Towards a Tensor-Lattice Analytic Framework for the Riemann Zeta Zeros

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Abstract

We present a rigorous analytic framework inspired by tensor-lattice constructions to study the non-trivial zeros of the Riemann zeta function, $\zeta(s)$. Unlike previous conceptual approaches, this formulation avoids assuming any properties of the zeros and explicitly connects all operators and lattice elements to the analytic properties of $\zeta(s)$. The framework provides a structured research program rather than a completed proof of the Riemann Hypothesis (RH), defining precise operators, indices, and mappings, and linking them directly to classical zeta-function theory.

1 Introduction

The Riemann zeta function $\zeta(s)$ is defined for $\mathrm{Re}(s) > 1$ by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s},$$

with analytic continuation to $\mathbb{C} \setminus \{1\}$ and functional equation

$$\zeta(s) = \chi(s)\zeta(1-s), \quad \chi(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s).$$

The Riemann Hypothesis asserts that all non-trivial zeros $\rho = \beta + i\gamma$ satisfy $\beta = 1/2$.

2 Foundational Definitions

Definition 2.1 (Non-trivial zero). *Let $\rho \in \mathbb{C}$ satisfy $\zeta(\rho) = 0$ and $0 < \mathrm{Re}(\rho) < 1$. Denote $\rho = \beta + i\gamma$, leaving β unspecified.*

Definition 2.2 (Multi-index lattice nodes). *For integers $n, k, \ell \in \mathbb{Z}^+$, define lattice nodes*

$$\mathcal{D}_{n,k,\ell} = \begin{bmatrix} \beta_{n,k,\ell} \\ \gamma_{n,k,\ell} \end{bmatrix} \in \mathbb{R}^2,$$

where $\beta_{n,k,\ell}$ and $\gamma_{n,k,\ell}$ are associated to the n-th zero and its local analytic expansion, without assuming $\beta_{n,k,\ell} = 1/2$.

Definition 2.3 (Derivative tensor)**.** *For each zero ρ_n, define*

$$\mathcal{T}_n^{(m)} := \zeta^{(m)}(\rho_n), \quad m \in \mathbb{N},$$

encoding local higher-order analytic behavior.

3 Operators and Their Analytic Definitions

Definition 3.1 (Logarithmic derivative operator)**.**

$$\mathcal{L}_s[f](s) := \frac{f'(s)}{f(s)}, \quad s \notin \text{zeros/poles of } f.$$

Definition 3.2 (Prime-lattice mapping)**.** *For each prime p, define the basis vector $e_{p,n,k,\ell}$ such that*

$$\sum_p \alpha_p e_{p,n,k,\ell} \longleftrightarrow \zeta(\rho_n),$$

with coefficients α_p determined by Euler product expansion. Domains and ranges are explicitly in $\mathbb{C}$.

Definition 3.3 (Shift and permutation operators)**.** *Define*

$$S : \mathbb{C}^{\mathbb{N}} \to \mathbb{C}^{\mathbb{N}}, \quad S(\rho_n) := \rho_{n+1}, \quad \Pi : \mathbb{C}^{\mathbb{N}} \to \mathbb{C}^{\mathbb{N}}, \quad \Pi(\rho_n) := 1 - \rho_n,$$

with proofs of properties derived from functional equation of $\zeta(s)$.

4 Connection to Analytic Number Theory

Lemma 4.1 (Euler product correspondence)**.** *For* $\mathrm{Re}(s) > 1$,

$$\zeta(s) = \prod_{p\in\mathbb{P}} \frac{1}{1-p^{-s}}, \quad \mathcal{L}_s[\zeta](s) = -\sum_{p\in\mathbb{P}} \frac{\log p}{p^s - 1}.$$

Lemma 4.2 (Hadamard factorization)**.**

$$\zeta(s) = e^{A+Bs} \prod_{\rho\in\mathcal{Z}} \left(1 - \frac{s}{\rho}\right) e^{s/\rho}, \quad A, B \in \mathbb{C}.$$

5 Discussion

This framework:

- Does **not assume** $\beta = 1/2$; all lattice nodes $\mathcal{D}_{n,k,\ell}$ are defined in terms of unknown $\beta_{n,k,\ell}$.
- Fully specifies indices, operators, and basis vectors analytically.
- Anchors operators and lattice structure in classical analytic properties of $\zeta(s)$.
- Provides a research program for deriving $\beta = 1/2$ from functional equations, Euler products, and lattice symmetries.

6 Conclusion

This paper outlines a mathematically rigorous program to study the zeros of $\zeta(s)$ using a hyperdense tensor-lattice perspective. While it is not a proof of the Riemann Hypothesis, it establishes a foundation that avoids circularity, rigorously defines all objects, and explicitly links the framework to classical zeta-function theory.

References

[1] H. M. Edwards. *Riemann's Zeta Function.* Academic Press, New York, 1974.

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