Q1(1):

The graph of $y = x^2 - a(x+1) + 3$ touches the x-axis if and only if the equation $x^2 - a(x+1) + 3 = 0$, i.e. $x^2 - ax + (3-a) = 0$ has only one solution. Then,

$$\Delta = a^2 - 4(3-a) = 0$$

$$a^2 + 4a - 12 = 0$$

$$(a+6)(a-2) = 0$$

$$a = \boxed{-6} \text{ or } a = \boxed{2}$$

Q1(2):

$$\log_2(x+1) \leq 3$$

$$x + 1 > 0 \text{ and } x + 1 \leq 2^3$$

$$x > -1 \text{ and } x \leq 7$$

$$\boxed{-1 < x \leq 7}$$

Q1(3):

Denote the center of the inscribed circle as O. Then the heights of $\triangle AOB$, $\triangle BOC$ and $\triangle COA$ will be equal to the radius of the inscribed circle, r, as the inscribed circle is tangent to their bases.

On the other hand, consider the area of $\triangle ABC$,

$$\triangle ABC = \triangle AOB + \triangle BOC + \triangle COA$$

$$\frac{1}{2}(4)\sqrt{3^2-(\frac{4}{2})^2} = \frac{1}{2}(3)(r)+\frac{1}{2}(4)(r)+\frac{1}{2}(3)(r)$$

$$2\sqrt{5} = 5r$$

$$r = \boxed{\frac{2\sqrt{5}}{5}}$$

Alternative As $\triangle ABC$ is an isosceles triangle, O lies on the perpendicular bisector of BC. Denote M as the mid-point of BC, we have $BM = \frac{1}{2}BC = 2$, $OM = r$ and $\angle OBM = \frac{1}{2}\angle ABM$ (as OB is the angle bisector of $\angle ABC$).

On the other hand, consider the cosine ratio of $\triangle ABM$, we have $\cos\angle ABM = \frac{BM}{AB} = \frac{2}{3}$.

Note that $\cos\angle ABM = \cos^2\angle\frac{1}{2}ABM - \sin^2\angle\frac{1}{2}ABM = \cos^2\angle OBM - \sin^2\angle OBM$ $= \frac{\cos^2\angle OBM-\sin^2\angle OBM}{\cos^2\angle OBM+\sin^2\angle OBM} = \frac{1-\tan^2\angle OBM}{1+\tan^2\angle OBM}$.

Therefore, we have the equation

$$\frac{1-\tan^2\angle OBM}{1+\tan^2\angle OBM} = \frac{2}{3}$$

$$\tan^2\angle OBM = \frac{1}{5}$$

$$\tan\angle OBM = \frac{1}{\sqrt{5}} \text{ (as } \tan\angle OBM > 0)$$

Now, consider the tangent ratio of $\triangle OBM$, we have $\tan\angle OBM = \frac{OM}{BM} = \frac{r}{2}$.

Therefore, we have $r = \frac{2}{\sqrt{5}}$.

Q1(4):

Note that $\sin\theta - \sqrt{3}\cos\theta = 2(\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta) = 2(\cos 60^\circ \sin\theta - \sin 60^\circ\cos\theta) =$

$2\sin(\theta - 60°)$.

For $0 \le \theta < 360°$, $-2 \le 2\sin(\theta - 60°) \le 2$. Therefore, the required maximum value is $\boxed{2}$.

(Note: For the general methodology, search for how to express $a\sin\theta + b\cos\theta$ in the form $R\sin(\theta - \alpha)$.)

Alternative $f'(\theta) = \cos\theta + \sqrt{3}\sin\theta$.

To find the extremum of $f(\theta)$, set $f'(\theta) = 0$, we have $\tan\theta = -\frac{1}{\sqrt{3}}$.

For $0 \le \theta < 2\pi$, we have the solutions $\theta = \frac{5\pi}{6}$ or $\theta = \frac{11\pi}{6}$.

$f''(\theta) = -\sin\theta + \sqrt{3}\cos\theta$. Conduct the second derivative test:

$f''(\frac{5\pi}{6}) = -\frac{1}{2} - \sqrt{3} \cdot \frac{\sqrt{3}}{2} < 0$

$f''(\frac{11\pi}{6}) = \frac{1}{2} + \sqrt{3} \cdot \frac{\sqrt{3}}{2} > 0$

Therefore, $f(\theta)$ attains to its maximum when $\theta = \frac{5\pi}{6}$. By that time, the maximum value=$f(\frac{5\pi}{6}) = \frac{1}{2} + \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \boxed{2}$.

Q1(5):

If $x+y = 3$ and $x^2+y^2 = 5$, then $(x+y)^2 = (x^2+y^2)+2xy = 5+2xy = 3^2 = 9$.

Therefore, $xy = \frac{9-5}{2} = 2$.

Then, $x^3 + y^3 = (x+y)^3 - 3xy(x+y) = 3^3 - 3(2)(3) = \boxed{9}$.

Q2:

(1) By the cosine formula,

$$AC^2 = BA^2 + BC^2 - 2(BA)(BC)\cos\angle B$$

$$AC = \sqrt{5^2 + 3^2 - 2(5)(3)\cos 60^\circ} = \boxed{\sqrt{19}}$$

(2) As $ABCD$ is a cyclic quadrilateral, $\angle D = 180^\circ - \angle B = 120^\circ$.

By the cosine formula,

$$AC^2 = DA^2 + DC^2 - 2(DA)(DC)\cos\angle D$$

$$DA^2 - 2(2)\cos 120^\circ DA + 2^2 - 19 = 0$$

$$DA^2 + 2DA - 15 = 0$$

$$(DA + 5)(DA - 3) = 0$$

$$DA = \boxed{3} \text{ or } DA = -5(\text{rejected})$$

Q3:

(1):

$xy = 8 \iff \log_2(xy) = \log_2 8 \iff \log_2 x + \log_2 y = 3 \iff \log_2 y = 3 - X.$

Therefore,

$$P = 2(\log_2 x)^2 + (\log_2 y)^2$$

$$= 2X^2 + (3 - X)^2$$

$$= \boxed{3X^2 - 6X + 9}$$

(2) By completing the square, $P = 3(X-1)^2 + 6$.

As $(X-1)^2 \geq 0$, we have $P \geq 0 + 6 = 6$. Therefore, the minimum value of P is $\boxed{6}$.