

Wzory na pochodne i całki

Pochodne

$$(a)' = 0 \quad a \in \mathbb{R}$$

$$(x)' = 1$$

$$(x^\alpha)' = \alpha \cdot x^{\alpha-1} \quad \alpha \in \mathbb{R}$$

$$(a^x)' = a^x \cdot \ln a$$

$$(e^x)' = e^x$$

$$(\log_a x)' = \frac{1}{x \cdot \ln a}$$

$$(\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$$

Własności pochodnych

$$(a \cdot f(x))' = a \cdot f'(x)$$

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

$$(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$(\ln(f(x)))' = \frac{f'(x)}{f(x)}$$

Całki

$$\int dx = x + C$$

$$\int x dx = \frac{1}{2}x^2 + C$$

$$\int x^\alpha dx = \frac{1}{\alpha+1}x^{\alpha+1} + C \quad \alpha \neq -1$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int e^x dx = e^x + C$$

$$\int \log_a x dx = \frac{x \cdot \ln x - x}{\ln a} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \operatorname{tg} x dx = -\ln|\cos x| + C$$

$$\int \operatorname{ctg} x dx = \ln|\sin x| + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 + q}} = \ln \left| x + \sqrt{x^2 + q} \right| + C$$

Własności całek

$$\int (a \cdot f(x)) dx = a \cdot \int f(x) dx$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int f(x) \cdot g'(x) dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x) dx$$