MATHEMATICS (B)   (2024)

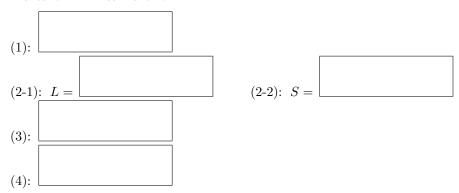| Nationality		No.			Marks	
Name	(Please print full name, underlining family name)					

Answer the following questions and fill in your responses in the correspond-ing boxed on the answer sheet.

1. Fill in the blanks with the correct numbers.

(1) The number of digits of an integer is 100 when shown in the senary (base 6) number system. By $\log_{10} 2 \approx 0.3010$ and $\log_{10} 3 \approx 0.4771$, the number of digits of the number when shown in a decimal (base 10) number system is ⬚.

(2) If $0 < x < \frac{\pi}{2}$, then $\frac{1}{\sin x} + \frac{1}{\cos x} \geq$ ⬚ with equality if and only if $x =$ ⬚.

(3) Denote $|S|$ as the number of elements of set S. For sets A and B, if $|\bar{A}\cap B| = 9$, $|A\cap\bar{B}| = 11$ and $|A\cup B| = 20$, then $|A\cap B| =$ ⬚.

(4) When the line $y = x + a$ cuts the circle $x^2 + y^2 = 1$, the length of the chord is $\sqrt{2}$. Then, the constant $a =$ ⬚.

(5) There are 3 positive numbers where the sum of their squares is equal to 39 and the sum of all combinations of multiplication between two distinct numbers among them is equal to 21. Then, the mean of them is ⬚ and the standard deviation of them is ⬚.

(6) The division of a natural number by 3 gives the remainder 1, and that by 5 gives the remainder 2. Thus, the remainder of the division of the

number by 15 is _____ .

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2. Let a, b be two real parameters. Let $\mathrm{O}(0,0,0)$ be the origin of the coordinate space. Consider a point $\mathrm{P}(a, b, \cos\theta)$ $(0 \le \theta \le \pi)$ and let Π be a square with side length l, centered at O and having the unit normal vector $\overrightarrow{OP}$. Fill in the blanks with the answers to the following questions.

(1) Express $a^2 + b^2$ in terms of θ.

(2) When the value of θ is fixed, the locus of P is an arc. Let L be the length of the arc. When the value of θ varies, the locus of P is a surface. Let S be the area of the surface. Express L and S in terms of θ.

(3) Π is projected to the plane $z = t$ $(t < -\frac{l}{2})$ as Ω. Express the area of Ω in terms of l, t, θ.

(4) When θ varies, the area of Ω varies. Express the expected value of the area of Ω in terms of l.

(1): _____

(2-1): $L = $ _____   (2-2): $S = $ _____

(3): _____

(4): _____

3. Define a real sequece C_n $(n = 0, 1, 2, ...)$ with the following:

(i) $C_0 = 1$

(ii) $C_{n+1} = \sum_{k=0}^{n} C_k C_{n-k}$

Let $f(x) = C_0 + C_1 x + C_2 x^2 + ...$. Fill in the blanks with the answers to the following questions. Here, the n-th order derivative of a differentiable function $h(x)$ is denoted as $h^{(n)}(x)$.

(1) Express the coefficient of x^n in $(f(x))^2$ in terms of C_k $(k = 0, 1, 2, ...)$.

(2) Express $f(x)$ $(x \neq 0)$ in terms of x.

(3) Let $g(x) = a_0 + a_1 x + a_2 x^2 + ...$, where a_n is the real coefficient of x^n. Express a_n in terms of $g^{(k)}(0)$ $(k = 0, 1, 2, ...)$ and n.

(4) Express C_n with binomial coefficient in terms of n.

(1):

(2):

(3):

(4):