General LPP - Simplex method

Simplex method is standard method for maximizing the linear function of several variables under the several constraints on other linear functions

Simplex method are Dantzig's simplex algorithm is an iterative procedure carried systematically to determine the optimal solution of an ~~to~~ LPP from a set of feasible solutions. ~~The~~ The concept and algorithm was introduced by T.S. Motzkin.

Definition :

A set of values $x_1, x_2 \dots x_n$ which satisfies constraints of an LPP is called as solution.

Feasible solution :

Any solution to LPP which satisfies non-negativity restriction of LPP is called its feasible solution.

Optimal feasible solution :

Any feasible solution which optimizes the objective of the LPP is called optimal feasible solution or optimum ~~feasible solution~~.

Definition :

If the constraints of general LPP be $\sum_{j=1}^{n} a_{ij} x_{ij} \leq b_i$ for $i = 1, 2 \dots K$.

Then the non-negativity variable S_i which are introduced to convert the inequalities to equalities called slack variables.

Definition :

If the constraints of LPP is

$\sum_{j=1}^{n} a_{ij}\, b_{ij} \geq b_i$ for $i = 1, 2 \ldots k$.

Then the non-negative variable S_i which are introduced to convert the inequalites to equalites $\sum_{j=1}^{n} a_{ij}\, b_{ij} - S_i = b_i$ is called surples variables.

Two Different Forms of LPP :

* canonical form
* standard form

canonical form :

Is expressed as Max $Z = CX$ (objective function)

S.T $AX \leq b$ (constrain)

& $X \geq 0$ (not - negative restriction)

where $C = (C_1, C_2 \ldots C_n)$

where $A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$ $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ $b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$

Characteristic of canonical form :

1. Objective function is of maximization type.
2. All the constraints are of less than or equal to type.

3. All variable x_i are non-negative

Standard Form of LPP:

$$\text{Max } Z = CX \text{ (objective function)}$$

$$\text{S.T } AX = b \text{ (constraints)}$$

$$\& \ X \geq 0$$

Characteristic of standard form:

1. The objective function is of maximization type.
2. All the constraints are expressed as equation.
3. Right-hand side of each constraints are non-negative.
4. All the variables are non-negative.

note:

If the problem is minimization problem then it is equivalent to maximization of negative expression of that function.

$$\text{Min } Z = -\text{Max } (Z)$$

(or)

$$\text{Min } Z = -\text{Max } Z^*$$

note 2 :

An inequality in one direction can be converted into an inequality to opposite direction by multiplying both sides by -1.

note 3 :

If the variable is unconstraint or unrestricted it can always be expressed as the difference of two non-negative variables. (ie) $x_2 = x_2' - x_2''$. where $x_2', x_2'' \geq 0$.

note 4 :

when the slack & surplus variables are introduced in the constraints they should also appear in objective functions with zero co-efficient.

Express the foll. LPP in canonical form

$$\text{Max } z = 2x_1 + 3x_2 + x_3$$

$$\text{S.T : } 4x_1 - 3x_2 + x_3 \leq 6$$

$$x_1 + 5x_2 - 7x_3 \geq -4$$

$$\& \ x_1, x_2, x_3 \geq 0, x_2 \text{ is unrestricted.}$$

Sol: G.T x_2 is unrestricted

$\therefore x_2 = x_2' - x_2''$.

$4x_1 - 3(x_2' - x_2'') + x_3 \leq 6$

$-x_1 - 5x_2 + 7x_3 \leq 4$.

$-x_1 - 5(x_2' - x_2'') + 7x_3 \leq 4$.

S.T:

$4x_1 - 3x_2' + 3x_2'' + x_3 \leq 6$

$-x_1 - 5x_2' + 5x_2'' + 7x_3 \leq 4$

& $x_1, x_2', x_2'', x_3 \geq 0$.

2. Express the foll. LPP in standard form

$\text{Max } z = 4x_1 + 2x_2 + 6x_3$

S.T:

$2x_1 + 3x_2 + 2x_3 \geq 6$

$3x_1 + 4x_2 = 8$

$6x_1 - 4x_2 + x_3 \leq 10$

& $x_1, x_2, x_3 \geq 0$.

Sol:

$\text{Max } z = 4x_1 + 2x_2 + 6x_3$

S.T: surplus slack

$2x_1 + 3x_2 + 2x_3 - S_1 + 0S_2 = 6$

$3x_1 + 4x_2 + 0S_1 + 0S_2 = 8$

$6x_1 - 4x_2 + x_3 + 0S_1 + S_2 = 10$

& $x_1, x_2, x_3, S_1, S_2 \geq 0$.

Express the foll. LPP as standard form

1. Min $z = 4x_1 + 3x_2$

S.T : $x_1 - x_2 \le 10$

$3x_1 + x_2 \ge 18$

$5x_1 + 2x_2 \le 20$

& $x_1, x_2 \ge 0$.

Sol :

Min $z = -\text{Max}(z)$.

Max $z^* = -4x_1 - 3x_2$.

S.T :

$x_1 - x_2 + S_1 + 0S_2 + 0S_3 = 10$

$3x_1 + x_2 - S_2 + 0S_1 + 0S_3 = 18$

$5x_1 + 2x_2 + S_3 + 0S_1 + 0S_2 = 20$

& $x_1, x_2, \cancel{x_3}, S_1, S_2, S_3 \ge 0$.

Matrix form above problem

Max $z^* = CX$

S.T $A \cdot X = b$ & $X \ge 0$.

where

$$A \begin{pmatrix} 1 & -1 & 1 & 0 & 0 \\ 3 & 1 & 0 & -1 & 0 \\ 5 & 2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 18 \\ 20 \end{pmatrix}$$

$C = (-4, -3, 0, 0, 0)$.

Basic Solution :

given a system of m linear equations with n variables (m < n) solution obtained by setting n-m variables is equal to zero and solving for the remaining m variables is called Basic solution. The m variables are called basic variables and they formed basic ~~variables and their~~ solution. m-n variables are called non-basic variables.

Non-Degenerate Basic Solution :

Basic solution is said to be non-degenerate basic solution if the none of the basic variable is 0.

A basic solution is said to be de-generate basic solution if one or more of basic variable are 0.

Feasible solution is also called basic and it is called basic feasible solution.