

Q1(1):

Let θ be the angle of projection and v be the initial speed.

We have $v_x = v \cos \theta$ and $v_y = v \sin \theta$.

Solving $v \sin \theta t - \frac{1}{2}gt^2 = 0$, we have the time that the ball travelled after projection $t = \frac{2v \sin \theta}{g}$.

Therefore, the horizontal range $= v \cos \theta \left(\frac{2v \sin \theta}{g} \right) = \frac{v^2 \sin 2\theta}{g}$, which is directly proportional to $\sin 2\theta$.

Therefore, the required ratio is $\sin 2 \cdot 45^\circ : \sin 2 \cdot 30^\circ = 1 : \left(\frac{\sqrt{3}}{2} \right) = \boxed{\frac{2}{\sqrt{3}}}$.

Q1(2):

By $C = \frac{\epsilon_r S}{d}$, we have $C \propto \frac{1}{d}$ for the action. Therefore, the capacitance after the action $= \frac{1}{3}C$.

As the internal energy of the capacitor is given by $U = \frac{1}{2}QV$, i.e. $U = \frac{Q^2}{2C}$, we have $\Delta U = \frac{Q^2}{2} \Delta \frac{1}{C}$.

As the only source for the change in internal energy is the work done from outside, we have $W = \Delta U = \frac{Q^2}{2} \left(\frac{3}{C} - \frac{1}{C} \right) = \boxed{\frac{Q^2}{C}}$.

Q1(3):

By $pV = nRT$, we have $(1 + 5 \cdot 1)(1) = nR(17 + 273)$.

Let V' be the volume on the surface, we have $(1)V' = nR(27 + 273)$.

Combine the two equations, we have $V' = \frac{300}{290} \cdot 6 \approx \boxed{6.2 \text{ cm}^3}$.

Q1(4):

The general form of Doppler effect is given by $f' = \frac{V - v_{observer}}{V - v_{source}} f$.

Note the the speed of the train is $72 \cdot \frac{1000}{3600} = 20 \text{ m/s}$.

When the train approaches the crossing signal, the observer (the passenger in the train) is travelling in the opposite direction to the propagation of the sound wave. Therefore, we have $720 = \frac{340+20}{340} f$, i.e. $f = 680 \text{ Hz}$.

After the train passes the crossing signal, the observer is travelling in the same direction to the propagation of the sound wave. Therefore, we have

$$f' = \frac{340-20}{340} \cdot 680 = \boxed{640 \text{ Hz}}.$$

Q1(5):

The equation of the decay is given by ${}_{92}^{238}\text{U} \rightarrow {}_2^4\alpha + {}_{90}^{234}\text{?}$ (here the element symbol of the element with atomic number 90 is denoted as ?).

Initially, the nucleus is at rest. Therefore, by the conservation of momentum, we have $4v + 234V = 0$, i.e. $|v/V| = \frac{234}{4} = \boxed{58.5}$.

Q2:

(1): The electric field from the charge at A on D is in the direction $S45^\circ E$ and that from the charge at D is in the direction $N45^\circ E$. Therefore, the net electric field on D is in the direction E , i.e. $\boxed{A \rightarrow B}$.

(2): The magnitudes of the electric fields from the charge at A and that at B are the same, where both are $k \cdot \frac{q}{2a^2}$ N/C as the distances between the two charges and D are $\sqrt{2}a$ by Pythagoras' theorem.

Consider the vector sum of the two electric fields, we have the magnitude of the electric field at D = $2 \cdot \frac{kq}{2a^2} \cos 45^\circ = \boxed{\frac{kq^2}{\sqrt{2}a^2}}$.

(3): The electric potential on D due to the charge on A = $\frac{kq}{\sqrt{2}a}$ and that due to the charge on B = $\frac{-kq}{\sqrt{2}a}$.

Therefore, we have $V_D = \frac{kq}{\sqrt{2}a} - \frac{kq}{\sqrt{2}a} = 0$ J/C .

Similarly, $V_C = \frac{kq}{a} - \frac{kq}{a} = 0$ J/C .

Therefore, we have $\boxed{V_C = V_D}$.

Note: In fact, the electric potential throughout the whole line is equal to 0 due to symmetry.

(4): Similar to (3), the electric potential on D = $\frac{kq}{\sqrt{2}a} + \frac{kq}{\sqrt{2}a} = \frac{\sqrt{2}kq}{a}$ J/C and that on C = $\frac{2kq}{a}$ J/C .

Therefore, the difference in electric potential = $\frac{2kq}{a} - \frac{\sqrt{2}kq}{a} = \frac{(2-\sqrt{2})kq}{a}$ J/C and the work done is equal to the difference in electric potential energy = $\boxed{\frac{(\sqrt{2}-2)kq^2}{a}}$ J .

Note: The answer is wrong unless the question is asking about the work done per unit charge.

(5): As calculated, the electric potential energy of the charge at $D = -\frac{\sqrt{2}kq^2}{a}$ J.

Once the charge is moved to the point of infinity, the EPE becomes 0.

Therefore, by the conservation of energy: $KE + EPE = KE + EPE$, we have

$$\frac{1}{2}mv^2 - \frac{\sqrt{2}kq^2}{a} = 0$$

$$v = \sqrt{\frac{2\sqrt{2}kq^2}{ma}}$$

Q3:

(1): Considering the cosine ratio, the height of A relative to the top $= R_0 \cos \theta_0 \approx$

$$R_0 - \frac{1}{2}R_0\theta_0^2.$$

$$\text{Therefore, } h \approx R_0 - (R_0 - \frac{1}{2}R_0\theta_0^2) = \frac{1}{2}R_0\theta_0^2.$$

(2): Set the GPE as 0 at the height of B.

Consider the conservation of energy: $KE + GPE = KE + GPE$, we have

$$\frac{1}{2}mv_{max}^2 + 0 = 0 + mgh$$

$$v_{max} = \sqrt{2gh}$$

(3): Note that θ_0 is in radian measure*. Therefore, by definition, we have

$$\widehat{AB} = R_0\theta_0.$$

*: Otherwise, the approximations $\sin \theta_0 \approx \theta_0$ and $\cos \theta_0 \approx 1 - \frac{1}{2}\theta_0^2$ are not valid.

(4): Combine the results of (1)-(3), we have $\bar{t} = \frac{R_0 \theta_0}{\sqrt{2g(\frac{1}{2}R_0 \theta_0^2)}} = \boxed{\sqrt{\frac{R_0}{g}}}$.

(5): The period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$.

Therefore, $t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{R_0}{g}}$.

Hence, we have the ratio $\boxed{t/\bar{t} = \frac{\pi}{2}}$.

Q4:

(1): The total mass = $0.2 \cdot 16 \cdot 2 + 0.3 \cdot 14 \cdot 2 = \boxed{14.8 \text{ g}}$.

Note: Oxygen and nitrogen are diatomic molecules.

(2): As oxygen molecules are diatomic, we have $k = \frac{7}{5}$.

As given, we have $c_p = \frac{\frac{7}{5}}{\frac{7}{5}-1}R = \frac{7}{2}R$.

Therefore, the heat capacity of oxygen = $nc_p = 0.2 \cdot \frac{7}{2}R = \boxed{0.7R}$.

(3): Similar to (2), the heat capacity of nitrogen = $0.3 \cdot \frac{7}{2}R = 1.05R$.

Therefore, the total heat capacity of the container = $1.05R + 0.7R = \boxed{1.75R}$.

(4): Consider the internal energy, we have $U = \frac{3}{2}nkT = \frac{1}{2}mv^2$. Therefore,

we have $v^2 \propto \frac{n}{m}$.

The higher the mole to mass ratio, the higher the speed of molecule.

As the ratio for oxygen = $\frac{0.2}{2 \cdot 16} = \frac{1}{160}$ and that for nitrogen = $\frac{0.3}{2 \cdot 14} = \frac{1}{\frac{280}{3}} > \frac{1}{160}$, we have $v_1 < v_2$.

Q5:

Background: Unlike standing waves on a string, standing water waves have antinodes at the end and nodes in the middle. Refer to this video for a visual insight: <https://youtu.be/xhtg-RosQHw?si=vLaKGXeBx2y12rdm>.

(1): As the frequency is fixed to minimum, the standing wave is of first harmonic. Therefore, there is $\boxed{1}$ node.

(2): There are $\boxed{2}$ nodes on the walls.

(3): As there is a $\frac{1}{2}$ wave between the two walls, the wave length is equal to twice the separation between walls, i.e. $\boxed{2L}$.