Q1(1):

$(x-1)^2 < x-1$

$(x-1)^2-(x-1) < 0$

$(x-1)(x-1-1) < 0$

$\boxed{1 < x < 2}$

Q1(2):

$2^{2x-1}+2\cdot 2^x-6=0$

$2^{-1}\cdot 2^{2x}+2\cdot 2^x-6=0$

$2^{2x}+4\cdot 2^x-12=0$

$(2^x+6)(2^x-2)=0$

$2^x=2$ or $2^x=-6$(rejected)

$x=\boxed{1}$

Q1(3):

$\log_2(4-x)-\log_4(x-1)=1$

$\log_2(4-x)-\frac{1}{2}\log_2(x-1)=1$

$\log_2(4-x)-\log_2\sqrt{x-1}=1$

$\log_2\frac{4-x}{\sqrt{x-1}}=1$

$\frac{4-x}{\sqrt{x-1}}=2$

$4-x=2\sqrt{x-1}$

$(4-x)^2 = 4(x-1)$

$x^2 - 8x + 16 = 4x - 4$

$x^2 - 12x + 20 = 0$

$(x-10)(x-2) = 0$

$x = \boxed{2}$ or $x = 10$(rejected)

(Note that the hidden condition for $\log_2(4-x)$ and $\log_2(x-1)$ to be defined is $1 < x < 4$.)

Q1(4):

$(2\sin x - \sqrt{3})(2\sin x - 1) < 0$

$\frac{1}{2} < \sin x < \frac{\sqrt{3}}{2}$

$\boxed{\dfrac{\pi}{6} < x < \dfrac{\pi}{3} \text{ or } \dfrac{2\pi}{3} < x < \dfrac{5\pi}{6}}$ (as $0 \le x \le 2\pi$)

Q1(5):

$\frac{1}{x-1} > \frac{1}{x+1}$

$$\iff \begin{cases} x+1 > x-1 \\ (x+1>0 \text{ and } x-1>0) \text{ or } (x+1<0 \text{ and } x-1<0) \end{cases}$$

$$\text{or } \begin{cases} x+1 < x-1 \\ (x+1<0 \text{ and } x-1>0) \text{ or } (x+1>0 \text{ and } x-1<0) \end{cases}$$

$\iff$ $(x+1>0$ and $x-1>0)$ or $(x+1<0$ and $x-1<0)$ (as $x+1$ always $> x-1$)

$\iff$ $(x>-1$ and $x>1)$ or $(x<-1$ and $x<1)$

$\iff \boxed{x > 1 \text{ or } x < -1}$

Q1(6):

Note that: $z = \frac{\sqrt{3}+3i}{\sqrt{3}+i} = \frac{(\sqrt{3}+3i)(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)} = \frac{3+3\sqrt{3}i-\sqrt{3}i+3}{\sqrt{3}^2-i^2} = \frac{6+2\sqrt{3}i}{3+1} = \frac{3}{2} + \frac{\sqrt{3}}{2}i.$

Therefore, $r = \sqrt{(\frac{3}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \frac{1}{2}\sqrt{9+3} = \boxed{\sqrt{3}}$.

$\theta = \tan^{-1}(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}) = \tan^{-1}(\frac{\sqrt{3}}{3}) = \boxed{30^\circ}$. (As z lies on the quadrant I of the complex plane.)

Q1(7):

By completing the square, $f(x) = ax^2 - 4ax + b = a(x^2 - 4x + 2^2 - 2^2) + b = a(x-2)^2 - 4a + b$. As $a > 0$, we have $a(x-2)^2 \geq 0$ and hence $f(x) \geq -4a + b$, where the equality holds when $x = 2$, which satisfies $1 \leq x \leq 5$. Therefore, the minimum value of $f(x)$ is $-4a + b$.

On the other hand, note that the axis of symmetry of the graph $y = f(x)$ is $x = 2$. As the line $x = 5$ is further away from the line $x = 2$ than the line $x = 1$, we have the maximum value of $f(x) = f(5) = 25a - 20a + b = 5a + b$.

By the given condition, we have the system of equations $\begin{cases} \frac{(-4a+b)+(5a+b)}{2} = 14 \\ (5a+b) - (-4a+b) = 18 \end{cases}$,

i.e. $\begin{cases} a + 2b = 28 \\ 9a = 18 \end{cases}$. By the second equation, we have $a = \boxed{2}$. Then, substitue it into the first equation, we have $2 + 2b = 28$, i.e. $b = \frac{28-2}{2} = \boxed{13}$.

Q1(8):

$\lim\limits_{x\to 1} \frac{x-1}{\sqrt{x+8}-3}$

$= \lim\limits_{x\to 1} \frac{(x-1)(\sqrt{x+8}+3)}{(\sqrt{x+8}-3)(\sqrt{x+8}+3)}$

$= \lim\limits_{x\to 1} \frac{(x-1)(\sqrt{x+8}+3)}{\sqrt{x+8}^2-3^2}$

$= \lim\limits_{x\to 1} \frac{(x-1)(\sqrt{x+8}+3)}{x+8-9}$

$= \lim\limits_{x\to 1} \frac{(x-1)(\sqrt{x+8}+3)}{x-1}$

$= \lim\limits_{x\to 1} (\sqrt{x+8}+3)$

$= \sqrt{1+8}+3$

$= 3+3$

$= \boxed{6}$

Q1(9):

$\int_0^{\sqrt{3}} 3x\sqrt{x^2+1}dx$

$= \int_0^{\sqrt{3}} \frac{3}{2}(x^2+1)^{\frac{1}{2}} d(x^2+1)$

$= (x^2+1)^{\frac{3}{2}}|_0^{\sqrt{3}}$

$= (3+1)^{\frac{3}{2}} - (0+1)^{\sqrt{3}2}$

$= 2^3 - 1^3$

$= 8-1$

$= \boxed{7}$

Q1(10):

Note that $\frac{-3+\sqrt{13}}{2}$ is a solution to the equation $x^2+3x-1=0$ (by the quadratic formula). Hence, we have $x^2+3x-1=\boxed{0}$.

Alternative: As $(\frac{-3+\sqrt{13}}{2})^2+3(\frac{-3+\sqrt{13}}{2})-1=\frac{9-6\sqrt{13}+13}{4}+\frac{-9+3\sqrt{13}}{2}-1=\frac{22-6\sqrt{13}-18+6\sqrt{13}-4}{4}=0$, we have $x^2+3x-1=\boxed{0}$.

Q1(11):

The number of ways to take out 2 blue balls$=C_2^2=1$. The total number of ways to take out 2 balls$=C_2^{3+2+5}=C_2^{10}=45$. Hence, the probability that 2 blue balls are taken out$=\boxed{\dfrac{1}{45}}$.

On the other hand, The number of ways to take out 2 red balls$=C_2^3=3$ and the number of ways to take out 2 green balls$=C_2^5=10$. Hence the total number of ways to take out 2 balls of the same colour$=3+1+10=14$. The probability that 2 balls of the same colour are taken out$=\boxed{\dfrac{14}{45}}$.

Q2:

(1)A^3

$$=\begin{bmatrix}\frac{1}{2} & 0\\ a & a\end{bmatrix}^3$$

$$= \begin{bmatrix} \frac{1}{2} & 0 \\ a & a \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ a & a \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ a & a \end{bmatrix}$$

$$= \begin{bmatrix} (\frac{1}{2})^2 & 0 \\ \frac{1}{2}a + a^2 & a^2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ a & a \end{bmatrix}$$

$$= \begin{bmatrix} (\frac{1}{2})^3 & 0 \\ (\frac{1}{2})^2 a + \frac{1}{2}a^2 + a^3 & a^3 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} \frac{1}{8} & 0 \\ \frac{1}{4}a + \frac{1}{2}a^2 + a^3 & a^3 \end{bmatrix}}$$

(2) In part (1), we have calculated that

$$A = \begin{bmatrix} \frac{1}{2} & 0 \\ a & a \end{bmatrix}$$

$$A^2 = \begin{bmatrix} (\frac{1}{2})^2 & 0 \\ \frac{1}{2}a + a^2 & a \end{bmatrix}$$

$$A^3 = \begin{bmatrix} (\frac{1}{2})^3 & 0 \\ (\frac{1}{2})^2 a + \frac{1}{2}a^2 + a^3 & a^3 \end{bmatrix}$$

By observing the pattern, we can deduce that $A^n = \boxed{\begin{bmatrix} (\frac{1}{2})^n & 0 \\ \sum\limits_{i=1}^{n} (\frac{1}{2})^{n-i} a^i & a^n \end{bmatrix}}$ for all $n \in \mathbb{N}$.

Proof: We should carry out the proof using induction. The case $n = 0$ (and $n = 1$) is obviously true. We may assume $A^k = \begin{bmatrix} (\frac{1}{2})^k & 0 \\ \sum\limits_{i=1}^{k} (\frac{1}{2})^{k-i} a^i & a^k \end{bmatrix}$ for some

$k \in \mathbb{N}$. Then, we are going to prove $A^{k+1} = \begin{bmatrix} (\frac{1}{2})^{k+1} & 0 \\ \sum_{i=1}^{k+1} (\frac{1}{2})^{k+1-i} a^i & a^{k+1} \end{bmatrix}$ under this circumstance.

As A^{k+1}

$$
\begin{aligned}
&= A^k A \\
&= \begin{bmatrix} (\frac{1}{2})^k & 0 \\ \sum_{i=1}^{k} (\frac{1}{2})^{k-i} a^i & a^k \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ a & a \end{bmatrix} \\
&= \begin{bmatrix} (\frac{1}{2})^{k+1} & 0 \\ \frac{1}{2} \sum_{i=1}^{k} (\frac{1}{2})^{k-i} a^i + a^{k+1} & a^{k+1} \end{bmatrix} \\
&= \begin{bmatrix} (\frac{1}{2})^{k+1} & 0 \\ \sum_{i=1}^{k} (\frac{1}{2})^{k+1-i} a^i + (\frac{1}{2})^{k+1-(k+1)} a^{k+1} & a^{k+1} \end{bmatrix} \\
&= \begin{bmatrix} (\frac{1}{2})^{k+1} & 0 \\ \sum_{i=1}^{k+1} (\frac{1}{2})^{k+1-i} a^i & a^{k+1} \end{bmatrix},
\end{aligned}
$$

we have proved that $A^n = \begin{bmatrix} (\frac{1}{2})^n & 0 \\ \sum_{i=1}^{n} (\frac{1}{2})^{n-i} a^i & a^n \end{bmatrix}$ for all $n \in \mathbb{N}$.

Q3:

By the condition $f(-2) = -10$, we have the equation $f(-2) = -8+4a-2b+c = -10$, i.e. $4a - 2b + c = -2$......(1).

On the other hand, as $f(x) = \frac{50}{27}$ when $x = \frac{2}{3}$, we have the equation $f(\frac{2}{3}) = \frac{8}{27} + \frac{4}{9}a + \frac{2}{3}b + c = \frac{50}{27}$, i.e. $4a + 6b + 9c = 14$......(2).

Moreover, as $f(x)$ attains to its extremum when $x = \frac{2}{3}$, we have $f'(\frac{2}{3}) = 0$. As

$f'(x) = 3x^2 + 2ax + b$, we have the equation $f'(\frac{2}{3}) = 3(\frac{2}{3})^2 + 2a(\frac{2}{3}) + b = 0$, i.e. $4a + 3b = -4......(3)$.

Therefore, we have the system of equation $\begin{cases} 4a - 2b + c = -2......(1) \\ 4a + 6b + 9c = 14......(2) \\ 4a + 3b = -4......(3) \end{cases}$. We are going to solve it by elimination:

By (2)-(1), we have $8b + 8c = 16$, i.e. $b + c = 2......(4)$.

By (2)-(3), we have $3b + 9c = 18$, i.e. $b + 3c = 6......(5)$.

By (5)-(4), we have $2c = 4$, i.e. $c = \boxed{2}$.

Substitute $c = 2$ into (2), we have $b = 2 - 2 = \boxed{0}$.

Substitute $(b, c) = (0, 2)$ into (1), we have $4a = -2 + 0 - 2 = -4$, i.e. $a = \boxed{-1}$.

Q4:

(1) When $x = k$, by the equation of C_1, we have $y = k - \frac{1}{2}k^2$. Hence, the coordinates of the point of tangent are $(k, k - \frac{1}{2}k^2)$.

Consider the derivative of C_1, $\frac{dy}{dx} = 1 - x$. Hence, the slope of tangent to C_1 when $x = k$ is equal to $\frac{dy}{dx}|_{x=k} = 1 - k$.

Then, by using the point-slope form of a straight line, we have the equation of tangent to C_1 when $x = k$ is $y - (k - \frac{1}{2}k^2) = (1 - k)(x - k)$, i.e. $\boxed{y = (1 - k)x + \frac{1}{2}k^2}$.

(2) When the line $y = (1 - k)x + \frac{1}{2}k^2$ is tangent to C_2, i.e the line $x = \frac{1}{1-k}y - \frac{k^2}{2(1-k)}$ $(k \neq 1)$ is tangent to C_2, we have the equation $\frac{1}{1-k}y - \frac{k^2}{2(1-k)} = y - \frac{1}{2}y^2$

has only one solution. Tidying up the equation, we have $(k-1)y^2-2ky+k^2=0$. When it has only one solution, we have

$$\Delta = (-2k)^2 - 4(k-1)(k^2) = 0$$

$$4k^2 - 4k^3 + 4k^2 = 0$$

$$4k^2(2-k) = 0$$

$$k = \boxed{0} \text{ or } k = \boxed{2}$$

Then, the equations of tangent are $y = (1-0)x + \frac{1}{2}(0)^2$, i.e. $\boxed{y = x}$ and $y = (1-2)x + \frac{1}{2}(2)^2$, i.e. $\boxed{y = -x+2}$ respectively.

(3) Let the points of tangent to C_2 be $A(0,0)$ and $B(2,0)$ respectively. Also, let the point of intersection of the two tangents in (2) be $C(1,1)$. We have the required area=(The area of $\triangle ABC$)-(The area bounded by the graph of C_2 and the y-axis).

Note that $\triangle ABC$ is a right-angled triangle. Therefore the area of $\triangle ABC = \frac{1}{2}(AC)(BC) = \frac{1}{2}(\sqrt{2})(\sqrt{2}) = 1$.

On the other hand, the area bounded by the graph of C_2 and the y-axis= $\int_0^2 (y - \frac{1}{2}y^2)dy = \frac{1}{2}y^2 - \frac{1}{6}y^3|_0^2 = \frac{2^2}{2} - \frac{2^3}{6} = 2 - \frac{4}{3} = \frac{2}{3}$.

Therefore, the required area=$1 - \frac{2}{3} = \boxed{\frac{1}{3}}$.