

Q1(1):

$$ax^2 - 3ax + 2a < 0$$

$$x^2 - 3x + 2 < 0$$

$$(x - 2)(x - 1) < 0$$

$$\boxed{1 < x < 2}$$

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Q1(2):

$$4^{3x-1} - 2^{5x-4}$$

$$2^{6x-2} = 2^{5x-4}$$

$$6x - 2 = 5x - 4$$

$$x = \boxed{-2}$$

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Q1(3):

$$10^{\log_{10} 5} = \boxed{5}$$

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Q1(4):

Sum of roots= $\alpha + \beta = -\frac{-5}{1} = 5$  and product of roots= $\alpha\beta = \frac{3}{1} = 3$ .

Then,  $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 5^2 - 2 \cdot 3 = \boxed{19}$ .

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 5^2 - 4 \cdot 3 = \boxed{13}$ .

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Q1(5):

As  $(\vec{a} - \vec{b})^2 = |\vec{a}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = 5 - 2\vec{a} \cdot \vec{b} = |\vec{a} - \vec{b}|^2 = 7$ , we have  $\vec{a} \cdot \vec{b} = -1$ .

Therefore,  $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = -\frac{1}{2}$ .

$\theta = \boxed{120^\circ}$

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Q1(6):

$\sin(\angle B + \angle C) = \sin(180^\circ - \angle A) = \sin \angle A = \sin 30^\circ = \boxed{\frac{1}{2}}$ .

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Q1(7):

The multiples of 3 are: 102, 105, ..., 198.

As  $198 = 102 + 3 \cdot 32$ , there are  $\boxed{33}$  multiples of 3.

Moreover, the sum =  $\frac{(102+198)(33)}{2} = \boxed{4950}$ .

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Q1(8):

By the factor theorem,  $1 + a + b + 5 = 0$ , i.e.  $a + b = -6 \dots (1)$ .

By the remainder theorem,  $8 + 4a + 2b + 5 = 5$ , i.e.  $2a + b = -4 \dots (2)$ .

(2)-(1):  $a = \boxed{2}$ .

Substitue it into (1), we have  $b = -6 - 2 = \boxed{-8}$ .

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Q1(9):

$$f(0) = |-1| = \boxed{1}.$$

Note that when  $0 < x < 1$ ,  $|x^2 - 1| = 1 - x^2$  and when  $1 < x < 2$ ,  $|x^2 - 1| = x^2 - 1$ .

$$\begin{aligned} & \text{Therefore, } \int_0^2 |x^2 - 1| dx \\ &= \int_0^1 (1 - x^2) dx + \int_1^2 (x^2 - 1) dx \\ &= \left[ x - \frac{1}{3}x^3 \right]_0^1 + \left[ \frac{1}{3}x^3 - x \right]_1^2 \\ &= 1 - \frac{1}{3} + \frac{8}{3} - 2 - \frac{1}{3} + 1 \\ &= \boxed{2}. \end{aligned}$$

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Q1(10):

Let the arithmetic progression be  $b + nd$ .

As  $a + b + c = 24$ , we have  $(b - d) + b + (b + d) = 24$ , i.e.  $b = \boxed{8}$ .

As  $abc = 440$ , we have  $(8 - d)(8 + d) = 55$ , i.e.  $d^2 = 9$ , i.e.  $d = 3$  (as the progression is increasing).

Therefore,  $a = \boxed{5}$  and  $c = \boxed{11}$ .

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Q2:

(1): By the two points form of straight line, we have AC is  $y - 0 = \frac{0-3}{4-0}(x - 4)$ ,

i.e.  $\boxed{3}x + \boxed{4}y - \boxed{12} = 0$ .

(2): The mid-point of AC is  $(2, \frac{3}{2})$  and the slope of the line perpendicular to AC is  $\frac{4}{3}$ .

Therefore, the perpendicular bisector of AC is  $y - \frac{3}{2} = \frac{4}{3}(x - 2)$ ,

i.e.  $8x - 6y = 7 \dots (1)$ .

The perpendicular bisector of AB is  $y = 0 \dots (2)$ .

As the circumcenter is the intersection of any two perpendicular bisectors, by solving (1),(2), we have the coordinates  $(\frac{\boxed{7}}{\boxed{8}}, \boxed{0})$ .

(3): By the angle bisector theorem, OD:DC=(The distance between B and the x-axis):BC= $\boxed{3 : 5}$ .

Note that the incentre lies on the x-axis. Therefore, the x-coordinate of it is  $\frac{3 \cdot 4 + 5 \cdot 0}{3+5} = \frac{3}{2}$  and the coordinates of it is  $(\frac{\boxed{3}}{\boxed{2}}, \boxed{0})$

Q3:

(1): As the line is tangent to the parabolas, the equations:

$$x^2 - 5x + 7 = x + k, \quad x^2 + 3x - 1 = x + k$$

have only one solution, i.e.

$$\Delta_1 = 36 - 4(7 - k) = 0 \text{ and } 4 - 4(-1 - k) = 0$$

$$k = \boxed{-2}$$

(2): Solving the equations

$$x^2 - 5x + 7 = x - 2, \quad x^2 + 3x - 1 = x - 2$$

$$x_P = \boxed{3} \text{ and } x_Q = \boxed{-1}$$

Moreover, find the intersection of the two parabolas

$$x^2 - 5x + 7 = x^2 + 3x - 1$$

$$x_R = \boxed{1}$$

$$\begin{aligned} (3): \text{ The area} &= \int_{-1}^1 ((x^2 + 3x - 1) - (x - 2))dx + \int_1^3 ((x^2 - 5x + 7) - (x - 2))dx \\ &= \left[ \frac{1}{3}x^3 + x^2 + x \right]_{-1}^1 + \left[ \frac{1}{3}x^3 - 3x^2 + 9x \right]_1^3 \\ &= \frac{1}{3} + 1 + 1 + \frac{1}{3} - 1 + 1 + 9 - 27 + 27 - \frac{1}{3} + 3 - 9 \\ &= \boxed{\frac{16}{3}} \end{aligned}$$