Q1(1):

We separate the situation into different cases:

1: The 4th move is "'return to O', then the first 3 moves will not matter. We have the probability $1 \cdot 1 \cdot 1 \cdot \frac{1}{6} = \frac{1}{6}$.

2: The 4th move is "move 1 in negative direction", then the coordinate of P after the 3rd move will be 1. We separate it into sub-cases:

2.1: The coordinate of P after the 2nd move is 2. Then, both the 1st and 2nd move moves P by 1 in the positive direction and the 3rd move moves P by 1 in the negative direction, the probability=$(\frac{1}{2})^2 \cdot (\frac{1}{3}) = \frac{1}{12}$.

2.2: The coordinate of P after the 2nd move is 0. Then, the 1st and 2rd moves move P in possitive and negative directions by 1 separately, or the 2rd move returns P to the origin no matter what the 1st move is. Following the 2rd move, the 3rd move moves P by 1 in the positive direction.

The probability=$(2 \cdot \frac{1}{2} \cdot \frac{1}{3} + 1 \cdot \frac{1}{6}) \cdot \frac{1}{2} = \frac{1}{4}$.

The total probability of case 2=$(\frac{1}{12} + \frac{1}{4}) \cdot \frac{1}{3} = \frac{1}{9}$.

3: The 4th move is "move 1 in possitive direction", then the coordinate of P after the 3rd move will be -1. We separate it into sub-cases:

3.1: The coordinate of P after the 2nd move is -2. Then, both the 1st and 2nd move moves P by 1 in the negative direction and the 3rd move moves P by 1 in the possitive direction, the probability=$(\frac{1}{3})^2 \cdot (\frac{1}{2}) = \frac{1}{18}$.

3.2: The coordinate of P after the 2nd move is 0. Then, same as case 2.2, with the 3rd move moves P by 1 in the negative direction instead,

the probability=$(2 \cdot \frac{1}{2} \cdot \frac{1}{3} + 1 \cdot \frac{1}{6}) \cdot \frac{1}{3} = \frac{1}{6}$.

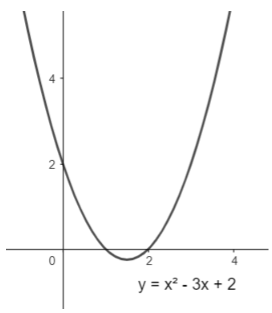
The total probability of case 3$=(\frac{1}{18}+\frac{1}{6})\cdot\frac{1}{3}=\frac{1}{9}$.

Therefore, the required probability$=\frac{1}{9}+\frac{1}{9}+\frac{1}{6}=\boxed{\dfrac{7}{18}}$.

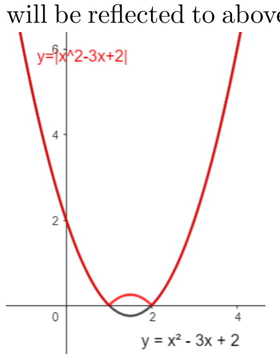
Q1(2):

We can sketch the graph of $y=x|x^2-3x+2|$:

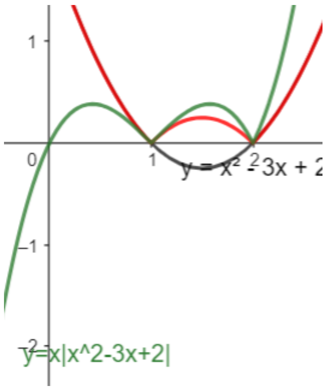
First, we sketch of the graph of $y=x^2-3x+2$. The two x-intercepts are 1 and 2 and hence the axis of symmetry is $x=\frac{3}{2}$:



Next, after the absolute value is taken, the part of the graph under the x-axis will be reflected to above the x-axis. We can sketch the graph of $y=|x^2-3x+2|$:



After we multiply an x to the function, the value turns to 0 when $x=0$ and turns to negative when $x<0$. We can sketch the graph of $y=x|x^2-3x+2|$:



When k takes a value greater than 0 and smaller than each local maximum, we have the maximum number of solutions $\boxed{5}$.

Since we have $y' = \begin{cases} 3x^2 - 6x + 2, \ x \leq 1 \text{ or } x \geq 2 \\ -(3x^2 - 6x + 2), \ 1 \leq x \leq 2 \end{cases}$, by setting $y' = 0$, we have the two local maximum are obtained when $x = \frac{3 \pm \sqrt{3}}{3}$.

By that time, the maximum values are both equal to $\frac{2\sqrt{3}}{9}$.

Therefore, the maximum number of solutions is obtained when $\boxed{0} < k < \boxed{\dfrac{2\sqrt{3}}{9}}$.

(Note: Let $f(x) = x(x^2 - 3x + 2) = x(x-1)(x-2)$, then $f(x+1)$ is an odd function. One can also notice the symmetry with respect to the point $(0, 1)$ and find the local maximum of $f(x)$ directly for the upper boundary of k.)

Q1(3):

Denote the centre of the unit circle as O. Then, $\angle BOA = \theta$ and $\angle COA = 2\theta$.

The area of $\triangle ABC$=(The area of quadrilateral $OABC$)-(The area of $\triangle OAC$)

=$(\frac{1}{2}(OA)(OB)\sin\theta + \frac{1}{2}(OB)(OC)\sin\theta) - \frac{1}{2}(OA)(OC)\sin 2\theta$

$= \boxed{\sin\theta - \frac{1}{2}\sin 2\theta}$.

To find the extremum, we set $(\sin\theta - \frac{1}{2}\sin 2\theta)'$, which is $\cos\theta - \cos 2\theta$, to be equal to 0. Then,

$$\cos\theta - \cos 2\theta = 0$$

$$\cos\theta - 2\cos^2\theta + 1 = 0$$

$$(2\cos\theta + 1)(\cos\theta - 1) = 0$$

$$\theta = \frac{2\pi}{3} \text{ (as } 0 < \theta < \pi)$$

Moreover, as $(\sin\theta - \frac{1}{2}\sin 2\theta)'' = -\sin\theta + 2\sin 2\theta$, which is equal to $-\sqrt{3} < 0$ when $\theta = \frac{2\pi}{3}$, we have the area attains to its maximum when $\theta = \boxed{\frac{2\pi}{3}}$ and the corresponding value is $\boxed{\frac{3\sqrt{3}}{4}}$.

Q1(4):

All divisors of the number $2^k p$ is in a form of $2^i p^j$, where $i, j \in \mathbb{N}$ and $0 \le i \le k$ and $0 \le p \le 1$.

Therefore, the sum of all divisors$=\sum\limits_{i=0}^{k}\sum\limits_{j=0}^{1} 2^i p^j$

$= (\sum\limits_{i=0}^{k} 2^i)(1+p)$

$= (\frac{2^{k+1}-1}{2-1})(1+p)$

$= (\boxed{2^{k+1}} - 1)(1 + \boxed{p})$.

Q1(5):

X is less than or equal to 3 means at least one card is numbered $1, 2$ or 3.

The required probability$=\frac{C_1^3 \cdot C_2^7 + C_2^3 \cdot C_1^7 + C_3^3}{C_3^{10}} = \frac{3 \cdot 21 + 3 \cdot 7 + 1}{120} = \boxed{\frac{17}{24}}$.

Alternative Consider the complementary event: All the three cards taken are not number $1, 2$ or 3.

The required probability$=1 - \frac{C_3^7}{C_3^{10}} = 1 - \frac{35}{120} = \boxed{\frac{17}{24}}$.

Q1(6):

By oberserving the pattern of the sequence, let the general term be a_n, we have $a_{n+1} = a_n + 3n$. Then,

$$\begin{aligned} a_{n+1} - a_n &= 3n \\ \sum_{i=1}^{n} (a_{n+1} - a_n) &= \sum_{i=1}^{n} (3n) \\ a_{n+1} - a_1 &= \frac{3n(n+1)}{2} \\ a_{n+1} &= \frac{3n(n+1)+2}{2} \\ a_n &= \frac{3(n-1)n+2}{2} = \boxed{\frac{3n^2 - 3n + 2}{2}} \end{aligned}$$

And the sum of the first n terms$=\sum_{i=1}^{n} \frac{3i^2 - 3i + 2}{2}$

$= \frac{3}{2} \cdot \frac{1}{6} n(n+1)(2n+1) - \frac{3}{2} \cdot \frac{1}{2} n(n+1) + n$

$= \frac{1}{4}(2n^3 + 3n^2 + n) - \frac{3}{4}(n^2 + n) + n$

$= \boxed{\frac{1}{2} n^3 + n}$

Q1(7):

$\frac{4a+b}{2a} + \frac{4a-3b}{b} = 2 + \frac{b}{2a} + \frac{4a}{b} - 3 = (\frac{b}{2a} + \frac{4a}{b}) - 1.$

By the AM-GM inequality, we have $\frac{b}{2a} + \frac{4a}{b} \geq 2\sqrt{(\frac{b}{2a})(\frac{4a}{b})} = 2\sqrt{2}$, where the equality holds when $\frac{b}{2a} = \frac{4a}{b}$, i.e. $b = 2\sqrt{2}a$.

Therefore, the expression is minimised when $b = \boxed{2\sqrt{2}}a$ and the minimum value is $\boxed{2\sqrt{2}-1}$.

Q1(8):

By the binomial expansion, $(x+1)^n = \sum\limits_{k=0}^{n} {}_nC_k \boxed{x}^{\boxed{k}}$.

Put $x = 2$, we have $\sum\limits_{k=0}^{n} {}_nC_k 2^k = \boxed{3}^{\boxed{n}}$.

Considering the derivative of the equality $(x+1)^n = \sum\limits_{k=0}^{n} {}_nC_k x^k$ with respect to x, we have $n(x+1)^{n-1} = \sum\limits_{k=0}^{n} {}_nC_k k x^{k-1}$.

Put $x = 2$, we have $\sum\limits_{k=0}^{n} {}_nC_k k 2^{k-1} = n \cdot 3^{n-1}$.

i.e. $\sum\limits_{k=0}^{n} {}_nC_k k 2^k = \frac{2n}{3} \cdot 3^n = \frac{\boxed{2n}}{\boxed{3}} \sum\limits_{k=0}^{n} {}_nC_k 2^k$.

Q1(9):

As $\sum\limits_{k=1}^{n} x_k \sum\limits_{l=0}^{k-1} 6^l = \sum\limits_{k=1}^{n} x_k (\frac{6^k-1}{6-1}) = \frac{1}{5}(\sum\limits_{k=1}^{n} x_k 6^k - \sum\limits_{k=1}^{n} x_k)$, we have

$\sum\limits_{k=1}^{n} x_k 6^k = \sum\limits_{k=1}^{n} x_k + 5 \sum\limits_{k=1}^{n} x_k \sum\limits_{l=0}^{k-1} 6^l$.

i.e. $\sum\limits_{k=0}^{n} x_k 6^k = x_0 + \sum\limits_{k=1}^{n} x_k 6^k$

$= x_0 + \sum\limits_{k=1}^{n} x_k + 5 \sum\limits_{k=1}^{n} x_k \sum\limits_{l=0}^{k-1} 6^l$

$= \boxed{\sum_{k=0}^{n} x_k} + \boxed{5} \sum_{k=1}^{n} x_k \sum_{l=0}^{k-1} 6^l.$

Therefore, a senary number can be divided by 5 with no remember if and only if the sum of all of its digits can be divided by $\boxed{5}$ with no remainder.

Q1(10):

$253x + 256y = 1 \iff 253(x+y) + 3y = 1$

Note that $3 \cdot 84 = 252$, we have $253 \cdot 1 - 3 \cdot 84 = 1$. Therefore, $(x+y, y) = (1, -84)$, i.e. $(x, y) = (85, -84)$ is a solution.

Therefore, we have $253 \cdot 85 - 256 \cdot 84 = 1$. Subtract it from the original equation, we have $253(x-85) = -256(y+84)$, which gives us the general solution $(x-85, y+84) = (-256k, 253k)$, where $k \in \mathbb{N}$,

i.e. $(x, y) = (85 - 256k, -84 + 253k)$.

To minimised x, we take $k = 0$. Therefore, we have $x = \boxed{85}$ and $y = \boxed{-84}$.

Q1(11):

The translation translated the point (x, y) to $(x+2, y-3)$. Using the original $x-y$ coordinates to descirbed the graph, we have to set the basis $(x-2, y+3)$.

Then, the equation becomes:

$y + 3 = 2(x-2)^2 + 3(x-2) + 1$, i.e. $y = \boxed{2}x^2 + \boxed{-5}x + \boxed{0}$.

Q2:

(1): $\cos\frac{\pi}{12} = \cos(\frac{\pi}{3} - \frac{\pi}{4})$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= (\frac{1}{2})(\frac{\sqrt{2}}{2}) + (\frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2})$

$= \frac{\boxed{\sqrt{6}}+\sqrt{2}}{4}$.

(2): As $\triangle ADC$ is a right-angled triangle, we can consider the cosine ratio of it:

$$\cos\angle DAC = \frac{AD}{AC}$$

$$AC = \frac{\sqrt{6}}{\cos\frac{\pi}{12}} = \frac{4\sqrt{6}}{\sqrt{6}+\sqrt{2}} = \boxed{6} - 2\sqrt{3}.$$

(3): By the cosine formula, we have $(BC)^2 = (AC)^2 + (AB)^2 - 2(AB)(AC)\cos\angle BAC$

$= (6-2\sqrt{3})^2 + (2\sqrt{2})^2 - 2(2\sqrt{2})(6-2\sqrt{3})\cos\frac{\pi}{12}$

$= 48 - 24\sqrt{3} + 8 - (6\sqrt{2} - 2\sqrt{6})(\sqrt{6}+\sqrt{2})$

$= \boxed{56} - 32\sqrt{3}$.

(4): $BC = \sqrt{56 - 32\sqrt{3}}$

$= 2\sqrt{2}\sqrt{7 - 2\sqrt{12}}$

$= 2\sqrt{2}\sqrt{(\sqrt{4})^2 - 2(\sqrt{4})(\sqrt{3}) + (\sqrt{3})^2}$

$= 2\sqrt{2}\sqrt{(\sqrt{4}-\sqrt{3})^2}$

$= 2\sqrt{2}(2-\sqrt{3})$

$= \boxed{4\sqrt{2}} - 2\sqrt{6}$.

Q3:

(1): By the fundamental theorem of calculus, we have $F'(x) = f(x)$.

As given, $F(x)$ has extreme values at $x = -2a, 2a$, we have $f(-2a) = f(2a) = 0$

and hence the axis of symmetry of the quadratic function $f(x)$ is $x = 0$. Then, $f(x)$ is an even function, i.e. $f(-x) = f(x)$ for all $x \in \mathbb{R}$.

Therefore, we have $F(-x) = \int_0^{-x} f(t)dt$

$= \int_0^x f(-t)d(-t)$

$= -\int_0^x f(t)dt$

$= \boxed{-1} \int_0^x f(t)dt$.

Alternative Geometrically, $F(k)$ represents the area bounded by the graph of $y = f(x)$ and the x-axis from $x = 0$ to $x = k$ (negative sign is taken if $k < 0$). As the graph of $y = f(x)$ is symmetric along $x = 0$, we have $F(-x) = \boxed{-1} F(x)$.

(2): By (1), one solution is $x = 2a$.

On the other hand, we have $f(x) = kx^2 - 4ka^2$, where $k \neq 0$.

Therefore, $F(x) = \frac{k}{3}x^3 - 4ka^2x$ and $F(2a) = \frac{-16k}{3}a^3$.

Solving $F(x) + F(2a) = 0$, we have

$$\frac{k}{3}x^3 - 4ka^2x - \frac{16k}{3}a^3 = 0$$

$$x^3 - 12a^2x - 16a^3 = 0$$

As we know that $x = -2a$ is a root, we can do the factorisation:

$$(x + 2a)(x^2 - 2ax + 8a^2) = 0$$

$$(x + 2a)^2(x - 4a) = 0$$

$$x = \boxed{-2a}, \boxed{4a}$$

(3): As (2), $F(x) = \frac{k}{3}x^3 - 4ka^2x$ and $F'(x) = kx^2 - 4ka^2$.

On the other hand, $(\frac{F(x)}{F'(0)})' = \frac{F'(x)}{F'(0)}$.

To find the extremum, we set the derivative to be 0. Then, by given, we have

$x = -2a$ or $x = 2a$.

Moreover, $(\frac{F(x)}{F'(0)})'' = \frac{2kx}{-4ka^2} = -\frac{x}{2a^2}$.

Therefore, the local maximum is obtained when $x > 0$.

Combine the above, the local maximum is $\frac{F(2a)}{F'(0)} = \frac{\frac{-16k}{3}a^3}{-4ka^2} = \boxed{\frac{4}{3}a}$.