

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

(1) The largest one among natural numbers that are less than

$$\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \dots \cdot \log_{2019} 2020$$

is .

(2) Let $f(x) = 1 + \frac{1}{x-1}$ ($x \neq 1$). The solution of the equation

$$f(f(x)) = f(x) \text{ is } x = \text{①} \text{ , } \text{②}.$$

(3) Let a and b be real numbers with $b \geq 0$. When the equation $x^4 + ax^2 + b = 0$ has exactly two real solutions, the minimum

value of $a+2b$ is , and the maximum value of $\lceil a-b \rceil$

is . Here, $\lceil r \rceil$ denotes the smallest integer that is larger than or equal to the real number r .

(4) The division of a polynomial function $f(x)$ by $(x-1)^2$ gives the remainder $x+1$, and that by x^2 gives the remainder $2x+3$. Thus, the remainder of the division of $f(x)$ by $x^2(x-1)$ is

$$\text{①} x^2 + \text{②} x + \text{③}.$$

- (5) The angle θ $\left(0 < \theta < \frac{\pi}{2}\right)$ between the two lines $y = (2 - \sqrt{3})x$ and $y = (\sqrt{3} - 2)x$ on the xy -plane is .

2. Cast a dice 3 times and let a, b, c be the number of dots on a side of the dice in order. Fill in the blanks with the answers to the following questions.

- (1) What is the probability P_1 that there exists a triangle with the side lengths $\sqrt{a}, \sqrt{b}, \sqrt{c}$?
- (2) Given that there exists a right triangle with the side lengths $\sqrt{a}, \sqrt{b}, \sqrt{c}$, what is the conditional probability P_2 that a, b, c are mutually different?
- (3) When there exists a triangle with the side lengths $\sqrt{a}, \sqrt{b}, \sqrt{c}$ and one of the angles is 60 degrees, what is M , which is the maximum value of $\frac{bc}{a}$?

(1) $P_1 =$

(2) $P_2 =$

(3) $M =$

3. Let C be a curve given by the parametric equations $x = \theta - \sin \theta$, $y = 1 - \cos \theta$ ($0 \leq \theta \leq 2\pi$). Fill in the blanks with the answers to the following questions.

- (1) Express $\frac{dy}{dx}$ in θ .
- (2) Find the area S of the figure A, which is bounded by C and the x -axis.
- (3) Find the volume V of the solid formed by rotation of A in (2) about the x -axis.

(1) $\frac{dy}{dx} =$

(2) $S =$

(3) $V =$