

Q1(1):

By completing the square, $2x^2 - 5x = 2(x - \frac{5}{4})^2 - \frac{25}{8}$. Therefore, when $x = \boxed{\frac{5}{4}}$, the expression takes its minimum $\boxed{-\frac{25}{8}}$.

On the other hand, as the graph of the expression convex downwards, the further the value of x from $\frac{5}{4}$, the greater the value of the expression. Therefore, the expression takes its maximum when $x = \boxed{4}$ and the value is $32 - 20 = \boxed{12}$.

Q1(2):

Note that the outcome of A 's coin and the outcome of B 's come at each round are independent. Therefore, the required probability is equal to the probability that the number of head that A got after three tosses is greater than that of B . Denote the numbers by the ordered pair (A, B) , we can exhaust all the cases:

$$(A, B) = (1, 0), (2, 0), (3, 0), (2, 1), (3, 1), (3, 2)$$

Therefore, the probability=

$$\begin{aligned} & (1 - (\frac{1}{2})^3)(\frac{1}{2})^3 + (C_1^3(\frac{1}{2})^2(\frac{1}{2}) + (\frac{1}{2})^3)(C_1^3(\frac{1}{2})(\frac{1}{2})^2) + (\frac{1}{2})^3(C_2^3(\frac{1}{2})^2(\frac{1}{2})) \\ &= \frac{7}{64} + \frac{3}{16} + \frac{3}{64} \\ &= \boxed{\frac{11}{32}}. \end{aligned}$$

Q1(3):

$$\begin{aligned} \frac{a}{b} + \frac{b}{a} &= \frac{a^2+b^2}{ab} \\ &= \frac{(a+b)^2}{ab} - 2 \end{aligned}$$

$$= \frac{(2\sqrt{5})^2}{5-3} - 2$$

$$= \boxed{8}.$$

Alternative $\frac{a}{b} + \frac{b}{a} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{8+2\sqrt{15}}{2} + \frac{8-2\sqrt{12}}{2}$$

$$= \boxed{8}.$$

Q1(4):

By De Morgan's law, $\neg(x \neq 0 \wedge y \neq 0) = (\boxed{x = 0 \vee y = 0})$.

Q1(5):

Denote the two circles as C_1 and C_2 and let the points of tangent of them be $(0, y_1)$ and $(0, y_2)$ respectively.

As they are tangent to the y-axis, the y-coordinates of their centres are equal to that of their points of tangent and the x-coordinates of their centres are equal to their radii.

Then, we have the equation of the two circles:

$$C_1 : (x - a)^2 + (y - y_1)^2 = a^2$$

$$C_2 : (x - b)^2 + (y - y_2)^2 = b^2$$

As they pass through points $(1, 3)$ and $(2, 4)$, by substituting the coordinates into their equations respectively, we have:

$$\begin{cases} (1-r)^2 + (3-\lambda)^2 = r^2 \\ (2-r)^2 + (4-\lambda)^2 = r^2 \end{cases}.$$

By symmetry, the two solutions of (r, λ) are (a, y_1) and (b, y_2) .

Combine the two equations, we have

$$(1-r)^2 + (3-\lambda)^2 = (2-r)^2 + (4-\lambda)^2$$

$$-(3-2r) = 7-2\lambda$$

$$\lambda = 5-r$$

Substitute this relation into the first equation, we have $(1-r)^2 + (r-2)^2 = r^2$,

i.e. $r^2 - 6r + 5 = 0$.

Now, we are going to find ab , which is the product of roots of this quadratic

equation, we have $ab = \frac{5}{1} = \boxed{5}$.

Q1(6):

We split the equation into three pieces:

$$\begin{cases} 2x-1+x-2=2 \\ x \geq 2 \end{cases} \quad \text{or} \quad \begin{cases} 2x-1-x+2=2 \\ \frac{1}{2} \leq x \leq 2 \end{cases} \quad \text{or} \quad \begin{cases} -2x+1-x+2=2 \\ x \leq \frac{1}{2} \end{cases}.$$

Solving each, we have two solutions $x = 1$ and $x = \frac{1}{3}$.

Therefore, the minimum solution is $\boxed{\frac{1}{3}}$ and the maximum solution is $\boxed{1}$.

Q1(7):

Rewrite $\omega = \sin 30^\circ + i \cos 30^\circ$.

By De Moivre's theorem, $\omega^5 = \sin 150^\circ + i \cos 150^\circ = \boxed{\frac{1}{2}} + \boxed{-\frac{\sqrt{3}}{2}}i$.

Alternative Note that ω is a root of $x^2 - x + 1 = 0^*$. Then, we have $\omega^5 = \omega^4 - \omega^3$

$$= (\omega^3 - \omega^2) - \omega^3$$

$$= -(\omega - 1)$$

$$= 1 - \frac{1 + \sqrt{3}i}{2}$$

$$= \boxed{\frac{1}{2}} + \boxed{-\frac{\sqrt{3}}{2}}i.$$

(*: Suppose ω is a root of a quadratic equation, then its conjugate is also a root. Therefore we have the sum of roots=1 and the product of roots=1 and the quadratic equation can hence be constructed.)

Alternative ω is the 3rd primitive root of unity, where $\omega^2 = \bar{\omega}$ and $\omega^3 = 1$.

Therefore, we have $\omega^5 = \bar{\omega} = \boxed{\frac{1}{2}} + \boxed{-\frac{\sqrt{3}}{2}}i$.

Q1(8):

Solve the recurrence using telescoping:

$$\sum_{i=1}^n (a_{i+1} - a_i) = \sum_{i=1}^n (2i)$$

$$a_{n+1} - a_1 = n(n+1)$$

$$a_n = \boxed{(n-1)n}$$

Q2:

(1): The angle at the centre is twice the angle at the circumference. Therefore, we have $\angle COB = 2a$, $\angle AOC = 2b$ and $\angle BOA = 2c$.

Suppose a, b, c are in radian measure*. Then, by definition, we have $AB = \boxed{2cr}$, $BC = \boxed{2ar}$ and $CA = \boxed{2br}$.

(*: The question did not state it clearly, but otherwise the answers will become $\frac{c}{90^\circ}r$ etc., which do not match the official answer key.)

(2): Suppose $\triangle ABC$ is an acute-angle triangle*.

Then, the area of $\triangle ABC = \triangle AOC + \triangle BOC + \triangle COA$

$$= \frac{1}{2}(r)(r)\angle AOC + \frac{1}{2}(r)(r)\angle BOC + \frac{1}{2}(r)(r)\angle COA$$

$$= \frac{r^2}{2}(\sin \boxed{2a} + \sin \boxed{2b} + \sin \boxed{2c}).$$

(*: Again, the question did not state it clearly, otherwise the formula will not necessarily hold. A counter example is the $30^\circ - 30^\circ - 120^\circ$ triangle.)

(3): By the sine formular, we have $\frac{AB}{\sin c} = \frac{BC}{\sin a} = \frac{AC}{\sin b} = 2r$.

$$\text{Therefore, } AB = 2 \sin 45^\circ = \boxed{\sqrt{2}}.$$

$$BC = 2 \sin 75^\circ = 2(\sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ)$$

$$= 2\left(\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right)$$

$$= \boxed{\frac{\sqrt{2} + \sqrt{6}}{2}}.$$

$$CA = 2 \sin 60^\circ = \boxed{\sqrt{3}}.$$

Q3:

By the fundamental theorem of calculus, we have $f(x) = (3x^2 + (a+8)x + 4)' =$

$6x + (a + 8)$. Substitue it back to the equation, we have

$$\int_a^x (6t + (a + 8))dt = 3x^2 + (a + 8)x + 4$$

$$[3t^2 + (a + 8)t]_a^x = 3x^2 + (a + 8)x + 4$$

$$3x^2 + (a + 8)x - 3a^2 - a(a + 8) = 3x^2 + (a + 8)x + 4$$

Compare the constant term, we have

$$-3a^2 - a(a + 8) = 4$$

$$a^2 + 2a + 1 = 0$$

$$a = \boxed{-1}$$

Therefore, $f(x) = \boxed{6x + 7}$ and $\int_a^x f(t)dt = 3x^2 + 7x + 4$.

To find the extremum, we set $(\int_a^x f(t)dt)' = 0$, then we have $x = -\frac{7}{6}$. As $(\int_a^x f(t)dt)'' = 6 > 0$, the extremum is a minimum. Therefore, the minimum of the integral obtained when $x = -\frac{7}{6}$, and the corresponding value is $\boxed{-\frac{1}{12}}$.

Alternative By the fundamental theorem of calculus, we have

$\int_a^x f(t)dt = g(x) - g(a)$, where $g(x)$ is a primitive function of $f(x)$. By taking the constant term of $g(x)$ as 0, we have $g(x) = 3x^2 + (a + 8)x$ and hence $-g(a) = -3a^2 - a(a + 8)$. Therefore, we have the equation $-3a^2 - a(a + 8) = 4$.