Q1(1):

$$2^x 4^y = 32 \iff 2^x 2^{2y} = 2^5 \iff 2^{x+2y} = 2^5 \iff x + 2y = 5......(1)$$

$$\frac{3^x}{9^y} = 3 \iff \frac{3^x}{3^{2y}} = 3 \iff 3^{x-2y} = 3 \iff x - 2y = 1......(2)$$

(1)+(2):

$$2x = 6$$

$$x = 3$$

Substitue $x = 3$ into (1), $y = \frac{5-3}{2} = 1$.

Then, $\frac{5^x}{125^y} = \frac{5^3}{125^1} = \boxed{1}$.

Q1(2):

$a^3 - 2a^2 - \frac{2}{a^2} - \frac{1}{a^3}$

$= (a^3 - (\frac{1}{a})^3) - 2(a^2 + (\frac{1}{a})^2)$

$= (a - \frac{1}{a})^3 + 3(a)(\frac{1}{a})(a - \frac{1}{a}) - 2((a - \frac{1}{a})^2 + 2(a)(\frac{1}{a}))$

$= (a - \frac{1}{a})^3 + 3(a - \frac{1}{a}) - 2((a - \frac{1}{a})^2 + 2)$

When $a - \frac{1}{a} = 2$, $a^3 - 2a^2 - \frac{2}{a^2} - \frac{1}{a^3} = 2^3 + 3(2) - 2(2^2 + 2) = \boxed{2}$.

Alternative When $a - \frac{1}{a} = 2$, $(a - \frac{1}{a})^2 = a^2 - \frac{1}{a^2} - 2(a)(\frac{1}{a}) = a^2 - \frac{1}{a^2} - 2 = 2^2 = 4$.

Therefore, $a^2 + \frac{1}{a^2} = 6$.

On the other hand, using the binomial theorem, $(a - \frac{1}{a})^3$

$= a^3 - \frac{1}{a^3} - 3(a^2)(\frac{1}{a}) + 3(a)(\frac{1}{a^2}) = a^3 - \frac{1}{a^3} - 3(a - \frac{1}{a}) = a^3 - \frac{1}{a^3} - 3(2)$

$= a^3 - \frac{1}{a^3} - 6 = 2^3 = 8$. Therefore, $a^3 - \frac{1}{a^3} = 14$.

Then, $a^3 - 2a^2 - \frac{2}{a^2} - \frac{1}{a^3} = 14 - 2(6) = \boxed{2}$.

Q1(3):

$4\log_2 \sqrt{2} - \frac{1}{2}\log_2 3 + \log_2 \frac{\sqrt{3}}{2}$

$= 2 - \log_2 \sqrt{3} + \log_2 \sqrt{3} - \log_2 2$

$= 2 - 1$

$= \boxed{1}$

Q1(4):

By the cosine formula, $BC^2 = AB^2 + CA^2 - 2(AB)(CA)\cos\angle A$

$$4^2 = 6^2 + 5^2 - 2(6)(5)\cos\angle A$$

$$\cos\angle A = \boxed{\frac{3}{4}}.$$

Alternative Construct the perpendicular foot of B on AC, denote the point of intersection as D. Note that $CD = CA - AD = 5 - AD$. Using Pythagoras' theorem twice, we have $AD^2 + BD^2 = AB^2$, i.e. $BD^2 = 36 - AD^2$ and $CD^2 + BD^2 = BC^2$, i.e. $BD^2 = 16 - (5 - AD)^2$. Combine the two equations:

$$36 - AD^2 = 16 - (5 - AD)^2$$

$$5(2AD - 5) = 20$$

$$AD = \frac{9}{2}.$$

Now, by considering the cosine ratio of $\triangle ADB$, we have $\cos A = \frac{AD}{AB} = \frac{\frac{9}{2}}{6} = \boxed{\frac{3}{4}}$.

Q1(5):

Sum of roots=$\alpha + \beta = -\frac{6}{3} = -2$ and product of roots=$\alpha\beta = \frac{7}{3}$.

Then, $(2\alpha - \beta)(2\beta - \alpha)$

$= 4\alpha\beta - 2\beta^2 - 2\alpha^2 + \alpha\beta$

$= 5\alpha\beta - 2(\alpha^2 + \beta^2)$

$= 5\alpha\beta - 2((\alpha + \beta)^2 - 2\alpha\beta)$

$= 5(\frac{7}{3}) - 2((-2)^2 - 2(\frac{7}{3}))$

$= \frac{35}{3} + \frac{4}{3}$

$= \boxed{13}$.

Alternative As $3x^2 + 6x + 7 = 0$, we have $x^2 = -\frac{6x+7}{3}$.

Then, $(2\alpha - \beta)(2\beta - \alpha)$

$= 4\alpha\beta - 2\beta^2 - 2\alpha^2 + \alpha\beta$

$= 5\alpha\beta + \frac{2(6\beta+7)}{3} + \frac{2(6\alpha+7)}{3}$

$= 5\alpha\beta + \frac{12(\alpha+\beta)+28}{3}$

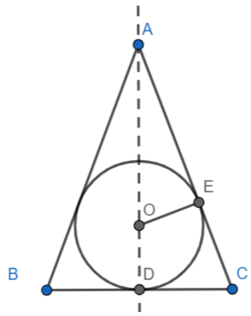
$= 5(\frac{7}{3}) + \frac{12(-2)+28}{3}$

$= \frac{35}{3} + \frac{4}{3}$

$= \boxed{13}$.

Q2:

(1): Consider the vertical section through the vertex of the cone, denote the points on it as shown below:



Note that $\triangle AOE \sim \triangle ACD$. Hence, $\frac{AE}{AD} = \frac{OE}{CD}$

$$\frac{\sqrt{(h-1)^2 - 1^2}}{h} = \frac{1}{r}$$

$$\boxed{r = \sqrt{\frac{h}{h-2}}}$$

(Note: Other equivalent answers are accepted)

(2): The volume of the cone=$V(h) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(\frac{h}{h-2})h = \frac{\pi}{3}\frac{h^2}{h-2}$, where $h > 2$.

$V'(h) = \frac{\pi}{3}\frac{2h(h-2)-h^2}{(h-2)^2} = \frac{\pi}{3}\frac{h^2-4h}{(h-2)^2}$.

To find the extremum of $V(h)$, we set $V'(h) = 0$, then $h = 4$ or $h = 0$(rejected).

Table of first derivative test:

h	$(2, 4)$	$(4, +\infty)$
$V'(h)$	$-$	$+$
$V(h)$	$\searrow$	$\nearrow$

Therefore, the volume attains to its minimum when $h = 4$ and the minimum volume by that time is $V(4) = \frac{\pi}{3}\frac{4^2}{4-2} = \boxed{\frac{8\pi}{3}}$.

Alternative to (2) (without calculus)

$$V(h) = \frac{\pi}{3}\frac{h^2}{h-2} \iff \frac{\pi}{3}h^2 - V(h)h + 2V(h) = 0$$

Regard it as a quadratic equation to solve $h \in \mathbb{R}$, we have

$$\Delta = V(h)^2 - \frac{8\pi}{3}V(h) \geq 0$$

$$V(h) \geq \frac{8\pi}{3}$$

Therefore, the minimum volume is $\boxed{\frac{8\pi}{3}}$.

Q3:

By long division, we have $\frac{n^2+8n+10}{n+9} = (n-1) + \frac{19}{n+9}$. As $n-1$ is an integer, a_n can be written as $a_n = (n-1) + [\frac{19}{n+9}]$.

Note that for $n = 1$ to $n = 10$, we have $1 \leq \frac{19}{n+9} < 2$, i.e. $[\frac{19}{n+9}] = 1$.

On the other hand, for $n = 11$ to $n = 30$, we have $0 < \frac{19}{n+9} < 1$, i.e. $[\frac{19}{n+9}]$.

Therefore, $\sum\limits_{n=1}^{30} a_n$

$= \sum\limits_{n=1}^{30} ((n-1) + [\frac{19}{n+9}])$

$= \sum\limits_{n=1}^{30} (n-1) + \sum\limits_{n=1}^{10} [\frac{19}{n+9}] + \sum\limits_{n=11}^{30} [\frac{19}{n+9}]$

$= \frac{(30+1)(30)}{2} - 30 + \sum\limits_{n=1}^{10} 1 + \sum\limits_{n=11}^{30} 0$

$= 31 \cdot 15 - 30 + 10 + 0$

$= \boxed{445}$.