Q1(1):

As $2 = \sqrt{4} < \sqrt{7} < \sqrt{9} = 3$, we have $a = 2$ and $b = \sqrt{7} - 2$.

Then, $\frac{a}{b} = \frac{2}{\sqrt{7}-2} = \frac{2(\sqrt{7}+2)}{3}$ by rationalisation.

As $3 = \frac{2(\sqrt{6.25}+2)}{3} < \frac{2(\sqrt{7}+2)}{3} < \frac{2(\sqrt{9}+2)}{3} = \frac{10}{3} < 4$, we have the integer part of $\frac{a}{b}$ is $\boxed{3}$.

Alternative By some ways (calculating square root by hand, Newton's method, Taylor expansion, etc.), we have the approximate value of $\sqrt{7} \approx 2.65$. Then, $\frac{a}{b} \approx 3.08$ and the integer part is $\boxed{3}$.

Q1(2):

Let r be the radius of the inscribed sphere.

Consider the vertical section of the cone, by similarity*, we have

$$\frac{8-r}{\sqrt{(\frac{12}{2})^2 + 8^2}} = \frac{r}{\frac{12}{2}}$$

$$\frac{8-r}{10} = \frac{r}{6}$$

$$r = 3.$$

Therefore, the volume of the sphere= $\frac{4}{3}\pi(3)^3 = \boxed{36\pi}$.

(*: See 2007 Mathematics (B) Q2(1) for detail.)

Q1(3):

5^{29} can be written as $a_0 \cdot 10^n + a_1 \cdot 10^{n-1} + ... + a_n$, where $n, a_0, a_1, ..., a_n \in \mathbb{N}$ and $0 < a_0, a_1, ..., a_n \leq 9$.

Then, the number of places will be $n+1$.

As $\log_{10}(a_0 \cdot 10^n + a_1 \cdot 10^{n-1} + ... + a_n) < \log_{10}(10^{n+1}) = n+1$, we can find out the value of $n+1$ by considering the value of $\log_{10} 5^{29}$:

$\log_{10} 5^{29} = 29(\log_{10} 5)$

$= 29(\log_{10} \frac{10}{2})$

$= 29(1 - \log_{10} 2)$

$\approx 29(1 - 0.3010)$

$= 20.271$

Therefore, $n+1 = 21$ and the number of places is $\boxed{21}$.

Q1(4):

Note that the required distance is minimised when the line passing through the two points is perpendicular to the tangent to the circle at the point on the circle, i.e. the line passes through the center of the circle (the origin).

Then, the minimum distance=(the distance between the line and the origin)-(the radius)

$=\frac{|3(0)+4(0)-12|}{\sqrt{3^2+4^2}} - 2$

$= \frac{12}{5} - 1$

$= \boxed{\frac{2}{5}}$.

Alternative The equation of the circle is $x^2 + y^2 = 4$. Then, the distance between a point on the circle is given by $\frac{|3a+4b-12|}{\sqrt{3^2+4^2}} = \frac{|3a+4b-12|}{5}$ with the constrain $a^2 + b^2 = 4$.

By the Cauchy–Schwarz inequality, we have

$-10 = -\sqrt{(3^2+4^2)(a^2+b^2)} \leq 3a + 4b \leq \sqrt{(3^2+4^2)(a^2+b^2)} = 10$

Therefore, the minimum distance is $\frac{|10-12|}{5} = \boxed{\frac{2}{5}}$.

Alternative Follow the previous alternative, we may introduce the parameter θ such that $a = 2\sin\theta$ and $b = 2\cos\theta$.

Then, $3a + 4b = 10(\frac{3}{5}\sin\theta + \frac{2}{5}\cos\theta) = 10\sin(\theta + \alpha)$, where $\tan\alpha = \frac{2}{3}$*.

Therefore, $-10 \leq 3a + 4b \leq 10$ and the minimum distance is $\frac{|10-12|}{5} = \boxed{\frac{2}{5}}$.

(*: Search for rewriting $a\sin\theta + b\cos\theta$ in a form of $R\sin(\theta+\alpha)$.)

Q1(5):

$$a_k - 4a_{k-1} + 3a_{k-2} = 0$$

$$a_k - 3a_{k-1} = a_{k-1} - 3a_{k-2}$$

Therefore,

$$a_k - 3a_{k-1} = a_{k-1} - 3a_{k-2} = a_{k-2} - 3a_{k-3} = ... = a_2 - 3a_1 = -1$$

$$a_k = 3a_{k-1} - 1$$

Suppose $a_k - n = 3(a_{k-1} - n)$, then we have $-3n + n = -1$, i.e. $n = \frac{1}{2}$.

Therefore, $a_k - \frac{1}{2}$ is a geometric sequence with common ratio 3, i.e.

$$a_k - \frac{1}{2} = (a_1 - \frac{1}{2})3^{k-1}$$
$$a_k = \frac{1 + \boxed{3^{k-1}}}{\boxed{2}}.$$

Alternative The charactoristic equation of the recurrence is $\lambda^2 - 4\lambda + 3\lambda = 0$.

By solving, we have $\lambda_1 = 1$ and $\lambda_2 = 3$.

Therefore, $a_k = A(1)^k + B(3)^k = A + B(3)^k$.

By the initial conditions, we have the system of equations $\begin{cases} A + 3B = 1 \\ A + 9B = 2 \end{cases}$.

By solving, we have $A = \frac{1}{2}$ and $B = \frac{1}{6}$.

Therefore, $a_k = \frac{1+\boxed{3^{k-1}}}{\boxed{2}}$.

Q1(6):

Substitue $f(x) = ax + b$ into the equation:

$$\int_{-m/2}^{m} (ax+b)dx = \frac{m(m+1)}{2}$$
$$\frac{a}{2}x^2 + bx|_{-m/2}^{m} = \frac{m^2}{2} + \frac{m}{2}$$
$$\frac{a}{2}m^2 + bm - \frac{a}{8}m^2 + \frac{b}{2}m = \frac{m^2}{2} + \frac{m}{2}$$
$$\frac{3a}{2}m^2 + \frac{3b}{2}m = \frac{1}{2}m^2 + \frac{1}{2}m$$

By comparing the coefficients, we have $a = \frac{4}{3}$ and $b = \frac{1}{3}$.

Therefore, $f(x) = \frac{\boxed{4}x+\boxed{1}}{3}$.

Q2:

(1): Note that the area attains to its maximum when C lies above the centre (as for fixed base AB, the altitude is maximised by that time). By that time, the area of $\triangle ABC = \frac{1}{2}(4)(\frac{4}{2}) = \boxed{4}$.

(2): By considering the trigonometric ratio of $\triangle ABC$, we have $AC = AB\cos\angle CAB$ and $BC = AB\sin\angle CAB$.

By Thales' theorem, $\angle ACB = 90°$.

Therefore, the area of $\triangle ABC = \frac{1}{2}(4\cos\angle CAB)(4\sin\angle CAB) = 4\sin 2\angle CAB$.

When the area of $\triangle ABC$=$\frac{4}{2} = 2$,

$$4\sin 2\angle CAB = 2$$

$$\sin 2\angle CAB = \frac{1}{2}$$

$$2\angle CAB = 30° \text{ or } 150° \text{ (as } 0 < \angle CAB < 90°)$$

$$\angle CAB = 15° \text{ or } 75°$$

As C is nearer to point A than point B, we have $\angle CAB = \boxed{75°}$.

Q3:

(1): $y = (x^3 + \frac{1}{x^3}) - 6(x^2 + \frac{1}{x^2}) + 3(x + \frac{1}{x})$

$= (x + \frac{1}{x})^3 - 3(x)(\frac{1}{x})(x + \frac{1}{x}) - 6(x + \frac{1}{x})^2 + 12(x)(\frac{1}{x}) + 3(x + \frac{1}{x})$

$= \boxed{1}t^3 + \boxed{(-6)}t^2 + \boxed{0}t + \boxed{12}$

Moreover, by the AM-GM inequality, $t = x + \frac{1}{x} \geq 2\sqrt{(x)(\frac{1}{x})} = \boxed{2}$.

Alternative As $x^2 - tx + 1 = 0$ for all $x \in \mathbb{R}$, we have

$$\Delta = t^2 - 4 \geq 0$$

$$t \geq \boxed{2} \text{ or } t \leq -4 (\text{rejected as } x > 0)$$

Alternative $t' = 1 - \frac{1}{x^2}$.

To find the extremum of t, we set $t' = 0$. Then, we have $x = 1$.

Moreover, $t'' = \frac{2}{x^3} > 0$ for $x > 0$.

Therefore, we have $t \geq 1 + \frac{1}{1} = \boxed{2}$.

(2): $y' = 3t^2 - 12t$.

To find the extremum of y, we set $y' = 0$. Then, we have $t = 4$.

Moreover, $y'' = 6t > 0$ as $t \geq 2$.

Therefore, when $t = \boxed{4}$, that is $x + \frac{1}{x} = 4$, or $x = \boxed{2 \pm \sqrt{3}}$, y has the minimum value $4^3 - 6(4)^2 + 12 = \boxed{-20}$.