Q1:

Put $(x, y) = (-2, 41)$ and $(x, y) = (5, 20)$ into the functions respectively, we have equations:

$4A + 2B + C = 41$......(1)

$25A - 5B + C = 20$......(2).

On the other hand, by completing the square, we have $y = A(x - \frac{B}{2A})^2 - \frac{B^2}{4A} + C$.

As the function is minimized at $x = 2$, we have $\frac{B}{2A} = 2$, i.e. $4A - B = 0$......(3).

(2)-(1): $21A - 7B = -21$, i.e. $3A - B = -3$......(4).

(3)-(4): $A = \boxed{3}$.

Substitue $A = 3$ into (3), $B = 4A = \boxed{12}$.

Substitue $(A, B) = (3, 12)$ into (1), $C = 41 - 4A - 2B = 41 - 12 - 24 = \boxed{5}$.

Moreover, when $x = 2$, we obtain the minimum value of the function, which is $y = -\frac{B^2}{4A} + C = -\frac{12^2}{12} + 5 = -12 + 5 = \boxed{-7}$.

(Note: The minimum value can also be calculated by putting $x = 2$ into the function.)

Alternative (with calculus) The equation (3) can also be obtained as the following:

$y' = 2Ax - B$.

When y attains to its extremum, $y' = 0$. Hence, by putting $x = 2$, we have $4A - B = 0$......(3).

Q2:

As x satisfies $x^2+2x-2=0$, we have $x^3=-2x^2+2x$. Therefore, P can be rewritten as $P=(-2x^2+2x)+x^2+ax+1=-x^2+(a+2)x+1$.

Moreover, as $x^2=-2x+2$, $P=-(-2x+2)+(a+2)x+1=(a+4)x-1$.

As P is independent on the value of x, we have $a+4=0$, i.e. $a=\boxed{-4}$.

In this case, the value of P is $\boxed{-1}$.

Q3:

(i):

$$x^2-3x-10<0$$

$$\iff (x-5)(x+2)<0$$

$$\iff \boxed{-2<x<5}$$

(ii):

$$|x-2|<a$$

$$\iff -a<x-2<a$$

$$\iff 2-a<x<a+2$$

In case $x^2-3x-10<0 \implies |x-2|<a$, we have

$$2-a\leq -2 \text{ and } 5\leq a+2$$

$$\iff 4\leq a \text{ and } 3\leq a$$

$$\iff \boxed{a\geq 4}$$

(iii) In case $|x-2|<a \implies x^2-3x-10<0$, we have

$$-2 \leq 2-a \text{ and } a+2 \leq 5$$

$$\iff a \leq 4 \text{ and } a \leq 3$$

$$\iff a \leq 3$$

On the other hand, for the inequality $|x-2|<a$ holds with a solution, we have the hidden condition $a>0$.

Combine the above, we have $\boxed{0<a\leq 3}$.

Q4:

(1): As a_n is an arithmetic series, we have $a_n = a_1 + (n-1)d$. Substitute it into the two given equation, we have:

$$(a_1+(5-1)d)(a_1+(7-1)d)-(a_1+(4-1)d)(a_1+(9-1)d)=60$$

$$(a_1^2+10a_1d+24d^2)-(a_1^2+11a_1d+24d^2)=60$$

$$a_1d=-60......(1)$$

and

$$a_1+(11-1)d=25$$

$$a_1=25-10d......(2)$$

Substitue (2) into (1), we have

$$(25-10d)d=-60$$

$$2d^2 - 5d - 12 = 0$$

$$(2d+3)(d-4) = 0$$

$$d = \boxed{4} \text{ or } d = \boxed{-\frac{3}{2}}$$

(2): Substitue $d = 4$ into (1), we have $a_1 = \frac{-60}{4} = \boxed{-15}$.

Then, $a_n = -15 + (n-1)(4) = \boxed{4}n - \boxed{19}$.

The sum of the first n terms can be written as $\frac{(a_1+a_n)n}{2}$, i.e. $\frac{(-15+4n-19)n}{2}$, i.e. $2n^2 - 17n$.

When the sum of the first n terms is 165, we have

$$2n^2 - 17n = 195$$

$$(2n+13)(n-15) = 0$$

$$n = \boxed{15} \text{ or } n = -\frac{13}{2} \text{(rejected)}$$

Q5:

As $\triangle ACP$ and $\triangle BDP$ are right angle triangles, by considering the tangent ratios, we have $\tan\alpha = \frac{AC}{PC} = \frac{\boxed{2}}{\boxed{1}-t}$ and $\tan\beta = \frac{BD}{DP} = \frac{1}{t-(-1)} = \frac{\boxed{1}}{\boxed{1}+t}$.

Hence, $\tan\theta - \tan(\pi - \alpha - \beta)$

$= -\tan(\alpha+\beta)$

$= -\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= -\frac{\frac{2}{1-t} + \frac{1}{1+t}}{1 - (\frac{2}{1-t})(\frac{1}{1+t})}$

$= -\frac{2+2t+1-t}{1-t^2-2}$

$= \frac{t+\boxed{3}}{t^2+\boxed{1}}$.

Moreover, $(\frac{t+3}{t^2+1})' = \frac{(1)(t^2+1)-(2t)(t+3)}{(t^2+1)^2} = -\frac{t^2+\boxed{6}t-1}{(t^2+\boxed{1})^2}$.

When θ is maximised, $\tan\theta$ is also maximised as $\tan\theta$ is monotonic increasing for $\theta \in (0, \frac{\pi}{2})$.

To find the extremum of $\tan\theta$, we set

$$(\frac{t+3}{t^2+1})' = 0$$

$$t^2 + 6t - 1 = 0$$

$$t = \frac{-6 \pm \sqrt{6^2 - 4(1)(-1)}}{2} = -3 \pm \sqrt{10}$$

As P lies on the segment DC, we have $t \in [-1, 1]$. Therefore only $t = -3+\sqrt{10}$ is a possible value for $\tan\theta$ to attain to its extremum. The table of first derivative test is shown:

t	$[-1, -3+\sqrt{10})$	$(-3+\sqrt{10}, 1]$
$(\frac{t+3}{t^2+1})'$	$+$	$-$
$\tan\theta$	$\nearrow$	$\searrow$

Therefore, $\tan\theta$ attains to its maximum when $t = -3 + \sqrt{10}$ and hence the coordinates of P are $(\boxed{-3+\sqrt{10}}, 0)$.

Q6:

As $f'(x) = 3x^2 + 2ax + b$ and $f''(x) = 6x + 2a$, we have

$f'(\alpha) = \boxed{3}\alpha^2 + \boxed{2}a\alpha + b$ and $f''(\alpha) = \boxed{6}\alpha + \boxed{2}a$.

By replacing x with $x+\alpha$ and y with $y+f(\alpha)$, we have

$$y+f(\alpha)=(x+\alpha)^3+a(x+\alpha)^2+b(x+\alpha)+c$$

$$y=x^3+(3\alpha+a)x^2+(3\alpha^2+2a\alpha)x+(\alpha^3+a\alpha^2+b\alpha+c)-f(\alpha)$$

$$y=x^3+\frac{6\alpha+2a}{2}x^2+f'(\alpha)x+f(\alpha)-f(\alpha)$$

$$y=x^3+\frac{f''(\alpha)}{\boxed{2}}x^2+f'(\alpha)x$$

For $f(x)=x^3-12x^2+48x-68$, we have $f'(x)=3x^2-24x+48$ and $f''(x)=6x-24$.

Solving $f'(x)=0$, we have $3(x-4)^2=0$, i.e $x=4$.

Solving $f''(x)=0$, we have $6(x-4)=0$, i.e. $x=4$.

Therefore, $f'(\boxed{4})=0$ and $f''(\boxed{4})=0$.

By using the result above, putting $\alpha=4$, when the point $(4,f(4))$, i.e. $(\boxed{4},\boxed{-4})$, on the graph is moved to the origin, we get the graph of $y=x^3$.

Q7:

The slope of tangent to the graph of $y=2\log x$ is given by $y'=\frac{2}{x}$.

As P lies on the graph of $y=2\log x$, the coordinates of P are $(t,2\log t)$.

Then, the equation of l will be $y-2\log t=\frac{2}{t}(x-t)$.

As l passes through the origin, we can put $(x,y)=(0,0)$, which gives

$-2\log t=\frac{2}{t}(-t)$, i.e. $\log t=\boxed{1}$ and $t=e$.

Hence the equation of l is $y-2=\frac{2}{e}(x-e)$, i.e. $y=\frac{\boxed{2}}{e}x$.

As $m\perp l$, the slope of $m=-\frac{1}{\text{slope of } l}=-\frac{e}{2}$. As P also lies on m, by using the point-slope form of straight line, we have the equation of m is $y-2=-\frac{e}{2}(x-e)$,

i.e. $y=-\frac{e}{\boxed{2}}x+\frac{e^2}{\boxed{2}}+\boxed{2}$.

The x-intercepts of $y=2\log x$ and m are 1 and $e+\frac{4}{e}$ respectively and the two graphs intersect each other when $x=e$.

Therefore, $S=\int_1^e 2\log x dx+\int_e^{e+\frac{4}{e}}(-\frac{e}{2}x+\frac{e^2}{2}+2)dx$

$=[2x\log x]_1^e-2\int_1^e xd(\log x)+[-\frac{e}{4}x^2+\frac{e^2}{2}x+2x]_e^{e+\frac{4}{e}}$

$=2e-2\int_1^e dx+(-\frac{e}{4}(e^2+8+\frac{16}{e^2})+\frac{e^2}{2}(e+\frac{4}{e})+2(e+\frac{4}{e})+\frac{e^3}{4}-\frac{e^3}{2}-2e)$

$=2e-[2x]_1^e+\frac{4}{e}$

$=2e-2e+2+\frac{4}{e}$

$=\boxed{2}+\frac{\boxed{4}}{e}$.