Q1(1):

$4x - 6 < 2x < 5x + 3$

$\iff \begin{cases} 4x - 6 < 2x \\ 2x < 5x + 3 \end{cases}$

$\iff \begin{cases} 2x < 6 \\ 3x > -3 \end{cases}$

$\iff \begin{cases} x < 3 \\ x > -1 \end{cases}$

$\iff \boxed{-1 < x < 3}$

Q1(2):

As $f(x)$ is divisible by $x - 2$, by the factor theorem, we have $f(2) = 0$, i.e. $8 + 4a + 2b + 2 = 0$, i.e. $2a + b = -5$......(1).

As the reminder is -3 when $f(x)$ is divided by $x + 1$, by the reminder theorem, we have $f(-1) = -3$, i.e. $-1 + a - b + 2 = -3$, i.e. $a - b = -4$......(2).

Combining the two equation, by $(1) + (2)$, we have $3a = -9$, i.e. $a = \boxed{-3}$.

Substitute $a = -2$ into (2), we have $b = -3 + 4 = \boxed{1}$.

Q1(3):

When the graph of $y = x^2 + ax + 1$ touches the x-axis, the equation $x^2 + ax + 1 = 0$

has only 1 solution. i.e.

$$\Delta = a^2 - 4(1)(1) = 0$$

$$a = \boxed{\pm 2}$$

Q1(4):

$(\log_2 3)(\log_3 4) + 3^{\log_3 5}$

$= (\log_2 3)(2\log_3 2) + 5$

$= 2\log_3(2^{\log_2 3}) + 5$

$= 2\log_3 3 + 5$

$= 2(1) + 5$

$= \boxed{7}$

Q1(5):

As $\frac{\sin A + \cos A}{\sin A - \cos A}$

$= \frac{\frac{\sin A + \cos A}{\cos A}}{\frac{\sin A - \cos A}{\cos A}}$ (as $\cos A \neq 0$)

$= \frac{\tan A + 1}{\tan A - 1}$,

when $\tan A = \sqrt{2}$, we have $\frac{\sin A + \cos A}{\sin A - \cos A} = \boxed{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}$.

(Note: Rationalisation is not necessary. After rationalisation, the answer is $\boxed{3 + 2\sqrt{2}}$.)

Q2:

(1): As C passes through the two fixed points, no matter what the value of k is, we may put certain value of k and find the common solutions of different equations (the solutions are exactly the required coordinates).

Putting $k=0$, we have $x^2+y^2+y-1=0$......(1)

Putting $k=-1$, we have $x^2+y^2-x=0$......(2)

By (1) − (2), we have $x+y-1=0$, i.e. $x=1-y$......(3)

Substitue (3) into (1), we have

$$(1-y)^2+y^2+y-1=0$$

$$2y^2-y=0$$

$$y(2y-1)=0$$

$$y=0 \text{ or } y=\frac{1}{2}$$

When $y=0$, by (3), we have $x=1$

When $y=\frac{1}{2}$, we have $x=\frac{1}{2}$

Therefore, the coordinates of the fixed points are $\boxed{(1,0)}$ and $\boxed{(\frac{1}{2},\frac{1}{2})}$.

(2): By completing the square, we can rewrite C as

$$(x+\frac{k}{2})^2+(y+\frac{1+k}{2})^2=(1+k)+(\frac{k}{2})^2+(\frac{1+k}{2})^2.$$

R.H.S.=(radius of $C)^2=\frac{1}{2}(k^2+3k)+\frac{5}{4}$.

By completing the square again, we have

(radius of $C)^2=\frac{1}{2}(k+\frac{3}{2})^2-\frac{1}{2}\cdot(\frac{3}{2})^2+\frac{5}{4}=\frac{1}{2}(k+\frac{3}{2})^2+\frac{1}{8}$. As $\frac{1}{2}(k+\frac{3}{2})^2\geq 0$, we have (radius of $C)^2\geq\frac{1}{8}$ and hence radius of $C\geq\frac{1}{2\sqrt{2}}$. The minimum radius of C is therefore $\boxed{\dfrac{1}{2\sqrt{2}}}$.

(Note: Rationalisation is not necessary. After rationalisation, the answer will be $\boxed{\dfrac{\sqrt{2}}{2}}$.)

Q3:

(1) By using the slope-intercept form of a straight line, the equation of BA is $y = \frac{7-0}{0-\frac{7}{2}}x+7$, i.e. $y = -2x+7$. As it is tangent to the parabola $y = -x^2+ax+b$, the equation $-x^2 + ax + b = -2x + 7$, i.e. $x^2 - (a + 2)x + (7 - b) = 0$ has only one solution. Then, we have $\Delta = (a + 2)^2 - 4(7 - b) = 0$......(1).

Similarly, by using the slope-intercept form of a straight line, the equation of BC is $y = \frac{7-0}{0+\frac{7}{6}}x+7$, i.e. $y = 6x+7$. As it is tangent to the parabola $y = -x^2+ax+b$, the equation $-x^2 + ax + b = 6x + 7$, i.e. $x^2 + (6 - a)x + (7 - b) = 0$ has only one solution. Then, we have $\Delta = (6 - a)^2 - 4(7 - b) = 0$......(2).

By (1) $-$ (2), we have

$$(a + 2)^2 - (6 - a)^2 = 0$$

$$(a + 2 + 6 - a)(a + 2 - 6 + a) = 0$$

$$a = \boxed{2}$$

Then, substituting $a = 2$ into (1), we have $4^2 - 4(7 - b) = 0$, i.e. $b = \boxed{3}$.

(2) By solving the equation of intersection of BA and the parabola found in part (1), $x^2 - (a + 2)x + (7 - b) = 0$, i.e. $x^2 - 4x + 4 = 0$, we have $x = 2$, i.e. the x-coordinate of the point of tangent of BA to the parabola is 2.

Note that the area of the region bounded by BA and the parabola=(The area under BA)-(The area under the parabola), from $x = 0$ to $x = 2$. Therefore, the area

$= \int_0^2 (-2x + 7)dx - \int_0^2 (-x^2 + 2x + 3)dx$

$= \int_0^2 (x^2 - 4x + 4)dx$

$= \frac{1}{3}x^3 - 2x^2 + 4x|_0^2$

$= \frac{8}{3} - 8 + 8$

$= \boxed{\dfrac{8}{3}}$