Q1 Fill in the blanks with the correct numbers.

(1) For triangle ABC with $AB = 4+\sqrt{3}$, $BC = 2\sqrt{3}$ and $AC = 12$, the radius of its circumcircle is $\boxed{}$.

(2) For triangle ABC with $AB : BC = 1 : 2$, $\angle BAC = 60^\circ$ and $BC = 6$, if the angle bisector of $\angle BAC$ intersect with BC at D, then $BD = \boxed{}$.

(3) For any convex quadilateral $ABCD$ with area S, we have $S \leq \boxed{}(AB + CD)(AD + BC)$.

(4) If AP is tangent to a circle at P, where $AP = 4$. If the shortest distance between A and the circle is 2, then the radius of the circle is $\boxed{}$.

(5) The volume of inscribed sphere in a regular tetrahedron with side length 1 is $\boxed{}$.

Q2 On a circular cone with slant heigh 6, having a circular base of radius 1. Choose an arbitrary diameter on its base and mark the end points A, B. Denote the vertex as O. Let P be a point lying on OB with $PB = x$. Fill in the blanks with the answer to the following questions.

(1) If $x = 1$, find the shortest distance between A and B on the surface of the cone.

(2) Express the distance between A and B on the surface of the cone in terms of x.

(3) A line is drawn from A, surrounded the cone once and back to A. Find the minimum length of this line.

(1) $\boxed{}$

(2) $\boxed{}$

(3) $\boxed{}$

Q3 For a regular pentagon $ABCDE$ with side length 1, let P be the point of intersection of diagonals AC and BD. Fill in the blanks with the answers to the following questions.

(1) Find $\angle CAD$ and $\angle ACD$

(2) Find CP and AC.

(3) Calculate $\sin 18^\circ$ and $\cos 36^\circ$.

(1) $\angle CAD = \boxed{}$ $\angle ACD = \boxed{}$

(2) $CP = \boxed{}$ $AC = \boxed{}$

(3) $\sin 18^\circ = \boxed{}$ $\cos 36^\circ = \boxed{}$

Brief Solutions and Comments

Q1(1) Ref: Not specific

Question related to measuration.

By cosine formula, $\angle ABC = 60°$.

By sine formula, $R = \frac{AC}{2\sin\angle ABC} = \boxed{4\sqrt{3}}$.

Very standard question on measuration. Judging from those appeared in Math A, if a measuration question appears in Q1, it is likely to be direct calculation like this question.

Q1(2) Ref: Not specific

Question related to plane geometry.

By the angle bisector thereom, $BD : BC = AB : AC = 1 : 2$. Therefore, $BD = \boxed{2}$.

This question is a direct application of the angle bisector theorem, which has never been asked explicitly but helpful in solving many Q1 questions.

Q1(3) Ref: Not specific

Question related to plane geometry.

We cut the quadilateral in two different ways, $\triangle ABD, \triangle CBD$ and $\triangle ADC, \triangle ABC$.

Consider the area of triangle, we have

$$S = \frac{1}{2}(AB)(AD)\sin\angle BAD + \frac{1}{2}(CB)(CD)\sin\angle BCD \leq \frac{1}{2}((AB)(AD)+(CB)(CD))$$

and

$$S \leq \frac{1}{2}((AD)(CD) + (AB)(BC))$$

Add them together and divide by 2, we have $S \leq \boxed{\frac{1}{4}}(AB + CD)(AD + BC)$.

(Note: A cheating way will be consider the square directly, which is often the extreme case for equality to hold.)

Solving geometry problem with considering the area is quite common in the early past papers. However, this kind of question is now not likely to appear.

Q1(4) Ref: Not specific

Question related to plane geometry.

By Pythagoras theorem, $4^2 + r^2 = (2 + r)^2$, we have $r = \boxed{3}$.

This question is a direct application of the Pythagoras theorem, where the same situation has been asked implicitly in past paper (hidden in a fake coordinate geometry question).

Q1(5) Ref: 2014 Math A Q1(2)

Question related to solid geometry.

It is clear that the area of each faces of the tetrahedron is $\frac{\sqrt{3}}{4}$ and the volume of the tetrahedral is $\frac{\sqrt{2}}{12}$. Then, as the inscribed sphere is tangent to each side, we have $4 \cdot \frac{1}{3} \cdot \frac{\sqrt{3}}{4} r = \frac{\sqrt{2}}{12}$, i.e. $r = \frac{\sqrt{6}}{12}$. Then, the volume of it is $\frac{4}{3}\pi r^3 = \boxed{\dfrac{\sqrt{6}}{216}\pi}$.

MEXT like to ask question related to inscribed circle (plane) or sphere (space) very much. Just bare in mind that most of this kind of question can be solved by considering the area (or volume). Do not ever try to solve them by brutal force as that will be very clumsy.

Q2 Ref: 2014 Math B Q2

Question related to measuration.

(1) We consider the curved surface of the cone in a plane. As the circumstance of the base is 2π, the surface is a sector with centre angle $\frac{2\pi}{6} = \frac{\pi}{3}$. Note that OB bisects the sector and hence $\angle AOB = \frac{\pi}{6}$. By cosine formula, $AB = \sqrt{6^2 + (6-1)^2 - 2(6)(6-1)\cos\frac{\pi}{6}} = \boxed{\sqrt{61 - 30\sqrt{3}}}$.

(2) Similar to (1), $AB = \sqrt{6^2 + (6-x)^2 - 2(6)(6-x)\cos\frac{\pi}{6}}$
$= \boxed{\sqrt{x^2 + 6(\sqrt{3}-2)x - 36(\sqrt{3}-2)}}$.

(3) The distance$=2AB$. AB attains to its minimum when $AB \perp OB$. By that time, $2AB = 2(6\sin\frac{\pi}{6}) = \boxed{3}$.

A very standard measuration question that will appear in Q2 that requires candidates to express something using a given variable and then find its extremum. In most cases, the calculus part (if involved) is the main character of the whole question. However, if one fail to figure out the first (few) subquestion(s), one will lose all the marks.

Q3 Ref: 2019 Math A Q2

Question related to trigonometry.

(1) Consider $\angle AED = \frac{540^\circ}{5} = 108^\circ$, $\angle EAD = \frac{180^\circ - 108^\circ}{2} = 36^\circ$ and similar for $\angle BAC$. Therefore, $\angle CAD = 108^\circ - 2 \cdot 36^\circ = \boxed{36^\circ}$.

Moreover, $\angle ADC - 108^\circ - 36^\circ = 72^\circ$ and hence

$\angle ACD = 180^\circ - 72^\circ - 36^\circ = \boxed{72^\circ}$.

(2) As $\triangle BAP, \triangle APD$ are isosceles, $AP = PD = 1$. Moreover, note that $\triangle ADC \sim \triangle DCP$ and $AC = AD$, we have $\frac{1}{CP} = \frac{1+CP}{1}$, i.e. $CP = \boxed{\dfrac{\sqrt{5}-1}{2}}$ and $AC = \boxed{\dfrac{\sqrt{5}+1}{2}}$.

(3) Note that $\triangle CDP$ is isosceles. Therefore, the angle bisector of $\angle CDP$ is perpendicular to CP. Consider the sine ratio, we have $\sin 18^\circ = \frac{\frac{\sqrt{5}-1}{4}}{1} = \boxed{\dfrac{\sqrt{5}-1}{4}}$.

One can use the double angle formula to obtain $\cos 36^\circ$. However, note that $\triangle APD$ is isosceles. Similarly we have $\cos 36^\circ = \frac{\frac{\sqrt{5}+1}{4}}{1} = \boxed{\dfrac{\sqrt{5}+1}{2}}$.

At the very beginning, MEXT required candidates to write down all the calculation steps for Q2 and Q3 so plane geometry was often asked. But know without steps, it is very hard to set a plane geometry question. Therefore, I think if a geometry question appears in Q3, it is likely to be a trigonometry question like the model question.