Q1(1):

$x + y = \frac{(3+\sqrt{3})^2+(3-\sqrt{3})^2}{3^2-\sqrt{3}^2} = 4$ and $x - y = \frac{(3+\sqrt{3})^2+(3-\sqrt{3})^2}{3^2-\sqrt{3}^2} = 2\sqrt{3}$.

$x^2 - y^2 = (x + y)(x - y) = \boxed{8\sqrt{3}}$

Q1(2):

By testing the potential rational roots given by the rational root theorem $\pm 1, \pm 2, \pm 4, \pm 8$, we have $x = -1$ is a root.

Therefore, we can do the factorisation by the long division:

$$x^3 - x^2 - 10x - 8 = 0$$

$$(x + 1)(x^2 - 2x - 8) = 0$$

$$(x + 1)(x + 2)(x - 4) = 0$$

$$x = \boxed{-1, -2, 4}$$

Q1(3):

$$2\sin^2 x - \cos x = 1$$

$$2 - 2\cos^2 x - \cos x = 1$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$\cos x = \frac{1}{2}, -1$$

$$x = \boxed{\frac{\pi}{3}, \pi, \frac{5\pi}{3}}$$

Q1(4):

$$2^{2x+2} + 3 \cdot 2^x - 1 = 0$$

$$4 \cdot 2^{2x} + 3 \cdot 2^x - 1 = 0$$

$$(4 \cdot 2^x - 1)(2^x + 1) = 0$$

$$2^x = \frac{1}{4}$$

$$x = \boxed{-2}$$

Q1(5):

$$(\log_3 x)^2 < \log_9 x^4$$

$$(\log_3 x)^2 - 2\log_3 x < 0$$

$$0 < \log_3 x < 2$$

$$\boxed{1 < x < 9}$$

Q1(6):

$$\sin 2x > \sqrt{2}\sin x$$

$$2\sin x\cos x > \sqrt{2}\sin x$$

$$\begin{cases} \cos x > \frac{\sqrt{2}}{2} \\ \sin x > 0 \end{cases} \quad \text{or} \quad \begin{cases} \cos x < \frac{\sqrt{2}}{2} \\ \sin x < 0 \end{cases}$$

$$\boxed{0 < x < \frac{\pi}{4} \text{ or } \pi < x < \frac{7\pi}{4}}$$

Q1(7):

$\vec{a} + t\vec{b} =< 1 + 3t, 2 + 2t, 3 + t >$.

$(\vec{a} + t\vec{b})//\vec{c}$ if and only if $(\vec{a} + t\vec{b}) = k\vec{c}$, i.e. $\frac{1+3t}{5} = \frac{2+2t}{4} = \frac{3+t}{3} = k$.

Therefore, $t = \boxed{3}$.

Q1(8):

Note that $\triangle OAA' = 2\triangle OAA_\perp$, where $AA_\perp$ is the perpendicular foot of A on the line $y = 2x$.

We have $AA_\perp = \frac{|2\cdot 3 - 1\cdot 1 + 0|}{\sqrt{2^2+1^2}} = \sqrt{5}$ and

$OA_\perp = \sqrt{OA^2 - AA_\perp^2} = \sqrt{3^2 + 1^2 - 5} = \sqrt{5}$.

Therefore, $\triangle OAA' = 2 \cdot \frac{1}{2}(\sqrt{5})(\sqrt{5}) = \boxed{5}$.

Q1(9):

$\{a_n\}$ is a geometric sequence with the first term 3 and the common ratio 2.

Therefore, $\sum_{n=1}^{5}(a_n-5)=\frac{3(2^5-1)}{2-1}-5\cdot 5=\boxed{68}$.

Q1(10):

$\lim_{x\to 0}(\sqrt{x^2+4x+5}-\sqrt{x^2+x})$

$=\lim_{x\to 0}\frac{3x+5}{\sqrt{x^2+4x+5}+\sqrt{x^2+x}}$

$=\lim_{x\to 0}\frac{3+\frac{5}{x}}{\sqrt{1+\frac{4}{x}+\frac{5}{x^2}}+\sqrt{1+\frac{1}{x}}}$

$=\boxed{\frac{3}{2}}$

Q1(11):

$$\ln f(x)=\ln\cos x-\frac{x}{2}$$
$$f'(x)=f(x)(-\frac{\sin x}{\cos x}-\frac{1}{2})$$
$$f'(0)=f(0)(-\frac{1}{2})=\boxed{-\frac{1}{2}}$$

Q1(12):

$\int_1^2(3x^2-4x)\ln x dx$

$=\int_1^2\ln x d(x^3-2x^2)$

$= (x^3 - 2x^2)\ln x|_1^2 - \int_1^2 (x^2 - 2x)dx$

$= -[\frac{1}{3}x^3 - x^2]_1^2$

$= \boxed{\dfrac{2}{3}}$

Q2:

1): $AB = \begin{bmatrix} x+9 & 21 \\ 3x+15 & 39 \end{bmatrix}$ and $BA = \begin{bmatrix} x+9 & 3x+15 \\ 21 & 39 \end{bmatrix}$.

For $AB = BA$, we have $x = \boxed{2}$.

2): $BA = \begin{bmatrix} 2x-2 & 4x-4 \\ 4+2y & 8+4y \end{bmatrix}$.

For $BA = O$, we have $x = \boxed{1}$ and $y = \boxed{-2}$.

3): $A^2 = A - I \iff A^3 = A^2 - A = A - I - A = -I \iff A^{3n} = (-1)^n I$.

Therefore, $A^{15} = (-1)^5 I = \boxed{-I}$

Q3:

1): $\int_0^{\frac{\pi}{4}} \cos^2 x dx$

$= \int_0^{\frac{\pi}{4}} \frac{1+\cos 2x}{2} dx$

$= \frac{1}{2}[x + \frac{1}{2}\sin 2x]_0^{\frac{\pi}{4}}$

$= \boxed{\dfrac{\pi}{8} + \dfrac{1}{4}}$

2): $\int_0^{\frac{\pi}{4}} \cos^3 x dx$

$= \int_0^{\frac{\pi}{4}} (1 - \sin^2 x) d(\sin x)$

$= [\sin x - \frac{1}{3} \sin^3 x]_0^{\frac{\pi}{4}}$

$= \boxed{\dfrac{5\sqrt{2}}{12}}.$

3): $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin x + 2\cos x)^2 dx$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin^3 x + 6\sin^2 x \cos x + 12 \sin x \cos^2 x + 8\cos^3 x) dx$

By the properties of even functions and odd functions, it is equal to

$2\int_0^{\frac{\pi}{4}} (6\sin^2 x \cos x + 8\cos^3 x) dx$

$= 2\int_0^{\frac{\pi}{4}} (6\cos x + 2\cos^3 x) dx$

$= 12[\sin x]_0^{\frac{\pi}{4}} + \frac{5\sqrt{2}}{3}$

$= \boxed{\dfrac{23\sqrt{2}}{3}}$