Stability Analysis - Continuous Time

The system is controlled by $v_R - v_L$, that we will set as Δv

$$\dot{\theta} = \frac{\Delta v}{2r} \Rightarrow \mathcal{L} \Rightarrow \Theta = \frac{\Delta v}{s * 2r} \tag{1}$$

Stability Analysis - Continuous Time

Since Δv will be treated as a constant from now and won't change on what sign the poles have we can set it as $\Delta v = 1$, and treat it as a constant. Giving us the open loop transfer function:

$$\Rightarrow G_{ol}(s) = \frac{1}{s * 2r} \tag{2}$$

And the closed loop transfer function:

$$\Rightarrow G_{cl}(s) = \frac{s * 2r}{s^2 * 4r^2 + s * 2r} \tag{3}$$

Stability Analysis - Continuous Time

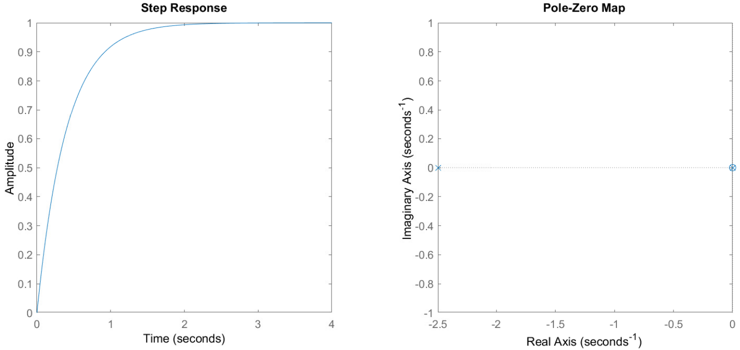

Figure: Step-response and "poles and zero" plot for the system without controller

Stability Analysis - Continuous Time

The **PD**-controller implementations goal is to improve on the closed-loop system. The transfer functions then become:

$$\Rightarrow G_{ol}(s) = \frac{k_p + k_d s}{s * 2r} \tag{4}$$

And the closed loop transfer function:

$$\Rightarrow G_{cl}(s) = \frac{s * 2r(kd * s + kp)}{s * 2r(s(k_d + 2r) + k_p)} \tag{5}$$

Stability Analysis - Continuous Time

From the closed-loop it's easy to see that $-2r < k_d$ in order to not have a pole in the positive right plane. The **PD** controller is therefore set to

$$\begin{cases} k_p = & 5 \\ k_d = & -r \end{cases} \tag{6}$$

Stability Analysis - Continuous Time

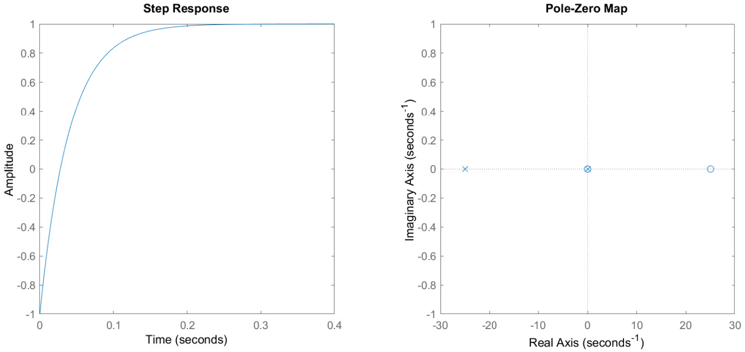

Figure: Step-response and "poles and zero" plot for the system with **PD**-controller

Stability Analysis - Continuous Time

Without controller

- RiseTime: 0.8788 s
- SettlingTime: 1.5648 s

PD Controller

- RiseTime: 0.0879 s
- SettlingTime: 0.1565 s

The system is approx 10x faster with the controller.

Stability Analysis - Discrete Time

The `c2d` command in MATLAB was used to discretize the transfer function with a sampling rate of 20 Hz and a input delay of 1/30 seconds

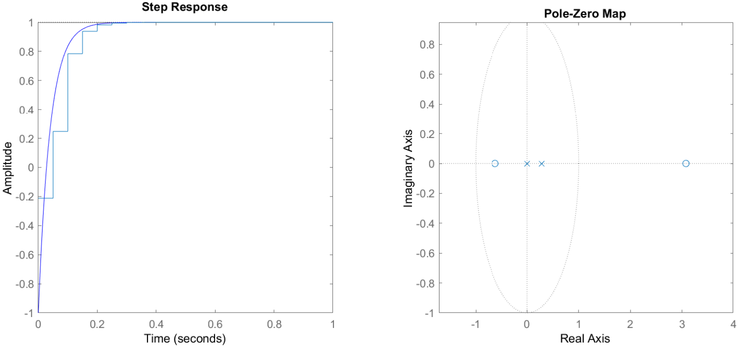

Figure: Step-response and "poles and zero" plot for the discrete system with a **PD**-controller