Q1(1):

Testing the potential rational roots given by the rational root theorem $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$,

we have $x = 1$ is a root.

Then, we can do the factorisation by the long division:

$$2x^3 + 3x^2 - 8x + 3 = 0$$

$$(x-1)(2x^3 + 5x - 3) = 0$$

$$(x-1)(2x-1)(x+3) = 0$$

$$x = \boxed{-3, 1, \frac{1}{2}}$$

Q1(2):

$$4\sin x\cos x - 1 = 0$$

$$2\sin 2x = 1$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \boxed{\frac{\pi}{12}, \frac{5\pi}{12}}$$

Q1(3):

$$4^x - 2^{x+1} > 48$$

$$2^{2x} - 2 \cdot 2^x - 48 > 0$$

$$(2^x - 8)(2^x + 6) > 0$$

$$2^x > 8$$

$$\boxed{x > 3}$$

Q1(4):

$$\log_4(2 - x) > \log_2 x$$

$$\log_4(2 - x) > \log_4 x^2$$

$$x^2 + x - 2 < 0$$

$$(x + 2)(x - 1) < 0$$

$$-2 < x < 1$$

On the other hand, the condition for $\log_4(2 - x)$ and $\log_2 x$ to be defined is $0 < x < 2$.

Finding the intersection, the solution is $\boxed{0 < x < 1}$.

Q1(5):

$\vec{a} + t\vec{b} =< 2 + t, 5 - t >$.

$|\vec{a} + t\vec{b}| = \sqrt{(2 + t)^2 + (5 - t)^2}$

$= \sqrt{2t^2 - 6t + 29}$

$= \sqrt{2(t - \frac{3}{2})^2 + \frac{49}{2}}$.

Therefore, $|\vec{a} + t\vec{b}|$ is minimised when $t = \boxed{\frac{3}{2}}$.

Q1(6):

The angle between the line $x - 2y = 3$ and the positive x-axis is $\arctan(\frac{1}{2})$ and that between the line $3x - y = 2$ and the positive x-axis is $\arctan(3)$.

Therefore, $\tan\theta = \tan(\arctan(3) - \arctan(\frac{1}{2}))$

$= \frac{3-\frac{1}{2}}{1+(3)(\frac{1}{2})}$

$= 1$.

And hence, $\theta = \boxed{\frac{\pi}{4}}$.

Alternative A vector in the direction of the line $x - 2y = 3$ is $< 3, \frac{3}{2} >$ and a vector in the direction of the line $3x - y = 2$ is $< \frac{2}{3}, 2 >$.

$\cos\theta = \frac{<3,\frac{3}{2}>\cdot<\frac{2}{3},2>}{|<3,\frac{3}{2}>||<\frac{2}{3},2>|} = \frac{1}{\sqrt{2}}$.

Therefore, $\theta = \boxed{\frac{\pi}{4}}$.

Q1(7):

By partial fraction, $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$.

Therefore, by the telescoping property, $\sum\limits_{k=1}^{100} \frac{1}{k(k+1)} = 1 - \frac{1}{101} = \boxed{\frac{100}{101}}$.

Q1(8):

$\lim\limits_{x\to\infty} \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{3x+5}-\sqrt{3x+1}}$

$= \lim\limits_{x\to\infty} \frac{\sqrt{3x+5}+\sqrt{3x+1}}{4(\sqrt{x+1}+\sqrt{x})}$

$= \lim\limits_{x\to\infty} \frac{\sqrt{3+\frac{5}{x}}+\sqrt{3+\frac{1}{x}}}{4(\sqrt{1+\frac{1}{x}}+\sqrt{1})}$

$= \frac{\sqrt{3}+\sqrt{3}}{4(1+1)}$

$= \boxed{\dfrac{\sqrt{3}}{4}}$

Q1(9):

$\lim\limits_{x\to 0} \frac{\log(1+3x)}{x}$

$= \lim\limits_{y\to 0} \frac{\log(1+3(\frac{e^y-1}{3}))}{\frac{e^y-1}{3}}$

$= 3\lim\limits_{y\to 0} \frac{y}{e^y-1}$

$= \boxed{3}$

Q1(10):

The probability of getting a 5=$\frac{1}{6}$ and the probability of not getting a 5=$\frac{5}{6}$.

For the binomial trial, the probability=$C^4_2(\frac{1}{6})^2(\frac{5}{6})^2 = \boxed{\dfrac{25}{216}}$.

Q1(11):

$y = \log\sqrt{\frac{1+\sin x}{1-\sin x}} = \frac{1}{2}(\log(1+\sin x) - \log(1-\sin x))$

$\frac{dy}{dx} = \frac{1}{2}(\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x}) = \frac{\cos x}{\cos^2 x} = \boxed{\frac{1}{\cos x}}$

Q1(12):

$\int_0^{\frac{\pi}{6}} \cos^3 x dx$

$= \int_0^{\frac{\pi}{6}} (1-\sin^2 x) d(\sin x)$

$= [\sin x - \frac{1}{3}\sin^3 x]_0^{\frac{\pi}{6}}$

$= \frac{1}{2} - \frac{1}{24}$

$= \boxed{\frac{11}{24}}$

Q2:

1): The characteristic polynomial of A is $|A - \lambda I| = \lambda^2 - 5\lambda - 2$.

By Cayley-Hamilton theorem, $A^2 - 5A - 2I = 0$, i.e. $A^2 - 5A = \boxed{2I}$.

Note: One can also calculate it directly.

2): $A^3 - 5A^2 + A + I$

$= (A^2 - 5A)A + A + I$

$= 3A + I$

$= \boxed{\begin{bmatrix} 4 & 9 \\ 6 & 13 \end{bmatrix}}$

3): $A^4 - 3A^3 - 10A^2 + A + I$

$= (A^3 - 5A^2 + A + I)A + 2A^3 - 11A^2 + I$

$= 2A^3 - 8A^2 + A + I$

$= 2(A^3 - 5A^2 + A + I) + 2A^2 - A - I$

$= 2A^2 + 5A + I$

$= 2(A^2 - 5A) + 15A + I$

$= 15A + 5I$

$= \boxed{\begin{bmatrix} 20 & 45 \\ 30 & 65 \end{bmatrix}}$

Alternative Refer to MEXT's official answer key, which regarded the expression as a polynomial and did the long division on it.

Q3:

1): $\frac{dy}{dx} = -2x$ and $\frac{dy}{dx}|_{x=a} = -2a$.

By the point-slope form of straight line, l is $y - (4 - a^2) = -2a(x - a)$, i.e. $y = \boxed{-2ax + a^2 + 4}$.

2): The x-intercept and y-intercept of l are $\frac{a^2+4}{2a}$ and $a^2 + 4$.

Therefore, $S(a) = \frac{1}{2}(a^2 + 4)(\frac{a^2+4}{2a}) = \boxed{\frac{(a^2 + 4)^2}{4a}}$.

3): $S'(a) = \frac{16a^2(a^2+4) - 4(a^2+4)^2}{16a^2}$.

To find the extremum of $S(a)$, we set $S'(a) = 0$, we have $a = \frac{2\sqrt{3}}{3}$ for $a > 0$.

The table of signs is given:

a	$(0, \frac{2\sqrt{3}}{3})$	$(\frac{2\sqrt{3}}{3}, +\infty)$
$S'(a)$	$-$	$+$
$S(a)$	$\searrow$	$\nearrow$

Therefore, $S(a)$ attains to its minimum when $a = \frac{2\sqrt{3}}{3}$ and the value is $S(\frac{2\sqrt{3}}{3}) = \boxed{\dfrac{32\sqrt{3}}{9}}$.