Q1(1):

$x + y = \frac{(\sqrt{5}+2)^2+(\sqrt{5}-2)^2}{\sqrt{5}^2-2^2} = 18$ and $xy = 1$.

$x^2 + y^2 = (x+y)^2 - 2xy = 18^2 - 2 = \boxed{322}$

Q1(2):

By testing the potential rational roots given by the rational root theorem $\pm1, \pm2, \pm3, \pm6$, we have $x = -1$ is a root.

Therefore, we can do the factorisation by the long division:

$$x^3 + 6x^2 + 11x + 6 = 0$$

$$(x+1)(x^2 + 5x + 6) = 0$$

$$(x+1)(x+2)(x+3) = 0$$

$$x = \boxed{-1, -2, -3}$$

Q1(3):

$$\sin x - \cos x - \frac{1}{\sqrt{2}} = 0$$

$$\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x = \frac{1}{2}$$

$$\sin(x - \frac{\pi}{4}) = \frac{1}{2}$$

$$x - \frac{\pi}{4} = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \boxed{\frac{5\pi}{12}, \frac{13\pi}{12}}$$

Q1(4):

$$2\log_9(x+2) + \log_3 x = 1$$

$$\log_3(x^2 + 2x) = 1$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = \boxed{1} \text{ (as } x > 0 \text{ for } \log_3 x \text{ to be defined)}$$

Q1(5):

$$2^{1-x} - 2^{x+2} < 7$$

$$4 \cdot 2^x + 7 - 2 \cdot 2^{-x} > 0$$

$$4 \cdot 2^{2x} + 7 \cdot 2^x - 2 > 0$$

$$(4 \cdot 2^x - 1)(2^x + 2) > 0$$

$$2^x > \frac{1}{4}$$

$$\boxed{x > -2}$$

Q1(6):

$$\cos 2x + 9\sin x - 5 < 0$$

$$1 - 2\sin^2 x + 9\sin x - 5 < 0$$

$$2\sin^2 x - 9\sin x + 4 > 0$$

$$(2\sin x - 1)(\sin x - 4) > 0$$

$$\sin x < \frac{1}{2}$$

$$\boxed{0 \le x < \frac{\pi}{6}, \frac{5\pi}{6} < x < 2\pi}$$

Q1(7):

The normal vector of the line $2x + y - 12 = 0$ is $< 2, 1 >$ and the unit normal vector is $< \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} >$.

The perpendicular distance between A and the line is $\frac{|2\cdot3+1\cdot2-12|}{\sqrt{2^2+1^2}} = \frac{4}{\sqrt{5}}$.

Therefore, the point after reflection is $(3+2\cdot\frac{4}{\sqrt{5}}\cdot\frac{2}{\sqrt{5}}, 2+2\cdot\frac{4}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}})$, i.e. $\boxed{(\frac{31}{5}, \frac{18}{5})}$.

Note: A lies in the region $2x + y - 12 < 0$.

Q1(8):

The probability that at least one coin shows heads$=C^2_1(\frac{1}{2})(\frac{1}{2}) + (\frac{1}{2})^2 = \frac{3}{4}$.

The probability that two coins shows heads=$(\frac{1}{2})^2 = \frac{1}{4}$.

Therefore, the conditional probability=$\frac{\frac{1}{4}}{\frac{3}{4}} = \boxed{\frac{1}{3}}$.

Q1(9):

$$a_{n+1} - a_n = n + 1$$
$$a_{n+1} - a_1 = \frac{n(n+1)}{2} + n$$
$$a_{n+1} = \frac{n(n+1)}{2} + (n+1)$$
$$a_n = \frac{(n-1)n}{2} + n = \boxed{\frac{1}{2}n^2 + \frac{1}{2}n}$$

Q1(10):

$\lim\limits_{x\to 0} \frac{\tan x - \sin x}{x^3}$

$= \lim\limits_{x\to 0} \frac{\sin x(1-\cos x)}{x^3 \cos x}$

$= \lim\limits_{x\to 0} \frac{2\sin x \sin^2 \frac{x}{2}}{x^3 \cos x}$

$= (\lim\limits_{x\to 0} \frac{2}{\cos x})(\lim\limits_{x\to 0} \frac{\sin x}{x})(\lim\limits_{x\to 0} \frac{\frac{1}{2}\sin\frac{x}{2}}{\frac{x}{2}})^2$

$= 2 \cdot 1 \cdot (\frac{1}{2})^2$

$= \boxed{\frac{1}{2}}$

Q1(11):

$y = \frac{1}{2}x \ln x$

$\frac{dy}{dx} = \boxed{\dfrac{1}{2}(1 + \ln x)}$

Q1(12):

$\int xe^{-2x} dx$

$= -\frac{1}{2} \int xd(e^{-2x})$

$= -\frac{1}{2}xe^{-2x} + \frac{1}{2} \int e^{-2x} dx$

$= \boxed{-\dfrac{1}{2}xe^{-2x} - \dfrac{1}{4}e^{-2x} + Constant}$

Q2:

1): $A^2 + A + I$

$$= \begin{bmatrix} -1 & -a \\ a & a^2 - 1 \end{bmatrix} + A + I$$

$$= \begin{bmatrix} 0 & -a - 1 \\ a + 1 & a^2 + a \end{bmatrix}$$

Solving $A^2 + A + I = O$, we have $a = \boxed{-1}$.

2): $A^3 = (A^2 + A + I)A - A^2 - A$

$= -(A^2 + A + I) + I$

$= \boxed{I}$

3): As $A^3 = 1$, we have $A^{2+3n} + A^{1+3n} + A^{3n} = O$ $(n = 0, 1, 2, ...)$.

Therefore, $I + A + A^2 + ... + A^{10}$

$= O + O + O + A^{3\cdot 3} + A^{3\cdot 3+1}$

$= I + A$

$= \boxed{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}$

Q3:

1): $\int_0^\pi \cos^2 x dx$

$= \int_0^\pi \frac{1+\cos 2x}{2} dx$

$= \frac{1}{2}[x + \frac{1}{2}\sin 2x]_0^\pi$

$= \boxed{\dfrac{\pi}{2}}$

2): $\int_0^\pi (a\cos x + b\sin x + 1)^2 dx$

$= \int_0^\pi (a^2\cos^2 x + b^2\sin^2 x + 1 + 2a\cos x + 2b\sin x + 2ab\sin x\cos x)dx$

$= \int_0^\pi ((a^2 - b^2)\cos^2 x + b^2 + 1 + 2a\cos x + 2b\sin x + ab\sin 2x)dx$

$= \boxed{\dfrac{\pi}{2}a^2 + \dfrac{\pi}{2}b^2 + \pi + 4b}$*

*: $\int_0^\pi \cos x = 0$ by symmetry and $\int_0^\pi \sin 2x dx = 0$ as $\sin 2x$ crossed a whole period.

3): $I = \frac{\pi}{2}a^2 + \frac{\pi}{2}(b + \frac{4}{\pi})^2 - \frac{8}{\pi} + \pi$ by completing the square.

Therefore, $\min I = \boxed{\pi - \dfrac{8}{\pi}}$, when $a = \boxed{0}$ and $b = \boxed{-\dfrac{4}{\pi}}$.