Q1(1):

Slope of the line $3x - 4y - 12 = 0$ is $-\frac{3}{-4} = \frac{3}{4}$.

For the line orthogonal to it, the solpe=$\frac{-1}{\frac{3}{4}} = -\frac{4}{3}$.

By using the point-slope form of straight line, we have the equation of it is

$$y - 2 = -\frac{4}{3}(x - (-3))$$

$$\frac{4}{3}x + y + 2 = 0$$

$$\boxed{\frac{2}{3}} + \boxed{\frac{1}{2}}y + 1 = 0$$

Q1(2):

$x^3 + x + 1 = (\frac{\sqrt{5}-1}{2})^3 + \frac{\sqrt{5}-1}{2} + 1$

$= \frac{5\sqrt{5}-3\cdot5+3\sqrt{5}-1}{8} + \frac{\sqrt{5}+1}{2}$

$= \frac{2\sqrt{5}-4}{2} + \frac{\sqrt{5}+1}{2}$

$= \boxed{\dfrac{3(\sqrt{5}-1)}{2}}$

Alternative If $x = \frac{\sqrt{5}-1}{2}$, then $x^2 + x - 1 = 0$*.

Therefore, $x^3 = -x^2 + x = -(-x+1) + x = 2x - 1$.

Then, $x^3 + x + 1 = (2x-1) + x + 1 = 3x = \boxed{\dfrac{3(\sqrt{5}-1)}{2}}$.

(*: If $\frac{\sqrt{5}-1}{2}$ is a root of a quadratic equation, then $\frac{-\sqrt{5}-1}{2}$ will be another root. Then, we have the sum of roots is -1 and the product of roots is -1. Hence the quadratic equation can be constructed.)

Q1(3):

$$\frac{2^x - 2^{-x}}{2^x + 2^{-x}} = \frac{1}{3}$$

$$3 \cdot 2^x - 3 \cdot 2^{-x} = 2^x + 2^{-x}$$

$$2^x = 2 \cdot 2^{-x}$$

$$(2^x)^2 = 2$$

$$2^x = 2^{\frac{1}{2}}$$

$$x = \boxed{\frac{1}{2}}$$

Q1(4):

If $\sin x + \cos x = \frac{4}{3}$,

$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + 2 \sin x \cos x = (\frac{4}{3})^2 = \frac{16}{9}$.

Therefore, $2 \sin x \cos x = \frac{16}{9} - 1 = \frac{7}{9}$.

For $0 \leq x \leq \frac{\pi}{4}$, $\sin x \leq \cos x$.

Then, $\sin x - \cos x = -\sqrt{(\cos x - \sin x)^2}$

$= -\sqrt{\sin^2 x + \cos^2 x - 2 \sin x \cos x}$

$= -\sqrt{1 - \frac{7}{9}}$

$= -\sqrt{\frac{2}{9}}$

$= \boxed{-\frac{\sqrt{2}}{3}}$.

Q1(5):

$$f(g(x)) = g(f(x))$$

$$-p(5x+1)+2 = 5(-px+2)+1$$

$$-p = 9$$

$$p = \boxed{-9}$$

Alternative Put $x = 0$,

$$f(g(0)) = g(f(0))$$

$$-p(1)+2 = 5(2)+1$$

$$p = \boxed{-9}$$

Q2:

By the angle bisector theorem, $BD : DC = BA : CA = 4 : 3$. Therefore, $BD = \frac{4}{4+3}BC = \frac{20}{7}$.

On the other hand, note the $\triangle ABC$ is a right angle triangle with $\angle A = 90°$. By considering the cosine ratio of $\triangle ABC$, we have $\cos\angle B = \frac{AB}{BC} = \frac{4}{5}$ By the cosine formula,

$$AD^2 = AB^2 + BD^2 - 2(AB)(BD)\cos\angle B$$

$$AD = \sqrt{4^2 + (\frac{20}{7})^2 - 2(4)(\frac{20}{7})(\frac{4}{5})} = \sqrt{\frac{288}{49}} = \boxed{\frac{12\sqrt{2}}{7}}.$$

Alternative By using the sine formula instead, note that $\angle DAB = 45°$ and $\sin\angle B = \frac{3}{5}$.

$$\frac{AD}{\sin\angle B} = \frac{BD}{\sin\angle DAB}$$

$$AD = \frac{\frac{20}{7}}{\sin 45°}(\frac{3}{5}) = \boxed{\frac{12\sqrt{2}}{5}}.$$

Alternative If one does not know the angle bisector theorem, the sine formula can still be used without BD found explicitly:

By using the sine formula, we have

$$\begin{cases} \frac{AD}{\sin\angle B} = \frac{BD}{\sin\angle DAB} \\ \frac{AD}{\sin\angle C} = \frac{5-BD}{\sin\angle DAC} \end{cases} \iff AD = \frac{3\sqrt{2}}{5}BD = \frac{4\sqrt{2}}{5}(5-BD)$$

Then, we have $BD = \frac{20}{7}$ and $AD = \boxed{\frac{12\sqrt{2}}{7}}$.

Alternative See MEXT's official solution, which considered the area of each triangle.

Alternative (coordinates) Set $A(0,0)$ be the origin and set $B = (-4,0)$ and $C = (0,3)$. As by the angle bisector theorem, $BD : DC = BA : CA = 4 : 3$, we have the coordinates of $D = (\frac{-4\cdot 3}{4+3}, \frac{3\cdot 4}{4+3}) = (-\frac{12}{7}, \frac{12}{7})$. Then, $AD = \sqrt{(-\frac{12}{7})^2 + (\frac{12}{7})^2} = \boxed{\frac{12\sqrt{2}}{7}}$.

Alternative (coordinates) With the setting given above, as $\angle DAB = 45°$, we have the slope of $DA = -1$. i.e. the equation of DA is $y = -x$.

On the other hand, using the intercept form of straight line, the equation of BC is $-\frac{x}{4} + \frac{y}{3} = 1$.

Solving the stimultaneous equation of them, we have $D = (x, y) = (-\frac{12}{7}, \frac{12}{7})$.

Then, $AD = \sqrt{(-\frac{12}{7})^2 + (\frac{12}{7})^2} = \boxed{\dfrac{12\sqrt{2}}{7}}$.

Q3:

(1): The equation can be rewritten as $x = n - 2y$. Then, for every positive integer y, a corresponding integer x can be found.

As $x \geq 1$, we have $n - 2y \geq 1$, i.e. $y \leq \frac{n-1}{2}$ ($\frac{n-1}{2}$ is an integer as n is an odd number, hence the equality can hold). Therefore, there are totally $\frac{n-1}{2}$ pairs:

x	y
$n-2$	1
$n-4$	2
$\vdots$	$\vdots$
1	$\frac{n-1}{2}$

(2): Note that $x + y \leq 2$. Then, by the result above, we have the pairs $(x + y, z)$ as the following:

$x+y$	z
$n-2$	1
$n-4$	2
$\vdots$	$\vdots$
3	$\frac{n-3}{2}$

As for the equation $x+y=k$, we can find $k-1$ pairs of (x,y):

$(x,y)=(1,k-1),(2,k-2),...,(k-1,1)$,

the number of pairs of (x,y,z)

=the sum of number of pairs of (x,y) for each value of z

$=((n-2)-1)+((n-4)-1)+...+((n-(n-3))-1)$

$= \underbrace{(n-1)+(n-1)+...+(n-1)}_{(\frac{n-3}{2})\text{ terms}} -(2+4+...+2(\frac{n-3}{2}))$

$=(n-1)(\frac{n-3}{2})-\frac{n-3}{2}\cdot(\frac{n-3}{2}+1)$

$=\boxed{\frac{(n-1)(n-3)}{4}}$.