

Q1(1):

Let  $a$  be the acceleration of the whole system. Consider each component separately, we have:

$$\begin{cases} F - T = Ma \\ T = ma \end{cases}$$

Therefore,  $T = \frac{m}{M+m}(F - T)$ , i.e.  $T = \boxed{\frac{m}{M+m}F}$ .

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Q1(2):

After the capacitor is fully charged, it has infinite resistance. Therefore the equivalent resistance of  $R_2 - C$  is  $R_2$ , where all the voltage are consumed by the capacitor.

Given the above, the voltage across the capacitor  $= \frac{R_2}{R_1+R_2}E$  and by the definition of capacitance, we have  $Q = CV = \boxed{\frac{R_2}{R_1+R_2}CE}$ .

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Q1(3):

The component of the weight in the direction as the cylinder  $= Mg \cos \theta$ .

The force due to the pressure difference  $= (p_0 - P)S$  (pointing at the opposite direction to the force above).

Consider the balance of forces, we have  $(p_0 - P)S = Mg \cos \theta$ , i.e.

$$P = \boxed{p_0 - \frac{Mg}{S} \cos \theta}.$$

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Q1(4):

Let  $\phi$  be the angle of refraction. By Snells' law, we have  $\sin \theta = n_2 \sin \phi$ .

On the other hand, the critical angle for total internal reflection to occur  $= \arcsin \frac{n_1}{n_2}$ .

Note that the angle of incidence  $= 90^\circ - \phi$ . For total internal reflection to occur,

we have the condition

$$90^\circ - \phi > \arcsin \frac{n_1}{n_2}$$

$$\cos \phi > \frac{n_1}{n_2}$$

$$\sqrt{1 - \sin^2 \phi} > \frac{n_1}{n_2}$$

$$\sqrt{1 - \left(\frac{\sin \theta}{n_2}\right)^2} > \frac{n_1}{n_2}$$

$$n_2^2 - \sin^2 \theta > n_1^2$$

$$\boxed{\sin \theta < \sqrt{n_2^2 - n_1^2}}$$

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Q1(5):

By Einstein's photoelectric equation, we have  $E = hf - \phi = \frac{hc}{\lambda} - \phi$ .

As the photoelectrons emit only when  $\lambda < 5.26 \times 10^{-7} \text{ m}$ , we have

$$\phi = \frac{hc}{5.26 \times 10^{-7}} \approx \boxed{3.78 \times 10^{-19}} \text{ J}.$$

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Q2:

(1): By Faraday's law, the induced EMF  $= \frac{d\Phi}{dt} = BL \frac{dx}{dt} = BLv$ .

Therefore, by Ohm's law, we have  $I = \frac{V}{R} = \boxed{\frac{BvL}{R}}$ .

(2): The weight of the coil =  $Mg$ .

The magnetic force due to the induced current =  $BIL = \frac{B^2vL^2}{R}$ .

As the coil is falling in a constant speed, the net force acting on it is 0. Therefore,

we have

$$Mg = \frac{B^2vL^2}{R}$$

$$v = \boxed{\frac{RMg}{B^2L^2}}$$

(3): The Joule heat produced is due to the magnetic force.

As  $W = Fs$ , we have  $P = Fv = (\frac{B^2vL^2}{R})v = \boxed{\frac{B^2v^2L^2}{R}}$ .

(4): When the coil is doubly turned, the induced EMF on the coil will be doubled. However, as the resistance of the coil is also doubled, the induced current remains unchanged. Hence, the magnetic force remains unchanged.

However, as the weight of the coil becomes  $2Mg$ , we have

$$2Mg = \frac{B^2v'L^2}{R}$$

$$v' = 2\frac{RMg}{B^2L^2} = \boxed{2}v$$

Q3:

(1): Set the GPE as 0 at the height of O. By the conservation of energy:

KE+GPE=KE+GPE, we have

$$0 + mg(2H) = \frac{1}{2}mv^2 + mgH$$

$$v = \boxed{\sqrt{2gH}}$$

(2): Let  $O$  be the origin, then the coordinates of  $B$  is given by  $(2H, H)$ .

Note that the equation of the curve is  $y = \frac{x^2}{4H}$ , we have  $\frac{dy}{dx} = \frac{x}{2H}$ .

At  $B$ , the slope is  $\frac{2H}{2H} = 1$ . Therefore, the slope angle =  $\arctan 1 = \boxed{\frac{\pi}{4}}$ .

(3): The horizontal velocity =  $v \cos \theta = \sqrt{gH}$ .

When the object level attains to its maximum, the vertical velocity becomes 0.

Therefore, by the conservation of energy: KE+GPE=KE+GPE, we have

$$0 + mg(2H) = \frac{1}{2}m(\sqrt{gH})^2 + mgy_{max}$$

$$y_{max} = \frac{3}{2}H$$

Therefore, we have  $\boxed{1.4H \leq y_{max} < 1.55H}$ .

(4): The initial vertical velocity =  $v \sin \theta = \sqrt{gH}$ .

As  $s_y = v_y t - \frac{1}{2}gt^2$ , when the object reaches the ground, we have

$$-H = \sqrt{gH}t - \frac{1}{2}gt^2$$

$$gt^2 - 2\sqrt{gH}t - 2H$$

$$t = \frac{2\sqrt{gH} + \sqrt{4gH + 8gH}}{2g} = \boxed{(1 + \sqrt{3})\sqrt{H/g}}$$

(5):  $s_x = v_x t = \sqrt{gH} \cdot ((1 + \sqrt{3})\sqrt{H/g}) = \boxed{(1 + \sqrt{3})H}$ .

(6): By the conservation of energy, KE+GPE=KE+GPE, we have

$$0 + mg(2H) = \frac{1}{2}mv^2 + 0$$

$$v = \boxed{2\sqrt{gH}}$$

Q4:

(1):  $\frac{14 \cdot 2 \cdot 78\%}{14 \cdot 2 \cdot 78\% + 16 \cdot 2 \cdot 21\% + 40 \cdot 1\%} \times 100\% \approx \boxed{76\%}$ .

Note: Nitrogen and oxygen are diatomic molecules whereas argon is monoatomic.

(2): By (1), the molar mass of air =  $14 \cdot 2 \cdot 78\% + 16 \cdot 2 \cdot 21\% + 40 \cdot 1\% = 28.96$ .

By  $pV = nRT$ , we have  $p = \frac{\rho}{m}RT$ , i.e.  $\rho = \frac{pm}{RT}$ .

Therefore, the density of air =  $\frac{0.1 \times 10^6 \cdot 28.96}{8.31 \cdot 273} \approx 1278 \text{ g/m}^3 \approx \boxed{1.29 \text{ kg/m}^3}$ .

Alternative At  $0^\circ\text{C}$ , 1 atm, 1 mol of gas is equivalent to  $22.4 \text{ dm}^3$ , i.e.  $0.0224 \text{ m}^3$ .

Therefore, the density of air =  $\frac{28.96}{0.0224} \approx 1293 \text{ kg/m}^3 \approx \boxed{1.29 \text{ kg/m}^3}$ .

(3): As air is a mixture of gas,  $c_p - c_v = R$  for air. Therefore,  $\frac{c_p - c_v}{R} = 1$

$\boxed{(b)}$ .

(4): Air is approximately a mixture of 99% diatomic gases and 1% of monoatomic gas. Therefore, we can simply regard air as a diatomic gas.

As  $c_p = \frac{7}{5}c_v$ , we have  $\frac{c_p}{c_v} = \frac{7}{5} \approx \boxed{1.4}$

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Q5:

(1): As the fan is giving a speed in the direction same as the propagation of the sound wave for some angle, Doppler effect occurs and hence the frequency varies. However, the fan is moving in a very slow speed when compared with the speed of sound wave. Hence, the Doppler effect is very weak and the answer will be  $\boxed{(c)}$ .

(2): By the deduction in part (1), the Doppler effect is very weak, which is even negligible. Therefore, the highest frequency will still be approximately 300 Hz  $\boxed{(d)}$ .

Note: If one want to do the calculation, the observed frequency is given by  $\frac{347}{347-r}300 \text{ Hz}$ , where  $r$  is the radius of the fan. Unless the fan is very big (which is nonsense), the frequency will be approximately 300 Hz.

(3): Similar to the above, the frequency is nearly constant with time  $\boxed{(a)}$ .

Note: The question's wording changed from "exactly" to "nearly". However, I

think option (c) should also be accepted.