Q1(1):

$k = \frac{\log_7 9}{\log_7 4} = \frac{2\log_7 3}{2\log_7 2} = \log_2 3$.

Then, $2^{5k} = 2^{5\log_2 3} = 2^{\log_2 3^5} = 3^5 = \boxed{243}$.

Q1(2):

$$\frac{2^x - 2^{-x}}{2^x + 2^{-x}} = \frac{1}{3}$$

$$3 \cdot 2^x - 3 \cdot 2^{-x} = 2^x + 2^{-x}$$

$$2^x = 2 \cdot 2^{-x}$$

$$(2^x)^2 = 2$$

$$2^x = 2^{\frac{1}{2}}$$

$$x = \boxed{\frac{1}{2}}$$

Q1(3):

$$f(g(x)) = g(f(x))$$

$$-p(5x+1) + 2 = 5(-px+2) + 1$$

$$-p = 9$$

$$p = \boxed{-9}$$

Alternative Put $x = 0$,

$$f(g(0)) = g(f(0))$$

$$-p(1) + 2 = 5(2) + 1$$

$$p = \boxed{-9}$$

Q1(4):

$$\cos 2x + 3\cos x - 1 = 0$$

$$2\cos^2 x - 1 + 3\cos x - 1 = 0$$

$$2\cos^2 x + 3\cos x - 2 = 0$$

$$(2\cos x - 1)(\cos x + 2) = 0$$

$$\cos x = \frac{1}{2} \text{ or } \cos x = -2 \text{(rejected)}$$

$$x = \boxed{\frac{\pi}{3}} \text{ as } 0 \leq x \leq \pi$$

Q1(5): By factorising p, we have $p = (n - 11)(n - 7)$.

If p is a prime, p has only one prime factor, i.e. $n - 11 = 1$ or $n - 7 = 1$.

As $p > 0$, we have $n - 11 = 1$, or $n = 12$. Then, $p = 12 - 7 = \boxed{5}$.

Q2:

(1): The slope of tangent to C is given by $y' = -e^{-x}$.

Therefore, the slope of the tangent to C at the point $P = -e^{-a}$.

By using the point-slope form of straight line, we have the equation of the tangent is

$$y - e^{-a} = -e^{-a}(x - a)$$

$$e^{-a}x + y - (1 + a)e^{-a} = 0$$

The, the x-intercept and y-intercept of it are $-\frac{-(1+a)e^{-a}}{e^{-a}} = 1 + a$ and $-\frac{-(1+a)e^{-a}}{1} = (1 + a)e^{-a}$ respectively.

As the triangle is a right-angled triangle, the area $S(a) = \boxed{\dfrac{1}{2}(1 + a)^2 e^{-a}}$.

(2): $S'(a) = (1 + a)e^{-a} - \frac{1}{2}(1 + a)^2 e^{-a} = \frac{1}{2}(1 + a)(1 - a)e^{-a}$.

To find the extremum of $S(a)$, we set $S'(a) = 0$, then $a = 1$ or $a = -1$(rejected as $a > -1$).

Table of first derivative test is given:

a	$(-1, 1)$	$(1, +\infty)$
$S'(a)$	$+$	$-$
$S(a)$	$\nearrow$	$\searrow$

Therefore, $S(a)$ attains to its maximum when $a = 1$ and the maximum value$=$ $S(1) = \boxed{\dfrac{2}{e}}$.

Q3:

(1): Define a sequence of ordered pairs with general term $S_i = (\frac{1}{i^2}, \frac{3}{i^2}, ..., \frac{2i-1}{i^2})$.

Note that the number of objects of $S_i = i$. Therefore, the total number of objects from S_1 to S_i will be $\frac{i(i+1)}{2}$.

Let m be the greatest integer satisfying $\frac{m(m+1)}{2} < n$, then a_n will be the $(n - \frac{m(m+1)}{2})$th object of S_{m+1}.

Put $n = 50$, then by the inequality $\frac{m(m+1)}{2} < 50$, we have $m = 9$. Therefore, a_{50} is the 5th object of S_{10}. i.e. $a_{50} = \frac{2(5)-1}{10^2} = \boxed{\dfrac{9}{100}}$.

(2): Note that the sum of all objects in S_i is equal to $\frac{\sum\limits_{k=1}^{i}(2k-1)}{i^2} = \frac{i^2}{i^2} = 1$.

As there are 9 pairs prior to S_{10}, we have

$\sum\limits_{n=1}^{50} a_n = 9 + (\frac{1}{100} + \frac{3}{100} + ... + \frac{9}{100})$

$= 9 + \frac{25}{100}$

$= \boxed{\dfrac{37}{4}}$.

(3): As the last object, $\frac{2i-1}{i^2}$ is the largest object in S_i, we are to find the maximal i satisfying $\frac{2i-1}{i^2} \geq \frac{1}{10}$, i.e. $i^2 - 20i + 10 \leq 0$ first.

Therefore, $i \leq \frac{20+\sqrt{(-20)^2-4(1)(10)}}{2} < 10 + 10 = 20$. Hence, we take $i = 19$.

Then, a_n will be the last, i.e. the 19th object in S_{19}.

Therefore, $n = \frac{18(18+1)}{2} + 19 = \boxed{190}$.

(Note: The introduction of S_i is to clarify the situation. One can introduce other terms like a sequence or a set (but need to be cautious that a set do **NOT** preserve an order), or define a new term such as a “group” or even an “apple” so as to make the pattern of the given sequence a_n clearer. However, it is not necessary as what MEXT’s official solution did is giving the above results by observation.)