Q1(1):

By AM-GM inequality, we have $x + 3y \geq 2\sqrt{3xy}$, i.e.

$$\sqrt{xy} \leq \frac{x+3y}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$xy \leq \frac{1}{3}$$

$$\frac{1}{xy} \geq \boxed{3}$$

The equality holds if and only if $x = 3y$, i.e. $x = \boxed{1}$ and $y = \boxed{\frac{1}{3}}$.

Alternative $\frac{1}{xy} = \frac{1}{(2-3y)y}$.

Consider the derivative $\frac{2-6y}{(2-3y)^2y^2}$. We set it to be 0 to find the extremum of the expression. For $\frac{2}{3} > y > 0$, we have $y = \frac{1}{3}$.

The table of signs is given by:

y	$(0, \frac{1}{3})$	$(\frac{1}{3}, \frac{2}{3})$
$(\frac{1}{xy})'$	$+$	$-$
$\frac{1}{xy}$	$\nearrow$	$\searrow$

Therefore, the minimum of the expression is obtained when $y = \boxed{\frac{1}{3}}$ and $x = 2 - 3(\frac{1}{3})\boxed{1}$.

On the other hand, as the minimum value is 3, we have $\frac{1}{xy} \geq \boxed{3}$.

Q1(2):

$$2^{x+2} - 2^{-x} + 3 = 0$$

$$2^2 \cdot 2^{2x} - 1 + 3 \cdot 2^x = 0$$

$$(4 \cdot 2^x - 1)(2^x + 1) = 0$$

$$2^x = \frac{1}{4} \text{ (as } 2^x > 0 \text{ for all } x \in \mathbb{R})$$

$$x = \boxed{-2}$$

Q1(3):

By the factor theorem, we have $1 + a + a + b - 6 = 0$, i.e. $b = 5 - 2a$.

Therefore, by the long division,

$x^4 + ax^3 + ax^2 + bx - 6 = (x-1)^2(x^2 + (a+2)x + (3a+3)) + (3a+9)x - (3a+9)$.

As the expression is divisible by $(x-1)^2$, we have $3a + 9 = 0$, i.e. $a = \boxed{-3}$ and hence $b = 5 - 2(-3) = \boxed{11}$.

Q1(4):

For $\frac{5}{18}\pi \leq x \leq \frac{2}{3}\pi$, we have $\frac{5}{6}\pi \leq 3x \leq 2\pi$.

For $\frac{5}{6}\pi \leq 3x \leq \pi$, $\sin 3x$ is decreasing with positive value taken.

For $\pi \leq 3x \leq 2\pi$, $\sin 3x$ takes negative value when minimum -1.

Given the abovr, the maximum is $\sin \frac{5}{6}\pi = \boxed{\frac{1}{2}}$ and the minimum is $\boxed{-1}$.

Q1(5):

$z + z^2 + ... + z^5 = \frac{z^6 - z}{z - 1}$.

On the other hand, by De Moivre's theorem, we have $z^6 = \cos\frac{6}{3}\pi + i\sin\frac{6}{3}\pi = 1$.

Therefore, the expression is equal to $\frac{1-z}{z-1} = \boxed{-1}$.

(Note: We usually use the summation of geometric sequence for real valued sequence, so it might sound a bit awkward when using here. However, if one goes into the proof (or derivation) of the formula, he should realise immediate that the formula can be extend to complex domain easily.)

Q1(6):

Note that the two sides with length 1 and the two sides with length $\sqrt{3}$ of T and R must be stuck together respectively.

Moreover, T and S are right-angled triangle.

To satisfy the above conditions, we will have the edge with length 1 perpendicular to the base.

Therefore, the volume of the pyramid will be $\frac{1}{3}(1)^2(1) = \boxed{\frac{1}{3}}$.

Q2:

(1): By symmetry, if a circle is tangent to a parabola with axis of symmetry $x = 0$, its x-coordinate must be 0, i.e. $a = \boxed{0}$.

Moreover, the slope of the tangent to the parabola is given by $y' = 2x$. When $x = t$, the slope is $2t$.

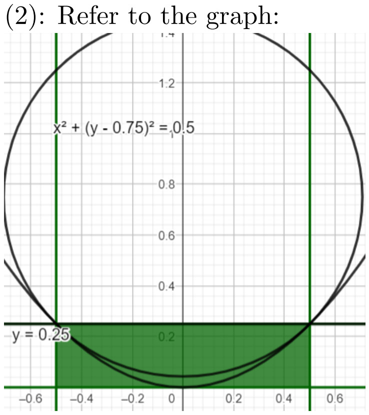
On the other hand, the slope of radius of the circle with respect to (t, t^2) is

$\frac{t^2-b}{t}$.

As the circle is tangent to the parabola, we have $(\frac{t^2-b}{t})(2t) = -1$, i.e. $b = \boxed{t^2 + \frac{1}{2}}$.

Thus, the radius$=\sqrt{(t^2-(t^2+\frac{1}{2}))^2+(t)^2} = \boxed{\sqrt{t^2+\frac{1}{4}}}$.

(2): Refer to the graph:



The required area=(The area of the green region)-(The area bounded by the line $y = \frac{1}{4}$ and the circle)-(The area bounded by the parabola and the x-axis).

First, the area of the green region$=\frac{1}{4}\cdot 1 = \frac{1}{4}$.

Second, the area of bounded by the parabola and the x-axis

$= \int_{\frac{1}{2}}^{\frac{1}{2}} x^2 dx = \frac{2}{3}[x^3]_0^{\frac{1}{2}} = \frac{1}{12}$.

Third, the area bounded by the line $y = \frac{1}{4}$ and the circle=(The area of the corresponding sector)-(The area of the triangle formed by the center and the two intersection points)

Note that the angle between the lines joining the center and the two intersection points is $90°$.

Therefore, the area$=\pi(\frac{1}{\sqrt{2}})^2\cdot(\frac{1}{4}) - \frac{1}{2}\cdot(\frac{1}{\sqrt{2}})^2 = \frac{\pi}{8} - \frac{1}{4}$.

Given the above, the required area$=\frac{1}{4}-\frac{1}{12}-\frac{\pi}{8}+\frac{1}{4}=\boxed{\dfrac{5}{12}-\dfrac{\pi}{8}}$.

Alternative The lower part of the circle is given by $y=-\sqrt{\frac{1}{2}-x^2}+\frac{3}{4}$.

Therefore, the area$=\int_{-\frac{1}{2}}^{\frac{1}{2}}(-\sqrt{\frac{1}{2}-x^2}+\frac{3}{4}-x^2)dx$

$=-\int_0^{\frac{\pi}{4}}\cos^2\theta d\theta+2[\frac{3}{4}x-\frac{1}{3}x^3]_0^{\frac{1}{2}}$

$=-[\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta]_0^{\frac{\pi}{4}}+\frac{2}{3}$

$=\boxed{\dfrac{5}{12}-\dfrac{\pi}{8}}$.

Q3:

(1): By the fundamental theorem of calculus, $f'(x)=\boxed{\dfrac{1}{1+x^2}}$.

Moreover, by chain rule, $g'(x)=f'(\frac{1}{x})(\frac{1}{x})'=(\frac{1}{1+(\frac{1}{x})^2})(-\frac{1}{x^2})=\boxed{-\dfrac{1}{1+x^2}}$.

(2): Note that $f(x)=\arctan x$. Therefore, $f(1)=\arctan 1=\frac{\pi}{4}$.

(3): As $f(1)=g(1)=\frac{\pi}{4}$, if $f(x)+g(x)$ is a constant, the constant will be equal to $f(1)+g(1)=\boxed{\dfrac{\pi}{2}}$.

Alternative $g(x)=\arctan\frac{1}{x}=\frac{\pi}{2}-\arctan x$. Therefore, $f(x)+g(x)=\boxed{\dfrac{\pi}{2}}$.

(4): $\lim\limits_{x\to\infty}f(x)=\lim\limits_{x\to\infty}\arctan x=\boxed{\dfrac{\pi}{2}}$.

Alternative As $\lim\limits_{x\to\infty}g(x)=0$, we have $\lim\limits_{x\to\infty}f(x)=\lim\limits_{x\to\infty}(f(x)+g(x))=\boxed{\dfrac{\pi}{2}}$.

(Fun fact: The question setting allowed us to prove step-by-step those properties of the arctangent function that we have regarded trivial when studying pre-calculus. However, it is a pity that only answers are required in the exam, which allowed candidates to write down the answers simply without realising how amazing the question is.)