Q1(1):

$$\log_3 6 - \log_9 x = \frac{1}{2}$$

$$\log_3 6 - \frac{1}{2}\log_3 x = \frac{1}{2}$$

$$\log_3 6^2 - \log_3 x = 1$$

$$\log_3 \frac{36}{x} = 1$$

$$\frac{36}{x} = 3$$

$$x = \boxed{12}$$

Q1(2):

If $\alpha + \beta = \frac{\pi}{4}$, then $\beta = \frac{\pi}{4} - \alpha$.

We have $\tan\beta = \tan(\frac{\pi}{4} - \alpha) = \frac{\tan\frac{\pi}{4} - \tan\alpha}{1 + \tan\frac{\pi}{4}\tan\alpha} = \frac{1 - \tan\alpha}{1 + \tan\alpha}$.

Therefore, $(\tan\alpha + 1)(\tan\beta + 1) = (\tan\alpha + 1)(\frac{2}{\tan\alpha + 1}) = \boxed{2}$.

Q1(3):

When $x + y = \frac{2\pi}{3}$, we have $y = \frac{2\pi}{3} - x$.

Then, $\sin y = \sin(\frac{2\pi}{3} - x) = \sin\frac{2\pi}{3}\cos x - \sin x\cos\frac{2\pi}{3} = \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$.

Moreover, $\sin x + \sin y = \frac{3}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$

$= \sqrt{3}(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x) = \sqrt{3}\sin(x + \frac{\pi}{6})$.*

For $0 \leq x \leq \frac{2\pi}{3}$, the maximum of the expression is $\boxed{\sqrt{3}}$ and the minimum is $\boxed{\dfrac{\sqrt{3}}{2}}$.

(*: Search for how to express $a\sin\theta + b\cos\theta$ in a form of $R\sin(\theta+\alpha)$.)

Alternative We have the expression is equal to $\frac{3}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$.

Consider the derivative, $\frac{3}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$. The find the extremum, we set the derivative to be 0. Then, for $0 \leq x \leq \frac{2\pi}{3}$, we have $x = \frac{\pi}{6}$.

Consider the second derivative, $-\frac{3}{2}\sin x - \frac{\sqrt{3}}{2}\cos x$, which valued less than 0 when $x = \frac{\pi}{6}$. Therefore, the expression takes maximum when $x = \frac{\pi}{6}$.

Check the boundaries:

When $x = 0$, the expression is valued $\frac{\sqrt{3}}{2}$.

When $x = \frac{2\pi}{3}$, the expression is valued $\frac{\sqrt{3}}{2}$.

Given the above, the maximum of the expression is $\boxed{\sqrt{3}}$ and the minimum is $\boxed{\dfrac{\sqrt{3}}{2}}$.

Q1(4):

As $M = 2 \implies N = 2$, we have $N \neq 2 \implies M \neq 2$, i.e. if $N = 1$ then $M = 1$.

Therefore the conclusion 1 is true.

The conclusions 2 and 3 are false as there is a counter exaple 211.

Thereofre, the answer is $\boxed{1}$, $\boxed{0}$, $\boxed{0}$.

Q1(5):

Denote the upper point in the middle as A and the lower point as B.

By counting directly, there are 2 routes from L to A and 3 routes from L to B.

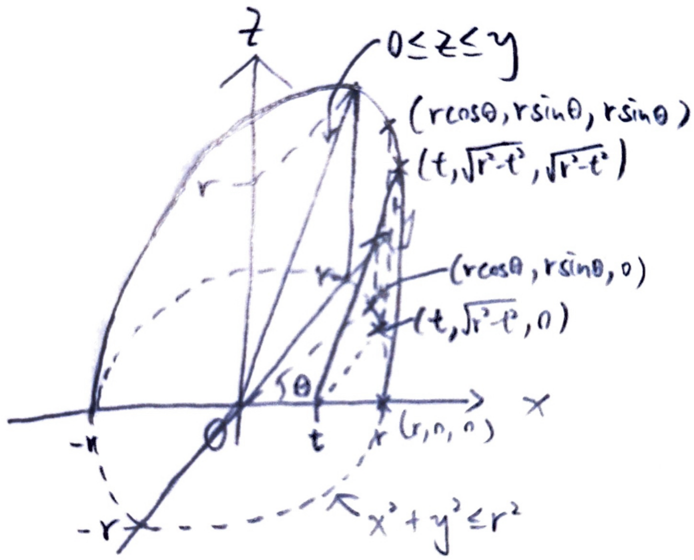
On the other hand, there are 3 routes from A to R and 2 routes from B to R.

The total number of routes from L to R is $2 \cdot 3 + 3 \cdot 2 = \boxed{12}$.

(Note: There are many ways to count the number of routes. Here I suggested a way that will make mistake less easily.)

Q2:

Refer to the sketch:



(1): Note that the intersection region is a triangle.

The plane $x = t$ cut the circle $x^2 + y^2 = r^2$ on the x-y plane with y-coordinate $\sqrt{r^2 - t^2}$. Therefore, the base length of the triangle is $\sqrt{r^2 - t^2}$.

Moreover, as C is bounded by $z \leq y$, in the space, when the x-coordinate is t and y-coordinate is $\sqrt{r^2-t^2}$, it represents a straight line joining $(t, \sqrt{r^2-t^2}, 0)$ and $(t, \sqrt{r^2-t^2}, \sqrt{r^2-t^2})$. Therefore, the hight of the triangle is $\sqrt{r^2-t^2}$.

Given the above, the area of the intersection is $\frac{1}{2}(\sqrt{r^2-t^2})(\sqrt{r^2-t^2}) = \boxed{\frac{1}{2}(r^2-t^2)}$.

(2): The volume of C equals to the integral of its cross section area over the bounaries.

Therefore, we have the volume$=\int_{-r}^{r}(\frac{1}{2}(r^2-t^2))dt$

$= \frac{1}{2}[r^2t - \frac{1}{3}t^3]_{-r}^{r}$

$= \boxed{\frac{2}{3}r^3}$.

(3): Note that θ represends the angle of the arc from $(r,0,0)$ to $(r\cos\theta, r\sin\theta, 0)$.

As θ is in radian measure, by definition, we have $a = \boxed{r\theta}$.

Moreover, $b = \boxed{r\sin\theta}$ as it is a straight line parallel to the z-axis.

(4): We separate the side of C into n parts with respect to θ. Let A_i be the area of the ith part from $\theta = 0$ to $\theta = \pi$. Moreover, let a_i be the arc length and of the ith part and let b_i^+, b_i^- be the height of the ith part measured closer to the $\theta = \pi$ side and the $\theta = 0$ side respectively. Then, we have:

$$a_i b_i^- \leq A_i \leq a_i b_i^+$$

By the results in (2), $a_i = r(\frac{i\pi}{n} - \frac{(i-1)\pi}{n}) = r\frac{\pi}{n}$,

$b_i^+ = r\sin(\frac{i\pi}{n})$ and $b_i^- = r\sin(\frac{(i-1)\pi}{n})$.

Therefore,

$$r^2\sin(\frac{(i-1)\pi}{n})\frac{1}{n} = (r\frac{\pi}{n})(r\sin(\frac{(i-1)\pi}{n})) \leq A_i \leq (r\frac{\pi}{n})(r\sin(\frac{i\pi}{n})) = r^2\sin(\frac{i\pi}{n})\frac{1}{n}$$

$$\sum_{i=1}^{n}(r^2\sin(\frac{(i-1)\pi}{n})\frac{1}{n}) \leq \sum_{i=1}^{n} A_n \leq \sum_{i=1}^{n}(r^2\sin(\frac{i\pi}{n})\frac{1}{n})$$

By taking $n \to +\infty$, we have the required area$=\int_0^\pi r^2 \sin\theta d\theta$

$= r^2[-\cos\theta]_0^\pi$

$= \boxed{2r^2}$.

(Note: Here I used the Riemann sum for precision, as no steps are required in the exam, one can evaluate $\int_0^\pi r\sin\theta d(r\theta)$ directly for the area without proof. However, note that $\int_0^\pi r\sin\theta d\theta$ is wrong as $d\theta$ is not the arc length we demanded for calculating the area.)

Q3:

(1): $f^n(x) = f(f^{n-1}(x)) = af^{n-1}(x) + b$.

To solve the recurrence, we suppose $f^n(x) - k = a(f^{n-1}(x) - k)$. Then, we have $k = \frac{b}{1-a}$.

Therefore, we have $f^n(x) - \frac{b}{1-a} = a^{n-1}(f^1(x) - \frac{b}{1-a})$, i.e.

$f^n(x) = a^{n-1}(ax + b - \frac{b}{1-a}) + \frac{b}{1-a} = \boxed{a^n x + \frac{(a^n-1)b}{a-1}}$.

(2): $\frac{f^n(x)-f^{n-1}(x)}{a^n} = \frac{a^n x + \frac{(a^n-1)b}{a-1} - a^{n-1}x - \frac{(a^{n-1}-1)b}{a-1}}{a^n}$

$= (1-\frac{1}{a})x + \frac{(1-\frac{1}{a})b}{a-1}$

$= \boxed{\frac{a-1}{a}x + \frac{b}{a}}$.

(3): Solving $ax + b = \frac{a-1}{a}x + \frac{b}{a}$, we have $x = \frac{b(1-a)}{a^2-a+1}$, i.e. $x_n = \boxed{\frac{b(1-a)}{a^2-a+1}}$.

Substitue it into $y = ax + b$, we have $y_n = \boxed{\frac{-b}{a^2-a+1}}$.

(4): As $-1 < a < 1$, we have $\lim_{n\to\infty} a^n = 0$.

Therefore, $\lim_{n\to\infty} f^n(x) = \lim_{n\to\infty}(a^n x + \frac{(a^n-1)b}{a-1}) = \boxed{\dfrac{b}{1-a}}$.