Visualization of Periodic Flows in a Continuously Stratified Fluid

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Introduction

The visualization the flow pattern of viscous continuously stratified fluid both experimental and computational methods is developed. Computational procedures are based on exact solutions of set of the fundamental equations. Solutions of the problems of flows producing by periodically oscillating disk (linear and torsion oscillations) are visualized with a high resolutions to distinguish small-scale the singular components on the background of strong internal waves.
Numerical algorithm of visualization allows to represent both the scalar and vector fields, such as velocity, density, pressure, vorticity, stream function.
Precision schlieren instrument is used to visualize the flow pattern produced by linear and torsion oscillations of strip and disk in a continuously stratified fluid. Uniform stratification is created by the continuous displacement method. The buoyancy period ranged from 7.5 to 14 s. In the experiments disks with diameters from 9 to 30 cm and a thickness of 1 mm to 10 mm were used. Different schlieren methods that are conventional vertical slit-Foucault knife, vertical slit-filament (Maksoutov's method) and horizontal slit-horizontal grating (natural rainbow schlieren method) help to produce supplementing flow patterns. Both internal wave beams and fine flow components were visualized in vicinity and far from the source. Intensity of high gradient envelopes increased proportionally the amplitude of the source. In domains of envelopes convergence isolated small scale vortices and extended mushroom - like jets. Experiments have shown that in the case of torsion oscillations pattern of currents more complicated than in case of forced linear oscillations. Comparison with known theoretical model have show that nonlinear interactions between the regular and singular flow components must be taken into account.

Theoretical solutions

Governing equations and boundary conditions

$$\frac{\partial\rho}{\partial t} + div\ (\rho\ \mathbf{v}) = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}, \nabla)\, \mathbf{v} = \frac{1}{\rho}\nabla p + \nu\Delta v + \rho\mathbf{g}, \quad \frac{\partial S}{\partial t} = \kappa_S \Delta S + \frac{v_z}{\Lambda},$$

$$v|_\Gamma = u_0 e^{-i\omega t}, \quad \kappa_S \left.\frac{\partial S}{\partial n}\right|_\Gamma = 0, \quad v = \nabla \times e_z\Phi + \nabla \times (\nabla \times e_z\Psi)$$

Exact solutions for oscillating 2D plane

$$v_\xi = \frac{u_0}{\pi^2}\int\limits_{-\infty}^{+\infty} \frac{1}{k_\xi(k_1-k_2)} \sin\frac{k_\xi a}{2}\left(k_1 e^{ik_1\zeta} - k_2 e^{ik_2\zeta}\right) e^{ik_\xi\xi} dk_\xi, \quad v_\eta = 0$$

$$v_\zeta = \frac{u_0}{\pi^2}\int\limits_{-\infty}^{+\infty} \frac{1}{k_2-k_1} \sin\frac{k_\xi a}{2}\left(e^{ik_1\zeta} + e^{ik_2\zeta}\right) e^{ik_\xi\xi} dk_\xi,$$

$$k_1 = k_\xi \cot(\varphi+\theta) \pm \frac{ik_\xi^3\delta_N^2}{2\cos\theta\sin^4(\varphi-\theta)}, \quad k_2 = \frac{1+i}{\delta_\varphi}$$

Exact solutions for oscillating 3D disk

$$v_z = u_0\ R \int\limits_0^{+\infty} \frac{k_2 e^{ik_1 z} - k_1 e^{ik_2 z}}{k_2-k_1} J_1(kR)\ J_0(kr)\ dk, \quad v_r = -iu_0\ R \int\limits_0^{+\infty} \frac{k_2 k_1}{k_2-k_1}\frac{e^{ik_1 z} - e^{ik_2 z}}{k}\ J_1(kR)\ J_1(kr)\ dk$$

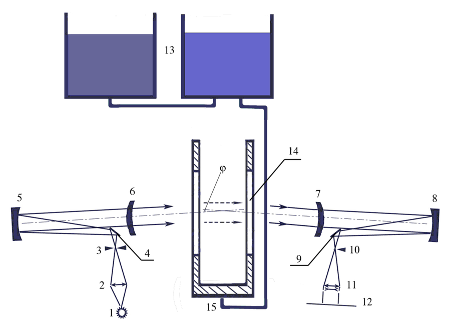
$$k_j^2 = -k^2 + \frac{i\sin\theta}{2\delta_N^2}\left[1 \mp \sqrt{1 + \frac{4ik^2\delta_N^2}{\sin^3\theta}}\right], \quad \delta_N = \sqrt{\frac{\nu}{N}}$$

Exact solutions for torsing disk

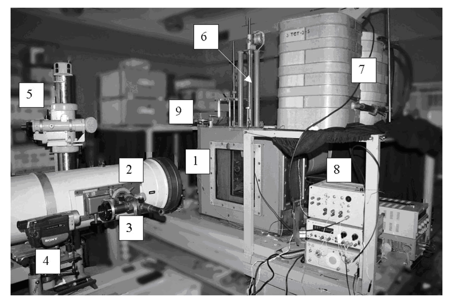
$$\Psi(r,z) = \frac{\nu u_0^2 e^{-i\omega t}}{4\omega^2 R tg\theta\left(\sqrt{2}+itg\theta\right)} \int\limits_{-\infty}^{+\infty} \frac{H_1^{(1)}(k_w r)}{H_1^{(1)}(k_w R)} \sin\frac{ka}{2} e^{ikz} dk$$

Experimental set

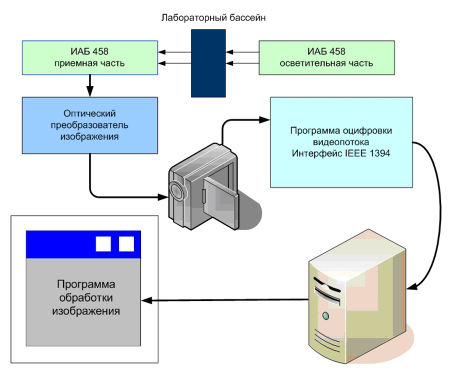
The laboratory set up include a tank 1 (70x25x70 cm) with windows made of optical glass, the device for creating stratification, system of the optical imaging, control device for determining position of the source, support mechanisms for oscillations of bodies, measuring system and system of registration information. Laboratory set installed on the massive metal table, based on antivibration mountings to reduce mechanical noise. Light and Shadow of the measuring instrument set on its own two separate bases. The registration device 4 is coupled with the shadow apparatus by auxiliary lens unit 3. Cathetometer 5 is used to determine the coordinates of the sensors and the position of the sources. On scanner 6 is placed sensor conductivity, coupled with power amplifiers and accessories 8. To crank-and-rod mechanism 9 is attached light source. To visualize the flow using schlieren instrument IAB-458 with a diameter of field view 230 mm. Owing to the dependence of the refractive index of the medium density, flat-walled tank filled with linearly stratified fluid, equivalent to an optical prism, which turns the light beams at an angle φ. Lighting and receiving parts are ajusted in vertical plane facing each other.



Optical scheme of the experimental set



Experimental set up



Workflow process

Numerical method of visualization

We used exact solution of linear problems of radiation internal waves for numerical visualization of flow in a viscous stratified fluid used the. The integral expressions are calculated by Lobbatto method for integrals on the infinite and semi-infinite intervals. Computation alogritm is implemented using the Intel Fortran compilers for Linux (www.intel.com) which free for non-commercial using. Visualization of the obtained values is carried out using GNUPlot (www.gnuplot.org). GNUPlot is part of the standard Linux distribution.

Exact solutions → Numerical calculations (ifort) → Visualization (GNUPlot)

References

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2. R. N. Bardakov, A. Yu. Vasil'ev and Yu. D. Chashechkin Calculation and measurement of conical beams of three-dimensional periodic internal waves excited by a vertically oscillating piston // Fluid Dynamics V.42, N4, PP 612-626
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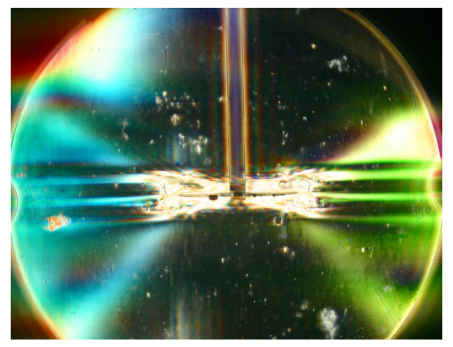
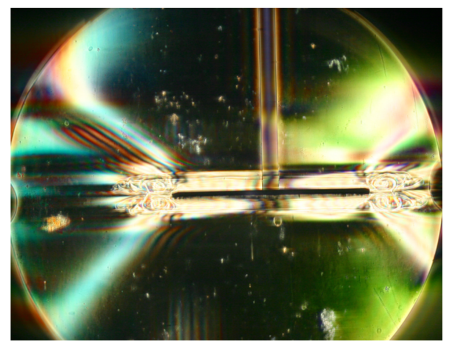
Torsion oscillations of disk

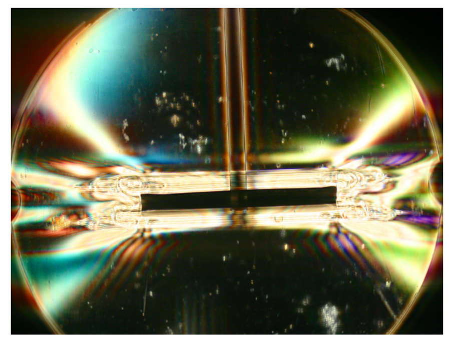
In the case of torsional oscillations of a horizontal disk in the shadow pattern traced the sequence of thin strips parallel to the plane of the disk, confined to the outer edge of ring structures, which adjoin inclined diffuse bands. Horizontal high-gradient layer visualize singular components, diffuse band - a section of the wave cone. The system of ring layers adjoining to outer edge of the disk visualizes the rotating in their plane horizontal toroidal vortices, which close the singular components of the flow on the upper and lower sides of the disc. High-gradient shell and spiral vortex elements inside them are formed in a continuously stratified fluid separated boundary layers.



$R = 3\ cm, \omega = 0.037\ s^{-1}$, $A = 2\pi, h = 1\ mm$



$R = 10\ cm, \omega = 0.037\ s^{-1}$, $A = \pi, h = 3\ mm$

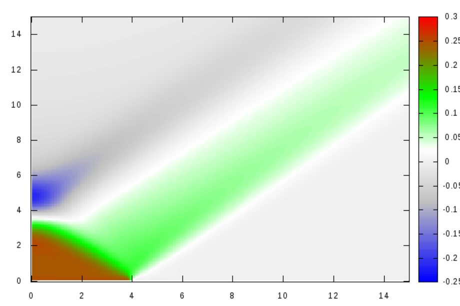


$R = 10\ cm, \omega = 0.037\ s^{-1}$, $A = 1.5\pi, h = 10\ mm$

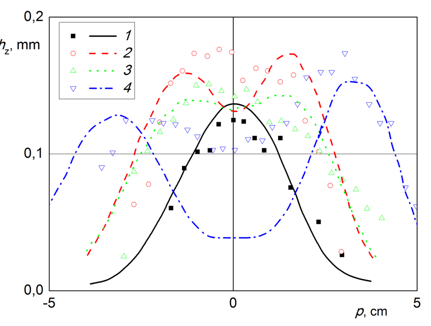
Linear oscillating of the disk

Shlieren visualizations allows to identify regular (waves) and singular components (boundary layers, vortices, jets) of the flow. The relationship between size of the radiator and universal micro-scale define modality of the waves beam. Bi-modal waves beam is transformed to the unimodal at certain distances from the source. Comparison of calculated wave beams with laboratory schlieren visualization shows their qualitatively and quantitatively compatibility. Agreement between calculations and experiments allows to extend this analysis of the flow patterns and predict the basic characteristics for majority forms of sources.

Shadow image of linear oscillating disk



Vizualisation of 3D oscillating disk (First quadrant, $r > 0, z > 0$)



Comparison of calculations and measurements. Plane 2D

Conclusions

1. The exact solution of the problem of generation internal waves in a continuously stratified fluid is qualitatively and quantitatively coincide with the results of visualization and measurement in a wide range of governing parameters of the problem.
2. Color shlieren method allows visualization of regular and singular components of the flow.
3. The obtained results show that it is necessary to consider dissipative factors (viscosity, stratification, diffusion) influence to improbe of sources movement in the viscous stratified fluid