

Q1(1):

For each element, we have 3 choices. Therefore, the total number of sequences is $3^3 = \boxed{27}$.

If we do not take into account the order, we have to separate into three cases:

- 1: There are no repeated elements, then the number of sequence is exactly 1.
- 2: There are two repeated elements, then we choose one among the three numbers as the repeated element and choose one among the remaining two numbers as the different elements, the number of such a sequence is $C_1^3 \cdot C_1^2 = 6$.
- 3: There are three repeated elements, then the number of sequence is 3.

Combine the above, there are total $1 + 6 + 3 = \boxed{10}$ sequences.

Alternative The latter part is asking about the number of combinations with repetition allowed. Therefore, we have the number of sequences $H_3^3 = C_2^5 = \boxed{10}$.

Q1(2):

The angle between the line segment of the two points and the horizon line is $\arctan \frac{3-1}{3-2} = \arctan 2$. After the transformation, the angle becomes $45^\circ + \arctan 2$ and the length of the line segment (i.e. $\sqrt{5}$) remains unchanged.

Therefore, the coordinates of the new point is given by

$$\begin{aligned} & (2 + \sqrt{5} \cos(45^\circ + \arctan 2), 1 + \sqrt{5} \sin(45^\circ + \arctan 2)) \\ &= (2 + \frac{\sqrt{10}}{2}(\cos \arctan 2 - \sin \arctan 2), 1 + \frac{\sqrt{10}}{2}(\cos \arctan 2 + \sin \arctan 2)). \end{aligned}$$

We have $\cos \theta = \pm \frac{1}{\sqrt{\tan^2 \theta + 1}}$.

As $90^\circ = 45^\circ + 45^\circ < 45^\circ + \arctan 2 < 45^\circ + 90^\circ = 125^\circ$, we have $45^\circ + \arctan 2$

lies on quadrant II and hence $\cos \arctan 2 = -\frac{1}{\sqrt{2^2+1}} = \frac{1}{\sqrt{5}}$.

Moreover, as $\sin \theta = \pm \sqrt{1 - \cos^2 \theta}$, we have $\sin \arctan 2 = \sqrt{1 - \frac{1}{5}} = \frac{2}{\sqrt{5}}$.

Therefore, the coordinates $= (2 + \frac{\sqrt{10}}{2}(\frac{-1}{\sqrt{5}}), 1 + \frac{\sqrt{10}}{2}(\frac{3}{\sqrt{5}}))$

$$= \left(2 - \frac{\sqrt{2}}{2}, 1 + \frac{3\sqrt{2}}{2} \right).$$

Alternative The column vector from $(2, 1)$ to $(3, 3)$ is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

After the rotation, the vector is given by $\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{-\sqrt{2}}{2} \\ \frac{3\sqrt{2}}{2} \end{bmatrix}$.

Therefore, the coordinates of the point after rotation is $\left(2 - \frac{\sqrt{2}}{2}, 1 + \frac{3\sqrt{2}}{2} \right)$.

Q1(3):

Let the equation be $y = Ax^3 + Bx^2 + Cx + D$. substitute the coordinates of

those given points into the equation, we have the system:

$$\begin{cases} D = 1 \dots (1) \\ -A + B - C + D = -2 \dots (2) \\ A + B + C + D = 2 \dots (3) \\ 8A + 4B + 2C + D = 9 \dots (4) \end{cases}.$$

By (2)+(3), we have $2B + 2D = 0$. Follow by (1), we have $B = -1$.

Substitute $(B, D) = (-1, 1)$ into (3) and (4) respectively, we have:

$$\begin{cases} A + C = 2 \\ 4A + C = 6 \end{cases}$$

Therefore, we have $(A, C) = (\frac{4}{3}, \frac{2}{3})$.

The equation is $y = \boxed{\frac{4}{3}}x^3 + \boxed{-1}x^2 + \boxed{\frac{2}{3}}x + \boxed{1}$.

Q1(4):

Note that $|x^2 - 2x| = |x(x - 2)|$. As for $x \in [0, 2]$, $x(x - 2) \leq 0$, it can be written

as $2x - x^2$.

$$\text{Therefore, we have } f(x) = \begin{cases} 1 + x - x^2, & x \in [0, 1] \\ 3x - 1 - x^2, & x \in [1, 2] \end{cases} = \begin{cases} -(x - \frac{1}{2})^2 + \frac{5}{4}, & x \in [0, 1] \\ -(x - \frac{3}{2})^2 + \frac{5}{4}, & x \in [1, 2] \end{cases}$$

by completing the square.

Therefore, the maximum of $f(x)$ is $\boxed{\frac{5}{4}}$.

Check the boundaries:

$$f(0)=1$$

$$f(1)=1$$

$$f(2)=1$$

Therefore, the minimum of $f(x)$ is $\boxed{1}$.

Q1(5):

Regard the condition as an equation for y , for $y \in \mathbb{R}$, we have

$$\Delta = 0^2 - 4(1)(2x^2 - 4) \geq 0$$

$$-\sqrt{2} \leq x \leq \sqrt{2}$$

Moreover, substitute $y^2 = 4 - 2x^2$ into the objective function, we have $4x + 4 - 2x^2$, which is equal to $-2(x - 1)^2 + 6$ by completing the square.

When $x = 1$, we have the maximum $\boxed{6}$.

Now, check the boundaries:

$$\text{When } x = -\sqrt{2}, 4x + 4 - 2x^2 = -4\sqrt{2}$$

$$\text{When } x = \sqrt{2}, 4x + 4 - 2x^2 = 4\sqrt{2}$$

Therefore, the minimum is $\boxed{-4\sqrt{2}}$.

Alternative The constrain $-\sqrt{2} \leq x \leq \sqrt{2}$ can also be obtained geometrically:

Rewrite the condition as $(\frac{x}{\sqrt{2}})^2 + (\frac{y}{2})^2 = 1$, we have the condition represents an ellipse with x-intercepts $\pm\sqrt{2}$. Therefore, we have $-\sqrt{2} \leq x \leq \sqrt{2}$.

Q1(6):

We use those white balls to separate those red balls. As there are 6 vacancies created with the 5 white balls (the beginning and the end included), we are going to pick 3 among of the 6 to place the red balls. Therefore, the number of arrangements is $C_3^6 = \boxed{20}$.

Next, without constrains, the arrangement is equalvalent to a permutation of 8 objects with 3 and 5 different indistinctive objects (red balls and white balls).

Therefore, the total number of arrangements is $\frac{8!}{3!5!} = 56$.

Now that red balls are adjoining, the case “no red balls are adjoining” has to be removed. Then, the number of arrangements $= 56 - 20 = \boxed{36}$.

Alternative For the latter part, we can separate it into 2 cases:

1: 2 red balls are adjoining, then we are going to place 2 distinctive objects (2 adjoining red balls and 1 single red ball) into the 6 vacancies created by the 5 white balls. The number of arrangements is hence $P_2^6 = 30$.

2: 3 red balls are adjoining, then we are going to place 1 object (3 adjoining red balls) into the 6 vacancies created by the 5 white balls. The number of arrangements is hence $C_1^6 = 6$.

Given the above, the total number of arrangements is $30 + 6 = \boxed{36}$.

Q1(7):

$$\begin{aligned} 10100101_2 &= 2^7 + 2^5 + 2^2 + 1 \\ &= 128 + 32 + 4 + 1 \\ &= \boxed{165}. \end{aligned}$$

Q1(8):

$$\begin{aligned} 2 - \log_{10} 2 - 2 \log_{10} 5 &= \log_{10} 10^2 - (\log_{10} 2 + \log_{10} 25) \\ &= \log_{10} 100 - \log_{10} 50 \\ &= \log_{10} \frac{100}{50} \end{aligned}$$

$$= \log_{10} \boxed{2}.$$

Q1(9):

The equation of the line passing through $A(2, 3)$ and $B(3, 5)$ is given by

$$y - 3 = \frac{5-3}{3-2}(x - 2), \text{ i.e. } 2x - y - 1 = 0.$$

$$\text{Therefore, the required distance is } \frac{|2(0)-(0)-1|}{\sqrt{2^2+(-1)^2}} = \boxed{\frac{1}{\sqrt{5}}}.$$

Alternative Let $P(p, 2p - 1)$ be a point on the line AB . Note that OP is minimised when $OP \perp AB$. By that time, considering the slopes, we have

$$\left(\frac{2p-1}{p}\right)(2) = -1, \text{ i.e. } p = \frac{2}{5}.$$

$$\text{Then, } OP = \sqrt{\left(\frac{2}{5}\right)^2 + \left(2\left(\frac{2}{5}\right) - 1\right)^2} = \boxed{\frac{\sqrt{5}}{5}}.$$

Alternative The vector \vec{OP} is given by $t\vec{OA} + (1-t)\vec{OB}$, i.e. $\langle 3-t, 5-2t \rangle$, where $t \in \mathbb{R}$.

When $\vec{OP} \perp \vec{AB}$, we have $\langle 3-t, 5-2t \rangle \cdot \langle 1, 2 \rangle = 0$, i.e. $3-t+10-4t=0$, i.e. $t = \frac{13}{5}$.

$$\text{Then, } |\vec{OP}| = \sqrt{\left(3 - \frac{13}{5}\right)^2 + \left(5 - 2\left(\frac{13}{5}\right)\right)^2} = \boxed{\frac{\sqrt{5}}{5}}.$$

Q1(10):

By the sine formula, we have $\frac{AB}{\sin \angle C} = \frac{BC}{\sin \angle A}$, i.e. $\sin \angle C = \frac{6}{7} \sin 60^\circ = \frac{3\sqrt{3}}{7}$.

Then, $\angle B = 180^\circ - \angle A - \angle C = 120^\circ - \arcsin \frac{3\sqrt{3}}{7}$.

$$\begin{aligned}
\cos \angle B &= \cos(120^\circ - \arcsin \frac{3\sqrt{3}}{7}) \\
&= \cos 120^\circ \cos \arcsin \frac{3\sqrt{3}}{7} + \sin 120^\circ \sin \arcsin \frac{3\sqrt{3}}{7} \\
&= -\frac{1}{2} \sqrt{1 - (\frac{3\sqrt{3}}{7})^2} + \frac{\sqrt{3}}{2} \frac{3\sqrt{3}}{7} \\
&= \frac{9 - \sqrt{22}}{14}.
\end{aligned}$$

$$\begin{aligned}
\text{By the cosine formula, } CA &= \sqrt{(AB)^2 + (BC)^2 - 2(AB)(BC) \cos \angle B} \\
&= \sqrt{36 + 49 - 6(9 - \sqrt{22})} \\
&= \sqrt{31 + 6\sqrt{22}} \\
&= \sqrt{3^2 + (\sqrt{22})^2 + 2(6)\sqrt{22}} \\
&= \sqrt{(3 + \sqrt{22})^2} \\
&= \boxed{3 + \sqrt{22}}.
\end{aligned}$$

Alternative $\cos \angle C = \cos \arcsin \frac{3\sqrt{3}}{7} = \sqrt{1 - (\frac{3\sqrt{3}}{7})^2} = \frac{\sqrt{22}}{7}.$

By the cosine formula,

$$AB^2 = CA^2 + CB^2 - 2(CA)(CB) \cos \angle C$$

$$\begin{aligned}
CA^2 - 2\sqrt{22}CA + 13 &= 0 \\
CA &= \frac{2\sqrt{22} + \sqrt{(2\sqrt{22})^2 - 4(1)(13)}}{2} = \boxed{3 + \sqrt{22}} \text{ (as } CA > 0)
\end{aligned}$$

Q2:

(1): Let $P(p, p^2)$ ($p > 0$) be the point of tangent.

As $y' = 2x$, we have the slope of the tangent at P is $2p$.

The line orthogonal to the tangent will hence have a slope of $-\frac{1}{2p}$.

On the other hand, the slope of the line passes through $(0, \frac{3}{2})$ and P is given by

$$\frac{p^2 - \frac{3}{2}}{p} = \frac{2p^2 - 3}{2p}.$$

Therefore, we have $\frac{2p^2 - 3}{2p} = -\frac{1}{2p}$, i.e. $p = 1$ (as $p > 0$).

Then, the slope of the orthogonal line is $-\frac{1}{2}$ and its equation is $y = \boxed{-\frac{1}{2}}x + \frac{3}{2}$

using the slope-intercept form of straight line.

Moreover, the x-coordinate of the intersection of the two lines is $p = \boxed{1}$.

$$(2): \int_0^1 x^2 dx = \frac{1}{3}x^3 \Big|_0^1 = \boxed{\frac{1}{3}}.$$

(3): Note that the region under the line $y = -\frac{1}{2}x + \frac{3}{2}$ and above the x-axis is a trapezium. Therefore, its area is $\frac{(\frac{3}{2}+1)(1)}{2} = \frac{5}{4}$.

$$S_2 = \frac{5}{4} - S_1 = \boxed{\frac{11}{12}}.$$

$$(4): \frac{S_2}{S_1} = \frac{\frac{11}{12}}{\frac{1}{3}} = \boxed{\frac{11}{4}}.$$

Q3:

(1): Consider the mean, $\frac{3+5+6+4+[3-1]+1+7+7+8+3}{10} = 5.0$, we have $[3-1] = \boxed{6}$.

Therefore, $[3-2] = [3-1] + 5 = \boxed{11}$.

For the mean of Question B, we have $[3-3] = \frac{3+5+6+4+5+8+6+5+5+3}{10} = \boxed{5.0}$.

Then, the mean of the total data $= [3-4] = \frac{5.0 \cdot 10 + 5.0 \cdot 10}{10} = \boxed{10.0}$.

Moreover, $[3-5] = \sqrt{\frac{(3-5)^2 + (5-5)^2 + (6-5)^2 + (4-5)^2 + (5-5)^2 + (8-5)^2 + (6-5)^2 + (5-5)^2 + (5-5)^2 + (3-5)^2}{10}}$

$$= \sqrt{\frac{4+0+1+1+0+9+1+0+0+4}{10}}$$

$$= \sqrt{2}$$

$$\approx \boxed{1.4}^*.$$

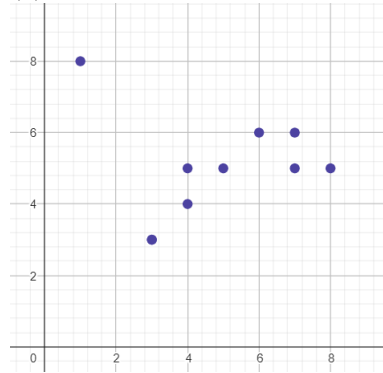
(*: Memorise $\sqrt{2} \approx 1.414$, or approximating using the method of calculating

square root by hand, Newton's method, Taylor's polynomial, etc.)

(2): Rearrange the data: 6, 6, 8, 9, 10, 11, 12, 12, 13, 13.

Therefore, the lowest quartile, median and the upper quartile are 8, 10.5 and 12 respectively. The boxplot matches it is $\boxed{(d)}$.

(3): A plot of scores of Question A against B is shown:



There is no a strong relation between two data set can be seen. Therefore, the condition should be $\boxed{(c)}$.

Alternative Refer to the table:

A	B	$(A - \bar{A})$	$(B - \bar{B})$	$(A - \bar{A})(B - \bar{B})$
3	3	-2	-2	4
5	5	0	0	0
6	6	-1	-1	1
4	4	-1	-1	1
6	5	1	0	0
1	8	-4	3	-12
7	6	2	1	2
7	5	2	0	0
8	5	3	0	0
3	3	-2	-2	4

The covariance= $4+0+1+1+0-12+2+0+0+1=0$.

Therefore, the correlation coefficient r is 0, so the answer is $\boxed{(c)}$.