Q1(1):

By testing the potential rational roots given by the rational root theorem $\pm1, \pm2$,

we have $x = 1$ is a root.

Therefore, we can do the factorisation by the long division:

$$x^3 + x^2 - 4x + 2 = 0$$

$$(x-1)(x^2 + 2x - 2) = 0$$

$$x = \boxed{-1, -1 \pm \sqrt{3}}$$

Q1(2):

$$\cos 2x + 3\cos x + 2 = 0$$

$$2\cos^2 x + 3\cos x + 1 = 0$$

$$(2\cos x + 1)(\cos x + 1) = 0$$

$$\cos x = -\frac{1}{2}, -1$$

$$x = \boxed{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}}$$

Q1(3):

$$3^{2x+1} + 5 \cdot 3^x - 2 = 0$$

$$3 \cdot 3^{2x} + 5 \cdot 3^x - 2 = 0$$

$$(3 \cdot 3^x - 1)(3^x + 2) = 0$$

$$3^x = \frac{1}{3}$$

$$x = \boxed{-1}$$

Q1(4):

$$4^{x+1} + 11 \cdot 2^x - 3 \geq 0$$

$$4 \cdot 2^{2x} + 11 \cdot 2^x - 3 \geq 0$$

$$(4 \cdot 2^x - 1)(2^x + 3) \geq 0$$

$$2^x \geq \frac{1}{4}$$

$$\boxed{x \geq -2}$$

Q1(5):

$$(\log_2 x)^2 = \log_4 x^4$$

$$(\log_2 x)^2 - 2\log_2 x = 0$$

$$\log_2 x = 0, 2$$

$$x = \boxed{1, 4}$$

Q1(6):

$$\log_3(3-x) + \log_3(x+1) < 1$$

$$(3-x)(x+1) < 3$$

$$x^2 - 2x > 0$$

$$x < 0 \text{ or } x > 2$$

Moreover, the condition for $\log_3(3-x)$ and $\log_3(x+1)$ to be defined is $-1 < x < 3$.

Finding the intersection, we have $\boxed{-1 < x < 0, 2 < x < 3}$.

Q1(7):

$|2\vec{a} - 3\vec{b}|$

$= \sqrt{(2\vec{a} - 3\vec{b})^2}$

$= \sqrt{4|\vec{a}|^2 + 9|\vec{b}|^2 - 12\vec{a} \cdot \vec{b}}$

$= \sqrt{4 \cdot 1 + 9 \cdot 9 - 12 \cdot 2}$

$= \sqrt{61}$

Q1(8):

The intersection point of the two lines is (1,2).

The slope of the line $x - 2y - 3 = 0$ is $\frac{1}{2}$, i.e. the slope of l is -2.

By the point-slope form of straight line, the equation of l is $y - 2 = -2(x - 1)$,

i.e. $\boxed{y = -2x + 4}$

Q1(9):

$a_n = S_n - S_{n-1} = 3^n + 2(n) - 1 - 3^{n-1} - 2(n-1) + 1 = \boxed{2 \cdot 3^{n-1} + 2}$

Q1(10):

$\lim\limits_{x \to \infty} (\sqrt{x^2 + 3x + 4} - x)$

$= \lim\limits_{x \to \infty} \frac{3x+4}{\sqrt{x^2+3x+4}+x}$

$= \lim\limits_{x \to \infty} \frac{3+\frac{4}{x}}{\sqrt{1+\frac{3}{x}+\frac{4}{x^2}}+1}$

$= \boxed{\dfrac{3}{2}}$

Q1(11):

$f(x) = \ln(x(x+e)) = \ln x + \ln(x+e)$

$f'(x) = \frac{1}{x} + \frac{1}{x+e}$

$f'(e) = \frac{1}{e} + \frac{1}{2e} = \boxed{\dfrac{3}{2e}}$

Q1(12):

$\int_0^{\frac{\pi}{2}} x \cos x dx$

$= \int_0^{\frac{\pi}{2}} x d(\sin x)$

$= x \sin x|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x dx$

$= \frac{\pi}{2} + [\cos x]_0^{\frac{\pi}{2}}$

$= \boxed{\dfrac{\pi}{2} - 1}$

Q2:

1): $B^2 = \begin{bmatrix} a^2 - 2 & -2a - 4 \\ a + 2 & 2 \end{bmatrix}.$

By $B^2 = B$, we have $a = \boxed{-1}$.

$BC = \begin{bmatrix} -b + 2 & 0 \\ b - 2 & 0 \end{bmatrix}.$

By $BC = 0$, we have $b = \boxed{2}$.

2): $xB + yC = \begin{bmatrix} -x + 2y & -2x + 2y \\ x - y & 2x - y \end{bmatrix}.$

If $A = xB + yC$, we have $x = \boxed{1}$ and $y = \boxed{2}$.

3): Note that $C^2 = C$.

$A^5 = (B + 2C)^5 = B^5 + 2^5 C^5 = B + 32C = \boxed{\begin{bmatrix} 63 & 62 \\ -31 & -30 \end{bmatrix}}$.*

*: As the multiplication between B and C are commutative, the binomial theorem can be applied. The expression is hence simplified by the given conditions.

Q3:

1): $f'(x) = \frac{1-\ln x}{x^2}$.

To find the extremum, we set $f'(x) = 0$, then $x = e$.

$f''(x) = \frac{-3+2\ln x)}{x^3}$.

As $f''(e) < 0$, $f(x)$ attains to its maximum when $x = e$ and $M = f(e) = \boxed{\frac{1}{e}}$.

2): Let $(k, \frac{\ln k}{k})$ be the point of tangency.

Then, the equation of tangent is $y - \frac{\ln k}{k} = \frac{1-\ln k}{k^2}(x-k)$.

As it passes through (0,0), we have

$$-\frac{\ln k}{k} = \frac{1-\ln k}{k^2}(-k)$$

$$\ln k = 1 - \ln k$$

$$k = \sqrt{e}$$

Therefore, the equation is $y - \frac{1}{2\sqrt{e}} = \frac{1}{2e}(x - \frac{2}{\sqrt{e}})$, i.e. $y = \boxed{\frac{1}{2e}x}$.

3): The area$=\int_0^1 (\frac{1}{2e}x)dx + \int_1^{\sqrt{e}} (\frac{1}{2e}x - \frac{\ln x}{x})dx$

$= [\frac{1}{4e}x^2]_0^{\sqrt{e}} - \int_1^{\sqrt{e}} \ln x d(\ln x)$

$= \frac{1}{4} - [\frac{1}{2}(\ln x)^2]_1^{\sqrt{e}}$

$= \boxed{\frac{1}{8}}$