

Q1(1):

By testing the potential rational roots given by the rational root theorem,

$\pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$ , we have  $x = 2$  is a root.

Then, do the factorisation by the long division, we have

$$3x^3 - 10x^2 + 10x - 4 = 0$$

$$(x - 2)(3x^2 - 4x + 2) = 0$$

$$x = \boxed{2, \frac{2 \pm \sqrt{2}i}{3}}$$

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Q1(2):

$$\begin{cases} 3x^2 - 8x - 3 \geq 0 \\ 2x^2 - 11x + 9 < 0 \end{cases}$$

$$\iff \begin{cases} (3x + 1)(x - 3) \geq 0 \\ (2x - 9)(x - 1) < 0 \end{cases}$$

$$\iff \begin{cases} x \leq -\frac{1}{3} \text{ or } x \geq 3 \\ 1 < x < \frac{9}{2} \end{cases}$$

$$\iff \boxed{3 \leq x < \frac{9}{2}}$$

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Q1(3):

$$\log_2(x-1) - \log_4(x+3) = \frac{1}{2}$$

$$\log_2 \frac{x-1}{\sqrt{x+3}} = \frac{1}{2}$$

$$\frac{x^2 - 2x + 1}{x + 3} = 2$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$x = \boxed{5}$  (Note the hidden condition for  $\log_2(x-1)$  to be defined:  $x > 1$ )

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Q1(4):

$$2 \sin^2 x > 3 \cos x$$

$$2 - 2 \cos^2 x > 3 \cos x$$

$$2 \cos^2 x + 3 \cos x - 2 < 0$$

$$(2 \cos x - 1)(\cos x + 2) < 0$$

$$-2 < \cos x < \frac{1}{2}$$

$$\boxed{\frac{\pi}{3} < x < \frac{5\pi}{3}}$$

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Q1(5):

Sum of roots= $\alpha + \beta = \frac{5}{2}$  and the product of roots= $\alpha\beta = \frac{1}{2}$ .

Then,  $a = -(\frac{1}{\alpha} + \frac{1}{\beta}) = -\frac{\frac{5}{2}}{\frac{1}{2}} = \boxed{-5}$ .

$b = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\frac{1}{2}} = \boxed{2}$ .

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Q1(6):

$$\frac{\alpha+\beta}{\gamma} = \frac{2+\sqrt{3}+i}{1+i} = \frac{2+\sqrt{3}+(1-2-\sqrt{3})i+1}{2} = \frac{3+\sqrt{3}}{2} + \frac{-1-\sqrt{3}}{2}i.$$

Therefore,  $r = \sqrt{(\frac{3+\sqrt{3}}{2})^2 + (\frac{-1-\sqrt{3}}{2})^2} = \sqrt{\frac{16+8\sqrt{3}}{4}} = \sqrt{4+2\sqrt{3}} = \boxed{1+\sqrt{3}}$ .

$$\theta = \arctan(\frac{\frac{-1-\sqrt{3}}{2}}{\frac{3+\sqrt{3}}{2}}) = -\arctan(\frac{\sqrt{3}}{3}) = \boxed{-\frac{\pi}{6}}.$$

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Q1(7):

$$\begin{aligned} & \sum_{k=1}^n (k+1)(k+2) \\ &= \sum_{k=1}^n (k^2 + 3k + 2) \\ &= \frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) + 2n \\ &= \boxed{\frac{1}{3}n^3 + 2n^2 + \frac{11}{3}n} \end{aligned}$$

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Q1(8):

$$|\vec{a} - 2\vec{b}|^2 = |\vec{a}|^2 + 4|\vec{b}|^2 - 4\vec{a} \cdot \vec{b} = 4.$$

Therefore,

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta = 3$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}$$

Q1(9):

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \\ &= \lim_{x \rightarrow 0} \frac{2}{\sqrt{1+x} + \sqrt{1-x}} \\ &= \frac{2}{1+1} \\ &= \boxed{1} \end{aligned}$$

Q1(10):

$$\text{As } \lim_{x \rightarrow \infty} \left(1 + \frac{1}{kx}\right)^x = \left(\lim_{kx \rightarrow \infty} \left(1 + \frac{1}{kx}\right)^{kx}\right)^{\frac{1}{k}} = e^{\frac{x}{k}}, \text{ we have } k = \boxed{2}.$$

Q1(11):

$$\begin{aligned} \log y &= \sin x \log x \\ \frac{1}{y} \cdot \frac{dy}{dx} &= \cos x \log x + \frac{\sin x}{x} \\ \frac{dy}{dx} &= \boxed{x^{\sin x} \left( \cos x \log x + \frac{\sin x}{x} \right)} \end{aligned}$$

Q1(12):

$$\begin{aligned}
& \int_0^1 x^2 e^x dx \\
&= \int_0^1 x^2 d(e^x) \\
&= x^2 e^x \Big|_0^1 - 2 \int_0^1 x d(e^x) \\
&= e - 2[xe^x]_0^1 + 2 \int_0^1 e^x dx \\
&= -e + 2[e^x]_0^1 \\
&= \boxed{e - 2}
\end{aligned}$$


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Q2:

1):  $(I + A)(I - A) = I^2 - A^2$  (as the multiplication operation involving  $I$  is commutative)

$$\begin{aligned}
&= I - O \\
&= \boxed{I}
\end{aligned}$$

2): By 1),  $(I + A)^{-1} = I - A$  and similarly  $(I + 2A)^{-1} = I - 2A$ .

Therefore, the expression  $= (I - A)(I - 2A)B$

$$\begin{aligned}
&= (I - 3A)B \\
&= \boxed{B}
\end{aligned}$$

$$3): \text{ As } A^2 = \begin{bmatrix} 1 + xy & x + xz \\ y + yz & xy + z^2 \end{bmatrix} = O \text{ and } AB = \begin{bmatrix} 2 + x & 4 + 2x \\ 2y + z & 4y + 2z \end{bmatrix} = O,$$

by solving, we have  $x = \boxed{-2}$ ,  $y = \boxed{\frac{1}{2}}$  and  $z = \boxed{-1}$ .

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Q3:

1): First, as A lies on C, we have  $2 = \sqrt{b}$ , i.e.  $b = \boxed{4}$ .

Moreover, as  $\frac{dy}{dx} = \frac{a}{2\sqrt{ax+4}}$ ,  $\frac{dy}{dx}|_{x=0} = \frac{a}{4}$ .

Therefore, the tangent at A is  $y = \frac{a}{4}x + 2$  and the x-intercept is  $\frac{-8}{a} = -8$ , i.e.

$$a = \boxed{1}.$$

2): Let the point of tangent be  $(p, \sqrt{4+p})$ , then  $\frac{dy}{dx}|_{x=p} = \frac{1}{2\sqrt{p+4}}$ .

Therefore, the equation of the tangent is  $y - \sqrt{4+p} = \frac{1}{2\sqrt{p+4}}(x - p)$ .

As the tangent passes through  $(-1, 2)$ , we have

$$2 - \sqrt{4+p} = \frac{1}{2\sqrt{p+4}}(-1 - p)$$

$$4\sqrt{p+4} - 2(p+4) = -1 - p$$

$$16(p+4) = p^2 + 14p + 49$$

$$(p+3)(p-5) = 0$$

$$p = -3, 5$$

Hence, the equations are  $y-1 = \frac{1}{2}(x+3)$ , i.e.  $y = \frac{1}{2}x + \frac{5}{2}$  and  $y-3 = \frac{1}{6}(x-5)$ ,

i.e.  $y = \frac{1}{6}x + \frac{13}{6}$ .

3): The point of intersection of the two tangents is P(-1,2).

Therefore, the area =  $\int_{-3}^{-1} ((\frac{1}{2}x + \frac{5}{2}) - \sqrt{x+4})dx + \int_{-1}^5 ((\frac{1}{6}x + \frac{13}{6}) - \sqrt{x+4})dx$

$$= [\frac{1}{4}x^2 + \frac{5}{2}x]_{-3}^{-1} + [\frac{1}{12}x^2 + \frac{13}{6}x]_{-1}^5 - [\frac{2}{3}(x+4)^{\frac{3}{2}}]_{-3}^5$$

$$= 3 + 15 - \frac{52}{3}$$

$$= \boxed{\frac{2}{3}}$$