

Q1(1):

$$2\sqrt{12} - 3\sqrt{6} \div \sqrt{18}$$

$$= 4\sqrt{3} - \frac{3}{\sqrt{3}}$$

$$= 4\sqrt{3} - \sqrt{3}$$

$$= \boxed{3\sqrt{3}}$$

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Q1(2):

$$\frac{x^2-x-6}{x^2+x-2} - \frac{2x-4}{x-1}$$

$$= \frac{(x-3)(x+2)}{(x+2)(x-1)} - \frac{2x-4}{x-1}$$

$$= \frac{(x-3)-(2x-4)}{x-1}$$

$$= \frac{1-x}{x-1}$$

$$= \boxed{-1}$$

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Q1(3):

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 2^2 - 2 \cdot 4 = \boxed{-4}.$$

$$\alpha^3 + \alpha\beta^2 = -4\alpha$$

$$\alpha^3 = -4\alpha - 4\beta = -4(\alpha + \beta) = -4 \cdot 2 = \boxed{-8}$$

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Q1(4):

$$x^4 - 13x^2 + 36 = 0$$

$$(x^2 - 9)(x^2 - 4) = 0$$

$$x^2 = 4, 9$$

$$x = \pm 2, \pm 3$$

Therefore, the smallest solution is  $\boxed{-3}$ .

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Q1(5):

$$\sin^2 x - \cos x + 1 = 0$$

$$1 - \cos^2 x - \cos x + 1 = 0$$

$$\cos^2 x + \cos x - 2 = 0$$

$$(\cos x + 2)(\cos x - 1) = 0$$

$$\cos x = 1$$

$$x = \boxed{0^\circ}$$


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Q1(6):

$$2\log_{10}(x - 4) - \log_{10} 4(x - 1) = 0$$

$$\log_{10} \frac{x^2 - 8x + 16}{x - 1} = \log_{10} 4$$

$$x^2 - 8x + 16 = 4x - 4$$

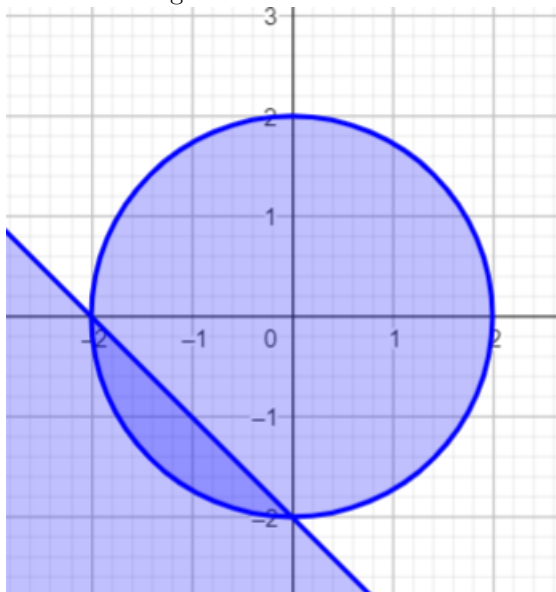
$$x^2 - 12x + 20 = 0$$

$$(x - 10)(x - 2) = 0$$

$x = \boxed{10}$  (note the hidden condition for  $\log_{10}(x - 4)$  to be defined:  $x > 4$ )

Q1(7):

Refer to the figure:



The area of the overlapping region is equal to the area of the  $\frac{1}{4}$  circle minus the area of the triangle bounded by the two radii and the chord.

Therefore, the area =  $\frac{1}{4}(\pi(2)^2) - \frac{1}{2}(2)(2) = \boxed{\pi - 2}$ .

Q1(8):

(i): The number of ways =  $C_3^{10} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = \boxed{120}$ .

(ii): The number of ways to choose 3 boys from 5 =  $C_3^5 = 10$ .

The number of ways to choose 2 girls from 5 =  $C_2^5 = 10$ .

Therefore, the required number of ways =  $10 \cdot 10 = \boxed{100}$ .

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Q1(9):

Note that  $a_n$  is an arithmetic sequence with initial term 1 and common difference 3.

Therefore, we have the general formula  $a_n = 1 + (n - 1)(3)$ .

$$a_{30} = 1 + 3 \cdot 29 = \boxed{88}.$$

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Q1(10):

When  $\vec{a}$  and  $\vec{b}$  are vertical,  $\vec{a} \cdot \vec{b} = 0$ , i.e.  $2x + 3 \cdot 2 = 0$ , i.e.  $x = \boxed{-3}$ .

When  $\vec{a}$  and  $\vec{b}$  are parallel, we have  $\vec{b} = k\vec{a}$  for some  $k$ .

Therefore,  $\frac{x}{2} = \frac{2}{3} = k$ , i.e.  $x = \boxed{\frac{4}{3}}$ .

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Q1(11):

(i):  $f'(x) = \boxed{3x^2 - 12x + 9}$ .

To find the extremum of  $f(x)$ , we set  $f'(x) = 0$ , then  $x = 3$  or  $x = 1$ .

$$f''(x) = 6x - 12.$$

As  $f''(1) = -6 < 0$ ,  $f(x)$  attains its maximum value when  $x = \boxed{1}$ .

(ii): To find the x-intercepts, we solve

$$x^3 - 6x^2 + 9x = 0$$

$$x(x - 3)^2 = 0$$

$$x = 0, 3$$

Therefore, the required area =  $\int_0^3 (x^3 - 6x^2 + 9x)$

$$= \left[ \frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2 \right]_0^3$$

$$= \frac{81}{4} - 54 + \frac{81}{2}$$

$$= \boxed{\frac{27}{4}}$$

Q2:

(1): When  $ODBA$  is a parallelogram,  $OD \parallel AB$  and  $OA \parallel DB$ .

Therefore, we have  $\frac{y-0}{x-0} = \frac{0-4}{3-2}$  and  $\frac{4-0}{2-0} = \frac{0-y}{3-x}$ , i.e.  $4x + y = 0 \dots (1)$  and

$2x - y = 6 \dots (2)$ .

By (1)+(2), we have  $6x = 6$ , i.e.  $x = \boxed{1}$ .

Substitute it into (1), we have  $y = -4(1) = \boxed{-4}$ .

**Alternative** When  $ODBA$  is a parallelogram,  $\vec{OD} = \vec{AB} = \langle 1, -4 \rangle$ .

Therefore,  $D = (0 + 1, 0 - 4) = \boxed{(1, -4)}$ .

(2): Note that the base  $OB$  is parallel to the x-axis.

The equation of  $OA$  is  $y = 2x$ .

Therefore, the intersection point between it and the line  $x = p$  is  $(p, 2p)$ .

The area of  $\triangle OAB = \frac{1}{2}(3 - 0)(4 - 0) = 6$ .

When the line  $x = p$  bisects  $\triangle OAB$ , we have

$$\frac{1}{2}(p - 0)(2p - 0) = \frac{6}{2}$$

$$2p^2 = 6$$

$$p = \boxed{\sqrt{3}}$$

(3): The equation of  $AB$  is  $y = -4x + 12$ .

The intersection points between  $y = p$  and  $OA, AB$  are  $(\frac{p}{2}, p)$  and  $(\frac{12-p}{4}, p)$  respectively.

Therefore, the length of the base of the upper triangle  $= \frac{12-p}{4} - \frac{p}{2} = \frac{12-3p}{4}$ .

The area of the upper triangle  $= \frac{1}{2}(\frac{12-3p}{4})(4 - p) = \frac{3(4-p)^2}{8}$ .

When  $y = p$  bisects  $\triangle OAB$ , we have

$$\frac{3(4-p)^2}{8} = \frac{6}{2}$$

$$4 - p = 2\sqrt{2}$$

$$p = \boxed{4 - 2\sqrt{2}}$$

(4): Note that  $L$  is the bisector of  $OA$ .

Therefore,  $L$  passes through the mid-point of  $OA$ ,  $(1, 2)$ .

Using the two points form of straight line, we have  $L: y = \frac{2-0}{1-3}(x-3) = \boxed{-x+3}$ .

(5): Note that the axis of symmetry of the parabola is  $x = \frac{3}{2}$ .

Let the parabola be  $y = a(x - \frac{3}{2})^2 + b$ .

Substituting the coordinates of  $O$  and  $A$  into it, we have  $9a + 4b = 0$ .....(1) and

$a + 4b = 16$ .....(2) respectively.

By (1)-(2), we have  $8a = -16$ , i.e.  $a = -2$ .

Substitue it into (2), we have  $b = \frac{16+2}{4} = \frac{9}{2}$ .

Therefore, the parabola is  $y = -2(x - \frac{3}{2})^2 + \frac{9}{2} = \boxed{-2x^2 + 6x}$

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Q3:

(1): For  $|x|$ , we have  $|-x| = |x| \boxed{(b)}$ .

(2): For  $x$ , we have  $(-x) = -x \boxed{(a)}$  and  $(kx) = kx \boxed{(d)}$ .

(3): For  $x^2$ , we have  $(-x)^2 = x^2 \boxed{b}$ .

(4): For  $x^3$ , we have  $(-x)^3 = -x^3 \boxed{(a)}$ .

(5): For  $x - \frac{1}{x}$ , we have  $(\frac{1}{x}) - \frac{1}{\frac{1}{x}} = \frac{1}{x} - x = -(x - \frac{1}{x})$   $\boxed{(c)}$ .

(6): For  $\sin x$ , we have  $\sin(-x) = -\sin x$   $\boxed{(a)}$ .

(7): For  $\cos x$ , we have  $\cos(-x) = \cos x$   $\boxed{(b)}$ .

(8): For  $2^x$ , we have  $2^x \cdot 2^y = 2^{x+y}$   $\boxed{(e)}$ .

(9): For  $2^{-x}$ , we have  $2^{-x} \cdot 2^{-y} = 2^{-(x+y)}$   $\boxed{(e)}$ .

(10): For  $\log_2 \frac{1}{x}$ , we have  $\log_2 \frac{1}{x} = \log_2 1 - \log_2 x = -\log_2 x$   $\boxed{(c)}$ .