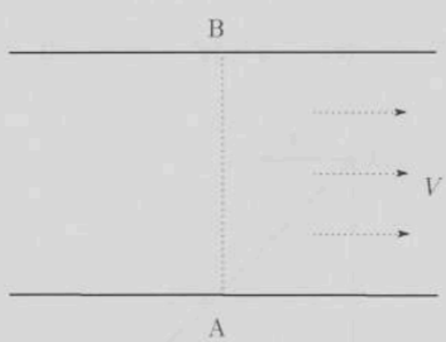


Fig. 1-1

-2015 1(1)

1. Answer the following questions.

(1) A point particle with mass m is thrown with speed of v at an angle θ above the horizontal axis as shown in Fig. 1-1. The acceleration due to gravity is denoted as g. Find the distance L to the point where the particle hits the ground.

(a) $\dfrac{2v^2\sin^2\theta}{g}$ (b) $\dfrac{2v^2\sin\theta\cos\theta}{g}$ (c) $\dfrac{2v^2\cos^2\theta}{g}$

(d) $\dfrac{v^2\sin^2\theta}{g}$ (e) $\dfrac{v^2\sin\theta\cos\theta}{g}$ (f) $\dfrac{v^2\cos^2\theta}{g}$

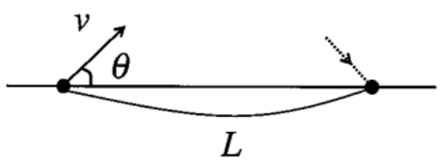


Fig. 1-1

-2016 1(1)

(1) A car accelerates uniformly from rest to a speed v for time t. Find the distance the car travels during the time t.

(a) vt (b) $\dfrac{1}{2}vt$ (c) $\dfrac{1}{3}vt$

(d) $\dfrac{1}{4}vt$ (e) $2vt$ (f) $3vt$

-2016 1(5)

(5) An object attached to a spring vibrates with a simple harmonic motion as described in Fig. 1-3. Here t is time and x is the position of the object. Find the maximum speed for this motion.

(a) $\dfrac{A}{2t_0}$ (b) $\dfrac{\pi A}{2t_0}$ (c) $\dfrac{2\pi A}{t_0}$

(d) $\dfrac{\pi A}{t_0}$ (e) $\dfrac{A}{\pi t_0}$ (f) $\dfrac{A}{t_0}$

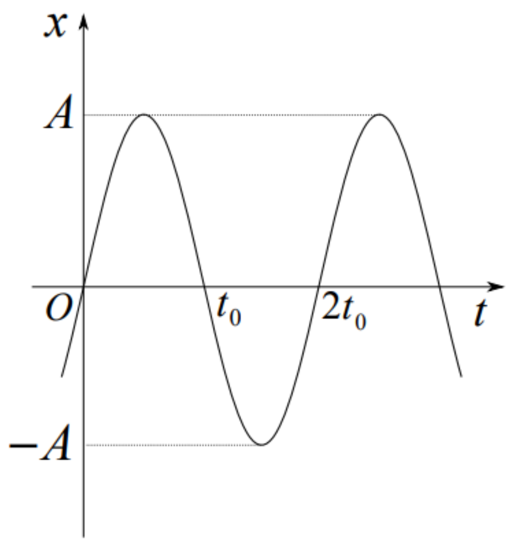


Fig. 1-3

-2016 3

3. A block P of mass M is on a smooth horizontal plane, and an object Q of mass m is always on top of the block P. Initially both P and Q are at rest. At a time $t = 0$, an initial speed v_0 is given to P in the rightward direction. Then Q also starts to move. When a time T is passed after P is given an initial speed, the velocity of P coincides with the velocity of Q. A coefficient of kinetic friction between P and Q is denoted as μ. Treat the rightward direction as positive, and the acceleration of gravity is denoted as g. Answer the following questions.

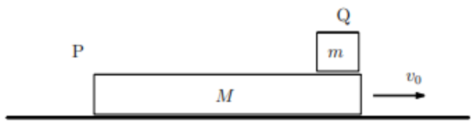


Fig. 3

(1) Find the force acting on Q at time t $(0 < t < T)$.

(a) μmg (b) $-\mu mg$ (c) μMg

(d) $-\mu Mg$ (e) $\mu(M+m)g$ (f) $-\mu(M+m)g$

(2) Find the force acting on P at time t $(0 < t < T)$.

(a) μmg (b) $-\mu mg$ (c) μMg

(d) $-\mu Mg$ (e) $\mu(M+m)g$ (f) $-\mu(M+m)g$

(3) Find the velocity of P at time t $(0 < t < T)$.

(a) μgt (b) $v_0 - \mu gt$ (c) $\frac{m}{M}\mu gt$

(d) $v_0 - \frac{m}{M}\mu gt$ (e) $\frac{m}{M+m}\mu gt$ (f) $v_0 - \frac{m}{m+M}\mu gt$

(4) Find the expression of T using some other suitable quantities.

(a) $\frac{v_0}{\mu g}$ (b) $\frac{\mu g}{v_0}$ (c) $\frac{m}{M}\frac{v_0}{\mu g}$

(d) $\frac{M}{m}\frac{v_0}{\mu g}$ (e) $\frac{m}{M}\frac{\mu g}{v_0}$ (f) $\frac{M}{m}\frac{\mu g}{v_0}$

(g) $\frac{M}{M+m}\frac{v_0}{\mu g}$ (h) $\frac{M+m}{M}\frac{v_0}{\mu g}$ (i) $\frac{M}{M+m}\frac{\mu g}{v_0}$

(j) $\frac{M+m}{M}\frac{\mu g}{v_0}$

(5) Find the distance which Q moved against P in the duration from $t = 0$ to $t = T$.

(a) $\frac{\mu g v_0^2}{2}$ (b) $\frac{v_0^2}{2\mu g}$ (c) $\frac{m\mu g v_0^2}{2M}$

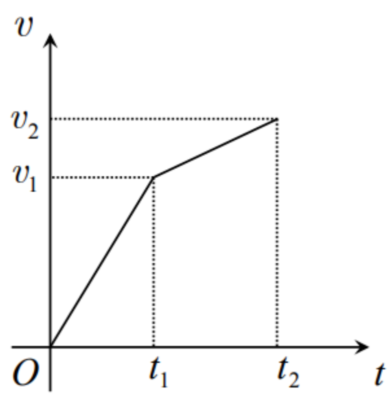
(d) $\frac{Mv_0^2}{2m\mu g}$ (e) $\frac{(M+m)\mu g v_0^2}{2M}$ (f) $\frac{Mv_0^2}{2(M+m)\mu g}$

-2017 1(1)

(1) A graph of velocity v versus time t for a mass particle moving along a straight line is shown in Fig. 1-1. From $t = 0$ to $t = t_1$ and from $t = t_1$ to $t = t_2$ the particle moves at constant acceleration. The velocity at $t = t_1$ is v_1 and the velocity at $t = t_2$ is v_2. Find the distance traveled from $t = 0$ to $t = t_2$.

(a) $\frac{1}{2}(v_1t_1 + v_2t_2)$ (b) $\frac{v_1t_2 - v_2t_1 + v_2t_2}{2}$ (c) $v_1t_1 + v_2t_2$

(d) $v_1t_2 - v_2t_1 + v_2t_2$ (e) $\frac{1}{2}(v_1t_2 + v_2t_1)$ (e) $(v_2 - v_1)(t_2 - t_1)$



-2017 1(3)

(3) A block of mass m attached to a spring with a force constant k is free to move on a frictionless horizontal surface. The angular frequency of the spring is denoted by $\omega = \sqrt{\dfrac{k}{m}}$. The block is released from a resting position after the spring is stretched a distance of d. The direction of the stretch is taken as the positive direction. Which is the correct graph of the velocity of the block versus time in Fig. 1-2 from (a) to (d)?

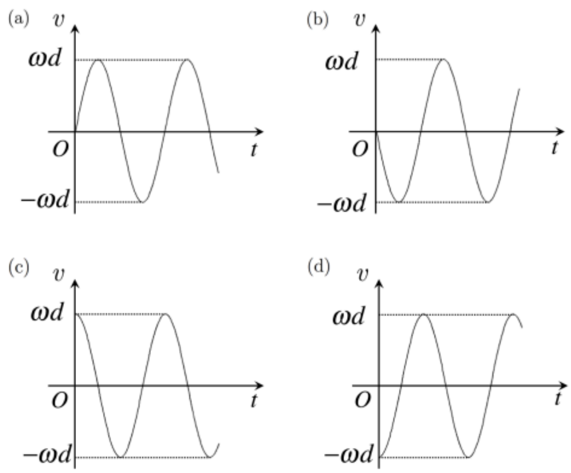


Fig. 1-2

-2017 3(5)

3. A hemisphere of radius R is fixed on a horizontal floor, as shown in Fig. 3. A small ball of mass m slides down from the top of the hemisphere at point A with zero initial speed. There is no friction between the ball and the surface of the hemisphere. After sliding down the surface, the ball leaves the surface of the hemisphere at point B and hits the horizontal floor at point C. As the ball slides between A and B on the surface of the hemisphere, the position of the ball is described by the angle θ shown in the figure. The acceleration of gravity is denoted as g. Answer the following questions.

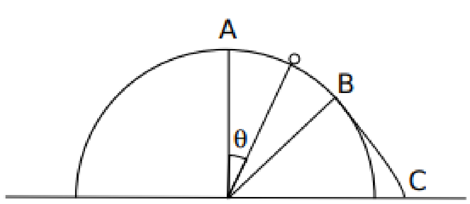


Fig. 3

The speed of the ball is given by $\sqrt{2gR(1-\sin\theta)}$ and the cosine value of the angle at B is given by $\frac{2}{3}$.

(5) Find the speed of the ball in the horizontal direction immediately before the ball reaches the point C.

(a) $\sqrt{\frac{2}{3}gR}$ (b) $\sqrt{\frac{3}{2}gR}$ (c) $\sqrt{\frac{8}{9}gR}$

(d) $\sqrt{\frac{9}{8}gR}$ (e) $\sqrt{\frac{8}{27}gR}$ (f) $\sqrt{\frac{27}{8}gR}$

(g) $\sqrt{\frac{7}{15}gR}$ (h) $\sqrt{\frac{15}{7}gR}$

-2018 1(1)

(1) An object of mass m is launched horizontally with a speed v at a height of h above the ground level as shown in Fig. 1-1. Let θ be the impact angle to the ground and g be the acceleration of gravity. Find the formula of $\tan\theta$.

(a) $\frac{\sqrt{2gh}}{v}$ (b) $\frac{\sqrt{gh}}{2v}$ (c) $\frac{v}{\sqrt{2gh}}$

(d) $\frac{2v}{\sqrt{gh}}$ (e) $\frac{\sqrt{gh}}{v}$ (f) $\frac{v}{\sqrt{gh}}$

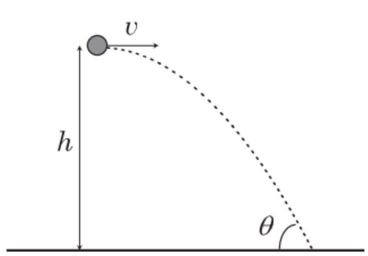


Fig. 1-1

-2019 1(1)

(1) A car at rest starts moving along a straight line and stops at time $t = 80$ s. The velocity v of the car changes as a function of time t as shown in Fig.1-1. Find the distance that the car travels.

(a)	5000 m	(b)	4000 m	(c)	3000 m
(d)	2000 m	(e)	1000 m	(f)	500 m

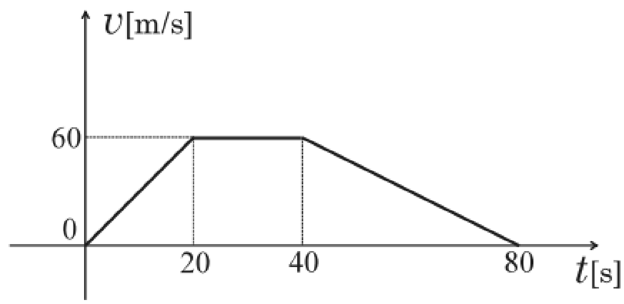


Fig. 1-1

-2019 1(2)

(2) A small ball at rest falls down from a height of h above the ground and bounces repeatedly. The coefficient of restitution between the ball and the ground is denoted as e, and the acceleration of gravity as g. Find the maximum height of the ball between the nth and the $(n+1)$th impact with the ground.

(a)	he	(b)	$h(1-e)^n$	(c)	he^n
(d)	he^{2n}	(e)	he^{2n+2}	(f)	he^{2n-2}

-2019 3

3. An object of mass m is moving with a speed v on a frictionless plane as shown in Fig. 3. The object reaches at the end of the plane, $x = 0$ and $y = h$, at time $t = 0$. and jumps into the air. There is a slope in the $x > 0$ region as shown in Fig. 3. The acceleration of gravity is denoted as g. Answer the following questions.

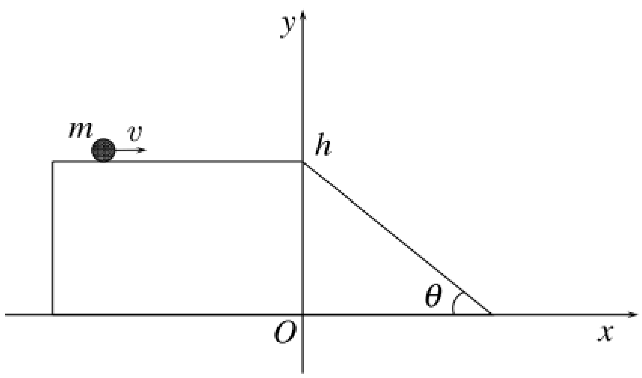


Fig. 3

(1) Find the x coordinate of the object at time $t(> 0)$ when the object is in the air.

(a) $\frac{1}{2}vt$ (b) $\frac{1}{3}vt$ (c) $2vt$

(d) vt (e) $\frac{1}{2}vt^2$ (f) $\frac{v}{t}$

(2) Find the y coordinate of the object at time $t(> 0)$ when the object is in the air.

(a) $-\frac{1}{2}gt^2$ (b) gt^2 (c) h

(d) $h - gt$ (e) $h - gt^2$ (f) $h - \frac{1}{2}gt^2$

(3) If v is larger than a speed v_c, the object does not hit the slope and directly drops to the horizontal plane at $y = 0$. Find the expression of v_c.

(a) $\frac{\sqrt{2gh}}{\tan\theta}$ (b) $\frac{\sqrt{2gh}}{2}$ (c) $\frac{\sqrt{2gh}}{2\tan\theta}$

(d) $\sqrt{gh}$ (e) $\frac{\sqrt{gh}}{2}$ (f) $\frac{\sqrt{gh}}{\tan\theta}$

(4) If v is less than v_c, the object hits the slope. Find the x coordinate of the impact point.

(a) $\frac{v^2}{g}\sin\theta$ (b) $\frac{v^2}{g}\tan\theta$ (c) $\frac{v^2}{g}\cos\theta$

(d) $\frac{2v^2}{g}\sin\theta$ (e) $\frac{2v^2}{g}\tan\theta$ (f) $\frac{2v^2}{g}\cos\theta$

(2) Mechanic Energy and Momentum

-2006 1(1)

A mass with mass m is hanging by a spring with spring constant k. At the beginning, it stretches the spring, and the system reaches an equilibrium. Now, holding the mass by a hand and slowly raising it to the natural length of the spring. What is the work done by hand in this process? The gravitational acceleration is g.

(a) $\frac{m^2g^2}{k}$ (b) $\frac{2m^2g^2}{k}$ (c) $\frac{m^2g^2}{2k}$ (d) $\frac{mg}{k}$ (e) $\frac{2mg}{k}$

(f) $\frac{mg}{2k}$ (g) kmg^2 (h) $2kmg^2$ (i) $\frac{kmg^2}{2}$

-2006 3(1)-(4)

A small object with mass m is going down from the top of a sphere of radius R with a small initial speed

v0 and finally jump out to the air at point P(X,Y). The frictions between the object and the sphere and between the object and the air are negligible. A coordinate system (x,y) is introduced, where y is the direction of gravity as shown in Figure 4. An angle θ is defined by the position of P as shown in the figure.

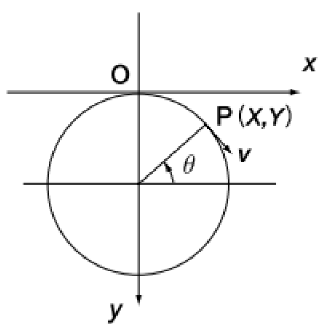


Figure 4

(1) Denote g as the gravitational acceleration, select the suitable equation representing the conservation of energy.

(a) $\frac{m}{2}v^2+mgY=\frac{m}{2}v_0^2$ (b) $\frac{m}{2}v^2+mgY=0$ (c) $\frac{m}{2}v^2-mgY=\frac{m}{2}v_0^2$

(d) $\frac{m}{2}v^2-mgY=0$ (e) $\frac{m}{2}v^2=\frac{m}{2}v_0^2$

(2) Select a suitable geometric relationship.

(a) $\sin\theta \fallingdotseq 1$ (b) $\sin\theta = \frac{R-Y}{R}$ (c) $\sin\theta = \frac{R+Y}{R}$

(d) $\sin\theta \fallingdotseq \frac{1}{2}$ (e) $\sin\theta \fallingdotseq 0$

(3) At point P, the following balance of forces is fulfilled:

$$mg\,\sin\theta = m\frac{v^2}{R}$$

Emitting v2 from the equation of conservation of energy, we have

$$Y = C_1 R + C_2\frac{v_0^2}{g}$$,

where C1 and C2 are constants. Select the suitable relationship of C1.

(a) $C_1 < \frac{1}{3}$ (b) $C_1 = \frac{1}{3}$ (c) $\frac{1}{3} < C_1 < \frac{1}{2}$

(d) $C_1 = \frac{1}{2}$ (e) $\frac{1}{2} < C_1$

(4) Select the suitable relationship of C2.

(a) $C_2 < -\frac{1}{2}$ (b) $C_2 = -\frac{1}{2}$ (c) $-\frac{1}{2} < C_2 < -\frac{1}{3}$

(d) $C_2 = -\frac{1}{3}$ (e) $-\frac{1}{3} < C_2$

-2007 1(1)

(1) A block of mass M is placed on a horizontal frictionless surface and is attached to a spring of force constant k as in Fig.1. Initially the spring is at its natural length and the block is at rest. Then a bullet of mass m and speed ν collides horizontally with the block and stops inside the block. The block with the bullet starts to move towards the right and compresses the spring. Find the maximum distance of compression.

(a) $\frac{m\nu}{k}$ (b) $\sqrt{\frac{m}{k}}\nu$ (c) $\frac{m\nu}{kM}$ (d) $\frac{m}{\sqrt{kM}}\nu$

(e) $\frac{m\nu}{k(M+m)}$ (f) $\frac{m\nu}{\sqrt{k(M+m)}}$ (g) $\frac{(M+m)\nu}{km}$ (h) $\frac{(M+m)\nu}{\sqrt{km}}$

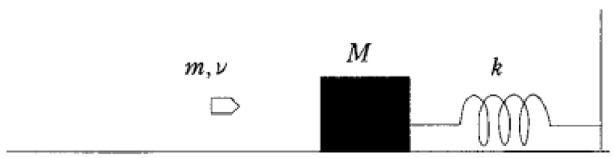


Fig. 1

-2008 1(5)

(5) In the α decay of a ${}^{238}_{92}U$ nucleus, what is the ratio, v/V, of the speed v of the emitted α particle to the speed V of the daughter nucleus?

(a) 58.5 (b) 118 (c) $\sqrt{58.5}$ (d) $\sqrt{118}$ (e) 1

-2009 3(2)(3)

3 A point object of mass m is connected to an inertialess string of length L, the other end of which is fixed to a point O. At time $t=0$, the object is assumed to begin to move horizontally in a vertical plane from the bottom point A ($\overline{OA}=L$) in the clockwise direction with an initial speed v_0 as shown in Fig. 5.

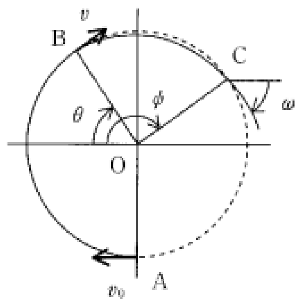


Fig. 5

If $\sqrt{2gL} < v_0 < \sqrt{5gL}$ (g : acceleration due to gravity), then at a point B (the angle between $\vec{OB}$ and the horizontal direction is designated θ as in Fig. 5) the magnitude of the force acting on the object from the string becomes zero, where $\overline{OB}=L$ and the velocity of the object is perpendicular to $\vec{OB}$, v being the magnitude of the velocity vector. We restrict ourselves to the case $0<\theta<\pi/2$.

Circle the correct answers to questions (1)–(5). The speed, v, is given by

$\sqrt{gL\sin\theta}$,

The initial speed, v_0, is

(2) (a) $\sqrt{(2+2\sin\theta)gL}$, (b) $\sqrt{(2+3\sin\theta)gL}$, (c) $\sqrt{(1+4\sin\theta)gL}$

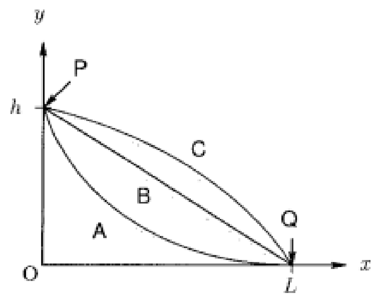
From the point B, for a while, the object takes a parabolic orbit till a point C, where $\overline{OC}=L$. The maximum elevation (with respect to the location B) is expressed as

(3) (a) $\dfrac{v_0^2}{2g}\sin^2\theta$, (b) $\dfrac{v_0^2}{2g}\cos^2\theta$, (c) $\dfrac{v^2}{2g}\sin^2\theta$,

(d) $\dfrac{v^2}{2g}\sin\theta\cos\theta$ (e) $\dfrac{v^2}{2g}\cos^2\theta$

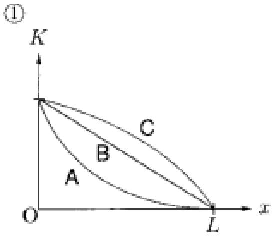
-2010 IE

E The figure below shows three different types of slides, A, B, and C. Their friction is negligible. The coordinate system is taken with the x-axis horizontal and the y-axis vertically upward. An object, initially at rest, is made to slide down each slide, from starting point P $(0,\, h)$ to lowest point Q $(L,\, 0)$.

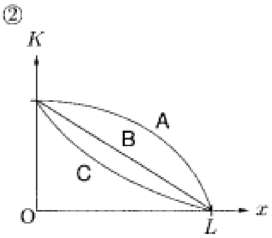


Q5 The object's x-coordinate is x, and its kinetic energy is K. How does K change with x when x changes from 0 to L? From ①-④ below choose the graph that best indicates this change for A, B, and C. **5**

①



②



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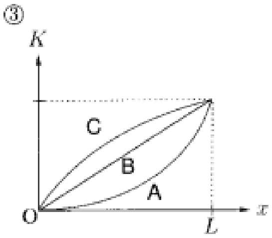
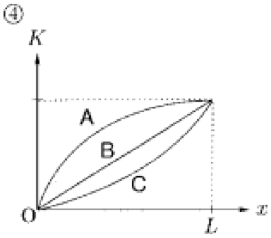


④



-2010 IF

F As shown in Figure 1, a thin, lightweight rubber string is suspended from a ceiling. When a weight (mass: 500 g) is attached to the lower end, it descends 10 cm and comes to rest, as shown in Figure 2. Next, as shown in Figure 3, the weight is pulled down 20 cm, where it touches the floor. The string's restoring force is proportional to the string's extension from its natural length. The restoring force is not exerted when the string is at less than its natural length.

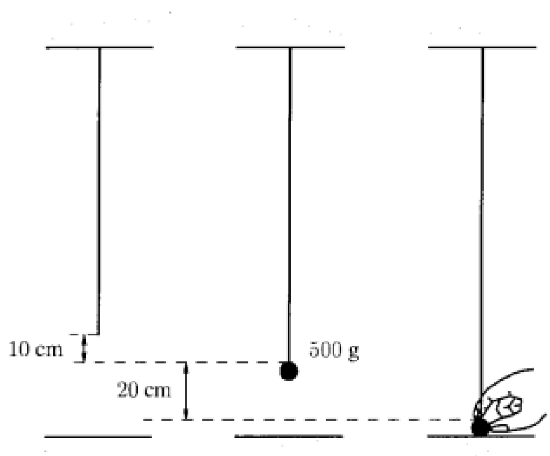


Figure 1 Figure 2 Figure 3

Q6 When the weight is gently released from its position shown in Figure 3, what maximum height (from the floor) does it attain? From ①-⑥ below choose the best answer, where $g = 9.8\text{m/s}^2$. $\boxed{6}$ cm

① 20 ② 30 ③ 40

④ 45 ⑤ 50 ⑥ 60

-2012 3

3 A sufficiently small steel smooth spherical object is initially at rest at point A on the smooth surface of a finite parabolic curve AOB in a vertical plane as shown in Fig.3−1, where the point O stands for the bottom point, the tangential line at O being horizontal, the vertical height at point A (relative to O) being $2H$, the vertical height at point B (relative to O) being H, the horizontal distance between B and O being $2H$. At time $t = 0$, the object is released slowly to move down frictionlessly along the curve toward point B, where it departs into the air. The effects of the rotation of the object around its center is assumed to be negligible. The speed of the object at B is

(1) (a) $\sqrt{0.5gH}$, (b) $\sqrt{gH}$, (c) $\sqrt{2gH}$, (d) $2\sqrt{gH}$, (e) $2\sqrt{2gH}$,

where g stands for acceleration of gravity. The slope angle of the object orbit measured from the horizontal level just after release in the air at B is

(2) (a) $\pi/8$, (b)$\pi/6$, (c) $\pi/5$, (d) $\pi/4$, (e) $\pi/3$.

The highest object level attained, $y_{\max}$, (based on point O) after release in the air is

(3) (a) $H < y_{\max} < 1.2H$, (b) $1.2H \le y_{\max} < 1.4H$, (c)$1.4H \le y_{\max} < 1.55H$, (d)$1.55H \le y_{\max} \le 1.75H$, (e)$1.75H < y_{\max} \le 2H$.

Finally the object will reach a point C with the same horizontal level as that at O. The necessary time to travel to C from B is

(4) (a) $2\sqrt{H/g}$, (b) $(1+\sqrt{3})\sqrt{H/g}$, (c) $2\sqrt{2}\sqrt{H/g}$, (d) $3\sqrt{H/g}$, (e) $2\sqrt{3}\sqrt{H/g}$.

The horizontal distance between C and B is

(5) (a)$2H$, (b) $(1+\sqrt{3})H$, (c) $2\sqrt{2}H$, (d) $3H$, (e) $2\sqrt{3}H$.

The speed of the object at C is

(6) (a)$\sqrt{gH}$, (b) $\sqrt{2gH}$, (c)$\sqrt{3gH}$, (d) $2\sqrt{gH}$, (e)$\sqrt{6gH}$.

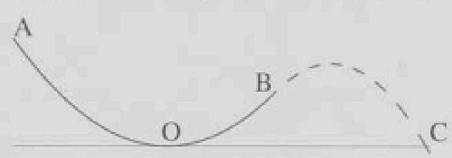


Fig. 3−1

-2014 1(1)

(1) Two objects with mass M and m are on a flat horizontal table. A horizontal spring with spring constant k is attached to the two objects as shown in Fig.1-1. If the system is released from rest when the spring is stretched by ℓ, find the maximum velocity of the object with mass m.

(a) $\sqrt{\frac{k\ell^2}{m}}$ (b) $\sqrt{\frac{k\ell^2}{(M+m)}}$ (c) $\sqrt{\frac{Mk\ell^2}{m(M+m)}}$

(d) $\sqrt{\frac{k\ell^2}{M}}$ (e) $\sqrt{\frac{k\ell^2}{|M-m|}}$ (f) $\sqrt{\frac{Mk\ell^2}{m(|M-m|)}}$

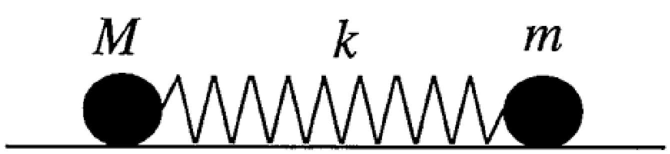


Fig. 1-1

-2014 1(2)

(2) An object is launched horizontally from the earth's surface. Let R be the radius of the earth and g the acceleration of gravity on the surface of the earth. Find the minimum speed to escape the earth's gravity.

(a) $\sqrt{gR}$ (b) $\sqrt{4gR}$ (c) gR

(d) $\sqrt{\frac{g}{R}}$ (e) $\sqrt{2gR}$

3. An object of mass M is hanging by a light spring of force constant k from the ceiling, as shown in Fig.3. A small ball of mass m which moves vertically upward collides with the object. After the collision, the object and the small ball stick together and oscillate in simple harmonic motion. Before the collision, the object is at rest. The speed of the small ball just before the collision is denoted as v. The acceleration of gravity is denoted as g. Answer the following questions.

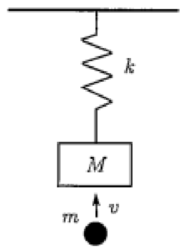


Fig. 3

(1) Find the amount of stretch of the spring from its natural length before the collision.

(a) Mk (b) $\frac{M}{k}$ (c) $\frac{k}{M}$
(d) Mgk (e) $\frac{Mg}{k}$ (f) $\frac{k}{Mg}$

(2) Find the speed of the object and the small ball just after the small ball collides with the object and they stick together.

(a) $\frac{m}{M}v$ (b) $\frac{M}{m}v$ (c) $\frac{m}{M+m}v$
(d) $\frac{M+m}{m}v$ (e) $\frac{M}{M+m}v$ (f) $\frac{M+m}{M}v$

(3) Find the amount of decrease of the sum of kinetic energies of the small ball and the object, before and after the small ball collides with the object.

(a) $\frac{1}{2}mv^2$ (b) $\frac{1}{2}Mv^2$ (c) $\frac{Mm}{2(M-m)}v^2$
(d) $\frac{Mm}{2(M+m)}v^2$ (e) $\frac{m(M+m)}{2M}v^2$ (f) $\frac{m(M-m)}{2M}v^2$

(4) Find the period of the simple harmonic oscillation after the small ball and the object stick together.

(a) $2\pi\sqrt{\frac{k}{M}}$ (b) $2\pi\sqrt{\frac{M}{k}}$ (c) $2\pi\sqrt{\frac{k}{M+m}}$
(d) $2\pi\sqrt{\frac{M+m}{k}}$ (e) $2\pi\sqrt{\frac{k}{Mg}}$ (f) $2\pi\sqrt{\frac{Mg}{k}}$
(g) $2\pi\sqrt{\frac{k}{(M+m)g}}$ (h) $2\pi\sqrt{\frac{(M+m)g}{k}}$

(5) During the simple harmonic oscillation of the small ball and the object, which are stuck together, the spring is at its natural length when the object is at its highest position. Find the kinetic energy of the small ball just before it collides with the object.

(a) $\frac{M(M+m)^2g^2}{km}$ (b) $\frac{M^2(M+m)g^2}{km}$ (c) $\frac{M(M+m)(M+2m)g^2}{km}$
(d) $\frac{M(M+m)^2g^2}{2km}$ (e) $\frac{M^2(M+m)g^2}{2km}$ (f) $\frac{M(M+m)(M+2m)g^2}{2km}$

-2017 1(4)

(4) A positive charge q on a frictionless, horizontal surface is attached to a spring with a force constant k. The charge and the spring are aligned along the x-axis as shown in Fig. 1-3 (a). The initial position of the charge q is $x = 0$ and the spring is in equilibrium. When a positive charge Q is placed on the x-axis at $x = \ell$, the spring is compressed by d and the charge q is stable as shown in Fig. 1-3 (b). The permittivity of free space is denoted by ε_0. Find the formula for the charge Q.

(a) $\frac{4\pi\varepsilon_0}{q} kd\,(\ell + d)^2$ (b) $\frac{4\pi\varepsilon_0}{q} kd\ell$ (c) $\frac{4\pi\varepsilon_0}{q} kd\,(\ell + d)$

(d) $\varepsilon_0 q$ (e) $\frac{4\pi\varepsilon_0}{q} kd^2\ell$ (f) $\frac{4\pi\varepsilon_0}{q} kd^2\,(\ell + d)$

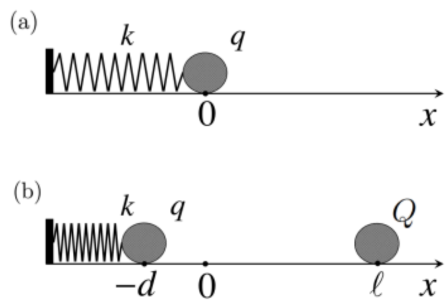


Fig. 1-3

-2017 3(1)(4)

3. A hemisphere of radius R is fixed on a horizontal floor, as shown in Fig. 3. A small ball of mass m slides down from the top of the hemisphere at point A with zero initial speed. There is no friction between the ball and the surface of the hemisphere. After sliding down the surface, the ball leaves the surface of the hemisphere at point B and hits the horizontal floor at point C. As the ball slides between A and B on the surface of the hemisphere, the position of the ball is described by the angle θ shown in the figure. The acceleration of gravity is denoted as g. Answer the following questions.

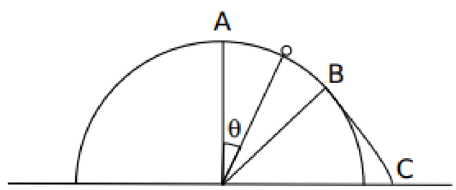


Fig. 3

(1) Find the speed of the ball when the ball is at a position on the hemisphere described by the angle θ.

(a) $\sqrt{2gR\sin\theta}$ (b) $\sqrt{2gR\cos\theta}$ (c) $\sqrt{2gR(1-\sin\theta)}$

(d) $\sqrt{2gR(1-\cos\theta)}$ (e) $\sqrt{\frac{gR}{2}\sin\theta}$ (f) $\sqrt{\frac{gR}{2}\cos\theta}$

(g) $\sqrt{\frac{gR}{2}(1-\sin\theta)}$ (h) $\sqrt{\frac{gR}{2}(1-\cos\theta)}$

(4) Find the speed of the ball immediately before the ball reaches the point C.

(a) $\sqrt{\frac{1}{2}gR}$ (b) $\sqrt{gR}$ (c) $\sqrt{2gR}$

(d) $\sqrt{5gR}$ (e) $\sqrt{\frac{1}{2}gR}$ (f) $\sqrt{\frac{1}{5}gR}$

-2020 1(2)

(2) In Fig. 1-2, block A of mass m is released from rest at a height of h, slides along a frictionless slope, and then collides with stationary block B of mass $3m$. After the collision, block A stops while block B moves to the right. Find the coefficient of restitution between block A and block B.

(a)	$\frac{1}{6}$	(b)	$\frac{1}{3}$	(c)	$\frac{1}{4}$
(d)	$\frac{1}{2}$	(e)	$\frac{2}{3}$	(f)	1

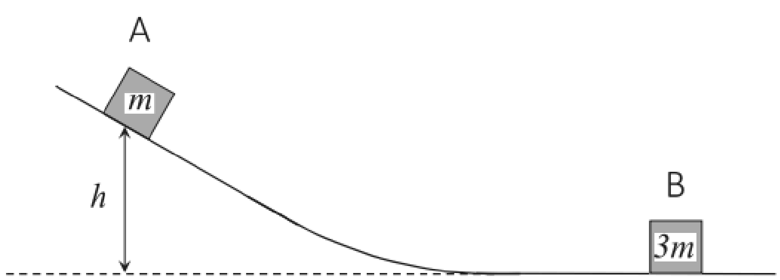


Fig. 1-2

-2020 3

3. As shown in Fig. 3, object A of mass m is moving with speed v on a frictionless surface. Object B of mass $3m$ is at rest and attached to the end of a spring with spring constant k. The other end of the spring is fixed to a wall. Objects A, B, and the spring are aligned on a straight line.

Fig. 3

At time $t = 0$, object A hits object B elastically.

(1) Find the velocity of object A after the collision.

(a) $\frac{v}{2}$ (b) $-\frac{v}{2}$ (c) $\frac{v}{3}$

(d) $-\frac{v}{3}$ (e) v (f) $-v$

(2) Find the velocity of object B after the collision.

(a) $\frac{v}{2}$ (b) $-\frac{v}{2}$ (c) $\frac{v}{3}$

(d) $-\frac{v}{3}$ (e) v (f) $-v$

After the collision, the spring is compressed. At time $t = T$, the spring takes the maximum compression, ℓ.

(3) Find the formula for ℓ.

(a) $\sqrt{\frac{mv^2}{9k}}$ (b) $\sqrt{\frac{3mv^2}{9k}}$ (c) $\sqrt{\frac{mv^2}{k}}$

(d) $\sqrt{\frac{3mv^2}{10k}}$ (e) $\sqrt{\frac{mv^2}{5k}}$ (f) $\sqrt{\frac{3mv^2}{4k}}$

(4) Find the formula for T.

(a) $2\pi\sqrt{\frac{3m}{k}}$ (b) $\pi\sqrt{\frac{3m}{k}}$ (c) $\frac{\pi}{2}\sqrt{\frac{3m}{k}}$

(d) $2\pi\sqrt{\frac{m}{k}}$ (e) $\pi\sqrt{\frac{m}{k}}$ (f) $\frac{\pi}{2}\sqrt{\frac{m}{k}}$

(5) Find the compression of the spring at time $t = T/2$.

(a) $\frac{\ell}{\sqrt{3}}$ (b) ℓ (c) 2ℓ

(d) $\frac{\ell}{2}$ (e) $\frac{\sqrt{3}\ell}{2}$ (f) $\frac{\ell}{\sqrt{2}}$

(3) Forces

-2009 1(1)

(1) A half sphere of radius r is fixed to a horizontal floor. The inner surface of the sphere is smooth. A small ball of mass m is at the bottom point P of the sphere and is given an initial velocity v as in Fig. 1. The initial velocity v is sufficiently large so that the small ball reaches a point Q at height r. Find the magnitude of the normal force exerted by the sphere on the small ball when the ball passes through the point Q. The acceleration due to gravity is denoted as g.

(a) $\frac{mv^2}{r}$ (b) $\frac{mv^2}{r} - mg$ (c) $\frac{mv^2}{r} - 2mg$

(d) mrv (e) $mrv - mg$ (f) $mrv - 2mg$

(g) $\frac{1}{2}mv^2$ (h) $\frac{1}{2}mv^2 - mgr$ (h) $\frac{1}{2}mv^2 - 2mgr$

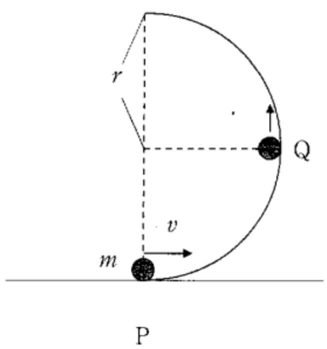


Fig. 1

-2009 1(2)

(2) What multiple of the distance between the center of the earth and that of the moon is the distance between the center of the earth and a geostationary satellite, which always stays above a fixed location on the equator? Take the cycle of revolution of the moon around the earth to be 27 days.

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{27}$ (d) 3

(e) 9 (f) 27

-2009 1(4)

(4) A cylinder with a frictionless piston of mass M and cross section S is placed vertically in an atmosphere of pressure p as shown in Fig. 3. The cylinder is then rotated 180 degrees so that the opening of the cylinder faces down. During the operation, the temperature of the gas inside the cylinder is kept fixed, and the volume of the gas is doubled. Which of the following is correct? The acceleration due to gravity is denoted as g.

(a) $p = \frac{3Mg}{S}$ (b) $p = \frac{2Mg}{S}$ (c) $p = \frac{Mg}{S}$

(d) $p = \frac{Mg}{2S}$ (e) $p = \frac{Mg}{3S}$

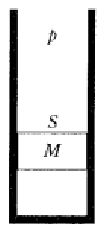


Fig. 3

-2009 3(1)

3 A point object of mass m is connected to an inertialess string of length L, the other end of which is fixed to a point O. At time $t = 0$, the object is assumed to begin to move horizontally in a vertical plane from the bottom point A ($\overline{OA} = L$) in the clockwise direction with an initial speed v_0 as shown in Fig. 5.

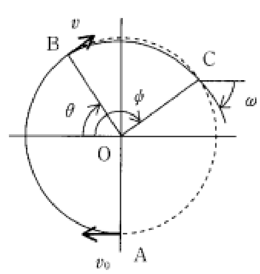


Fig. 5

If $\sqrt{2gL} < v_0 < \sqrt{5gL}$ (g : acceleration due to gravity), then at a point B (the angle between $\overrightarrow{OB}$ and the horizontal direction is designated θ as in Fig. 5) the magnitude of the force acting on the object from the string becomes zero, where $\overline{OB} = L$ and the velocity of the object is perpendicular to $\overrightarrow{OB}$, v being the magnitude of the velocity vector. We restrict ourselves to the case $0 < \theta < \pi/2$.

Circle the correct answers to questions (1)–(5). The speed, v, is given by

(1) (a) $\sqrt{gL}$, (b) $\sqrt{2gL}$, (c) $\sqrt{gL \sin\theta}$, (d) $\sqrt{2gL \sin\theta}$

-2010 ID

D Consider two springs of negligible mass that have the same natural length (50 cm) and the same spring constant. As shown in Figure 1, the two springs are joined together, and then are suspended vertically, with the lower end also fixed in place. Next, as shown in Figure 2, when small object A (mass: 500 g) is attached at the point where the springs are joined, object A descends 20 cm and comes to rest.

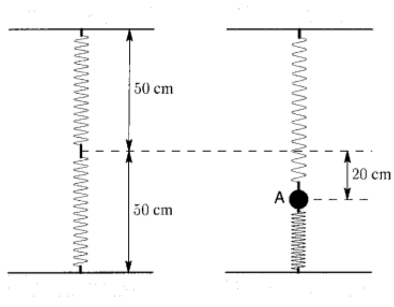


Figure 1 Figure 2

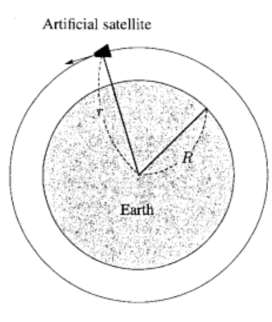
Q4 A is pulled downward and released gently. How much time elapses from release until A reaches its maximum height? From ①-⑥ below choose the best answer, where $g = 9.8\text{m/s}^2$. $\boxed{4}$ seconds

① 0.15 ② 0.30 ③ 0.45

④ 0.60 ⑤ 0.90 ⑥ 1.2

-2010 IG

G As shown in the figure below, an artificial satellite is traveling in a circular orbit around the earth. The radius of the orbit from the earth's center is r. Assume that the earth is a uniform sphere with a radius of R. The magnitude of acceleration due to gravity at the earth's surface is g.



Q7 What is the period of the satellite's orbital motion? From ①-④ below choose the correct answer. $\boxed{7}$

① $2\pi\dfrac{r}{\sqrt{gR}}$ ② $2\pi\dfrac{R}{\sqrt{gr}}$

③ $2\pi\dfrac{r}{R}\sqrt{\dfrac{r}{g}}$ ④ $2\pi\dfrac{R}{r}\sqrt{\dfrac{R}{g}}$

-2012 1(1)

(1) Object A of mass m and object B of mass M are tied by a string and are on a smooth and flat floor, as shown in Fig. 1-1. A force of magnitude F is applied to object B in the horizontal direction. Find the magnitude of the tension in the string.

(a)	F	(b)	$\frac{m}{M}F$	(c)	$\frac{M}{M+m}F$
(d)	$\frac{m}{M+m}F$	(e)	$\frac{M}{M-m}F$	(f)	$\frac{m}{M-m}F$

-2012 1(3)

(3) A cylinder with a cross section S with a frictionless piston with a mass of M is fixed at the angle θ in a vertical direction, as shown in Fig. 1-3. Find the pressure inside the cylinder. The atmospheric pressure is denoted as p_0 and the acceleration of gravity is denoted as g.

(a)	$p_0 + MgS\cos\theta$	(b)	$p_0 - MgS\cos\theta$	(c)	$p_0 + MgS\sin\theta$
(d)	$p_0 - MgS\sin\theta$	(e)	$p_0 + \frac{Mg}{S}\cos\theta$	(f)	$p_0 - \frac{Mg}{S}\cos\theta$
(g)	$p_0 + \frac{Mg}{S}\sin\theta$	(h)	$p_0 - \frac{Mg}{S}\sin\theta$		

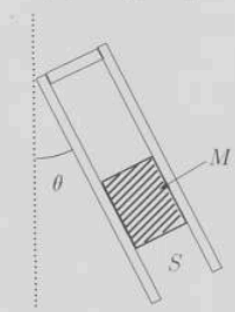


Fig. 1-3

-2013 1(2)

(2) Kepler's third law, $T^2 = ka^3$, holds in the revolution of planets in the solar system, where T is the revolution period, a is the semimajor axis of the elliptical orbit, and k is a constant of proportionality. Choose the appropriate formula for k from below. The gravitation constant is G and the mass of the Sun is M.

(a)	GM	(b)	$\frac{G}{M}$	(c)	$\frac{M}{G}$
(d)	$\frac{1}{GM}$	(e)	$4\pi^2 GM$	(f)	$\frac{4\pi^2 G}{M}$
(g)	$\frac{4\pi^2 M}{G}$	(h)	$\frac{4\pi^2}{GM}$	(i)	$\frac{GM}{4\pi^2}$
(j)	$\frac{G}{4\pi^2 M}$	(k)	$\frac{M}{4\pi^2 G}$	(l)	$\frac{1}{4\pi^2 GM}$

-2014 3

3. A small ball of mass m is connected to a fixed point O by a light string of length a, as shown in Fig. 3. The small ball is initially at rest at A and is released gently. The height of A is the same as that of O. At C which is located right below O, there is a thin nail directing perpendicularly to a vertical plane which includes OA. The distance OC is denoted as b. a and b satisfy $a/2 < b < a$. Air resistance to the ball and to the string may be ignored. The acceleration of gravity is denoted as g. Answer the following questions.

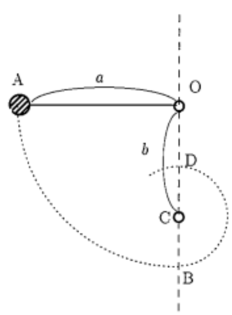


Fig. 3

(1) Find the speed of the small ball when it reaches B, which is located right below O.

(a)	$\sqrt{\frac{3g}{a}}$	(b)	$\sqrt{\frac{2g}{a}}$	(c)	$\sqrt{\frac{g}{a}}$
(d)	$\sqrt{\frac{g}{2a}}$	(e)	$\sqrt{\frac{g}{3a}}$	(f)	$\sqrt{3ga}$
(g)	$\sqrt{2ga}$	(h)	$\sqrt{ga}$	(i)	$\sqrt{\frac{ga}{2}}$
(j)	$\sqrt{\frac{ga}{3}}$	(k)	$\sqrt{\frac{3a}{g}}$	(l)	$\sqrt{\frac{2a}{g}}$
(m)	$\sqrt{\frac{a}{g}}$	(n)	$\sqrt{\frac{a}{2g}}$	(o)	$\sqrt{\frac{a}{3g}}$

(2) Find the magnitude of the tension in the string just before the small ball reaches B.

(a)	mg	(b)	$2mg$	(c)	$3mg$
(d)	$5mg$	(e)	$\frac{1}{2}mg$	(f)	$\frac{1}{3}mg$
(g)	$\frac{1}{5}mg$				

(3) After passing B, the small ball moves around in a circular motion centered at C. The string does not bend in the circular motion. Find the magnitude of the tension in the string when the small ball is at D which is located on OB.

(a)	$\frac{3a-5b}{a+b}mg$	(b)	$\frac{-3a+5b}{a+b}mg$	(c)	$\frac{5a-3b}{a+b}mg$
(d)	$\frac{-5a+3b}{a+b}mg$	(e)	$\frac{3a-5b}{a-b}$	(f)	$\frac{-3a+5b}{a-b}mg$
(g)	$\frac{5a-3b}{a-b}$	(h)	$\frac{-5a+3b}{a-b}$		

(4) Find the relation which a and b must satisfy so that the small ball reaches D without any bending of the string.

(a)	$a > \frac{3}{5}b$	(b)	$a < \frac{3}{5}b$
(c)	$a > \frac{5}{3}b$	(d)	$a < \frac{5}{3}b$

(5) Consider the case of $b = a/2$. In this case, the string bends before the small ball reaches D after passing through B. Find the speed of the small ball when the string starts to bend.

(a)	$\sqrt{\frac{3g}{a}}$	(b)	$\sqrt{\frac{2g}{a}}$	(c)	$\sqrt{\frac{g}{a}}$
(d)	$\sqrt{\frac{g}{2a}}$	(e)	$\sqrt{\frac{g}{3a}}$	(f)	$\sqrt{3ga}$
(g)	$\sqrt{2ga}$	(h)	$\sqrt{ga}$	(i)	$\sqrt{\frac{ga}{2}}$
(j)	$\sqrt{\frac{ga}{3}}$	(k)	$\sqrt{\frac{3a}{g}}$	(l)	$\sqrt{\frac{2a}{g}}$
(m)	$\sqrt{\frac{a}{g}}$	(n)	$\sqrt{\frac{a}{2g}}$	(o)	$\sqrt{\frac{a}{3g}}$

-2015 1(2)

(2) There is a planet with a moon. The moon has an orbital radius of R and a period of T. Assuming the orbit is circular, find the correct formula for the mass of the planet. Here the gravitational constant is denoted as G.

(a)	$\frac{4\pi^2 R}{GT^2}$	(b)	$\frac{4\pi^2 R^2}{GT^2}$	(c)	$\frac{4\pi^2 R^3}{GT^2}$
(d)	$\frac{R}{GT^2}$	(e)	$\frac{R^2}{GT^2}$	(f)	$\frac{R^3}{GT^2}$

-2015 4(1)

4. A cylinder is placed vertically in an atmosphere fitted with a frictionless piston of mass M, as shown in Fig. 4. One mole of a monatomic gas is contained in the cylinder. The cross-sectional area inside the cylinder is denoted as S, and the pressure of the atmosphere is denoted as p_0. Initially, the height of the gas in the cylinder is h, and the pressure of the gas is twice the pressure of the atmosphere, $2p_0$. The cylinder and the piston do not conduct heat. The gas may be regarded as an ideal gas. The acceleration of gravity is denoted as g, and the universal gas constant is denoted as R. Answer the following questions.

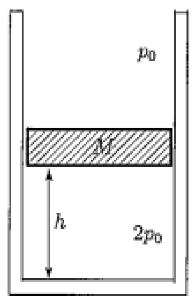


Fig. 4

(1) Express the mass M of the piston using other quantities.

(a)	p_0Shg	(b)	p_0Sg	(c)	$\frac{p_0Sh}{g}$
(d)	$\frac{p_0S}{g}$	(e)	$\frac{p_0}{Shg}$	(f)	$\frac{p_0}{Sg}$

-2016 1(4)

(4) In Fig. 1-2, a box of mass m is on an inclined plane. The angle of incline is θ. The coefficient of static friction between the box and the incline is μ. Find the condition where the box will slide.

(a)	$\mu < \tan\theta$	(b)	$\mu = \tan\theta$	(c)	$\mu > \tan\theta$
(d)	$\mu < \cot\theta$	(e)	$\mu = \cot\theta$	(f)	$\mu > \cot\theta$

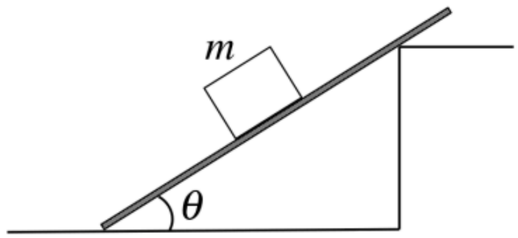


Fig. 1-2

-2017 3(3)(4)

3. A hemisphere of radius R is fixed on a horizontal floor, as shown in Fig. 3. A small ball of mass m slides down from the top of the hemisphere at point A with zero initial speed. There is no friction between the ball and the surface of the hemisphere. After sliding down the surface, the ball leaves the surface of the hemisphere at point B and hits the horizontal floor at point C. As the ball slides between A and B on the surface of the hemisphere, the position of the ball is described by the angle θ shown in the figure. The acceleration of gravity is denoted as g. Answer the following questions.

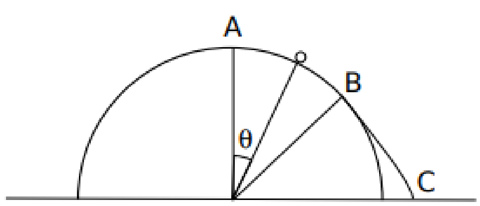


Fig. 3

The speed of the ball is $\sqrt{2gR(1-\sin\theta)}$

(2) Find the normal force acting from the hemisphere to the ball when the ball is on the hemisphere at a position described by the angle θ.

(a) $mg\cos\theta$ (b) $mg\sin\theta$ (c) $mg(2-\cos\theta)$

(d) $mg(2-\sin\theta)$ (e) $mg(3\cos\theta-2)$ (f) $mg(3\sin\theta-2)$

(g) $mg(3-4\cos\theta)$ (h) $mg(3-4\sin\theta)$

(3) The position B at which the ball leaves the hemisphere is described by the angle θ_B. Find $\cos\theta_B$.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2}{3}$

(d) $\frac{\sqrt{5}}{3}$ (e) $\frac{3}{4}$ (f) $\frac{\sqrt{7}}{4}$

-2018 1(2)

(2) An object of mass m is attached to a light spring with a force constant k and a natural length l_0. The object is moving on a frictionless flat horizontal table with a uniform circular motion as shown in Fig. 1-2. The center of the circle O is at the other end of the spring. During this motion, the length of the spring is extended by αl_0 ($\alpha > 0$) from the natural length. Find the speed v of the object.

(a) $\sqrt{\frac{(1+\alpha)\alpha m}{k}}l_0$ (b) $\sqrt{\frac{m}{k}(1+\alpha)}l_0$ (c) $\sqrt{\frac{m}{k}}\alpha l_0$

(d) $\sqrt{\frac{k}{m}(1+\alpha)}l_0$ (e) $\sqrt{\frac{(1+\alpha)\alpha k}{m}}l_0$ (f) $\sqrt{\frac{k}{m}}\alpha l_0$

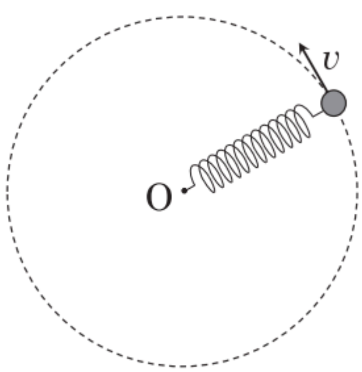


Fig. 1-2

-2018 3

3. At the surface of the earth the acceleration of gravity has the value $g = 9.8$ m/s^2. The constant of universal gravitation is given by $G = 6.67 \times 10^{-11}$ N$\cdot$m^2/kg^2. Answer the following questions.

(1) The radius of the earth is 6.4×10^3 km. Find the mass of the earth using the values of g and G.

(a) 6.0×10^{24} kg (b) 6.0×10^{18} kg (c) 6.0×10^{30} kg

(d) 2.0×10^{30} kg (e) 2.0×10^{36} kg (f) 2.0×10^{24} kg

(2) An object can escape from the gravitational attraction of a planet if the object has a large enough speed. The minimum value of this speed is called the escape speed. Find the escape speed of the earth.

(a) 1.1×10^2 m/s (b) 1.1×10^3 m/s (c) 1.1×10^4 m/s

(d) 7.9×10^2 m/s (e) 7.9×10^3 m/s (f) 7.9×10^4 m/s

(3) The mass of Jupiter is about 320 times larger than the earth and the radius of Jupiter is about 11 times larger than the earth. What is the ratio of the escape speed from Jupiter to that of the earth?

(a) 2.1 (b) 5.4 (c) 11

(d) 18 (e) 29 (f) 320

(4) A satellite moves in a circular orbit around the earth. If the satellite's orbital period is equal to the Earth's rotational period, what is the radius of the satellite's orbit?

(a) 4.2×10^6 m (b) 4.2×10^7 m (c) 4.2×10^8 m

(d) 6.4×10^6 m (e) 6.4×10^7 m (f) 6.4×10^8 m

-2020 1(1)

(1) In Fig. 1-1, three blocks with masses of m, $2m$, and $3m$ are connected and pulled to the right on a horizontal frictionless table by a force with a magnitude of F. Find the magnitudes of the tensions T_1 and T_2 in the interconnecting cords.

(a) $T_1 = F,\ T_2 = F$ (b) $T_1 = \frac{1}{2}F,\ T_2 = \frac{1}{2}F$

(c) $T_1 = \frac{1}{2}F,\ T_2 = \frac{1}{6}F$ (d) $T_1 = \frac{1}{6}F,\ T_2 = \frac{1}{2}F$

(e) $T_1 = \frac{1}{2}F,\ T_2 = \frac{1}{3}F$ (f) $T_1 = \frac{1}{3}F,\ T_2 = \frac{1}{2}F$

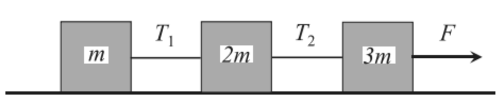


Fig. 1-1

-2020 4(2)-(4)

4. A hot air balloon of mass M is shown in Fig. 4. The volume of the air bag is V. Mass M does not include the mass of the air inside the bag. The bottom of the bag is open, and the air pressure inside the bag is equal to the surrounding air pressure, P. The burner sitting on the basket is used to heat the air inside the bag. The molar mass of air is denoted by m and the acceleration of gravity is denoted by g. The air is an ideal gas and the universal gas constant is denoted by R. The temperature of the surrounding air is T_0.

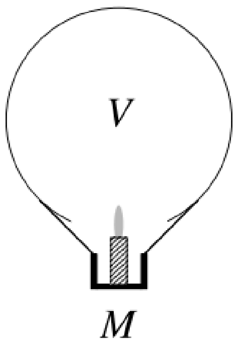


Fig. 4

After the air inside the bag is heated by the burner, the temperature of the air becomes T.

The density of air is given by $\frac{mP}{RT}$ (T is a variable)

(2) Find the buoyancy force, namely the upward force created by the air surrounding the balloon, acting on the balloon.

(a) Mg (b) $\frac{PVmg}{RT_0}$ (c) $\frac{2PVmg}{RT_0}$

(d) $\frac{PVmg}{2RT_0}$ (e) $\frac{5PVmg}{RT_0}$ (f) $\frac{mg}{RT_0}$

The balloon starts rising from the ground when the temperature of the air inside the bag is T_1.

(3) Find the formula for T_1.

(a) $\frac{mPVT_0}{mPV - MRT_0}$ (b) $\frac{mPVT_0}{mPV - 2MRT_0}$ (c) $\frac{mPV}{MR}$

(d) $\frac{mPV - MRT_0}{mPVT_0}$ (e) T_0 (f) $2T_0$

(4) Using the values $m = 2.9 \times 10^{-2}\,\mathrm{kg \cdot mol^{-1}}$, $M = 5.0 \times 10^2\,\mathrm{kg}$, $P = 1.0 \times 10^5\,\mathrm{Pa}$, $V = 3.0 \times 10^3\,\mathrm{m^3}$, $T_0 = 3.0 \times 10^2\,\mathrm{K}$, and $R = 8.3\,\mathrm{J \cdot K^{-1} \cdot mol^{-1}}$, find the approximate value of T_1.

(a) 3.0×10^2 K (b) 7.0×10^2 K (c) 2.5×10^3 K

(d) 3.5×10^2 K (e) 4.5×10^2 K (f) 4.0×10^3 K

(4) Equilibrium of Objects

-2007 3

3 A uniform thin straight bar AE is at rest inside a hemisphere in the configuration shown in Fig.6, under the assumption that friction between the bar and the hemisphere is negligible. This configuration is possible as long as the length of the bar remains within a limited range. The center of the hemisphere is on the vertical plane containing the two points A and B. The upper plane BC of the hemisphere is kept horizontal. The directions $\vec{AD}$ and $\vec{BD}$ mean the direction of the force acting on the bar (from the hemisphere) at point A and that on the bar at point B respectively. $\vec{DG}$ is the direction of the force of gravity acting on the bar, where G is the center of gravity of the bar. The angle θ ($\equiv \angle ABC$) means the angle between the bar and the horizontal line, and $\alpha \equiv \angle ABD$, $\beta \equiv \angle BAD$.

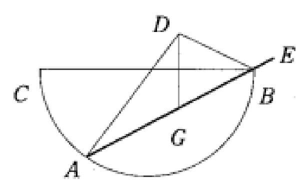


Fig. 6 Configuration

(1) For α, which of the following is correct?

(a) $\alpha \approx 0$ (b) $\alpha = \theta$ (c) $\alpha = 2\theta$ (d) $\alpha = \pi/2$

(2) For β, which of the following is correct?

(a) $\beta \approx 0$ (b) $\beta = \theta$ (c) $\beta = 2\theta$ (d) $\beta = \pi/2$

(3) Is the case $\theta = \pi/4$ possible or impossible?

(a) Possible (b) Impossible

(4) Is the case $\theta = 5\pi/24$ possible or impossible?

(a) Possible (b) Impossible

(5) In case of $\theta = \pi/6$, choose the suitable ratio of the length of the bar to the diameter of the hemisphere from the following.

(a) $3/2$ (b) $\sqrt{2}$ (c) $3\sqrt{3}/4$ (d) $5/4$ (e) $2/\sqrt{3}$

-2010 IB

B As shown in Figure 1, two people of the same height are holding up a box attached to a pole. The center of gravity of the pole and the box is located at point G. The pole is supported at points A and B. The pole and the box do not change form.

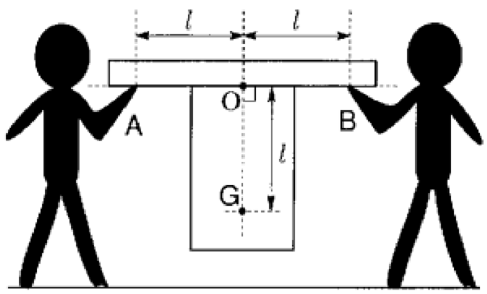


Figure 1

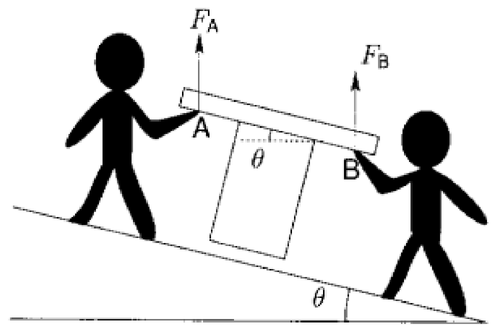


Figure 2

Q2 As shown in Figure 2, the two people are now standing on a slope inclined by angle θ ($< \frac{\pi}{4}$) from the horizontal. The person supporting the upper end at A exerts a force of magnitude of F_A vertically upward. The person supporting the lower end at B exerts a force of magnitude of F_B vertically upward. What is the value of the ratio $\frac{F_A}{F_B}$? From ①-⑥ below choose the correct answer. $\boxed{2}$

① $\dfrac{1+\cos\theta}{1-\cos\theta}$ ② $\dfrac{1-\cos\theta}{1+\cos\theta}$ ③ $\dfrac{1+\sin\theta}{1-\sin\theta}$

④ $\dfrac{1-\sin\theta}{1+\sin\theta}$ ⑤ $\dfrac{1+\tan\theta}{1-\tan\theta}$ ⑥ $\dfrac{1-\tan\theta}{1+\tan\theta}$

3 A thin stiff uniform rectangular plate with width L ($L \equiv \overline{\mathrm{AB}}$) is lying on two inclined surfaces as shown in Fig. 3−1. The angle between the horizontal surface and the left inclined surface is α, and that between the horizontal surface and the right inclined surface is β. It is assumed that the plate length is sufficiently larger than L and both edges of the plate are smooth enough to neglect friction.

(1) In case of $\alpha + \beta = \pi/2$, the static balance shown in Fig. 3−1 is

(a) possible only provided $\alpha \neq \beta$.

(b) always possible.

(c) always impossible.

(2) In case of $\alpha = \beta$, the static balance shown in Fig. 3−1 is

(a) possible only provided $\alpha < \pi/4$.

(b) possible only provided $\alpha \geq \pi/4$.

(c) always possible.

(d) always impossible.

(3) In case of $\alpha + \beta = \pi/4$, the static balance shown in Fig. 3−1 is

(a) possible only provided $1/2 > \tan\alpha$.

(b) possible only provided $\tan\alpha > 1/3$.

(c) possible only provided $\tan\alpha \geq 1/2$.

(d) possible only provided $1/3 \geq \tan\alpha$.

(e) possible only provided $1/2 > \tan\alpha > 1/3$.

(f) always impossible.

(4) In case of $\alpha = \pi/4$ and $\beta = \pi/3$, the static balance shown in Fig. 3−1 is

(a) impossible.

(b) possible.

(5) In case of $\alpha = \pi/4$ and $\beta = \pi/20$, the static balance shown in Fig. 3−1 is

(a) impossible.

(b) possible.

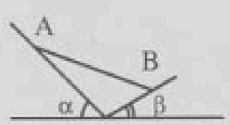


Fig. 3−1