

Q1(1):

By the conservation of momentum, we have $mv + 0 = (m + M)V$, i.e. the speed after collision $V = \frac{m}{m+M}v$.

Now, consider the conservation of energy: KE+EPE=KE+EPE

$$\frac{1}{2}(m + M)\left(\frac{m}{m + M}v\right)^2 + 0 = 0 + \frac{1}{2}kl^2$$

$$l = \boxed{\frac{mv}{\sqrt{k(M + m)}}}$$

Note: The kinetic energy before the collision $\frac{1}{2}mv^2$ can't be used here as the collision is inelastic, i.e. there is energy loss.

Q1(2):

Set the potential as E at the top of the circuit, then the potential at the bottom is 0.

Regard the four resistors as two in series resistors $R_1 - R_3$ and $R_2 - R_4$ linked in parallel. Then, the voltages passes through $R_1 - R_3$ and $R_2 - R_4$ will be the same, E .

As R_1 and R_3 are in series, the voltage consumed by $R_1 = \frac{R_1}{R_1+R_3}E$. Hence, the potential at the point between the two resistors $= E - \frac{R_1}{R_1+R_3}E = \frac{R_3}{R_1+R_3}E$.

Similarly, the potential at the point between the resistors R_2 and $R_4 = \frac{R_4}{R_2+R_4}E$.

As the voltmeter connected between the two points, the voltage across the voltmeter is equal to the potential difference between the two points,

$$\text{i.e. } \frac{R_4}{R_2+R_4}E - \frac{R_3}{R_1+R_3}E = \boxed{\frac{R_1R_4 - R_2R_3}{(R_1 + R_3)(R_2 + R_4)}E}.$$

Note: In fact, $\frac{R_2 R_3 - R_1 R_4}{(R_1 + R_3)(R_2 + R_4)} E$ can also be the answer as the relationship of values and the poles of the voltmeter is not provided. However, there is no such an option so we can just ignore it.

Q1(3):

As the pressure is constant, we have $V \propto T$. After the cylinder is heated, the volume of it becomes $\frac{400}{300} \cdot 6.0 \times 10^{-3} = 8 \times 10^{-3} \text{ m}^3$.

As the work done by the gas is given by $p\Delta V$, we have

$$W = 1.0 \times 10^5 (8 \times 10^{-3} - 6.0 \times 10^{-3}) = \boxed{200 \text{ J}}.$$

Q1(4):

By Pythagoras' theorem, $S_1 A = \sqrt{12^2 + 16^2} = 20 \text{ cm}$.

Therefore, the path difference of the waves = $20 - 16 = 4 \text{ cm}$.

As the path difference = $\frac{1}{2}\lambda$, we have destructive interference happens.

Q1(5):

We have the equation ${}_0^1n + {}_{92}^{235}\text{U} \rightarrow {}_{56}^{144}\text{Ba} + {}_{36}^{89}\text{Kr} + x {}_0^1n$.

As the mass number is conserved, we have $1 + 235 = 144 + 89 + x$, i.e. $x = \boxed{3}$.

Q2:

(1): Before the conducting plate is inserted, the distance for electrons to travel between the parallel plates is d .

As the conducting plate conduct electrons to the other end immediately, after the conducting plate is inserted, the distance is $\frac{2}{3}d$.

As $C \propto \frac{1}{d}$, the capacitance after the conducting plate is inserted = $\boxed{\frac{3}{2}C}$.

(2): By the definition of capacitance, we have $Q = CV$. As the voltage across the parallel plates remains unchanged during the process, we have $\Delta Q = V\Delta C =$

$$E(\frac{3}{2}C - C) = \boxed{\frac{1}{2}CE}.$$

(3): By $U = \frac{1}{2}CV^2$, we have $\Delta U = \frac{1}{2}V^2\Delta C = \frac{1}{2}E^2(\frac{3}{2}C - C) = \boxed{\frac{1}{4}CE^2}$.

(4): We have $W = V\Delta Q$. By (2), $\Delta Q = \frac{1}{2}CE$. Therefore, $W = \boxed{\frac{1}{2}CE^2}$.

(5): As $\Delta U = W_{force} + W_{battery}$, we have $W_{force} = \frac{1}{4}CE^2 - \frac{1}{2}CE^2 = \boxed{-\frac{1}{4}CE^2}$.

(6): After the switch is opened, the number of charges stored in the capacitor becomes constant while the voltage across it becomes variable.

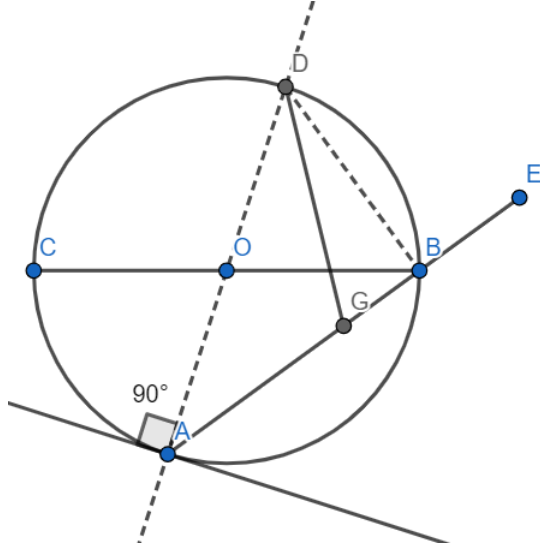
By the definition of capacitance, $Q = CV = \frac{3}{2}CE$.

As $U = \frac{Q^2}{2C}$, for fixed Q , we have $\Delta U = \frac{Q^2}{2}\Delta C = \frac{(\frac{3}{2}CE)^2}{2}(C - \frac{2}{3}C) = \frac{3}{8}CE^2$.

As the force provided the energy entirely, we have $W = \Delta U = \boxed{\frac{3}{8}CE^2}$.

Q3:

(1): The normal reaction acted on A is perpendicular to the tangent to the circle on A and the normal reaction acted on B is perpendicular to the bar. We have the graph:



Therefore, we have $\alpha = \frac{\pi}{2}$.

(2): As AD passes through O , we have $OB = OA$ and therefore $\beta = \theta$.

(3): Let $AB : BE = t : (1 - t)$, denote the length of the bar $AE = l$, then $AB = tl$ and $AG = \frac{l}{2}$.

Denote the mass of the bar as m , then the weight $= mg$.

Let the normal reaction at A and B be R_A and R_B respectively, consider the balance of forces, we have:

$$\begin{cases} R_A \sin 2\theta + R_B \cos \theta = mg \\ R_A \cos 2\theta = R_B \sin \theta \end{cases}.$$

Moreover, consider the moment at G , we have $R_A \sin \theta(\frac{l}{2}) = R_B(tl - \frac{l}{2})$, i.e.

$$t = \frac{1}{2R_B}(R_A \sin \theta + R_B).$$

When $\theta = \frac{\pi}{4}$, By the second equation, we have $R_B = 0$, which is impossible.

(4): Continue (3), solving the system, we have $(R_A, R_B) = (\frac{mg \sin \theta}{\sin 3\theta}, \frac{mg \cos 2\theta}{\sin 3\theta})$.

Therefore, $t = \frac{\sin \theta + \cos 2\theta}{2 \cos 2\theta}$.

When $\theta = \frac{5\pi}{24}$, $t = \frac{\sin \frac{5\pi}{24} + \cos \frac{5\pi}{12}}{2 \cos \frac{5\pi}{12}}$.

As we have $t \leq 1$, we have $\sin \frac{5\pi}{24} < \cos \frac{5\pi}{12} = 1 - 2 \sin^2 \frac{5\pi}{24}$.

However, the inequality $\sin \theta \leq 1 - 2 \sin^2 \theta$ holds only when $\sin \theta \leq \frac{1}{2}$. Since we have $\sin \frac{5\pi}{24} > \sin \frac{\pi}{6} = \frac{1}{2}$, the inequality does not hold. Therefore, the case is

impossible.

(5): By our calculation in (4), the bar merely supported by the edge of the hemi-sphere when $\theta = \frac{\pi}{6}$. By that time, as $\angle CAB = \frac{\pi}{2}$ (Thales' theorem), the

required ratio can be found by the trigonometric ratio $\frac{1}{\cos \frac{\pi}{6}} = \frac{2}{\sqrt{3}}$.

Q4:

(1): As $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$, we have $x = \frac{5}{3}$, i.e. $1 < x < 2$.

(2): By $pV = nRT$, we have $p = \frac{\rho}{m}RT$. For the same pressure and volume, we have $\rho \propto m$.

Therefore, as $m_{O_2} > m_{N_2}$, we have $\boxed{(\rho)_{O_2} > (\rho)_{N_2}}$.

(3): By $v = \sqrt{\frac{\gamma p}{\rho}}$ (provided), we have $v \propto \frac{1}{\rho}$. Therefore, we have $\boxed{v_{O_2} < v_{N_2}}$ as $(\rho)_{O_2} > (\rho)_{N_2}$.

(4): The density of water is much higher than that of air. As the propagation of sound wave requires vibration of particles, the denser the medium the faster the sound wave propagate. Therefore, we have $\boxed{v_w > v_{N_2}}$.

Note: The formula provided is in fact a bit confusing. Generally, the speed of sound wave should increase with the density of the medium. However, for the cases of ideal gases, gas molecules are moving randomly, an increase in density turns out increased the randomly of gas molecules' motions and make the propagation of sound more difficult.

Q5:

(1): Note that $1 \text{ km/h} = \frac{5}{18} \text{ m/s}$. The magnitude of the component of the velocity of the vehicle in the direction of the observer = $60 \cdot \frac{5}{18} \cos 45^\circ \approx 12$.

The general form of Doppler effect is given by $f_{observed} = \frac{V - v_{observer}}{V - v_{source}} f$.

Therefore, $\frac{\nu_A}{\nu_B} - 1 \approx \frac{340+12}{340-12} - 1 \approx \boxed{0.07}$.

(2)-(3): The effect by the wind is very insignificant. Therefore, the answer is still approximately $\boxed{0.07}$.

Note: I don't think MEXT did expect candidates to really do the calculation for (2) and (3) as the question stressed "approximate" and the error allow for the options are quite large (even the approximation I made in (1), $60 \cdot \frac{5}{18} \cos 45^\circ \approx 12$, does not affect the value in a large extend). If one really want to do the calculation once, subtract (or add) the component of wind speed in the direction of the wave's propagation to the velocity of sound wave will do.