Into
oo

Math. model
oooo

Theoretical analyze
ooo

Numerical analyze
ooooo

Results
oooooo

Conclusion
oo

Modeling flow in a viscous continuously stratified fluid taking into account diffusivity effects

Vasiliev Alexey

Laboratory of Fluid Mechanics
Institute for Problems in Mechanics of the RAS
Moscow, Russia

Firth International Scientific School for Young Scientists.
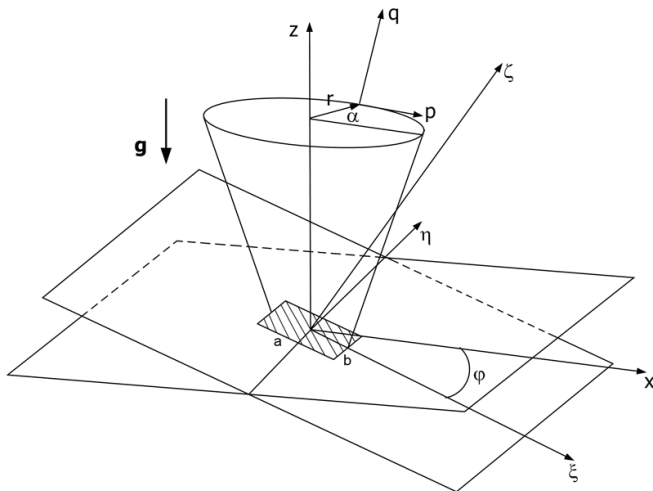"Waves and vortices in complex media", 2014

Review

1. J.V.S. Rayleigh
2. J. Lighthill
3. Yu.D. Chashechkin
4. V.A. Gorodtsov
5. T.N. Stevenson
6. D.G. Hurley, G.J. Keady
7. B.R. Sutherland
8. Yu.V. Kistovich
9. A.V. Kistovich
10. B. Voisin

Color schlirien images oscillation of disk

System coordinate frame for analytical analyze

Governing equations and boundary conditions

Governing equations

$$\frac{\partial \rho}{\partial t} + \mathbf{v}\nabla\rho = 0, \quad \text{div } \mathbf{v} = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}, \nabla)\,\mathbf{v} = \frac{1}{\rho}\nabla p + \nu\Delta v + \rho\mathbf{g}$$

$$\frac{\partial S}{\partial t} = \kappa_S\Delta S + \frac{v_z}{\Lambda}$$

Boundary conditions

$$\mathbf{v}|_\Gamma = \mathbf{u}_0 e^{-i\omega t}, \quad \kappa_S \left.\frac{\partial S}{\partial n}\right|_\Gamma = 0$$

$$v \to 0, \quad \rho \to \rho_0, \quad \partial P/\partial z \to \rho_0\,(z)\,g, r \to \infty$$

Toroidal-poloidal decomposition

$$\mathbf{v} = \nabla \times \mathbf{e}_z\Phi + \nabla \times (\nabla \times \mathbf{e}_z\Psi)$$

System equations for functions Φ and Ψ

$$\left(\left(\frac{\partial}{\partial t} - D\Delta \right) \left(\frac{\partial}{\partial t} - \nu\Delta \right) \Delta + N^2 \Delta_\perp \right) \Phi = 0$$

$$\left(\frac{\partial}{\partial t} - \nu\Delta \right) \Psi = 0$$

$$\left(\left(\frac{\partial}{\partial t} - D\Delta \right) \left(\frac{\partial}{\partial t} - \nu\Delta \right) \Delta + N^2 \Delta_\perp \right) S = 0$$

where

$$\Delta_\perp = \partial_x^2 + \partial_y^2 \quad \Delta = \partial_x^2 + \partial_y^2 + \partial_z^2 \quad N^2 = \sqrt{\frac{g}{\Lambda}}$$

Construction solution for Φ and Ψ in Fourier transform

$$\Phi = e^{-i\omega t} \sum_{j=1}^{3} \int\limits_{-\infty}^{+\infty} A_j\left(k_\xi, k_\eta\right) E_j \ dk_\xi dk_\eta$$

$$S = -\frac{\rho_0}{\Lambda} e^{-i\omega t} \sum_{j=1}^{3} \int\limits_{-\infty}^{+\infty} \frac{\left(k_\xi \cos\varphi - k_j \sin\varphi\right)^2 + k_\eta^2}{i\omega - \kappa_S \ k^2} A_j\left(k_\xi, k_\eta\right) E_j dk_\xi dk_\eta$$

$$\Psi = e^{-i\omega t} \int\limits_{-\infty}^{+\infty} B\left(k_\xi, k_\eta\right) \ E_4 dk_\xi dk_\eta$$

where

$$E_j = \exp\left(ik_j\zeta + ik_\xi\xi + ik_\eta\eta\right), \quad k = \sqrt{k_j^2 + k_\xi^2 + k_\eta^2}$$

| Into | Math. model | Theoretical analyze | Numerical analyze | Results | Conclusion |
| oo | oooo | ●oo | ooooo | oooooo | oo |

Viscous stratified fluid take into diffusion

Dispersion equation

$$\left(\nu\kappa_S\tilde{k}^6 - i\omega\left(\nu + \kappa_S\right)\tilde{k}^4 - \omega^2\tilde{k}^2 + N^2 k_\perp^2\right)\left(\tilde{k}^2 + \frac{\omega}{i\nu}\right) = 0$$

$$\tilde{k}^2 = 2k_\zeta^2 + k_\perp^2, \quad k_\perp^2 = k_\xi^2 + k_\eta^2$$

Viscous stratified fluid take into diffusion

Regular solution(waves)

$$k_1 = \frac{k_\xi \sin\varphi \cos\varphi \pm \kappa \cos\theta}{\mu_\theta} \pm \delta_N^2 (1+\varepsilon) \frac{i \tan\theta \mu_\theta^4}{2\kappa\mu^4} + ...$$

$$\mu = \sin^2\varphi - \sin^2\theta, \quad \mu_\theta = (k_\xi \sin\varphi \cos\varphi \pm \kappa \cos\varphi),$$

$$\varepsilon = Sc^{-1} = \frac{\kappa_S}{\nu}, \quad \delta_N = \sqrt{\frac{\nu}{N}}$$

Singular solution

$$k_{2,3} \approx \sqrt{\frac{i\omega(\varepsilon + 1 \pm \lambda_{\nu\kappa})}{\varepsilon}}, \quad \lambda_{\nu\kappa} = \frac{2}{\sin\theta}\sqrt{(1+\varepsilon)^2 - \frac{4\varepsilon\mu}{\sin^2\theta}}$$

$$k_4 = \sqrt{\frac{2i}{\delta_\nu^2} - k^2}, \quad \delta_\nu = \delta_N\sqrt{\frac{2}{\sin\theta}}, \quad \delta_\varphi = \delta_N\sqrt{\frac{2\sin\theta}{|\mu|}}, \quad \delta_\kappa = \delta_N\sqrt{\frac{2\varepsilon}{\sin\theta}}$$

Viscous stratified fluid take into diffusion.Vetrical component of the velocity

$$v_\zeta \approx \int\limits_{-\infty}^{+\infty} A_1 \left(k_\eta^2 \sin\varphi - k_\xi \beta_1\right) E_1 dk_\xi dk_\eta -$$

$$-ie^{\frac{i-1}{\delta_\nu}\zeta} \sin\varphi \int\limits_{-\infty}^{+\infty} B E_{\xi\eta} \, dk_\xi dk_\eta - \frac{i+1}{\delta_\varphi} e^{\frac{i-1}{\delta_\varphi}\zeta} \cos\varphi \int\limits_{-\infty}^{+\infty} A_2 k_\xi E_{\xi\eta} dk_\xi dk_\eta -$$

$$-\frac{1+i}{\delta_\kappa}\sqrt{\frac{\sin\theta}{2}} e^{-\frac{\sqrt{\sin\theta}}{\delta_\kappa\sqrt{2}}\zeta + \frac{i\zeta}{\delta_\kappa\sqrt{2}}} \cos\varphi \int\limits_{-\infty}^{+\infty} A_3 k_\xi E_{\xi\eta} dk_\xi dk_\eta$$

where

$$E_{\xi\eta} = exp\left(ik_\xi\xi + ik_\eta\eta\right)$$

Why OpenFOAM?

Pluses:

1. OpenFOAM free and open source, under the GNU general public licence (GPL).

2. Support of community
   http://www.cfd-online.com/Forums/openfoam/,
   http://openfoamwiki.net/

3. Support open-source Linux platform (openSUSE, Ubuntu, RHEL)

Minuses:

1. No GUI to create grids, but can use other applications such as: GMSH (**http://geuz.org/gmsh/**), Salome (**http://www.salome-platform.org/**) or commercial mesh generators such as: Icem CFD (**www.ansys.com**), Gambit (**www.ansys.com**), pro*star Star-CD (**www.cd-adapco.com**)

Into
○○

Math. model
○○○○

Theoretical analyze
○○○

Numerical analyze
○●○○○

Results
○○○○○○

Conclusion
○○

Hardware, software and workflow process

**Platform: 8 cpu, 32 Gb
Software: CentOS 6,
OpenFOAM-2.2.2**

**Platform: 8 cpu, 32 Gb
Software: CentOS 6,
OpenFOAM-2.2.2**
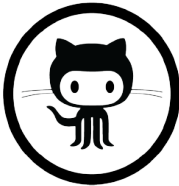
GitHub
(solvers, models)

Supercomputer
"Lomonosov"

Structure of the solver. IGWFoam.C

Navier - Stokes equations

```
fvVectorMatrix UEqn (
   fvm::ddt(U) + fvm::div(phi, U)
 - fvm::laplacian(nu, U) - S*g
);
solve(UEqn = = -fvc::grad(p)/dens0);
```

Equations for salinity S and density $dens$

```
fvScalarMatrix SEqn (
   fvm::ddt(S) + fvm::div(phi, S)
 - fvm::laplacian(DS, S)
 - U.component(vector::Z)/Lambda
);
SEqn.solve();
dens = dens0*(1.0-Z/Lambda+S);
```

Into	Math. model	Theoretical analyze	Numerical analyze	Results	Conclusion
oo	oooo	ooo	ooo●o	oooooo	oo

Create O-grid model

To construct the mesh used a standard utility of OpenFOAM
blockMesh or **pyFoam** (Python for OpenFOAM)

Into	Math. model	Theoretical analyze	Numerical analyze	Results	Conclusion
oo	oooo	ooo	ooooo●	oooooo	oo
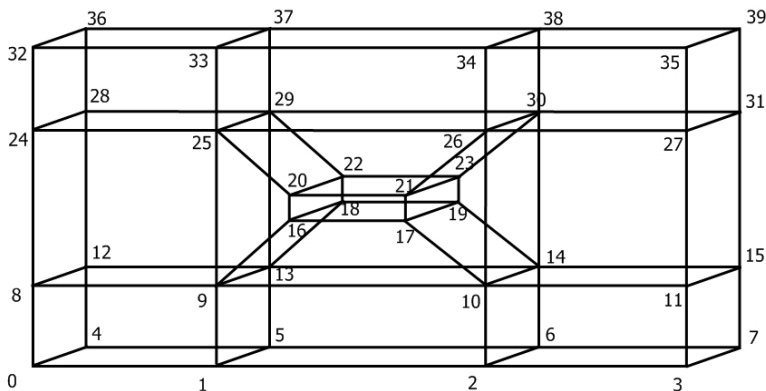
Create mesh

Create mesh using blockMesh utility:
[user@server]$ blockMesh -case name-of-model
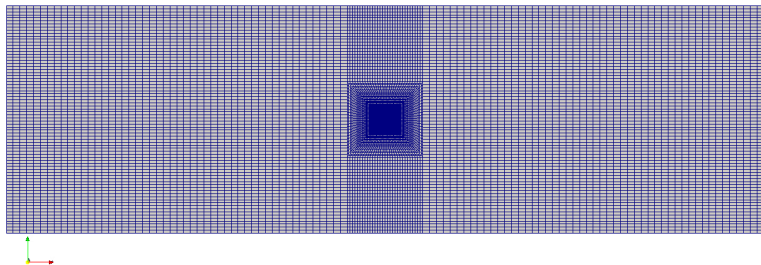
Boundary conditions:
Top, Botton, Left, Right: freeSteram (U, p), zeroGradient (S)
Plate: oscillating (U), fixedGradient (S)

Into
○○

Math. model
○○○○

Theoretical analyze
○○○

Numerical analyze
○○○○○

Results
●○○○○○

Conclusion
○○

Different velocities $L_x = 1$ cm, $N = 0.9 \ s^{-1}$, $\omega = 0.54 \ s^{-1}$
Module of velocity. Source - horizontal plate. Type - piston
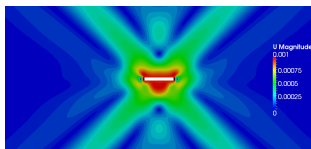
$u_0 = 0.001 \ m \ s^{-1}$
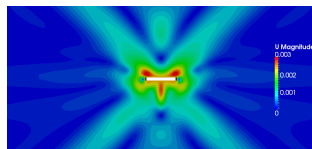
$u_0 = 0.0025 \ m \ s^{-1}$
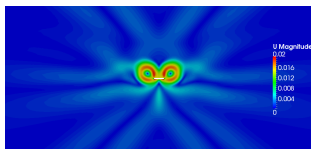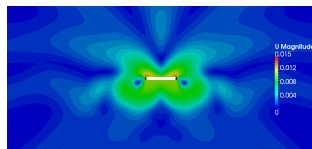
$u_0 = 0.025 \ m \ s^{-1}$
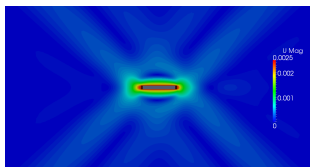

$u_0 = 0.01 \ m \ s^{-1}$

Different velocities $L_x = 1$ cm, $N = 0.9$ s^{-1}, $\omega = 0.54$ s^{-1}
Module of velocity. Source - horizontal plate. Type - friction
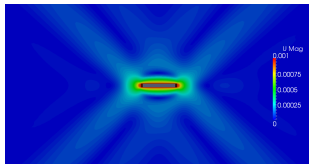
$u_0 = 0.001$ m s^{-1}
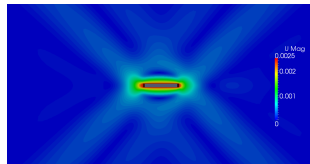
$u_0 = 0.0025$ m s^{-1}
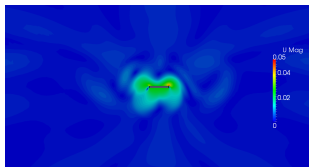
$u_0 = 0.025$ m s^{-1}
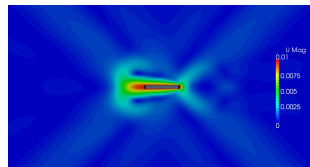
$u_0 = 0.01$ m s^{-1}

Different viscosity $L_x = 1$ cm, $N = 0.9\ s^{-1}$, $\omega = 0.54\ s^{-1}$
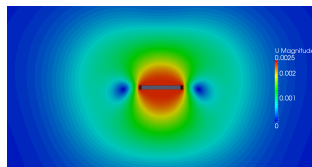Module of velocity. Source - horizontal plate. Type - piston
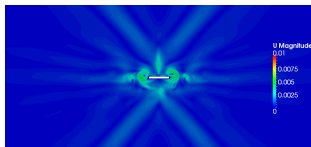
$\nu = 10^{-8}\ m^2\ s^{-1}$
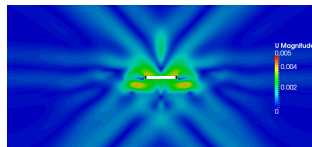
$\nu = 10^{-7}\ m^2\ s^{-1}$
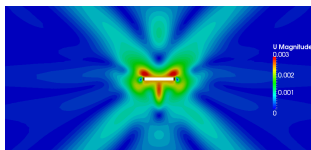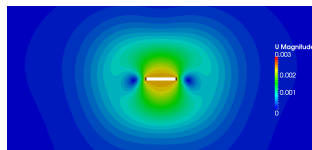
$\nu = 10^{-6}\ m^2\ s^{-1}$
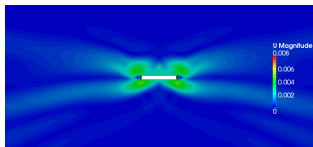

$\nu = 10^{-5}\ m^2\ s^{-1}$

Different viscosity $L_x = 1$ cm, $N = 0.9$ s^{-1}, $\omega = 0.54$ s^{-1}
Module of velocity. Source - horizontal plate. Type - friction
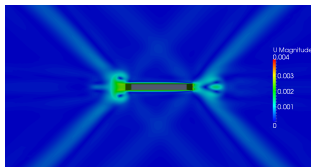
$\nu = 10^{-8}$ m^2 s^{-1}
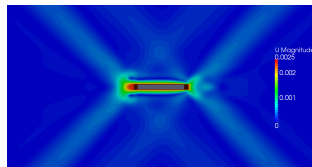
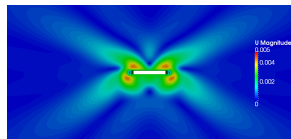$\nu = 10^{-7}$ m^2 s^{-1}

$\nu = 10^{-6}$ m^2 s^{-1}
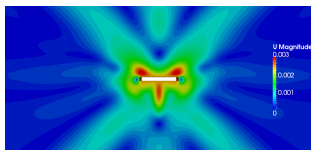

$\nu = 10^{-5}$ m^2 s^{-1}

Different stratification $L_x = 1$ cm, $\nu = 10^{-2}$ cm^2 s^{-1}, $\omega = 0.54$ s^{-1}
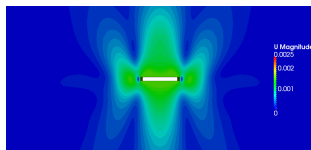Module of velocity. Source - horizontal plate. Type - piston

$N = 2.8$ s^{-1}

$N = 1.46$ s^{-1}

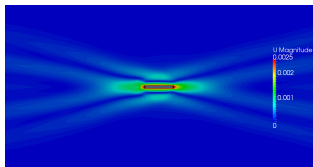

$N = 2.8$ s^{-1}

$N = 0.58$ s^{-1}

Into
○○

Math. model
○○○○

Theoretical analyze
○○○

Numerical analyze
○○○○○

Results
○○○○○○●

Conclusion
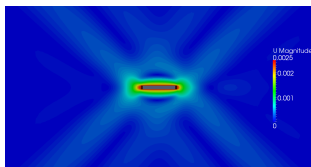○○

Different stratification $L_x = 1$ cm, $\nu = 10^{-2}$ cm^2 s^{-1}, $\omega = 0.54$ s^{-1}
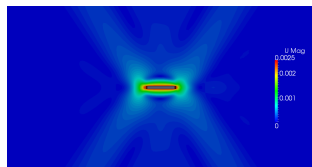Module of velocity. Source - horizontal plate. Type - friction
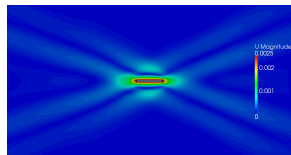
$N = 2.8$ s^{-1}

$N = 1.46$ s^{-1}

$N = 2.8$ s^{-1}

$N = 0.58$ s^{-1}

Conclusion

1. In the general case, in a viscous stratified fluid there are two types of solutions: regular (waves) and three type singular solutions. Two of them don't have analogues in homogeneous fluid. Their properties are defined viscosity, stratification, diffusion and the geometry of the problem;

2. For a complete description of the flow of fluid you must consider all parameters (viscosity, stratification, diffusion);

3. Create solver for calculation of the internal gravity waves in a continuesly stratified fluid;

4. Calculations case horizontal plate for two types of the sources: friction and piston.

Into
○○

Math. model
○○○○

Theoretical analyze
○○○

Numerical analyze
○○○○○

Results
○○○○○○

Conclusion
○●

Thank you