

Q1(1):

$$\begin{aligned} & (-2x^2y)^3 \div x^5y \div (-2x^2y^2) \\ &= \frac{-8x^6y^3}{-2x^7y^3} \\ &= \boxed{\frac{4}{x}} \end{aligned}$$

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Q1(2):

$$\begin{aligned} & \frac{\sqrt{5}}{\sqrt{3}+1} - \sqrt{\frac{30}{8}} + \frac{\sqrt{45}}{2} \\ &= \frac{\sqrt{15}-\sqrt{5}}{2} - \frac{\sqrt{15}}{2} + \frac{3\sqrt{5}}{2} \\ &= \boxed{\sqrt{5}} \end{aligned}$$

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Q1(3):

Note that  $(x+1)(2x+3) = 2x^2 + 5x + 3$ .

Therefore, the identity is equivalent to  $a(2x+3) - b(x+1) = x+4$ .

Putting  $x = -1$  to cancel the  $b$  term, we have  $a = \boxed{3}$ .

Putting  $x = -\frac{3}{2}$  to cancel the  $a$  term, we have  $\frac{1}{2}b = \frac{5}{2}$ , i.e.  $b = \boxed{5}$ .

**Alternative** By  $(2a-b)x + (3a-b) = x+4$ , comparing the coefficients, we

have:

$$\begin{cases} 2a - b = 1 \\ 3a - b = 4 \end{cases}$$

Solving, we have  $a = \boxed{3}$  and  $b = \boxed{5}$

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Q1(4):

Let  $n$  be the largest one among the four numbers, we have

$$n + (n - 2) + (n - 4) + (n - 6) = 224$$

$$4n = 236$$

$$n = \boxed{59}$$

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Q1(5):

$$8^x < \frac{1}{4}$$

$$2^{3x} < 2^{-2}$$

$$3x < -2$$

$$x < -\frac{2}{3}$$

Therefore, the largest integer satisfying it is  $\boxed{-1}$ .

$$2 < \log_x 45 < 3$$

$$x^2 < 45 < x^3$$

$$3 = \sqrt[3]{27} < \sqrt[3]{45} < x < \sqrt{45} < \sqrt{49} = 7$$

Therefore, there are  $\boxed{3}$  integers 4, 5, 6 satisfying it.

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Q1(6):

As  $x = 1$  is a root, we have  $1 + a + 1 + 1 = 0$ , i.e.  $a = \boxed{-3}$ .

Moreover, we can do the factorisation by the long division.

$$x^3 - 3x^2 + x + 1 = 0$$

$$(x - 1)(x^2 - 2x - 1) = 0$$

$$x = 1, \boxed{-1 \pm \sqrt{2}}$$

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Q1(7):

When  $\cos \theta = -\frac{1}{2}$  for  $90^\circ \leq \theta \leq 180^\circ$ , we have  $\theta = \boxed{120^\circ}$ .

By  $\sin^2 \theta + \cos^2 \theta = 1$ , we have  $\cos^2 \theta = 1 - \sin^2 \theta = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25}$ .

As  $\cos \theta \leq 0$  for  $90^\circ \leq \theta \leq 180^\circ$ , we have  $\cos \theta = -\sqrt{\frac{16}{25}} = \boxed{-\frac{4}{5}}$ .

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Q1(8):

To choose three numbers and arrange them to a three-digit integer, there are

$P_3^5 = \boxed{60}$  ways.

If the integer is greater than 400, the leading digit is either 4 or 5 (2 choices).

To choose the remaining two digits among the 4 numbers, there are  $P_2^4 = 12$

ways. Therefore, there are together  $2 \cdot 12 = \boxed{24}$  different integers.

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Q1(9):

$$\vec{x} - \vec{a} = \vec{b} - \vec{x}$$

$$2\vec{x} = \vec{a} + \vec{b}$$

$$\vec{x} = \frac{1}{2}(< 2, 3 > + < -1, 4 >) = \frac{1}{2} < 1, 7 > = \boxed{< \frac{1}{2}, \frac{7}{2} >}$$

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Q1(10):

(i):  $f'(x) = \boxed{-3x^2 + 12x - 9}$ .

(ii): To find the extremum of  $f(x)$ , we set  $f'(x) = 0$ . For  $0 \leq x \leq 3$ , we have  $x = 1$  or  $x = 3$ .

We test the minimum by exhaustion:

$$f(0) = 1$$

$$f(1) = -3$$

$$f(3) = -3$$

Therefore, the minimum value of  $f(x)$  is  $\boxed{-3}$ .

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Q2:

(1): By completing the square, we have  $y = (x - 2)^2 - 9$ .

Therefore, the vertex of  $A$  is  $\boxed{(2, -9)}$ .

To find the x-intercepts, we are going to solve

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$

$$x = \boxed{-1, 5}$$

(2): After the transformation,  $(x, y)$  becomes  $(-x, -y)$ .

Therefore, the equation becomes  $y = -((-x)^2 - 4(-x) - 5) = \boxed{-x^2 - 4x + 5}$ .

(3): When  $y = 2x + k$  touches  $A$ , the equation  $2x + k = x^2 - 4x - 5$  has only one solution. Therefore,

$$\Delta = 36 + 4(5 + k) = 0$$

$$k = \boxed{-14}$$

By that time, the x-coordinate of the point of tangent can be found

$$x^2 - 6x + 9 = 0$$

$$x = 3$$

Therefore, the area =  $\int_0^3 ((x^2 - 4x - 5) - (2x - 14)) dx$

$$= \left[ \frac{1}{3}x^3 - 3x^2 + 9x \right]_0^3$$

$$= 9 - 27 + 27$$

$$= \boxed{9}$$

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Q3:

(1): The graph is a straight line with positive slope and positive y-intercept.

The symmetric graph will have a negative slope. It can hence be deduced that the equation of it is  $\boxed{(4)}$ .

(2): The graph is a exponential function  $y = a^x$  with  $a > 1$  as the graph is increasing. The symmetric graph will be decreasing, i.e.  $a < 1$ . Therefore, the equation is  $\boxed{(14)}$ .

(3); The graph is a circle with the y-coordinate of centre 0 and the x-coordinate of centre is negative. The symmetric graph will have a positive x-coordinate of centre. It can hence be deduced that the equation of it is  $\boxed{(11)}$ .

(4): The graph is a sine wave that passes through the origin and increasing for the first half period, which has the equation  $y = \sin x$ . Therefore, the equation of the symmetric graph will be  $y = \sin(-x) = -\sin x$   $\boxed{(3)}$ .

(5): The graph is a parabola with the right part taken the absolute value. The symmetric graph will have the left part taken the absolute value. Therefore, the equation is  $\boxed{(8)}$ .