Q1(1):

By testing the potential rational roots given by the rational root theorem $\pm 1, \pm 2$,

we have $x = 1$ is a root.

Therefore, we can do the factorisation by the long division:

$$x^3 + 2x^2 - x - 2 = 0$$

$$(x-1)(x^2 + 3x + 2) = 0$$

$$(x-1)(x+2)(x+1) = 0$$

$$x = \boxed{-1, -2, 1}$$

Q1(2):

$$\sin 2x = \sqrt{3} \cos x$$

$$2 \sin x \cos x = \sqrt{3} \cos x$$

$$\sin x = \frac{\sqrt{3}}{2} \text{ or } \cos x = 0$$

$$x = \boxed{\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}}$$

Q1(3):

$$2 \log_3(x+2) = \log_3(10 - x)$$

$$(x+2)^2 = 10 - x$$

$$x^2 + 5x - 6 = 0$$

$$(x+6)(x-1) = 0$$

$x = \boxed{1}$ (Note that $-2 < x < 10$ for the two logarithms to be defined)

Q1(4):

$$3^{2x+1} - 2 \cdot 3^x - 1 < 0$$

$$3 \cdot 3^{2x} - 2 \cdot 3^x - 1 < 0$$

$$(3 \cdot 3^x + 1)(3^x - 1) < 0$$

$$0 < 3^x < 1$$

$$\boxed{x < 0}$$

Q1(5):

$$(\log_3 x)(\log_{27} x) - \log_9 x + \frac{1}{6} < 0$$

$$\frac{1}{3}(\log_3 x)^2 - \frac{1}{2}\log_3 x + \frac{1}{6} < 0$$

$$2(\log_3 x)^2 - 3\log_3 x + 1 < 0$$

$$(2\log_3 x - 1)(\log_3 x - 1) < 0$$

$$\frac{1}{2} < \log_3 x < 1$$

$$\boxed{\sqrt{3} < x < 3}$$

Q1(6):

By the cosinie formula,

$$AB^2 = AO^2 + BO^2 - 2(AO)(BO)\cos\angle AOB$$

$$(x-1)^2 + (-1-1)^2 = 1^2 + 1^2 + x^2 + (-1)^2 - 2\sqrt{(1^2+1^2)(x^2+(-1)^2)}(\frac{1}{2})$$

$$-2x + 2 = -\sqrt{2(x^2+1)}$$

$$4x^2 - 8x + 4 = 2x^2 + 2$$

$$x^2 - 4x + 1 = 0$$

$$x = \boxed{2+\sqrt{3}} \text{ (Note that } 2-\sqrt{3} \text{ does not satisfy the equation)}$$

Note: $-2(2-\sqrt{3}) + 2 = 2\sqrt{3} - 2 > 0$ whereas the R.H.S. < 0.

Q1(7):

$x^2 + y^2 + z^2 = 2x + 6y - 1 \iff (x-1)^2 + (y-3)^2 + z^2 = -1 + 1 + 3^2 = 3^2$

by completing the square.

Therefore, the radius of the sphere is $\boxed{3}$.

Q1(8):

$\frac{dy}{dx} = x$.

Let the point of tengency be $(k, \frac{1}{2}k^2 + 3)$, then the equation of the tangent is

$y - \frac{1}{2}k^2 - 3 = k(x - k)$.

As it goes through (0,0), we have

$$-\frac{1}{2}k^2 - 3 = k(-k)$$

$$k^2 = 6$$

$$k = \sqrt{6} \text{ (as } k \leq 0)$$

Therefore, the equation is $y - 3 - 3 = \sqrt{6}(x - \sqrt{6})$, i.e. $y = \boxed{\sqrt{6}x}$

Q1(9):

$\sum\limits_{k=1}^{100} \frac{1}{k(k+1)}$

$= \sum\limits_{k=1}^{100} (\frac{1}{k} - \frac{1}{k+1})$

$= 1 - \frac{1}{101}$

$= \boxed{\dfrac{100}{101}}$

Q1(10):

$\lim\limits_{x \to \frac{1}{2}} \frac{16x^4 - 1}{2x - 1}$

$= \lim\limits_{x \to \frac{1}{2}} \frac{(4x^2+1)(4x^2-1)}{2x-1}$

$= \lim\limits_{x \to \frac{1}{2}} (4x^2 + 1)(2x + 1)$

$= (1+1)(1+1)$

$= \boxed{4}$

Q1(11):

$f'(x) = -3\cos^2 x \sin x$

$f'(\frac{\pi}{6}) = -3(\frac{\sqrt{3}}{2})^2(\frac{1}{2}) = \boxed{-\dfrac{9}{8}}$

Q1(12):

$\int_1^{e^2} x \log x dx$

$= \int_1^{e^2} \log x d(\frac{1}{2}x^2)$

$= \frac{1}{2}x^2 \log x|_1^{e^2} - \frac{1}{2}\int_1^{e^2} xdx$

$= e^4 - \frac{1}{2}[\frac{1}{2}x^2]_1^{e^4}$

$= \boxed{\dfrac{3}{4}e^4 + \dfrac{1}{4}}$

Q2:

1): As $A^2 - tr(A)A + det(A)I = 0$ (applying the Cayley-Hamilton theorem to an order 2 square matrix), we have $tr(A) = 1 + b = 1$, i.e. $b = \boxed{0}$ and $det(A) = b - 2a = -2$, i.e. $a = \boxed{1}$.

Note: It is not necessarily the only solutions. A more precise way to solve this question requires direct calculation.

2): A^3

$= A^2 + 2A$

$= (A + 2I) + 2A$

$= 3A + 2I$

$= \boxed{\begin{bmatrix} 5 & 6 \\ 3 & 2 \end{bmatrix}}$

3): $A^6 - A^5 + A^4 - A^3 + A^2 - A$

$= 2A^4 + 2A^2 + 2I$

$= 2(A^3 + 2A^2) + 2A^2 + 2I$

$= 2A^3 + 6A + 14I$

$= \boxed{\begin{bmatrix} 30 & 24 \\ 12 & 18 \end{bmatrix}}$

Q3:

1):

$$\ln f(x) = \ln x - x^2$$

$$f'(x) = xe^{-x^2}(\frac{1}{x} - 2x) = \boxed{e^{x^2}(1 - 2x^2)}$$

2): $\int xe^{-x^2}dx$

$= -\frac{1}{2}\int e^{-x^2}d(-x^2)$

$$= \boxed{-\frac{1}{2}e^{-x^2} + Constant}$$

3): $\lim\limits_{a\to\infty} \int_0^a f(x)dx$

$= \lim\limits_{a\to\infty} (-\frac{1}{2}e^{-a^2} + \frac{1}{2})$

$$= \boxed{\frac{1}{2}}$$