UOPL-6.4f^{+}

1.A.N

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Signature

$$\left(u, \mathbb{T}, \S, \text{Traise}, \|\cdot\|, M\right) \oplus \Big(\text{U}\forall, \text{U}\exists\Big)$$

1 Axioms

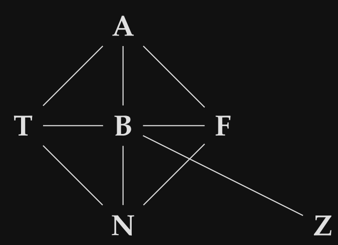
Axiom 6.1 (Universal Quantifier).

$$\text{U}\forall_x \varphi(x) = \bigwedge_{x_i \in D} \text{Val}\big(\varphi(x_i)\big)$$

Axiom 6.2 (Existential Quantifier).

$$\text{U}\exists_x \varphi(x) = \bigvee_{x_i \in D} \text{Val}\big(\varphi(x_i)\big)$$

The lattice $\mathbb{T} = \{\mathbf{T}, \mathbf{F}, \mathbf{N}, \mathbf{B}, \mathbf{A}, \mathbf{Z}\}$ is ordered as follows:



2 Derived Results

Test Case 1

$$\forall x\, P(x), \quad P(x) := \text{“}x \text{ is true and false”}$$

Assume $D = \{a, b, c\}$ with $P(a) = P(b) = P(c) = \mathbf{B}$. Then

$$\text{U}\forall_x P(x) = \mathbf{B}$$

Test Case 2

$$\exists x\, Q(x), \quad Q(x) := \text{“}x \neq x\text{”}$$

Truth table of $Q(x)$:

x **Z**	**T**	**F**	**N**	**B**
$Q(x)$ **T**	**F**	**F**	**F**	**B**

Over $D = \{x_1, x_2, x_3\}$ with $x_1 = \mathbf{Z},\ x_2 = \mathbf{F},\ x_3 = \mathbf{B}$:

$$\mathrm{U}\exists_x Q(x) = \mathbf{T} \vee \mathbf{F} \vee \mathbf{B} = \mathbf{A}$$

Test Case 3

$$\forall x \exists y\, R(x, y), \quad R(x, y) := \text{“}(x \Rightarrow y) \wedge (y \Rightarrow \neg x)\text{”}$$

Using the UOPL implication

$$x \Rightarrow y := \S(\neg x, y) \otimes u$$

yields a value cycle that collapses to **A** under both quantifiers.

3 Visual Echoes

4 Final Signature

$$\boxed{\left(\mathrm{U}\forall, \mathrm{U}\exists\right) \oplus (u, \mathbb{T}, \S, \text{Traise}, \|\cdot\|, M)}$$