

Q1(1):

$$\begin{aligned}\log_{10} \frac{4}{5} + 2 \log_{10} 5\sqrt{5} &= \log_{10} 4 - \log_{10} 5 + 3 \log_{10} 5 \\&= 2(\log_{10} 2 + \log_{10} 5) \\&= 2 \log_{10} 10 \\&= \boxed{2}.\end{aligned}$$

Q1(2):

$$2 \cos x - \sqrt{3} < 0 \iff \cos x < \frac{\sqrt{3}}{2}.$$

$\cos x$ is decreasing from $x = 0$ to $x = \pi$ and increasing from $x = \pi$ to $x = 2\pi$.

$$\text{Note that } \cos \frac{\pi}{6} = \cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2}.$$

Therefore, we have the solution to the inequality: $\boxed{\frac{1}{6}}\pi < x < \boxed{\frac{11}{6}}\pi$.

Q1(3):

$$\sqrt{2-2a} = a$$

$$2-2a = a^2 \text{ and } 2-2a \geq 0 \text{ and } a > 0$$

$$a^2 + 2a - 2 = 0 \text{ and } 0 < a \leq 1$$

$$a = \boxed{-1 + \sqrt{3}}$$

(Note: One can also calculate both roots of the quadratic equation and reject the root $-1 - \sqrt{3}$ by root checking.)

Q1(4):

The equation of the line through the two points is given by $y = -tx + t$ using the point-slope form of straight line.

Substitute it to $x^2 + y^2 = 1$ so as to solve a :

$$x^2 + (-tx + t)^2 = 1$$

$$(1 + t^2)x^2 - 2t^2x + t^2 - 1 = 0$$

$$(x + 1)((1 + t^2)x + (t^2 - 1)) = 0^*$$

$$x = -1, \frac{1 - t^2}{1 + t^2}$$

Therefore, $a = \frac{\boxed{1} + \boxed{0}t + \boxed{-1}t^2}{1 + t^2}$.

(*: As we know $x = -1$ is a root, we know $(x + 1)$ is a factor and hence we can do the factorisation easily.)

Q1(5):

By the binomial expansion, we have $(1 + x)^{10} = \sum_{k=0}^{10} \binom{10}{k} x^k$.

Thereofre, put $x = 1$, we have $\sum_{k=0}^{10} \binom{10}{k} = 2^{10} = \boxed{1024}$.

Alternative Consider the combinatoric meaning of $\sum_{k=0}^{10} \binom{10}{k}$: The number of ways to picks 1,2,...,9, or 10 objects among 10 different objects.

We can calculate the number in another way: For each object, we can decide

whether or not pick it, i.e. there are 2 possibilities. As there are 10 objects, the number of possibilities is 2^k .

Given the above, we have $\sum_{k=0}^{10} \binom{10}{k} = 2^{10} = \boxed{1024}$.

Q1(6):

$$1 = \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \iff xyz = xy + yz + zx.$$

As $x < y < z$, we have $xy, xz < yz$. Therefore, $xyz < yz + yz + yz = 3yz$, i.e. $x < 3$.

Note that the case $x = 1$ is impossible. Therefore, we have $x = \boxed{2}$.

Now, the equation becomes $yz = 2y + 2z$, i.e. $z(y - 2) = 2y$. Obviously, $(y, z) = (\boxed{3}, \boxed{6})$.

Q2:

(1): The area under the parabola $= \int_a^b x^2 dx$

$$= \left[\frac{1}{3} x^3 \right]_a^b$$

$$= \frac{1}{3} (b^3 - a^3).$$

Moreover, the region under the line is a tripazium, the area $= \frac{(a^2 + b^2)(b - a)}{2}$.

As $y = x^2$ convex downwards, we have the required area $= \frac{(a^2 + b^2)(b - a)}{2} - \frac{1}{3} (b^3 - a^3)$

$$= \frac{-a^3 + b^3 - ab^2 + a^2b}{6}$$

$$= \boxed{\frac{(b - a)^3}{6}}.$$

(2): The slope of $l = \frac{b^2 - a^2}{b - a} = a + b$.

The slope of the tangent to the parabola is given by $y' = 2x$. When $x = b$, the slope = $2b$.

As l is perpendicular to the tangent, we have $(a + b)(2b) = -1$, i.e. $a =$

$$\boxed{-\frac{1 + 2b^2}{2b}}.$$

(3): Substitue the result of (2) into the result of (1), we have

$$S = \frac{1}{6} \left(b - \left(-\frac{1+2b^2}{2b} \right) \right)^3 = \frac{1}{48} \left(\frac{1+4b^2}{b} \right)^3.$$

$$\text{Then, } S' = \frac{1}{48} (3) \left(\frac{1+4b^2}{b} \right)^2 \left(-\frac{1}{b^2} + 4 \right).$$

To find the extremum of S , we set $S' = 0$, then $b = \pm \frac{1}{2}$.

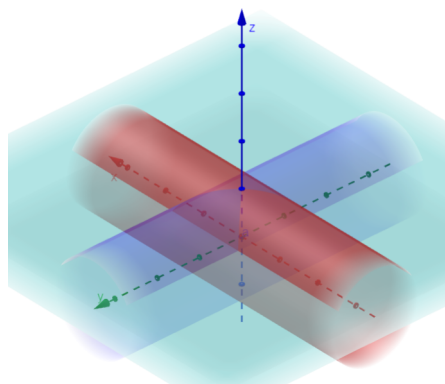
The table of signs is given:

b	$(-\infty, -\frac{1}{2})$	$(-\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, +\infty)$
S'	+	-	+
S	\nearrow	\searrow	\nearrow

Therefore, S attains to its minimum when $b = \boxed{\frac{1}{2}}$. By that time, $m = \boxed{\frac{4}{3}}$.

Q3:

A sketch for reference:



(1): The horizontal cross section of a cylinder is a rectangle.

When $z = t$, the two equations become $\sqrt{r^2 - t^2} \leq x \leq \sqrt{r^2 - t^2}$ and $\sqrt{r^2 - t^2} \leq y \leq \sqrt{r^2 - t^2}$, which are two rectangles with width $2\sqrt{r^2 - t^2}$ extending on the x-direction and the y-direction respectively.

As the two rectangles are perpendicular to each other, the intersection of them is a square with side length $2\sqrt{r^2 - t^2}$.

Therefore, the area is $(2\sqrt{r^2 - t^2})^2 = \boxed{4(r^2 - t^2)}$.

(2): The integration of the cross section area of B among the boundaries gives its volume.

Therefore, the volume $= \int_{-r}^r 4(r^2 - t^2) dt$

$$= 4[r^2 t - \frac{1}{3} t^3]_{-r}^r$$

$$= \boxed{\frac{16}{3} r^3}.$$