Q1(1):

By completing the square,

$x^2 + y^2 - 4x + 6y - 12 = 0 \iff (x-2)^2 + (y+3)^2 = 12 + 2^2 + 3^2 = 25 = 5^2.$

Therefore, the radius= $\boxed{5}$.

Q1(2):

Solving the simultaneous equation $\begin{cases} x + 2y - 1 = 0 \\ x - y + 2 = 0 \end{cases}$, we have $(x, y) = (-1, 1)$,

i.e. the two straight lines meet at $(-1, 1)$.

If $ax - y + 3 = 0$ meet the two straight lines at one point, then it should pass through $(-1, 1)$. i.e. $-a - 1 + 3 = 0$, i.e. $a = \boxed{2}$.

Alternative By Gaussian elimination:

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & -1 & -2 \\ a & -1 & -3 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & -3 & -3 \\ 0 & -1-2a & -3-a \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2+a \end{array}\right).$$

If the system has a unique solution, we have $-2 + a = 0$, i.e. $a = \boxed{2}$.

Q1(3):

$\sqrt{5 - x} < x + 1$

$5 - x < (x+1)^2$

$x^2 + 3x - 4 > 0$

$(x+4)(x-1) > 0$

$x < -4$ or $x > 1$

Besides, as $\sqrt{5-x} > 0$, we have $x + 1 > 0$, i.e. $x > -1$.

On the other hand, for the expression $\sqrt{5-x}$ to be defined, we have $5 - x \geq 0$, i.e. $x \leq 5$.

Finding the intersection of the above two results, we have the solution $\boxed{1 < x \leq 5}$.

Q1(4):

Sum of roots=$\alpha + \beta = -\frac{-1}{1} = 1$ and product of roots=$\alpha\beta = \frac{4}{1} = 4$.

Then, $\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{1^2 - 2(4)}{4}$

$= \boxed{-\dfrac{7}{4}}$.

Alternative $\alpha^2 + \beta^2$ can also be computed as the following:

As $x^2 = x - 4$, we have $\alpha^2 + \beta^2 = (\alpha - 4) + (\beta - 4) = (\alpha + \beta) - 8 = 1 - 8 = -7$.

Q1(5):

We have $a + b = \sqrt{10}$ and $ab = \frac{10-2}{4} = 2$.

Then, $\log_2(a^2+ab+b^2) = \log_2((a+b)^2-ab)$

$= \log_2(10-2)$

$= \boxed{3}$.

Alternative Substitue the values of a and b into the expression and evaluate directly:

$\log_2(a^2+ab+b^2) = \log_2((\frac{\sqrt{10}+\sqrt{2}}{2})^2 + (\frac{\sqrt{10}+\sqrt{2}}{2})(\frac{\sqrt{10}-\sqrt{2}}{2}) + (\frac{\sqrt{10}-\sqrt{2}}{2})^2)$

$= \log_2(\frac{12+2\sqrt{20}}{4} + \frac{8}{4} + \frac{\frac{12-2\sqrt{20}}{4}}{)}$

$= \log_2(6+2)$

$= \boxed{3}$.

Q2:

(1): By the given conditions, we have:

$\int_0^2 (ax+b)dx = 2 \iff [\frac{a}{2}x^2 + bx]_0^2 = 2 \iff a+b = 1......(1)$

$\int_0^2 (ax+b)^2 dx = 4 \iff \int_0^2 (a^2x^2 + 2abx + b^2)dx = 4$

$\iff [\frac{a^2}{3}x^3 + abx^2 + b^2x]_0^2 = 4 \iff \frac{4}{3}a^2 + 2ab + b^2 = 2......(2)$

Substitue (1) into (2):

$$\frac{4}{3}a^2 + 2a(1-a) + (1-a)^2 = 2$$

$$a^2 = 3$$

$$a = \pm\sqrt{3}$$

Then, $b = 1 \mp \sqrt{3}$.

As $f(0) = b > 0$, we have $b = 1+\sqrt{3}$ and $a = -\sqrt{3}$.

Therefore, $f(x) = \boxed{-\sqrt{3}x + (1+\sqrt{3})}$.

(2): $\int_0^2 (g(x))^2 dx = \int_0^2 (f(x)+c)^2 dx$

$= \int_0^2 (f(x))^2 dx + 2c\int_0^2 f(x)dx + c^2 \int_0^2 dx$

$= 4 + 4c + 2c^2$.

Therefore, $\frac{d(\int_0^2 (g(x))^2 dx)}{dc} = 4c + 4$.

To find the extremum of the integral, we set $\frac{d(\int_0^2 (g(x))^2 dx)}{dc} = 0$, then $c = -1$.

As $\frac{d^2(\int_0^2 (g(x))^2 dx)}{dc^2} = 4 > 0$, the integral attains to its minimum when $c = -1$ and the corresponding value$=4-4+2 = \boxed{2}$.

Alternative to check the minimum Conduct the first derivative test:

c	$(-\infty, -1)$	$(-1, \infty)$
$\frac{d(\int_0^2 (g(x))^2 dx)}{dc}$	$-$	$+$
$\int_0^2 (g(x))^2 dx$	$\searrow$	$\nearrow$

Therefore, the integral attains to its minimum when $c = -1$.

Q3:

(1): By Thales's theorem, $\angle APB = 90°$.

Consider the cosine ration, we have $\cos x = \frac{PB}{AB}$, i.e. $PB = 6\cos x$.

Therefore, the area$=\frac{1}{2}(AB)(BP)\sin x = \frac{1}{2}(6)(6\cos x)\sin x = 18\sin x\cos x = \boxed{9\sin 2x}$.

(2): Solving $9\sin 2x \geq \frac{9\sqrt{2}}{2}$ for $x \in (0, 180^\circ)$, we have

$$\sin 2x \geq \frac{\sqrt{2}}{2}$$

$$45^\circ \leq 2x \leq 135^\circ$$

$$\boxed{25.5^\circ \leq x \leq 67.5^\circ}$$

(3): As $\angle APB = 90^\circ$, by Pythagoras' theorem, we have $AP^2 + BP^2 = AB^2$.

Given that $AP = 3\sqrt{6} - BP$, we have

$$(3\sqrt{6} - BP)^2 + BP^2 = 6^2$$

$$BP^2 - 3\sqrt{6}BP + 9 = 0$$

$$BP = \frac{3(\sqrt{6} \pm \sqrt{2})}{2}$$

Then, $AP = \frac{3(\sqrt{6} \mp \sqrt{2})}{2}$.

The area of $\triangle APB = \frac{1}{2}(AP)(BP)$

$= \frac{1}{2}(\frac{3(\sqrt{6} \pm \sqrt{2})}{2})(\frac{3(\sqrt{6} \mp \sqrt{2})}{2}) = \frac{1}{2}(\frac{9(6-2)}{4}) = \boxed{\frac{9}{2}}$.