

Electromagnetism

(1) Point Charges

-2006 1(2)

Two point charges $+q$ and $-q$ are placed on the x -axis with a separation d as shown in Figure 1. Point P forms a right-angle triangle with the two charges. What is the direction and the magnitude of the electric field acting on P? The Coulomb constant is k .

Direction: (a) **A** (b) **B** (c) **C** (d) **D**

Magnitude: (a) $k \frac{q}{d}$ (b) $k \frac{2q}{d}$ (c) $k \frac{\sqrt{3}q}{d}$ (d) $k \frac{q}{d^2}$ (e) $k \frac{2q}{d^2}$ (f) $k \frac{\sqrt{3}q}{d^2}$

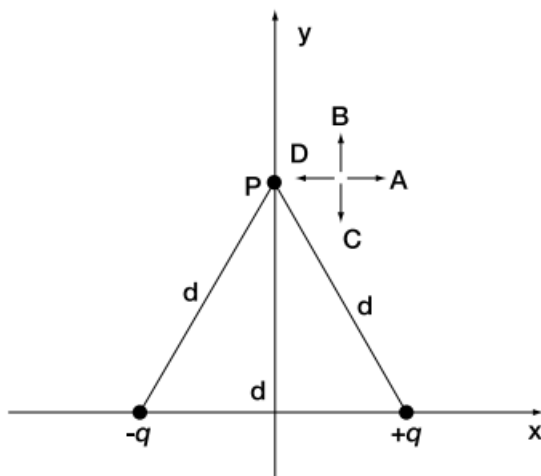


Figure 1

- 2 There are 4 points, A, B, C, and D on a smooth horizontal plane as shown in figure 1. $AC=BC=CD=a$, and AB and CD are mutually perpendicular. First we assume that a charge $+q$ is placed at A and a charge $-q$ is placed at B. $q > 0$ and the Coulomb constant is k .

- (1) Find the direction of the electric field at point D.
 (a) $A \rightarrow B$ (b) $B \rightarrow A$ (c) $C \rightarrow D$ (d) $D \rightarrow C$
- (2) Find the magnitude of the electric field at point D.
 (a) $\frac{kq}{a^2}$ (b) $\frac{kq}{\sqrt{2}a^2}$ (c) $\frac{kq}{2a^2}$
 (d) $\frac{kq}{a}$ (e) $\frac{kq}{\sqrt{2}a}$ (f) $\frac{kq}{2a}$
- (3) What is the relation between the electric potential V_C at point C and the electric potential V_D at point D?
 (a) $V_C > V_D$ (b) $V_C = V_D$ (c) $V_C < V_D$

Next, we assume that a charge $+q$ is placed at both points A and B, and a particle of mass m , charge $-q$ moves on the straight line including CD.

- (4) The particle is initially at rest at point C. Find the work required to move the particle slowly from point C to D.
 (a) $\frac{2kq}{a^2}$ (b) $\frac{(2-\sqrt{2})kq}{a^2}$ (c) $\frac{(2+\sqrt{2})kq}{a^2}$
 (d) $\frac{2kq}{a}$ (e) $\frac{(2-\sqrt{2})kq}{a}$ (f) $\frac{(2+\sqrt{2})kq}{a}$
- (5) The particle at point D starts to move with initial speed v toward the point C and reaches the point of infinity. Find the minimum speed v that is required to make the particle reach the point of infinity.

- (a) $\sqrt{\frac{2kq^2}{ma^2}}$ (b) $\sqrt{\frac{2\sqrt{2}kq^2}{ma^2}}$ (c) $\sqrt{\frac{4kq^2}{ma^2}}$
 (d) $\sqrt{\frac{2kq^2}{ma}}$ (e) $\sqrt{\frac{2\sqrt{2}kq^2}{ma}}$ (f) $\sqrt{\frac{4kq^2}{ma}}$

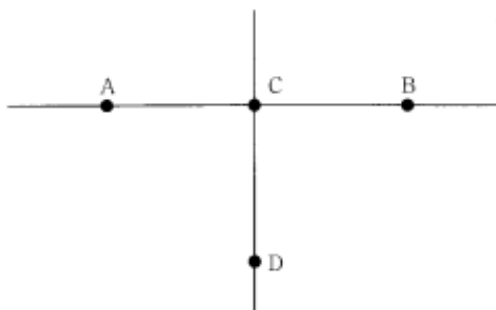


Fig. 1

(3) A charge Q is placed at the center of a spherical-shell conductor as in Fig. 2.

Both the radius of the inner shell and the thickness of the conductor are R .

Which of the following is correct?

- (a) A charge of amount $-Q$ is distributed uniformly on the inner surface of the conductor.
- (b) A charge of amount $-Q$ is distributed uniformly on the outer surface of the conductor.
- (c) A charge of amount Q is distributed uniformly on the inner surface of the conductor.
- (d) A charge of amount Q is distributed uniformly on the outer surface of the conductor.
- (e) Charges of amount $-Q/2$ are distributed uniformly both on the inner and outer surfaces of the conductor.
- (f) Charges of amount $Q/2$ are distributed uniformly both on the inner and outer surfaces of the conductor.
- (g) Charges of amount $-Q$ and Q are distributed uniformly on the inner and outer surfaces of the conductor, respectively.
- (h) Charges of amount Q and $-Q$ are distributed uniformly on the inner and outer surfaces of the conductor, respectively.
- (i) No charge appears in the conductor.

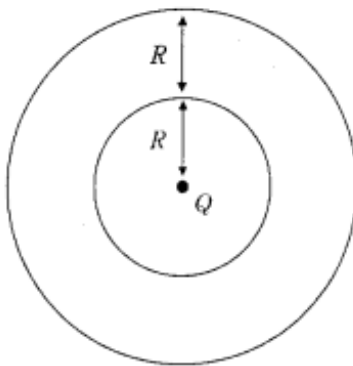
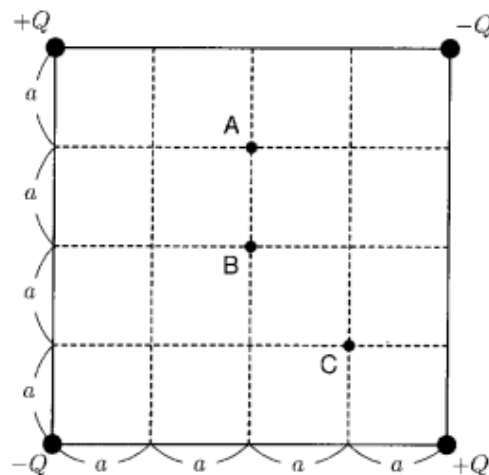


Fig. 2

- A** As shown in the figure below, four electrical point charges ($+Q$, $-Q$, $+Q$, and $-Q$, where $Q > 0$) are fixed to the corners of a square insulator plate (side length: $4a$).



- Q1 What is the relationship between the level of electrical potential V_A , V_B , and V_C at points A, B, and C in the figure? From ①-⑦ below choose the correct answer. 14

- | | |
|---------------------|---------------------|
| ① $V_A = V_B < V_C$ | ② $V_A = V_B > V_C$ |
| ③ $V_A < V_B = V_C$ | ④ $V_A > V_B = V_C$ |
| ⑤ $V_A = V_C < V_B$ | ⑥ $V_A = V_C > V_B$ |
| ⑦ $V_A < V_B < V_C$ | |

-2013 1(3)

(3) In Fig. 1-3, ABC is a rectangular equilateral triangle. An electric charge q is placed at A and an electric charge q' at B. The electric field at C is found to be parallel to the side AB, as shown by the arrow in the figure. Choose the appropriate formula from below which shows the relation between q and q' correctly.

- | | | |
|----------------------------------|--------------------------------|---------------------------------|
| (a) $q = q'$ | (b) $q = -q'$ | (c) $q = \sqrt{2}q'$ |
| (d) $q = -\sqrt{2}q'$ | (e) $q = \frac{1}{\sqrt{2}}q'$ | (f) $q = -\frac{1}{\sqrt{2}}q'$ |
| (g) $q = \frac{1}{2}q'$ | (h) $q = -\frac{1}{2}q'$ | (i) $q = \frac{1}{2\sqrt{2}}q'$ |
| (j) $q = -\frac{1}{2\sqrt{2}}q'$ | (k) $q = \frac{1}{4}q'$ | (l) $q = -\frac{1}{4}q'$ |

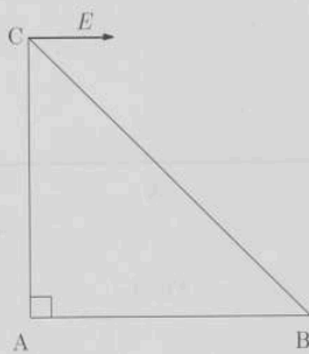


Fig. 1-3

-2015 1(4)

(4) A point particle of mass m and charge $q(>0)$ approaches a point particle of charge $Q(>0)$ at a fixed position. When the distance between the two particles is L , the speed of the moving particle is v . The permittivity of the vacuum is denoted as ϵ_0 . Find the minimum distance between the two particles.

- | | | |
|-------------------------------------|---|--|
| (a) $\frac{Q}{q}L$ | (b) $\frac{qQL}{qQ + 2\pi\epsilon_0mv^2}$ | (c) $\frac{qQL}{qQ + 2\pi\epsilon_0mv^2L}$ |
| (d) $\frac{qQ}{2\pi\epsilon_0mv^2}$ | (e) $\frac{2qQL}{2qQ + \epsilon_0mv^2L}$ | (f) $\frac{2qQ}{\epsilon_0mv^2}$ |

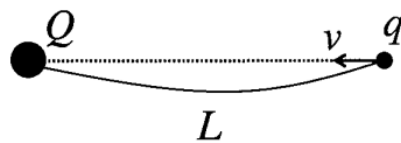


Fig. 1-2

-2018 1(3)

- (3) A charged particle of mass m and charge q is in a uniform electric field E . Initially the particle is at rest, and then accelerated by the electric field. Find the time for the particle to travel at a distance of d from the initial location.

(a) $\frac{md}{2qE}$	(b) $\sqrt{\frac{md}{qE}}$	(c) $\frac{2md}{qE}$
(d) $\sqrt{\frac{md}{2qE}}$	(e) $\frac{md}{qE}$	(f) $\sqrt{\frac{2md}{qE}}$

-2020 2

2. As shown in Fig. 2, two particles, each of charge q ($q > 0$), are fixed at A $(-a, 0)$ and B $(a, 0)$ ($a > 0$) in the x - y plane. A proportionality constant of the Coulomb's law is denoted as k .

- (1) Find the magnitude of the electric field at C $(0, a)$.

(a) $\frac{kq}{\sqrt{2}a^2}$	(b) $\frac{kq}{\sqrt{2}a}$	(c) $\frac{\sqrt{2}kq}{a}$
(d) $\frac{\sqrt{2}kq}{a^2}$	(e) $\frac{kq}{a^2}$	(f) $\frac{kq}{a}$

- (2) Find the electric potential at C. Note that the potential is zero at infinity.

(a) $\frac{kq}{\sqrt{2}a^2}$	(b) $\frac{kq}{\sqrt{2}a}$	(c) $\frac{\sqrt{2}kq}{a}$
(d) $\frac{\sqrt{2}kq}{a^2}$	(e) $\frac{kq}{a^2}$	(f) $\frac{kq}{a}$

Then, a particle with a charge $-q$ and a mass m is placed at C.

- (3) Find the magnitude of the force acting on the particle at C.

(a) $\frac{kq^2}{a}$	(b) $\frac{\sqrt{2}kq^2}{a}$	(c) $\frac{kq^2}{\sqrt{2}a}$
(d) $\frac{kq^2}{a^2}$	(e) $\frac{\sqrt{2}kq^2}{a^2}$	(f) $\frac{kq^2}{\sqrt{2}a^2}$

- (4) The particle at C starts to move from rest and passes through the origin O. Find the speed of the particle at O.

(a) $\sqrt{\frac{\sqrt{2}kq^2}{ma}}$	(b) $\sqrt{\frac{2\sqrt{2}kq^2}{ma}}$	(c) $\sqrt{\frac{(2-\sqrt{2})kq^2}{ma}}$
(d) $\sqrt{\frac{2(2-\sqrt{2})kq^2}{ma}}$	(e) $\sqrt{\frac{(\sqrt{2}-1)kq^2}{2ma}}$	(f) $\sqrt{\frac{2(\sqrt{2}-1)kq^2}{ma}}$

- (5) When the particle at C has an initial speed v_0 in the negative y -direction, it escapes from the influence of the Coulomb forces from the particles at A and B and moves to infinity. Find the minimum value of v_0 .

$$\begin{array}{lll}
 \text{(a)} \quad \sqrt{\frac{\sqrt{2}kq^2}{ma}} & \text{(b)} \quad \sqrt{\frac{2\sqrt{2}kq^2}{ma}} & \text{(c)} \quad \sqrt{\frac{(2-\sqrt{2})kq^2}{ma}} \\
 \text{(d)} \quad \sqrt{\frac{2(2-\sqrt{2})kq^2}{ma}} & \text{(e)} \quad \sqrt{\frac{(\sqrt{2}-1)kq^2}{2ma}} & \text{(f)} \quad \sqrt{\frac{2(\sqrt{2}-1)kq^2}{ma}}
 \end{array}$$

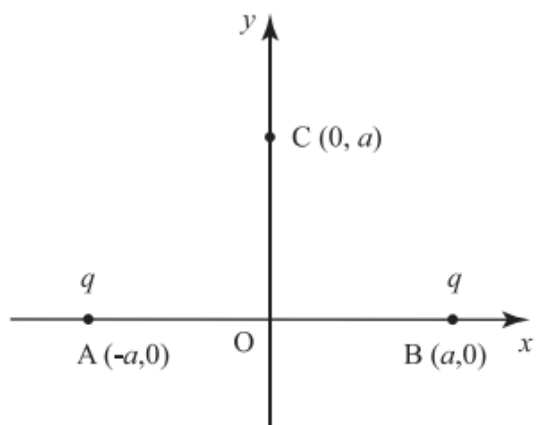


Fig. 2

(2) Circuit

-2006 2

As shown in Figure 3, there is a circuit consisted of a battery with $E=6[V]$, two switches S_1 and S_2 , two resistors $R_1=4[\Omega]$ and $R_2=2[\Omega]$, and a capacitor with $C=2[\mu F]$. The internal resistance of the battery is negligible. Initially, both the two switches are opened, and the capacitor contains no charges. The switch S_1 is closed at a certain moment. After a sufficient time, the capacitor is fully charged, and the circuit becomes stable.

(1) Right after the switch S_1 is closed, what is the current passes through the resistor R_1 ?

- (a) $1.5[A]$ (b) $3[A]$ (c) $6[A]$ (d) $12[A]$ (e) $24[A]$

(2) How many charges are stored in capacitor C ?

- (a) $6[\mu C]$ (b) $12[\mu C]$ (c) $36[\mu C]$ (d) $72[\mu C]$ (e) $144[\mu C]$

(3) Find the work done by the battery during the charging of capacitor C .

- (a) $6[\mu J]$ (b) $12[\mu J]$ (c) $36[\mu J]$ (d) $72[\mu J]$ (e) $144[\mu J]$

(4) Find the Joule heat generated in the resistor R_1 during the charging of capacitor C .

- (a) $6[\mu J]$ (b) $12[\mu J]$ (c) $36[\mu J]$ (d) $72[\mu J]$ (e) $144[\mu J]$

While the switch S_1 is closed, the switch S_2 is also closed. After the switch S_2 has been closed for a sufficient time, the circuit becomes stable again. After the switch S_2 is closed for a sufficient time, how many charges are stored in capacitor C ?

- (a) $4[\mu C]$ (b) $6[\mu C]$ (c) $12[\mu C]$ (d) $24[\mu C]$ (e) $36[\mu C]$

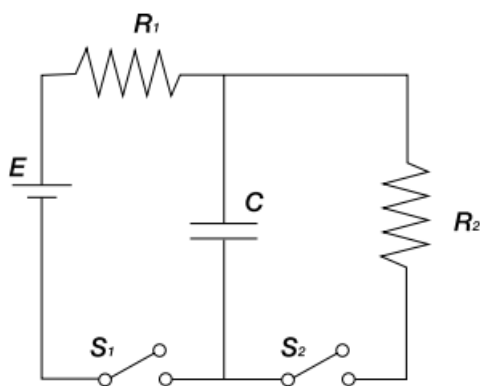


Figure 3

-2006 4(4)

Let E and R_0 be the voltage across the heater and the resistance of the heat. How much heat is evolved per unit time? (Choose the suitable option)

(a) E

(b) E/R_0

(c) E^2/R_0^2

(d) E^2/R_0

(e) E/R_0^2

-2007 1(2)

(2) Consider the circuit shown in Fig.2, consisting of four resistors of resistances R_1, R_2, R_3, R_4 and a battery of voltage E . Find the voltage that the high-resistance voltmeter V shows.

(a) $\frac{R_1 R_2 - R_3 R_4}{(R_1 + R_2)(R_3 + R_4)} E$

(b) $\frac{R_1 R_4 - R_2 R_3}{(R_1 + R_3)(R_2 + R_4)} E$

(c) $\frac{R_1 R_3 - R_2 R_4}{(R_1 + R_4)(R_2 + R_3)} E$

(d) $\frac{R_1 R_2 + R_3 R_4}{(R_1 + R_2)(R_3 + R_4)} E$

(e) $\frac{R_1 R_4 + R_2 R_3}{(R_1 + R_3)(R_2 + R_4)} E$

(f) $\frac{R_1 R_3 + R_2 R_4}{(R_1 + R_4)(R_2 + R_3)} E$

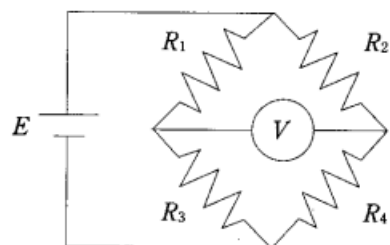


Fig. 2

2 Consider the circuit shown in Fig.5, consisting of a battery of voltage E , a switch S , and a parallel-plate capacitor with capacitance C . The capacitor consists of two parallel conducting plates of equal area A separated by a distance d . After the switch S is closed and the capacitor is fully charged, a conducting plate of thickness $d/3$ and area A is inserted slowly between the plates of the capacitor. The inserted conducting plate is kept parallel to the conducting plates of the capacitor. Select answers to the questions from (a) to (z) below, and write the symbol of the answer in the box.

- (1) Find the capacitance of the capacitor after the conducting plate is inserted.
- (2) How much is the increase in the charge stored in the capacitor caused by inserting the conducting plate?
- (3) How much is the increase in the energy stored in the capacitor caused by inserting the conducting plate?
- (4) How much work is done by the battery during the insertion of the conducting plate?
- (5) How much work is done by the force applied to the conducting plate during its insertion?
- (6) Next, the switch S is opened, and the conducting plate is removed slowly. How much work is done by the force applied to the conducting plate to remove it?

- (a) $\frac{1}{4}C$ (b) $\frac{3}{8}C$ (c) $\frac{1}{2}C$ (d) $\frac{3}{4}C$ (e) $\frac{3}{2}C$
 (f) $\frac{1}{8}CE$ (g) $\frac{1}{4}CE$ (h) $\frac{3}{8}CE$ (i) $\frac{1}{2}CE$ (j) CE
 (k) $\frac{3}{2}CE$ (l) $2CE$ (m) $-\frac{1}{8}CE^2$ (n) $-\frac{1}{4}CE^2$ (o) $-\frac{3}{8}CE^2$
 (p) $-\frac{1}{2}CE^2$ (q) $-CE^2$ (r) $-\frac{3}{2}CE^2$ (s) $-2CE^2$ (t) $\frac{1}{8}CE^2$
 (u) $\frac{1}{4}CE^2$ (v) $\frac{3}{8}CE^2$ (w) $\frac{1}{2}CE^2$ (x) CE^2 (y) $\frac{3}{2}CE^2$
 (z) $2CE^2$

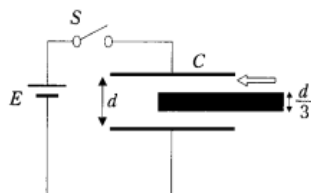


Fig. 5

-2008 1(2)

- (2) An electric charge Q is stored in a parallel-plate capacitor with capacitance C . The distance between the plates in this capacitor is tripled, keeping the electric charge unchanged. How much work is done from outside in this process?

- (a) $3CQ$ (b) $2CQ$ (c) CQ (d) $\frac{1}{2}CQ$ (e) $\frac{1}{3}CQ$
(f) $\frac{3Q^2}{C}$ (g) $\frac{2Q^2}{C}$ (h) $\frac{Q^2}{C}$ (i) $\frac{Q^2}{2C}$ (j) $\frac{Q^2}{3C}$

-2010 IVB

- B** The parallel-plate capacitor shown in Figure 1 consists of two plates (each with area S) separated by a distance of $2d$. A charge of Q is stored in the capacitor. Next, as shown in Figure 2, an uncharged conductor of thickness d and area S is inserted between the capacitor's plates, parallel to them.

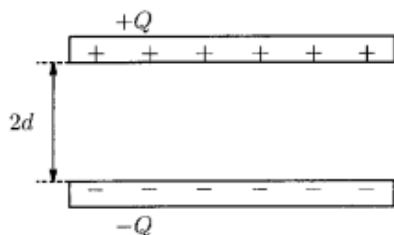


Figure 1

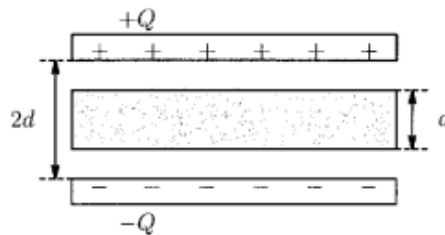


Figure 2

- Q2 What is the ratio of the electrostatic energy in Figure 2 to the electrostatic energy in Figure 1? From ①-⑤ below choose the correct value.

15

- ① $\frac{1}{4}$ ② $\frac{1}{2}$ ③ 1
④ 2 ⑤ 4

-2010 IVC

- C** Figure 1 shows the relationship between voltage and current when voltage is applied to a certain light bulb. Next, as shown in Figure 2, the bulb is connected in series with a 12-V battery and a $5\text{-}\Omega$ resistor.

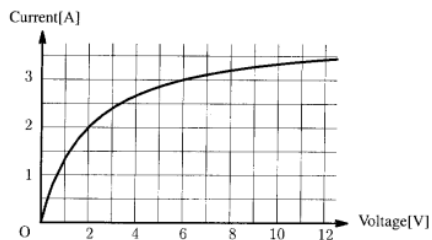


Figure 1

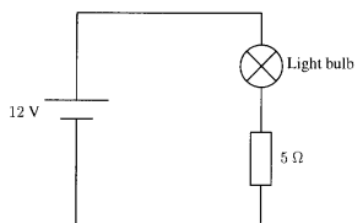


Figure 2

- Q3 What is the amount of power consumed by the light bulb? From ①-⑥ below choose the best answer. 16 W

- | | | |
|------|------|------|
| ① 1 | ② 4 | ③ 10 |
| ④ 18 | ⑤ 34 | ⑥ 42 |

-2012 1(2)

- (2) Consider the circuit shown in Fig. 1-2, consisting of two resistors of resistances R_1 and R_2 , a capacitor of capacitance C , a battery of voltage E , and a switch S . Find the charge accumulated in the capacitor after the switch has been closed for a sufficient period of time.

- | | | |
|--------------------------|-----------------------------|-----------------------------|
| (a) $\frac{CE}{R_1+R_2}$ | (b) $\frac{CE}{R_1}$ | (c) $\frac{CE}{R_2}$ |
| (d) $\frac{CE}{R_1+R_2}$ | (e) $\frac{R_1}{R_1+R_2}CE$ | (f) $\frac{R_2}{R_1+R_2}CE$ |

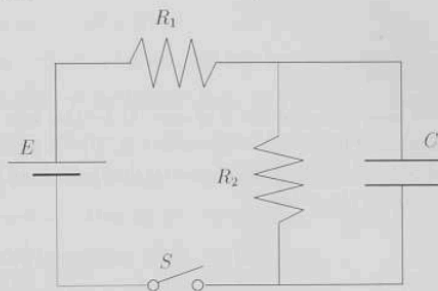


Fig. 1-2

2 Consider the circuit shown in Fig. 2-1, consisting of three capacitors C_1 , C_2 , C_3 , two resistors R_1 , R_2 , two switches S_1 , S_2 , and a battery V . The capacitance of the capacitors C_1 and C_3 is 2×10^{-6} [F], and the capacitance of the capacitor C_2 is 4×10^{-6} [F]. The voltage of the battery V is 6 [V]. The internal resistance of the battery and the resistance of the switches may be ignored. Find numerical values appropriate for the open boxes from (1) to (5) from a list shown below. The same numerical value may be used more than once.

At the beginning, three capacitors were fully discharged. The switch S_1 was then closed while the switch S_2 was kept open. A long time after the switch S_1 was closed, the charge accumulated in the capacitor C_1 was $\times 10^{-6}$ [C], and the difference in voltage between the two plates of the capacitor C_1 was [V]. Next, the switch S_2 was closed while the switch S_1 was opened. A long time after the switch S_2 was closed, the charge accumulation in the capacitor C_3 was $\times 10^{-6}$ [C]. The sum of the energy stored in the capacitors C_2 and C_3 was $\times 10^{-6}$ [J]. The total Joule heat generated in the resistor R_2 after the switch S_2 was closed amounts to $\times 10^{-6}$ [J].

- | | | |
|--------------------|--------------------|--------------------|
| (a) 2 | (b) 4 | (c) 8 |
| (d) 16 | (e) 32 | (f) $\frac{2}{3}$ |
| (g) $\frac{4}{3}$ | (h) $\frac{8}{3}$ | (i) $\frac{16}{3}$ |
| (j) $\frac{32}{3}$ | (k) $\frac{2}{9}$ | (l) $\frac{4}{9}$ |
| (m) $\frac{8}{9}$ | (n) $\frac{16}{9}$ | (o) $\frac{32}{9}$ |

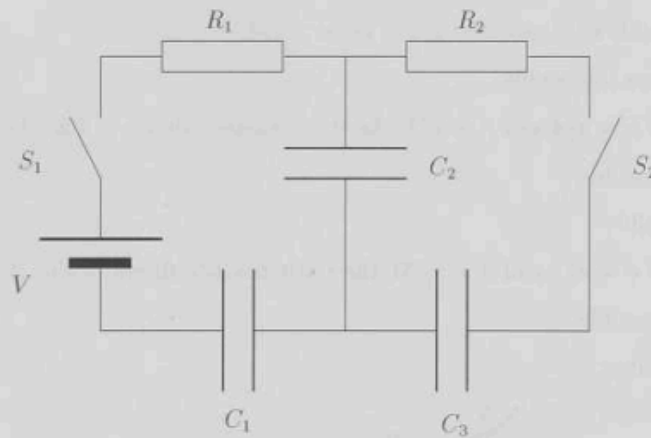


Fig. 2-1

-2014 1(4)

- (4) Consider the circuit shown in Fig. 1-4, consisting of three capacitors C_1 , C_2 , and C_3 , a battery of voltage E , and a switch S . Initially the capacitors are uncharged. Find the charge that has accumulated in the capacitor C_1 after the switch has been closed for a sufficient period of time.

- (a) $(C_1 + C_3)E$ (b) $\frac{C_1 C_3}{C_1 + C_2 + C_3}E$ (c) $\frac{C_1 C_2}{C_1 + C_2 + C_3}E$
 (d) $(C_1 + C_2 + C_3)E$ (e) $\frac{C_1 C_3}{C_1 + C_2}E$ (f) $\frac{C_1 C_2}{C_3}E$

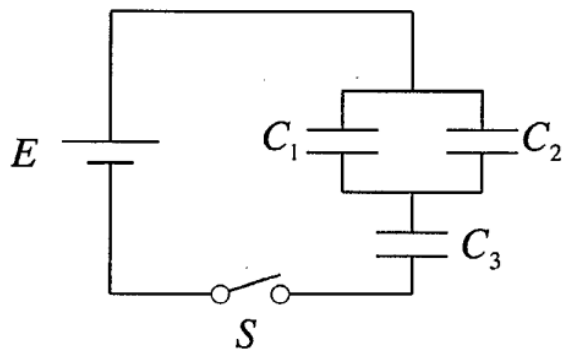


Fig. 1-4

-2016 1(3)

- (3) A parallel-plate capacitor with a plate separation d has a capacitance C . The capacitor is charged to Q and then disconnected from the battery as shown in Fig. 1-1. Find the electric field between the plates.

- (a) $\frac{Q}{C}$ (b) $\frac{C}{Qd}$ (c) Cd
 (d) $\frac{C}{d}$ (e) $\frac{Q}{d}$ (f) $\frac{Q}{Cd}$

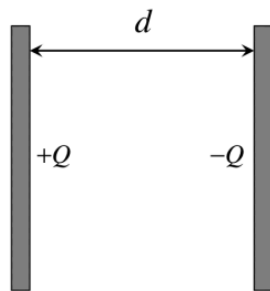


Fig. 1-1

2. A parallel-plate capacitor with a plate separation of d as shown in Fig. 2 (a) has a capacitance C in the absence of a dielectric. Answer the following questions:

- (1) When the potential difference between the two plates is V , how much charge is on each capacitor.

- (a) $4CV$ (b) $\frac{1}{2}CV$ (c) CV^2
 (d) CV (e) $\frac{1}{2}CV^2$ (f) $\frac{1}{2}C^2V$

- (2) Find the electric field created between the two plates in the case of (1).

- (a) V^2/d (b) V/d (c) V/d^2
 (d) CV/d (e) C/d (f) C/d^2

- (3) A slab of dielectric of relative permittivity ε and thickness $d/2$ is inserted between plates as shown in Fig. 2 (b). This capacitor is equivalent to two capacitors C_1 and C_2 connected in a series as shown in Fig. 2 (c). Find the capacitance of C_1 in Fig. 2 (c).

- (a) εC (b) $(\varepsilon + 1)C$ (c) C/ε
 (d) $2C/\varepsilon^2$ (e) $2\varepsilon C$ (f) $2C/(\varepsilon + 1)$

- (4) Find the capacitance of the capacitor shown in Fig. 2 (b).

- (a) $\frac{2\varepsilon}{\varepsilon + 1}C$ (b) $4\varepsilon C$ (c) $2(\varepsilon + 1)C$
 (d) $\frac{\varepsilon + 1}{2\varepsilon}C$ (e) $\frac{\varepsilon}{\varepsilon + 1}C$ (f) $\frac{2\varepsilon + 1}{\varepsilon}C$

- (5) Two dielectrics with relative permittivity ε_1 and ε_2 each fill half the space between the two plates as shown in Fig. 2 (d). Find the capacitance of this capacitor.

- (a) $(\varepsilon_1 + \varepsilon_2)C$ (b) $\frac{\varepsilon_1 \varepsilon_2}{2(\varepsilon_1 + \varepsilon_2)}C$ (c) $\frac{\varepsilon_1 + \varepsilon_2}{2}C$
 (d) $\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_2}C$ (e) $\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 \varepsilon_2}C$ (f) $\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1}C$

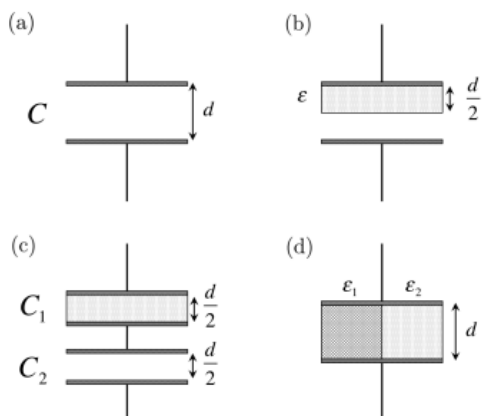


Fig. 2

2. Consider the circuit shown in Fig. 2-1, consisting of a resistor R , a capacitor C , an inductor (coil) L , a switch S , and a battery with voltage V . At the beginning, the switch is in position **b** and the capacitor is uncharged. Then the switch is changed to position **a**. After having been in position **a** for a long time, the switch is changed to position **c**. Answer the following questions.

(1) Find the current I_0 through the resistor just after the switch is changed to position **a**.

- (a) CV (b) $\frac{1}{2}CV$ (c) $\frac{V}{3R}$
 (d) $\frac{V}{2R}$ (e) $\frac{V}{R}$ (f) 0

(2) Find the charge on the capacitor after having been in position **a** for a long time.

- (a) $C(V + RI_0)$ (b) $C(V - RI_0)$ (c) $\frac{1}{2}CV^2$
 (d) CV^2 (e) $\frac{1}{2}CV$ (f) CV

(3) Find the Joule heat generated in the resistor by the time the capacitor is fully charged.

- (a) 0 (b) $\frac{1}{2}CV^2$ (c) $\frac{1}{3}CV^2$
 (d) $\frac{1}{4}CV^2$ (e) $\frac{2}{3}CV^2$ (f) CV^2

(4) At time $t = 0$, the switch is moved from position **a** to position **c**. How does the current I_L , which flows into the inductor (coil), change as a function of time? Find one appropriate graph in Fig. 2-2.

(5) Find the maximum value of I_L .

- (a) $\sqrt{\frac{C}{L}}V$ (b) $\sqrt{\frac{L}{C}}V$ (c) $\sqrt{LC}V$
 (d) $\frac{1}{\sqrt{LC}}V$ (e) $\frac{V}{R}$ (f) $\frac{V}{2R}$

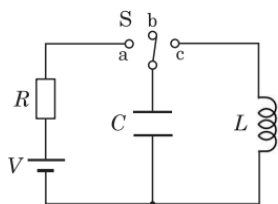


Fig. 2-1

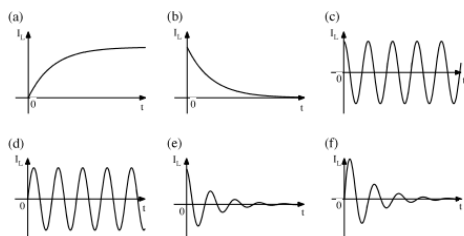


Fig. 2-2

- (3) A parallel-plate capacitor has plates of area S and separation d and is charged to a potential difference V . The charging battery is then disconnected, and the plates are pulled apart until their distance becomes $d + \Delta d$. The vacuum permittivity is denoted as ε_0 . Find the work required to separate the plates.

(a) $\frac{V^2 d}{\varepsilon_0 S} \Delta d$

(b) $\frac{V^2 d}{2\varepsilon_0 S} \Delta d$

(c) $\frac{\varepsilon_0 S V}{d^2} \Delta d$

(d) $\frac{\varepsilon_0 S V}{2d^2} \Delta d$

(e) $\frac{\varepsilon_0 S V^2}{d^2} \Delta d$

(f) $\frac{\varepsilon_0 S V^2}{2d^2} \Delta d$

(3) Magnetism

-2009 2

2 An overview of a mass spectrometer is shown in Fig. 4. The whole spectrometer is placed in a vacuum. A positive ion (ion with positive charge) of mass M , charge q , and speed v emitted from the ion source passes through the slits S_1 , S_2 , S_3 , keeping a constant velocity. Between the slits S_2 and S_3 , a uniform electric field of magnitude E from a parallel plate capacitor PQ and a uniform magnetic field of magnitude B_1 apply to the ion. After passing through the slit S_3 , the ion moves circularly under a uniform magnetic field of magnitude B_2 and collides with the photographic plate.

- (1) Which of the following is a correct explanation for the force acting on a charged particle under a uniform magnetic field?
- (a) The direction of the force lies in the plane composed by the magnetic field and the velocity, and the magnitude of the force is independent of the speed.
 - (b) The direction of the force lies in the plane composed by the magnetic field and the velocity, and the magnitude of the force is proportional to the speed.
 - (c) The direction of the force is perpendicular to the plane composed by the magnetic field and the velocity, and the magnitude of the force is independent of the speed.
 - (d) The direction of the force is perpendicular to the plane composed by the magnetic field and the velocity, and the magnitude of the force is proportional to the speed.

- (2) Which is the direction of the uniform magnetic field acting in the region of parallel plate capacitor PQ where the positive ion moves straight with a constant speed?
- In the same direction as the electric field.
 - In the opposite direction to the electric field.
 - Perpendicular to and into the plane of the paper.
 - Perpendicular to and out of the plane of the paper.
 - From up to down in the plane of the paper.
 - From down to up in the plane of the paper.
- (3) Which of the following is the correct relationship among the speed v of the positive ion passing through the slit S_0 , the magnitude of the electric field E , and the magnitude of the magnetic field B_1 ?
- $v = EB_1$
 - $v = \frac{EB_1}{M}$
 - $v = \frac{E}{B_1}$
 - $v = \frac{E}{MB_1}$
 - $v = \frac{B_1}{E}$
 - $v = \frac{B_1}{ME}$
 - $v = \frac{1}{EB_1}$
 - $v = \frac{1}{MEB_1}$
- (4) What is the radius of the circular motion of the positive ion after passing through the slit S_0 ?
- $r = \frac{qB_2}{mv}$
 - $r = \frac{mv}{qB_2}$
 - $r = \frac{qvB_2}{m}$
 - $r = \frac{m}{qvB_2}$
 - $r = \frac{qB_2}{mv^2}$
 - $r = \frac{mv^2}{qB_2}$
 - $r = \frac{qv^2B_2}{m}$
 - $r = \frac{m}{qv^2B_2}$
- (5) Consider the case where a proton and an alpha particle are emitted from the ion source. What multiple of the radius of the circular motion of the proton is that of the alpha particle?
- $\frac{1}{4}$
 - $\frac{1}{2}$
 - 1
 - 2
 - 4

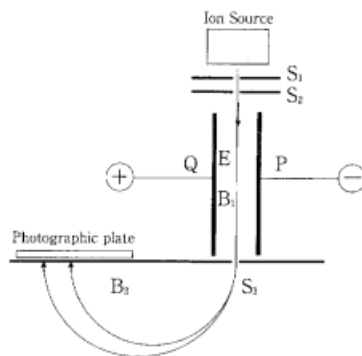
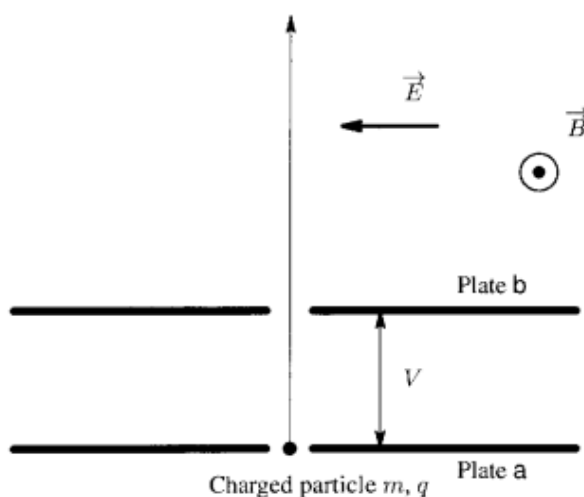


Fig. 4

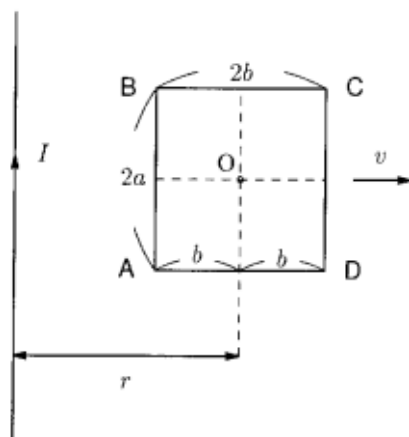
- D** As shown in the figure below, potential difference V exists between two parallel plates (a, b) that have a small hole. Uniform electric field \vec{E} and magnetic flux density \vec{B} exist in the region above the plates, and are perpendicular to each other. The direction of \vec{E} is parallel to this page and plate b. The direction of \vec{B} is perpendicular to this page, from back to front. A positively charged particle of charge q and mass m , initially at rest in the hole of plate a, is accelerated by potential difference V so that it enters the region above the plate perpendicularly to \vec{E} and \vec{B} and travels straight through the region.



- Q4 What is the ratio of the magnitude E of the electric field to the magnitude B of the magnetic flux density $\frac{E}{B}$? From ①-⑥ below choose the correct answer. 17

- | | | |
|--------------------------|--------------------------|--------------------------|
| ① $\sqrt{\frac{m}{2qV}}$ | ② $\sqrt{\frac{2qV}{m}}$ | ③ $\sqrt{\frac{m}{qV}}$ |
| ④ $\sqrt{\frac{qV}{m}}$ | ⑤ $\sqrt{\frac{2m}{qV}}$ | ⑥ $\sqrt{\frac{qV}{2m}}$ |

- E** As shown in the figure below, a current I passes through a straight conducting wire located within this page. Rectangular circuit **ABCD** on the page is moving away from the straight conducting wire with speed v , while side **AB** remains parallel with current I . The length of side **AB** is $2a$, and the length of side **AD** is $2b$. The resistance of the circuit is R .

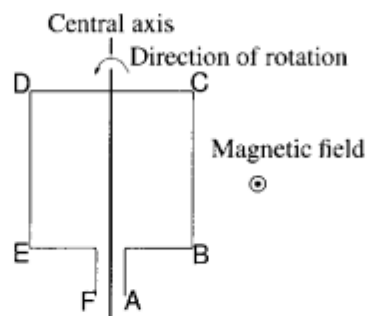


- Q5** How much current flows through circuit **ABCD** when the center of the circuit (**O**) is distance r from the straight wire? From ①-⑥ below choose the correct answer. The magnetic permeability of a vacuum is μ_0 , and the magnetic flux density created by the circuit's current is negligible.

18

- | | | |
|---|--|-------------------------------------|
| ① $\frac{\mu_0 abvI}{\pi R(r^2 - b^2)}$ | ② $\frac{2\mu_0 abvI}{\pi R(r^2 - b^2)}$ | ③ $\frac{\mu_0 avI}{\pi R(r - b)}$ |
| ④ $\frac{2\mu_0 avI}{\pi R(r - b)}$ | ⑤ $\frac{\mu_0 avI}{\pi R(r + b)}$ | ⑥ $\frac{2\mu_0 avI}{\pi R(r + b)}$ |

F As shown in the figure below, a uniform magnetic field exists perpendicular to this page, in the direction from back to front. A coil is placed in the field, parallel to the page. At time $t = 0$, the coil (initially in the state shown) begins to rotate about its central axis at a constant angular velocity with period T . The direction of rotation is so that at $t = 0$ side BC moves from back to front of this page, and side DE moves from front to back.



Q6 From ①-⑥ below choose the graph that best represents the change over time of the electromotive force generated in the coil. The positive direction of the electromotive force is the direction $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F$. **19**

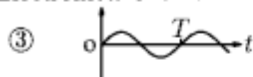
Electromotive force



Electromotive force



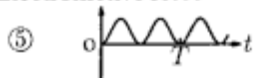
Electromotive force



Electromotive force



Electromotive force



Electromotive force



2. The right half of the x - y plane is filled with a uniform magnetic field of magnitude B pointing out of the page as shown in Fig. 2-1. A charged particle of mass m enters the magnetic field region along the negative x axis in the positive x direction with speed v . The magnitude of the charge is denoted by q . The trajectory of the particle describes a semi-circle as shown in Fig. 2-1. Answer the following questions.

(1) Is the charge of the particle positive or negative?

- (a) Positive (b) Negative

(2) Find the magnitude of the force on the particle.

- (a) B (b) vB (c) qB
 (d) $\frac{vB}{q}$ (e) qvB (f) $\frac{qB}{v}$

(3) Find the radius of the circular orbit.

- (a) $\frac{mv}{B}$ (b) $\frac{v}{qB}$ (c) $\frac{m}{qB}$
 (d) $\frac{mv}{qB}$ (e) $\frac{2mv}{qB}$ (f) $\frac{mv}{2qB}$

(4) Find the time it takes the particle to travel from O to P .

- (a) $\frac{2\pi m}{qB}$ (b) $\frac{\pi m}{qB}$ (c) $\frac{m}{\pi qB}$
 (d) $\frac{m}{qB}$ (e) $\frac{m}{2qB}$ (f) $\frac{4\pi m}{qB}$

(5) Now we change the magnitude of the magnetic field from B to $2B$ and change the value of the particle speed from v to $3v$. What multiple of the radius of the original circular orbit is the new radius now after these changes. Choose the correct answer from the following.

- (a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$
 (d) 3 (e) $\frac{3}{4}$ (f) $\frac{9}{2}$

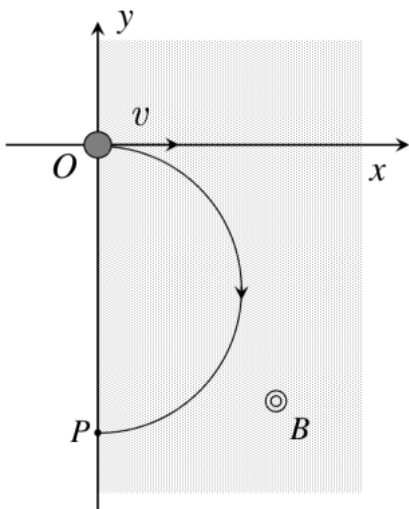


Fig. 2-1

2. The region on the right hand side of line XY is filled with a uniform magnetic field of flux density B pointing out from back to front as shown in Fig. 2. A square coil abcd with a side length of l enters the magnetic field region with a constant speed v . Line XY and side ab are parallel to each other. The resistance of the coil is R . At a time $t=0$, side ab of the coil passes line XY. Answer the following questions in the case of $0 < t < l/v$.

(1) Find the magnitude of the magnetic flux which passes through the coil at the time t .

- (a) $Bl t$ (b) $Bl^2 t$ (c) $vBl t$
 (d) $vB^2 l t$ (e) $vBl^2 t$ (f) $vB t$

(2) Find the magnitude of the electromotive force induced in the coil.

- (a) vB (b) vBl (c) $vB^2 l$
 (d) Bl (e) Bl^2 (f) vBl^2

(3) Find the induced current which flows in the coil.

- (a) $\frac{vBl^2}{R}$ (b) $\frac{Bl}{R}$ (c) $\frac{vB^2 l}{R}$
 (d) $\frac{vB}{R}$ (e) $\frac{vBl}{R}$ (f) $\frac{Bl^2}{R}$

(4) Find the direction of the induced current which flows in the coil.

- (a) $a \rightarrow b \rightarrow c \rightarrow d$ (b) $a \rightarrow d \rightarrow c \rightarrow b$

(5) Find the magnitude of the external force to maintain the constant speed v of the coil.

- (a) $\frac{B^2 l}{R}$ (b) $\frac{vBl^2}{R}$ (c) $\frac{Bl^2}{R}$
 (d) $\frac{vB^2 l^2}{R}$ (e) $\frac{vB^2 l}{R}$ (f) $\frac{B^2 l^2}{R}$

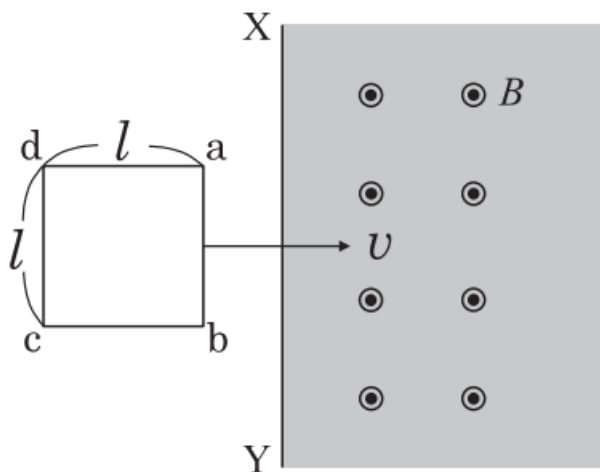


Fig. 2

-2019 1(3)

- (3) Two long straight wires, with the same current I flowing in the opposite direction, are placed parallel to each other with a distance of $2d$ as shown in Fig. 1-2. Find the magnitude of magnetic field H at point P .

(a) $\frac{I}{2\pi d}$

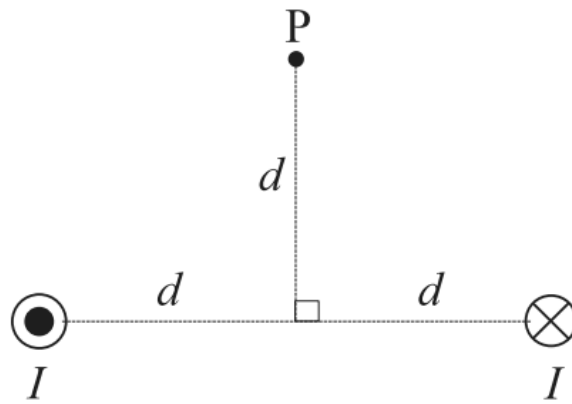
(b) $\frac{I}{\pi d}$

(c) $\frac{\pi I}{d}$

(d) $\frac{2\pi I}{d}$

(e) $\frac{I}{d}$

(f) $\frac{I}{2d}$



(4) Miscellaneous (cross topics)

-2012 2

2. A single-turn coil of a rectangular shape falls down in a space where a uniform magnetic field of magnitude B in a horizontal direction exists above a certain height, as shown in Fig. 2-1. At a sufficiently later time after the bottom of the coil entered in the space without a magnetic field, the coil falls down at a constant speed v . The resistance, the mass, and the horizontal width of the single-turn coil is R , M , and L , respectively. The face of the coil is perpendicular to the magnetic field. The length of the coil in vertical direction is sufficiently long. The acceleration of the gravity is denoted as g . Answer the following questions.

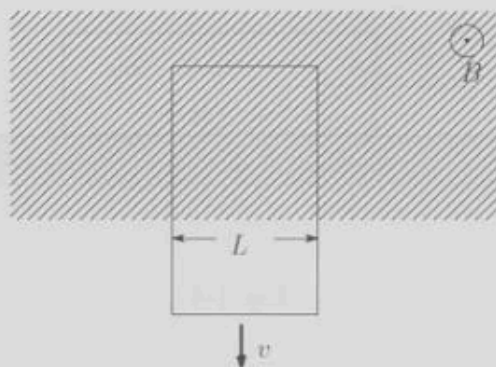


Fig. 2-1

- (1) Find the electric current passing through the coil.

(a) $\frac{BvR}{R}$	(b) $\frac{BvL}{vL}$	(c) $\frac{BvR}{L}$
(d) $\frac{BvL}{R}$	(e) $\frac{BR}{vL}$	(f) $\frac{BL}{vR}$

- (2) Express the speed of the falling coil in terms of other quantities.

(a) $\frac{RMg}{BL}$	(b) $\frac{R^2Mg}{B^2L}$	(c) $\frac{RMg}{B^2L^2}$
(d) $\frac{LMg}{BR}$	(e) $\frac{L^2Mg}{B^2R}$	(f) $\frac{LMg}{B^2R^2}$

- (3) Find the Joule heat produced in the coil per unit time.

(a) $\frac{BvL}{R}$	(b) $\frac{B^2vL}{R}$	(c) $\frac{B^2v^2L^2}{R}$
(d) $\frac{BvR}{L}$	(e) $\frac{B^2vR}{L}$	(f) $\frac{B^2v^2R^2}{L^2}$

- (4) When the coil is doubly turned, what multiple of the falling speed is that of the single-turn coil.

(a) $\frac{1}{4}$	(b) $\frac{1}{2}$	(c) 1	(d) 2	(e) 4
-------------------	-------------------	-------	-------	-------

2. In Fig. 2-1, a current I flows through a thin strip of metal. The strip has a thickness a , width b , and length c . We assume that there are n electrons with a charge of $-q(< 0)$ per unit volume and they all move with speed v .

- (1) Find the number of electrons passing through the strip in time t .
- | | | |
|-------------|-------------|--------------|
| (a) $nabct$ | (b) $nabt$ | (c) nc^2vt |
| (d) $nabvt$ | (e) $nbcvt$ | (f) $ncavt$ |
- (2) Which of the following is the correct formula for the magnitude of the current I ?
- | | | |
|-------------|-------------|------------|
| (a) $qnabc$ | (b) $qnabv$ | (c) $qnab$ |
| (d) $nabc$ | (e) $nabv$ | (f) nab |

Now we apply a magnetic field B perpendicular to the strip as shown in Fig. 2-2. A voltage V , which is perpendicular to the current and the magnetic field, is produced by charge accumulation on sidewalls.

- (3) Find the magnitude of the magnetic force on the electrons moving with speed v .
- | | | |
|------------|-----------|------------|
| (a) qB | (b) vB | (c) bcB |
| (d) $bcvB$ | (e) qvB | (f) $qbcB$ |
- (4) Find the formula for V .
- | | | |
|-----------|------------|-----------|
| (a) Bbc | (b) Bab | (c) Bvc |
| (d) Bva | (e) $Bbcv$ | (f) Bvb |
- (5) Find the formula for n that contains B , I , and V .
- | | | |
|------------------------|------------------------|------------------------|
| (a) $\frac{BI}{qbV^2}$ | (b) $\frac{BI}{qeV^2}$ | (c) $\frac{BI}{qcV^2}$ |
| (d) $\frac{BI}{aV^2}$ | (e) $\frac{BI}{bV^2}$ | (f) $\frac{BI}{cV^2}$ |

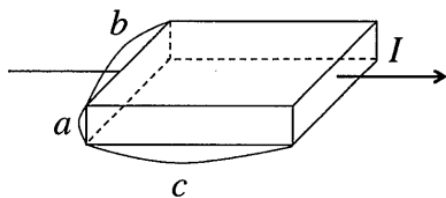


Fig. 2-1

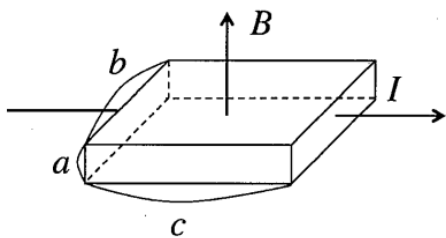


Fig. 2-2

2. Figure 2-1 shows a conducting bar of mass m that can slide without friction on a pair of conducting rails separated by a distance d and located on an inclined plane that makes an angle θ with respect to the ground. The two rails are connected by a resistor. The resistance of the resistor is denoted as R . A uniform magnetic field of magnitude B is directed upward, perpendicular to the ground over the entire region through which the bar moves. The bar is released from rest and slides down. The acceleration due to gravity is denoted as g .

(1) Which way does the current flow, from a to b or b to a ?

- (a) From a to b (b) From b to a

Now the bar is moving along the rails at speed v . The bar is always perpendicular to the rails.

(2) Which of the following is the correct formula for the induced current I ?

- (a) $\frac{vdB \sin \theta}{R}$ (b) $\frac{vdB \tan \theta}{R}$ (c) $\frac{vdB}{R \sin \theta}$
 (d) $\frac{vdB}{R}$ (e) $\frac{vdB}{R \cos \theta}$ (f) $\frac{vdB \cos \theta}{R}$

(3) Besides gravity, the magnetic force acts on the bar. Find the correct formula for the component of this magnetic force along the inclined plane.

- (a) $\frac{vdB^2 \cos^2 \theta}{R}$ (b) $\frac{vd^2 B \cos^2 \theta}{R}$ (c) $\frac{vd^2 B^2 \cos^2 \theta}{R}$
 (d) $\frac{vdB^2}{R}$ (e) $dB \cos \theta$ (f) $\frac{vd^2 B^2}{R}$

After a sufficiently long time the bar moves at a constant speed. At this terminal velocity u the gravity force is balanced by the magnetic force along the inclined plane.

(4) Find the correct formula for u .

- (a) $\frac{mgR}{dB \cos \theta}$ (b) $\frac{mgR \cos \theta}{d^2 B^2 \cos^2 \theta}$ (c) $\frac{mgR \sin \theta}{d^2 B^2}$
 (d) $\frac{mgR}{d^2 B^2 \sin \theta}$ (e) $\frac{mgR \sin \theta}{d^2 B^2 \cos^2 \theta}$ (f) $\frac{mgR \sin^2 \theta}{d^2 B^2 \cos^2 \theta}$

(5) Which of the following is the correct formula for the rate of work done by gravity on the bar?

- (a) $\frac{m^2 g^2 R \tan^2 \theta \sin \theta}{d^2 B^2}$ (b) $\frac{m^2 g^2 R \sin \theta}{d^2 B^2 \cos^2 \theta}$ (c) $\frac{mgR \tan^2 \theta}{d^2 B^2}$
 (d) $\frac{mgR \tan^2 \theta}{d^2 B^2}$ (e) $\frac{m^2 g^2 R \tan^2 \theta}{d^2 B^2}$ (f) $\frac{mgR \sin^2 \theta}{d^2 B^2}$

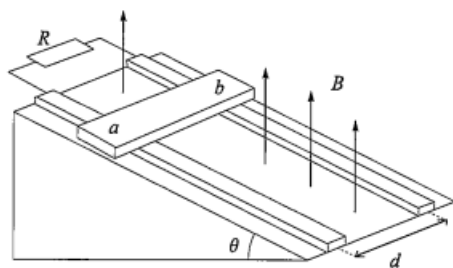


Fig. 2-1