Q1(1):

$x+y=\frac{(3+\sqrt{5})^2+(3-\sqrt{5})^2}{3^2-\sqrt{5}^2}=\frac{28}{4}=7.$

$xy=1.$

$x^3+y^3=(x+y)^3-3xy(x+y)=7^3-3(1)(7)=\boxed{322}.$

Q1(2):

By testing the potential roots given by the rational root theorem $\pm1,\pm2,\pm3,\pm6$,

we have $x=-3$ is a root.

Therefore, we can do the factorisation by the long division:

$$x^3+5x^2+8x+6=0$$

$$(x+3)(x^2+2x+2)=0$$

$$x=\boxed{-3,-1\pm i}$$

Q1(3):

$$3^{2x+1}+2\cdot3^x-1=0$$

$$3\cdot3^{2x}+2\cdot3^x-1=0$$

$$(3\cdot3^x-1)(3^x+1)=0$$

$$3^x=\frac{1}{3}$$

$$x = \boxed{-1}$$

Q1(4):

$$\cos^2 x - \sin x \cos x = 1$$

$$-\sin x \cos x = \sin^2 x$$

$$\sin x = 0 \text{ or } \tan x = -1$$

$$x = \boxed{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}}$$

Q1(5):

$$x^2 - 4x + 2 < 0$$

$$(x - (2 + \sqrt{2}))(x - (2 - \sqrt{2})) < 0$$

$$\boxed{2 - \sqrt{2} < x < 2 + \sqrt{2}}$$

Q1(6):

$$(\log_2 x)^2 - 2\log_4 x^3 + 2 < 0$$

$$(\log_2 x)^2 - 3\log_2 x + 2 < 0$$

$$(\log_2 x - 2)(\log_2 x - 1) < 0$$

$$1 < \log_2 x < 2$$

$$\boxed{2 < x < 4}$$

Q1(7):

$\vec{c} = < 2 + t, 1, -3 - 2t >$.

$\vec{c} \perp \vec{a}$ if and only if

$$\vec{c} \cdot \vec{a} = 0$$

$$4 + 2t + 1 + 9 + 6t = 0$$

$$t = \boxed{-\frac{7}{4}}$$

Q1(8):

The area$=\frac{1}{2}|\vec{OA} \times \vec{OB}|$

$= \frac{1}{2}\left\| \begin{vmatrix} i & j & k \\ 3 & 1 & 4 \\ -1 & 4 & 3 \end{vmatrix} \right\|$

$= \frac{1}{2}| < -13, -13, 13 > |$

$= \boxed{\frac{13\sqrt{3}}{2}}$

Q1(9):

Suppose $a_{n+1} + k = \frac{2}{3}(a_n + k)$, then $k = -\frac{3}{4}$.

Therefore,

$$\frac{a_{n+1} - \frac{3}{4}}{a_n - \frac{3}{4}} = \frac{2}{3}$$
$$\frac{a_{n+1} - \frac{3}{4}}{a_1 - \frac{3}{4}} = (\frac{2}{3})^n$$
$$a_{n+1} = (\frac{2}{3})^{n-2} + \frac{3}{4}$$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} = \boxed{\dfrac{3}{4}}$.

Q1(10):

$\lim_{x\to\infty} (\sqrt{x^2 + 3x + 1} - x)$

$= \lim_{x\to\infty} \frac{3x+1}{\sqrt{x^2+3x+1}+x}$

$= \lim_{x\to\infty} \frac{3+\frac{1}{x}}{\sqrt{1+\frac{3}{x}+\frac{1}{x^2}}+1}$

$= \boxed{\dfrac{3}{2}}$

Q1(11):

$\frac{dy}{dx} = 3x^2 + k$.

As the slope of $y = 2x + 1$ is 2, when $y = 2x + 1$ is tangent to $y = x^3 + kx + 3$,

we have:

$$\begin{cases} 3x^2 + k = 2......(1) \\ x^3 + kx + 3 = 2x + 1......(2) \end{cases}$$

From (1), we have $x = \sqrt{\frac{2-k}{3}}$.

Substitue it into (2), we have

$$(\sqrt{\frac{2-k}{3}})^3 + (k-2)\sqrt{\frac{2-k}{3}} = -2$$

$$(2-k)\sqrt{2-k} - 3(2-k)\sqrt{2-k} = -6\sqrt{3}$$

$$(2-k)^{\frac{3}{2}} = 3^{\frac{3}{2}}$$

$$2-k = 3$$

$$k = \boxed{-1}$$

Q1(12):

$f'(x) = 1 + \frac{x}{\sqrt{1-x^2}}$.

To find the extremum of $f(x)$, we set $f'(x) = 0$, then $x = \pm\frac{1}{\sqrt{2}}$.

The table of signs is given:

x	$(-1, -\frac{1}{\sqrt{2}})$	$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$	$(\frac{1}{\sqrt{2}}, 1)$
$f'(x)$	$-$	$+$	$-$
$f(x)$	$\searrow$	$\nearrow$	$\searrow$

Therefore, $f(x)$ attain to its minimum when $x = \boxed{-\frac{1}{\sqrt{2}}}$.

Q2:

1): $AB = \begin{bmatrix} 2+2b & a+4 \\ 4+4b & 2a+8 \end{bmatrix}$.

Solving $AB = O$, we have $a = \boxed{-4}$ and $b = \boxed{-1}$.

2): $X = A^{-1}B$

$$= -\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 8 & 2 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}}$$

3): $A^2 + A - 6I$

$$= \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} + A - 6I$$

$$= \begin{bmatrix} a^2 + bc + a - 6 & ab + bd + b \\ ac + cd + c & bc + d^2 + d - 6 \end{bmatrix}.$$

Therefore, we have:

$$\begin{cases} a^2 + bc + a - 6 = 0......(1) \\ ab + bd + b = 0......(2) \\ ac + cd + c = 0......(3) \\ bc + d^2 + d - 6 = 0......(4) \end{cases}.$$

By (2) and (3), we have $b = c = 0$ or $a + d = -1$.

As $a + d > 0$, we have $b = c = 0$.

Then, by (4), $d^2 + d - 6 = 0$, i.e. $d = 2$ or $d = -3$.

By (1), we have $a^2 + a - 6 = 0$, i.e. $a = 2$ or $a = -3$.

As $a + d > 0$, we have $(a, d) = (2, 2)$, then $a + d = \boxed{4}$ and $ad - bc = \boxed{4}$.

Q3:

1): $\int_0^\pi \sin^2 t dt$

$= \int_0^\pi \frac{1-\cos 2t}{2} dt$

$= [\frac{1}{2}t - \frac{1}{4}\sin 2t]_0^\pi$

$= \boxed{\frac{\pi}{2}}$

2):

$$\int_0^\pi t\sin^2 t dt = \int_0^\pi (\pi - t)\sin^2(\pi - t)dt = \pi \int_0^\pi \sin^2 t dt - \int_0^\pi t\sin^2 t dt$$

$$\int_0^\pi t\sin^2 t dt = \frac{1}{2}(\frac{\pi^2}{2}) = \boxed{\frac{\pi^2}{4}}$$

3): Let $c = \frac{1}{\pi}\int_0^\pi f(t)\sin^2 t dt$.

Then, $f(x) = x + c$ and

$$c = \frac{1}{\pi}\int_0^\pi (x+c)\sin^2 t dt$$

$$c = \frac{1}{\pi}(\frac{\pi^2}{4} + \frac{c\pi}{2}) = \frac{\pi}{4} + \frac{c}{2}$$

$$c = \frac{\pi}{2}$$

Therefore, $f(x) = \boxed{x + \frac{\pi}{2}}$.