Q1(1):

From the indentity $\sin^2\theta + \cos^2\theta = 1$, we have $(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = 1$.

For the equation $\sqrt{2}x^2 - \sqrt{3}x + k = 0$, sum of roots$=-\frac{-\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}$ and product of roots$=\frac{k}{\sqrt{2}}$.

If the two roots can be written as $\sin\theta$ and $\cos\theta$, we have

$$(\sqrt{\frac{3}{2}})^2 - 2\frac{k}{\sqrt{2}} = 1$$

$$l = \boxed{\frac{\sqrt{2}}{4}}$$

Alternative Subtitue $x = \sin\theta$ and $x = \cos\theta$ into the two equations respectively, we have:

$\sqrt{2}\sin^2\theta = \sqrt{3}\sin\theta - k$

$\sqrt{2}\cos^2\theta = \sqrt{3}\cos\theta - k$

Adding two equations together, we have $\sqrt{2} = \sqrt{3}(\sin\theta + \cos\theta) - 2k$.

On the other hand, as the sum of roots$=\sqrt{\frac{3}{2}}$, we have $\sqrt{2} = \frac{3}{\sqrt{2}} - 2k$, i.e. $k = \boxed{\frac{1}{2\sqrt{2}}}$.

Q1(2):

Consider the binomial expansion of $(x^3 + \frac{a}{x^2})^5$, which is

$\sum\limits_{k=0}^{5} C_k^5 (x^3)^k (\frac{a}{x^2})^{5-k} = \sum\limits_{k=0}^{5} C_k^5 (a^{5-k}) x^{5k-10}$, where C_r^n is the binomial coefficient.

For the constant term, the index of x equals to 0. Solving $5k - 10 = 0$, we have $k = 2$, i.e. the constant term is obtained when $k = 2$.

When $k=2$, the term$=C^5_2 a^3 = 10a^3$.

Therefore, the have $10a^3 = -270$, i.e. $a = \boxed{-3}$.

Q1(3):

As the relation provided holds independent on the value of x, we can put any values of x we want expect $-\frac{1}{2}$ and $\frac{3}{2}$.

Note that $p = 2$ and $p = -\frac{2}{3}$ does not satisfy the relation $f(g(x)) = x$.

Put $x = -\frac{1}{p}$, then $g(x) = 0$, the relation becomes

$$f(0) = -\frac{1}{p}$$

$$1 = -\frac{1}{p}$$

$$p = \boxed{-1}$$

Alternative

$$f(g(x)) = x$$

$$\frac{3(\frac{px+1}{2x-3})+1}{2(\frac{px+1}{2x-3})+1} = x$$

$$\frac{(3p+2)x}{(2p+2)x-1} = x$$

$$(2p+2)x^2 - (3p+3)x = 0$$

As the relation holds independent on the value of x, we have $2p+2 = 0$ and $3p+3 = 0$, which gives us $p = \boxed{-1}$.

Q1(4):

Note that the hidden condition for $\log_2 x$ and $\log_2(x-2)$ to be defined is $x > 2$.

On the other hand,

$$\log_2 x + \log_2(x-2) < 4\log_{16} 8$$

$$\log_2(x^2 - 2x) < \log_2 8$$

$$x^2 - 2x - 8 < 0$$

$$(x-4)(x+2) < 0$$

$$-2 < x < 4$$

Combine the above, we have $\boxed{2 < x < 4}$.

Q1(5):

Consider the prime factorisation of $600 = 2^3 \times 3 \times 5^2$.

Therefore, all positive divisors of 600 are in a form of $2^i \times 3^j \times 5^k$, where $i, j, k \in \mathbb{N}$

satisfy $0 \leq i \leq 3$, $0 \leq j \leq 1$ and $0 \leq k \leq 2$.

By finding the number of combinations of i, j, k, we have the total number of

positive divisors of $600 = 4 \cdot 2 \cdot 3 = \boxed{24}$.

On the other hand, the sum of all divisors$=\sum_{i=0}^{3}\sum_{j=0}^{1}\sum_{k=0}^{2} 2^i \times 3^j \times 5^k$

$= (\sum_{i=0}^{3}\sum_{j=0}^{1} 2^i \times 3^j)(5^0 + 5^1 + 5^2)$

$= (\sum_{i=0}^{3} 2^i)(3^0 + 3^1)(5^0 + 5^1 + 5^2)$

$= (2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1)(5^0 + 5^1 + 5^2)$

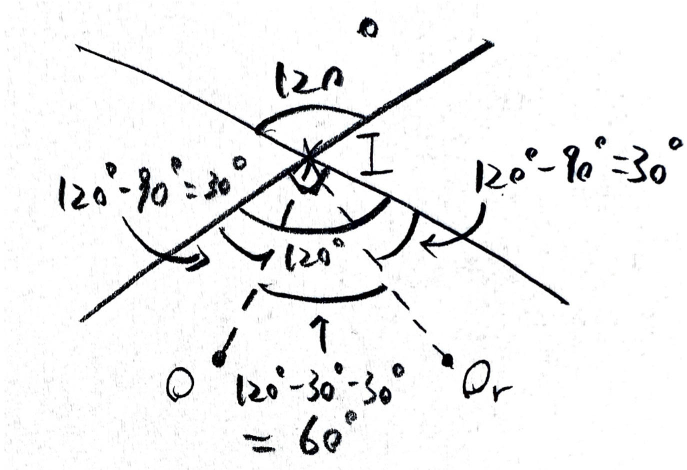
$= (15)(4)(31)$

$= \boxed{1860}$.

Q2:

(1): Denote the centres of C and C_r as O and O_r respectively. In addition, denote the point of intersection as I.

Refer to the sketch:



The angle between two radii is equal to 60°. By the cosine formula, the distance between two centres$=\sqrt{1^2+r^2-2(1)(r)\cos 60°} = \boxed{\sqrt{r^2-r+1}}$.

(2): By completing the square, $d = \sqrt{(r-\frac{1}{2})^2+\frac{3}{4}}$.

Therefore, when $(r-\frac{1}{2})^2 = 0$, i.e. $r = \boxed{\frac{1}{2}}$, the distance is minimised.

Alternative $d' = \frac{2r-1}{2\sqrt{r^2-r+1}}$.

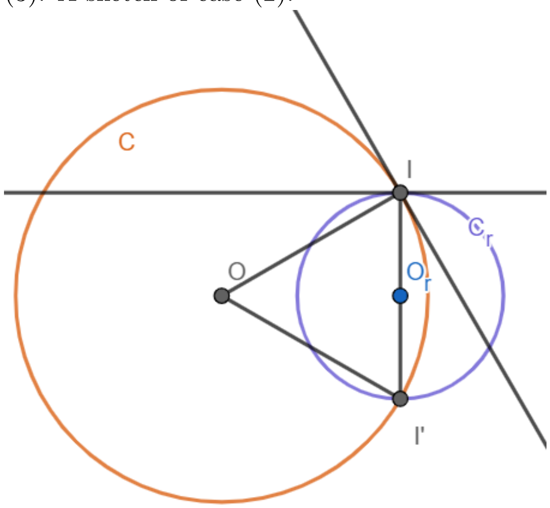
The find the extremum of d, we set $d' = 0$. Then, we have $r = \frac{1}{2}$.

The table of first derivative test is given:

r	$(0, \frac{1}{2})$	$(\frac{1}{2}, +\infty)$
d'	$-$	$+$
d	$\searrow$	$\nearrow$

Therefore, d attains to its minimum when $r = \boxed{\dfrac{1}{2}}$.

(3): A sketch of case (2):



The required area=(The area of sector (semi-circle) $\widehat{IO_rI'}$)+(The area of sector $\widehat{IOI'}$)-(The area of $\triangle IOI'$)

Note that every interior angle of $\triangle IOI'$ equals to $60°$.

Then, the area=$(\frac{1}{2}\pi(\frac{1}{2})^2) + (\pi(1)^2 \cdot \frac{60°}{360°}) - (\frac{1}{2}(1)(1)\sin 60°)$

$= \frac{\pi}{8} + \frac{\pi}{6} - \frac{\sqrt{3}}{4}$
$= \boxed{\dfrac{7\pi}{24} - \dfrac{\sqrt{3}}{4}}$.

Alternative Introduce coordinates: Set $O(0,0)$ be the orgin, then $O_r = (\frac{\sqrt{3}}{2}, 0)$ by (1) and (2).

Now, the equation of C is $x^2 + y^2 = 1$ and that of C_r is $(x - \frac{\sqrt{3}}{2})^2 + y^2 = \frac{1}{4}$.

The x-coordinate of their point of intersections will be given by

$x^2 - 1 = (x - \frac{\sqrt{3}}{2})^2 - \frac{1}{4}$. By solving, we have $x = \frac{\sqrt{3}}{2}$.

Moreover, the x-intercepts of C are ± 1 and that of C_r are $\frac{\sqrt{3}}{2} \pm \frac{1}{2}$.

Now, we consider only the upper part (i.e. that above the x-axis) of the region.

Then, by symmetry, the total area will be twice of it.

Therefore, the area$=2(\int_{\frac{\sqrt{3}}{2}-\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \sqrt{\frac{1}{4} - (x - \frac{\sqrt{3}}{2})^2} dx + \int_{\frac{\sqrt{3}}{2}}^{1} \sqrt{1 - x^2} dx)$

$= 2(\frac{1}{4} \int_{-\frac{\pi}{2}}^{0} \cos^2 \theta d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 \theta d\theta)$

$= 2(\frac{1}{4}[\frac{1}{2}\theta + \frac{\sin 2\theta}{4}]_{-\frac{\pi}{2}}^{0} + [\frac{1}{2}\theta + \frac{\sin 2\theta}{4}]_{\frac{\pi}{3}}^{\frac{\pi}{2}})$

$= 2(\frac{\pi}{16} + \frac{\pi}{12} - \frac{\sqrt{3}}{8})$

$= \boxed{\dfrac{7\pi}{24} - \dfrac{\sqrt{3}}{4}}$.

Q3:

(1): $y = 8^x - 9 \cdot 4^x + 15 \cdot 2^x$

$= (2^x)^3 - 9 \cdot (2^x)^2 + 15 \cdot 2x$

$= \boxed{X^3 - 9X^2 + 15X}$.

(2): $y' = 3X^2 - 18X + 15 = 3(X - 5)(X - 1)$.

To find the extremum of y, we set $y' = 0$, then $X = 1$ or $X = 5$.

$y'' = 6X - 18$.

Conduct the second derivative test:

When $X = 1$, $y'' = -12 < 0$

When $X = 5$, $y'' = 12 > 0$

Therefore, the local maximum is obtained when $X = \boxed{1}$, and the value of it is

$1 - 9 + 15 = \boxed{7}$.

The local minimum is obtained when $X = \boxed{5}$, and the value of it is

$125 - 225 + 75 = \boxed{-25}$.

(3): $0 \leq x \leq \log_2 7 \iff 1 \leq X \leq 7$

When $X = 1$, maximum is obtained by the result of (2).

When $X = 7$, $y = 7(49 - 63 + 15) = 7$

Combine the above with the result of (2), we have:

The global maximum is $\boxed{7}$, at $X = 1$ or 7, i.e. $x = \boxed{0 \text{ or } \log_2 7}$.

The global maximum is $\boxed{-25}$, at $X = 5$, i.e. $x = \boxed{\log_2 5}$.