

Q1(1):

$$x^2 - 5x + 1 < 0$$

$$0 < \frac{5 - \sqrt{21}}{2} < x < \frac{5 + \sqrt{21}}{2} < 5$$

Therefore, the integers that satisfying the equation are 1, 2, 3, 4, where there are totally $\boxed{4}$ integers.

Q1(2):

$$\sqrt{a^2 + 2a + 1} + \sqrt{a^2 - 4a + 4}$$

$$= \sqrt{(a+1)^2} + \sqrt{(a-2)^2}$$

As for $-1 < a < 2$, $a+1 > 0$ and $a-2 < 0$, we have

$$\sqrt{a^2 + 2a + 1} + \sqrt{a^2 - 4a + 4}$$

$$= (a+1) - (a-2)$$

$$= \boxed{3}$$

Q1(3):

$$2^{2x} + 2^{-2x} = (2^x - 2^{-x})^2 + 4 = 4^2 + 2 = \boxed{18}.$$

$$2^{3x} - 2^{-3x} = (2^x - 2^{-x})^3 + 3(2^x - 2^{-x}) = 4^3 + 3(4) = \boxed{76}.$$

Q1(4):

$$\log_3(x-3) - \log_9(x-1) = 0$$

$$\log_3(x-3) = \log_3(x-1)^{\frac{1}{2}}$$

$$(x-3)^2 = x-1$$

$$x^2 - 7x + 10 = 0$$

$$(x-5)(x-2) = 0$$

$$x = \boxed{5} \text{ (Note the hidden condition for } \log_3(x-3) \text{ to be defined: } x > 3)$$

Q1(5):

By the cosine formula, we have

$$(x+2)^2 = x^2 + (x-2)^2 - 2x(x-2)\cos 120^\circ$$

$$x^2 + 4x + 4 = 2x^2 - 4x + 4 + x^2 - 2x$$

$$x^2 - 5x = 0$$

$$x = \boxed{5} \text{ (as } x > 2)$$

Q1(6):

For a four digit number, there are 4 choice for the leading digits and P_3^4 permutations for the remaining digits. Therefore, there are totally $4 \cdot P_3^4 = \boxed{96}$.

We separate it into 2 cases for a four digit odd number:

-The leading digit is an odd number (2 choices) and the last digit is the remained odd number. Moreover, there are P_2^3 permutations for the remaining digits. Therefore, there are totally $2 \cdot P_2^3 = 12$ such numbers.

-The leading digit is an even number (2 choices) and the last digit is an odd number (2 choices). Moreover, there are P_2^3 permutations for the remaining digits. Therefore, there are totally $2 \cdot 2 \cdot P_2^3 = 24$ such numbers.

Given the above, there are totally $12 + 24 = \boxed{36}$ four digit odd numbers.

Q1(7):

$$1^2 + 2^2 + \dots + 5^2 = \frac{1}{6}(5)(6)(11) = \boxed{55}.$$

$$6^2 + 7^2 + \dots + 13^2 = (1^2 + 2^2 + \dots + 13^2) - (1^2 + 2^2 + \dots + 5^2) = \frac{1}{6}(13)(14)(27) - 55 = \boxed{764}.$$

Q1(8):

$$2\vec{a} + 3\vec{b} = \langle 1, 4 + 3x \rangle \text{ and } \vec{a} - 2\vec{b} = \langle -3, 2 - 2x \rangle.$$

$$\text{If } 2\vec{a} + 3\vec{b} // \vec{a} - 2\vec{b}, \text{ then } k(2\vec{a} + 3\vec{b}) = \vec{a} - 2\vec{b}, \text{ i.e. } \frac{2-2x}{4+3x} = \frac{-3}{1} = k, \text{ i.e. } x = \boxed{-2}.$$

Q1(9):

(i):

$$f(x) = g(x)$$

$$x^2 + 2x - 1 = x + 1$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$x = \boxed{1}, \boxed{-2}$$

(ii): By completing the square, $f(x) = (x + 1)^2 - 2$. Therefore, the coordinates of vertex are $\boxed{(-1, -2)}$.

(iii): As $f'(x) = 2x + 2$, the slope of the tangent $= f'(0) = 2$.

Therefore, by the point-slope form of straight line, the equation is $y = \boxed{2x - 1}$.

(iv): The area $= \int_{-2}^1 ((x + 1) - (x^2 + 2x - 1)) dx$

$$= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \right]_{-2}^1$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 - \frac{8}{3} + 2 + 4$$

$$= \boxed{\frac{9}{2}}$$

Q2:

(1): $BP = BR = 13 - AR$ and similarly $CP = 13 - AR$.

Therefore, $BC = 26 - 2AR = 10$, i.e. $AR = \boxed{8}$.

(2): Note that $\angle ARO = 90^\circ$. Therefore, we have $\sin \angle AOR = \cos \angle OAR$.

On the other hand, note that $\angle APB = 90^\circ$. Consider the cosine ratio, we have

$$\cos \angle OAR = \frac{AP}{AB} = \frac{\sqrt{13^2 - 5^2}}{13} = \boxed{\frac{12}{13}}.$$

$$\begin{aligned} (3): \tan \angle AOR &= \sqrt{\frac{1}{\cos^2 \angle AOR} - 1} \\ &= \sqrt{\frac{1}{1 - \sin^2 \angle AOR} - 1} \\ &= \sqrt{\frac{1}{1 - (\frac{12}{13})^2} - 1} \\ &= \boxed{\frac{12}{5}}. \end{aligned}$$

$$(4): \text{Consider the tangent ratio, } \tan \angle AOR = \frac{AR}{RO} = \frac{12}{5}, \text{ we have } RO = \boxed{\frac{10}{3}}.$$

$$(5): \vec{AB} \cdot \vec{AO} = AB \cdot AO \cos \angle OAR = 13 \cdot 8 = \boxed{104}.$$

$$\vec{AB} \cdot \vec{AC} = -AB \cdot AC \cos \angle BAC = \frac{AB^2 + BC^2 - AC^2}{2} = \boxed{-50}.$$

Q3:

$$(1): \text{As the vertex is } (-2, 1), \text{ the equation is } y = a(x+2)^2 + 1 = ax^2 + 4ax + 4a + 1.$$

As the y-intercept is 5, we have $4a + 1 = 5$, i.e. $a = 1$.

$$\text{Therefore, the equation is } y = \boxed{1}x^2 + \boxed{4}x + \boxed{5}.$$

(2): By the intercept form,

$$\frac{x}{-3} + \frac{y}{9} = 1$$

$$y = \boxed{0}x^2 + \boxed{3}x + \boxed{9}$$

(3): As the two x-intercepts are -1 and 3, the equation is

$$y = k(x + 1)(x - 3) = kx^2 - 2kx - 3k.$$

As the y-intercept is 6, we have $-3k = 6$, i.e. $k = -2$.

Therefore, the equation is $y = \boxed{-2}x^2 + \boxed{4}x + \boxed{6}$.