

# Domácí Úkol 00

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22. února 2026

## 1. Úvod

Úkol jsem zpracoval do sešitu a ofotil. Případné grafy, simulink modely, atd. jsem doplnil jako přílohy do pdf, jména kapitol odpovídají úkolu.

## 2. Fotografie výpočtů

Hw 0

$$1. \quad G(s) = \frac{-0.2e^{-s}}{(s+3)(s+2)(s+7)} \quad \text{DC gain: } L = \lim_{s \rightarrow 0} G(s) = -\frac{5}{3 \cdot 2 \cdot 7} = \frac{5}{6} \leftarrow \text{rechts für } G(s)$$

$$2. \quad \mu(s) = 5 \Rightarrow U(s) = \frac{5}{s} \rightarrow U(s) \cdot G(s) = \frac{-0.25}{(s+3)(s+2)(s+7)}$$

$$\text{keine offlat} = \frac{25}{6}$$

$$3. \quad x(N=2)(t) \Rightarrow x(t)=? \Rightarrow \lim_{s \rightarrow 0} s \cdot G(s) = 0$$

$$2x_1(t) = -6x_1(t) + 26 \cdot x_2(t), \quad x_1(0) = 2 \quad \xrightarrow{\text{Laplace}} \quad \left\{ \begin{array}{l} x_1(t) = -2x_1(t) \\ x_2(t) = -2x_1(t) \end{array} \right. \quad \left\{ \begin{array}{l} x_1(t) - x_2(t) = -6x_2(t) + 26x_1(t) \\ x_2(t) - x_1(t) = -\frac{2}{2}x_1(t) \end{array} \right. \Rightarrow$$

$$\left\{ \begin{array}{l} x_1(t) = -2x_2(t) \\ x_2(t) = -\frac{13}{2}x_1(t) \end{array} \right. \quad \left\{ \begin{array}{l} x_1(t) = \frac{1}{2} \\ x_2(t) = -\frac{13}{2}x_1(t) \end{array} \right. \quad \left\{ \begin{array}{l} x_1(t) = -20 \\ x_2(t) = -\frac{13}{2}x_1(t) \end{array} \right. \quad \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} x_1(t) + 6x_2(t) + \frac{13}{2}x_1(t) = 2 \\ x_1(t) = -\frac{1}{20}x_1(t) \end{array} \right. \Rightarrow \left\{ \begin{array}{l} x_1(t) = -\frac{2}{s^2 + 6s + 73} = \frac{20}{(s+3)^2 + 4} \\ x_2(t) = -\frac{1}{2}x_1(t) \end{array} \right. \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} x_1(t) = -\frac{2}{(s+3)^2 + 4} - 3 \cdot \frac{2}{(s+3)^2 + 4} - x_1(t) \\ x_1(t) = -\frac{2}{(s+3)^2 + 4} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} x_1(t) = -\frac{2}{(s+3)^2 + 4} = 2e^{-3t} \cos(2t) - 3e^{-3t} \sin(2t)/2t \\ x_2(t) = -\frac{1}{2}x_1(t) = -\frac{1}{2}e^{-3t} \sin(2t)/2t \end{array} \right. \Rightarrow$$

$$x_2(t) = -\frac{1}{2}e^{-3t} \sin(2t)/2t = -\frac{1}{2}e^{-3t} \sin(2t)/2t$$

$$3. \quad x_i = x_2$$

$$\left\{ \begin{array}{l} x_2 = -2 \sin(x_1) - \frac{9}{10}x_2 \cdot x_1' \\ y = x_2 \end{array} \right. \quad \left. \begin{array}{l} P = [x_{1p}, x_{2p}, x_{1p}'] ; x_p(t) = 2 \\ 0 = x_{2p} \\ 0 = -2 \sin(x_1p) - \frac{9}{10}x_2p \cdot x_1p' = -2 \sin(x_1p) - 2 \\ x_2p = x_{1p} \end{array} \right. \quad \left. \begin{array}{l} \Rightarrow P = \left[ \frac{1}{2} + 2t, 0, \frac{1}{2} + 2t, 0 \right] \end{array} \right.$$

$$2. \quad \Delta \vec{x} = A \Delta \vec{x} + B \Delta u \quad A = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$0 \vec{x} = C_0 \vec{x} + D_0 u$$

$$4. \quad G(s) = \frac{1}{(s+1)(s+10)(s+8)} = \frac{1}{s+1} = \frac{1}{8000} \quad \frac{1}{s+10} = \frac{1}{10000} \quad \frac{1}{s+8} = \frac{1}{8000}$$

$$G_1(t) = \frac{1}{(s+1)(s+10^3)(s+10^3)}$$

Obrázek 1: Výpočty - strana 1

5)  $\begin{pmatrix} 1 & 1 \\ -4 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2+0,5 \\ -3 & 2 & -2 & 0 \end{pmatrix} \text{ m}$   $M(a) = C(aI - A)^{-1} \cdot B$

$$\begin{pmatrix} 1 & 1 \\ -4 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ -3 & 2 & -2 & 0 \end{pmatrix} \quad (aI - A) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 2 & -2 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow$  dle  $x_3 \neq 0$  je výřešení možné, kdežto  $A' = \begin{pmatrix} -4 & 2 \\ 0 & 4 \end{pmatrix}$ ,  $B' = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $C' = \begin{pmatrix} 1 & 0 \end{pmatrix}$

$$\|aI - A'\| = \sqrt{(-4)^2 + 0^2} = (a+4)(a-4) + 16 = a^2 + 4$$

$$(aI - A')^{-1} = \frac{1}{a^2 + 4} \cdot \begin{pmatrix} 0 & -2 \\ 2 & -4 \end{pmatrix} = H$$

$$C \cdot H \cdot B' = \frac{1}{a^2 + 4} \cdot \begin{pmatrix} 1 & 0 \\ -4 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{a^2 + 4} \cdot (1 - 4) = \frac{-3}{a^2 + 4}$$

$$6) \begin{pmatrix} 1 & 0 & 0 & 1 \\ 2 & -2 & -1 & 2 & -1 \\ 1 & 2 & 0 & 1 & 1 \end{pmatrix} \text{ m} \quad M(a) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ -2 & 0 & 2 & -1 \\ -1 & -2 & 0 & 1 \end{pmatrix} = (a-2) \cdot \begin{pmatrix} 1 & 0 & 0 & 1 \\ -2 & 0 & 2 & -1 \\ -1 & -2 & 0 & 1 \end{pmatrix}$$

$$x = \begin{pmatrix} 1 & 0 & 0 & 1 \\ -2 & 0 & 2 & -1 \\ -1 & -2 & 0 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm i\sqrt{3} = (a-2)(1 \mp i\sqrt{3})$$

$\rho f: 1, -1 \pm i\sqrt{3}, -1 - i\sqrt{3} \rightarrow$  některá

$\Rightarrow G(a) = \frac{a-3}{(a+1)(a-2)} \rightarrow \rho f: 1, -2 \rightarrow$  dle výpočtu perioda cca 10x menší, než  $\frac{1}{2}$

$\Rightarrow T_{0,1} = 0,01 \text{ s}$ , některá  $\approx 0,5$

7)  $G(a) = -\frac{a-3}{(a+1)(a-2)(a-3)} \therefore$  obecný obdob. obdob.  $\in \rho f: -1, -2, -3$

8)  $G_1(a) = \frac{a-3}{(a+1)(a-2)} \quad \text{obecný obdob. obdob.}$

9)  $G_2(a) = \frac{1}{a-2} \quad \text{... obecný obdob. obdob. obdob.}$

10)  $G_3(a) = \frac{a-1}{a-1} \quad \text{... obecný obdob. obdob. obdob. obdob.}$

11)  $G_4(a) = \frac{a-1}{a-1} \quad G_2 = \frac{1}{a-2}$

12)  $G_1 = G_1 + G_2 = \frac{a-3}{a+1} + \frac{1}{a-2} = \frac{a^2-a+7}{(a+1)(a-2)} = \frac{a^2+7}{a^2-1}$

13)  $H(a) = G_1 \cdot G_2 = \frac{a-3}{a+1} \cdot \frac{1}{a-2} = \frac{a-3}{a^2-1}$

14)  $g(a) = G_1 \cdot n(a) - G_2 \cdot g(a)$

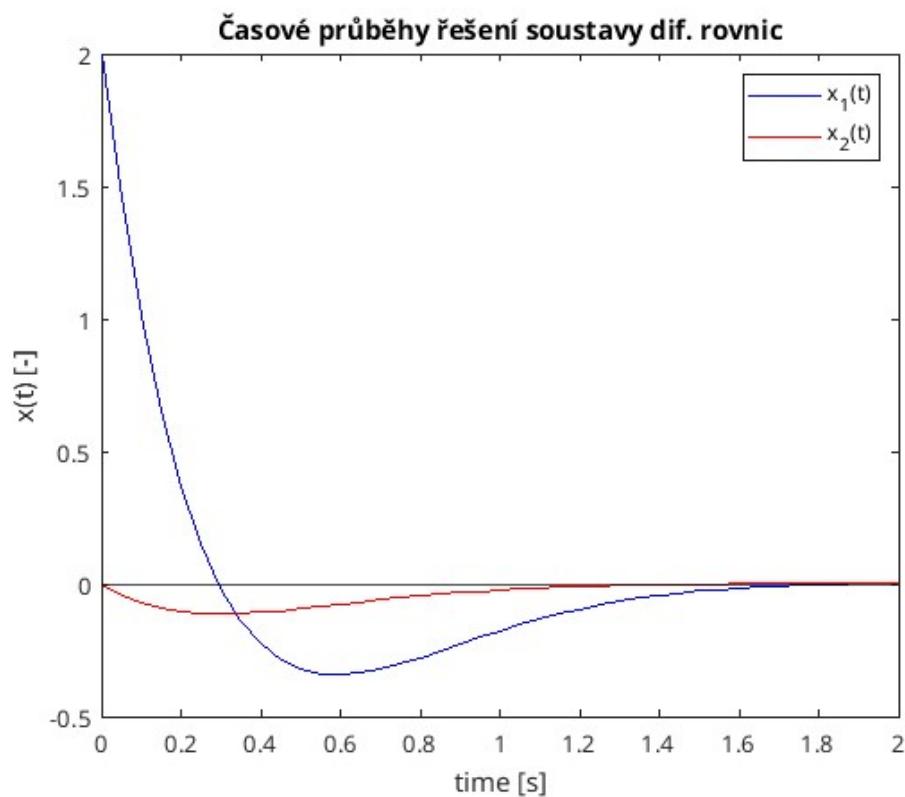
15)  $g(a) = \frac{G_1 \cdot n(a)}{n(a) + G_2 \cdot g(a)} \Rightarrow H(a) = \frac{G_1}{n(a) + G_2 \cdot g(a)}$

Obrázek 2: Výpočty - strana 2

$\text{M} \cdot 3r^2 - 2(2-r) + \frac{7}{2}r(r-2) = r(2-r) - r(2-r)$   
 $3Mr^2 - r^2M(r) + \frac{7}{2}r^2(r-2) = r^2M(r) - r^2M(r)$   
 $M(r)(3r^2 - r^2 + \frac{7}{2}r^2 - 2r) = M(r)(r^2 - r^2 - 2r)$   
 $M(r) = \frac{r^2 - r^2 + \frac{7}{2}r^2}{3r^2 - r^2 + \frac{7}{2}r^2} \quad M(r) \Rightarrow M(r) = \frac{r-2}{3r^2 - r^2 + \frac{7}{2}}$   
 $2 \cdot 3r^2 - r^2 + \frac{7}{2}r^2 = 0 \quad r = \frac{2 \pm \sqrt{1+4 \cdot 3 \cdot \frac{7}{2}}}{6} = \frac{2 \pm \sqrt{5}}{6}$   
 $\left| \frac{2 + \sqrt{5}}{6} \right| = \sqrt{\frac{1}{36} + \frac{5}{36}} = \sqrt{\frac{6}{36}} = \sqrt{\frac{1}{6}} < 1 \Rightarrow \text{obě kořeny}$   
 Vyř. A-F B-D C-E  
 $\text{I. } M(r) = \dots \rightarrow M(r) = \int_0^r \dots dr; M(r) = \dots$   
 Vyř. A-F B-D C-E Z = -3dB

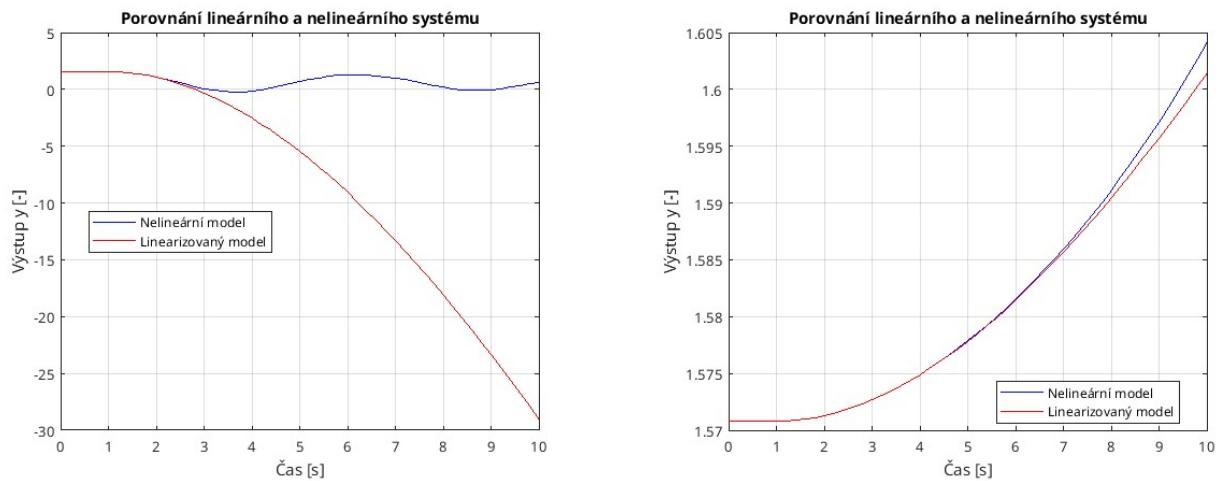
Obrázek 3: Výpočty - strana 3

### 3. Úkol 02

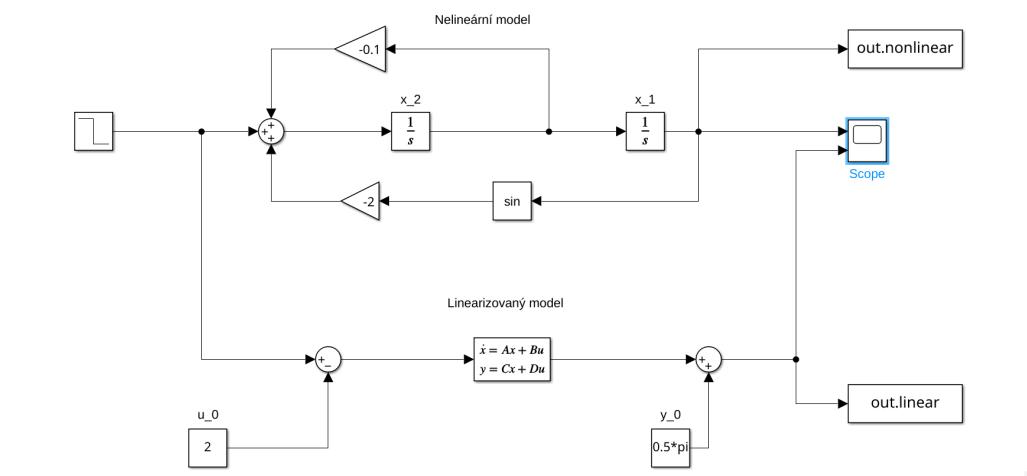


Obrázek 4: Úkol 02 - graf

## 4. Úkol 03

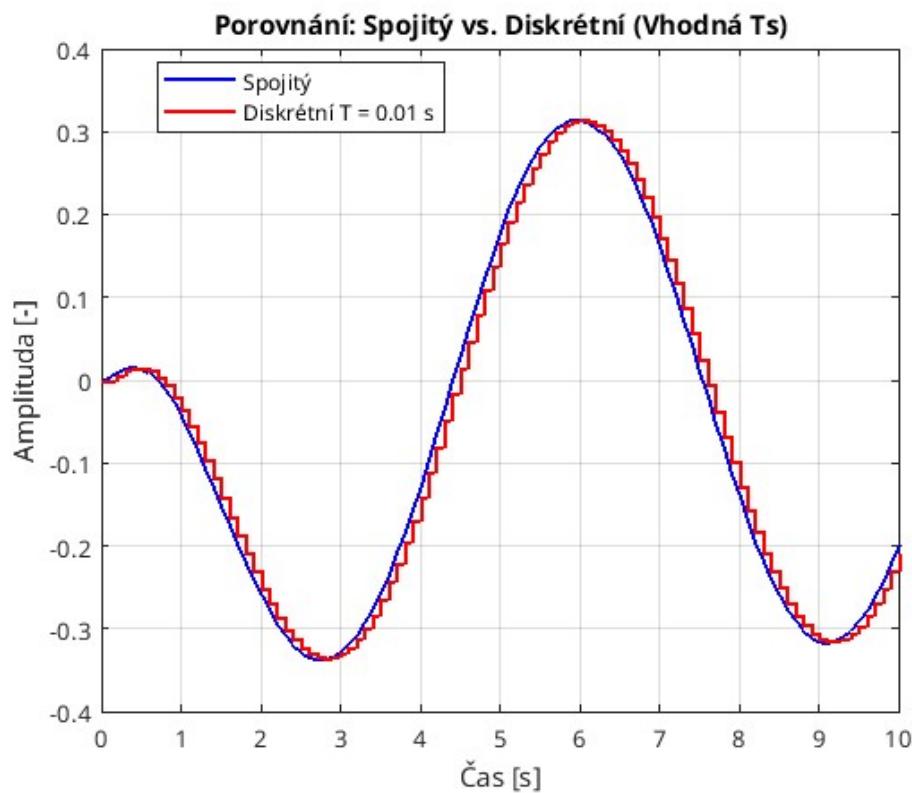


Obrázek 5: Úkol 03 - Vstupní signály  $u = 1$  (vlevo) a  $u = 2.0001$  (vpravo)

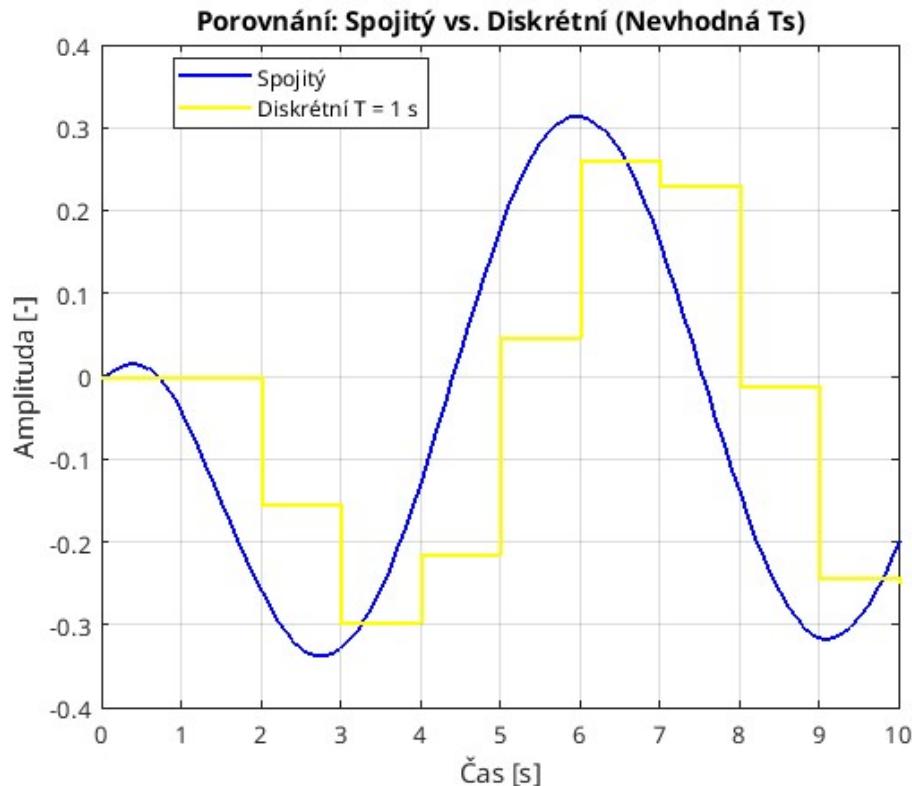


Obrázek 6: Úkol 03 - Simulink model

## 5. Úkol 07

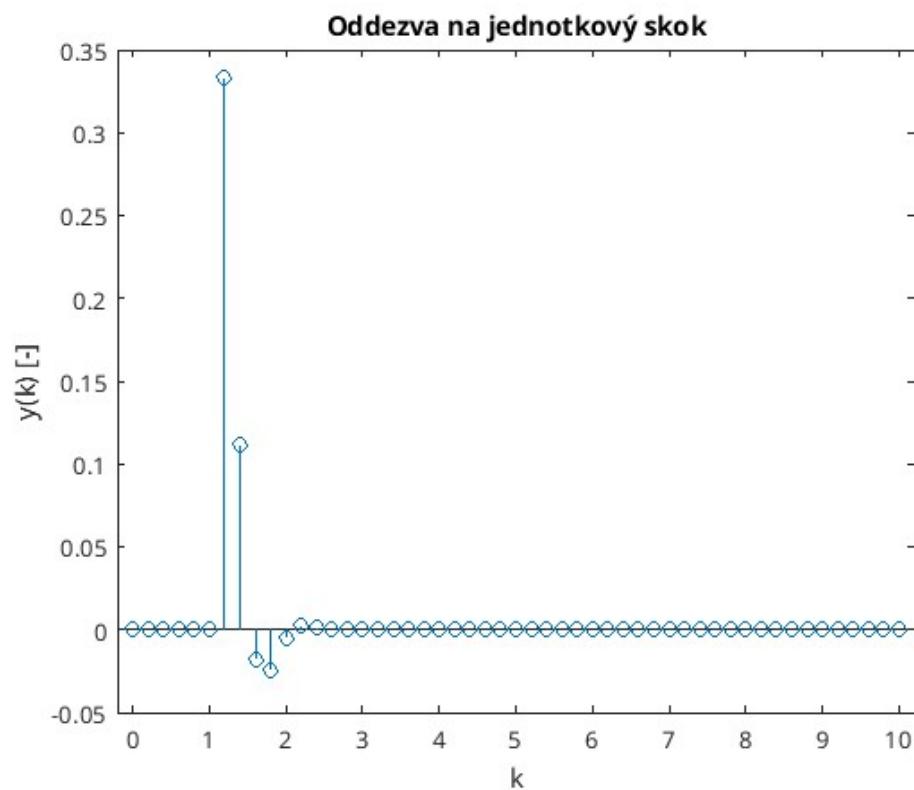


Obrázek 7: Úkol 07 - Vhodná Ts



Obrázek 8: Úkol 07 - Nevhodná Ts

## 6. Úkol 10



Obrázek 9: Úkol 10 - Graf