

Q1(1):

$$\begin{aligned} & \sqrt{5 - 2\sqrt{6}} - \frac{1}{\sqrt{2+\sqrt{3}}} \\ &= \sqrt{\sqrt{3}^2 - 2\sqrt{3}\sqrt{2} + \sqrt{2}^2} - \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} \\ &= \sqrt{(\sqrt{3} - \sqrt{2})^2} - \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2} \\ &= |\sqrt{3} - \sqrt{2}| - \frac{\sqrt{2}-\sqrt{3}}{2-3} \\ &= \sqrt{3} - \sqrt{2} + \sqrt{2} - \sqrt{3} \\ &= \boxed{0} \end{aligned}$$

Q1(2):

$$\begin{aligned} & (-2x^2y^3)^2 \div (-xy^2)^3 \\ &= \frac{2x^4y^6}{x^3y^6} \\ &= \boxed{2x} \end{aligned}$$

Q1(3):

$$\begin{aligned} & 4^x - 2^{x+1} - 15 = 0 \\ & 2^{2x} - 2 \cdot 2^x - 15 = 0 \\ & (2^x - 5)(2^x + 3) = 0 \\ & 2^x = 5 \text{ or } 2^x = -3(\text{rejected}) \\ & x = \boxed{\log_2 5} \end{aligned}$$

Q1(4):

$$2 \cos^2 x + 3 \sin x - 3 = 0$$

$$2(1 - \sin^2 x) + 3 \sin x - 3 = 0$$

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

$$(2 \sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{2} \text{ or } \sin x = 1$$

$$x = 30^\circ, 90^\circ \text{ or } \boxed{150^\circ} \text{ (as } 0^\circ \leq x \leq 180^\circ)$$

Q1(5):

Note that $a_2 - a_1 = 2 - 1 = 1$, $a_3 - a_2 = 5 - 2 = 3$, $a_4 - a_3 = 10 - 5 = 5$ and $a_5 - a_4 = 17 - 10 = 7$. It can hence be deduced that $a_{n+1} - a_n = 2n - 1$ for all $n \in \mathbb{Z}^+$, or $a_{n+1} = a_n + (2n - 1)$. Then, $a_8 = a_7 + 13 = a_6 + 11 + 13 = a_5 + 9 + 24 = 17 + 33 = \boxed{50}$.

Q1(6):

(i) By completing the square, $f(x) = x^2 - 2x + 1 - 1 - 3 = (x - 1)^2 - 4$. Hence, the coordinates of the vertex are $(\boxed{1}, \boxed{-4})$.

(ii) $f(x) = 0$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = \boxed{-1} \text{ or } x = \boxed{3}$$

(iii) When the graph of $y = f(x)$ is translated along the x-axis by 1 unit and the y-axis by 2 units, the graph becomes $y = f(x - 1) + 2$. The graph is same as that of $y = g(x)$ if and only if $g(x) = f(x - 1) + 2$. Then, we have $x^2 + ax + b = (x - 1)^2 - 2(x - 1) - 3 + 2 = x^2 - 2x + 1 - 2x + 2 - 1 = x^2 - 4x + 2$, which implies $a = \boxed{-4}$ and $b = \boxed{2}$.

(iv) Let α and β ($\alpha < \beta$) be the two roots of $g(x)$, i.e. $g(x) = (x - \alpha)(x - \beta)$.

Solving the system of inequalities $\begin{cases} f(x) < 0 \\ g(x) < 0 \end{cases}$, we have:

$$\begin{aligned} & \begin{cases} x^2 - 2x - 3 < 0 \\ (x - \alpha)(x - \beta) > 0 \end{cases} \\ \Leftrightarrow & \begin{cases} (x - 3)(x + 1) < 0 \\ x < \alpha \text{ or } x > \beta \end{cases} \\ \Leftrightarrow & \begin{cases} -1 < x < 3 \\ x < \alpha \text{ or } x > \beta \end{cases} \\ \Leftrightarrow & -1 < x < \alpha \text{ or } \beta < x < 3 \end{aligned}$$

When $-1 < x < 1$ or $2 < x < 3$, we have the two roots of $g(x)$ are 1 and 2,

i.e. $g(1) = g(2) = 0$. By solving the system of equations $\begin{cases} g(1) = 0 \\ g(2) = 0 \end{cases}$, we have

$$\begin{cases} 1 + a + b = 0 \dots (1) \\ 4 + 2a + b = 0 \dots (2) \end{cases}.$$

Subtract (1) from (2), we have $3 + a = 0$, i.e. $a = \boxed{-3}$. Then, substitute $a = -3$

into (1), we have $1 - 3 + b = 0$, i.e. $b = \boxed{2}$.

(v) As $f'(x) = 2x - 2$, we have $f'(2) = 2 \cdot 2 - 2 = \boxed{2}$.

On the other hand, $\int_0^3 f(x)dx = \int_0^3 (x^2 - 2x - 3)dx = \frac{1}{3}x^3 - x^2 - 3x \Big|_0^3 = \frac{3^3}{3} - 3^2 - 3 \cdot 3 - \frac{0^3}{3} + 0^2 + 3 \cdot 0 = 9 - 9 - 9 = \boxed{-9}$.

Q2:

(1) $\tan \alpha = \text{Slope of AB} = \boxed{3}$.

(2) **Lemma:** From the equality $\sin^2 x + \cos^2 x = 1$ for all $x \in \mathbb{R}$, dividing both sides by $\cos^2 x$, we can get the equality $\tan^2 x + 1 = \frac{1}{\cos^2 x}$ for all $x \in \mathbb{R}$.

We have $\sin^2 \alpha = 1 - \cos^2 \alpha = 1 - \frac{1}{\frac{1}{\tan^2 \alpha}}$. Then, by **Lemma**, we have $\sin^2 \alpha = 1 - \frac{1}{\tan^2 \alpha + 1} = 1 - \frac{1}{3^2 + 1} = 1 - \frac{1}{10} = \frac{9}{10}$. Hence, $\sin \alpha = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} = \boxed{\frac{3}{10}\sqrt{10}}$.

Alternative: Geometrically, consider the tangent ratio of $\triangle ABC$, we have $\tan \alpha = AC : AB = 3 : 1$. Then, we may let $AC = 3k$ and $AB = k$. By Pythagoras's theorem, we have $BC = \sqrt{k^2 + (3k)^2} = \sqrt{10}k$. Hence, by considering the sine ratio of $\triangle ABC$, we have $\sin \alpha = AC : BC = (3k) : (\sqrt{10}k) = \frac{3}{\sqrt{10}} = \boxed{\frac{3}{10}\sqrt{10}}$.

(3) We have $\cos \alpha = \frac{\sin \alpha}{\tan \alpha} = \frac{\frac{3}{10}\sqrt{10}}{3} = \frac{\sqrt{10}}{10}$.

On the other hand, we have the y-coordinate of A=the y-intercept of AB=4 and the x-coordinate of B=the x-intercept of AB= $-\frac{4}{3}$. Hence, we have $AB = \sqrt{(-\frac{4}{3})^2 + 4^2} = \frac{4}{3}\sqrt{1+9} = \frac{4}{3}\sqrt{10}$.

Combine the above two results, using the cosine ratio of $\triangle ABC$ we have

$$\cos \alpha = \frac{AB}{BC} = \frac{\sqrt{10}}{10}. \text{ Hence, we have } BC = \frac{10AB}{\sqrt{10}} = \frac{10 \cdot \frac{4}{3}\sqrt{10}}{\sqrt{10}} = \frac{40}{3}.$$

Then, as (the x-coordinate of C) - (the x-coordinate of B) = BC, we have the

$$\text{x-coordinate of C} = BC + (\text{the x-coordinate of B}) = \frac{40}{3} + \left(-\frac{4}{3}\right) = \frac{36}{3} = 12. \text{ Hence,}$$

the coordinates of C are $\boxed{(12, 0)}$.

Alternative: By doing part (4) first using the alternative method suggested,

we can get the equation of AC, which is $y = -\frac{1}{3}x + 4$, without using the coordinate of C. Then, the x-coordinate of C = the x-intercept of AC = 12. Hence, we

have the coordinates of C are $\boxed{(12, 0)}$.

(4) Consider the points A and C, we have the slope of AC = $\frac{4-0}{0-12} = -\frac{1}{3}$.

Using the slope-intercept form of straight line with the point A, we have the

equation fo AC is $\boxed{y = -\frac{1}{3}x + 4}.$

Alternative: As AB and AC are orthogonal to each other, we have the relation

$$(\text{Slope of AB})(\text{Slope of AC}) = -1. \text{ Then, we have the slope of AC} = -\frac{1}{\text{Slope of AB}} =$$

$-\frac{1}{3}$. Using the slope-intercept form of straight line with the point A, we have

the equation fo AC is $\boxed{y = -\frac{1}{3}x + 4}.$

(5) By using the mid-point theorem of vector, we have $\vec{AM} = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AC}.$

$$\text{Now, } \vec{MA} \cdot \vec{AC} = -\vec{AM} \cdot \vec{AC} = -\left(\frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AC}\right) \cdot \vec{AC} = -\frac{1}{2}(\vec{AB} \cdot \vec{AC} + \vec{AC} \cdot \vec{AC}).$$

As $\vec{AB} \perp \vec{AC}$, we have $\vec{AB} \cdot \vec{AC} = 0$. On the other hand, $\vec{AC} \cdot \vec{AC} = |\vec{AC}|^2 =$

$$\sqrt{3^2 + 12^2}^2 = 9 + 144 = 153. \text{ Therefore, we have } \vec{MA} \cdot \vec{AC} = -\frac{1}{2}(0 + 153) =$$

$$\boxed{-\frac{153}{2}}.$$