

Q1(1):

$$|x + 3| < 4x$$

$$-4x < x + 3 < 4x$$

$$x > -\frac{5}{2} \text{ and } x > 1$$

$$\boxed{x > 1}$$

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Q1(2):

The solutions are:

$$(x, y, z) = (0, 0, 4), (0, 1, 3), (0, 2, 2), (0, 3, 1), (0, 4, 0),$$

$$(1, 0, 3), (1, 1, 2), (1, 2, 1), (1, 3, 0),$$

$$(2, 0, 2), (2, 1, 1), (2, 2, 0),$$

$$(3, 0, 1), (3, 1, 0),$$

$$(4, 0, 0)$$

Where there are totally  $\boxed{15}$  solutions.

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Q1(3):

The centre of the circle is the mid-point of OA, i.e. (3,4).

The radius of the circle =  $\frac{OA}{2} = \frac{\sqrt{6^2+8^2}}{2} = 5$ .

Therefore, the equation of the circle is  $(x - \boxed{3})^2 + (y - \boxed{4})^2 = \boxed{5}^2$ .

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Q1(4):

$$\log_4 9 = \frac{\log_2 9}{\log_2 4} = \log_2 9^{\frac{1}{2}} = \log_2 \boxed{3}.$$

Similarly,  $\log_9 4 = \log_3 \boxed{2}.$

$$\text{Hence, } (\log_2 3 + \log_4 9)(\log_3 2 + \log_9 4) = (2 \log_2 3)(2 \log_3 2) = \boxed{4}.$$

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Q1(5):

$$\sqrt[6]{25} \times \sqrt[3]{25} \div \sqrt{5} = \frac{\sqrt[3]{5} \cdot \sqrt[3]{5^2}}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \boxed{\sqrt{5}}.$$

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Q1(6):

We have:

$$\begin{cases} a + ar + ar^2 = a \frac{r^3-1}{r-1} = 14.....(1) \\ ar + ar^2 + ar^3 = ar \frac{r^3-1}{r-1} = -42.....(2) \end{cases}$$

By (2)÷(1), we have  $r = \boxed{-3}.$

Substitue it into (1), we have  $7a = 14$ , i.e.  $a = \boxed{2}.$

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Q1(7):

By  $|\vec{c}| = 3$ , we have  $x^2 + y^2 + z^2 = 9.....(1).$

As  $\vec{c} \perp \vec{a}$ , we have  $\vec{c} \cdot \vec{a} = x - z = 0.....(2).$

Similarly, we have  $\vec{c} \cdot \vec{b} = -2x + 2y + z = 0.....(3).$

Substitue (2) and (3) into (1), we have  $x^2 + (\frac{x}{2})^2 + x^2 = 9$ , i.e.  $x = \boxed{2}$ .

And then,  $y = \frac{x}{2} = \boxed{1}$  and  $z = x = \boxed{2}$

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Q1(8):

By the cosine formula,  $\cos \angle ABC = \frac{BA^2 + BC^2 - AC^2}{2(BA)(BC)} = \frac{36 + 64 - 16}{96} = \boxed{\frac{7}{8}}$ .

$$AM^2 = BA^2 + BM^2 - 2(BA)(BM) \cos \angle ABC$$

$$AM = \sqrt{36 + 16 - 2(6)(4)(\frac{7}{8})} = \boxed{\sqrt{10}}$$

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Q1(9):

As  $f'(x) = -2x + 1$ , the slope of the tangent  $= f'(0) = 1$ .

By the point-slope form of straight line, the tangent is  $y - 2 = 1(x - 0)$ , i.e.

$$y = \boxed{x + 2}.$$

The x-intercepts of the parabola are -1 and 2.

The x-intercept of the tangent is -2.

Therefore, the area  $= \int_{-2}^{-1} (x + 2) dx + \int_{-1}^2 ((x + 2) - (-x^2 + x + 2)) dx$

$$= [\frac{1}{2}x^2 + 2x]_{-2}^{-1} + [\frac{1}{3}x^3]_{-1}^2$$

$$= \frac{1}{2} + \frac{1}{3}$$

$$= \boxed{\frac{5}{6}}$$

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Q2:

(1): As  $\angle ACD = 90^\circ$ , the diameter is AD.

Therefore, the radius  $= \frac{AD}{2} = \frac{\frac{DC}{\cos \angle ADC}}{2} = \boxed{1}$ .

(2):  $BC = AC = AD \sin \angle ADC = 2 \sin 60^\circ = \sqrt{3}$ .

Therefore,  $AB = \sqrt{\sqrt{3}^2 + \sqrt{3}^2} = \sqrt{6}$  and the radius  $= \frac{AB}{2} = \boxed{\frac{\sqrt{6}}{2}}$ .

(3): Consider the area of  $\triangle ABC$ , we have:

$$\frac{1}{2}(AC)(BC) = \frac{1}{2}(AC)(r) + \frac{1}{2}(BC)(r) + \frac{1}{2}(AB)(r)$$

$$\frac{3}{2} = \left( \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{6}}{2} \right) r$$

$$r = \frac{3}{2\sqrt{3} + \sqrt{6}} = \boxed{\frac{2\sqrt{3} - \sqrt{6}}{2}}$$

(4): Note that  $\angle ABC = 45^\circ$ .

Therefore,  $DH = BD \sin \angle ABC = (\sqrt{3} - 1) \sin 45^\circ = \boxed{\frac{\sqrt{6} - \sqrt{2}}{2}}$ .

(5): Consider the sine ratio, we have  $\sin \angle DAH = \frac{DH}{DA} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$ .

(6):  $\cos \angle DAH = \sqrt{1 - \sin^2 \angle DAH}$

$$= \sqrt{1 - \left( \frac{8 - 2\sqrt{12}}{16} \right)}$$

$$= \frac{1}{4} \sqrt{8 + 2\sqrt{12}}$$

$$= \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

Q3:

(1): A straight line with positive slope and x-intercept 2. The equation matches it is  $\boxed{(9)}$ .

(2): A straight line with positive slope and y-intercept 2. The equation matches it is  $\boxed{(5)}$ .

(3): A parabola convex downwards and with y-intercept 2. The equation matches it is  $\boxed{(13)}$ .

(4): A parabola convex upwards and with x-intercept 2. The equation matches it is  $\boxed{(15)}$ .

(5): A circle with centre (2,2) and radius 2. The equation is  $(x-2)^2 + (y-2)^2 = 4$ , i.e.  $x^2 - 4x + y^2 - 4y + 4 = 0$   $\boxed{(7)}$ .