

Q1(1):

$$\log_5 0.008 = \log_5 (8 \times 10^{-3}) = \log_5 5^{-3} = \boxed{-3}.$$

$$(\sqrt[6]{16})^3 = \sqrt{16} = \boxed{4}$$

---

Q1(2):

$$\sin 75^\circ + \sin 120^\circ - \cos 150^\circ + \cos 165^\circ$$

$$= \cos 15^\circ + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} - \cos 15^\circ$$

$$= \boxed{\sqrt{3}}$$

---

Q1(3):

$$\text{By partial fraction, } \sum_{i=1}^4 \frac{1}{(3i)(3(i+1))} = \frac{1}{9} \sum_{i=1}^4 \left( \frac{1}{i} - \frac{1}{i+1} \right).$$

$$\text{Therefore, by the telescoping property, } \sum_{i=1}^4 \frac{1}{(3i)(3(i+1))} = \frac{1}{9} \left( 1 - \frac{1}{5} \right) = \boxed{\frac{4}{45}}.$$

---

Q1(4):

$$-x < x^2 < 6$$

$$x(x+1) > 0 \text{ and } (x - \sqrt{6})(x + \sqrt{6}) < 0$$

$$(x < -1 \text{ or } x > 0) \text{ and } (-\sqrt{6} < x < \sqrt{6})$$

$$-\sqrt{6} < x < -1 \text{ or } 0 < x < \sqrt{6}$$

As  $2 < \sqrt{6} < 3$ , the integers satisfying the inequality are  $-2, 1, 2$ , totally  $\boxed{3}$  integers.

---

Q1(5):

The first digit is choosed among 5,6,7,8,9 (totally 5 choices) and the remaining digits are the permutation of 3 numbers among the remaining 9 numbers (totally  $P_3^9=504$  choices). Therefore, there are  $5 \cdot 504 = \boxed{2520}$  such integers.

---

Q1(6):

Consider the dot products between  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{a} + \vec{b} + \vec{c}$ , we have:

$$\begin{cases} 1 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0 \\ \vec{a} \cdot \vec{b} + 1 + \vec{b} \cdot \vec{c} = 0 \\ \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} + 1 = 0 \end{cases}$$

By solving, we have  $\vec{a} \cdot \vec{b} = -\frac{1}{2}$ .

Therefore, the angle between  $\vec{a}$  and  $\vec{b} = \arccos \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = 120^\circ$ .

$$|\vec{a} - \vec{b}| = \sqrt{(\vec{a} - \vec{b})^2} = \sqrt{2 - 2\vec{a} \cdot \vec{b}} = \boxed{\sqrt{3}}.$$


---

Q1(7):

The progression is given by the recurrence

$$a_{n+1} - a_n = 2^{n-1}$$

$$a_{n+1} - a_1 = \frac{2^n - 1}{2 - 1}$$

$$a_{n+1} = 2^n + 2$$

$$a_n = 2^{n-1} + 2$$

Therefore,  $a_8 = 2^{n-1} + 2 = \boxed{130}$ .

Solving  $2^{n-1} + 2 = 1026$ , we have  $n = \boxed{11}$ .

Q1(8):

(i):  $f(-2) = 4 + 8 + 1 = \boxed{13}$ .

(ii):

$$x^2 - 4x + 1 = 0$$

$$x = \boxed{2 \pm \sqrt{3}}$$

(iii): Let  $\alpha = 2 - \sqrt{3}$  and  $\beta = 2 + \sqrt{3}$ .

Then, the area  $= -\int_{\alpha}^{\beta} (x^2 - 4x + 1) dx$

$$= -\left[\frac{1}{3}x^3 - 2x^2 + x\right]_{\alpha}^{\beta}$$

$$= -\left(\frac{1}{3}((\beta - \alpha)^3 + 3\alpha\beta(\beta - \alpha)) - 2(\beta + \alpha)(\beta - \alpha) + (\beta - \alpha)\right)$$

$$= -\left(\frac{1}{3}(24\sqrt{3} + 6\sqrt{3}) - 16\sqrt{3} + 2\sqrt{3}\right)$$

$$= \boxed{4\sqrt{3}}$$

Q1(9):

$\vec{AB} = \langle -1, -2, 1 \rangle$  and  $\vec{AC} = \langle 2, 2, 0 \rangle$ .

The area  $= \frac{1}{2} |\vec{AB} \times \vec{AC}|$

$$= \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -2 & 1 \\ 2 & 2 & 0 \end{vmatrix}$$

$$= \frac{1}{2} | \langle -2, 2, 2 \rangle |$$

$$= \frac{1}{2} (2\sqrt{3})$$

$$= \boxed{\sqrt{3}}$$

---

Q2:

(1): Consider  $\triangle ABC$ , by the sine formula, we have

$$2R = \frac{BC}{\sin \angle BAC}$$

$$2R = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}}$$

$$R = \boxed{1}$$

(2): By the cosine formula, we have

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos \angle BAC$$

$$3 = 5AB^2 - 2AB^2$$

$$AB = 1$$

Therefore,  $AC = \boxed{2}$ .

(3):  $\angle BDC = \angle BAC = \boxed{60^\circ}$ .

(4): By the cosine formula, we have

$$BC^2 = BD^2 + DC^2 - 2(BD)(DC) \cos \angle BDC$$

$$BD^2 = 3$$

$$\text{Therefore, } \triangle BDC = \frac{1}{2}(BD)(DC) \sin \angle BDC = \frac{1}{2}(BD^2) \sin 60^\circ = \boxed{\frac{3\sqrt{3}}{4}}.$$

(5): Note that AC is the diameter. We have  $\angle ADC = 90^\circ$  and  $DA^2 = AC^2 - DC^2 = 1$ .

Therefore,  $\vec{DC} \cdot \vec{CA} = -CD \cdot CA \cos \angle CDA$

$$= -\frac{CD^2 + CA^2 - DA^2}{2}$$

$$= -\frac{3 + (2)^2 - 1}{2}$$

$$= \boxed{-3}$$

Q3:

(1): As the parabola is convex upwards, we have  $a \boxed{<} 0$ .

(2): As the parabola has two x-intercepts, we have  $\Delta = b^2 - 4ac > 0$ .

Therefore,  $4ac - b^2 \boxed{<} 0$ .

(3): When  $x = 1$ , the parabola takes value smaller than 0.

Therefore, we have  $a + b + c \leq 0$ .

(4): When  $x = -2$ , the parabola takes value smaller than 0.

Therefore, we have  $4a - 2b + c \leq 0$ .

(5): As the y-intercept of the parabola is smaller than 0, we have  $c < 0$ .

Therefore,  $\frac{c}{a} \geq 0$ .

(6): By completing the square,  $y = a(x + \frac{b}{2a})^2 + c - \frac{b^2}{4a}$ .

As the axis of symmetry  $x = -\frac{b}{2a}$  lies in the region  $x > 0$ , we have  $-\frac{b}{2a} > 0$ ,

i.e.  $\frac{b}{a} \leq 0$ .

(7): As the axis of symmetry is  $x = -\frac{b}{2a} > 2$ , we have  $b + 4a \geq 0$ .

(8): As  $b + 4a > 0$  and  $a < 0$ , we have  $b + 2a > -2a \geq 0$ .