Q1(1):

$f(x) = -\sin^2 x + 3\sin x + 10 = -(\sin x - \frac{3}{2})^2 + \frac{49}{4}$ by completing the square.

The axis of symmetry of the graph $f(x)$ is $\sin x = \frac{3}{2}$, although it is impossible to find a $x \in \mathbb{R}$ satisfies it, as the graph convex upwards, the further the value of $\sin x$ away from $\frac{3}{2}$, the smaller the value of $f(x)$.

As $-1 \leq \sin x \leq 1$, $f(x)$ attains to its minimum when $\sin x = -1$, and the corresponding minimum is $-(-1-\frac{3}{2})^2 + \frac{49}{4} = \boxed{6}$.

Alternative $f'(x) = -(2+\sin x)\cos x + (5-\sin x)\cos x = \cos x(3-2\sin x)$.

To the the extremum of $f(x)$, we set $f'(x) = 0$, then $x = \frac{\pi}{2} + k\pi$, $k \in \mathbb{N}$.

$f''(x) = -\sin x(3-2\sin x) - 2\cos^2 x = -3\sin x - 2\cos(2x)$.

Conduct the second derivative test:

$f''(\frac{\pi}{2} + 2k\pi) = -3 < 0$

$f''(\frac{3\pi}{2} + 2k\pi) = 3 > 0$

Therefore, $f(x)$ attains to its minimum value when $x = \frac{3\pi}{2} + 2k\pi$ and the minimum value=$(2-1)(5+1) = \boxed{6}$.

(Note: One can also use the first derivative test to test for the minimum, but that will be more complicated.)

Q1(2):

$(2k+1)x - (k-2)y + 3k - 1 = 0 \iff (2x - y + 3)k + (x + 2y - 1) = 0$.

If the equation holds independent on the value of k, then we have $\begin{cases} 2x - y + 3 = 0 \\ x + 2y - 1 = 0 \end{cases}$.

By solving the simultaneous equation, we have $x = \boxed{-1}$ and $y = \boxed{1}$.

Alternative As the equation holds independent on the value of k, we may any value of k we want:

Put $k = 2$ to cancel the y term: $5x + 6 - 1 = 0$, i.e. $x = \boxed{-1}$.

Put $k = -\frac{1}{2}$ to cancel the x term: $\frac{5}{2}y - \frac{3}{2} - 1 = 0$, i.e. $y = \boxed{1}$.

Q1(3):

Solving the simultaneous equation $\begin{cases} x + 2y - 1 = 0 \\ x - y + 2 = 0 \end{cases}$, we have $(x, y) = (-1, 1)$, i.e. the two straight lines meet at $(-1, 1)$.

If $ax - y + 3 = 0$ meet the two straight lines at one point, then it should pass through $(-1, 1)$. i.e. $-a - 1 + 3 = 0$, i.e. $a = \boxed{2}$.

Alternative By Gaussian elimination:

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & -1 & -2 \\ a & -1 & -3 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & -3 & -3 \\ 0 & -1-2a & -3-a \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2+a \end{array}\right).$$

If the system has a unique solution, we have $-2 + a = 0$, i.e. $a = \boxed{2}$.

Q1(4):

By rationalisation and binomial expansion,

$\frac{(\sqrt{3}+\sqrt{2})^3}{\sqrt{3}-\sqrt{2}} = (\sqrt{3}+\sqrt{2})^4 = 9 + 4(3)\sqrt{6} + 6(3)(2) + 4(2)\sqrt{6} + 4 = 49 + 20\sqrt{6}$.

Therefore, $a = \boxed{49}$ and $b = \boxed{20}$.

Q1(5):

If $3^x = 2^y = 5$, then $x = \frac{\log 5}{\log 3}$ and $y = \frac{\log 5}{\log 2}$.

Then, $\frac{1}{x} + \frac{1}{y} = \frac{\log 3}{\log 5} + \frac{\log 2}{\log 5}$

$= \log_5 3 + \log_5 2$

$= \log_5 \boxed{6}$.

Q2:

(1): By the fundamental theorem of calculus, we have

$f(x) = F'(x) = 3x^2 - 4x + 1$.

Substitue it back to the relation $\int_a^x f(t)dt = x^3 - 2x^2 + x - a$, we have

$$\int_a^x (3t^2 - 4t + 1)dt = x^3 - 2x^2 + x - a$$

$$t^3 - 2t^2 + t|_a^x = x^3 - 2x^2 + x - a$$

$$x^3 - 2x^2 + x - (a^3 - 2a^2 + a) = x^3 - 2x^2 + x - a$$

Therefore, by comparing the constant terms, we have

$$a^3 - 2a^2 + a = a$$

$$a^2(a-2) = 0$$

As $a \neq 0$, we have $a = \boxed{2}$.

Alternative By the fundamental theorem of calculus, we have $\int_a^x f(t)dt = g(x) - g(a)$, where $g(x)$ is a primitive function of $f(x)$.

Without lost of generality, set the constant term of $g(x)$ be 0 and we have $g(x) = x^3 - 2x^2 + x$.

Then, we have $g(a) = a^3 - 2a^2 + a = a$, i.e. $a = \boxed{2}$.

(2): Note that $x - 2$ is a factor of $F(x)$*.

Then, by the long division, we have $F(x) = (x-2)(x^2+1)$.

As $x^2 + 1 > 0$ for all $x \in \mathbb{R}$, we have $F(x) > 0$ when $\boxed{x > 2}$.

(*: Exhaust the rational root out by using the rational root theorem.)

(3): To find the x-intercepts of the graph of $f(x)$, solve

$$f(x) = 0$$

$$3x^2 - 4x + 1 = 0$$

$$(3x-1)(x-1) = 0$$

$$x = \frac{1}{3} \text{ or } x = 1$$

Therefore, the area=$|\int_{\frac{1}{3}}^{1} f(x)dx|$

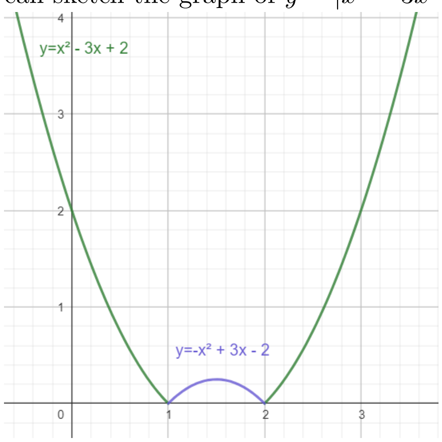
$= |[x^3 - 2x^2 + x]_{\frac{1}{3}}^{1}|$

$= |1 - 2 + 1 - \frac{1}{27} + \frac{2}{9} - \frac{1}{3}|$

$= \boxed{\frac{4}{27}}$

Q3:

(1): As $|x^2 - 3x + 2| = |(x-1)(x-2)| = \begin{cases} x^2 - 3x + 2, \ x \leq 1 \text{ or } x \geq 2 \\ \\ -x^2 + 3x - 2, \ 1 \leq x \leq 2 \end{cases}$, we can sketch the graph of $y = |x^2 - 3x + 2|$:



Then, we have $m > 0$.

On the other hand, the graph of $y = mx$ intersects the graph of $y = -x^2 + 3x - 2$ at two distinct points. i.e. the equation $-x^2 + (3-m)x - 2 = 0$ has two distinct solution. Then,

$$\Delta = (3-m)^2 - 4(-1)(-2) > 0$$

$$m^2 - 6m + 1 > 0$$

$$m < 3 - 2\sqrt{2} \text{ or } m > 3 + 2\sqrt{2}$$

Note that $m > 3 + 2\sqrt{2}$ is not applicable in this case.

Combine the above, we have $\boxed{0 < m < 3 - 2\sqrt{2}}$.

Alternative The relation $m < 3 - 2\sqrt{2}$ can also be obtained as following:

Suppose the line $y = mx$ tangent to the graph $y = -x^2 + 3x - 2$ at the region $1 \leq x \leq 2$. i.e. the equation $-x^2 + (3 - m)x - 2 = 0$ has only one solution. Then,

$$\Delta = (3 - m)^2 - 8 = 0$$

$$m = 3 \pm 2\sqrt{2}$$

Substitue the value of m back to the equation to check:

For $m = 3 + 2\sqrt{2}$, $x = -\sqrt{2} \notin [1, 2]$.

For $m = 3 - 2\sqrt{2}$, $x = \sqrt{2} \in [1, 2]$.

Therefore, only $m = 3 - 2\sqrt{2}$ is applicable.

Note that m represents the slope. For a straight line intersect the graph $y = |x^2 - 3x + 2|$ at 4 distinct points, the slope of it should be less than $3 - 2\sqrt{2}$. Therefore, we have $m < 3 - 2\sqrt{2}$.

Alternative (with calculus) Using the same logic as the previous alternative, the relation $m < 3 - 2\sqrt{2}$ can also be obtained as following with calculus:

Suppose the line $y = mx$ tangent to the graph of $y = -x^2 + 3x - 2$ at the point $P(p, -p^2 + 3p - 2)$ ($p \in [1, 2]$), then we have $m = \frac{-p^2+3p-2}{p}$.

On the other hand, the slope of a tangent to the graph of $y = -x^2 + 3x - 2$ is given by $y = -2x + 3$. Therefore, we have $m = -2p + 3$.

Combining the two equation, we have

$$\frac{-p^2+3p-2}{p} = -2p+3$$

$$p^2 = 2$$

$$p = \sqrt{2} \text{ (as } p \in [1,2])$$

By that time, $m = -2p+3 = 3-2\sqrt{2}$.

Therefore, we have $m < 3-2\sqrt{2}$.

(2): Without lost of generality, we let α, β and δ, γ be the two distinct roots of the equation $x^2-3x+2 = mx$ and $x^2-3x+2 = -mx$ respectively. By considering the sum of roots and product of roots of quadractic equation, we have:

$\alpha+\beta = 3+m$, $\alpha\beta = 2$, $\delta+\gamma = 3-m$, and $\delta\gamma = 2$

Then, $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\delta^2}+\frac{1}{\gamma^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}+\frac{\delta^2+\gamma^2}{(\delta\gamma)^2}$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{2^2}+\frac{(\delta+\gamma)^2-2\delta\gamma}{2^2}$

$= \frac{(3+m)^2-4+(3-m)^2-4}{4}$

$= \boxed{\dfrac{m^2+5}{2}}$.

Alternative $\alpha^2+\beta^2$ can also be evaluated using the fact that $\alpha^2 = (m+3)\alpha-2$ and $\beta^2 = (m+3)\beta-2$, which gives $\alpha^2+\beta^2 = (m+3)(\alpha+\beta)-4 = (m+3)^2-4$. That for $\delta^2+\gamma^2$ is similar.

(3):

$$0 < m < 3-2\sqrt{2}$$

$$0 < m^2 < 17-12\sqrt{2}$$

$$5 < m^2 + 5 < 22 - 12\sqrt{2}$$

$$\boxed{\frac{5}{2} < \frac{m^2 + 5}{2} = s(m) < 11 - 6\sqrt{2}}$$