

Q1(1):

$$x^2 - 6x + 3 < 0$$

$$0 < 3 - \sqrt{6} < x < 3 + \sqrt{6} < 6$$

Therefore, the integers satisfying the inequality are 1, 2, 3, 4, 5, where there are totally $\boxed{5}$ integers.

Q1(2):

$$\sin 30^\circ + \cos 120^\circ + \tan 45^\circ$$

$$= \frac{1}{2} - \frac{1}{2} + 1$$

$$= \boxed{1}$$

Q1(3):

$$2^{3x-2} = 128 = 2^7$$

$$3x - 2 = 7$$

$$x = \boxed{3}$$

Q1(4):

By completing the square, we have $y = (x - 1)^2 + 2$. Therefore, the minimum

value is $\boxed{2}$.

Now, test the boundaries:

When $x = 0$, $x^2 - 2x + 3 = 3$.

When $x = 3$, $x^2 - 2x + 3 = 6$.

Therefore, the maximum value is $\boxed{6}$.

Q1(5):

By the cosine formula,

$$BC = \sqrt{AB^2 + AC^2 - 2(AB)(AC)\cos \angle A} = \sqrt{12 + 9 - 18} = \boxed{\sqrt{3}}.$$

By the sine formula,

$$\frac{AB}{\sin \angle C} = \frac{BC}{\sin \angle A}$$
$$\frac{2\sqrt{3}}{\sin \angle C} = \frac{\sqrt{3}}{\sin 30^\circ}$$

$$\sin \angle C = 1$$

$$C = \boxed{90^\circ}$$

Q1(6):

As the prime factorisation of $108=2^2 \times 3^3$, there are $(2 + 1) \cdot (3 + 1) = \boxed{12}$

positive divisors with 108.

Q1(7):

When $x^2 - 2x + a$ is divisible by $x + 1$, by the factor theorem, $1 + 2 + a = 0$,

i.e. $a = \boxed{-3}$.

Q1(8):

$$f(2) = 3 \cdot 4 - 2 \cdot 2 + 1 = \boxed{9}.$$

$$f'(x) = 6x - 2, f'(1) = 6 - 2 = \boxed{4}.$$

$$\int_0^2 (3x^2 - 2x + 1) dx = [x^3 - x^2 + x]_0^2 = 8 - 4 + 2 = \boxed{6}.$$

Q1(9):

Note that the progression is defined by the recurrence $a_{n+1} = n + a_n$.

Therefore, $a_4 = 3 + a_3 = 3 + 4 = \boxed{7}$.

Q1(10):

The slopes of the two straight lines are $\frac{3}{a-3}$ and $-(a+1)$ respectively.

When they are perpendicular to each other, we have

$$\frac{-3(a+1)}{a-3} = -1$$

$$3a + 3 = a - 3$$

$$a = \boxed{-3}$$

Q1(11):

By the AM-GM inequality, $a + \frac{9}{a} \geq 2\sqrt{a(\frac{9}{a})} = \boxed{6}$.

Q2:

(1): $x^2 + y^2 - 4x + 6y + 8 = 0 \iff (x-2)^2 + (y+3)^2 = -8 + 2^2 + 3^2 = 5$.

Therefore, the centre of P is $\boxed{(2, -3)}$ and the radius is $\boxed{\sqrt{5}}$.

(2): The slope of radius to Q = $\frac{-3+5}{2-3} = -2$. As the tangent is perpendicular to the radius, we have the slope of the tangent $\frac{-1}{-2} = \frac{1}{2}$.

By the point-slope form of straight line, we have the equation of tangent

$$y + 5 = \frac{1}{2}(x - 3), \text{ i.e. } x - \boxed{2}y - \boxed{13} = 0.$$

(3): We have $\vec{QP} = \langle -1, 2 \rangle$ and $\vec{QR} = \langle -2, -1 \rangle$.

Therefore, $\vec{QP} \cdot \vec{QR} = 2 - 2 = \boxed{0}$.

Hence, $\angle PQR = 90^\circ$. Considering the tangent ratio, we have

$$\tan \angle PRQ = \frac{PQ}{QR} = \frac{\sqrt{(3-2)^2 + (-5+3)^2}}{\sqrt{(1-3)^2 + (-6+5)^2}} = \boxed{1}.$$

Q3:

(1): As the vertex of the parabola is $(1, -3)$, the equation of it is

$y = a(x-1)^2 - 3 = ax^2 - 2ax + a - 3$, where $a > 0$ as the parabola is convex downwards.

The option that match this fact is $\boxed{(3)}$.

(2): As the point (x, y) is moved to $(-x, -y)$, the equation becomes $-2(-x)^2 - 4(-x) - 1$, i.e. $-2x^2 - 4x + 1$ $\boxed{(5)}$.

(3): The graph is an exponential function a^x that is decreasing (i.e. $0 < a < 1$).
The correct option is $\boxed{(9)}$.

(4): When shift by -1, (x, y) becomes $(x - 1, y)$. Therefore, $x' = x + 1$ and the equation is $(\frac{1}{2})^{x+1}$, i.e. 2^{-x-1} $\boxed{(12)}$.

(5): The transformation will obtain the graph of the inverse function, i.e.
 $y = \log_{\frac{1}{2}} x$ $\boxed{(7)}$.