

Q1(1):

Let  $v$  be the maximum velocity of the object with mass  $m$ . Then, by the conservation of momentum, the magnitude of the maximum velocity of the object with mass  $M = \frac{mv}{M}$ .

As all the elastic potential energy are transferred to kinetic energy, we have the equation of conservation of energy:

$$\frac{1}{2}kl^2 = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2$$
$$v = \sqrt{\frac{Mkl^2}{m(M+m)}}$$

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Q1(2):

After escape the gravitational field from the Earth, the potential energy becomes zero. Therefore, we have the following equation of conservation of energy:

$$KE + GPE = KE + GPE$$

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 + 0$$
$$v = \sqrt{\frac{2GM}{R}}$$

As the gravitational field strength of the Earth is given by  $g = \frac{GM}{R^2}$ , we have the escape speed  $v = \sqrt{2gR}$ .

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Q1(3):

By the state equation, we have  $pV = nRT$ , which can be rewritten as  $p = \frac{\rho}{m}RT$  ( $m$  refers to the molecular mass).

Therefore, undering constant pressure, we have  $\rho \propto \frac{1}{T}$ .

When the temperature increases from 270K to 300K, the density of air becomes

$$1.3 \cdot \frac{270}{300} = 1.17 \text{ kg } m^{-3}.$$

The volume of air escaped=(The volume of air before)-(The volume of air after)

$$= 100(1.3 - 1.17)$$

$$= \boxed{13 \text{ kg}}$$

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Q1(4):

Let  $Q$  be the number of charges accumulated in the capacitor  $C_1$ . Then, the voltage across  $C_1$  will be  $\frac{Q}{C_1}$  by the defination of capacitance.

Moreover, as  $C_2$  is in parallel with  $C_1$ , the voltage across it will also be  $\frac{Q}{C_1}$ .

Hence, the number of charges accumulated in  $C_2 = \frac{QC_2}{C_1}$ .

As  $C_3$  is in series with  $C_1 - C_2$ , the number of charges acculmulated in  $C_3$  is equal to the total number of charges accumulated in  $C_1$  and  $C_2$ , i.e.

$$Q + \frac{QC_2}{C_1} = \frac{Q(C_1+C_2)}{C_1}. \text{ Then, the voltage across } C_3 = \frac{Q(C_1+C_2)}{C_1C_3}.$$

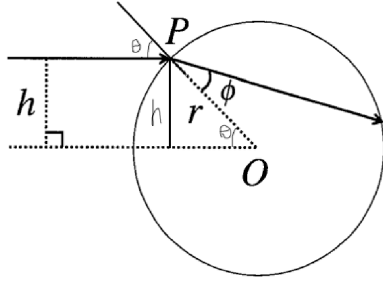
Consider the total voltage consumed in the circuit, we have  $E = \frac{Q}{C_1} + \frac{Q(C_1+C_2)}{C_1C_3}$ .

$$\text{Therefore, } Q = \boxed{\frac{C_1C_2}{C_1 + C_2 + C_3}E}.$$

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Q1(5):

Refer to the figure:



Consider the trigonometric ratio, we have  $\sin \theta = \frac{h}{r}$ .

On the other hand, by the Snell's law, we have  $\sin \theta = n \sin \phi$ .

Combine the above, we have  $\sin \phi = \boxed{\frac{h}{nr}}$ .

Q2:

(1): The cross section area of the strip= $ab$ .

Across the period of time  $t$ , the electrons travelled by a distance of  $vt$ .

Therefore, the volume of electrons travelled across the period of time  $t=abvt$ .

As the electron density is  $n$ , the number of electrons is given by  $\boxed{nabvt}$ .

(2): Following (1), we have  $q = nabvt$ .

As  $I = \frac{dq}{dt}$ , we have  $I = \boxed{qnabv}$ .

(3): Total magnetic force received by the electrons= $BIl = B(qnabv)c = (nabc)qvB$ .

As there are totally  $nabc$  electrons, the average magnetic force received by each

$$\text{electron} = \frac{F}{nabc} = \boxed{qvB}.$$

(4): As the magnetic force is balanced by the electric force, we have

$$qE = qvB$$

$$\frac{V}{b} = vB$$

$$V = \boxed{Bvb}$$

(5): By (2), we have  $n = \frac{I}{qabv}$ .

By (4), we have  $b = \frac{V}{Bv}$ . Therefore,  $n = \boxed{\frac{BI}{qaV}}$ .

Q3:

(1): Setting the potential energy as 0 at the point B.

By the conservation of energy, we have

$$KE + GPE = KE + GPE$$

$$0 + mga = \frac{1}{2}mv^2 + 0$$

$$v = \boxed{\sqrt{2ga}}$$

(2): Consider the balance of force: Centripetal force=(Tension)-(Weight)

$$\frac{mv^2}{a} = T - mg$$

$$t = \frac{m(\sqrt{2ga})^2}{a} + mg = \boxed{3mg}$$

(3): By the conservation of energy, we have

$$mga = \frac{1}{2}mv^2 + mg(2(a-b))$$

$$mv^2 = (-2a + 4b)mg$$

Moreover, consider the balance of force: Centripetal force=(Tension)+(Weight)

$$\frac{mv^2}{a-b} = T + mg$$

$$T = \left(\frac{-2a + 4b}{a-b} - 1\right)mg = \boxed{\frac{-3a + 5b}{a-b}mg}$$

(4): The small ball reaches  $D$  without any bending if the tension at  $D$  is greater than 0, i.e.

$$\frac{-3a + 5b}{a-b}mg > 0$$

$$a < \boxed{\frac{5}{3}b}$$

(5): Suppose the string bend at the height  $h$ . By the conservation of energy, we have

$$mga = \frac{1}{2}mv^2 + mgh$$

$$mv^2 = 2mg(a-h)$$

Moreover, as the string bended, the tension of it is equal to 0. Therefore, we have the component of the centripetal force at the direction of weight=weight.

The component of the centripetal force at the direction of weight

$$= \frac{mv^2}{r} \cos \theta = \frac{2mg(a-h)}{\frac{a}{2}} \cdot \frac{\frac{a}{2}}{h-\frac{a}{2}} = \frac{4mg(a-h)}{2h-a}, \text{ by considering the cosine ratio.}$$

Therefore, we have  $\frac{4mg(a-h)}{2h-a} = mg$ , i.e.  $h = \frac{5}{6}a$ .

Now, substitute it back to  $mv^2 = 2mg(a-h)$ , we have  $v = \boxed{\sqrt{\frac{ga}{3}}}$ .

Q4:

(1): As  $\frac{pV}{T}$  is constant for fixed number of moles, we have

$$\frac{p_0 V_0}{T_A} = \frac{4p_0 3V_0}{T_C}$$

$$T_C = \boxed{12} T_A$$

(2): By  $W = p\Delta V$ ,  $W_{A \rightarrow B} = p_0(3V_0 - V_0) = \boxed{2p_0 V_0}$ .

(3): By the first law of thermodynamics, we have  $Q = \Delta U + W_{gas}$ . Therefore, we have the table ( $\Delta U$  is given by  $\frac{3}{2}\Delta pV$ ):

	$\Delta U$	$W_{gas}$	$Q$
$A \rightarrow B$	$3p_0 V_0$	$2p_0 V_0$	$5p_0 V_0$
$B \rightarrow C$	$\frac{27}{2}p_0 V_0$	0	$\frac{27}{2}p_0 V_0$
$C \rightarrow D$	$-12p_0 V_0$	$-8p_0 V_0$	$-20p_0 V_0$
$D \rightarrow A$	$-\frac{9}{2}p_0 V_0$	0	$-\frac{9}{2}p_0 V_0$

Therefore, the thermal heat the gas receives from outside in process  $\boxed{B \rightarrow C}$  is at a maximum.

(4):  $\boxed{\frac{27}{2}p_0V_0}$ .

(5): The net thermal heat received =  $5p_0V_0 + \frac{27}{2}p_0V_0 - 20p_0V_0 - \frac{9}{2}p_0V_0 = -6p_0V_0$ .

Therefore, the net thermal heat emitted =  $\boxed{6p_0V_0}$ .

**Alternative** The net change in internal energy = 0 after the full cycle.

The net work done to the surrounding by the gas

= The area bounded by the cycle =  $(3p_0)(2V_0) = 6p_0V_0$ .

Therefore, the net thermal heat emitted =  $\boxed{6p_0V_0}$ .

Q5:

Background: The general formula of Doppler effect is  $f_{observed} = \frac{V - v_{observer}}{V - v_{source}} f$ .

(1):  $\boxed{f \frac{V}{V - v}}$

(2): Note that the reflected wave travelled in the opposite direction as the

source before the reflection. Therefore, the observed frequency =  $\boxed{f \frac{V}{V + v}}$ .

(3): Number of beats per second =  $|f_1 - f_2| = |f \frac{V}{V - v} - f \frac{V}{V + v}| = \boxed{f \frac{2Vv}{V^2 - v^2}}$ .

(4): Note that the wave travels in the opposite direction as the observer. There-

fore, the observed frequency =  $\boxed{f \frac{V + v}{V}}$ .

(5): The frequency before reflection= $f \frac{V+v}{V}$ .

The frequency after reflection (i.e. heard)= $(f \frac{V+v}{V})(\frac{V}{V-v}) = \boxed{f \frac{V+v}{V-v}}$ .