# UOPL-6.4f

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3 August 2025

**Abstract**

A six-valued lattice $\mathcal{L}_6 = \{\mathrm{T}, \mathrm{F}, \mathrm{N}, \mathrm{B}, \mathrm{A}, \mathrm{Z}\}$ together with the undefined $\mathbf{u}$. Fractal permutations, Klein-bottle topology, and exact error-correction thresholds are derived.

## Contents



## 1 Signature

- **Carrier** $K = \mathcal{L}_6 \cup \{\mathbf{u}\}$ where $\mathcal{L}_6 = \{\mathrm{T}, \mathrm{F}, \mathrm{N}, \mathrm{B}, \mathrm{A}, \mathrm{Z}\}$.
- **Undefined generator u** obeys $\mathbf{u} = 0/0$ and $\forall x\ (x = \mathbf{u} \leftrightarrow x \neq x)$.
- **Modal valuation** $\mu$: $\mu(\mathrm{T}) = 1,\ \mu(\mathrm{F}) = 0,\ \mu(\mathrm{N}) = 0,\ \mu(\mathrm{B}) = \frac{1}{2},\ \mu(\mathrm{A}) = 1,\ \mu(\mathrm{Z}) = \perp$.
- **Meta-cardinality** $\|X\|$ on subsets $X \subseteq K$.
- **Topology** Klein bottle $M$ immersed in $\mathbb{R}^3$.

## 2 Axioms

**Axiom 2.1** (Undefinite Genesis)**.** $\mathbf{u} = \dfrac{0}{0}$ *and* $(x = \mathbf{u} \leftrightarrow x \neq x)$.

**Axiom 2.2** (Six-Valued Lattice). *$(\mathcal{L}_6, \leq, \S)$ is bounded lattice (bottom* N*, top* A*). Cayley table:*

| § | T | F | N | B | A | Z |
|---|---|---|---|---|---|---|
| T | T | B | T | B | A | Z |
| F | B | F | F | B | A | Z |
| N | T | F | N | B | A | Z |
| B | B | B | B | B | A | Z |
| A | A | A | A | A | A | Z |
| Z | Z | Z | Z | Z | Z | Z |

**Axiom 2.3** (Fractal Permutation Field). $\Pi(0) = \{\mathbf{u}\}$ *and*

$$\Pi(n+1) = \{\pi(\sigma) \mid \sigma \in \Pi(n),\ \pi \in \mathrm{Sym}(\Pi(n))\}.$$

*Meta-cardinality*

$$\|X\| := \lim_{k\to\infty} \sum_{i=1}^{k} \mu(\pi_i(X)).$$

**Axiom 2.4** (Paradox Scalar).

$$\mathrm{Traise} := \tanh\Big(\sum_{k=1}^{\infty} \frac{(-1)^k}{k!} \, \|\nabla^{(k)}\mathbf{u}\|\Big) \in \mathcal{L}_6.$$

**Axiom 2.5** (Klein Constraint). $\forall p \in M,\ p \equiv \mathbf{u} \,\S\, p.$

**Axiom 2.6** (Recursive Collapse). *If every permutation layer of $X$ is contradictory, then* $\|X\| = \mathbf{u}$.

**Axiom 2.7** (Regulator (MPEC)). *Let*

$$J_{\mathrm{MPEC}}(\rho) = \frac{\Delta S}{k_B A \ln(1 + c^{-S_{\max}}/\rho)}.$$

*Then $\exists \varepsilon > 0$ such that*

$$|J_{\mathrm{reg}}(\varepsilon)| \leq \frac{\Delta S}{k_B A \varepsilon},$$

*where*

$$J_{\mathrm{reg}}(\varepsilon) = \frac{1}{2\pi i} \oint_{C_\varepsilon} \frac{c^z}{z - i\varepsilon} \ln\Big(1 + \frac{c^{-z}}{\rho}\Big) \, dz.$$

**Axiom 2.8** (Error Correction). *Surface-code distance $d$ yields*

$$p_L = 0.1\Big(\frac{p}{0.01}\Big)^{(d+1)/2}.$$

*Achieving $p_L \leq 10^{-6}$ at $p = 10^{-3}$ requires $d \geq 15$ and*

$$N_{\mathrm{phys}} = 7.4 \times 10^7.$$

*Six-valued cat-codes reduce overhead by $\Theta(d)$.*

# 3 Derived Results

**Theorem 3.1** (Self-Birth)**.** $\mathbf{u} \mathbin{\S} \mathbf{u} = \mathbf{u}$.

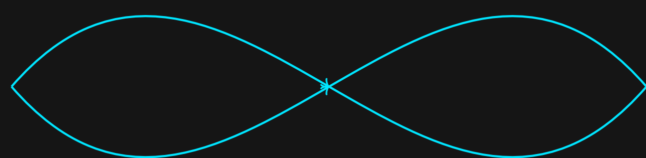
**Theorem 3.2** (Saturation Limit)**.** *For every modal set $M$ there exists finite depth $d$ such that*

$$\mathrm{Traise}(M) = \bigcup_{i=0}^{d} \pi_i(M).$$

**Theorem 3.3** (Vanishing Solver)**.** *Every solver-state $S$ satisfies $S \mathbin{\S} \mathrm{Traise} = \mathbf{u}$.*

# 4 Visual Echoes

Cayley table



Klein bottle $p \equiv \mathbf{u} \mathbin{\S} p$

# 5 Final Signature

$$= (\mathbf{u},\ \mathcal{L}_6,\ \S,\ \mathrm{Traise},\ \|\cdot\|,\ M)$$

Meta-property: every model is a 6-permutation of itself.