Q1(1):

$2x^2 - 3x - 2 \leq 0$

$(x-2)(2x+1) \leq 0$

$\boxed{\frac{1}{2} \leq x \leq 2}$

Q1(2):

By the cosine formula, $BC^2 = AB^2 + CA^2 - 2(AB)(CA)\cos\angle A$

$$4^2 = 6^2 + 5^2 - 2(6)(5)\cos\angle A$$

$$\cos\angle A = \boxed{\frac{3}{4}}.$$

Moreover, using the identity $\sin^2\theta + \cos^2\theta = 1$, we have $\sin^2\angle A = 1 - \cos^2\angle A = 1 - \frac{9}{16} = \frac{7}{16}$.

As $0^\circ < \angle A < 180^\circ$, we have $\sin\angle A > 0$ and hence $\sin\angle A = \boxed{\frac{\sqrt{7}}{4}}$.

Alternative: Construct the perpendicular foot of B on AC, denote the point of intersection as D. Note that $CD = CA - AD = 5 - AD$. Using Pythagoras' theorem twice, we have $AD^2 + BD^2 = AB^2$, i.e. $BD^2 = 36 - AD^2$ and $CD^2 + BD^2 = BC^2$, i.e. $BD^2 = 16 - (5 - AD)^2$. Combine the two equations:

$$36 - AD^2 = 16 - (5 - AD)^2$$

$$5(2AD - 5) = 20$$

$$AD = \frac{9}{2}.$$

Substitute it back to the former equation, we have $BD = \sqrt{36 - AD^2} = \sqrt{36 - \frac{81}{4}} = \frac{3\sqrt{7}}{2}$. Now, by considering the sine ratio and cosine ratio of $\triangle ADB$, we have $\cos A = \frac{AD}{AB} = \frac{\frac{9}{2}}{6} = \boxed{\dfrac{3}{4}}$ and $\sin A = \frac{BD}{AB} = \frac{\frac{3\sqrt{7}}{2}}{6} = \boxed{\dfrac{\sqrt{7}}{4}}$

Q1(3):

$$2^x 4^y = 32 \iff 2^x 2^{2y} = 2^5 \iff 2^{x+2y} = 2^5 \iff x + 2y = 5......(1)$$

$$\frac{3^x}{9^y} = 3 \iff \frac{3^x}{3^{2y}} = 3 \iff 3^{x-2y} = 3 \iff x - 2y = 1......(2)$$

(1)+(2):

$$2x = 6$$

$$x = 3$$

Substitue $x = 3$ into (1), $y = \frac{5-3}{2} = 1$.

Then, $\frac{5^x}{125^y} = \frac{5^3}{125^1} = \boxed{1}$.

Q1(4):

$5\log_2\sqrt{2} - \frac{1}{2}\log_2 3 + \log_2\frac{\sqrt{3}}{2}$

$= \frac{5}{2} - \log_2\sqrt{3} + \log_2\sqrt{3} - \log_2 2$

$= \frac{5}{2} - 1$

$= \boxed{\dfrac{3}{2}}$

Q1(5):

Note that $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{(\sqrt{x}+\sqrt{y})^2}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} = \frac{(x+y)+2\sqrt{xy}}{x-y}$.

When $x > y$, $x - y = \sqrt{(x-y)^2} = \sqrt{(x+y)^2 - 4xy}$. Therefore, $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{(x+y)+2\sqrt{xy}}{\sqrt{(x+y)^2-4xy}}$.

If $x + y = 5$ and $xy = 1$, $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{5+2\sqrt{1}}{\sqrt{5^2-4(1)}} = \boxed{\dfrac{7}{\sqrt{21}}}$.

(Note: Retionalisation is not necessary. After rationalisation, the answer will become $\boxed{\dfrac{\sqrt{21}}{3}}$.)

Alternative: If $x + y = 5$ and $xy = 1$, we have

$\sqrt{x} + \sqrt{y} = \sqrt{(\sqrt{x}+\sqrt{y})^2} = \sqrt{(x+y)+2\sqrt{xy}} = \sqrt{5+2(1)} = \sqrt{7}$.

Moreover, as $x > y$, we have

$\sqrt{x} - \sqrt{y} = \sqrt{(\sqrt{x}-\sqrt{y})^2} = \sqrt{(x+y)-2\sqrt{xy}} = \sqrt{5-2(1)} = \sqrt{3}$.

Then, $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \boxed{\dfrac{\sqrt{7}}{\sqrt{3}}}$.

Q2:

(1): Note that $\triangle ABH \sim \triangle MAH$. Therefore,

$\triangle ABH : \triangle AHM = (AB : AM)^2 = (AB : (\frac{1}{2}AC))^2 = (2 : 1)^2 = \boxed{4 : 1}$

(2): As $\triangle ABP$ and $\triangle APC$ share a common altitude, the ratio of their areas equal to the ratio of their base lengths, i.e.

$BP : PC = \triangle ABP : \triangle APC = (\frac{1}{2}(AB)(AP)\sin\angle PAB) : (\frac{1}{2}(AP)(AC)\sin\angle PAC)$

$= \sin\angle PAB : \sin\angle PAC$.

On the other hand, $\triangle ABH : \triangle AHM = (\frac{1}{2}(AB)(AH)\sin\angle HAB) : (\frac{1}{2}(AH)(AM)\sin\angle HAC)$

$= ((AB)\sin\angle PAB) : ((\frac{1}{2}AC)\sin\angle PAC) = 2(\sin\angle PAB : \sin\angle PAC)$.

Therefore, $BP : PC = \sin\angle PAB : \sin\angle PAC = \frac{1}{2}(\triangle ABH : \triangle AHM)$

$= \frac{1}{2}(4 : 1) = \boxed{2 : 1}$.

Alternative to (2) (pure geometry) See MEXT's official solution.

Alternative (coordinates) Introduce the coordinate system such that $A(0,0)$ is the origin, and set the other points be $B(2,0)$, $C(0,2)$ and $M(0,1)$.

(1): As $\triangle ABH$ and $\triangle AHM$ share a common altitude, the ratio of their areas equal to the ratio of their base lengths, i.e. $\triangle ABH : \triangle AHM = BH : HM$.

The slope of line $MB = \frac{0-1}{2-0} = -\frac{1}{2}$. As $AH \perp MB$, the slope of line $AH = \frac{-1}{(\text{Slope of } MB)} = \frac{-1}{-\frac{1}{2}} = 2$.

The equation of MB is $y = -\frac{1}{2}x + 1$ and the equation of AH is $y = 2x$. Solving the simultaneous equations give the coordinates of H, which are $(\frac{2}{5}, \frac{4}{5})$.

Therefore, $BH : HM = \sqrt{(2 - \frac{2}{5})^2 + (0 - \frac{4}{5})^2} : \sqrt{(0 - \frac{2}{5})^2 + (1 - \frac{4}{5})^2}$

$= \frac{4\sqrt{5}}{5} : \frac{\sqrt{5}}{5} = 4 : 1$.

Hence, the required ratio$= \boxed{4 : 1}$.

(2) The requation of line CB is $y = -x + 2$. Solving the simutaneous equation of it and the equation of line AH, we have the coordinates of P are $(\frac{2}{3}, \frac{4}{3})$.

Therefore, $BP : PC = \sqrt{(2 - \frac{2}{3})^2 + (0 - \frac{4}{3})^2} : \sqrt{(0 - \frac{2}{3})^2 + (2 - \frac{4}{3})^2}$

$= \frac{4\sqrt{2}}{3} : \frac{2\sqrt{2}}{3} = \boxed{2 : 1}$.

Alternative (vector) Set $\vec{AB} = \vec{i}$, $\vec{AC} = \vec{j}$ and $\vec{k}$ be three orthogonal unit vectors (as the basis vectors). Then, $\vec{AM} = \frac{1}{2}\vec{j}$ and $\vec{BM} = \vec{AM} - \vec{AB} = \frac{1}{2}\vec{j} - \vec{i}$.

(1) As H lies on line BM, $\vec{AH} = t\vec{AB} + (1-t)\vec{AM} = t\vec{i} + \frac{1-t}{2}\vec{j}$ for a constant $t \in \mathbb{R}$.

As $\vec{AH} \perp \vec{BM}$, we have $\vec{AH} \cdot \vec{BM} = 0$, i.e. $-t + \frac{1-t}{4} = 0$, $t = \frac{1}{5}$. Therefore, $\vec{AH} = \frac{1}{5}\vec{i} + \frac{2}{5}\vec{j}$.

$\triangle ABH : \triangle AMH = \frac{1}{2}|\vec{AB} \times \vec{AH}| : \frac{1}{2}|\vec{AH} \times \vec{AM}|$

$= \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ \frac{1}{5} & \frac{2}{5} & 0 \end{vmatrix} \right\| : \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & \frac{1}{2} & 0 \end{vmatrix} \right\|$

$= \frac{2}{5} : \frac{1}{10}$

$= \boxed{4:1}$

(2) Let $BP : PC = \lambda : (1-\lambda)$ for a $\lambda \in \mathbb{R}$.

Then, $\vec{AP} = (1-\lambda)\vec{AB} + \lambda\vec{AC} = (1-\lambda)\vec{i} + \lambda\vec{j}$.

As $\vec{AP} // \vec{AH}$, we have $\frac{1-\lambda}{\frac{1}{5}} = \frac{\lambda}{\frac{2}{5}}$, i.e. $\lambda = \frac{2}{3}$.

Therefore, $BP : PC = \frac{2}{3} : (1 - \frac{2}{3}) = \boxed{2:1}$.

Q3:

By long division, we have $\frac{n^2+8n+10}{n+9} = (n-1) + \frac{19}{n+9}$. As $n-1$ is an integer, a_n can be written as $a_n = (n-1) + [\frac{19}{n+9}]$.

Note that for $n = 1$ to $n = 10$, we have $1 \leq \frac{19}{n+9} < 2$, i.e. $[\frac{19}{n+9}] = 1$.

On the other hand, for $n = 11$ to $n = 30$, we have $0 < \frac{19}{n+9} < 1$, i.e. $[\frac{19}{n+9}]$.

Therefore, $\sum\limits_{n=1}^{30} a_n$

$$
\begin{aligned}
&= \sum_{n=1}^{30} ((n-1) + [\tfrac{19}{n+9}]) \\
&= \sum_{n=1}^{30} (n-1) + \sum_{n=1}^{10} [\tfrac{19}{n+9}] + \sum_{n=11}^{30} [\tfrac{19}{n+9}] \\
&= \tfrac{(30+1)(30)}{2} - 30 + \sum_{n=1}^{10} 1 + \sum_{n=11}^{30} 0 \\
&= 31 \cdot 15 - 30 + 10 + 0 \\
&= \boxed{445}.
\end{aligned}
$$