

Q1(1):

$$8x^2 - 10x + 3 = 0$$

$$(4x - 3)(2x - 1) = 0$$

$$x = \boxed{\frac{1}{2}, \frac{3}{4}}$$

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Q1(2):

$$\begin{cases} 2x^2 - 5x - 3 < 0 \\ 3x^2 - 4 - 11x \geq 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} (2x + 1)(x - 3) < 0 \\ (3x + 1)(x - 4) \geq 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} -\frac{1}{2} < x < 3 \\ x \leq -\frac{1}{3} \text{ or } x \geq 4 \end{cases}$$

$$\Leftrightarrow \boxed{-\frac{1}{2} < x \leq -\frac{1}{3}}$$

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Q1(3):

$$2 \cos^2 x + 3 \sin x - 3 = 0$$

$$2 - 2 \sin^2 x + 3 \sin x - 3 = 0$$

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

$$(2 \sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{2}, 1$$

$$x = \boxed{30^\circ, 90^\circ, 150^\circ}$$

Q1(4):

$$\log_3(2x - 1) + \log_3(x - 1) < 1$$

$$\log_3(2x - 1)(x - 1) < 1$$

$$2x^2 - 3x + 1 < 3$$

$$(x - 2)(2x + 1) < 0$$

$$-\frac{1}{2} < x < 2$$

Note the hidden condition for  $\log_3(2x - 1)$  and  $\log_3(x - 1)$  to be defined is  $x > 1$ .

Finding the intersection, the solution is  $\boxed{1 < x < 2}$ .

Q1(5):

As  $(\vec{a} - \vec{b}) \perp (6\vec{a} + \vec{b})$ , we have

$$(\vec{a} - \vec{b}) \cdot (6\vec{a} + \vec{b}) = 0$$

$$6|\vec{a}|^2 - 5\vec{a} \cdot \vec{b} - |\vec{b}|^2 = 0$$

$$\vec{a} \cdot \vec{b} = 3$$

$$|\vec{a}||\vec{b}| \cos \theta = 3$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \boxed{60^\circ}$$

Q1(6):

Sum of roots= $a + 3a = -\frac{8}{3}$ , i.e.  $a = -\frac{2}{3}$ .

Product of roots= $a(3a) = \frac{4}{3} = \frac{k}{3}$ , i.e.  $k = \boxed{4}$ .

Q1(7):

As  $\frac{1}{2-\sqrt{3}} = 2 + \sqrt{3}$ , we have  $a = 3$  and  $b = \sqrt{3} - 1$ .

Therefore,  $a - b + \frac{2}{b} = 3 - \sqrt{3} + 1 + \frac{2}{\sqrt{3}-1} = \boxed{5}$ .

Q1(8):

We have  $\log_{10} A = a = -b$ , i.e.  $A = 10^{-b}$ , i.e.  $A^{\frac{1}{b}} = 10^{-1}$ .

Similarly,  $B^{\frac{1}{a}} = 10^{-1}$  and  $A^{\frac{1}{b}} B^{\frac{1}{a}} = \boxed{10^{-2}}$ .

Q1(9):

Taking the square of both sides, we have  $\sin^2 \alpha + \cos^2 \beta + 2 \sin \alpha \cos \beta = \frac{1}{4}$  and  $\cos^2 \alpha + \sin^2 \beta + 2 \cos \alpha \sin \beta = \frac{3}{4}$ .

Adding the two equations together, we have

$$2 + 2 \sin(\alpha + \beta) = 1$$

$$\sin(\alpha + \beta) = -\frac{1}{2}$$

$$\alpha + \beta = \boxed{\frac{7\pi}{6}} \text{ (as } \frac{\pi}{2} < \alpha + \beta < \frac{3\pi}{2} \text{)}$$

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Q1(10):

$$\alpha + \alpha^{-1} = (\alpha^{\frac{1}{2}} + \alpha^{-\frac{1}{2}})^2 - 2 = \boxed{3}.$$

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Q1(11):

$$\sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n = \frac{\frac{9}{10}}{1 - \frac{9}{10}} = \boxed{9}.$$

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Q1(12):

$$\begin{aligned} & \lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4} \\ &= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x}+2} \\ &= \frac{1}{2+2} \\ &= \boxed{\frac{1}{4}} \end{aligned}$$

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Q1(13):

$$\frac{dy}{dx} = \sqrt{x} + \frac{1}{2\sqrt{x}}(x+2) = \boxed{\frac{3}{2}\sqrt{x} + \frac{1}{x}}$$

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Q1(14):

$$\begin{aligned} & \int_0^2 (x-1)^3 dx + 2 \int_{-1}^2 x(x-1) dx \\ &= \left[ \frac{1}{4}(x-1)^4 \right]_0^2 + 2 \left[ \frac{1}{3}x^3 - \frac{1}{2}x^2 \right]_{-1}^2 \\ &= \frac{1}{4} - \frac{1}{4} + 2 \left( \frac{8}{3} - 2 + \frac{1}{3} + \frac{1}{2} \right) \\ &= \boxed{3} \end{aligned}$$

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Q2:

$$\begin{aligned} & 1): A^2 + pA + qE \\ &= \begin{bmatrix} 14 & -10 \\ 5 & -1 \end{bmatrix} + \begin{bmatrix} 4p & -2p \\ p & p \end{bmatrix} + \begin{bmatrix} q & 0 \\ 0 & q \end{bmatrix} \\ &= \begin{bmatrix} 14 + 4p + q & -10 - 2p \\ 5 + p & -1 + p + q \end{bmatrix} \end{aligned}$$

Solving  $14 + 4p + q = 0$  and  $5 + p = 0$ , we have  $p = \boxed{-5}$  and  $q = \boxed{6}$ .

**Check:** When  $p = -5$  and  $q = 6$ ,  $-1 + p + q = -1 - 5 + 6 = 0$ .

2): As  $x^2 - 5x + 6 = (x-2)(x-3)$ , the two roots are 2 and 3.

Therefore, we have  $2^n = 2a + b$  and  $3^n = 3a + b$ .

Solving, we have  $a = \boxed{3^n - 2^n}$  and  $b = \boxed{3 \cdot 2^n - 2 \cdot 3^n}$ .

3): As  $A^2 - 5A + 6E = 0$ , we have

$$A^n = (3^n - 2^n)A + (3 \cdot 2^n - 2 \cdot 3^n)E = \begin{bmatrix} 2 \cdot 3^n - 2^n & 2^{n+1} - 2 \cdot 3^n \\ 3^n - 2^n & 2^{n+1} - 3^n \end{bmatrix}.$$


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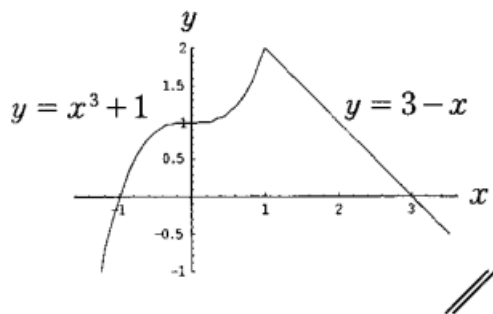
Q3:

1): Solving  $x^3 + 1 = 3 - x$  by testing the potential rational roots given by the rational root theorem  $\pm 1$ , we have  $x = 1$  is a root.

Do the factorisation by the long division,  $(x - 1)(x^2 + x + 2) = 0$ , i.e.  $x = 1$  or  $x^2 + x + 2 = 0$ . For the latter, as  $\Delta = -7 < 0$ , it has no real roots.

Therefore,  $\min\{x^3 + 1, 3 - x\} = \begin{cases} x^3 - 1, & x \leq 1 \\ 3 - x, & x \geq 1 \end{cases}$  and the graph can be

sketched:



2):  $F'(x) = f(x)$  by the fundamental theorem of calculus.

To find the extrema, we set  $F'(x) = 0$ . Then  $x = -1$  and  $x = 3$ .

$$\text{As } F''(x) = \begin{cases} 3x^2, & x \leq 1 \\ -1, & x \geq 1 \end{cases}, \text{ we have } F''(-1) > 0 \text{ and } F''(3) < 0.$$

Therefore, the maximal value of  $F(x)$  is  $F(3) = \int_1^3 (3-x)dx = [3x - \frac{1}{2}x^2]_1^3 = \boxed{2}$

and the minimum value of  $F(x)$  is  $F(-1) = \int_1^{-1} (x^3+1)dx = [\frac{1}{4}x^4+x]_1^{-1} = \boxed{-2}$ .