Q1(1):

$$16^x - 4^x - 2 = 0$$

$$4^{2x} - 4^x - 2 = 0$$

$$(4^x - 2)(4^x + 1) = 0$$

$$4^x = 2$$

$$x = \boxed{\frac{1}{2}}$$

Q1(2):

$$\sin x + 2\cos^2 x = 1$$

$$\sin x + 2 - 2\sin^2 x = 1$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2}, 1$$

$$x = \boxed{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

Q1(3):

$$x + \frac{1}{x} < \frac{1}{2}(7 - x)$$

$$2x^2 + 2 < 7x - x^2 \text{ or } x < 0$$

$$3x^2 - 7x + 2 < 0 \text{ or } x < 0$$

$$(3x - 1)(x - 2) < 0 \text{ or } x < 0$$

$$\boxed{\frac{1}{3} < x < 2 \text{ or } x < 0}$$

Q1(4):

$$\log_2(x + 2) < 2$$

$$x + 2 < 4$$

$$x < 2$$

On the other hand, for $\log_2(x + 2)$ to be defined, we have $x > -2$.

Therefore, the solution is $\boxed{-2 < x < 2}$.

Q1(5):

Suppose $3(a_{n+1} + k) = 2(a_n + k)$, then $k = -1$.

Therefore,

$$\frac{a_{n+1} - 1}{a_n - 1} = \frac{2}{3}$$

$$\frac{a_{n+1}-1}{a_1-1}=(\frac{2}{3})^n$$

$$a_{n+1}=1+(\frac{2}{3})^n$$

$$a_n=\boxed{1+(\frac{2}{3})^{n-1}}$$

Q1(6):

$\lim\limits_{h\to 0}\frac{\cos(x-2h)-\cos(x+h)}{\sin(x+3h)-\sin(x-h)}$

$=\lim\limits_{h\to 0}\frac{2\sin(x-\frac{h}{2})\sin\frac{3h}{2}}{2\cos(x+h)\sin 2h}$

$=(\lim\limits_{h\to 0}\frac{\sin(x-\frac{h}{2})}{\cos(x+h)})(\frac{3}{2}\lim\limits_{h\to 0}\frac{\sin\frac{3h}{2}}{\frac{3h}{2}})(\frac{1}{2}\lim\limits_{h\to 0}\frac{2h}{\sin 2h})$

$=(\frac{\sin x}{\cos x})(\frac{3}{2})(\frac{1}{2})$

$=\boxed{\dfrac{3}{4}\tan x}$

Q1(7):

$\frac{d}{dx}e^{x\sin x}=(x\sin x)'e^{x\sin x}=(\sin x+x\cos x)e^{x\sin x}$

Q1(8):

$\int_{\frac{1}{e}}^{e}\ln x dx$

$=x\ln x|_{\frac{1}{e}}^{e}-\int_{\frac{1}{e}}^{e}xd(\ln x)$

$=e+\frac{1}{e}-[x]_{\frac{1}{e}}^{e}$

$=\boxed{\dfrac{2}{e}}$

Q1(9):

By exhausting, the pairs (the order is omitted) do not satisfy the condition are:

(1,1),(1,2),...,(1,6),(2,2)

Totally 7 pairs.

Therefore, $21-7=14$ pairs statisfy the condition.

The probability is $\frac{14}{21}=\boxed{\frac{2}{3}}$.

Note: Without the order omitted, there are 12 pairs do not satisfy the condition.

Q1(10):

By the binomial expansion,

$(ax^2-\frac{1}{ax})^9=\sum_{i=0}^{9}C_i^9(ax^2)^i(-\frac{1}{ax})^{9-i}=\sum_{i=0}^{9}(C_i^9(-1)^{9-i}a^{2i-9})x^{3i-9}$.

The x^9 term is obtained when $3i-9=9$, i.e. $i=6$.

By that time, the coefficient$=C_6^9(-1)^3a^3=-84a^3$.

Solving $-84a^3=\frac{21}{3}$, we have $a=\boxed{-\frac{1}{2}}$.

Q1(11):

The position vector of P is given by

$t<2,0,1>+(1-t)<0,1,2>=<2t,1-t,2-t>$ where t is a real parameter.

On the other hand, $\vec{AB} =< -2, 1, 1 >$.

When $\vec{OP} \perp \vec{AB}$, we have

$$\vec{OP} \cdot \vec{AB} = 0$$

$$-4t + 1 - t + 2 - t = 0$$

$$t = \frac{1}{2}$$

Therefore, P is $\boxed{(1, \frac{1}{2}, \frac{3}{2})}$.

Q1(12):

$x^3 = 8 \iff (x-2)(x^2+2x+4) = 0$

Therefore, α and β are the roots of $x^2 + 2x + 4 = 0$.

Sum of roots=$\alpha + \beta = -2$ and product of roots=$\alpha\beta = 4$.

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 4 - 8 = \boxed{-4}$.

Q2:

1): $AX = \begin{bmatrix} 2 \\ \\ -2 \end{bmatrix}$

Therefore, $a = \boxed{2}$.

2): $AY = \begin{bmatrix} 5 \\ \\ 10 \end{bmatrix}$

Therefore, $b = \boxed{5}$.

3): $cX + dY = \begin{bmatrix} c+d \\ -c+2d \end{bmatrix}$.

Solving, we have $c = \boxed{\dfrac{1}{3}}$ and $d = \boxed{\dfrac{8}{3}}$.

4): $A^n Z$

$= A^n(\frac{1}{3}X + \frac{8}{3}Y)$

$= \frac{1}{3}A^n X + \frac{8}{3}A^n Y$

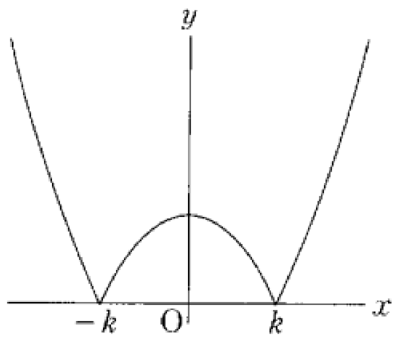
$= \frac{1}{3} \cdot 2^n X + \frac{8}{3} \cdot 5^n Y$

$= \boxed{\begin{bmatrix} \frac{2^n + 8\cdot 5^n}{3} \\ \frac{-2^n + 16\cdot 5^n}{3} \end{bmatrix}}$

Q3:

(Note: The conditions in parts 2) and 3) should be $|k| < 1$ and $|k| > 1$ respectively.)

1): We sketch the graph of $y = x^2 - k^2$ and reflect the negative region along the x-axis:



2): As $|k| < 1$, $f(x) = k^2 - x^2$ for $x \in (-k, k)$ and $f(x) = x^2 - k^2$ for $x \in (-1, k) \cup (k, 1)$.

Therefore, $I(k) = \int_{-k}^{k}(k^2 - x^2)dx + \int_{-1}^{k}(x^2 - k^2)dx + \int_{k}^{1}(x^2 - k^2)dx$

$= 2[k^2 x - \frac{1}{3}x^3]_0^k + 2[\frac{1}{3}x^3 - k^2 x]_k^1$

$= \boxed{\dfrac{8}{3}k^3 - 2k^2 + \dfrac{2}{3}}.$

3): As $|k| > 1$, $f(x) = k^2 - x^2$ for $x \in (-1, 1)$.

Therefore, $I(k) = \int_{-1}^{1}(k^2 - x^2)dx = 2[k^2 x - \frac{1}{3}x^3]_0^1 = \boxed{2k^2 - \dfrac{2}{3}}.$

4): Combine the results of 2) and 3), we have:

$$I(k) = \begin{cases} \frac{8}{3}k^3 - 2k^2 + \frac{2}{3}, \ |k| < 1 \\ \\ 2k^2 - \frac{2}{3}, \ |k| > 1 \end{cases}$$

$$I'(k) = \begin{cases} 8k^2 - 4k, \ |k| < 1 \\ \\ 4k, \ |k| > 1 \end{cases}$$

To find the extremum, we set $I'(k) = 0$, then $k = 0, \frac{1}{2}$.

$$I''(k) = \begin{cases} 16k - 4, \ |k| < 1 \\ \\ 4, \ |k| > 1 \end{cases}$$

Therefore, $I''(0) < 0$ and $I''(\frac{1}{2}) > 0$.

Moreover, test the boundaries:

$I(1) = I(-1) = \frac{4}{3}$.

Given the above, the minimum is obtained when $k = \boxed{\dfrac{1}{2}}$ and the value of it is $I(\frac{1}{2}) = \boxed{\dfrac{1}{2}}$.